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Investment-Specific Technological Change and the Length of Patents

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Implementation Cycles: Investment-Specific Technological Change and the Length of Patents *

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Abstract

This paper shows that implementation cycles, introduced in Shleifer (1986), are possible in the presence of capital and the absence of borrowing constraints. In a two-sector economy, patents on cost-saving ideas which take the form of investment-specific technological change arrive exogenously at a sequential, perfectly smooth rate: in odd-numbered periods, they reach a firm producing capital of type 1 and, in the even-numbered ones, a firm producing capital of type 2. Firms can make profits out of these once. While the immediate appropriation (henceforth, "implementation") of patents is always a possibility, for accordingly formed expectations, firms can alternatively implement their patents simultaneously. This is because investment-specific technological change naturally introduces a one-period discrepancy between the time firms implement their patents and the time they receive revenue out of them. The implementation of a patent implies a sharp fall in investment which, in turn, causes a boom in current consumption. As a result, the consumption boom takes place before the wealth boom. This not only eliminates the need to smooth consumption away from the wealth boom to the period before it as conjectured, but, further, it implies that the interest rate paid when revenue is realized -and wealth expands- falls. Consequently, present discounted profits rise and implementation cycles can become a possibility. In a policy extension, I show that prolonging patent rights to two periods rules out "implementation cycles" and may lead to a welfare improvement.

JEL Classification: D42, D51, E21, E22, E32, O33, O34.

Keywords: Implementation cycles, capital, savings, monopoly, demand externalities, multiple equilibria, patent rights

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1 Introduction

When it comes to the release of new products, companies, especially technology ones, are particularly concerned about two things: its timing and being secretive. The latter suggests that potential gains are short-lived; the former suggests that timing affects them.

This paper attempts to address three questions, related to the above remarks. First, can companies coordinate the launch of new (improved) products even though they may develop them time-separately in the presence of capital (or any storable commodity) and the absence of borrowing constraints? Second, can regulators affect this possibility by extending rights over improved technologies? And if so, will this necessarily lead to welfare improvements? Yes, yes, and perhaps.

A natural starting point in this attempt is Shleifer (1986) which introduced "implementation cycles." However, it did so in an economy where storable commodities are absent and argued that their absence is indeed indispensable.[1] This paper builds on it, allows for capital and savings and shows that, in sharp contrast with Shleifer's conjecture, implementation cycles are still possible.

More analytically, the economy consists of a representative agent who consumes a final good produced by a respective firm which he owns and to which he supplies his labor. The final-good firm is competitive and, besides labor, uses two different types of capital. For each type of capital, there is a respective sector comprising a number of Bertrand-competing capital makers. Capital makers use foregone consumption (investment) to produce the capital good they specialize in.

Suppose that in odd-numbered periods, a patent reaches randomly a firm (capital maker) in sector 1 and, in the even-numbered ones, a firm in sector 2 . Patents are on cost-reducing technologies which imply that a unit of capital requires less resources in order to be produced. Initially, as in Shleifer (1986) , I let firms make profits out of a patent only for one period; once a patent is utilized, the innovating firms' competitors costlessly copy the idea the patent was on and drive sector profits to zero -until a new patent arrives to the sector. As competitors cannot reverse-engineer an idea a patent is on before it is actually implemented, I will henceforth use the terms patent and idea interchangeably.

Firms need to decide when they will implement their patents. I show that, if they share expectations about future and have perfect foresight, multiple "sunspot" equilibria can arise: firms can either implement their patents as soon as they receive them, which implies that patents are in place at the same -perfectly smooth- rate as that of their arrival, or they may instead coordinate their

[1] See (Shleifer, 1986, page 1183) .

implementation, in which case "implementation cycles" with capital are generated.

To see this, note that imperfect competition invites demand externalities among capital-good sectors. Since a capital maker can postpone implementation of a patent, for instance, to the following period, when with certainty no improved technology will arrive, it needs to decide whether to implement it immediately or in the following period. As Matsuyama (1995) notes, it is precisely the intertemporal decision that firms face in combination with the presence of intratemporal demand externalities that can result in multiple equilibria which can be Pareto-ranked.

Nevertheless, and despite this intuition, implementation cycles in the presence of capital and the absence of borrowing constraints (or constrained investment volatility) is something that Shleifer (1986) conjectured against: anticipating future profits, agents would attempt to reduce their current savings in order to smooth out consumption. In turn, that would lower production and hence profits in a hypothetical implementation boom. For consumption smoothing to be mitigated, higher real interest rates would be necessary, which would in turn imply that firms discount future profits more. Both effects could rule out implementation cycles.

Why is this not so here? The reason is that patents are on investment-specific technological change, in the spirit of Greenwood et al. (1997, 2000). Investment-specific technological change introduces a one-period discrepancy between the consumption boom and the wealth boom. To see this, note that the implementation of a patent in the technology of a capital good reduces its current production cost, whereas the revenue out of it becomes realized in the following period. The latter implies that the wealth boom occurs one period *after* the coordinated implementation of patents takes place. The former implies that investment is substantially reduced in the implementation periods -in fact, it can even undershoot- and drives consumption above trend. As a result, the interest rate paid then is higher than the interest rate paid in the following, "wealth-boom," period. This increases investment in implementation booms, smoothing out consumption in the opposite direction from the conjectured one -without overturning the result on consumption which is a general equilibrium one-, and implies that more capital is installed in the following period which, given the elastic demand for it, leads to greater profits. Taking all into account, discounted profits after a conjectured coordinated implementation of patents become greater and, therefore, implementation cycles with capital possible.

In a policy extension, I let firms appropriate a patent for two periods. It turns out that implementation cycles become impossible. To see this, note that, in that case, postponing implementation to the following period is equivalent to postponing implementation to two periods afterwards in the one-period monopoly case. In a stationary equilibrium, every other period is the same. As a result, as long as firms discount future at a positive rate, which turns out to be always the case, the possibility of implementation cycles is ruled out.

A natural question is whether the immediate implementation of patents when patent rights last two periods is welfare-improving over the equilibria when patent rights last one period. Relative to the immediate implementation equilibrium of the latter case the answer is negative: patents diffuse faster to the implementing firms' competitors which implies that the economy reaches a certain consumption level faster. Nevertheless, relative to the cyclical equilibrium the answer can be positive. There are two effects which push in opposite directions. The first one is the one already described: patents (only of sector 2 though) diffuse faster in the cyclical equilibrium. Nevertheless, in the "two-period patent" equilibrium, two firms appropriate a patent in each period. In other words, two firms cut down on current investment every period as opposed to -them doing so- every other period in the cyclical "one-period patent" equilibrium. This effect favors consumption in the two-period patent equilibrium. For patents being on not too drastic (cost-reducing) ideas, the foregone cost of them being diffused faster does not exceed the consumption benefit of having two sectors implementing in each period, rendering, thereby, a prolongation of patent rights potentially desirable.

Related literature. The closest paper to the present one is Shleifer (1986) , the main differences with which I highlighted above. In the remaining parts of the paper, I frequently refer to how the two papers relate to each other in greater detail.[2,3]

Francois and Lloyd-Ellis (2008) also generates implementation cycles with capital which, further, can be sustained as a unique equilibrium outcome. Although, their model is quite different from the one in this paper, I will restrict attention to two key differences. A central assumption the authors make is that patents arrive after firms have incurred an endogenous search cost. A consequence of this assumption is that patents arrive simultaneously in all sectors, which is in sharp contrast with the perfectly smooth rate of their arrival in Shleifer (1986) and here: if patents arrive in cycles, they are more easily implemented in cycles as well. A second key difference is that, in

[2] A simplified version of Shleifer (1986) which could serve as an intermediate step between Shleifer's model and mine can be found on Lawrence Christiano's teaching webpage. The link to this is `http://faculty.wcas.northwestern.edu/~lchrist/d11/d1101/implement.pdf` .

[3] With expectations arbitrarily supporting one of the possible multiple equilibria, this paper relates to the "sunspots" literature, which, includes, among other articles, Azariadis (1981) , Cass and Shell (1983) and Grandmont (1985) . Benhabib and Farmer (1999) offers an overview of this literature. A complementary earlier survey with an emphasis on endogenous cycles can be found in Boldrin and Woodford (1990) .

Francois and Lloyd-Ellis (2008), patents do not affect the technology of capital but they instead affect the technology of intermediate goods which requires only labor. Consequently, the wealth and the consumption booms coincide. Here, patents affect the investment technology which naturally introduces a one-period discrepancy between the consumption and the wealth boom.[4]

Turning to the literature related to the paper's result on patent policy, this paper differs from a recent and growing literature on patent protection and intellectual property rights which includes, among other articles, Boldrin and Levine (2002, 2008b) and Henry and Ponce (2011).[5,6] This literature focuses on the incentives to innovate and analyzes whether markets for patents can substitute for the absence of patent rights. Here, innovation is exogenous, whereas allowing for a market for patents would leave the results intact.

Hopenhayn and Squintani (2010) also considers the effects of patent rights on the timing of patent releases. However it does so in a model of sequential innovation which allows for preemptive entry by an innovating firm's competitors. Consequently, the timing of patents balances the possibility of preemption and the generation of future patents. I abstract from such considerations here: firms can utilize a patent immediately or with delay without the fear of preemption and without the fear that the implementation of a patent affects their future generation of patents, which happens independently over time, randomly and costlessly.

These abstractions allow the paper to concentrate solely on the effects of patent rights on the implementation -rather than the generation- of patents and, thereby, offer a clean argument from a different -and, to the best of my knowledge, new- perspective to the ongoing -and lively- discussion about the length of patent rights.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes the equilibria when patent rights last one period. Section 4 discusses welfare. Section 5 extends patent rights to two periods and performs welfare comparisons with the equilibria which can prevail when patent rights last one period. Section 6 concludes.

[4] Certainly this paper shares features, which I will not attempt to review here, with the growth literature, prominent contributions to which include Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992), the literature on strategic delay (for instance, Chamley and Gale (1994)), as well as Matsuyama (1999) and, more recently, Jovanovic (2009). Needless to say, this list is by no means exhaustive.

[5] See also the discussion in Boldrin and Levine (2008a).

[6] This literature dates back to Schumpeter (1942) and Arrow (1962). Chapter 12 in Acemoglu (2009) offers an excellent overview of the early literature. Holmes and Schmitz (2010) offers a more general discussion on how competition affects productivity.

2 The Model

The economy is populated by an infinitely-lived (representative) agent. The agent consumes a single storable commodity (final good) produced by a (representative) final-good firm to which he supplies labor. Further, the final-good firm uses two storable (composite) capital goods in an additively-separable way. For each capital good, there is a respective sector which comprises at least two capital-good firms Bertrand-competing for its production. It takes one period to produce a capital good and foregone consumption (investment) is used as input in its production. Capital-good firms become the recipients of patents on cost-saving ideas (henceforth, simply “patents”) and, once they implement these, they can make temporary monopoly profits before being imitated by their competitors. I elaborate more on this last -and central- feature of the economy below.

There is no uncertainty; agents and firms share expectations about the future and have perfect foresight. Agents (firms) can perfectly borrow against their future profits (revenue). Time is discrete and infinite and commences in period 1.

2.1 Representative agent

The preferences of the agent are given by

$$\sum_{t=1}^{\infty} \beta^{t-1} U(x_t\,,\, l_t) \tag{1}$$

with

$$U(x_t\,,\, l_t) \,=\, \log x_t \,+\, \chi \log l_t\,, \tag{2}$$

where x_t denotes consumption of the final good, l_t denotes leisure, $\beta \in (0,1)$ parametrizes the agent's time preference, and $\chi > 0$ parametrizes the relative weight on leisure within the period utility of the agent.[7]

The agent is endowed with one unit of time, owns all firms in the economy and can freely borrow against his perfectly foreseen future profits. This last assumption is essentially an assumption of perfect capital markets and allows me to use the agent's intertemporal budget constraint given by

$$\sum_{t=1}^{\infty} m_t\, x_t \,\leq\, \sum_{t=1}^{\infty} m_t \left[w_t\,(1\,-\,l_t) \,+\, \Pi_t^f \,+\, \sum_{i=1}^{2} \Pi_{t,i}\right], \tag{3}$$

[7] The intertemporal elasticity of substitution is 1. However, the implementation cycles that I specify below are more easily generated the greater the intertemporal elasticity of substitution is.

where $m_t = \frac{1}{R_1 \ldots R_{t-1}}$ for $t > 1$, with $m_1 = 1$; R_t denotes the gross real interest rate paid in period $t+1$, w_t denotes the real wage paid by the final-good firm, and Π_t^f and $\Pi_{t,i}$ denote the profits that accrue to the agent by the final-good firm and capital-good firm i for $i = 1, 2$, respectively. All prices and the real interest rate are expressed in units of the final good.

The agent chooses $\{(x_t\,,\, l_t)_{t=1}^{\infty}\}$, where $x_t > 0$ and $l_t \in (0,1]$ to maximize his lifetime utility given by (1)-(2) subject to his intertemporal budget constraint given by (3). The first order conditions with respect to x_t and l_t imply the following relations:

$$\frac{x_{t+1}}{x_t} = \beta\, R_t \tag{4}$$

$$\frac{x_{t+1}}{x_{t-1}} = \beta^2\, R_t\, R_{t-1} \tag{5}$$

$$\frac{l_{t+1}}{l_t} = \beta\, R_t\, \frac{w_t}{w_{t+1}} \tag{6}$$

$$\frac{l_{t+1}}{l_{t-1}} = \beta^2\, R_t\, R_{t-1}\, \frac{w_{t-1}}{w_{t+1}}\,. \tag{7}$$

2.2 Final-good firm

The final-good firm is competitive in both the final-good and the input markets. Its (neoclassical) technology is given by

$$F\,(n_t\,,\, k_{t,1}\,,\, k_{t,2}) \;=\; n_t^{\alpha}\,(k_{t,1}^{1-\alpha} \,+\, k_{t,2}^{1-\alpha})\,, \tag{8}$$

where n_t denotes employed labor, $k_{t,1}$ and $k_{t,2}$ denote capital of types 1 and 2, respectively, rented[8] in period t and $\alpha \in (0,1)$ measures the labor share and the inverse elasticity of substitution between the two types of capital.[9] Note that the marginal products of the two capital types are conditionally -on labor- independent of each other as in Romer (1990). I assume that capital depreciates fully

[8] I assume that final-good firms rent rather than buy capital in order to avoid possible problems with the final-good firm buying capital in advance in order to use it in the future.

