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Exergy Optimisation for Cascaded Thermal Storage

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Abstract
Cascaded thermal storage, consisting of multiple Phase Change Materials (PCMs) with different melting temperatures, has been proposed to solve the problem of poor heat transfer caused by unavoidable decrease of temperature differences during heat exchange process. This paper conducts a theoretical study of the overall thermal performance for a cascaded thermal storage system. Both heat transfer rate and exergy efficiency are taken into account. The main findings are: the cascaded arrangement of PCMs enhances the heat transfer rate by up to 30%, whilst it does not always improve the exergy efficiency (-15 to +30%). Enhanced heat transfer and reduced exergy efficiency can both be attributed to the larger temperature differences caused by the cascaded arrangement. A new parameter $h_{ex}$ (exergy transfer rate) has been proposed to measure the overall thermal performance. It is defined as the product of heat transfer rate and exergy efficiency, representing the transfer rate of the utilisable thermal energy. The simulation results indicate that the cascaded thermal storage has higher overall thermal performance than the single-staged storage despite of higher exergy efficiency loss.

Nomenclature
Bi Biot number, $hd/\lambda$ (dimensionless)
c_p specific heat of HTF (kJ/kg)
c_s specific heat of PCMs (kJ/kg)
d characteristic length (m)
dA differential thermal area (m²)
dq differential heat flow (kJ)
m mass flow rate (kg/s)
h system dimension (m)
h heat transfer coefficient (W/m² K)
h_e enthalpy of PCMs (kJ/kg)
H L latent heat (kJ/kg)
L system length (m)
Nu Nusselt number, $hd/\lambda$ (dimensionless)
P pressure (Pa)
Pr Prandtl number, $C_p\mu/\lambda$ (dimensionless)
q overall heat exchange rate (W/m²)
Re Reynolds number, $ud/\nu$ (dimensionless)
R_g ideal gas constant (kJ/kg K)
s specific entropy (kJ/kg ℃)
T temperature (℃)
T_0 ambient temperature (℃)
T_10 inlet HTF temperature (℃)
In a Thermal Energy Storage (TES) system, the temperature differences undergo an unavoidable decrease during heat exchange process, which worsens the heat transfer. In order to tackle this problem, a new concept of cascaded thermal storage has been proposed [1–3]. A cascaded thermal storage system consists of multiple Phase Change Materials (PCMs) with staged melting temperatures, so that a relatively high temperature difference can be maintained to achieve higher heat transfer rate during the charging/discharging process. The concept of cascaded thermal storage was tested by Michelsa and Pitz-Paal [4] for high-temperature molten salt storage system, and their results indicated that cascaded arrangement of PCMs increased the charging/discharging rate. Watanabe et al. [5] also identified a significant heat transfer enhancement in their 'three-type' storage system.

However, most previous studies on cascaded storage have focused on heat transfer rate, and therefore failed to reflect an important energy conversion factor – exergy. Exergy is the useful part of thermal energy in PCMs which can be converted into electricity. Krane [6] employed the ε–NTU analysis to conduct an exergy study of a TES system, but only sensible heat was considered. It is necessary to make an overall thermal performance analysis of a TES system, not only considering sensible heat, but also latent heat. This study aims to investigate the overall thermal performance of a cascaded thermal storage system, considering both heat transfer performance and exergy efficiency.

2. Problem description
For comparison, two systems are presented in this study: the Cascaded Storage (CS) and the Single-staged Storage (SS). Fig. 1 (a) illustrates the CS system, and Fig. 1 (b) illustrates the SS system. The CS system was formed by staging three PCMs (different physical properties) along the flow direction of HTF (heat transfer fluid), whilst the SS system was formed by using only one PCM.

In Fig. 1, \( H_L \) (kJ/kg) and \( T_m (^\circC) \) denote latent heat and melting temperature, respectively; \( h \) and \( L \) show the dimensions of the two systems. Heat transfer fluid (HTF) enters each system from the left (inlet temperature \( T_{in} = 100 ^\circ C \)), and exits each system from the right (temperature \( T(t) \), varying from time). The thermal properties of PCMs used in this study are shown in Table 1. The melting point of PCM 4 (the SS system) was roughly chosen to have the average value of three PCMs used in the Cascaded Storage (CS) system, and thus a comparison made between two systems can be justified. The initial temperatures of two systems were both 20°C, and the ambient temperature was 20°C.
3. Mathematical description

3.1. Exergy analysis

The entropy change [3] of a system from state ‘1’ to state ‘2’ can be written as:

\[ s_2 - s_1 = c_p \ln(T_2 / T_1) - R_g \ln(P_2 / P_1) \]  

