Customers' Complaints and Quality Regulation

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Customers' Complaints and Quality Regulation *

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Abstract

By studying a monopoly investment decision, this paper considers the informativeness of customers complaints in contexts characterised by the absence of direct benefits and free riding incentives. Neither the consumer nor the regulator observe the firm's investment, they only observe a realisation of quality that is related to investment in a first order stochastically dominance sense. After observing quality, consumers decide whether to complain based on the difference between the realised quality and a reference point defined by their rational expectations. If a high proportion of consumers complain, the regulator punishes the firm. The paper shows that the absence of a reference point results either in no complaints in equilibrium or in the proportion of complaints being independent of the realised level of quality. The main result is that complaints are not always informative about the level of quality being delivered by the firm. Indeed, a firm might be punished despite of investment levels being high if consumers expected high quality or, on the contrary, not being punished when investing is low if consumers expected low quality. Furthermore, this lack of informativeness can be worsened by a repeated interaction between the firm and the consumers.

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1 Introduction

A variety of economic agents use customers' complaints or feedback as a source of information when making their decisions. Some online markets, for example, rely in the feedback provided by past consumers to signal the quality of a product or service to future buyers.[1] In a similar way, certain governmental agencies use complaints to obtain information about products or markets. Consumers' complaints are used by some regulators to get information about the level of unobservable/unverifiable quality delivered by regulated firms[2] or to evalu-

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[1] Empirical studies on online auctions suggest that negative feedback decreases both, the final price and the probability of sale. See Bajari and Hortacsu (2004), Resnick, Zeckhauser, Swanson, and Lockwood (2006), Houser and Wooders (2006), Menlik and Alm (2002), Waterson and Doyle (2012), among others.

[2] For example, the Argentinean natural gas regulator relies in customers' complaints to regulate the quality of customers' services delivered by the regulated natural gas distributor monopoly.

ate the functioning of markets.[3] In all these cases it is implicitly assumed that the better the market performs the smaller the number of customers that complain. However, the theoretical foundations for that assumption are not clear. Consumers' incentives to complain may vary from market to market. In particular, if the consumer does not have a direct benefit out of his complaint or the benefits of complaining can not be individualised, incentives to lodge a complain or complete a review are not obvious.[4] In this case, it is not clear how informative complaints are about the firm's behaviour.[5]

By studying a monopoly investment decision, this paper considers the informativeness of customers complaints in contexts characterised by the absence of direct benefits and free riding incentives. It considers a situation in which some aspects of quality are relatively well-perceived by the consumers, but it is very costly for the regulator to observe them and so he relies in the police power of consumers to infer the true quality. As in the examples above, I look at contexts in which the consumer does not receive a direct benefit from complaining. Therefore, I assume they complain because they feel "disappointed" with the level of quality they received and consider the firm should be punished for its "poor performance".[6] Consumers' disappointment is determined by comparing the realised quality with some prior expectation.[7] The presence of this "reference point" in consumers' complaining decision has an empirical counterpart in Forbes (2008), who studies the effects of service quality and expectations on customer complaints in the context of an airline industry, where customers can observe information on past quality. She finds that the number of complaints decreases with actual quality and that, after controlling for actual quality, consumers complain more often when they would have expected to receive higher quality.

The main result of the paper is that consumers' complaints are not always informative about the level of quality being delivered by the firm and that this lack of informativeness can be worsened by a repeated interaction between the firm and the consumers. The model prescribes some comparative statics that coincide with the empirical findings in Forbes (2008) -namely, that an increase in the realised quality decreases the (expected) proportion of complaints only when the higher quality was not anticipated by the consumers. It further delivers comparative statics results on consumers' optimal complaining decision as a function of other parameters such as the cost of making a complain and the regulatory rule. The paper shows

[3]The series of studies "Monitoring Consumers Market in the European Union" contains information about the functioning of 51 consumers markets in the European Union. The 2011 report states that the complaints measurement is considered "a key metric to evaluate the functioning of a market" (page 12). This statement relies on the assumption that complaints reflect the severity of the problems consumers face in a market, and so complaints are one of the elements the study takes into consideration in order to derive conclusions about the market's performance.

[4]For example, empirical studies on eBay show that most of the times the customer is not likely to buy again from the same seller, implying that he does not receive a direct benefit out of the review. Yet, Resnick and Zeckhauser (2002) report that 52.1% of the buyers on eBay actually provide voluntary feedback about their sellers.

[5]Standard microeconomic theory is enough to explain complaints or feedback when there are direct benefits (like monetary compensations because of electricity shortcuts or reimbursements of incorrectly high bills, for example), but it may not be helpful to understand consumers' incentives when those benefits do not exist.

[6]As it will become clear in Section 2.2, the combination of free riding incentives and a lack of direct benefits imply that without this assumption there would be no complaints in equilibrium. Furthermore, this result also holds if the game is repeated $T > 1$ periods, as simple regulatory rules as the one considered here induce a myopic behaviour in the consumers (see Section 4).

[7]The idea that complaints are related with consumers' intention to punish the firm is related with the literature on psychological games (Geanakoplos, Pearce, and Stacchetti (1989) and Battigalli and Dufwenberg (2009)). In a different context, Akerlof (2010) shows how norms may be followed because a failure to do so provokes anger and (potentially) punishment.

that the absence of a reference point results either in no complaints in equilibrium or in the proportion of complaints being independent of the realised level of quality. An important consequence of the above results is that when consumers evaluate quality relative to their rational expectations, a policy based on customers' complaints may result in the firm being punished in spite of investment levels being high or, on the contrary, the firm not being punished when it is not investing.

I consider a finite horizon model. In every period a monopoly decides whether to make a costly investment that increases quality in a first order stochastic dominance sense and the consumers decide, after observing a realisation of quality, whether to complain about the firm's "poor performance". A central aspect of the model is the way in which the regulatory agency uses complaints to infer something about the realised quality. Even though there are potentially many ways in which the regulator could extract information from consumers' complaints, I consider a very simple yet widely used regulatory rule. I assume that if the number of complaints is higher than some threshold the regulator punishes the firm because of its "poor performance". The punishment consists of a fine proportional to the firm's revenues, with a probability that depends on the level of complaints.[8] Apart from having very low informational requirements, this regulatory rule has the advantage of making very clear what it means by moving the "police power" from the regulator to the consumers.

The firm can be "good" (G) or "bad" (B). While the bad type always delivers relatively low quality, the good one can, at a cost, deliver high quality. The firm has private information about its type and chosen action. The firm's total costs equal the investment cost plus a fine that depends on the proportion of customers that complained. The cost of providing quality is independent from any other cost faced by the company and is public information. Consumers demand one unit of the firm's good per period and so the firm's revenues are deterministic, independent of the its investment decisions, and equal to that period's price. These assumptions aim to isolate the effect of customers' complaints in the firm's investment decision and imply that, without the presence of the fine, the firm has no incentives to invest in quality.

After observing the realised quality, every consumer decides wether to complain by comparing the quality realisation with his reference point. Complaining is costly and these costs are considered to be heterogeneous across agents. The greater the difference between expected and realised quality, the higher is the cost the consumer is willing to face in order to have the firm "punished". I assume that the level of quality consumers expect to receive in any period is determined by their lagged rational expectations. This means that disappointment and poor performance are endogenously defined and depend on the context. However, the existence of free riding incentives means that disappointment is not sufficient for consumers to complain in equilibrium: every consumer would prefer the firm to be punished (if he feels disappointed), but he would also prefer others to face the cost.[9] Therefore, as proposed by Harsanyi (1980), and formalised by Feddersen and Sandroni (2006a, 2006b), I assume

[8] Footnote 21 shows that this is not a strong assumption because, under very mild conditions about consumers' preferences, the setting is equivalent to one in which the regulator punishes the firm with probability one if the number complaints is above the threshold. Alternatively, it could be the case that the agency is not allowed to punish the firm unless it has "hard evidence" that it has performed poorly, and that complaints constitute that evidence; if the punishment is an increasing function of the proportion of complaints the problem is analogous to the one stated in the text.

[9] In this way, the complaining decision has some similarities with a more standard game in which players decide whether to contribute to the provision of a public good. In the public good case, free riding incentives arise because each player benefits when the good is provided, but each would prefer the other players to

that consumers are "group utilitarians" and so they are willing to follow a social rule that maximises consumers' expected utility if followed by everyone else.[10] I assume that, given their disappointment, consumers have preferences about the probability with which the firm should be punished and the cost of complaining. These preferences are not identical across consumers because complaining costs are heterogeneous. However, fixing the probability of the firm being punished, all consumers prefer to minimise the cost of complaining. Then, every consumer independently decides that the optimal rule is the one that maximises the group's expected utility, taken as given the behaviour of every other consumer. This rule consists of a cut off cost of complaining below which a consumer lodges a complain.[11]

The payoff functions of both, the consumers and the firm, depend not only on what they do but also on what consumers were *expecting* from the firm, and this should be considered by the equilibrium concept.[12] Therefore, any equilibrium of the complaining game satisfies three requirements.[13] First, the firm choses the investment level that maximises its expected profits given its beliefs about consumers' cut off rule and expected quality. Second, consumers choose the social rule optimally given their disappointment with the quality they received. And third, the firm correctly anticipates consumers' expected quality, which is in turn consistent with the firm's strategy and the consumers' prior beliefs. Using this definition, the static version of model may have two different equilibria: a "high quality equilibrium" in which consumers expect the firm to invest and the firm optimally invests, and a "low quality equilibrium" in which consumers do not expect the firm to invest and the firm optimally fulfils those expectations.

By studying those equilibria the model derives some conclusions regarding the degree in which customers' complaints are informative about the firm's investment decision. In a static setting it shows that complaints are informative about the equilibrium being played only if the cost of complaining is neither too high nor too low, and the probability consumers assign to the firm being good is not very high. Complaints are not informative when complaining is very "cheap" because consumers maximise the probability of having the firm fined without the restriction of the cost. As a result, the expected proportion of complaints becomes independent of the realised quality and so they contain no information about the firm's behaviour. A similar result holds when complaining is very costly: consumers are not willing to face such a high cost even if they are very disappointed and so there is no information transmission. Finally, when the probability they assign to the firm being of the good type

incur the costs of supplying it. It is worth noting that the free riding problem arises whenever the benefits of complaining can not be individualised and so it is independent of the existence of a reference point. It would also arise if consumers complain in order to affect the firm's future behaviour. If future investment increases with current complaints but all the consumers receive the resulting higher quality, independently of whether they made a complaint or not, then they would prefer other players to face the cost of complaining.

[10]The notion of *utilitarian agents* has been proposed to explain the so-called "paradox of no-voting": if voting is costly then, since the likelihood of a vote being pivotal is very small, standard game-theoretic models predict low levels of turnout (Downs 1957). The free riding problem in the voting context is analogous to the one in the complaining game in that in both contexts the agent takes a costly action with no apparent benefit.

[11]In the context of a voting game, Feddersen and Sandroni (2006a) show that a behaviour rule profile that defines rules such that each agent decides he must follow given a proper anticipation of the behaviour of other agents can be described by cutoff points. Their result extends to the application in this paper.

[12]The players' payoffs at the end of the period are endogenous: consumers' disappointment with the received quality determines their willingness to complaint and so their payoff and the firm's expected profits. That disappointment, in turns, depends on the beliefs consumers hold in advance about the firm's type and strategy. Thus, a given level of investment may lead to different final payoffs for different pre-play beliefs -i.e., the same distribution of quality might lead to different expected payoffs for different levels of consumers expectations.

[13]Geanakoplos, Pearce, and Stacchetti (1989)

is very high, the increase in the expected quality when consumers expect the firm to invest is higher (on expectations) than the increase in the realised quality and so the (expected) proportion of complaints is higher in an equilibrium with investment than in an equilibrium without investment. The results also confirms the idea that only if the punishment is strong enough the firm has the incentives to make a costly investment to increase the average quality. However, in the context of this paper, a "strong enough punishment" is not only related with the size of the fine but also with how easy it is for the consumers to make a complaint. Because the model assumes that the regulator relies on consumer's complaints to get information about realisation of quality, if consumers have no incentives to complain, then the firm has no incentives to invest.

When the game is played repeatedly, consumers could complaint in order to affect the firm's future behaviour. It is shown, however, that when the regulator uses a regulatory rule as the one considered in this paper, consumers' optimal behaviour in the repeated game is the myopic best response to the quality they received, given they prior expectations. This myopic behaviour creates the conditions for the firm to try to "keep expectations low" making complaints less informative in the repeated game than in the one shot game.

The model in this paper contributes to the extensive literature on quality provision by a monopoly firm. Starting by the seminal papers of Spence (1975) and Shesinski (1982), the literature suggests that an unregulated monopoly will over or under supply quality according to whether the marginal consumer values additional quality more or less highly than do the infra-marginal consumers on average.[14] It has further been shown that regulation of service prices can compound, ameliorate or otherwise complicate the already existing market failure (see for example, Spence (1975), Mussa and Rosen (1978) and D. Besanko and White (1987); Sappington (2005) surveys the literature).[15] The main conclusion of much of the existing literature is that when quality is not verifiable the regulator needs to have a great deal of information before even know in which direction he should intervene. By studying the informational content of complaints, this paper considers whether the "police power" could be moved from the regulatory agency to the consumers.

This paper is also related to the literature on reference dependence utility[16] and with some models in marketing research on customers satisfaction.[17] In both cases, it is suggested that consumers utility depends not only on the actual product quality that was received but also on whether that quality was above or below some reference level. This paper adds to the first branch of literature because, in spite of being widely accepted, the effect of that reference point on consumers' complaining decision and on the firm's incentives to invest have not been studied yet. It differs from the second branch in that they do not require consumers'

[14]The difference depends on whether quantity and quality are seen as substitutes or as complements by consumers. In the former case, consumers willingness to pay a higher price for an increase in quality decreases with the quantity that he buys (i.e., the demand curve becomes less elastic as quality increases), while in the latter the elasticity of the demand increases with quality. Thus, the monopoly is more likely to undersupply quality if quality and quantity are substitutes, and to oversupply it if they are complements.

