A Thesis Submitted for the Degree of PhD at the University of Warwick

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Essays in International Finance

by Gino Cenedese

A thesis submitted in partial fulfilment
of the requirements for the degree of
Doctor of Philosophy in Finance

Warwick Business School
The University of Warwick

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Declaration

I declare that any material contained in this thesis has not been submitted for a degree to any other university. I further declare that one paper titled “Average Variance, Average Correlation and Currency Returns”, drawn from Chapter One of this thesis, is co-authored with Lucio Sarno and Ilias Tsiakas. Also, the paper “Currency Fair Value Models”, drawn from Chapter Three of this thesis and co-authored with Thomas Stolper, is forthcoming in the Handbook of Exchange Rates, edited by Jessica James, Ian W. Marsh, and Lucio Sarno.

Gino Cenedese

September 2011
Abstract

This thesis consists of three essays in international finance, with a focus on the foreign exchange market. The first chapter provides an empirical investigation of the predictive ability of average variance and average correlation on the return to carry trades. Using quantile regressions, we find that higher average variance is significantly related to large future carry trade losses, whereas lower average correlation is significantly related to large gains. This is consistent with the carry trade unwinding in times of high volatility and the good performance of the carry trade when asset correlations are low. Finally, a new version of the carry trade that conditions on average variance and average correlation generates considerable performance gains net of transaction costs.

In the second chapter I study the evolution over time of the response of exchange rates to fundamental shocks. Using Bayesian time-varying-parameters VARs with stochastic volatility, I provide empirical evidence that the transmission of these shocks has changed over time. Specifically, currency excess returns tend to initially underreact to interest rate differential shocks for the whole sample considered, undershooting the level implied by uncovered interest rate parity and long-run purchasing power parity. In contrast, at longer horizons the previously documented evidence of overshooting tends to disappear in recent years in the case of the euro, the British pound and the Canadian dollar. Instead, overreaction at long horizons is a persistent feature of the excess returns on the Japanese yen and the Swiss franc throughout the whole sample.

In the third chapter we provide a comprehensive review of models that are used by policymakers and international investors to assess exchange rate misalignments from their fair value. We survey the literature and illustrate a number of models by means of examples and by evaluating their strengths and weaknesses. We analyse the sensitivity of underlying balance (UB) models with respect to estimated trade elasticities. We also illustrate a fair value concept extensively used by financial markets practitioners but not previously formalised in the academic literature, and dub it the indirect fair value (IFV). As case studies, we analyse the models used by Goldman Sachs and by the International Monetary Fund’s Consultative Group on Exchange Rate Issues (CGER).
Overview

This thesis consists of three essays in international finance, with a focus on the foreign exchange market. The first chapter provides an empirical investigation of the predictive ability of average variance and average correlation on the return to carry trades. The carry trade is a popular currency trading strategy that invests in high-interest currencies by borrowing in low-interest currencies. This strategy is designed to exploit deviations from uncovered interest parity (UIP). If UIP holds, the interest rate differential is on average offset by a commensurate depreciation of the investment currency and the expected carry trade return is zero. There is extensive empirical evidence dating back to Bilson (1981) and Fama (1984) that UIP is empirically rejected. In practice, it is often the case that high-interest rate currencies appreciate rather than depreciate.\(^1\) As a result, over the last 35 years, the carry trade has delivered sizeable excess returns and a Sharpe ratio more than twice that of the US stock market (e.g., Burnside et al., 2011). It is no surprise, therefore, that the carry trade has attracted enormous attention among academics and practitioners.

An emerging literature argues that the high average return to the carry trade is no free lunch in the sense that high carry trade payoffs compensate investors for bearing risk. The risk measures used in this literature are specific to the foreign exchange (FX) market as traditional risk factors used to price stock returns fail to explain the returns to the carry trade (e.g., Burnside, 2010). In a cross-sectional study, Menkhoff et al. (2011) find that the large average carry trade payoffs are compensation for exposure to global FX volatility risk. In times of high unexpected volatility, high-interest currencies

\(^1\)The empirical rejection of UIP leads to the well-known forward bias, which is the tendency of the forward exchange rate to be a biased predictor of the future spot exchange rate (e.g., Engel, 1996).
Overview

deliver low returns, whereas low-interest currencies perform well. This suggests that investors should unwind their carry trade positions when future volatility risk increases. Christiansen et al. (2011) further show that the risk exposure of carry trade returns to the stock and bond markets depends on the level of FX volatility. Lustig et al. (2011) identify a slope factor in the cross-section of FX portfolios based on the excess return to the carry trade itself constructed in similar fashion to the Fama and French (1993) “high-minus-low” factor. Burnside et al. (2011) propose that the high carry trade payoffs reflect a peso problem, which is a low probability of large negative payoffs. Although they do not find evidence of peso events in their sample, they argue that investors still attach great importance to these events and require compensation for them. Brunnermeier et al. (2009) suggest that carry trades are subject to crash risk that is exacerbated by the sudden unwinding of carry trade positions when speculators face funding liquidity constraints. Similar arguments based on crash risk and disaster premia are put forth by Farhi et al. (2009) and Jurek (2009).

This chapter investigates the intertemporal tradeoff between FX risk and the return to the carry trade. We contribute to the recent literature cited above by focusing on four distinct objectives. First, we set up a predictive framework, which differentiates this study from the majority of the recent literature that is primarily concerned with the cross-sectional pricing of FX portfolios. We are particularly interested in whether current market volatility can predict the future carry trade return. Second, we evaluate the predictive ability of FX risk on the full distribution of carry trade returns using quantile regressions, which are particularly suitable for this purpose. In other words, we relate changes in FX risk with large future gains and losses to the carry trade located in the tails of the return distribution. Predicting the full return distribution is useful for the portfolio choice of investors (e.g., Cenesizoglu and Timmermann, 2010), and can also shed light on whether we can predict currency crashes (Farhi et al., 2009; Jurek, 2009). Third, we define a set of FX risk measures that capture well the movements in aggregate FX volatility and correlation. These measures have recently been studied in the equities literature but are new to FX. Finally, we assess the economic gains of our
analysis by designing a new version of the carry trade strategy that conditions on these FX risk measures.\(^2\)

The empirical analysis is organized as follows. The first step is to form a carry trade portfolio that is rebalanced monthly using up to 33 US dollar nominal exchange rates. Our initial measure of FX risk is the market variance defined as the variance of the returns to the FX market portfolio. We take a step further by decomposing the market variance in two components: the cross-sectional average variance and the cross-sectional average correlation, implementing the methodology applied by Pollet and Wilson (2010) to predict equity returns. Then, using quantile regressions, we assess the predictive ability of average variance and average correlation on the full distribution of carry trade returns. Quantile regressions provide a natural way of assessing the effect of higher risk on different parts (quantiles) of the carry return distribution.\(^3\) Finally, we design an augmented carry trade strategy that conditions on average variance and average correlation. This new version of the carry trade is implemented out of sample and accounts for transaction costs.

We find that the product of average variance and average correlation captures more than 90% of the time-variation in the FX market variance, suggesting that this decomposition works very well empirically. More importantly, the decomposition of market variance into average variance and average correlation is crucial for understanding the risk-return tradeoff in FX. Average variance has a significant negative effect on the left tail of future carry trade returns, whereas average correlation has a significant negative effect on the right tail. This implies that: (i) higher average variance is significantly related to large losses in the future returns to the carry trade, potentially leading investors to unwind their carry trade positions, and (ii) lower average correlation is significantly

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\(^2\)There is a well-established literature that relates exchange rate returns to volatility (e.g., Diebold and Nerlove, 1989; and Bekaert, 1995). This literature differs from our study in that it focuses on individual exchange rates and uses conventional measures of individual exchange rate volatility. In general, these papers cannot detect a meaningful link between volatility and exchange rate movements, and we provide evidence that this is partly due to the way risk is measured.

\(^3\)Cenesizoglu and Timmermann (2010) estimate quantile regressions and relate them to the intertemporal capital asset pricing model of Merton (1973, 1980). Their results show that predictive variables (such as average variance and average correlation) have their largest effect on the tails of the return distribution.
related to large future carry trade returns by enhancing the gains of diversification. Market variance is a weaker predictor than average variance and average correlation because, by aggregating information about the latter two risk measures into one risk measure, market variance is less informative than using average variance and average correlation separately. Finally, the augmented carry trade strategy that conditions on average variance and average correlation performs considerably better than the standard carry trade, even accounting for transaction costs. Taken together, these results imply the existence of a meaningful predictive relation between average variance, average correlation and carry trade returns: average variance and average correlation predict currency returns when it matters most, namely when returns are large (negative or positive), whereas the relation may be non-existent in normal times.

In addition, we find that average variance is a significant predictor of the left tail of the exchange rate component to the carry trade return. We then show that the predictive ability of average variance and average correlation is robust to the inclusion of additional predictive variables. It is also robust to changing the numeraire from the US dollar to a composite numeraire that is based on the US dollar, the euro, the UK pound and the Japanese yen. We further demonstrate that implied volatility indices, such as the VIX for the equities market and the VXY for the FX market, are insignificant predictors of future carry returns, and hence cannot replicate the predictive information in average variance and average correlation. Finally, the predictive quantile regression framework allows us to compute a robust measure of conditional skewness, which is predominantly positive at the beginning of the sample and predominantly negative at the end of the sample.

Our analysis is partly motivated by the intertemporal capital asset pricing model (ICAPM) of Merton (1973, 1980), which implies a positive linear relation between the expected excess return on the risky market portfolio and the conditional market variance. The ICAPM may be applied to the FX market as it holds for any risky asset in any market. In this model, the coefficient on the market variance reflects the investors’ risk aversion. As systematic risk increases, risk-averse investors require a higher
risk premium to hold aggregate wealth and the expected return must rise. There is an extensive literature investigating the intertemporal risk-return tradeoff in equity markets, but the empirical evidence on the sign and statistical significance of the relation is inconclusive. Often the relation between risk and return has been found insignificant, and sometimes even negative.\footnote{See, among others, French et al. (1987); Chan et al. (1992); Glosten et al. (1993); Goyal and Santa-Clara (2003); Ghysels et al. (2005); Bali (2008). In a recent study of the FX market, Christiansen (2011) finds a positive contemporaneous risk-return tradeoff in exchange rates but no evidence of a predictive risk-return tradeoff.}

Our chapter is related to Bali and Yilmaz (2011), who estimate two types of predictive regressions based on the ICAPM: first, of individual FX returns on individual variances for which they find a positive but statistically insignificant relation; and second, of individual FX returns on the covariance between individual exchange rates and the FX market variance for which they find a positive and statistically significant relation. Our analysis, however, substantially deviates from Bali and Yilmaz (2011) in a number of ways: (i) we focus on the carry trade portfolio, not on individual exchange rates; (ii) we analyze a larger number of currencies (33 versus 6 exchange rates) and a longer sample (34 years versus 7 years); (iii) we decompose the market variance into average variance and average correlation; (iv) we assess predictability across the full distribution of carry trade returns using quantile regressions; and (v) we design a new carry trade strategy that conditions on average variance and average correlation leading to substantial gains over the standard carry trade.

The risk measures employed in our analysis have been the focus of recent intertemporal as well as cross-sectional studies of the equity market. The intertemporal role of average variance is examined by Goyal and Santa-Clara (2003) and Bali et al. (2005). These studies show that average variance reflects both systematic and idiosyncratic risk and can be significantly positively related to future equity returns. The intertemporal role of average correlation is examined by Pollet and Wilson (2010), who find that average correlation is a significant positive predictor of future stock market returns. If individual stocks share a common sensitivity to aggregate (market) shocks, then an increase in average correlations reflects an increase in aggregate systematic risk and
a corresponding increase in expected returns. In the cross-section of equity returns, the negative price of risk associated with market variance is examined by Ang et al. (2006, 2009). They find that stock portfolios with high sensitivities to innovations in aggregate volatility have low average returns. Similarly, Krishnan et al. (2009) find a negative price of risk for equity correlations. Finally, Chen and Petkova (2010) examine the cross-sectional role of average variance and average correlation. They find that for portfolios sorted by size and idiosyncratic volatility, average variance has a negative price of risk, whereas average correlation is not priced.

In the second chapter I study the evolution over time of the response of exchange rates to fundamental shocks. In a frictionless and risk-neutral economy, asset prices should react instantaneously to fundamental shocks to ensure that expected excess returns are zero. In the case of the foreign exchange market, this implies that a sudden increase in interest rate differentials should lead to an impact appreciation of the high-interest currency, followed by a depreciation so that uncovered interest rate parity (UIP) holds. A carry trader, who invests in a high-interest currency (the investment currency) by funding her position in a low-interest currency (the funding currency), would therefore face only an impact positive excess return, but this would then become zero on average as implied by UIP.

However, empirical evidence seems to be at odds with the exchange rate behaviour outlined above. The “forward premium puzzle” implies that UIP is systematically violated as future currency excess returns are predictable (Fama, 1984; Bilson, 1981; Engel, 1996), and that carry trade strategies tend to be profitable (Della Corte et al. 2009, Burnside et al. 2011). These results violate unconditional UIP—the response of the exchange rate to all shocks on average. Moreover, UIP is also violated conditionally: conditional on monetary policy shocks, cumulative excess returns on foreign exchange tend to be sizable and persistent.\footnote{In distinguishing between unconditional and conditional UIP violations, I follow Faust and Rogers (2003) and Scholl and Uhlig (2008).} This latter evidence has been studied in much of the literature on the “delayed overshooting puzzle”: contractionary monetary policy shocks lead to a persistent appreciation of the domestic currency before starting to
depreciate (Eichenbaum and Evans, 1995; Scholl and Uhlig, 2008). These dynamics stand in stark contrast with Dornbusch (1976) classical hypothesis of an immediate appreciation and subsequent persistent depreciation following a monetary policy shock, a hypothesis which follows from the assumption of UIP and long-run purchasing power parity (PPP). Similarly, Brunnermeier et al. (2009) find that exchange rates initially underreact to interest rate differential shocks: when the foreign interest rate increases relative to the domestic interest rate, the investment currency appreciates sluggishly, with cumulative excess returns reaching the level implied by UIP and PPP only after a few quarters. At longer horizons, instead, they find evidence of possible overreaction of the exchange rate to interest rate differential shocks.

This chapter re-examines these issues in light of the recent literature on nonlinearities in the foreign exchange market. I do not consider UIP and PPP in general, but conditional on interest rate differential shocks. Previous studies that document sizable conditional excess returns (violating UIP) and a sluggish reaction of the exchange rate do not generally consider the possibility that either or both the volatility of the shocks and the transmission mechanism may have changed over time. Therefore, previous results may not reflect the current state of the economy but just an average over the past. Given a simple present-value model for the currency excess return which assumes UIP and long-run PPP, the research questions are therefore the following: how large the deviations from the present value of future fundamentals should one expect following an interest rate shock, given the current state of the economy? Do these conditional deviations converge to the level implied by fundamentals, and, if so, how does this behaviour evolve over time as the state of the economy changes?

A number of previous studies have already documented how nominal and real exchange rate dynamics may have changed over time. Moreover, these studies have shown how allowing for nonlinearities may shed light, and possibly explain, apparent devia-

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6 The present-value approach adopted in this chapter is inspired by e.g. Froot and Ramadorai (2005), Brunnermeier et al. (2009), Engel (2010), and Engel and West (2010).

7 In this chapter, I consider as fundamentals only those strictly implied by the assumptions of UIP and long-run PPP, i.e. real interest rate differentials, as discussed in Section 2.2. Therefore, I do not consider other “classic” fundamentals such as relative money supplies and outputs, as in e.g. Engel and West (2005).
tions from the parity relations which form the basis of much of the international finance literature—namely, UIP and PPP. For example, Taylor et al. (2001) show that real exchange rates (or equivalently, PPP deviations) are well characterized by a nonlinear mean-reverting processes leading to time-varying half-lives in which larger shocks mean-revert much faster than those previously reported for linear models, therefore potentially explaining the PPP puzzle (Rogoff, 1996). Sarno et al. (2006) find that deviations from UIP display significant nonlinearities, consistent with theories based on transaction costs (e.g. Dumas, 1992) or limits to speculation (Lyons, 2001). This evidence leads them to conclude that UIP deviations may be less indicative of major market inefficiencies than previously thought. Christiansen et al. (2011) show that carry trade returns display time-varying risk exposure to the stock and bond markets depending on switching regimes characterized by the level of foreign exchange volatility. Mumtaz and Sunder-Plassmann (2010) find that the transmission of demand, supply and nominal shocks on the real exchange rate displays significant time variation, with an increasing impact of demand shocks over the years. However, none of these studies analyse the evolution of conditional violations of UIP over time.

Therefore, the importance of analysing nonlinearities in exchange rate dynamics seems to be undisputed. Similarly to the empirical studies above, I approximate nonlinearities by allowing for time variation in the parameters linking fundamentals to exchange rates. In the context of this chapter, in which I analyse conditional violations of UIP, this translates into estimating the time-varying impulse response functions of the currency excess return to interest rate differential shocks. A natural framework to estimate these impulse responses is to use a Bayesian time-varying-parameters vector-autoregression (TVP-VAR) with stochastic volatility. Particularly, I adopt the methodology from recent advances in the macroeconometric literature which has fruitfully applied this technique in other contexts, see e.g. Cogley and Sargent (2005), Primiceri

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8Bacchetta and van Wincoop (2004, 2010) provide a theory of exchange rate determination which rationalizes parameter instability in empirical exchange rate models. They show that foreign exchange market participants can optimally choose to change the weight attached to different economic fundamentals in the context of rational expectation models.
Allowing for time variation both in the VAR coefficients and the covariance matrix leaves it up to the data to determine whether the time variation of the linear structure derives from changes in the size of the shocks (impulse) or from changes in the propagation mechanism (response).

I provide empirical evidence that the transmission of the interest rate differential shocks has changed over time. However, even if to a varying degree over the years, some of the puzzling results previously documented with linear models remain. I show that currency excess returns tend to initially underreact to interest rate differential shocks for the whole sample considered, undershooting the level implied by UIP and long-run PPP. At longer horizons, the previously documented evidence of overshooting tends to disappear in recent years in the case of the euro, the British pound and the Canadian dollar. Instead, overreaction at long horizons is a persistent feature of the excess returns on the Japanese yen and the Swiss franc throughout the whole sample.

These results suggest that previously documented conditional violations of UIP may have secularly declined over time, at least for the euro, the British pound and the Canadian dollar. However, the results for the Japanese yen and the Swiss franc—two currencies which have been traditionally used for funding carry trade positions—may hint that speculation in the foreign exchange market may constitute a destabilizing force, driving exchange rates away from fundamentals.

In the third chapter we provide a comprehensive review of models that are used by policymakers and international investors to assess exchange rate misalignments from their fair value. Policymakers need to assess the possible misalignment of currencies for a number of reasons. Exchange rates play a crucial role in a country’s external adjustment process, particularly as economies become more and more integrated. At the time of writing, advanced economies have faced some degree of exchange rate realignment since the onset of the recent global financial crisis, whereas this realignment has been limited for emerging market economies, creating tensions and constituting a threat to the global

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9See also Koop and Korobilis (2010) for a recent survey of the methodologies used in this chapter.
recovery (IMF, 2011, Chapter 1). More generally, substantial misalignments can have severe consequences, as exchange rates may abruptly adjust when the misalignment becomes unsustainable, leading to currency crises generally associated with large output contractions, especially in emerging markets (Dornbusch et al., 1995; Gupta et al., 2007; Gourinchas and Obstfeld, 2011). In a theory paper, Engel (2011) shows that currency misalignments are inefficient, lower world welfare, and should be targeted by monetary policymakers in a model in which firms price to market and prices are sticky.

In Transition Economies, especially for countries of Central and Eastern Europe, the apparent trend appreciation of the real exchange rates of some of these countries raised the question of whether this appreciation reflected an adjustment to fair value or not (Égert et al., 2006). De Broeck and Sløk (2006) show how real exchange rates were generally misaligned at the onset of the transition and how most of the misalignment was eliminated over a relatively short period. In developing countries an overvalued currency can represent a major obstacle for a successful development strategy (Johnson et al., 2007).

For currency unions it is critically important to get a sense of fair value to assess the subsequent adjustment needs via relative inflation rates. And for heavily managed or pegged exchange rates, a fair value estimate may help establish policy targets. However, because exchange rates are a policy tool for the authorities of a country, and because there is the potential to use the currencies value to gain an advantage over another country, the political debate of fair value has always been contentious. The most recent example are the attempts to determine fair value for the Chinese currency (e.g., Cline and Williamson, 2008, 2011).

Investors and other agents engaging in international transactions, including trade, are interested in estimating the fair value of the currency as an input in hedging and investment strategies. For example, fair value models are useful to assess crash risks in popular currency speculation strategies. Exchange rates may be pushed away from fundamentals by carry trades, occasionally reverting back abruptly and leading to sudden losses (Brunnermeier et al., 2009; Plantin and Shin, 2011).
A number of investment strategies try to exploit long-run reversion to fair value by taking a long position in undervalued currencies and a short position in overvalued currencies. They typically provide lower risk-adjusted returns than carry strategies, but they seem to be less prone to crash risk (Jordà and Taylor, 2009; Nozaki, 2010). Major financial institutions recently introduced fully investable and tradable indices that track the performance of such strategies, such as Goldman Sachs FX Valuation Current (Goldman Sachs, 2009) and Deutsche Bank Valuation Index (Deutsche Bank, 2007).

Strategic Foreign Direct Investments (FDI) decisions with very long investment horizons may be affected by currency values. Variable real exchange rates may influence the location of production facilities chosen by multinationals (see e.g. Goldberg and Kolstad, 1995) and a fair value estimate may be useful as a long-term forecast.

Given the diverse use of currency fair value models highlighted above, it is important to understand which models are more suitable for a given context. In this chapter we analyse this issue in detail by surveying and critically assessing a number of fair value models proposed in the literature.\footnote{As highlighted below, we focus on the practical implementation of these models. For their theoretical foundations, see e.g. Chinn (2011).} We intentionally avoid an extensive discussion of PPP, as this literature is covered in detail in many surveys: see for example Sarno and Taylor (2003, Chapter 3) and Taylor and Taylor (2004).

After providing a short history of fair value models in the literature, we discuss the basic characteristics of fair value models with a particular focus on how implicit or explicit design choices typically affect the results, the robustness and the general usability of these models.

We provide an exposition of a number of fair value and equilibrium exchange rate models that are widely used in practice. In particular, we focus on the two main families of fair value models, namely the behavioural equilibrium exchange rate (BEER) and the underlying balance (UB) models. As case studies we then discuss in more detail the IMF framework, as well as Goldman Sachs Dynamic Equilibrium Exchange Rate (GSDEER) model. In both cases we highlight how the estimates of fair value are af-
fected by some typical implementation choices. We also illustrate a fair value concept extensively used by financial markets practitioners but not previously formalised in the academic literature. This model, which we dub Indirect Fair Value (IFV), relies on indirect estimation of fair value of the currency by “removing” the speculative components that drive exchange rates in the short run.

We argue that there is no explicit answer regarding which model delivers the correct fair value of a currency, because each model has its own individual strengths and weaknesses. We illustrate this point by means of examples, focusing on the practical implementation of the models. For instance, we discuss the sensitivity of UB models with regard to variations in import and export elasticities, and show how the different specifications of productivity can affect the results in “adjusted-PPP” models. Moreover, we discuss how the treatment of external balance in different models appears responsible for discrepancies between estimation results. Researchers are therefore left with a wide range of estimates, and many use a set of models or a combination of these in order to assess exchange rate misalignments.
Chapter 1

Average variance, average correlation and currency returns

1.1 Introduction

The carry trade is a popular currency trading strategy that invests in high-interest currencies by borrowing in low-interest currencies. This strategy is designed to exploit deviations from uncovered interest parity (UIP). If UIP holds, the interest rate differential is on average offset by a commensurate depreciation of the investment currency and the expected carry trade return is zero. There is extensive empirical evidence dating back to Bilson (1981) and Fama (1984) that UIP is empirically rejected. In practice, it is often the case that high-interest rate currencies appreciate rather than depreciate.\(^1\)

As a result, over the last 35 years, the carry trade has delivered sizeable excess returns and a Sharpe ratio more than twice that of the US stock market (e.g., Burnside et al., 2011). It is no surprise, therefore, that the carry trade has attracted enormous attention among academics and practitioners.

An emerging literature argues that the high average return to the carry trade is no free lunch in the sense that high carry trade payoffs compensate investors for bearing risk. The risk measures used in this literature are specific to the foreign exchange (FX)\(^1\)

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\(^1\)The empirical rejection of UIP leads to the well-known forward bias, which is the tendency of the forward exchange rate to be a biased predictor of the future spot exchange rate (e.g., Engel, 1996).
market as traditional risk factors used to price stock returns fail to explain the returns to the carry trade (e.g., Burnside, 2010). In a cross-sectional study, Menkhoff et al. (2011) find that the large average carry trade payoffs are compensation for exposure to global FX volatility risk. In times of high unexpected volatility, high-interest currencies deliver low returns, whereas low-interest currencies perform well. This suggests that investors should unwind their carry trade positions when future volatility risk increases. Christiansen et al. (2011) further show that the risk exposure of carry trade returns to the stock and bond markets depends on the level of FX volatility. Lustig et al. (2011) identify a slope factor in the cross-section of FX portfolios based on the excess return to the carry trade itself constructed in similar fashion to the Fama and French (1993) “high-minus-low” factor. Burnside et al. (2011) propose that the high carry trade payoffs reflect a peso problem, which is a low probability of large negative payoffs. Although they do not find evidence of peso events in their sample, they argue that investors still attach great importance to these events and require compensation for them. Brunnermeier et al. (2009) suggest that carry trades are subject to crash risk that is exacerbated by the sudden unwinding of carry trade positions when speculators face funding liquidity constraints. Similar arguments based on crash risk and disaster premia are put forth by Farhi et al. (2009) and Jurek (2009).

This chapter investigates the intertemporal tradeoff between FX risk and the return to the carry trade. We contribute to the recent literature cited above by focusing on four distinct objectives. First, we set up a predictive framework, which differentiates this study from the majority of the recent literature that is primarily concerned with the cross-sectional pricing of FX portfolios. We are particularly interested in whether current market volatility can predict the future carry trade return. Second, we evaluate the predictive ability of FX risk on the full distribution of carry trade returns using quantile regressions, which are particularly suitable for this purpose. In other words, we relate changes in FX risk with large future gains and losses to the carry trade located in the tails of the return distribution. Predicting the full return distribution is useful for the portfolio choice of investors (e.g., Cenesizoglu and Timmermann, 2010), and can
also shed light on whether we can predict currency crashes (Farhi et al., 2009; Jurek, 2009). Third, we define a set of FX risk measures that capture well the movements in aggregate FX volatility and correlation. These measures have recently been studied in the equities literature but are new to FX. Finally, we assess the economic gains of our analysis by designing a new version of the carry trade strategy that conditions on these FX risk measures.\(^2\)

The empirical analysis is organized as follows. The first step is to form a carry trade portfolio that is rebalanced monthly using up to 33 US dollar nominal exchange rates. Our initial measure of FX risk is the market variance defined as the variance of the returns to the FX market portfolio. We take a step further by decomposing the market variance in two components: the cross-sectional average variance and the cross-sectional average correlation, implementing the methodology applied by Pollet and Wilson (2010) to predict equity returns. Then, using quantile regressions, we assess the predictive ability of average variance and average correlation on the full distribution of carry trade returns. Quantile regressions provide a natural way of assessing the effect of higher risk on different parts (quantiles) of the carry return distribution.\(^3\) Finally, we design an augmented carry trade strategy that conditions on average variance and average correlation. This new version of the carry trade is implemented out of sample and accounts for transaction costs.

We find that the product of average variance and average correlation captures more than 90% of the time-variation in the FX market variance, suggesting that this decomposition works very well empirically. More importantly, the decomposition of market variance into average variance and average correlation is crucial for understanding the risk-return tradeoff in FX. Average variance has a significant negative effect on the left

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\(^2\)There is a well-established literature that relates exchange rate returns to volatility (e.g., Diebold and Nerlove, 1989; and Bekaert, 1995). This literature differs from our study in that it focuses on individual exchange rates and uses conventional measures of individual exchange rate volatility. In general, these papers cannot detect a meaningful link between volatility and exchange rate movements, and we provide evidence that this is partly due to the way risk is measured.

\(^3\)Cenesizoglu and Timmermann (2010) estimate quantile regressions and relate them to the intertemporal capital asset pricing model of Merton (1973, 1980). Their results show that predictive variables (such as average variance and average correlation) have their largest effect on the tails of the return distribution.
tail of future carry trade returns, whereas average correlation has a significant negative effect on the right tail. This implies that: (i) higher average variance is significantly related to large losses in the future returns to the carry trade, potentially leading investors to unwind their carry trade positions, and (ii) lower average correlation is significantly related to large future carry trade returns by enhancing the gains of diversification. Market variance is a weaker predictor than average variance and average correlation because, by aggregating information about the latter two risk measures into one risk measure, market variance is less informative than using average variance and average correlation separately. Finally, the augmented carry trade strategy that conditions on average variance and average correlation performs considerably better than the standard carry trade, even accounting for transaction costs. Taken together, these results imply the existence of a meaningful predictive relation between average variance, average correlation and carry trade returns: average variance and average correlation predict currency returns when it matters most, namely when returns are large (negative or positive), whereas the relation may be non-existent in normal times.

In addition, we find that average variance is a significant predictor of the left tail of the exchange rate component to the carry trade return. We then show that the predictive ability of average variance and average correlation is robust to the inclusion of additional predictive variables. It is also robust to changing the numeraire from the US dollar to a composite numeraire that is based on the US dollar, the euro, the UK pound and the Japanese yen. We further demonstrate that implied volatility indices, such as the VIX for the equities market and the VXY for the FX market, are insignificant predictors of future carry returns, and hence cannot replicate the predictive information in average variance and average correlation. Finally, the predictive quantile regression framework allows us to compute a robust measure of conditional skewness, which is predominantly positive at the beginning of the sample and predominantly negative at the end of the sample.

Our analysis is partly motivated by the intertemporal capital asset pricing model (ICAPM) of Merton (1973, 1980), which implies a positive linear relation between the
expected excess return on the risky market portfolio and the conditional market variance. The ICAPM may be applied to the FX market as it holds for any risky asset in any market. In this model, the coefficient on the market variance reflects the investors’ risk aversion. As systematic risk increases, risk-averse investors require a higher risk premium to hold aggregate wealth and the expected return must rise. There is an extensive literature investigating the intertemporal risk-return tradeoff in equity markets, but the empirical evidence on the sign and statistical significance of the relation is inconclusive. Often the relation between risk and return has been found insignificant, and sometimes even negative.⁴

Our chapter is related to Bali and Yilmaz (2011), who estimate two types of predictive regressions based on the ICAPM: first, of individual FX returns on individual variances for which they find a positive but statistically insignificant relation; and second, of individual FX returns on the covariance between individual exchange rates and the FX market variance for which they find a positive and statistically significant relation. Our analysis, however, substantially deviates from Bali and Yilmaz (2011) in a number of ways: (i) we focus on the carry trade portfolio, not on individual exchange rates; (ii) we analyze a larger number of currencies (33 versus 6 exchange rates) and a longer sample (34 years versus 7 years); (iii) we decompose the market variance into average variance and average correlation; (iv) we assess predictability across the full distribution of carry trade returns using quantile regressions; and (v) we design a new carry trade strategy that conditions on average variance and average correlation leading to substantial gains over the standard carry trade.

