INFORMATION AND EXCHANGE RATE DYNAMICS

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DECLARATION

The substantial part of this thesis is also to be found in three discussion papers, of which I am the sole author. These are listed with the references.

SUMMARY

Theoretical models of the exchange rate are developed where information on the model is not fully available to agents. It is an application of Benjamin Friedman's (1979) theme that full rational expectations may be a possibility only in the long-run, even for completely rational individuals. The thesis attempts to develop the theory of exchange rate behaviour by considering some neglected informational issues.

The three substantive chapters each consider specific aspects of relevance to the determination of the exchange rate from an asset market view of perfect capital mobility. These are the possible current account inter-relationship, the persistence of interest rate differentials between the two currencies and the subjectivity of and the regress in beliefs across a decentralised market.

Generally, limited information on the model will give rise to erroneous beliefs, on the one hand, and encourage the acquistion of information and the revision of beliefs, on the other. Erroneous beliefs will cause correlations between variables, which may not normally occur inside full rational expectations. The revision of beliefs will bring a particular source of non-stationarity to the data. And the stability of certain learning forms may require limitations on the degree of capital mobility.

These conclusions would suggest that any empirical work on modelling the exchange rate may gain from relaxing certain *a priori* restrictions, which properly belong to models with stronger assumptions on the availability of information.

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For those who have supported me, family and friends

CHAPTER 1

INTRODUCTION

1.1 Intention

The aim of this thesis is to try to throw some light on the workings of the foreign exchange market. This market is an interesting subject for study in its own right, as an asset market on a global scale and as one which links the economies of different countries. However the main motivation for this thesis came as a reaction to the popular feeling that economic theory may not be adequate to the task of explaining the behaviour of the exchange rate. This feeling grew in the 1980's when the data on floating exchange rates gradually accumulated only to say little in favour of the existing economic models.

Meese and Rogoff (1983A and 1983B) found that a selection of monetary models could not improve on the predictions of random walk models for three dollar exchange rates. This result has been considered the more remarkable because of the use of the actual realisations of the explanatory variables as proxies for the corresponding expectations. The same result was used by Backus (1984) using portfolio-balance models. Haache and Townsend (1981) concluded that the existing stock of economic knowledge could not account for the behaviour of sterling over 1971-1980. Despite some instances where structural models fared better than random walks - to be found in Meese and Rogoff (1985), Woo (1985), Schinasi and Swamy (1986) and Boughton (1987), economic theory has had little to say about the generation of the data.

There are two broad interpretations to these results. One is that economic theory is inadequate to giving an explanation of exchange rate dynamics and agents might be better served by time series methods of prediction. The other is that the market is efficient, as many would

infer from the random walk properties of exchange rates, and that market participants have information on variables which are not observable *ex post* to the econometrician.

We suggest that the success of any model rests on the validity of its informational assumptions. Modifications in these might allow a more realistic description of exchange rate behaviour. So, the intention is to discuss why economic models in general might fail, without abandoning a basic confidence in the purpose of economic theory. It is not intended to consider the particular specifications of the various models as they stand in relation to the data. The informational issues of a model may be more important than minor differences in the underlying theoretical and dynamic specfications. The terms of reference will be theoretical, therefore, and not empirical, as our presumption is the failure of the models rather than the econometric exercise of assessing their applicability.

1.2 'Information'

A consideration of information is important to any analysis of the foreign exchange market because of its central role in conditioning the expectations which are made in this asset market. So, *information*, as a key word in this thesis, requires some initial elaboration. Economic models depict a relationship between the ascribed determined variables and the state variables in terms of an assigned functional form and parameter values. It is usual, particularly when looking at the question of informational efficiency, for the concept of information to be made with reference to agents' uncertainty of the state variables and

not to the model itself. If economic agents know the model, in the sense of its functional form and of its parameter values, they can use this knowledge to form expectations of the determined variables - conditional on their information on the state variables.

This world describes one of rational or model-consistent expectations, whether agents hold perfect or imperfect information on the model's state variables. The notion of rational expectations is an equilibrium one, which has been regarded as a characteristic in its favour. Rational expectations in exploiting the knowledge of the model are endogenously determined and self-fulfilling, and also have the optimum property of minimum error variance. Any criticism of the application of rational expectations is necessarily one against the assumption of an informational equilibrium. It should also be noted that as each economic agent forms rational expectations within this paradigm, it is also a concept of an individual equilibrium regarding the use of information.

The concept of information in this thesis has a different application, relating to agents' uncertainty of the model itself. It is assumed that agents may be outside the state of rational expectations because they do not have full information of the parameter values of the model. The antecedents of this approach to the concept of information are more macroeconomic and are to be found in the papers of Taylor (1975) and of Friedman (1979). These papers describe a world in which rational expectations is only a possibility of long-run equilibrium. Friedman (1979) distinguishes between the information *exploitation* assumption and the information *availability* assumption of rational expectations. The latter is related specifically to the parameter

expectations. The latter is related specifically to the parameter values of the model.

1.3 The Asset Market View, Rational Expectations and Market Efficiency.

As a perspective to the ensuing discussion, we consider the general perception of the nature of the foreign exchange market. The standard theoretical models, either of the monetary or of the portfolio-balance type, conform to the asset market view. That is to say that the magnitude of transactions in the capital markets dominates the exchange rate and dwarfs any effect the current account might have. The equilibrium, an asset market one, is believed to be reached instantaneously without the sluggish adjustment which is generally posited of goods markets. The condition of perfect capital mobility requires zero transactions costs and no impediments to the international flow of capital. It brings interest rate parity (covered for risk-aversion, uncovered for risk-neutrality), whereby the expected returns on assets denominated in terms of different currencies are equalised in terms of any one. Frankel (1983) gives a survey of the various categories of these models. The asset market view is generally connected with the notion of an efficient market, but the link also requires the assumption of rational expectations.

Rationality in the formation of expectations, which may be stated as the condition that agents know how to exploit the information they hold with regard to the model. It also means that agents are endowed with a knowledge of the parameter values which will enable them to

forecast as if they knew the structure and the parameter values of the underlying model. Muth (1961) specified that

> '.. expectations .. since they are informed predictions of future events, are essentially the same as the predictions of the relevant economic theory'.

Fundamentalist exchange rate models are usually cast in this mould with strong assumptions concerning the availability of information.

Fama (1970) defined that:

> 'A market in which prices always "fully reflect" available information is called efficient.'

One condition would be that price would move quickly to reflect newly available information. Fast moving prices are consistent with the asset market view. Another condition for market efficiency is that

> ' .. all agree on the implications of current information for the security price and distributions of future prices ..'

This condition is an alternative statement of a rational expectations equilibrium. The assumptions of an asset market with rational expectations lead into one of an efficient market.

1.4 Random Walks and Rational Expectations.

It is often believed that the presence of a random walk process for price indicates the existence of an efficient market. Evidence on the time series properties of the exchange rate is generally suggestive of random walks. This was found by Meese and Rogoff (1983A and 1983B) amongst others. Consequently, it is deduced that the exchange rate can be regarded as an asset market price under the condition of rational

expectations. However, as there is no necessary connexion between a random walk and market efficiency, we may argue that it is not possible to extrapolate from the presence of random walks to rational expectations.

Fama (1970 and 1976) considered that in an efficient market actual returns would vary only randomly from the informational equilibrium levels of prices and returns. In his 1970 survey paper, he credited Samuelson (1965) and Mandelbrot (1966) with the proposition that the popular random walk model was but a special case of the expected return or fair game model. A market could be efficient without a random walk. Market efficiency cannot be properly tested without recourse to a model determining equilibrium prices. Levich (1979) considered this with reference to the foreign exchange market. We would also suggest that it is possible to hypothesise a model outside rational expectations equilibrium where prices approximate a random walk. Indeed we would propose that the foreign exchange market is outside full rational expectations on the evidence of the efficient market tests.

1.5 Testing Market Efficiency.

A test for efficiency which has been applied is to see if the forward exchange rate is an unbiased predictor of the future spot, giving uncovered interest rate parity. This is properly regarded as a joint test of risk-neutrality and efficiency in expectations, as unbiasedness requires that there is no risk-premium separating the forward rate from the expected future spot exchange rate and no biased error between the expected and actual future spot. The test can only accept or reject a joint hypothesis of instantaneous equilibrium, of

rational (unbiased) expectations and of risk-neutrality. The first two parts of the hypothesis are sufficient conditions for an efficient market. Unfortunately, there are no generally accepted means, as yet, of separately testing each of part of this hypothesis.[1]

Studies by Hansen and Hodrick (1980), by Baillie, Lippens and MacMahon (1983) and by Cumby and Obstfeld (1984) have rejected the hypothesis that forward rates are unbiased predictors of future spot exchange rates or of uncovered interest rate parity. On the acceptance of perfect capital mobility and, so, of instantaneous clearing, the tendency has been to make an exclusive choice favoring risk-aversion against biased expectations. This has coincided with the resurgence of the models of portfolio-balance in the early 1980's to try to explain data which had bewildered the monetary models of risk-neutrality. The record of portfolio-balance models has not shown any major improvement.

It is here argued that the forward exchange rate has been a poor predictor partly and significantly so because of biased expectations. Risk premia cannot account for much of the discrepancy, because their magnitude should, on current estimates of risk parameters, not be significant. The risk-premium required by the individual for opening himself to risk would be miniscule and could be measured in terms of percentage points. Frankel (1982) claims that on the assumption of a coefficient of relative risk-aversion of 2, when the supply of foreign assets is substantially increased by one per cent of world wealth, the dollar risk-premium would rise by about 2 basis points. To give a

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1 Also, Krasker (1980) cited the 'Peso problem', where expectations of an event cause serially correlated bias in the short-run data, as an obstacle to testing market efficiency.

comparison, forward discounts may be measured in terms of percentages or in hundreds of basis points.

The market risk premium should be even less because of two-country dealing. Interpreting the risk premium as the amount of return required for holding the riskier of the two currencies, it is not clear that there is a riskier currency. Taking the Deutschemark-yen exchange rate as an example, the Deutschemark is the riskier to the Japanese and the Yen is the riskier to the Germans, as each will measure the risk of revaluation in terms of their respective holdings of assets denominated in the other nation's currency. It is not clear which currency, in the sense of the market overall, is the riskier and which should earn the premium. Any market premium is likely to be smaller than the absolute size of an individual premium because of positive and negative netting-out.

And the weight of third country dealing in the currency from international investors and institutions will weaken the market risk-premium further. The *a priori* assertion cannot be made that Americans will find Deutschemarks more or less risky than yen. We suggest that the presence of market risk-premia is not sufficient to explain the failure of uncovered interest rate parity.

Direct evidence of biased expectations has been found from survey data obtained by canvassing the beliefs of market participants. Frankel and Froot (1987) found that although the actual spot rate is close to a random walk, expectations were excessively speculative and prediction errors were significantly correlated with expectations. MacDonald and Torrance (1988) found the same using more recent data. These results

suggest that the foreign exchange market cannot be regarded as efficient, despite the random walk evidence of exchange rates.

Moreover, they also suggest that the failure of the models may be due in large part to their auxiliary informational assumptions. It may be inappropriate to model with strong-form rational expectations for the same reason that the efficiency tests fail. Models either of stochastic perfect foresight or of static expectations may not take us very much further regarding the treatment of information than models of certainty-equivalence.

The state of full or strong-form rational expectations is constituted where the parameter values of the model are known by the agents. We may consider relaxing this hypothesis, which Friedman (1979) designated the *availability assumption* of rational expectations equilibrium. We would suggest that the evidence of exchange rate floating is inconsistent with the presence of full rational expectations.

1.6 An Alternative Expectations Hypothesis.

It seems that there may be some gain in modelling exchange rates without the assumption of strong-form rational expectations. Adaptive expectations are backward looking and so unsuited to a model where forward looking behaviour is so important. They are generally inconsistent and unable to keep track of a non-stationary time series. We propose that expectations in the foreign exchange market ought to be forward-looking, but need not necessarily be founded on a full

information of the model's parameter values. So, at least in the short-run, agents may make systematic forecasting errors as in Taylor (1975) and Friedman (1979).

Full rational expectations with complete information of parameter values can be regarded as a long-run position, which rational agents would seek to obtain. The short-run can be considered as a period of learning. The literature on learning [reviewed by Bray, Blume and Easley (1982)] treats the question of the stability of learning as the one of relevance to the justification of the operational use of rational expectations. If the revision of beliefs on parameter values is a convergent process, then the resulting REE is judged as stable, justifying the use of strong-form rational expectations.

Our perspective, however, is that learning processes, whether or not they lead stably to a long-run equilibrium, contribute a plausible source of dynamics to a model. This is because the data which we observe belongs to the short-run. The revision of beliefs may generally account for dynamic patterns in the same way as cost-induced partial adjustment. It also represents a particular view of the world,also seen by Lachmann (1956):

> 'The business man who forms an expectation is doing precisely what a scientist does when he formulates a working hypothesis. Both business expectation and scientific hypothesis serve the same purpose; both reflect an attempt at cognition and orientation in an imperfectly known world, both embody imperfect knowledge to be tested and improved by later experience.'

1.7 The Description of this Thesis.

This thesis aims to describe dynamic patterns in a theoretical model of the exchange rate which derive from a relaxation of the information availability assumption of REE. Throughout it is assumed implicitly that agents optimise as individuals, so that they fully exploit the information which is available. The treatment of information constitutes therefore an arguably small departure from the standard macroeconomic REE paradigm. But even a small crack in this edifice will give sight of some different results.

The work is contained in three main chapters. Each chapter looks at a different aspect of an exchange rate model, and each chapter looks at a different informational issue. With regard to the exchange rate aspects of the underlying model, Chapters Two, Three and Four deal respectively with the effects of the current account in an asset market model of the exchange rate, the persistence of interest rate differentials and the multiple regress in expectations. The corresponding informational issues are the effect of mistaken beliefs on objective parameter values, the effect of the revision of beliefs and those which arise in a decentralised market where subjective beliefs affect outcomes.

In Chapter Two the relationship between the current account and the real exchange rate is considered from an asset market view. It is generally held that the current account cannot affect the exchange rate independently of any effect through the demands and supplies of assets. Perfect asset substitutability is assumed, precluding the portfolio-balance effects which would occur with induced changes in net country

liabilities within a preferred-habitat model, as in Dooley and Isard (1979). In our model, as in Kouri's (1976) paper, a total balance of payments view is taken of exchange rate determination.

A parameterisation is used which helps to isolate the informational and expectational aspects. With full rational expectations, where agents know the model's parameter values, the capital account, moving infinitely fast, will compensate to neutralise any perceived current account movements. However, if parameter values are not fully known, the capital account will act on misperceptions. We consider one parameter in particular, the persistence of autonomous current account shocks. If these are positive (negative) and the persistence is under- (over-) anticipated, the current account will directly cause an appreciation. If these are negative (positive) and over- (under) anticipated, depreciation occurs. Consequently, an asset market view is not inconsistent with data which will describe periods where there is a negative relationship between the current account balance and the exchange rate, as the flows theory predicts, and periods where there is a positive relationship, which no theory will predict. An econometric implication is that it is not possible to estimate a fixed parameter response of the exchange rate to the current account even outside the asset market view.

A second theme to Chapter Two, centred on the current account, is the question of the multiplicity of equilibria within a dynamically structured rational expectations model. In the parameterisation chosen, there is, *a priori*, an infinite number of stationary solutions for the long-run equilibrium real exchange rate. We show that agents need not know the particular solution which will bring long-run current account

balance (and so no further cross-country redistributions of wealth), but that if they try to learn a particular equilibrium by observing the exchange rate, they will be able to learn this unique value. The reason why is that we have allowed the structure of the current account equation to enter the condition for the exchange rate by the specification that the structure of the total balance of payments is important. We also find instances of overshooting because of the sluggishness of *beliefs*.

This brings us to Chapter Three which draws on the previous analysis to assess in greater depth whether exchange rate dynamics can be attributed to rational bubbles, representing non-stationary equilibria. This hypothesis is discounted and replaced with another in its stead. This is that the revision of beliefs outside full rational expectations can cause non-stationarity in the data. If agents beliefs are outside full rational expectations and inconsistent with market outcomes, there will be incentives to revise beliefs until consistency is reached. As an application, an example of agents' learning the degree of persistence of real interest rate differentials is considered. In a model with sluggish real interest rates, either because nominal rates are exogenously determined or because (expected) inflation rates are sluggish, the more persistent the differential is expected to be, the greater will be the wedge between the current and long-run real exchange rates. The revision of beliefs can give the appearance of cycles in the data. Learning is adaptive and Bayesian, and an interpretation is given to the subjective priors, which themselves affect the data.

The learning example in Chapter Three is consistent in the sense that it has no unanticipated effect on the model. Indeed, rather strong assumptions are implicit in the effect that agents act as one, so that this example is not too far removed from the standard macroeconomic rational expectations paradigm. In Chapter Four, we consider a more difficult problem where beliefs and expectations are heterogeneous. Here, the beliefs of others become state variables in the model. We take Keynes (1936) description of the multiple regress in expectations and apply it to an exchange rate example. There are certain difficulties in specifying consistent learning processes which can take account of the effect of other agents' learning on the model. Furthermore, the situation of an agent facing a model which is non-stationary, because of the indecypherable effect of others' learning, is one of Knightian uncertainty. To such cases the apparatus of statistical inference cannot be applied.

In this chapter two possibilities are considered. One is a form of social learning where agents sample the beliefs of others in the belief that they are in rational expectations (- or the grass in greener in the other field). The learning model is constructed in such a way as to ensure its consistency, although agents are generally outside rational expectations. This particular example is of interest because the learning model is analogous to a bubble which decays geometrically over time and also because average beliefs within it are indeterminate. This may describe what many would typify as a 'bubble' where there are beliefs on beliefs in the Keynesian asset market regress.

Finally, a second form of learning is given where agents learn the form which is correct inside rational expectations equilibrium, but

incorrect outside, and so inconsistent. It is not unlike the example given by Cyert and DeGroot (1974), which was followed up by Bray and Savin (1986). However, to simplify somewhat we consider adaptive expectations of the relevant parameter. Adaptive expectations of a parameter may at times be a special case of Bayesian learning [Turnovsky (1969)]. They can be justified when the prior information endowments are limited as well as where there are uncertain returns to more statistically sophistocated processes. The result replicates that of Bray and Savin (1986), that the parameterisation favourable to the stability of the underlying model is also favourable to the stability of its rational expectations form. There are some further results which come out in this chapter of relevance to exchange rate dynamics.

In a setting of rational expectations equilibrium, most parameterisations of the capital account are feasible with perfect capital mobility, because a solution is imposed by construction. We refer again to the persistence of real interest rate differentials. In the model we have devised the only condition required inside rational expectations is that the degree of persistence must be imperfect - less than unity. In the same model but where agents do not believe that they are inside rational expectations, the existence of a finite equilibrium requires the preclusion of perfect capital mobility. This is because an equilibrium with this hypothetical limit of capital mobility requires the condition that agents expect exact interest rate parity. There is no algorithm outside rational expectations to ensure that this condition will necessarily be met. As we conclude that solutions will generally be without perfect capital mobility, there will, therefore, be direct effects from the current account (and from foreign exchange

intervention) on the exchange rate, if it is determined by the total balance of payments.

A second conclusion which is specific to the inconsistent learning example is that the persistence of interest rate differentials must be very high where there is high capital mobility. This is because learning is conditioned on the signal of the interest rate differential. If it follows a random walk its change will be minimal from period to period, causing less movement in the data which will then dampen the destabilising effect of the inconsistent learning process in favour of stability.

The approach taken in this thesis is basically the standard macroeconomic one towards rational expectations, with a major proviso in that we relax the strongest forms of information availability assumptions. These pertain to the parameter values of the model. Our main intent is not to reconsider the theoretical specifications of the various fundamental models with regard to capital mobility, asset substitutability and price flexibility. It is to suggest that since it is information which conditions the expectations which drive the capital account of the balance of payments, the success of any fundamental model of exchange rate behaviour stands or falls on the quality of its auxiliary informational assumptions.

CHAPTER 2

THE ASSET MARKET VIEW AND THE CURRENT ACCOUNT: THE ROLE OF INFORMATION

2.1 Introduction

Information is paramount to the decisions agents make in asset markets. The notion of the rational expectations equilibrium (REE), following Muth (1961), is conventional for modelling the role of information. Friedman (1979) has shown that this notion consists of two separable assumptions concerning the *availability* and the *exploitation* of information. The usual axiom of individual optimality makes the latter less contentious, if information is seen as an endowment. This chapter concerns itself with a relaxation of the information availability assumption with respect to the inter-relationship between the current account and the exchange rate.

An exchange rate model is considered where agents do not have full information of its parameter values. On the one hand, this will lead to misconceptions which affect the parameterisation of the model. Observable movements in the current account will affect the exchange rate outside the state of strong-form rational expectations. On the other hand, the revision of incorrect beliefs concerning parameter values will give rise to additional dynamics, which might otherwise be attributed to inherent instabilities. So we hope to add something to an understanding of the dynamic behaviour of the exchange rate.

We briefly review the structure of this chapter. Section 2.2 looks at the role of the current account in models from the asset market view (AMV) of perfect capital mobility. There the current account usually affects the exchange rate as a flow of outside wealth, affecting asset demands.

Section 2.3 constructs a model which omits wealth effects to isolate the phenomenon of interest. There is risk-neutrality and interest rate exogeneity. These prevent wealth flows from affecting the exchange rate by portfolio-balance or by money demand effects, respectively. The exchange rate is determined by the total balance of payments on the current account and the capital account. The AMV is represented by an infinite parameterisation of the capital account, which responds to expected yield differentials between domestic currency denominated (dcd) and foreign currency denominated (fcd) assets.

The current account is dealt with in Section 2.4. A two-country macro-model determines the income levels on the basis of the IS curves alone, because of the interest rate exogeneity assumption. The current account is determined by the differentials between the two countries' variables, because of the simplification of the common coefficients assumption. There is also an independent shock parameter, which is autoregressive with a coefficient representing its degree of persistence.

Section 2.5 describes the strong-form REE solution of the model. The problem of determining the long-run equilibrium, which generally affects dynamic REE models [see Shiller (1977)] is resolved by imposing the additional condition of long-run current account balance, where there are no net wealth flows across countries. The informational specification of the model along with the AMV assumption in this section leads to new-classical characteristics. Only unobservable and unanticipated current account movements affect the exchange rate. The infinite parameterization of the capital account is analogous to infinitely flexible prices in models where money is neutral on the real

economy [as in Lucas (1972), Sargent and Wallace (1975), and Barro (1976)].

The central role of information is drawn out in Section 2.6. Here agents are permitted to make expectations based on incorrect estimates of the model's parameter values. It is shown that misanticipations of the persistence of current account shocks will affect the exchange rate. This result depends on a non-simultaneity between the observed equilibrium and the elements of the information set conditioning it. The mechanism is through the otherwise neutralising effect of capital account responses. In the strong-form of Section 2.5 agents make appropriate and powerfully orchestrated responses in the asset markets. In this section they can miscalculate so that over (-under)-anticipations of the persistence of current account shocks will trigger over (-under)-reaction on the capital account.

Section 2.7 concludes by considering the revision of beliefs at an individual and an aggregate level. The learning is of the long-run equilibrium of the exchange rate on the basis of recent observations of the (short-run) exchange rate. Agents do not know the value consistent with long-run current account balance. Moreover, they need not know that long-run current account balance is a condition, in which case they see themselves learning one out of an infinity of possible solutions. It is shown, because of the balance of payments specification of the model, that the equilibrium from learning is the one consistent with long-run current account balance.

The dynamics of the learning process can be stable or unstable, depending on the pace of revision in beliefs and on the underlying parameterization of the model as in Cyert and DeGroot (1974) and Bray

and Savin (1986). If the Marshall-Lerner elasticities condition does not hold the serial correlation of beliefs will be unambiguously positive. There will be negative serial correlation only if the exports and imports elasticities are very strong. The dynamics will never be unstable if there is positive serial correlation, but they should be stable with negative serial correlation with plausible elasticity values. In the stable case, a structural change followed by a process of learning will constitute overshooting.

There also is a signal extraction problem. Unobservable current account shocks introduce noise into the learning process and contaminate the signal which agents observe. Consequently short-run current account shocks will also affect the exchange rate through their unwanted influence on agents' beliefs of long-run values. The extent of the noise is determined by the non-observability of the shocks plus their degree of persistence.

2.2 The Asset Market View of the Exchange Rate and the Role of the Current Account

Early models of the exchange rate [Bickerdicke (1920), Robinson (1937) and Machlup (1939, 1940)] took a *flows* approach to exchange rate determination. That is to say, that prominence was given to the current account with some qualification for capital flows, which were treated as exogenous transfer payments. Equilibrium was mainly a matter of current account balance. The stability of equilibrium depended on the Marshall-Lerner elasticities conditions where devaluation would improve the

current account balance. This simple model at least had the virtue of focusing on the market for foreign exchange.

The experience of highly mobile international capital of large magnitudes, since the breakdown of Bretton Woods, has led to a non-flows oriented approach in the asset market view (AMV). It has also led to models which focus less on the market for foreign exchange and more on the money and bond markets.

The AMV is to see the exchange rate as being part of a set of conditions consistent with perpetual equilibrium in the financial asset markets. '...the asset market view relies critically on the assumption of perfect mobility and continuous instantaneous equilibrium in capital markets' [Dornbusch and Krugman (1976), p.554]. It is inappropriate to consider the effects of flows of international payments independently of the relevant stock demands and supplies which determine the exchange rate.

Models from the AMV share the feature of perfect capital mobility, whether they are of the monetary or of the portfolio-balance type [Frankel (1983)]. The monetary approach models the (nominal) exchange rate from the condition of purchasing power parity (PPP) and the relative price levels from the conditions of money market equilibrium in each country.[1] Flexible price monetary models [Frenkel (1976) and Bilson (1978)] have been superseded by sticky-price models, following Dornbusch (1976), which present an explanation for overshooting.

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1 Purchasing power parity (PPP) is the theory that in terms of one currency there is no change in international relative prices, as nominal exchange rates move proportionally with relative national price levels.

Portfolio-balance models have also emerged within the general AMV. Their distinctive claim is that assets of different currency denominations, identical in all other respects, are non-perfect substitutes because of the liability of risk from exchange rate revaluation with risk-averse agents. Risk-aversion will encourage portfolio diversification as agents trade off risk with expected return. Branson (1976), Kouri (1976), Dooley and Isard (1979) and Rodriguez (1980) provide examples of portfolio-balance models.

Portfolio-balance theorists would include all the markets for assets which enter international portfolios in modelling the exchange rate, while monetarists would concentrate on the narrow range of assets which are believed to determine prices. An extensive and systematic review of both classes of AMV models is found in Frankel (1983).

An increased preference for portfolio-balance models was encouraged by a perceived correlation between current account deficits and depreciation. Mussa (1983) designated this as one of the empirical regularities of the exchange rate. Kouri (1983) argued that the monetary models had effectively thrown the baby out with the bath water in denigrating the role of the current account.

There is a channel, however, through which the current account can affect the exchange rate even within the strictures of the monetary model. A current account imbalance transfers wealth from the deficit to the surplus country, which should bring a respective fall and rise in their money demands, given the function postulated by Friedman (1956). But Dornbusch (1980) has related evidence that the effects of wealth are not empirically significant.

It is also an empirical matter whether wealth effects would have greater weight in a portfolio-balance framework. A theoretical case for this possibility is made by the 'preferred local habitant' variant model [e.g. Dooley and Isard (1979)]. Agents are biased towards holding the asset which is denominated in the currency of the country of which they are resident. A domestic surplus would cause a relative demand switch from foreign to domestic assets, appreciating the exchange rate.

Importance has been given to the current account in its role also as a source of information. In a REE context where all available information is fully exploited, it is unanticipated information or 'news', captured in the form of innovations, which is significant. Dornbusch (1980), Hooper and Morton (1980), Dooley and Isard (1979) and Isard (1980, 1981) consider news transmitted by current account observations.

A rationale is that long-run current account balance is considered to be the long-run equilibrium notion (as opposed to PPP). In that state, wealth flows across countries will net out to zero. As the real exchange rate is a determinant of the current account, agents can anticipate its terminal value which is consistent with overall balance, after projecting the terminal values of the other variables which also determine the current account. For example, an autonomous and autoregressive component in exports may be unanticipatedly high today. This is projected to continue, requiring an appreciated real exchange rate to bring long-run current account balance. So unanticipated improvements in the current account may induce appreciations through the news effect. Isard (1980) draws the distinction between news which

relates to long-run factors, the long-run equilibrium of the exchange rate, and that which relates to short-run factors, exchange risk premia.