[15]Price ceilings that are independent of the firm's realised costs limits its incentives to supply quality, because they prevent the firm from capturing any of the incremental consumer's surpluses that would result from the higher service quality. However, as noted by Laffont and Tirole (1993), even under pure cost-of-service regulation, the regulated firm does not gain from providing costly services either, so a low perceived cost of supplying quality does not imply a high incentive to provide quality.

[16]Kahneman and Tversky (2001), Koszegi and Rabin (2006), among others.

[17]See for example, Singh (1988), V. Zithaml and Parasuraman (1996), Boulding W. and V. (1993), Oliver (1977), Oliver (1980).

expectations to be rational and, as a result, they are not able to make clear predictions about the firm's strategic response to consumers' complaints.

Apart from the application in this paper and the voting literature, Harsanyi's (1980) type arguments have also been used to explain household's response to conservation appeals during the California's energy crisis in 2000 and 2001. Reiss and White (2008)) find empirical evidence that consumers do respond to voluntary appeals provided the costs of a collective action failure are tangible and that the public is well aware of it. In this case, each household faces private costs of reducing consumption, a virtually zero possibility of bringing about any tangible benefit with respect to the crisis through individual effort, and a considerable incentive to free-ride on whatever efforts are made by others. The nature of individuals' free-rider problem here and the lack of private incentives for electricity conservation leave largely "moral suasion"-type arguments to explain their behaviour: consumers individually wanting to"do their part" to mitigate the crisis.

The rest of the paper is organised as follows. Section 2 presents the details of the model and describes the players action spaces and payoff functions. It also discusses how consumers' prior expectations about the level of quality they may receive in any given period are formed. I then analyse the implications of complaints in the firm's investment decision in a simple one shot game. This is a useful exercise because it highlights most of the strategic considerations that will shape the equilibrium when the game is repeated a larger (but finite) number of periods. Therefore, apart from the definition and existence of the equilibrium in the one shot game, section 3 analyses how the equilibrium proportion of complaints is affected by changes in the various parameters of the model and how informative is that proportion about the firm's investment decision. Section 4 extends the model to the case in which $T > 1$. Finally, section 5 concludes.

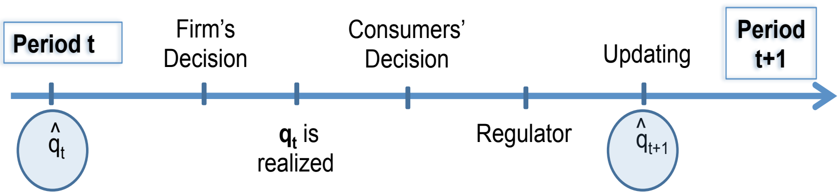
2 The Model

This section develops a dynamic model of quality regulation based on customers' complaints. In each period the firm decides whether to invest in quality and consumers decide, after observing a quality realisation, whether to complaint about the firm's "poor performance". The firm can be of any of two types, a "bad" (B) type that always delivers relatively low quality, and a "good" (G) type that is able to make a costly investment that increases expected quality. Nature selects the firm's type once and forever at the beginning of the game. Neither consumers nor the regulator observe it, but they assign a prior probability τ to the firm being good. The sequence of events within any period t is as follows. At the beginning of the period consumers form an expectation about the level of quality they may receive $(\hat{z}_t)$. This expected level acts as a "reference point" against which they evaluate actual quality. The firm makes its unobserved investment decision and quality is realised; quality is random but it is related with investment in a first order stochastic dominance sense (higher investment increases the probability of higher quality draws). Both, the consumers and the firm observe the realisation of quality and then consumers decide whether to complain and how strongly.[18] Finally, consumers update their beliefs about the level of quality they may receive in the next period. It is assumed that every consumer receives the same draw of quality.[19] The timing is showed in Figure 1.

[18] Quality is not verifiable, so that contracts contingent on outcomes cannot be written.

[19] In this way, $\hat{z}_t$ is the same for every consumer.

Figure 1: Timing



A singular feature of the model is related with consumers' complaining decision. It is assumed that consumers complain because they are "disappointed" with the level of quality they received and consider the firm should be punished for its "poor performance". Consumers' disappointment is defined as the difference between the expected level of quality and the actual one. The level of quality consumers expect to receive in any period is determined by their beliefs about the firm's type and investment strategy.

Also central to the model is the way in which the regulatory agency uses complaints.[20] It is assumed that if the proportion of complaints is higher than some threshold $\bar{\delta}$ the regulator punishes the firm because of its "poor performance". The punishment consists of a fine equal to m times the firm's revenues, with a probability that is proportional to the level of complaints. A regulatory rule is then defined as a pair $(\bar{\delta}, m) \in [0,1]^2$ of parameters that are public information.[21]

The remainder of this section describes the players payoffs and beliefs. I first describe the firm's expected payoff (section 2.1) and then the utility function of consumers (section 2.2). Finally, section 2.3 explains how consumers' expectations are formed.

2.1 The Firm

The firm is a regulated monopoly. The regulator sets a binding price cap, which does not depend on the realised level of quality. Consumers have an inelastic demand of one unit of the firm's product in every period.[22] In this way, the firm's revenues in period t are deterministic, independent of its investment decision and equal to that period's price (p_t). This means that, without the presence of the fine the firm has no incentives to invest in quality. The cost of providing quality is independent from any other cost faced by the company and is public information.[23] The firm's total costs equal the cost of investing in quality plus the expected fine. While the first one is a fixed cost $h > 0$, the second one depends on the realisation of quality, which is a stochastic function of the firm's type and investment.

[20]The regulatory agency is not an strategic player, it only applies a rule that determines the fine the firm must pay on the basis of the proportion of consumers which lodged a complaint.

[21]A different setting in which the regulator punishes the firm with probability one if the proportion of complaints is higher than $\bar{\delta}$ would generate the same qualitative results if it is also assumed that the proportion of utilitarian consumers is stochastic and the distribution from which that proportion is drawn is public information.

[22]This assumption eliminates the possibility that the firm wants to invest in quality in order to increase future demand. For a model in which investment in quality aims to affect future demand see Shapiro (1982), for example.

[23]Results would not change if we assume that the regulator cannot observe the firm's costs.

The firm is risk neutral and seeks to maximise expected profits. In every period t, the good firm's investment is a zero-one decision $I_t \in \{0,1\}$. Suppose that $I \in \{0,1\}^T$ is a sequence of investment decisions made by the good type of the firm through periods 0 to T (the bad firm has an empty action space)[24] and that $\sigma \in [0,1]^T$ is a sequence of proportions of complaints observed along the same T periods. The good firm's profits at the end of the game are then:

$$\Pi(I,\sigma) = \Sigma_{t=0}^{T} \beta^t \Pi_t(I_t, \sigma_t)$$

where each element of this sum is the firm's profits in a given period:

$$\Pi_t(I_t, \sigma_t) = p_t - \mathbf{1}(I_t = 1) \cdot h - m p_t \sigma_t \tag{1}$$

$\mathbf{1}(I_t = 1)$ is an indicator function which takes the value of one if the firm invests in period t, and zero otherwise, and $m p_t \sigma_t$ is the fine payed by the firm in peirod t. In order to keep the model as tractable as possible, it is assumed that quality is uniformly distributed over $[0, 1/2]$ if the firm is bad or if it is of the good type but it does not invest, and $q_t \sim U[0,1]$ if the good firm invests -i.e., investment increases quality in a first order stochastic dominance sense.[25]

2.2 Consumers' Expected Utility

There is a large number of consumers approximated by a continuum that is normalised to size one. Each consumer's utility in period t is the sum of the "consumption utility" he derives from the realised quality (q_t) and, if he makes a complaint, his payoff from complaining.

It is assumed that complaining is costly. The cost of complaining refers, for example, to the costs and time incurred in going to the regulatory agency to lodge a complaint. These costs are heterogeneous across agents: each consumer i faces a cost of complaint $\sigma_i c$, where σ_i is the realisation of a random variable uniformly distributed over $[0,1]$, and c is a positive constant. σ_i is independent of any other random variable in the model. Consumers do not observe the cost of other consumers, but do know the distribution from which they are drawn.

The model introduces reference dependence in the consumer's complaining decision by assuming that his willingness to complain is positively related with the degree in which he feels "disappointed" with the firm. Disappointment is defined as the difference between the expected level of quality ($\hat{z}_t$) and the level actually received (q_t). Consumers expectations depend on what they believe about the firm's type and strategy, and so the reference point is endogenous and evolves according to those beliefs. In this way, consumers' willingness to complain does not depend only on what the firm does, but also on what consumers *were expecting* it to do.[26] I consider $\hat{z}$ as given along this section; in sub-section 2.3 I discuss how

[24] Alternatively, it could be assumed that investment costs are so high for the bad type that it never invests.

[25] The assumption of uniform distributions allows closed form solution for all the results. However, the qualitative results would not change if quality were distributed according to a more general cumulative function $F(\cdot)$, as long as $F(q; I = 1) \leq F(q; I = 0)\ \forall\ q$ (with strict inequality for some q).

[26] The fact that consumers' utility function (and hence their complaining decision and the firm's expected profits) depends on the outcomes but also on the beliefs means that the complaining game belongs to the class of psychological games. Psychological games differ from standard games in that the domain of the utility function includes explicitly the beliefs a player holds about the other players' strategies. As a result, payoffs at a given endnode are endogenous: beliefs determine the player's utility and they are explained/predicted

consumers' expectations are formed.

In any period t, the action space of an individual consumer is {Complain, Don't Complain}, and his utility depends on his type and chosen action, but also on the realised quality and its difference with the reference point ($\hat{z}_t$). Denote by $\mathbb{C} \in \{0,1\}$ consumers' action ($\mathbb{C} = 1$ meaning the consumer makes a complaint). The utility of an individual consumer i with cost σ_i, who was expecting $\hat{z}_t$ and received q_t is:

$$U_{i,t}(\mathbb{C}_t; q_t, \hat{z}_t, \sigma_i) = q_t + \theta(\hat{z}_t - q_t)\mathbf{1}(F) - \mathbb{C}_t c\sigma_i \tag{2}$$

The first term of the utility function is the consumption utility the consumer derives from quality, while the second and third terms are his payoff from complaining. $\theta \in (0,1)$ can be interpreted as the consumer's marginal utility from complaining and the indicator function $\mathbf{1}(F)$ takes the value 1 if the firm is fined and zero otherwise.[27] The utility function in (2) reflects the assumption that the consumer complains in order to "punish the firm's poor performance": consumer i considers the firm performed poorly if $\hat{z}_t > q_t$, and the higher the difference, the higher is his willingness to lodge a complaint. The consumer's payoff from complaining is increasing in his disappointment if the firm is punished for its poor performance ($\mathbf{1}(F) = 1$) but it is zero otherwise, meaning that consumers derive utility from complaining only when the firm's poor performance is punished. Implicit in the utility function is the additional assumption that consumers heterogeneity is restricted to individual costs of complaining (σ_i); this means that all the consumers have the same willingness to complaint and the same intensity of preferences over quality. The assumption is relevant in that it sidesteps the question of how the burden of complaining should be shared among consumers with different intensities of preferences.

The utility function highlights the agent's free riding incentives: every consumer receives a payoff $\theta(\hat{z}_t - q_t)$ if the firm is punished, independently of whether he made a complain or not, but only those agents who actually made a complaint ($\mathbb{C} = 1$) face the costs. Furthermore, the effect of any single complaint in the probability of fine becomes negligible as the number of consumers becomes large, accentuating the free riding incentives.

The free riding problem that arises in consumers' complaining decision is analogous to the one that originates the so-called "paradox of no-voting": if voting is costly then, since the likelihood of a vote being pivotal is very small, standard game-theoretic models predict low levels of turnout (Downs 1957). Feddersen and Sandroni (2006a, 2006b) and Coate and Conlin (2004) propose a rational choice model of voting in large elections with cost to vote which predicts positive levels of turnout and is consistent with strategic behaviour. The model, based on Harsanyi's (1980) notion of utilitarian agents, assumes that individuals follow the voting rule that would maximise aggregate utility if everybody followed it. The intuition is that, if a consumer believes that all the other utilitarian agents will use the same strategy as he does himself, he will independently decide that the right strategy is the one that maximises aggregate utility. In this way, a consumer will be willing to face the cost of complaining even though he understands that his single complaint has no effect on the final outcome. In solving the free riding problem in the complaining stage of the game, I borrow from that literature and assume that consumers are *"group - utilitarians"* -i.e., they follow

via some solution concept (Battigalli and Dufwenberg (2009), Geanakoplos, Pearce, and Stacchetti (1989)). The standard assumption is that beliefs are correct in equilibrium, and that is the condition I impose in the equilibrium definitions of sections 3 and 4.

[27] $\theta < 1$ implies that, everything else constant, individual utility is an increasing function of quality.

the rule that, if followed by all the other consumers, maximises consumers' aggregate utility.[28]

Let a *social rule* σ' be a cut off that specifies a critical cost level below which a consumer makes a complaint.[29] The utilitarian assumption implies that the group's expected utility from such a rule, when the realised quality was q_t and they were expecting $\hat{z}_t$, is:[30]

$$\mathbb{E}U(\sigma'; q_t, \hat{z}_t) = \begin{cases} q_t + \theta(\hat{z}_t - q_t)\sigma' - \frac{c}{2}\sigma'^2 & \text{if } \sigma' \geq \bar{\delta} \\ q_t - \frac{c}{2}\sigma'^2 & \text{if } \sigma' < \bar{\delta} \end{cases} \tag{3}$$

The expected utility is discontinuous at the minimum level of complaints required by the regulator ($\bar{\delta}$). This is because the probability of the firm being fined is positively related to the quantity of complaints (σ') as long as it reaches (or exceeds) $\bar{\delta}$, but becomes zero if the amount of complaints is slightly bellow that threshold.