The risk measures employed in our analysis have been the focus of recent intertemporal as well as cross-sectional studies of the equity market. The intertemporal role of average variance is examined by Goyal and Santa-Clara (2003) and Bali et al. (2005). These studies show that average variance reflects both systematic and idiosyncratic risk and can be significantly positively related to future equity returns. The intertempo-

⁴See, among others, French et al. (1987); Chan et al. (1992); Glosten et al. (1993); Goyal and Santa-Clara (2003); Ghysels et al. (2005); Bali (2008). In a recent study of the FX market, Christiansen (2011) finds a positive contemporaneous risk-return tradeoff in exchange rates but no evidence of a predictive risk-return tradeoff.
1.2. Measures of Return and Risk for the Carry Trade

This section describes the FX data set and defines our measures for: (i) the excess return to the carry trade for individual currencies, (ii) the excess return to the carry trade for a portfolio of currencies, and (iii) three measures of risk: market variance, average variance and average correlation.

1.2.1 FX Data

We use a cross-section of US dollar nominal spot and forward exchange rates by collecting data on 33 currencies relative to the US dollar: Australia, Austria, Belgium, Ca-
1.2. Measures of Return and Risk for the Carry Trade

nada, Czech Republic, Denmark, Euro area, Finland, France, Germany, Greece, Hong Kong, Hungary, India, Ireland, Italy, Japan, Mexico, Netherlands, New Zealand, Norway, Philippines, Poland, Portugal, Saudi Arabia, Singapore, South Africa, South Korea, Spain, Sweden, Switzerland, Taiwan and United Kingdom. The sample period runs from January 1976 to February 2009. Note that the number of exchange rates for which there are available data varies over time; at the beginning of the sample we have data for 15 exchange rates, whereas at the end we have data for 22. The data are collected by WM/Reuters and Barclays and are available on Thomson Financial Datastream. The exchange rates are listed in Table 1.1.\(^5\)

1.2.2 The Carry Trade for Individual Currencies

An investor can implement a carry trade strategy for either individual currencies or, more commonly, a portfolio of currencies. In practice, the carry trade strategy for individual currencies can be implemented in one of two equivalent ways. First, the investor may buy a forward contract now for exchanging the domestic currency into foreign currency in the future. She may then convert the proceeds of the forward contract into the domestic currency at the future spot exchange rate. The excess return to this currency trading strategy for a one-period horizon is defined as:

\[ r_{j,t+1} = s_{j,t+1} - f_{j,t}, \]

for \( j = \{1, ..., N_t\} \), where \( N_t \) is the number of exchange rates at time \( t \), \( s_{j,t+1} \) is the log of the nominal spot exchange rate defined as the domestic price of foreign currency \( j \) at time \( t+1 \), and \( f_{j,t} \) is the log of the one-period forward exchange rate \( j \) at time \( t \), which is the rate agreed at time \( t \) for an exchange of currencies at \( t+1 \). Note that an increase in \( s_{j,t+1} \) implies a depreciation of the domestic currency, namely the US dollar.

Second, the investor may buy a foreign bond while at the same time selling a domestic bond. The foreign bond yields a riskless return in the foreign currency but a risky return.

\(^5\)Note that our data includes no more than 33 currencies to avoid having exchange rate series with short samples and non-floating regimes.
1.2. Measures of Return and Risk for the Carry Trade

in the domestic currency of the investor. Hence the investor who buys the foreign bond is exposed to FX risk. In this strategy, the investor will earn an excess return that is equal to:

\[ r_{j,t+1} = i_{j,t}^* - i_t + s_{j,t+1} - s_{j,t}, \]  

(1.2)

where \( i_{j,t}^* \) and \( i_t \) are the one-period foreign and domestic nominal interest rates respectively. The carry trade return in Equation (1.2) has two components: the interest rate differential \( i_{j,t}^* - i_t \), which is known at time \( t \), and the exchange rate return \( s_{j,t+1} - s_{j,t} \), which is the rate of depreciation of the domestic currency and will be known at time \( t + 1 \).

The returns to the two strategies are exactly equal due to the covered interest parity (CIP) condition: \( f_{j,t} - s_{j,t} = i_t - i_{j,t}^* \) that holds in the absence of riskless arbitrage. As a result, there is an equivalence between trading currencies through spot and forward contracts and trading international bonds.\(^6\) The return \( r_{j,t+1} \) defined in Equations (1.1) and (1.2) is also known as the FX excess return.

If UIP holds, then the excess return in Equations (1.1) and (1.2) will on average be equal to zero, and hence the carry trade will be unprofitable. In other words, under UIP, the interest rate differential will on average be exactly offset by a commensurate depreciation of the investment currency. However, it is extensively documented that UIP is empirically rejected so that high-interest rate currencies tend to appreciate rather than depreciate (e.g., Bilson, 1981; Fama, 1984). The empirical rejection of UIP implies that the carry trade for either individual currencies or portfolios of currencies tends to be highly profitable (e.g., Della Corte et al., 2009; Burnside et al., 2011).

1.2.3 The Carry Trade for a Portfolio of Currencies

There are many versions of the carry trade for a portfolio of currencies. In this chapter, we implement one of the most popular versions. We form a portfolio by sorting at the

\(^6\)There is ample empirical evidence that CIP holds in practice for the data frequency examined in this chapter. For recent evidence, see Akram et al. (2008). The only exception in our sample is the period following Lehman’s bankruptcy, when the CIP violation persisted for a few months (e.g., Mancini-Griffoli and Ranaldo, 2011).
1.2. Measures of Return and Risk for the Carry Trade

beginning of each month all currencies according to the value of the forward premium $f_{j,t} - s_{j,t}$. If CIP holds, sorting currencies from low to high forward premium is equivalent to sorting from high to low interest rate differential. We then divide the total number of currencies available in that month in five portfolios (quintiles), as in Menkhoff et al. (2011). Portfolio 1 is the portfolio with the highest interest rate currencies, whereas portfolio 5 has the lowest interest rate currencies. The monthly return to the carry trade portfolio is the excess return of going long on portfolio 1 and short on portfolio 5. In other words, the carry trade portfolio borrows in low-interest rate currencies and invests in high-interest rate currencies. We denote the monthly return to the carry trade portfolio from time $t$ to $t + 1$ as $r_{C,t+1}$.

1.2.4 FX Market Variance

Our first measure of risk is the FX market variance, which captures the aggregate variance in FX. Note that this measure of market variance focuses exclusively on the FX market, and hence it is not the same as the market variance used in equity studies (e.g., Pollet and Wilson, 2010). Specifically, FX market variance is the variance of the return to the FX market portfolio. We define the excess return to the FX market portfolio as the equally weighted average of the excess returns of all exchange rates:  

$$r_{M,t+1} = \frac{1}{N_t} \sum_{j=1}^{N_t} r_{j,t+1}. \tag{1.3}$$

This can be thought of as the excess return to a naive $1/N_t$ currency trading strategy, or an international bond diversification strategy that buys $N_t$ foreign bonds by borrowing domestically.  

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7We use equal weights as it would be difficult to determine time-varying “value” weights on the basis of monthly turnover for each currency over our long sample range. Menkhoff et al. (2011) weigh the volatility contribution of different currencies by their share in international currency reserves in a given year and find no significant differences relative to equal weights.

8Note that the direction of trading does not affect the FX market variance. If instead the US investor decides to lend 1 US dollar by buying a domestic US bond and selling $N_t$ foreign bonds with equal weights, the excess return to the portfolio would be $r^*_{M,t+1} = -r_{M,t+1}$. However, the market variance would remain unaffected: $V(r^*_{M,t+1}) = V(-r_{M,t+1}) = V(r_{M,t+1})$. 


We estimate the monthly FX market variance (MV) using a realized measure based on daily excess returns:

$$MV_{t+1} = \sum_{d=1}^{D_t} r_{M,t+d/D_t}^2 + 2 \sum_{d=2}^{D_t} r_{M,t+d/D_t} r_{M,t+(d-1)/D_t}, \quad (1.4)$$

where $D_t$ is the number of trading days in month $t$, typically $D_t = 21$. Following French et al. (1987), Goyal and Santa-Clara (2003), and Bali et al. (2005), among others, this measure of market variance accounts for the autocorrelation in daily returns.\(^9\)

### 1.2.5 Average Variance and Average Correlation

Our second set of risk measures relies on the Pollet and Wilson (2010) decomposition of MV into the product of two terms, the cross-sectional average variance (AV) and the cross-sectional average correlation (AC), as follows:

$$MV_{t+1} = AV_{t+1} \times AC_{t+1}. \quad (1.5)$$

The decomposition would be exact if all exchange rates had equal individual variances, but is actually approximate given that exchange rates display unequal variances. Thus, the validity of the decomposition is very much an empirical matter. Pollet and Wilson (2010) use this decomposition for a large number of stocks and find that the approximation works very well. As we show later, this approximation works remarkably well also for exchange rates.

We can assess the empirical validity of the decomposition by estimating the following regression:

$$MV_{t+1} = \alpha + \beta (AV_{t+1} \times AC_{t+1}) + u_{t+1}, \quad (1.6)$$

where $E[u_{t+1} \mid AV_{t+1} \times AC_{t+1}] = 0$. The coefficient $\beta$ may not be equal to one because exchange rates do not have the same individual variance and there may be measurement

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\(^9\)This is similar to the heteroskedasticity and autocorrelation consistent (HAC) measure of Bandi and Perron (2008), which uses linearly decreasing Bartlett weights on the realized autocovariances. Our empirical results remain practically identical when using the HAC market variance, and hence we use the simpler specification of Equation (1.4) for the rest of the analysis.
error in $MV_{t+1}$, $AV_{t+1}$ and $AC_{t+1}$. However, the $R^2$ of this regression will give us a good indication of how well the decomposition works empirically.

We estimate $AV$ and $AC$ as follows:

$$AV_{t+1} = \frac{1}{N_t} \sum_{j=1}^{N_t} V_{j,t+1},$$

$$AC_{t+1} = \frac{1}{N_t(N_t - 1)} \sum_{i=1}^{N_t} \sum_{j \neq i} C_{ij,t+1},$$

where $V_{j,t+1}$ is the realized variance of the excess return to exchange rate $j$ at time $t+1$ computed as

$$V_{j,t+1} = \sum_{d=1}^{D_t} r_{j,t+d/D_t}^2 + 2 \sum_{d=2}^{D_t} r_{j,t+d/D_t} r_{j,t+(d-1)/D_t},$$

and $C_{ij,t+1}$ is the realized correlation between the excess returns of exchange rates $i$ and $j$ at time $t+1$ computed as

$$C_{ij,t+1} = \frac{V_{ij,t+1}}{\sqrt{V_{i,t+1}} \sqrt{V_{j,t+1}}},$$

$$V_{ij,t+1} = \sum_{d=1}^{D_t} r_{i,t+d/D_t} r_{j,t+d/D_t} + 2 \sum_{d=2}^{D_t} r_{i,t+d/D_t} r_{j,t+(d-1)/D_t}.$$

Note that we do not demean returns in calculating variances. This allows us to avoid estimating mean returns and has very little impact on calculating variances (see, e.g., French et al., 1987).

### 1.2.6 Systematic and Idiosyncratic Risk

Define $V_{j,d}$ as the variance of the excess return to exchange rate $j$ on day $d$. In this section only, for notational simplicity we suppress the monthly index $t$. Then, $V_{j,d}$ is a measure of total risk that contains both systematic and idiosyncratic components. Following Goyal and Santa-Clara (2003), we can decompose these two parts of total risk as follows. Suppose that the excess return $r_{j,d}$ is driven by a common factor $\mu_d$ and an idiosyncratic zero-mean shock $\varepsilon_{j,d}$ that is specific to exchange rate $j$. For simplicity, further assume that the factor loading for each exchange rate is equal to one, the common
1.2. Measures of Return and Risk for the Carry Trade

and idiosyncratic factors are uncorrelated, and ignore the serial correlation adjustment in Equation (1.9). Then, the data generating process for daily returns is:

$$r_{j,d} = \mu_d + \varepsilon_{j,d},$$

(1.12)

and the return to the FX market portfolio for day $d$ in a given month $t$ is:

$$r_{M,d} = \frac{1}{N_t} \sum_{j=1}^{N_t} r_{j,d} = \mu_d + \frac{1}{N_t} \sum_{j=1}^{N_t} \varepsilon_{j,d},$$

(1.13)

where the second term becomes negligible for large $N_t$.

It is straightforward to show that in a given month $t$:

$$MV = \sum_{d=1}^{D_t} r_{M,d}^2 = \sum_{d=1}^{D_t} \left[ \mu_d^2 + \frac{2}{N_t} \mu_d \sum_{j=1}^{N_t} \varepsilon_{j,d} + \left( \frac{1}{N_t} \sum_{j=1}^{N_t} \varepsilon_{j,d} \right)^2 \right],$$

(1.14)

$$AV = \sum_{d=1}^{D_t} \left[ \frac{1}{N_t} \sum_{j=1}^{N_t} r_{j,d}^2 \right] = \sum_{d=1}^{D_t} \left[ \mu_d^2 + \frac{2}{N_t} \mu_d \sum_{j=1}^{N_t} \varepsilon_{j,d} + \frac{1}{N_t} \sum_{j=1}^{N_t} \varepsilon_{j,d}^2 \right].$$

(1.15)

The first two terms of MV and AV are identical and capture the systematic component of total risk as they depend on the common factor. The third term that depends exclusively on the idiosyncratic component is different for MV and AV. For a large cross-section of exchange rates, this term is negligible for MV, and hence MV does not reflect any idiosyncratic risk. For AV, however, the third term is not negligible and captures the idiosyncratic component of total risk.

To get a better idea of the relative size of the systematic and idiosyncratic components, consider the following example based on the descriptive statistics of Table 1.2. In annualized terms, the expected monthly variances are:

$$E[MV] \times 10^3 = 5 = \underbrace{\text{Systematic}}_{5} + \underbrace{\text{Idiosyncratic}}_{0},$$

(1.16)

$$E[AV] \times 10^3 = 10 = \underbrace{\text{Systematic}}_{5} + \underbrace{\text{Idiosyncratic}}_{5}. $$

(1.17)
Therefore, half of the risk captured by AV in FX is systematic and the other half is idiosyncratic, whereas all of the risk reflected in MV is systematic.

Similarly, the standard deviations are:

\[ STD [MV] \times 10^3 = 2, \]  \hspace{1cm} (1.18)

\[ STD [AV] \times 10^3 = 3. \]  \hspace{1cm} (1.19)

As a result, the \( t \)-ratio of mean divided by standard deviation is 2.5 for MV and 3.3 for AV. In other words, AV is measured more precisely than MV, which can possibly make AV a better predictor of FX excess returns.

### 1.3 Predictive Regressions

Our empirical analysis begins with ordinary least squares (OLS) estimation of two predictive regressions for a one-month ahead horizon. The first predictive regression provides a simple way for assessing the intertemporal risk-return tradeoff in FX as follows:

\[ r_{C,t+1} = \alpha + \beta MV_t + \varepsilon_{t+1}, \]  \hspace{1cm} (1.20)

where \( r_{C,t+1} \) is the return to the carry trade portfolio from time \( t \) to \( t + 1 \), and \( MV_t \) is the market variance from time \( t - 1 \) to \( t \). This regression will capture whether, on average, the carry trade has low or negative returns in times of high market variance.

The second predictive regression assesses the risk-return tradeoff implied by the variance decomposition of Pollet and Wilson (2010):

\[ r_{C,t+1} = \alpha + \beta_1 AV_t + \beta_2 AC_t + \varepsilon_{t+1}, \]  \hspace{1cm} (1.21)

where \( AV_t \) and \( AC_t \) are the average variance and average correlation from time \( t - 1 \) to \( t \). For notational simplicity, we use the same symbol \( \alpha \) for the constants in the two regressions. The second regression separates the effect of AV and AC in order to
determine whether the decomposition provides a more precise signal on future carry returns.

The simple OLS regressions focus on the effect of the risk measures on the conditional mean of future carry returns. We go further by also estimating two predictive quantile regressions, which are designed to capture the conditional effect of either MV or AV and AC on the full distribution of future carry trade returns. It is possible, for example, that average variance is a poor predictor of the conditional mean return but predicts well one or both tails of the return distribution. After all, higher variance implies a change in the tails of the distribution and here we investigate whether this is true in a predictive framework. Using quantile regressions provides a natural way of assessing the effect of higher risk on different parts of the distribution of future carry returns. It is also an effective way of dealing with outliers. For example, the median is a quantile of particular importance that allows for direct comparison to the OLS regression that focuses on the conditional mean. It is well known that outliers may have a much larger effect on the mean of a distribution than the median. Hence the quantile regressions can provide more robust results than OLS regressions even for the middle of the distribution. In our analysis, we focus on deciles of the distribution of future carry returns.

The first predictive quantile regression estimates the conditional quantile function:

\[ Q_{r_{C,t+1}} (\tau \mid MV_t) = \alpha (\tau) + \beta (\tau) MV_t, \]  

where \( \tau \) is the quantile of the cumulative distribution function of one-month ahead carry returns.\(^{10}\)

The second predictive quantile regression yields estimates of the conditional quantile function:

\[ Q_{r_{C,t+1}} (\tau \mid AV_t, AC_t) = \alpha (\tau) + \beta_1 (\tau) AV_t + \beta_2 (\tau) AC_t. \]  

\(^{10}\) We obtain estimates of the quantile regression coefficients \( \{\alpha (\tau), \beta (\tau)\} \) by solving the minimization problem \( \{\alpha (\tau), \beta (\tau)\} = \arg \min_{\alpha,\beta} \mathbb{E} [\rho_\tau (r_{C,t+1} - \alpha (\tau) - \beta (\tau) MV_t)], \) using the asymmetric loss function \( \rho_\tau (r_{C,t+1}) = r_{C,t+1} (\tau - I (r_{C,t+1} < 0)). \) We formulate the optimization problem as a linear program and solve it by implementing the interior point method of Portnoy and Koenker (1997). See also Koenker (2005, Chapter 6).
1.3. Predictive Regressions

In addition to statistical reasons, there is an economic argument that makes the use of quantile regressions appealing in this context. Cenesizoglu and Timmermann (2010) provide the economic intuition based on the Merton (1973, 1980) ICAPM model applied to equity markets, although the same intuition extends to FX markets. Suppose that the return to the carry trade follows the process:

\[ r_{C,t+1} = \mu + \kappa \sigma^2_{t+1} + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N \left( 0, \sigma^2_{t+1} \right), \]

\[ \sigma^2_{t+1} = \varphi_0 + \varphi_1 AV_t + \varphi_2 AC_t. \]  

Then, the conditional quantile function has the form:

\[ Q_{rC,t+1} (\tau \mid AV_t, AC_t) = \mu + \varphi_0 (\kappa + Q^N_{\tau}) + (\kappa + Q^N_{\tau}) \varphi_1 AV_t + (\kappa + Q^N_{\tau}) \varphi_2 AC_t = \alpha (\tau) + \beta_1 (\tau) AV_t + \beta_2 (\tau) AC_t, \]

where Equation (1.26) is the same as Equation (1.23), and \( Q^N_{\tau} \) is the \( \tau \)-quantile of the normal distribution, which has a large negative value deep in the left tail and a large positive value deep in the right tail. If, as suggested by the Merton (1973, 1980) ICAPM model, \( \kappa > 0 \) and also \( \varphi_1, \varphi_2 > 0 \), then we expect AV and AC to have a negative slope in the left tail and positive in the right tail. This provides further justification for the use of quantile regressions to separate the effect of AV and AC on different return quantiles.

The standard error of the quantile regression parameters is estimated using a moving block bootstrap (MBB) that provides inference robust to heteroskedasticity and autocorrelation of unknown form (Fitzenberger, 1997). Specifically, we employ a circular MBB of the residuals as in Politis and Romano (1992). The optimal block size is selected using the automatic procedure of Politis and White (2004), as amended by Patton et al. (2009). The bootstrap algorithm is detailed in Appendix 1.A.
1.4 Empirical Results

1.4.1 Descriptive Statistics

Table 1.2 reports descriptive statistics on the following variables: (i) the return to the carry trade portfolio; (ii) the return to the exchange rate and interest rate components of the carry trade return; (iii) the excess return to the FX market portfolio; (iv) the FX market variance (MV); and (v) the FX average variance (AV) and average correlation (AC). Assuming no transaction costs, the carry trade delivers an annualized mean return of 8.6%, a standard deviation of 7.8% and a Sharpe ratio of 1.092.\textsuperscript{11} The carry trade return is primarily due to the interest rate differential across countries, which delivers an average return of 13.7%. The exchange rate depreciation component has a return of −5.1%, indicating that on average high-interest rate currencies do not depreciate enough to offset the interest rate differential. The carry trade return displays negative skewness of −0.967 and kurtosis of 6.043. These statistics confirm the good historical performance of the carry trade and are consistent with the literature (e.g., Burnside et al., 2011). Finally, the average market return is low at 1.0% per year, and its standard deviation is the same as that of the carry trade return at 7.8%.

Turning to the risk measures, the mean of MV is 0.005. The mean of AV is double that of MV at 0.010, and the mean of AC is 0.471. MV and AV exhibit high positive skewness and massive kurtosis. The time variation of AV and AC together with the cumulative carry trade return are displayed in Figure 1.1.

Panel B of Table 1.2 shows the cross-correlations. The correlation between the excess returns on the carry and the market portfolio is 9.1%. The three risk measures are highly positively correlated with each other but are negatively correlated with the carry and market returns. This is a first indication that there may be a negative risk-return relation in the FX market at the one-month horizon.

\textsuperscript{11}We fully account for the effect of transaction costs in a later section.
1.4. Empirical Results

1.4.2 The Decomposition of Market Variance into Average Variance and Average Correlation

The three FX risk measures of MV, AV and AC are related by the approximate decomposition of Equation (1.5). We evaluate the empirical validity of the decomposition by presenting regression results in Table 1.3. The first regression is for MV on AV alone, which delivers a slope coefficient of 0.493 for AV and \( R^2 = 76.8\% \). The second regression is for MV on AC alone, which delivers a slope of 0.015 for AC and \( R^2 = 23.5\% \). The third regression is for MV on AV and AC (additively, not using their product), which raises \( R^2 \) to 86.8\%. Finally, the fourth regression is for MV on the product of AV and AC, which is consistent with the multiplicative nature of the decomposition, and delivers a slope coefficient of 0.939 and \( R^2 = 93.0\% \). In all cases, the coefficients are highly statistically significant. In conclusion, therefore, the MV decomposition into AV and AC captures almost all of the time variation in MV.

1.4.3 Predictive Regressions

We examine the intertemporal risk-return tradeoff for the carry trade by first discussing the results of OLS predictive regressions, reported in Table 1.4. The first regression is for the one-month ahead carry trade return on the lagged MV. The table shows that overall there is a significant negative relation. In other words, high market variance is related to low future carry trade returns. This clearly indicates a negative risk-return tradeoff for the carry trade and suggests that in times of high volatility the carry trade delivers low (or negative) returns. It is also consistent with the cross-sectional results of Menkhoff et al. (2011), who find that there is a negative price of risk associated with high FX volatility.12

We refine this result by estimating a second regression for the one-step ahead carry trade return on AV and AC. We find that AV is also significantly negatively related to future carry trade returns. AC has a negative but insignificant relation. The \( R^2 \) is

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12It is important to emphasize, however, that our result is set up in a predictive framework, not in a cross-sectional contemporaneous framework.
1.2% in the first regression and rises to 1.8% in the second regression. At first glance, therefore, there is at best a slight improvement in using the decomposition of MV into AV and AC in a predictive regression.

These results explore the risk-return tradeoff only for the mean of carry returns. It is possible, however, that high market volatility has a different impact on different quantiles of the carry return distribution. We explore this possibility by estimating predictive quantile regressions. We begin with Figure 1.2 which plots the parameter estimates of the predictive quantile regressions of the one-month ahead carry trade return on MV. These results are shown in more detail in Table 1.5. MV has a consistently negative relation to the future carry trade return but this relation is statistically significant only for a few parts of the distribution. The significant quantiles are all in the left tail: 0.05, 0.3, 0.4 and 0.5. Also note that the constant is highly significant, being negative below the 0.3 quantile and positive above it.

The results improve noticeably when we move to the second quantile regression of the future carry trade return on AV and AC. As shown in Figure 1.3 and Table 1.6, AV has a strong negative relation to the carry trade return, which is highly significant in all left-tail quantiles. The lower the quantile, the more negative the value of the coefficient. Above the median, the AV coefficient revolves around zero (positive or negative) and is not significant. Furthermore, it is interesting to note that AC has a negative and significant relation to the future carry trade return in the right tail of the distribution, and especially for quantiles 0.7 and higher.

The pseudo-$R^2$ reported in Table 1.6 ranges from 0.1% for the quantile regressions describing the 0.6-quantile, to 4.1% for the quantile regression describing the extreme left tail of the return distribution. This result adds to the evidence that different parts of the return distribution present different degrees of predictability. In most cases, the pseudo-$R^2$ is below 2%, in line with the modest predictability of FX excess returns typically found in the literature. However, we show in Section 6 how this modest statistical predictability leads to significant economic gains by designing trading

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13We compute the pseudo-$R^2$ as in Koenker and Machado (1999).
strategies that condition on AV and AC.

These results lead to three important conclusions. First, it is very informative to look at the full distribution of carry trade returns to better assess the impact of high volatility and get a more precise signal. High MV has a significant negative impact only in the left tail.

Second, the decomposition of MV into AV and AC is helpful in understanding the risk-return tradeoff in FX. AV has a much stronger and significant negative impact than MV on the left tail of the carry trade return. As shown in Figure 1.3, this establishes clearly that high volatility in FX excess returns is strongly related to low future carry returns in the left tail. This is a new result that is consistent with the large negative returns to the carry trade in times of high volatility that typically lead investors to unwind their carry trade positions. It is also consistent with the empirical result in equity studies that idiosyncratic risk captured by (equally weighted) AV is significantly negatively related to the conditional mean of future returns (Goyal and Santa-Clara, 2003).

Third, AC is significantly negatively related to the right tail of future carry returns. This is also a new result. When FX return correlations are low, the carry trade is expected to perform well over the next period. The lower the correlations on average, the stronger the diversification effect arising from a given set of currencies, which tends to produce high carry trade returns. This is consistent with Burnside et al. (2008), who show that diversification (i.e., trading a larger set of currencies) can substantially increase the Sharpe ratio of carry trade strategies. However, our result adds to previous empirical evidence in that we show that this diversification benefit tends to have an asymmetric effect on the return distribution: AC significantly affects the probability of large gains on carry trades, but its relation to large losses is insignificant. We do not have a theoretical explanation for this asymmetric effect, but believe that it is an intriguing result that warrants further research.
1.5 Robustness and Further Analysis

1.5.1 The Components of the Carry Trade

The carry trade return has two components: (i) the exchange rate component, which on average is slightly negative for our sample; and (ii) the interest rate component, which on average is highly positive.\(^\text{14}\) Note that the exchange rate component is the uncertain part of the carry trade return as it is not known at the time that the carry trade portfolio is formed. In contrast, the interest rate component is known and actually taken into account when the carry trade portfolio is formed. Therefore, predicting the exchange rate component (i.e., whether high-interest currencies will depreciate and vice versa) effectively allows us to predict the carry trade return.

Figure 1.4 illustrates that when AV is high, the returns of the left-tail exchange rate component become lower. This negative relation is highly significant for the left tail of the exchange rate component up to the median. This result is consistent with high-interest currencies depreciating sharply (i.e., the forward bias diminishing) when AV is high and we are in the left tail of the distribution.\(^\text{15}\) More importantly, it also implies that to some extent exchange rates are predictable. In other words, this constitutes evidence against the well-known result that exchange rates are unpredictable. In short, our results establish that AV is a significant predictor of the large negative returns to the exchange rate component of the carry trade.

1.5.2 Additional Predictive Variables

As a robustness test, we use two additional predictive variables to determine whether they affect the significance of AV and AC. These are the average interest rate differential (AID) and the lagged carry return (LCR). AID is equal to the average interest rate differential of the quintile of currencies with the highest interest rates minus the average interest rate of the quintile of currencies with the lowest interest rates.

\(^\text{14}\)Recall the descriptive statistics in Table 1.2.

\(^\text{15}\)This case is also consistent with the hypothesis of flight to quality, safety (e.g., Ranaldo and Söderlind, 2010) or liquidity that may explain why high-interest currencies depreciate and low-interest currencies appreciate in times of high volatility (Brunnermeier et al., 2009).
interest rate differential of the quintile of currencies with the lowest interest rates. All interest rates are known at time $t$ for prediction of the carry trade return at time $t+1$. LCR is simply the carry trade return lagged by one month.

The predictive quantile regression results are in Figure 1.5 and Table 1.7, and can be summarized in three findings: (i) AID has a significant positive effect on future carry trade returns in the middle of the distribution;\(^{16}\) (ii) LCR has a significant positive effect in the left tail; and, more importantly, (iii) the effect of AV and AC remains qualitatively the same (although their significance diminishes slightly in the relevant parts of the distribution). The $R^2$ now improves to 4.8% for the 0.05 quantile. Overall, the effect of AV and AC remains significantly negative in the left and right tails, respectively, even when we include other significant predictive variables.

### 1.5.3 The Numeraire Effect

A unique feature of the FX market is that investors trade currencies but all exchange rates are quoted relative to a numeraire. Consistent with the vast majority of the FX literature, we have used data on exchange rates relative to the US dollar. It is interesting, however, to check whether using a different numeraire would meaningfully affect the predictive ability of AV and AC. This is an important robustness check since it is straightforward to show analytically that the carry trade returns and risk measures are not invariant to the numeraire.\(^{17}\) In essence, the question we want to address is: given that changing the numeraire also changes the carry returns and the risk measures, does the relation between risk and return also change?

We answer this question by reporting predictive quantile regression results using a composite numeraire that weights the carry trade return, AV and AC across four different currencies. The weights are based on the Special Drawing Rights (SDR) of

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\(^{16}\)This is consistent with Lustig et al. (2010), who find that the average forward discount is a good predictor of FX excess returns. In their study, the average forward discount is equal to the difference between the average interest of a basket of developed currencies and the US interest rate.