[9] The technology of the final-good firm can be alternatively expressed as $F\,(n_t, K_t) \;=\; n_t^{\alpha}\, K_t^{1-\alpha}$, where $K_t \equiv (k_{t,1}^{\frac{\epsilon_\alpha - 1}{\epsilon_\alpha}} + k_{t,2}^{\frac{\epsilon_\alpha - 1}{\epsilon_\alpha}})^{\frac{\epsilon_\alpha}{\epsilon_\alpha - 1}}$ with $\epsilon_\alpha = \frac{1}{\alpha}$.

within a period.[10]

In each period t, the final-good firm chooses $\{n_t, k_{t,1}, k_{t,2}, y_t\}$ (all in non-negative quantities) to maximize its temporal profits given by

$$\Pi_t^f = y_t - w_t n_t - q_{t,1} k_{t,1} - q_{t,2} k_{t,2},$$

where $y_t \leq F(n_t, k_{t,1}, k_{t,2})$ with $F(\cdot)$ given by (8); $q_{t,i}$ denotes the real rental price of capital type i for $i = 1, 2$. In period 1, the firm is endowed with quantities of capital $k_{1,1}$ and $k_{1,2}$, on which I elaborate below and in Section 4.1.

The firm's maximization problem yields

$$w_t = \alpha n_t^{\alpha-1} (k_{t,1}^{1-\alpha} + k_{t,2}^{1-\alpha}) \tag{9}$$

$$q_{t,i} = (1-\alpha) n_t^{\alpha} k_{t,i}^{-\alpha}, \text{ for i} = 1, 2. \tag{10}$$

Observe that both demand functions are elastic ($\epsilon_{w,n} = -\frac{1}{1-\alpha}$ and $\epsilon_{q,k_i} = -\frac{1}{\alpha}$, respectively) and that profits are zero in each period, an implication of constant returns to scale.

2.3 Capital-good firms

I consider two types of capital goods.[11] For each capital good, there is a number of capital makers which Bertrand-compete for its production with no capacity constraints. They are indexed by j and j' for firms 1 and 2, respectively, where $j = 1, 2, \ldots, J$ and $j' = 1, 2, \ldots, J'$ with $J, J' \geq 2$.

Capital-good firms operate a constant-returns-to-scale technology. For instance, the technology of capital maker j manufacturing capital of type 1 is given by

$$k_{t,1,j} = \psi_{t-1,1,j}\, i_{t-1,1,j}, \tag{11}$$

[10] Since capital depreciates fully within a period and with demand for it being positive in equilibrium, it may be pointed out that this is essentially an irreversibility constraint since it effectively rules out disinvestment. That may be true at face value, nevertheless, at the same time, it implies that investment has the highest possible volatility as each period the economy needs new capital to be produced. See also fn. 33.

[11] This is an abstraction. In fact, I need two *baskets* of capital goods with each basket including a sufficiently high number of capital goods so that each capital-good firm is small enough within its "basket." The desired implication of this assumption is that capital-good firms take aggregate outcomes as given; put differently, any strategic interactions among firms within and across baskets are ruled out. For simplicity, let each basket contain a continuum of unit measure of capital goods. That is $k_{t,i} = \int_{i' \in [0,1]} k_{t,i,i'}\, di'$, where $k_{t,i,i'}$ denotes capital good i' in basket i in t. Assuming a symmetric treatment of capital goods within a composite capital good allows me henceforth, for expositional clarity, to mention capital-good type i and actually refer to the representative capital good (i, i') in basket i.

where $k_{t,1,j}$ denotes capital of type 1 produced by capital maker j in period t and $i_{t-1,1,j}$ denotes capital maker j's investment, which is foregone consumption, in period $t-1$ used as input in the production of capital type 1 . Likewise for capital good of type 2 (see also fn. 10) .

Aggregate investment in t is

$$i_t = \sum_{j=1}^{J} i_{t,1,j} + \sum_{j'=1}^{J'} i_{t,2,j'} . \tag{12}$$

Supposing that, before the economy starts, firms j and j' in sectors 1 and 2 , respectively, have no inferior technology relative to that of their competitors, the initial level of investment required is given by

$$i_0 = \frac{k_{1,1,j}}{\psi_{0,1,-j}} + \frac{k_{1,2,j'}}{\psi_{0,2,-j'}} , \tag{13}$$

where $\psi_{0,1,-j}$ and $\psi_{0,2,-j'}$ denote the technology level of their competitors before the economy starts.

Since I focus on the balanced growth path (steady state) of the economy, I treat time as if had commenced in $-\infty$. This implies that capital in period 1 summarizes the state of an economy which started in $-\infty$. Equivalently, an economy which starts in period 1 must do so with the "right" levels of capital. I resume this discussion in Section 4.1 .

2.3.1 Pattern of patents

I make two assumptions on the arrival of patents which are as in Shleifer (1986) .

Assumption 1. Patents on improved technologies arrive exogenously. They reach sectors sequentially, at a perfectly smooth rate. With no loss of generality, a patent reaches randomly a firm in capital sector 1 in odd periods and a firm in capital sector 2 in the even ones. Patents are on ideas that affect the technology of capital of type i in the following way:

$$\frac{\psi_{t+1,i,j}}{\psi_{t-1,i,j''}} = \mu ,$$

where $\mu > 1$ and $\psi_{\cdot,i,j}$ denotes the state-of-the-art technology (inverse marginal cost of capital) in sector i possessed by firm j .[12] As time commences in period 1 , an odd period, imposing $\psi_{0,1,j} = 1$ and $\psi_{0,2,j'} = \mu^{\frac{1}{2}}$ removes the first-mover advantage of sector 1, thereby ensuring the symmetric treatment of capital-good sectors: the lead in the technology race alternates between sectors with

[12] Observe that the economy encompasses only investment-specific technological change as in Greenwood et al. (1997) . For expositional reasons, I completely abstract from total factor productivity (TFP) .

their relative technology "distance" remaining fixed at $\mu^{\frac{1}{2}}$. Put differently, it is as if time commenced in $-\infty$.[13]

Assumption 2. A firm can appropriate an idea a patent is on only for *one* period: in the period following its implementation, this idea becomes publicly disclosed and imitators enter driving prices down to marginal cost and profits to zero.[14]

2.3.2 Profits

Profits of capital maker j [15] which produces capital good i are given by

$$\Pi_{t,i,j} = q_{t,i,j}\, k_{t,i,j} - R_{t-1}\, i_{t-1,i,j}\, .$$

Capital maker j of capital good i chooses $\{k_{t,i,j}\, ,\, i_{t-1,i,j}\}$ for each t to maximize its profits subject to the technology given by (11). Since revenue is realized one period after investment is made, I allow capital-good firms to be able to perfectly borrow against their future revenue.

Below I distinguish between two cases: in the first case, all firms within sector i operate the same technology in which case a firm, say j , is randomly selected to produce capital of type i . In the second case, firm j has a technological advantage over its competitors which allows it to enjoy monopoly profits.

Perfect competition. When $\psi_{t,i,j} = \psi_{t,i,-j}$, where $\psi_{\cdot,\cdot,-j}$ denotes the inverse marginal cost of the competitors of capital maker j , there is perfect competition which, given the constant returns to scale, implies zero profits.

Firm j in sector i supplies the capital good i at price

$$q^{*}_{t,i,j} = \frac{R_{t-1}}{\psi_{t-1,i,-j}}\, . \tag{14}$$

[13] Although I have not explored this case, presumably the assumption that patents arrive periodically could be partially relaxed; what really matters is that the probability with which a patent reaches a particular sector before the indicated time is sufficiently low. The fact that patents reach the economy at a perfectly smooth rate renders the generation of cycles harder and crystallizes the forces which underpin them. If patents arrived in a cyclical fashion, then their cyclical implementation would perhaps come as no surprise. In fact, this is what underpins the result in Francois and Lloyd-Ellis (2008), in which, though, the timing of the patent arrival is endogenous.

[14] The fact that firms make temporary monopoly profits is an assumption in line with Shleifer (1986) as well as the Schumpeterian growth literature originating in Aghion and Howitt (1992), which also focuses on quality improvements ("process innovations"); it is in contrast with Romer (1990) in which firms' rights over the *use* of an idea last forever. Further, unlike the endogenous growth literature, this paper entirely abstracts from issues concerning the generation of patents. This will prove useful in Section 5 where I extend the horizon of patent rights to two periods, as it will enable me to focus on the "implementation" effects of patent rights and to entirely abstract from their effect on the generation of patents, an issue which the literature traditionally studies.

[15] With a slight abuse of notation, henceforth I will make no distinction between j and j' .

Inverse demand for capital from the final-good firm (10) pins down the competitive quantity given by

$$k^*_{t,i,j} \;=\; \left(\frac{(1-\alpha)\,\psi_{t-1,i,-j}}{R_{t-1}}\right)^{\frac{1}{\alpha}} n_t\,. \tag{15}$$

Monopoly in the presence of a competitive fringe. Capital maker j implementing a patent in the production of capital type i, for instance, in period $t-1$ enjoys monopoly profits in the following period, t, as it takes one period to build capital. Capital maker j chooses $k_{t,i,j}$ to maximize its profits given by

$$\Pi_{t,i,j} \;=\; q_{t,i,j}\,k_{t,i,j} \;-\; R_{t-1}\,\frac{k_{t,i,j}}{\mu\,\psi_{t-1,i,-j}}\,, \tag{16}$$

subject to the (inverse) demand for capital given by (10). Since demand for capital is elastic, the solution is well defined:

$$q^m_{t,i,j} \;=\; \frac{R_{t-1}}{(1-\alpha)\,\mu\,\psi_{t-1,i,-j}} \quad \text{and} \quad k^m_{t,i,j} \;=\; \left(\frac{(1-\alpha)^2\,\mu\,\psi_{t-1,i,-j}}{R_{t-1}}\right)^{\frac{1}{\alpha}} n_t\,.$$

Henceforth, I restrict attention to the case in which $q^m_{t,i,j} \;\geq\; q^*_{t,i,j}$. In this case, the "limit" price $q^*_{t,i,j}$ is set: a capital maker which implements a patent cannot charge more than the price its competitors would set, $q^*_{t,i,j}$, else its competitors would undercut it and capture the whole market. Limit pricing takes place (i.e. $q^m_{t,i,j} \;\geq\; q^*_{t,i,j}$) when

$$\mu\,(1-\alpha) \;\leq\; 1\,. \tag{17}$$

The lower the innovation rate, μ, and the less elastic the demand for capital is, the more easily (17) is satisfied. For a certain level of the elasticity of the demand for capital, condition (17) imposes an upper bound on the innovation rate. Likewise, for a certain innovation rate, (17) imposes an upper bound on the elasticity of capital demand.[16]

Then, a monopolist sells a quantity given by (15), which corresponds to the technology level of its competitors, at the price its competitors would set, given by (14), and makes profits because of its lower -by μ relative to its competitors- marginal cost of producing a unit of capital. Combining (16) with (14) and (15) implies that the monopolist's profits are given by

[16] For $\alpha \,=\, 2/3$, this implies that $\mu \,\leq\, 3$. In R&D theory, the complementary case in which $q^m_{t,i,j} \,<\, q^*_{t,i,j}$ refers to "drastic innovations." See also Chapter 12 in Acemoglu (2009).

$$\Pi_{t,i,j} \;=\; (1-\alpha)^{\frac{1}{\alpha}}\, \psi_{t-1,i,-j}^{\frac{1}{\alpha}-1} \left(\frac{\mu-1}{\mu}\right) \frac{n_t}{R_{t-1}^{\frac{1}{\alpha}-1}}\,. \tag{18}$$

Profits depend negatively on the interest rate paid in the period they are made. This is because demand for capital is elastic and a higher real interest rate implies that capital becomes more costly. That the demand for capital is elastic also explains the increase of profits in the technology level of a monopolist's competitors. In addition, profits depend proportionally on contemporaneous employment by virtue of the technology of the final-good firm.

2.3.3 The implementation decision

As I have already argued, the implementation of a patent results in profits in the following period. In the presence of two capital-good sectors, suppose that the recipient of a patent needs to decide whether to implement it immediately or in the following period.

A capital maker j in sector i receiving a patent in, say, period $t-1$ will implement it immediately rather than in the following period as long as its present discounted period t profits exceed its present discounted $t+1$ profits. That is (superscripts denote the date a patent arrives and time in subscripts refers to the date profits are made) , it must be that

$$\frac{\Pi_{t,i,j}^{t-1}}{R_{t-1}} \;\geq\; \frac{\Pi_{t+1,i,j}^{t-1}}{R_{t-1}\,R_t}\,.$$

By (18) , this boils down to

$$\frac{n_t}{n_{t+1}} \left(\frac{R_t}{R_{t-1}}\right)^{\frac{1}{\alpha}-1} \;\geq\; \frac{1}{R_t}\,. \tag{19}$$

In the complementary case in which

$$\frac{n_t}{n_{t+1}} \left(\frac{R_t}{R_{t-1}}\right)^{\frac{1}{\alpha}-1} \;<\; \frac{1}{R_t}\,, \tag{20}$$

capital maker j prefers to postpone implementation to the following period.

Implicit in these is that a capital maker cannot affect the real interest rate by its actions alone (see also fn. 11) .

Last, note that, even though profits depend on the competitors' technology level at the period of implementation, the implementation decision is independent of it.

3 Balanced Growth Path Equilibria

I restrict attention to perfect-foresight, balanced growth path equilibria at which the period of cycles is constant.

On the balanced growth path (BGP) , consumption, production, and investment grow all at the same long-run rate. However, since, by construction, a patent reaches a sector every other period, the growth rate of the economy's variables may differ between odd and even periods within a cycle. I impose the following stationarity conditions on consumption:

$$\frac{x_{\tau+1}}{x_{\tau}} = v \tag{21}$$

$$\frac{x_{\tau+1}}{x_{\tau-1}} = \lambda\,, \tag{22}$$

where τ denote an even period, with no loss of generality (see also fn. 19) . Combining (21) and (22) with (4) and (5) implies

$$R_t = \begin{cases} \frac{v}{\beta} & \text{if } t = \tau \\ \frac{\lambda}{v\,\beta} & \text{if } t = \tau + 1 \end{cases} . \tag{23}$$

Combining (22) with (5) leads to the following remark:

Remark 1. $R_\tau\, R_{\tau+1} = \frac{\lambda}{\beta^2}$.

Next, I provide the equilibrium definition:

Definition 1 (BGP Equilibrium)**.** *A perfect-foresight (periodic) balanced growth path equilibrium is a set of interest rates* $\{R_t\}_{t=1}^{\infty}$ *and prices* $\{w_t\,,\,\{q_{t,i}\}_{i=1}^{2}\}_{t=1}^{\infty}$ *, an allocation* $\{l_t,\,x_t\}_{t=1}^{\infty}$ *for the representative agent, an allocation* $\{n_t\,,\,\{k_{t,i}^{d}\}_{i=1}^{2}\,,\,y_t\}_{t=1}^{\infty}$ *for the final-good firm, and an allocation* $(\{k_{t,i}^{s}\,,\,i_{t,i}\}_{t=1}^{\infty})_{i=1,2}$ *for the technology-leading firms in the capital-good sectors,*[17] *such that*

1. *(Stationarity) Stationarity (steady-state) conditions (21) and (22) are satisfied.*

2. *(Optimality) The allocations of the agent, the final-good firm and the leaders in the capital-good sectors solve their problems, laid out in Section 2 , at the stated prices.*

3. *(Market clearing)* $k_{t,i}^{d} = k_{t,i}^{s} \equiv k_{t,i}$ *for all* i *,* $n_t + l_t = 1$ *and* $y_t = x_t + i_t$ *where* $i_t = i_{t,1} + i_{t,2}$ *, for all* t *.*

[17] $k_{t,i}$ refers to capital produced by *sector* i . Furthermore, as I have already mentioned, if all firms within a sector have the same technology level, a capital-good firm is randomly chosen.