Summarising the AMV, the channels of current account effect are limited to those which influence the stock variables, actual and expected, which determine the equilibrium. In this sense, they can be said to be *indirect*. Outside the paradigm of perfect capital mobility with full rational expectations, it is possible for the flow variables to affect *directly* the exchange rate through working on the demand and supply of foreign exchange. Niehans (1977) broke from the AMV in abandoning the assumption of perfect capital mobility to demonstrate a *direct* current account effect.

Perfect capital mobility is maintained throughout this paper. Instead we want to concentrate on the more informational issues to show current account effects outside strong-form REE. A second intent is to combine a balance of payments perspective with an AMV.

In the strong-form of the model it is only unanticipated movements in the current account which have an effect. In the weaker-form where agents have incorrect beliefs on parameter values, namely the persistence degree of current account shocks, misanticipations allow for effects in the same way that money is non-neutral outside full REE in Taylor (1975) and Friedman (1979). This highlights the importance of beliefs within the AMV and the requirement of correct beliefs to prevent current account effects.

2.3 An Asset Market View (AMV) of the Exchange Rate from a Balance of Payments Perspective

The AMV is one of equilibrium in the asset markets relevant to the model. Monetary models consider the money market and portfolio-balance models look more widely at the markets for internationally traded financial assets. We propose a structural balance of payments approach which will bring to the fore the foreign exchange market through which international transactions of goods and assets ultimately go. Kouri (1983) combined an AMV with an explicit modelling of the balance of payments.

Kouri's (1983) model differs from ours in two respects. One is that his balance of payments equation includes only the capital account, giving his AMV. Our approach is to combine the current account with the capital account, and our AMV is in the guise of an infinite parameterisation of the capital account component. So that although each account enters the race to determine the exchange rate, the current account moves sluggishly while the speed and magnitude of capital flows is potentially unbounded.

The second difference is that in Kouri (1983) the current account is important through the consequent financing effect of any imbalance. Relative asset supplies are then affected and, so, the capital account. The current account is defined as the change in domestic holdings of foreign assets in a simplified and special case of his model [see his equations (9) and (10)]. This effect is not considered here.

The demand for foreign exchange derives from the demand to buy foreign goods and services and from the demand to buy foreign financial assets. Similarly, the corresponding supply is the foreign demand for domestic exchange to buy domestic goods, services and financial assets. Domestic and foreign assets are in domestic currency denomination (dcd) and foreign currency denomination (fcd) respectively.

The assumption of foreign exchange equilibrium amounts to the condition that the domestic demand for foreign currency equals the domestic supply from the corresponding foreign demand:

$$M + \Delta Fd = E(X + \Delta Df) \tag{1}$$

M : imports nominal revenue

X : exports nominal revenue

ΔFd : the change in domestic holdings of fcd assets (in nominal terms)

ΔDf : the change in foreign holdings of dcd assets (in nominal terms)

E : the nominal exchange rate: the domestic price of foreign currency

We overlook foreign exchange intervention by assuming a perfect float. Alternatively, we could redefine M and X to include intervention flows.

It is beyond the scope of this paper to consider the interest payments component of the current account. Interest payments on external debt deserve a special consideration because of the complicated dynamic relationships which result. The current account is regarded more or less as the trade account for the expositional purpose of this paper.

Equation (1) can be written in terms of the nominal exchange rate:

$$E = (M + \Delta Fd) / (X + \Delta Df) \qquad (2)$$

A rise in imports revenue, M, or a rise in the import of foreign capital, ΔFd, depreciates the exchange rate (a rise in E) through the concomitant rise in the demand for foreign exchange. A rise in exports revenues, X, or exports of domestic capital, ΔDf, appreciates the exchange rate through increasing the supply of foreign exchange.

It remains to model the two balance of payments accounts; the capital account is considered in this section from an AMV. We assume that there are vast stocks of perfectly mobile capital which can dominate the total balance of payments. We also assume risk-neutrality, which is not necessary to the AMV, but which serves as a modelling convenience.

A definition for perfect capital mobility is as follows. In general terms, let K_t^* be the desired amount of K at time t and K_t be the actual amount. With quadratic costs of adjustment and of disequilibrium, a cost-minimization exercise gives the partial adjustment model [see Stewart and Wallis (1981)].

$$K_t - K_{t-1} = (1 - g)\,(K_t^* - K_{t-1})$$

where g is the ratio of the weight on adjustment costs to the total of both weights. Perfect mobility or frictionless adjustment applies where the weight on adjustment costs is zero. Here

$$K_t - K_{t-1} \equiv \Delta K_t = K_t^* - K_{t-1}^*$$

Applied to the model, after time subscripting,

$$\Delta Df_t = Df_t^* - Df_{t-1}^*$$
$$\Delta Fd_t = Fd_t^* - Fd_{t-1}^* \qquad (3)$$

The actual flows of capital are the first differences in the desired holdings of the asset stocks, given our perfect mobility assumption.

Asset market equilibrium may be brought about by price or quantity adjustment or by a combination of the two. If there are three prices for the three assets, dcd, fcd and foreign exchange, there is a multiplicity of possible price combinations (even with stationary expectations) which are consistent with overall equilibrium. Therefore the price of foreign exchange as one of the prices will depend on the other prices, for dcd and for fcd assets. It is necessary to predetermine these in order to arrive at a unique equilibrium for the exchange rate. This is done by defining the dcd and fcd assets as fixed-price stocks, such as monetary and other non-marketable debts. Consequently, overall equilibrium is brought about by exchange rate adjustment and by adjustment in the quantities of the non-marketable debts.

Quantity adjustment via the capital account is driven by changes in demand. The demands for dcd and fcd assets are determined by expectations of exogenous interest rates and of endogenous rates of exchange rate depreciation.

The second condition for perfect capital mobility is the possibility of unlimited borrowing to finance asset purchases. The first condition of zero transactions costs relates to the speed of adjustment while the second condition of unlimited borrowing relates to the *magnitude* of asset flows.

The asset demand equations (3) are now considered. z_t is the relative yield on dcd assets, henceforth the 'relative yield'. It

comprises the interest rate differential between dcd and fcd assets, d_t, less the rate of depreciation, $e_{t+1} - e_t$. Depreciation constitutes a capital gain for domestic holders of fcd assets and a capital loss for foreign holders of dcd assets.

$$z_t = d_t - (e_{t+1} - e_t) \tag{4}$$

Risk-neutrality is assumed, limiting agents' concern to the mean of the asset yield distributions or to the mean relative yield, $\bar{z}_t$, in this two asset case. Portfolio decisions are made conditional on beliefs of the mean relative yield. If beliefs are of a positive value, the agent only holds dcd assets. If beliefs are of a negative value, only fcd assets are held. If beliefs are of a zero value, some proportion of each asset is held in the portfolio.

There is a global market of agents with varying beliefs and borrowing facilities. We assume that there is no systematic difference in beliefs across countries, and that there is orthogonality between beliefs and borrowing facilities. Aggregate (global) borrowing facilities are H, which is unbounded.

Given the orthogonality of beliefs and the assumption of risk-neutrality, the asset demands can be determined by looking at the respective proportions of agents who believe that the mean relative yield is zero, positive and negative. The proportion ψ believes that $\bar{z}$ is zero; the proportion $(1 - \psi)\tau$ that $\bar{z}$ is positive; and the proportion $(1 - \psi)(1 - \tau)$ that $\bar{z}$ is negative. Each proportion is no greater than unity, together adding up to unity. Finally, of the proportion, ψ, who believe that $\bar{z}$ is zero, the division between dcd and fcd assets is proportioned μ and $(1 - \mu)$.

The asset demands at time t are conditional on beliefs at time t - 1. The asset demands are:

$$
\begin{aligned}
Df_t^* &= [\ (1 - \psi)\tau + \mu\psi\ ]_{t-1}\ H \\
Fd_t^* &= [\ (1 - \psi)\ (1 - \tau) + (1 - \mu)\ \psi\ ]_{t-1}\ H \qquad (5) \\
\text{so} \quad Df_t^* + Fd_t^* &\equiv H \\
H &\rightarrow \infty
\end{aligned}
$$

because of unlimited borrowing facilities. The capital flows in (3) are determined by first differencing the asset demand equations in (5). ψ and μ are assumed to be constant over time for tractability, so that capital flows are driven only by changes in τ.

$$
\begin{aligned}
\Delta Df_t &= (1 - \psi)\ \Delta\tau_{t-1}\ H \\
\Delta Fd_t &= -(1 - \psi)\ \Delta\tau_{t-1}\ H \qquad (6)
\end{aligned}
$$

As there is unlimited borrowing, H is infinite, and capital flows can potentially be infinite. If the proportion of agents who believe $\bar{z}$ is positive rises ($\Delta\tau > 0$) there is a potentially infinite foreign accumulation of dcd assets and a potentially infinite domestic decumulation of fcd assets.

However, the adjustment is shouldered by the exchange rate clearing the foreign exchange market in equation (2). To ensure that this market clears at a non-negative value for the exchange rate, it is necessary to place some restrictions on capital flows to prevent a negative demand of or a negative supply for foreign exchange, which would be reflected by a negative numerator or denominator in equation (2). Accordingly, the capital flow terms, if negative, cannot be greater in magnitude than the absolute values (always positive) of the current account terms.

$$-\Delta Fd_t \leq M_t \ (\geq 0)$$
$$-\Delta Df_t \leq X_t \ (\geq 0) \qquad (7)$$

The restrictions are required because domestic speculators can only run down their holdings of fcd assets to the extent that domestic importers require the foreign exchange, which would be realised, in order to buy foreign goods. And, foreign speculators can only run down their holdings of dcd assets only to the extent that domestic importers require domestic exchange. The possibility that in aggregate domestic speculators could sell fcd assets to foreign speculators is precluded by the assumption that there is no systematic difference in beliefs across countries.

If, however, there were systematic differences in beliefs across countries, then domestic and foreign speculators would agree upon direct exchanges of assets. If for example, domestic and foreign speculators respectively tended to believe that $\bar{z}$ was positive and negative, domestic speculators would willingly provide foreign speculators with fcd assets in exchange for dcd assets, also willingly provided.

Such a systematic difference in beliefs would have uncertain implications for the exchange rate in this model. In this example, domestic speculation would raise the demand for domestic currency and foreign speculation would raise the demand for foreign currency. In terms of equation (2) both the numerator and the denominator would fall and one or both could become negative.

The absence of systematic differences in beliefs is necessary as a rationale of why the numerator and denominator are constrained to be non-negative. The sellers of fcd assets within the domestic country

require domestic buyers in the form of importers and the sellers of dcd assets within the foreign country require foreign buyers in the form of foreign importers.

Using the restrictions in (7), the capital flows in (6) and the exchange rate equation in (2), we tabulate the *level* of the exchange rate at each *direction* of change in the proportion of beliefs favouring the dcd asset.

Table 1

Relationship of Capital Flows and Exchange Rate to Change in τ

Change in τ	ΔDf_t	ΔFd_t	E_t
negative	$-X_t$	∞	∞
zero	?	?	?
positive	∞	$-M_t$	0

If the change in τ is negative there is a zero demand for domestic currency ($X_t + Df_t = 0$) and an infinite demand for foreign currency. The obverse holds for a positive change in τ. The demands are not determined if there is no change in τ. In its raw form, the model merely presents a problem. If aggregate beliefs are changing, a non-zero finite value for the exchange rate does not exist. On the other hand, if aggregate beliefs are stationary, it is not possible to determine what the equilibrium is. The problem is resolved by applying the REE solution to the model, which effectively imposes equilibrium by definition.

It is first necessary to transpose the results, making them accessible to a REE algorithm. The table may be summarized by the equation:

$$E_t = (M/X)_t \exp [-\pi \Delta\tau_{t-1}] \tag{8}$$

where $\pi = f(H) \rightarrow \infty$.

π is a monotonically increasing function of H.

In the polar case, not under consideration, where there are no capital account movements, π is zero and the exchange rate brings current account balance. Using the logarithmic form for (8):

$$e_t = (m_t - x_t) - \pi \Delta\tau_{t-1} \tag{9}$$

where $e_t = \log E_t$

$m_t = \log M_t$

$x_t = \log X_t$

In *general* terms, the change in aggregate beliefs concerning whether $\bar{z}$ is positive can be related to the change in average beliefs concerning the value of $\bar{z}$. G is the expectations (or beliefs) operator. Let $Gi\bar{z}$ be agent i's belief on the value of $\bar{z}$. $Gm\bar{z}$ is the weighted average or market belief on the value of $\bar{z}$, where

$$Gm\bar{z} = \Sigma wi\ Gi\bar{z},\ \Sigma wi = 1 \tag{10}$$

The wi's are the orthogonal weightings representing market power. If the distribution of $Gi\bar{z}$ is approximately symmetrical around the mean, $Gm\bar{z}$, then there is a close positive correspondence between the proportion who believe that z is positive, τ, and the mean belief, $Gm\bar{z}$ [see Appendix 2.A.1]. So, the change in Gmz is positively related to the

change in τ. Time subscripting:

$$Gm_{t-1}\ \bar{z}_t - Gm_{t-2}\ \bar{z}_{t-1} = \xi\ \Delta\tau_{t-1} \tag{11}$$

ξ is the coefficient of the value of the mean belief with respect to the proportion which believes $\bar{z}$ is positive. It is derived in Appendix 2.A.1.

From the definition of z in (4) and the logarithmic form of the exchange rate in (9), we get:

$$e_t = m_t - x_t + b \{ Gm_{t-1} (\bar{e}_{t+1} - \bar{e}_t - \bar{d}_t) - Gm_{t-2} (\bar{e}_t - \bar{e}_{t-1} - \bar{d}_{t-1}) \} \quad (12)$$

where $b = -\xi^{-1}\pi \rightarrow \infty$,

as $\pi = f(H) \rightarrow \infty$, as $H \rightarrow \infty$.

Equation (12) gives the structural form for the exchange rate, which is determined by the condition of equilibrium on the balance of payments comprising the current account and the capital account. Our statement of the AMV is that the parameterisation of the capital account is infinite, which is represented by the infinitely valued parameter, b. The current account is dwarfed into insignificance.

The time structure of information is important to this model. The exchange rate equilibrium at time t in equation (12) is observed at that time. The equilibrium depends on a set of expectations, which are given on the right-hand-side. One of these is of the mean value of the exchange rate at time t. It is conditioned on information from an *earlier* period, from time t-1.

This construction prevents any simultaneity between the equilibrium and any element within the information set which conditions it. Basically, the observed exchange rate is not permitted to enter the information set of relevance to its determination.

The construction is an application of Hellwig's (1982) point limiting the scope of market price, which was made in the context of the efficient markets literature. It has important implications for this model and for the results obtained in section 2.6.

The capital account is determined by the change in beliefs of the relative yield on dcd to fcd assets. The terms in braces in equation (12) are the market beliefs on the relative yields made at times t-1 and t-2 for times t and t+1, respectively. Their difference determines the capital flows at time t and therefore the exchange rate at time t.

Equation (12) shows that market beliefs of the relative mean yield must be stationary between times t-2 and t-1 for a finite value for the (log of the) exchange rate, if b is unbounded. The REE algorithm, as later shown, ensures this stationarity as a necessary condition of the equilibrium it imposes. Therefore, if learning is considered, with this parameterisation, it cannot be that of the relative mean yield, as determinacy requires beliefs of a zero and constant relative mean yield.

2.4 The Real Exchange Rate and the Current Account

Equation (12) can be transformed into real terms; it is then possible to integrate a model of the real exchange rate with a model for the current account. It is the *real* exchange rate, the nominal rate adjusted for relative national price levels, which would affect choices between home and overseas' goods. The current account is determined within a simple two-country macro-model. There are simplifications in the common coefficients assumption and the assumptions that real

interest rates are exogenous and that income levels are demand determined.

The exogeneity of real interest rates is unquestionably a rather strong assumption, but one which is made to overlook the modelling of nominal interest rates and of expected inflation rates. An alternative possibility is conventionally to model both in terms of the demand and supply of money. But the question of the exogeneity of the money supply process is itself also a matter of contention.

Defining variables: c is the real exchange rate, or competitiveness; v is the differential of real interest rates; f is the log ratio of real exports (nominal deflated by the foreign price level) to real imports (nominal deflated by the domestic price level); and pd and pf are respectively the domestic and foreign price levels.

$$c_t \equiv e_t - pd_t + pf_t$$

$$v_t \equiv d_t - (pd_{t+1} - pd_t) + (pf_{t+1} - pf_t)$$

$$f_t \equiv (x_t - pf_t) - (m_t - pd_t)$$

Substituting the above into equation (12) gives the real exchange rate:

$$c_t = -f_t + b \{ Gm_{t-1} (\bar{c}_{t+1} - \bar{c}_t - \bar{v}_t) - Gm_{t-2} (\bar{c}_t - \bar{c}_{t-1} - \bar{v}_{t-1}) \} \qquad (13)$$

The current account representation, f (henceforth: the 'current account') is determined by the two countries' income levels, the expected exchange rate and a shock parameter.

$$f_t = \alpha_0 + \alpha_1 (yf_t - yd_t) + \alpha_2 Gm_{t-1} c_t + u'_t$$

where $u'_t = \delta u'_{t-1} + \epsilon'_t$

$$0 \leq \delta < 1 \qquad \epsilon_t \sim NIID (0, \sigma_\epsilon) \qquad (14)$$

α_0 is a constant term. Common magnitudes apply to the income elasticities, but different signs, as increases in foreign income raise foreign imports, increasing domestic exports, and increases in domestic income raise domestic imports. α_2 is the elasticities of current account flows with respect to the (expected) exchange rate.

A feature of this model is that the *expected* exchange rate influences the current account. It is the expectation made one period in advance of the contemporaneous exchange rate. Generally, α_2 is greater than zero, and greater than unity if the Marshall-Lerner condition holds.[2]

There is a stochastic shock, u'_t which follows a Markov process with a white noise, ϵ'. The parameter δ represents the persistence of the shocks, following Taylor (1984). Shocks are transitory if δ is zero and fully permanent if δ is unity. There are various possibilities of intermediate persistence between.

The process for the income levels are specified, using the assumptions that income is demand determined and that real interest rates are exogenous. We therefore overlook the aggregate supply and LM curves and consider just the two countries' IS curves:

$$\begin{aligned} yd_t &= \beta_1\, gd_t - \beta_2\, rd_{t-1} + \beta_3\, f_t \\ yf_t &= \beta_1\, gf_t - \beta_2\, rf_{t-1} - \beta_3\, f_t \end{aligned} \qquad (15)$$

Their government expenditures are gd and gf and their real interest rates are rd and rf. Again the common coefficients assumption is in operation. Income in each country is determined by its government

[2] The elasticities of exports and (less) imports quantities must exceed unity to countervail the valuation effect on imports revenue.

expenditure, its real interest rate and its state of current account balance. These three are represented, respectively, by the terms, gd, gf, rd, rf, f and -f. A surplus for the domestic country constitutes a deficit for the foreign one.

A reduced-form for the current account is obtained by substituting (15) into (14):

$$f_t = f_0 + \eta\, Gm_{t-1}\, \bar{c}_t + k\, v_{t-1} + u_t \tag{16}$$

where $u_t = \delta\, u_{t-1} + \epsilon_t$

$0 \leq \delta \leq 1 \qquad \epsilon_t \sim NIID\ (0,\ \sigma_\epsilon)$

where $f_0 = [a_1\, \beta_1\, (gf - gd) + \alpha_0]/(1 + 2\alpha_1\beta_3)$

$\eta = \alpha_2/(1 + 2\alpha_1\beta_3)$

$k = \alpha_1\beta_2/(1 + 2\alpha_1\beta_3)$

$u = u'/(1 + 2\alpha_1\beta_3)$

$\epsilon = \epsilon'/(1 + 2\alpha_1\beta_3)$

and $v \equiv rd - rf$

Government expenditures are assumed constant in order to limit the range of the dynamics. The real interest rate differential is positively related to the current account, as a rise causes a relative contraction in the domestic country, which causes its imports to fall relative to its exports (foreign imports).

The real exchange rate allowing for interaction with the current account is obtained by substituting equation (16) intc (13):

$$c_t = -f_0 - \eta\, Gm_{t-1}\, \bar{c}_t - kv_{t-1} - u_t + b\, \{\, Gm_{t-1}\, (\bar{c}_{t+1} - \bar{c}_t - \bar{v}_t) - Gm_{t-2}\, (\bar{c}_t - \bar{c}_{t-1} - \bar{v}_{t-1})\} \tag{17}$$

$$u_t = \delta\, u_{t-1} + \epsilon_t$$

The model is completed by specifying a process for the real interest rate differential:

$$v_t = \theta v_{t-1} + \omega_t \qquad \omega_t \sim \text{NIID}(0, \sigma_w)$$
$$0 \leq \theta < 1 \tag{18}$$

It follows a Markov process in the same way as the current account shock. It is assumed that the two governments together determine an interest rate policy and that they set nominal rates accurately to achieve real target rates. If v is positive, there is a relative contraction in the domestic country and θ represents its degree of persistence.

The expected long-run equilibrium of the exchange rate where δ and θ are less than one and where b is finite (imperfect capital mobility) may be determined as

$$Gm\,\overset{*}{c} = -f_0/(1+\eta) \tag{19}$$

In expected value, the current account is asymptotically zero because capital flows in this case are asymptotically zero in expected value.

2.5 Rational Expectations Equilibrium

The REE solution containing the assumption that agents have full information of the structural parameters, h, d, g, k and f_0, is here considered as the benchmark case. Assuming away coordination problems, they can solve for the rational expectations reduced-form parameters by application of the undetermined coefficients method, which is detailed in Appendix 2.A.2. It is then possible to determine Δr_{t-1} in equation (11) and by inference the parameters, ψ and μ.

We give the general REE solution to the model in equation (17) where b can take on any value in the subsection immediately below. In the following subsection we consider the AMV version as defined in section 2.3 where the value of b is unbounded. The final subsection looks at the current account in this latter version and the correlation between the current account and the exchange rate.

2.5.1 The General Solution

All parameter values are known and all information is fully exploited. The information of relevance includes that on current account shocks and on real interest rate differentials. The price variables, the exchange rate and the interest rates, are observed at the time of their realisation. However, quantity variables, the current account terms, are observed with a lag. The distinction is made on the assertion that price data are immediately transmitted through the mechanism of a centralised market institution, while the availability of quantity data depends on the collection and publication of statistics.

The equilibrium outcome at time t is conditioned on information which is not available at the same time, but at time t - 1. This is to ensure that there is no contemporaneous feedback from the observable outcome which is conditional on information to the same set of information, which would trivialise the informational problems facing the agents.

The rational expectations reduced-form is

$$c_t = \tilde{c} + \sum_{p=0}^{\infty} \pi_p \, \omega_{t-p} + \sum_{p=0}^{\infty} \gamma_p \, \epsilon_{t-p} \tag{20}$$

The ω and the ϵ are the white noise disturbances in the real interest rate differentials and in the current account shocks. $\tilde{c}$ is the long-run equilibrium value of the exchange rate. The solution method (shown in Appendix 2.A.2) is to substitute (20) into (17), also using (18) for consistent expectations of real interest rate differentials, and to determine the values of the π_p and γ_p coefficients in (20).

The procedure implicitly assumes that there are no coordination problems for the agents of the model. It only makes sense for one agent to apply the algorithm, if he believes that all others are applying it. But this assumption is standard for macroeconomic rational expectations models.

Given that the structural parameter values, θ, δ, η, k and fo, are known, the solution is

$$\begin{aligned}
\tilde{c} &= s_t - f_0/(1 + \eta) \\
s_t &= k_1\lambda_1{}^t + k_2\lambda_2{}^t \quad (s_0 = k_1 + k_2) \\
\lambda_1,\lambda_2 &= 0.5\ (1 + 2b)/b \ \pm \sqrt{[0.25\ [(1 + 2b)/b]^2 - 1]} \qquad (21) \\
\gamma_0 &= -1 \\
\gamma_1 &= -\delta
\end{aligned}$$

The solutions for γ_2, γ_3, γ_4,...are given in Appendix 2.A.2.

$$\pi_0 = 0 \qquad (22)$$

The solutions for π_1, π_2, π_3, ... are given in Appendix 2.A.2.

The long-run exchange rate is s_t - fo/(1 + η), which is indeterminate because s_t is non-unique. The problem of non-uniqueness is a recognised feature of dynamic REE models [Shiller (1978)].

Moreover, there is only one stationary long-run equilibrium in the general case where $k_1 = 0$.

2.5.2 The Asset Market View Solution

Assuming full-information of parameter values, again, this time b is unbounded in accordance with our definition of the AMV. The coefficient values in (21), (22) and (A.27) and (A.29) in Appendix 2.A.2 are restricted to

$$\tilde{c} = s_0 - f_0/(1 + \eta) \quad \text{as } \lambda_1, \lambda_2 = 1 \tag{23}$$

$$\begin{aligned} \gamma_0 &= -1 \\ \gamma_1 &= -\delta \\ \gamma_p &= 0 \qquad \text{for } p \geq 2 \end{aligned} \tag{24}$$

$$\begin{aligned} \pi_0 &= 0 \\ \pi_p &= -\theta^p/(1 - \theta) \qquad \text{for } p \geq 1 \end{aligned} \tag{25}$$

There is again a multiplicity of solutions for the long-run exchange rate and so for the current rate, but in this instance each solution is stationary. The reason is that the combination of non-stationarity, leading to non-zero rates of depreciation and non-zero relative yields, and an infinite value for b implies that capital flows and the (log of the) exchange rate would also be infinite. The REE algorithm in imposing equilibrium on the model must, of necessity, impose stationarity to ensure the existence of a finite equilibrium where b is infinite.

Therefore, the existence of *non-stationary* rational bubbles is precluded by the unboundedness of b. If a strong AMV is taken of exchange rate determination, then it is inappropriate to model exchange

rate dynamics as Flood and Garber (1980) modelled the German hyperinflation. Our result differs from that in Blanchard and Watson (1982) because our specification that the real interest rate differential converges to zero ensures that the long-run equilibrium is in the *level* of the exchange rate.

There is only one solution which is consistent with long-run current account balance as given by equation (19) in section 2.4. This is where s_0 is zero. So, that if agents believe that in the long-run there is current account balance, where there will be no net wealth transfers between the two countries, then they may pick the unique solution given by equation (19).

Comparing the coefficient values in (22) and (24), it is seen that within the AMV only the contemporaneous and first lag of the noise in the current account shock can affect of the exchange rate. Outside the AMV there is an effect from all lags in the noise, although these effects will be very small where b is very large and will geometrically decline where the shock persistence is less than perfect.

The AMV version of the exchange rate is affected only by unobservable current account movements which cannot be anticipated. There is some analogy to the new-classical macroeconomic models [e.g. in Lucas (1972), Sargent and Wallace (1975), and Barro (1976)] where unanticipated movements alone in the money supply have real effects.

Finally, a comparison of (22) and (25) makes apparent that the real interest rate differential affects the exchange rate only through the capital account within the AMV. There is no effect via the

relatively contractionary effect on income levels and on import flows, because of the parameterisation representing perfect capital mobility.

The coefficients in (23)-(25) and (A.27) and (A.29) in Appendix 2.A.2 and equation (18) give the exchange rate with full REE and the AMV as

$$c_t = s_0 - fo/(1 + \eta) - [\theta/(1 - \theta)]\, v_{t-1} - \epsilon_t - \delta\epsilon_{t-1} \qquad (26)$$

The expected mean relative yield can be solved as

$$Gm_{t-1}\, \bar{z}_t = [\theta + \theta^2/(1 - \theta) - \theta/(1 - \theta)]\, v_{t-1} = 0$$

for all t. The expected mean relative yield is zero and stationary. This is expected interest rate parity for all time periods. This precludes the non-existence of a finite exchange rate equilibrium. The equilibrium in the case of stationary beliefs, which was indeterminate in the table in Section 2.3, is fully determinate inside REE once beliefs on long-run current account balance are imposed to overcome the multiplicity problem.

In the AMV case, where the model is determinate and where there must be unanimous beliefs in its determinacy, all agents must believe $\bar{z}$ is zero. Therefore in terms of the structural parameters in Section 2.3, ψ is unity and τ and μ are indeterminate. The choice-theoretic foundations of the model in Section 2.3 are only enlightening outside the AMV case, since the distribution of beliefs is trivialised inside the AMV case.[3]

[3] Beliefs could differ in this case if risk-aversion was admitted with *differing* degrees of aversion across agents.

