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Essays on Financial Networks, Systemic Risk and Policy

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A thesis submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy in Economics

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Last but not the least, I would like to commemorate my grandparents, may they rest in peace.

Declaration

I declare the following:

The material contained in this thesis is my own work.

The thesis has not been submitted for a degree at any another university.

Abstract

This essay consists of three chapters.

Chapter one extends Allen and Gale's (2000) model to a core-periphery network structure. We identify that the financial contagion in core-periphery structure is different to Allen and Gale (2000) in two aspects. Firstly, the shocks to the periphery bank and to the core bank have different contagion processes. Secondly, contagion not only depends on the amount of claims a bank has on a failed bank, but also on the number of links the failed neighbour has.

Chapter two studies the policy effect on financial network formation when the government has time-inconsistency problem on bailing out systemically important bank. We show that if interbank deposits are guaranteed, the equilibrium network structure is different from the one under market discipline. We show that under market discipline individual banks can collectively increase the component size using interbank intermediation in order to increases the severity of systemic risk and hence trigger the bailout. If interbank intermediation is costly the equilibrium network has core-periphery structure.

Chapter three follows Acharya and Yorulmazer's (2007) study of the "too many to fail" problem in a two-bank model. They argue that in order to reduce the social losses, the financial regulator finds it *ex post* optimal to bail out every troubled bank if they fail together, because the acquisition of liquidated assets by other investors result in a high misallocation cost. In contrast to their paper, we argue that there is no "too many to fail" bailout, unless banking capital is costly and market price sensitive. We argue that market price sensitive capital can induce banks herding and high social cost.

Introduction

Early banking theories, such as those of Diamond (1984) and Diamond and Dybvig (1983), mostly result from representative agent model. They show that, on the one hand, banks play an important role in completing the market and providing special expertise in monitoring the loans, on the other hand, banks tend to have risk-shifting incentive, as in principle-agent models. This type of banking theory gives strong support for regulating individual banks, so-called micro-prudential financial policies, such as Basel 1 and 2, which argue that the prudent behaviour and risk management practices of each individual bank ensures the stability and soundness of the financial system as a whole (Kashyap, Rojan and Stein 2008). In other words, the financial system works fine as long as each individual bank is well behaved.

However, this type of financial policies fails to internalize individual banks' strategic interactions, In particular, the strategic herding behaviour and network interactions which could lead to systemic failure of financial system[1]. A prominent example regarding to the ineffectiveness of individual regulatory policy is the turmoil of recent sub-prime mortgage crisis. There are two distinct features in the crisis. First, banks' balance sheets exhibit high similarity before the crisis; many banks possess Mortgage Backed Securities (MBS). Second, the crisis is highly contagious stem from interbank connection (see Brunnermeier (2008) and Hellwig (2009)). As a result, the governments have to bail out the troubled banks.

In this essay we examine the effect of financial policy on individual banks' interaction and the resulting systemic risk. The essay focuses on two issues. Firstly we

[1] In this essay we impose two definitions on systemic failure. In Chapter 1 and 2, systemic risk refers to the propagation of an agent's economic distress to other agents linked to that agent through interbank connections. To the extent that interbank lendings are neither collateralised nor insured against, a bank's failure may trigger a chain of subsequent failures and therefore force the central bank to intervene to bail out the troubled banks, see Craig and Peter (2010). In Chapter 3 we define systemic failure as the state where all banks fail simultaneously. See De Bandt and Hartmann (2000) for detailed discussion on defining systemic failures.

show how network theory can contribute to the analysis of systemic risk. Secondly we examine how financial policy affects banks' strategic interaction.

The general concept of a network is quite intuitive: a network describes a collection of nodes and the links connecting them. In the context of financial systems, the nodes can represent individual banks, while the links are created through, for example, credit exposures between banks. As Haldane (2009) stresses, a network approach to financial system is particularly important for evaluating contagious failure such that the failure of a single institution may create for the financial system as a whole, and can be instrumental in examining individual banks' interaction, in particular network externalities. A better understanding of network externalities could thus be helpful for the reform of the current financial policy.

The current financial policy, as we have discussed above, focuses heavily on regulating individual banks. We argue that this type of policy may be inefficient in a decentralized banking system, if the government suffers from time-inconsistency problems of bailout. In particular we show that market price sensitive capital regulation and market discipline may induce banks to coordinate to high probability of systemic failure. This argument is similar to Torre and Ize (2009) who suggest that a successful financial policy needs to integrate the paradigms of agency (principle-agent problems) and externalities (contagious problems). If financial policy tries to address the central problem under agent paradigm, it may made the problems under the externalities paradigms worse.

The essay consists of three chapters. Chapter one extends Allen and Gale's (2000) model of financial contagion in regular network to a core-periphery network structure. Empirical evidence shows that the financial network structure is heterogeneous, with a few nodes having many links and many nodes having a few links- so called core-periphery structure. We identify that the contagion process under this structure is different from Allen and Gale (2000). Firstly, in Allen and Gale (2000) a shock to any one of the banks has the same contagious effect on the others, whereas we show that shocks to the peripheral bank and to the core bank have different contagion processes. A shock to the core bank affects many other banks in the economy, whereas a shock to a peripheral bank is less powerful; this is similar to the robustness of network resilience to a disturbance resulted from statistical physics in the complex network literature. Secondly, in contrast to Allen and Gale (2000), who argue that once the neighbour of a failed bank becomes bankrupt it has to be true that all the banks in the component fail, our finding suggests that this is not necessarily the case, because contagion not only depends on the amount of claims a bank has on a failed bank, but also on the number of links this failed neighbour has; the more neighbours it has, the greater liquidation value it has and the less likely that its neighbour(s) fail. We conclude that regular networks and core–periphery networks react differently to idiosyncratic shock.

Chapter two examines the policy effect on financial network formation when government has time-inconsistency problem of bailing out systemically important banks. We show that if interbank deposits are guarantee, the equilibrium network structure is different to the one under market discipline. Under market discipline individual banks can collectively increase the component size by using interbank intermediation in order to increases the severity of systemic risk and hence trigger the government bailout. If intermediation is costly we show that the equilibrium network has core-periphery structure. The chapter suggests that if financial regulation is based on representative agent model or treats the network structure as given, they may ignore the effect of a new regulation on banks' interaction. A new equilibrium network could emerge which lead to this regulatory policy ineffective.

Chapter three argues that market price sensitive capital regulation combined with the government's bailout policy can induce banks to herd by investing in highly correlated assets. The chapter follows Acharya and Yorulmazer's (2007) study of the "too many to fail" problem in a two-bank model. They argue that in order to reduce social losses due to the systemic risk of banks failing together, the financial regulator finds it *ex post* optimal to bail out every troubled bank, because the acquisition of liquidated assets by other investors result in a high misallocation cost. In contrast to their paper, we argue that as long as it is profitable for banks to purchase the liquidated assets at the cash-in-the-market price, in the state of systemic failure, the regulator can always commit to randomly bailing out only one bank and letting the bailed out bank purchase the failed bank's assets– there is no "too many to fail" bailout. We then show that market price sensitive capital regulation can remove banks' incentive to purchase liquidated assets at the cash-in-the-market price, given the high cost of banking capital. Thus, in the state of systemic failure, the regulator has to bail out every troubled bank. A "too many to fail" rescue arises. We then argue that market price sensitive capital can increase systemic failure.

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Chapter 1

Financial Contagion in a Core–Periphery Network

Summary

Empirical evidence shows that the financial network is asymmetric, with a few nodes having many links and many nodes having a few links- so called core-periphery structure. We identify that the contagion process under this structure is different from Allen and Gale (2000). Firstly, in Allen and Gale (2000) a shock to any one of the banks has the same contagious effect on the others, whereas we show that shocks to the peripheral bank and to the core bank have different contagion processes. A shock to a core bank affects many other banks in the economy, whereas a shock to a peripheral bank is less powerful; this is similar to the robustness of network resilience to a disturbance resulted from statistical physics in the complex network literature. Secondly, in contrast to Allen and Gale (2000), which shows that once the neighbour of a failed bank becomes bankrupt it has to be true that all the banks in the connected network fail, our finding suggests that this is not necessarily the case, because contagion not only depends on the amount of claims a bank has on a failed bank, but also on the number of links this failed neighbour has; the more neighbours it has, the greater liquidation value it receives and the less likely that its neighbour fails. Thus, regular networks and core–periphery networks react differently to shocks.

1. Introduction

The existence of interbank market formed by financial institutions plays a key role in financial stability. On the one hand, argued by Cocco et al. (2004), it plays a crucial role in monitoring banks, conducting monetary policy and most importantly the provision of liquidity for a troubled bank facing a liquidity shock. On the other hand, the interbank market can transmit shocks from one bank to another which triggers the problem of contagious failure, thereby increasing the likelihood of systemic risk. Since one of the targets for the central banks is to ensure financial stability, this spillover effect implies that the central banks needs to avoid the systemic risk that would otherwise undermine the system as a whole, and make the system stronger by reducing such risk. In this vein, it is important for central banks to understand how shocks propagate across the financial system.

In order to do so, one can use network topology to model the financial system. In general, a network is any system that can have mathematical representation as a graph in which the nodes identify the elements of the system and the set of connecting links represents the relation or interaction among those elements. In this regard, the financial system is a very good candidate for a network in which the nodes are financial institutions and the links represent the interaction among them, such as credit exposures between banks due to liquidity risk sharing.

Remarkably, many empirical evidences found that financial networks in many developed countries share a common feature – they exhibit statistically heterogeneous connectivity (see Boss et al. (2004), Degryse and Nguyen (2004), Upper and Worms (2004), and Becher et al. (2008)). The high level of heterogeneity of many networks is simply provided by the fact that many nodes have just a few connections, while a few hubs collect hundreds or even thousands of edges– a core–periphery structure. The presence of hubs and connectivity ordering in many cases yields a degree distribution with heavy tails[2]. Figure 1 gives the illustration of a core-periphery financial network structure in Austria.

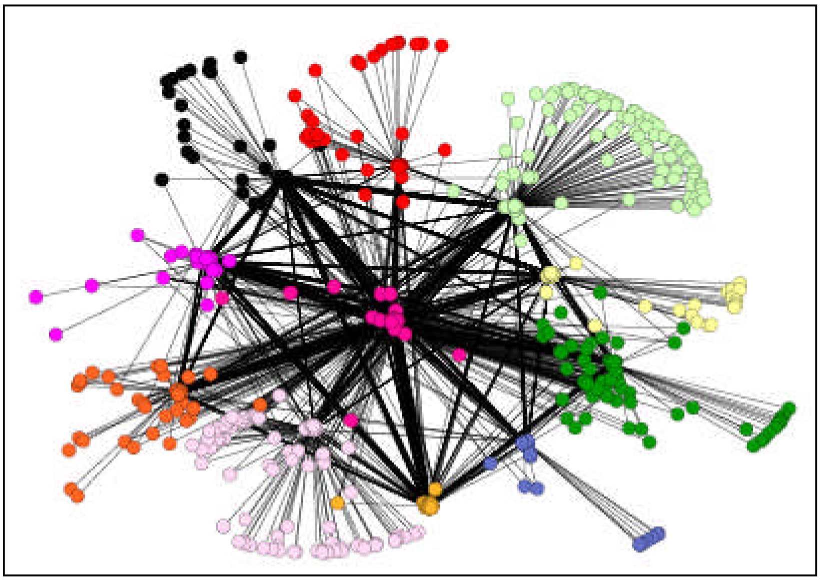


Figure 1

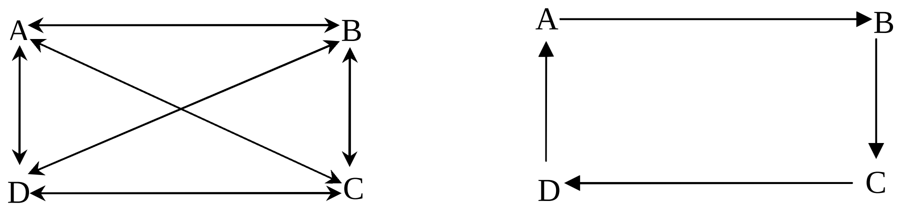
Network structure of the Austrian interbank market. See Boss et al. (2004) figure 1

However, despite the vigorous empirical evidence on the structure of financial networks, there is little attention focusing on the effect of core-periphery structure on financial stability in the theory of economics, and there are just a few papers that try to model financial contagion in a network context. Perhaps the most influential paper in these studies is that of Allen and Gale (2000). They focus on contagion through overlapping claims, which are held by *ex ante* identical banks on one another.

Allen and Gale (2000) assume that the aggregate demand for liquidity is known in the economy, but the number of early consumers is random in each bank with the

[2] Most real-world networks have skewed degree distribution in the sense that the degrees vary over a broad range. This behaviour is very different from the case of bell-shaped, exponentially decaying distributions and in several cases the heavy tail can be approximated by a power law decay, which results in linear behaviour on the double logarithmic scale. In the theory of complexity, this implies that nodes with degrees much larger than the average are found with a non-negligible probability. In other words, the average behaviour of the system is not typical, which means all the intermediate values are present and the average degree does not represent any special value for the distribution.

same variance. Since the banks are otherwise identical, banks with liquidity surpluses have an incentive to provide liquidity for banks with liquidity shortages through the exchange of interbank deposits. Thus, this interbank relationship works well as long as there is no shock to the aggregate liquidity demand. They mainly consider two cases in which all the banks have the same number of neighbours and each link has the same amount of overlapping deposits. First, the deposits of each bank are evenly distributed to every bank (see figure 2.a): a so-called completely connected network. Second, each bank only has a deposit in its negative correlated bank: an incomplete network (see figure 2.b).



a **b**

Figure 2

Bank A and C are negatively correlated with B and D

The first-best allocation can be achieved in both cases. However, when there is a probability zero liquidity shock at the beginning of the first period, two different structures give different threshold values of an outbreak of financial contagion. Since the aggregate liquidity supply in period 1 is fixed by period 0 allocation and banks are linked through overlapping claims, if one of the regions faces bankruptcy, the neighbour of this region suffers a loss because the value of claims on the troubled bank falls. It is intuitive that the fewer claims one bank has on the troubled bank, the less spillover effect it suffers from the troubled bank. Allen and Gale (2000) conclude that, at a given value of a probability zero liquidity shock, it is less likely to have contagion in a complete market than in an incomplete market.

Inspired by the study of Allen and Gale (2000), we extend their model to a core–periphery network by imposing heterogeneous volatility of liquidity shocks among banks. This heterogeneity produces some interesting results. Firstly, unlike Allen and Gale's study in which a shock to any one of the banks has the same effect on the others, we show that a shock to a peripheral bank has a different contagion process from a shock to a core bank. This finding is similar to the robustness of network resilience to a disturbance result from statistic physics in a complex network; the comparison between Allen and Gale's model and our model is backed by evidence put forward by Albert et al. (2000), who argue that heterogeneous and homogeneous topologies react very differently to damage. Secondly, in contrast to Allen and Gale (2000), who argue that once the neighbour of a failed bank becomes bankrupt it has to be true that all the banks in the economy fail, our finding suggests that this is not necessarily the case, because contagion not only depends on the amount of claims a bank has on a failed bank, but also on the number of links this failed neighbour has; the more neighbours it has, the greater liquidation value it will have and the less likely that its neighbour will fail. The chapter thus implies that the core banks play an important role in financial network stability. The argument is very similar to that of Gai and Kapadia (2010) where they show that- with a stochastically generated complex network- the financial system exhibits a “robust-yet-fragile” property. This feature indicates that core banks, which have greater connectivity, can reduce the likelihood of financial contagion; but they can have highly significant impact on the system if they fail. Their argument is consistent with our proposition in section 4, as we will see later. Note however that this chapter examines a fixed core-periphery network, unlike the stochastically generated networks of Gail and Kapadia (2010).

The structure of this chapter is as follows. Section 2 reviews the theoretical and empirical literature on financial networks. Section 3 presents the model. We show that the decentralized economy with heterogeneous regional liquidity volatility can reach first-best solution as in Allen and Gale (2000). Section 4 examines financial contagion under core-periphery structure. Section 5 analyses the model result and Section 6 concludes.

2. Literature review

The theory of interbank market and systemic risk is first discussed by Bhattacharya and Gale (1987). They examine the effect of customers' preference shocks on a multi-bank system. However, they do not examine the network effect. Allen and Gale (2000) model financial contagion based on Diamond and Dybivg (1983) in a regular network[3]. They argue that if a bank has more links with other banks the initial impact of a financial crisis in that bank may be attenuated since each of its neighbours takes a small hit when the bank has a run. In other words, a "complete" financial network is more resilient than an "incomplete" one. Freixas, Parigi and Rochet (2000) consider financial contagion in payment systems. Banks face liquidity shocks because depositors are uncertain about the location where they are going to consume in the future. Banks create financial interconnections by extending credit lines to one another in order to deal with depositors' consumption needs. In their model, a financial crisis can arise as a result of a coordination failure among depositors. If depositors believe that there will not be enough resources in their future location, they will liquidate their investment in their original location, making it optimal for depositors in other regions to do the same. Banks are exposed to a positive probability

[3] For detailed discussion of Diamond and Dybig (1983) and Allen and Gale (2000) see also Allen and Gale (2007).

of contagion if the failure of one bank triggers a domino effect. In the last part of their paper they have a general discussion on “money centre” banks and its systemic importance[4]. However, they do not examine the effect of core periphery structure on the financial contagion in detail.

Some other researchers applied statistical physics to study contagion in complex network as in epidemiological literature (see Gai and Kapadia, 2010, Gai et al, 2010 and Haldane and May, 2011). These studies point out that high connectivity could reduce the likelihood of contagion. However, shocks could have significant impact if the failed banks are “super-spreaders”. They hence emphasize the important role of core banks in ensuring the stability of financial network.

As for the empirical evidence on financial network, Upper and Worm (2004) find that the current German banking system exhibits two-tier characteristic. Lower tier banks connect only one of the upper tier banks and upper tier banks have transactions with a variety of other banks. Craig and Peter (2010) have similar finding. In addition, they show that The German interbank network fits the core-periphery model eight times better than Erdos-Renyi random graphs and about two times better than scale free networks. Boss et al (2004) find that the degree distribution of Austrian interbank link follows power law and the clustering coefficient is relatively small compared with other social network, such as network of actors’ collaboration or sexual contact. This means that two banks that have interbank relations with a third bank have very low probability of having interbank connection with one another. Bech and Atalay (2008) also find in the U.S. the federal fund network follows a fat-tailed degree distribution, with most banks having few counterparties and a small number having many.

[4] In their model “money centre” banks are core banks that suffer high liquidity shock relative to periphery banks. In this sense, our paper combines Freixas, Parigi and Rochet (2000) and Allen and Gale (2000) in order to examine the systemic risk in core-periphery network.

3. The Model

The model extends Diamond and Dybvig (1983) and Allen and Gale (2000). The economy consists of four regions, i = A, B, C, and D. Each region consists of four sectors, $i1$, $i2$, $i3$ and $i4$[5]. There are three dates, $t = 0, 1, 2$. In each sector there are a large number of *ex ante* identical consumers each endowed with one unit of a homogeneous consumption good at date 0 but nothing at 1 and 2. Consumers have different liquidity preferences at date 1. Early consumers only value consumption at date 1, C_1, whereas late consumers only consume at date 2, C_2. They are uncertain about their type at date 0.

There are short-term assets and long-term assets. One unit of consumption good invested in a short asset in period t will result in one unit of consumption good in period t+1. The long asset has a high return, $R > 1$, but requires two periods to mature. The long asset can be liquidated, however, at date 1 at scrap value $0 < r < 1$. In each sector there are many banks, so that banks maximize depositors' *ex ante* expected utility.

Consumers have Diamond–Dybvig preferences: with probability ω they are early consumers, with utility of $u(C_1)$; with 1-ω they are late consumers, with utility of $u(C_2)$. The utility function is assumed to be twice continuously differentiable, increasing and strictly concave. The probability ω varies within and across regions. The realization of the probabilities depends on the state of nature.

There are two equally likely states: S_1 and S_2. The aggregate early and late demand for liquidity is constant in each state, but the liquidity shock fluctuates in each sector and region. There are two different types of banks in terms of the variance of the early consumers in the economy. Bank $i1$ faces four times higher volatility of early and late

[5] Note that this assumption does not affect the model result as long as there are heterogeneous levels of liquidity shock in different region.

consumers than banks *i*2, *i*3 and *i*4 *ex ante*. Bank *i*1s are negatively correlated with the rest of the banks in the same region, and each bank *i*1 is perfectly positively correlated with one of the other bank *i*1s and perfectly negatively correlated with the other two bank *i*1s.