The unusable part of the thermal energy (i.e. anergy X), depends on the irreversible entropy increase, which is shown in Eq. (2).
\[ X = T_0 \left( s_2 - s_1 \right) = T_0 \left[ c_p \ln\left( \frac{T_2}{T_1} \right) - R_s \ln\left( \frac{P_2}{P_1} \right) \right] \]  

Thus the percentage of the usable energy can be calculated by using Eq. (3):

\[ \eta_{ex} = \frac{c_p \left( T_2 - T_1 \right) - X}{c_p \left( T_2 - T_1 \right)} \]  

Substituting Eq. (2) into Eq. (3), exergy efficiency is given in Eq. (4):

\[ \eta_{ex} = \frac{c_p \left( T_2 - T_1 \right) - R_s \left[ c_p \ln\left( \frac{T_2}{T_1} \right) - R_s \ln\left( \frac{P_2}{P_1} \right) \right]}{c_p \left( T_2 - T_1 \right)} \]  

Eq. (4) can be reduced to Eq. (5), under the assumption of PCM incompressibility.

\[ \eta_{ex} = \frac{c_p \left( T_2 - T_1 \right) - c_p T_0 \ln\left( \frac{T_2}{T_1} \right)}{c_p \left( T_2 - T_1 \right)} \]  

In this study, Eq. (5) has been used to obtain the exergy efficiency for both the Cascade Storage and Single-staged Storage system.

### 3.2. Heat transfer analysis on the HTF side

Considering the charging process of each system, heat transfer comes from high-temperature heat transfer fluid (HTF) to low-temperature PCMs. Thermal resistance of heat transfer consists two parts: HTF-side resistance and PCMs-side resistance. The effective heat transfer coefficient on the HTF side can be obtained by simply employing the Dittus–Boelter Equation [7].

\[ Nu = 0.023 \Re^{0.8} \Pr^{0.4} \]  

In this study, water was used as HTF: \( u = 0.5 \) (flow velocity), \( v = 0.553 \times 10^{-6} \text{ m}^2/\text{s} \) (kinetic viscosity at 50°C), \( \Pr = 3.56 \) (Prandtl number at 50°C), characteristic length \( d = (h_2 - h_1)/2 = 0.01 \text{m} \). By employing the Dittus–Boelter Equation shown in Eq. (6), the effective heat transfer coefficient \( h \) was calculated as 1117.7W/m² (\( h = 0.023 \Re^{0.8} \Pr^{0.4}/d \)).

Biot number was also obtained as follows:

\[ Bi = \frac{hd}{\lambda} = 55.9 >> 1 \]  

Biot number [7] roughly represents how many times bigger the thermal resistance on the PCMs side is than that on the HTF side. Since Biot number is far more than 1, the thermal resistance on the HTF can be reasonably neglected, which gives much convenience to the following analyses.

### 3.3. Heat transfer analysis

Perfect thermal insulation was assumed in the study, so the heat transfer equations can be established by employing the Energy Conservation Law: PCMs absorb the same amount of thermal energy as HTF releases, which is reflected in Eq. (8).

\[ dq_f = dq_s = h(T_f - T_s) \text{d}A \]  

\( dq_f \) and \( dq_s \) in Eq. (8) can be expanded as follows:

\[ dq_f = c_j m_f \frac{\partial T_f}{\partial t} \frac{\partial}{\partial x} \]  

\[ dq_s = \rho \text{h}_s \text{d}A \frac{\partial h_s}{\partial t} \]
The last term on the right hand side of Eq. (9) is equal to the flow velocity of HTF, which is shown in Eq. (11).

\[
\frac{\partial x}{\partial t} = u
\]

Thus Eq. (8) can be rewritten as Eq. (12).

\[
-c_j m_j \frac{\partial T_j}{\partial x} u = \rho_j h dA \frac{\partial h}{\partial t} = h(T_j - T_s)\frac{dA}{dt}
\]

To tackle the phase change problem, the Enthalpy Method [8] has been employed. The PCM enthalpy \( h_s \) shown in Eq. (10) has the following relationship with the PCM temperature \( T_s \) [8–10].

\[
T_s = \begin{cases} 
\frac{h_s}{c_s}, & h_s \in (0, c_s T_m) \\
T_m, & h_s \in (c_s T_m, c_s T_m + H_L) \\
\frac{h_s - H_L}{c_s}, & h_s \in (c_s T_m + H_L, +\infty)
\end{cases}
\]

4. Numerical procedure

As stated in the earlier section, Eq. (12) and (13) are the equations governing such particular heat transfer phenomena. These equations are solved simultaneously by the Finite Difference Method (FDM) under the workspace of Matlab®. Three thousand uniform meshes were used in the \( x \)-direction to ensure the simulation accuracy. Mesh independency was also examined, and it was found that 6000 meshes could only improve the accuracy by 0.17% compared to the case of 3000 meshes. The Implicit Iteration was adopted as the Difference Scheme, because the simulation indicated that Explicit Iteration made the results divergent whilst Implicit Iteration made the results convergent and accurate. Numerical simulations were set to stop when the error between two consecutive iterations was less than \( 10^{-4} \) (i.e., 0.01%).