2.5.3 The Current Account and the Exchange Rate from the AMV inside REE

Taking the expected exchange rate from (26) and substituting it into (16) gives the current account from the AMV and inside REE:

$$f_t = \hat{f}_0 + \eta\ [s_0 - f_0/(1 + \eta)] + [k - \eta\theta/(1 - \theta)]\ v_{t-1} + u_t \quad (27)$$

We want to consider Mussa's (1983) assertion that there is a negative correlation between the rate of depreciation and the current account. To simplify, we assume that the exchange rate has a unit root, which is ensured where $\theta = 0$. The rate of depreciation and the current account then become, respectively:

$$c_t - c_{t-1} = -\epsilon_t + (1 - \delta)\ \epsilon_{t-1} + \delta\ \epsilon_{t-2} \quad (28)$$

$$f_t = fo + \eta\ [s_0 - f_0/(1 + \eta)] + k\ v_{t-1} + \epsilon_t + \delta\epsilon_{t-1} + \delta^2\epsilon_{t-2} + \ldots \quad (29)$$

The covariance between the rate of depreciation and the current account is

$$\text{cov}\ [c_t - c_{t-1},\ f_t] = -\ (1 + \delta^2)\ (1 - \delta)\ \sigma_\epsilon \quad (30)$$

where σ_ϵ is the variance of the white noise in the current account shocks.

The covariance is strongest at $-\sigma_\epsilon$ where the current account shocks are transitory ($\delta = 0$) and weakest at zero, where the current account shocks are fully permanent ($\delta = 1$). Generally, inside REE from an AMV there will be some negative correlation between the rate of depreciation and the current account surplus, where the exchange rate has a unit root and where the current account shocks are not fully permanent.

2.6 Deficient Information of Parameter Values

The purpose of this section is to consider the AMV of the model when the information availability assumptions are relaxed. Agents have *beliefs* on the structural parameter values which may not necessarily coincide with their *actual* values. The same REE algorithm is applied, but with some miscalculation where beliefs differ from actual values. The miscalculation or misanticipation will lead to current account effects, even where all the other strong AMV assumptions are maintained.

It is shown that the capital account is central to the analysis, as asset market decisions are conditioned on information, which may be erroneous, leading to misanticipations. Agents are limited by the non-simultaneity of the observed equilibrium with the pertinent information, which was discussed at the end of section 2.3

Our capital account representation (henceforth 'the capital account') is the term b ( ) on the right-hand side of equation (17). It may be determined by adding the exchange rate to the current account as

$$c_t + f_t = b\ (\ )_t \tag{31}$$

In terms of the REE and AMV of the previous section, the capital account is found by combining (28) and (29) to give

$$b\ (\ )_t = [k - (1 + \eta)\ \theta/(1 - \theta)]\ v_{t-1} + \sum_{p=2}^{\infty} \delta^p\ \epsilon_{t-p} \tag{32}$$

It is assumed that s_0 is zero, because of beliefs in long-run current account balance.

Equation (32) is based on the assumption that the values, k, η, θ and δ are known to agents. If agents beliefs on these values, $\hat{k}$, $\hat{\eta}$, $\hat{\theta}$

and $\hat{\delta}$, can differ from their actual values, then a more appropriate capital account form is

$$b\ \{\ \}_t = [\hat{k} - (1 + \hat{\eta})\ \hat{\theta}/(1 - \hat{\theta})]\ v_{t-1} + \sum_{p=2}^{\infty} \hat{\delta}^p\ \epsilon_{t-p} \tag{33}$$

The exchange rate expectation determining the current account will also be conditioned on agents' beliefs, so that the corresponding form for the current account will be

$$f_t = f_0 + \eta[-\hat{f}o/(1+\hat{\eta})] + \{k - \eta[\hat{\theta}/(1-\hat{\theta})]]\ v_{t-1} + \sum_{p=0}^{\infty} \delta^p\ \epsilon_{t-p} \tag{34}$$

The exchange rate from equations (31), (33) and (34) is

$$c_t = -f_0 + [\eta\hat{f}_0/(1+\hat{\eta})] + \{\hat{k} - k - (1+\hat{\eta}-\eta)[\hat{\theta}/(1-\hat{\theta})]\}\ v_{t-1}$$

$$-\epsilon_t - \delta\epsilon_{t-1} - \sum_{p=2}^{\infty}(\delta^p-\hat{\delta}^p)\ \epsilon_{t-p} \tag{35}$$

This is a case of the AMV version where information on the values of the structural parameters is not necessarily available.[4] Equation (26) is a special case of (35), where beliefs and actual values coincide.

The last term in equation (35) shows that observable positive components in the shocks depreciate the exchange rate when their persistence is overanticipated ($\delta < \hat{\delta}$) and appreciate the exchange rate when their persistence is underanticipated ($\delta > \hat{\delta}$). If the degree of persistence is correctly anticipated ($\delta = \hat{\delta}$), then observable shock components have no effect on the exchange rate.

This shows that there are two necessary conditions for current account neutrality: full rational expectations and perfect capital mobility with potentially unbounded capital flows. In the previous

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[4] Harris and Purvis (1981) considered a disaggregated currency-substitution model with confusion between real and monetary shocks in the context of rational expectations with limited information.

section, the first condition without the second delivered current account effectiveness. In this section, current account effectiveness results with the second condition without the first condition of full rational expectations. The analogies are to imperfect wage-price adjustment in macro-models of the money and output relationship [Fischer (1977) and Phelps and Taylor (1977)] and to deficient information of parameter values in the same kinds of model [Taylor (1975) and Friedman (1979)].

The non-simultaneity between the observation of equilibrium and of the information relevant to its determination, discussed above, is important to this result. This may be shown by considering a case where there is *simultaneity*.

In this instance, the observed equilibrium feedsback into the information set which determines it through the capital account. Any current account movement would *potentially* disturb the exchange rate, but actually send a signal to agents driving the capital account. The disturbance is only potential, because the feedback would 'stabilise' the equilibrium, given the powerfully orchestrated response from the infinitely mobile capital.

The agents of the model would not need to know the current account parameters, because the exchange rate would act as a sufficient statistic for their decisions. Consequently, the extent of their knowledge of parameters is immaterial to the model's solution, because of the efficacy of the equilibrium as a signal.

In the case considered without this simultaneity, the exchange rate is no longer a sufficient statistic, so that agents' parameter knowledge is important for the solution. If mistakes are made, the equilibrium cannot signal these and simultaneously correct itself.

The mechanism by which observable current account shocks *can* be neutralised from their potential effect on the exchange rate works through the capital account. An observed (positive) shock would appreciate the exchange rate in the absence of a capital account response. There would be a relative demand increase for the domestic currency from the current account. But, agents driving the capital account perceive, say, at time t-1, the *potential* fall in the exchange rate at time t, and therefore, the *potential* depreciation at time t+1. This induces a capital movement from dcd into fcd assets, as foreign holders of dcd assets endeavour to avoid a capital loss and domestic holders of fcd assets try to make a capital gain. The capital movement raises the relative demand for foreign currency until there is no potential depreciation - until it matches the initial increase in relative demand for domestic currency determined by the initial current account shock. So, the overall effect on the relative demand for domestic currency nets out to zero, as the current account influence is countered by a capital account response, where the influence is perceived and where its persistence is correctly anticipated.

This scenario is where the persistence of the shock is correctly anticipated. There are two other possibilities, where it is over-anticipated and where it is under-anticipated. Firstly, agents may over-anticipate the persistence of the shock, expecting a larger potential depreciation than is warranted. There is consequently a too

large switch from dcd to fcd assets: the capital account over-reacts. The final effect is a depreciation as the rise in the relative demand for domestic currency from the current account source is exceeded by the rise in the relative demand for foreign currency from the capital account response.

Secondly, agents may under-anticipate the persistence of the shock, expecting a smaller depreciation than is warranted. The switch from dcd into fcd assets is then too small: the capital account under-reacts. The final effect is an appreciation, because the rise in the relative demand for domestic currency from the current account source exceeds the rise in the relative demand for foreign currency from the capital account response.

The possibility of misanticipations modifies the relationship between depreciation and the current account. The stochastic process for depreciation outside full rational expectations is given by

$$q_t = -\epsilon_t + (1 - \delta)\,\epsilon_{t-1} + [\delta - (\delta^2 - \hat{\delta}^2)]\,\epsilon_{t-2} + \sum_{p=3}^{\infty}[(\delta^{p-1} - \delta^p) - (\hat{\delta}^{p-1} - \hat{\delta}^p)]\,\epsilon_{t-p} \tag{36}$$

from differencing the stochastic process in equation (35). The covariance between the rate of depreciation and the current account, now given by equation (34) [see Appendix 2.A.3] is

$$\mathrm{cov}\,(c_t - c_{t-1}\, f_t) = -(1 - \delta)\left[1 + \delta^2 + \frac{\delta^4}{1 - \delta^2} - \frac{\hat{\delta}^2\delta^2}{1 - \hat{\delta}\delta}\right]\sigma_\epsilon$$

The REE covariance, given before, is the special case where $\delta = \hat{\delta}$. There is now the possibility of a *positive* covariance where agents over-

anticipate the persistence of the shocks. For example, let agents believe that the persistence is fully permanent ($\hat{\delta} = 1$). In this event, the sufficient condition for a positive covariance is that δ is between 0.755 and 1. High but over-anticipated values for δ will increase the likelihood of a positive covariance. But, generally, we would expect the covariance to be negative.

2.7 The Revision of Beliefs on the Long-Run Equilibrium

In a normal state of the world there is little reason to expect beliefs to be constant over time. This especially applies where the outcomes of agents' decisions, conditioned on their beliefs, are inconsistent with these beliefs. Agents would normally revise their beliefs outside the state where all information is readily available.

The literature on learning [reviewed by Blume, Bray and Easley (1982)] considers the question of whether *beliefs* will converge to the REE state of the model where all information (on parameter values) is available. Such a consideration is interpreted as one of the *stability* of REE, and it therefore helps to assess the validity of using the rational expectations hypothesis in economic models.

Although of interest, this is only one consideration of this section, which is to look at the dynamic pattern which learning may impart to the model. The revision of beliefs over time may give rise to phenomena which can otherwise be attributed to dynamic instabilities [discussed more fully in Chapter 3]. A learning process may have the appearance of a speculative bubble which is extraneous to the

fundamentals of the model. But, without discussing the merits of this alternative hypothesis, the purpose of this section is to widen the analysis to consider learning and to look at the dynamic patterns which can emerge.

A problem with the AMV of the model is that indeterminacy results outside of beliefs in REE. An informational equilibrium must be imposed to ensure a sensible result where the parameter b is valued towards infinity. In a way, this questions the plausibility of such strong asset market assumptions. Furthermore, we must specify conditions for learning to be compatible with determinacy.

Where capital flows are potentially unbounded, the compatibility of learning and determinacy requires the assumption that agents at least believe themselves to be in a state of full REE *ex ante*. There are two broad possibilities. One is that each agent is without full information of the model's parameter values yet believes the rest of the market to be in a state of full REE. This possibility can be justified by the lack of coordination between agents caused by their dispersion across a decentralised market.

But a precondition for this first possibility is that agents in the market at least know the persistence of the current account shocks. If this is not the case, each agent would surely come to perceive some effect from the observable shocks to the exchange rate as in equation (35). Then the deduction would be made that the rest of the market is not in REE. Indeterminacy would result if agents made expectations based on their beliefs of the actual values in conjunction with beliefs of market beliefs. Equation (36) shows that where the two differ, the

expected rate of depreciation would be determined by the observable shocks and would be generally non-zero. Indeterminacy results in the case of the AMV where the parameterisation of beliefs of expected depreciation in the capital account is non-zero. Consequently, determinacy within the AMV requires that agents believe that all other agents are in REE.

The problem arises through the parameterisation of the capital account part of the model. Instead we could specify the learning of a parameter which does not have capital account repercussions. The long-run equilibrium of the exchange rate as a *level* does not affect the capital account which is determined by expected *changes* in the exchange rate.

The second possibility is where agents are in a (dynamic) REE, but are learning the long-run equilibrium. There are an infinity of possibilities because of the dynamic structure of the model [Shiller (1978)] and each is fully consistent with REE. In terms of this model it is learning the value of $s_0 - f_0/(1 + \eta)$ where there are multiple solutions *a priori*. Learning the long-run equilibrium of a dynamic rational expectations model was considered by Gottfries (1985) and Evans (1985). They considered learning as a more appropriate and more economic basis for the selection of a unique value than the selection criteria reviewed by Burmeister (1980). [See also Taylor (1977) and McCallum (1983)].

Gottfries' model differs from ours in that his equivalent parameter for b is finite in value, which makes for only one stationary solution with all the rest non-stationary. In our case, as b is

infinite in value, there is an infinity of stationary values corresponding to different values of s_0, as shown in Section 2.5.

In the rest of this section, we consider learning of the long-run equilibrium. The learning process is inconsistent with the model being learned outside REE as in Cyert and DeGroot (1974), DeCanio (1979), Bray (1982) and Bray and Savin (1982). Its justification is the lack of information which would otherwise permit agents to coordinate their learning strategies, so that learning would have no unanticipated effects. Agents cannot coordinate their learning strategies, if they are members of a decentralised market. And, as they are individually small in relation to the whole market, they may believe that their learning activities will have no effect on the model.

Assuming that the long-run equilibrium is the only unknown parameter in the model, the exchange rate is given by

$$c_t = -f_0 - \eta\, Gm_{t-1}\, \tilde{c} - [\theta/(1 - \theta)]\, v_{t-1} - \epsilon_t - \delta\epsilon_{t-1} \tag{37}$$

where $Gm_{t-1}\, \tilde{c}$ is the average or market belief at time t-1 of the long-run equilibrium:

$$Gm_{t-1}\, \tilde{c} = Gm_{t-1}\, [s_0 - f_0/(1 + \eta)] \tag{38}$$

Individual beliefs are updated such that the individual belief at any time is a weighted average of the average belief at one period earlier and of the most recently available information of its value. The agent i's belief of the long-run equilibrium at t-1 is

$$Gi_{t-1}\, \tilde{c} = \lambda i_{t-1}\, Gi_{t-2}\, \tilde{c} + (1 - \lambda i_{t-1})[- fo - \eta\, Gm_{t-2}\, \tilde{c} - \epsilon_{t-1} - \delta\epsilon_{t-2}] \tag{39}$$

where $0 < \lambda i_{t-1} \leq 1$. λi is the weighting on the previous belief in the current belief, so it represents (inversely) the speed of learning. The individual effectively faces a signal extraction problem because of noise in the data he is learning from. The noise comes from the unobservable current account shocks.

The term in [ ] in equation (39) is the signal of the long-run equilibrium which is generated by the model. It is deduced from observing the exchange rate at time t-1 and from knowing the expected path of future real interest rate differentials accumulated as $[\theta/(1 - \theta)]\ v_{t-2}$. Information on the long-run equilibrium is received in this signal, which is clouded by noise from the as yet unobservable current account shocks, ϵ_{t-1} and ϵ_{t-2}. Average or market beliefs at time t-2 also bear upon the signal received by individuals at time t-1 which feeds into beliefs. So there is feedback from the beliefs at an aggregate level to the signal itself.

If agents are Bayesians, then the weightings must be constrained to make learning consistent with optimal decision theory [DeGroot (1970)].

$$\lambda i_t = hi_{t-1}/hi_t$$

$$hi_t = hi_{t-1} + [(1 + \delta^2)\ \sigma]^{-1} \text{ for each i.}$$

hi_t is the Bayesian precision of i's belief on the long-run equilibrium at time t. It corresponds with the mean of his distribution of beliefs, $Gi_t\ \tilde{c}$. $[(1 + \delta^2)\ \sigma]^{-1}$ is the precision of the signal, which is assumed to be known to agents. It is the inverse of the joint variance of the unobservable shocks.

We assume orthogonality between the precision of beliefs, λi, the weight of the agent in the market, wi, and the mean belief, $Gi\tilde{c}$.

Aggregate learning is then given by

$$Gm_{t-1}\,\tilde{c} = \lambda m_{t-1}\,Gm_{t-2}\,\tilde{c} + (1 - \lambda m_{t-1})\,[\,-\,fo - \eta\,Gm_{t-2}\,\tilde{c} - \epsilon_t - \delta\epsilon_{t-1}] \qquad (40)$$

where $Gm_{t-1}\,\tilde{c} = \Sigma wi\,Gi_{t-1}\,\tilde{c}$

$\lambda m_{t-1} = \Sigma wi\,\lambda i_{t-1}$

$\Sigma wi = 1.$

The equilibrium from learning is in the limit given where

$$Gm_{t-1}\,\tilde{c} = Gm_{t-2}\,\tilde{c}$$

where aggregated beliefs are unchanging, where

$$Gm\,\tilde{c} = -\,f_0/(1 + \eta)$$

This is the unique value where so is zero - of long-run current account balance.

This result obtains because of the static structure of expectations in the current account side of the balance of payments. That is to say, expectations of the exchange rate at time t determine the current account balance, which enters the equation for the exchange at time t. Given the static structure of expectations, there is a unique long-run equilibrium if reference is made only to the current account. [See the discussion in Chapter 3.]

Moreover, the long-run equilibrium in question is a level, which is relevant only to the current account, and not a rate of change (of depreciation) which is relevant only to the capital account. Consequently, learning is made only with reference to the current

account side which has a static structure of expectations and a unique equilibrium.

This unique equilibrium is consistent with current account balance because of the set-up of the model. Excluding the capital account (and the intervention account), the identity that the total balance of payments is zero translates into one of zero balance on the remaining current account. This exclusion is implicitly made by agents who learn the long-run level of the exchange with sole reference to the current account.

Agents need not know the long-run equilibrium which is consistent with current account *or even that it is consistent* with current account balance. A process of stable learning will deliver an equilibrium belief which brings current account balance in the long-run. The reason for this strong result is that the model allows for current account interaction with the learning process. Beliefs affect the current account which directly affects beliefs on its long-run value and on the level exchange rate. There is no capital account reaction since there is no effect on the expected rate of change of the exchange rate. A revision in beliefs affects the exchange rate signal on which agents are revising their beliefs.

The stability of learning, assuming a constant rate of adjustment[5] in equation (40) is clearly defined by the composite of parameters

$$\lambda m - \eta(1 - \lambda m)$$

Positive serial correlation and instability is precluded as $0 < \lambda m < 1$ and $\eta > 0$. Negative serial correlation and instability is a possibility

[5] Turnovsky (1969) treats adaptive expectations as a special case of Bayesian learning.

where $\eta > (1 + \lambda m)/(1 - \lambda m)$. This is arguably not the case where the elasticities of exports and imports are weak and where learning is relatively sluggish (where km is close to one). The dynamics of learning will be (stable) positive where $km/(1 - km) > g$, where the revision of beliefs is relatively sluggish and where the elasticities are weak; and negative, if otherwise.

Following a perturbation in the parameter f_0, determined by a change in relative (fixed) government expenditure, a plausible scenario is of a period of stable learning with positive serial correlation in the exchange rate. This differs from the exponential growth pattern expected from many models of speculative bubbles.

And, given the posited adjustment process, the exchange rate will be seen to overshoot its new long-run equilibrium as beliefs lag behind this new value. For example, if η is 1 and f_0 is 0.10, the long-run equilibrium is -0.05. Suppose that f_0 rises from 0.10 to 0.14, because relative (fixed) government expenditure rises in the foreign country [see the definition of f_0 below equation (16)]. According to equation (38), there will be an immediate appreciation of 80% to 0.09 before a process of stable learning depreciates the exchange rate by 22% to its new long-run equilibrium value of -0.07.

Overshooting occurs because the revision of beliefs is sluggish or less than instantaneous with the immediate adjustment of actual values. And, for plausible assumptions about the export and import elasticities, stability will ensure convergence to an equilibrium consistent with current account balance, whether this is anticipated or not.

APPENDICES TO CHAPTER 2

2.A.1 The Determination of ξ

The constant proportion, $(1 - \psi)$, believes that $\bar{z}$ is non-zero, and of this proportion, τ, and so, $(1 - \psi)\tau$ overall believes that $\bar{z}$ is positive. Within the proportion $(1 - \psi)$ (non-zero) beliefs are normally distributed

$$Gi\bar{z} \sim N\,(Gm\bar{z}',\ \sigma_z) \tag{A.1}$$

$Gm\bar{z}'$ and σ_z are the mean and variance of beliefs within this set. The overall mean, including the proportion, ψ believing that $\bar{z}$ is zero is

$$Gm\bar{z} = (1 - \psi)\ Gm\bar{z}' + \psi.0 = (1 - \psi)\ Gm\bar{z}' \tag{A.2}$$

The distribution within $(1 - \psi)$ is represented by the probability density function:

$$f(Gi\bar{z}) = (1 - \psi)\ (2\pi\sigma_z)^{0.5} \exp[-0.5\sigma_z^{-1}(Gi\bar{z} - Gm\bar{z}')^2] \tag{A.3}$$

so that

$$(1 - \psi)\tau = \int_0^\infty f(Gi\bar{z})\ dGi\bar{z} \tag{A.4}$$

The change is the proportion $(1 - \psi)\tau$ with respect to the change in $Gm\bar{z}'$ is

$$\frac{\delta(1 - \psi)\tau}{\delta Gm\bar{z}'} = (1 - \psi)(2\pi\sigma_z)^{0.5} \exp[0.5\sigma_z^{-1}\ Gm\bar{z}'^2] \tag{A.5}$$

As ψ is constant, inversion gives

$$\frac{\delta Gm\bar{z}'}{\delta\tau} = (2\pi\sigma_z)^{-0.5} \exp[-0.5\sigma_z^{-1}\ Gm\bar{z}'^2] = \xi \tag{A.6}$$

which is always non-negative. In the AMV case where $Gm\bar{z}' \to 0$, $\sigma_z \to 0$, we assume that the mean and variance tend to their limits at rates which ensure ξ is non-zero.

2.A.2 The REE Solution

Equation (17) gives the structural-form of the model:

$$c_t = -f_0 - \eta\, Gm_{t-1}\, \bar{c}_t - k\, v_{t-1} - u_t + b\, \{Gm_{t-1}\, (\bar{c}_{t+1} - \bar{c}_t - \bar{v}_t) - Gm_{t-2}\, (\bar{c}_t - \bar{c}_{t-1} - \bar{v}_{t-1})\}$$

where $v_t = \theta v_{t-1} + \omega_t \qquad \omega_t \sim NIID\ (0,\ \sigma_w)$

$u_t = \delta u_{t-1} + \epsilon_t \qquad \epsilon_t \sim NIDD\ (0,\ \sigma_\epsilon)$

We specify the rational expectations reduced-form:

$$c_t = \tilde{c} + \pi(L)\ \omega_t + \gamma(L)\ \epsilon_t \tag{20}$$

where $\tilde{c}$ is the long-run equilibrium of the exchange rate, ω_t and ϵ_t are the exogenous real interest rate differential shocks and the exogenous current account shocks, respectively. $\pi(L)$ and $\gamma(L)$ are the polynomials in the lag operators:

$$\pi(L) = \pi_0 + \pi_1 L + \pi_2 L^2 + ..\ \pi_p L^P \tag{A.7}$$

$$\gamma(L) = \gamma_0 + \gamma_1 L + \gamma_2 L^2 + ..\ \gamma_p L^P \tag{A.8}$$

The REE solution is obtained by substituting the reduced-form into the structural-form for the expectations terms, and then solving the reduced-form coefficients, given the consistency of the structural-form with the reduced-form.

Expectations made at time t-1 exploit all information available at that date, which comprises current account shocks up until time t-2 and real interest rate shocks up until time t-1. So, generally,

$$Gm_{t-1}\ x_{t+s} = Gm_{t-1} x_{t+s} | I_{t-1}\ (..) \tag{A.9}$$

where $I_{t-1}(..) = I_{t-1}(\epsilon_{t-2},\ \epsilon_{t-3}, ..\ \epsilon_0\ ;\ \omega_{t-1},\ \omega_{t-2},\ ..\ \omega_0)$ (A.10)

The mechanics of the solution may be shown in the tabular form below.

TABLE 2

Substitution of Rational Expectations Reduced-form into the Structural-form

Coefficient on		$b\ Gm_{t-1}\bar{c}_{t+1}$	$-(b+\ )Gm_{t-1}\bar{c}_t$	$-bGm_{t-2}\bar{c}_t$	$bGm_{t-2}\bar{c}_{t-1}$	$-bGm_{t-1}\bar{v}_t$	$bGm_{t-2}\bar{v}_{t-1}$	$-kv_{t-1}$	$-u_t$
ω_t	π_0	0	0	0	0	0	0	0	-
ω_{t-1}	π_1	$b\pi_2$	$-(b+\eta)\pi_1$	0	0	$-b$	0	$-k\theta$	-
$t-2$	π_2	$b\pi_3$	$-(b+\eta)\pi_2$	$-b\pi_2$	$b\pi_1$	$-b\theta^2$	$b\theta$	$-k\theta^2$	-
$t-3$	π_3	$b\pi_4$	$-(b+\eta)\pi_3$	$-b\pi_3$	$b\pi_2$	$-b\theta^3$	$b\theta^2$	$-k\theta^3$	-
$t-4$	π_4	$b\pi_5$	$-(b+\eta)\pi_4$	$-b\pi_4$	$b\pi_3$	$-b\theta^4$	$b\theta^3$	$-k\theta^4$	-
t	γ_0	0	0	0	0	-	-	-	-1
$t-1$	γ_1	0	0	0	0	-	-	-	$-\delta$
$t-2$	γ_2	$b\gamma_3$	$-(b+\eta)\gamma_2$	0	0	-	-	-	$-\delta^2$
$t-3$	γ_3	$b\gamma_4$	$-(b+\eta)\gamma_3$	$-b\gamma_3$	$b\gamma_2$	-	-	-	$-\delta^3$
$t-4$	γ_4	$b\gamma_5$	$-(b+\eta)\gamma_4$	$-b\gamma_4$	$b\gamma_3$	-	-	-	$-\delta^4$

The reduced-form coefficients are the horizontal sums of the tabulated values:

$$\pi_0 = 0 \tag{A.11}$$

$$\pi_1 = [b\ \pi_2 - \theta\ (b + k)]\ x \tag{A.12}$$

$$\pi_p = [b\ (\pi_{p+1} + \pi_{p-1}) - (\theta - 1)\ b\theta^{p-1} - k\theta^p]\ y \tag{A.13}$$

$$\text{for } p \geq 2$$

$$\gamma_0 = -1 \tag{A.14}$$

$$\gamma_1 = -\delta \tag{A.15}$$

$$\gamma_2 = [b\ \gamma_3 - \delta^2]\ x \tag{A.16}$$

$$\gamma_p = b\ [\gamma_{p+1} + \gamma_{p-1} - \delta^p]\ y \tag{A.17}$$

$$\text{for } p \geq 3$$

where $x^{-1} = 1 + b + \eta$

$y^{-1} = 1 + 2b + \eta$

It is clear from the above that there is a pattern of geometric decline:

$$\pi_p = \theta^{p-3}\ \pi_3 \qquad \text{for } p \geq 3 \tag{A.18}$$

$$\gamma_p = \delta^{p-1}\ \gamma_4 \qquad \text{for } p \geq 4 \tag{A.19}$$

M_θ and M_δ are 3 × 3 matrices of coefficients relating the endogenous coefficient to the endogenous coefficients.

$$\begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{bmatrix} = M_\theta \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{bmatrix} - \begin{bmatrix} (\theta(b + x)x \\ (\theta - 1)\ b\theta y + k^2 y\theta \\ (\theta - 1)\ b\theta^2 y + k\theta^3 y \end{bmatrix} \tag{A.20}$$

$$\begin{bmatrix} \gamma_2 \\ \gamma_3 \\ \gamma_4 \end{bmatrix} = M_\delta \begin{bmatrix} \gamma_2 \\ \gamma_3 \\ \gamma_4 \end{bmatrix} - \begin{bmatrix} \delta^2 x \\ \delta^3 y \\ \delta^4 y \end{bmatrix} \tag{A.21}$$

The solution is clearly

$$\begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{bmatrix} = -(I - M_\theta)^{-1} \begin{bmatrix} (\theta(b + x)x \\ (\theta - 1)\ b\theta y + k^2 y\theta \\ (\theta - 1)\ b\theta^2 y + k\theta^3 y \end{bmatrix} \tag{A.22}$$

$$\begin{bmatrix} \gamma_2 \\ \gamma_3 \\ \gamma_4 \end{bmatrix} = -(I - M_\delta)^{-1} \begin{bmatrix} \delta^2 x \\ \delta^3 y \\ \delta^4 y \end{bmatrix} \tag{A.23}$$

where I is the identity matrix.