4. (Consistency) For expectations arbitrarily centered around an equilibrium ("sunspots"), capital-good firms must find it optimal to implement their patents as conjectured.

5. (No storage) No storage takes place.

I start with the no-storage condition (requirement (5) in Definition 1):[18]

Condition 1 (No storage)**.** *$R_t > 1$ for all t rules out storage in equilibrium.*

Proof. See the Appendix. □

As my focus is on stationary equilibria, I will restrict attention to just period τ, an even period, and the periods before and after it. Hence, following the analysis in Section 2.3.1, in period $\tau - 1$, an odd period, a patent reaches a firm in sector 1 and the state-of-the-art technology, irrespectively of whether the patent is implemented or not, becomes $\mu\,\psi$, greater by by μ compared to its assumed previous level ψ, whereas in sector 2 it remains $\mu^{1/2}\,\psi$; in period τ a patent reaches a firm in sector 2 in which the state-of-the-art technology becomes $\mu^{3/2}\,\psi$, whereas in sector 1 it remains $\mu\,\psi$. As I have already pointed out, the ratio of leading technologies in the two sectors equals $\mu^{1/2}$ in odd periods and $\frac{1}{\mu^{1/2}}$ in even ones; that is, the lead of the patent race alternates between sectors ad infinitum.

I analyze two perfect-foresight equilibria: an acyclical, immediate implementation equilibrium and a cyclical, synchronized implementation one. In the former, capital makers implement a patent as soon as they receive it. In the latter, the capital maker receiving a patent first (henceforth, "firm 1") waits and implements it together with the capital maker receiving a patent second (henceforth, "firm 2"); that is patents are implemented in even periods.[19]

For each equilibrium, I first center expectations around it and ensure that requirements (1) - (3) in Definition 1 are satisfied. Next, I specify the conditions under which the conjectured timing of the patents' implementation is optimal for the capital-good firms (requirement (4) in Definition 1). Last, I confirm that the no-storage condition is met and pin down the transversality condition.

[18] The storage technology I assume is one-to-one.

[19] Although I have not explored this possibility explicitly, I find no reason for why there cannot be a symmetric equilibrium in which patents are implemented in odd periods, which, in fact, could well be the case under the premise that period 1 summarizes the state of an economy starting in $-\infty$. This would require simply letting τ denote an odd period. Since time however starts in an odd period, welfare in the two cyclical equilibria will be different. See my conjecture on that in fn. 42.

3.1 Immediate implementation equilibrium

In the immediate (acyclical) implementation equilibrium, firms expect each other to implement their patents immediately.

Period $\tau-1$ **.** Firm 1 receives a patent which it immediately implements. Since I assume throughout that condition (17) holds, firm 1 sets the same price as its competitors would, given by (14), and produce the quantity which the technology level of their competitors justifies, given by (15). In line with the analysis above, the technology levels in the two sectors are $\psi_{\tau-1,1,-j} = \psi_{\tau-2,1} = \psi < \psi_{\tau-1,1,j} = \mu\,\psi$, since $\mu > 1$, and $\psi_{\tau-1,2} = \mu^{\frac{1}{2}}\,\psi$, respectively.[20] Then,

$$k_{\tau,1} = \left(\frac{(1-\alpha)\,\psi}{R_{\tau-1}}\right)^{\frac{1}{\alpha}}(1-l_\tau) \tag{24}$$

$$k_{\tau,2} = \left(\frac{(1-\alpha)\,\mu^{\frac{1}{2}}\,\psi}{R_{\tau-1}}\right)^{\frac{1}{\alpha}}(1-l_\tau)\,. \tag{25}$$

Investment by (11) and (12) is

$$i_{\tau-1} = \frac{k_{\tau,1}}{\mu\,\psi} + \frac{k_{\tau,2}}{\mu^{\frac{1}{2}}\,\psi}\,. \tag{26}$$

Combined with (24) and (25), (26) becomes

$$i_{\tau-1} = \left[\mu^{-1} + \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right]\psi^{\frac{1}{\alpha}-1}(1-\alpha)^{\frac{1}{\alpha}}\,\frac{(1-l_\tau)}{R_{\tau-1}^{\frac{1}{\alpha}}}\,. \tag{27}$$

Observe that investment depends negatively on the interest rate paid in the following period, whereas it is proportionally related to the following period's employment, an implication of the constant-returns-to-scale technology of the final good.

Similarly, since $\psi_{\tau-2,1,-j} = \psi$ and $\psi_{\tau-2,2,-j} = \mu^{-\frac{1}{2}}\,\psi$,

$$k_{\tau-1,1} = \left(\frac{(1-\alpha)\,\psi}{R_{\tau-2}}\right)^{\frac{1}{\alpha}}(1-l_{\tau-1}) \tag{28}$$

$$k_{\tau-1,2} = \left(\frac{(1-\alpha)\,\mu^{-\frac{1}{2}}\,\psi}{R_{\tau-2}}\right)^{\frac{1}{\alpha}}(1-l_{\tau-1})\,. \tag{29}$$

[20] Whenever I omit j or $-j$ from the technology subscript, I refers to all firms within a sector.

Combining (8) with (28) and (29) yields

$$y_{\tau-1} \;=\; \left[1+\mu^{-\frac{1}{2}(\frac{1}{\alpha}-1)}\right]\,\psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}-1}\,\frac{(1-l_{\tau-1})}{R_{\tau-2}^{\frac{1}{\alpha}-1}}\,. \tag{30}$$

Output depends negatively on the interest rate paid currently: the higher the current interest rate the lower the investment in capital and, thus, the lower current production is.

Market clearing in the final-good market implies that consumption is given by

$$x_{\tau-1} \;=\; \psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\;\left[\frac{\left(1+\mu^{-\frac{1}{2}(\frac{1}{\alpha}-1)}\right)(1-l_{\tau-1})}{(1-\alpha)\,R_{\tau-2}^{\frac{1}{\alpha}-1}}\;-\;\frac{\left(\mu^{-1}\;+\;\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right)(1-l_{\tau})}{R_{\tau-1}^{\frac{1}{\alpha}}}\right]\,. \tag{31}$$

Period τ . Firm 2 receives and immediately implements a patent. In period τ, technology in the two sectors is $\psi_{\tau,1,} \;=\; \mu\,\psi$ and $\psi_{\tau,2,-j} \;=\; \mu^{\frac{1}{2}}\,\psi \;<\; \psi_{\tau,2,j} \;=\; \mu^{\frac{3}{2}}\,\psi$. Capital in the following period is given by

$$k_{\tau+1,1} \;=\; \left(\frac{(1-\alpha)\,\mu\,\psi}{R_\tau}\right)^{\frac{1}{\alpha}}\,(1-l_{\tau+1}) \tag{32}$$

$$k_{\tau+1,2} \;=\; \left(\frac{(1-\alpha)\,\mu^{\frac{1}{2}}\,\psi}{R_\tau}\right)^{\frac{1}{\alpha}}\,(1-l_{\tau+1})\,. \tag{33}$$

By (11) and (12) , investment is

$$i_\tau \;=\; \frac{k_{\tau+1,1}}{\mu\,\psi} \;+\; \frac{k_{\tau+1,2}}{\mu^{\frac{3}{2}}\,\psi}\,,$$

which combined with (32) and (33) becomes

$$i_\tau \;=\; \left[\mu^{\frac{1}{\alpha}-1}+\mu^{\frac{1}{2\alpha}-\frac{3}{2}}\right]\,\psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\,\frac{(1-l_{\tau+1})}{R_\tau^{\frac{1}{\alpha}}}\,. \tag{34}$$

Substituting (24) and (25) in (8) yields

$$y_\tau \;=\; \left[1+\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right]\,\psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}-1}\,\frac{(1-l_\tau)}{R_{\tau-1}^{\frac{1}{\alpha}-1}}\,.$$

Market clearing in the final-good market implies that consumption is

$$x_\tau \;=\; \psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\;\left[\frac{\left(1+\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right)\,(1-l_\tau)}{(1-\alpha)\,R_{\tau-1}^{\frac{1}{\alpha}-1}}\;-\;\frac{\left(\mu^{\frac{1}{\alpha}-1}\;+\;\mu^{\frac{1}{2\alpha}-\frac{3}{2}}\right)\,(1-l_{\tau+1})}{R_\tau^{\frac{1}{\alpha}}}\right]\,. \tag{35}$$

Period $\tau + 1$**.** Firm 1 receives and implements a patent. Technology in the two sectors is $\psi_{\tau+1,1,-j} = \mu\,\psi < \psi_{\tau+1,1,j} = \mu^2\,\psi$ and $\psi_{\tau+1,2} = \mu^{\frac{3}{2}}\,\psi$. Then,

$$k_{\tau+2,1} = \left(\frac{(1-\alpha)\,\mu\,\psi}{R_{\tau+1}}\right)^{\frac{1}{\alpha}} (1 - l_{\tau+1}) \tag{36}$$

$$k_{\tau+2,2} = \left(\frac{(1-\alpha)\,\mu^{\frac{3}{2}}\,\psi}{R_{\tau+1}}\right)^{\frac{1}{\alpha}} (1 - l_{\tau+2})\,. \tag{37}$$

Proceeding in the same way as before, investment is given by

$$i_{\tau+1} = \frac{k_{\tau+2,1}}{\mu^2\,\psi} + \frac{k_{\tau+2,2}}{\mu^{\frac{3}{2}}\,\psi}\,,$$

which combined with (36) and (37) becomes

$$i_{\tau+1} = \left[\mu^{\frac{1}{\alpha}-2} + \mu^{\frac{3}{2}(\frac{1}{\alpha}-1)}\right] \psi^{\frac{1}{\alpha}-1} (1-\alpha)^{\frac{1}{\alpha}} \frac{(1-l_{\tau+2})}{R_{\tau+1}^{\frac{1}{\alpha}}}\,.$$

Substituting (32) and (33) into (8) yields

$$y_{\tau+1} = \left[\mu^{\frac{1}{\alpha}-1} + \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right] \psi^{\frac{1}{\alpha}-1} (1-\alpha)^{\frac{1}{\alpha}-1} \frac{(1-l_{\tau+1})}{R_{\tau}^{\frac{1}{\alpha}-1}}\,.$$

Market clearing in the final-good market implies that consumption is

$$x_{\tau+1} = \psi^{\frac{1}{\alpha}-1} (1-\alpha)^{\frac{1}{\alpha}} \left[\frac{\left(\mu^{\frac{1}{\alpha}-1} + \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right)(1-l_{\tau+1})}{(1-\alpha)\,R_{\tau}^{\frac{1}{\alpha}-1}} - \frac{\left(\mu^{\frac{1}{\alpha}-2} + \mu^{\frac{3}{2}(\frac{1}{\alpha}-1)}\right)(1-l_{\tau+2})}{R_{\tau+1}^{\frac{1}{\alpha}}}\right]. \tag{38}$$

The above satisfy optimality and market clearing, that is requirements (2) - (3) of the equilibrium definition. Next, I impose stationarity and check for consistency and no storage in turn.

For accordingly formed expectations, an immediate implementation (acyclical) equilibrium is sustained as long as each firm which receives a patent finds it optimal to implement it immediately. I split this into two steps, as in Shleifer (1986), which I label "Profit Condition 1 (IPC1)" and "Profit Condition 2 (IPC2)."

Profit condition 1. In the acyclical equilibrium the economy grows at a constant rate. By symmetry, employment and the real interest rate remain constant across periods, that is $l_\tau = l_{\tau+1} \equiv l$ and $R_{\tau-1} = R_\tau \equiv R$ (see also fn. 21).

A firm prefers to implement a patent immediately rather than in the following period if and only if condition (19) is satisfied. Given that employment and the real interest rate are constant, condition (19) simplifies to

$$R \geq 1\,. \tag{IPC1}$$

Profit condition 2. I look for the condition under which no firm receiving a patent has an incentive to wait for two periods irrespectively of the fact that a new patent will arrive in its sector rendering the one in question obsolete. That is interest rates must be such that no firm has an incentive to wait "too much." It must be then that

$$\frac{\Pi_{t,i}^{t-1}}{R_{t-1}} \geq \frac{\Pi_{t+2,i}^{t-1}}{R_{t-1}\,R_t\,R_{t+1}}\,.$$

Since $R_{t-1} = R_t \equiv R$, the above condition boils down once again to

$$R \geq 1\,. \tag{IPC2}$$

Profit condition 2 ensures that firm 1 does not postpone implementation to the next odd period and firm 2 to the next even one.

Combining profit conditions (IPC1) and (IPC2) implies that no firm postpones implementation to any period after the next odd (for firm 1) or even (for firm 2) one either. To see this, note that profit condition (IPC2) implies that, for instance, firm 1 does not wait until the next odd period, whereas by profit condition (IPC1), it does not wait until the even period following this, and so forth, and likewise for firm 2.

3.1.1 Balanced growth path

As I have already noted, by symmetry, $l_{odd} = l_{even} \equiv l$ and $R_{\tau-1} = R_\tau \equiv R$.[21] Then, it follows from (23) that $v = \lambda^{\frac{1}{2}}$ and from (22), (31) and (38) together that $\lambda = \mu^{\frac{1}{\alpha}-1}$. Then, by (23), the real interest rate along the balanced growth path is $R = \frac{\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}}{\beta}$. Since $\mu > 1$, $\alpha \in (0\,,1)$ and $\beta \in (0\,,1)$, it follows that $R > 1$, which satisfies profit conditions (IPC1) and (IPC2) as well as the no-storage condition (Condition 1).[22]

[21] One can confirm that this is in fact a unique stationary solution by taking the same steps as in the synchronized implementation equilibrium. The steps in the case of the synchronized implementation equilibrium are collected in the Appendix.

[22] Since $R > 1$, the transversality condition, which I show for the synchronized implementation equilibrium in the Appendix, always holds.