2.3 Consumers' Beliefs and Expectations

The level of quality consumers expect to receive is determined by their beliefs about the type and strategy of the firm, and the equilibrium condition requires those beliefs to be correct in equilibrium.[31] Denote by $\bar{q}_B = \int_B q \cdot dq$ the average quality that is delivered by the bad type of the firm, and by $\bar{q}_G(I_t)$ the average quality that would be delivered in period t by the good firm if it invests I_t:

$$\bar{q}_G(I_t) = \begin{cases} \bar{q}_{G,H} & \text{if } I_t = 1 \\ \bar{q}_{G,L} & \text{if } I_t = 0 \end{cases}$$

Denote by τ_t the probability consumers assign to the firm being good at the beginning of period t given the history of the game up to that moment. Then, the level of quality consumers expect to receive in that period can be written as:[32]

$$\hat{z}(\tau_t) = \begin{cases} \tau_t \bar{q}_{G,H} + (1-\tau_t)\bar{q}_B & \text{if consumers believe } I_t = 1 \\ \tau_t \bar{q}_{G,L} + (1-\tau_t)\bar{q}_B & \text{if consumers believe } I_t = 0 \end{cases}$$

[28]The qualitative results would not change if only a proportion $\gamma \in (0,1)$ of consumers were group utilitarians, as long as either γ or the distribution from which it is drawn, is public information.

[29]Given a social rule σ', a (utilitarian) consumer's action in period t can be written as:

$$\mathbb{C}_t(\sigma_i, \sigma') = \begin{cases} 1 & \text{if } \sigma_i < \sigma'(q_t; \hat{z}_t) \\ 0 & \text{otherwise} \end{cases}$$

[30]The probability that an agent makes a complaint is $Prob(\sigma_i \leq \sigma') = \sigma'$. The expected cost of complaining, conditional on the consumer effectively making a complaint is $E(\sigma_i | \sigma_i \leq \sigma') = (1/\sigma') \int_0^{\sigma'} x dx = \sigma'/2$.

[31]This assumption rules out beliefs structures in which, for example, the consumer reduces his prior expectations so that he does not feel disappointed if the realisation of quality is low. For models of belief-dependent preferences in which the agents can choose beliefs see Akerlof and Dickens (1982) or Brunnermeier and Parker (2005), for example.

[32]Consumers' beliefs about the firm's strategy are they "first order beliefs", defined as a probability distribution over the firm's action space (Battigalli and Dufwenberg 2009). We are considering only pure strategies, so consumers' first order beliefs assign probability one or zero to the event in which the good firm invests.

3 One Shot Game

This section presents a one shot version of the complaining game. Looking at this simpler version of the model is useful in that it makes clear the impact of consumers' disappointment on the firm's investment decision and so, it highlights the strategic considerations that will shape the equilibrium in the repeated version of the game.

When the game between the firm and the consumers is played only once, there is no updating of beliefs. Consumers' complaining decision is made after they observed quality. Given his disappointment with the quality delivered by the firm, each consumer's problem is to choose the cut off rule σ^* that maximises (3) if followed by every other consumer. In this way, the utilitarian assumption implies that the group's problem is strategically equivalent to a one person decision problem with payoff function (3) and strategy consisting of a mapping from their disappointment $(\hat{z}-q)$ into a cutoff point between zero and one: $\sigma(q;\hat{z}) : [0,1]^2 \to [0,1]$.

The good firm's strategy in the one shot game is an investment level $I \in \{0,1\}$. The firm does not observe consumers' disappointment but it does have some beliefs about the level of quality consumers' expect to receive, $\hat{\hat{z}}$.[33] Given those beliefs, the good firm's expected payoff is the expectation of (1) with respect to the probability measure induced by its investment strategy:

$$\mathbb{E}_{G,I}(\Pi;\hat{\hat{z}},\sigma^*) = p - \mathbf{1}(I=1)\cdot h - mp\mathbb{E}_{G,I}(\sigma(q;\hat{\hat{z}})) \tag{4}$$

Analogously, the expected profits of the bad type of the firm are:

$$\mathbb{E}_B(\Pi;\hat{\hat{z}},\sigma^*) = p - mp\mathbb{E}_B(\sigma^*(q;\hat{\hat{z}})) \tag{5}$$

The firm and the consumers choose their actions according to pre-play beliefs without observing each other's action, and so the game is equivalent to a strategic game with asymmetric information. The equilibrium concept I use is, therefore, Bayesian Nash Equilibrium. In this application, such an equilibrium needs to satisfy three requirements.[34] First, the firm choses the investment level that maximises its expected profits given its beliefs about consumers' cut off rule and expected quality. Second, consumers choose the complaining rule optimally given their disappointment with the quality they received (i.e., given $\hat{z}$ and q). And third, the firm correctly anticipates consumers' expected quality, which is in turn consistent with the firm's strategy and consumers' prior about its type, τ.[35] The firm investment decision I^* maximises its expected profits if and only if $\mathbb{E}_{G,I^*}(\Pi;\hat{\hat{z}},\sigma^*) \geq \mathbb{E}_{G,I}(\Pi;\hat{\hat{z}},\sigma^*)$, $I \in \{0,1\}$; consumers' optimal cut off point maximises the group's expected utility: $\sigma^* \in \arg\max \mathbb{E}U(\sigma;q,\hat{z})$. Definition 1 formalises those requirements

Definition 1. Equilibrium in the Static Game. *An equilibrium of the complaining game*

[33] Consumers' complaining decision depends on their prior expectations, so the firm needs to form some beliefs about them in order to decide the level of investment that maximises its profits; $\hat{\hat{z}}_t$ denotes the firm's belief about $\hat{z}_t$.

[34] Geanakoplos, Pearce, and Stacchetti (1989)

[35] The game differs from standard bayesian games in that payoffs at the end of the period are endogenous: consumers' disappointment with the received quality determines their willingness to complaint and so their payoff and the firm's expected profits. That disappointment, in turns, depends on the beliefs consumers hold in advance about the firm's type and strategy. Hence, a given level of investment may lead to different final payoffs for different pre-play beliefs -i.e., the same distribution of quality might lead to different expected payoffs for different levels of consumers expectations.

when $T = 1$ is a pair of strategies (I^*, σ^*), such that there exist beliefs $(\hat{\hat{z}}, \hat{z})$ for which the following conditions are satisfied:

1. *I^* maximises the firm's expected profits given $\hat{\hat{z}}$ and σ^**
2. *σ^* maximizes consumers' expected utility given $\hat{z}$*
3. *$\hat{\hat{z}} = \hat{z} = \tau \bar{q}_H(I^*) + (1-\tau)\bar{q}_B$.*

The game is solved backwards in sections 3.1, 3.2 and 3.3. The first section describes consumers' optimal behaviour given the realisation of quality they observed and their beliefs about the firm's strategy, the second one defines the firm's optimal strategy and the latter presents the equilibrium. Finally, subsections 3.4 and 3.5 discuss the type of information contained in the equilibrium expected proportion of complaints and present some comparative statics about consumers' optimal complaining behaviour.

3.1 Consumers' Equilibrium Strategy

The cut off rule that maximises consumers' expected utility, given a realisation of quality and consumers' expectations, is:[36] [37]

$$\sigma^*(q;\hat{z}) = \begin{cases} 1 & \text{if } q \leq \hat{z} - \frac{c}{\theta} \\ \frac{\theta(\hat{z}-q)}{c} & \text{if } \hat{z} - \frac{c}{\theta} < q \leq \hat{z} - \frac{\bar{\delta}c}{\theta} \\ \bar{\delta} & \text{if } \hat{z} - \frac{\bar{\delta}c}{\theta} < q \leq \hat{z} - \frac{\bar{\delta}c}{2\theta} \\ 0 & \text{Otherwise} \end{cases} \tag{6}$$

The cut off rule reflects the assumption that consumers' complaining decision is driven by their disappointment with the quality they received: given $\hat{z}$, the smaller is q the higher is consumers' disappointment and so is the cost they are willing to face in order to have the firm punished. The relationship between $\sigma^*(\cdot)$ and q is shown in Figure 2. When consumers are very disappointed with the firm, they maximise the probability of punishment by setting $\sigma^* = 1$, but as the realised quality increases the optimal cut off rule decreases. Eventually, quality reaches a level in which consumers disappointment is small enough so as to set the social rule at the minimum level for which the probability of fine is positive (i.e., $\sigma^*(q;\hat{z}) = \bar{\delta}$).[38] [39]

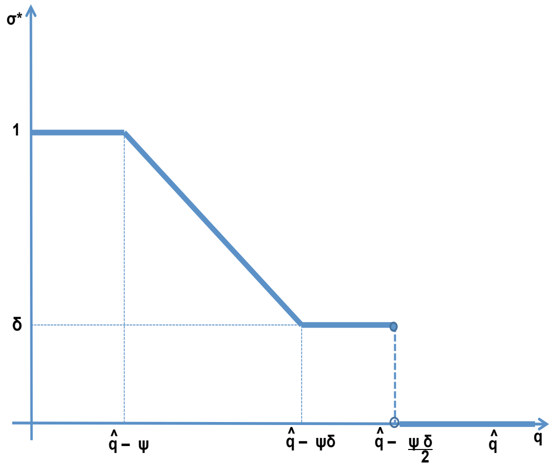
[36] $\sigma^*(q;\hat{z})$ could in principle take any value in the interval $[0,1]$; however, it is clear from (3) that values of σ^* different from zero but smaller than $\bar{\delta}$ cannot be optimal. Consumers' optimisation problem can then be written as: $\text{Max}_\sigma\{\mathbb{E}U(q,0); \text{Max}_{\sigma\in[\bar{\delta},1]}\, \mathbb{E}U(q,\sigma(q;\hat{z}))\}$.

[37] It is worth noting that, even though the utilitarian consumers story sounds appealing, there is nothing in the mathematical formulation of the problem that differentiates it from another story that explains the existence of complaints in equilibrium on the basis of consumers collective decision. This alternative story could be the basis of the role of consumers' associations or consumers' protection agencies. Coate and Conlin (2004) analyse the relationship between individual and collective decision making in the voting context.

[38] Consumers' optimal strategy in (6) implicitly assumes that when the realised quality is exactly $\hat{z} - \frac{\bar{\delta}c}{2\theta}$ consumers do complaint, even though they are indifferent between complaining in a proportion $\bar{\delta}$ and not complaining at all. The exact way in which this indifference is broken does not affect the results.

[39] As quality increases with in the range $[\hat{z} - \frac{\bar{\delta}c}{\theta}, \hat{z} - \frac{\bar{\delta}c}{2\theta}]$ consumers would prefer to reduce the proportion of complaints but this would result in $\sigma^*(\cdot) < \bar{\delta}$ and the payoff of complaining would jump to zero. Thus, they prefer to direct the minimum required level of consumers to complaint.

Figure 2: Relationship between σ^* and q



When $q \geq \hat{z}$, consumers receive no benefit from complaining and $\sigma^*(q; \hat{z}) = 0$. However, the optimal social rule is also zero for some quality realisations below $\hat{z}$ for which the benefits from complaining are not enough to compensate the costs, and so not even utilitarian agents are willing to complaint.[40]

Property 1. *There exists a* "zone of tolerance" *in which consumers do not complain even though the realised quality is smaller than their prior expectations.*

Proof. When quality is slightly higher than $\hat{z} - (\frac{\bar{\delta}c}{2\theta})$, the group's expected utility for $\sigma^*(q; \hat{z}) = \bar{\delta}$, is $\hat{z} - (\frac{\bar{\delta}c}{2\theta}) + \epsilon - (\theta\bar{\delta}\epsilon)$, whether if they do not complaint at all it is $\hat{z} - (\frac{\bar{\delta}c}{2\theta}) + \epsilon$. Expected utility is higher in the second case as complaining by the minimum proportion required by the regulator is too costly for such a small level of disappointment. A rule that directs any proportion of complaints in the interval $(0, \bar{\delta})$ is not optimal either, as it would result in a zero probability of the firm being fined (zero payoff of complaining) but a positive cost. Thus, in this case $\sigma^*(q; \hat{z}) = 0$ maximises consumers' expected utility. □

This "zone of tolerance" increases whenever it becomes more difficult for the consumers to have the firm punished. Whenever $\bar{\delta}$ or c increase or θ decreases, the level of disappointment necessary for consumers to be willing to face the cost of a positive probability of fine (i.e., $\sigma^*(\cdot) \geq \bar{\delta}$) becomes higher and so there exist a higher range of quality realisations for which consumers are not willing to complain even though $\hat{\hat{z}} > q$.

[40]This result supports some arguments made in marketing that define a "zone of tolerance" within which "the company is meeting customer expectations" (Singh 1988) and that is delimited by the *desired service level* and the *adequate service level* (i.e., the level of service the customer will accept). Consumers' optimal rule in (6) implies that a "region of tolerance" is defined by levels of quality within the range $(\hat{z} - (\frac{\bar{\delta}c}{2\theta}), \hat{z})$. Furthermore, it also identifies the parameters affecting the "width" of this region.

3.2 Firm's Equilibrium Strategy

The firm maximises its expected profits given the level of quality it believes consumers expect to receive ($\hat{\hat{z}}$). Its optimal action depends on the trade-off between the cost of investment and the expected fine: a lower investment reduces the firm's costs by h, but it also makes it less likely that quality meets consumers' expectations, increasing the expected fine. The firm optimally invests if and only if $\mathbb{E}_{I=1}(\Pi; \hat{\hat{z}}, \sigma^*) \geq \mathbb{E}_{I=0}(\Pi; \hat{\hat{z}}, \sigma^*)$, which implies $[\mathbb{E}_{I=0}(\sigma^*(q; \hat{\hat{z}})) - \mathbb{E}_{I=1}(\sigma^*(q; \hat{\hat{z}}))] \geq \frac{h}{mp}$. Denote by $\hat{\hat{z}}^*$ the level of $\hat{\hat{z}}$ at which the firm is indifferent between investing and not investing; $\frac{h}{mp}$ is constant and independent of consumers' expectations, but the change in the expected proportion of complaints when the firm's investment changes is an increasing function of $\hat{\hat{z}}$.[41] Then, the firm's optimal strategy is a cut-off of the form:

$$I^* = \begin{cases} 1 & \text{if } \hat{\hat{z}} \geq \hat{\hat{z}}^* \\ 0 & \text{if } \hat{\hat{z}} < \hat{\hat{z}}^* \end{cases}$$

The firm's strategy is increasing in consumers' expectations. When consumers expect too much from the firm, the firm's best reply is to fulfil those expectations, as otherwise the fine becomes too heavy. However the firm also fulfils consumers prior expectations when they are low, because if consumers do not expect much, their disappointment is not very high and the (expected) proportion of complaints is not enough to compensate the cost of investment (h).