\(^{17}\)For example, consider taking the point of view of a European investor and hence changing the numeraire currency from the US dollar to the euro. Then, all previous bilateral exchange rates become cross rates and $N_t$ of the previous cross rates become bilateral. Furthermore, converting dollar excess returns into euro excess returns replaces the US bond as the domestic asset by the European bond.
1.5. Robustness and Further Analysis

The International Monetary Fund (IMF) and are as follows: 41.9% on the US dollar-denominated measures, 37.4% on the Euro-denominated measures, 11.3% on the UK pound-denominated measures, and 9.4% on the Japanese yen-denominated measures. The SDR is an international reserve asset created by the IMF in 1969 to supplement its member countries’ official reserves that is based on a basket of these four key international currencies. The IMF (and other international organizations) also use SDRs as a unit of account and effectively that is what we do in this exercise.

The advantage of this approach is that: (i) we capture the numeraire effect in a single regression as opposed to estimating multiple regressions for each individual numeraire; (ii) it is popular among practitioners who often measure FX returns using a composite numeraire across these four main currencies; (iii) it provides the interpretation of generating a new weighted carry trade portfolio that is effectively a composite numeraire; and (iv) the weighted AV and weighted AC are straightforward to compute.

The results shown in Figure 1.6 and Table 1.8 confirm that this exercise does not affect qualitatively our main result: the weighted AV still has a significant negative effect on the future weighted carry trade return in the lower tail, and the weighted AC still has a significant negative effect on the future weighted carry trade return in the upper tail. This is clear evidence that there is a strong statistical link between average variance, average correlation and future carry returns for certain parts of the distribution even when we consider a broad basket of numeraire currencies.

1.5.4 VIX, VXY and Carry Trade Returns

Our analysis quantifies FX risk using realized monthly measures of market variance, average variance and average correlation based on daily FX excess returns. An alter-

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18 Based on our experience, the typical weights adopted by practitioners in measuring returns relative to a composite numeraire are: 40% on the US dollar, 30% on the euro, 20% on the Japanese yen and 10% on the UK pound.

19 The weighted carry trade return is computed as follows: \( r_{C,t+1}^W = \sum_{p=1}^{P} w_p r_p,C,t+1 \), where \( p = 1, ...P = 4 \) is the number of numeraires. The weighted average variance is: \( AV_{t+1}^W = \sum_{p=1}^{P} w_p AV_{p,t+1} = \frac{1}{N_t} \sum_{j=1}^{N_t} \sum_{p=1}^{P} w_p V_{p,j,t+1} \) for \( j \) currencies. The weighted average correlation is: \( AC_{t+1}^W = \sum_{p=1}^{P} w_p AC_{p,t+1} = \frac{1}{N_t(N_t-1)} \sum_{i=1}^{N_t} \sum_{j \neq i}^{N_t} \sum_{p=1}^{P} w_p C_{p,ij,t+1} \) for \( i, j \) currencies.
native way of measuring risk is to use implied volatility (IV) indices based on the IVs of traded options that can be thought of as the market’s expectation of future realized volatility. As a further robustness check, we estimate predictive quantile regressions using two IV indices: the VIX index, which is based on the 1-month model-free IV of the S&P 500 equity index and is generally regarded as a measure of global risk appetite (e.g., Brunnermeier et al., 2009); and the VXY index, which is based on the 3-month IV of at-the-money forward options of the G-7 currencies. The sample period for the VIX begins in January 1990 and for the VXY in January 1992, whereas for both it ends in February 2009.

We begin with Table 1.9, which reports OLS results for simple contemporaneous regressions of each of the two IV indices on MV, AV and AC. These results will help us determine the extent to which the VIX and VXY are correlated with the FX risk measures we use. We find that the VIX is significantly positively related to AV and significantly negatively related to AC. Together AV and AC account for 37.6% of the variation of VIX. The VXY is also significantly related to AV but the relation to AC is low and insignificant. AV accounts for 54.5% of the variation in VXY.

The predictive quantile regression results for VIX and VXY, reported in Figure 1.7 and Table 1.10, suggest that neither the VIX nor the VXY are significantly related to future carry trade returns for any part of the distribution. Although the coefficients are predominantly negative, there is no evidence of statistical significance. Therefore, the predictive ability of AV and AC is not captured by the two IV indices and further justifies the choice of AV and AC as risk measures. The lack of predictive ability for the VIX in one-month ahead predictive regressions is consistent with the results of Brunnermeier et al. (2009), who find that the VIX has a strong contemporaneous impact on the carry return but is an insignificant predictor.
1.5.5 Conditional Skewness

The carry trade return is well known to exhibit negative skewness due to large negative outliers.\(^{20}\) This has led to an emerging literature that investigates whether the high average carry trade returns: reflect a peso problem, which is the low probability of large negative outliers (e.g., Burnside et al., 2011); and are compensation for crash risk associated with the sudden unwinding of the carry trade (e.g., Brunnermeier et al., 2009).

The predictive quantile regression approach has the advantage that it can be used to compute a measure of conditional skewness that is robust to outliers. As in Kim and White (2004), we use the Bowley (1920) coefficient of skewness that is based on the interquartile range. Using quantile regression (1.23), we estimate skewness conditionally period-by-period as follows:

\[
SK_t = \frac{\hat{Q}_{0.75,t} + \hat{Q}_{0.25,t} - 2\hat{Q}_{0.5,t}}{\hat{Q}_{0.75,t} - \hat{Q}_{0.25,t}},
\]

(1.27)

where \(\hat{Q}_{0.75,t}\) is the forecast of the third conditional quartile at time \(t\) for next period, \(\hat{Q}_{0.25,t}\) is the forecast of the first conditional quartile at time \(t\) for next period, and \(\hat{Q}_{0.5,t}\) is the forecast of the conditional median at time \(t\) for next period.

For any symmetric distribution, the Bowley coefficient is zero. This measure allows us to explore whether an increase in total risk (e.g., a rise in average variance) typically coincides with an increase in downside risk (e.g., lower conditional skewness). For example, when the lower tail conditional quantiles decline more than the upper tail conditional quantiles, this leads to negative conditional skewness and an increase in downside risk.

Figure 1.8 plots the conditional skewness and illustrates that it tends to be positive at the beginning of the sample and negative from the mid-nineties onwards. This is indicative of higher crash risk in the last part of the sample.

\(^{20}\)Indeed, it is often said in industry speak that the carry trade payoffs “go up the stairs and down the elevator” or that the carry trade is like “picking up nickels in front of a steam roller.”
1.6 Augmented Carry Trade Strategies

We further evaluate the predictive ability of average variance and average correlation on future carry trade returns by assessing the economic gains of conditioning on average variance and average correlation out of sample for three augmented carry trade strategies. These strategies are then compared to the benchmark strategy, which is the standard carry trade. Our discussion begins with a description of the strategies, and then reports results with and without transaction costs.

1.6.1 The Strategies

The first augmented carry trade strategy conditions on AV only and implements the following rule at each time period $t$: for the carry trade returns that are lower than the $\tau$-quantile of the distribution, if AV has increased from $t - 1$ to $t$, we close the carry trade positions and thus receive an excess return of zero; otherwise we execute the standard carry trade. This strategy is designed to exploit the negative relation between current AV and the one-month ahead carry return. The focus of the AV strategy is the left-tail quantiles of the carry trade return distribution, where the negative effect of AV is the strongest.

The second strategy conditions on AC only and implements the following rule at each time period $t$: for the carry trade returns that are higher than the $1 - \tau$ quantile, if AC has decreased from $t - 1$ to $t$, we double the carry trade positions and thus receive twice the carry return; otherwise we execute the standard carry trade. This strategy is designed to exploit the negative relation between current AC and the one-month ahead carry return. However, the focus of the AC strategy is the right-tail quantiles, where the negative effect of AC is the strongest.

Third, the combined AV and AC strategy makes the following decision at each time period $t$: for the carry trade returns that are lower than the $\tau$-quantile, if AV has increased from $t - 1$ to $t$, we close the carry trade positions and thus receive an excess return of zero; and for the carry returns that are higher than the $1 - \tau$ quantile, if AC has decreased from $t - 1$ to $t$, we double the carry trade positions and thus receive twice
the carry trade return; otherwise we execute the standard carry trade. The combined AV and AC strategy focuses at the same time on both the low and the high quantiles of the carry trade return distribution. For example, this strategy is first applied to the 0.1 (for AV) and 0.9 (for AC) quantiles for the full sample, then to 0.2 and 0.8 quantiles and so on.

It is important to note that all three strategies are implemented out of sample. Specifically, all strategies move forward recursively starting 3 years after the beginning of the sample. The strategies do not directly use the parameter estimates from the quantile regressions but simply try to exploit the negative relation between future carry returns and current AV and AC separately for low and high quantiles of the distribution.

The economic evaluation of the three strategies focuses on the Sharpe ratio. We also report the mean and standard deviation of the augmented carry trade returns. All these measures are reported in annualized units. We assess the practical applicability of the strategies by computing the turnover ratio as the percentage of the currencies that on average are traded every period. The turnover ratio provides us with a sense of how much more trading and rebalancing is required to implement an augmented strategy relative to the standard carry trade.

### 1.6.2 No Transaction Costs

Panel A of Table 1.11 reports the results for no transaction costs. The AV strategy performs very well for most quantiles and, as expected, does increasingly better as we move to the lower quantiles. For example, at the 0.1 quantile, the Sharpe ratio of the AV strategy is 1.314 compared to 1.070 for the standard carry trade. These large economic gains require only slightly higher trading as the turnover ratio rises from 17.1% in the benchmark case to 20.8%. By design, the turnover ratio remains reasonably low for the lowest quantiles where the conditioning on AV is implemented less often.

In contrast to the AV strategy that performs best in the lowest quantiles, the AC
1.6. Augmented Carry Trade Strategies

strategy performs well across all quantiles. It appears, therefore, that low average correlations are economically beneficial to the carry trade regardless of the quantile we focus on. For example, at the 0.1 quantile the Sharpe ratio of the AC strategy is 1.208, whereas at the 0.9 quantile it is 1.134 compared to 1.070 of the standard carry trade. These economic gains require only slightly higher trading for the high quantiles as the turnover ratio rises from 17.1% in the benchmark case to 21.4% at the 0.9 quantile. By design, for the AC strategy the turnover ratio remains reasonably low for the highest quantiles.

Finally, the combined AV and AC strategy performs better than the standard carry trade for all quantiles. For example, at the highest and lowest quantiles (0.1 for AV and 0.9 for AC) the Sharpe ratio is 1.224 and the turnover ratio is 0.257. In short, therefore, there are sizeable economic gains in implementing augmented carry trade strategies that condition on average variance and average correlation. In addition to showing tangible out-of-sample economic gains, these results also highlight the negative predictive relation of AV and AC to future carry trade returns across different quantiles of the distribution.

1.6.3 The Effect of Transaction Costs

A realistic assessment of the profitability of the carry trade strategies needs to account for transaction costs in trading spot and forward exchange rates. Every month that we form a new carry trade portfolio, we take a position in one forward and one spot contract for each currency that belongs to either portfolio 1 (highest interest rate currencies) or portfolio 5 (lowest interest rate currencies). At the end of the month, the contracts expire and new contracts are entered (on the same or different currencies). Define $c_{j,t}^S$ and $c_{j,t}^F$ as the one-way proportional transaction cost at time $t$ for trading at the spot and forward exchange rate $j$, respectively. These values are equal to half of the spot and forward proportional bid-ask spread.$^{22}$ It is straightforward to show that the return to

$^{22}$The proportional bid-ask spread is the ratio of the bid-ask spread to the mid rate.
the carry trade for an individual currency $j$ net of transaction costs is equal to:

$$r_{j,t+1}^{\text{net}} = s_{j,t+1} - f_{j,t} - c_{j,t+1}^S - c_{j,t}^F. \tag{1.28}$$

Our analysis implements the carry trade strategies using the transaction costs listed in Table 1.1. Due to data availability, we use the median transaction costs across time, which are different for each currency.\footnote{Using the median as opposed to the mean of transaction costs mitigates the effect of few large outliers in the time series of bid-ask spreads. The results remain largely unchanged when using the average proportional bid-ask spread. Note also that generally the effective spread is lower than the quoted spread, since trading will take place at the best price quoted at any point in time, suggesting that the worse quotes will not attract trades (e.g., Mayhew, 2002). Although some studies consider effective transaction costs in the range of 50% to 100% of the quoted spread (e.g., Goyal and Saretto, 2009), we use the full spread, which will likely underestimate the true returns.} These are taken from Datastream and are computed using the longest bid and ask times series available for each exchange rate for the sample range of January 1976 to February 2009. The cross-currency average of the median one-way transaction costs is 6.10 basis points for the spot rates, and 9.42 basis points for the forward rates. Our transaction costs are consistent with the values discussed in Neely et al. (2009).

The performance of the carry trade strategies with transaction costs is shown in Panel B of Table 1.11. As expected, when accounting for transaction costs the Sharpe ratios of all strategies are lower. For example, the standard carry trade has a Sharpe ratio of 1.070 before transaction costs and 0.741 after transaction costs. More importantly, we find that the augmented strategies still perform substantially better than the benchmark. In particular, the AV strategy still dominates the standard carry trade in the left tail. The AC as well as the combined AV and AC strategies outperform the benchmark in all quantiles. For example, the AV strategy can deliver a Sharpe ratio net of transaction costs as high as 0.983, the AC strategy as high as 0.971, and the combined strategy as high as 0.959, compared to 0.741 for the benchmark. We conclude, therefore, that the improvement in the performance of the carry trade when conditioning on the movements of AV and AC is robust to transaction costs.
1.7 Conclusion

The carry trade is a currency investment strategy designed to exploit deviations from uncovered interest parity. Its profitability is based on the empirical observation that the interest rate differential across countries is not, on average, offset by a depreciation of the investment currency. Hence, investing in high-interest currencies by borrowing from low-interest currencies tends to deliver large positive excess returns.

This chapter fills a gap in the literature by demonstrating empirically the existence of an intertemporal risk-return tradeoff between the return to the carry trade and risk in a predictive setting. We measure FX risk by the variance of the returns to the FX market portfolio. We then take a step further by decomposing the market variance into the cross-sectional average variance and the cross-sectional average correlation of exchange rate returns. Our empirical analysis is based on predictive quantile regressions, which provide a natural way of assessing the effect of higher risk on different quantiles of the return distribution.

Our main finding is that average variance has a significant negative effect on the left tail of the distribution of future carry trade returns, whereas average correlation has a significant negative effect on the right tail. We take advantage of this finding by forming a new version of the carry trade that conditions on average variance and average correlation, and show that this strategy performs considerably better than the standard carry trade. These results imply that to some extent exchange rates are predictable, especially when it matters most: when the carry trade produces large gains or large losses. In other words, if the carry trade is about “going up the stairs and down the elevator,” then average variance and average correlation can tell us something valuable about when the elevator is likely to go up or down. In the end, by focusing on the tails of the return distribution of carry trades, we uncover a negative risk-return tradeoff in foreign exchange.
Appendix 1.A  Notes on the bootstrap procedure

1.A.1 Bootstrap Standard Errors

We estimate the standard error of the parameters of the predictive quantile regressions using a moving block bootstrap (MBB), which provides inference that is robust to heteroskedasticity and autocorrelation of unknown form (Fitzenberger, 1997). Specifically, we employ a circular MBB of the model residuals as in Politis and Romano (1992). The optimal block size is selected using the automatic procedure of Politis and White (2004), as amended by Patton et al. (2009). The bootstrap algorithm implements the following steps:

1. Estimate the coefficients of the $\tau$-th conditional quantile function:

$$Q_{y_{t+1}}(\tau \mid X_t) = X_t' \beta(\tau),$$

for $t = 1, \ldots, T$, where $y_{t+1}$ is the dependent variable (e.g., the carry trade return), $X_t$ is a $K \times 1$ matrix of regressors (e.g., a constant, AV and AC), and $\beta(\tau)$ is the $K \times 1$ vector of coefficients. We denote the estimates as $\hat{\beta}(\tau)$ and obtain the residuals associated to the $\tau$-th quantile as $\hat{\varepsilon}_{t+1} = y_{t+1} - X_t' \hat{\beta}(\tau)$.

2. “Wrap” the residuals $\{\hat{\varepsilon}_1, \ldots, \hat{\varepsilon}_T\}$ around a circle, i.e., define the new series $\hat{\varepsilon}_t = \hat{\varepsilon}_{t+l}$ for $t = 1, \ldots, T$, and $\hat{\varepsilon}_t = \hat{\varepsilon}_{t-l}$ for $t = T + 1, \ldots, T + l - 1$, where $l$ is the length of the block defined below. This “circular” structure, specified by Politis and Romano (1992), guarantees that the first and last few observations have the same probability of being selected as observations in the middle of the series.

3. Construct a bootstrap pseudo-series $\{e^*_1, \ldots, e^*_T\}$ by resampling with replacement of overlapping blocks of size $l$. The block size $l$ is computed using the automatic procedure of Politis and White (2004). In our application, the estimated optimal block length ranges from 1 to 4 monthly observations.

4. Form the dependent variable $y^*_{t+1}$ using the bootstrap residual series $e^* \equiv \{e^*_1, \ldots, e^*_T\}'$
1.A. Notes on the bootstrap procedure

and the estimates $\hat{\beta}(\tau)$ as follows:

$$y_{t+1}^* = X_t' \hat{\beta}(\tau) + e_{t+1}^*,$$

and then estimate the conditional quantile function $Q_{y_{t+1}^*}(\tau | X_t) = X_t' \hat{\beta}(\tau)$, obtaining the bootstrap estimate $\hat{\beta}^*(\tau)$.

5. Repeat steps 3 and 4 for $B = 10,000$ times. Denoting $\hat{\beta}_{j}^*(\tau)$ as the estimate of the $j^{th}$ bootstrap, for $j = 1, \ldots, B$, estimate the variance-covariance matrix of $\hat{\beta}(\tau)$ as follows:

$$\text{Var}(\hat{\beta}(\tau)) = \frac{1}{B} \sum_{j=1}^{B} \left( \hat{\beta}_{j}^*(\tau) - \bar{\beta}_{j}^*(\tau) \right) \left( \hat{\beta}_{j}^*(\tau) - \bar{\beta}_{j}^*(\tau) \right)' .$$

1.A.2 Bootstrap Hypothesis Testing

In order to test the null hypothesis of no predictability (i.e., $\beta(\tau) = 0$), we perform a double bootstrap to compute the bootstrap $p$-values (see, e.g., MacKinnon, 2007). The procedure is as follows:

I. Obtain the estimate $\hat{\beta}(\tau)$.

II. Obtain the estimate $\text{Var}(\hat{\beta}(\tau))$ using the block bootstrap described above for $B_2 = 500$ bootstrap samples.

III. Compute the $t$-statistic for each coefficient $\beta_i(\tau), i = 1, \ldots, K$ as follows:

$$t_i = \frac{\hat{\beta}_i(\tau)}{\sqrt{\text{Var}(\hat{\beta}_i(\tau))}}.$$

IV. Generate $B_1 = 1,000$ bootstrap samples using the bootstrap DGP as in step 4 above, but this time imposing the null $\beta_i(\tau) = 0$. Use each of the new samples to calculate $\hat{\beta}_{i,j}^{**}(\tau), j = 1, \ldots, B_1$. 

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V. For each of the $B_1$ bootstrap samples, perform steps II and III above in order to generate $B_1$ bootstrap test statistics $t_{i,j}^{**}$.

VI. Calculate the bootstrap $p$-values for $\hat{t}_i$ using

$$\hat{p}_i^*(\hat{t}_i) = \frac{1}{B_1} \sum_{j=1}^{B_1} I(|t_{i,j}^{**}| > |\hat{t}_i|),$$

where $I(\cdot)$ denotes the indicator function, which is equal to 1 when its argument is true and 0 otherwise.
1. Tables

Table 1.1. Exchange Rates

The table lists the 33 US dollar nominal exchange rates used to construct the FX market and carry trade portfolios. The start date and end date of the data sample is shown for each exchange rate. The transaction costs reported below are the median one-way proportional costs for the spot and forward exchange rates, defined as half of the bid-ask spread divided by the mid rate, and are reported in basis points. The transaction costs are computed using the longest bid and ask times series available for each exchange rate for the sample range of January 1976 to February 2009.

<table>
<thead>
<tr>
<th>Exchange Rate</th>
<th>Start of Sample</th>
<th>End of Sample</th>
<th>Spot</th>
<th>Forward</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Australian Dollar</td>
<td>December 1984</td>
<td>February 2009</td>
<td>5.42</td>
<td>7.31</td>
</tr>
<tr>
<td>4. Canadian Dollar</td>
<td>January 1976</td>
<td>February 2009</td>
<td>2.72</td>
<td>4.75</td>
</tr>
<tr>
<td>5. Czech Koruna</td>
<td>January 1997</td>
<td>February 2009</td>
<td>5.97</td>
<td>6.75</td>
</tr>
<tr>
<td>6. Danish Krone</td>
<td>January 1976</td>
<td>February 2009</td>
<td>4.16</td>
<td>7.03</td>
</tr>
<tr>
<td>7. Euro</td>
<td>January 1999</td>
<td>February 2009</td>
<td>2.64</td>
<td>2.77</td>
</tr>
<tr>
<td>12. Hong Kong Dollar</td>
<td>October 1983</td>
<td>February 2009</td>
<td>0.64</td>
<td>1.92</td>
</tr>
<tr>
<td>13. Hungarian Forint</td>
<td>October 1997</td>
<td>February 2009</td>
<td>5.05</td>
<td>8.36</td>
</tr>
<tr>
<td>18. Mexican Peso</td>
<td>January 1997</td>
<td>February 2009</td>
<td>4.35</td>
<td>5.25</td>
</tr>
<tr>
<td>23. Polish Zloty</td>
<td>February 2002</td>
<td>February 2009</td>
<td>6.52</td>
<td>7.41</td>
</tr>
<tr>
<td>24. Portuguese Escudo</td>
<td>January 1976</td>
<td>December 1998</td>
<td>18.76</td>
<td>34.43</td>
</tr>
<tr>
<td>25. Saudi Arabian Riyal</td>
<td>January 1997</td>
<td>February 2009</td>
<td>0.53</td>
<td>0.93</td>
</tr>
<tr>
<td>28. South Korean Won</td>
<td>February 2002</td>
<td>February 2009</td>
<td>2.24</td>
<td>6.44</td>
</tr>
<tr>
<td>31. Swiss Franc</td>
<td>January 1976</td>
<td>February 2009</td>
<td>11.61</td>
<td>18.60</td>
</tr>
<tr>
<td>32. Taiwanese Dollar</td>
<td>January 1997</td>
<td>February 2009</td>
<td>2.95</td>
<td>8.28</td>
</tr>
<tr>
<td>33. United Kingdom Pound</td>
<td>January 1976</td>
<td>February 2009</td>
<td>2.67</td>
<td>3.54</td>
</tr>
</tbody>
</table>
Table 1.2. Descriptive Statistics

The table reports descriptive statistics for the monthly excess returns of two FX portfolios: the carry trade and the market; for the two components of the carry trade: the exchange rate depreciation and the interest rate differential; and for three monthly risk measures: market variance, average variance and average correlation. The sample of 33 US dollar nominal exchange rates runs from January 1976 to February 2009. The return to the FX market portfolio is an equally weighted average of all exchange rate excess returns. The carry trade portfolio is constructed every month by going long on the quintile of currencies with the highest interest rates and going short on the bottom quintile. Market variance is the variance of the monthly returns to the FX market portfolio. Average variance is the equally weighted cross-sectional average of the monthly variances of all exchange rate excess returns. Average correlation is the equally weighted cross-sectional average of the pairwise monthly correlations of all exchange rate excess returns. The mean, standard deviation and the Sharpe ratio are annualized and assume no transaction costs. AR(1) is the first order autocorrelation.

<table>
<thead>
<tr>
<th>Panel A: Summary Statistics</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Sharpe Ratio</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>AR(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Portfolio Returns</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Carry Trade</td>
<td>0.086</td>
<td>0.078</td>
<td>1.092</td>
<td>−0.967</td>
<td>6.043</td>
<td>0.132</td>
</tr>
<tr>
<td>Market</td>
<td>0.010</td>
<td>0.078</td>
<td>0.131</td>
<td>−0.120</td>
<td>3.195</td>
<td>0.097</td>
</tr>
<tr>
<td><strong>Carry Trade Components</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exchange Rate</td>
<td>−0.051</td>
<td>0.079</td>
<td>−1.133</td>
<td>6.232</td>
<td>0.140</td>
<td></td>
</tr>
<tr>
<td>Interest Rate</td>
<td>0.137</td>
<td>0.023</td>
<td>2.506</td>
<td>15.578</td>
<td>0.655</td>
<td></td>
</tr>
<tr>
<td><strong>Variances and Correlations</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market Variance</td>
<td>0.005</td>
<td>0.002</td>
<td>3.380</td>
<td>19.367</td>
<td>0.512</td>
<td></td>
</tr>
<tr>
<td>Average Variance</td>
<td>0.010</td>
<td>0.003</td>
<td>5.174</td>
<td>47.780</td>
<td>0.539</td>
<td></td>
</tr>
<tr>
<td>Average Correlation</td>
<td>0.477</td>
<td>0.182</td>
<td>0.028</td>
<td>2.241</td>
<td>0.796</td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Cross-Correlations</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Portfolio Returns</th>
<th>Carry Return</th>
<th>Market Return</th>
<th>Market Variance</th>
<th>Average Variance</th>
<th>Average Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carry Trade Return</td>
<td>1.000</td>
<td>0.091</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market Return</td>
<td>0.091</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Variances and Correlations</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market Variance</td>
<td>−0.250</td>
<td>−0.145</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Variance</td>
<td>−0.371</td>
<td>−0.150</td>
<td>0.877</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>Average Correlation</td>
<td>−0.048</td>
<td>−0.054</td>
<td>0.478</td>
<td>0.191</td>
<td>1.000</td>
</tr>
</tbody>
</table>
1. Tables

Table 1.3. Market Variance Decomposition

The table presents the ordinary least squares results for regressions on alternative decompositions of the FX market variance. The dependent variable is the market variance defined as the variance of the monthly returns to the FX market portfolio, which is an equally weighted average of the excess returns of 33 US dollar nominal spot exchange rates. Average variance is the equally weighted cross-sectional average of the variances of all exchange rate excess returns. Average correlation is the equally weighted cross-sectional average of the pairwise correlations of all exchange rate excess returns. All variables are contemporaneous and, with the exception of average correlation, they are annualized. Newey-West (1987) $t$-statistics with five lags are reported in parentheses. The sample period runs from January 1976 to February 2009.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.000</td>
<td>−0.002</td>
<td>−0.004</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.056)</td>
<td>(−2.324)</td>
<td>(−11.079)</td>
<td>(1.473)</td>
</tr>
<tr>
<td>Average Variance</td>
<td>0.493</td>
<td>0.456</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(9.993)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Correlation</td>
<td>0.015</td>
<td>0.010</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(8.960)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(\text{Average Variance}) \times$</td>
<td></td>
<td></td>
<td></td>
<td>0.939</td>
</tr>
<tr>
<td>$(\text{Average Correlation})$</td>
<td></td>
<td></td>
<td></td>
<td>(24.281)</td>
</tr>
<tr>
<td>$R^2$ (%)</td>
<td>76.8</td>
<td>23.5</td>
<td>86.8</td>
<td>93.0</td>
</tr>
</tbody>
</table>

Regressions for the Market Variance
The table presents the ordinary least squares results for two predictive regressions. The first regression is: $r_{C,t+1} = \alpha + \beta_{MV} M_{V,t} + \varepsilon_{t+1}$, where $r_{C,t+1}$ is the one-month ahead carry trade return and $M_{V,t}$ is the lagged market variance. The second regression is: $r_{C,t+1} = \alpha + \beta_1 AV_t + \beta_2 AC_t + \varepsilon_{t+1}$, where $AV_t$ is the lagged average variance and $AC_t$ is the lagged average correlation. The return to the carry trade portfolio is constructed every month by going long on the quintile of currencies with the highest interest rates and going short on the bottom quintile. Market variance is the variance of the return to the market portfolio constructed as the equally weighted average of the excess returns on 33 US dollar nominal exchange rates. Average variance is the equally weighted cross-sectional average of the monthly variances of all exchange rate excess returns. Average correlation is the equally weighted cross-sectional average of the pairwise monthly correlations of all exchange rate excess returns. With the exception of average correlation, all variables are annualized. Newey-West (1987) $t$-statistics with five lags are reported in parentheses. Bootstrap $p$-values generated using 10,000 bootstrap samples are in brackets. The sample period runs from January 1976 to February 2009.

### Regressions for the Carry Trade Return

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.115</td>
<td>0.156</td>
</tr>
<tr>
<td></td>
<td>(5.939)</td>
<td>(3.888)</td>
</tr>
<tr>
<td></td>
<td>[0.122]</td>
<td>[0.064]</td>
</tr>
<tr>
<td>Market Variance</td>
<td>-5.999</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-2.006)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.046]</td>
<td></td>
</tr>
<tr>
<td>Average Variance</td>
<td>-3.781</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-1.972)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.051]</td>
<td></td>
</tr>
<tr>
<td>Average Correlation</td>
<td>-0.070</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-0.936)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.350]</td>
<td></td>
</tr>
<tr>
<td>$\bar{R}^2$ (%)</td>
<td>1.2</td>
<td>1.8</td>
</tr>
</tbody>
</table>

[48]
Table 1.5. Market Variance

The table presents the regression results for the conditional quantile function: $Q_{rC,t+1} (\tau \mid MV_t) = \alpha(\tau) + \beta(\tau) MV_t$, where $\tau$ is a quantile of the one-month ahead carry trade return $r_{C,t+1}$ and $MV_t$ is the lagged market variance. The return to the carry trade portfolio is constructed every month by going long on the quintile of currencies with the highest interest rates and going short on the bottom quintile. Market variance is the variance of the return to the market portfolio constructed as the equally weighted average of the excess return on 33 US dollar nominal exchange rates. All variables are annualized. Bootstrap $t$-statistics generated using 1,000 bootstrap samples are reported in parentheses. Bootstrap $p$-values using the double bootstrap are in brackets. The pseudo-$R^2$ is computed as in Koenker and Machado (1999). The sample period runs from January 1976 to February 2009.