	*A*1	*A*2	*A*3	*A*4
S_1	$\omega^4{}_H= \lambda+4\Delta$	$\omega_L=\lambda-\Delta$	$\omega_L=\lambda-\Delta$	$\omega_L= \lambda-\Delta$
S_2	$\omega^4{}_L= \lambda-4\Delta$	$\omega_H=\lambda+\Delta$	$\omega_H=\lambda+\Delta$	$\omega_H=\lambda+\Delta$
	*B*1	*B*2	*B*3	*B*4
S_1	$\omega^4{}_L= \lambda-4\Delta$	$\omega_H=\lambda+\Delta$	$\omega_H=\lambda+\Delta$	$\omega_H= \lambda+\Delta$
S_2	$\omega^4{}_H= \lambda+4\Delta$	$\omega_L=\lambda-\Delta$	$\omega_L=\lambda-\Delta$	$\omega_L=\lambda-\Delta$
	*C*1	*C*2	*C*3	*C*4
S_1	$\omega^4{}_H= \lambda+4\Delta$	$\omega_L=\lambda-\Delta$	$\omega_L=\lambda-\Delta$	$\omega_L= \lambda-\Delta$
S_2	$\omega^4{}_L= \lambda-4\Delta$	$\omega_H=\lambda+\Delta$	$\omega_H=\lambda+\Delta$	$\omega_H=\lambda+\Delta$
	*D*1	*D*2	*D*3	*D*4
S_1	$\omega^4{}_L= \lambda-4\Delta$	$\omega_H=\lambda+\Delta$	$\omega_H=\lambda+\Delta$	$\omega_H= \lambda+\Delta$
S_2	$\omega^4{}_H= \lambda+4\Delta$	$\omega_L=\lambda-\Delta$	$\omega_L=\lambda-\Delta$	$\omega_L=\lambda-\Delta$

TABLE 1: **Liquidity shock in the economy**

Let $\omega^4{}_H$ and $\omega^4{}_L$ denote the probabilities of a liquidity shock at date 1 for bank *i*1, and ω_H and ω_L denote the probabilities of a liquidity shock for banks *i*2, *i*3 and *i*4. Let the average demand for the liquidity be λ, which is also the average fraction of early consumers, we then have

$$\lambda = \omega_H + \omega_L / 2 = \omega_H^4 + \omega_L^4 / 2$$

and

$$\Delta = \omega_H - \lambda = \lambda - \omega_L = \frac{\omega_H^4 - \lambda}{4} = \frac{\lambda - \omega_L^4}{4}$$

(see Table 1). All uncertainty is resolved at date 1. A consumer's type is not observable, so late consumers can always mimic the actions of early consumers.

3.1 Central Planner's Allocation

Since there is no aggregate uncertainty, the incentive for efficient allocation is the same as the first-best allocation for a social planner. At date 0 each consumer has an equal probability of being an early or a late consumer, the expected utility is:

$$E\left[\lambda u(C_1) + (1-\lambda)u(C_2) \middle| S_i\, i = 1,2\right]$$

The central planner chooses a portfolio subject to the feasibility constraint:

$$x + y \le 1 \qquad (1)$$

where x and y denote the per capita amounts invested in the long and short assets, respectively. Then the feasibility constraint in period 1 is:

$$\lambda C_1 \le y \qquad (2)$$

and the feasibility constraint in period 2 is:

$$(1-\lambda)C_2 \le Rx \qquad (3)$$

The central planner maximizes consumers' expected utility subject to three constraints: (1), (2) and (3). We can realize that the first-best allocation satisfies the first-order condition $u'(C_1) \ge u'(C_2)$. Otherwise, the objective function could be increased by using the short asset to shift some consumption from early to late consumers. Thus, the first-best allocation satisfies the incentive constraint $C_1 \le C_2$, which says that late consumers find it weakly optimal to reveal their true type rather than pretend to be early consumers. In order to achieve the first best, the social planner has to transfer liquidities across regions in periods 1 and 2.

To illustrate, in state S_1, there is a ω^4_H fraction of early consumers in banks A1 and C1 and ω^4_L in regions B1 and D1. Each bank has λC_1 units of short assets. So, banks A1 and C1 each have an excess demand for $(\omega^4_H - \lambda)C_1$ units of consumption and banks B1 and D1 each have an excess supply of $(\lambda - \omega^4{}_L)C_1 = (\omega^4{}_H - \lambda)C_1 = 4\Delta$ units of consumption. There is a ω_H fraction of early consumers in banks B2, B3, B4, D2, D3 and D4 and each has an excess demand for $(\omega_H - \lambda)C_1$ units of consumption. There are ω_L early consumers in banks A2, A3, A4, D2, D3 and D4 and each has an excess supply of $(\lambda - \omega_L)C_1 = (\omega_H - \lambda)C_1 = \Delta$. In period 2 the transfers flow in the opposite direction.

3.2 Decentralized Allocation

We can show that the first-best allocation can be decentralized by a competitive banking sector. The aggregate early and late demand for liquidity is constant, but the number of early and late consumers fluctuates randomly in each region. This motivates banks to have interbank deposits as insurance so that banks with liquidity surpluses provide liquidity for regions with liquidity shortages. Here we assume that banks form links within the same region first, before they form regional links[6]. Thus, in the first round of network formation, banks *i*2, *i*3 and *i*4 have an incentive to deposit in bank *i*1, since they are negatively correlated with *i*1; see figure 2. In this case, banks *i*2, *i*3 and *i*4 each deposit Δ amount in bank *i*1, and bank *i*1 also deposit Δ

[6] This priority formation rule can be justified in a richer model where interbank insurance requires the monitoring cost; in this case, banks in the same region are preferred since they are locally adjacent which requires relatively low cost to monitor than banks in other regions.

in banks *i*2, *i*3 and *i*4. In the network topology, we have a star network in each region in which *i*1 is the hub or core in each region which connects every other bank in the region, whereas other banks only connect to *i*1.

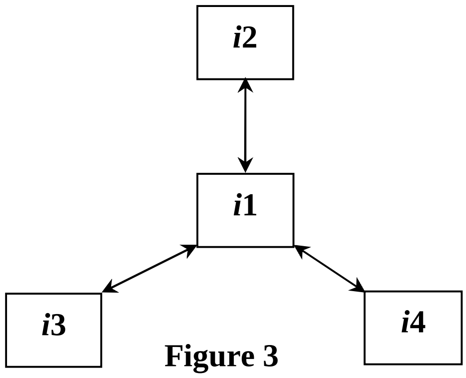


Figure 3

After the first-round exchange of deposits, banks with low volatility of early consumers are completely insured. Banks with high volatility can achieve complete insurance by forming links with banks in other regions which also have high volatility. Let us assume that core banks in different regions do not know with which banks they are positively or negatively correlated. In this case, the only equilibrium is that core banks will exchange $\Delta/2$ amount of deposit with each other in order to reach complete insurance. Figure 4 shows the liquidity risk sharing network with heterogeneous liquidity shocks. In terms of the network topology, it is a standard core–periphery network consisting of two types of nodes: hub nodes connect to each other and to all the peripheral nodes in their region, whereas peripheral nodes have a single link each and this link is with a hub node.

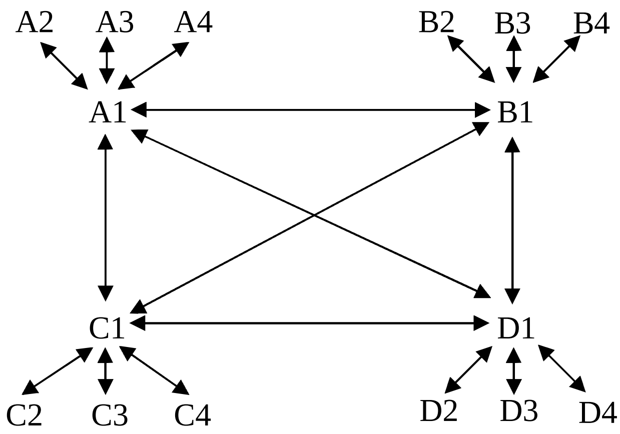


Figure 4

Without loss of generality, suppose we face state 1, in which banks A1 and C1 have a high demand for liquidity ω^4_H, and banks B2, B3, B4, D2, D3 and D4 have a high liquidity demand of ω_H. They liquidate all their deposits in other regions to satisfy the liquidity demand. Banks A1 and C1 in period 1 have *y* units of the short asset, claims of Δ deposits from each bank in their own region and Δ/2 claims from banks in other regions. They must pay C_1 to the fraction ω^4_H of early consumers and also Δ/2 to each other. Their budget constraint in period 1 should be:

$$(\omega_H^4 + \Delta/2)C_1 = y + 3\Delta C_1 + 3\Delta C_1/2 \qquad (4)$$

Given $\Delta = \omega_H - \lambda = \lambda - \omega_L$ and $\omega_H^4 = \lambda + 4\Delta$, the equation can be simplified to the social planner's budget constraint at date 1:

$$\lambda C_1 = y$$

Banks B2, B3, B4, D2, D3 and D4 also have *y* units of the short asset and claims of Δ from banks B1 and D1. They have to pay C_1 to the fraction ω_H of early consumers. Thus, the budget constraint should be:

$$\omega_H C_1 = y + \Delta C_1 \qquad (5)$$

This again implies:

$$\lambda C_1 = y$$

As for banks B1 and D1, they each face a demand of ω^4_L fraction of early consumers from their own sector, Δ amount of deposit from their own region and $\Delta/2$ from each of their negatively correlated regions. The supply of the liquidity is just the short asset *y*. The budget constraint has to satisfy:

$$(\omega_L^4 + 3\Delta + 2\Delta/2)C_1 = y \qquad (6)$$

Since $\omega_L^4 = \lambda - 4\Delta$, we then have:

$$\lambda C_1 = y$$

Banks A2, A3, A4, C2, C3 and C4 have to pay C_1 to the fraction ω_L of early consumers in their own sector, and also Δ to bank A1 or bank C1. They have *y* units of the short asset. The budget constraint has to be:

$$(\omega_L + \Delta)C_1 = y \qquad (7)$$

This also implies:

$$\lambda C_1 = y$$

At date 2, banks A1 and C1 pay C_2 to the fraction (1- ω^4_H) of late consumers in their own sector, pay back the Δ deposits from each of the three banks in their own region and face the withdrawal of $\Delta/2$ fraction of deposits from each negatively correlated region. They have *Rx* units of the long asset to meet these demands. So, the budget constraint for banks A1 and C1 in period 2 is:

$$\left[(1-\omega_H^4)+3\Delta+2\Delta/2\right]C_2 = Rx \qquad (8)$$

This again can be simplified into the social planner's constraint:

$$(1-\lambda)C_2 = Rx$$

Banks B2, B3, B4, D2, D3 and D4 in period 2 each faces the demand of (1- ω_H) fraction of late consumers in their own sector and Δ from B1 or D1, valued at C_2.

They have, again, Rx units of the long asset to meet this demand. The budget constraint has to satisfy:

$$(1-\omega_H+\Delta)C_2 = Rx \qquad (9)$$

We then have:

$$(1-\lambda)C_2 = Rx$$

As for Banks B1 and D1, they each have to pay C_2 to $(1-\omega^4_L)$ fraction of early consumers from their own sector. The supply of the liquidity is the long asset Rx, withdrawal of Δ amount of deposit from each bank in its own region and $\Delta/2$ from each negatively correlated region. The budget constraint has to satisfy:

$$(1-\omega^4{}_L)C_2 = Rx + 3\Delta C_2 + 2\Delta C_2/2 \qquad (10)$$

We again have:

$$(1-\lambda)C_2 = Rx$$

Finally, banks A2, A3, A4, C2, C3 and C4 have to pay C_2 to the fraction $(1-\omega_L)$ of early consumers in their own sector. They have Rx units of the long asset and claims of Δ from their regional core bank. The budget constraint is:

$$(1-\omega_L)C_2 = Rx + \Delta C_2 \qquad (11)$$

This also implies:

$$(1-\lambda)C_2 = Rx$$

4. Financial Contagion

Interbank deposit exchange works well as long as there is enough liquidity in the banking system as a whole. The financial linkages caused by these cross holdings can turn out to be disastrous if there is an excess demand for liquidity. In this section, we examine financial contagion by assuming a failure of one bank. Before doing so, we have to note that banks have a particular order in which to liquidate assets, a pecking order that says that in order to meet the liquidity demand, a bank liquidates short assets first, then deposits from other banks and finally long assets. Intuitively, liquidating short assets is the least costly since one unit of the short asset is worth one unit of consumption today, and also one unit of consumption tomorrow. By liquidating one unit of deposits, a bank gives up C_2 units of period 2 consumption and receives C_1 units of period 1consumption. Thus, the cost of liquidating is C_2 / C_1. Given the first-order condition $u'(C_1) = Ru'(C_2)$, we have $C_2 / C_1 > 1$. By liquidating long assets, a bank obtains r units of present consumption but gives up R units of future consumption. Given that r is small enough, we then have:

$$R / r > C_2 / C_1 > 1$$

A bank becomes bankrupt if it liquidates all its long assets and still cannot meet the liquidity demand at date 1. If a bank fails, the value of a deposit in period 1 is no longer C_1; instead, it is equal to the liquidation value of all the bank's assets. Let q denote the value of the representative bank's deposits in period 1. If $q < C_1$, then all the depositors (early and late customers, and deposit banks in other regions) withdraw at date 1. Every depositor receives q from the failed bank for each unit invested at date 0.

4.1 Shock on Core Bank

We first examine the propagation of a liquidity shock originating from the failure of one of the core banks. Suppose there is third state in which the average demand for liquidity in period 1 is greater than that in states 1 and 2. In this state, every region has the previous average demand for liquidity λ except for bank A1, in which the demand for liquidity is $\lambda + \varepsilon$. Assuming that this state occurs with zero probability, the allocation in period 0 does not change[7]. Bank A1 has $\lambda C_1 = y$ units of the short asset, but there is excess demand for εC_1. In accordance with the pecking order, no bank wants to liquidate the long assets if it can be avoided; the only equilibrium is one in which bank A1 will liquidate its deposit from its neighbours A2, A3, A4, B1, C1 and D1, and its neighbours will redeem the claim from A1 in order to meet their liquidity demand. The mutual withdrawal simply cancels out the effect of interbank insurance when there is an aggregate shock. Bank A1 has to meet the excess demand by liquidating some of its long assets. A bank could meet the excess demand of liquidity in period 1 by liquidating some of its long asset up to the point at which $C_1 = C_2$ (if $C_1 > C_2$ late consumers would be better off withdrawing in period 1). Therefore, a bank with a fraction of λ of early consumers must keep at least $(1-\lambda)C_1 / R$ units of long assets to satisfy the late consumers in period 2. The capital buffer is:

$$CB = r\left[x - \frac{(1-\lambda)C_1}{R}\right]$$

The capital buffer for A1 is:

$$CB_{A1} = r\left[x - \frac{(1-\lambda-\varepsilon)C_1}{R}\right]$$

Bank A1 will not become bankrupt if and only if the capital buffer in bank A1 is greater than or equal to the excess demand of liquidity εC_1:

$$CB_{A1} \geq \varepsilon C_1$$

In this case, bank A1 is safe but the late consumers in A1 are worse off because the value of period 2 consumption is less than C_2. If that this condition is violated, all its consumers and neighbouring banks withdraw in period 1. Since the liquidity shock happened at the beginning of date 1, it is reasonable to assume that the banks' action

---

[7] Note that the assumption of probability zero event is crucial for the model, because otherwise the *ex ante* interbank risk sharing can be different if the probability is positive. See Allen and Gale (2000) for the discussion on the difficulty of imposing positive probability.

is consistent with the first best in period 0. The value of deposits in bank A1 is then q^{A1}, which is less than C_1. The total demand for liquidity is 1 from its own depositors, 3Δ from its neighbours in its own region and 3Δ/2 from neighbours in other regions. The total liability demand in region A is then (1+3Δ+3Δ/2) q^{A1}. The total assets in A1 are short assets y, x units of long assets, 3Δ from its neighbours and 3Δ/2 deposits in each region. The assets are valued at

$$y + rx + \Delta q^{A2} + \Delta q^{A3} + \Delta q^{A4} + \Delta / 2 q^{B1} + \Delta / 2 q^{C1} + \Delta / 2 q^{D1}$$

Assuming that all banks, except A1, are safe, in equilibrium we then have:

$$q^{A1} \leq q_U^{A1} = \frac{y + rx + 3\Delta C_1 + 3\Delta C_1 / 2}{1 + 3\Delta + 3\Delta / 2} \qquad (12)$$

where bank q_U^{A1} is the upper bound on the value of the deposits in A1 under.[8] A1's neighbours in the same region (A2, A3 and A4) will be safe if and only if:

$$\begin{aligned}&(\lambda + \Delta) C_1 \leq y + CB + \Delta q^{A1} \\ &\Rightarrow \\ &(\lambda + \Delta) C_1 \leq y + CB + \Delta q_U^{A1}\end{aligned}$$

where the left-hand side of the inequality is the demand of date 1 consumption in its own sector plus the redeemed deposit from bank A1, and the right-hand side is the total short assets available, consisting of short asset y, capital buffer $CB = r\left[x - \frac{(1-\lambda)C_1}{R}\right]$ and claims from bank A1 valued at q_U^{A1}.[9] Given $\lambda C_1 = y$, and substituting out q_U^{A1}, this inequality implies that

[8] In what follows, we use the upper-bound value to derive the contagion threshold. It would be easier than working out all the liquidation values, which are determined by the equilibrium condition when there is bankruptcy in period 1. As we will see later, this assumption does not affect the model result.

[9] Note that all banks, except the failed bank, have the same capital buffer since they face the same liquidity demand, λ, in the first period.

$$
\begin{aligned}
&\Delta(C_1 - q_U^{A1}) \leq CB \\
&\Rightarrow \\
&\Delta\left(\frac{C_1 - y - rx}{1 + 3\Delta + 3\Delta/2}\right) \leq CB
\end{aligned}
\qquad (13)
$$

If condition 13 fails then all the banks in region A will fail.

We then consider the contagion threshold on bank A1's neighbours across regions, B1, C1 and D1. The only difference between regional and cross-regional neighbours is the amount of deposits they have in bank A1. They will not become bankrupt if and only if:

$$(\lambda + 3\Delta + 3\Delta/2)C_1 \leq y + CB \ + 3\Delta C_1 + 2\Delta C_1/2 + \Delta q_U^{A1}/2$$

This again implies:

$$
\begin{aligned}
&\Delta/2(C_1 - q_U^{A1}) \leq CB \\
&\Rightarrow \\
&\Delta/2\left(\frac{C_1 - y - rx}{1 + 3\Delta + 3\Delta/2}\right) \leq CB
\end{aligned}
\qquad (14)
$$

Supposing the inequality does not hold, then all the core banks fail. The liquidity shock will continue to propagate to all the banks in the economy. To illustrate this situation, let bank B1's liquidation value be q^{B1}. The total demand of liquidity is $(1+3\Delta+3\Delta/2)q^{B1}$, and the total liquidity supply is $y+rx+3\Delta C_1+\Delta q_U{}^{A1}/2+2\Delta C_1/2$. The liquidation value of bank B1 is determined by:

$$q^{B1} \leq q_U^{B1} = \frac{y + rx + 3\Delta C_1 + \Delta q_U{}^{A1}/2 + 2\Delta C_1/2}{1 + 3\Delta + 3\Delta/2} \qquad (15)$$

where q_U^{B1} is the upper-bound liquidation value. Banks B2, B3 and B4 are safe if and only if:

$$
\begin{aligned}
&(\lambda + \Delta)C_1 \leq y + CB \ + \Delta q_U^{B1} \\
&\Rightarrow \\
&\Delta(C_1 - q_U^{B1}) \leq CB
\end{aligned}
\qquad (16)
$$

However, this inequality cannot hold. We can show that as long as condition 14 is violated, it must be true that condition 13 cannot hold. Given that q_U^{B1} in equation 15 is always less than q_U^{A1} in equation 12, it implies that the inequality 16 cannot hold as long as condition 14 fails.