The expected proportion of complaints when the good type of the firm invests is $\sigma_H^*(q; \hat{\hat{z}}) = \hat{z} - \frac{c}{2\theta}$, and when the good firm does not invest (or when the firm is bad) $\sigma_L^*(q; \hat{\hat{z}}) = 2\hat{z} - \frac{c}{\theta}$. Then, the cut off point in the firm's strategy is $\hat{\hat{z}}^* = \frac{h}{mp} + \frac{\psi}{2}$, where $\psi = \frac{c}{\theta}$; $\hat{\hat{z}}^*$ is determined by the magnitude of the "punishment" (mp) relative to the investment cost and by consumers' relative cost of complaining (ψ). The less harsh the punishment or the more difficult it is for consumers to complain, the higher is the $\hat{\hat{z}}$ required for the firm's optimal action to be $I = 1$.

Finally, note that given the firm's investment, expected profits decrease whenever consumers find it easier or more profitable to complain or when the level of quality they expect to receive increases.[42]

3.3 Equilibrium

The static game has a separating and a pooling equilibrium. In the first case, the good type of the firm invests and differentiates itself from the other type with a positive probability. In the second case, the firm does not invest and so it camouflages itself with the bad type. Define a *"High Quality Equilibrium"* (HQE) as one in which consumers expect the good type of the firm to invest and the firm's optimal action is indeed to invest. Analogously, define a *"Low Quality Equilibrium"* (LQE) as one in which consumers expect the firm not to invest and the firm does not invest. Denote by $\hat{z}_H(\tau)$ consumers' expected quality when they assign probability τ to the firm being good and expect the firm's action to be $I = 1$ ($\hat{\hat{z}}_H(\tau)$ denotes the firm's beliefs about $\hat{z}_H$), and by $\hat{z}_L$ consumers' expectations when they believe $I = 0$ ($\hat{\hat{z}}_L$ denote the firm's beliefs about $\hat{z}_L$). Given the equilibrium definition above,

[41] Given $\hat{\hat{z}}$, the expected proportion of complaints when the firm invests is $\hat{\hat{z}} - \frac{c}{2\theta}$, and when it does not invest $2\hat{\hat{z}} - \frac{c}{\theta}$ (See Appendix A.2); the difference $\mathbb{E}_{I=0}(\sigma^*(q; \hat{\hat{z}})) - \mathbb{E}_{I=1}(\sigma^*(q; \hat{\hat{z}})) = \hat{\hat{z}} - \frac{c}{2\theta}$ is an increasing function of $\hat{\hat{z}}$.

[42] Given a realisation of quality (i.e., keeping constant the investment level): $\frac{\partial E_{j,I_t}(\Pi; \hat{\hat{z}}, \sigma)}{\partial \hat{\hat{q}}} = -mp\frac{\partial \sigma}{\partial \hat{\hat{z}}} \leq 0$ because $\frac{\partial \sigma}{\partial \hat{\hat{z}}} \geq 0$.

a HQE exists if and only if $\mathbb{E}_{I=1}(\Pi;\hat{\hat{z}}_H,\sigma^*) \geq \mathbb{E}_{I=0}(\Pi;\hat{\hat{z}}_H,\sigma^*)$, while a LQE exists if and only if $\mathbb{E}_{I=0}(\Pi;\hat{\hat{z}}_L,\sigma^*) \geq \mathbb{E}_{I=1}(\Pi;\hat{\hat{z}}_L,\sigma^*)$.[43]

Proposition 1. Equilibrium in the Static Game. *Given $\psi < \frac{1}{4}$ and $\tau \in (0,1)$:*

1. *If $\hat{\hat{z}}^* \geq \frac{1}{2}$ the game has a unique low quality equilibrium.*
2. *If $\hat{\hat{z}}^* \in (\frac{1}{4},\frac{1}{2})$ there exists $\tau^* \in (0,1)$ such that there is a unique low quality equilibrium for $\tau \in [0,\tau^*)$, but high and low quality equilibria coexist for $\tau \in [\tau^*,1)$*
3. *If $\hat{\hat{z}}^* \leq \frac{1}{4}$ there is a unique high quality equilibrium.*

In a low quality equilibrium, consumers' expectations are $\hat{z}_L = \tau\bar{q}_H(I^*=0) + (1-\tau)\bar{q}_B$ and in a high quality equilibrium they are $\hat{z}_H = \tau\bar{q}_H(I^*=1) + (1-\tau)\bar{q}_B$.

Proof. In equilibrium the firm has correct beliefs about the level of quality consumers expect to receive, so $\hat{\hat{z}} = \hat{z}$. Given that the distributions of quality are public information, $\hat{z}_H(\tau) = \frac{1}{4} + \frac{1}{4}\tau$ and $\hat{z}_L = \frac{1}{4}$. The firm's optimal strategy depends on whether $\hat{z}$ is greater than or smaller than $\hat{\hat{z}}^*$. There are three possibilities:

- When $\hat{\hat{z}}^* > 1/2$, the cost of investing in quality is high relative to the (expected) punishment, $\hat{z}_L \leq \hat{z}_H(\tau) \leq \hat{\hat{z}}^*$, and as a result $\mathbb{E}_{I=0}(\Pi;\hat{\hat{z}}_L,\sigma^*) > \mathbb{E}_{I=1}(\Pi;\hat{\hat{z}}_L,\sigma^*)$ and $\mathbb{E}_{I=0}(\Pi;\hat{\hat{z}}_H(\tau),\sigma^*) > \mathbb{E}_{I=1}(\Pi;\hat{\hat{z}}_H(\tau),\sigma^*)$. From Sub-Section 3.2, the firm's dominant strategy is $I=0$ (independently of consumers expectations) and so rational consumers should not expect something different from low quality. Consumers expect $\hat{z}_L$ and the firm exerts no effort. There is a unique low quality equilibrium.
- If $\hat{\hat{z}}^* \in [\frac{1}{4},\frac{1}{2}]$, there exists a unique τ^* such that $\hat{z}_H(\tau^*) = \hat{\hat{z}}^*$. Uniqueness is given by the fact that $\hat{z}_H(\tau) \in [\frac{1}{4},\frac{1}{2}]$ and is an increasing function of τ, while $\hat{\hat{z}}^*$ belongs to the same interval but is exogenously given and independent of τ. For $\tau < \tau^*$, $\hat{\hat{z}}^* > \hat{z}_H(\tau) > \hat{z}_L$, implying $\mathbb{E}_{I=0}(\Pi;\hat{\hat{z}}_L,\sigma^*) > \mathbb{E}_{I=1}(\Pi;\hat{\hat{z}}_L,\sigma^*)$ and $\mathbb{E}_{I=0}(\Pi;\hat{\hat{z}}_H(\tau),\sigma^*) > \mathbb{E}_{I=1}(\Pi;\hat{\hat{z}}_H(\tau),\sigma^*)$. The firm's dominant strategy is $I=0$, independently of consumers' prior expectations, and there exists a unique low quality equilibrium. As τ increases so does $\hat{z}_H(\tau)$ -the level of quality consumers expect to receive increases as they become more convinced they are facing a good firm. For $\tau > \tau^*$, $\hat{z}_H(\tau) > \hat{\hat{z}}^* > \hat{z}_L$ and $\mathbb{E}_{I=0}(\Pi;\hat{\hat{z}}_L,\sigma^*) > \mathbb{E}_{I=1}(\Pi;\hat{\hat{z}}_L,\sigma^*)$ and $\mathbb{E}_{I=1}(\Pi;\hat{\hat{z}}_H(\tau),\sigma^*) > \mathbb{E}_{I=0}(\Pi;\hat{\hat{z}}_H(\tau),\sigma^*)$ and so there are two equilibria: the firm optimally invest if consumers expect high quality (HQE) and the firm does not invest if consumers' expected quality is $\hat{z}_L$ (LQE).
- If $\hat{\hat{z}}^* < 1/4$, the (expected) punishment is harsh compared with h and $\hat{\hat{z}}^* \leq \hat{z}_L \leq \hat{z}_H(\tau)$ $\forall\tau \in (0,1)$. In this case $\mathbb{E}_{I=1}(\Pi;\hat{\hat{z}}_H(\tau),\sigma^*) > \mathbb{E}_{I=0}(\Pi;\hat{\hat{z}}_H(\tau),\sigma^*)$ and $\mathbb{E}_{I=1}(\Pi;\hat{\hat{z}}_L,\sigma^*) > \mathbb{E}_{I=0}(\Pi;\hat{\hat{z}}_L,\sigma^*)$. Then, investing is a strictly dominant strategy. Consumers anticipate this and expect high quality ($\hat{z}_H(\tau)$). There is a unique high quality equilibrium.

□

[43] $\mathbb{E}_{I=0}(\Pi;\hat{\hat{z}}_H,\sigma^*)$ and $\mathbb{E}_{I=1}(\Pi;\hat{\hat{z}}_L,\sigma^*)$ can not be the firm's profits in any equilibrium of the game, because those profits imply that what consumes' expect the firm to do is different from what the firm actually does (i.e., consumers' beliefs about the firm's strategy are wrong), and the equilibrium definition requires beliefs to be correct in equilibrium.

The equilibrium results show how the existence of the fine affects the firm's incentives to invest. Without the possibility of being punished the firm would have no incentives to invest in quality. The existence of a punishment based on consumers complaints create the possibility of an equilibrium in which the firm invests as long as (1) the size of the fine, as determined by the proportion of revenues lost in case of a fine (mp) is high relative to the cost of investment (h) and (2) consumers do transmit their dissatisfaction to the regulatory agency. Both conditions are summarised by the parameter $\hat{\hat{z}}^*$: a high value of $\hat{\hat{z}}^*$ reflects a reduced effectiveness of the punishment, either because the cost of investment is high relative to the fine or because consumers' relative cost of complaining is high ($\psi = \frac{c}{\theta}$). In any case, the result is that the firm maximises its profits by paying the fine and not by investing in higher quality. Therefore, as $\hat{\hat{z}}^*$ increases the game moves towards a low quality equilibrium.[44]

The quality consumers expect to receive is higher when they believe the firm's strategy is to invest but also when they assign a higher probability to the firm being of the good type -i.e., consumers expect more from a good firm. As a result, the firm's payoff in a HQE is a decreasing function of τ: the more convinced consumers are that they are facing a good firm, the more they expect and so the higher is the (expected) proportion of complaints and the smaller the firm's (expected) profits for every investment. Furthermore, the closer is τ to one, the smaller is the size of the fine required to induce investment (m).

It is worth noting, however, that the change in the set of equilibria resulting from the introduction of the fine does not necessarily imply an increase in total welfare. The (average) quality in a low quality equilibrium is the same that would be delivered without the regulatory rule. The firm's expected profits, however, are smaller after the introduction of the regulation because it faces a positive probability of fine. An equivalent statement about the change in consumers' welfare with and without the regulatory rule is less clear because I am assuming that they derive some positive utility out of complaining. However, if the introduction of the fine creates an inefficiency, then total welfare could be reduced.

In a high quality equilibrium the firm optimally invests because the cost of investing is smaller than the additional fine it would have to pay if a low realisation of quality results in a high proportion of complaints. Even though the level of quality consumers receive in this case is higher than without the regulation, the cost of that quality exceeds the cost of investing by the expected fine (because there is a positive probability of fine). As this creates an inefficiency, the result is only a "second best" result. Furthermore, the cost of delivering a higher quality is increasing in consumers' expectations and so the more convinced are consumers that the firm is of the good type, the higher is the cost of the extra quality.

3.4 Informativeness of Complaints

I consider that complaints are informative about the equilibrium being played if the (expected) proportion of complaints is higher in the low quality equilibrium than in the high quality equilibrium. This would be a relevant definition would the regulatory agency be interested in punishing the firm more harshly when it is not investing. According to this definition, complaints are informative if $\sigma_L^*(q; \hat{z}_L) > \sigma_H^*(q; \hat{z}_H)$. Given a level of quality consumers expect to receive, the (expected) proportion of complaints is higher when the

[44] Note that, as the fine is over revenues, it is irrelevant whether the regulatory agency knows the firm's investment costs. All that is required is that he is able to observe the proportion of complaints and the firm's revenues.

firm does not invest: $\sigma_L^*(q;\hat{z}) = 2\sigma_H^*(q;\hat{z})$. However, in equilibrium consumers have correct beliefs about the firm's strategy and they modify their expectations accordingly. Because consumers expectations are higher in a high quality equilibrium, it is not clear whether they will complain more when the firm is not investing. Lemma 1 presents the conditions under which complaints are informative in the one shot game.

Lemma 1. *Take τ^* from Proposition 1. Given $\hat{\hat{z}} \in (\frac{1}{4}, \frac{1}{2})$, there exists $\bar{\tau}$ such that complaints are informative about the equilibrium being played if and only if (1) $\tau \in (\tau^*, \bar{\tau})$ and (2) complaining is neither "too cheap" nor "too costly".*

Proof. First, note that the definition of informativeness of complaints is based in the existence of multiple equilibria, so the analysis is restricted to $\tau > \tau^*$ and $\hat{\hat{z}} \in (\frac{1}{4}, \frac{1}{2})$. If consumers believe the firm is not going to invest, the level of quality they expect to receive is $\hat{z}_L = \frac{1}{4}$, but if they believe its optimal strategy is $I_G = 1$ their expected quality goes up to $\hat{z}_H(\tau) = \frac{1}{4} + \frac{1}{4}\tau$, which is an increasing function of τ. Then, $\sigma_L^*(q;\hat{z}_L) > \sigma_H^*(q;\hat{z}_H)$ if and only if $2\hat{z}_L - \hat{z}_H(\tau) > \frac{\psi}{2}$ (where $\psi = \frac{c}{\theta}$), and complaints in the LQE are higher than in a HQE as long as $\tau < 1 - 2\psi$. Denoting $1 - 2\psi = \bar{\tau}$, the expected proportion of complaints is higher in a LQE than in a HQE for values of $\tau \in (\tau^*, \bar{\tau})$, and part (1) of the Lemma holds.