<table>
<thead>
<tr>
<th>Quantile</th>
<th>0.05</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.024</td>
<td>-0.019</td>
<td>-0.005</td>
<td>0.003</td>
<td>0.007</td>
<td>0.011</td>
<td>0.015</td>
<td>0.020</td>
<td>0.025</td>
<td>0.034</td>
<td>0.044</td>
</tr>
<tr>
<td></td>
<td>(-6.227)</td>
<td>(-4.742)</td>
<td>(-2.100)</td>
<td>(1.424)</td>
<td>(4.711)</td>
<td>(7.297)</td>
<td>(8.886)</td>
<td>(10.660)</td>
<td>(17.106)</td>
<td>(15.022)</td>
<td>(15.569)</td>
</tr>
<tr>
<td></td>
<td>[0.000]</td>
<td>[0.001]</td>
<td>[0.058]</td>
<td>[0.169]</td>
<td>[0.001]</td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
</tr>
<tr>
<td></td>
<td>(-3.256)</td>
<td>(-0.731)</td>
<td>(-1.577)</td>
<td>(-2.265)</td>
<td>(-2.364)</td>
<td>(-2.192)</td>
<td>(-1.533)</td>
<td>(-1.139)</td>
<td>(-1.109)</td>
<td>(-1.546)</td>
<td>(-1.598)</td>
</tr>
<tr>
<td></td>
<td>[0.006]</td>
<td>[0.495]</td>
<td>[0.120]</td>
<td>[0.042]</td>
<td>[0.035]</td>
<td>[0.039]</td>
<td>[0.147]</td>
<td>[0.296]</td>
<td>[0.259]</td>
<td>[0.132]</td>
<td>[0.137]</td>
</tr>
<tr>
<td>$\overline{R^2}$ (%)</td>
<td>0.9</td>
<td>0.4</td>
<td>0.4</td>
<td>0.8</td>
<td>1.1</td>
<td>0.5</td>
<td>-0.1</td>
<td>0.0</td>
<td>0.3</td>
<td>0.7</td>
<td>0.8</td>
</tr>
</tbody>
</table>
Table 1.6. Average Variance and Average Correlation

The table presents the regression results for the conditional quantile function: $Q_{r_{C,t+1}}(\tau \mid AV_t, AC_t) = \alpha(\tau) + \beta_1(\tau) AV_t + \beta_2(\tau) AC_t$, where $\tau$ is a quantile of the one-month ahead carry trade return $r_{C,t+1}$. $AV_t$ is the lagged average variance and $AC_t$ is the lagged average correlation. The return to the carry trade portfolio is constructed every month by going long on the quintile of currencies with the highest interest rates and going short on the bottom quintile. Average variance is the equally weighted cross-sectional average of the monthly variances of all exchange rate excess returns. Average correlation is the equally weighted cross-sectional average of the pairwise monthly correlations of all exchange rate excess returns. With the exception of average correlation, all variables are annualized. Bootstrap $t$-statistics generated using 1,000 bootstrap samples are reported in parentheses. Bootstrap $p$-values using the double bootstrap are in brackets. The pseudo-$R^2$ is computed as in Koenker and Machado (1999). The sample period runs from January 1976 to February 2009.

<table>
<thead>
<tr>
<th>Quantile</th>
<th>0.05</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>$-0.017$</td>
<td>$-0.009$</td>
<td>$-0.005$</td>
<td>$0.004$</td>
<td>$0.010$</td>
<td>$0.014$</td>
<td>$0.020$</td>
<td>$0.025$</td>
<td>$0.032$</td>
<td>$0.040$</td>
<td>$0.055$</td>
</tr>
<tr>
<td></td>
<td>$(-2.246)$</td>
<td>$(-0.988)$</td>
<td>$(-1.125)$</td>
<td>$(1.064)$</td>
<td>$(3.349)$</td>
<td>$(4.686)$</td>
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<td>$(8.705)$</td>
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<tr>
<td>Average Variance</td>
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<td>$-6.558$</td>
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<td>$-2.616$</td>
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<td>$(-1.551)$</td>
<td>$(-1.713)$</td>
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<td>0.9</td>
<td>1.6</td>
<td>1.8</td>
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Table 1.7. Additional Predictive Variables

The table presents the regression results for the conditional quantile function: $Q_{RC,t+1} (\tau \mid AV_t, AC_t, AID_t, r_{C,t}) = \alpha (\tau) + \beta_1 (\tau) AV_t + \beta_2 (\tau) AC_t + \beta_3 (\tau) AID_t + \beta_4 (\tau) r_{C,t}$. where $\tau$ is a quantile of the one-month ahead carry trade return $r_{C,t+1}$, $AV_t$ is the lagged average variance, $AC_t$ is the lagged average correlation, $AID_t$ is the average interest rate differential and $r_{C,t}$ is the lagged carry trade return. The return to the carry trade portfolio is constructed every month by going long on the quintile of currencies with the highest interest rates and going short on the bottom quintile. Average variance is the equally weighted cross-sectional average of the monthly variances of all exchange rate excess returns. Average correlation is the equally weighted cross-sectional average of the pairwise monthly correlations of all exchange rate excess returns. AID is the average interest rate differential of the quintile of currencies with the highest interest rates minus the average interest rate differential of the quintile of currencies with the lowest interest rates. With the exception of average correlation, all variables are annualized. Bootstrap $t$-statistics generated using 1,000 bootstrap samples are reported in parentheses. Bootstrap $p$-values using the double bootstrap are in brackets. The pseudo-$\hat{R}^2$ is computed as in Koenker and Machado (1999). The sample period runs from of January 1976 to February 2009.

<table>
<thead>
<tr>
<th>Quantile</th>
<th>0.05</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.162</td>
<td>0.100</td>
<td>0.185</td>
<td>0.554</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
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<td>-2.819</td>
<td>-1.394</td>
<td>-1.808</td>
<td>-1.136</td>
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<td>-0.946</td>
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<td>-0.012</td>
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<td>0.003</td>
<td>-0.004</td>
<td>-0.003</td>
<td>-0.006</td>
<td>-0.010</td>
<td>-0.016</td>
<td>-0.019</td>
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<td>-2.597</td>
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<td>Average Interest Differential</td>
<td>-0.398</td>
<td>0.218</td>
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<td>0.499</td>
<td>0.443</td>
<td>0.440</td>
<td>0.502</td>
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<td>2.515</td>
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<td>0.015</td>
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<tr>
<td></td>
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<td>2.002</td>
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<td>1.953</td>
<td>1.461</td>
<td>0.663</td>
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<td>-0.810</td>
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<td>$\hat{R}^2$ (%)</td>
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<td>2.4</td>
<td>2.9</td>
<td>2.0</td>
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<td>1.6</td>
<td>1.1</td>
<td>1.6</td>
<td>1.4</td>
</tr>
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</table>
Table 1.8. Weighted Average Variance and Weighted Average Correlation

The table presents the regression results for the conditional quantile function: $Q_{1_{C,t+1}}(\tau | AV^W_t, AC^W_t) = \alpha(\tau) + \beta_1(\tau) AV^W_t + \beta_2(\tau) AC^W_t$, where $\tau$ is a quantile of the one-month ahead weighted carry trade return $1_{C,t+1}$. $AV^W_t$ is the lagged weighted average variance and $AC^W_t$ is the lagged weighted average correlation. All variables are weighted in order to account for the effect of numeraire in both the carry trade return and the risk measures. The weights are based on the Special Drawing Rights (SDR) of the International Monetary Fund and are as follows: 41.9% on the US dollar-denominated measures, 37.4% on the Euro-denominated measures, 11.3% on the UK pound-denominated measures and 9.4% on the Japanese yen-denominated measures. The weighted return to the carry trade portfolio uses the SDR weights across the four numeraires, and for a given numeraire is constructed every month by going long on the quintile of currencies with the highest interest rates and going short on the bottom quintile. Weighted average variance is the weighted average across the four numeraires, where for a given numeraire average variance is the equally weighted cross-sectional average of the monthly variances of all exchange rate excess returns. Weighted average correlation is the weighted average across the four numeraires, where for a given numeraire average correlation is the equally weighted cross-sectional average of the pairwise monthly correlations of all exchange rate excess returns. With the exception of average correlation, all variables are annualized. Bootstrap $t$-statistics generated using 1,000 bootstrap samples are reported in parentheses. Bootstrap $p$-values using the double bootstrap are in brackets. The pseudo-$R^2$ is computed as in Koenker and Machado (1999). The sample period runs from of January 1976 to February 2009.

<table>
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<tr>
<th>Quantile</th>
<th>0.05</th>
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<th>0.2</th>
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<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
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<th>0.9</th>
<th>0.95</th>
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<td>$0.003$</td>
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<td>$0.018$</td>
<td>$0.026$</td>
<td>$0.037$</td>
<td>$0.041$</td>
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<td>$0.060$</td>
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<td>($-1.011$)</td>
<td>($-1.151$)</td>
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<td>(2.742)</td>
<td>(3.717)</td>
<td>(4.449)</td>
<td>(7.676)</td>
<td>(7.786)</td>
<td>(6.618)</td>
<td>(5.114)</td>
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<td>[0.237]</td>
<td>[0.650]</td>
<td>[0.016]</td>
<td>[0.001]</td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
<td>[0.000]</td>
</tr>
<tr>
<td></td>
<td>[0.013]</td>
<td>[0.007]</td>
<td>[0.011]</td>
<td>[0.006]</td>
<td>[0.013]</td>
<td>[0.028]</td>
<td>[0.093]</td>
<td>[0.227]</td>
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<td>[0.723]</td>
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<td>$0.011$</td>
<td>$0.017$</td>
<td>$0.004$</td>
<td>$-0.012$</td>
<td>$-0.016$</td>
<td>$-0.024$</td>
<td>$-0.038$</td>
<td>$-0.038$</td>
<td>$-0.048$</td>
<td>$-0.045$</td>
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<tr>
<td></td>
<td>(0.143)</td>
<td>(0.331)</td>
<td>(0.293)</td>
<td>(0.304)</td>
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<td>($-1.485$)</td>
<td>($-1.834$)</td>
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<td>[0.008]</td>
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<td>1.1</td>
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<td>1.1</td>
<td>0.7</td>
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<td>1.4</td>
<td>1.7</td>
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</table>
Table 1.9. VIX, VXY and FX Risk Measures

The table presents ordinary least squares results for regressions of the VIX and VXY indices on monthly FX risk measures. The VIX index is based on the 1-month model-free implied volatility of the S&P 500 equity index. The VXY index is based on the 3-month implied volatility of at-the-money-forward options on the G-7 currencies. Market variance is the variance of the return to the market portfolio constructed as the equally weighted average of the excess returns on 33 US dollar nominal exchange rates. Average variance is the equally weighted cross-sectional average of the variances of exchange rate excess returns. Average correlation is the equally weighted cross-sectional average of the pairwise correlations of exchange rate excess returns. With the exception of average correlation, all variables are annualized. Newey-West (1987) $t$-statistics with five lags are reported in parentheses. The sample period for the VIX begins in January 1990 and for the VXY in January 1992. The sample period for all indices ends in February 2009.

### Panel A: Regressions for the VIX

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<th>(4)</th>
<th>(5)</th>
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<tr>
<td>(Average Variance)$\times$ Average Correlation</td>
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</tr>
<tr>
<td>$R^2$ (%)</td>
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### Panel B: Regressions for the VXY

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<tr>
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</table>
Table 1.10. VIX and VXY

The table presents the regression results for two conditional quantile functions. The first one is:
$$Q_{RC,t+1}(\tau \mid VIX_t) = \alpha(\tau) + \beta(\tau) VIX_t,$$
where $\tau$ is a quantile of the one-month ahead carry trade return $r_{C,t+1}$ and $VIX_t$ is the lagged VIX index. The second one is:
$$Q_{RC,t+1}(\tau \mid VXY_t) = \alpha(\tau) + \beta(\tau) VXY_t,$$
where $VXY_t$ is the lagged VXY index. The return to the carry trade portfolio is constructed every month by going long on the quintile of currencies with the highest interest rates and going short on the bottom quintile. The VIX index is based on the 1-month model-free implied volatility of the S&P 500 equity index. The VXY index is based on the 3-month implied volatility of at-the-money-forward options on the G-7 currencies. All variables are annualized. Bootstrap $t$-statistics generated using 1,000 bootstrap samples are reported in parentheses. Bootstrap $p$-values using the double bootstrap are in brackets. The pseudo-$R^2$ is computed as in Koenker and Machado (1999). The sample period runs from January 1976 to February 2009.

### Panel A: The Carry Trade Return on VIX

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<td>0.023</td>
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<tr>
<td></td>
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<td>(0.929)</td>
<td>(2.013)</td>
<td>(3.670)</td>
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<td>(4.077)</td>
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<tr>
<td></td>
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<td>[0.090]</td>
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<td>[0.006]</td>
<td>[0.003]</td>
<td>[0.001]</td>
<td>[0.006]</td>
</tr>
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<td>−0.023</td>
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<td>0.026</td>
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<tr>
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<td>(−0.580)</td>
<td>(−0.605)</td>
<td>(0.283)</td>
<td>(0.379)</td>
<td>(−0.029)</td>
<td>(0.731)</td>
<td>(1.057)</td>
<td>(0.870)</td>
<td>(1.988)</td>
</tr>
<tr>
<td></td>
<td>[0.140]</td>
<td>[0.605]</td>
<td>[0.584]</td>
<td>[0.511]</td>
<td>[0.745]</td>
<td>[0.699]</td>
<td>[0.957]</td>
<td>[0.520]</td>
<td>[0.341]</td>
<td>[0.444]</td>
<td>[0.065]</td>
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<tr>
<td>$R^2$ (%)</td>
<td>0.8</td>
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<td>−0.2</td>
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<td>−0.4</td>
<td>−0.4</td>
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### Panel B: The Carry Trade Return on VXY

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<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>0.95</th>
</tr>
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<tbody>
<tr>
<td>Constant</td>
<td>−0.051</td>
<td>−0.028</td>
<td>0.003</td>
<td>0.009</td>
<td>0.012</td>
<td>0.023</td>
<td>0.028</td>
<td>0.039</td>
<td>0.027</td>
<td>0.041</td>
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</tr>
<tr>
<td></td>
<td>(−1.646)</td>
<td>(−1.660)</td>
<td>(0.241)</td>
<td>(0.953)</td>
<td>(1.347)</td>
<td>(2.379)</td>
<td>(3.239)</td>
<td>(4.258)</td>
<td>(3.093)</td>
<td>(4.029)</td>
<td>(1.414)</td>
</tr>
<tr>
<td></td>
<td>[0.150]</td>
<td>[0.098]</td>
<td>[0.795]</td>
<td>[0.356]</td>
<td>[0.199]</td>
<td>[0.028]</td>
<td>[0.004]</td>
<td>[0.001]</td>
<td>[0.010]</td>
<td>[0.003]</td>
<td>[0.173]</td>
</tr>
<tr>
<td>VXY</td>
<td>0.183</td>
<td>0.073</td>
<td>−0.096</td>
<td>−0.102</td>
<td>−0.082</td>
<td>−0.123</td>
<td>−0.140</td>
<td>−0.190</td>
<td>0.005</td>
<td>−0.081</td>
<td>0.160</td>
</tr>
<tr>
<td></td>
<td>(0.628)</td>
<td>(0.447)</td>
<td>(−0.777)</td>
<td>(−1.139)</td>
<td>(−0.959)</td>
<td>(−1.354)</td>
<td>(−1.704)</td>
<td>(−2.209)</td>
<td>(0.061)</td>
<td>(−0.828)</td>
<td>(0.956)</td>
</tr>
<tr>
<td></td>
<td>[0.591]</td>
<td>[0.615]</td>
<td>[0.439]</td>
<td>[0.282]</td>
<td>[0.355]</td>
<td>[0.185]</td>
<td>[0.105]</td>
<td>[0.044]</td>
<td>[0.916]</td>
<td>[0.373]</td>
<td>[0.352]</td>
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<tr>
<td>$R^2$ (%)</td>
<td>−0.2</td>
<td>−0.2</td>
<td>0.1</td>
<td>0.1</td>
<td>0.4</td>
<td>1.0</td>
<td>1.0</td>
<td>0.4</td>
<td>−0.5</td>
<td>−0.3</td>
<td>1.2</td>
</tr>
</tbody>
</table>
The following rule at each time $t$: for the carry trade returns that are lower than the $1\tau$-quantile, if average correlation has decreased from $t-1$ to $t$, we close the carry trade positions and thus receive a return of zero; and for the carry trade returns that are higher than the $1\tau$-quantile, if average correlation has decreased from $t-1$ to $t$, we double the carry trade positions and thus receive twice the carry return; otherwise we execute the standard carry trade. The combined average variance and average correlation strategy makes the following decision at each time $t$: for the carry trade returns that are lower than the $\tau$-quantile, if average variance has increased from $t-1$ to $t$, we close the carry trade positions and thus receive a return of zero; otherwise we execute the standard carry trade. Average correlation is the equally weighted cross-sectional average of the pairwise monthly correlations of all exchange rate excess returns. The average variance strategy implements the following rule at each time $t$: for the carry trade returns that are lower than the $\tau$-quantile of the distribution, if average variance has increased from $t-1$ to $t$, we close the carry trade positions and thus receive a return of zero at $t+1$; otherwise we execute the standard carry trade. Average variance is the equally weighted cross-sectional average of the monthly variances of all exchange rate excess returns.

### Table 1.11. Out-of-Sample Augmented Carry Trade Strategies

The table presents the out-of-sample performance of augmented carry trade strategies that condition on the movement of average variance and/or average correlation with and without transaction costs. Panel A assumes no transaction costs, whereas Panel B implements the carry trade strategies using the transaction costs listed in Table 1. Average variance is the equally weighted cross-sectional average of the average variance strategy implements the following rule at each time $t$: for the carry trade returns that are lower than the $\tau$-quantile of the distribution, if average variance has increased from $t-1$ to $t$, we close the carry trade positions and thus receive a return of zero at $t+1$; otherwise we execute the standard carry trade. Average correlation is the equally weighted cross-sectional average of the pairwise monthly correlations of all exchange rate excess returns. The average correlation strategy implements the following rule at each time $t$: for the carry trade returns that are lower than the $1\tau$-quantile, if average correlation has decreased from $t-1$ to $t$, we double the carry trade positions and thus receive twice the carry return at $t+1$; otherwise we execute the standard carry trade. The combined average variance and average correlation strategy makes the following decision at each time $t$: for the carry trade returns that are lower than the $\tau$-quantile, if average variance has increased from $t-1$ to $t$, we close the carry trade positions and thus receive a return of zero; and for the carry trade returns that are higher than the $1\tau$-quantile, if average correlation has decreased from $t-1$ to $t$, we double the carry trade positions and thus receive twice the carry return; otherwise we execute the standard carry trade. The mean, standard deviation, and Sharpe ratio are reported in annualized terms. The turnover ratio is equal to the percentage of the currencies that on average are traded every month. The sample period runs from of January 1976 to February 2009. All strategies move forward recursively starting 3 years after the beginning of the sample so that the first observation is for January 1979.

### Panel A: No Transaction Costs

<table>
<thead>
<tr>
<th>Quantile</th>
<th>Carry Trade</th>
<th>Average Variance Strategy</th>
<th>Average Correlation Strategy</th>
<th>Combined Average Variance and Average Correlation Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.084</td>
<td>0.094 0.091 0.085 0.079 0.073 0.069 0.066 0.059 0.054</td>
<td>0.079 0.071 0.069 0.066 0.065 0.064 0.061 0.060 0.057 0.056</td>
<td>1.070 1.314 1.326 1.279 1.213 1.142 1.120 1.098 1.022 0.968</td>
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<tr>
<td></td>
<td>0.171</td>
<td>0.208 0.264 0.310 0.346 0.387 0.421 0.444 0.485 0.514</td>
<td>0.171 0.121 0.115 0.111 0.107 0.104 0.100 0.096 0.091 0.085</td>
<td>0.171 0.557 0.511 0.454 0.412 0.370 0.340 0.294 0.252 0.214</td>
</tr>
<tr>
<td>Mean</td>
<td>0.084</td>
<td>0.146 0.147 0.139 0.130 0.122 0.118 0.114 0.107 0.097</td>
<td>0.079 0.121 0.115 0.111 0.107 0.104 0.100 0.096 0.091 0.085</td>
<td></td>
</tr>
<tr>
<td>St. Dev.</td>
<td>0.079</td>
<td>0.121 0.115 0.111 0.107 0.104 0.100 0.096 0.091 0.085</td>
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<tr>
<td>Sharpe</td>
<td>1.070</td>
<td>1.208 1.284 1.262 1.212 1.174 1.186 1.186 1.182 1.134</td>
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<td></td>
</tr>
<tr>
<td>Turnover</td>
<td>0.171</td>
<td>0.557 0.511 0.454 0.412 0.370 0.340 0.294 0.252 0.214</td>
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(continued)
### Panel B: With Transaction Costs

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<tr>
<td>Mean</td>
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<td>0.068</td>
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<td>0.055</td>
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<td>0.063</td>
<td>0.061</td>
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<td>Sharpe ratio</td>
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<td>0.983</td>
<td>0.941</td>
<td>0.892</td>
<td>0.866</td>
<td>0.869</td>
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<td>0.761</td>
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<td>0.109</td>
<td>0.107</td>
<td>0.097</td>
<td>0.091</td>
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<td>Turnover</td>
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<tr>
<td>Mean</td>
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<tr>
<td>Turnover</td>
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<td>0.438</td>
<td>0.347</td>
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</table>

### Combined Average Variance and Average Correlation Strategy

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<td>0.895</td>
<td>0.926</td>
<td>0.959</td>
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<tr>
<td>Turnover</td>
<td>0.171</td>
<td>0.594</td>
<td>0.521</td>
<td>0.438</td>
<td>0.347</td>
<td>0.347</td>
<td>0.259</td>
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</tr>
</tbody>
</table>
Figure 1.1. Carry Trade Return, Average Variance and Average Correlation

This figure displays the time series of the cumulative carry trade return, FX average variance and FX average correlation from January 1976 to February 2009.
This figure shows the parameter estimates of the predictive quantile regression of the one-month-ahead carry trade return on the lagged market variance. The dashed lines indicate the 95% confidence interval based on bootstrap standard errors.
Figure 1.3. Average Variance and Average Correlation

This figure illustrates the parameter estimates of the predictive quantile regression of the one-month-ahead carry trade return on the lagged average variance and average correlation. The dashed lines indicate the 95% confidence interval based on bootstrap standard errors.
Figure 1.4. The Exchange Rate and Interest Rate Components of the Carry Trade

This figure exhibits the parameter estimates of two sets of predictive quantile regressions. The left panel shows the results for the one-month-ahead exchange rate component of the carry trade return on the lagged average variance and lagged average correlation. The right panel shows the results for the interest rate component of the carry trade return on the lagged average variance and average correlation. The dashed lines indicate the 95% confidence interval based on bootstrap standard errors.
This figure illustrates the parameter estimates of the predictive quantile regression of the one-month-ahead carry trade return on the lagged average variance, lagged average correlation and two additional predictive variables: the interest rate differential and the lagged carry trade return. The dashed lines indicate the 95% confidence interval based on bootstrap standard errors.
Figure 1.6. The Numeraire Effect

This figure displays the parameter estimates of the predictive quantile regression of the weighted one-month-ahead carry trade return on the lagged weighted average variance and lagged weighted average correlation. The weighted variables account for the numeraire effect using the IMF weights for Special Drawing Rights: 41.9% on the US dollar, 37.4% on the Euro, 11.3% on the UK pound and 9.4% on the Japanese yen. The dashed lines indicate the 95% confidence interval based on bootstrap standard errors.
This figure exhibits the parameter estimates of two sets of predictive quantile regressions. The left panel shows the results for the one-month-ahead carry trade return on the lagged VIX index. The right panel shows the results for the one-month-ahead carry trade return on the lagged VXY index. The dashed lines indicate the 95% confidence interval based on bootstrap standard errors.
This figure shows the time-variation of the robust conditional skewness measure of Kim and White (2004) based on the predictive quantile regressions. The measure uses the inter-quartile range ($\alpha = 0.25$).
Chapter 2

On the evolution of the exchange rate response to fundamental shocks

2.1 Introduction

In a frictionless and risk-neutral economy, asset prices should react instantaneously to fundamental shocks to ensure that expected excess returns are zero. In the case of the foreign exchange market, this implies that a sudden increase in interest rate differentials should lead to an impact appreciation of the high-interest currency, followed by a depreciation so that uncovered interest rate parity (UIP) holds. A carry trader, who invests in a high-interest currency (the investment currency) by funding her position in a low-interest currency (the funding currency), would therefore face only an impact positive excess return, but this would then become zero on average as implied by UIP.

However, empirical evidence seems to be at odds with the exchange rate behaviour outlined above. The “forward premium puzzle” implies that UIP is systematically violated as future currency excess returns are predictable (Fama, 1984; Bilson, 1981; Engel, 1996), and that carry trade strategies tend to be profitable (Della Corte et al. 2009, Burnside et al. 2011). These results violate unconditional UIP—the response of the exchange rate to all shocks on average. Moreover, UIP is also violated conditionally: conditional on monetary policy shocks, cumulative excess returns on foreign exchange
tend to be sizable and persistent.\footnote{In distinguishing between unconditional and conditional UIP violations, I follow Faust and Rogers (2003) and Scholl and Uhlig (2008).} This latter evidence has been studied in much of the literature on the “delayed overshooting puzzle”: contractionary monetary policy shocks lead to a persistent appreciation of the domestic currency before starting to depreciate (Eichenbaum and Evans, 1995; Scholl and Uhlig, 2008). These dynamics stand in stark contrast with Dornbusch (1976) classical hypothesis of an immediate appreciation and subsequent persistent depreciation following a monetary policy shock, a hypothesis which follows from the assumption of UIP and long-run purchasing power parity (PPP). Similarly, Brunnermeier et al. (2009) find that exchange rates initially underreact to interest rate differential shocks: when the foreign interest rate increases relative to the domestic interest rate, the investment currency appreciates sluggishly, with cumulative excess returns reaching the level implied by UIP and PPP only after a few quarters. At longer horizons, instead, they find evidence of possible overreaction of the exchange rate to interest rate differential shocks.

This chapter re-examines these issues in light of the recent literature on nonlinearities in the foreign exchange market. I do not consider UIP and PPP in general, but conditional on interest rate differential shocks. Previous studies that document sizable conditional excess returns (violating UIP) and a sluggish reaction of the exchange rate do not generally consider the possibility that either or both the volatility of the shocks and the transmission mechanism may have changed over time. Therefore, previous results may not reflect the current state of the economy but just an average over the past. Given a simple present-value model for the currency excess return which assumes UIP and long-run PPP,\footnote{The present-value approach adopted in this chapter is inspired by e.g. Froot and Ramadorai (2005), Brunnermeier et al. (2009), Engel (2010), and Engel and West (2010).} the research questions are therefore the following: how large the deviations from the present value of future fundamentals\footnote{In this chapter, I consider as fundamentals only those strictly implied by the assumptions of UIP and long-run PPP, i.e. real interest rate differentials, as discussed in Section 2.2. Therefore, I do not consider other “classic” fundamentals such as relative money supplies and outputs, as in e.g. Engel and West (2005).} should one expect following an interest rate shock, given the current state of the economy? Do these conditional deviations converge to the level implied by fundamentals, and, if so, how does this
behaviour evolve over time as the state of the economy changes?

A number of previous studies have already documented how nominal and real exchange rate dynamics may have changed over time. Moreover, these studies have shown how allowing for nonlinearities may shed light, and possibly explain, apparent deviations from the parity relations which form the basis of much of the international finance literature—namely, UIP and PPP. For example, Taylor et al. (2001) show that real exchange rates (or equivalently, PPP deviations) are well characterized by a nonlinear mean-reverting processes leading to time-varying half-lives in which larger shocks mean-revert much faster than those previously reported for linear models, therefore potentially explaining the PPP puzzle (Rogoff, 1996). Sarno et al. (2006) find that deviations from UIP display significant nonlinearities, consistent with theories based on transaction costs (e.g. Dumas, 1992) or limits to speculation (Lyons, 2001). This evidence leads them to conclude that UIP deviations may be less indicative of major market inefficiencies than previously thought. Christiansen et al. (2011) show that carry trade returns display time-varying risk exposure to the stock and bond markets depending on switching regimes characterized by the level of foreign exchange volatility. Mumtaz and Sunder-Plassmann (2010) find that the transmission of demand, supply and nominal shocks on the real exchange rate displays significant time variation, with an increasing impact of demand shocks over the years. However, none of these studies analyse the evolution of conditional violations of UIP over time.

Therefore, the importance of analysing nonlinearities in exchange rate dynamics seems to be undisputed. Similarly to the empirical studies above, I approximate nonlinearities by allowing for time variation in the parameters linking fundamentals to exchange rates. In the context of this chapter, in which I analyse conditional violations of UIP, this translates into estimating the time-varying impulse response functions of the currency excess return to interest rate differential shocks. A natural framework to estimate these impulse responses is to use a Bayesian time-varying-parameters vector-

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4Bacchetta and van Wincoop (2004, 2010) provide a theory of exchange rate determination which rationalizes parameter instability in empirical exchange rate models. They show that foreign exchange market participants can optimally choose to change the weight attached to different economic fundamentals in the context of rational expectation models.
autoregression (TVP-VAR) with stochastic volatility. Particularly, I adopt the methodology from recent advances in the macroeconometric literature which has fruitfully applied this technique in other contexts, see e.g. Cogley and Sargent (2005), Primiceri (2005), Benati (2008), and Mumtaz and Sunder-Plassmann (2010).\(^5\) Allowing for time variation both in the VAR coefficients and the covariance matrix leaves it up to the data to determine whether the time variation of the linear structure derives from changes in the size of the shocks (impulse) or from changes in the propagation mechanism (response).

I provide empirical evidence that the transmission of the interest rate differential shocks has changed over time. However, even if to a varying degree over the years, some of the puzzling results previously documented with linear models remain. I show that currency excess returns tend to initially underreact to interest rate differential shocks for the whole sample considered, undershooting the level implied by UIP and long-run PPP. At longer horizons, the previously documented evidence of overshooting tends to disappear in recent years in the case of the euro, the British pound and the Canadian dollar. Instead, overreaction at long horizons is a persistent feature of the excess returns on the Japanese yen and the Swiss franc throughout the whole sample.

These results suggest that previously documented conditional violations of UIP may have secularly declined over time, at least for the euro, the British pound and the Canadian dollar. However, the results for the Japanese yen and the Swiss franc—two currencies which have been traditionally used for funding carry trade positions—may hint that speculation in the foreign exchange market may constitute a destabilizing force, driving exchange rates away from fundamentals.