4.2 Shock on Peripheral Bank

We now consider financial contagion given a failure of one of the peripheral banks, say A2. Assuming that there is a state four, in which every bank has the average demand for liquidity λ except for bank A2, which faces liquidity demand $\lambda+\varepsilon$. As before, bank A2 becomes bankrupt if and only if its capital buffer is less than the value of excess liquidity demand εC_1. The value of the period 1 deposit in bank A2 becomes q^{A2}, which is again less than C_1. The value of deposits is determined by the equilibrium condition in which the demand is 1 from its own consumers and Δ from neighbouring bank A1 and the supply is $y + rx + \Delta q^{A1}$. Assuming that bank A1 is safe, we have:

$$q^{A2} \le q_U{}^{A2} = \frac{y + rx + \Delta C_1}{1 + \Delta} \tag{17}$$

where $q_U{}^{A2}$ is the upper bound of the liquidation value. A1will be safe if and only if:

$$\begin{aligned} &(\lambda + 3\Delta + 3\Delta/2)C_1 \le y + CB + \Delta q^{A2} + 2\Delta C_1 + 3\Delta C_1/2 \\ &\Rightarrow \\ &\Delta\left(\frac{C_1 - y - rx}{1 + \Delta}\right) \le CB \end{aligned} \tag{18}$$

If condition 18 fails, the liquidity shock will propagate to all the banks in region A and all the other regions' central node in the economy. Bank A1's upper-bound liquidation value, if it fails, is:

$$q^{A1} \leq q_U^{A1} = \frac{y + rx + \Delta q_U^{A2} + 2\Delta C_1 + 3\Delta C_1 / 2}{1 + 3\Delta + 3\Delta / 2} \tag{19}$$

Let us first consider the contagion within region A. Banks A3 and A4 will be safe if and only if:

$$\begin{aligned} &(\lambda + \Delta) C_1 \leq y + CB \; + \Delta q_U^{A1} \\ &\Rightarrow \\ &\Delta \left[\frac{C_1 + \Delta(C_1 - q_U^{A2}) - y - rx}{1 + 3\Delta + 3\Delta / 2} \right] \leq CB \end{aligned} \tag{20}$$

Substituting (17) into condition (20) we have:

$$\Delta \left[\frac{(C_1 - y - rx)(1 + 2\Delta)}{(1 + \Delta)(1 + 3\Delta + 3\Delta / 2)} \right] \leq CB \tag{21}$$

If condition 21 fails then all the banks in region A will fail.

As for regional contagion, banks B1, C1 and D1 will not fail as long as:

$$\begin{aligned} &(\lambda + 3\Delta + 3\Delta / 2) C_1 \leq y + CB \; + \Delta q_U^{A1} / 2 + 3\Delta C_1 + 2\Delta / 2 C_1 \\ &\Rightarrow \\ &\Delta / 2 \left[\frac{C_1 + \Delta(C_1 - q_U^{A2}) - y - rx}{1 + \Delta} \right] \leq CB \end{aligned} \tag{22}$$

Again, substituting (17) into (22), we have:

$$\frac{\Delta}{2} \left[\frac{(C_1 - y - rx)(1 + 2\Delta)}{(1 + \Delta)(1 + 3\Delta + 3\Delta / 2)} \right] \leq CB \tag{23}$$

Supposing this condition does not hold, then all the peripheral banks will suffer from a liquidity shock. For example, the failure of B1 means its liquidation value is

$$q^{B1} \leq q_U^{B1} = \frac{y + rx + 3\Delta C_1 + 2\Delta C_1 / 2 + \Delta q_U^{A1} / 2}{1 + 3\Delta + 3\Delta / 2} \tag{24}$$

The peripheral banks will all fail unless:

$$\begin{aligned} &(\lambda + \Delta) C_1 \leq y + CB \; + \Delta q_U^{B1} \\ &\Rightarrow \\ &\Delta \left[\frac{C_1 + \Delta / 2(C_1 - q^{A1}) - y - rx}{1 + 3\Delta + 3\Delta / 2} \right] \leq CB \end{aligned} \tag{25}$$

Substituting (19) and (17) into condition (25), we have:

$$\frac{\Delta}{2}\left[\frac{(C_1 - y - rx)(2+12\Delta+11\Delta^2)}{(1+\Delta)(1+3\Delta+3\Delta/2)^2}\right] \le CB$$

Rearranging the condition by multiplying $(1+2\Delta)$ in the numerator and denominator, we have:

$$\frac{\Delta}{2}\left[\frac{(C_1 - y - rx)(1+2\Delta)}{(1+\Delta)(1+3\Delta+3\Delta/2)}\frac{(2+12\Delta+11\Delta^2)}{(1+\frac{13}{2}\Delta+9\Delta^2)}\right] \le CB \qquad (26)$$

5. Analysis

When one of the core banks, A1, fails, it affects a considerable proportion of the banks in the economy simply because it has many neighbours in which only the later consumers all suffer from the shock, assuming that all the banks are safe. Otherwise, since condition (14) is more likely to hold relative to (13), the core banks in other regions are relatively more likely to survive than the peripheral banks in region A. Given two types of bank facing the same liquidation value from bank A1, it implies that the lower the amount of deposits put into the failed bank, the lower the likelihood of being affected by the liquidation. The central nodes in other regions can act as a buffer for their peripheral nodes to be immune from the shock. Note that once condition (14) fails, then all the core banks will fail and in turn all the banks in the economy will fail. This is because the liquidation value of B1- q_U^{B1} is less than q_U^{A1}, thus condition (13) is more likely to hold relative to condition (16). We then have:

Proposition: Once all the core banks fail, all banks in this connected network fail.

When one of the peripheral banks, A2, fails, the core bank, A1, will not fail if and only if condition (18) holds, otherwise A1 would become bankrupt and its neighbour would receive its liquidation value, which is q_U^{A1} at the upper bound. Given $0 < \Delta < 1$, conditions (18) and (21) imply that $q_U^{A2} < q_U^{A1}$. This shows that the failure of A1 does not necessarily lead to the failure of the whole region. This result implies that the worth of liquidation not only depends on the amount of deposits a bank has in the failed bank, but it also depends on the number of links the failed bank has. Essentially, the more links a bank has, the higher the liquidation value it yields relative to low-degree banks, when it is bankrupt.

Conditions (21) and (23) again imply that even if all the banks in region A fail, it is still possible for the other regions to be safe. Core banks act like a buffer to ensure the safety of their peripheries. Given $0 < \Delta < 1$, condition (26) is less likely to hold relative to condition (23), which implies that once the core fails, all the banks in the economy will fail. Note that if we order the liquidation values, given that all the values are less than C_1, we can find that $q_U^{A2} < q_U^{A1} < q_U^{B1}$, which means the liquidation value that banks B2, B3 and B4 receive is higher than the value B1 receives from A1. However, since B1 has a smaller proportion of deposits in A1 than the peripheries have in B1, the total worth of liquidation is less than B1 received from A1.

Our model result, to some extent, is similar to the finding of robustness of network resilience to a disturbance in a complex network. Albert et al. (2000) and Crucitti et al. (2004) find that core–periphery networks are robust to random failures, but vulnerable to targeted attacks, due to their heterogeneous topologies. In other words, core–periphery networks display great stability even if they are confronted by a large number of repeated small failures on peripheral nodes, while at the same time major damage can be triggered by attacking central nodes. Apparently, given that the capital buffer is big enough, the failure of a core bank, say A1, will trigger a considerable amount of late consumers suffering from the initial shock simply because the hub has many neighbours relative to the periphery. In contrast, the failure of a peripheral bank only affects its regional core bank, given a big enough capital buffer.

6. Conclusion

This chapter extends Allen and Gale's (2000) model to a core–periphery network structure. In Allen and Gale's (2000) study, a shock to any of the banks has the same effect on other banks, due to homogenous characteristics of liquidity shocks. We show that in a core–periphery network, a shock to the periphery has a different contagion process from a shock to one of the hubs. In their paper, Proposition 2 shows that as long as the neighbour of the failed bank becomes bankrupt, then all the banks in the connected network fail. In other words, a shock to a single bank can either bring down the entire banking system or no bank collapses. This is because all the banks in Allen and Gale's research (2000) hold the same amount of interbank deposits and have the same number of neighbours. In our model, we show that the liquidation value not only depends on the amount of claims one has on the troubled bank, but also on how many neighbours the troubled bank has; the more neighbours it has, the higher the liquidity value it produces when it bankrupts and the higher the contagion threshold. Thus, financial contagion behaves differently in a core–periphery network – even if the neighbour of a failed bank fails, it is not necessarily the case that all banks fail. The combination of our result and Allen and Gale's result is backed by evidence put forward by Albert et al. (2000), who argue that heterogeneous and homogeneous topologies react very differently to increasing levels of damage.

However, the model does not include much strategic behaviour. Since we assume no aggregate shock and cost of forming and maintaining a link, the formation of a financial network does not seem so interesting. We could adopt an equilibrium concept from a network theory in economics, such as pair-wise stability or bilateral equilibrium, to analyse the incentive of network formation.

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Chapter 2

Financial networks, "Too Systemic to Fail" and Regulatory Policy

Summary

This chapter studies how financial regulation can affect network formation when the government suffers from time-inconsistency problem of bailing out systemically important bank. We argue that if interbank deposits are guaranteed, the equilibrium structure of the network is different from the one under market discipline. We show that under market discipline individual banks can collectively increase the interconnectedness of the financial network by using interbank intermediation. The new equilibrium network can effectively increase the severity of systemic risk *ex post*, hence increasing the possibility that the government will bail out the insolvent bank. If intermediation is costly the network has core-periphery property. The chapter suggests that if the financial regulatory policy is based on representative agent model or a given network structure, it may ignore the effect of banks' interaction such that the resulting new equilibrium network can make this policy ineffective.

1. Introduction

Financial networks, formed by the connections between banks, are crucial for reallocating the liquidity from banks having cash in excess to those facing liquidity in demand[10]. However, these connections can also induce contagious failure and systemic risk such that the failure of one bank can lead to the failure of many other banks in a sequential fashion.

In this chapter, we study how financial regulation can affect bank behaviour in forming financial networks when the government suffers from a time-inconsistency problem in bailing out systemically important banks[11]. In particular, we show how the implementation of market discipline[12] - by subordinating interbank debt - can give individual banks the incentive to form an equilibrium network that is "too systemic to fail".

To study how network formation can be affected by regulatory policy, we assume banks suffer from independent liquidity and solvency shocks. The liquidity shock gives banks the incentive to form a risk-sharing network by exchanging interbank deposits ex ante, as in Allen and Gale (2000). However, banks in the network could also suffer from contagious failure if any one bank is hit by an idiosyncratic solvency

[10] We study a financial network induced by credit extension between banks, rather than a payment and settlement system, such as CHAPS in the UK, in which the determinants of network structure involve legal and technological factors. Credit extensions, unlike payments, do not cease to exist after they have been made, so the structure of the resulting network is of greater relevance for financial stability (see Craig and Peter (2010) and Kahn and Roberts (2009)).

[11] In this chapter a bailout refers to the government's act of injecting capital to the bank hit by a solvency shock. The government in this context refers to the authority who implements the bailout, such as FDIC in the US or the Treasury in the UK.

[12] Market discipline refers to a "market based" incentive scheme in which investors in bank liabilities, such as subordinated debt or uninsured deposits, monitor banks for their risk taking by demanding higher yields on these liabilities. The reason market discipline is needed is that banks are believed to engage in moral hazard behaviour. Banks collect deposits and invest these funds in risky assets. But the bank's own solvency target may not take into account the interests of depositors, nor of society as a whole. As a result, banks may engage in excessive risk taking. Market discipline is deemed to be a mechanism that can potentially curb the incentive to take excessive risk, by making risk taking more costly for banks, see Nier and Baumann (2006). In this paper we do not model the effect of market discipline on individual bank's risking taking behaviour explicitly. However, our model result implies that the market discipline may not be effective for limiting banks' risk taking incentives if the government suffers from time-inconsistency problems.

shock; and, if the systemic risk triggered by the insolvency of an individual bank is severe, the government may well bail out the insolvent bank *ex post*[13].

Before the early 1990s interbank deposits were implicitly or explicitly guaranteed (as discussed later in the next section), so that no bank would suffer from contagious failure. Because such guarantees ensure that an individual solvency shock does not impose any negative externalities to other banks, this implies that there are effectively no systemically important banks. In these circumstances banks have every incentive to form a network that shares the liquidity risks; and we show that, for *ex ante* identical banks, the most stable structure is a symmetric bipartite network with many components.

If the interbank deposit guarantee is removed, however, government's no bailout commitment is no longer credible due to the risk of contagious failure. In this case, individual banks must take three issues into account when forming a network: liquidity risk sharing, contagious failure due to an individual solvency shock and the government's incentive to bail out systemically important banks. We show that banks may act strategically so as to increase the interconnectedness of the network via interbank intermediation, so that an individual solvency shock will cause systemic risk and trigger a bailout of the insolvent bank. Moreover, if interbank intermediation is costly, there would be an agglomeration effect that generates a core-periphery network.

Our model is supported by the recent empirical findings on the financial network structure. They show that before early 1990s, the financial network structures were

[13] See Mishkin (1995), Freixas, Parigi and Rochet (1998), and Freixas (1999), Hoggarth, Reidhill and Sinclair (2004), they emphasize that the rationale behind "Too Big to Fail" rescue is that governments bail out financial institutions on the grounds that it eliminates negative externalities to other financial institutions who have credit exposure with the troubled bank. If they do not do so, contagious failure can leads to systemic risk which disrupt the proper functioning of the financial system as a whole. We will discuss this issue in detail later.

symmetric in many developed economies where each individual bank possesses more or less similar number of links with other banks for interbank liquidity insurance[14]. Banks were structurally equivalent with each other. In particular, banks either lend or borrow at a time with no apparent role of interbank intermediaries (see Craig and Peter (2010)). They also show that there is network structure evolution happened during late 1990s where the financial network evolved from symmetric one to the current network which exhibits core-periphery structure where core bank intermediates between peripheral banks that do not extend credit among themselves directly. The core bank not only acts as a financial intermediary transferring fund from depositors to the real economy, but also as an interbank intermediary transferring liquidities for periphery banks[15].

This chapter implies that *ex ante* identical agents can form an asymmetric risk sharing network structure. It is because the equilibrium network with asymmetry is not necessarily the result of some underlying heterogeneities among players, rather it can be the result of the interaction under certain network externalities and regulations. The financial network structural change can be the result of the change of individual banks' interacting behaviour due to the change of regulatory policy. Our result thus has a normative implication for financial regulation. If financial regulation is derived from representative agent models or by treating the network structure as exogenously given, it may ignore the effect of banks' interaction such that a new equilibrium

[14] See Degryse and Nguyen (2004), Mistulli (2005) and Manna and Iazzetta (2009).
[15] See Upper and Worm (2004), and Craig and Peter (2010) for the analysis of German financial network, Boss et al (2004) and Degryse and Nguyen (2004) for Belgium, Mistulli (2005) and Manna and Iazzetta (2009) for Italy, and Bech and Atalay (2008) for the U.S.

network can make this regulatory policy ineffective and high probability of systemic risk[16].

This chapter contributes to the existing literatures in three aspects. First, it is the first to examine financial network with interbank intermediaries. Second, this is the first to analyse the effect of financial regulatory policy on the network formation. Third, the chapter gives one possible explanation on the financial network evolution during 1990s.

The chapter is organized as follows. Section 2 discusses the intuition behind market discipline and the government's bailout policy. Section 3 discusses the related literature. Section 4 presents the network and network equilibrium concept. Section 5 analyses the model. Section 6 examines the network formation game under interbank deposit guarantee. Section 7 studies the network formation if the guarantee is removed, but the government has time-inconsistency problem. Section 8 discusses the model results, and section 9 concludes. The appendices and references are included in the last two sections respectively.

2. Market Discipline and Bailout Policy

Banking theories in the 1980s were mostly based on representative agent models, such as those of Bryan (1980), Diamond and Dybvig (1983), Diamond (1984) and Jacklin and Bhattacharya (1988). They consider the delegated monitoring role played by financial institutions in minimizing financial intermediation costs and curbing fund borrowers' moral hazard problems. These models demonstrate that financial crises

[16] This argument is akin to the second best theory in which it is not necessarily welfare improving if one distortion is removed, given there are many distortions in the economy. The regulator wanted to solve for principle-agent problem whereas accentuated the systemic risk due to time-inconsistent bailout. See also Torre and Ize (2009)

can be induced by a panic-based bank run. They hence support the idea that deposit insurance can help the banking system to prevent bank runs which can severe the economy's ability to channel funds to those with productive investment opportunities.

However, the deposit insurance accentuates the bank owners' moral hazard problem, because losses were incurred that were not borne by them. The moral hazard problem provides a powerful rationale for financial regulation. In 1988, Basel 1 was introduced in order to regulate individual banks, so that the expected losses to the government insurer were minimized. The fundamental principle of the Basel Capital Accord is that bank failures are bad for the economy, so that the goal of financial regulation and the associated principle of prompt corrective action are meant to ensure that such failures are avoided (see Kashyap, Rojan and Stein 2008). The Basel Capital Accord does not consider regulation for interbank lending. Nevertheless, according to the Basel's principle, since bank failures are socially costly, interbank defaults can cause uncertainties about the solvency of other financial institutions, which can cause contagious failure, so it is better to avoid them.

During the 1980s and early 1990s, governments often made implicit or explicit guarantees of repayments for depositors in the interbank market. In the U.S., sellers of fed funds to insolvent institutions were often protected by the Federal Deposit Insurance Corporation (FDIC). King (2008) shows that banks recovered high percentage of their principal if their counterparties failed before early 1990s. Stern and Feldman (2004) show that, between 1979 and 1989, when roughly 1,100 commercial banks failed, 99.7% of all deposit liabilities were fully protected through the discretionary actions of U.S. policymakers. Benston and Kaufman (1998) show that uninsured depositors were protected in nearly 90% of the bank failures until early

1990s[17]. Sironi (2001) shows the same situation in the European banking industries: most bank failures never gave rise to direct losses to their creditors. Uninsured interbank deposits were often paid off through implicit government bailout policies or through explicit government guarantees.

One could argue that the government guarantee on interbank credit risk eliminates the incentive for banks to monitor each other and that this encourages moral hazard behaviour in the interbank market. In the early 1990s, Rochet and Tirole (1996) developed a theory on interbank lending in decentralized system. They suggest that a decentralized operation of interbank lending must be motivated by peer monitoring. In the meantime, criticisms of Basel 1 also surged regarding to the non-risk based, so-called 'standard approach' of financial regulation which ignores market discipline. The implementation of market discipline is often deemed to increase banks' credit risk disclosure in order to encourage peer monitoring between banks[18]. While the first two pillars in Basel 2 focus on credit risk capital requirements and on the future role of national supervisors, the third pillar aims at strengthening the role of market

[17] Note that, during this period, it was not just the large banks whose uninsured creditors received guarantees on their deposits. Additional evidence that 'too big to fail' has not played the key role in producing banking crises is provided by Thorsten Beck, Demirguc-Kunt and Ross Levine (2005), who do not find a positive relationship between banking system concentration and the likelihood of a banking crisis. Although bank bailouts have been the source of serious moral hazard risk taking on the part of banks that has led to very costly banking crises throughout the world, 'too big to fail' has not played a dominant role in most banking crises. It was not dominant in most of the banking crises in developed countries, with the U.S. S&L crisis being one notable example. The savings and loan crisis was not caused to fail: none of these thrift institutions were sufficiently large to pose systemic risk from one of their failures being too large. Instead, it was the result of the willingness of the Federal Home Loan Bank Board and its deposit insurance agency, the Federal Savings and Loan Insurance Corporation, to prop up the entire savings and loan industry, including almost all small S&Ls (Kane 1989). It has been the political process which bails out almost all banking institutions that has been the driving force behind banking crises (see Mishkin, 2006).

[18] The emphasis on market discipline is based on two concerns: one is due to the financial innovation that makes financial regulators difficult to monitor and control individual banks' risk taking behaviour; another one is that in order to reduce the incentives for financial regulation arbitrage, the Basel Committee has proposed an 'Internal Ratings Based' (IRB) approach to capital requirements. This capital adequacy regime gives banks liberty to determine their own capital ratio, but also leads to possible risk taking behaviour, due to information asymmetry. The growing independence of bank management in determining their own capital adequacy must therefore be accompanied by an increasing role of market forces in monitoring banks' risk profiles and influencing their management decisions, thus creating market discipline. The relevance of this role to be played by each individual bank and other private investors has been recognized by the Basel Committee itself.

discipline by encouraging greater bank disclose on credit risk by means of such as subordinating the interbank debt (Basel Committee on Banking Supervision (2001)).

In the meantime, in the U.S., the Federal Deposit Insurance Corporation's Improved Act (FDICIA) was introduced to impose a greater credit risk on uninsured bank liability holders. The introduction of FDICIA in 1991 and National Depositor Preference (NDP) legislation in 1994 mandated least-cost failure resolution and subordinated fed funds to all domestic deposit liabilities and thus increased the expected loss to fed funds sellers in the event of bankruptcy. In particular, prior to this legislation, the pro rata share of a failed bank's assets that was recovered by federal funds lenders was the same as that received by other general creditors. NDP decreased this share by subordinating fed-funds claims to uninsured depositors. Benston and Kaufman (1998) show that uninsured depositors were protected in nearly 90% of the bank failures in the 6 years preceding FDICIA; whereas in the 6 years following, the figure dropped to around 30%.