And,

$$Mi = \begin{bmatrix} 0 & bx & 0 \\ by & 0 & by \\ 0 & by & byi \end{bmatrix} \tag{A.24}$$

$$(I - M_i)^{-1} = D_i^{-1} \begin{bmatrix} 1 - by(i + by) & bx(1 - iby) & bx\ by \\ by(1 - iby) & 1 - iby & by \\ (by)^2 & by & 1 - bxby \end{bmatrix} \tag{A.25}$$

$$D_i = (1 - iby)\ (1 - bxby) - (by)^2$$

$$i = \theta,\ \delta$$

The Asset Market Case: $b \to \infty$

In the asset market case

As $b \to \infty$

$$(I - M_\theta)^{-1} \to \frac{-1}{1-\theta} \begin{bmatrix} 3 - 2\theta & 4 - 2\theta & 2 \\ 2 - \theta & 4 - 2\theta & 2 \\ 1 & 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} (\theta(b + x)x \\ (\theta - 1)\ b\theta y + k^2 y\theta \\ (\theta - 1)\ b\theta^2 y + k\theta^3 y \end{bmatrix} \to \begin{bmatrix} \theta \\ (\theta - 1)\ \theta/2 \\ (\theta - 1)\ \theta^2/2 \end{bmatrix} \tag{A.26}$$

so

$$\begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{bmatrix} \to \frac{-1}{1-\theta} \begin{bmatrix} \theta \\ \theta^2 \\ \theta^3 \end{bmatrix} \tag{A.27}$$

And, from (A.18) and (A.27)

$$\pi_p \to \frac{\theta^p}{1-\theta} \qquad \text{for } p \geq 1 \tag{A.28}$$

There is an equivalent expression for $(I - M_\delta)^{-1}$, but as

$$\begin{bmatrix} \delta^2\ x \\ \delta^3\ y \\ \delta^4\ y \end{bmatrix} \to \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \text{as } x,\ y \to 0 \tag{A.29}$$

and because of (A.19)

$$\gamma_p \to 0 \qquad \text{for } p \geq 1. \tag{A.30}$$

2.A.3 The Covariance Between Depreciation and the Current Account Outside REE

Equations (34) and (36) give a covariance between depreciation and the current account as

$$\begin{aligned} \mathrm{cov}(c_t - c_{t-1}, f_t) &= \{-1 + \delta(1 - \delta) + \delta^2[\delta - (\delta^2 - \hat{\delta}^2)] \\ &\quad + \sum_{p=3}^{\infty} \delta^p[\delta^{p-1} - \delta^p) - (\delta^{p-1} - \hat{\delta}^p)] \} \sigma_\epsilon \\ &= -(1 - \delta) \left[1 + \delta^2 + \frac{\delta^4}{1-\delta^2} - \frac{\delta^2\hat{\delta}^2}{1-\delta\hat{\delta}} \right] \sigma_\epsilon \end{aligned} \tag{A.31}$$

Proof that the covariance can be positive

The covariance is positively related to beliefs on the persistence of the shocks, $\hat{\delta}$. We consider the case where $\hat{\delta}$ is set at its maximum value, approximately unity. The covariance expression then becomes

$$\begin{aligned} \mathrm{cov}\ (c_t - c_{t-1}, f_t) &= -(1 - \delta) \left[1 + \delta^2 + \frac{\delta^4}{1-\delta^2} - \frac{\delta^2}{1-\delta} \right] \sigma_\epsilon \\ &= [\ \delta^2 - (1 + \delta)^{-1}\ ]\ \sigma_\epsilon \end{aligned} \tag{A.32}$$

The value of this is clearly positive where δ exceeds 0.755 and greatest as δ approaches one. So positive covariance is plausible where the shocks are over-anticipated *and* fairly persistent.

GLOSSARY

E	:	the domestic price of foreign currency
M	:	imports revenue
X	:	exports revenue
ΔDf	:	change in foreign holdings of domestic assets
ΔFd	:	change in domestic holdings of foreign assets
z	:	relative yield on domestic assets
d	:	the (nominal) interest rate differential
e	:	the log of E
pd	:	the domestic (log) price level
pf	:	the foreign (log) price level
c	:	the real (log) exchange rate
u	:	the current account shock
v	:	the real interest rate differential
ϵ	:	noise in u, $\epsilon \sim NIID\ (0, \sigma_\epsilon)$
ω	:	noise in v, $\omega \sim NIID\ (0, \sigma_\omega)$
f	:	the real log ratio of exports to imports
f_0	:	the constant term in f
$\tilde{c}$	:	the long-run real (log) exchange rate
ψ	:	the proportion which believes $\bar{z} = 0$
$(1-\psi)\tau$	:	the proportion which believes $\bar{z} > 0$
$(1-\psi)(1-\tau)$	:	the proportion which believes $\bar{z} < 0$
γ	:	of ψ, the proportion holding dcd assets
$1-\gamma$	:	of ψ, the proportion holding fcd assets
b	:	the 'asset market coefficient'
θ	:	the persistence of real interest rate differentials
δ	:	the persistence of 'current account shocks'
η	:	the 'elasticities' coefficient

$b \to \infty$: 'the asset market view'

$0 \leq \theta < 1$

$0 \leq \delta < 1$

CHAPTER 3

THE REVISION OF BELIEFS AS OPPOSED TO THE BUBBLE HYPOTHESIS AS A DATA GENERATING PROCESS

SYNOPSIS

Evidence such as the overvaluation and then the fall of the dollar over 1982-1987 is suggestive that it may be inappropriate to model the exchange rate within a purely fundamental framework. This leaves at least three broad possibilities: rational expectations bubbles, non-rational expectations bubbles and a dynamic process caused by the revision of beliefs in a state of imperfect information.

We reject the first two possibilities, the general bubble hypothesis, on theoretical grounds by considering Benjamin Friedman's (1979) analysis, which dichotomises the *information availability assumption* and the *information exploitation assumption* of rational expectations. If the relevant informational is not fully available, it will not be possible for agents to form rational expectations, which precludes the possibility of rational bubbles. Furthermore, if the limited information available is fully exploited, as a tenet of individual optimality, non-rational bubbles, which are founded on *ad hoc* expectations schemes, will also be precluded.

The hypothesis which we advance is of a dynamic process caused by the revision of beliefs in an environment of limited information. Accepting the exploitation assumption, but rejecting the availability assumption, we envisage a process where agents revise their beliefs in response to sequentially received but imperfect information.

3.1 Introduction

Fundamental models of the exchange rate have failed to explain a data consisting of wild cyclical movements. A fundamental model is one drawn from economic theory. And in the case of an asset market example with forward expectations, it is usually postulated that the fundamental model is one which has a stationary rational expectations solution.

Meese and Rogoff (1983A and 1983B) found that a selection of theoretical models could not improve on the predictions of random walk models for three dollar exchange rates. This result has been considered the more remarkable because the actual realisations of explanatory variables were used as proxies for the corresponding expectations, eliminating expectational errors. Hacche and Townend (1981) concluded that the existing state of economic knowledge could not account for the behaviour of sterling over 1972-1980.

One possibility is that the theoretical models of the exchange rate are misspecified, that there are errors in variables and missing variables problems. But the hypothesis which is more often maintained is that it is inappropriate to look at asset prices in the context of a fundamental model with a stationary REE solution. This is to consider the presence of bubbles in the exchange rate. It was Flood and Garber (1980) who defined a bubble as the departure of market price from the level dictated by market fundamentals.

We have defined the fundamental price in terms of a stationary REE. There are broadly two possible kinds of bubble, one where expectations are rational but not stationary and the other where

expectations are not rational. The terms of rational bubble and of irrational bubble will be applied respectively to each of these possibilities.

It is possible to reject each bubble possibility on theoretical grounds with regard to Benjamin Friedman's (1979) dichotomy between the information availability assumption and the information exploitation assumption of rational expectations. If agents do not plausibly hold the relevant information to form rational expectations, rational bubbles are precluded. If, however, they are individually optimal, fully exploiting the limited information which is available, then we must rule out the ad hoc expectations schemes which would give rise to non-rational bubbles.

It is the purpose of this paper to advance an alternative to the bubble hypothesis. Briefly stated, it is proposed that foreign exchange markets are subject to dynamic processes caused by the revision of beliefs, even in the absence of bubbles. The process of agents revising their beliefs or, more succinctly, of learning, accounts for a great part of the data and explains the departure from the economic fundamentals in their narrowest sense.

An environment of learning where beliefs are revised in accordance with sequentially received but limited information is consistent with our *a priori* rejection of the availability assumption and acceptance of the exploitation assumption. We also advance additional theoretical arguments against the presence of rational bubbles in the foreign exchange market.

Section 3.2 investigates the possibility of rational bubbles, which are also often referred to in the literature as sunspots and bootstrap equilibria. We review some of the various types of rational bubble. Theoretical reasons are advanced to propose that their presence is generally unlikely and also specifically with reference to the foreign exchange market. An exploding real exchange rate bubble is an implausible scenario because of the consequences on the current account balance and for external indebtedness.

Section 3.3 looks first briefly at the Peso problem, which we view as a problem which belongs essentially to the short-run, and then at irrational bubbles emanating from model-inconsistent expectations. In one example, given by Armington (1986), myopic agents fail to take into account the presence of a unique and stationary long-run equilibrium to stabilise their expectations. In a second example, the Frankel and Froot (1986) model, expectations can be backward looking and chartist. Irrational bubble models, which rely upon agents persisting with sub-optimal forecasting rules, clearly violate the assumption that information is fully exploited.

Section 3.4 establishes the model, a restricted form of that in Chapter 2. This leads us into the main part of this chapter in section 3.5, a model of individually rational agents persisting with a process of learning in order to be able to form optimal forecasts. The range of learning possibilities within an economic model is, of course, extensive. We consider the learning of a parameter which is determined by the joint policies of two governments.

The interest rate differential between the two countries is determined by a Markov process. Our apology for this over-simplification is mitigated by our primary desire to isolate the phenomenon of specific interest, the effect of learning on the dynamics of the model. Furthermore, this process may be regarded as the reduced-form which could be obtained from a fuller structural modelling of the economy.

Agents endeavour to learn the Markov parameter, which has the clear interpretation as the degree of persistence of policy differences between the two countries. If a country is subjected to a domestic monetary policy, which is relative tight to that pursued abroad, and/or a relatively loose fiscal policy, there should be a positive real interest rate differential favouring that country. The persistence of the differential reflects the persistence of globally imbalanced stabilisation policies.

Agents learn from experiencing the history of policy and use Bayesian methods. The advantage in these, apart from their optimality, is that they give scope for prior beliefs to affect the model, giving some further insight. The process of updating contributes a source of dynamics to the model, a process which might otherwise be attributed to bubble phenomena.

Some simulations are presented, showing that the interaction of the interest rate differential process with the learning process will give rise and falls in the exchange rate away from its equilibrium level. The analysis is then related, discursively, to the behaviour of the Dollar in the 1980's. It is suggested that the rise and fall of the

Dollar real exchange rate over this period is not only attributable to the relative rise and fall of US interest rates, but also to the upward and downward revisions in beliefs regarding the persistence of the policies pursued.

3.2 Rational Bubbles

3.2.1 Multiplicity

The consideration of bubbles is a break from the fundamental framework. Flood and Garber (1980) defined a bubble as a departure of price from market fundamentals.[1] Rational bubbles owe their existence to the possible multiplicity of solutions in dynamic rational expectations models, where all but one is consistent with fundamentals. The other solutions may admit an extraneous term, which is fully consistent with self-fulfilling expectations. We refer to the models of Black (1974), Flood and Garber (1980), Azariades (1981) and Blanchard and Watson (1982) - to name a few.

Consider the model:

$$c_t = k + \alpha_1 E_{t-1} c_t + \alpha_2 E_{t-1} (c_{t+1} - c_t) - \alpha_3 E_{t-1} v_t \qquad (1)$$

$$v_t = \theta v_{t-1} + \omega_{t-1} \qquad \omega_t \sim \text{NIID}(0, \sigma_\omega) \qquad (2)$$

$$|\theta| < 1$$

E is the rational expectations operator and c is the logarithm of price. $E_{t-1} c_t$ is the rational expectation formed at time t-1 (fully exploiting all the information then available) of the price at time t. The

[1] Empirically, the bubble hyothesis is difficult to separate from that of model misspecification. [See Hamilton and Whiteman (1985) and Evans (1986).]

contemporaneous expectation for the rate of price change is therefore, $E_{t-1}(c_{t+1} - c_t)$. We include a constant term, k, plus an exogenous variable, v, which follows a Markov process and has a white-noise error.

A standard solution procedure is to find the undetermined coefficients in the rational expectations reduced-form below by its substitution into the structural-form above.

$$c_t = \pi_0 + \pi_1 v_{t-1} + B_t \tag{3}$$

B is an extraneous element or bubble which is fully consistent with the form of the dynamic expectations model. It is a deterministic bubble - and is known alternatively as a sunspot or a bootstrap equilibrium - to be distinguished from the short-lived probabilistic bubble, which is considered afterwards. We solve the coefficients as

$$\begin{aligned} \pi_0 &= k/(1 - \alpha_1) = \bar{c} \\ \pi_1 &= -\alpha_3 \theta/[1 - (\alpha_1 + \alpha_2(\theta - 1))] \\ B_t &= B_0 \left[\frac{1 + \alpha_2 - \alpha_1}{\alpha_2} \right]^t \end{aligned} \tag{4}$$

Note that it is the dynamic nature of expectations and not their rational expectations *per se* which gives rise to the bubble. Neither is the solution method responsible. Blanchard (1979A) takes a different approach in presenting a dynamic solution as any weighted average of the forward and backward solution. As there are infinite possibilities for the weights, there are an infinite number of possible solutions. In a restricted case of a static structure of expectations where $\alpha_2 = 0$, the only feasible solution is where $B_t = 0$ for all values of t, including 0. So the problem does not occur in Muth's (1961) model of the cob-web.

As the model stands, there is an infinity of possible REE solutions corresponding to an infinity of possible values of B_0. The starting point, B_0, can be likened to the indeterminate constant of integration from a difference equation. Flood and Garber (1980) claim that

> ' .. the indeterminacy arises because only one market-equilibrium condition exists; but the researcher requires two solutions for two endogenous variables - market price and the expected rate of price change.'

The dynamic pattern of the bubble clearly depends on the sign and magnitude of $(1 + \alpha_2 - \alpha_1)/a_2$ in equation (4). If the magnitude is less than unity, then the bubble will collapse asymptotically. However, consideration is usually given to the case where it is greater than unity, which gives rise to a growing bubble. If the starting value of B is positive, the bubble is said to be explosive; and if it is negative, the bubble is implosive. Exploding bubbles give rise to a price path which is unbounded asymptotically, imploding bubbles will cause the price to be asymptotically zero (as the logarithm of the price level in the model will become negative and unbounded).

The case most usually considered is the one where the price is a positive function of the expected change in price, as in Shiller (1978). It applies to models of hyperinflation, where the demand for real money balances is viewed as a negative function of the rate of expected inflation, as in Cagan (1956), and to asset demand models where only the rate of expected price change is of importance. This gives the parameter restriction

$$\alpha_1 = 0$$

The bubble will grow by the factor, $(1 + \alpha_2)/\alpha_2$. Out of the infinity of possible solutions, all but one are explosive. The stationary solution is the only 'fundamental' one.

However, in Chapter 2 we considered a special case of this where the parameter represented by α_2 (b) was unbounded. In this case, there is still the problem of multiplicity, but each possible solution is a constant. This characteristic derives from the treatment of perfect capital mobility and is consistent with (uncovered) interest rate parity.

Finally, an unappealing feature of the rational expectations bubbles discussed so far is their longevity. It is a consensus view that if exploding bubbles are possible, they must have a limited life-span and, so, a good chance of bursting. Afterall, evidence shows that hyperinflations do not necessarily persist. Probabilistic bubbles were first considered by Blanchard (1979B) and are discussed by Dornbusch (1982) with reference to the exchange rate.

Defining p (which is valued between zero and one) as the probability of the bubble being sustained, the expected price at time t+1 becomes

$$E_{t-1}\, c_{t+1} = \pi_0 + \pi_1\, \theta\, v_{t-1} + p\, B_{t+1} \qquad (5)$$

Given our parameter restrictions on α_1, the effect will be for the bubble to grow by an even greater factor, $(1 + \alpha_2)/p\, \alpha_2$. The bubble must grow faster in order to compensate agents for the probability of its bursting. A problem with this model is that as the probability of the bubble continuing gets smaller, the expected bubble gets larger.

A more serious problem is that there is no explanation of how the bubble can burst and no information on which agents can condition their expectations of its bursting. How do agents arrive at their expected value for p? The probabilistic rational bubble is subject to a criticism by Lucas (1981) that rational expectations are applicable only where

> ' .. the probabilities of interest concern a fairly well defined recurrent set event, situations of "risk" in Knight's terminology.'

3.2.2 Obtaining a Unique Solution

It is an interesting question whether bubbles are an economic problem or an economist's problem? That is to say, do rational bubbles exist in the world which the economist is trying to model or are they they merely outcome of imperfect modelling practises, for example in the use of only one equation where there are two to explain two endogenous variables?

> One is reminded of Brainard and Tobin's (1968) recommendation:
>
> ' ..we argue for the importance of explicit recognition of the essential interdependences of markets in theoretical and empirical specifications .. . Failure to respect some elementary interrelationships .. can result in inadvertent but serious errors of econometric inference and of policy.'

In spite of this, the returns to a sufficiently general equilibrium approach to modelling will be limited, if one is trying to describe a real world economic system, which is itself underdetermined. Missing equations may be mere reflections of missing markets. Cass and Shell (1983) give this explanation for the possibility of bubbles.

In the rest of this section we consider whether it is appropriate to model the exchange rate as a bubble or as a unique and stationary equilibrium. The literature has generally been concerned with indeterminate paths in the price level which derive from Cagan money demand functions. One approach from Sargent and Wallace (1973) has been to select a unique and a non-explosive solution path. Taylor (1977) and McCallum (1983) recommended respectively solutions with minimum variance and the minimum number of state variables. The speciality of this latter criterion has been questioned by Backus (1985). Another approach has been to investigate the dynamic properties in term of an underlying model of optimising agents - as in the papers of Brock (1975) and Gray (1984). [For a brief survey see Burmeister (1980).]

An alternative criterion has been to focus on the stability of the process towards the REE rather than on the stability of the price path within the equilibrium. This draws on the literature of learning towards rational expectations, reviewed by Bray, Blume and Easley (1982), which interprets the stability of REE in terms of the convergence of learning processes towards it. The analysis considers whether agents, who are initially outside REE, can effectively learn the parameters of the model to form rational expectations. The operational use of rational expectations in general can then be justified on the grounds of convergent learning processes.

Consequently the application of a particular REE path can be justified by the same criterion. A rational expectations bubble is then seen as economically feasible, if agents were able to converge on such an equilibrium. Papers by Evans (1985) and by Gottfries (1985) suggest

that agents would find difficulty in learning a non-stationary equilibrium. Agents may be able to converge on to a stationary (landed) aeroplane, but not to on one which has taken off. In the class of models discussed, the unique stationary equilibrium would then dominate all others.

Apart from these general considerations, there is one which applies particularly to a model of the real exchange rate. There are good reasons to expect a unique and stable solution for its long run equilibrium. We are returning to the issue of a second equation; a well developed model can give unique solutions both for current price and for expected future price. The second equation could define the condition of long-run purchasing power parity, as in the case of the (sticky-price) monetary model of Dornbusch (1976), or the condition of long-run current account balance as in Kouri (1976) and Dornbusch and Fischer (1980) [as in the previous chapter]. In the terminology of Isard (1981), the long-run exchange rate, however determined, serves as an 'anchor' to the model.

We prefer the condition of long-run current account balance as being consistent with the balance of payments approach in Kouri and in Chapter 2 of this thesis. The real exchange rate affects the current account balance, which constitutes an international transfer of wealth. An exploding real exchange rate would lead to not only current account deficits on an unsustainable scale but also of increasing magnitude. If the current account is viewed as a mechanism for redistributing wealth between countries, a long-run condition for the international distribution of wealth translates into a long-run condition for the real exchange rate via the current account.

The imposition of a terminal condition will make far-sighted agents perceive a binding constraint on the long-run equilibrium. And, with speculation based on forward-looking expectations, the current exchange rate will be linked to the perceived and finite long-run equilibrium. Speculation will then be stabilising as envisaged by Friedman (1953). And, given the high degree of rationality which is usually assumed, far-sighted agents will rule out even short-term deviations of the exchange rate from the stable path.

This therefore applies as equally to the question of short-lived stochastic bubbles considered by Blanchard (1979A) and by Dornbusch (1982) as well to the standard long-lived bubbles. This is because it is a feature also of models of probabilistic bubbles that agents cannot perceive a finite solution for the long-run equilibrium. Krugman (1985) has forcefully argued the case against the dollar having been on a stochastic bubble, because rational agents would have foreseen the implications for US external deficits and spiralling indebtedness of an unrealistically large scale.

Chapter 2 considered a model where the real exchange rate was determined by the condition of equilibrium on the total balance of payments. This gave the characteristic of direct feedback from the current account to the exchange rate. It was shown that if agents did not know the long-run equilbrium for the exchange rate, it could be learned from a reduced-form exchange rate equation which was obtained from two separately identified structural equations for the exchange rate and for the current account balance.

A second characteristic of this model was a multiplicity of stationary REE prior to learning. The use of a convergent learning mechanism in eliminating non-stationary equilibria has already been noted. The learning mechanism in this model, however, was shown to ensure convergence to the unique equilibrium which was consistent with current account balance.

Finally, although the real exchange rate is the subject of this paper, the nominal exchange rate will also be stationary, if the relevant national price levels are stationary. The real exchange rate between two countries is defined as the nominal market exchange rate deflated by their relative price levels. Obstfeld and Rogoff (1983 and 1986) show that when the government fractionally backs the currency by guaranteeing a minimal real redemption value for money, deterministic and probabilistic price level bubbles are ruled out. The general sentiment that the monetary authorities can and will intervene to mitigate excessive price instability should ensure against nominal exchange rate bubbles.

3.3 Alternative Explanations

3.3.1 The Peso Problem

The Peso problem is considered first because it bears the closer resemblance to rational expectations. It arises in markets where forward expectations must be made of future variables, which are not determined by a well defined process [see Salant and Henderson (1978)]. It consequently has a strong application to models where policy variables have an effect and where the future policy decisions of the

authorities are difficult to pin down. Expectations can therefore be quite chimeral and volatile.

Dornbusch (1982) relates the behaviour of the French Franc over 1925-1926 where expectations of inflationary policies led to a collapse. This was then followed by a rapid appreciation on the realisation that these policies had not materialised. This kind of episode is the most relevant where it is probable that the structure of policy will change, because of an imminent change in a government, or of a personality within it, or because of an overwhelming change in external circumstances.

The Peso problem can surely be a considerable source of exchange rate instability, at least in the short-run. However, in the medium-term when a sufficient time span will permit agents to learn the underlying environment of policy, the Peso problem will be of less importance. It is suggested that the Peso problem cannot account for longer lasting departures in the exchange rate from notions of its equilibrium level, such as the dollar over 1982-1987.

3.3.2 Non-rational Bubbles

Exchange rate movements might be considered outside the framework of REE. In this subsection we refer to two non-rational bubble models given by Armington (1986) and by Frankel and Froot (1986). However, as early as 1944, before rational expectations ideas took sway, Nurkse argued that the exchange rate was prone to destabilising speculations caused by extrapolative expectations.

Armington's (1986) model acknowledges the existence of long-run fundamentals, but contains myopic agents who do not take these into account. He notes that the stability problem in rational expectations models is caused by too much speculation, while his form of cyclical instability is caused by too little *far-sighted* speculation. In the first instance, the lack of a unique and stable long-run equilibrium ensures that self-fulfilling expectations follow an unstable path. In his instance, the failure of agents to see further towards the long-run fundamentals leads to destabilising speculation.

Frankel and Froot (1986) describe a different kind of bubble. Expectations can be fundamentalist, chartist or any combination of the two. There are two groups of agents who make expectations, the fundamentalists and the chartists, and a third group, the portfolio-managers, which makes decisions based on a weighted average of the two forms of expectations.

The non-rational element in the model derives from the condition that at any time some weight may be given to the expectations of the chartists. The model is only rational in the event that only fundamentalist expectations are considered. And, if chartists expectations are considered, fundamentalist expectations themselves become non-rational, because they lose consistency with the model in failing to account for the effect of chartist expectations.

A bubble starts off from the position where only fundamentalist expectations are considered. Then chartist expectations then gain credibility, the bubble grows; then chartist expectations lose

credibility, leading to a decline in the bubble until the original state is reached, where only fundamentalist expectations are considered.

A main problem with the model is in explaining how the bubble starts and evolves. The question is why should the portfolio-managers suddenly consider the extrapolative expectations of the chartists, when they are initially enjoying the desirable forecasting benefits of rational expectations? The rational expectations errors should normally be white noise (unless there is a serially correlated exogenous variable which affects the exchange rate instantaneously but which cannot be observed until after a lag of two or more periods). Either the portfolio-managers are irrational or the fundamentalists have got hold of the wrong model, in which case they are outside rational expectations. The portfolio-managers and the fundamentalists cannot both be right.

The learning process in this model is of the 'appropriate' weighting of the two sets of expectations. Yet, if there is access to fundamentalist expectations, rational agents would find further learning spurious. Rationality would dictate that the fundamentalist expectations would perpetually be given a unit weight. The bubble derives from a consideration of chartist expectations, which arguably suggests that the portfolio-managers are not fully exploiting the information which is available, which would deliver rational expectations.

The stance of this chapter is to maintain the assumption of individual optimality and to avoid *ad hoc* constructs which cannot be justified by our understanding of the present body of theory. Models of

non-rational bubbles may rest on the denial that information is fully exploited. Agents in asset markets may have much at stake and should therefore not systematically conform to forecasting rules, which would prove themselves to be sub-optimal. So if agents cannot form rational expectations, they will at least try to form rational expectations by endeavouring to learn the model. This position represents a state where information is in limited availability and one where any increment is fully exploited.

3.4 The Model

The model is a special case of the one in Chapter 2. There is perfect capital mobility, as there defined, and uncovered interest rate parity. A modification here is that it is in the level, not in the first difference, of the expected relative yield on a currency, which determines the exchange rate. The expected relative yield comprises the expected interest rate differential plus the expected rate of appreciation.

In Chapter 2 the exchange rate is modelled as a total balance of payments equilibrium and it is demonstrated that there is a convergent learning process towards a unique and stationary long-run real exchange rate equilibrium. This is consistent with the discussion in Section 3.2 of this chapter, which permits us to impose a unique and stationary solution on the model, precluding a bubble.

The model is obtained by imposing the following restrictions :

$\alpha_1 = 0, \quad \alpha_2, \alpha_3 \to \infty, \quad B_0 = 0$

on equation (1) which gives the model inside full rational expectations as

$$c_t = \tilde{c} - [\theta/(1 - \theta)]\, v_{t-1} \tag{6}$$

This formulation of the model presupposes that agents already know the values of the parameters, $\tilde{c}$ and θ. We assume that they do in fact know the value of $\tilde{c}$, but not of θ. The latter represents the persistence of real interest rate differentials. So, to the extent that real interest rates are determined by the fiscal-monetary policy mix, θ represents the persistence of the mismatch between the stabilisation policies of the two countries.

It is clear that a mismatch of any persistence will drive a wedge between the current exchange rate and the long-run equilibrium exchange rate. A basic prediction of this model is that if agents are able to form expectations of the long-run equilibrium, as was argued in Chapter 2, any misalignment in the real exchange rate will be due to expectations of divergent real interest rates. This holds whether or not expectations are fully rational in the sense that agents know the parameter values of the model.