These lead to the following corollary:

Corollary 1 (Steady-growth)**.** *An acyclical (steady-growth) equilibrium is always possible for accordingly formed expectations.*

Further, the endogenous variables evolve as follows on the balanced growth path:[23]

$$\frac{y_{t+1}}{y_t} = \frac{i_{t+1}}{i_t} = \frac{x_{t+1}}{x_t} = \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\ , \text{ for all } t \tag{39}$$

$$l = \left[\frac{\alpha}{\chi}\frac{\mu\,(1 + \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)})}{\mu - \beta(1-\alpha) + (1-\beta(1-\alpha))\mu^{1+\frac{1}{2}(\frac{1}{\alpha}-1)}} + 1\right]^{-1}. \tag{40}$$

That is, in the acyclical (baseline) equilibrium, output, consumption, and investment grow at the same constant rate, whereas employment remains constant across time.

A higher innovation rate, μ, sets the economy onto a steeper growth path and results in a higher real interest rate, while a lower subjective discount factor β also calls for a higher interest rate. Turning to leisure, it increases in μ and the relative taste parameter for leisure χ and decreases in β.

3.2 Synchronized implementation equilibrium

I focus on the synchronized (*cyclical*) implementation equilibrium at which, firm 1, which receives a patent in an odd period, finds it optimal to save it and implement it in the following even period together with firm 2 which receives a patent then (see also fn. 19).

Period $\tau - 1$. A patent reaches firm 1 and is not implemented but is instead stored and implemented in τ. Effectively, from the viewpoint of $\tau - 1$, a not implemented patent is as if it had never arrived. As (17) holds, firm 1 (likewise, firm 2) sets the same price and produces the same quantity as its competitors would, given by (14) and (15), respectively, and it produces at the same marginal cost as they would. The technology with which the following period's capital is produced is $\psi_{\tau-1,1,-j} = \psi$ and $\psi_{\tau-1,2} = \mu^{\frac{1}{2}}\psi$. Then, capital in the following period is given by

[23] To find leisure, I use the intratemporal optimality condition $\chi\,\frac{x_t}{l_t} = w_t$, where, from the final-good firm's problem, $w_t = \alpha\,\frac{y_t}{1-l_t}$.

$$k_{\tau,1} \;=\; \left(\frac{(1-\alpha)\,\psi}{R_{\tau-1}}\right)^{\frac{1}{\alpha}}(1-l_\tau) \tag{41}$$

$$k_{\tau,2} \;=\; \left(\frac{(1-\alpha)\,\mu^{\frac{1}{2}}\,\psi}{R_{\tau-1}}\right)^{\frac{1}{\alpha}}\;(1-l_\tau). \tag{42}$$

Since

$$i_{\tau-1} \;=\; \frac{k_{\tau,1}}{\psi} \;+\; \frac{k_{\tau,2}}{\mu^{\frac{1}{2}}\,\psi}\,, \tag{43}$$

investment in period $\tau-1$ is

$$i_{\tau-1} \;=\; \left[1+\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right]\,\psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\,\frac{(1-l_\tau)}{R_{\tau-1}^{\frac{1}{\alpha}}}\,. \tag{44}$$

Likewise, as the technology of firm j's competitors in $\tau-2$ is given by $\psi_{t-2,1,-j} \;=\; \mu^{-1}\,\psi$ and $\psi_{t-2,2,-j} \;=\; \mu^{-\frac{1}{2}}\,\psi$, capital in $\tau-1$ is given by

$$k_{\tau-1,1} \;=\; \left(\frac{(1-\alpha)\,\mu^{-1}\,\psi}{R_{\tau-2}}\right)^{\frac{1}{\alpha}}\;(1-l_{\tau-1}) \tag{45}$$

$$k_{\tau-1,2} \;=\; \left(\frac{(1-\alpha)\,\mu^{-\frac{1}{2}}\,\psi}{R_{\tau-2}}\right)^{\frac{1}{\alpha}}\;(1-l_{\tau-1})\,. \tag{46}$$

The above imply

$$y_{\tau-1} \;=\; \left[\mu^{-\,(\frac{1}{\alpha}-1)} \;+\; \mu^{-\frac{1}{2}\,(\frac{1}{\alpha}-1)}\right]\,\psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}-1}\,\frac{(1-l_{\tau-1})}{R_{\tau-2}^{\frac{1}{\alpha}-1}}\,. \tag{47}$$

Market clearing in the final-good market implies that consumption in $\tau-1$ is

$$x_{\tau-1} = \left[1 \;+\; \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right]\,\psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\;\left[\frac{1-l_{\tau-1}}{\mu^{\frac{1}{\alpha}-1}(1-\alpha)\,R_{t-2}^{\frac{1}{\alpha}-1}} \;-\; \frac{1-l_\tau}{R_{t-1}^{\frac{1}{\alpha}}}\right]\,. \tag{48}$$

Period τ . Implementation takes place in both sectors. Technology across capital makers in the two sectors is $\psi_{\tau,1,-j} \;=\; \psi \;<\; \psi_{\tau,1,j} \;=\; \mu\,\psi$ and $\psi_{\tau,2,-j} \;=\; \mu^{\frac{1}{2}}\,\psi \;<\; \psi_{\tau,2,j} \;=\; \mu^{\frac{3}{2}}\,\psi$, which implies that in period $\tau + 1$ capital is

$$k_{\tau+1,1} \;=\; \left(\frac{(1-\alpha)\,\psi}{R_\tau}\right)^{\frac{1}{\alpha}} \,(1 - l_{\tau+1}) \tag{49}$$

$$k_{\tau+1,2} \;=\; \left(\frac{(1-\alpha)\,\mu^{\frac{1}{2}}\,\psi}{R_\tau}\right)^{\frac{1}{\alpha}} \,(1 - l_{\tau+1})\,. \tag{50}$$

Since

$$i_\tau \;=\; \frac{k_{\tau+1,1}}{\mu\,\psi} \;+\; \frac{k_{\tau+1,2}}{\mu^{\frac{3}{2}}\,\psi}\,,$$

investment in period τ is

$$i_\tau \;=\; \left[\frac{1+\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}}{\mu}\right]\,\psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\,\frac{(1-l_{\tau+1})}{R_t^{\frac{1}{\alpha}}}\,. \tag{51}$$

The production function (8) combined with (41) and (42) yields

$$y_\tau = \left[1+\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right]\,\psi^{\frac{1}{\alpha}-1}(1-\alpha)^{\frac{1}{\alpha}-1}\,\frac{(1-l_\tau)}{R_{t-1}^{\frac{1}{\alpha}-1}}\,. \tag{52}$$

Market clearing in the final-good market then implies that

$$x_\tau = \left[1+\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right]\,\psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\,\left[\frac{1-l_\tau}{(1-\alpha)\,R_{\tau-1}^{\frac{1}{\alpha}-1}} \;-\; \frac{1-l_{\tau+1}}{\mu\,R_\tau^{\frac{1}{\alpha}}}\right]. \tag{53}$$

Period $\tau + 1$. A patent reaches firm 1 but it is kept stored until period $\tau + 2$, when the next implementation boom takes place. Since it is not implemented, effectively it is as if it had never arrived. Effective technology in the two sectors is $\psi_{\tau+1,1,-j} \;=\; \mu\,\psi$ and $\psi_{\tau+1,2} \;=\; \mu^{\frac{3}{2}}\,\psi$, which implies that capital in $\tau + 2$ is

$$k_{\tau+2,1} \;=\; \left(\frac{(1-\alpha)\,\mu\,\psi}{R_{\tau+1}}\right)^{\frac{1}{\alpha}} \,(1 - l_{\tau+2})$$

$$k_{\tau+2,2} \;=\; \left(\frac{(1-\alpha)\,\mu^{\frac{3}{2}}\,\psi}{R_{\tau+1}}\right)^{\frac{1}{\alpha}} \,(1 - l_{\tau+2})\,.$$

Taking familiar steps, since

$$i_{\tau+1} \;=\; \frac{k_{\tau+2,1}}{\mu\,\psi} \;+\; \frac{k_{\tau+2,2}}{\mu^{\frac{3}{2}}\,\psi}\,,$$

investment in period $\tau + 1$ is given by

$$i_{\tau+1} \;=\; \mu^{\frac{1}{\alpha}-1}\,\left[1+\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right]\,\psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\,\frac{(1-l_{\tau+2})}{R_{\tau+1}^{\frac{1}{\alpha}}}\,. \tag{54}$$

Combining the production function (8) with (49) and (50) implies

$$y_{\tau+1} = \left[1+\mu^{\frac{1}{2}\left(\frac{1}{\alpha}-1\right)}\right] \psi^{\frac{1}{\alpha}-1} (1-\alpha)^{\frac{1}{\alpha}-1} \frac{(1-l_{\tau+1})}{R_{\tau}^{\frac{1}{\alpha}-1}} . \tag{55}$$

Market clearing then implies

$$x_{\tau+1} = \left[1+\mu^{\frac{1}{2}\left(\frac{1}{\alpha}-1\right)}\right] \psi^{\frac{1}{\alpha}-1} (1-\alpha)^{\frac{1}{\alpha}} \left[\frac{1-l_{\tau+1}}{(1-\alpha) R_{\tau}^{\frac{1}{\alpha}-1}} - \frac{\mu^{\frac{1}{\alpha}-1} (1-l_{\tau+2})}{R_{\tau+1}^{\frac{1}{\alpha}}}\right] . \tag{56}$$

A two-period implementation cycle -of the type that I focus on- requires firm 1 , which receives a patent in an odd period to find it optimal to wait exactly *one* period before implementing it, and firm 2, which receives a patent in the following even period to find it optimal to implement it immediately. I will take the same steps as in the case of the acyclical equilibrium. Starting with consistency and after imposing stationarity, I first derive the condition under which firm 1 prefers to postpone implementation to the following period rather than implement immediately, which I label "Profit Condition 1 (SPC1) ;" subsequently, I explore the condition under which firm 1 prefers not to postpone implementation from the following implementation boom to the one after that, which I label "Profit Condition 2 (SPC2) ." The two conditions combined imply that firm 1 prefers to wait exactly one period to implement and firm 2 prefers to implement immediately than in any future period. Omitted derivations in what follows are collected in the Appendix.

Profit condition 1 . Firm 1 prefers to implement in the following period rather than immediately as long as its present discounted profits in the former case exceed its present discounted period in the latter (recall profits are realized one period after implementation) . Of course, expectations are centered around the synchronized implementation equilibrium in both cases.[24] Given stationarity and τ being an even period, the implementation decision, made in $\tau - 1$, must then satisfy

$$\frac{\Pi_{\tau+1,1}^{\tau-1}}{R_{\tau-1} R_{\tau}} > \frac{\Pi_{\tau,1}^{\tau-1}}{R_{\tau-1}} . \tag{SPC1}$$

Given (18) , (SPC1) becomes

$$\frac{1-l_{\tau+1}}{1-l_{\tau}} > \frac{R_{\tau}^{\frac{1}{\alpha}}}{R_{\tau-1}^{\frac{1}{\alpha}-1}} , \tag{57}$$

[24] For the lack of strategic interactions among firms see fn. 11 .

which as I show in the Appendix boils down to

$$\frac{\mu\,[1\,+\,\beta\,(1-\alpha)]}{\mu\,+\,\beta\,(1-\alpha)}\;>\;\frac{1}{\beta}\,. \tag{58}$$

We can see that (58) is more easily satisfied as μ and β increase and as α decreases. A higher innovation rate implies higher profits, hence a greater incentive for firms to coordinate in the presence of demand externalities. A higher β implies that firm 1 is more likely to wait for a certain level of profits; in the limit as $\beta \to 1$, (58) is always satisfied. Turning to α, it parametrizes both the capital share $(1-\alpha)$ as well as the elasticity of substitution between the two types of capital $(\frac{1}{\alpha})$. Greater values of both imply that implementation cycles are more easily sustained. As α falls, the former increases whereas the latter increases. However, the former effect dominates, hence (58) is more easily met. Last, observe that (58) is independent of the relative weight of leisure χ in the agent's preferences which is due to the separability of leisure and consumption within the flow utility and, given this, to preferences being logarithmic in consumption.[25]

Profit condition 2. For a synchronized implementation (cyclical) equilibrium to be sustained, no firm must find it optimal to postpone implementation past the two-period cycle. For this to be the case, it suffices to show that no firm has an incentive to wait until the next implementation period, i.e. period $\tau+2$, an argument also appearing in Shleifer (1986).

To see this in the case of firm 1, note that, given the stationary structure of the economy, if firm 1 prefers to implement in τ rather than in $\tau-1$, i.e. when condition (SPC1) holds, then it also finds it optimal to postpone implementation from $\tau+1$ to $\tau+2$. In other words, condition (SPC1) effectively implies that implementation can only take place in even periods. Thus, showing that firm 1 opts to implement in τ as opposed to doing so in $\tau+2$ or, by the same token, any future even period, is what I need to complete the consistency requirement of Definition 1. Note that this argument is independent of the fact that a new patent will reach sector 1 in $\tau+1$ rendering the patent received in $\tau-1$ obsolete. An analogous reasoning applies to firm 2.

Then, in the case of firm 1, the following condition must be satisfied:

$$\frac{\Pi^{\tau-1}_{\tau+1,1}}{R_{\tau-1}\,R_{\tau}}\;\geq\;\frac{\Pi^{\tau-1}_{\tau+3,1}}{R_{\tau-1}\,R_{\tau}\,R_{\tau+1}\,R_{\tau+2}}\,. \tag{SPC2}$$

Given that $R_{\tau-1} = R_{\tau+1}$ and $R_{\tau} = R_{\tau+2}$, condition (SPC2) simplifies to

$$R_{\tau}\,R_{\tau+1}\;\geq\;1\,. \tag{59}$$

[25] For $\alpha = 2/3$, which by (17) requires $\mu \leq 3$, $\chi = 1.5$ and $\beta = 0.97$, (58) requires approximately that $\mu > 1.14$.

Condition (SPC2) implies that profits are discounted at an on average positive net real interest rate and we can think of it as a weak version of the transversality condition which I analyze below.

One can confirm that conditions (57) and (59) combined imply that firm 2 finds it optimal to implement immediately.[26]

Combining (22) and (23) with (48) and (56) implies that

$$\lambda = \mu^{\frac{1}{\alpha}-1} > 1 \ , \text{since } \mu > 1 \, . \tag{60}$$

Lemma 1. *Remark 1 and (60) imply that (59) always holds.*

3.2.1 Balanced growth path

Leisure. On the balanced growth path of the synchronized implementation equilibrium, leisure takes values which alternate between odd and even periods and remain constant every other period, that is $l_{\tau-1} = l_{\tau+1} = l_{odd}$ and $l_\tau = l_{\tau+2} = l_{even}$.[27] As I show in the Appendix, leisure is given by

$$l_{odd} = \left[\frac{\alpha}{\chi} \frac{\mu \left(1 + \beta \left(1-\alpha\right)\right)}{\mu - \beta^2 (1-\alpha)^2} + 1 \right]^{-1} \tag{61}$$

$$l_{even} = \left[\frac{\alpha}{\chi} \frac{\mu + \beta \left(1-\alpha\right)}{\mu - \beta^2 (1-\alpha)^2} + 1 \right]^{-1} . \tag{62}$$

Remark 2. *It is $l_{odd} < l_{even}$.*

Remark 2 implies that employment falls when implementation takes place.