### 2.2 Foreign Exchange Excess Returns

The objective of this chapter is to study the evolution over time of conditional deviations from UIP, i.e. the reaction of currency excess returns to an unexpected shock to interest

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\(^5\)See also Koop and Korobilis (2010) for a recent survey of the methodologies used in this chapter.
2.2. Foreign Exchange Excess Returns

rate differentials. Therefore, we first need to compute the level of the unexpected excess return that would be predicted under the assumption of UIP. By definition, the log excess return on foreign exchange is equal to the interest rate differential plus the appreciation rate of the foreign currency

\[ z_t = i_t^* - i_{t-1} + (s_t - s_{t-1}), \]  

(2.1)

where \( s_t \) is the natural logarithm of the exchange rate (defined as the domestic price of foreign currency, so that an increase in \( s_t \) denotes appreciation of the foreign currency), and \( i_t \) and \( i_t^* \) are the continuously compounded domestic and foreign riskless interest rates, respectively. In terms of the log real exchange rate, defined as \( q_t = s_t + p_t^* - p_t \) where \( p_t^* \) and \( p_t \) are the logs of the foreign and domestic price levels, Equation (2.1) can be rewritten as

\[ z_{t+1} = (i_t^* - \pi_{t+1}^*) - (i_t - \pi_{t+1}) + (q_{t+1} - q_t), \]  

(2.2)

where \( \pi_{t+1}^* \) and \( \pi_{t+1} \) are the foreign and domestic inflation rates. Assuming UIP, the expected excess return for a risk-neutral investor should be equal to zero, i.e., \( E_t z_{t+j} = 0 \), implying that an interest rate differential gain from borrowing abroad and lending domestically should be offset by the expected appreciation rate of the foreign currency. As in Froot and Ramadorai (2005) and Brunnermeier et al. (2009), I also make the relatively milder assumption that long-run PPP holds in expectation, i.e. \( \lim_{j \to \infty} E_t q_{t+j} = 0 \).

We can now solve Equation (2.2) forward and, by taking the expectation conditional on time-\( t \) information, we have

\[ q_t = \sum_{i=1}^{\infty} E_t [(i_{t+i+1}^* - \pi_{t+i+1}^*) - (i_{t+i-1} - \pi_{t+i})]. \]  

(2.3)

Therefore, under UIP and long-run PPP, an increase in expected future real interest rate differentials should be reflected in a real appreciation of the foreign currency. Following a surprise increase in foreign interest rates relative to the domestic ones (i.e. \( i_t^* - i_{t+1} - E_t [i_{t+1}^* - i_{t+1}] > 0 \)), we can use Equation (2.3) together with the definition of excess return in Equation (2.2) in order to express the unexpected one-period excess return as
the sum of all future innovations in expected future real interest differentials:

\[
z_{t+1} - E_t z_{t+1} = \sum_{i=1}^{\infty} \left[ E_{t+1} (i_{t+i}^* - i_{t+i}) - i_{t+1}^* - i_{t+1} - \left( E_{t+1} (\pi_{t+i}^* - \pi_{t+i+1}) - E_t (\pi_{t+i+1}^* - \pi_{t+i+1}) \right) \right]. \tag{2.4}
\]

Note also that, under UIP, \( E_t z_{t+1} = 0 \), so that \( (z_{t+1} - E_t z_{t+1}) = z_{t+1} \). Therefore, UIP predicts that a higher foreign interest rate leads to a jump in the excess return which reflects the present value of changes in expectations of future fundamentals. At longer horizons (i.e. after the unexpected shock) UIP also implies that the cumulative return should remain flat, as the expected future interest rate differentials are exactly offset by exchange rate depreciations. The present value in Equation (2.4) can be estimated using standard results (see e.g. Campbell, 1991, and Hamilton, 1994) using the long-run responses of a VAR which includes \( i_t^* - i_t, \pi_t^* - \pi_t \), and \( z_t \).

Comparing this present value to the actual cumulative response of the excess return to interest rate differential shocks, Brunnermeier et al. (2009) find that exchange rates initially underreact to the interest rate differential shock for all the currencies considered: when the foreign interest rate increases relative to the domestic interest rate, the investment currency appreciates sluggishly, with cumulative excess returns reaching the level implied by UIP and PPP only after a few quarters.

### 2.3 Empirical Approach

#### 2.3.1 Model

I analyse the evolution over time of the deviations from UIP conditional on interest rates shocks as implied by the present-value relationship in Equation (2.4), and re-examine the results obtained by the previous literature (especially Brunnermeier et al., 2009).

A natural way to do so is to allow for the parameters of the VAR used in the estimation of Equation (2.4) to vary over time, and then compare the UIP-implied levels of the unexpected excess return shock to the actual (time-varying) impulse responses.
However, as noted by Sims (2001) and Stock (2001), the possible presence of time-varying covariance matrix in the dynamics of the variables considered may incorrectly inflate the time-variation of the coefficients of a TVP-VAR which does not explicitly take this into account.

Therefore, I use the TVP-VAR model proposed by Primiceri (2005), which allows for time-variation both in the coefficients and in the covariance matrix of the VAR innovations. As stressed by Primiceri (2005), this feature of the model leaves it up to the data to determine whether the time variation of the linear structure derives from changes in the size of the shocks (impulse) or from changes in the propagation mechanism (response).

I specify a time-varying parameter VAR(p) with stochastic volatility as follows:

$$y_t = B_{0,t} + \sum_{j=1}^{p} B_{j,t} y_{t-j} + \epsilon_t,$$

where

$$y_t = \begin{bmatrix} i^*_t - i_t \\ \pi^*_t - \pi_t \\ z_t \end{bmatrix},$$

with $i^*_t - i_t$ being the interest rate differential, $\pi^*_t - \pi_t$ being the inflation rate differential, and $z_t$ being the excess return on foreign exchange as defined in Equation (2.1). The vector $\epsilon_t$ includes heteroskedastic unobservable shocks with covariance matrix $\Omega_t$. Using the Bayesian Information Criterion with quarterly data, I set the number of lags $p$ equal to 1. Following Cogley and Sargent (2001, 2005) and Benati (2008), I model the time-varying parameters as driftless random walks subject to reflecting barriers to ensure the stability of the VAR system. Let $\beta_t$ denote the vector stacking all right-hand-side coefficients of Equation (2.5), and let $\beta^T = \{\beta'_1, \ldots, \beta'_T\}'$, then the joint prior distribution of the VAR coefficients is

$$p(\beta^T, Q) \propto I(\beta^T)f(Q)f(\beta^T|\beta_0, Q) = I(\beta^T)f(Q)\prod_{t=1}^{T} p(\beta_t|\beta_{t-1}, Q),$$

(2.6)
2.3. Empirical Approach

where

$$\beta_{t+1} | \beta_t, Q \sim N(\beta_t, Q).$$

(2.7)

The reflecting barrier \( I(\beta^T) = \prod_{s=1}^{T} I(\beta_s) \) ensures the stability of the system by using the indicator function \( I(\beta_s) \) which takes a value of zero when the roots of the associated VAR polynomials are inside the unit circle, and it is equal to one otherwise.

Following Primiceri (2005), I model the conditional covariance matrix \( \Omega_t \) of the reduced-form innovations in Equation (2.5) using the following decomposition:

$$A_t \Omega_t A'_t = \Sigma_t \Sigma'_t,$$

(2.8)

where \( \Sigma_t \) is a diagonal matrix with diagonal elements \( \sigma_{j,t} \), that is

$$\Sigma_t = \begin{bmatrix} \sigma_{1,t} & 0 & \ldots & 0 \\ 0 & \sigma_{2,t} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \ldots & 0 & \sigma_{M,t} \end{bmatrix},$$

(2.9)

and \( A_t \) is the lower triangular matrix

$$A_t = \begin{bmatrix} 1 & 0 & \ldots & \ldots & 0 \\ a_{21,t} & 1 & \ldots & \ldots & . \\ . & . & \ldots & \ldots & 1 \\ \vdots & \vdots & \ldots & \ldots & 1 \\ a_{M1,t} & \ldots & a_{M(M-1),t} & 1 \end{bmatrix},$$

(2.10)

which captures the time variation of the simultaneous interactions among the \( M \) variables in the VAR (in my application, \( M = 3 \)). As discussed by Primiceri (2005), it is important to allow for time variation of \( A_t \) because a constant \( A_t \) would imply that the innovation to one variable has a time invariant contemporaneous effect on other variables of the system.

Stacking the diagonal elements of \( \Sigma_t \) in the column vector \( \sigma_t \) and the off-diagonal
and non-zero elements of $A_t$ (by rows) in the column vector $\alpha_t$, the dynamics of the time-varying parameters of the model are specified as

$$
\beta_t = \beta_{t-1} + \nu_t, \tag{2.11}
$$

$$
\alpha_t = \alpha_{t-1} + \xi_t, \tag{2.12}
$$

$$
\ln \sigma_t = \ln \sigma_{t-1} + \eta_t. \tag{2.13}
$$

For the sake of parsimony and to allow a structural interpretation of the innovations, all the innovations of the model are jointly normal with block-diagonal covariance matrix

$$
V = \text{Var} \begin{bmatrix}
    u_t \\
    \nu_t \\
    \xi_t \\
    \eta_t
\end{bmatrix} = \begin{bmatrix}
    I_3 & 0 & 0 & 0 \\
    0 & Q & 0 & 0 \\
    0 & 0 & S & 0 \\
    0 & 0 & 0 & W
\end{bmatrix}, \tag{2.14}
$$

where $u_t$ is such that $\epsilon_t = A_t^{-1}\Sigma_t u_t$. In order to simplify inference and improve the efficiency of the estimation algorithm, I assume $S$ to have a block diagonal structure, with blocks corresponding to parameters belonging to separate equations. That is, the coefficients of the contemporaneous relations among variables are assumed to evolve independently in each equation. In the present application with three equations in the system, we have

$$
S = \text{Var}(\xi_t) = \begin{bmatrix}
    S_1 & 0_{1\times 2} \\
    0_{2\times 1} & S_2
\end{bmatrix}, \tag{2.15}
$$

where $S_1 = \text{Var}(\xi_{21,t})$ and $S_2 = \text{Var}(\xi_{31,t}, \xi_{32,t})$.

### 2.3.2 Estimation

I estimate the model via Bayesian methods, using the Gibbs sampling procedure developed by Primiceri (2005). Here I only sketch the Markov Chain Monte Carlo (MCMC) algorithm used for the estimation of the model, whereas the priors and a more detailed description of the algorithm are outlined in Appendix 2.A and 2.B, respectively.
2.4. Data and Stability Tests

The MCMC algorithm developed by Primiceri (2005) is a Gibbs sampling procedure which sequentially draws from the full conditional distributions of the parameters of interest in order to generate a sample from the joint distribution of \((\beta^T, A^T, \Sigma^T, V)\), where the superscript \(T\) denotes the whole history of the variable of interest. This means that the Gibbs sampling draws sequentially the time varying coefficients of \(\beta^T\), the simultaneous relations \(A^T\), the volatilities \(\Sigma^T\), and the hyperparameters \(V\), conditional on the observed data and the rest of the parameters.

Conditional on \(A^T\) and, \(\Sigma^T\), the state space form of the model is linear and Gaussian, so that we can draw from the conditional posterior of \(\beta^T\) by using standard algorithms such as the one developed by Carter and Kohn (1994). We can also draw \(A^T\) in a similar way, whereas drawing \(\Sigma^T\) relies mostly on an adaptation of the method of Kim et al. (1998), which transforms a nonlinear and non-Gaussian state space model in a linear and approximately Gaussian form, so that one can still apply the algorithm of Carter and Kohn (1994). Finally, conditional on the data, \(\beta^T, A^T,\) and \(\Sigma^T\) the innovations of the state equations—\(\nu_t, \xi_t\), and \(\eta_t\)—are observable so that we can draw the hyperparameters contained in \(V\) from their respective distributions.

The MCMC algorithm therefore simulates the posterior distribution of the states and the hyperparameters by iterating the steps outlined above. I use 200,000 iterations from which I discard a burn-in period of 80,000 iterations in order to assure convergence to the ergodic distribution.

2.4 Data and Stability Tests

I collect quarterly exchange rates against the US dollar and three-month LIBOR rates for the following major currencies: Canadian dollar (CAD), Japanese yen (JPY), British pound (GBP), euro (EUR), and Swiss franc (CHF). I also collect quarterly CPI data from the IMF International Financial Statistics Database. The data span is from the first quarter of 1975 to the third quarter of 2009, except for the JPY which starts in the third quarter of 1978. For the euro before 1999, I use data for Germany and the German mark, as is standard in the literature.
Table 2.1 shows summary statistics, unit-root and stability tests for the series. The summary statistics present standard stylised facts in the international finance literature, such as the much higher volatility of currency excess returns relative to fundamentals (interest rate and inflation differentials in our case), and the positive skewness of the excess returns on typical funding currencies such as the JPY and the CHF (which translates in a negative skewness of the returns on a short position in the funding currencies).

The Phillips-Perron test statistics reject the null hypothesis of a unit root for most of the series at standard significance levels. Notable exceptions are the interest rate differentials for the CHF and the EUR: the Phillips-Perron test does not reject the unit-root hypothesis at any standard significance level for the CHF, and only at the 10% level for the EUR. However, a reason for the test not to reject the unit-root hypothesis may well be that interest rate differentials may follow an unstable process over the sample considered, a possibility which is explicitly taken in to account in the time-varying parameter model used in this chapter.

To test for the stability of the parameters of the model specified in Equation (2.5), I run the $\hat{q}LL$ efficient test statistic of Elliott and Müller (2006) for each of the equations of the VAR. The null hypothesis of a stable regression model, $y_t = X_t'\gamma + \epsilon_t$, is tested against the unstable model $y_t = X_t'\gamma_t + \epsilon_t$ in which, notably, the coefficient vector $\gamma_t$ is time-varying. Elliott and Müller (2006) show that the precise form of the breaking process $\{\gamma_t\}$, which is generally unknown, is irrelevant for the asymptotic power of their test. The $\hat{q}LL$ test is calculated using a six-step procedure, which involves running a number of auxiliary OLS regressions using an appropriate transformation of the OLS residuals of the stable model, see Elliott and Müller (2006). I report results of the test in Table 2.1: with few exceptions, the null of stable parameters of the equations in the VAR is rejected. This result corroborates my argument for explicitly allowing for time-variation in the coefficients of the VAR in order to properly capture the evolution of the propagation mechanism of the shocks in the model.
2.5 Empirical Results

2.5.1 Volatilities

Figures 2.1 to 2.5 show the posterior mean of the standard deviations (i.e., the square root of the diagonal elements of $\Omega_t$) of the VAR equations for all country pairs. The estimates indicate that the time variation of the covariance matrix $\Omega_t$ is an important feature of the data. For some country pairs, particularly for the US relative to the European block (UK, euro zone, and Switzerland), volatility patterns share some common features. The standard deviation of the UK-US interest rate differential displays a clear downward pattern from the start of the sample in the early eighties, with a temporary peak in the early nineties which coincides with the exit of the British pound from the Exchange Rate Mechanism. Notably, the volatility reaches a new high during the onset of financial crisis of the late 2000s. The standard deviation of the inflation rate differential exhibits a similar pattern, but with the downward trend starting only in the early nineties. These results are consistent with the “Great Moderation”, i.e. the increased stability experienced by the US economy during the Volcker and Greenspan chairmanships of the Federal Reserve (see e.g. Stock and Watson, 2002; Cogley and Sargent, 2001, 2005) and by the UK economy after the start of the inflation targeting regime in 1992 (see Benati, 2008).

A similar volatility pattern characterizes also the interest rate and inflation rate differentials in the case of the euro zone-US and Switzerland-US country pairs. Interest rate differential movements show a pattern of increasing stability in the early nineties, but display significant peaks in volatility during the burst of the dot-com bubble and the credit crisis. The inflation rate differential between the US and the euro zone also experiences a period of stability in the early nineties, followed by an increase in volatility in the 2000s.

As shown in Figures 2.1 and 2.2, the volatilities of the currency excess returns for the British pound and the euro (relative to the US dollar) increase from the start of the sample until a first peak in the early nineties, followed by a sharp drop and a stability
period before a second and more significant peak during the financial markets turbulence after the Lehman Brothers default in 2008. These results are broadly consistent with those of Mumtaz and Sunder-Plassmann (2010) for the pound and the euro real exchange rate volatility. The currency excess return on the third European currency, the Swiss franc, does not show clear trends in volatility, but we can still distinguish periods of sharp market instability during the recent crises.

Analysing the results for the US relative to the non-European countries (Canada and Japan in Figures 2.3 and 2.5, respectively), I do not find evidence of downward trends in the volatility of the interest and inflation rate differential as it was instead the case for the European countries. It is worth noting that the interest rate and inflation volatilities for the Japan-US pair, even though remarkably low and stable for much of the sample considered, exhibit three distinct spikes at the peak of the Japanese asset market bubble in 1989–1990, during the Asian financial crisis of 1997, and during the credit crisis of the late 2000s.

Regarding the volatility of the excess returns on the non-European currencies, notice that, consistent with the findings of Mumtaz and Sunder-Plassmann (2010), the volatility of the Canadian dollar excess return remains almost flat for most of the sample but increases sharply in 2000 and reaches its peak in recent quarters. The volatility of the Japanese yen excess return is fairly stable, with notable peaks during the Asian and 2000s crises.

### 2.5.2 Impulse Responses

Figures 2.6 to 2.10 display the estimated accumulated time-varying impulse responses of the currency excess return to a 1% shock to the interest rate differential \((i^*-i)\). I use a 1% shock instead of the usual one-standard-deviation shock because the latter is time-varying and would not allow to distinguish between the evolution of the transmission mechanism of the shocks and the time variation of the shocks themselves.

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6Note that the results of Mumtaz and Sunder-Plassmann (2010) are not directly comparable to those of the present study, as they analyse the time variation of the unconditional volatility of the real exchange rate change.
The impulse response is usually interpreted as a difference in conditional expectations such as

\[ E_{t+1}y_{t+h} - E_{t}y_{t+h}, \]

for any horizon \( h \). In a nonlinear model such as the one used in this chapter, these expectations can be calculated using simulation methods that produce what are called generalised impulse responses (see e.g. Koop, 1996). However, this may be computationally demanding. Due to limitations in computer memory, I use the structural VAR coefficients at each point in time and compute conventional impulse responses. As noted by Koop et al. (2009), these can be interpreted as impulse response functions calculated assuming all shocks to the model (including the shocks to the state equations) between time \( t+1 \) and \( t+h \) are simply set to their expected values of zero.

The shocks underlying the impulse responses are based on a Cholesky decomposition with recursive ordering \([i_t - i_t, \pi_t^* - \pi_t, z_t]\). This decomposition has the potential limitation of imposing that shocks to the interest rate differential cause contemporaneous changes in the other two variables but shocks to the other variables do not affect the innovations of the interest rate differential. Nevertheless, there are two main reasons why I use this identification scheme. First, and more importantly, it permits a direct comparability of my empirical results with those of Brunnermeier et al. (2009) who use the same identification scheme, therefore letting me focus the analysis on the effects of allowing for time variation of the dynamics of the system.

Second, previous literature has shown that the apparent conditional UIP deviations are not likely to be due to identifying assumptions. For example, Faust and Rogers (2003) show that conditional deviations of UIP are a robust finding even when one suspends what they call “dubious” identifying assumptions.\(^7\) Moreover, Scholl and Uhlig (2008), using an identification procedure that involves sign restrictions on the impulse responses but which leaves the response of the exchange rate “agnostically open”, find also strong evidence on conditional deviations from UIP (even when they

\(^7\)In Faust and Rogers (2003), this is not the case instead for the delayed overshooting puzzle, which they find to be sensitive to dubious identifying assumptions.
rule out delayed overshooting by construction).\(^8\)

**Impulse Responses for Selected Dates**

Figures 2.6 to 2.10 display the time-varying impulse responses in four dates, i.e. at the end of 1985, 1995, 2005, and 2008. This choice of dates is arbitrary and made only for illustrative purposes. Together with the posterior median of the impulse responses, I plot their 16th and 84th percentiles.\(^9\) The dashed horizontal line in each graph is the estimated cumulative excess return implied by UIP (Equation 2.4), reflecting the present value of all the future real interest rate differentials as predicted by the VAR.\(^10\)

For the sake of clarity in the figure, I only plot the posterior median of the UIP-implied present value. For the moment, I limit the analysis to the dates considered.

It is worth stressing that a cumulative excess return which is persistently different from zero does not constitute *per se* evidence of conditional UIP deviations, as most of the empirical literature on the delayed overshooting puzzle seem to infer (e.g., Eichenbaum and Evans, 1995; Faust and Rogers, 2003; Scholl and Uhlig, 2008). I interpret the results as evidence of conditional deviation from UIP only when the cumulative excess return is different from the present value of all future fundamentals, as described in the analysis of Section 2.2 and consistently with Brunnermeier et al. (2009). This approach can also help to understand whether exchange rates underreact or overreact to interest rate shocks.

As in Brunnermeier et al. (2009), Figures 2.6 to 2.10 seem to provide evidence that exchange rates initially underreact to the interest rate differential shock for all the currencies considered: when the foreign interest rate increases relative to the domestic interest rate, the investment currency appreciates sluggishly, with cumulative excess

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\(^8\)It should be noted however that both Faust and Rogers (2003) and Scholl and Uhlig (2008) use a different VAR specification: they use a constant-parameters VAR and they analyse UIP deviations conditional on *monetary policy* shocks rather than *interest rate differential* shocks as this chapter and Brunnermeier et al. (2009) do.

\(^9\)Under normality, the 16-th and 84-th percentiles correspond to the bounds of a one-standard-deviation confidence interval.

\(^10\)This present value is equal to the difference of the long-run response of the interest rate differential and the long-run response of the inflation rate differential to a shock to the interest rate differential. For each quarter \(t\) and each iteration of the Gibbs sampler, we can use the draws of the VAR parameters to calculate the long-run responses using standard formulas, see e.g. Lütkepohl (2005), p. 56.
returns reaching the level implied by UIP and PPP only after a few quarters. At longer horizons, I find evidence of overreaction: the investment currency continues to appreciate over the level implied by UIP, consistent with the “bubble” view (see e.g. Abreu and Brunnermeier, 2003), pushing exchange rates away from fundamentals. However, when considering parameter uncertainty, the overreactions of GBP, EUR, and CAD seem not to be statistically significant in most of the dates considered, given the wide posterior distribution of their accumulated impulse responses; instead, the long-run overreactions of CHF and JPY seem to be present at all the dates considered.

**Evolution over Time**

To investigate further whether these results—short-run underreaction and long-run overreaction—are sample-specific, and to try to identify whether there are any trends or significant patterns in the evolution of the responses of exchange rates to fundamental shocks, consider Figures 2.11 to 2.15. In these figures, in order to focus on the evolution over time of the exchange rate responses, I only consider posterior medians and will turn back to evaluate parameter uncertainty and statistical significance in the next subsection.

Each figure displays, for each point in time,\(^{11}\) the conditional UIP deviations defined as the difference between the UIP-implied response and the actual accumulated response of the excess return, conditional on a 1% positive shock to the interest rate differentials. The four lines represent the posterior medians of the simultaneous conditional UIP deviations, and the posterior medians of the conditional UIP deviations after 4, 12, and 20 quarters. A positive conditional UIP deviation indicates that the exchange rate undershoots the level implied by fundamentals, while a negative conditional deviation indicates overshooting.

The responses for all five currencies (with the exception maybe of the JPY) exhibit great variation over time, but they also show that exchange rates tend to consistently underreact to interest-rate shocks at short horizons. Figure 2.11 shows that the long-

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\(^{11}\)I sample the results every four quarters because of computer memory limitations.
horizon overreaction for the GBP displays a cyclical pattern but also evidence of a somewhat secular declining trend in the size of the conditional UIP deviations. The case of the EUR (Figure 2.12) is particularly interesting because it shows that the size of the conditional UIP deviations decreased rapidly between the beginning of the sample and the ERM crisis, but increased thereafter. The long-horizon deviations for the CAD (Figure 2.13) display wide swings over time, whereas the CHF and the JPY (Figures 2.14 and 2.15, respectively) show relatively stable long-horizon overreaction over the whole sample.

**Parameter Uncertainty**

To assess the statistical significance of the results described above, I take parameter uncertainty explicitly into account in Figures 2.16 to 2.20. The four charts of each of these figures represent, as a function of time, the posterior medians of the simultaneous conditional UIP deviations, and the posterior medians of the the conditional UIP deviations after 4, 12 and 20 quarters, together with the 16-th and 84-th percentiles.

In all five currencies, the simultaneous conditional UIP deviations tend to be significantly positive across the whole sample considered, meaning that the short-term underreaction of exchange rates to interest rate differential shocks is a pervasive feature across currencies and time. The only case in which initial underreaction does not seem to be statistically significant is for the CAD at the beginning of the sample in the early 1980s, but then it becomes significant and mostly increases during the whole period.

Focusing on longer horizons, I find evidence of greater parameter uncertainty. For the GBP (Figure 2.16), the long-run deviations are mostly not significantly different from zero. The long-run responses for the EUR (Figure 2.17) present an interesting pattern: between the 1980s and the early 1990s, the responses show evidence of statistically significant long-run overreaction (i.e. significantly negative deviations), but after the ERM crisis in 1992 the actual long-run responses are hardly distinguishable from those implied by UIP. Similarly, Figure 2.18 displays how the CAD shows also significant long-run overreaction only in the first part of the sample before the nineties, but this
disappears more gradually than in the case of the EUR. The results for the CHF and JPY in Figures 2.19 and 2.20, respectively, are also interesting: they stand in stark contrast with the other currencies: significant overreactions at long horizons are statistically and economically significant during the whole period.

Overall, these results seem to indicate that the short-run underreaction of exchange rates to interest-rate is a robust finding even when allowing for nonlinearities in the dynamics of the exchange rates. However, this conditional deviation does not seem to persist for long, as cumulative excess returns at longer horizons are often not distinguishable from those implied by UIP and long-run PPP, at least for the case of GBP, EUR, and CAD. Instead, in the case of JPY and CHF—two currencies which have been traditionally used for funding carry trade positions—I find evidence of persistent conditional violations of UIP at long horizons. Particularly, these violations take the form of overreaction, in that exchange rates tend to overshoot the level implied by fundamentals.

2.6 Conclusion

In this chapter, I analyse the reaction of exchange rates to interest rate shocks. Previous literature has shown that exchange rates tend to react sluggishly to shocks, so that an unexpectedly higher foreign interest rate generates a slow and persistent appreciation of the foreign currency for several quarters. This finding implies a violation of a conditional version of UIP and long-run PPP: the excess return on foreign exchange should react instantaneously to an unexpected interest rate shock and jump to the level implied by the present value of changes in expectations of future real interest rate differentials.

I argue that previous empirical findings, by mostly ignoring the role of nonlinearities and the evolution of exchange rates dynamics over time, may have represented only an average of the past, and not reflected the current state of the economy. Therefore, previous results may be biased and may not provide useful insights for a currency investor betting on violations of UIP in a given point in time.

In order to re-examine the evidence of conditional violations of UIP, I employ a Baye-
sian time-varying-parameter VAR which allows for time-variation both in the transmis-
sion mechanism of the shocks and the volatility of the shocks themselves. This frame-
work allows me to estimate the time-varying responses of the excess returns on foreign
exchange and to compare them to those implied by UIP and long-run PPP, at the same
time explicitly taking into account parameter uncertainty.

I find that the transmission of the interest rate differential shocks has changed over
time. However, even if to a varying degree over the years, some of the puzzling results
previously documented with linear models remain. I show that currency excess returns
tend to initially underreact to interest rate differential shocks for the whole sample
considered, undershooting the level implied by fundamentals. At longer horizons, the
previously documented evidence of overshooting tends to disappear in recent years in
the case of the euro, the British pound and the Canadian dollar. Instead, overreaction
at long horizons is a persistent feature of the excess returns on the Japanese yen and
the Swiss franc throughout the whole sample.

These results suggest that previously documented conditional violations of UIP may
have secularly declined over time, at least for euro, the British pound and the Canadian
dollar. However, the results for the Japanese yen and the Swiss franc—two currencies
which have been traditionally used for funding carry trade positions—may hint that
speculation in the foreign exchange market may constitute a destabilizing force, driving
exchange rates away from fundamentals.
Appendix 2.A  Priors

I follow Primiceri (2005) in setting the prior distributions for the parameters of the model outlined in Section 2.3.1, the only difference being the length of the sample size used to calibrate the prior distributions. I use the first six years of data (i.e. $T_0=24$ observations) in order to calibrate the prior distributions of the initial states of the time varying coefficients, the simultaneous relations and log standard errors—$\beta_0$, $\alpha_0$, and $\sigma_0$—which are assumed to be normal and independent both from one another and from the prior distributions of the hyperparameters.

For example, I set the mean of $\beta_0$ equal to the OLS estimates of the coefficients of a time-invariant VAR estimated using the 6-year subsample, $\hat{\beta}_{OLS}$, and the variance of $\beta_0$ equal to four times the variance of $\hat{\beta}_{OLS}$. The prior for $A_0$ can be calibrated in a similar way, whereas the prior for log $\sigma_0$ has mean equal to the logarithm of the OLS estimates of the standard errors of the innovations of the time-invariant VAR. The covariance matrix of log $\sigma_0$ is arbitrarily set to be equal to four times the identity matrix. The degrees of freedom and scale matrices for the inverse-Wishart prior distributions of the hyperparameters are set to minimize the impact of the prior and maximize the influence of sample information, with the only exception (as suggested by Primiceri (2005)) of a slightly tighter prior on $Q$. The degrees of freedom of the latter are set equal to the size of the initial subsample. Note that this choice is anyway less informative to the one of Primiceri (2005) who uses a longer subsample of ten years. What follows summarizes the choice of the priors:

$$B_0 \sim N(\hat{B}_{OLS}, 4V(\hat{B}_{OLS})), \quad (2.17)$$

$$A_0 \sim N(\hat{A}_{OLS}, 4V(\hat{A}_{OLS})), \quad (2.18)$$

$$\log \sigma_0 \sim N(\log \hat{\sigma}_{OLS}, 4I_3), \quad (2.19)$$

$$Q \sim IW(k_Q^2 \times T_0 \times V(\hat{B}_{OLS}, T_0)), \quad (2.20)$$

$$W \sim IW(k_W^2 \times 4 \times I_3, 4), \quad (2.21)$$

$$S_1 \sim IW(k_S^2 \times 2 \times I_3, 9), \quad (2.22)$$
\[ S_1 \sim IW(k_S^2 \times 3 \times I_3, 9), \quad (2.23) \]

where \( k_Q = 0.01, \ k_S = 0.1, \ k_W = 0.01 \), which are the standard values chosen in the literature (see e.g. Primiceri, 2005, Cogley and Sargent, 2005, and Benati, 2008) so that the priors are not flat, but diffuse and uninformative.

**Appendix 2.B \ Posterior s**

In order to estimate the model, I use the MCMC algorithm developed by Primiceri (2005). This algorithm is a Gibbs sampling procedure which sequentially draws from the full conditional distributions of the parameters of interest in order to generate a sample from the joint distribution of \( (\beta^T, A^T, \Sigma^T, V) \), where the superscript \( T \) denotes the whole history of the variable of interest. This means that the Gibbs sampling draws sequentially the time varying coefficients of \( \beta^T \), the simultaneous relations \( A^T \), the volatilities \( \Sigma^T \), and the hyperparameters \( V \), conditional on the observed data and the rest of the parameters.