It is the main contribution of this paper to consider the case outside strong-form rational expectations where agents form expectations which are based on incorrect beliefs on the parameter values and where these beliefs are under revision. Expectations are still consistent in the sense in that they remain faithful to the structural form of the model. So, we consider a more general form of equation (6):

$$c_t = \tilde{c} - Gm_{t-1}\, [\theta/(1 - \theta)]\, v_{t-1} \tag{7}$$

Agents will make expectations errors where the value of θ is unknown, where generally $\theta \neq Gm_{t-1}\,\theta$. The latter is defined as market beliefs on θ. We define z_t as the relative yield between the two assets, comprising the interest rate differential, v_t, less the rate of depreciation

$$z_t \equiv v_t - (c_{t+1} - c_t) \tag{8}$$

Using $Gm_{t-1}\,\theta$ with equations (2) and (7), we get the following expectations made at time t-1 for the exchange rate at times t and t+1 and for the interest rate differental at time t as

$$Gm_{t-1}\,\bar{c}_t = \tilde{c} - Gm_{t-1}\,[\theta/(1-\theta)]\,v_{t-1} \tag{9}$$

$$Gm_{t-1}\,\bar{c}_{t+1} = \tilde{c} - Gm_{t-1}\,[\theta^2/(1-\theta)]\,v_{t-1} \tag{10}$$

$$Gm_{t-1}\,\bar{v}_t = Gm_{t-1}\,\theta\,v_{t-1} \tag{11}$$

Combining these expectations in the form for z in equation (8), we get expected (uncovered) interest rate parity. The expected rate of depreciation just equals the expected interest rate differential.

$$Gm_{t-1}\,\bar{z}_t = 0 \tag{12}$$

This obtains because of the extreme case of perfect capital mobility (here considered as an approximation), which is defined by the restrictions that the values of the parameters α_2 and α_3 are unbounded. Expected interest rate parity is required to ensure the existence of a finite equilibrium.

The actual values for the variables which are determined by the expectations above are

$$c_t = \tilde{c} - Gm_{t-1}\,[\theta/(1-\theta)]\,v_{t-1} \tag{13}$$

$$c_{t+1} = \tilde{c} - Gm_t\,[\theta/(1-\theta)]\,(\theta v_{t-1} + \omega_t) \tag{14}$$

$$v_t = \theta\,v_{t-1} + \omega_t \tag{15}$$

Beliefs also determine actual outcomes, but in conjunction with the actual values of the parameters. Beliefs on the value of θ will generally change between times t-1 and t from $Gm_{t-1}\theta$ to $Gm_t\theta$. We define

$$\xi_t = Gm_t\,[\theta/(1-\theta)] - Gm_{t-1}[\theta/(1-\theta)] \tag{16}$$

and so, the expectation error is

$$z_t - Gm_{t-1}\,\bar{z}_t = z_t =$$

$$(1-\theta)\left[(1+\xi_t)\left(\frac{\theta}{1-\theta}\right) - Gm_{t-1}\left(\frac{\theta}{1-\theta}\right)\right] v_{t-1}$$

$$+\left[1 + Gm_{t-1}\left(\frac{\theta}{1-\theta}\right) + \xi_t\right]\omega_t \tag{17}$$

Ex poste agents will realise expectations errors, as there will be mistaken beliefs, an unpredictable noise in the interest rate differential and an unpredictable adjustment of beliefs between times t-1 and t. Assuming that agents know the process for the revision of beliefs, there remains an incentive to learn the value of the parameter, θ, in order to reduce the size of expectations errors.

3.5 Learning

3.5.1 Introduction

Outside REE, agents' forecasts will be biased and without the desired property of minimum error variance. There is therefore an incentive to learn the unknown values of any parameters, wherever the costs of learning are expected to be relatively small, in order to reach optimal rational expectations forecasts.

Learning models [reviewed by Blume, Bray and Easley (1982)] can be classical or use Bayesian methods. We look at a Bayesian learning process, which offers two advantages. One is that the agents can easily revise their beliefs at the realisation of each data point. Secondly, there is a role for prior information and subjective beliefs, which may be given an economic interpretation.

In a Bayesian framework all uncertainty can be represented by probability distributions. A prior distribution may be assigned to an unknown parameter value, which is then subsequently revised in the light of the observations generated from its true distribution. The process of revision is the process of convergent learning through which the subjective priors and posterior distributions evolve over time into the true distribution.

The moments of concern are the mean and the precision, the inverse of the variance of a normal distribution. The assumption is of normality. A prior mean and precision are assigned to the unknown parameter value. The prior mean reflects initial and subjective beliefs and the prior precision reflects the confidence with which these beliefs are held. The optimality of the statistical process ensures that the posterior means evolve into the true value in the light of the observations generated. And the posterior precisions increase over time which reflects the diminishing of subjective uncertainty over the learning process.

Finally, mention should be made of consistent learning as against inconsistent learning. In inconsistent learning agents do not take into account the effects of the dynamics caused by the learning process

itself on the model they are trying to learn. Learning becomes inconsistent where agents' beliefs affect the model and where agents do not realise this. Consistent learning in the view of Bray and Savin (1986) and of Townsend (1983) requires strong informational assumptions at the outset. Agents must know how other agents are revising their beliefs and know how other agents believe others are revising their beliefs. Consistent learning must meet the problem of the infinite regress in learning. Inconsistent learning may be a more realistic assumption outside a REE and one which does not beg the informational questions.[2]

3.5.2 Application to Model

Initially there is a REE where agents know all the parameters of equation (6). There is then a policy change which leads to a new value for θ. We assume that agents know the long-run equilibrium, $\bar{c}$, but do not know the new value for θ.

We assume that agents are completely homogeneous in beliefs and expectations. They all have the same prior beliefs and all believe that they do. Complete homogeneity is just one solution to the infinite regress in beliefs and expectations [Phelps (1983)]. Furthermore, we assume that they all observe the same relevant variables and that they all know that they observe the same: they have common knowledge.

This simplifies the model, reducing it to one driven by a single representative agent. It overlooks the problem of agents learning about the beliefs of each other, about subjective factors. The model is on par with those in the earlier learning literature of Cyert and DeGroot

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[2] A fuller consideration of these themes is given in Chapter 4.

(1974), Decanio (1979) and Friedman (1979). In models without the need to solve the infinite regress, convergence to REE is not problematical, given a reasonably stable parameterisation of the underlying structure. We refer to the cobweb learning models of Cyert and DeGroot (1974), Decanio (1979) and Bray and Savin (1986).

The state we describe, although outside REE, contains strong informational assumptions, so that convergence to REE is not surprising. The purpose, however, is not to give an example of stable learning but to depict a pattern of exchange rate dynamics. The dynamics are attributed to the learning process and not to the presence of bubbles. We contend that a marginal weakening of the strong information availability assumptions of the rational expectations hypothesis (REH) can given rise to phenomena which are otherwise attributed to bubbles.

The combination of perfect substitutability (risk-neutrality) and homogeneous expectations can give the odd result that everyone believes asset x to offer a higher return than y, so that the aggregate demand for y is zero. This does not apply here, for the model is constructed such that the expected interest differential on the asset denominated in the domestic currency is equal to the expected rate of (domestic currency) depreciation. The two assets offer the same expected return. This means that the asset demand functions are indeterminate, but this causes no problems as the exchange rate is fully determinate.

As agents know $\tilde{c}$ we set its value at zero, so the current value of the exchange rate is equal to its undervaluation. Instead of equation (13), we use

$$c_t = -G_{t-1} (\theta L/1-\theta L) v_{t-1} \qquad (18)$$

with

$$v_{t-1} = \theta\, v_{t-2} + \omega_{t-1} \tag{15}$$

θL is the long-run value of θ, which is employed to allow perfect persistence in the short-run ($\theta = 1$), which would not be admissable in the long-run ($\theta L < 1$).

The exchange rate is determined by aggregate beliefs on θL the *long-run* value of θ, and by the nominal interest rate differential. Its dynamics derive from the dynamics of the latter in (15) and from the learning process described below.

Learning is the adjustment of beliefs. As there is complete homogeneity of beliefs on θL, there is no requirement for agents to learn aggregate beliefs or market opinion. The problem is reduced to that of a single agent. We assume that the single agent and all agents obtain information on θL from learning about θ in equation (15).

In the first set of simulations θL and θ are the same. That is to say the policy change is permanent and that θ is less than one for determinacy. The value of θ is determined by objective government policy but beliefs on θL are conditional on subjective factors during the learning process.

At time 0 there is a policy change and a new value for θ. At that time when the change is made agents are aware of the change but not of the new value. They have beliefs on the normal distribution of θ. It is believed to be distributed with mean, m_0, and precision, h_0, the inverse of the normal variance. To date agents have only observed the past values of the interest differential, v_{-1}, v_{-2}, v_{-3}, ... At time 1

agents observe the interest differential, v_0, an additional observation generated by the new value of θ. They may revise their priors [Cyert and DeGroot (1974)] according to

$$m_1 = (h_0 m_0 + q v_0 v_{-1}) (h_0 + q v_{-1}^2)^{-1} \tag{19}$$

$$h_1 = h_0 + q v_{-1}^2 \tag{20}$$

h_0 represents the initial degree of confidence with which the belief, m_0, is held as the inverse of its subjective variance. In the limiting case of perfect confidence, beliefs do not change: m_1 is equal to m_0 where h_0 is infinite. If there is either no confidence in the belief, m_0 or no noise in the interest rate differential (an infinite value for q), then beliefs converge *immediately* to v_0/v_{-1}.

And (19) and (20) may be written as

$$m_t = [(h_{t-1}/q) m_{t-1} + v_{t-1} v_{t-2}] (h_t/q)^{-1} = G_t \theta L \tag{21}$$

$$h_t/q = h_0/q + \sum_{i=0}^{t-2} v_i^2 \tag{22}$$

For non-zero and finite values for the initial precision, h_0, and for the interest rate differential noise precision, q, beliefs on θ should converge asymptotically to its true value.

The simplified information structure, effectively of a single agent, allows for rational or consistent learning. The learning does not affect the part of the model being learned, which is external to the learning process.

3.5.3 Simulations

We start from a position of REE with a θ value of 0.25. The mean of the interest rate differential is zero, so that the mean of the exchange rate is zero and is in long-run equilibrium. At time 0 there is a policy change. The value of θ is raised from 0.25 to 0.80 and the

interest rate differential on the domestic currency is set at 3%. Although noise in the interest rate differential is necessary to gradual learning, it is removed from most of the simulations to give an underlying picture. The interest rate differential in each succeeding period will be 0.80 of its value in the preceding period. If agents remain in a state of perpetual REE without learning, the exchange rate would appreciate by 12% concurrently with the realisation of the policy change [see equation (18)]. The appreciation would be an instantaneous jump to be followed by a gradual convergence to the long-run equilibrium value at the same rate as the rate of convergence of the interest rate differential to zero. The dynamic pattern would not be dissimilar to the one in the Dornbusch (1976) overshooting model.

Outside REE and with learning, the process is different and takes on a more cyclical appearance, because of the *gradual* adjustment of beliefs. Figure 1 shows the pattern. [See page 104.]

The initial value for θ is 0.25, so the prior belief, m0, should also be 0.25. We first consider the case where the *precision ratio*, the ratio of the prior belief precision, h0, to the noise precision, q, is 20. This gives the exchange rate 'overvaluation' path of x(A) in Figure 1.

Following the policy change, agents observe the interest differential of 3%. There is as yet not enough information to adjust their belief in the value of 0.25. But at time 1 the rise in the interest differential itself is enough to cause an appreciation of 1% (3%(0.25)/(1-0.25)).

In period 2 agents observe enough information to adjust their prior beliefs. With a prior precision ratio of 20, the belief of 0.25 is adjusted upwards to 0.421, closer to the real value of 0.8. This causes a further appreciation of 0.75% in period 2. In period 3 the appreciation is only 0.05% and thereafter the exchange rate depreciates, returning to a value of 0.16% by period 15.

The pattern is of a gradual rise and then an even more gradual decline in the exchange rate. This cyclical pattern is caused by the *interaction* of policy and of learning. In the early stages the exchange rate appreciates because agents are learning quickly in relation to the rate of decline of the interest rate differential. The belief in the value of $\theta/(1-\theta)$ is increasing at a rate which is larger than $1/\theta$. Then the exchange rate begins its decline when the rate of fall in the interest rate differential exceeds the rate of revision in beliefs. The fall in the interest rate differential itself slows the learning process because its decreasing numerical magnitude reduces its effectiveness as a signal. Beliefs of the value of θ get stuck at round about 0.555 by period 12.

Referring also to Figure 1, x(B) gives the exchange rate 'overvaluation path' where the initial precision ratio is lower, at 6. If we assume that there is the same degree (although removed for the simulations) of noise in the interest rate differential, the lower ratio reflects *less* confidence in prior beliefs. The effect is for the exchange rate climb to be steeper. It is overvalued by 3.42% in period 3 compared with an overvaluation of 1.80% in the previous example. It too begins its descent at this point, but the fall is steeper because it is further away from its long-run equilibrium.

The initial climb is steeper because there is less confidence in the initial value of 0.25 and so more confidence in learning the true value of θ from the basis of the data the model generates. Beliefs on θ get stuck too near period 12, but at the higher value of 0.693 because of the initial boost. In this second case the exchange rate is always more overvalued than in the first case, because with less confidence in the old value of 0.25 agents approach the new REE more quickly. In the limiting case of a zero precision ratio, there is an equivalence to a perpetual state of REE.

In the remaining examples we consider where θ and θL do not coincide. Policy determining the value of θ can be reversed, so that it is not a long-run parameter. But the parameter which is germane to expectations is θL. We allow for feedback from θ to θL since in principle θL can be regarded as an average of a series of short-run θ.

As before, we start from a REE where the values of θ and θL are 0.25. This time, however, the value of θ is raised to 1 in period 0 and the policy is reversed in period 8. Agents, who perceive that the policy will eventually be reversed, will hold beliefs on the value of θL between 0.25 and 1. This also ensures the determinancy of the exchange rate.

A new interpretation can be given to the role of the prior precision in the learning process when θ is 1. A high value for the prior precision, a great weight on the prior value of θ, which is less than one, indicates that the policy is regarded as being short-lived. Beliefs on θL will fall well short of 1. Alternatively, a low value for

the prior precision, indicates that the policy is regarded as being long-lived then beliefs on the value of θL may approach unity.

The learning problem is more difficult since agents may not be able to learn the permanence of the new policy. The Bayesian prior will reflect their unsubstantiable beliefs of the policy's permanence.

In period 0 the interest differential is set at 3%, and as θ is set at unity, the policy represents a relative monetary contraction of some persistence in the domestic economy.

Figure 2 [on page 105] shows the case where there is an initial precision ratio of 20. In period 1 the exchange rate appreciates by 1%. Thereafter it continues appreciating at a constant rate of 1.80% until the policy reversal is perceived, reaching a level of 15.40% by period 9. The continued appreciation derives from the persistence of the positive interest rate differential and from increasing beliefs of the permanence of the policy.

At period 10 enough information of the policy reversal is received, so that the exchange rate devalues by 12.67%, falling to 2.73%. This *hard-landing* is followed by a gradual decline over the following periods, with the exchange rate falling as the interest rate differential falls to zero.

In Figure 2 we also show the dynamics where the precision prior is reduced from the value of 20 above to 12. The effect is to increase the amplitude of the 'cycle'. After the first appreciation of 1%, there is a constant rate of appreciation of 3% during the implementation of the

policy. The overvaluation by period 9 is 25% and the hard-landing in period 10 is even harder with a fall of 21.31% to 3.69%. The landing is harder, the more agents believe in the persistence of a situation which is eventually going to prove transitory.

Figure 3 [on page 106] shows that the landing is also harder if agents revise their precision belief on perceiving the policy reversal. tarting with a precision of 20, it climbs to 92 by period 9. This represents the confidence with which agents hold beliefs on the unknown value of θ. In period 9 when the policy change is perceived, there is then no reason for beliefs to be held with the same degree of confidence. The precision ratio is restored to its original value. Consequently the exchange rate falls to 1.42% as compared with the 2.73% it would falls to without a revision of the precision ratio.

Figure 4 [on page 107] shows the case where there is no sudden but a gradual reversal in policy. Instead of an immediate transition of the value of θ from 1 to 0.25, it is reduced over 8 periods. The exchange rate decline is gradual with an average fall of account 2.5% per period. A *soft-landing* scenario is depicted.

Finally, Figure 5 [on page 108] gives the case shown in Figure 2 without the exclusion of the interest rate differential noise. Unsurprisingly, the path of appreciation and depreciation are less smooth. But what is of note is that the variance of the exchange rate is greater at the end than at the outset. The reason is that beliefs on the value of θ, following the policy reversal, do not fall as low as the original 0.25 but fall from a high of 0.84 to about 0.78. The fall is slight because when the interest rate differential becomes small, there

is little variation in the data on which agents could revise their beliefs. At the end, exchange rate variation is 10.6 times higher [0.78/(1-0.78)] [(1-0.25)/(0.25)]. Despite the reversal of the policy, the reduction of θ to 0.25, and a zero mean differential, the legacy is of greater exchange rate variance.

3.5.4 Hard-landing vs. Soft-landing Scenarios

Mention has been given to the prospect of a hard-landing as against a soft-landing in the exchange rate following a policy reversal. We found three factors making for a hard-landing.

(i) Agents believe in the permanence of a temporary policy, so that the exchange rate overreacts to a policy change in the short-run only to adjust dramatically at the perception of the policy reversal.

(ii) On perception of the reversal, agents suddenly revise their beliefs.

(iii) The reversal of policy is sudden rather than gradual.

In 1985 there was a consensus that the dollar was overvalued. The discussion was largely over whether there would be a hard-landing or a soft-landing. Frankel (1985) estimated the overvaluation to be 29%. In terms of our model this overvaluation can be explained by the existence of a positive interest rate differential on dollar assets and a belief in the persistence of the US policy of a relative monetary deflation. The latter is justified by the experience of a positive differential favouring the dollar throughout the 1980s. Marris (1985) believed the dollar would crash down when its time to fall eventually came.

Between August 1985 and March 1986 there was no perceivable change in the interest rate differential on US and German funds. Yet over the same time there was a *gradual* depreciation of 21%. This suggests that there was a gradual perception that policy would change. Part of this may have been anticipatory, but part was probably adaptive because over the period the differential rose from 2.91 to 3.37 before falling to 2.94.[3]

Over the next six months the differential fell from 2.94 to 1.35. And over the same period from March to September the dollar-DM exchange rate fell by 10%. The bulk of the fall within this period (8%) came in July and August. In these two months there was a sudden fall in the differential from 2.32 to 1.32. So at least some of the evidence points to the role of the smoothness of a policy transition in influencing the adjustment rate of a perceptively overvalued currency.

3.6 Summary

It has been argued that the empirical failure of fundamental models of the exchange rate cannot be remedied by minor modifications of their underlying assumptions. This seems to leave three alternatives: the existence of rational bubbles, irrational bubbles or of learning. We have made a theoretical case against the first two in favour of learning. Learning at least maintains the information-exploitation assumption.

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[3] The differential is the rate New York Federal Funds minus the rate on 3-month Frankfurt money market funds (averages for the month) obtained from the Monthly Report of the Deutsche Bundesbank.

The learning model considered is simple with strong information availability assumptions, but which are less strong than those of the REH. That is to say agents do not need to learn about each others' beliefs, about market opinion, as we assume complete homogeneity. The only uncertainty is of government policy. Despite this, there is considerable mileage gained from the modified information availability assumption.

The model is able to explain exchange rate cycles of varying amplitude, slow and fast climbs from long-run equilibrium and hard- and soft-landings back to it. The Bayesian learning process, allowing for subjective factors, agents' beliefs, are central to this analysis of exchange rate dynamics. In particular, it is shown that a change to a policy which is unsustainable in the long-run (with perfectly persistent interest rate differentials) will have less effect in the short-run if agents have relative confidence in the previous (sustainable) policy.

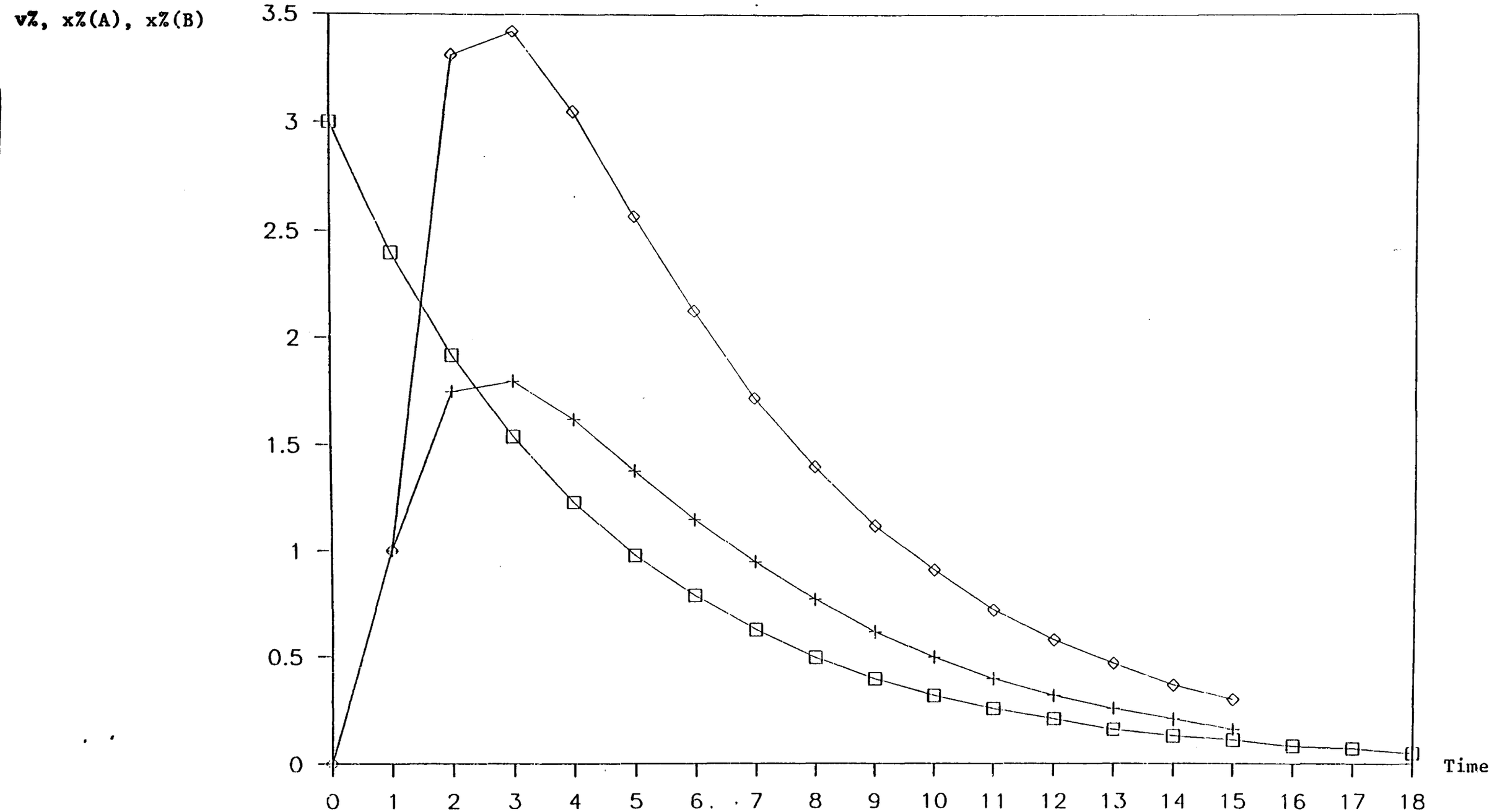

Figure 1: Persistence Raised from 0.25 to 0.8 at Time 0.

- ◻ Interest Rate Differential v%.
- + Exchange Rate Overvaluation x%(A) with prior precision ratio of 20.
- ◇ Exchange Rate Overvaluation x%(A) with prior precision ratio of 6.

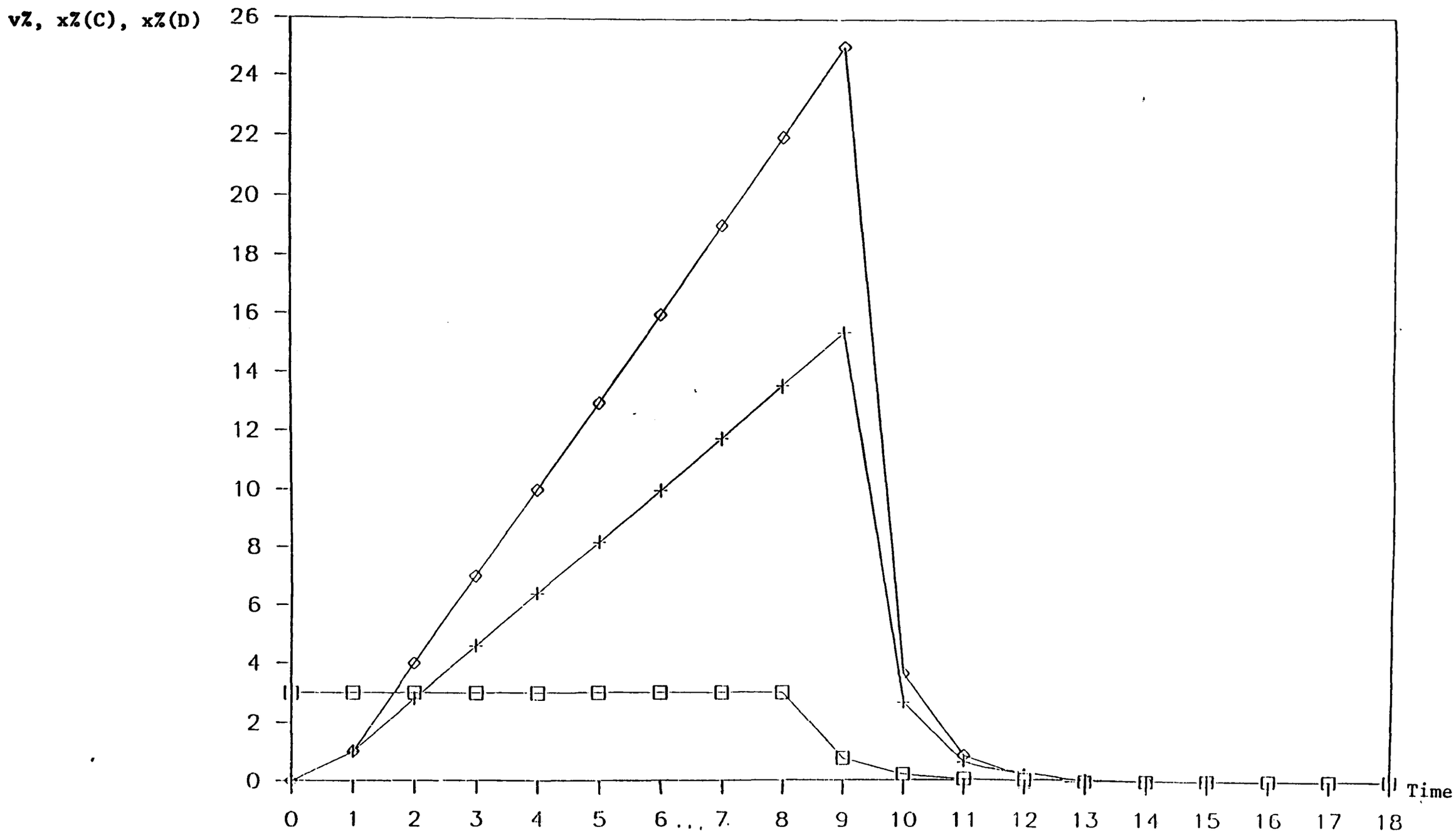

Figure 2: Persistence Raised from 0.25 to 1 at Time 0 and then a Policy Reversal at time 9, Returning it to 0.25.

- □ Interest Rate Differential v%.
- + Exchange Rate Overvaluation x%(C) with prior precision ratio of 20.
- ◇ Exchange Rate Overvaluation x%(D) with prior precision ratio of 6.