King (2008), Furfine (2001) and Flannery and Rangan (2004) have found a statistically significant yield response to credit risk in interbank market since mid-1990s. In European banking industries, Sironi (2001, 2003) finds a significant increase in subordinated debt since the mid 1990s[19]. However, these papers also show that a sensitive credit risk will decrease if the borrowing banks are perceived to

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[19] Although there was no obvious regulation change in the European banking industry at that time, Sironi points out, two effects could give rise to increasing concern of market discipline for individual banks. One such effect was the loss that continental European countries' central banks suffered as a consequence of the EMU's monetary policy. A lower degree of freedom in fiscal policy and a transfer of monetary policy to the European Central Bank (ECB) meant that banks became less likely to receive government guarantees. Secondly, the introduction of FDICIA in the U.S. and the criticism of risk based Basel 1 with regard to its lack of market discipline gave rational investors the perception that financial regulators seek to impose greater credit risks on uninsured bank liability holders, by withdrawing from implicit government guarantees. Regulatory and legislative changes in the early 1990s may have reduced the market's perceived probability that a failed bank's creditors would be insured.

receive the government bailout (see King (2008) and Sironi (2003)). The 'Too Big to Fail' problem limits the incentive effects of market discipline.

However, as argued by Mishkin (1995), Freixas, Parigi and Rochet (1998), Freixas (1999) and Hoggarth, Reidhill and Sinclair（2004）the fundamental reason to bail out a bank is that it can eliminate the potential problem of contagion spreading to connected financial institutions, which can lead to systemic risks otherwise. Rochet and Tirole (1996) also argue that size of an individual bank per se cannot be the cause of TBTF. It is more likely to be related to the individual banks' systemic importance in the financial system[20]. FDIC (1998) explains that the primary reason for its decision to bail out Continental Illinois is to avoid systemic risks, such as occurred with the failure of MCorp and the Bank of New England.[21] Paul Volcker, then Chairman of the Federal Reserve, argued that if they had not stepped in, the ultimate systemic crisis that had affected so many other financial institutions would have occurred and wiped out the Western financial system. The 'Too Big to Fail' problem is better described as the "Too interconnected to Fail" (*TITF*) or 'Too Systemic to Fail' (*TSTF*) problem.

Stern and Feldman (2004) argue that FDICIA and other regulations have a systemic risk exception. A bank can be, in effect, too interconnected so that both an insured and uninsured deposit can be fully protected, which would otherwise have serious adverse effects on financial stability. The lack of credibility of the government's commitment to bailing out a systemically important bank manifests itself as a time-inconsistency problem. The following comment on the FDICIA by

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[20] Rochet and Tirole (1996) argue that large institutions such as Drexel and BCCI were allowed to fail because their failure created little systemic risk, as they were somewhat disconnected from the rest of the system.

[21] The 1989 failure of MCorp led to the direct demise of 14 affiliated institutions, and the 1991 failure of the Bank of New England directly triggered the failure of Connecticut Bank and Trust through losses on federal funds sold.

Alan Greenspan (2001) expresses the concern of central banks about the time consistency problem: *“expanded credit risk disclosure will be critical to enhanced market discipline, but the additional information will be irrelevant unless counterparties believe that they are, in fact, at risk.”* Some empirical evidence found by Nier and Baumann (2006) and Sironi (2001) also shows that the effect of disclosure and uninsured funding is reduced when banks enjoy a high degree of government support, pointing to the limits of the effectiveness of market discipline for banks.

3. Related Literature

The first paper on network analysis of the interbank market is by Allen and Gale (2000). The financial linkages between banks arise from the mutual insurance arrangements against liquidity shock. Financial contagion can occur when there is an excessive demand for liquidity in the economy. Since banks’ insurance arrangements make them mutually dependent, this implies that a loss of value in one bank can cause sufficient loss of value in other banks, so precipitating a run. They conclude that, if a bank has more links with other banks, the initial impact of a financial crisis in that bank may be attenuated since each of its neighbours takes a small hit when the bank experiences a run. In other words a ‘complete’ financial network is more resilient than an ‘incomplete’ one. Babus (2007) extends Allen and Gale’s model (2000) using a network formation game. She shows that banks have an incentive to form links with each other in order to insure against the liquidity shock and to reduce the risk of contagion. Leitner (2005) considers the possibility of private bailouts in the financial network. He shows that a bank’s investment return depends positively on the investments of the banks connected to it. Hence, a private bailout is possible when there is a threat of contagion. The idea behind Leitner's model is that banks can be surprised by randomly distributed endowments from agents which can make at least one bank in the network bankrupt. Therefore, an efficient financial network needs to minimize the trade-off between the potential for contagion and risk sharing.

In other research studies, the network effect on financial stability is considered. Freixas, Parigi and Rochet (2000) consider financial contagion in the payment system. Banks face liquidity shocks because depositors are uncertain about where their future locations for consumption are likely to be. Financial linkages are formed by extending credit lines to one another in order to deal with regional liquidity shocks. In their model, financial crisis arise as a result of a coordination failure among depositors. If depositors believe that there will not be enough resources in their future location, they will liquidate their deposits in their original location, making it necessary for depositors in other regions to do the same. Banks are exposed to the positive probability of contagion if the failure of one bank triggers a domino effect.

Cifuentes et al. (2005) developed a model of contagion in which price effect plays an important role. They argue that the market for illiquid assets has a downward sloping residual demand curve, so that more of the illiquid asset will sold by the banks if the price is lower. There are two channels for contagion in their model. One is through the usual bilateral exposures in the interbank market; the other is through the effect of asset price changes on bank capital. Cifuentes et al. (2005) show that for appropriate parameter values the price effect greatly amplifies the extent of contagion.

Other researchers, such as Gai and Kapadia. (2010) and Gai et al. (2010), have applied statistical physics to study contagion in complex networks, as in epidemiological literature. These studies focus on highly interconnected financial networks, where shocks to "super-spreaders" can have significant systemic impact.

With regard to empirical evidence in financial networks, Upper and Worm (2004) show that the current German banking system can be divided into two tiers. Lower tier banks connect with only one of the upper tier banks and upper tier banks intermediate with a variety of other banks. Craig and Peter (2010) report similar findings. Boss et al (2004) find that the degree of distribution of Austrian interbank links follows a power law and its clustering coefficient is relatively small. This means that two banks that have interbank relations with a third bank will have a very low probability of having established interbank connections with each another. Bech and Atalay (2008) also find that, in the U.S., the federal fund network follows a fat-tailed degree distribution, with most banks having few counterparties and a small number having many.

In addition to the above evidence, some researchers have examined the evolution of the financial networks. Perhaps the most interesting findings are by Degryse and Nguyen (2004), Manna and Iazzetta (2009) and Mistulli (2005). Degryse and Nguyen (2004), report that, since the 1990s, the Belgian banking system has shifted away from a symmetric network in which all banks have more or less symmetric exposure to an incomplete network exhibiting core periphery characteristics. Manna and Iazzetta (2009) and Mistulli (2005) also found that evidence for the evolution of the banking system in Italy from symmetric structures towards an asymmetric core periphery pattern since the 1990s.

The network formation game was first considered in terms of non-cooperative game theory by Aumann and Myerson (1988) and Myerson (1977). However, the drawback of this approach is that an empty network is always in a state of Nash equilibrium. To address this issue, one has to consider the effect of coordinated actions and coalitions. The concept of pairwise stability of Jackson and Wolinsky

(1996) formalizes this idea: a network is pairwise stable if every link which is present in the network is profitable for the players involved in it, and for every link which is absent from the network it cannot be that both players are better off by forming the link. While pairwise stability is a useful check for strategic stability, it only allows for the deletion of a single link or the addition of a single link. This consideration leads to many refinements of pairwise stability. Goyal and Joshi (2006) consider examples of pairwise Nash stability whereby an individual can delete multiple links at the same time, rather than just one at a time. Goyal and Vega-Redondo (2007) consider issues related to coordination between two players and propose the establishment of a bilateral equilibrium and strictly bilateral equilibrium. In this chapter we follow the equilibrium concept by Goyal and Vega-Redondo (2007).

4. Network and Equilibrium Concept

Every player makes an announcement of intended links. An intended link $\delta_{ij} \in \{0,1\}$, is where $\delta_{ij} = 1$ indicates that player i intends to form a link with player j, while $\delta_{ij} = 0$ means that player i does not want to form such a link. A strategy of player i is given by $\delta_i = \{\{\delta_{ij}\}_{j \in N \setminus i}\}$ where N represents the set of the players. Let Δ_i denote the strategy set of player i. We consider bilateral agreement of network formation, hence a link between two players i and j will be formed if and only if $\delta_{ij} = \delta_{ji} = 1$. We denote the formed link by $g_{ij} = 1$ and the absence of a link by $g_{ij} = 0$. A strategy profile $\delta = \{\delta_1, \delta_2, ..., \delta_n\}$ therefore induces a network, denoting as $g(\delta)$. Let G denote the set of all networks, and $N_i(g) = \{j = N : j \neq i, g_{ij} = 1\}$ represent the set of players with whom player i has a link in network g and $\eta_i(g) = |N_i(g)|$ denote the cardinality of the set.

In network g, $g+g_{ij}$ denotes the network obtained by replacing $g_{ij}=0$ in network g by $g_{ij}=1$, while $g-g_{ij}$ denotes the network obtained by replacing $g_{ij}=1$ in network g by $g_{ij}=0$. A path between i and j in network g exists if either $g_{ij}=1$ or if there is a distinct set of players $\{i_1, i_2, ..., i_n\}$ such that $g_{ii_1}=g_{ii_2}=,...,=g_{ii_n}$. A network is connected if a path exists between the pair $i, j \in N$. A network, $g' \subset g$ is a component of g if for all $i, j \in g', i \neq j$, there exists a path in g' connecting i and j, and for all $i \in g'$ and $k \in g$, $g_{ik}=1$ implies $k \in g'$.

A network is said to be regular or symmetric if every node has the same number of links, i.e. $\eta_i(g)=\eta, \quad \forall i \in N$. A complete network is a symmetric network in which $\eta = n-1$. A bipartite network is one for which N can be partitioned into the two sets N^A and N^B, where $N = N^A \cup N^B$, so that if $g_{ij}=1$, then one of the nodes comes from N^A and other comes from N^B. A minimally connected network is where there is at most one path connecting any of the two nodes. A core-periphery network structure is minimally connected which describes the following situation. Let $\mathrm{N}_1(g)$ and $\mathrm{N}_k(g)$ represent a division of nodes into two distinct groups, a node belongs to the same group if and only if it has the same number of links to 1 or *k*. The nodes in $\mathrm{N}_1(g)$ constitute the periphery and have a single link with a node in $\mathrm{N}_k(g)$; nodes in the set $\mathrm{N}_k(g)$ constitute the cores and are linked with a subset of nodes in $\mathrm{N}_1(g)$.

Let $u_i(\delta_i, \delta_j)$ denote the payoff to player i. The strategy profile $\delta^* = \left\{\delta_1^*, \delta_2^*, ...\delta_n^*,\right\}$ can be said to have achieved a state of Nash equilibrium if

$$u_i(\delta_i^*, \delta_{-i}^*) \geq u_i(\delta_i, \delta_{-i}^*), \quad \forall \delta_i \in \Delta_i, \quad \forall i \in N$$

In the model a link requires that both players agree to the formation of the link. It is easy to see that an empty network is always a Nash. More generally, for any pair i and j, it is always the best mutual response for a players to offer to form no link. To avoid this potential coordination problem, in this chapter we use the concept of bilateral equilibrium network and strictly bilateral equilibrium network, by Goyal and Vega-Redondo (2007). A bilateral equilibrium network is a network where no agent or pair of agents has the incentive to change its links. No single agent or pair of agents can improve its situation by breaking a link, and for any pair of agents, if one agent could benefit from a new link, the second agent would not and hence the link would not be formed. The bilateral equilibrium network is strict if the existence of deviations that affect the network structure must also affect the payoff for each individual agent involved[22].

Definition 1: A network g is a bilateral equilibrium network if the following conditions hold:

1. There is a Nash strategy profile which supports g so that

$$u_i(\delta_i^*, \delta_{-i}^*) \geq u_i(\delta_i, \delta_{-i}^*), \quad \forall \delta_i \in \Delta_i, \quad \forall i \in N$$

2. For every pair of players $i, j \in N$ and every strategy pair (δ_i, δ_j), we have:

$$u_i(\delta_i, \delta_j, \delta_{-i-j}^*) \geq u_i(\delta_i^*, \delta_j^*, \delta_{-i-j}^*) \Rightarrow u_j(\delta_i, \delta_j, \delta_{-i-j}^*) < u_j(\delta_i^*, \delta_j^*, \delta_{-i-j}^*)$$

Definition 2: A network g is a strict bilateral equilibrium network if the following conditions hold:

[22] The reason we impose two equilibrium concepts is because that strictly bilateral equilibrium yields more stable network. In a dynamic network formation game strictly bilateral equilibrium is more resilient to deviation than bilateral equilibrium. See Jackson (2007) Chapter 11 and Goyal (2008) Chapter 7 for the discussion on the stability of network equilibrium concept.

1. For any $i \in N$ and every $\delta_i \in \Delta_i$ such that

$$g(\delta_i, \delta^*_{-i}) \neq g(\delta^*), \quad u_i(\delta_i, \delta^*_{-i}) < u_i(\delta^*_i, \delta^*_{-i})$$

2. For every pair of players $i, j \in N$ and every strategy pair (δ_i, δ_j), with $g(\delta_i, \delta_j, \delta^*_{-i-j}) \neq g(\delta^*)$ we have

$$u_i(\delta_i, \delta_j, \delta^*_{-i-j}) \geq u_i(\delta^*_i, \delta^*_j, \delta^*_{-i-j}) \Rightarrow u_j(\delta_i, \delta_j, \delta^*_{-i-j}) < u_j(\delta^*_i, \delta^*_j, \delta^*_{-i-j})$$

From the definition we can see that strictly bilateral equilibrium is a bilateral equilibrium.

5. The Model

Our model is a simplified version of Allen and Gale (2000) which explains the premises for a network formation game[23]. The central aim of this section is to provide a micro-foundation for the role of interbank market in reallocating liquidity in the financial system and to construct a model in which a shock within a single bank can propagate to other banks through interbank connections. Although the assumptions seem stylized and restrictive, the model captures the nature of the interactions in a financial network.

[23]In Allen and Gale's (2000) setting, consumers have different liquidity preferences as in Diamond and Dybvig (1983). Each region experiences random fluctuations of liquidity needs for early consumers. They intend to show that the interbank market can decentralize the social planner's solution if banks with a high proportion of early consumers can borrow the liquidity from the banks that have a low proportion of early consumers. Banks can insure themselves against liquidity shocks by exchanging interbank deposits. In our model, we mainly focus on the structural change of financial network due to incentive misalignment between individual banks and the government. We assume liquidity shocks is due to the asset side of the economy, which gives similar insight to Allen and Gale (2000) regarding to liquidity risk sharing.

5.1 Consumers, Government, and Banks

Consider an economy which consists of n regions. In each region there is continuum of consumers of measure 1. There are three time periods: $t = 0,1,2$. At date 0, each consumer is endowed with one unit of consumption good. Consumers have consumption needs at date 1 and 2. Let q denote the amount of consumption good needed at date 1 and 1-q denote the amount they need to consume at date 2. Each Consumer is endowed with a storage technology which allows her to consume their endowment at date 1 and 2. In each region there is a regional bank i. Let $N = \{1,2,...,n\}$ denote the set of the regions, we then have $i \in \{1,2,...,n\}$. Consumers can choose either to invest their endowment in storage technology or lend it to their regional bank.

There is a government in the economy that insures consumers' deposits. We assume the government only plays a passive role in the model. This assumption can be justified if the economy is an evolving, complex system in which no agent can precisely predict the state of nature due to the complex interactions between individual agents. The government can instead only adaptively design a financial policy which is optimal for the present state[24]. Let $\Pi(n)$ denote the government's disutility on bailing out an individual bank, we have:

$$\Pi(n) = \pi(n) - \theta \qquad (1)$$

where θ denotes the cost of bailing out an individual bank i[25], $\pi(n)$ denotes the government's disutility stemming from the failure of n banks triggered by the failure

[24] In this sense our model is akin to the notion of 'self-organizing to criticality' in complex system where for a given financial policy (e.g. 'environment'), in our model market discipline, individual banks can coordinate to the critical point of systemic failure, which induce the 'Too Systemic to Fail' bailout.

[25] The cost we have in mind is not only the fiscal costs of providing funds which induce a distorting effect of tax increases and huge government deficits in the economy, but also the reputational cost of the government in reneging on its *ex ante* commitment not to bail out individual banks.

of bank *i*. This disutility can be interpreted such that the government is concerned with the functionality of the financial system as a whole. The more banks fail the more disruptive for the financial system[26]. We assume that $\theta > \pi(1)$ and $\pi'(n) > 0$. Let n^* denote the number of failed banks such that $\theta = \pi(n^*)$. The government will bail out bank *i* if its failure leads to contagious failure of other $n^* - 1$ banks. For simplicity, we assume that both n and n^* are even numbers, and that n / n^* is divisible.

The banks have no endowment, and they only consume at date 2. Each bank has two investment opportunities: a risk free liquid asset with return of 1 after one period, denoting as short asset, and a risky illiquid asset which need two periods to mature, denoting as long asset. The long asset pays a return of R at date 2 if it is successful, where $R > 1$. At date 1, each long asset suffers from random fluctuations of liquidity shocks with a probability of $(1-\varepsilon)$. Each long asset either faces a liquidity deficit of $\omega(x_i)$, or a liquidity surplus of $\omega(x_i)$, where x_i denotes the amount of funds that bank i invests in long term asset. (A liquidity surplus may arise because part of the long asset matures early: a liquidity deficit may reflect additional investment needs from a long asset.) We assume $x_i > \omega(x_i)$ and $\omega'(x_i) > 0$. The realization of a liquidity shock depends on the state of nature, there being two such states S_1 and S_2 occurring with equal probability $(1-\varepsilon)/2$. In each state, there are *n/*2 banks experiencing liquidity deficit, the other half facing a liquidity surplus. These shocks do not affect solvency of the bank as they are offset by corresponding changes in the final date payoff and do not therefore change the overall return on the long asset. For example, if a long asset generates a surplus (deficit) of $\omega(x_i)$ at date 1, its date 2 return will be $Rx_i - \omega(x_i)$

[26] This assumption thus reflects the argument by Mishkin (1995), Freixas, Parigi and Rochet (1998), Freixas (1999), Rochet and Tirole (1996) and Hoggarth, Reidhill and Sinclair（2004）as we mentioned above. In addition, the more banks failed, the more costly for the government to pay back consumers' deposit which creates more incentive for the government to bail out the initially failed bank.

($Rx_i + \omega(x_i)$) likewise. Nevertheless interim financing of negative liquidity shocks is essential: if a long asset faces a liquidity deficit but cannot receive the required liquidity injection, it will fail with a return equal to zero.

A long asset could also be subject to an idiosyncratic solvency shock which is not insurable by liquidity risk sharing. With probability ε, let one of the long assets suffer a liquidity deficit denoted as $\gamma(x_i)$ at date 1, the rest of the *n*-1 long assets being unaffected. Assume that, if it receives the liquidity injection of $\gamma(x_i)$ from the government, it will generate a return of Rx_i in period 2; and, if not, the return is zero. Since the increased return expected next period will be less than the current liquidity injection, i.e. $Rx_i < \gamma(x_i)$, this idiosyncratic shock involves a reduction of net worth i.e. it is a solvency shock. The question arises is the willingness of the government to supply funds in the face of such a shock. The central bank would normally require good collateral which will afford it from the solvency shock. If the collateral is not available, however, the Treasury or FDIC may be willing to act even in the expectation of financial loss. However, liquidity injection accompanied by solvency risk would only be provided for systemically important banks[27]. Each bank suffers from such idiosyncratic shocks, with probability ε / n. We assume $(1-\varepsilon) > \varepsilon$. We denote the set of idiosyncratic shocks as $\tilde{S}_i \in \{\tilde{S}_1, \tilde{S}_2, ..., \tilde{S}_n\}$, where $\tilde{S}_i$ denotes the state in which bank i suffers from the idiosyncratic shock. The detailed liquidity shocks and idiosyncratic shocks are shown in table 1. If the bank does not suffer from either of these two shocks, the long asset can be liquidated at date 1, with return $r < 1$[28].