Interest rates. Equilibrium interest rates are given by (23), with λ given by (60) and υ given by

[26] Starting with profit condition 2, it would be like (SPC2) but without $R_{\tau-1}$ in the denominators, which plays no role anyway. Then, (59) is what we need.

Turning to profit condition 1, firm 2 implements immediately if

$$\frac{\Pi^{\tau}_{\tau+1,i}}{R_\tau} \geq \frac{\Pi^{\tau}_{\tau+2,i}}{R_\tau \, R_{\tau+1}} \, .$$

After substituting for profits, given by (18), and exploiting stationarity, which implies $l_\tau = l_{\tau+2}$, the above condition boils down to

$$R_{\tau+1} \left(\frac{R_{\tau+1}}{R_\tau} \right)^{\frac{1}{\alpha}-1} \geq \frac{1 - l_\tau}{1 - l_{\tau+1}} \, .$$

This holds if conditions (57) and (59) hold together which we can confirm by multiplying the LHS of (57) by $R_\tau \, R_{\tau+1}$.

[27] I use interchangeably throughout $l_{\tau-1}$, $l_{\tau+1}$ and l_{odd} for leisure in odd periods and l_τ, $l_{\tau+2}$ and l_{even} for leisure in the even ones.

$$v = \left[\mu^{(\frac{1}{\alpha}-1)^2} \frac{\alpha\,(\mu + \beta\,(1-\alpha)) + \chi\,(\mu - \beta^2\,(1-\alpha)^2)}{\alpha\,\mu\,(1 + \beta\,(1-\alpha)) + \chi\,(\mu - \beta^2\,(1-\alpha)^2)} \right]^{\frac{\alpha}{2-\alpha}} . \tag{63}$$

Claim 1. *It is* $v < \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}$.

Proof. See the Appendix. □

It follows from (60) that the geometric average growth rate is $\mu^{\frac{1}{2}\,(\frac{1}{\alpha}-1)}$, which is the one that prevails every period in the immediate implementation (steady-growth) equilibrium. Controlling for deviations from it and using Claim 1 implies that consumption booms when implementation takes place.

Letting R_{even} and R_{odd} denote the real interest rate paid in odd and even periods in the synchronized equilibrium, respectively,[28] and, with R denoting the real interest rate in the immediate implementation equilibrium, Claim 1 leads to the following remark:

Remark 3. *It is* $R_{even} < R < R_{odd}$.

Controlling for variations in employment, Remark 3 bears witness to the presence of demand externalities between the two capital-good sectors: since $R_{even} < R$, profits (both discounted and current-valued) of the following firm (firm 2) , which implements a patent immediately in both equilibria, are greater in the synchronized than in the immediate implementation one (see also (18)) . Allowing for variations in employment does not overturn this observation for all the parametrizations that I have considered.

Transversality condition. In what is the last step, I check the transversality condition. The transversality condition requires the present discounted value of the agent's lifetime wealth to converge. In other words, the present discounted value of the labor income and the capital-good firms' profits needs to converge. In the Appendix I show that the transversality condition is always satisfied.[29]

With (SPC2) always satisfied, the following proposition can be generated:

Proposition 1. *A two-period synchronized implementation equilibrium prevails as a perfect-foresight equilibrium as long as condition (SPC1) and the no-storage condition (1) are satisfied.*

[28] Throughout I use interchangeably R_{even} , R_τ and $R_{\tau+2}$ for the interest rate paid in odd periods and R_{odd} , $R_{\tau-1}$ and $R_{\tau+1}$ for the interest rate paid in the even ones.

[29] This result is due to preferences being logarithmic in consumption. For general CRRA preferences, the transversality condition is more easily satisfied the lower the intertemporal elasticity of substitution is.

This proposition is central to the paper. In sharp contrast with the conjecture in (Shleifer, 1986, page 1183), implementation cycles with capital can be generated and they do so for plausible values of the parameters. Importantly, this happens in the presence of storable commodities and in the absence of borrowing constraints and investment irreversibilities (on the latter see also fn. 33). But how so? Following Shleifer's line of thought, one would expect that, in the prospect of future profits, agents would reduce current savings and, thereby, future capital stock in order to smooth out their consumption across periods. At the same time, a (real) interest rate increase would be necessary to prevent agents from borrowing in the period before the wealth expansion. Both effects combined imply that firms' present discounted profits in an implementation boom would fall which could eliminate their incentives to postpone implementation until then.

This intuition does not apply here. This is because, in sharp contrast with Shleifer (1986) in which innovations are sector-neutral, that is they enhance total factor productivity (TFP), innovations here are investment-specific as in Greenwood et al. (1997, 2000). This difference in modeling technological change is important. Unlike changes in TFP, investment-specific technological change introduces a one-period discrepancy between the date firms invest and the date they receive their revenue. As a result, a coordinated implementation of patents implies a concurrent considerable fall in savings/investment -in fact, investment can even undershoot- due to the reduced cost of producing capital and a considerable increase in the wealth of agents in the period following it.[30] The former implies that consumption grows considerably in an implementation boom even after taking into account the effects on output.[31] The latter implies that the consumption boom takes place before the wealth boom. This not only eliminates the need to smooth consumption away from the wealth boom to the period before it, but, additionally, it implies that the interest rate linking the implementation period to the one when revenue is realized and wealth expands actually falls (Remark 3). As a result, firms discount future profits less rather than more. The fall in the real interest rate, in turn, causes two, tied to each other, effects. First, it increases investment in an implementation period which thereby smooths out consumption in the opposite direction from the conjectured one, though (without overturning results; Claim 1 is a general equilibrium result). Second, it leads to an increase of the capital stock in the period revenue is realized relative to that

[30] In fact, output and, consequently, labor income also boosts with a one-period lag (see the analysis below). This need not be always true. For instance, in the extreme case in which labor is inelastically supplied, labor income booms when implementation takes place. Nevertheless, for all the parametrizations that I have considered, variations in labor income prove insufficient to make the wealth boom happen simultaneously with the consumption boom and, hence, potentially overturn the intuition in the main text.

[31] More precisely, consumption grows above trend as Claim 1 attests.

in the implementation period.[32] With the demand for capital being elastic, this implies that the profits capital-good firms make following an implementation boom grow rather than fall (see also (18)) relative to the profits they would have made had they implemented alone. Taking everything into account, present discounted profits increase which eventually makes implementation cycles possible (Proposition 1).[33]

Below I discuss the role of parameters in the generation of cycles and the balanced growth path. I set $\alpha = \frac{2}{3}$, which by (17) implies that $\mu \leq 3$ and, based on Greenwood et al. (2000), $\chi = 1.5$ and consider values of β close to 1.[34] A consequence of setting $\alpha = \frac{2}{3}$ is that υ, given by (63), is greater than one. In turn, this implies that R_τ is greater than one, whereas by Claim 1, $R_{\tau+1}$ is also greater than one. As a result, the no-storage condition is always satisfied.

Generation of cycles. A greater innovation rate sets the economy onto a steeper growth path which is accompanied with higher interest rates, R_τ and $R_{\tau+1}$. However, $R_{\tau+1}$ increases sufficiently more than the interest rate paid after an implementation boom, R_τ, does so that the RHS of (57) falls. In other words, controlling for changes in employment which as I argue next actually reinforce this effect, as the innovation rate increases, discounted profits in an implementation boom become greater relative to profits that would be realized if firm 1 instead opted to implement alone. This is because a greater innovation rate results in considerably reduced savings/investment in an implementation period. In turn, this implies a substantial increase in contemporaneous consumption both in absolute and, crucially, in relative to trend terms, where I define as trend the geometric average growth rate $\lambda^{\frac{1}{2}} = \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}$ which characterizes the immediate implementation (steady-growth) equilibrium.[35]

Turning to leisure/employment, a greater innovation rate, μ, results in an increase in leisure in both periods; the leisure ratio $\frac{l_{\tau+1}}{l_\tau}$ falls, whereas the employment ratio $\frac{1-l_{\tau+1}}{1-l_\tau}$, which is on the LHS

[32] Simply compare equations (41) - (42) with (49) - (50). The result follows from Remarks 2 and 3.

[33] As I noted in fn. 10, that capital depreciates fully effectively rules out disinvestment and, therefore, it could be argued that I impose investment irreversibilities which according to (Shleifer, 1986, page 1183) could render implementation cycles possible. The way I interpret his argument, even though such an interpretation may be susceptible to criticism, is that he expects that too volatile investment would rule implementation cycles with capital out. By allowing for full capital depreciation, I indeed maximize the volatility of investment as new capital needs to be produced every period. In fact, as it may have already become apparent, it is the excessive investment volatility -actually investment can even undershoot- that renders implementation cycles with capital possible here.

[34] The numerical values that I report below correspond to $\beta = 0.97$, which is again based on Greenwood et al. (2000). In fact, many results below hold for a much wider range of parametrizations, however I abstract from such considerations.

[35] The trend of output, consumption and investment is $\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}$, that of capital is $\mu^{\frac{1}{2\alpha}}$ and that of employment is 1.

of (57) increases. Taking both effects into account, the greater the innovation rate μ, the more attractive an implementation boom is to firm 1.

A greater β lowers v, that is consumption in the implementation periods becomes higher both in absolute and relative to trend terms, and both interest rates. This is because agents become more patient. However, $R_{\tau+1}$ falls less relative to R_τ and the RHS of (57) decreases. Parallel to this, a greater discount factor β decreases leisure in all periods as well as the leisure ratio $\frac{l_{\tau+1}}{l_\tau}$, whereas it increases the employment ratio $\frac{1-l_{\tau+1}}{1-l_\tau}$, which is on the LHS of (57). Once again both effects imply that a higher β leads more easily to implementation cycles.

Next, I analyze the balanced growth path.

Balanced growth path. Output, consumption and investment grow by $\mu^{\frac{1}{\alpha}-1}$ every two periods:

$$\frac{y_{t+1}}{y_{t-1}} = \frac{x_{t+1}}{x_{t-1}} = \frac{i_{t+1}}{i_{t-1}} = \mu^{\frac{1}{\alpha}-1}. \tag{64}$$

Within a cycle, we know that $\frac{x_{\tau+1}}{x_\tau} = v$, whereas output and investment's evolution is given by

$$\begin{aligned}\frac{y_{\tau+1}}{y_\tau} &= \left(\frac{1-l_{\tau+1}}{1-l_\tau}\right)\left(\frac{R_{\tau+1}}{R_\tau}\right)^{\frac{1}{\alpha}-1}\\ \frac{i_{\tau+1}}{i_\tau} &= \left(\frac{1-l_\tau}{1-l_{\tau+1}}\right)\left(\frac{\mu R_\tau}{R_{\tau+1}}\right)^{\frac{1}{\alpha}},\end{aligned}$$

which follow from (52) and (55), and (51) and (54), respectively.
After substituting for the interest rates, given by (23), and using eq. (96) in the Appendix, the above expressions become

$$\frac{y_{\tau+1}}{y_\tau} = v\left(\frac{\mu(1+\beta(1-\alpha))}{\mu+\beta(1-\alpha)}\right) \tag{65}$$

$$\frac{i_{\tau+1}}{i_\tau} = v\left(\frac{\mu+\beta(1-\alpha)}{1+\beta(1-\alpha)}\right), \tag{66}$$

where v is given by (63).

For the considered parametrization, output grows above trend in the period following the implementation of patents. Investment is procyclical and undershoots: it falls when implementation takes place, as fewer resources need to be directed towards the production of capital goods, and rises sharply in the period following implementation. Further, by Remark 2 employment is procyclical, whereas, most notably, by Claim 1 consumption is countercyclical. As I argued above, it is necessary for the generation of cycles that consumption booms in the implementation periods.

The above and (65) and (66) imply that investment is more volatile than output since $\frac{i_{\tau+1}}{i_\tau} > \frac{y_{\tau+1}}{y_\tau}$. In turn, output is more volatile than employment. Notably (for the considered parametrization), consumption is also more volatile than output[36] but less volatile than investment. The interpretation for this is simple: with output relatively stable, a very volatile investment implies a very volatile consumption.

As the innovation rate increases, investment, consumption and employment become more volatile, whereas the volatility of output responds non-monotonically increasing at low values of μ and falling at higher ones.[37]

4 Welfare

From a planner's viewpoint, both equilibria are suboptimal which is due to the (periodic) presence of monopolies in the capital-good markets.[38] Nevertheless, the equilibria can be Pareto ranked.

In the immediate implementation equilibrium, the lifetime utility of the representative agent, given by (1) - (2), is equal to

$$U_i = \log x_1^i + \chi \log l + \beta \left(\log \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)} x_1^i + \chi \log l\right) + \beta^2 \left(\log \mu^{\frac{1}{\alpha}-1} x_1^i + \chi \log l\right) + \ldots ,$$

where I have taken into account that leisure is constant across time and that consumption grows each period by $\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}$ (see (39) - (40)). This expression boils down to

$$U_i = \frac{1}{1-\beta}\left(\frac{1}{2}\frac{\beta}{1-\beta} \log \mu^{\frac{1}{\alpha}-1} + \log x_1^i + \chi \log l\right), \tag{67}$$

where l is given by (40).

[36] As consumption is countercyclical, I compare $\frac{y_{\tau+1}}{y_\tau}$ given by (65) with $\frac{x_\tau}{x_{\tau-1}}$, where $\frac{x_\tau}{x_{\tau-1}} = \frac{\mu^{\frac{1}{\alpha}-1}}{v}$.

[37] In addition to these, note that, for the considered parametrization, capital is countercyclical. To see this, recall that, because of "limit-pricing," implementing firms produce the quantity of capital that their competitors would produce. This implies that, controlling for variations in interest rates and leisure, patents will affect the quantity of capital installed after two periods. Further, capital is more volatile than output, which should not come as a surprise given that it depreciates fully within a period, is less volatile than investment and its volatility increases in μ.

[38] Since the implementation of a patent improves the technology of an implementing firm's competitors, one would suggest that externalities is an additional source of inefficiency. My argument for why this is not indeed an issue is the same as the one in Shleifer (1986) (in particular, see (Shleifer, 1986, page 1178) and fn. 10 there). In principle, this potential problem could be corrected by allowing for decentralized markets on a one-to-one basis between the firm endowed with a patent and one of its competitors with the former setting the price in exchange for sharing the rights to its patent (we can equivalently think in terms of contracts on an individual, "take it or leave it" basis). The presence of constant returns to scale in the capital-good's technology implies that competitors, which behave symmetrically, will demand zero at any positive price since, afterwards, they will Bertrand-compete at least with the firm owning the patent which, in turn, is not willing to suggest a zero price. Hence, markets would clear at positive prices small enough so that demand and supply are equal to zero in each period. Therefore, externalities is not an additional source of inefficiency.