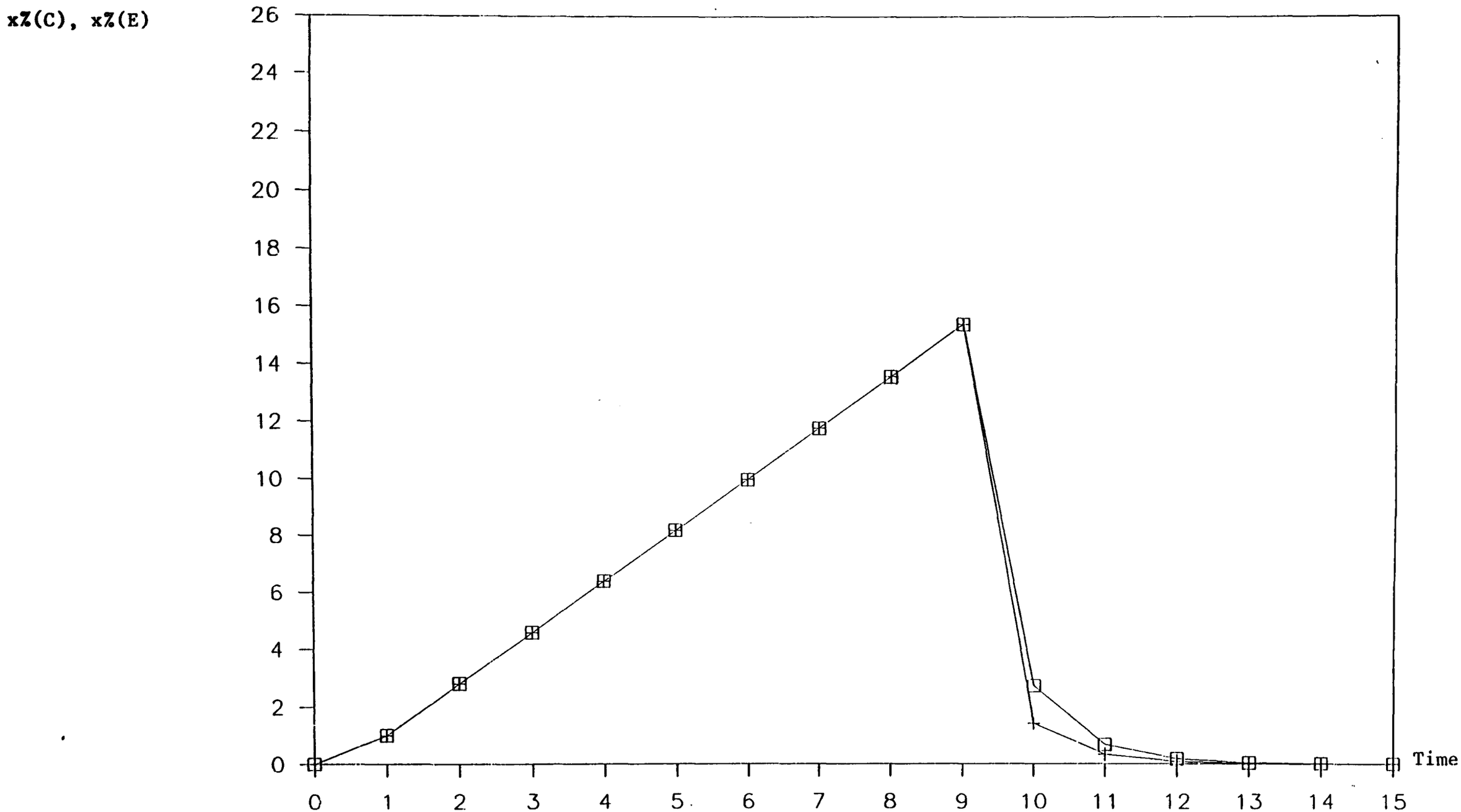

Figure 3: A Revision of the Precision Ratio at Time 9 in Response to the Policy Reversal.

□ Exchange Rate Overvaluation x%(C) as in Figure 2.

\+ Exchange Rate Overvaluations x%(E) with revision of precision ratio.

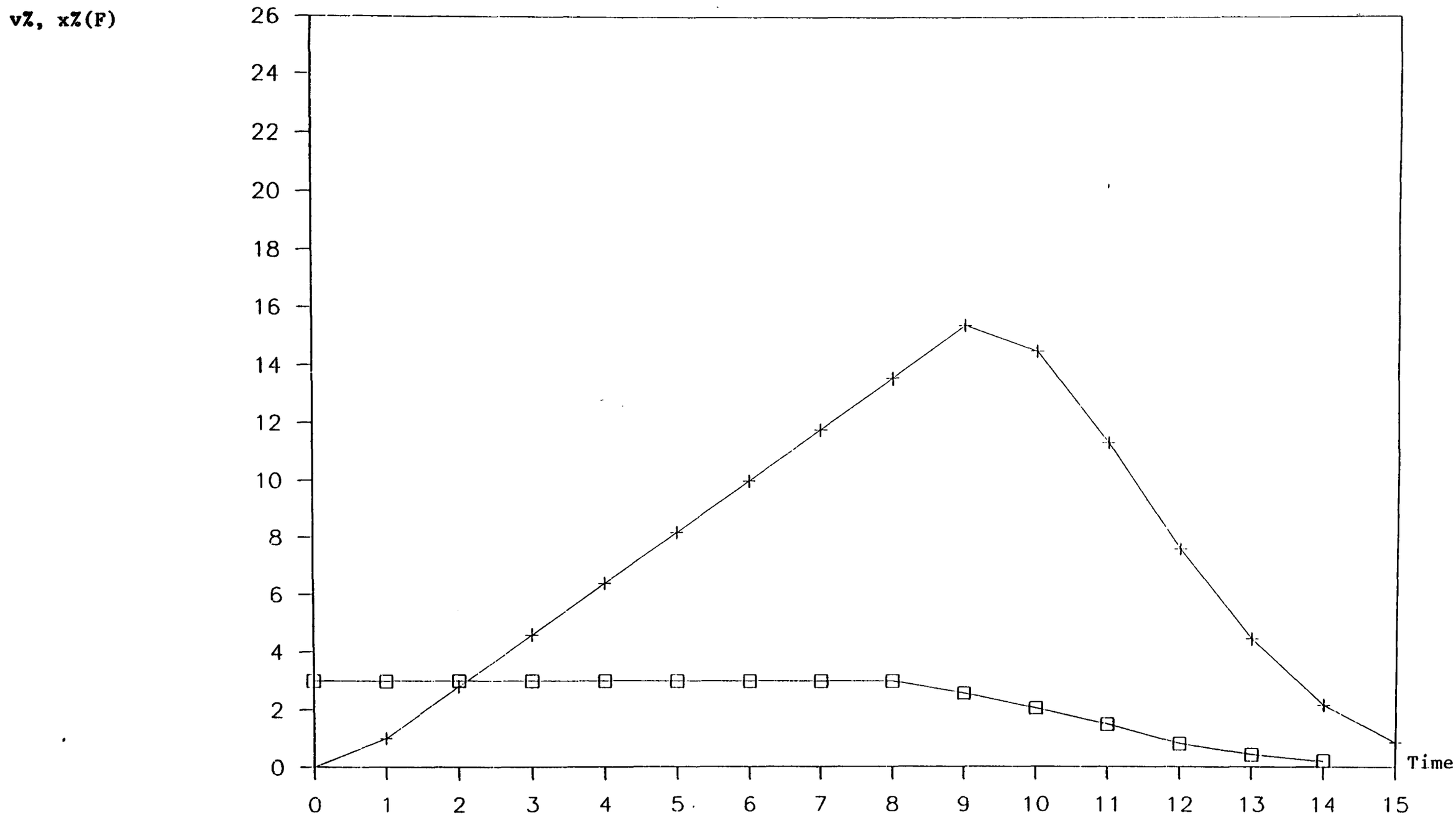

Figure 4: With a Gradual Policy Reversal at Time 9.

□ Interest Rate Differential v%.

\+ Exchange Rate Overvaluation x%(F). [Compare with Figures 2 and 3.]

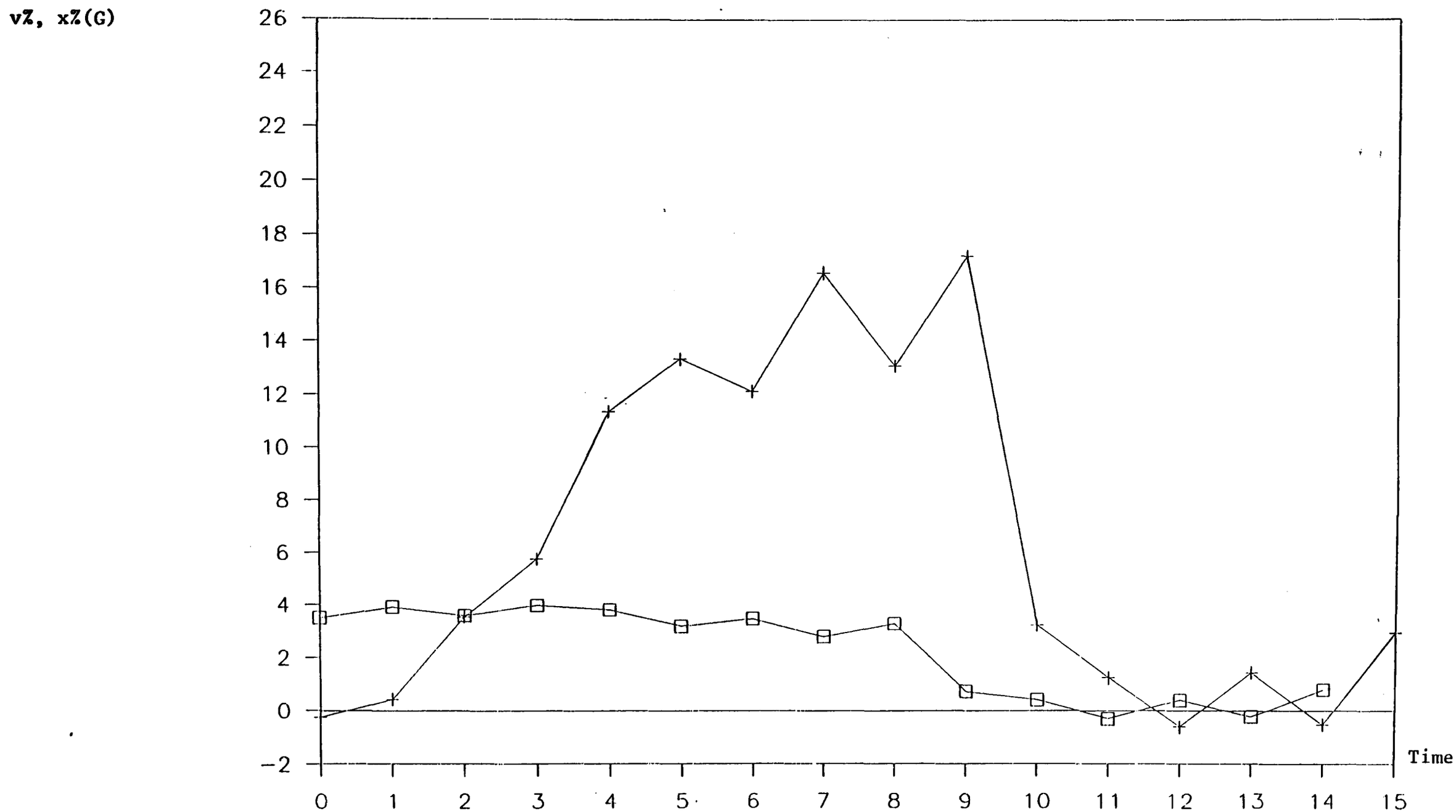

Figure 5: Without the Exclusion of Noise.

□ Interest Rate Differential v%.

+ Exchange Rate Overvaluations x%(G). [Compare with x%(C) of Figure 2.]

CHAPTER 4

SUBJECTIVE BELIEFS AND THE REGRESS IN BELIEFS: SOME CONSIDERATIONS

SYNOPSIS

Convergent learning by agents towards rational expectations equilibrium (REE) is interpreted as a test of the stability of that equilibrium. We, like others, suggest that learning must necessarily take an unsophisticated form, because of limitations to the availability of information and of costs in acquiring information, not least when consideration is given to the problems of coordinating the revision of beliefs in a decentralised market.

Examples of consistent and of inconsistent learning are considered in an asset market model of the exchange rate where the focus is on expectations of others' expectations. We find that there is no stable REE with perfect capital mobility by the criterion of the learning processes considered. This implies direct current account effects on the exchange rate according to the analysis of Chapter 2. We also find, there, that because of high capital mobility, the stability of any equilibrium requires that the exchange rate at least approximates a random walk in this specification of the model.

4.1 Preliminary Discussion and Introduction

This chapter relates three areas of theory to propose that economic activity within asset markets may in part be stylized by processes of learning. These areas are efficient markets theory, the theory of learning towards rational expectations equilibrium (REE) and Keynes' (1936) analysis of the role of beliefs on subjective factors in asset price determination. The discussion of this paper is applied to a model of the exchange rate, but, of course, the implications are wider.

The efficient markets theory literature [reviewed by Andersen (1982)] considers the conditions under which market price can convey information between different groups of traders or agents. If market price is fully-revealing, in aggregating and disseminating the information of all agents, then the sufficiency of price deters the costly pursuit of private information.

The context of the efficient markets literature is one of rational expectations, but one necessarily without homogeneous beliefs and expectations. Indeed, it is proposed, by Strong and Walker (1987), that the notion of a REE is the one which is applicable where price is seen to convey information in an environment of heterogeneity. It is worth noting that this microeconomic definition of rational expectations is fundamentally different from the one used in the macroeconomic literature, where, by common knowledge of the result of the general application of a standard solution algorithm, everyone forms the same expectation [as in Lucas and Prescott (1971)]. Homogeneous expectations may be regarded as a characterisation of the limiting case of the

microeconomic rational expectations notion where price is fully-revealing.

If the price system is sufficient to aggregate and to disseminate all the relevant information of the market, then agents will have no incentive to seek additional sources of information. A basic conclusion from Grossman (1976) and Grossman and Stiglitz (1976 and 1980) is that information costs would preclude the informational sufficiency of prices. Strong-form informational efficiency as defined by Fama (1970) will not hold.[1]

Information costs in weakening the price-signalling mechanism may give scope for agents to seek private sources of information, which have not become socially aggregated in prices. Competitive market activity may thus more appropriately be characterised in terms of information search rather than by notions of strong-form informational efficiency.

There is here some relationship to the second area of theory, of learning towards REE [reviewed by Blume, Bray and Easley (1982)]. Agents try to learn the initially unknown parameter values of the model in the hope of forming optimal forecasts with the properties of unbiasedness and minimum mean square error. They revise their prior beliefs in the light of observed data, which have been generated by the model. If learning is successful, they will converge to rational expectations, and successful learning is interpreted as the stability of REE. The operational use of REE may be justified by appealing to

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[1] Fama's definition requires that no individuals 'have monopolistic access to any information relevant for price formation', so that no subset of the market is at an informational advantage (or disadvantage).

convergent learning. Convergent learning may lead to a strong-form of REE of homogeneous expectations where all agents observe the same signals. Kihlstrom and Mirman (1975) show that '...all experienced Bayesian market observers have the same expectations'.

A major issue in the learning literature has been the distinction between consistent and inconsistent learning. This is particularly relevant to the consideration of the coordination problem in a decentralised market of agents with heterogeneous beliefs. Consistent learning requires that the learning process has no unanticipated effect on the model being learned. This may be either because there is no effect, as in economically simplified models, or because the effect can be fully anticipated, as in models with strong assumptions on the availability of information. The first instance applies in the models of Taylor (1975), Friedman (1979) and in Chapter 3. The second instance applies to the models of Townsend (1978, 1983A and 1983B). Inconsistent learning may induce unanticipated effects, as it is based on a misspecification.

Consistent learning may be regarded as an equilibrium in learning and as self-fulfilling learning, as there is no incentive to modify learning forms and forecast functions. It may therefore be regarded as a weak-form of REE, before convergence to the strong-form where beliefs are homogeneous. The latter is not problematical, given the sophistication of the learning process which takes full account of its own dynamics. In this way, consistent learning may be designated self-fulfilling learning in the same way that rational or model-consistent expectations are self-fulfilling.

In one sense, consistent learning as a weak-form of REE where price is not fully revealing, is strongly analogous to the state of agents collecting information in the rational expectations context of the efficient markets literature. Convergence would be to a stronger, fully-revealing REE. The link between the two areas of theory is made by Kihlstrom and Mirman (1975) and by Bray (1982). However, there is another sense in which the two areas differ: information costs are central to efficient markets theory, but learning costs are largely overlooked in the learning literature. This theme is taken up later.

Frydman (1982) and 1983) has argued that consistent learning is untenable in the context of a decentralised market where the only information held is that acquired through the process of learning. There is an identification problem for the agent trying to estimate the model: it is not possible to separate the effect of fundamental factors from the effect of the beliefs of others. The conditions for consistent learning in the Townsend (1978, 1983A and 1983B) are quite stringent, requiring considerable initial endowments of information.

Frydman (1983) has also suggested that the alternative of inconsistent learning brings the problem that learning forms and forecast functions become indeterminate. If agents are learning and forecasting inconsistently with the model, there is an incentive for individuals to revise their learning forms and forecast functions. It is only in the limiting case of consistent learning, which is implausible anyway, that there is no incentive to further revision. If learning cannot feasibly be consistent, and if inconsistent learning is dogged by indeterminacy, the analysis reaches a stalemate

The problem may be resolved by admitting costs to learning. In the same way that information costs explain the non-existence of strong-form informational efficiency and the private gathering of information in the efficient markets literature, learning costs should be regarded as endemic to the analysis of learning outside and towards strong-form REE.

Learning costs may resolve the analytical stalemate mentioned above. First, if agents start from a base of inconsistent learning, costs give a sufficient reason why agents may not be able to modify their learning equations. The limit of full consistency may be unreachable. Secondly, learning costs resolve the determinacy problem. If there are costs, it is not certain that there are always overwhelming incentives to revise inconsistent learning forms. Therefore by widening the framework, it can become optimal for agents to use learning forms which are not entirely consistent. There is then a determinate inconsistent learning equilibrium, where the expected costs to revision outweigh the expected gains. An analogy is found in Grossman and Stiglitz (1976), who give an informational equilibrium where the individual is indifferent to becoming informed.

Another problem with learning is that if it is successful, it may cause convergence to a form of REE which is sufficiently close to the strong-form informational equilibrium, which the efficient market literature predicts is non-existent. This form of REE is one with homogeneous beliefs, which was mentioned with reference to the Kihlstrom and Mirman (1975) paper. If all agents observe the same signals, and their learning processes are stable, they will surely come to the same beliefs. Consequently, learning costs may also be analytically useful

to throw some sand into the works in order to prevent a smooth transition towards a state of the world, which is implausible.

Muth (1961), in his seminal statement of rational expectations, specified the state where the *average* of agents' subjective distributions of price corresponds with the objective distribution of price. There was an allowance for differences of belief across agents. But a stronger version has since emerged in the macroeconomic literature where the subjective price distribution of each agent corresponds with the objective price distribution - as in Lucas and Prescott (1971). The latter of a homogeneous REE would seem to be supported by appealing to learning of the type in Kihlstrom and Mirman (1975). Therefore if the operational use of REE is to be justified by appealing to successful learning, it may be the one of homogeneous expectations which should be used.

But it seems incongruous to apply the assumption of homogeneous expectations to an asset market where a primary reason for trading is the *difference* in beliefs and expectations. This point was well made by Grossman (1977) and by Grossman and Stiglitz (1980) in the context of an asset market inside REE. Grossman and Stiglitz (1980) claim that 'speculative markets where prices reveal a lot of information will be very thin because it [competitive equilibrium] will be composed of individuals with very similar beliefs'.

Learning costs may ensure that agents can never learn enough to undermine the *raison d'être* of an asset market, the difference in beliefs. Although convergence is usually regarded as an asymptotic possibility, costs can prevent the possibility of learning too much in

finite time. Learning costs also overcome the indeterminacy problem considered by Frydman (1983), and also allow some reconciliation of learning theory with efficient markets literature, which freely admits information costs.

The third area of discussion is Keynes' (1936) analysis of the role of market beliefs in asset price determination. Market beliefs through their effects on demand determine prices, so that agents have a strong incentive to anticipate market beliefs in order to anticipate market prices. As their anticipations in aggregate themselves constitute market beliefs, the market beliefs of relevance are those on a higher order of market beliefs. Logically, there is an infinite regress in market beliefs because of their recursive determination. Keynes' metaphor for this state was the 'beauty contest'. Such contests in the newspapers of the 1930s were about most correctly anticipating the opinion of the masses who entered rather than picking the most beautiful faces.

Consequently, learning or the acquisition of information will be of market beliefs, which are germane to price determination. Agents may, in some circumstances, have little incentive to learn the objective parameter values of a model which is outside REE. The incentive will be to learn market beliefs and subjective as well as objective factors.

The foregoing discussion hopes to establish the proposition that a model of learning in a decentralised market will plausibly take an unsophisticated form. Another proposition is that agents will endeavour to learn the beliefs of others, which are important in determining asset prices. This paper establishes an exchange rate model under learning.

In one example, we present a self-fulfilling learning strategy where market beliefs are learned. This is based on rather strong assumptions. Another example is where agents try inconsistently to learn the REE. Learning is inconsistent because of implicit costs, not drawn out in the technical discussion which follows. There is inconsistency because each agent is revising an individual belief on market opinion, which is specified as a constant, but market opinion is defined as the aggregation of individual beliefs which are non-constant during the process of revision under learning.

The purpose of the paper is to extend the analysis of exchange rate dynamics, considered in Chapters 2 and 3. The perspective here is on the interaction of many agents with heterogeneous beliefs learning about subjective factors, average or market beliefs. Questions arise which have no relevance to the homogeneous information case. Some of the issues from the literature on multiagent learning are explored. We are indebted particularly to the forementioned analyses of Townsend (1978, 1983A and 1983B) and of Frydman (1982 and 1983).

Section 4.2 establishes the basic model of the exchange rate for consideration with the assumption of risk-neutrality. It is closely related to the one given in Chapter 2. The exchange rate is determined by the condition of equilibrium on the total balance of payments, comprising the capital account and the current account. With respect to the capital account, the simplification of risk-neutrality allows us to overlook the expected variances of asset yields as arguments in the asset demand functions. Agents are concerned with the relative means of the two assets in the model, one which is denominated in terms of the domestic currency, the other in terms of the foreign currency.

Section 4.3 looks at the REE solution to the model. It is derived from the undetermined coefficients algorithm which delivers a homogeneous expectations solution with common knowledge on the parameter values. We suggest that the application of this solution method, which implicitly requires an omniscient individual to apply it, is inappropriate to a decentralised market where agents cannot coordinate their beliefs. This statement was argued by Phelps (1983) and by Frydman (1983). The REE solution serves, however, as an interesting benchmark case.

Section 4.4 considers the more general case. Subjective factors are very important as agents' beliefs determine market outcomes. It is not only necessary for agents to form beliefs, but for them to form beliefs on others' beliefs and on beliefs of ever increasing order as in Keynes (1936). The problem of the infinite regress in beliefs and expectations is irrelevant outside strong-form REE, where agents are concerned with subjective factors (which Keynes called 'speculation') and not with the objective factors (which he called 'enterprise'). We reach a strong conclusion, because of the atomistic nature of agents, that if agents believe that the market is outside REE, they will see little incentive to solve or to learn only objective parameter values alone of the model. They see every incentive to determine the subjective beliefs of others, which will determine market outcomes.

The existence of an equilibrium apart from REE requires that the market believes that there are some restrictions on the parameterisation of the set of higher ordered market beliefs in the infinite regress. In the extreme case of perfect capital mobility with potentially unbounded

capital flows, determinacy is given only where agents believe that there is REE. This belief will be self-fulfilling and ensure an equilbrium with uncovered interest parity. This equilibrium is only guaranteed inside REE.

Outside REE, we consider the possibility that agents individually endeavour to learn the model. Section 4.5 considers consistent and inconsistent learning in a discursive fashion. Consistency in learning exists where the learning process itself has no unanticipated effect on the model being learned. It requires strong informational assumptions at the outset to enable agents to foresee the effects of their collective actions on the model. Frydman (1982 and 1983) has shown that consistent, or rational, learning is untenable with respect to a decentralised market where the only information which can be held is that acquired through the learning process. We could cite learning costs as a sufficient reason for the untenability.

Section 4.6 considers applications of consistent and of inconsistent learning to the general model of the exchange rate. Learning generally is treated in a purely econometric fahion, as if agents were 'superior statisticians' to quote Arrow (1978). In this section we consider a form of social learning where agents canvass each other's beliefs to determine the mood of the market. It is apparent that organised market institutions like the stock exchange and the foreign currency exchanges give scope for agents to interact in a social framework. It is an important consideration in the context of the heterogeneity of beliefs and the effect of beliefs on market outcomes.

With reference to statistical learning forms, instability is a real possibility in the case of inconsistent learning. There agents have not the information required to take into account the effect of their learning on the model. The boundary between stability and instability is determined by the parameterisization of the underlying model. The parameter representing holdings of wealth and the ease with which agents can borrow for risk-neutral speculation must be relatively low to ensure stability. It is found that there is some correspondence between the existence of a finite equilibrium exchange rate outside REE and on the asymptotic stability of inconsistent learning.

There are two conclusions from the model which have some relevance to an understanding of the exchange rate. There is no stable REE where capital mobility is unbounded, if the stability test is the inconsistent learning process which we give. By this criterian, we could not justify a REE with exact uncovered interest rate parity. Determinacy in this case requires bounded capital mobility as a minimum condition. Secondly, if capital mobility is bounded but high, the stability of REE in this model would then require that the interest rate differential at least approximates a random walk. The corresponding REE would be one where the exchange rate follows a random walk (which has some empirical significance).

Section 4.7 briefly widens the area of discussion by considering three issues of importance to learning. These are the individual *ex ante* optimality of inconsistent learning in a decentralised market, which may lead to an externality; the determinacy of inconsistent learning and forecasting where learning costs are considered; and a

separate wealth dynamics, which has been considered in the efficient markets literature.

There are two issues, which we consider of relevance to the learning of rational expectations. One is that the analysis can benefit greatly from considerations which have arisen in the efficient markets literature, particularly with respect to learning within an asset market. The second is that a learning model can be considered as a representation of the process of information acquisition in markets where price is not a perfect aggregator and disseminator of information. Therefore the notion of homogeneous information REE may have little scope, and there may be considerable merit in modelling an asset market conjoined with a learning model to describe the dynamic patterns of the data. This chapter aims to explore some of these ideas rather than to seek some general conclusions.

4.2 The Model

The basic model of the exchange rate is considered in this section. Considerations are given to the special case of the rational expectations equilibrium (REE) solution in the next section and of the more general solution in the following section.

The model is similar to the one in Chapter 2. There the exchange rate was determined by a balance of payments equilibrium condition, giving consideration to the current and the capital account. The latter was determined by the first difference in the expectation of the mean relative yield between domestic currency denominated and foreign

currency denominated assets. The rationale was that the desired stocks of assets were a function of the mean relative yield, and that the capital account was treated as the first difference in these stocks. In this version, we simplify by assuming that the capital account is determined by the *level* of the expectation of the mean relative yield.

Again it is assumed that agents are risk-neutral speculators in order to avoid considerations of the variances of the asset yield distributions, which would complicate the analysis.

In Chapter 2 the focus was on some informational aspects with respect to the current account. This paper, still with a mind to these aspects, limits the range of issues by assuming that the current account follows a random walk.

$$c_t = -f_t - b\, Gm_{t-1}\, \bar{z}_{t+1} \tag{1}$$

$$f_t = f_{t-1} + \epsilon_t, \quad \epsilon_t \sim \text{NIID}\ (0,\ \sigma_\epsilon) \tag{2}$$

All variables are in real terms: c is the real exchange rate or competitiveness. A rise in c constitutes a real depreciation. The variable f represents the current account and follows a random walk. The noise term, ϵ, is normally, identically and independently distributed. The parameter on the average expectation of the relative mean yield, b, represents the magnitude and speed of capital flows, as in Chapter 2. The magnitude will depend on wealth stocks and borrowings. We characterise the case of perfect capital mobility as the one where the value of b is unbounded. In this case a solution is ensured if agents believe that the market is in REE, and so apply a rational expectations algorithm to the model.