For the second part of the Lemma, note that when consumers have no cost of complaining ($c = 0$) the expected utility in (3) is maximised when all the consumers make a complaint if $\hat{z} > q$ (because $\sigma^* = 1$ maximises the probability that the firm is fined) and when nobody complaints if $\hat{z} \leq q$.[45] Then, the proportion of complaints becomes constant and independent of consumers' disappointment. Finally, when complaining is very costly, complaints are not informative either because the set of τ's determined in the previous paragraph is empty. $\tau \in (\tau^*, 1 - 2\psi)$ is not an empty set if $1 - 2\psi \geq \tau^*$. From Proposition 1, $\tau^* = 4\hat{\hat{z}}^* - 1$. Then, the second condition in the Lemma implies that complaints are informative only for values of ψ in the set $(0, 1 - 2\frac{h}{mp})$. □

The Lemma shows that complaints are not always a good signal of the firm's investment. Consumers complaints are not informative of the equilibrium being played whenever the change in their disappointment between the low and the high quality equilibria is driven by a change in their expectations and not by a change in the (average) quality being delivered by the firm. When consumers are reasonably convinced that the firm is "good" (high τ), $\hat{z}$ increases more than the (average) realisations of quality and so complaints are (on expectations) higher when the firm invests and consumers anticipate this behaviour. In this case, complaints are informative about how disappointed consumers are with the quality they received but not about the firm's investment. As a result, the firm might be punished more harshly when it is investing than when it is not.

The condition $\tau \leq 1 - 2\psi$ means that, given τ, the informativeness of complaints decreases if ψ increases -i.e., if the cost of making a complaint is higher relative to consumers' willingness to complain. Hence, the informativeness of complaints depends also on how easy it is for consumers to complain. The result in the Lemma shows that if complaining is too costly, the level of disappointment required for consumers to be willing to face the cost of "informing" the regulator about the problem is too high and so the regulator observes only a small proportion of complaints -i.e., consumers do not complain enough so as to transmit information to the regulator. On the other extreme, if $c = 0$ the optimal social norm becomes independent of the size of the difference between expected and realised quality, and

[45] Consumers' optimal strategy in (6) shows that when $c = 0$, $\sigma^*(q;\hat{z}) = 1$ if and only if $q < \hat{z}$.

the (expected) proportion of complaints is the same in both equilibria.[46] When complaining is very cheap, the proportion of consumers that lodge a complain is so high that complaints become meaningless.

The limited informativeness of complaints is due to the fact that consumers' complaining decision does not depend solely on the realisation of quality but also on their prior expectations. As a result, there is not a unique relationship between the proportion of complaints and the firm's investing behaviour. However, the existence of a reference point is necessary for the existence of a positive proportion of complaints in any equilibrium of the game.

3.5 Comparative Static of the Optimal Complaining Rule

The optimal rule in (6) can be used to derive predictions about the way in which complaints depend on the exogenous variables in both, consumers' individual utility and the regulatory rule. Those predictions are summarised in the following properties.

Property 2. *The (expected) proportion of complaints is increasing in consumers' prior expectations.*

A higher $\hat{z}$ increases consumers' disappointment with every realisation of quality (q), and so consumers are willing to accept the higher social cost that results from an increase in the cutoff point. This effect is showed in Figure 3a, where the the continuous line represents consumers' optimal strategy before the increase in $\hat{z}$ and the dashed line the optimal strategy after that increase.

Property 3. *Given consumers' disappointment, the optimal social rule decreases when $\psi = \frac{c}{\theta}$ increases.*

An increase in c or a decrease in θ reduces consumers' expected utility from any social rule and thus, they are less willing to complaint. Figure 3b shows how the optimal cut off rule changes as the value of $\psi = c/\theta$ increases. The continuous line in the figure shows consumers' optimal strategy before the increase in the cost, while the dashed one shows the strategy after that increase. The figure shows that the optimal social rule directs a smaller proportion of consumers to complaint for most realisations of quality. The only exception are very low realisations for which it is still optimal to direct every ethical agent to complaint. Furthermore, expected utility decreases more for higher values of $\sigma^*(q)$, so the impact is higher for higher cutoff points. The figure also shows that the region of tolerance increases when c/θ increases.

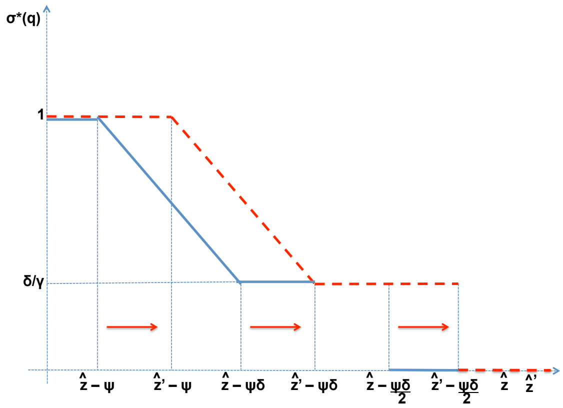
Property 4. *An increase $\bar{\delta}$ makes it more costly for consumers to punish the firm when quality realisations are relatively high.*

The change in $\bar{\delta}$ affects the optimal rule for relatively high realisations of quality but not when consumers' disappointment is high. A higher $\bar{\delta}$ makes it more difficult for consumers to meet the regulator's requirements and so it increases the group's cost of punishing the firm for high levels of quality, when the payoff of complaining is relatively low. For realisations

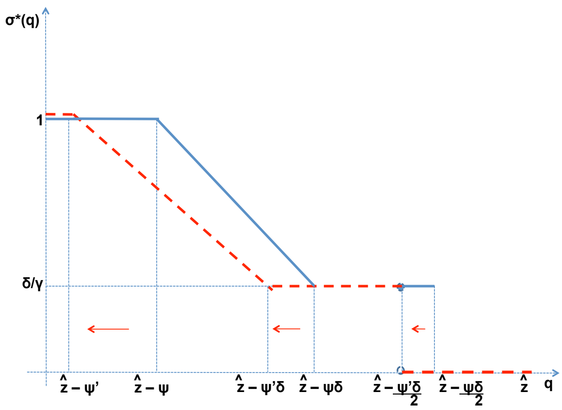
[46]When $c = 0$, $\hat{z}^* = \frac{h}{mp}$ and the firm's incentives to invest depend solely on the relative magnitude of the investment cost and the fine.

Figure 3: Changes in the Optimal Cutoff as the Parameters Change

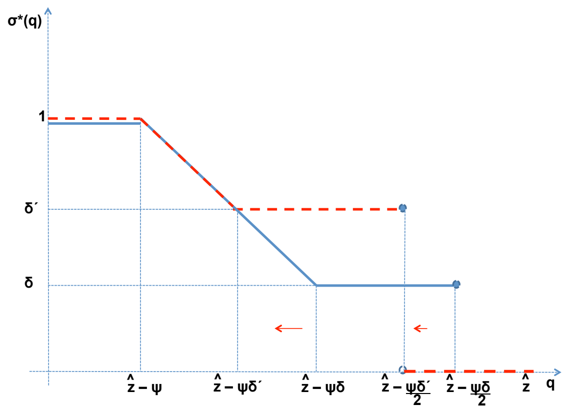
(a) Increase in $\hat{z}$



(b) Increase in ψ



(c) Increase in $\bar{\delta}$



of quality relatively high, consumers optimal strategy moves upwards (a higher proportion of complaints is needed to have the firm punished) and to the left (because the social cost of complaining is higher), as shown in Figure 3c.

4 Repeated Game

In this section I explore how the repetition of the game affects the informativeness of consumers' complaints and the firm's incentives to invest in quality. When the complaining game is played repeatedly, consumers' beliefs are updated at the end of every period and so their reference point changes over time. The firm's strategy depends on consumers current expectations, but also on how today's investment affects the level of quality they expect to receive in the future: higher investment in a given period reduces that period's expected fine but it increases consumers' future expectations and so it increases the probability of being fined in the future. In this way, the repetition of the game generates incentives for the good firm to induce particular beliefs in the consumers in order to elicit beneficial best replies.[47] The main result in this section is that a repeated interaction between consumers and the firm may reduce the firm's incentives to invest and the informativeness of complaints. The later is due to the fact that the adverse effect on investment is stronger for values of the parameters for which complaints would have been informative in the one shot game: by not investing in any of the T periods, the firm keeps consumers' expectations (and the proportion of complaints) low, and so the regulator can not infer the lack of investment from the observed σ_t.

In order to define an equilibrium for the repeated game, I need to make clear the information available to each player. The public information at the beginning of period t is the history of past quality realisations and consumers' complaints, $h^t = (q_1, \sigma_1; q_2, \sigma_2; \dots; q_{t-1}, \sigma_{t-1})$. The set of public histories is then $\mathcal{H} = \bigcup_{t=1}^{T}[0,1]^{2t}$. When making a complaining decision, consumers know the public history up to that moment but they also have private information about the level of quality they expect to receive (and the level they expected in any previous period). Their private history is $\mathcal{H}_C = \bigcup_{t=1}^{T}[0,1]^{3t}$. Consumers' optimal rule maximises the present value of their expected utility, $\sigma_h^* \in \arg\max \Sigma_{h=t}^{T}\beta^{h-1}\mathbb{E}U(\sigma_h; q_h, \hat{z}_h)$ and so, their strategy in the repeated game is a sequence of complaining decisions $\{\sigma_h\}_{h=t}^{T}$, each of which maps their private history and the current period quality into a cutoff between zero and one, $\sigma_t : \mathcal{H}_C \times [0,1] \to [0,1]$.[48]

The firm has information about past quality and complaints, but also about its own past actions and its beliefs about the level of quality consumers expect to receive. Then a private history for the good type of the firm includes both the public history and the history of its investment decisions and beliefs: $\mathcal{H}_G = \bigcup_{t=1}^{T}\{0,1\}^t \times [0,1]^{3t}$. The strategy of the good type of the firm is a sequence $\{I_h\}_{h=t}^{T}$ that assigns, in each period, an investment level for

[47] The psychological aspect of the game means that current actions affect future play like in standard dynamic games, but they also affect players' beliefs and, because beliefs affect payoffs, current actions affect future payoff for any possible action.

[48] Consumers make their period-t complaining decision after observing the realisation of q_t, so this last quality realisation also forms the history of the game by the moment in which they decide how strongly to complain. Also note that individual consumers have private information about their costs of complaining and actions. However, the group utilitarian assumption implies that their strategy is the same that would result if there were only one "big" consumer. As a result, the only relevant information is the distribution of the costs of complaining (which is public information) and the distribution of consumers' actions (σ_t), which is also public information. The latter holds because, as there is a continuous of anonymous consumers, each of them can do no better than myopically follow the complaining rule.

any possible private history, $I_t : \mathcal{H}_G \to \{0,1\}$. The firm's investment decision in period t, I_t^* maximises the present value of its profits, which is the expectation with respect to the probability distribution induced by the current investment; this implies that I_t^*, maximises

$$\mathbb{E}_{I_t^*}[(\Pi_t(I_t^*; \hat{\hat{z}}_t, \sigma_t^*) + \Sigma_{h=t+1}^T \beta^{h-t} \mathbb{E}_{I_{h-1}^*}(\Pi_h | q_{h-1})] \tag{7}$$

The strategy for the bad type of the firm is trivial and consists of $I_B^* = 0$, $\forall t = 0, 1, \ldots, T$.

The equilibrium of the repeated game requires players behaviour to be optimum in every period given their beliefs about the other players' type and strategy but also their understanding of the way in which current behaviour affects future payoffs. At the end of every period consumers update their beliefs about the type of the firm and form some expectations about the level of quality they should receive in the following period. The equilibrium requires consumers' beliefs to be correct in the sense of being consistent with the firm's strategy in the repeated game. Beliefs about the type of the firm are required to be consistent in a bayesian way: the probability that the firm is good is exogenous, and consumers update that probability at the end of every period according to Bayes' rule (and the firm's strategy). Furthermore, the probability distributions I am assuming imply that quality realisations between zero and 1/2 can not be off the equilibrium path, while realisations higher than 1/2 can be out of the equilibrium path but they are fully revealing of the firm's type. Then, Bayesian updating defines consumers' beliefs about the firm's type everywhere. Definition 2 formalises the requirements for an equilibrium.

Definition 2. *An equilibrium of the complaining game when $T > 1$ is a sequence of strategies $\{I_t^*, \sigma_t^*\}_{t=0}^T$ such that there exist beliefs $(\tau_t, \hat{\hat{z}}_t, \hat{z}_t)$ for $t = 1, 2, \ldots, T$, for which:*

1. *$\{I_h^*\}_{h=t}^T$ maximises the present value of the firm's expected profits, given $\mathcal{H}_G$.*
2. *$\{\sigma_h^*\}_{h=t}^T$ maximises the present value of consumers expected utility, given $\mathcal{H}_C$*
3. *In every period, after any history arising with positive probability under $(I_h^*, \sigma_h^*)_{h=t}^T$, the firm has correct beliefs about the level of quality consumers expect ($\hat{\hat{z}}_t = \hat{z}_t$) and consumers expectations are consistent with the equilibrium strategies in the repeated game and Bayes' Rule.*
4. *After a realisation of quality that has zero probability under the equilibrium strategy, $\tau_{t+1} = 1$ and consumers expect the firm to invest.*

The definition states that, when the game is played $T > 1$ periods, consumers' optimal strategy should maximise the sum of current and future expected utility. However, the specific regulatory rule I am studying implies that current complaints do not affect the firm's future behaviour and so, consumers only reason for complaining is to punish the firm's current "poor performance". As a result, consumers behave *as if* they were myopic (even though they are not). This result is shown in Lemma 2.

Lemma 2. *Consumers behave as if they were myopic: their optimal complaining strategy in the repeated game is the same as in the static game: $\sigma_t^*(q_t; \hat{z}_t) = \sigma^*(q_t; \hat{z}_t)$.*

Consumers myopically respond to current quality if the firm's future behaviour is not affected by current complaints (and consumers anticipate this), meaning that $\mathbb{E}(q_{t+1}|\sigma_t) =$

$\mathbb{E}(q_{t+1})$ and $\mathbb{E}(\hat{z}_{t+1}|\sigma_t) = \mathbb{E}(\hat{z}_{t+1})$.