In the synchronized implementation equilibrium, the lifetime utility of the agent is

$$U_s = \log x_1^s + \chi \log l_{odd} + \beta \left(\log \frac{\mu^{\frac{1}{\alpha}-1}}{v} x_1^s + \chi \log l_{even} \right) + \beta^2 \left(\log \mu^{\frac{1}{\alpha}-1} x_1^s + \chi \log l_{odd} \right) + \dots ,$$

where I have taken into account that leisure is constant controlling for the period being odd or even and that consumption grows as (21) and (22) prescribe. The above expression simplifies to

$$U_s = \frac{1}{1-\beta} \left[\frac{\beta}{1-\beta^2} \log \mu^{\frac{1}{\alpha}-1} - \frac{\beta}{1+\beta} \log v + \log x_1^s + \frac{\chi}{1+\beta} \left(\log l_{odd} + \beta \log l_{even} \right) \right] , \quad (68)$$

where v is given by (63) and l_{odd} , l_{even} are given by (61) and (62) , respectively.

To make welfare comparisons, simply subtract (68) from (67) to get

$$U_i - U_s = \frac{1}{1-\beta} \left[\frac{\beta}{1+\beta} \left(\log v - \log \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)} \right) + \log x_1^i - \log x_1^s + \chi \left(\log l - \frac{\log l_{odd} + \beta \log l_{even}}{1+\beta} \right) \right] . \quad (69)$$

For the considered parametrization, welfare is greater in the immediate implementation equilibrium than in the synchronized implementation one.[39] To analyze this result, I will start with the last terms in (69) , which reflect differences in welfare due to differences in leisure levels. For the considered values of α , χ and β and sufficiently high values of μ , the leisure component of lifetime utility is greater in the synchronized implementation equilibrium. However, the effect of leisure in welfare comparisons is typically negligible.

What is crucial is differences in lifetime consumption. To analyze these, I will draw a distinction between the difference in the initial consumption levels, captured by the third and the fourth term in (69) combined, and the difference in the consumption growth rates between the two equilibria, captured by the first two terms. The latter takes a negative value by Claim 1 . To see this, recall that the growth rate every two periods is the same across equilibria. Hence, the difference in consumption growth is due to the difference in the growth rate within a cycle, captured by the first two terms in (69) . Since consumption grows faster in the synchronized implementation equilibrium (Claim 1) , the first two terms combined take a negative value.

Turning to the initial level of consumption, it is higher in the immediate implementation equilibrium. To see this compare (31) and (48) .[40] It turns out that in the immediate implementation

[39]In fact, I have failed to obtain the opposite result for a wide range of parametrizations.

[40]Since what matters is relative consumption, any odd-period consumption would do for the comparison between equilibria.

equilibrium output is greater and investment lower compared to the synchronized implementation equilibrium.[41] Controlling for the interest rates which do not overturn the result for the considered parametrization, investment is lower in the immediate implementation equilibrium since one firm (firm 1) implements a patent as opposed to none doing so in the synchronized implementation one (compare (26 with (43)). As for output, once again controlling for the interest rates, it is greater in the immediate implementation equilibrium because patents in sector 1 are implemented faster which leads to a greater level of capital of type 1 (compare (28) with (45)) and, hence, a greater level of output.[42]

4.1 Initial levels of capital/investment

Section 3 analyzed the two equilibria independently of each other. By this I mean that, in each equilibrium, the economy starts with the "right" quantity of capital-goods.

Below, I find the initial (period-0) investment required in each equilibrium. I suppose that the level of initial investment is the one that would have prevailed if time had started in $-\infty$. Therefore, I set $\psi = 1$ and divide the RHS of both (34) and (51) by $\mu^{\frac{1}{\alpha}-1}$.[43] This yields, respectively,

$$i_0^i = \left[1 + \mu^{-\frac{1}{2}\left(\frac{1}{\alpha}+1\right)}\right] (1-\alpha)^{\frac{1}{\alpha}} \frac{1-l}{R^{\frac{1}{\alpha}}} \tag{70}$$

$$i_0^s = \left[\mu^{-\frac{1}{\alpha}} + \mu^{-\frac{1}{2}\left(\frac{1}{\alpha}+1\right)}\right] (1-\alpha)^{\frac{1}{\alpha}} \frac{1-l_{\tau+1}}{R_\tau^{\frac{1}{\alpha}}}, \tag{71}$$

where l is given by (40), $R = \frac{\mu^{\frac{1}{\alpha}-1}}{\beta}$, $l_{\tau+1}$ is given by (61) and $R_\tau = \frac{v}{\beta}$ with v given by (63).

For the considered parametrization,[44] initial investment is greater in the immediate implementation equilibrium. As a result, the two cases can be Pareto ranked only conditional on this difference in initial investment.

[41] The result for investment is true for the standard parametrization but not in all the ones that I have considered, whereas the results for output and, crucially, consumption hold for all the parametrizations that I have considered.

[42] I have not explored welfare in the case of a synchronized implementation equilibrium in which implementation booms take place in odd periods (see also fn. 19). Nevertheless, since that cyclical equilibrium is symmetric with the considered one and given that time commences in an odd period, my conjecture is that results would possibly be overturned. Such an argument might apply to the welfare considerations in Section 5 as well.

[43] I opted for initial investment rather than initial levels of capital in order to facilitate comparisons between equilibria. To find initial levels of capital one needs to set $\psi = 1$ in $k_{\tau-1,1}$ and $k_{\tau-1,2}$.

[44] In particular, this is true for all the parametrizations that I have considered.

5 The Desirability of Extending Patent Rights

In this section I explore whether extending the patent horizon is potentially welfare-improving. Therefore, the only assumption that I relax concerns the duration of patent rights. In particular, I let firms make monopoly profits out of a patent for *two* periods instead of one. Everything else remains unchanged.

5.1 Immediate implementation equilibrium

As in Section 3.1 , leisure and the real interest rate remain constant across periods with values given by $\tilde{l}$ and R , respectively, which I find subsequently.[45]

Period $\tau - 1$ **.** Firm 1 receives a patent which it immediately implements, whereas firm 2 , which implemented its patent in the previous period, enters its second and last period as a monopolist. The technology levels in the two sectors are $\psi_{\tau-1,1,-j} = \psi < \psi_{\tau-1,1,j} = \mu\,\psi$ and $\psi_{\tau-1,2,-j} = \mu^{-\frac{1}{2}}\,\psi < \psi_{\tau-1,2,j} = \mu^{\frac{1}{2}}\,\psi$, respectively. Then,

$$k_{\tau,1} = \left(\frac{(1-\alpha)\,\psi}{R}\right)^{\frac{1}{\alpha}} (1-\tilde{l}) \tag{72}$$

$$k_{\tau,2} = \left(\frac{(1-\alpha)\,\mu^{-\frac{1}{2}}\,\psi}{R}\right)^{\frac{1}{\alpha}} (1-\tilde{l})\,. \tag{73}$$

Investment is given by

$$i_{\tau-1} = \frac{k_{\tau,1}}{\mu\,\psi} + \frac{k_{\tau,2}}{\mu^{\frac{1}{2}}\,\psi}\,.$$

Combining the above expression with (72) and (73) yields

$$i_{\tau-1} = \left[\mu^{-1} + \mu^{-\frac{1}{2}(\frac{1}{\alpha}+1)}\right] \psi^{\frac{1}{\alpha}-1} (1-\alpha)^{\frac{1}{\alpha}} \frac{(1-\tilde{l})}{R^{\frac{1}{\alpha}}}\,. \tag{74}$$

Similarly, since $\psi_{\tau-2,1,-j} = \mu^{-1}\,\psi$ and $\psi_{\tau-2,2,-j} = \mu^{-\frac{1}{2}}\,\psi$,

[45] As it will become evident below, the real interest rate in an immediate implementation (steady-growth) equilibrium is the same irrespectively of the duration of patent rights, hence the use of R as opposed to, for instance, $\hat{R}$.

$$k_{\tau-1,1} \;=\; \left(\frac{(1-\alpha)\,\mu^{-1}\,\psi}{R}\right)^{\frac{1}{\alpha}}(1-\tilde{l})$$

$$k_{\tau-1,2} \;=\; \left(\frac{(1-\alpha)\,\mu^{-\frac{1}{2}}\,\psi}{R}\right)^{\frac{1}{\alpha}}(1-\tilde{l})\,.$$

I proceed in a familiar way to find output and consumption:

$$y_{\tau-1} \;=\; \left[\mu^{-\left(\frac{1}{\alpha}-1\right)}+\mu^{-\frac{1}{2}\left(\frac{1}{\alpha}-1\right)}\right]\,\psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}-1}\,\frac{(1-\tilde{l})}{R^{\frac{1}{\alpha}-1}} \tag{75}$$

$$x_{\tau-1} \;=\; \psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\,(1-\tilde{l})\,R^{-\frac{1}{\alpha}}\left[\frac{R\left(\mu^{-\left(\frac{1}{\alpha}-1\right)}+\mu^{-\frac{1}{2}\left(\frac{1}{\alpha}-1\right)}\right)}{1-\alpha}\;-\;\left(\mu^{-1}\;+\;\mu^{-\frac{1}{2}\left(\frac{1}{\alpha}+1\right)}\right)\right]. \tag{76}$$

Period τ . Firm 2 receives and immediately implements a patent. In period τ , technology in sector 1 remains as in $\tau-1$, that is $\psi_{\tau,1,-j} \;=\; \psi \;<\; \psi_{\tau,1,j} \;=\; \mu\,\psi$, whereas in sector 2 it becomes $\psi_{\tau,2,-j} \;=\; \mu^{\frac{1}{2}}\,\psi \;<\; \psi_{\tau,2,j} \;=\; \mu^{\frac{3}{2}}\,\psi$. Capital in the following period is given by

$$k_{\tau+1,1} \;=\; \left(\frac{(1-\alpha)\,\psi}{R}\right)^{\frac{1}{\alpha}}(1-\tilde{l}) \tag{77}$$

$$k_{\tau+1,2} \;=\; \left(\frac{(1-\alpha)\,\mu^{\frac{1}{2}}\,\psi}{R}\right)^{\frac{1}{\alpha}}(1-\tilde{l})\,. \tag{78}$$

Investment is given by

$$i_\tau \;=\; \frac{k_{\tau,1}}{\mu\,\psi}\;+\;\frac{k_{\tau,2}}{\mu^{\frac{3}{2}}\,\psi}\,, \tag{79}$$

which combined with (77) and (78) yields

$$i_\tau \;=\; \left[\mu^{-1}+\mu^{\frac{1}{2\alpha}-\frac{3}{2}}\right]\,\psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\,\frac{(1-\tilde{l})}{R^{\frac{1}{\alpha}}}\,. \tag{80}$$

Output and consumption are given by

$$y_\tau \;=\; \left[1+\mu^{-\frac{1}{2}\left(\frac{1}{\alpha}-1\right)}\right]\,\psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}-1}\,\frac{(1-\tilde{l})}{R^{\frac{1}{\alpha}-1}}$$

$$x_\tau \;=\; \psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\,(1-\tilde{l})\,R^{-\frac{1}{\alpha}}\left[\frac{R\left(1+\mu^{-\frac{1}{2}\left(\frac{1}{\alpha}-1\right)}\right)}{1-\alpha}\;-\;\left(\mu^{-1}\;+\;\mu^{\frac{1}{2\alpha}-\frac{3}{2}}\right)\right].$$

Period $\tau + 1$ **.** Firm 1 receives and implements a patent. Technology in the two sectors is $\psi_{\tau+1,1,-j} = \mu\,\psi \;<\; \psi_{\tau+1,1,j} \;=\; \mu^2\,\psi$ and $\psi_{\tau+1,2,-j} \;=\; \mu^{\frac{1}{2}}\,\psi \;<\; \psi_{\tau+1,2,j} \;=\; \mu^{\frac{3}{2}}\,\psi$. Then,

$$k_{\tau+2,1} \;=\; \left(\frac{(1-\alpha)\,\mu\,\psi}{R}\right)^{\frac{1}{\alpha}}\,(1-\tilde{l})$$

$$k_{\tau+2,2} \;=\; \left(\frac{(1-\alpha)\,\mu^{\frac{1}{2}}\,\psi}{R}\right)^{\frac{1}{\alpha}}\,(1-\tilde{l})\,.$$

Investment is given by

$$i_{\tau+1} \;=\; \frac{k_{\tau,1}}{\mu^2\,\psi} \;+\; \frac{k_{\tau,2}}{\mu^{\frac{3}{2}}\,\psi}\,,$$

which is equal to

$$i_{\tau+1} \;=\; \left[\mu^{\frac{1}{\alpha}-2} + \mu^{\frac{1}{2\alpha}-\frac{3}{2}}\right]\,\psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\,\frac{(1-\tilde{l})}{R^{\frac{1}{\alpha}}}\,.$$

Output and consumption are given by

$$y_{\tau+1} \;=\; \left[1 + \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right]\,\psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}-1}\,\frac{(1-\tilde{l})}{R^{\frac{1}{\alpha}-1}}$$

$$x_{\tau+1} \;=\; \psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\,(1-\tilde{l})\,R^{-\frac{1}{\alpha}}\,\left[\frac{R\,\left(1+\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right)}{1-\alpha} \;-\; \left(\mu^{\frac{1}{\alpha}-2} + \mu^{\frac{1}{2\alpha}-\frac{3}{2}}\right)\right]\,.$$

5.1.1 Profit conditions.

As each firm can make profits for two periods out of a patent, profit condition 1 in the case of a firm receiving a patent in period $t-1$ becomes

$$\frac{\Pi_t}{R_{t-1}} \;+\; \frac{\Pi_{t+1}}{R_{t-1}\,R_t} \;\geq\; \frac{\Pi_{t+1}}{R_{t-1}\,R_t} \;+\; \frac{\Pi_{t+2}}{R_{t-1}\,R_t\,R_{t+1}}\,.$$

The LHS refers to discounted profits as of $t-1$ made when a patent is immediately implemented, whereas the RHS refers to discounted profits made when a patent is implemented in the following period. The above condition does not take into account that, in the latter case, after one period a new patent will render the one in question obsolete, which would imply that the condition would be more easily satisfied.

Likewise, profit condition 2 becomes

$$\frac{\Pi_t}{R_{t-1}} \;+\; \frac{\Pi_{t+1}}{R_{t-1}\,R_t} \;\geq\; \frac{\Pi_{t+2}}{R_{t-1}\,R_t\,R_{t+1}} \;+\; \frac{\Pi_{t+3}}{R_{t-1}\,R_t\,R_{t+1}\,R_{t+2}}\,.$$

Since in the steady-growth equilibrium, employment and interest rates are constant, so are temporal profits. Then, both profit conditions boil down to $R \geq 1$.[46]

5.1.2 Balanced growth path

Along the balanced growth path, output, consumption and investment grow all by $\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}$ and the real interest rate is $R = \frac{\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}}{\beta}$, which implies that both profit conditions are always met. Leisure is given by (see also fn. 23)

$$\tilde{l} = \left[\frac{\alpha\,\mu}{\chi\,(\mu - \beta(1-\alpha))} + 1\right]^{-1} . \tag{81}$$

5.2 Synchronized implementation equilibrium

In the synchronized implementation equilibrium (of the type I considered in Section 3), firm 1 postpones implementation until the following even period when it implements together with firm 2. Let me start with the profit conditions and, subsequently, show a synchronized implementation equilibrium is not possible when rights over a patent last two periods.