Model (2.5) can be rewritten as

\[
y_t = Z_t \beta_t + \epsilon_t \quad (2.24)
\]

where \( Z_t = I_M \otimes X_t \), \( X_t = (1, y_{t-1}, \ldots, y'_{t-p}) \) is a \( 1 \times K \) vector of variables, \( K = 1 + M_p \), \( k = KM \). The vector \( \epsilon_t \) contains heteroskedastic errors with covariance matrix \( \Omega_t \) such that \( A_t \Omega_t A'_t = \Sigma_t \Sigma' \) as specified in section 2.3.1.

The draws from the full conditional posteriors are described in the following steps:

- **Draw \( \beta^T \) from** \( p(\beta^T \mid y^T, A^T, \Sigma^T, V) \):
  
  Conditional on \( A^T, \Sigma^T \) and \( V \), (2.24) is the measurement equation of linear and Gaussian state-space model, with state equation \( \beta_t = \beta_{t-1} + \nu_t \) as in (2.11). Therefore, we can draw \( \beta^T \) by using standard Gibbs sampling algorithms for state-space models. In my application I use the Carter and Kohn (1994) algorithm.

- **Draw \( A^T \) from** \( p(A^T \mid y^T, \beta^T, \Sigma^T, V) \):
The system (2.24) can be rewritten as

\[ A_t(y_t - Z_t \beta_t) = A_t \hat{y}_t = \Sigma_t u_t, \]  

(2.25)

where, given \( \beta_t \), \( \hat{y}_t \) is observable. The previous equation can be rewritten as

\[ \hat{y}_t = C_t \alpha_t + \Sigma_t u_t. \]  

(2.26)

A general definition of \( C_t \) is given by Primiceri (2005). In the three-variables case \( C_t \) is defined as

\[
C_t = \begin{bmatrix}
0 & 0 & 0 \\
-\hat{y}_{1t} & 0 & 0 \\
0 & -\hat{y}_{1t} & -\hat{y}_{2t}
\end{bmatrix}.
\]  

(2.27)

Conditional on \( y^T, \beta^T, \Sigma^T, V \), and together with (2.12), model (2.26) is in form of a Normal linear state space model, and therefore we can draw \( A_t \) by using the Carter and Kohn (1994) algorithm.

- **Draw** \( \Sigma^T \) from \( p(\Sigma^T | y^T, A^T, \beta^T, V, s^T) \):

We can rewrite (2.24) as

\[ y^*_t = A_t(y_t - Z_t \beta_t) = A_t \epsilon_t, \]

so that \( \text{var}(y_t^*) = \Sigma_t \Sigma_t' \). Let \( y_{j,t}^{**} = \ln [(y_{j,t}^*)^2 + c] \) where \( y_{j,t}^* \) denotes the \( j \)-th element of \( y_t^* \) and \( c = 0.001 \) is an offsetting constant. Denoting \( y_t^{**} \) the vector stacking all \( y_{j,t}^{**} \) for \( j = 1, \ldots, M \), we can write the state space form of the model using the measurement equation

\[ y_t^{**} = 2 \ln(\sigma_t) + \epsilon_t, \]  

(2.28)

and state equation (2.13). Since the innovations \( \epsilon_t \) are independent of one another by construction and are distributed as \( \ln \chi^2(1) \), we can use the results of Kim et al.
(1998) to draw the volatility states equation by equation. Kim et al. (1998) show that a \( \ln \chi^2(1) \) distribution is well approximated by a mixture of seven normal distributions with component probabilities \( q_j \), means \( m_j - 1.2704 \), and variances \( v_j^2 \), for \( j = 1, \ldots, 7 \), where the values of these constants are given in their Table 4. Conditional on the component indicator variables \( s^T \), which select at each point in time which member of the mixture of normals to be used, the model has an approximately linear Gaussian state space form so that we can use the Carter and Kohn (1994) to draw the volatility states.

- **Draw** \( s^T \) from \( p(s^T | y^T, A^T, \Sigma^T, V) \):

  As in Kim et al. (1998), the component indicator variables \( s^T \) which used for the mixture distribution can be independently sampled from the discrete density defined by

  \[
  \Pr(s_{i,t} | y_{i,t}^{**}, \ln \sigma_{i,t}) \propto q_j f_N(y_{i,t}^{**} | 2 \ln \sigma_{i,t} + m_j - 1.2704, v_j^2),
  \]

  \[
  (2.29)
  \]

  for \( j = 1, \ldots, 7 \), \( i = 1, \ldots, M \), and \( t = 1, \ldots, T \), and where \( q_j, m_j \), and \( v_j^2 \) are given in Table 4 of Kim et al. (1998).

- **Draw hyperparameters:**

  The diagonal blocks of \( V \), i.e. \( Q, W, \) and \( S \), all have an inverse-Wishart posterior distribution when conditioning on \( \beta^T, \Sigma^T, A^T, \) and \( y^T \). It is easy to draw from these distributions, as they are independent of one another and the errors are observable conditional on \( \beta^T, \Sigma^T, A^T, \) and \( y^T \). For details on how to draw from the inverse-Wishart posteriors, see e.g. Koop and Korobilis (2010).

  The MCMC algorithm therefore simulates the posterior distribution of the states and the hyperparameters by iterating the steps outlined above. I use 200,000 iterations from which I discard a burn-in period of 80,000 iterations in order to assure convergence to the ergodic distribution.
Table 2.1. Descriptive Statistics and Preliminary Tests

The table presents descriptive statistics, unit-root tests, and stability tests for a number of currencies using quarterly annualised data. The data span is from the first quarter of 1975 to the third quarter of 2009, except for the Japanese yen which starts in the third quarter of 1978. \( i_t^* - i_t \), \( \pi_t^* - \pi_t \), and \( z_t \) denote the interest rate differential, the inflation rate differential, and the excess return on foreign exchange, where the domestic country is the US. PP-stat denotes the Phillips-Perron test statistics of the null hypothesis of a unit root. qLL is the efficient test statistic of Elliott and Müller (2006) for each of the equations of the VAR. The null hypothesis of a stable regression model, \( y_t = X_t' \gamma + \epsilon_t \), is tested against the unstable model \( y_t = X_t' \gamma_t + \epsilon_t \). The asterisks *, **, *** denote significance at the 10%, 5%, and 1% level, respectively.

<table>
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<tr>
<th>Currency</th>
<th>( i_t^* - i_t )</th>
<th>( \pi_t^* - \pi_t )</th>
<th>( z_t )</th>
<th>( i_t^* - i_t )</th>
<th>( \pi_t^* - \pi_t )</th>
<th>( z_t )</th>
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<td><strong>British pound</strong></td>
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</tr>
<tr>
<td>Mean</td>
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<td>0.007</td>
<td>0.000</td>
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<td>0.008</td>
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</tr>
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<td>0.160</td>
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<td>3.938</td>
<td>3.243</td>
<td>4.430</td>
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<td></td>
</tr>
<tr>
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<td>0.016</td>
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<td>0.012</td>
<td>0.014</td>
<td>0.132</td>
</tr>
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<td>-0.187</td>
<td>0.781</td>
<td>0.409</td>
</tr>
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<td>2.875</td>
<td>3.152</td>
<td>4.914</td>
<td>2.999</td>
</tr>
<tr>
<td>qLL</td>
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<td>-40.328***</td>
<td>-270.741***</td>
<td>-379.266***</td>
<td>-76.287***</td>
</tr>
<tr>
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<td></td>
</tr>
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<tr>
<td>qLL</td>
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<td>-203.480***</td>
<td>-23.887*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Japanese yen</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
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<td>0.002</td>
<td>-0.033</td>
<td>-0.025</td>
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<tr>
<td>Stdev</td>
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<td>0.017</td>
<td>0.120</td>
<td>0.012</td>
<td>0.014</td>
<td>0.132</td>
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<td>-164.361***</td>
<td>-40.328***</td>
<td>-270.741***</td>
<td>-379.266***</td>
<td>-76.287***</td>
</tr>
</tbody>
</table>

88
Figure 2.1. Standard deviations for GBP

Each chart displays the standard deviation of the innovation in the corresponding VAR equation. The standard deviation is computed as square root of the corresponding diagonal element of $\Omega_t$. 
Figure 2.2. Standard deviations for EUR

Each chart displays the standard deviation of the innovation in the corresponding VAR equation. The standard deviation is computed as square root of the corresponding diagonal element of $\Omega_t$. 
Each chart displays the standard deviation of the innovation in the corresponding VAR equation. The standard deviation is computed as square root of the corresponding diagonal element of $\Omega_t$. 

Figure 2.3. Standard deviations for CAD
Figure 2.4. Standard deviations for CHF

Each chart displays the standard deviation of the innovation in the corresponding VAR equation. The standard deviation is computed as square root of the corresponding diagonal element of $\Omega_t$. 
Figure 2.5. Standard deviations for JPY

Each chart displays the standard deviation of the innovation in the corresponding VAR equation. The standard deviation is computed as square root of the corresponding diagonal element of $\Omega_t$. 
Figure 2.6. GBP Impulse responses

The charts display the estimated accumulated time-varying impulse responses of the currency excess return to a 1% shock to the interest rate differential \( (i^* - i) \). The bold line indicates the posterior median, whereas the dashed lines indicate the 16-th and 84-th percentiles. The dashed horizontal line in each graph is the estimated cumulative excess return implied by UIP (Equation 2.4), reflecting the present value of all the future real interest rate differentials as predicted by the VAR.
Figure 2.7. EUR Impulse Responses

The charts display the estimated accumulated time-varying impulse responses of the currency excess return to a 1% shock to the interest rate differential \((i^* - i)\). The bold line indicates the posterior median, whereas the dashed lines indicate the 16-th and 84-th percentiles. The dashed horizontal line in each graph is the estimated cumulative excess return implied by UIP (Equation 2.4), reflecting the present value of all the future real interest rate differentials as predicted by the VAR.
Figure 2.8. CAD Impulse Responses

The charts display the estimated accumulated time-varying impulse responses of the currency excess return to a 1% shock to the interest rate differential \((i^* - i)\). The bold line indicates the posterior median, whereas the dashed lines indicate the 16-th and 84-th percentiles. The dashed horizontal line in each graph is the estimated cumulative excess return implied by UIP (Equation 2.4), reflecting the present value of all the future real interest rate differentials as predicted by the VAR.
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Figure 2.10. JPY Impulse Responses

The charts display the estimated accumulated time-varying impulse responses of the currency excess return to a 1% shock to the interest rate differential ($i^* - i$). The bold line indicates the posterior median, whereas the dashed lines indicate the 16-th and 84-th percentiles. The dashed horizontal line in each graph is the estimated cumulative excess return implied by UIP (Equation 2.4), reflecting the present value of all the future real interest rate differentials as predicted by the VAR.
Figure 2.11. Conditional UIP deviations, GBP

The four lines represent the posterior medians of the simultaneous conditional UIP deviations, and the posterior medians of the conditional UIP deviations after 4, 12 and 20 quarters. A positive conditional UIP deviation indicates that the exchange rate undershoots the level implied by fundamentals, while a negative conditional deviation indicates overshooting.
2. Figures

Figure 2.12. Conditional UIP deviations, EUR

The four lines represent the posterior medians of the simultaneous conditional UIP deviations, and the posterior medians of the conditional UIP deviations after 4, 12 and 20 quarters. A positive conditional UIP deviation indicates that the exchange rate undershoots the level implied by fundamentals, while a negative conditional deviation indicates overshooting.
Figure 2.13. Conditional UIP deviations, CAD

The four lines represent the posterior medians of the simultaneous conditional UIP deviations, and the posterior medians of the conditional UIP deviations after 4, 12 and 20 quarters. A positive conditional UIP deviation indicates that the exchange rate undershoots the level implied by fundamentals, while a negative conditional deviation indicates overshooting.
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Figure 2.16. Uncertainty and Conditional UIP deviations, GBP

The four charts in the figure represent, as a function of time, the simultaneous conditional UIP deviations and the conditional UIP deviations after 4, 12 and 20 quarters. Bold lines represent posterior medians whereas dashed lines represent the 16-th and 84-th percentiles.
Figure 2.17. Uncertainty and Conditional UIP deviations, EUR

The four charts in the figure represent, as a function of time, the simultaneous conditional UIP deviations and the conditional UIP deviations after 4, 12 and 20 quarters. Bold lines represent posterior medians whereas dashed lines represent the 16-th and 84-th percentiles.
Figure 2.18. Uncertainty and Conditional UIP deviations, CAD

The four charts in the figure represent, as a function of time, the simultaneous conditional UIP deviations and the conditional UIP deviations after 4, 12 and 20 quarters. Bold lines represent posterior medians whereas dashed lines represent the 16-th and 84-th percentiles.
Figure 2.19. Uncertainty and Conditional UIP deviations, CHF

The four charts in the figure represent, as a function of time, the simultaneous conditional UIP deviations and the conditional UIP deviations after 4, 12 and 20 quarters. Bold lines represent posterior medians whereas dashed lines represent the 16-th and 84-th percentiles.
Figure 2.20. Uncertainty and Conditional UIP deviations, JPY

The four charts in the figure represent, as a function of time, the simultaneous conditional UIP deviations and the conditional UIP deviations after 4, 12 and 20 quarters. Bold lines represent posterior medians whereas dashed lines represent the 16-th and 84-th percentiles.
Chapter 3

Currency fair value models

3.1 Introduction

What is the fair value of a currency? Policymakers and international investors have always been asking this question, and economists have been trying to find answers, proposing many different methodologies to estimate fair value. Simple measures have ranged from relative national price indices (Cassel, 1918) to relative hamburger prices (The Economist, 6 September 1986). More elaborate models take into consideration economic factors, ranging from productivity differentials (Harrod, 1933; Balassa, 1964; Samuelson, 1964) to men-women sex ratios (Du and Wei, 2011). In this chapter we provide a comprehensive review of models that are used by policymakers and international investors in order to assess exchange rate misalignments from their fair value.

Policymakers need to assess the possible misalignment of currencies for a number of reasons. Exchange rates play a crucial role in a country's external adjustment process, particularly as economies become more and more integrated. At the time of writing, advanced economies have faced some degree of exchange rate realignment since the onset of the recent global financial crisis, whereas this realignment has been limited for emerging market economies, creating tensions and constituting a threat to the global recovery (IMF, 2011, Chapter 1). More generally, substantial misalignments can have severe consequences, as exchange rates may abruptly adjust when the misalignment becomes unsustainable, leading to currency crises generally associated with large output
contractions, especially in emerging markets (Dornbusch et al., 1995; Gupta et al., 2007; Gourinchas and Obstfeld, 2011). In a theory paper, Engel (2011) shows that currency misalignments are inefficient, lower world welfare, and should be targeted by monetary policymakers in a model in which firms price to market and prices are sticky.

In Transition Economies, especially for countries of Central and Eastern Europe, the apparent trend appreciation of the real exchange rates of some of these countries raised the question of whether this appreciation reflected an adjustment to fair value or not (Égert et al., 2006). De Broeck and Sløk (2006) show how real exchange rates were generally misaligned at the onset of the transition and how most of the misalignment was eliminated over a relatively short period. In developing countries an overvalued currency can represent a major obstacle for a successful development strategy (Johnson et al., 2007).

For currency unions it is critically important to get a sense of fair value to assess the subsequent adjustment needs via relative inflation rates. And for heavily managed or pegged exchange rates, a fair value estimate may help establish policy targets. However, because exchange rates are a policy tool for the authorities of a country, and because there is the potential to use the currencies value to gain an advantage over another country, the political debate of fair value has always been contentious. The most recent example are the attempts to determine fair value for the Chinese currency (e.g., Cline and Williamson, 2008, 2011).

Investors and other agents engaging in international transactions, including trade, are interested in estimating the fair value of the currency as an input in hedging and investment strategies. For example, fair value models are useful to assess crash risks in popular currency speculation strategies. Exchange rates may be pushed away from fundamentals by carry trades, occasionally reverting back abruptly and leading to sudden losses (Brunnermeier et al., 2009; Plantin and Shin, 2011).

A number of investment strategies try to exploit long-run reversion to fair value by taking a long position in undervalued currencies and a short position in overvalued currencies. They typically provide lower risk-adjusted returns than carry strategies,
but they seem to be less prone to crash risk (Jordà and Taylor, 2009; Nozaki, 2010). Major financial institutions recently introduced fully investable and tradable indices that track the performance of such strategies, such as Goldman Sachs FX Valuation Current (Goldman Sachs, 2009) and Deutsche Bank Valuation Index (Deutsche Bank, 2007).

Strategic Foreign Direct Investments (FDI) decisions with very long investment horizons may be affected by currency values. Variable real exchange rates may influence the location of production facilities chosen by multinationals (see e.g. Goldberg and Kolstad, 1995) and a fair value estimate may be useful as a long-term forecast.

Given the diverse use of currency fair value models highlighted above, it is important to understand which models are more suitable for a given context. In this chapter we analyse this issue in detail by surveying and critically assessing a number of fair value models proposed in the literature.\(^1\) We intentionally avoid an extensive discussion of purchasing power parity (PPP), as this literature is covered in detail in many surveys: see for example Sarno and Taylor (2003, Chapter 3) and Taylor and Taylor (2004).

We start by providing a short history of fair value models in the literature in the next section. In Section 3.3, we discuss the basic characteristics of fair value models with a particular focus on how implicit or explicit design choices typically affect the results, the robustness and the general usability of these models.

Section 3.4 provides an exposition of a number of fair value and equilibrium exchange rate models that are widely used in practice. In particular, we focus on the two main families of fair value models, namely the behavioural equilibrium exchange rate (BEER) and the underlying balance (UB) models. As case studies we then discuss in more detail the IMF framework, as well as Goldman Sachs Dynamic Equilibrium Exchange Rate (GSDEER) model. In both cases we highlight how the estimates of fair value are affected by some typical implementation choices. We also illustrate a fair value concept extensively used by financial markets practitioners but not previously formalised in the academic literature. This model, which we dub Indirect Fair Value (IFV), relies on indi-

\(^1\)As highlighted below, we focus on the practical implementation of these models. For their theoretical foundations, see e.g. Chinn (2011).
rect estimation of fair value of the currency by “removing” the speculative components that drive exchange rates in the short run.

We argue that there is no explicit answer regarding which model delivers the correct fair value of a currency, because each model has its own individual strengths and weaknesses. We illustrate this point by means of examples, focusing on the practical implementation of the models. For instance, we discuss the sensitivity of UB models with regard to variations in import and export elasticities, and show how the different specifications of productivity can affect the results in “adjusted-PPP” models. Moreover, we discuss how the treatment of external balance in different models appears responsible for discrepancies between estimation results. Researchers are therefore left with a wide range of estimates, and many use a set of models or a combination of these in order to assess exchange rate misalignments.

3.2 Currency Fair Value in the Historical Context

Modelling the fair value of a currency is a relatively young field of economics with the bulk of the research published in the last 30 years. However, a “primitive” notion of a fair currency value has probably been around for as long as currencies exist. Vaughan (1675) makes the following observations in the context of governments reducing the precious metal content of their currency:

There are two causes of the raising of Money: […] But the […] most frequent cause hath been, an Art which States have used to rob one another of their Money, by setting on higher prices upon it; so that some States being induced, by an unjust device, to draw to themselves the Money of their Neighbours, and others by a necessity to keep their own.

Vaughan continues with an example going back to ancient times:

But first to shew the Antiquity of the practice of raising of Monies we will begin with the Romans. The As, which was originally coyned of a pound
weight, was, during the first Punick War for help of publick necessities, brought to 2 ounces. [...] 

This historic reference is particularly interesting because it links a notion of fairness to a currency value qualifying the exchange rate as “an unjust device”. In fact, Vaughan describes currencies, which are overvalued because their precious metal content has been reduced relative to a previous norm, but unknowingly to the users of the currency. In the same book, he also discusses the “intrinsical value” of money and describes a way to “equalize the exchange”, which consists of comparing the precious metal content of two coins.

A closely related “primitive” example of fair value model is Purchasing Power Parity (PPP), dating back to the Salamanca School in 16th-century Spain (for a history of PPP, see Officer, 2011). According to PPP, the fair value of an exchange rate between two countries is determined by the two countries’ relative price levels (Cassel, 1918). In the same context, early scholars of exchange rates also noted that the increase in money supply tends to lead to an increase in the general price level.

All these “primitive” examples of fair value go back to the stylised fact that there seem to be shared long-run trends between monetary fundamentals and the nominal exchange rate (Frankel and Rose, 1995). In that respect, the basic monetary model of the exchange rate can be seen as a modern formalised version of the “primitive” notions, which have been around for centuries. Accumulated changes in money supply could give valuable hints regarding the fair value of a currency.

Another important concept of fair value was linked to the balance of payment dynamics and in particular to situations which led to a persistent drain of finite foreign exchange reserves. Again this analytical approach has to be seen in the historical context. Governments and rulers had to respond to dwindling gold reserves and tinkering with the exchange rates had always been one possible option. The related fair value concept of foreign exchange reserve stability, or some broadly defined external balance, became visible in the open economy extensions of the IS-LM framework (see for example Mundell, 1963, or Fleming, 1962). Another example is the underlying balance (UB) approach,
3.2. Currency Fair Value in the Historical Context

which focuses on the requirements for achieving internal and external balance simultaneously (for an overview see for example Isard, 2007).

Interestingly, the collapse of Bretton Woods followed signs of misalignment on the basis of most of these early fair value concepts. Starting in the 1950s it became increasingly clear that the supply of US dollar grew substantially faster than the amount of gold that backed the same currency. Triffin (1960) highlighted the linkages between monetary policy and unsustainable external imbalances in the context of a reserve currency.

With the advent of flexible exchange rates in the post-Bretton Woods area, the initial basic assumption was that balance of payment imbalances could no longer persist on a permanent basis. The exchange rate was now able to freely adjust to fair value without the explicit input from policymakers. Determining fair value was therefore less of a policy priority. Indeed, the dollar initially weakened from overvalued Bretton Woods levels and the US external balance improved. However, it became quickly clear that nominal and in particular real exchange rates displayed far more volatility than under the previous fixed regime (Mussa, 1986). Research focus shifted to exchange rate determination, often using variations of the monetary model. However, the difficulty of beating the Meese and Rogoff (1983) random walk yardstick not only undermined the usefulness of these models, it also implicitly suggested that the concept of fair value may be meaningless under floating exchange rates. A further blow to the concept of fair value came from the lack of empirical support for the PPP hypothesis in the first years of floating exchange rates (Frenkel, 1981, Taylor and Taylor, 2004).

The interest in being able to assess the fair value of currency started to re-emerge from the late 1980s onwards. Three developments supported this trend. First, time series covering the period of floating exchange became longer, which helped empirical models to pick up long-run regularities (Mark, 1995). Second, advances in econometrics, in particular the development of unit-root tests (Dickey and Fuller, 1979), cointegration analysis (Engle and Granger, 1987) and panel data gradually led to the realisation that the earlier rejection of the PPP hypothesis may have been pre-emptive. Third, the US dollar appreciated rapidly in response to very tight monetary policy under the
Fed chairmanship of Paul Volcker, which in turn led to significant and rapid widening of the US current account deficit. Policymakers quickly realised that exchange rates misalignment once again started to have a severe impact on the economic situation. As a result, the focus on the fair value of currencies increased in the run-up to the Plaza Accord (Funabashi, 1988).

These three developments essentially prepared the ground for a completely new generation of fair value models, which were no longer focused on the precious metal content of money. The last 20–30 years have seen a proliferation of models and tools—some grounded on economic theory, others of largely statistical nature. With every specification implicit choices are being made, which affect the usefulness for the potential user of the fair value model. And not all potential issues can be overcome. It is therefore increasingly becoming common practice to use a combination of fair value models, knowing that each of them may have potential strengths and shortcomings in the context of individual user’s needs (IEO, 2007).

Among the large number of different approaches, there are two dominating families of currency fair values, which are widely used today. The first group are so-called Behavioural Equilibrium Exchange Rate (BEER) models, which typically attempt to directly estimate the reduced-form long-run relationships between a set of macroeconomic variables and the exchange rate. These models typically require long datasets, use cointegration analysis and rely on the assumed stability of the underlying equilibrium relations.

The second family of models retains as core feature the idea that external imbalances are unsustainable in the long run and have to be seen as variations of the old Underlying Balance (UB) models adopted for flexible exchange rates. The underlying approach of these UB models can be summarised in a hypothetical question from a policymaker: “By how much do I have to move the currency to bring my current account back to a sustainable level assuming full employment?” Variants of UB models are widely used by economists advising policymakers and were initially developed by the International Monetary Fund (IMF) and by the Washington DC-based Peterson Institute of Inter-
3.3 Characteristics of Fair Value Models

In this section, we will try to highlight the most important generic characteristics of different fair value models. Given the applied nature of this branch of economic research it is useful to be explicit on how these chosen characteristics potentially affect the model results. In later sections, it will also be easier to make reference to these characteristics when discussing specific fair value models.

3.3.1 Horizon/Frequency

A major dimension of fair value models is the time horizon of the analysis. A number of authors have implicitly or explicitly assumed that the observed exchange rate will converge to different fair values depending on the horizon. The speed of convergence is also one of the most important features for the applied use of fair value models.

Driver and Westaway (2004), in a framework similar to that of Clark and MacDonald (1998), start from the following reduced-form relation which relates the observed exchange rate to a number of explanatory variables:

\[ e_t = \beta'Z_t + \theta'T_t + \epsilon_t, \]  

(3.1)

where \( e_t \) is the exchange rate, broadly defined,\(^2\) \( Z_t \) is a vector of medium and long-term economic fundamentals, \( T_t \) is a vector of short-term, transitory factors, \( \epsilon_t \) is a residual term, and \( \beta \) and \( \theta \) are coefficient vectors.

Driver and Westaway define short-run equilibrium as the exchange rate which would pertain when its fundamentals are at their current (e.g. observed) values at time \( t \),

\(^2\)For illustrative purposes, we do not specify here if the exchange rate is defined in nominal or real terms, bilateral or effective.
abstracting from the influence of asset market bubbles. In a notation similar to Clark and MacDonald, they define the short-term equilibrium as

\[ e_t = \beta'Z_t + \theta'T_t. \] (3.2)

This is what Williamson (1983) and Clark and MacDonald (1998) call the *current equilibrium exchange rate*. The half-life of misalignments in applied models containing observed values of cyclical variables would likely be measured in weeks or months.

Industry practitioners often attempt to use direct measures of speculative positioning to decompose the exchange rate into an “observed” speculative component and an unobserved fair value. Fair value models of this kind tend to display half-lives that can typically be measured in days or weeks, and therefore can be considered as an attempt to identify the fair value of a currency for the very short term. We will discuss in more detail these models in Section 3.4.5.

The medium-run fair value can be described by the following reduced-form equation,

\[ \hat{e}_t = \beta'\hat{Z}_t, \] (3.3)

where the hat indicates that the variables abstract from cyclical components. In most empirical applications, this often translates in (i) excluding fundamentals which are typically thought to characterize only cyclical deviations from the equilibrium level and in (ii) the fundamentals being set at their trend values. Though consistent with a flow equilibrium, the medium-run fair value may still allow adjusting towards a long-run, stock equilibrium. For example the net foreign asset position may still be changing.

Using the Driver and Westaway (2004) notation again, a long-term equilibrium is

3Driver and Westaway (2004) define the medium-run equilibrium as the exchange rate which is compatible with the economy being at internal and external balance. For the concept of internal and external balance, see Sections 3.4.2 and 3.5.1.

4For example by excluding interest rate differentials from the analysis, as exchange rates tend to inherit the cyclical properties of interest rate differentials (see e.g. Lustig et al. 2010).
3.3. Characteristics of Fair Value Models

defined as the point where a stock equilibrium is achieved for all agents in the economy:

$$\bar{e}_t = \beta' \bar{Z}_t,$$

where the overbar denotes the long-run values of variables. Long-run fair values models are therefore mostly used when studying structural sources of misalignment. For example, a reserve currency will tend to be overvalued in the long run as its economy will tend to run current account deficits, and therefore a depreciation will be needed for restoring the long-run equilibrium. Other examples of structural misalignment are commodity-exporting countries with growing sovereign wealth funds, or central banks following a policy of deliberate undervaluation and systematic intervention.

Fair value estimates will therefore reflect the choice in the selection of fundamentals. Particularly, models which try to directly estimate reduced form equations such as (3.2) to (3.4) will be particularly affected: as they use the fitted value as an estimate of fair value, they tend to inherit the statistical properties of the explanatory variables used. This fact will in turn affect the estimated size and duration of exchange rate misalignments.

3.3.2 Direct econometric estimation versus “methods of calculation”

One may attempt to compute currency’s fair value by directly estimating a reduced form equation relating the level of the exchange rate to a set of fundamental variables. This reduced form equation takes generally the form of a long-run cointegrating relationship, whose short-run dynamics may be estimated using an Error Correction Model (ECM). The BEER models discussed in Section 3.4 belong to this family.

These models are useful in order to characterise the time variation properties of exchange rates, and therefore may provide some forecasting power. However, as these models rely on the assumption of stable long-run relations, their estimation may be plagued by the limited availability and poor quality of historical data. Especially in the
case of emerging markets and less developed economies, the presence of small samples, systematically managed or pegged exchange rates, and data from unreliable sources may severely bias the estimated coefficients.

Another approach, e.g. the one followed by the fundamental equilibrium exchange rate (FEER) model (see Section 3.4.2) and by the IMF’s External Sustainability approach (see Section 3.5.3), is to start from a simple macroeconomic relationship between the exchange rate and an economic policy objective (such as target capital flows or current accounts). Then, calculate the implied exchange rate change (from prevailing levels) that would be required in order to reach that objective. These models can be so considered as a “method of calculation” (Wren-Lewis, 1992), since they require no (or minimal) econometric estimation.

Among the advantages of this latter family of models are the fact that their simple structure makes them less reliant on data availability issues, and that they take policy objectives explicitly into consideration. In general, they also provide a useful reference point that can be compared to exchange rate assessments obtained using more complicated econometric models. On the other hand, they often rely on relatively strong assumptions, making them less robust in empirical applications in which these assumptions are likely to be violated (see e.g. Section 3.5.4).