The firm's optimal investment decision in period t is independent of past complaints because they constitute a sunk cost. To see that this is indeed the case, consider the firm's optimal action in the last period. In period T there are no inter temporal incentives and so, the firm's optimal strategy is a cut off analogous to the one in the one-shot game: the firm invests as long as $\hat{\hat{z}}_T > \hat{\hat{z}}^*$ and does not invest if the inequality is reversed. In equilibrium, $\hat{\hat{z}}_T = \hat{z}_T$, which depends on consumers' beliefs about the firm's type (τ_T) and investment strategy. τ_T is a function of past quality realisations (through Bayesian updating) and consumers beliefs about the firm's strategy reflect common knowledge of the strategy profile. Therefore, the firm's investment in period T is determined by past realisations of quality but is not affected by the fact that the firm was fined in the previous period. A similar argument explains why past fines (and hence past complaints) do not affect current or future investment in periods before the last one. Consumers anticipate the firm's best response and so they do not expect current complaints to affect future quality. Consumers' problem in the repeated game, $\max_{\sigma_h} \Sigma_{h=t}^{T} \beta^{h-1} \mathbb{E}U(\sigma_h; q_h, \hat{z}_h)$ is then equivalent to $\Sigma_{h=t}^{T} \beta^{h-1} \max_{\sigma_h} \mathbb{E}U(\sigma_h; q_h, \hat{z}_h)$ and so $\sigma_t^*(q_t; \hat{z}_t) = \sigma^*(q_t; \hat{z}_t)$.

The intuition behind this result is that, as the firm is punished in the same period in which complaints occur, past fines (and hence past complaints) become a sunk cost when the firm decides its current (and future) investment. Rational consumers understand that future quality (and future firm's behaviour) is not affected by current complaints and so their complaining strategy is simple the myopic best response to the quality realisation they received, given their prior expectations -i.e., they complain in order to punish the firm's current "poor performance". An important consequence of Lemma 2 is that without the presence of the reference point in consumers' utility function, the optimal complaining strategy would be $\sigma_t^* = 0$ for every $t = 1, 2, ...T$, because not even utilitarian consumers would receive a positive payoff from complaining.

The formal implication of consumers' myopic behaviour is that the repeated game is strategically equivalent to a game in which a long-lived firm faces a sequence of short-lived "consumers", each of which plays only once but observes all previous realisations of quality and complaints.

4.1 Equilibrium

The definition of informativeness of complaints I introduced in section 3 is based on the existence of multiple equilibria, as it compares the equilibrium (expected) proportion of complaints when consumers expect low quality relative to the case in which they expect high quality. Therefore, in this section I consider only the set of parameters for which high and low quality equilibria coexist in the stage game and look at how that set is affected by the repetition of the game. This means that I focus on values of $\hat{\hat{z}}^* \in [1/4, 1/2]$ and $\tau \geq \tau^*$ (see Proposition 1). Recall that $\hat{\hat{z}}^* = \frac{h}{mp} + \frac{\psi}{2}$ summarises the main parameters determining the strength of the punishment, namely, the relative size of the fine and consumers' complaining cost. In a way analogous to the one in section 3, I say that there is a *high quality equilibrium* (HQE) when consumers expect the firm to invest in every period and the firm optimally does so, and a *low quality equilibrium* (LQE) when consumers expect the firm not to invest in any period and the firm's best reply is not to invest. Furthermore, as two periods are enough to prove the main results, this section presents only the case in which $T = 2$. The case with

$T \to \infty$ is presented in Appendix B.[49] Proposition 2 summarises the main result of the repeated game.

Proposition 2. *Take τ^* from Proposition 1. Given $\hat{\hat{z}}^* \in [1/4, 1/2]$, $\psi < \frac{1}{4}$ and $T = 2$, there exist $\tau^{**} \leq 1 - 2\psi$ such that for all $\tau \in (\tau^*, \tau^{**})$, the one shot game has a high quality and a low quality equilibrium, but only the second one survives in the repeated game. As a result, complaints are less informative in the repeated game than in the one shot game.*

The proof of this proposition is divided in three parts. Lemmas 3 and 4 below characterise the set of parameters for which a high and a low quality equilibria exist in the repeated game, while Lemma 5 relates those results to the degree of informativeness of complaints.

Lemma 3. *Given $\hat{\hat{z}}^* \in (1/4, 1/2)$ and $T = 2$, there exists τ^{**} such that for $\tau \geq \tau^{**}$, the firm optimally invests in $t = 1, 2$ if consumers expect it to do so.*

Proof. This can be shown by solving the game backwards. The level of quality consumers expect to receive in the second period is a function of their beliefs about the firm's type and strategy. If they believe the firm's strategy in the repeated game is to invest in both periods, the probability they assign at the beginning of period two to the firm being good is $\tau_1 = 1$ if $q_1 > 1/2$ and $\tau_1 = \frac{\tau}{2-\tau} \leq \tau$ if $q_1 \leq 1/2$. On the equilibrium path the firm invests in both periods and so consumers expect $I_1 = I_2 = 1$. If $\tau_1 = 1$, the firm's best reply is to invest as long as $\hat{\hat{z}}^* < 1/2$, which is always true in the region I am considering. If the first period quality was low and $\tau_1 < \tau$, the firm's best reply is $I = 1$ only if consumers expectations are high enough, i.e., if $\hat{\hat{z}}^* < \hat{\hat{z}} = \frac{1}{4} + \frac{1}{4}\tau_1$.[50]

The firm's optimal investment in the first period is a dynamic decision becasue it considers the effect of q_1 on the level of quality consumers expect to receive in the second period. If $I_1 = 1$ there is a probability of 1/2 that consumers find out it is good and, as a result they expect a very high quality in the second period ($\hat{z}_H(1)$), and a probability of 1/2 that $q_1 \leq 1/2$ and so consumers expected quality for the second period is $\hat{z}_H(\tau_1) \leq \hat{z}_H(1)$ (with strict inequality if $\tau < 1$). However, if the firm deviates in the first period, the realisation of quality is smaller than 1/2 for sure; consumers do not realise the deviation, but the low quality induces them to reduce the probability they assign to the firm being good and to lower their second period expectations accordingly. The trade-off faced by the firm is that the deviation reduces current profits because, given consumers expectations, a smaller realisation of quality increases the expected fine, but it increases expected future profits because the level of quality consumers expect for the second period is smaller. The firm optimally invests if, given $\frac{1}{4} + \frac{1}{4}\tau_1 > \hat{\hat{z}}^*$, the expected profits from investing in $t = 1, 2$ are higher than the profits from deviating in the first period, given that consumers expect $I_t = 1$, $t = 1, 2$. The profits on equilibrium path are:

$$p - h - mp\sigma^*(q_1; \hat{z}_H(\tau)) + \beta[p - h - mp(\frac{1}{2}\sigma^*(q_2; \hat{z}_H(1)) + \frac{1}{2}\sigma^*(q_2; \hat{z}_H(\tau_1))]$$

to $\tau_1 = \frac{\tau}{2-\tau} \leq \tau$. The expected profits if the firm deviates in the first period are:

$$p - mp\sigma^*(q_1; \hat{z}_H(\tau)) + \beta[p - h - mp\sigma^*(q_2; \hat{z}_H(\tau_1))]$$

[49]The infinite horizon game shows that the results of this Section do not depend upon the existence of a final period.

[50]As this is the last period, both conditions are identical to those in the stage game, and so the reader is referred to the proof of Proposition 1.

The profits on equilibrium path are higher if and only if $\frac{1}{4}+\frac{1}{4}\tau-\frac{\beta}{8}(1-\tau_1)>\hat{\hat{z}}^*$. Denote by τ^{**} the value of τ for which the firm is indifferent between investing and not in the repeated game. Then, $\tau^{**}=4\hat{\hat{z}}^*-1+\frac{\beta}{2}(1-\tau_1)$ and the high quality equilibrium holds for $\tau\geq\tau^{**}$.[51] □

The Lemma shows that when $T=2$ and $\hat{\hat{z}}^*\in(1/4,1/2)$, there exists τ^{**} such that for $\tau>\tau^{**}$ the firm's optimal strategy is $\{I_1^*,I_2^*\}=\{1,1\}$ and the level of quality consumers expect to receive in each period is $\hat{z}_1=\hat{z}_H(\tau)$ and $\hat{z}_2=\hat{z}_H(1)$ if $q_1>1/2$ and $\hat{z}_2=\hat{z}_H(\tau_1)$ if $q_1\leq 1/2$. This means that, if the probability consumers assign to the firm being good is high enough (and so the level of quality they expect to receive is relatively high), there exists an equilibrium of the repeated game in which the firm invests in every period and consumers expect that behaviour.

Lemma 4. *Given $\hat{\hat{z}}^*\in(1/4,1/2)$ and $T=2$, there exists an equilibrium in which consumers expect the firm not to invest in any of the two periods and the firm's best reply is to fulfil those expectations.*

Proof. When the firm does not invest in any of the two periods $q_t\leq 1/2$ for $t=1,2$. If consumers anticipate the firm is not going to invest, the low quality realisation in the first period is not informative about the firm's type and $\tau_1=\tau$. By solving the game backwards, it can be shown that $(I_1^*,I_2^*)=(0,0)$ constitutes an equilibrium when $\hat{z}_1=\hat{z}_2=\hat{z}_L(\tau)$ and $\hat{\hat{z}}^*\in(1/4,1/2)$.

If the firm did not invest in the first period, $q_1<1/2$ and $\tau_1=\tau$. If consumers expect $I_2=0$ too, the firm's optimal action in $t=2$ is not to invest as long as $\hat{\hat{z}}^*\geq\frac{1}{4}$.[52] In the first period, if consumers expect $I_1=0$ but the firm deviates to $I_1=1$, there is 1/2 probability that the realisation of quality is small and so consumers do not find out the deviation, but there is another 1/2 of probability that the realisation of quality is high and consumers find out they are facing a good firm. After observing $q_1\geq 1/2$, consumers know the firm is good ($\tau=1$) and expect to receive $\hat{z}_H(1)$ in the next period. The trade-off faced by the firm when consumers' expectations are low is that a higher first period investment increases current profits but it also reduces future (expected) profits as it may result in consumers expecting a higher level of quality in the second period. Expected profits under the deviation are smaller than under the equilibrium strategy if $\hat{\hat{z}}^*>\frac{1}{2(2+\beta)}$.[53] So, if the firm is very impatient ($\beta=0$), we are back in the one shot game and the LQE holds for $\hat{\hat{z}}^*>\frac{1}{4}$, but if $\beta\to 1$, the firm will not invest in any of the two periods as long as $\hat{\hat{z}}^*>\frac{1}{6}$. Therefore, the region of parameters in which the firm does not invest is greater in the repeated game. □

Lemma 5. *Complaints are less informative in the repeated game than in the one shot game.*

Proof. Lemmas 3 and 4 show that when $T=2$ and $\hat{\hat{z}}^*\in(1/4,1/2)$, there exists an equilibrium in which the firm does not invest in any of the two periods and consumers expect low quality in both periods for every $\tau\in(0,1)$, but that an equilibrium in which the firm invests in $t=1,2$ and consumers expect high quality in both periods exists only for $\tau\geq\tau^{**}$. Proposition 1 presents the equilibrium results for the one shot game. It shows that for $\hat{\hat{z}}^*\in(1/4,1/2)$,

[51] In the first period, the firm is willing to invest as long as $\tau\geq\tau^{**}$. In the second period, it invests if $\tau_1\geq 4\hat{\hat{z}}^*-1$. Given that $\tau_1\leq\tau$, the firm invests in the second period given that it invested in the first period and consumers expect it to invest in both periods.

[52] As this is the last period of the game, the result is the same as the one in Proposition 1.

[53] Expected game profits under the equilibrium strategy are $p-mp\sigma^*(q_1;\hat{z}_L)+\beta[p-mp(q_2;\hat{z}_L)]$, and under the deviation: $p-h-mp(q_1;\hat{z}_L)+\beta[\frac{1}{2}(p-mp(q_1;\hat{z}_L))+\frac{1}{2}(p-h-mp(q_2;\hat{z}_H(1)))]$.

the static game also has a low quality equilibrium for any τ, but a high quality one exists only for $\tau \geq \tau^*$. Furthermore, as long as $\beta > 0$ and $\tau < 1$, $\tau^{**} \geq \tau^*$ and so there exists a set $\tau \in (\tau^*, \tau^{**})$, for which the HQE exists in the one shot game but not in the repeated game. $\tau^{**} - \tau^* = \frac{\beta}{2}(1 - \tau_1)$, which implies that the more patient is the firm and the smaller is the initial τ, the greater is the set of values of τ for which the repetition of the game eliminates the high quality equilibrium.

From Lemma 1, complaints are informative in the one shot game when $\tau \in (\tau^*, 1 - 2\psi)$. The upper bound is the result of the condition that the (expected) proportion of complaints is smaller in a LQE than in a HQE. This upper bound is the same when the game is repeated, because if $\tau > 1 - 2\psi$, the level of quality consumers expect to receive when they believe the firm is going to invest increases faster, relative to $\hat{z}_L$, than the change in actual quality when the firm changes its investment strategy form 0 to 1. The lower bound on τ is the minimum level at which the firm is willing to invest in the one shot game. Complaints are informative about the firm's investment when $T = 2$ if $\tau \in (\tau^{**}, 1 - 2\psi)$. Therefore, the set $\tau \in (\tau^*, \tau^{**})$ for which the equilibrium with high quality ceases to exist when the game is repeated contains values of τ for which complaints are informative in the one stage game, meaning that the repetition of the game also reduces the degree of informativeness of complaints. □

Lemma 5, together with Lemmas 3 and 4, show that the set of parameters for which the firm invests in equilibrium is reduced by the repetition of the game, which in turn reduces the degree of informativeness of complaints. The intuition behind this result is that when the firm invests it faces the risk that consumers find out its type. When that happens in the last period (or when the game is played only once) it does not affect the firm's continuation value and so it optimally invests if the decrease in the (expected) fine compensates the cost of investment. However, when consumers find out the type of the firm before the end of the game, they increase the level of quality they expect to receive in the future, reducing the firm's future profits. The cost of reducing the current fine is higher in the repeated game as it includes not only the cost of a higher investment but also the cost of smaller profits in the future. The change in the firm's expected profits due to higher consumers expectations is a decreasing function of τ and so the firm's incentives to keep consumers' expectations low are higher for smaller values of τ. When τ is small, the inter temporal trade-off is more relevant for its investment decision than the intra temporal trade-off and the firm's optimal action is to keep future expected quality low by not investing today. On the contrary, when the probability consumers assign to the firm being of the good type is very high, there is only a small scope to "manage" consumers expectations, what makes the intra temporal trade-off more relevant. In this case, the firm's profit maximisation strategy is to invest if consumers expect high quality and not to invest if their expect so. The relevance of the inter temporal trade off also depends on how patient is the firm: the higher β the more value the firm assigns to future profits and the more it cares about keeping consumers expectations low.[54]

5 Conclusions

The model in this paper considers the role of customers complains as a regulatory tool in contexts in which quality is relatively well-perceived by the consumers, but it is very costly for the regulator to observe it. By studying a monopoly's investment decision, the paper

[54]In the case in which the game is repeated two periods, the lower bound of τ goes up from $4\hat{\hat{z}}^* - 1$ to $4\hat{\hat{z}}^* - 1 + \frac{\beta}{2}(1 - \tau_1)$.

derives some conditions under which complaints are informative about the firm's behaviour and studies how the informativeness of complaints is related to the regulatory rule.