For firm 1 which receives a patent, say, in $\tau - 1$ to prefer to implement in period τ rather than immediately, it must be that

$$\frac{\Pi_{\tau+1}}{R_{\tau-1}\,R_\tau} + \frac{\Pi_{\tau+2}}{R_{\tau-1}\,R_\tau\,R_{\tau+1}} > \frac{\Pi_\tau}{R_{\tau-1}} + \frac{\Pi_{\tau+1}}{R_{\tau-1}\,R_\tau} . \tag{82}$$

The first term on the LHS and the second on the RHS cancel out so that (82) becomes

$$\frac{\Pi_{\tau+2}}{R_{\tau-1}\,R_\tau\,R_{\tau+1}} > \frac{\Pi_\tau}{R_{\tau-1}} . \tag{83}$$

By juxtaposing (83) with (SPC1), we can see that postponing implementation to the following period when the monopoly horizon is two periods is equivalent to postponing implementation to two periods afterwards in the context of the synchronized equilibrium in the one-period monopoly case.

Since in a stationary equilibrium interest rates and employment remain constant controlling for the period being odd or even, it follows that $\Pi_\tau = \Pi_{\tau+2}$. This implies that (83) becomes[47]

$$R_\tau\,R_{\tau+1} < 1 . \tag{84}$$

[46] As in Section 3, the no-storage condition requires that $R > 1$ and the TVC always holds.

[47] As it has been so far in the paper, whenever it comes to profit condition 2, I ignore the possibility that new patents can render the ones in question obsolete, which would imply that profit condition 2 is more easily met.

Turning to profit condition 2 , this is

$$\frac{\Pi_{\tau+1}}{R_{\tau-1}\,R_{\tau}} + \frac{\Pi_{\tau+2}}{R_{\tau-1}\,R_{\tau}\,R_{\tau+1}} \geq \frac{\Pi_{\tau+3}}{R_{\tau-1}\,R_{\tau}\,R_{\tau+1}\,R_{\tau+2}} + \frac{\Pi_{\tau+4}}{R_{\tau-1}\,R_{\tau}\,R_{\tau+1}\,R_{\tau+2}\,R_{\tau+3}}\,. \tag{85}$$

By stationarity, one can confirm that (85) simplifies to

$$\left(1 - \frac{1}{R_{\tau+1}\,R_{\tau+2}}\right)\left(\Pi_{\tau+1} + \frac{\Pi_{\tau+2}}{R_{\tau+1}}\right) \geq 0.$$

Given that it is not possible that firms make negative profits in equilibrium since in that case they would rather not implement their patents, this expression becomes (recall that, by stationarity, $R_{\tau} = R_{\tau+2}$)

$$R_{\tau}\,R_{\tau+1} \geq 1\,. \tag{86}$$

Juxtaposing (84) and (86) implies that synchronized implementation is *not* possible when patent rights last two periods.

Below I explore whether extending patent rights to two periods can lead to a welfare improvement.

5.3 Welfare

I showed above that only the immediate implementation equilibrium is possible when patent rights last two periods.[48] Then the agent's lifetime utility is

$$\tilde{U}_i = \frac{1}{1-\beta}\left(\frac{1}{2}\frac{\beta}{1-\beta}\,\log\mu^{\frac{1}{\alpha}-1} + \log\tilde{x}_1^i + \chi\,\log\tilde{l}\right)\,, \tag{87}$$

where $\tilde{l}$ is given by (81) . I will compare (87) with the lifetime utility in the two equilibria which can prevail when firms profit out of a patent once. That is I will compare (87) with (68) and (67) in that order.

5.3.1 Welfare comparison with the synchronized implementation equilibrium

The difference in lifetime utilities is given by

$$U_s - \tilde{U}_i = \frac{1}{1-\beta}\left[\frac{\beta}{1+\beta}\left(\log\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)} - \log v\right) + \log x_1^s - \log\tilde{x}_1^i + \chi\left(\frac{\log l_{odd} + \beta\,\log l_{even}}{1+\beta} - \log\tilde{l}\right)\right]\,. \tag{88}$$

[48] To be precise, I have failed to find an equilibrium besides this one.

I will take the same steps as in Section 4 . Starting with the leisure terms, one can confirm that $\tilde{l} > l_{even} > l_{odd}$. The first inequality follows by comparing (81) with (62) , whereas the second follows from Remark 2 . Then, although its role is non-pivotal in welfare comparisons, leisure utility is greater in the two-period patent equilibrium.

Turning to consumption, by Claim 1 and as explained in Section 4 the growth terms together take a negative value. However, what is once again crucial for the welfare comparison outcome is the distance between the initial levels of consumption in the two equilibria given by (48) and (76) . It turns out that for sufficiently low values of the innovation rate, μ , initial consumption is greater in the two-period patent equilibrium (see also Figure 1) . I will proceed into two steps.

Starting with initial output levels, we can see from (47) and (75) for $\psi = 1$ that

$$\Delta y \equiv y_1^s - \tilde{y}_1^i = \left[\mu^{-\left(\frac{1}{\alpha}-1\right)} + \mu^{-\frac{1}{2}\left(\frac{1}{\alpha}-1\right)}\right] (1-\alpha)^{\frac{1}{\alpha}-1} \left(\frac{1-l_{odd}}{R_{even}^{\frac{1}{\alpha}-1}} - \frac{1-\tilde{l}}{R^{\frac{1}{\alpha}-1}}\right) . \tag{89}$$

Taking into account that $\tilde{l} > l_{odd}$ and that, by Remark 3 , $R_{even} < R$ implies that the last term is positive, hence initial output is greater in the synchronized implementation, one-period patent equilibrium than in the two-period patent one. Furthermore, for the considered parametrization, the initial output difference increases in μ .

Turning to initial investment levels, we can see from (44) and (74) for $\psi = 1$ that

$$\Delta i \equiv i_1^s - \tilde{i}_1^i = \left[\frac{1}{\mu} + \mu^{-\frac{1}{2}\left(\frac{1}{\alpha}+1\right)}\right] (1-\alpha)^{\frac{1}{\alpha}} \left[\mu^{\frac{1}{2}\left(\frac{1}{\alpha}+1\right)} \frac{1-l_{even}}{R_{odd}^{\frac{1}{\alpha}}} - \frac{1-\tilde{l}}{R^{\frac{1}{\alpha}}}\right] . \tag{90}$$

For the considered parametrization, the last term is positive and the initial (positive) investment difference grows in μ .[49]

It follows from the above that, for the considered parametrization, in the synchronized implementation (one-period patent) equilibrium both output and investment are greater compared with the two-period patent equilibrium. These follow from the fact that, in the latter, patents in sector 2 become available with one-period lag relative to the former. To inspect things more, after controlling for the differences in interest rates and employment, output is the same in the considered equilibria, whereas investment in greater in the synchronized implementation one. The former is because firms use the same level of capital; to see this, observe that in even periods, the implementing firms competitors' technology level is the same in both equilibria. The latter is because in odd periods no firm implements (or, appropriates) a patent in the synchronized implementation equilibrium, as opposed to *both* in the two-period patent one.

[49] In fact, the initial investment difference is positive for all the parametrizations that I have considered.

For the considered parametrization, at sufficiently low values of μ the "investment" effect dominates the "output" effect $(\Delta y < \Delta i)$, hence initial consumption is greater in the two-period patent equilibrium. However, $\Delta y \,/\, \Delta i$ increases in μ and becomes greater than one for high enough values of it. The threshold value is $\mu^* \simeq 2.14$.

Since the level of initial consumption is pivotal in the welfare comparisons, the latter will exhibit the same pattern: for sufficiently low values of μ, the two-period patent equilibrium is Pareto-superior to the synchronized implementation equilibrium which can prevail when patent rights last one period.[50] The threshold value in the welfare comparison is $\mu^{**} \simeq 1.95$ (see also Figure 1), which is lower than μ^* since the combination of the growth and the leisure effects favors the synchronized implementation equilibrium.

5.3.2 Welfare comparison with the immediate implementation equilibrium

The difference in lifetime utilities is given by

$$U_i - \tilde{U}_i = \frac{1}{1-\beta}\left[\log x_1^i - \log \tilde{x}_1^i + \chi\,(\log l - \log \tilde{l})\right]. \tag{91}$$

Starting with the leisure terms, we can confirm by comparing (40) and (81) that $\tilde{l} > l$. However, the role of leisure in the welfare comparison is negligible.

Turning to consumption, I show in the Appendix that, for all parameter values, $x_1^i > \tilde{x}_1^i$. There are two forces underlying differences in consumption. On the one hand, patents may be first implemented at the same time in the two equilibria, however, in the two-period patent equilibrium, firms can profit out of these for one additional period. This implies that -in the two-period patent equilibrium- patents become available to the implementing firms' competitors and, hence, to the economy with one period delay. One can actually confirm that, controlling for differences in employment, $y_t = \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\,\tilde{y}_t$, for all t.[51] On the other hand, in the two-period patent equilibrium, two capital-good firms implement a patent in each period as opposed to one in the immediate, one-period patent, equilibrium. As a result, and once again controlling for differences in employment, we can see (for instance, by (27) and (74)) that $i_t > \mu^{\frac{1}{\alpha}-1}\,\tilde{i}_t$, for all t. That is extending patent rights to two-periods implies that investment falls more compared to output. Nevertheless, these are in relative terms; in *absolute* terms the "output" effect always dominates the "investment" effect and consumption falls. Furthermore, the initial consumption difference grows in μ.[52]

[50] Of course, one needs to make sure first that condition (SPC1) is met so that a synchronized implementation equilibrium is possible.

[51] Taking into account differences in employment would imply that $y_t > \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\,\tilde{y}_t$ since $1 - l > 1 - \tilde{l}$.

[52] This result holds for all the parametrizations that I have considered.

With initial consumption's role being pivotal in the welfare comparison, welfare is greater in the one-period patent immediate implementation equilibrium.

Combining the results above leads to the central policy implication of the paper:

Proposition 2. *For a sufficiently low innovation rate, extending patent rights to two periods can lead to a welfare improvement.*

5.3.3 Initial level of investment

Proceeding as in Section 4.1 , to find the initial level of investment in the two-period patent equilibrium, I set $\psi = 1$ and divide the RHS of (80) by $\mu^{\frac{1}{\alpha}-1}$. This yields

$$\tilde{i}_0^i = \left[\mu^{-\frac{1}{\alpha}} + \mu^{-\frac{1}{2}(\frac{1}{\alpha}+1)}\right] (1-\alpha)^{\frac{1}{\alpha}} \frac{1-\tilde{l}}{R^{\frac{1}{\alpha}}} . \tag{92}$$

For all considered parametrizations, initial investment in the two-period patent equilibrium is lower than in both one-period patent equilibria. Hence, the welfare improvement that Proposition 2 considers is unconditional on the initial level of investment.

6 Conclusion

This paper showed that implementation cycles in the presence of capital and the absence of borrowing constraints or constraints on investment volatility are possible. The reason is that patents are on investment-specific technological change which introduces a one-period discrepancy between the time a new patent is implemented and the time revenue out of it is realized.

Furthermore, the exogenous generation of patents permitted it to view patent rights from a different perspective. The "default" one concerns the incentives of innovators which, ultimately, affect the generation of patents. This paper abstracted from this, otherwise important and heated, debate and, instead, focused on the effects that the length of patent rights can have on the implementation of patents. In particular, it showed that a prolongation of patent rights eliminates implementation cycles and may lead to welfare improvements.

The model, arguably highly stylized, certainly has its limitations. From a macro-perspective, consumption booms before output and is too volatile. From a more theoretical one, the analysis was silent about transitional dynamics, whereas, what is a related issue, the initial conditions were assumed to be the "right" ones. Resolving these issues could be part of future research.

A Appendix: Proofs

Proof of Condition 1 (No storage): There are two kinds of storable commodities in the economy: the capital goods and the final good. I deal with these in turn. The storage technology I assume in both cases is one-to-one.

Capital goods. Suppose that capital-good firm j produces an additional unit of capital good i in period $t-1$ and, instead of selling it to the final-good firm in period t, it instead stores and sells it in period $t+1$ (I assume that capital depreciates only if used). The cost of producing it as of $t-1$ is, say, $\frac{1}{\psi}$, whereas the revenue generated out of it as of $t-1$ is $\frac{q_{t+1}}{R_t\, R_{t-1}}$, where q is the competitive price offered by the final-good firm given by (14). No storage takes place if

$$\frac{R_t}{\psi\, R_t\, R_{t-1}} < \frac{1}{\psi}\,,$$

which is equivalent to

$$R_{t-1} > 1\,. \tag{93}$$

If, instead, the capital-good firm considers selling the additional unit of capital in period $t+2$, then, ignoring the possibility that a new idea will render the one in question obsolete, the no-storage condition becomes

$$R_t\, R_{t-1} < 1\,.$$

It follows then that (93) suffices to rule out storage in this case as well. Of course, if a new patent renders the one in question obsolete and supposing that the firm in question receives the new patent (see also the last paragraph in this proof), discounted revenue will be even lower and the no-storage condition will hold even more easily.

Proceeding in this way, (93) suffices to rule out storage and sale of a capital good in any period after period $t+2$. Therefore, positive net interest rates rule out storage in equilibrium.

Let me underline that, in the above argument, I have implicitly assumed that a capital-good firm sells at least an infinitesimally small quantity of the capital good it specializes in in t. This implies that in case it possesses and makes use of a superior technology (patent), that becomes publicly available in t so that its competitors copy it and the competitive price prevails in $t+1$. And, of course, I rule out the possibility that a firm uses two different technologies at the same time.

I deal with the possibility that a capital-good firm possesses a superior technology and does not implement it in later sections of the main text and I label the respective conditions profit conditions 1 and 2 . In other words, the profit conditions and the no-storage condition act in a somewhat complementary way. The former specify that a capital-good firm implements a patent and, since it maximizes profits, meets the whole demand for the type of capital it specializes in *when* it is conjectured to do so, whereas the latter rules out the possibility that it produces an additional amount of capital which it stores in order to sell it in a future period.