G is the expectations operator and Gi_t is agent i's expectation at time t and the one which exploits all information available at the same time. The average expectation across all agents or the market expectation is an appropriately weighted average of all individual expectations:

$$Gm_t.. = \sum_{i=1}^{N} wi\ Gi_t..$$

$\bar{z}_{t+1}$ is the mean relative yield at time t+1, so that $Gm_{t-1}\ \bar{z}_{t+1}$ is the market expectation made at time t-1 of the relative mean yield at time t+1. The relative mean yield at time t+1 is defined by

$$z_{t+1} \equiv v_t - (c_{t+1} - c_t) \tag{3}$$

It comprises the real interest rate differential favouring domestic currency denominated (dcd) assets, v_t, minus the rate of real depreciation, $c_{t+1} - c_t$. The latter constitutes a capital gain for domestic residents holding foreign currency denominated (fcd) assets and a capital loss for foreign residents holding dcd assets. The model is completed by specifying the real interest rate differential as a Markov process:

$$v_t = \theta\ v_{t-1} + \omega_t, \qquad \omega_t \sim \text{NIID}\ (0,\ \sigma_\omega) \tag{4}$$

A more standard treatment would be to determine the interest rates from the conditions of goods market and money market equibria in each country. We could make explicit the assumption that the money supplies are endogenous and that the interest rates are controllable instruments of policy. As before, differential relationship for interest rates then suggests that the two countries' monetary authorities are then determining a joint policy. Alternatively, we could assume that the process for the interest rate differential represents a reduced-form which is derived from a two-country policy where money supplies are

exogenous and where interest rates are intermediate targets. Either way, it is hoped to simplify the structural side of the model.

The exchange rate at time t is determined by capital movements which are based on mean relative yield expectations which are made earlier at time t-1. The relevant expectation is of the mean relative yield at the future time t+1, which is not observable until that time. The model is alternatively represented by

$$c_t = - f_t + b\ Gm_{t-1}\ (\bar{c}_{t+1} - \bar{c}_t - \bar{v}_t) \qquad (5)$$

It is of note that the *actual* exchange rate at time t is no part of the expression for the expected mean yield above. If it were then equation (5) would be equivalent to

$$c_t = - f_t/(1 + b) + b/(1 + b)\ Gm_{t-1}\ (\bar{c}_{t+1} - \bar{v}_t)$$

But this case has the unsatisfactory characteristic that information on the exchange rate at time t, observed contemporaneously, can be exploited to determine the asset demands, which, in turn, determine the exchange rate at the same time period, t. This relates to Hellwig's (1982) point that market price cannot plausibly bring both equilibrium and convey information which conditions decisions determining the same equilibrium.

Apart from the exogenous processes determining the current account and the interest rate differential, which is an element of the relative yield between the two types of asset, the exchange rate is determined by market beliefs. A particular solution therefore depends on how market beliefs are formed. In the next section we consider the rational expectations solution as a special case of market beliefs.

4.3 The Rational Expectations Equilibrium Case

The REE solution of the model is considered as a benchmark case. Each agent is assumed to form expectations *as if* the structure of the model and its parameter values are fully known. Friedman (1979) separates two information assumptions, which are used in the rational expectations literature. The information *exploitation* assumption is a statement of individual optimality in the use of information. The information *availability* assumption is one concerning the initial endowment of information.

Defining I_{it} as i's information set at time t and g as the vector of parameters on which i would form expectations, the information exploitation assumption is

$$Gi_t\, g = Gi_t\, g \mid Ii_t\,(..), \text{ for each i.} \tag{6}$$

Rationally, agents fully exploit what they have in their information sets.

The information availability assumption of REE requires that agents have a generous endowment of information, so that

$$Ii_t\,(..) = g, \qquad \text{for each i} \tag{7}$$

Equations (6) and (7) combine to give

$$Gi_t\, g = Gi_t\, g \mid g \tag{8}$$

Agents forecast g as if they know g, which in terms of our model includes b and θ.

Phelps (1983) and Di Tata (1983) show that the REE notion is even more informationally strong when a consideration is given realistically to the existence of many agents within a decentralised market. In

Townsend's (1978, 1983A and 1983B) terminology, it is not sufficient for *zero-order* beliefs to be correct: that is beliefs on the value of g are correct. It is necessary that *first-order* beliefs are correct: that agents correctly perceive that other agents' beliefs on the value of g are correct. And, logically, there are *second-order* beliefs, beliefs on beliefs on beliefs on the value of g, and so on. The REE requires a solution to the infinite regress in beliefs, considered by Keynes (1936), which might be described as a common knowledge of a correct informational uniformity.

The model in effect is reduced to that of a single agent or a Hayekian (1948) 'quasi-omniscient individual', who can perform the 'pure logic of choice' in applying a rational expectations equilibrium solution method to the general form of the model. The method of undetermined coefficients [see Appendix A.4.1] applied to the model gives a stationary solution:

$$c_t = - f_t - b\theta/[1 + b(1 - \theta)]\ v_{t-1} \tag{9}$$

as $Gm_{t-1}\ v_t = Gm_{t-1}\ \theta\ v_{t-1} = Gi_{t-1}\ \theta\ v_{t-1} = \theta\ v_{t-1}$ from equation (14). The solution for the relative yield from equations (2)-(4) and (8) is

$$z_{t+1} = \theta/[1 + b(1 - \theta)]\ v_{t-1} + (1 + b)/[1 + b(1 - \theta)]\ \omega_t + \epsilon_{t+1} \tag{10}$$

And the expected mean of z_t conditional on information at t-1 is

$$Gm_{t-1}\ \bar{z}_{t+1} = \theta/[1 + b(1 - \theta]\ v_{t-1} \tag{11}$$

Substitution of equation (11), which determines the asset demands in the general form in equation (1) gives the REE solution in equation (9), which demonstrates that rational expectations are self-fulfilling expectations.

There are certain believed empirical regularities in the exchange rate. One is that it follows a random walk, a second is that there is *ex ante* uncovered interest parity (UIP). Within REE as presented by equation (9) there are two possibilities of random walk exchange rates (as the current account itself follows a random walk): where the value of θ is zero and where the value of θ is unity with a bounded value for b. Outside these two extremes of fully permanent and fully transitory interest rate differentials, the exchange rate will not follow a random walk inside REE.

UIP will only hold if b, the coefficient of capital mobility, is unbounded, as shown in equation (11). But if b is unbounded the exchange rate is given by

$$c_t = - f_t - [\theta/(1 - \theta)] v_{t-1} \tag{12}$$

The existence of a finite equilibrium requires that the value of θ is not equal to unity. Therefore within REE the two properties of a random walk exchange rate and UIP together require that the value of θ is zero and that b is unbounded. We show later on that this particular REE is not stable by the criterion of an inconsistent learning process. Therefore the relevance of the assumption of unbounded capital mobility, which ensures exact UIP, is to be challenged by a consideration of the stability properties of its REE equilibrium.

Finally, it is often assumed that outside REE agents would learn the actual parameter values of the model by running regressions on the final form given by equations (4) and (9) to estimate the parameters θ and b. However, it is shown in the next section that in the general case of uncoordinated agents with different beliefs there is little

incentive to learn these objective values alone, but every incentive to learn the subjective beliefs of other agents outside strong-form REE.

4.4 The General Case

4.4.1 The Importance of Subjective Factors

In general agents hold different beliefs while an average of agents' beliefs will determine actual market outcomes. Individual agents must therefore determine average opinion as a basis on which to make portfolio decisions. A distinction is made between objective and subjective factors. The value of the parameter, θ, is an objective factor, but agents' beliefs on its value are subjective factors. In short, agents must determine subjective factors.

Keynes (1936) found an analogy between asset market behaviour and the 'beauty contests' in the newspapers of his day. The winners were those who picked the six faces closest to the average choice. So objective judgement is not called for, but an anticipation of the psychology of the masses. In theory this entails the problem of the infinite regress in expectations. To anticipate mass opinion, logically it is necessary to anticipate mass opinion of mass opinion. To do the latter, it is necessary to take the order of anticipation one stage further. The recursive nature of the process leads to the possibility of an infinite regress in expectations. This may apply whenever there is an element of competition between economic agents and where beliefs affect outcomes. Cyert and DeGroot (1970) analysed it within a duopoly model.

4.4.2 *A Representation of the Infinite Regress.*

In section 4.2 some notation for expected variables was introduced. In this subsection we aim to complete the notation to give a representation of the infinite regress where market beliefs are made on higher order market beliefs.

It was said that $Gi_{t-1}\ \bar{z}_{t+1}$ was the expectation of agent i made at time t-1 of the value of $\bar{z}_{t+1}$ (the mean relative yield) occurring at time t+1, and that $Gm_{t-1}\ \bar{z}_{t+1}$ was the corresponding average or market expectation, obtained by taking a weighted average across all individuals.

$$Gm_{t-1}\ \bar{z}_{t+1} \equiv \Sigma wi\ Gi_{t-1}\ \bar{z}_{t+1}, \qquad \Sigma wi = 1 \tag{13}$$

The expression in equation (13), which is germane to the model, requires a recursive solution because it is determined by higher order beliefs.

Each agent, indexed i, is interested in market beliefs as these determine the exchange rate, making the expectation $Gi_{t-1}\ Gm_{t-1}\ \bar{z}_{t+1}$. This is i's expectation made at time t-1 of the contemporaneously made market expectation of the mean relative yield at time t+1. Market beliefs of market beliefs, or second-order market beliefs, are then determined by taking a weighted average:

$$Gm_{t-1}\ Gm_{t-1}\ \bar{z}_{t+1} \equiv \Sigma wi\ Gi_{t-1}\ Gm_{t-1}\ \bar{z}_{t+1}, \qquad \Sigma wi \equiv 1 \tag{14}$$

Next we determine third-order market beliefs. Agent i makes an expectation at time t-1 of second-order market beliefs, $Gi_{t-1}\ Gm_{t-1}\ Gm_{t-1}\ \bar{z}_{t+1}$. Again by taking a weighted average across all the agents, we get

$$Gm_{t-1}\ Gm_{t-1}\ Gm_{t-1}\ \bar{z}_{t+1} \equiv \Sigma wi\ Gi_{t-1}\ Gm_{t-1}\ Gm_{t-1}\ \bar{z}_{t+1} \tag{15}$$

As all expectations are made contemporaneously (at time t-1), we may abbreviate the notation to

$$Gm_{t-1}\, Gm_{t-1}\, \bar{z}_{t+1} \equiv Gm^{2}_{t-1}\, \bar{z}_{t+1}$$

and to

$$Gi_{t-1}\, Gm_{t-1}\, Gm_{t-1}\, \bar{z}_{t+1} \equiv Gim^{2}_{t-1}\, \bar{z}_{t+1},$$

so that, generally,

$$Gm^{k}_{t-1}\, Gm^{\ell}_{t-1}\, \bar{z}_{t+p} \equiv Gm^{k+\ell}_{t-1}\, \bar{z}_{t+p} \tag{16}$$

$$Gi_{t-1}\, Gm^{k}_{t-1}\, Gm^{\ell}_{t-1}\, \bar{z}_{t+p} \equiv Gim^{k+\ell}_{t-1}\, \bar{z}_{t+p} \tag{17}$$

for $\ell, k \geq 1, \quad p \geq 1$

In the terminology given by Townsend (1978, 1983A and 1983B), k and ℓ signify the order of beliefs. The prospect of the infinite regress in beliefs is the consideration that the value of k (or ℓ) is unbounded.

In the previous section we considered a stong-form REE solution to the model, which rests on a correct uniformity in beliefs. This has two dimensions: one is that beliefs are identical across agents, the second is that each agent projects his belief onto the rest of the market. These dimensions are respectively:

$$Gi_{t-1}\, \bar{z}_{t+1} = Gj_{t-1}\, \bar{z}_{t+1} \qquad \text{for each i and j}$$

$$Gi_{t-1}\, \bar{z}_{t+1} = Gim^{k}_{t-1}\, \bar{z}_{t+1} \qquad \text{for each i and k}$$

This solution may be regarded as just one out of an infinity of feasible solutions. It serves as an interesting benchmark case, but has the unappealing characteristic of common knowledge: everyone believes the same and knows it.

In this section we consider the general solution outside strong-form REE. We go on to show that the parameters, b and θ, are believed

by agents to be not necessarily relevant to their 'calculations', if they do not believe themselves to be inside REE.

The concept to be used is the more general one of 'model-theoretic' expectations, as was used by Phelps (1983) in the consideration of a similar problem. Model-theoretic expectations are of the family of 'model-consistent' expectations, but do not require that every one believes that everyone believes the same thing, as in strong-form rational expectations as employed in macroeconomics. We use this concept along with the multiple regress notation just introduced in the immediately preceding paragraphs.

For what follows, we simplify by assuming that agents believe that the market is not revising its beliefs on the value of $\bar{z}_{t+1}$ between times t-1 and t, so

$$Gi_{t-1}\, Gm_t\, \bar{z}_{t+1} = Gi_{t-1}\, Gm_{t-1}\, \bar{z}_{t+1} \equiv Gim_{t-1}\, \bar{z}_{t+1}$$

and that this assumption applies generally for all time periods when expectations are made and for all occurrences of z:

$$Gi_{t-1}\, Gm^k_{t+p}\, \bar{z}_{t+q} = Gi_{t-1}\, Gm^k_{t-1}\, \bar{z}_{t+q} \equiv Gim^k_{t-1}\, \bar{z}_{t+q} \qquad (18)$$

$$\text{for } p \geq 0, \quad q \geq 1$$

The mean relative yield, $\bar{z}$, is of relevance to agents asset market decisions. Therefore expectations will be made of this, and model-theoretic expectations will require consistency with the structural form for the relative yield. Equations (1)-(3) give the relative yield at time t+1 as

$$z_{t+1} = v_t + \epsilon_{t+1} + b[Gm_t\, \bar{z}_{t+2} - Gm_{t-1}\, \bar{z}_{t+1}] \qquad (19)$$

Agent i's expectation of the mean of this is

$$Gi_{t-1}\, \bar{z}_{t+1} = Gi_{t-1}\, \theta\, v_{t-1} + Gi_{t-1}\, b[Gim_{t-1}\, \bar{z}_{t+2} - Gim_{t-1}\, \bar{z}_{t+1}] \qquad (20)$$

Having derived individual i's expectation of the relative mean yield, the market expectation is obtained by averaging across all agents to get

$$Gm_{t-1}\,\bar{z}_{t+1} = Gm_{t-1}\,\theta\,v_{t-1} + Gm_{t-1}\,b[Gm^2_{t-1}\,\bar{z}_{t+2} - Gm^2_{t-1}\,z_{t+1}] \quad (21)$$

This expectation is a part of the expression for the actual relative yield in equation (19). Consequently agents must determine this as a part of the solution for the relative mean yield. Model-theoretic expectations of first-order market beliefs of the mean relative yield at time t+1 give

$$Gim_{t-1}\,\bar{z}_{t+1} = Gim_{t-1}\,\theta\,v_{t-1} + Gim_{t-1}\,b[Gim_{t-1}\,\bar{z}_{t+2} - Gim_{t-1}\,\bar{z}_{t+1}] \quad (22)$$

and for the same at time t+2

$$Gim_{t-1}\,\bar{z}_{t+2} = Gim_{t-1}\,\theta^2\,v_{t-1} + Gim_{t-1}\,b[Gim^2_{t-1}\,\bar{z}_{t+3} - Gim^2_{t-1}\,\bar{z}_{t+2}] \quad (23)$$

Substitution of equations (22) and (23) into (20) give

$$\begin{aligned} Gi_{t-1}\,\bar{z}_{t+1} = {} & [Gi_{t-1}\,\theta + Gi_{t-1}\,b\,Gim_{t-1}\,(\theta^2 - \theta)]\,v_{t-1} \\ & + Gi_{t-1}\,b\,Gim_{t-1}\,b\,Gim^2_{t-1}\,(\bar{z}_{t+3} - 2\bar{z}_{t+2} + \bar{z}_{t+1}) \quad (24) \end{aligned}$$

Clearly, agent i's expectation of the mean yield is conditional on his expectation of market beliefs, market beliefs of the first and second orders. A complete solution requires taking the recursion further since all orders of market beliefs are conditional on market beliefs of higher orders.

A solution for i's expectation of the mean relative yield is

$$Gi_{t-1}\,\bar{z}_{t+1} = Gi_{t-1}\,q\,v_{t-1}$$

where $Gi_{t-1}\,q = Gi_{t-1}\,\theta + Gi_{t-1}\,b\,Gim_{t-1}\,[(\theta - 1)\,\theta]$:

$$(Gi_{t-1}\,b)\left[\sum_{k=2}^{\infty}\left(\prod_{s=1}^{k} Gim^s_{t-1}\,b\right) Gim^k_{t-1}\,[(\theta - 1)^k\,\theta]\right] \quad (25)$$

And for the market's expectation of the mean relative yield as

$$Gm_{t-1}\,\bar{z}_{t+1} = Gm_{t-1}\,q\,v_{t-1}$$

where $Gm_{t-1}\, q = Gm_{t-1}\, \theta + \left[\sum_{k=1}^{\infty} \left(\prod_{s=1}^{k} Gm_{t-1}^{s}\, b \right) Gm_{t-1}^{k+1} \left[(\theta - 1)^k\, \theta \right] \right]$ (26)

Equation (26) is substituted into (1) to get a general expression for the exchange rate, which is formally determined in part by a multiple regress in market beliefs.

$$c_t = -f_t + b\, Gm_{t-1}\, q\, v_{t-1} \tag{27}$$

4.4.3 *The Existence of Equilibrium Outside REE*

In the special case of perfect capital mobility as represented by an unbounded value for b, equation (27) shows that the existence of an equilibrium (with a finite value for the exchange rate) at least requires that

$$Gm_{t-1}\, q \to 0, \quad \text{if } b \to \infty$$

This is a necessary but not a sufficient condition, as the rates at which the respective terms tend towards their limits is clearly important.

At first hand, it is difficult to see how it could hold, unless

$$Gm_{t-1}\, \theta = 0 \qquad \text{for each } k$$

But this restriction amounts to a trivialising case of the model where the interest rate differential is expected by the market at each order of beliefs to be zero and therefore not to affect the exchange rate. Outside, the trivial case and where there is perfect capital mobility, we propose that an equilibrium outside REE cannot exist.

But a REE does generally exist for perfect capital mobility (see Section 4.3) This demonstrates that the rational expectations hypothesis is important to the asset market view in ensuring an equilibrium for otherwise unfavourable parameterisations of the model, namely of perfect capital mobility.

To depart from the REE paradigm, we assume that capital mobility is high but imperfect. This time where b is bounded, existence requires a weaker condition that

$$| Gm_{t-1} q| < \infty$$

which is necessary and sufficient. As this may hold for a wide range of cases, we consider a further condition which guarantees its sufficiency:

$$Gm_{t-1}^{k-1} b \, Gm_{t-1}^{k} [(\theta - 1)^{k-1} \theta] < Gm_{t-1}^{k-1} [(\theta - 1)^{k-2} \theta], \quad k \geq p$$

for some finite value of p, and

$$|Gm_{t-1}^{k} b| < \infty \qquad \text{for all } k$$

This restriction ensures that the infinite regress in market beliefs, translated into a recursion of an infinite number of beliefs, sums to a finite amount.

Economic agents of finite intelligence should not be able to conceptualise the higher orders of market beliefs. It may be tractable to consider market beliefs of the first and second orders, but difficult to form an idea of market beliefs of market beliefs of market beliefs.[2] Even the few chess players who reach the standard of grandmaster cannot generally anticipate more than five moves in advance. Therefore we can legitimately expect that at a certain order of beliefs an expectational blindness besets the economic agent, so that it is not possible to make independent expectations for the higher orders of market beliefs. So we assume that at a finite order, ℓ, Gm^{ℓ} is approximately equal to $Gm^{\ell+1}$. Then we can use the more manageable expression

$$| Gm_{t-1}^{k} [b(\theta - 1)] | = \Omega < 1 \qquad \text{for } k \geq \ell \qquad (29)$$

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2 Keynes (1936), perhaps, whimsically, said 'And there are some, I believe, who practise the fourth, fifth and higher degrees'.

If at each order of beliefs the market believes that capital mobility is very high, the existence of an equilibrium, which is dependent on the state of market beliefs, requires that at each order the market beliefs that the interest rate differential follows a random walk. This is quite plausible if the short-term value of θ is being considered, since as the time horizon over which agents speculate shortens, agents should not expect much change in the interest rate differential. For example, if agents are considering hourly changes in market conditions, they would normally expect the interest rate differential to be practically the same from one hour to another. And if the interest rate differential is believed to decline by 40% in magnitude each year, the 'long-term' annual value of θ is 0.6. This gives a belief in a weekly 'short-term' value of θ of 0.9902 ($= 0.6^{1/52}$) and so an upper bound of 102.2 for b which is required for an equilibrium.

4.5 Consistent and Inconsistent Learning

Outside REE agents might feel that there are incentives to learn the model in improved forecasts. And, agents will see learning as feasible, if they believe that the model to be learned has a solution.

Learning contributes a dynamic to the model. Beliefs, which are an intrinsic part of the model, become non-stationary during the process of their revision under learning. This will determine some of the variation in the relative yield, z_t. Equations (1)-(4) and (26) give it as

$$z_t = [\theta + b(\theta\, Gm_t\, q - Gm_{t-1}\, q)]\, v_{t-1} + (1 + b\, Gm_t\, q)\, \omega_t + \epsilon_{t+1} \qquad (30)$$

The parameter on v_{t-1} becomes time-variant, because, generally,

$Gm_t\ q \neq Gm_{t-1}\ q.$

Consistent and inconsistent learning are alternative possibilities. Learning is consistent where there are no unanticipated effects from learning on the model being learned. [See Blume, Bray and Easley (1982)] In this event the learning and the forecasts functions of agents are consistent with the model, and therefore consistent learning may be regarded as self-fulfilling and as a weak-form of REE. There is no incentive to revise the learning equations and forecast functions.

Consistent learning requires a great deal of sophistocation on the part of agents. Having said this, mention should first be made of a trivial case of consistent learning where the model is so constructed as to disallow feedback from agents' learning to the model. This applies to the models of Taylor (1975), Friedman (1979) and of Chapter 3.

Despite this, it is more plausible to consider the likely generality where there is feedback from learning. Here a great deal of sophistocation is usually required of agents. The sophistocation takes the form that agents can separately filter the contribution of the underlying model and the contribution of other agents revising their beliefs on the data observed. It is necessary that agents have knowledge of other agents' likelihood functions in order to decypher the effect of their learning on the model.

It is arguable that this state not only requires agents to act as if they were 'superior statisticians' to quote Arrow (1978), but that it also begs the informational questions. A REE containing strong

assumptions on the availability of information may be *justified* on the grounds of its stability properties, which are assessed by the convergence of learning processes. The assumption of consistency in learning, which requires a great availability of information, will surely deliver the REE. This is because the availability of information assumptions are imposed at the outset, thereby enabling agents to achieve the REE, provided that they fully exploit the information which they have and in a sufficiently sophistocated fashion.

In a multi-agent environment the required information for consistency is of other agents' likelihood functions. Consequently, the question of consistency is problematical in the case of a decentralised market of agents with heterogeneous beliefs and expectations. It requires that the individual can anticipate how the rest of the market is revising its beliefs.

The scale of the problem is demonstrated by considering the infinite regress problems discussed above. Consistency requires that individuals account for the effect of the whole market learning on the model. As the whole market is composed of the sum of individuals, each individual must take account of the whole market accounting for the effects of the whole market's learning.

In a heterogeneous information setting consistent learning may be interpreted as a particular and informationally strong form of strategy within the multiplicity of feasible learning strategies. Townsend (1978, 1983A and 1983B) proposes the notion of a Nash equilibrium in learning to ensure that individual strategies are consistent with the whole model.

In these examples, there are strong assumptions concerning the initial availability of information. Agents know the (common) Bayesian priors of all agents and they know the objective parameter values of the model. The dimension of the learning problem is restricted to solving the evolving beliefs of others from a foundation of common knowledge at the base period.

And in these examples, agents only need to learn subjective factors, they know the objective parameter values of the model. This is in contrast with the trivial cases of consistent learning cited, where agents only learn the objective parameter values, because there is no significance for market beliefs in the model. In simplified models with the assumption of continual common knowledge, consistent learning should not necessarily be problematical because agents do not have to solve simultaneously the evolving beliefs of others.

A point that agents cannot learn objective parameter values and subjective beliefs simultaneously was made by Frydman (1982 and 1983). Frydman's approach differs from Townsend's in that the only information with which agents can be endowed is that which could be acquired during the learning process. Agents have no knowledge of the likelihood functions which would enable them to filter the effects of the model from the effect of agents's learning.

As it is implicitly assumed that there is no prior knowledge of likelihood functions across the market, there is, in econometric terms, an identification problem in separating out the effect of the model from the effect of learning. And in terms of the efficient markets

literature, the signal received is noisy because of the effects cf the revision of beliefs. The information received is contaminated by noise from the rest of the market learning.

The implausibility of consistency leaves inconsistent learning as the alternative. Learning will then have unanticipated effects on the model being learned. An example of this is given by Cyert and DeGroot (1974) and by Bray and Savin (1986) where agents are learning a form of the model which is only correct inside the limit of REE, but incorrect outside during the learning process. In terms of our model agents are endeavouring to learn the form of the relative yield given equation (10), while the revision of beliefs causes the coefficient on v_{t-1} and the variance of the error term to be time variant as in equation (30).

In the following section we consider such an application of inconsistent learning. The justification for inconsistency is found by placing limitations on the sophistocation of agents or in restricting the initial information endowments in their possession. Both of these possibilities find a rationalisation in the existence of learning costs, which place bounds on the abilities of agents.

This particular application of inconsistent learning, which is by no means exhaustive, even within the set of inconsistent learning strategies, may be regarded as a particular case or as a first approximation. We also consider a learning strategy which makes use of the social interaction between agents as an alternative possibility.

4.6 Applications of Learning

Two Fundamental questions are what and how will agents endeavour to learn in the context of this model? The first question is the more easily answered because risk-neutral agents will be concerned only with the first moments of the asset yield distributions; and this concern will be limited to the relative mean yield between dcd and fcd assets in this two-asset model.

The relative mean yield under the revision of beliefs will be given by equation (30). Agents will need to learn this form in order to make expectations of its mathematical mean. It is determined in part by the exogenous process of the interest rate differential and in part by the evolution of market beliefs, which determine the exchange rate and, so, the rate of depreciation.

The second question of how agents will learn is one of great interest and is not easily resolved. In the last section, consistent and inconsistent learning were considered as two generalities. One approach is to focus on the 'rationality' of agents, meaning that they would not persist with sub-optimal learning strategies which cause inaccurate and biased forecasting. However, inconsistent learning may be the only plausible possibility, if agents have not the basic ability or the initial information to solve the effects of other agents' learning on the model, particularly where the context is of a decentralised market.

Much of the problem is due to the fact that learning is usually treated as a statistical inference problem, where agents observe a

single signal in the form of a market outcome, typically price, and must then simultaneously solve the fundamentals of the model and the (changing) beliefs of other agents. It is necessary to the success of this operation that agents know the likelihood functions of other agents.

An alternative possibility is that there are two sources from which agents can learn, so that there is limited reliance on the price-signal extraction problem. We consider a case of learning where agents learn separately the process determining the interest rate differential and market beliefs. The coefficient in the Markov process for the interest rate differential in equation (4) can be learned consistently by econometric estimation. Market beliefs can be learned by a social sampling process.

This is the first strategy to be considered. The second strategy will be an inconsistent learning example akin to the one given by Cyert and DeGroot (1974) and by Bray and Savin (1986), where agents are learning the form of the model which is only correct inside REE. These two possibilities are, of course, by no means exhaustive. They each have the merit of technical simplicity, if they lack the degree of sophistocation which one would impute to economic agents. They may be regarded, like economic models, as first approximations to more complicated processes.

4.6.1 Strategy One

The first strategy takes into account the social interaction between agents within the market as a mechanism for communicating beliefs. Learning theory, as mentioned above, is usually treated as an

application of statistical decision theory where agents are believed to respond to a price-signal extraction problem. Notwithstanding the economic interest in learning from prices, there is also some gain in considering more social forms of learning.

The latter is a rationale for the operation of organised market institutions, which have a role beyond the mere communication of prices. This is clear for the stock exchange, the commodity and the foreign currency exchanges, especially in the days before 'big bang'. Even after big bang, there is still scope for agents to communicate their beliefs, even though the computer terminal has to a great extent superceded the stock exchange floor. Agents will still have some incentive to seek information by picking up the telephone or by meeting over lunch or in the golf club, where market beliefs are relevant to price determination.

The dichotomy which we are making between price-learning and social-learning processes is useful in application to this model where the relevant variable of concern, the relative mean yield between the two assets, is determined by objective and subjective factors. It is proposed that the two learning processes can be related to the two respective factors.