The paper starts by noting that in some contexts consumers' receive no direct benefits from complaining or those benefits can not be individualised. Therefore, and following some empirical evidence, it is assumed that they complain because they feel "disappointed" with the level of quality they received and consider the firm should be punished for its "poor performance". Consumers' disappointment is determined by comparing the realised quality with their prior rational expectation. As a result, "disappointment" and "poor performance" are endogenously determined. Due to the lack of direct benefits, the reference point is a necessary assumption in order to explain consumers' complaining decision. The absence of a reference point results either in no complaints in equilibrium or in the proportion of complaints being independent of the realised level of quality. While this seems natural in the static game, it becomes particularly interesting in the dynamic case. When the game is played repeatedly, consumers could complaint in order to affect the firm's future behaviour. It is shown, however, that when the regulator uses a regulatory rule as the one considered in this paper, consumers' optimal behaviour in the repeated game is the myopic best response to the quality they received, given they prior expectations. This myopic behaviour creates the conditions for the firm to try to "keep expectations low" making complaints less informative in the repeated game than in the one shot game.

The model derives some conditions under which complaints are informative about the firm's investment decision. It is shown that complaints are informative when complaining is neither too cheap nor too costly and when consumers assign a relatively low probability to the firm being good. The fact that consumers' reference point is determined by the rational expectations about the quality they might receive implies that they expect more from a good firm and so the expected proportion of complaints may be higher in a high quality equilibrium than in a low quality one if consumers believe they are facing a good firm. An immediate implication is that, when the above conditions do not hold, the regulator may observe more complaints when the firm is investing than when it is not and so the firm might be punished more harshly in the first case.

The cost of making a complaint also affects the level of information that is transmitted to the regulatory agency. An interesting result of the model is that the lack of informativeness of complaints can be caused by complaining being either too cheap or too costly. In the first case, it is shown that complaints become independent of the level of consumers' disappointment and so the agency can not identify the firm's investment from a difference in the (expected) proportion of complaints. In the second case, complaints are not informative just because consumers do not lodge enough complaints so as to transmit information.

Furthermore, the set of parameters for which complaints are informative about the firm's behaviour is reduced when the game is played repeatedly. The repetition of the game reduces the firm's incentives to invest in quality because it introduces an inter temporal trade-off: high current investment reduces the current fine but increases the level of quality consumers expect to receive in the future. By not investing, the firm keeps consumers' expectations (and the proportion of complaints) low, and so it indirectly reduces the information they transmit to the regulatory agency.

The main results in this paper are positive in the sense that they study the effect of a particular regulatory rule on both, consumers incentives to complain and the firm's incentives to invest in quality. An important next step is to study the characteristics a regulation based on customers' complaints should have in order to induce a better transmission of information from the consumers to the regulatory agency. When consumers receive no direct benefit out of their complaints their only reason to lodge a complaint is to transmit their dissatisfaction. Even more, as the fine acts as a sunk cost from the firm's perspective and consumers are rational, they behave as if they were myopic, in the sense that their complaining decision depends only on the current quality. This limitation is a consequence of the punishment depending only on current complaints and opens the possibility for the firm to strategically "manage" consumers expectations. An optimal regulatory rule would need to trade-off the necessity of breaking the consumers' myopic behaviour against the costs of a more complicated regulation.

Finally, it is worth noting that the results in this paper hold in a more general context than the one studied here. Whenever consumers find switching supplier difficult or very costly, market structure becomes, from the their perspective, similar to the one analysed here. As a result, firms might also have incentives to keep consumers' expectations low even in oligopoly markets.

A Expected Proportion of Complaints and Updating

A.1 Expected Proportion of Complaints

For a firm type $j \in \{G, B\}$, that invested $I \in \{0, 1\}$, the expected proportion of complaints in period t is:

$$\begin{aligned}\mathbb{E}_{j,I}(\sigma_t^*(q_t;\hat{z}_t)) &= [P_{j,I}(\sigma_t^*(q_t;\hat{z}_t)=1)*1+\\ &+ P_{j,I}(\sigma_t^*(q_t;\hat{z}_t)=\frac{\theta}{c}(\hat{z}_t-q_t))\mathbb{E}_{j,I}(\sigma_t^*(q_t;\hat{z}_t)|\sigma_t^*(q_t;\hat{z}_t)\in(\bar{\delta},1))+\\ &+ P_{j,I}(\sigma_t^*(q_t;\hat{z}_t)=\bar{\delta})*\bar{\delta}]\end{aligned}$$

Using consumer' optimal strategy in (6), the expectation can be rewritten in terms of the realised level of quality:

$$\begin{aligned}\mathbb{E}_{j,I}(\sigma_t^*(q_t;\hat{z}_t)) &= [P_{j,I}(q_t\leq\hat{z}_t-\frac{c}{\theta})+\\ &+ P_{j,I}(\hat{z}_t-\frac{c}{\theta}\leq q_t\leq\hat{z}_t-\frac{\bar{\delta}c}{\theta})*\\ &* \mathbb{E}_{j,I}[\sigma_t^*(q_t;\hat{z}_t)|\hat{z}_t-\frac{c}{\theta}\leq q_t\leq\hat{z}_t-\frac{\bar{\delta}c}{\theta}]+\\ &+ P_{j,I}(\hat{z}_t-\frac{\bar{\delta}c}{\theta}\leq q_t\leq\hat{z}_t-\frac{\bar{\delta}c}{2\theta})*\bar{\delta}]\end{aligned}\qquad(8)$$

where:

$$\mathbb{E}_{j,I}[\sigma_t^*(q_t;\hat{z}_t)|\hat{z}_t-\frac{c}{\theta}\leq q_t\leq\hat{z}_t-\frac{\bar{\delta}c}{\theta}]=\frac{\theta}{c}(\hat{z}_t-E_{j,I}[q_t|\hat{z}_t-\frac{c}{\theta}\leq q_t\leq\hat{z}_t-\frac{\bar{\delta}c}{\theta}])$$

$$\mathbb{E}_{j,I}[q_t|\hat{z}_t-\frac{c}{\theta}\leq q_t\leq\hat{z}_t-\frac{\bar{\delta}c}{\theta}]=\frac{1}{P_{j,I}(\hat{z}_t-\frac{c}{\theta}\leq q_t\leq\hat{z}_t-\frac{\bar{\delta}c}{\theta})}\int_{\hat{z}_t-\frac{c}{\theta}}^{\hat{z}_t-\frac{\bar{\delta}c}{\theta}}xf(x)dx$$

The expected proportion of complaints is then a function of the distribution of quality, which in turns depend on the type and investment decision of the firm. For a good firm which invests $q_t \sim U[0,1]$. The probabilities in the above expressions, become:

$$P_{G,I=1}(q_t\leq\hat{z}_t-\frac{c}{\theta})=\hat{z}_t-\frac{c}{\theta}$$

$$P_{G,I=1}(\hat{z}_t-\frac{c}{\theta}\leq q_t\leq\hat{z}_t-\frac{\bar{\delta}c}{\theta})=\frac{c}{\theta}(1-\bar{\delta})$$

$$P_{G,I=1}(\hat{z}_t-\frac{\bar{\delta}c}{\theta}\leq q_t\leq\hat{z}_t-\frac{\bar{\delta}c}{2\theta})=\frac{\bar{\delta}c}{2\theta}$$

$$\mathbb{E}_{G,I=1}(q_t|\hat{z}_t-\frac{c}{\theta}<q_t<\hat{z}_t-\frac{\bar{\delta}c}{2\theta})=\hat{z}_t-\frac{c}{2\theta}(1+\bar{\delta})$$

$$\mathbb{E}_{G,I=1}[\sigma_t^*(q_t;\hat{z}_t)|\hat{z}_t-\frac{c}{\theta}\leq q_t\leq\hat{z}_t-\frac{\bar{\delta}c}{\theta}]=\frac{1+\bar{\delta}}{2}$$

All the above probabilities are equal than or greater than zero if $\tau > 4\psi - 1$. Substituting these results in (8), the expected proportion of complaints when the goof type of the firm invests becomes $\mathbb{E}_{G,I=1}(\sigma_t^*(q_t;\hat{z}_t)) = \hat{z}_t - \frac{\psi}{2}$, where $\psi = \frac{c}{\theta}$.

When the good type of the firm does not invest, or when the firm is bad, $q_t \sim U(0,1/2)$. The above probabilities in this case are:

$$P_{G,I=0}(q_t \leq \hat{z}_t - \frac{c}{\theta}) = P_B(q_t \leq \hat{z}_t - \frac{c}{\theta}) = 2(\hat{z}_t - \frac{c}{\theta})$$

$$P_{G,I=0}(\hat{z}_t - \frac{c}{\theta} \leq q_t \leq \hat{z}_t - \frac{\bar{\delta}c}{\theta}) = P_B(\hat{z}_t - \frac{c}{\theta} \leq q_t \leq \hat{z}_t - \frac{\bar{\delta}c}{\theta}) = 2\frac{c}{\theta}(1-\bar{\delta})$$

$$P_{G,I=0}(\hat{z}_t - \frac{c}{\theta} \leq q_t \leq \hat{z}_t - \frac{\bar{\delta}c}{\theta}) = P_B(\hat{z}_t - \frac{c}{\theta} \leq q_t \leq \hat{z}_t - \frac{\bar{\delta}c}{\theta}) = \frac{\bar{\delta}c}{\theta}$$

$$\mathbb{E}_{G,I=0}(q_t|\hat{z}_t - \frac{c}{\theta} < q_t < \hat{z}_t - \frac{\bar{\delta}c}{2\theta}) = \mathbb{E}_B(q_t|\hat{z}_t - \frac{c}{\theta} < q_t < \hat{z}_t - \frac{\bar{\delta}c}{2\theta}) = \hat{z}_t - \frac{c}{2\theta}(1+\bar{\delta})$$

$$\mathbb{E}_{G,I=0}[\sigma_t^*(q_t;\hat{z}_t)|\hat{z}_t - \frac{c}{\theta} \leq q_t \leq \hat{z}_t - \frac{\bar{\delta}c}{\theta}] = \mathbb{E}_B[\sigma_t^*(q_t;\hat{z}_t)|\hat{z}_t - \frac{c}{\theta} \leq q_t \leq \hat{z}_t - \frac{\bar{\delta}c}{\theta}] = \frac{1+\bar{\delta}}{2}$$

The expected proportion of complaints when a good firm does not invest or when the firm is of the bad type is $\mathbb{E}_{G,I=0}(\sigma_t^*(q_t;\hat{z}_t)) = \mathbb{E}_B(\sigma_t^*(q_t;\hat{z}_t)) = 2\hat{z}_t - \psi$, where $\psi = \frac{c}{\theta}$.

$\frac{c}{\theta} \geq \frac{1}{4}$, guarantees that all the above probabilities are greater than zero. In fact, this condition, together with $\tau > 0$ implies that the probabilities of $\sigma_t^* = 1$, $\sigma_t^* = \frac{\theta\gamma}{c}(\hat{q}_t - q_t)$ and $\sigma_t^* = \frac{\bar{\delta}}{\gamma}$ are all greater than zero, whichever the type of the firm. That is the reason why the condition is sufficient but not necessary for the equilibria described in the text.

A.2 Bayesian Updating

When consumers expect the good type of the firm to invest, a low realisation of quality increases the evidence in favour of the firm being bad. Bayesian updating implies:

$$\begin{aligned}
\tau_1 = P(G|q_1 < \frac{1}{2}) &= \frac{P(q_1 < \frac{1}{2}|G)P(G)}{P(q_1 < \frac{1}{2}|G)P(G) + P(q_1 < \frac{1}{2}|B)P(B)} \\
&= \frac{\frac{\tau}{2}}{\frac{\tau}{2} - (1-\tau)} \\
&= \frac{\tau}{2-\tau} \qquad (9)
\end{aligned}$$

If consumers do not expect the good type of the firm to invest, a low realisation of quality does not give them any additional information about the firm's type and so there is no updating of beliefs.

In both cases, a high (expected or unexpected) realisation of quality is fully revealing of the firm type, and implies $\tau_1 = 1$.

B Equilibria of the Repeated Game when $T \to \infty$

This Appendix extends the results of section 4 to the case in which the number of periods goes to infinity.

When the game between the firm and the consumers is repeated an infinity number of periods, the qualitative results are the same as when T is finite, namely that the region in which there is multiplicity of equilibria is reduced because the high quality equilibrium is sustainable under a smaller set of values of τ. Furthermore, the set of values of τ for which the firm optimally invests in the static game but not in the dynamic one is the set for which complaints are informative. To see that this is indeed the case, I first prove the existence of both equilibria (Claims 1 and 2) and then compare the results with those of the static game (Claim 3).