But still there is the possibility that a firm with a superior technology prefers to implement it in the following period, but considers using it immediately in secrecy aiming to sell the capital it produces using it in two periods. Given that it faces a given demand from the final-good firm and acts as a profit maximizer -that is it sells a certain profit-maximizing quantity- , for positive net real interest rates the discounted cost of producing it tomorrow is lower than today. Given that the price per unit is constant and common under both scenarios, once again positive net real interest rates rule this possibility out. As for the case in which that firm considers selling capital in any other future period (from the third period following the considered one onwards) , see my analysis above.

Last, note that in some of the above arguments I assumed that the capital-good firm deciding whether to store capital or not will with certainty have the chance to produce capital in the future which, of course, need not be the case and which would render some of my above arguments irrelevant.

Final good. It is easy to confirm that condition (93) rules out storage of the final good as well.

Derivations in Section 3.2 :

Profit condition (58) . As stationarity requires $\frac{x_{\tau+1}}{x_\tau} = v$, combining (53) and (56) , and given that, again by stationarity, $R_{\tau-1} = R_{\tau+1}$ and $l_\tau = l_{\tau+2}$ one can get

$$\frac{1-l_{\tau+1}}{(1-\alpha)\, R_\tau^{\frac{1}{\alpha}-1}} - \frac{\mu^{\frac{1}{\alpha}-1}(1-l_\tau)}{R_{\tau-1}^{\frac{1}{\alpha}}} = v\left(\frac{1-l_\tau}{(1-\alpha) R_{\tau-1}^{\frac{1}{\alpha}-1}} - \frac{1-l_{\tau+1}}{\mu\, R_\tau^{\frac{1}{\alpha}}}\right). \tag{94}$$

Rearranging terms in (94) yields

$$\left(\frac{1-l_{\tau+1}}{1-l_\tau}\right)\left(\frac{1}{1-\alpha}+\frac{v}{\mu R_\tau}\right) \;=\; \left(\frac{R_\tau}{R_{\tau-1}}\right)^{\frac{1}{\alpha}-1}\left(\frac{\mu^{\frac{1}{\alpha}-1}}{R_{\tau-1}}+\frac{v}{1-\alpha}\right). \tag{95}$$

Combining (95) with (23) implies that

$$\frac{1-l_{\tau+1}}{1-l_\tau} \;=\; \left(\frac{R_\tau}{R_{\tau-1}}\right)^{\frac{1}{\alpha}-1} R_\tau \left[\frac{\beta\,\mu\,(1+\beta(1-\alpha))}{\mu+\beta(1-\alpha)}\right]. \tag{96}$$

The profit condition (57) requires that

$$\frac{1-l_{\tau+1}}{1-l_\tau} \;>\; R_\tau \left(\frac{R_\tau}{R_{\tau-1}}\right)^{\frac{1}{\alpha}-1}.$$

Substituting in the LHS of (57) the RHS of (96), taking into account that R_τ and $R_{\tau+1}$ are both positive (since λ and v, given by (60) and (63), respectively, are positive) and rearranging yields the profit condition (58).

Leisure equations (61) and (62). Combining (6) with (9) and the production function (8), we get

$$\beta \;=\; \frac{y_{\tau+1}}{y_\tau}\,\frac{1-l_\tau}{1-l_{\tau+1}}\,\frac{l_{\tau+1}}{l_\tau}\,\frac{1}{R_\tau}. \tag{97}$$

Equations (52) and (55) together imply that

$$\frac{y_{\tau+1}}{y_\tau} \;=\; \frac{1-l_{\tau+1}}{1-l_\tau}\left(\frac{R_{\tau-1}}{R_\tau}\right)^{\frac{1}{\alpha}-1}. \tag{98}$$

Substituting (98) into (97) yields

$$\frac{l_{\tau+1}}{l_\tau} \;=\; \beta\, R_\tau \left(\frac{R_\tau}{R_{\tau-1}}\right)^{\frac{1}{\alpha}-1}. \tag{99}$$

Combining (96) and (99) results in

$$\frac{1-l_{\tau+1}}{l_{\tau+1}} \;=\; \frac{1-l_\tau}{l_\tau}\left[\frac{\mu\,(1+\beta(1-\alpha))}{\mu+\beta(1-\alpha)}\right]. \tag{100}$$

The intratemporal optimality condition of the household in period τ is

$$\chi\,\frac{x_\tau}{l_\tau} \;=\; w_\tau\,, \tag{101}$$

where x_τ is given by (53).

Labor demand from the final-good firm (see (9)) is $w_\tau = \frac{\alpha\, y_\tau}{1-l_\tau}$, with y_τ given by (52). Combining these and substituting on the RHS of (101) yields

$$\frac{1}{l_\tau}\left[\frac{1-l_\tau}{(1-\alpha)R_{\tau-1}^{\frac{1}{\alpha}-1}}-\frac{1-l_{\tau+1}}{\mu R_\tau^{\frac{1}{\alpha}}}\right]=\frac{\alpha}{\chi\,(1-\alpha)}\,\frac{1}{R_{\tau-1}^{\frac{1}{\alpha}-1}}\,. \tag{102}$$

Multiplying both sides of (102) by $R_{\tau-1}^{\frac{1}{\alpha}-1}$ and using (99) yields

$$\frac{1-l_\tau}{l_\tau}\,\frac{1}{1-\alpha}-\frac{1-l_{\tau+1}}{l_{\tau+1}}\frac{\beta}{\mu}=\frac{\alpha}{\chi\,(1-\alpha)}\,. \tag{103}$$

Using (100) to substitute for $\frac{1-l_{\tau+1}}{l_{\tau+1}}$ in (103) results in (62). In turn, inserting (62) into (100) yields (61).

Derivation of (63). Substituting for the interest rates given by (23) into (99) and rearranging implies

$$v=\left(\mu^{(\frac{1}{\alpha}-1)^2}\,\frac{l_{\tau+1}}{l_\tau}\right)^{\frac{\alpha}{2-\alpha}}\,. \tag{104}$$

To find $\frac{l_{\tau+1}}{l_\tau}$, divide (61) by (62) across sides which yields

$$\frac{l_{\tau+1}}{l_\tau}=\frac{\frac{\alpha}{\chi}\,\frac{\mu+\beta(1-\alpha)}{\mu-\beta^2(1-\alpha)^2}+1}{\frac{\alpha}{\chi}\,\frac{\mu\,(1+\beta(1-\alpha))}{\mu-\beta^2(1-\alpha)^2}+1}\,. \tag{105}$$

Inserting (105) into (104) results in (63).

Proof of Claim 1: As the fraction term in (63) is lower than one, it suffices to show that

$$\mu^{(\frac{1}{\alpha}-1)^2\,\frac{1-\alpha}{2-\alpha}}<\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\,.$$

It is straightforward to confirm that this is true.

Transversality Condition: The transversality condition (TVC) requires the agent's present discounted lifetime wealth to converge. I explore it for the synchronized implementation equilibrium.

The first step is to find the present discounted lifetime wealth of the agent. The agent's wealth consists of his labor income (DLI) and profits ($D\Pi$) out of the ownership of capital-good firms. I check both in turn.

The present discounted lifetime labor income is

$$DLI=w_1\,n_1+\frac{w_2\,n_2}{R_1}+\frac{w_3\,n_3}{R_1\,R_2}+\ldots\;\;.$$

Using the fact that, from the final-good firm's problem, $w_t = \alpha \frac{y_t}{n_t}$ and that, by stationarity, $n_1 = n_3 = \ldots$, $n_2 = n_4 = \ldots$, $R_1 = R_3 = \ldots$ and $R_2 = R_4 = \ldots$ the above expression simplifies to

$$DLI = n_1 \left[\frac{\alpha\, y_1}{n_1} + \frac{\alpha\, y_3}{n_1\, R_1\, R_2} + \frac{\alpha\, y_5}{n_1\, (R_1\, R_2)^2} \cdots \right] + \frac{n_2}{R_1} \left[\frac{\alpha\, y_2}{n_2} + \frac{\alpha\, y_4}{n_2\, R_1\, R_2} + \frac{\alpha\, y_6}{n_2\, (R_1\, R_2)^2} \cdots \right].$$

Since output grows by $\mu^{\frac{1}{\alpha}-1}$ every two periods, the above expression boils down to

$$DLI = \alpha\, (y_1 + \frac{y_2}{R_1}) \left[1 + \frac{\mu^{\frac{1}{\alpha}-1}}{R_1\, R_2} + \left(\frac{\mu^{\frac{1}{\alpha}-1}}{R_1\, R_2} \right)^2 + \ldots \right].$$

Turning to present discounted profits and noting that they grow by $\mu^{\frac{1}{\alpha}-1}$ every two periods,

$$D\Pi = \frac{\Pi_{3,1} + \Pi_{3,2}}{R_1\, R_2} \left[1 + \frac{\mu^{\frac{1}{\alpha}-1}}{R_1\, R_2} + \left(\frac{\mu^{\frac{1}{\alpha}-1}}{R_1\, R_2} \right)^2 + \ldots \right].$$

Given that $R_1\, R_2 = \frac{\mu^{\frac{1}{\alpha}-1}}{\beta^2}$, the above expressions converge, hence, the TVC is always satisfied.

Proceeding in the same way, it is straightforward to check that the TVC is also always satisfied in the case of the immediate implementation equilibrium in which $R = \frac{\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}}{\beta}$.

Proof in Section 5.3.2 : Subtracting (76) from (31) -having set $\psi = 1$ in both- and taking into account that $1 - l > 1 - \tilde{l}$ implies that

$$x_1^i - \tilde{x}_1^i > (1-\alpha)^{\frac{1}{\alpha}}\, R^{-\frac{1}{\alpha}}\, (1-\tilde{l}) \left(\frac{R}{1-\alpha} \left[1 - \mu^{-(\frac{1}{\alpha}-1)} \right] - \left[\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)} - \mu^{-\frac{1}{2}(\frac{1}{\alpha}+1)} \right] \right)$$

$$= (1-\alpha)^{\frac{1}{\alpha}-1}\, R^{-\frac{1}{\alpha}}\, (1-\tilde{l}) \left(R \left[1 - \mu^{-(\frac{1}{\alpha}-1)} \right] - (1-\alpha)\, \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)} \left(1 - \mu^{-\frac{1}{\alpha}} \right) \right)$$

$$> (1-\alpha)^{\frac{1}{\alpha}-1}\, R^{-\frac{1}{\alpha}}\, (1-\tilde{l})\, \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)} \left[1 - \mu^{-(\frac{1}{\alpha}-1)} - (1-\alpha) \left(1 - \mu^{-\frac{1}{\alpha}} \right) \right]$$

$$= (1-\alpha)^{\frac{1}{\alpha}-1}\, R^{-\frac{1}{\alpha}}\, (1-\tilde{l})\, \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)} \left[\alpha - \mu^{-(\frac{1}{\alpha}-1)} + (1-\alpha)\, \mu^{-\frac{1}{\alpha}} \right].$$

In the third line, I use the fact that $\beta \in (0,1)$ taking into account that $\mu > 1$ and $\alpha \in (0,1)$, which imply that $1 - \mu^{-(\frac{1}{\alpha}-1)} > 0$.

The next step is to show that

$$\alpha - \mu^{-(\frac{1}{\alpha}-1)} + (1-\alpha)\, \mu^{-\frac{1}{\alpha}} \geq 0,$$

or equivalently that

$$\alpha\,\mu^{\frac{1}{\alpha}} + 1 - \alpha \geq \mu\,. \tag{106}$$

With $\mu > 1$ and $\alpha \in (0,1)$, one can confirm that eq. (106) is always true. Hence, $x_1^i > \tilde{x}_1^i$, as desired.

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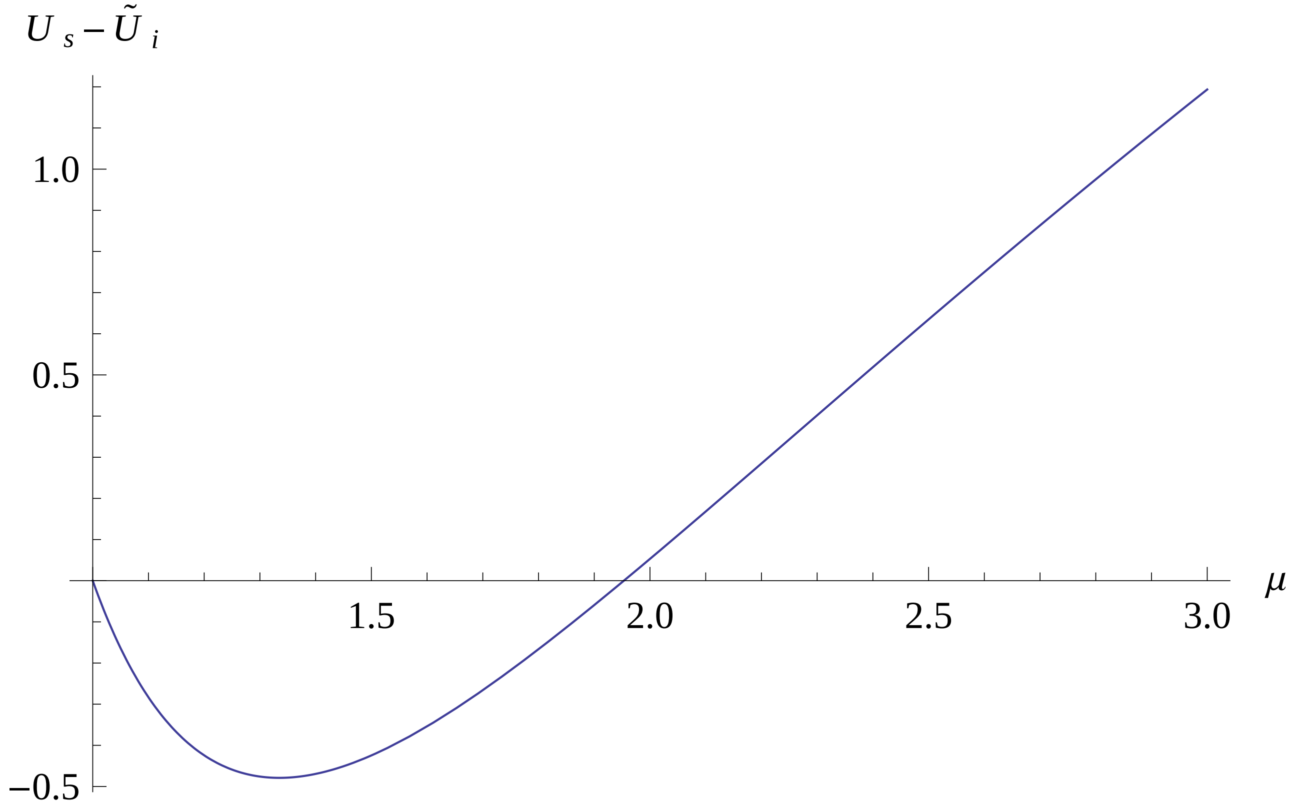

Figure 1: Welfare comparison between the synchronized implementation equilibrium and the two-period patent equilibrium