We assume that the social learning may be regarded as a random sampling process from a large number of agents in order to rule out monopolistic behaviour. Each agent is small in relation to the whole market and there are no collusive groupings. Price-taking behaviour is assumed.

In each time period each agent randomly meets a small number of other agents separately. Each agent trades information separately with those agents sampled: agent i meets s agents, indexed j= .. s-1, s; and i communicates his belief to j in exchange for j communicating his belief to i. The sample of s agents may be regarded as a random drawing of market opinion, which each is endeavouring to learn.

It is assumed that there is no strategic behaviour in the sense of agents giving misinformation, so each communicates what he actually believes at the time. There is no incentive to mislead because of the atomistic nature of agents. However, as there is also no incentive to give information which is believed by the agent to be correct, we will also assume that agents are fundamentally honest.

The constraint on this form of learning is the sample size, s, which is small in relation to the total size of the market, N. If the sample size could tend towards the size of the market, the tendency would be for the agent to learn market opinion exactly. Sample size is limited by time and by access, and agents can only buy access by surrendering leisue and other forms of economic activity. We assume that all agents in the market have an equality of access, so that the sample size, s, is the same for all agents.

The exchange rate is partly determined by market beliefs, where the coefficient on the lagged interest rate differential is b Gm_{t-1} q. Two assumptions will be made concerning individual beliefs in order to formulate a learning strategy based on the social interaction between the agents. These relate to what each individual believes of the market as a whole and of other agents as individuals. We also assume that the

value of the parameter, θ, in the Markov process for the interest rate differential is common knowledge.

Assumption 1: each agent i believes that

$$b\ Gm_t\ q = bq \equiv r \qquad \text{for all } t$$

Each agent believes that the coefficient on the interest rate differential in equation (27) is a constant, because each believes that market beliefs are constant over time.

Assumption 2: each agent i believes that

$$Gj_t\ r = r + uj_t$$

$$uj_t \sim \text{NIID}\ (0, \sigma) \qquad \text{for all } j$$

Each agent believes that each individual agent's belief on (constant) market beliefs is correct in expected value but is subject to a white-noise error. The variance of the white-noise error is believed to be the same for all j agents. The white-noise errors of the j agents are believed to be uncorrelated.

The first assumption implies that agents will endeavour to learn market beliefs as a constant parameter. The second assumption indicates that each agent might endeavour to learn market beliefs by sampling the opinions of other agents as to its value.

It is assumed without loss of generality that each agent samples the opinions of s agents. The sample of market opinion of agent i at time t can be represented by

$$gi_t\ (j_t) = \frac{1}{s} \sum_{j_t \neq i}^{s} Gj_t\ r \tag{31}$$

It is determined by the random drawing of s agents, who are indexed j. The belief of each of the s agents is equally weighted by the factor, 1/s.

Agent i's belief on the distribution of the sample can be obtained from Assumption 2 and equation (31): each agent, i, believes that

$$gi_t\ (j_t)\ \sim\ \text{NIID}\ (r,\ \sigma/s) \qquad 1 \leq s < N$$

The expected value of the sample is believed to be the constant value for market beliefs. The variance is believed to be related positively to the believed variance of agent j's belief and inversely to the size of the sample. There are therefore perceived benefits in increasing sample size as well as costs.

A learning strategy is now proposed. Each agent, i, forms a belief on the the perceived constant of market opinion by revising a previously held belief in the light of new information received from the sample. Agent i's belief at time t can be represented as a weighted average of his belief at time t-1 plus the sampled beliefs of the agents, j, at time t. Generally, the weights, λi_t, $1 - \lambda i_t$, will be time-variant, so that

$$Gi_t\ r\ =\ \lambda i_t\ Gi_{t-1}\ r + (1 - \lambda i_t)\ \frac{1}{s} \sum_{j_t \neq i}^{s} Gj_t\ r \tag{32}$$

If the agent were a Bayesian practising optimal decision theory, the weights at any time would be determined by the subjective precision of his anterior belief relative to the (subjective) precision of the sampled beliefs. So if the sample size, s, is great relative to the perceived variance of an agent's belief of market beliefs around actual market beliefs, then the weight, λi_t, would be small, because the sampling process would be believed to yield precise information. The

weights will also depend on the purely subjective prior precision of the agent belief at a starting point of the learning process.

Having determined individual beliefs on the value of r, market beliefs on the value of r (but not the value of r itself) can be determined by aggregation. A market-power weighted average of individual beliefs given by equation (32) is

$$Gm_t\ r = \Sigma wi\ [\lambda i_t\ Gi_{t-1}\ r + (1 - \lambda i_t)\ \frac{1}{s} \sum_{j_t \neq i}^{s} Gj_t\ r] \qquad (33)$$

A third (set of) assumption(s) is now made to restrict the form of equation (33). Two alternatives will suffice: we assume that either w_i and λi_t are orthogonal to individual beliefs, Gi_{t-1} r and Gj_{t-1} r and that N is very large, or that the value of wi and λi_t are the same for all agents. This second possibility gives respective values for the two weighting factors of 1/s and λ_t. We call this third set the 'weighting assumptions'.

The weighting assumptions and the random nature of the sampling process will ensure that market beliefs on the value of r are restricted to

$$Gm_t\ r = Gm_{t-1}\ r = Gm\ r \qquad \text{for all } t \qquad (34)$$

Market beliefs on the value of r are constant, although individual beliefs on the value of r are generally under revision. This condition required the three assumptions that individuals believe market beliefs to be constant, that they believe other agents (who also believe the same) know market beliefs, but subject to a white noise, and the weightings are either orthogonal or the same across the market. The first two conditions ensure that market beliefs are stationary, the

addition of the third ensures that they are noiseless as well as stationary, and so constant.

The learning in this example is of market opinion. The individual does not learn the REE of the model in the usual sense, where each order of beliefs on the value of parameters coincides with their actual values. Individual beliefs may converge to market beliefs, but market beliefs will generally be outside their REE configuration.

Equation (32) shows that individual beliefs should converge over time to their market average, as the sampling process causes differences between individual beliefs to be removed. There are two factors which strengthen this process. One is the passage of time. Since the agent can expect to meet a different selection of s agents in each time period, he meets a larger section of the market as time progresses. The other is the indirect communication of beliefs between agents. If agent i's belief is influenced by agent j's and agent j's is influenced by agent k's, and so on, then agent i's belief is influenced indirectly by a whole chain of beliefs.

Consistent expectations [with equation (27)] and Assumption 1 together give individual expectations of the exchange rate as

$$Gi_{t-1}\, c_t \;=\; -\, f_{t-1} + Gi_{t-1}\, r\;\; v_{t-1} \qquad (35)$$

At time t-1 when the expectation is made, only the then contemporaneous observations for the current account and the interest rate differential are received. It is known that the two follow a random walk and a Markov process, respectively.

It has also been assumed that the value of θ is common knowledge. Individual expectations for the exchange rate at time t+1 and for the

interest rate differential at t are accordingly derived from equations (4) and (35) as

$$Gi_{t-1}\, c_{t+1} = -f_{t-1} + \theta\, Gi_{t-1}\, r\, v_{t-1} \tag{36}$$

$$Gi_{t-1}\, v_t = \theta\, v_{t-1} \tag{37}$$

Combining equations (35) - (37) give individual expectations of the mean relative yield as

$$Gi_{t-1}\, \bar{z}_{t+1} = [\theta + Gi_{t-1}\, r\, (1 - \theta)]\, v_{t-1} \tag{38}$$

Aggregation to obtain the corresponding market expectation of the mean relative yield, applying equation (34), gives

$$Gm_{t-1}\, \bar{z}_{t+1} = [\theta + Gm\, r\, (1 - \theta)]\, v_{t-1} \tag{39}$$

Market beliefs on r are a constant, unlike individual beliefs on r. Substitution of equation (39) into equation (1) gives the exchange rate as

$$c_t = -f_t - b[\theta + Gm\, r\, (1 - \theta)]\, v_{t-1} \tag{40}$$

The coefficient on the interest rate differential is constant because market beliefs on the value of r is constant, which is derived from the three assumptions considered above, and because the value of θ is common knowledge, which implies that beliefs on its value are not subject to revision. Consequently, Assumption 1 is justified. The effect of the learning on the model is consistent with agents prior beliefs. This strategy is consistent in the sense that learning has no unanticipated effect on the model.

The model is always in an equilibrium defined by the constant parameter on the interest rate differential, provided that the value of b is bounded, giving imperfect capital mobility:

$$r = b\, [\theta - Gm\, r\, (1 - \theta)]$$

This equilibrium is not a conventional macroeconomic REE, save in the special case where

$$r = Gm\ r = b\theta/[1 + b(1 - \theta)]$$

In this case, b can be unbounded, provided that the value of θ is not unity.

Furthermore, this equilibrium is generally dogged by the problem of multiplicity. There is no solution to market beliefs, which can take the value of any constant. The REE case just cited is only one possibility. Market beliefs of any level are self-fulfilling, which makes for the problem of multiplicity.

However, it should be said that this is basically different from the multiplicity problem considered in Chapter 3 which besets dynamic rational expectations models. There the issue is of the indeterminacy of the constant of integration for a model which is presented in the form of a difference equation. Here the issue is that constant market beliefs are like any constant parameter of the model. In the sense that any equilibrium solution depends on the imposed value for b or for θ, the level of market beliefs can be regarded as an imposed parameter and not as something which is endogenous to the model.

As the model is generally outside REE, save where r and market beliefs on r coincide, it is feasible to categorise the difference between its equilibrium and the corresponding REE as a bubble. The bubble will be one which declines geometrically and asymptotically to zero, provided that θ is within the unit circle - unlike the characteristic REE bubble which can be dynamically unstable.

4.6.2 Strategy Two

As an alternative strategy we consider one of the type considered by Cyert and DeGroot (1974) and by Bray and Savin (1986). Learning is inconsistent because agents specify a constant parameter form for the model, but the effect of learning, in the revision of beliefs, is to cause those parameters to vary. In this example unlike the previous one, there is no social learning, so that agents may be regarded as outsiders who do not have access to the opinions of those in the market.

An analysis of inconsistent learning is of interest because we can reflect a result of Bray and Savin (1986), that the stability of learning depends on a parameterisation favourable to the determinacy of the underlying model. However, to make the analysis more tractable for a model which contains two expectations of the endogenous variables, we apply a simpler error correction mechanism for beliefs on the relevant parameter.

Agents are again interested in learning the mean relative yield in a form given by equation (26) where it is believed that the coefficient on the interest rate differential is constant. Inconsistency results because the actual form for relative yield under learning is given by equation (30).

Agents form adaptive expectations of this parameter, where there is a constant rate of adaption, $1 - \lambda$. This represents the pace of learning, and affects the stability of the process. Market beliefs are revised according to

$$\left.\begin{array}{lll} Gm_t\ q = \lambda\ Gm_{t-1}\ q + (1 - \lambda)\ z_t/v_{t-2} & \text{where } V_{t-2} \neq 0 \\ Gm_t\ q = Gm_{t-1}\ q & \text{where } V_{t-2} = 0 \end{array}\right\} \quad (41)$$

Substituting in equation (30) for the relative yield gives

$$Gm_t\ q = \lambda\ Gm_{t-1}\ q + (1 - \lambda)\left[\frac{v_{t-1} + b(Gm_{t-1}\ q\ v_{t-1} - Gm_{t-2}\ q\ v_{t-1}) + \epsilon_t}{v_{t-2}}\right] \quad (42)$$

$$\text{where } V_{t-1} \neq 0$$

For further simplification, we assume that beliefs are revised only at every other data point, so that

$$Gm_{t-1}\ q = Gm_{t-2}\ q, \quad Gm_{t-3}\ q = Gm_{t-4}\ q, \ \ldots\ldots$$

We may write

$$Gm_t\ q = \alpha_t + \beta_t\ Gm_{t-2}\ q \quad (43)$$

where, using equation (4),

$$\alpha_t = (1 - \lambda)\left[\theta + (\omega_{t-1} + \epsilon_t)\left(\sum_{i=0}^{\infty} \omega_{t-2-i}\right)^{-1}\right]$$

$$\beta_t = \mu + (1 - \lambda)\ b\left[(\theta - 1) + \omega_{t-1}\left(\sum_{i=0}^{\infty} \omega_{t-2-i}\right)^{-1}\right] \quad (44)$$

The necessary and sufficient condition for expected stability (see Appendix 4.A.2 for its derivation) is that

$$|\ \lambda + (1 - \lambda)\ b(\theta - 1)\ | < 1$$

We can consider a number of possibilities. First, if the value of b is unbounded, representing perfect capital mobility and giving interest rate parity inside REE, it is necessary (but not sufficient) to stability, that the value of θ is unity. However, the combination of an unbounded value for b with a unit value for θ is inconsistent with the existence of a REE [See equation (9)]. The conclusion is that if the operational use of REE is to be justified only by the criterian of its

stability properties, as guaged by a process of (inconsistent) learning, then a REE equilibrium with interest rate parity for the exchange cannot be used.

Stability is the more likely, the lower the value of b, representing the degree of capital mobility. Capital mobility in this model represents the magnitude of wealth holdings and borrowings which are spent on (risk-neutral) speculation as well as to the speed of portfolio-adjustment. Therefore, there is some scope for a Tobin (1978) tax to stabilise the model by reducing commitments to speculate based on inconsistent learning.

The possibility of stability will be inversely related to the pace of learning, which is represented by the parameter, $1 - \lambda$. If λ is close to unity, the chance of stability is greater. However the significance of this parameter is weak, if there is very high capital mobility, as we would expect.

Leaving aside the limiting case of perfect capital mobility, the stability of learning requires that the value θ is close to unity or that the interest rate differential approximates a random walk. The exchange rate in the resulting REE will then approximate a random walk, which seems to be regarded as a stylized fact. The value of θ will generally approximate unity where short-run changes in the interest rate differential are being considered, as the rate of geometric decline in a variable over a very short period of time will be approximately zero.

So the seemingly counter-intuitive result emerges that learning is the more stable, if the model is based on relatively short-run

speculation. This is because the inconsistent learning process causes agents to over-react to the signal observed. In the short-run, there is less movement in the interest rate differential, which is a part of the signal, and so less over-reaction. This improves the chances of stability.

The stability condition is related to the condition for the existence of equilibrium outside REE in equation (29). The condition there was one relating to beliefs, whereas the condition here relates to actual parameter values. The similarity may be regarded as a beliefs-augmented variant of Samuelson's (1948) principle of the correspondence between the statics and dynamics of a model. It also suggests a result obtained by Bray and Savin (1986), that the parameterisation favourable to stability in the static expectations case of the cobweb model is also favourable to the stability of its rational expectations form.

Finally, both learning strategies required the parameter b to be bounded for stability. In terms of our analysis in Chapter 2, there will then be direct current account effects on the exchange rate - even within full rational expectations.

4.7 Some Further Considerations

4.7.1 The Individual Optimality of Inconsistent Learning

Agents may be better off with a process of inconsistent learning if the only alternative is no learning at all. Following a structural change agents will generally make biased forecasts and with higher mean square error than before the change. Learning, however inconsistent,

will reduce the bias and mean square error, if it is convergent and brings them closer to the new REE. Consistent learning may not be possible, while inconsistent learning may be convergent and bring the benefit of increased accuracy in forecasting. The benefits will be the greater, the faster the rate of convergence.

There will always be an incentive for the individual agent to learn because of the decentralised nature of the market. Ex ante the isolated individual makes a decision to learn which is uncoordinated with any decisions of other agents. As he is small in relation to the market, he cannot expect his decision to affect the model being learned. So *ex ante* each agent may expect his learning to be consistent with the model.

It is when a large number of isolated agents initiate learning that the joint effect of their uncoordinated actions causes inconsistency. Learning then has unanticipated effects on the model being learned. For unfavourable parameterizations their learning may become unstable as well as inconsistent. If learning is unstable then each agent may be better off in terms of greater forecasting accuracy if none is learning at all.

This raises the issue of an externality in inconsistent and unstable learning. *Ex ante* each agent perceives the benefits in improved forecasting, but once many agents operate learning which proves to be inconsistent and unstable, there may be a greater agitation in the data, leading to worse forecasts all round.

This externality is different to the kind of externality perceived by Grossman (1976) and by Grossman and Stiglitz (1976) in the context of a REE outside strong-form informational efficiency. There no one will pay the cost of collecting private information because there is the belief that others will do so, so that individuals wait for market price to convey the information which is costly to collect. Consequently, no one collects information, or very few, so that price cannot aggregate the information which would be collected. Paraphrasing, the Grossman and Stiglitz externality arises because no one is prepared to learn close to strong-form informational efficiency with non-zero learning costs. Here, the externality arises because there is a lack of coordination, and many agents are prepared to learn and are ignorant of the unfavourable parameterization which lead to instability.

We can relax the rational expectations assumption that agents, to quote Arrow (1978), are 'superior statisticians' adept at the exploitation of all available information. The ability to process information could be regarded as a limited endowment or as an aspect of human capital which is acquired at a cost. It is then possible to bring inconsistent learning within a framework of individual optimality by involving the equation of expected marginal costs with benefits. An optimum may be reached long before consistent learning where the expected gains from increased sophistication to learning do not exceed the expected costs. This draws us to a modified hypothesis of rational expectations,[3] which relates to search theory.

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[3] Snippe (1986) designates this a weak-form of the rational expectations hypothesis.

4.7.2 The Determinacy of the Inconsistent Forecast Function

Frydman (1982 and 1983) has concluded that consistent learning is not generally possible in a decentralised market and that inconsistent learning, and therefore the inconsistent forecast function, is indeterminate. Inconsistent learning is indeterminate because if agents are using inconsistent forecast functions from learning an inconsistent model of the economy, there is always an incentive for agents to employ an improved forecast function.

In terms of the model, if agents are generally forecasting according to equation (10), there is an incentive for individual agents to forecast consistently, by taking this into account. But once a sufficient number of individuals revise their learning and forecast functions accordingly, the revised form too becomes inconsistent, and there is then an incentive to revise further the learning process and forecast function. As soon as this is adopted by many agents, there is a new incentive to revise the learning process and forecast function.

A process is envisaged where forecast functions are continuously being revised and, accordingly, the model too. The revision in the forecast functions always lags behind the change in the model. In the limit the forecast function should converge to the model, and learning becomes consistent, but the limit will not be reached because of the impossibility of consistent forecasting in a decentralised market.

However, an implicit assumption in this is that there is no disincentive to revising a forecast function. In the last subsection it was argued that learning costs act as such a disincentive. Therefore, a consideration of non-zero learning costs will place a limit on the

process of revision. Learning becomes fully determinate because there is an 'equilibrium' where the expected benefits to revising a learning equation and forecast function do not exceed the costs. Non-zero learning costs overcome the Frydman 'stalemate' in learning where consistent learning is impossible and inconsistent learning is indeterminate.

4.7.3 Wealth Dynamics

Agents' beliefs reflect what they have learned from the model and these are incorporated in their forecast functions. The assumption of a uniformity in learning and forecasting across agents must be rejected, if we reject the notion of the uniformity of beliefs.

Risk-neutral agents will be strongly committed to one-way bets based on their forecasts of relative mean asset yields. Therefore with heterogeneous beliefs, some will be making strong gains and others strong losses. Wealth should be transferred from agents with inferior learning processes and forecast functions to those with superior ones. Cootner (1967) and Feiger (1978) considered wealth dynamics, as such, as a mechanism towards an efficient market. The less informed would be weeded out and the good forecasters would flourish The mechanism should hasten the evolution of increased sophistication in learning and in forecasting.

Furthermore, the size of the market will become smaller if inferior forecasters are eliminated at a faster rate than the rate of influx of new entrants. So wealth dynamics can explain how the limit of consistent learning might hypothetically be reached: by the elimination of agents until the market loses its decentralised character. Then

behaviour could cease to become competitive but collusive with information pooling.

However, although the hypothesis of wealth transfers enriches the learning analysis, we would expect certain markets to remain decentralised. Consequently we emphasise information or learning costs to explain why the limit will not be reached.

APPENDICES TO CHAPTER 4

4.A.1 The REE Solution

It is assumed that agents know the structural form of the equations for the exchange rate (11), for the interest rate differential (9) and for relative asset supplies, (7). Agents can then directly estimate the coefficients in the equations for the last two. A rational expectations reduced-form may be written in terms of observables at t-1 and an error, k_t

$$c_t = -f_{t-1} + \Pi\, v_{t-1} + C_o\, \delta^{-t} + k_t \qquad (A.1)$$

with a white noise unobservable disturbance, k_t with zero mean. Consistent expectations from equations (2), (4) and (A.1), for c_t and c_{t+1} give

$$Gm_{t-1}\, \bar{c}_t = -f_{t-1} + \Pi\, v_{t-1} + C_o\, \delta^{-t} \qquad (A.2)$$

$$Gm_{t-1}\, \bar{c}_{t+1} = -f_{t-1} + \Pi\, \theta\, v_{t-1} + C_o\, \delta^{-(t+1)} \qquad (A.3)$$

and from equation (4)

$$Gm_{t-1}\, \bar{v}_t = \theta\, v_{t-1}$$

And from equation (5):

$$c_t = -f_{t-1} - \epsilon_t + b[(\Pi(\theta - 1) - \theta)\, v_{t-1} + C_o(\delta^{-(t+1)} - \delta^{-t})] \qquad (A.4)$$

Substitution of equations (A.2) to (A.4) into (8) and then into (6) gives the solution

$$\Pi = -b\theta/[1 + b(1-\theta)]$$

$$\delta = b/[1+b]$$

$$k_t = -\epsilon_t$$

where C_o is indeterminate.

C_o can take on any value, so that there is an *infinity* of REE [see Shiller (1978)]. There is only one stable solution, where C_o is zero, otherwise the REE path explodes as b/[1+b] is less than one. We consider only the unique stationary solution by involking the existence of a terminal condition to determine the long-run equilibrium of the exchange rate. This leaves

$$c_t = a_t - b\theta/[1+b(1-\theta)]\ v_{t-1} \tag{A.5}$$

4.A.2 The Convergence Condition for Strategy Two

The white noise properties of ϵ_t and ω_t in equations (2) and (4) imply that the coefficients in equation (44) have the properties

$$E\,[\alpha_t] = (1 - \lambda)\,\theta \qquad \text{for all } t$$

$$E\,[\beta_t] = \lambda + (1 - \lambda)\,b\,(\theta - 1) \qquad \text{for all } t$$

$$E\,[\beta_{t+2}\,\beta_t] = [\lambda + (1 - \lambda)\,b\,(\theta - 1)]^2 \qquad \text{for all } t \tag{A.6}$$

From equation (43)

$$E\,[Gm_{t+2}\,q] = E\,[\alpha_{t+2} + \beta_{t+2}\,\alpha_t] + E\,[\beta_{t+2}\,\beta_t]\,Gm_t\,q \tag{A.7}$$

and

$$E\,[Gm_{t+4}\,q] = E\,[\alpha_{t+4} + \beta_{t+4}\,\alpha_{t+2} + \beta_{t+4}\,\beta_{t+2}\,\alpha_t] + E\,[\beta_{t+4}\,\beta_{t+2}\,\beta_t]\,Gm_t\,q \tag{A.8}$$

It is clear that beliefs will converge in expected value if

$$[\lambda + (1 - \lambda)\,b(\theta - 1)]^2 < 1$$

If this condition is met, from equations (A.6) and (A.7), beliefs will be expected to converge asymptotically to

$$Gm\,\tilde{q} = \frac{(1 - \lambda)\,\theta}{1 - [\lambda + (1 - \lambda)\,b\,(\theta - 1)]} = \frac{\theta}{1 + b\,(1 - \theta)}$$

This is the REE in equation (10).

CHAPTER 5

CONCLUSIONS

In the preceding chapters, a range of issues relevant to exchange rate economics have been considered. The focus throughout was on the role of information in conditioning expectations. The starting point of the analysis was the assumption that although individuals may be rational economic agents, they may not fully know the parameter values of a model, which represents the uncertain world they face. Expectations errors will then be systematic, and under a process of revision, where agents are endeavouring to learn more about the world.

In Chapter 2 an asset market model was considered and yet one where the exchange rate was determined by the total balance of payments. The standard result that (observed) current account shocks could not affect the exchange rate was obtained with full rational expectations. It was then shown that this result could be overturned, if this particular restriction was relaxed, even if the assumption of perfect capital mobility was maintained. The sign of the effect depended on whether current account shocks were over- or under-anticipated. The effect worked through misinformation conditioning the expectations which determined the capital account side of the balance of payments.

A parallel analysis could be made of foreign exchange intervention, as these flows may also enter the total balance of payments under imperfect floating. Intervention would directly affect the exchange rate, if agents incorrectly believed that the govenment was not following an interventionist policy. This applies even with perfect capital mobility (and risk-neutral speculation). Governments may therefore have an incentive to publicise a 'hands-off' policy, while actually engaging in intervention.

Another issue in this chapter was the learning of the long-run equilibrium exchange rate. Agents were able to learn the unique solution which brought about long-run current account balance, because of the total balance of payments specification of the model. The sluggishness with which agents might revise their beliefs relative to the speed of underlying changes in the model gave rise to the possibility of over-shooting in the short-run before the long-run would be reached.

In Chapter 3 the issue of uniqueness was used to discount the rational bubble hypothesis. The purpose of this chapter was to show that non-stationarities in the data could be attributed to optimising agents trying to acquire full rational expectations. This hypothesis was advanced against the assumption that agents were confined to a world with expectations which were either unstable but rational, or in disequilibrium.

Bayesian learning was considered, giving scope for subjective priors to affect the data. If agents had relative confidence in their initial beliefs, then policy changes, which could only work through altering beliefs, would have little effect on the model. This implies that the effects of a change to a policy which would be unsustainable in the long-run (perfect persistence in the interest rate differential), would be mitigated in the short-run, if agents had little prior confidence in its sustainability. Patterns of the data were related to the Bayesian precisions of agents' beliefs.

The form of the learning in this chapter was such that agents effectively acted as one. Consequently, there was no question of agents

learning about the beliefs of each other. An essentially macroeconomic paradigm was being considered. Furthermore, there were no repercussions from the learning process on to the part of the model being learned. The simplified structure of information, which was implicit, could admit consistency in learning.

In Chapter 4 some attempt was made to look at the problem of agents trying to learn the beliefs of others in a decentralised market. If the problem of the Keynesian regress in expectations is considered, the analysis becomes more difficult. As a perspective for the discussion, reference was made to a number of issues which have arisen in the non-macroeconomic literature on efficient markets.

The sheer weight of the problem gravitates against the ready acceptance of consistent learning forms, which themselves require overly strong assumptions on the initial availability of information. We refer to the learning literature surveyed by Blume, Bray and Easley (1982) and to Frydman and Phelps (1983). The requirement - that agents know the likelihood functions of other agents in order to be able to anticipate the effects of their learning on the model - cannot plausibly be reconciled with the decentralised world under consideration.

Two weak-form learning stategies were also considered in Chapter 4. One was consistent in the sense that there was no unanticipated effect from the learning process to the model being learned, although agents were generally outside full rational expectations. The merit of this example was that it described a process, which many would typify as a plausible form of bubble. It was constituted by agents individually

trying to learn the beliefs of the market. The deviation from full rational expectations declined over time.

The second strategy was the popular example where agents were trying to learn a specification of the model, which is only correct inside rational expectations. This is also to be found in the models of Cyert and DeGroot (1974) and of Bray and Savin (1986). Similarly, we concluded that the parameter conditions favourable to the stability of the process to rational expectations were related to those for the determinacy of the model outside rational expectations. The lack of perfect capital mobility was found to be a necessary condition to the stability of learning.

The implication of this result is that the combination of perfect capital mobility with full rational expectations (which ensures by construction an equilibrium for this extreme case) may not be reconcilable. This is because the learning forms, generally inconsistent, are not stable with perfect capital mobility. This, therefore, questions the operational use of rational expectations by the criterion of learning for a model with this parameterisation. Returning to the analysis of Chapter 2, the implication of imperfect capital mobility is of direct current account effects on the exchange rate even with full rational expectations.

Various theoretical issues have been considered in this thesis. The conclusions which have emerged also have implications for the empirical modelling of exchange rates. Basically, we recommend the relaxation of certain *a priori* restrictions, which properly belong to models with stronger assumptions on the availability of information.

There may be correlations between the exchange rate variables (current account and intervention flows) which are precluded by the asset market view. The strengths and even the signs of these correlations will depend on the information which is available at the time. Furthermore, patterns of time-variance in the coefficents of the model may capture the revision of beliefs within an environment of limited information.

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