[27] In recognition of the solvency risk, the Treasury could make an explicit capital injection to the insolvent bank. For further discussion see Wickens (2012).
[28] See Shleifer and Vishny (1992) and Allen and Gale (1998) for a discussion of costly premature liquidation.

Probability	***State***	***1***	***2***	***…***	***n/2***	***(n+2)/2***	***(n+4)/2***	***…***	***n***
(1-ε)/2	S_1	$-\omega(x_1)$	$-\omega(x_2)$	$-\omega(x_i)$	$-\omega(x_{n/2})$	$+\omega(x_{(n+2)/2})$	$+\omega(x_{(n+4)/2})$	$+\omega(x_j)$	$+\omega(x_n)$
(1-ε)/2	S_2	$+\omega(x_1)$	$+\omega(x_i)$	$+\omega(x_n)$	$+\omega(x_{n/2})$	$-\omega(x_{(n+2)/2})$	$-\omega(x_{(n+4)/2})$	$-\omega(x_j)$	$-\omega(x_n)$
ε/n	$\tilde{S}_1$	$\gamma\,(x_1)$	0	0	0	0	0	0	0
ε/n	$\tilde{S}_2$	0	$\gamma\,(x_2)$	0	0	0	0	0	0
ε/n	$\tilde{S}_3$	0	0	$\gamma\,(x_3)$	0	0	0	0	0
ε/n	**…**	**…**	**…**	**…**	**…**	**…**	**…**	**…**	**…**
ε/n	$\tilde{S}_n$	0	0	0	0	0	0	0	$\gamma\,(x_n)$

Table 1: Liquidity shocks and idiosyncratic shocks at date 1.

5.2 Bank Asset Investment

If consumers lend their endowment to the regional bank, in exchange they receive a non-state contingent debt contract that guarantees them an amount of consumable goods at date 1 and date 2. Since consumers' deposits are insured by the government, each bank finds it optimal to offer *q* at date 1 and 1-*q* at date 2, in order to just meet consumers' reservation utility. Consumers are thus indifferent between investing their endowment in storage technology and lending to the regional bank. Bank fails if it cannot fulfil the contract. Each bank then has 1 unit of endowment to invest in short and long assets. Let y_i denote the amount of deposits that bank i invested in the short asset, we then have banks' budget constraint:

$$x_i + y_i = 1 \qquad (2)$$

Bank receives a positive payoff only if it invests in a long asset, and its expected payoff increases with x_i. However, banks have to invest an amount of funds in short asset in order to satisfy the liquidity demand at date 1. In state S_1 and S_2, bank has to satisfy deposit withdraws q and face either a liquidity deficit or a surplus of $\omega(x_i)$ from the long asset. The liquidity demand at date 1 is thus either $q+\omega(x_i)$ or $q-\omega(x_i)$ for each bank. The expected liquidity need is then q. We assume that for all $x_i > (1-q)$ we have $q < \omega(x_i) + y_i$. This assumption imposes the condition that bank always fails at date 1 if it invests more than $(1-q)$ endowment in long term asset. The assumption implies that the bank always finds it better off to share liquidity risk with other banks, rather than remain in autarky. Since the banks want to economize short assets and the average level of liquidity demand at date 1 is q, we assume that banks can reach an individually optimal equilibrium in which every bank invests $(1-q)$ in long asset, and invests q in liquid asset[29]. We then have:

$$(x_i, y_i) = [(1-q), q] \qquad (3)$$

Equation 3 implies that $\omega(x_i) = \omega(1-q) = \omega$ and $\gamma(x_i) = \gamma(1-q) = \gamma$.

In this case, in state S_1 and S_2, although the number of early and late consumers in each region fluctuates randomly, the aggregate demand for liquidity is constant. In each state there are two different types of banks due to liquidity demands. In particular, at date 1 there are $n/2$ banks each facing a liquidity shortage of ω , whereas the other $n/2$ banks each facing a liquidity surplus of ω .

[29] Alternative to the assumption we can achieve this allocation by imposing the condition that banks do not realise the opportunity for interbank insurance until they finish allocating their endowment for short assets and long assets. In this case, suppose there is a value of ω^* such that bank can hedge the liquidity shock completely by themselves, thus as long as $\omega^* \geq \frac{n(1-\varepsilon)}{2(n-\varepsilon)}(1-q)$, the bank's expected payoff under $(x_i, y_i) = [(1-q), q]$ will always be greater than the expected payoff under $(x_i, y_i) = [(1-q-\omega^*), q+\omega^*]$. It is then optimal for a bank to choose $(x_i, y_i) = [(1-q), q]$ in autarky. After the allocation, when bank realize the opportunity for interbank insurance, they find it optimal to exchange interbank deposits with other bank(s).

5.3 Balance Sheet Linkages

Although banks do not know their type *ex ante*, they have information on how the liquidity shocks are correlated. The different liquidity shocks thus allow for interbank insurance as banks with liquidity surpluses provide liquidity for banks with liquidity shortages at date 1. The provision of insurance can be organized through an exchange of interbank deposits at date 0 before the liquidity shocks are observed[30]. If bank *i* has a higher than average liquidity demand at date 1 it can liquidate its interbank deposits in bank(s) *j*. On the other hand, bank *j* is happy to retain the interbank deposits it holds in bank *i* if bank *j* has enough liquidity to satisfy its own liquidity demand at date 1. At date 2 the process is reversed, as bank *j* liquidates the interbank deposits in bank *i* to meet its last period of deposit withdrawal.

Banks can carry out interbank insurance through two mechanisms: direct liquidity risk sharing and indirect risk sharing. Direct liquidity risk sharing refers to interbank insurance only with bank's immediate neighbour, whereas indirect liquidity risk sharing allows banks to exchange their interbank deposit with all banks in its component through interbank intermediation. However intermediation incurs a small transaction cost *c* for each intermediary, so that any bank that needs interbank intermediation has to pay each of its intermediary *c* in order to compensate the cost[31]. We consider these two mechanisms in the next two subsections respectively.

[30] An important feature of our model is that the swap of deposits occurs *ex ante*, before the state of the world is realized. This prevents cases when lenders having a monopoly of power. For instance, Acharya et al. (2008) emphasizes the problem of *ex post* liquidity transfer: in an *ex post* market of deposits, lenders might take advantage of their position as liquidity providers to extract profit from banks with a shortage of liquidity. To avoid this unfavourable situation, banks prefer to close firm contacts that set the price of liquidity ex ante. Also note that this setting mimics the model where liquidity shock and idiosyncratic shock happens in the same state. For example bank *i* lend to its neighbour bank *j* at date 1, bank *j* fails after the liquidity injection due to idiosyncratic shock, which is unable to pay back the fund to bank *i* at date 2, hence triggers contagious failure.

[31] For the discussion of network intermediation cost see Goyal and Vega-Redondo (2007), Galeotti and Melendez (2004) and Gilles et al (2006).

5.3.1 Direct Liquidity Risk Sharing

Let d_{ij} denote the amount of deposit exchanged between bank *i* and *j* at date 0. We consider that interbank deposit contracts are bilateral, hence $d_{ij} = d_{ji}$. We denote N_i as the set of bank *i*'s neighbour, N_i^{neg} as bank *i*'s set of neighbours with a negative correlation, and $N_i^{pos} = N_i \setminus N_i^{neg}$ as the set of *i*'s neighbours with a positive correlation. We present two examples below to show how banks can insure for liquidity shocks under different network structures.

Example 1: Bipartite network with *n*/2 components: *n*=8, $\eta_i = 1$ and $d_{ij} = \omega$ (see figure 1.*a*)

Each bank is only connected with one negatively correlated bank. They exchange interbank deposit of ω with each other at date 0. Without loss any generality, we can denote the two connected banks as Bank *A* and *B*. Suppose that Bank *A* experiences a high liquidity demand and Bank *B* experiences a low liquidity demand at date 1, Bank *A* has to pay *q* to the depositors and inject ω into the long asset. The total liquidity demand is therefore

$$(q + \omega)$$

On the supply side, Bank *A* has $(q - \omega)$ amount of its own liquid asset, ω amount of liquid assets in the form of Bank *B*'s interbank deposits in *A* and claims to ω amount of deposits in Bank *B*. We then have:

$$q + \omega = (q - \omega) + \omega + \omega$$

The liquidity demand is equal to the liquidity supply, thus the excess demand of liquidity at date 1 is zero for Bank *A*.

As for Bank *B*, it has to pay q to the depositors, and ω to Bank *A*. Its total liquidity demand is also

$$(q+\omega)$$

Bank *B* has $(q-\omega)$ amount of liquid assets, ω amount of interbank deposits and ω amount of payoff from early maturity of long assets. We then have:

$$q+\omega=(q-\omega)+\omega+\omega$$

The excess demand of liquidity at date 1 is also zero for Bank *B*.

At the last date, Bank *A*'s demand for withdraws is $(1-q)$ from its own depositors, ω from Bank *B*. The total payoff from long assets is $R(1-q)+\omega$. Since $R>1$, we then have

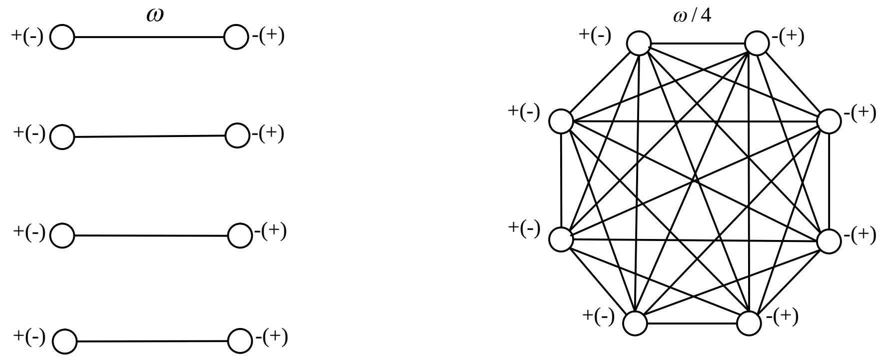
$$(1-q)+\omega<R(1-q)+\omega$$

Bank *A* can satisfy the consumers and interbank deposit withdrawals at date 2 and receive a payoff of $(R-1)(1-q)$.

As for Bank *B* we have:

$$(1-q)<[R(1-q)-\omega]+\omega$$

where the left-hand side is the demand for consumers' deposits and the right-hand side is the total payoff received from long asset and interbank deposits in Bank *A*. Bank *B* receives the same payoff as Bank *A*.



a. Bipartite Network: Each bank deposits ω in one negatively correlated bank.

b: Complete Network: Each bank deposits $\omega/4$ in every bank.

Figure 1

Example 2: Complete connected network: n=8, $\eta_i = 7$ and $d_{ij} = \frac{1}{4}\omega$ (see figure 1.*b*)

Each bank i exchanges interbank deposit of $\frac{1}{4}\omega$ with $n-1$ banks. We then have

$$\sum_{j \in N_i} d_{ij} = \frac{1}{4}\omega \cdot \sum_{h \in N_i^{pos}} g_{ih} + \frac{1}{4}\omega \cdot \sum_{k \in N_i^{neg}} g_{ik} = \frac{1}{4}\omega \cdot 3 + \frac{1}{4}\omega \cdot 4 = \frac{7}{4}\omega$$

Consider a bank that has a high liquidity demand at date 1. It has to pay q to the depositors, inject ω into the long assets, and has to pay $\frac{1}{4}\omega$ to each of its neighbours who also have high liquidity demands. On the supply side, bank i has its own liquid asset $(q - \frac{1}{4}\omega \cdot 7)$, and can withdraw $\frac{1}{4}\omega$ from each bank and use $\frac{1}{4}\omega \cdot 7$ amount of liquid assets in the form of interbank deposits. We then have:

$$q + \omega + \frac{1}{4}\omega \cdot 3 = (q - \frac{1}{4}\omega \cdot 7) + \frac{1}{4}\omega \cdot 7 + \frac{1}{4}\omega \cdot 7$$

where the left-hand side is bank i's total liquidity demand. $\frac{1}{4}\omega \cdot 3$ is the amount of interbank withdrawals by banks with a positive correlation. The right-hand side is the supply of the liquidity. There is no excess demand for liquidity at date 1.

As for a bank with low liquidity demand, we have:

$$q+\frac{1}{4}\omega\cdot 3+\frac{1}{4}\omega\cdot 4=(q-\frac{1}{4}\omega\cdot 7)+\frac{1}{4}\omega\cdot 7+\frac{1}{4}\omega\cdot 3+\omega$$

where the left-hand side is the liquidity demand, which is the sum of consumer's deposit withdrawal q plus the liquidity demand by three banks with a positive correlation $\frac{1}{4}\omega\cdot 3$ and the liquidity demand from four banks with negative correlations. The right-hand side is the liquidity supply, which is the sum of bank's own liquid asset $(q-\frac{1}{4}\omega\cdot 7)$, the liquid asset in the form of the other 7 banks' interbank deposit $\frac{1}{4}\omega\cdot 7$, withdrawals of interbank deposits from the bank with positive correlation $\frac{1}{4}\omega\cdot 3$, and the return from the early maturity of long asset ω. There is again no excess demand for liquidity at date 1.

At date 2, banks with high liquidity demand at date 1 face consumers' withdrawals of $(1-q)$ and $\frac{1}{4}\omega\cdot 4$ interbank deposit withdrawals from banks with a negative correlation at date 1. We then have:

$$(1-q)+\omega<R(1-q)+\omega$$

The bank's payoff at date 2 is $(R-1)(1-q)$.

As for Banks with low liquidity demand at date 1, we have:

$$(1-q)<[R(1-q)-\omega]+\omega$$

The bank's payoff is also $(R-1)(1-q)$ at date 2.

5.3.2 Indirect Liquidity Risk Sharing

Banks can carry out liquidity risk sharing through interbank intermediaries. This means that banks can exchange interbank deposits not only with their immediate neighbours but also with their neighbour's neighbours, and so on. Let $C_i(g)$ denote the set of banks that belongs to the same component as bank *i*, and $C_i^{neg}(g)$ is the set of banks in *i*'s component which have a negative correlation with bank *i*, we thus have $C_i^{pos}(g) = C_i(g) \setminus C_i^{neg}(g)$ as the set of *i*'s neighbours with a positive correlation. At date 0 each bank uses ω amount of funds as an interbank deposit.

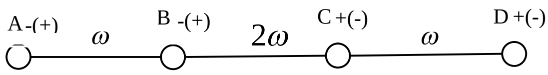


Figure 2

Example 3: String network: $|C_i(g)| = 4$ $|C_i^{pos}(g)| = |C_i^{neg}(g)| = 2$ (see figure 2)

There are four banks in the component in which $i \in \{A, B, C, D\}$. Bank *B* and *C* are interbank intermediaries which transfer interbank deposits not only for each other but also for Bank *A* and *D*. Without losing any generality we assume that Bank *A* and *B* have a negative correlation with Bank *C* and *D*[32].

At date 0, banks arrange interbank insurance by exchange interbank deposits. The process of interbank deposit exchange can be specified as follows. Firstly, Bank *A* and *D* deposit ω in Bank *B* and *C*. Secondly, Bank *B* and *C* exchange 2ω with each other, using their own interbank deposit plus the deposit from *A* and *D* respectively. Finally, Bank *B* and *C* deposit ω back in Bank *A* and *D*. Bank *A* and *B*, and Bank *C* and *D*

[32] Note that banks' type and location in the network do not affect the amount of interbank deposit exchanges at date 0.

have interbank deposit exchange of ω and Bank *B* and *C* have interbank deposit exchange of 2ω as interbank intermediaries, see figure 2. Each bank has interbank deposit of ω in its balance sheet.

Suppose Bank *A* and *B* experience high liquidity demand and Bank *C* and *D* experiences low liquidity demand at date 1. Bank *A* has to pay its consumers *q* and inject ω into the long asset. The total liquidity demand is:

$$q+\omega$$

On the supply side, Bank *A* has $(q-\omega)$ amount of its own liquid asset, ω amount of liquid asset in form of Bank *B*'s deposits and claims of ω amount of interbank deposits in Bank *B*. The total liquidity supply for Bank *A* is thus:

$$(q-\omega)+\omega+\omega=q+\omega$$

Bank A can thus satisfy the liquidity demand at date 1.

As for Bank *B*, it has to pay *q* to the depositors and inject ω into the long asset. In addition, it has to meet the withdrawal of the interbank deposit of ω by Bank *A*. The liquidity demand will then be:

$$q+\omega+\omega$$

On the supply side, Bank *B* has $(q-\omega)$ amount of its own liquid asset, ω amount liquid asset in form of interbank deposits and claims to 2ω interbank deposits in Bank *C*. We then have:

$$(q-\omega)+\omega+2\omega$$

Bank *B* can satisfy the liquidity demand for Bank *A* and its long asset.

Bank *C* has to pay consumer *q*, and it has to transfer 2ω to Bank *B*. Bank *C*'s liquidity demand will then be:

$$q+2\omega$$

On the supply side, Bank *C* has $(q-\omega)$ amount of its own liquid asset, ω amount of funds from early mature of long asset, ω liquid asset in form of interbank deposits, and claims to ω interbank deposits in Bank *D*. Its liquidity supply is:

$$q+2\omega$$

There is no excess liquidity demand for Bank *C*.

As for Bank *D*, it has to pay consumer *q*, and meet the interbank deposit withdrawal from Bank *C*. we have:

$$q+\omega$$

Its liquidity supply consists of $(q-\omega)$ amount of its own liquid asset, ω amount of funds from early maturity of its long assets, and ω liquid assets in form of interbank deposits. The sum is also:

$$q+\omega$$

The total excess liquidity demand in this network is zero at date 1.

At date 2, Bank *A*'s demand for withdraws is $(1-q)$ from its own consumers, ω amount of interbank deposits from Bank *B*, and the total payoff from long assets is $R(1-q)+\omega$. Since $R>1$, we then have

$$(1-q)+\omega < R(1-q)+\omega$$

Bank *A* can satisfy the consumers and interbank deposit withdrawals at date 2 and receive the payoff of $(R-1)(1-q)-2c$, where $2c$ is the cost of interbank intermediation it has to pay to Bank *B* and *C*.

As for Bank *B*, in addition to consumers' demand for $(1-q)$, it also has to meet Bank *C's* interbank deposit withdrawals of 2ω. The total payoff from the long asset is $R(1-q)+\omega$. Bank *B* can liquidate the interbank deposit of ω from Bank *A*. We then have:

$$(1-q)+2\omega < R(1-q)+\omega+\omega$$

Bank *B*'s payoff is $(R-1)(1-q)-c$, where c is the intermediation cost Bank *B* has to pay to Bank *C*.

Bank *C* also has demand of $(1-q)$ from its consumers. In addition it has to return ω amount of interbank deposits to Bank *D*. The total payoff is $R(1-q)-\omega$ from long assets and claims of 2ω from Bank *B*. We then have:

$$(1-q)+\omega < R(1-q)-\omega+2\omega$$

Bank *C*'s payoff is also $(R-1)(1-q)-c$, where c is the intermediation cost Bank *C* has to pay to Bank *B*.

Bank *D* also faces $(1-q)$ withdrawals from its consumers. The total payoff is $R(1-q)-\omega$, from long asset and claims of ω from Bank *C*. We then have:

$$(1-q) < [R(1-q)-\omega]+\omega$$

where the left-hand side is the demand for withdrawals and the right-hand side is the total payoff received from the long asset and interbank deposit in Bank *A*. Bank *D*'s payoff is $(R-1)(1-q)-2c$, since it has to pay c to Bank *C* and Bank *B* for intermediation service.

5.4 Contagious Failure

In the previous section we have shown that liquidity shocks in states S_1 and S_2 create incentives for banks to form connections by exchanging interbank deposits at date 0. However, interbank deposit exchange works well only if there is no excess demand for liquidity in the financial system as a whole. If there are non-insurable idiosyncratic shock in states $\tilde{S}_1,...,\tilde{S}_n$, the cross holdings of interbank deposits can cause problems of contagion. That is, the shock may affect initially only one institution and then spread to other bank(s) through interbank deposit exchange. In order to evaluate the contagion risk we introduce the concept of loss given default (*LDG*) which is the loss of value a bank incurs on its interbank deposits when its counterpart fails at date 1. We illustrate the process of contagious failure using examples 1 and 3.