5. Results and discussion

Figure 2 shows the comparison of heat transfer rates between the CS and SS system. It indicates that the cascaded arrangement of PCMs (CS) enhanced heat transfer rate by up to 30% (overall). However, it needs to be noted that the heat transfer rate of CS system was lower than that of SS system when PCM 2 finished melting process. The low heat transfer rate (CS) can be attribute to two reasons: firstly, the temperatures in the CS system increased rapidly (sensible heat only) when PCM 2 (50°C) finished phase change; secondly, at the same time when the temperatures in the CS system rose rapidly, the temperatures in the SS system kept relatively constant because PCM 4 (55°C) is still melting (latent heat). The rapidly-rising temperatures caused the decrease of temperature differences between PCMs and HTF, resulting in a lower heat transfer rate.

The phase change regions of PCM 1, PCM 2, PCM 3 and PCM 4 can also be seen in Fig. 2. The figure also indicates that PCM 3 used the most time to finish phase change, because temperature differences for PCM 3 were much smaller than others. As temperature differences decreased, PCM 2 and PCM 3 took more time to be melted than PCM 1.
Fig. 2. A comparison of heat transfer rate $q$ (W/m$^2$)

The comparison of exergy efficiency between the two systems is shown in Fig. 3. The CS system does not always have higher exergy efficiency than the SS system (-15% to +30%). The exergy efficiency of the CS system was lower than that of the SS system in early stages before PCM 4 started to melt, because PCM 1 and PCM 2 in the CS system delayed the increase of temperature rise due to their latent heat. Since lower temperatures mean lower quality of energy, the CS system had lower exergy efficiency at this time. However, the situation was changed when PCM 2 finished phase change and PCM 4 started its phase change. From this time on, the temperatures in the CS system began to increase rapidly (sensible heat) whilst the temperatures in the SS system kept relatively constant (latent heat), which led to a higher exergy efficiency of the CS system than the SS system.

Fig. 3. A comparison of Exergy Efficiency $\eta$ (%)
The Second Law of Thermodynamics states that not 100% of $q$ (shown in Eq. (8)) can be converted into electricity, meaning that heat transfer rate cannot reflect the real efficiency of a TES system. Thus, a new concept of exergy transfer rate $h_{ex}$ was proposed in the study to evaluate the overall thermal performance of the CS and SS systems.

$$h_{ex} = q \times \eta_{ex} \text{ (W/m}^2\text{)} \quad (14)$$

$h_{ex}$ denotes the effective exergy transfer rate, representing how much useful thermal energy is transferred from HTF to PCMs during charging processes.

Figure 4 shows the comparison of effective exergy transfer rates ($h_{ex}$) between the CS and SS system, it can be concluded that the CS system nearly always produced higher exergy flow rate (up to 23%) than the SS system. It needs to be noted that the CS system gave slightly lower exergy flow rate than the SS system, only when PCM 1 started phase change and after PCM 4 finished phase change. The probable reasons are as follows: when PCM 1 started its phase change, the CS system had lower exergy efficiency than the SS system although it had slightly higher heat transfer rate than the SS system; after PCM 4 finished its phase change, the heat transfer rate of the SS system was higher than the CS system due to the long-time delay of temperature rise (latent heat of PCM 4), but the exergy efficiency of the SS system was much lower than the CS system (shown in Fig. 3) due to its low temperatures after phase change.

6. Conclusions
An overall comparative study for the Cascaded Storage system and the Single-staged Storage system was carried out. In order to take energy conversion efficiency into account, effective exergy transfer rate was first introduced to measure the real energy efficiency of a Thermal Energy Storage system. The main findings are: although the Cascaded Storage system shows lower exergy efficiency (-15%), it has much higher heat transfer performance (30%), making its overall thermal performance still superior (23%) to the Single-staged Storage system.

7. Future work
This work has neglected natural convection and the heat conduction in vertical direction (y-direction). It is necessary to conduct further work that incorporates a multi-dimensional numerical simulation on the coupled natural convection and heat conduction.
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