### 3.3.3 Treatment of External Imbalances

External imbalances are an implicit or explicit part of most fair value models and often are the very core of the model. However, defining an external imbalance is in practice more difficult than one may expect and there are a number of issues that have to be considered when using external imbalances in fair value models. We discuss here some of these issues:

- Defining which part of an external imbalance is of cyclical nature and which part is of structural nature is not trivial. Policy choices can drive structural imbalances: for example, the choice of pegged or managed exchange rate regimes, the existence of sovereign wealth funds or persistent fiscal policy differentials. Structural
imbalances can also be driven by the choice of reserve currency, or depend on the natural endowment of commodities. These factors have implications for assessing both the observed and the equilibrium levels of the current account, and therefore have to be recognized explicitly when modelling currency fair values. Sub-optimal modeling choices with regards to these issues could for example lead to foreign exchange policy recommendations that would condemn a country to a state of permanent Dutch disease as a result of commodity driven current account surpluses. As we discuss in Sections 3.4 and 3.5, these choices are explicitly reflected in many fair value models, while others simply assume that external imbalances will correct themselves over time.

- Fair value models based on the idea of sustainable current account positions should respect the “N-1” global consistency requirement (Faruqee, 1998), given that the sum of global current account balances should add up to zero.

- Many models use trade elasticities to calculate the necessary exchange rate changes needed to reach an external balance target. However, as we show in Section 3.5.4, this approach is highly sensitive to errors in the estimation of trade elasticities. Even very small changes in estimated trade elasticities can create substantial changes in fair value estimates. Moreover, the very idea of using exchange rates to address external imbalances is based on the assumption that the Marshall-Lerner condition holds, an assumption that should ideally be tested.

- When using measures of external imbalances in time-series-based fair values, the estimated coefficients on the external variables often show the “wrong” (i.e., counterintuitive) signs. For example, the estimates may show that the fair value appreciates with growing current account deficits. Obviously, the idea of external sustainability would suggest the opposite, namely that growing current account deficits lead to a depreciated fair value estimate. A weaker currency in a fair value sense would then help reduce the external imbalances. There could be several reasons for this empirical problem. For example the Marshall-Lerner condition may
not hold as already discussed above. More likely, however, is a contamination of the estimates by cyclical effects. Most models of exchange rate determination would suggest that a small open economy facing a positive domestic demand shock will likely experience an appreciating currency at the same time as a deterioration of the current account balance. Fair value models with wrong signs may therefore simply pick up these cyclical forces. Explicitly correcting for these cyclical factors may be one solution. Alternatively, dropping the external variable is a very simple solution, which is based on the assumption that over the long run external imbalances will mean-revert to equilibrium levels.

Therefore, in terms of modelling choices, the treatment of external imbalances tends to have a notable impact on fair value estimates and careful consideration of the explicit or implicit choices is important. As will be discussed more in subsequent sections, estimates of fair value based on variants of the Underlying Balance (UB) model tend to be far more sensitive to the extent of current account imbalances than those from most BEER or adjusted-PPP models.

### 3.3.4 Real versus Nominal Exchange Rates

In most cases, it will be reasonably easy to map the results of a nominal fair value model into real exchange rates and vice versa. But data limitations may play a role. There is a wide variety of inflation data available but not necessarily comparable across countries. Moreover, inflation data may be available at weekly, monthly or only quarterly intervals and hence affect the underlying horizon/frequency of the model. The vast majority of theory-driven models rely on real exchange rates.

### 3.3.5 Bilateral versus Effective Exchange Rate

The choice between bilateral and effective (i.e. multilateral) exchange rates is crucial when assessing exchange rate misalignments empirically. Effective exchange rates have the advantage that they allow making a fair value assessment for a single currency, whereas a bilateral fair value calculation always depends on influences by two countries.
Bilateral fair value signals are often misinterpreted when the anchor currency itself is misaligned on a broad basis—an issue frequently encountered in the last few years when the US dollar has been undervalued according to many fair value models. In that case assessing multilateral misalignment may be more useful but at the same time it may suffer more from constraints on data availability, as this approach requires collecting data from many countries.

Effective exchange rates also depend on the weighting scheme used to calculate the basket. Many different trade-weighted baskets are used in practice, including some alternative weighting schemes based on capital flows or even volatility. Weighting schemes with static weights suffer from lack of representativeness after a period of changing trade patterns in the global economy (see Chinn, 2006).

Bilateral fair value calculations are less demanding on data availability and less influenced by the choice of weighting scheme. In particular for descriptive modelling approaches, bilateral exchange rates have the conceptual advantage of being actually observable in the markets.

In general, it is possible to transform bilateral exchange rate fair value estimates into effective exchange rate estimates, simply by applying the trade weights and calculating the geographic average. The reverse is also possible. Alberola et al. (1999) suggest a procedure to extract bilateral misalignments from a vector of effective misalignments. See also the approach used by Cline (2008).

### 3.3.6 Time Series versus Cross-Section or Panel

The choice of the estimation procedure is largely an econometric issue, which we will not discuss here, but its seems worthwhile highlighting an important trade-off when choosing between a single-equation estimate and a panel with homogeneous coefficients for all cross-sectional units.

With many countries having different economic structures, it is possible that there

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5 Clearly, bilateral real exchange rates will still depend on the weighting scheme of the chosen price index.
are different relations between macro variables and the respective currencies. Imposing
the same coefficients across currencies in a panel may therefore not fully reflect the
country-specific characteristics. One possible solution is to break a panel into more
homogeneous sub-panels and ultimately to estimate fair values for individual currencies.

On the other hand, a panel allows assessing the potential influence on fair value of a
factor that has not been relevant for a currency in the past. For example, the discovery
of a previously unknown natural resource in a country may affect fair value, but it will
be impossible to quantify the impact in a single-equation approach because of the lack
of historical data. On the contrary, a panel estimate allows quantifying the impact if
other currencies in the panel have been affected by a similar discovery in the past.

One possible constraint for using a panel however is data availability. A comparable
dataset for each cross section unit is necessary and may seriously restrict the number
of possible explanatory variables and the size of the dataset. A frequently used panel
cointegration technique is the panel dynamic ordinary least squares (DOLS), see e.g.

3.3.7 Model Maintenance

As highlighted above, one interesting feature of fair value models is that they are often
an important input for policy decisions or investments. This creates special demands
on the timeliness of estimates, data availability, and data quality.

General model maintenance is an important aspect in applied work when more than
a point estimate is needed. The frequency of updates and re-estimation needs to be
determined. Validation procedures may need to be implemented to assess the impact of
data revisions on the fair value estimates. Fitted fair values may change when statistical
agencies change the frequency, base year or calculation of input data. The quality of
estimates may deteriorate as the sample increases over time. Additional cross sections
may have to be included in a panel as more data becomes available. Cross-sectional
units may vanish as countries enter currency unions. All these events will likely have a
more or less severe impact on misalignment and hence need to be assessed carefully and
quickly, before policy or investment decisions start to be influenced by issues related to model maintenance.

It is important not to underestimate this factor as complex models covering a large number of countries may rely on thousands of data series from diverse sources. A simple model that relies on a few standard input variables may prove more robust over time than a state-of-the-art model with excessively complex and time consuming updating procedures, which may then result in frequent maintenance errors.

This trade-off between complexity and robustness on the implementation side is an important factor for any institution planning to use a fair value model on a regular basis.

3.4 Models/Taxonomy

3.4.1 “Adjusted PPP”: Harrod-Balassa-Samuelson and Penn Effects

Many studies have tried to measure exchange rate misalignment by exploiting the positive relation between real per capita income and relative prices across countries, that is, the fact that rich countries tend to have higher price levels than poor countries. This empirical relation is also known as the “Penn effect” after the Penn World Table of Summers and Heston (1991), and has been explained usually by appealing to productivity differentials between the tradeable and non-tradeable sectors—the Harrod-Balassa-Samuelson effect (see e.g. Rogoff, 1996). This effect provides a structural interpretation of long-run deviations from PPP (hence the “adjusted PPP” terminology used in many studies) based on real factors that can be exploited for measuring exchange rate misalignments. The exchange rate misalignment can therefore be measured as the residual of a cross sectional regression such as

\[ q_t = a + b \ln(GDP_t) + \nu_t, \]  

(3.5)
where $q$ is the log real exchange rate, $a$ and $b$ are coefficients, $\nu$ is the residual term, and GDP is measured in per capita terms, usually relative to the United States and converted using PPP-based exchange rates. Frankel (2006) uses this kind of regression in order to evaluate the renminbi misalignment at two different time points, the years 1990 and 2000. De Broeck and Sløk (2006) adopt a similar methodology for measuring the misalignment of the real exchange rates for a number of transition economies at the onset of the transition period. Cheung et al. (2007, 2009) extend the cross-sectional approach of Frankel (2006) by using panel data techniques for a number of specifications, e.g. allowing for country fixed effects, random effects, and adding different control variables. Rodrik (2008) builds an index of currency undervaluation by taking the residuals of an estimated panel regression of real exchange rates on real per capita income allowing for time fixed effects.

In Section 3.6 we show in a simple example that estimated fair values based on the Penn effect can be quite sensitive to the exact specification of productivity in the tradable goods sector.

### 3.4.2 The Underlying Balance approach

The underlying balance (UB) approach asserts that the fair value of a currency is the level of the exchange rate which is consistent with a country’s internal and external balance. Most applications of the underlying balance approach identify internal balance as a country’s economic activity being at full potential output (i.e. zero output gap). External balance can be broadly defined as country’s current account position being at equilibrium or sustainable levels, but different interpretations of what “equilibrium” or “sustainable” exactly mean have given rise to different implementations of this approach in the literature. The underlying balance approach has it roots in the pioneering work of Nurkse (1945), Metzler (1951), Meade (1951), and Swan (1963), in their attempts to characterize the equilibrium in an open economy. More than most other fair value models, the UB approach explicitly considers external imbalances to derive fair value estimates. We discussed the implications of this important choice in section 3.3.3.
The underlying balance approach starts from the balance of payments identity which expresses the current account \( CA \) in terms of either a change in net foreign assets \( \Delta NFA \) or of the excess domestic savings \( S \) over domestic investment \( I \), i.e.

\[
CA = \Delta NFA = S - I.
\]  

The equilibrium exchange rate is then found as the level of the exchange rate that closes the gap between the current account set to prevail in the medium run when countries are at internal balance—the underlying current account (UCUR)—and some equilibrium or sustainable value of \( S - I \) or \( \Delta NFA \)—the saving-investment/current account “norm” or “target” net capital flows.

Figure 3.1 provides a summary of the approach. The UCUR line is downward sloping because an appreciation of the real effective exchange rate (corresponding here to an increase in its level) tends to be associated with a declining current account, for a given level of output. The current account norm, instead, is generally assumed to be independent of the level of the exchange rate, so that the line is vertical. The equilibrium level of the real effective exchange rate is therefore associated with the intersection of the two lines, \( REER^* \).

The estimation of the underlying current account and the current account norm are distinguishing features of the empirical applications of the underlying balance approach. We will consider down here the two most prominent cases, i.e. the “fundamental equilibrium exchange rate” in Section 3.4.2 and the IMF Macroeconomic Balance approach in Section 3.5.1.

**The Fundamental Equilibrium Exchange Rate (FEER)**

The term “fundamental equilibrium exchange rate” (FEER) is often used as a synonym for the exchange rate which is consistent with internal and external balance, and therefore falls into the broad category of models which follow the underlying balance approach discussed above.

However, here we will use the term FEER to indicate more narrowly the model first
developed by Williamson (1983, 1994), given its major influence in the development of exchange rate fair value models. Williamson (1983) defines the FEER as the real effective exchange rate “generating for every country a current-account surplus or deficit equal to the underlying capital flow over the cycle, given that the country is pursing internal balance as best it can and not restricting trade for balance of payments reasons” (Chapter 2, p. 14).

The main distinguishing feature of FEER with respect to other underlying balance models (especially, the CGER Macroeconomic Balance model discussed in Section 3.5.1) is the definition of external balance in terms of “underlying” capital flows. The focus here is on computing the real effective exchange rate which equates the underlying current account to an equilibrium level of capital flows. This level is derived not through estimation procedures but mostly by judgmental considerations, and is often assumed to be a constant proportion of GDP (Williamson and Mahar, 1998). This is why Wren-Lewis (1992) defines the FEER model as a “method of calculation” (as opposed to an econometric estimation) of the equilibrium exchange rate.

Highlighting the normative content of the FEER approach, Bayoumi et al. (1994) name the approach as “desired equilibrium exchange rate” (DEER). The DEER is therefore the exchange rate consistent with internal balance and a target current account explicitly set (or “desired”) by policymakers. In their application, Bayoumi et al. (1994) calculate the DEER for a number of countries in 1970 in order to analyse the break-up of the Bretton-Woods system. They use a one-percent target current account surplus, as this was the approximate stated objective of the US government during the discussions that led to the Smithsonian Agreement in 1971 (in general, a one-percent target current account surplus was also widely supported by the IMF for industrial countries in the 1960s, see also Polak, 1995, p. 749).

In a more recent example of judgmental current account targets, Cline and Williamson (2008, 2011) discuss the appropriate set of current account targets for a number of developed and emerging economies. They start from the “presumption” that external imbalances should not exceed 3 percent of GDP in the medium run for any country,
and then refine the current account target based on judgmental considerations for each country.

By contrast, the CGER MB model discussed below focuses on the behaviour of the current account balance as implied by its medium-run determinants, and relies on less ad hoc assumptions (see the Section 3.5 on the CGER models).

As such, the FEER model is generally considered to be a method of assessment of currency misalignment, rather than a model of exchange rate determination. However, it implicitly assumes that the actual exchange rate will exhibit a tendency to revert to its fair value, at least in the medium run.

### 3.4.3 The Behavioural Equilibrium Exchange Rate Family of Models

Clark and MacDonald (1998) propose the estimation of a reduced-form equation in order to explain the behaviour of the real effective exchange rate both in the short and medium run—what they call the behavioural equilibrium exchange rate (BEER). As is the case for the FEER model above, the acronym BEER is often used to indicate, by extension, a whole family of models which follow similar methodologies.

#### The Standard BEER Model

The BEER model can be used to estimate what Clark and MacDonald (1998) call the current misalignment and the total misalignment of a currency. The current misalignment is defined as the difference between actual values of the exchange rate and the estimated level of the fair value given the current values of the fundamentals. The total misalignment is instead the difference between actual values of the exchange rate and the estimated level of the fair value given a measure of the sustainable or long-run fundamentals.

The theoretical basis of the BEER model is the risk-adjusted uncovered interest parity condition in real terms. By definition, the log excess return on foreign exchange
3.4. Models/Taxonomy

is equal to the interest rate differential plus the appreciation rate of the foreign currency

\[ z_{t+k} = i_t^* - i_t + (s_{t+k} - s_t), \tag{3.7} \]

where \( s_t \) is the natural logarithm of the exchange rate (defined as the domestic price of foreign currency, so that an increase in \( s_t \) denotes appreciation of the foreign currency), and \( i_t \) and \( i_t^* \) are the continuously compounded \( k \)-period domestic and foreign riskless interest rates, respectively. In terms of the log real exchange rate, defined as \( q_t = s_t + p_t^* - p_t \), where \( p_t^* \) and \( p_t \) are the logs of the foreign and domestic price levels, Equation (3.7) can be rewritten as

\[ z_{t+k} = r_{t+k} - r_{t+k} + q_{t+k} - q_t, \tag{3.8} \]

where \( r_{t+k} \) and \( r_{t+k}^* \) denote domestic and foreign real interest rates, respectively. In general, the expected excess return, \( E_t z_{t+k} \), will be equal to a time-varying risk premium, \( \rho_t \), so that

\[ q_t = E_t(q_{t+k}) + E_t(r_{t+k}^* - r_{t+k}) - \rho_t. \tag{3.9} \]

That is, the equilibrium real exchange rate reflects expectations of future real exchange rates, expectations of future real interest rate differentials, and a time-varying risk premium.

To make their model empirically tractable, Clark and MacDonald make the further assumption that the unobservable expectations of the exchange rate are a function of long-run economic fundamentals, i.e. \( E_t(q_{t+k}) = \beta'Z_t \), where \( Z_t \) denotes the vector of fundamentals. They identify the latter as the terms of trade, the relative price of nontraded to traded goods (proxying for Harrod-Balassa-Samuelson effects), and net foreign assets. Moreover, they proxy the time-varying risk premium \( \rho_t \) with the relative supply of domestic and foreign debt, arguing that an increase in the relative supply of outstanding domestic debt relative to foreign debt will increase the domestic risk premium, thereby requiring a depreciation of the current equilibrium exchange rate (see e.g. Giorgianni, 1997).
Empirically, the BEER is generally estimated using the fitted values of a cointegration relationship between the real effective exchange rate and a set of fundamentals such as those estimated above. For example, Clark and MacDonald (1998) use the Johansen (1988) method which allows for the existence of multiple cointegrating vectors. Extensions of the BEER approach are among the most popular fair value models among policy institutions and in the financial industry. For example, see the IMF equilibrium real exchange rate (ERER) approach and Goldman Sachs’s GSDEER model discussed later in this chapter.

A related approach is the so-called capital enhanced equilibrium exchange rate (CHEER), introduced by Johansen and Juselius (1992) and MacDonald and Marsh (1997), and later extended by MacDonald and Marsh (2004). The starting point is the view that nominal exchange rates may be misaligned from their PPP-implied level because of non-zero interest rates differentials (what MacDonald and Marsh (1997) call the “Casselian view” of PPP). A cointegration relation is therefore estimated between nominal exchange rates, domestic and foreign price levels, and domestic and foreign interest rates. In this approach, the estimated speed of convergence tends to be faster than the typical PPP adjustment based on univariate models, and the inferred nominal exchange rate forecasts have some degree of short-term predictive ability when compared to the random walk benchmark.

The Permanent Equilibrium Exchange Rate (PEER)

Even though the BEER model explicitly recognises the distinction between current and total misalignment (see above), most of the actual implementations of BEER models generally focus only on the former. As the current values of fundamentals may depart substantially from sustainable or long-run levels, a number of researchers have been investigating the fair value of the real exchange rate consistent with its long-run fundamentals.

Huizinga (1987) and Cumby and Huizinga (1990) use respectively univariate and multivariate Beveridge-Nelson decompositions in order to decompose the real exchange
rate into the sum of permanent and transitory components. The permanent component is then considered to be the permanent equilibrium exchange rate.

More recently, the permanent equilibrium exchange rate (PEER) model of Clark and MacDonald (2004) is a direct extension of the BEER models outlined above. Clark and MacDonald use the method developed by Gonzalo and Granger (1995) in order to decompose the fundamentals in permanent and transitory components, where the former are used to identify the long-run value of the fundamentals necessary to estimate the total misalignment defined above. The fundamentals are the same as in the BEER approach, but the terms of trade and the government debts ratio are dropped in the empirical analysis.

3.4.4 The natural real exchange rate (NATREX)

Stein (1994) defines the natural real exchange (NATREX) as “the exchange rate that would prevail if speculative and cyclical factors could be removed while unemployment is at its natural rate” (Stein, 1994, p. 135). As for the FEER model, the NATREX model is based on the underlying balance approach.

The NATREX model explicitly recognizes different dynamics for medium and long-run equilibrium exchange rates. Speculative and cyclical factors influence the exchange rate at the short horizon, whereas at the medium term is dominated by the stock of capital, the stock of foreign debt, and a number of fundamentals. The long-term equilibrium exchange rate is determined solely by the fundamentals, as the stock of capital and foreign debt are assumed to set at their long-run, steady state values. A number of different fundamentals have been proposed in empirical work, the most important of which are identified as domestic and foreign productivity, and domestic and private propensity to save (the so-called “social thrift”), both at home and abroad.

In the empirical implementations, the NATREX is generally estimated similarly to the BEER, that is, by identifying a cointegrating relation between the real exchange rate and a number of fundamentals. Some of these fundamentals are not directly observable and thus must be proxied. For example, Stein (1994) uses real GNP growth rates in
order to proxy the growth of capital stock at home and abroad. Moreover, the rate of change of foreign debt is proxied using the current account to GNP ratio, and propensity to save is proxied by private and public consumption as a ratio of GNP. The ratios are used in order to abstract from cyclical factors, the same reason for which most of the variables are taken as twelve-quarter moving averages. For a detailed exposition of the NATREX approach, see also Stein (2006).

3.4.5 The Indirect Fair Value

In this section, we discuss an indirect approach to modelling fair value, which is frequently used in financial markets. Relatively few macroeconomic assumptions are being made with regard to the drivers of the fair value. Instead, the approach depends on the assumption that speculative activity is the principal cause for misaligned exchange rates. To our knowledge, this Indirect Fair Value (IFV) approach has not been previously formalised in the academic literature.

Market participants use this kind of model as a way to assess where the exchange rate would be had speculative activity not pushed it away from a loosely defined fair value concept. The idea that speculative activity can create these deviations is based on two assumptions. First, speculative order flow has an impact on exchange rates (Lyons, 2001). Second, one has to assume that speculative order flow is mean reverting over the medium term. The latter is a corollary of the definition of speculative activity, which is based on the assumption that speculators will at some stage reverse their position and realize either a profit or a loss.

Two measures of speculative positioning often employed in this approach are risk reversals or International Money Market (IMM) positioning (Mogford and Pain, 2006). The former is defined as the difference between the implied volatility between comparable out-of-the-money call and put options. When the majority of speculative investors expect appreciation, demand for call options will likely rise relative to the demand for puts. As a result the relative price, and implied volatility, will increase for the call options relative to the put options. The second measure is based on the weekly Com-
mitments of Traders (COT) Report, which contains information about the positioning size of so-called non-commercial traders on the IMM futures exchange, part of the Chicago Mercantile Exchange (CME). The report is restricted to data for the most liquid exchange rates against the US dollar.

These measures of speculative positioning tend to be stationary and highly correlated with spot exchange rates (Campa et al., 1998; Mogford and Pain, 2006). Moreover, indicators of speculative positioning also tend to be strongly autocorrelated (Dunis and Lequeux, 2001), which implies that periods of speculatively driven misalignments tend to persist for a certain time, but typically not more than a few months.

Most measures of speculative positioning have a clearly defined neutral point. For example risk reversals are equal to zero when the implied volatilities of equivalent out-of-the-money call and put options are identical. A similar argument applies to the net positions of non-commercial traders on the IMM. In practice, however, indicators of speculative positioning tend to oscillate around a non-zero mean. Speculative investors may on average perceive that the appreciation of a currency is more likely than the depreciation, or vice versa. Moreover, these indicators can display structural breaks, or trend stationarity.

In practice, these factors tend to affect the choice of sample size. On one hand the sample has to be large enough to guarantee stationarity and the mean reverting properties of the indicators of speculative positioning. On the other hand, longer samples create the risk of having to deal with trends or structural breaks in the positioning variable. Practitioners tend to look at daily or weekly data with sample sizes between 6 months and 3 years, which emphasizes the more trading-oriented concept of fair value underlying this approach. This approach also highlights that the statistical properties of the input variables play a far bigger role than in most other concepts of fair value.

More formally, we can express the relation between the level of the exchange rate

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6See www.cftc.gov/marketreports/commitmentsoftraders/index.htm for details on the COT reports.
and the variables of speculative positioning in an equation similar to (3.1):

\[ e_t = \beta' Z_t + \theta' S_t + \epsilon_t, \quad (3.10) \]

where \( e_t \) is the spot exchange rate observed in the market, \( Z_t \) is a vector of broadly defined fundamentals, \( S_t \) contains variables reflecting speculative activity, \( \epsilon_t \) is a residual term, and \( \beta \) and \( \theta \) are coefficient vectors. Given that the focus is on relatively short-term deviations from fair value, and daily or weekly data, some of the fundamental variables in \( Z_t \) can potentially be approximated by linear and higher-order time trends.

The exchange rate \( e_t \) is typically expressed in nominal terms and expected to display some form of long-run relationship with \( Z_t \). Equation (3.10) is estimated using cointegration techniques, hence \( e_t \) and \( Z_t \) are expected to display unit roots, whereas \( S_t \) is expected to be stationary around a constant mean, as mentioned above. When these criteria are not satisfied, the model fails to produce a fair value estimate.

Having estimated Equation (3.10) it is possible to use the parameter estimates to calculate fair value in the following way:

\[ \bar{e}_t = \hat{\beta}' \bar{Z}_t + \hat{\theta}' \bar{S}, \quad (3.11) \]

with the overbar denoting the value of \( S \) that is consistent with neutral speculative positioning. As we discussed above, neutral speculative positioning is not well defined in the presence of a non-zero mean in the \( S \), though in most cases the natural choice would be to simply use the sample mean.

With most of the focus on modeling the transitory forces, users of these fair value models are typically agnostic with regards to the choice of fundamental variables in \( Z_t \). Moreover, as practitioners tend to focus on very short-term deviations from fair value, there is a strong preference for financial and macro data available at daily or weekly frequency.

We illustrate the concept using the exchange rate of the Canadian dollar against the US dollar from January 2004 to January 2007. The daily spot exchange rate is
regressed on the difference between US and Canadian 2-year swap rates, as well as linear, quadratic, and cubic time trends. We use 3-month 25-delta risk reversals as a measure of speculative positioning in the regression. We conducted several unit-root tests to confirm that the exchange rate and interest rate differential likely display a unit root over the sample, while risk reversals are likely stationary. Indeed, Figure 3.2 shows that risk reversals behave as a stationary time series with a sample mean which is very close but not equal to zero.

Figure 3.3 illustrates the results. The black line represents the observed daily exchange rate between the US and Canadian dollars. The light grey line displays the fitted value of the regression using the observed values of all right hand variables. Finally, the dark grey line represents the IFV, which describes where the exchange rate would have been without the influence of speculative activity. This fair value is computed using Equation (3.11), i.e. as the fitted exchange rate but using the sample mean of the risk reversals instead of the observed values.

As Figure 3.3 shows, misalignment from the IFV of up to ten percent occurred in the sample considered. Moreover, it seems that much of the short-term swings in the exchange rate seem to be related to speculative positioning. However, as discussed earlier in this section, the IFV concept does not shed light on the macroeconomic factors driving medium and longer-term trends. Therefore, we have to rely on other fair value concepts in order to understand the determinants of events such as the trend appreciation of the Canadian dollar’s fair value estimate visible in the graph.

3.5 IMF CGER - Consultative Group on Exchange Rate Issues

One of the most prominent responsibilities of the International Monetary Fund (IMF) is that of exchange rate surveillance. Under Article IV of IMF’s Articles of Agreement (“Obligations Regarding Exchange Arrangements”), member countries must adopt policies that can ensure stability, both at country and systemic levels. Therefore, the
surveillance role of the IMF has taken the form of both bilateral (single-country) and multilateral (global, cross-country) surveillance. Particularly following the collapse of the Bretton-Woods system of fixed exchange rates in the late 1970s, the IMF has put much emphasis on the surveillance of its member countries’ exchange rate policies. In this framework, the IMF constantly monitors the external sustainability\(^7\) of its member countries by providing exchange rate assessments to understand to what extent currency misalignments can constitute a matter of concern.

The IMF Consultative Group on Exchange Rate Issues (CGER), established in 1995 as an interdepartmental working group, has provided systematic exchange rate and current account assessments for a number of countries since then, fostering discipline and consistency in the IMF staff’s judgments about currency misalignments.

CGER methodologies for exchange rate assessments have evolved since they were made first publicly available (see Isard and Faruqee, 1998, Isard et al., 2001, and Lee et al., 2008), most notably in order to include more and more countries in the analysis over the years. Particularly in 2006, the CGER has revised and extended its main methodologies for exchange rate assessments covering not only advanced countries but also emerging market countries, as illustrated in Lee et al. (2008).

At present, the CGER uses three complementary methodologies: the Macroeconomic Balance (MB) approach, the Equilibrium Exchange Rate (ERER) approach, and the External Sustainability (ES) approach. Even though the methodologies for exchange rate assessment are publicly available (see the references above), the assessments themselves are not. The assessments are indeed classified by the IMF as “Strictly Confidential”, due to potential market (and, perhaps, political) sensitivity of IMF views on exchange rate misalignments.

The three methodologies will, in general, give three different point estimates of currency misalignment. If the average of these three estimates is within the zero to five percent range, or if all the three estimates are less than ten percent in absolute value, then the currency is assessed to be broadly at its equilibrium value (i.e. consistent with

\(^7\)“External stability refers to a balance of payments position that does not, and is not likely to, give rise to disruptive exchange rate movements”, (IMF, 2007).
fundamentals). If instead the three estimated misalignments are more substantial, the assessment is based either on the midpoint of the range of the estimates (when the three estimates don’t differ more than ten per cent from one another), or on the range of the estimates (when the three estimates differ more than ten percent from one another). For more detail on how to combine information from these estimates, see also Abiad et al. (2009).

3.5.1 The Macroeconomic Balance (MB) Approach

The Macroeconomic Balance (MB) approach is based on the notion of underlying balance, in which external and internal balances (see Section 3.4.2) are achieved over the medium run. The medium run is defined by the CGER as the “horizon over which domestic and partner-country output gaps are closed and the lagged effects of past exchange rate changes are fully realized” (Lee et al., 2008).

The MB approach is probably one of the oldest methods to estimate a currency’s fair value by the IMF. Early attempts date back to at least 1967, when an IMF team led by Marcus Fleming computed the required magnitude of the devaluation of the sterling to bring the UK balance of payments in equilibrium (Polak, 1995). This approach has been subsequently developed over the years until its current form (see e.g. Artus, 1978, for a formal exposition of an early version of the model, far before the establishment of the CGER).

In this approach, the fair value of a currency is identified by calculating the exchange rate adjustment that would eliminate the difference between the current account balance projected over the medium term and the “current account norm” (also called “saving-investment norm” in earlier versions), i.e. an estimated equilibrium current account balance over the same horizon.

The MB approach, in its most recent version (Lee et al., 2008), is implemented in three steps. First, the CGER estimates the equilibrium relationship between the current account, expressed as a ratio to GDP, and a set of fundamentals. These fundamentals include the fiscal balance, demographics, net foreign assets, oil balance, economic
growth, and dummies for economic crises and financial centers (for extensive studies of the medium-term determinants of current accounts, see also Debelle and Faruqee, 1996, and Chinn and Prasad, 2003). The medium-term equilibrium relationship is estimated via panel data regressions, using four-year averages of the data.

The second step is to compute current account norms by applying the estimated coefficients of the panel regression to the medium-term values of the regressors. These medium-term values are mostly taken from the IMF’s World Economic Outlook (WEO) database, and are generated by IMF’s country experts.

The third step involves computing the real exchange rate adjustment that would close the gap between the current account norm (estimated in the first two steps) and the underlying current account (UCUR) balance. In the CGER setting, the IMF WEO medium-term projection of the level of the current account is taken as the estimate of the underlying current account balance. This projection assumes that economies operate at full potential output and the effect of lagged exchange rates has vanished. Projections for different countries are based on different models, reflecting country-specific views of IMF experts. Therefore, a drawback of this approach is that it may lack of global consistency.