In example 1, Bank *A* connects with Bank *B* and the amount of interbank deposit exchange is ω. Suppose that Bank *A* suffers from idiosyncratic shock at date 1, it then faces an excessive liquidity demand of γ. Bank A's liquidity demand is thus $(q+\gamma)$, whereas other banks face a liquidity demand of *q* only. Bank *A* has liquid asset which is worth $(q-\omega)$, and it can use ω amount of liquid assets in the form of Bank *B*'s claims and liquidate ω amount of its interbank claims on Bank *B*. However, since Bank *B* also has the liquidity need of *q*, it needs to liquidate the interbank deposit from Bank *A*[33]. We then have:

$$(q-\omega)+\omega+\omega<q+\gamma+\omega$$

where the right-hand side is Bank *A*'s liquidity demand, which is less than the liquidity supply in the left-hand side. Bank *A* cannot meet the liquidity need and the long asset fails. Given that interbank deposits are subordinated debt relative to consumers' deposits, Bank *A* has to satisfy consumers' deposit withdraw of $(1-q)$ before meeting the interbank deposit withdrawal. Since $(1-q)>\omega$, Bank *B* then suffers from a loss given default of ω. To meet the liquidity deficit, Bank *B* has to liquidate an amount of the long asset that equals ω. Liquidating the illiquid asset prematurely, however, incurs the penalty rate $r<1$. The maximum amount of a long

[33] Note that bank finds it optimal to liquidate the interbank deposits before liquidating the long asset, because the return of interbank deposit is one at date 1 and 2, whereas the return of long asset is *r* at date 1 and *R* at date 2. Since $R>r$; the pecking order of asset liquidation is liquidating short asset first, liquidating interbank deposit second and liquidating long asset last, see also Allen and Gale (2000).

asset that can be liquidated without causing bankruptcy is that Bank *B* has to meet consumers' deposit withdrawals $(1-q)$ at date 2. We then have:

$$CB = r \cdot \left[\frac{(R-1)(1-q)}{R} \right] \qquad (4)$$

where *CB* denotes Banks' capital buffer which depends on the rate of early liquidation of long asset r, the amount of consumer's deposit withdraw at date 2 and the return of a long asset at date 2. If the capital buffer is less than the loss given default, i.e. $CB < \omega$, Bank *B* fails due to contagion. In general, bank *i* suffers from contagious failure as long as

$$CB < LGD_i \qquad (5)$$

With regard to example 3, if Bank *A* suffers from idiosyncratic shock at date 1, by the same argument, Bank *A* cannot meet the liquidity need and the long asset fails. Bank *B* then suffers from a loss given default of ω. Hence, to meet its liquidity deficit, Bank *B* has to liquidate its interbank deposit 2ω from Bank *C*. However, Bank *C* also needs to liquidate the interbank deposit of 2ω from Bank *B*. We then have:

$$(q-\omega) + 2\omega < q + 2\omega$$

where the left-hand side is Bank *B*'s liquidity supply and the right-hand side its liquidity demand. Bank *B* faces an excess liquidity demand of ω. Again, since the capital buffer is less than the loss given default, Bank *B* fails. It has to liquidate all its long assets at date 1. Bank *B*'s liquidity supply consist of early liquidation of its long assets at date 1 $r(1-q)$, short assets $(q-\omega)$, and interbank deposits, 2ω, from Bank C. Its liquidity demand is 1 from consumer deposit withdrawal and 2ω from Bank *C*'s interbank deposit withdraw. If *r* is small, we then have:

$$r(1-q) + (q+\omega) \leq 1 \qquad (6)$$

Under this condition, Bank C suffers from a loss given default of 2ω. Bank C's liquidity supply is the sum of own short asset $(q-\omega)$, interbank deposit ω, and its interbank deposit ω in Bank D, subtracting the loss given default 2ω.

$$(q-\omega)+\omega+\omega-2\omega$$

The total liquidity supply for Bank C is then $(q-\omega)$, which consists of its liquidity demand q from consumers' deposits withdraw and ω from Bank D's interbank deposit withdraw. Bank C has to liquidate an amount of long asset equal to 2ω. Since $CB<\omega$, Bank C then fails. Its total liquidity supply is then

$$r(1-q)+(q-\omega)$$

which is less than 1 given condition 6. Bank D thus suffers from a loss given default of ω, which also fails given that the same argument as described above applies.

6. Network and Interbank Deposit Guarantee

We first consider the network formation game where interbank deposits are guaranteed by the government. This implies that the failure of an individual bank's neighbour does not impose any contagious failure on itself. Hence the loss given default is equal to zero. In other words, equilibrium network is independent of idiosyncratic shock in state $\tilde{S}_i$. The network formation incentive for each individual bank is only to share the liquidity risk. Since interbank intermediation incurs costs for banks that need the service, it is intuitive that no bank is willing to pay intermediation cost if there is opportunity of direct risk sharing. We show in the appendices that it is always optimal to have direct risk sharing if there is interbank deposit guarantee.

Under direct liquidity risk sharing, banks can form a bilateral link with each other by exchanging interbank deposit at date 0. The total amount of deposits that i exchanged with its neighbours should balance out its liquidity shortage or excess. In other word, bank *i* has to ensure that there are always at least ω amount of interbank deposit retained in its balance sheet if it faces a liquidity deficit at date 1.

Since interbank insurance against liquidity shocks is provided only through links with banks of a different type, sufficient condition for complete liquidity risk sharing should satisfy:

$$\sum_{j \in N_i^{neg}} d_{ij} \geq \omega \qquad (7)$$

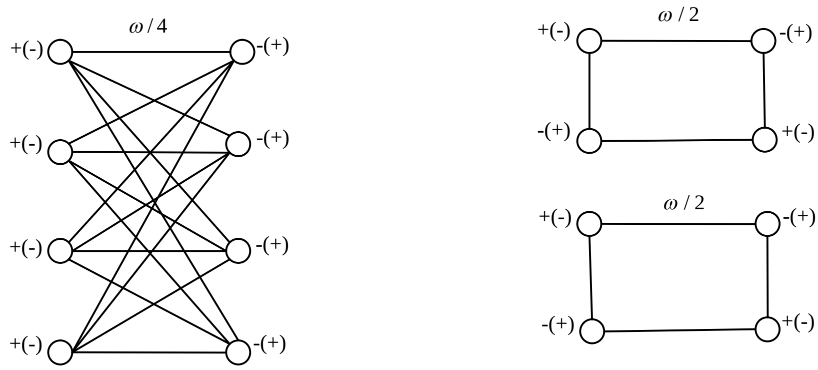
Let $I_{\{\sum_{j \in N_i^{neg}} d_{ij} \geq \omega\}} \in \{0,1\}$ denote the indicator function specifying whether bank *i* has interbank deposit exchange satisfying condition (7). The indicator function is equal to 1 if condition (7) is satisfied and 0 otherwise. The individual bank's expected payoff function can be written as:

$$u_i(g) = (1-\varepsilon)[\frac{1}{2}(R-1)(1-q) + \frac{1}{2}(R-1)(1-q) \cdot I_{\{\sum_{j \in N_i^{neg}} d_{ij} \geq \omega\}}] + \frac{\varepsilon(n-1)}{n}(R-1)(1-q) \quad (8)$$

where the first term is the bank's expected return in state S_1 and S_2. The first term in the square bracket is bank *i*'s expected payoff if it has a liquidity surplus and the second term is its expected payoff if it has a liquidity deficit. The liquidity deficit can be satisfied only if there are at least ω amount of interbank deposits can be retained in bank *i*, as in the examples we presented above. The second term is the expected payoff in the state of idiosyncratic shock $\tilde{S}_i$. Since there is interbank deposit guarantee, the banks are safe from their neighbour's failure. Also, because of $\theta > \pi(1)$, there will be no bailout on individual banks. Bank *i* thus suffers from idiosyncratic shock with probability $\frac{\varepsilon}{n}$. We can show that:

Proposition 1: Under interbank deposit insurance with positive intermediation costs, the bilateral equilibrium networks do not have interbank intermediaries. The bilateral equilibrium networks satisfy $\sum_{j \in N_i^{neg}} d_{ij} = \omega$ and include symmetric structure. The only strictly bilateral equilibrium network is bipartite network with $n/2$ components.

Proof: see Appendices



a: Each bank deposits $\omega/4$ in each negatively correlated bank.

d: Each bank deposits $\omega/2$ in two negatively correlated banks.

Figure 3

The intuition is the following. In state S_1 and S_2 the aggregate liquidity demand is equal to the aggregate liquidity supply. In any network, if there is one bank that has $\sum_{j \in N_i^{neg}} d_{ij} < \omega$, there must be another bank that is negatively correlated with this bank also has $\sum_{j \in N_i^{neg}} d_{ij} < \omega$. This then violates the definition of bilateral equilibrium because these two banks can coordinate to drop all their links and form a link with each other which gives higher expected payoff. Since the bank's link with positive correlated banks has no effect on liquidity risk sharing, banks are indifferent when connecting with these banks. We then have that symmetric network with $\sum_{j \in N_i^{neg}} d_{ij} = \omega$ satisfies bilateral equilibrium, such as the complete network in Figure 1.*b*. The other regular network structures, as shown in Figure 1.*a* and Figure 3, are also bilateral equilibrium networks. The bipartite network with $n/2$ components, as in Figure 1.*a*, is the only strictly bilateral equilibrium network, because in any other bilateral equilibrium network any two banks with negative correlation can coordinate to delete all their links and form a link with each other which gives them the same expected payoffs.

7. Network, Market Discipline, and Bailout

In this section, we examine the equilibrium network under market discipline where interbank deposit guarantee is removed. As a result, the non insurable idiosyncratic shocks bear the risk of contagion, as we shown in section 5.4. However, the government suffers from the 'Too Systemic to Fail' bailout. Individual banks know their interbank deposit will be implicitly guaranteed if the bank that suffers from idiosyncratic shock can receive a bailout. The *TSTF* bailout is determined by two factors. Firstly, as we have shown in section 5.4, it depends on the value of capital buffer and the amount of loss given default. The contagious failure happens as long as condition 5 is satisfied, which implies that the contagious failure increases with amount of interbank deposit exchange. In this chapter we impose the condition $CB < \omega$. Secondly, the *TSTF* bailout depends on government's disutility for bailing out individual banks, which is the function of number of failed banks induced by contagion, as shown in equation 1. Individual banks take account of the government's *ex post* incentive problem when forming a liquidity risk sharing network. In order to be able to write an individual bank's payoff function explicitly, we assume[34]:

$$r(1-q) + q + (n^* - 2)\omega \leq 1 \qquad (9)$$

[34] This assumption is in line with condition 6. Note that this assumption is not crucial for our general results. We will discuss this assumption in the next section.

We show how the network with interbank intermediaries can be induced by market discipline and a *TSTF* bailout. Let $I_{\{C_i^{pos}(g)\cup i=C_i^{neg}\}}\in\{0,1\}$ denote the indicator function which is equal to 1 if the number of banks in bank *i*'s component with positive correlation is the same as the number of banks with negative correlation, and 0 otherwise[35], $I_{\{|C_i|\geq n^*\}}$ denote the indicator function which is 1 if the number of banks in bank *i*'s component is no less than n^*, and 0 otherwise, and $I_{\{|C_i|<n^*\}}$ denotes the indicator function which is 0 if the number of banks in bank *i*'s component is no less than n^*, and 1 otherwise. Finally we denote $e(i, j\in C_i(g))$ as the number of bank *i*'s interbank intermediaries. We can then express the individual bank's expected payoff under interbank intermediation as:

$$\begin{aligned} u_i(g) &= (1-\varepsilon)[\frac{1}{2}(R-1)(1-q)+\frac{1}{2}(R-1)(1-q)\cdot I_{\{C_i^{pos}(g)\cup i=C_i^{neg}\}}] \\ &\quad +\varepsilon(R-1)(1-q)[\frac{n-|C_i(g)+i|}{n}I_{\{|C_i|<n^*\}}+I_{\{|C_i|\geq n^*\}}]-e(i,j\in C_i(g))\cdot c \end{aligned} \tag{10}$$

where the first term in equation 10 is the bank's expected payoff in state S_1 and S_2. The first term in the first square bracket is the expected payoff if the bank has a liquidity surplus at date 1, and the second term is the expected payoff if the bank is in liquidity deficit. The value depends on the number of negatively and positively correlated banks in the component. The second term in equation 10 refers to the expected payoff for banks in the state of idiosyncratic shock $\tilde{S}_i$. If the number of banks in *i*'s component is less than n^*, the government will not initiate the bailout.

[35] This indicator function rules out the possibility that there is unequal number of banks with different types in equilibrium. This is quite intuitive because since there is no aggregate liquidity shock in state S_1 and S_2, and also since $(1-\varepsilon)>\varepsilon$, if there is one component with unequal number of banks of different types, it must be true that there is another component which also has unequal number of banks of different types. This means that any two banks that are negatively correlated in these two components respectively can both reach better expected payoffs by deleting all their links and form a link with each other. The detailed discussion is in the appendices.

Given $CB < \omega$ and condition 9, all banks in the component will fail, if any one of them goes bankrupt. The probability of bank *i*'s contagious failure is thus $\varepsilon \cdot \frac{|C_i(g)|}{n}$. If the number of banks in *i*'s component is no less than n^*, then the failure of any one of the banks in the component will trigger the bailout. For $I_{\{|C_i| \geq n^*\}}$, the bank's payoff in state $\tilde{S}_i$ will then be $(R-1)(1-q)$. The last term represents the amount bank *i* has to pay to its intermediaries. We can show that:

Proposition 2: under market discipline and *TSTF* bailout, given $CB < \omega$ and condition 9, if $0 < c < \frac{2}{n}(R-1)(1-q)$, the strictly bilateral equilibrium network with interbank intermediaries exhibits core-periphery structure.

Proof: see Appendix

The intuition is as follows. If the network consists of several components it must be true that in each component there are equal number of negatively and positively correlated banks. This is intuitive given equation 10 and $(1-\varepsilon) > \varepsilon$, because, otherwise, there are always two negatively correlated banks, each belonging to different components with both having unequal amount of banks with different types, finding optimal to delete all their links and reach a higher expected payoff by forming a link with each other.

Next, we show that in a component with interbank intermediaries the number of banks must be at least n^*. Otherwise the probability of contagious failure is greater than the bank only has one connection with its negatively correlated bank, which only has probability of $\varepsilon \cdot \frac{1}{n}$ of contagious failure.

We then show that in each component with interbank intermediaries it must be minimally connected because, intuitively, if there are more than two shortest paths, the total interbank deposit exchange ω is divided among the paths. If condition 5 satisfies $\omega - \xi < CB < \omega$ where ξ is infinitely small, then the failure of its neighbour does not necessarily impose failure to the bank hence does not trigger the bailout. Banks have incentive to avoid having more than two paths. A more careful proof on this argument regarding to saving the cost of interbank intermediation is discussed in the appendices.

Finally we can show that the strictly bilateral equilibrium network exhibits core periphery structure because any periphery bank has incentive to disconnect its neighbour bank(s) and connect with other intermediary which has high centrality. This is because the deviation can increase the number of periphery nodes which by definition decreases number of intermediaries, thus deceases the cost of intermediation for each bank. We then have agglomeration effect which generates core periphery structure. In equilibrium, the peripheral banks' the expected payoffs equal to

$$(R-1)(1-q)-c \qquad (11)$$

The core banks' expected payoff is then:

$$(R-1)(1-q) \qquad (12)$$

8. Discussion

8.1 Robustness of the Model

This section demonstrates that condition 9 is not a necessary condition to derive our result. Suppose condition 9 is violated, that is $r(1-q)+q+(n^{*}-2)\omega > 1$ and capital buffer satisfies $\omega-\xi < CB < \omega$, where ξ is infinitely small. We can show that the string network for each component having n^{*} number of banks can also trigger the bailout, as long as condition 6 is satisfied, that is $r(1-q)+q+\omega \geq 1$. Suppose that $n^{*}=6$, a string network with 6 banks is illustrated in figure 4. The process of interbank deposit exchange at date 0 is the same as in example 3, and the amount of interbank exchange is specified in figure 4.

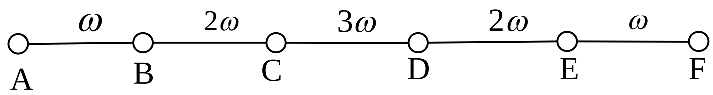


Figure 4

Without loss of any generality, we assume that Bank *A* suffers from idiosyncratic shock and failure[36]. Bank *B* then suffers from a loss given default of ω . Bank *B* fails with a total liquidity supply equal to $r(1-q)+(q-\omega)+2\omega$, where 2ω is Bank *B*'s interbank deposit withdraw from Bank *C*. Given condition 6, Bank *C* thus suffers from loss given default of 2ω, and by the same argument Bank *C* fails with a total liquidity supply equal to $r(1-q)+(q-\omega)-\omega+3\omega$, which again satisfies condition 6. Bank *D* thus suffers from a loss given default of 3ω. By the same argument Bank *E* fails and so does Bank *F*. This then triggers the bailout of Bank *A*.

[36] Note that the failure of any bank in the string network can trigger the same effect.

In the next example we show that a core-periphery structure can still be an equilibrium, even if $r(1-q)+q(n^{*}-2)\omega > 1$.

Example 4: Supposing we have $n^{*} = 12$ and $r(1-q)+q+5\omega = 1$, we can show that a core-periphery network with 12 banks can trigger the bailout.

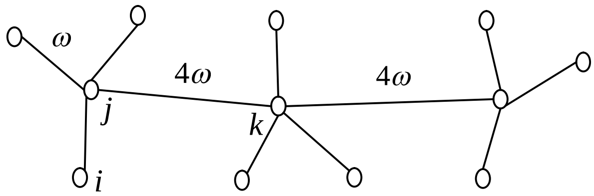


Figure 5

Considering Figure 5, where there is a core-periphery network with three core banks, each possessing three periphery banks. Each periphery bank deposits ω in its core bank, and the core banks exchange 4ω with each other, and then deposit ω back to its periphery banks. If one of the periphery banks *i* fails (same argument holds for the failure of one of the core banks), its core bank *j* will fail and the total liquidity supply is $r(1-q)+q+5\omega$, which is equal to 1. Thus all other periphery banks fail. The other core bank *k* will also fail, with a total liquidity supply equal to $r(1-q)+q+4\omega$ which is also less than 1. By the same argument, all the rest of banks in the network fail. The government will then have to bail out the initially failed bank. Figure 5 is the strictly bilateral equilibrium network, if we have $c < \frac{2}{3n}(R-1)(1-q)$.

8.2 Circuit Breaker and Network Formation

Our result of core-periphery structure as strictly bilateral equilibrium relies on two assumptions: first, the government's time-inconsistency on bailing out systemically important bank, second, banks have the incentive to avoid high interbank intermediation costs which they have to pay in order to compensate for the cost of deposit transfer by their intermediaries. This then induces an agglomeration effect on generating core-periphery structure. In this section we show that the core-periphery structure can still be a strict equilibrium, even if the cost of interbank intermediation has to be paid for by intermediaries themselves.

A smart government would implement a bailout policy as a circuit breaker so that it only bail out those banks who are about to fail due to contagion, as argued by Rochet and Tirole (1996). In other words, the government only bails out banks who suffer from contagious failure rather than direct idiosyncratic shocks[37]. By doing so, the government on the one hand prevents the systemic risk, and on the other hand could decrease the bank's incentive for risk shifting because there is positive probability of failure. Considering the string network in figure 2, if Bank *A* fails, the government bails out Bank *B* instead. However, as argued by Rochet and Tirole (1996) the use of bailout as circuit breaker suffers from transaction costs. They point out that, to operate the rescue, the government must have a clear picture of mutual positions, priority rules in bankruptcy and solvent banks' needing cash infusions. The government must exercise difficult judgements on each bank over a short time span in order to operate this selective rescue properly. If these costs increase with number of banks they are dealing with, it is less costly to rescue the bank which triggered the systemic risk instead. In this situation, in the core periphery network, a core bank always gets the

[37] Note that the circuit breaker bailout is not the same as interbank deposit guarantee since the former is conditional on systemic risk, hence network structure.

bailout if the transaction cost of bailing out all periphery banks is greater than the cost of rescuing the core bank. Each periphery bank is likely to fail due to idiosyncratic shock, with a probability of $\varepsilon \cdot \frac{1}{n}$. The core periphery structure is strictly bilateral equilibrium in a size n^* component with equal number of different type of banks, if the cost of intermediation for the core banks is no more than the benefit it gets from the bailout, that is $c < \frac{2}{(n^*-1)n}(R-1)(1-q)$, and it is strictly beneficial for peripheral banks not to deviate from core-periphery structure. In this case the payoff for core bank is

$$(R-1)(1-q)-(n^*-1)c \qquad (13)$$

And the expected payoffs for periphery banks are:

$$(1-\varepsilon)(R-1)(1-q)+\varepsilon\frac{(n-1)}{n}(R-1)(1-q) \qquad (14)$$

8.3 Market Discipline and Risk-Shifting Problems

The government's desire to encourage market discipline is because market discipline is deemed to be an incentive scheme in which investors in subordinated debt can take account of the probability of contagious failure hence be able to limit the greater risk taking behaviour of the borrowers. However, effectiveness of market discipline depends on the extent of the government's bailout. We showed that given time-inconsistency problem banks can coordinate to a network structure with more number of banks in the component and more susceptible to systemic risk.