Claim 1. *Given $\hat{\hat{z}}^* \in (1/4, 1/2)$ and $T \to \infty$, there exists an equilibrium in which the good type of the firm never invests and consumers expect this behaviour.*

When $\hat{\hat{z}}^* \in (1/4, 1/2)$, there exists an equilibrium of the one shot game in which the firm does not invest and consumers expect low quality. If the game is repeated and the firm's strategy in the repeated game is $I^*_t = 0$, $\forall t = 0, 1, 2...$ (and this is the behaviour consumers anticipate), on equilibrium path there is no updating of beliefs and so every period is strategically identical to previous one. If the firm deviates an invests in any period t, there is a probability of 1/2 that the realisation of quality is high and consumers find out its true type (and expect the firm to invest after that), and another half that the the realisation is small an consumers do not find out the deviation. The condition for the existence of an equilibrium in this case is:

$$V(I^* = 0; \tau_t) \geq \mathbb{E}_{I^*=1}\Pi_t(\hat{\hat{z}}_L(\tau_t); \sigma^*) + \frac{\beta}{2}[V(I^* = 0; \tau_t) + V(I^* = 1; 1)] \tag{10}$$

where $V(I^* = 0; \tau_t) = \Sigma_h^\infty \beta^h \mathbb{E}_{I^*=0}\Pi_t(\hat{\hat{z}}_L(\tau_t); \sigma^*) = \frac{\mathbb{E}_{I^*=0}\Pi_t(\hat{\hat{z}}_L(\tau_t);\sigma^*)}{1-\beta}$ is the present value of the firm's expected profits if it never invests and consumers expect no investment. $V(I^* = 1; 1)$ is analogously defined for the case in which consumers assign probability one to the firm being good and expect the firm to invest in every period, and the firm does invest. The inequality above can be re written in the following way:

$$\mathbb{E}_{I^*=0}\Pi_t(\hat{\hat{z}}_L(\tau_t); \sigma^*) - \mathbb{E}_{I^*=1}\Pi_t(\hat{\hat{z}}_L(\tau_t); \sigma^*) \geq \beta[V(I^* = 1; 1) - V(I^* = 0; \tau_t)]$$

The left hand side of the above inequality is positive because no investing is an equilibrium of the stage game when $\hat{\hat{z}}^* \geq 1/4$ (i.e., given that consumers expect the firm not to invest, period-t profits are higher if the firm does not invest that if it invests):

$$\begin{aligned}
[p - mp\sigma^*(q; \hat{z}_L(\tau_t))] - [p - h - mp\sigma^*(q; \hat{z}_L(\tau_t))] &\geq 0 \\
\frac{h}{mp} + \frac{\psi}{2} - \frac{1}{4} &\geq 0 \\
\hat{\hat{z}}^* &\geq \frac{1}{4}
\end{aligned}$$

The right hand side of (10) is always negative: the present value of the firm's profits are always higher in an equilibrium in which consumers do no expect much than in one on which they know the firm is good and expect a good firm to invest in every period. Then, the left hand side of (10) is always greater than the right hand side and the Claim is proved.

Claim 2. *Given $\hat{\hat{z}}^* \in (1/4, 1/2)$ and $T \to \infty$, there exists τ^{***} such that if $\tau \geq \tau^{***}$ there is an equilibrium in which the firm invests in every period and consumers expect it to invest.*

Suppose the firm invested in every period before t and consumers expected it. Under the equilibrium path, quality realisations above 1/2 are fully revealing of the firm's type, while those below 1/2 constitute evidence in favour of the firm being bad. Then, if any realisation of quality up to $(t-1)$ was higher than 1/2, consumers know they are facing a good firm, but if every previous realisation was below 1/2, the probability they assign to the firm being good is $\tau_t \leq \tau$. Consumers' beliefs about the type of the firm in period t can be summarised as:

$$\tau_t = \begin{cases} 1 & \text{if } q_h > 1/2 \text{ for any } h \leq t-1 \\ \frac{\tau_{t-1}}{2-\tau_{t-1}} \leq \tau_{t-1} & \text{if } q_h \leq 1/2 \text{ for every } h \leq t-1 \end{cases}$$

If the firm invests in period t, its expected profits for that period are:

$$\mathbb{E}_{I=1}\Pi_t(\hat{\hat{z}}_H(\tau_t), \sigma_t^*) = p - h - mp\sigma^*(q_t; \hat{z}_H(\tau_t))$$

If $\tau_t < 1$, the firm's continuation value under the equilibrium strategy is:

$$\begin{aligned} V(I^* = 1; \tau_t) &= \mathbb{E}_{I_t^*=1}[\Sigma_{h=1}^{\infty}\beta^h \mathbb{E}_{I_{t+h}^*}(\Pi_{t+h}(\hat{\hat{z}}_{t+h}(\tau_t), \sigma_{t+h}^*)|q_t)] \\ &= \frac{1}{2}V(I^* = 1; 1) + \frac{1}{2}[\beta\mathbb{E}_{I_{t+1}^*=1}\Pi_{t+1}(\hat{\hat{z}}_H^*(\tau_{t+1}), \sigma^*) + \beta^2 V(I^* = 1, \tau_{t+1})] \end{aligned}$$

If the firm invests in every period after t, $V(I^* = 1; \tau_t)$ converges to $V(I^* = 1; 1)$ because the realisation of quality will eventually be high enough so that consumers find out the firm is good. Once the value function has converged, the game becomes a standard repeated game: there is no more updating of beliefs, and the payoffs do not change anymore as consumers expect the firm to invest in any following period.[55] Then, the existence of a (subgame perfect) equilibrium in which the firm invest in every period, given that consumers expect it to invest, can be proved using the one-deviation principle.[56] The firm will attach to its investment strategy if the (expected) profits from investing in every period are higher than the (expected) profits from not investing in the current period but investing in every future period:

[55]The players payoffs depend on their believes about the level of quality consumers expect to receive, which in turn depends on what consumers believe about the firm's type and strategy. According to the definition of equilibrium, consumers expect the good firm to invest in every period, so if they find out the firm is good for sure (and so τ_t does not change anymore), their beliefs about the firm's strategy are $I_t^* = 1$ for every t.

[56]The equilibrium is subgame perfect as long as the firm can not increase its payoff by changing her action at the start of any subgame given the level of quality consumers expect to receive and their optimal complaining rule, and the rest of the firm's investment strategy.

$$\frac{V(I^*=1;1)}{1-\beta} \geq \mathbb{E}_{I^*_{t+1}}\Pi_{t+1}(\hat{\hat{z}}^*(\tau_{t+1}),\sigma^*) + \frac{\beta}{1-\beta}V(I^*=1;1)$$
$$\frac{1}{1-\beta}V(I^*=1;1) \geq p - mp\sigma^*(q;\hat{z}_H(1)) + \frac{\beta}{1-\beta}V(I^*=1;1)$$
$$p - h - mp(\frac{1-\psi}{2}) \geq p - mp(1-\psi)$$

where $V(I^*=1;1) = p - h - mp\sigma^*(q;\hat{z}_H(1))$. The inequality holds as long as $\frac{1}{2} > \hat{\hat{z}}^*$, which is always the case in the range of parameters I am considering. Then, if consumers' assign probability $\tau = 1$ to the firm being good and expect a good firm to invest, no matter how many periods the game will still be played, the firm optimally invests in every period (and consumers correctly anticipate it). Once consumers know the type of the firm, there is no more possibility of "managing" consumers' future expectations and so, there are no more inter temporal incentives: if one period the firm does not invest, that period's fine is higher (because of the lowest quality realisation) but consumers' future expectations are not affected (and neither are the firm's future profits). Consumers interpret the low realised quality as "bad luck".[57]

It remains to see whether the firm would deviate from the investment strategy before the value function converges to $V(I^*=1;1)$ (i.e., at any point before a high quality is realised). If the firm invests in period t, given that all the previous quality realisations have been smaller than $1/2$ and consumers expected the firm to invest in every period, $\tau_t < \tau < 1$. Its current and future profits are:

$$p - h - mp\sigma^*(q_t;\hat{z}_H(\tau_t)) + \beta[\frac{1}{2}V(I^*=1;1) + \frac{1}{2}V(I^*=1;\tau_{t+1})]$$

If $\tau_t < 1$ and the firm does not invest, that period's low realisation increases the probability consumers assign to the bad type of the firm, and so $\tau_{t+1} < \tau_t$. If it deviates in period t and then goes back to the investment strategy, it delays by one period the moment in which consumers find out its type. The firm's current and future profits under the deviation are:

$$p - mp\sigma^*(q_t;\hat{z}_H(\tau_t)) + \beta V(I^*=1;\tau_{t+1})$$

The firm will attach to the investment strategy if the profits in the first case are higher:

$$\frac{1}{4} + \frac{1}{4}\tau_t - \frac{\beta}{2mp}[V(I^*=1,\tau_{t+1}) - V(I^*=1,1)] > \hat{\hat{z}}^* \tag{11}$$

$[V(I^*=1,\tau_{t+1}) - V(I^*=1,1)]$ is positive because the firm's continuation value is a decreasing function of τ. The difference, however, is decreasing in τ: if τ is very close to one, the scope for "managing" expectations is small and so is the expected gain from deviating in the current period. The left hand side is then an increasing function of τ, while $\hat{\hat{z}}^*$ is a constant. Then, there exists a unique τ^{***} such that for $\tau \geq \tau^{***}$ the firm invests in every period of consumers expect that.

Claim 3. *Given $\hat{\hat{z}}^* \in (1/4, 1/2)$, there exist $\tau \in (\tau^*, \tau^{***})$ such that the game has a HQE when $T = 1$ but it does not if $T \to \infty$. Furthermore, the parameters for which only the low*

[57]A different interpretation would violate perfect recall.

quality equilibrium survives the infinite repetition of the game are those for which complaints would have been informative in the one shot game. The difference ($\tau^{***} - \tau^*$) increases with β and the number of periods before the type of the firm is discovered and decreases with the initial τ.

The Claim implies that an equilibrium in which the firm invests in every period (and consumers expect it to invest) is more difficult to sustain when $T \to \infty$ than when $T = 1$ and that the degree in which the equilibrium can not be sustained is greater the more patient is the firm and the less convinced are consumers, at the beginning of the game, that the firm is good. In both cases, the result implies that the firm is more interested in attach to a high investment equilibrium the less the scope to manage consumers' future expectations. However, the less the scope to manage consumers expectations the less informative are complaints about the equilibrium being played.

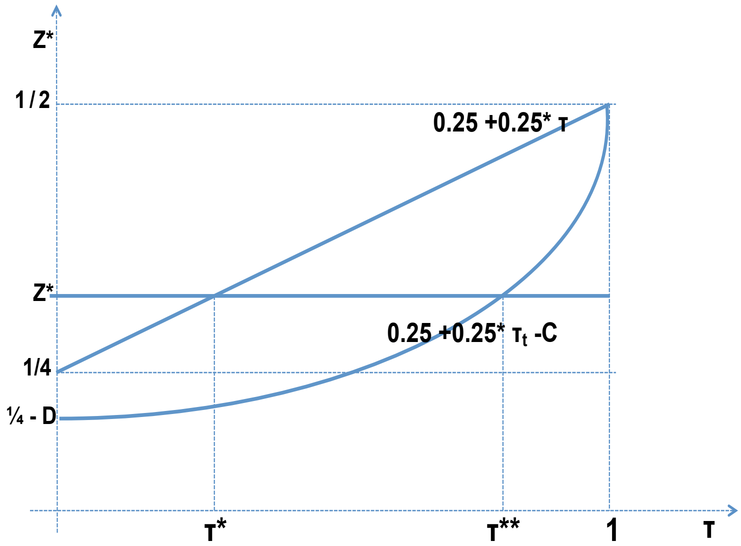
From Proposition 1, there exist $\tau > \tau^* = 4\hat{\hat{z}}^* - 1$ such that the one shot game has an equilibrium in which the good type of the firm invests and consumers expect $\hat{z}_H(\tau)$. Analogously, from Claim 2, there exist $\tau^{***} = 4\hat{\hat{z}}^* - 1 + \frac{2\beta}{mp}[V(I^* = 1, \tau_1) - V(I^* = 1, 1)]$ such that for $\tau > \tau^{***}$ and $T \to \infty$ there exists an equilibrium in which the firm invests in every period and consumers expect $\hat{z}_H(\tau_t)$ in every period. As $[V(I^* = 1, \tau_1) - V(I^* = 1, 1)] > 0$, $\tau^{***} \geq \tau^*$. The difference in the lower bound of τ between the one shot game and the infinitely repeated one is an increasing function of β and a decreasing function of the initial τ. The more patient is the firm and/or the smaller is the initial τ, the more difficult is to sustain an equilibrium in which the firm invests in every period (and consumers expect that behaviour). The difference between the two cases is shown in Figure 4. The figure shows that the dynamic incentives introduced by the repetition of the game move the right hand side of the inequality downwards and to the right, to $\frac{1}{4} + \frac{1}{4}\tau_t - C$, where $C = \frac{\beta}{2mp}[V(I^* = 1, \tau_{t+1}) - V(I^* = 1, 1)]$.[58] Thus, for every $\hat{\hat{z}}^* \in (1/4, 1/2)$, there exists a set of values of $\tau \in (\tau^*, \tau^{**})$ for which the HQE was sustainable in the one shot game but is not sustainable anymore if the game is repeated an infinity number of times.[59]

Furthermore, the same as when the game is repeated only a finite number of times, the set of $\tau \in (\tau^*, \tau^{**})$ corresponds to values of the parameters for which complaints are informative in the one shot game. When $T = 1$, complaints are informative if $1 - 2\psi \geq \tau \geq 4\hat{\hat{z}}^* - 1$, which implies $1 - 2\psi \geq \tau \geq \tau^*$. In each period of the repeated game this condition becomes $1 - 2\psi \geq \tau_t \geq \tau^{**}$. The set of values of τ for which the second condition holds is smaller because $\tau^{**} \geq \tau^*$.

[58] In the graph, $D = \frac{1}{4} - \frac{\beta}{2mp}[V(I^* = 1, 0) - V(I^* = 1, 1)]$.

[59] If after observing a low realisation of quality consumers expect the firm not to invest anymore, then the firm's incentives to attach to the investment strategy are even lower. The firm will not deviate if $\frac{1}{4} + \frac{1}{4}\tau_t - \frac{\beta}{mp}[V(I = 0, \tau_{t+1}) - \frac{V(I=1,1)+V(I=1,\tau_{t+1})}{2}] > \hat{z}^*$. The left hand side in this case is smaller and is also a decreasing function of τ.

Figure 4: Equilibria with investment when $T = \infty$ and $\hat{\hat{z}}^* \in (1/4, 1/2)$



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