An alternative to the WEO projections would be to employ a standard trade model in order to estimate the UCUR balance, as in previous versions of the MB approach (Isard and Faruqee, 1998; Isard et al., 2001). Typically, such (multiple-equation) trade model would specify imports as a function of domestic GDP and past real exchange rates, and exports as a function of a trade-weighted average of foreign GDP and past real exchange rates. Underlying exports and imports (and therefore the UCUR as measured as net underlying exports) would then be calculated by applying the estimated coefficients to the value of domestic and foreign activity at potential output levels, once the effect of exchange rates has fully realised (see also Isard, 2007). Even though CGER assessments rely now only on WEO projections in order to estimate the underlying current account balance, a similar trade model is still used in order to estimate the trade elasticities which are needed to calculate the exchange rate adjustment.
With the underlying current account balance and the current account norm at hand, the degree of misalignment can be calculated using import and export elasticities as we discuss in Section 3.5.4.

### 3.5.2 The Equilibrium Real Exchange Rate (ERER) Approach

The Equilibrium Real Exchange Rate (ERER) approach obtains the fair value of a country’s currency by directly estimating a reduced-form equation, which models the equilibrium exchange rate as a function of medium-term fundamentals.

The medium-term adjustment of the real exchange rate is then calculated simply as the difference between the current, projected value of the exchange rate and the corresponding estimated, equilibrium value.

In its most recent version (Ricci et al., 2008; Lee et al., 2008), the ERER approach is employed for estimating equilibrium CPI-based real effective exchange rates for 48 countries. The choice of fundamentals is partly driven by theories of real exchange rate determination and partly by data availability issues. The medium-term fundamentals most notably include measures for net foreign asset positions, relative productivity differentials between the tradable and non-tradeable sectors, and terms of trade. These measures have been widely employed in reduced-form estimation of equilibrium exchange rates, see e.g. Lane and Milesi-Ferretti (2004).

Net foreign assets are included because countries which are net debtors will need more depreciated real exchange rates in order to stimulate future trade surpluses to service their external liabilities. Productivity differentials are used to proxy for Harrod-Balassa-Samuelson effects: higher productivity in the tradable sector relative to the nontradables sector would imply an appreciating real exchange rate. The measure of terms of trade used in the ERER reflects only the prices of imported and exported commodities, and therefore is different from the usual terms of trade variable based on exports and imports of all goods and services. Higher commodity prices tend to imply an appreciating real exchange rate through income or wealth effects. Other fundamentals included in the analysis are government consumption, a trade restriction index, and a
proxy for price controls.

The ERER approach uses panel dynamic ordinary least squares (DOLS) in order to estimate the following long-run cointegrating relationship between the log of the real effective exchange rate, \( q \), and the set of fundamentals:

\[
q_{i,t} = \alpha_i + \beta'Z_{i,t} + \sum_{j=-p}^{j=p} \gamma_j \Delta Z_{i,t+j} + \epsilon_{i,t}, \tag{3.12}
\]

where \( Z \) is the vector of fundamentals, \( \Delta \) denotes the first-difference operator, \( \beta \) and \( \gamma_j \) are coefficients vectors, \( \alpha_i \) are country fixed effects, \( \epsilon_{i,t} \) denotes the residuals, and \( i \) and \( t \) denote the country and time, respectively. Given that real effective exchange rate are index numbers, their levels are not comparable across countries, so that country fixed effects are used.

The panel DOLS specification (3.12) is used because inference in a panel fixed effect cointegrating relationship would be flawed in the presence of correlation between the residuals and the stationary component of the unit-root processes of the regressors. Adding leads and lags of first differences of the regressors automatically removes this correlation, see Stock and Watson (1993) and Mark and Sul (2003).

The ERER approach then uses an error-correction-mechanism (ECM) specification in order to assess the speed of adjustment of the real exchange rate to its long-run equilibrium value:

\[
\Delta q_{i,t} = c_i + \delta(q_{i,t-1} - \alpha_i - \beta'Z_{i,t-1}) + \lambda q_{i,t-1} + \phi'\Delta Z_{i,t} + \psi'\Delta Z_{i,t-1} + \eta_{i,t}. \tag{3.13}
\]

Ricci et al. (2008), in their analysis of 48 industrial countries and emerging markets for the period 1980–2004, estimate an adjustment coefficient \( \delta \) which implies a half life for deviations from the equilibrium level of the exchange rate of around two and a half years.
3.5.3 The External Sustainability (ES) Approach

The CGER External Sustainability (ES) approach calculates the equilibrium real exchange rate that would bring the current account or trade balance from its projected medium-term level to the level that would stabilise the net foreign assets position of a country. Unlike the MB and ERER approaches, the ES approach requires only few inputs (such as the prevailing growth and rates of return on external assets and liabilities) to be implemented, without the need of any direct econometric estimations of equilibrium relations.

The concept of external sustainability is analogous to the one of public debt sustainability, but with the object of the analysis being the whole economy instead of the public sector alone. According to the IMF, external sustainability is reached when a country meets its intertemporal budget constraint (see e.g. IMF, 2002, 2008), which implies that the net present value (NPV) of future current account or trade surpluses balances must be equal or greater than the NPV of that country’s external liabilities. This condition is automatically met when the debt-to-GDP ratio is either stable or declining.

The intertemporal budget constraint is

\[ B_t - B_{t-1} = CA_t + KG_t + E_t, \]

where \( B \) denotes net foreign assets, \( CA \) is the current account, \( KG \) is the net capital gain on the existing holding of foreign assets and liabilities, and \( E \) represents capital account transfers and errors and omissions. Assuming that the latter two factors are negligible and therefore setting them equal to zero, and denoting ratios to nominal GDP with lower case letters, we can write Equation (3.14) as

\[ b_t - b_{t-1} = ca_t - \frac{g_t + \pi_t(1 + g_t)}{(1 + g_t)(1 + \pi_t)}b_{t-1}, \]

where \( g \) is the growth rate or real GDP and \( \pi \) is the rate of change of the GDP deflator. We can express current account \( ca^* \) which stabilises NFA positions to a predetermined...
level $b^*$ by setting $b_t - b_{t-1}$ to zero, so that from the previous equation

$$ca^S = \frac{g + \pi(1 + g)}{(1 + g)(1 + \pi)} b^S.$$ (3.16)

Analogously, we can apply the same approach to compute the NFA stabilising trade balance. Denoting the gross real interest rates as $(1 + r) = \frac{(i + \delta)}{(1 + \pi)}$, we can write the NFA-stabilizing trade balance (inclusive of services and transfers) as

$$tb^S = \frac{-r - g}{1 + g} b^S.$$ (3.17)

Therefore, given the assumed values for $g$, $\pi$, and $r$, the NFA-stabilizing current accounts and trade balances can be readily computed without the econometric estimation of any equilibrium relation. Clearly, a drawback of this approach is that the choice of benchmark level of NFA, $b^S$, will be to some extent arbitrary.

We can compare the NFA-stabilising values of $CA^S$ and $TB^S$ obtained using equations (3.16) and (3.17) to their actual values, and apply the same trade elasticities as for the MB approach in order to calculate the required change in the real effective exchange rate (see also Section 3.5.4).

Given their simplicity, equations (3.16) and (3.17) have straightforward implications. For example, the NFA-stabilizing current account is proportional to the GDP growth rate, so that a faster-growing economy can afford to run larger current account deficits. Moreover, if the rate of return on external assets and liabilities is greater than the GDP growth rate, an increase in the former implies, ceteris paribus, a larger trade surplus for a debtor country whereas a creditor country can afford larger trade deficits (with this relation being inversed if the rate of return is less than the growth rate).

3.5.4 The Importance of Trade Elasticities

The MB approach and the ES approach really only differ in the way they estimate—or calculate—the current account target. In a second step, both models use an identical procedure to calculate the needed exchange rate adjustment for the current account to
reach this target.

Countries with high ratios of exports and imports to GDP (i.e., countries which are more open to trade), will require smaller exchange rate adjustments in order to achieve current accounts consistent with macroeconomic balance. This effect can be seen from Figure 3.1: an increase in trade openness, other things being equal, will tend to flatten the UCUR line, resulting in a smaller movement in the real effective exchange rate to move from the current level $REER^t$ to the equilibrium level $REER^*$. Analytically, denoting the trade-balance-to-GDP ratio as $tb$, we have that

$$tb = \frac{P_X X}{GDP} - \frac{P_M M}{GDP}, \quad (3.18)$$

where $M$ and $X$ are import and export volumes, and $P_M$ and $P_X$ are prices of imports and exports in local currency.\(^8\) The total differential of $tb$ with respect to the real exchange rate $Q$ is

$$\frac{\partial tb}{\partial Q} = \frac{\partial X}{\partial Q} \frac{P_X X}{GDP} + \frac{\partial P_X}{\partial Q} \frac{X}{GDP} - \frac{\partial M}{\partial Q} \frac{P_M M}{GDP}, \quad (3.19)$$

Assuming that exports are priced in local currency (so that $\frac{\partial P_X}{\partial Q} = 0$) and that imports are priced in foreign currency (so that they are unit elastic with respect to $Q$, $\frac{\partial P_M}{\partial Q} = -\frac{P_M}{Q}$), we have that Equation (3.19) can be rewritten as

$$\frac{\partial tb}{(\partial Q)/Q} = \eta_X \frac{P_X X}{GDP} - (\eta_M - 1) \frac{P_M M}{GDP}, \quad (3.20)$$

where $\eta_X = \frac{\partial X}{\partial Q} X$ is the export elasticity and $\eta_M = \frac{\partial M}{\partial Q} M$ is the import elasticity, with $\eta_X < 0$ and $\eta_M > 0$. This implies that, for given export and import elasticities, the impact of a change in the exchange rate will be roughly proportional to trade openness.

This calculation tends to be sensitive with respect to the estimated trade elasticities—a small error in the estimation of the trade elasticities can potentially lead to calculated exchange rate adjustment which may differ significantly to the true value, or even have

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\(^8\)In what follows, we assume that the trade balance is the sole source of current account adjustments, consistently with the IMF approach.
3.6 Goldman Sachs GSDEER

a counterintuitive sign. This is a particularly important issue, as trade elasticities tend to be difficult to estimate. For example, Cheung et al. (2010) report a wide range of estimated exports and imports elasticities for China, based both on previous studies and their own calculations.

Furthermore, the empirical evidence regarding the Marshall-Lerner condition is mixed. Several studies find supporting evidence for selected countries over longer horizons, but considerable doubts remain in particular with regard to shorter term dynamics (see e.g. Rose, 1991; Hsing, 2010).

Table 3.1 illustrates the point and presents, for given trade elasticities, the percentage change in the real effective exchange rate which is required to bring China’s underlying current account in line with the current account target. The required change is computed using formula (3.20). We use the current account projections for 2016 from the IMF WEO of April 2011 as an estimate of the underlying current account balance. We estimate the 2010 China’s exports/GDP and imports/GDP ratios as 29.8 and 25.9 percent, respectively. Data for these ratios are from China’s national statistics and the IMF. For illustrative purposes, we do not estimate here the current account target (as opposed to the CGER approach) but assume two different levels. Panel A shows the results assuming a current account target of zero percent, whereas Panel B shows the results assuming a current account target of three percent. The table clearly shows that MB calculations are extremely sensitive with respect to the estimated trade elasticities, in particular when they approach the region where the Marshall-Lerner condition is no longer satisfied. In this simple example, the required exchange rate adjustments can differ for figures as high as 2300 percent for only a 0.1 change in the estimated elasticities.

3.6 Goldman Sachs GSDEER

Similar to the work of the IMF on exchange rates, Goldman Sachs has been using a family of fair value models, which have followed different concepts and which have changed over time. In this section, we first describe the models and then discuss
more detail the specific adjustments applied to the latest generation of the GSDEER model.

### 3.6.1 The Evolution of the GSDEER Model

The first version of GSDEER was introduced in the mid 1990s (Goldman Sachs, 1996) and was a simple adjusted-PPP model allowing for Harrod-Balassa-Samuelson effects. The coefficient on productivity was assumed to be unity. The model was not estimated but in fact calculated by applying inflation and productivity differentials to an initial reference period, at which the bilateral exchange rates were assumed to be in equilibrium. The latter was determined by judgment with strong focus on the size of current account imbalances. Calculated misalignment values where available for the currencies of advanced economies.

At the same time, Goldman Sachs developed a second fair value model for Emerging Market currencies, which was essentially a BEER model with a relatively large number of model inputs reflecting external sustainability—a choice driven by the fact that EM currency crises were frequent at the time. Specifically, the following variables were used: terms of trade, the degree of openness to foreign trade, the share of long-term capital inflows as a percentage of GDP, the amount and composition of government spending and the level of international interest rates (Goldman Sachs, 1996).

These two models were merged into a unified BEER-style approach estimated with panel DOLS cointegration techniques with country fixed effects (O’Neill et al., 2005). More specifically, the real bilateral exchange rate was estimated as a function of terms of trade differentials, productivity differentials and the relative net foreign asset position between two countries. The real exchange rate was calculated using CPI indices. The coefficient estimates were highly significant except for the net foreign asset position.

Given that the model is estimated in a panel for approximately 30 currencies, the choice of variables is partly driven by the availability of data. To allow out-of-sample

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9This model was called the Goldman Sachs Dynamic Equilibrium Emerging Markets Exchange Rate (GSDEEMER) model.
projections of fair value estimates, Goldman uses its in-house forecasts for the input variables.

This generation of GSDEER model has been re-estimated about every two years with some modifications at each iteration. Recently the net foreign asset variable has been dropped as its significance dropped further and given that the signs flipped and became counterintuitive, indicating that rising external liabilities were associated with an appreciating fair value (Fuentes and Meechan, 2007). The authors faced the challenges linked to external balance variables discussed in Section 3.3.3 and decided to drop the variable altogether. As a result, the GSDEER fair value estimates now do not depend on variables of external imbalances.

### 3.6.2 Level Adjustments Based on the Penn effect

Another major issue relates to what we discussed in Section 3.3.2: short samples for some countries or a history of managed exchange rates for others may severely bias the estimation of the intercept of reduced form fair value models. In turn, errors in the estimation of the intercept will translate in a biased estimate of the fair value level of the exchange rate. In the latest version of the GSDEER model, Goldman’s economists (see Stolper et al., 2009) argue that most of the data problems leading to a biased estimate of the intercept will also lead to the violation of the assumption of cointegration between the exchange rates and the fundamentals. They test this assumption by testing the stationarity of the cross-sectional residuals. Their results show that the hypothesis of a unit root in the residuals is particular unlikely to get rejected for:

- currencies with a history of managed exchange rates, typically in developing Asia;
- currencies which have been subject to periods of hyperinflation and the related data quality issues, typically in Latin America;
- currencies with short samples, in particular in Central and Eastern Europe.

For those cases in which the unit-root hypothesis cannot be rejected, Stolper et al. (2009) adjust the country fixed effects of the GSDEER model by estimating a cross-
sectional model based on the Penn effect, as the one described in Section 3.4.1. Figure 3.4 shows time series of nominal GSDEER fair values (against the US dollar) for the euro, the British pound, the Japanese yen, and the Chinese renminbi. Of these four fair values, only the renminbi has been subject to a level correction based on the Penn effect correction.

### 3.6.3 Penn effects and the Size of Agricultural Sector

When estimating the Penn effect, Stolper et al. (2009) make the explicit choice to adjust the income-per-capita variable for non-agricultural sectors of the economy. This adjustment is worth a more detailed explanation.

As discussed in Section 3.4.1, the Penn effect is based on productivity differentials between the non-tradable and the tradable sectors. Assuming that productivity in the non-tradable sector is comparable across countries, the deviations from PPP can be shown to be a function of cross country differential in tradable sector productivity.

In typical empirical estimates of the Penn effect, it is assumed that GDP-per-capita differentials are proportional to tradable sector productivity differentials. However, this makes the implicit assumption that the tradable sector is of comparable size across countries—an assumption that is frequently violated, in particular in countries with large agricultural sectors.

To illustrate the importance of this adjustment in the GSDEER, we estimate the Penn effect as in Equation (3.5), but calculating GDP-per-capita variable in three different ways: (i) unadjusted, (ii) adjusted for output and employment in primary and secondary sector, i.e. excluding agriculture, and (iii) adjusted for the industrial sector alone. To make the adjustment, we use World Bank data on the employment and output shares of the individual sectors.

The three estimates are quite comparable in fit (see Table 3.2), but when calculating fitted values for the currencies of countries with large agricultural sectors, substantial differences arise. Figure 3.5 illustrates this point for the case of the Chinese renminbi, and shows differences in estimated misalignments varying by up to nine percentage.
points. The estimated undervaluation for the renminbi reached 15 percent in 2003 when using the adjusted measure of GDP-per-capita for the industrial sector. When we consider instead the unadjusted measure, the estimates show at most six percent undervaluation during the same period.

3.7 Conclusion

A primitive notion of currency misalignment linked to the precious metal content of coins has probably been around since the antiquity. The notion that prices of tradable goods are unlikely to diverge substantially across countries can be traced back several centuries and remains a key building block of modern adjusted-PPP or BEER models. Similarly, the link between currency valuation and external imbalances—in particular the impact on foreign exchange reserves—has been observed in Roman times and remains an important building block of the many variants of modern underlying balance models.

In this chapter we review the most important families of fair value models currently in use. Most of these models have been developed over the last 20 years. Moreover, we introduce the concept of Indirect Fair Value (IFV), a notion of fair value frequently used by financial market participants for short-term investment decisions, but to our knowledge not previously formalised in the academic literature.

Currency fair value modelling has always been a field of interest for policymakers and investors and fair value estimates are frequently an input for important political or financial decisions. As a result, it is important to highlight the implicit or explicit modelling and implementation choices. Given important trade-offs when choosing a fair value model, many practitioners now combine several models and approaches to correct or compensate for individual weaknesses. Both our case studies, the IMF GER framework and the Goldman Sachs GSDEER models, follow that path.

In the context of these case studies, we illustrate empirically how some of the modelling choices affect fair value estimates. Specifically, we show the sensitivity of underlying balance models to import and export elasticities. Moreover, we estimate three variants of a Penn-Effect model and show the sensitivity to implicit assumptions about the re-
lative size of the non-tradable sector in the economy.
Table 3.1. Real Effective Exchange Rate Adjustment for China, Sensitivity Analysis

The table presents, for given trade elasticities, the percentage change in the real effective exchange rate which is required to bring China’s underlying current account in line with the current account target. The required change is computed using Equation (3.20) in the main text. We use the current account projections for 2016 from the IMF WEO of April 2011 as an estimate of the underlying current account balance. We estimate the 2010 China’s exports/GDP and imports/GDP ratios as 29.8 and 25.9 percent, respectively. Data for these ratios are from China’s national statistics and the IMF. Panel A shows the results assuming a current account target of zero percent, whereas Panel B shows the results assuming a current account target of 3 percent.

### Panel A: 0% Current Account Target

<table>
<thead>
<tr>
<th>Export Elasticities</th>
<th>-0.1</th>
<th>-0.2</th>
<th>-0.3</th>
<th>-0.4</th>
<th>-0.5</th>
<th>-0.6</th>
<th>-0.7</th>
<th>-0.8</th>
<th>-0.9</th>
<th>-1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>38.4</td>
<td>45.0</td>
<td>53.3</td>
<td>66.8</td>
<td>92.7</td>
<td>143.6</td>
<td>318.4</td>
<td>171.7</td>
<td>222.2</td>
<td>120.2</td>
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<tr>
<td>0.2</td>
<td>44.0</td>
<td>52.8</td>
<td>66.2</td>
<td>86.6</td>
<td>134.0</td>
<td>274.6</td>
<td>557.1</td>
<td>250.0</td>
<td>127.9</td>
<td>85.9</td>
</tr>
<tr>
<td>0.3</td>
<td>51.5</td>
<td>64.1</td>
<td>84.9</td>
<td>125.6</td>
<td>241.5</td>
<td>3120.0</td>
<td>285.7</td>
<td>136.6</td>
<td>89.8</td>
<td>66.8</td>
</tr>
<tr>
<td>0.4</td>
<td>62.1</td>
<td>81.4</td>
<td>118.2</td>
<td>215.5</td>
<td>1218.8</td>
<td>333.3</td>
<td>146.6</td>
<td>94.0</td>
<td>69.1</td>
<td>54.7</td>
</tr>
<tr>
<td>0.5</td>
<td>78.2</td>
<td>111.6</td>
<td>194.5</td>
<td>757.3</td>
<td>400.0</td>
<td>158.2</td>
<td>98.6</td>
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<td>56.2</td>
<td>46.3</td>
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<tr>
<td>0.6</td>
<td>105.7</td>
<td>177.3</td>
<td>549.3</td>
<td>500.0</td>
<td>171.8</td>
<td>103.7</td>
<td>74.3</td>
<td>47.9</td>
<td>40.1</td>
<td></td>
</tr>
<tr>
<td>0.7</td>
<td>162.8</td>
<td>430.9</td>
<td>666.7</td>
<td>188.0</td>
<td>109.4</td>
<td>77.2</td>
<td>59.6</td>
<td>48.5</td>
<td>35.4</td>
<td></td>
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<tr>
<td>0.8</td>
<td>234.5</td>
<td>1000.0</td>
<td>207.4</td>
<td>115.7</td>
<td>80.2</td>
<td>61.4</td>
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<td>41.8</td>
<td>36.0</td>
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<tr>
<td>0.9</td>
<td>2000.0</td>
<td>231.5</td>
<td>122.8</td>
<td>83.6</td>
<td>63.4</td>
<td>51.0</td>
<td>42.7</td>
<td>36.7</td>
<td>32.2</td>
<td></td>
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<tr>
<td>1.0</td>
<td>261.7</td>
<td>130.9</td>
<td>87.2</td>
<td>65.4</td>
<td>52.3</td>
<td>43.6</td>
<td>37.4</td>
<td>32.7</td>
<td>29.1</td>
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### Panel B: 3% Current Account Target

<table>
<thead>
<tr>
<th>Export Elasticities</th>
<th>-0.1</th>
<th>-0.2</th>
<th>-0.3</th>
<th>-0.4</th>
<th>-0.5</th>
<th>-0.6</th>
<th>-0.7</th>
<th>-0.8</th>
<th>-0.9</th>
<th>-1.0</th>
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</thead>
<tbody>
<tr>
<td>0.1</td>
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<td>27.7</td>
<td>33.4</td>
<td>42.1</td>
<td>57.1</td>
<td>88.4</td>
<td>195.9</td>
<td>905.7</td>
<td>136.8</td>
<td>74.0</td>
</tr>
<tr>
<td>0.2</td>
<td>27.1</td>
<td>32.5</td>
<td>40.7</td>
<td>54.5</td>
<td>82.5</td>
<td>169.0</td>
<td>3428.6</td>
<td>153.8</td>
<td>78.7</td>
<td>52.9</td>
</tr>
<tr>
<td>0.3</td>
<td>31.7</td>
<td>39.4</td>
<td>52.2</td>
<td>77.3</td>
<td>148.6</td>
<td>1920.0</td>
<td>175.8</td>
<td>84.1</td>
<td>55.2</td>
<td>41.1</td>
</tr>
<tr>
<td>0.4</td>
<td>38.2</td>
<td>50.1</td>
<td>72.7</td>
<td>132.6</td>
<td>750.0</td>
<td>205.1</td>
<td>90.2</td>
<td>57.8</td>
<td>42.6</td>
<td>33.7</td>
</tr>
<tr>
<td>0.5</td>
<td>48.1</td>
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<td>119.7</td>
<td>466.0</td>
<td>246.2</td>
<td>97.4</td>
<td>60.7</td>
<td>44.1</td>
<td>34.6</td>
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</tr>
<tr>
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<td>65.0</td>
<td>109.1</td>
<td>338.0</td>
<td>307.7</td>
<td>105.7</td>
<td>63.8</td>
<td>45.7</td>
<td>35.6</td>
<td>29.2</td>
<td>24.7</td>
</tr>
<tr>
<td>0.7</td>
<td>100.2</td>
<td>265.2</td>
<td>410.3</td>
<td>115.7</td>
<td>67.3</td>
<td>47.5</td>
<td>36.7</td>
<td>29.9</td>
<td>25.2</td>
<td>21.8</td>
</tr>
<tr>
<td>0.8</td>
<td>218.2</td>
<td>615.4</td>
<td>127.7</td>
<td>71.2</td>
<td>49.4</td>
<td>37.8</td>
<td>30.6</td>
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</tr>
<tr>
<td>0.9</td>
<td>1230.8</td>
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<td>75.6</td>
<td>51.4</td>
<td>39.0</td>
<td>31.4</td>
<td>26.3</td>
<td>22.6</td>
<td>19.8</td>
<td>17.6</td>
</tr>
<tr>
<td>1.0</td>
<td>161.1</td>
<td>80.5</td>
<td>53.7</td>
<td>40.3</td>
<td>32.2</td>
<td>26.8</td>
<td>23.0</td>
<td>20.1</td>
<td>17.9</td>
<td>16.1</td>
</tr>
</tbody>
</table>
Table 3.2. Penn effect adjustment in GSDEER, different sectors

The table presents the regression results for the Penn effect adjustment in the GSDEER for the Chinese renminbi against the US dollar.

<table>
<thead>
<tr>
<th></th>
<th>All sectors</th>
<th>Industry</th>
<th>Industry plus Services</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient</td>
<td>0.343</td>
<td>0.428</td>
<td>0.421</td>
</tr>
<tr>
<td>$t$-statistic</td>
<td>25.745</td>
<td>24.637</td>
<td>27.135</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.478</td>
<td>0.455</td>
<td>0.504</td>
</tr>
</tbody>
</table>
Figure 3.1. Underlying Balance Approach

The figure illustrates the Underlying Balance approach described in the main text.
Figure 3.2. USD/CAD 3-month 25-Delta Risk reversals

Notes: The figure shows risk reversals for 3-month 25-Delta out-of-the-money options for the Canadian dollar against the US dollar.
Figure 3.3. Indirect Fair Value for USD/CAD

The figure illustrates the Indirect Fair Value (IFV) for the daily nominal spot exchange rate of the Canadian dollar against the US dollar. The time span is between January 2004 and January 2007. The black line displays the actual observed value of the exchange rate. The light grey line displays the fitted value of a regression of the spot exchange rate on: the difference between US and Canadian 2-year swap rates; linear, quadratic, and cubic time trends; and 3-month 25-Delta risk reversals. Finally, the dark grey line displays the IFV of the Canadian dollar, using the sample mean of the risk reversals as a proxy for neutral speculative positioning.
The figure shows the observed nominal exchange rates and the GSDEER fair value estimates for a number of currencies.

Figure 3.4. GSDEER Fair Values
The figure illustrates the misalignment of the Chinese renminbi against the US dollar for different Penn effect specifications as described in the main text. The solid line displays the misalignment considering all sectors; the dashed line exhibits the misalignment considering only the industrial sector; finally, the dash-dot line shows the misalignment considering the industrial and services sectors together.
Chapter 4

Concluding Remarks

The carry trade is a currency investment strategy designed to exploit deviations from uncovered interest parity. Its profitability is based on the empirical observation that the interest rate differential across countries is not, on average, offset by a depreciation of the investment currency. Hence, investing in high-interest currencies by borrowing from low-interest currencies tends to deliver large positive excess returns.

The first chapter of this thesis fills a gap in the literature by demonstrating empirically the existence of an intertemporal risk-return tradeoff between the return to the carry trade and risk in a predictive setting. We measure FX risk by the variance of the returns to the FX market portfolio. We then take a step further by decomposing the market variance into the cross-sectional average variance and the cross-sectional average correlation of exchange rate returns. Our empirical analysis is based on predictive quantile regressions, which provide a natural way of assessing the effect of higher risk on different quantiles of the return distribution.

Our main finding is that average variance has a significant negative effect on the left tail of the distribution of future carry trade returns, whereas average correlation has a significant negative effect on the right tail. We take advantage of this finding by forming a new version of the carry trade that conditions on average variance and average correlation, and show that this strategy performs considerably better than the standard carry trade. These results imply that to some extent exchange rates are predictable, especially when it matters most: when the carry trade produces large gains or large
losses. In other words, if the carry trade is about “going up the stairs and down the elevator,” then average variance and average correlation can tell us something valuable about when the elevator is likely to go up or down. In the end, by focusing on the tails of the return distribution of carry trades, we uncover a negative risk-return tradeoff in foreign exchange.

In the second chapter, I analyse the reaction of exchange rates to interest rate differentials shocks. Previous literature has shown that exchange rates tend to react sluggishly to shocks, so that an unexpectedly higher foreign interest rate generates a slow and persistent appreciation of the foreign currency for several quarters. This finding implies a violation of a conditional version of UIP and long-run PPP: excess return on foreign exchange should react instantaneously to an unexpected interest rate shock and jump to the level implied by the present value of changes in expectations of future real interest rate differentials.

I argue that previous empirical findings, by mostly ignoring the role of nonlinearities and the evolution of exchange rates dynamics over time, may have represented only an average of the past, and not reflected the current state of the economy. Therefore, previous results may be biased and may not provide useful insights for a currency investor betting on violations of UIP in a given point in time.

In order to re-examine the evidence of conditional violations of UIP, I employ a Bayesian time-varying-parameter VAR which allows for time-variation both in the transmission mechanism of the shocks and the volatility of the shocks themselves. This framework allows me to estimate the time-varying responses of the excess returns on foreign exchange and to compare them to those implied by UIP and long-run PPP, at the same time explicitly taking into account parameter uncertainty.

I find that the transmission of the interest rate differential shocks has changed over time. However, even if to a varying degree over the years, some of the puzzling results previously documented with linear models remain. I show that currency excess returns tend to initially underreact to interest rate differential shocks for the whole sample considered, undershooting the level implied by fundamentals. At longer horizons, the
previously documented evidence of overshooting tends to disappear in recent years in the case of the euro, the British pound and the Canadian dollar. Instead, overreaction at long horizons is a persistent feature of the excess returns on the Japanese yen and the Swiss franc throughout the whole sample.

These results suggest that previously documented conditional violations of UIP may have secularly declined over time, at least for euro, the British pound and the Canadian dollar. However, the results for the Japanese yen and the Swiss franc—two currencies which have been traditionally used for funding carry trade positions—may hint that speculation in the foreign exchange market may constitute a destabilizing force, driving exchange rates away from fundamentals.

In the third chapter we review the most important families of fair value models currently in use. Most of these models have been developed over the last 20 years. Moreover, we introduce the concept of Indirect Fair Value (IFV), a notion of fair value frequently used by financial market participants for short-term investment decisions, but to our knowledge not previously formalised in the academic literature.

Currency fair value modelling has always been a field of interest for policymakers and investors and fair value estimates are frequently an input for important political or financial decisions. As a result, it is important to highlight the implicit or explicit modelling and implementation choices. Given important tradeoffs when choosing a fair value model, many practitioners now combine several models and approaches to correct or compensate for individual weaknesses. Both our case studies, the IMF GER framework and the Goldman Sachs GSDEER models, follow that path.

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