Although we do not model banks' individual risk-shifting problem explicitly, we can still see that the withdraw of interbank deposit guarantee and the present of *TSTF* bailout leads to even greater risk-shifting incentive since the banks' expected payoff is independent of the idiosyncratic shock and contagious failure as shown in equation 11 and 12. Also if individual bank's expected payoff is increasing with level of the risk-shifting, then equation 13 is easier to hold since the core bank can always compensate the cost of intermediation by investing in risky asset than periphery banks, which makes the financial system even more fragile.

9. Conclusion

This chapter examines how financial regulation based on individual banks can affect the interconnectedness of financial network and the systemic risk using network formation game approach. We suggest that any regulation targets only on individual banks or takes financial network structure as given could ignore the effect of this very regulation on the strategic interactions between banks which could end up with a new equilibrium network structure which make the *ex ante* regulatory policy ineffective.

The chapter thus implies that the financial authorities, such as FDICIA, needs to prompt corrective action on monitoring systemic risk rather than only focus on individual risk. After all, it is the functionality of financial system as a whole that really matters. Key policy issue facing FDICIA today is the need for systemic regulation on regulating individual banks' interacting behaviour.

Our analysis supports the recent proposal by the Independent Commission for Banking chaired by Sir John Vickers (see ICB, 2011). The ICB's financial report introduces the idea of "ring fencing" banks that supply essential commercial banking services. By limiting their investment banking activities, this is equivalent to breaking up or disentangling the highly interconnected financial network. Ring fencing, as Miller and Zhang (2012) suggest, is similar to imposing *ex ante* circuit-breakers, thus limiting the spread of systemic risks *ex post.* In this chapter a "ring fence" would separate the large financial network component into several small ones, thus decreasing the systemic importance of individual banks. By lowering the incentive for systemic bailout, it can help restore market discipline.

Haldane and May's (2011) suggest that regulatory capital and/or liquidity ratios should be designed so as to limit the potential for network spillovers. They recommend that high ratios being imposed on banks deemed systemically important. Our model implies that such systemic regulation can have two possible effects. First, if a systemically important bank chooses to hold higher ratio of capital or liquid assets, it is less likely to trigger the contagious failure. Second, if high cost of capital or low return on liquid assets make it unattractive to be systemically important, the regulation can induce banks to break up the highly interconnected network, yielding a bipartite network with many components, which as we have shown in section 6, will decrease the number banks suffering from contagious failure.

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Appendices

Proposition 1*:* Under interbank deposit insurance with positive intermediation costs, the bilateral equilibrium networks do not have interbank intermediaries. The bilateral equilibrium networks satisfy $\sum_{j \in N_i^{neg}} d_{ij} = \omega$ and include symmetric structure. The only strictly bilateral equilibrium network is bipartite network with $n/2$ components.

Proof: In order to make the proof tractable, we impose the condition on the amount of interbank deposit exchange between two banks. The amount of deposit exchange between two negatively correlated banks is determined by the bank that has the most number of links of negatively correlated banks. We then have:

$$d_{ij} = \min\{\frac{\omega}{\eta_i^{neg}}, \frac{\omega}{\eta_j^{neg}}\} \quad \forall i \in N^{pos}, j \in N^{Neg}, g_{ij} = 1 \qquad \text{(A.1)}$$

where η_i^{neg} denotes the number of i's neighbours with negative correlation, N^{pos} denotes the set of banks with liquidity deficits (surpluses) and N^{neg} denotes the number of banks with liquidity surpluses (deficits). We then have $|N^{neg}| = |N^{pos}| = n/2$.

We can first show that bipartite network with $n/2$ components is a bilateral equilibrium network as in figure 1.a with $\sum_{j \in N_i^{neg}} d_{ij} = \omega$. Each bank's expected payoff in this network is thus:

$$u_i(g) = (1-\varepsilon)[(R-1)(1-q)] + \frac{\varepsilon(n-1)}{n}(R-1)(1-q) \qquad \text{(A.2)}$$

By the definition of strictly bilateral equilibrium no pair of banks can coordination to reach higher or equivalent expected payoffs by deviating from this network structure.

We then show that bilateral equilibrium network must satisfy $\sum_{j \in N_i^{neg}} d_{ij} = \omega$. The proof is based on the argument that there is no aggregate liquidity demand and supply in state S_1 and S_2. Suppose there is a bank i which has $\sum_{j \in N_i^{neg}} d_{ij} < \omega$, then under condition A.1 there must be at least one bank, say bank *j*, which is negatively correlated with bank *i* also has $\sum_{j \in N_i^{neg}} d_{ij} < \omega$. By the definition of bilateral equilibrium, banks *i* and *j* can coordinate with each other to reach the payoff of A.2, which is better for both banks, either by forming a link with each other, if $g + g_{ij}$ implies $\sum_{j \in N_i^{neg}} d_{ij} \geq \omega$ and $\sum_{k \in N_j^{neg}} d_{jk} \geq \omega$, the additional link gives higher expected payoff for both *i* and *j*, or deleting all their links and forming a link with each other which can both have $\sum_{j \in N_i^{neg}} d_{ij} = \omega$ and reaches the payoff in A.2.

Since there is no aggregate shock in state S_1 and S_2, if there is one bank with $\sum_{j \in N_i^{neg}} d_{ij} > \omega$, it must be true that there are two other banks with $\sum_{j \in N_i^{neg}} d_{ij} < \omega$ which are negatively correlated. By the same argument above, this is not a bilateral equilibrium network. The equilibrium network has to be $\sum_{j \in N_i^{neg}} d_{ij} = \omega$.

Links between two banks with positive correlations do not have an effect on liquidity risk sharing, so that banks are indifferent in forming a link with positively correlated banks. We then have that any symmetric network that satisfies condition $\sum_{j \in N_i^{neg}} d_{ij} = \omega$ is a bilateral equilibrium network. In any bilateral equilibrium network, any two banks with negative correlations can delete all their other links and form a link with each other. This is a payoff equivalent to the original equilibrium network with some structural changes applied. By the definition of a strictly bilateral equilibrium, the only network that satisfies the criteria is the bipartite network with $n/2$ components.

With regard to network with interbank intermediaries, by the definition of intermediation it must be true that there are two negatively correlated banks, say *i* and *j*, that are path connected. Each bank has to pay a cost of at least *c* to compensate the intermediary. These two banks can reach the payoff of A.2, by deleting all their links and forming a link with each other, which violate the definition of bilateral equilibrium.

Proposition 2: under market discipline and *TSTF* bailout, given $CB < \omega$ and condition 9, if $0 < c < \frac{2}{n}(R-1)(1-q)$, then the strictly bilateral equilibrium network with interbank intermediaries exhibits core-periphery structure.

Proof: first we show that in equilibrium network, each component contains an equal number of negatively and positively correlated banks. Contrary to what was asserted, suppose there is one component, denoting *A*, with unequal number of different types of banks, without any loss of generality, we assume $\left|C_A^{neg}\right| > \left|C_A^{pos}\right|$. Since on aggregate there is no excess demand for liquidity, it must be true that there is another component, say *B*, which has $\left|C_B^{neg}\right| < \left|C_B^{pos}\right|$. It must be true that in component *A* and *B* there are two banks *i* and *j* that are negatively correlated which have the maximum expected payoffs of:

$$(1-\varepsilon)\frac{1}{2}(R-1)(1-q)+\varepsilon(R-1)(1-q) \qquad \text{(A.3)}$$

However, if they could delete all their links and form a link with each other, then their minimum payoffs are equal to:

$$(1-\varepsilon)(R-1)(1-q)+\varepsilon\cdot\frac{n-2}{n}(R-1)(1-q) \qquad \text{(A.4)}$$

Therefore, for $\varepsilon \leq \dfrac{n}{n+4}$, the argument holds.

Next, we show that in a component with interbank intermediaries, the number of banks must be at least n^*. The proof above implies that the number of banks in a component must be even. If we suppose there is one component containing interbank intermediaries with $n^* - 2$ number of banks, then the bank's maximum payoff for this component is:

$$(1-\varepsilon)(R-1)(1-q) + \varepsilon \cdot \frac{n-(n^*-2)}{n}(R-1)(1-q) \qquad \text{(A.5)}$$

By the definition of interbank intermediary, we must have $n^* - 2 \geq 4$. The proof above argues that there must be an equal number of banks with different types, and then implies that in the component any two banks with negative correlations can delete all their links to connect with each other. The expected payoffs for these two banks are given by A.4, which is better than A.5.

Next we show that any component with interbank intermediation has to be minimally connected. There are two ways to prove this argument. The first one is intuitive, and is based on condition 5. It is natural to impose the condition that interbank intermediation can only take place on the shortest path, if there are more than two short paths, and the interbank deposit exchange is divided among the shortest paths. Suppose condition 5 satisfies $\omega - \xi < CB < \omega$ where ξ is infinitely small, then the failure of its neighbour does not necessarily impose failure on the bank, hence does not trigger the bailout. If there are two shortest paths connecting negatively correlated bank *i* and *j*. either of these Banks have an incentive unilaterally break one path. Alternatively, since it does not trigger the bailout, they can coordinate to delete all their links and reach better payoff of A.4 by forming a link with each other.

The second proof relies on the definition of interbank intermediaries. Let $E(i, j; C_i(g))$ denotes the set of banks connecting bank *i* and *j* in the shortest path(s), then bank *i*'s total interbank intermediaries can be expressed as:

$$e(i, j \in C_i(g)) = \left| \bigcup_{j \in N \setminus i} E(i, j; C_i(g)) \right| \qquad \text{(A.6)}$$

We can first consider the cycle with an even number of banks. Let bank *i* and bank *j* denote two banks who are the furthest away from each other. Let χ_i denotes the set of banks which belong to the cycle, there are then $\frac{|\chi_i| - 2}{2}$ number of bank on each side of bank *i*. There are two shortest path connecting *i* and *j*. According to A.6, The total interbank intermediaries for bank *i* are $|\chi_i| - 2$. Bank *i* and *j*, can strictly increase their payoff if they can coordinate with each other, by deleting only one link and forming a link with each other. This deviation can increase the number of periphery nodes which by definition decrease the number of intermediaries, thus decreasing the cost of intermediation for each bank. The marginal payoff for bank *i* and *j* is at least *c*. (see Figure 6 below).

Figure 6

A similar argument for a cycle with odd numbers can also be constructed. Suppose the component contains a cycle with *m* number of banks, where *m* is an odd number and where $m \geq 5$, since for *m* to be equal to 3, there are no interbank intermediaries in the cycle. If bank *i* belongs to the cycle, then it must be true that there are $m-3$ banks intermediating for bank *i*. Bank *i* can coordinate with one of the banks which is the furthest distance away to *i*, say Bank *j*, to each delete one link and link up with each other. The marginal payoff for bank *i* is then zero, and *c* for bank *j* which again violates the concept of bilateral equilibrium. See figure 7 for an example.

Figure 7

The minimally connected component implies equation A.6 can be written as:

$$e(i, j \in C_i(g)) = |C_i(g)| - |N_{j \in N}^{\eta_j = 1}| - I(\eta_i \geq 2) \qquad \text{(A.7)}$$

where $|N_{j \in N}^{\eta_j = 1}|$ is number of banks with only one link, and $I(\eta_i \geq 2)$ is an indicator function which is equal to one if *i* has at least 2 links. In any minimally connected component, there must be at least two periphery banks, because a string network contains the maximum number of interbank intermediaries with only two banks on either side of a string which does not intermediate. Let bank *i* denote a periphery bank, and bank *k* denote bank *i*'s neighbour bank. Bank *i* can delete the link with bank *k* and link with bank *j*, where $\eta_j \geq 2 \;\; \forall j \notin N_i$. This deviation can give bank *i* marginal payoff of *c*, since under the new structure the total number of banks with one link increases to $|N_j^{\eta_j = 1}| + 1$. Bank *j* is also willing to link with *i*, since the marginal payoff for bank j is also *c*, see Figure 8. The repetition of the process is the agglomeration effect which generates the core periphery structure.

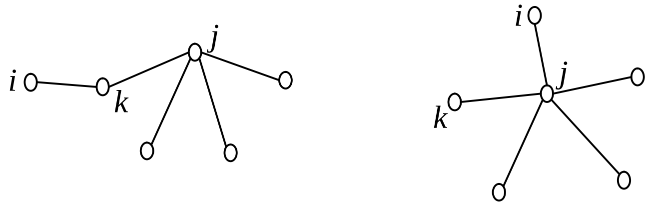

Figure 8

Chapter 3

Capital Requirement, “Too Many to Fail” and Systemic Risk

Summary

This chapter argues that market price sensitive capital regulation combined with the government’s bailout policy can induce banks to herd by investing in highly correlated assets. We follow Acharya and Yorulmazer’s (2007) study of the “too many to fail” problem in a two-bank model. They argue that in order to reduce social losses due to the systemic risk of banks failing together, the financial regulator finds it *ex post* optimal to bail out every troubled bank, because the acquisition of liquidated assets by other investors result in a high misallocation cost. In contrast to their paper, we argue that as long as it is profitable for banks to purchase the liquidated assets at the cash-in-the-market price, in the state of systemic failure, the regulator can always commit to randomly bailing out only just one bank and letting the bailed out bank purchase the failed bank’s assets– there is no “too many to fail” bailout. We then show that market price sensitive capital regulation can remove banks’ incentive to purchase liquidated assets at the cash-in-the-market price, if the cost of banking capital is high. Therefore, in the state of systemic failure, the regulator has to bail out every troubled bank. A “too many to fail” rescue arises. Thus, the market price sensitive capital can induce highly correlated banking assets.

1. Introduction

This chapter studies the joint effect of the official bailout policy and market-price sensitive capital regulation on a bank's strategic choice of investment correlation and the resulting systemic risk. We argue that this joint effect could give rise to the "too many to fail" problem[38] for the financial regulator.[39] This can increase banks' incentive to herd by investing in highly correlated assets *ex ante*, and accentuate the problem of systemic risk *ex post*.

The chapter follows Acharya and Yorulmazer's (2007) study of financial regulators' "too many to fail" problem in a two-bank, two-period and two-asset banking model. Banks are assumed to be more efficient users of their loans than other financial institutions, called outside investors, because of their special expertise and relationship-specific skills with their loans, as in Diamond and Rajan (2001). Their specialities imply that the financial regulator could suffer from the "too many to fail" problem: in order to reduce the social losses due to the systemic risk of banks failing together, the financial regulator finds it *ex post* optimal to bail out every troubled bank, because the acquisition of liquidated assets by outside investors could result in a high misallocation cost.

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[38] The rationale for this bailout is that many banks will fail simultaneously due to the correlation of their assets. One might say that the motivation is that the banks are "Too Correlated to Fail". But we follow the terminology of Acharya and Yorulmazer (2007) for reasons of consistency.

[39] There are three reasons why regulators' forbearance increases with the number of failed banks. First, the banking service is special for both depositors and firms that rely on bank loans in the economy, compared with other financial institutions (Diamond and Rajan, 2001); therefore, the social cost of losing the banking service is increasing with the number of failed banks. Second, a domestic regulator is prone to liquidate failed banks' asset to domestic banks rather than foreign banks because of the competition in the global financial market (Acharya and Yorulmazer, 2008). As the number of failed domestic banks increases, other safe domestic banks are incapable to purchase all the failed assets. the financial regulator has to implement the bailout. Third, the regulator is concerned with the systemic disruption of the financial system. Korinek (2011) shows that the more banking assets being liquidated, the larger the decline in asset prices, due to limited liquidity, which requires further sales to make repayment obligation or regulatory obligation. This financial amplification effects can trigger the systemic risk. Hoggarth, Reidhill and Sinclair (2004) point out network effects: the more bank failures, the more likely that the contagious effect through financial network could reach the tipping point of phase transition, which leads to the collapse of the financial sector as a whole.

There are three dates, *t*=0, 1, 2. At *t*=0, banks choose whether to invest in the same or different industries, which subsequently determines the level of correlation of failing together. If the banks' return from the asset is high at *t*=1, they will operate for one more period. If their return is low, they could be bailed out or liquidated by the regulator. The regulator insures depositors and maximizes social welfare. At *t*=1, the financial regulator decides whether to close the failed banks or to bail the bank(s) out, if their banking assets fail.

If only one bank fails at *t*=1, the regulator will pay off the failed bank's depositors and choose one of the following strategies: 1. allowing the failed bank's assets to be liquidated to outside investors, 2. allowing the assets to be liquidated to the surviving bank, 3. bailing out the failed bank. Acharya and Yorulmazer (2007) show that liquidating the failed bank's asset to the surviving bank is optimal for both the regulator and the surviving bank. For the regulator, bailing out the failed bank entails the high cost of paying off the depositors, liquidating the failed bank's asset to outside investors yields a lower cost of deposit insurance cover but with the cost of asset misallocation and liquidating the assets to the surviving bank could result in the same lower cost of deposit insurance cover regarding the outside investors but with no misallocation cost. For the surviving bank, since outside investors are inferior users of the assets, the price of the assets, set by the outside investors' limited fund in hand (cash-in-the-market pricing), is always lower than the surviving bank's willingness to pay.

If both banks fail at *t*=1, Acharya and Yorulmazer (2007) argue that because their assets have to be liquidated to outside investors, which generates a high misallocation cost, it is then *ex post* optimal for the regulator to bail out both banks if the bailout cost of deposit insurance cover is lower than the cost of asset misallocation. This “too-many-to-fail” guarantee could then induce banks to herd *ex ante* by investing in highly correlated assets, in order to receive the bailout *ex post*.

This chapter argues however that Acharya and Yorulmazer do not consider the full set of strategies from which the regulator can choose in the state of systemic failure. In the benchmark model, we show that if both banks fail, the regulator can always commit to bailing out just one bank randomly and letting the bailed-out bank purchase the other failed bank’s assets at the cash-in-the-market price set by the outside investors. We show that this is an optimal strategy for the regulator to deal with systemic failure, because the regulator can minimize both the cost of deposit insurance and the misallocation cost. Also, the bailed out bank finds it profitable to purchase the liquidated assets at the cash-in-the-market price. A “too many to fail” rescue will not arise as long as it is profitable for one bank to purchase the other bank’s liquidated assets. Thus, upon bailout, the regulator can still use the market solution to decrease the cost of deposit insurance cover, hence decreasing the social cost.

We then include the market price sensitive capital regulation in the model. The universal agreement on capital regulation is that it could curb individual banks’ risk-shifting incentive, hence protecting the depositors and the regulator. A coarse risk classification-based capital regulation, i.e. Basel 1, however, could cause several problems, as we will discuss later. According to the fundamental principle of capital regulation, since the degree to which solvency can be ensured depends on the riskiness of the assets chosen by banks, the regulatory capital ratio should be a measure of the risk of asset holding. Thus, unlike the fixed capital ratio in the first Basel Accord, Basel 2 includes the idea of market price sensitive capital regulation in which the measured risk of the assets is marked to the market price movement. If the asset price drops, the risk measurement, e.g. Value-at-Risk, increases, which means that the bank has to increase its capital holding in order to acquire the assets. We show that given market price sensitive capital regulation and the high cost of capital holding by bank owners, banks would have no incentive to purchase liquidated assets at the cash-in-the-market price. Thus, in the state of systemic failure, the regulator has to bail out every troubled bank if the asset misallocation cost is high. A "too many to fail" rescue arises.

Our argument is somewhat similar to Rajan (2010). In Chapter 7 of his book "Fault Lines", he argues that high banking assets correlation is one of the key features of the recent crisis. The reason behind it is that the prospective and actual government intervention creates incentives for banks to coordinate in investing in highly correlated assets. By doing so, banks make the realization of losses more likely. But the government helped make the high correlation more attractive than they should have been, or maybe even making it applaud such behaviour. In addition to Rajan, our chapter shows that the current market price sensitive capital regulation could further distort banks' incentive in investing in highly correlated assets.

Many researchers have criticized the current capital regulation for focusing only on the micro-prudential policy of individual banks, whereas it fails to prevent the banking system from systemic risk. This chapter argues, however, that the current capital regulation may even increase the systemic risk; it could distort banks' incentive to herd *ex ante*, resulting in a high likelihood of systemic failure. By curbing the risk-shifting incentive of the individual banks, the market price sensitive capital regulation gives rise to a problem similar to debt overhang, which restrains banks from purchasing assets at a low price because of the high cost of capital.

The chapter is organized as follows. Section 2 provides an overview of capital regulation. Section 3 discusses the related literature. Section 4 presents the benchmark model and the analysis. Section 5 analyses the effect of market price-sensitive capital regulation on the benchmark model. Section 6 discusses the result and policy implication, and section 7 concludes. The references and appendices are in section 8 and 9 respectively.

2. An Overview of Capital Regulation

Early banking theories, such as those of Diamond (1984) and Diamond and Dybvig (1983), argue that banks play an important role in completing the market and providing special expertise and relationships in monitoring the loans. These theories also suggest the possibility of bank runs, such as those that occurred in the Great Depression, which then gives the justification for the government to implement deposit insurance. However, deposit insurance encourages banks' risk-shifting incentives. Banks' moral hazard problem related to the choice of investment provides the main rationale for implementing prudential regulation.

From the 1930s to the 1970s, financial authorities regulated the banking sector using several instruments, such as regulation of the market structure, assets allocation rules, interest rate rules or a mixture of these instruments. During this period, banking activities rarely crossed national borders and it was easy for domestic regulators to manage the competition in the banking sector. In the late 1970s, these regulatory instruments were largely dismantled. Hellwig (2008) points out that the trend towards deregulation in this period was because the liberalization of international capital flows, the globalization of financial activities, the financial innovations and the revolution in information and communication technologies intensified the competition in financial sectors all over the world. These regulatory instruments weakened banks' ability to cope with increased risk in a world of wide fluctuations in interest rates and exchange rates, and most importantly weakened the position of domestic banks competing with foreign banks. Thus, during the early 1980s, there was an incentive for countries to compete in deregulation.

During the period of increasing deregulation and financial market globalization, in 1988 bank regulators from G-10 countries agreed on the bank capital regulation following the terms recommended by the Basel Committee on Banking Supervision (BCBS) of the Bank of International Settlements (BIS). Aiming to harmonize the international competition on domestic banking deregulation, the Basel Accord of 1988 stipulated minimum capital requirements for banks. Charging the capital amounted to 8% of risky loans was believed to reduce their incentive to gamble and also to reduce the cost of deposit insurance for regulators once a bank became bankrupt. From then on, capital regulation has been the cornerstone of financial regulation for banks.

The first Basel, however, received several criticisms in the late 1980s and early 1990s. Two main drawbacks attracted most of the attention. First, coarse risk buckets in which assets are classified induce capital regulation arbitrage that could allow banks to hold riskier assets with a relatively low capital ratio. Second, the non-risk-based capital regulation on market risk, the so-called "standard approach", was said to mark a step back from the quality of risk management. It brings potential distortion, because it imposes the same cost of capital mark-up on different types of assets. The Basel Accord ignores the fact that banks possess special expertise in managing their loans through the development of quantitative models with an empirical and conceptual foundation. Risk management on the basis of banks' own quantitative models was deemed to be much closer to the true risks that different assets posed for the banks than the rigid Basel Accord capital regulation.

In 1996 the Amendment to the Basel Accord, “Basel 2”, was proposed. Basel 2 allows banks to use a model-based approach, known as the Internal Ratings Based Approach (IRB), in which banks’ capital holding is an increasing function of banks’ estimates of the probability of default (PD) and loss given default (LGD) of the assets. This approach encourages banks to use their special expertise to create their own internal systems to manage risks. The modification of “Basel” was designed to improve the risk calibration of capital requirements – regulatory capital aims to be more closely attuned to the “actual” risks in banking. However, the IRB approach implies that the financial regulators give more freedom to banks in charge of their own risk management activity, which could again lead to moral hazard behaviour. To tackle this problem, Basel 2 introduces marking to market or fair value pricing on the riskiness of assets.

Under marking to market the level of the capital ratio banks are required to hold depends on the market value of the assets. As asset prices drop, risk measures increase, not only leading to higher capital ratio costs (a high margin requirement and haircut) for the banks holding these assets, but also reducing the risk appetite for other banks to purchase such assets. The haircut is typically obtained from risk measures like Value at Risk. While the definitions of these measures have their own shortcomings, the bigger problem is how they are estimated. Typically these risk measures are estimated naively using past data. Hence, a sharp price drop leads to a sharp increase in the estimates of these risk measures. The direct result of this risk calibration of regulatory capital is that banks’ ability to sell or buy the assets is very susceptible to the market condition. This so-called “market discipline” works well only if the market in question is functioning. If the financial market is illiquid, the Value at Risk measurement of assets' value may not be beneficial for banks and the regulator. In times of bank failure, the illiquid market leads to a situation in which the prices do not reflect the future payoffs but rather reflect the amount of cash available to the buyers in the market- a cash-in-the-market pricing.

3. Related Literature

As mentioned above, the idea of regulators' "too many to fail" problem is formally discussed by Acharya and Yorulmazer (2007). Brown and Dinc (2009) find evidence that the "too-many-to-fail" effect is robust in many emerging markets. In addition, they argue that regulators' intervention in the recent crisis can also be a very good example of a "too-many-to-fail" rescue in developed countries. Kasa and Spiegel (1999) show that regulators appear to practice excessive regulatory forbearance when there are severe problems for many banks. Similar findings are presented by Barth, Caprio and Levine (2006) and Reidhill and Sinclair (2004).

The "too many to fail" problem also relates to bank herding and systemic risk. Wagner (2006) shows that portfolio diversification could exacerbate the systemic risk if there is fire-sale externality. Acharya (2009) shows that individual bank failure could induce both positive and negative externalities in the surviving banks. If the negative externality of a higher cost of the interest rate triggered by the failure of other banks dominates *ex post*, then *ex ante* banks tend to herd in order to avoid failing individually. Acharya and Yorulmazer (2005) show that the failure of one bank conveys adverse information about the systematic factor in bank loan returns and increases the cost of borrowing for the surviving banks relative to the case with no bank failures. Hence, banks herd *ex ante* to increase the likelihood of joint survival.

The analysis of capital regulation's effect on banks' behaviour has mainly been based on a representative banking model. The literature mostly examines the consequences of capital rules for banks' choice of asset riskiness. Furlong and Keeley (1989) demonstrate that capital requirements reduce risk-taking incentives. Koehn and Santomero (1980, 1988) and Rochet (1992) show that improperly chosen risk weights may increase the riskiness of banks. Repullo and Suarez (2007) analyse the cyclical effects of Basel 2 in a dynamic equilibrium model. They show that there could be a significant contraction in the supply of credit when the economy enters a recession, an effect that does not occur under Basel 1 capital requirements. In a similar vein, Guillaume et al. (2005) argue that marking to market pricing could inject excessive volatility into financial markets. They argue that in times of crisis market prices are not accurate measures of value. Some discussion articles by Brunnermeier (2008), Brunnermeier et al. (2009) and Hellwig (2009) all criticize the effect of market price-sensitive capital regulation on financial market volatility. They argue that the current Basel 2 is excessively focused on seeking to improve the behaviour and risk management practices of individual banks, but not on wider systemic issues. In contrast to these papers, we focus on banks' *ex ante* interaction behaviour under market price sensitive capital regulation.

4. The Benchmark Model

We first consider the benchmark model in which no capital requirement is imposed on banks. An economy, with three dates t=0, 1, 2, consists of four different types of agents: bank owners A and B, the regulator, the depositors and the outside investors. We assume that all the agents are risk neutral.

Each bank can borrow from a continuum of depositors of measure 1, each endowed with 1 unit of consumption goods at t=0 and t=1. The depositors have access to a reservation investment opportunity that gives a return of 1 per unit of investment at t+1. They choose to invest their endowment in the reservation investment opportunity or in their bank at t=0 and t=1. A deposit is in a simple form of a non-state contingent debt contract with maturity of one period; the contract is independent of the realized return and the banks' investment strategy.

The banks are endowed with a risky investment technology that needs 1 unit of consumption good at t=0 and t=1.[40] The performance of banks' loans determines the return at t+1. For simplicity, we assume with probability π that the return from the investment is high, $R>1$, and with probability $1-\pi$ that the return is zero. The returns in the two periods are independent. The banks are also endowed with their own funds *K*. They can decide whether to invest using deposits only or both the deposits and their own capital. However, there are private costs for banks to invest using their own capital. These costs can be due to manager–shareholder conflicts (Dewatripont and Tirole, 1993), asymmetric information of the equity holders (Froot, Scharfstein and Stein, 1993) or informed trading in capital markets (Froot and Stein, 1998). Here we simply treat the capital cost as the cost of dilution: an equity issue results in a transfer of value from the existing shareholders.[41] This then implies that if there is no capital regulation banks only use deposits to invest.

We assume that banks can choose their investment in one of the different industries, say real estate or manufacturing, denoted as 1 and 2. Bank *A* (*B*) can lend to firms *A*1

[40] The risky technology is to be thought of as a portfolio of loans to firms in a corporate sector.
[41] See Lee et al. (1996) on the empirical evidence of underpricing the costs of outside equity, and the theoretical justifications for diluting cost of outside equity by Leland and Pyle (1977).

or $A2$ ($B1$ or $B2$) in industries 1 and 2, respectively.[42] The two banks' investment choice determines the correlation of bank returns. If in equilibrium the banks choose to lend to firms in the same industry, specifically they either lend to $A1$ and $B1$ or to $A2$ and $B2$, then they are assumed to be perfectly correlated, that is, $\rho = 1$. Conversely, if Banks A and B choose different industries, then for simplicity we assume that their returns have no correlation at all: $\rho = 0$.

Let $E(u(\rho))$ be the bank's expected profit from the investment given the correlation. In a Nash equilibrium, if the banks invest in the same industry, that is, in ($A1$ and $B1$) or ($A2$ and $B2$), they then receive $E(u(0))$. Thus, for the same level of correlation, the identity of the industries in which the banks invest does not matter in terms of bank returns. In other words, while there may be multiple Nash equilibria resulting in the same level of correlation, they are the payoff equivalent. Thus, we only focus on the correlation, rather than on individual industries, for the banks' choice. Specifically, given the symmetry in our basic model, banks invest in the same industry if $E(u(1)) > E(u(0))$, and invest in different industries otherwise. This gives us the joint distribution of banks' returns in Table 1. Bank owners and depositors obtain a time additive utility u_t where u_t is the expected wealth at time t.

$\rho = 1$

A/B	High	Low
High	π	0
Low	0	$1-\pi$

[42] We assume that the information costs of investing in both industries are very high so that each bank invests in only one industry; see Acharya (2009).

$\rho = 0$

A/B	High	Low
High	π^2	$\pi(1-\pi)$
Low	$(1-\pi)\pi$	$(1-\pi)^2$

Table 1: Joint distribution of bank returns under different correlations

The regulator provides deposit insurance to the depositors at *t*=1 only, because *t*=2 is the last period of the economy. The regulator's objective is to maximize the social welfare.

Upon bailout, the regulator can punish the bank by acquiring a fraction β of the banks' profit, where $0 < \beta < 1$.[43] However, when a bank is bailed out, the regulator must bear the entire cost of the deposit insurance cover. This is because the regulator's share of the bailed-out bank's profit is not pledgeable in capital markets, and it does not reduce the immediacy costs for providing deposit insurance.[44] Hence, bailouts are associated with an opportunity cost for the regulator relative to bank sales. These opportunity costs are also part of the regulator's objective function.

At *t*=1, outside investors are endowed with a limited amount of funds. They can consume their endowment either at *t*=1 or at *t*=2 with no discounting. Outside investors are inefficient users of banking assets; they can only generate $R-\Delta$ from the assets, similar to investors in Diamond and Rajan's study (2001).[45] Thus, when the

[43] One could argue that the regulator can exploit a high fraction of the profit upon bailout, $\beta = 1$, in order to prevent banks from herding. However, Acharya and Yorulmazer (2007) argue that bank owners have an incentive to shirk if they have a low fraction of the asset return. In equilibrium, the regulator will never choose the fraction that violates bank owners' IC constraint. In our model, we impose β at the level of the binding IC constraint, as in Acharya and Yorulmazer (2007).

[44] The cost of providing funds with immediacy can be linked to a variety of sources, such as the distortionary effects of a tax increase required to fund deposit insurance and bailouts and the likely effect of government deficits on the country's exchange rate, manifested in the fact that twin crises have often occurred in many countries. For a detailed discussion on the costs associated with banking collapses and bailouts, see Calomiris (1998) and Hoggarth, Reis and Saporta (2002).

[45] Diamond and Rajan (2001) assume relationship-specific skills of bank managers compared with depositors, whereas we assume a distinction in skills between bank managers and outside investors.

banking assets are liquidated to outside investors, there is a social welfare loss due to the misallocation of the assets. Let p denote the outside investors' maximum willingness to pay for the liquidated asset. We assume that the outside investors' endowment at t=1 is no more than their willingness to pay for the assets. This then implies that the price of the liquidated assets is subject to the limited amount of cash held by outside investors– a cash-in-the-market pricing.[46] In addition, we assume that the cash-in-the-market price of liquidated assets decreases as the number of liquidated assets increases.[47] Let p^I denote the cash-in-the-market price of the liquidated assets when only one bank fails – individual failure – and p^S denote the price of the liquidated assets if there is systemic failure. We then have $p \geq p^I > p^S$. Note that because outside investors are risk neutral, they are indifferent between consuming at t=1 and lending to banks if there is no investment opportunity, as long as the expected return is equal to 1.

4.1 Analysis

We start with the analysis of the banks' and the regulator's strategies and payoffs at t=1. There are four possible states: 1) both Bank A and Bank B survive and receive a high return on the loans, denoted SS; 2) Bank A survives whereas Bank B fails,

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The notion that outsiders may not be able to use the banking assets as efficiently as the existing bank owners is also akin to the notion of asset specificity, first introduced into the corporate finance literature by Shleifer and Vishny (1992) and Wllianmson (1988). This literature suggests that firms whose assets tend to be specific, that is, whose assets cannot be readily redeployed by firms outside the industry, are likely to experience lower liquidation values because they may suffer from fire-sale discounts in cash auctions for asset sales. James (1991) shows that there is significant going concern value that is preserved if the failed bank is liquidated by the FDIC.

[46] For a detailed discussion of cash-in-the-market pricing, see Allen and Gale (1994, 1998). They develop a theory that under incomplete market participation of investors and market illiquidity, the need for liquidity at short notice will lead to an inelastic supply of liquidity in the short run. It then causes asset price fluctuation.

[47] Shleifer and Vishny (1992) argue that if there are two types of potential buyers of a firm's asset – the same industry and industry outsiders – when the same-industry firms are highly correlated, then the failure of many firms causes a higher inefficiency cost and a lower liquidation price. This idea is also employed by Morris and Shin (2004), who assert that correlated failure is more costly, since it will cause more risk discounts due to higher levels of uncertainty.

receiving zero return and facing the resolution by the regulator, denoted *SF*; 3) Bank *B* survives whereas Bank *A* receives zero return and faces the resolution by the regulator, denoted *FS*; 4) both banks fail at t=1, *FF*. Let *j* denote the possible state where $j \in \{SS, SF, FS, FF\}$.

1. Both banks survive (SS)

Because of the deposit insurance in the first period, the banks' promised rate of interest to depositors at *t*=0 is $r_0 = 1$. In the second period, without loss of any generality, the deposit rate is denoted as r_1. We then have $R > r_1 \geq r_0$. In this state, both banks have enough return to pay off the depositors and invest again. The depositors receive 0 if the return is low and r_1 if the return is high. Banks' expected payoff in state *SS*, denoted $E(u_2^{SS})$, is:

$$E(u_2^{SS}) = \pi(R - r_1) = \pi R - 1 \tag{1}$$

Note that the expected payoff is independent of bank correlation, because *t*=2 is the last period of the economy.

2. Only one bank fails (*SF* or *FS*)

These two states happen if and only if the banks' correlation is $\rho = 0$. Since these two states are symmetric, we only consider the case in which *A* has a positive return and *B* has zero return. Bank *B* cannot continue to operate for one more period because it cannot pay back the depositor r_0. Its assets will be liquidated, unless there is a bailout. The liquidation price is determined by the cash outside investors are willing to pay p^I. The regulator has three strategies: 1) to liquidate the assets to outside investors, 2) to liquidate the assets to Bank *A*, 3) to bail out Bank *B*. If the regulator lets Bank *A* purchase Bank *B*'s assets, Bank *A* can access *B*'s depositors, and borrow from outside investors. Thus, Bank *A* can borrow one unit from its own depositors, one unit from Bank *B*'s depositors and p^I units from the outside investors for second-period investments. We can show that:

Lemma 1: It is optimal for the regulator to liquidate Bank *B*'s asset to Bank *A*, and it is also profitable for Bank *A* to purchase the assets at the cash-in-the-market price p^I by accessing Bank *B*'s depositors and borrowing from the outside investors.

Proof: See appendix.

3. Both banks fail (FF)

In state *FF* the regulator's objective is to maximize the total expected output of the banking sector, subtracting any cost of bailout or liquidation. We denote $E(U_2^{FF})$ as the regulator's objective function in *FF*. There are four strategies from which regulator can choose:

a) *The regulator can bail out both banks*

The regulator's objective function takes the value:

$$E(U_2^{FF}) = 2(\pi R - 1) - 2r_0 \tag{2}$$

In this case the only cost for the regulator is to pay off the two banks' depositors. The cost of the deposit insurance cover requires $2r_0$.

b) *The regulator liquidates both banks' assets to outside investors*

The regulator's objective function takes the value:

$$E(U_2^{FF}) = 2[\pi(R-\Delta)-1] + 2p^S - 2r_0 = 2(\pi R - 1) + 2p^S - 2r_0 - 2\pi\Delta \qquad (3)$$

The required fund to provide deposit insurance covers is $2r_0$ net of the proceeds from the sale of the liquidated assets at $2p^S$, and $2\pi\Delta$ is the cost of misallocation incurred by liquidating the assets to outside investors.

c) *The regulator randomly bails out one bank and liquidates another bank's assets to outside investors*

The regulator's objective function in this case is:

$$E(U_2^{FF}) = (\pi R - 1) + [\pi(R-\Delta)-1] - (2r_0 + p^S) = 2(\pi R - 1) + p^S - 2r_0 - \pi\Delta \qquad (4)$$

d) *The regulator randomly bails out one bank and lets the bailed-out bank acquire the failed bank's assets at the cash-in-the-market price*

The regulator's objective function takes the value:

$$E(U_2^{FF}) = 2(\pi R - 1) - r_0 - (r_0 - p^S) = 2(\pi R - 1) - 2r_0 + p^S \qquad (5)$$

where the second term in the first equality is the cost of the deposit insurance for the failed bank and the third term is the cost to pay off the depositor for the bailed-out bank. Note that given Lemma 1 we know that the bailed-out bank is willing to purchase the liquidated assets at the cash-in-the-market price.

Comparing these four expected payoffs, we can obtain the following bailout policy for the regulator:

Lemma 2: In state *FF*, if $p^S \geq 2\pi\Delta$, the regulator liquidates both banks' assets to outside investors. If $p^S < 2\pi\Delta$, the regulator always randomly bails out one bank and lets this bailed-out bank acquire the failed bank's assets at the cash-in-the-market price.

Let $E(u_2^{FF})$ denote the banks' expected payoff in state *FF*. If $p^S < 2\pi\Delta$, the regulator randomly bails out one bank, each bank have a probability of ½ of being bailed out. After the bailout the regulator takes β fraction of the bank's profit. We then have

$$E(u_2^{FF}) = \begin{cases} \frac{1}{2}(1-\beta)\left[2(\pi R - 1) - p^S\right] & if\ p^S < 2\pi\Delta \\ 0 & if\ p^S > 2\pi\Delta \end{cases} \qquad (6)$$

First-period (t=0) investment correlation

We now characterize the banks' strategy in the first period. At *t*=0, the banks choose whether to invest in the same assets or different assets, i.e. the banks choose the correlation ρ. Since Bank *A* and Bank *B* are *ex ante* identical, we only consider a representative bank, say Bank *A*. The bank's objective function, at *t*=0, is to maximize:

$$E(u_1(\rho)) + E(u_2(\rho)).$$

We ignore the discount factor since it does not contribute anything to the result. If the banks invest in the same industry, then their correlation is $\rho = 1$, otherwise it is equal to 0. Since the banks pay the depositors the promised return r_0 only if the return on loans is high, the expected payoff of each bank from its first-period investment is:

$$E(u_1) = \pi(R - r_0) = \pi(R - 1) \qquad (7)$$

The bank's first-period expected payoff is independent of the correlation level. The banks only take into account the second-period profits when choosing ρ.

The second-period expected return needs to take account of different states; we then have:

$$E(u_2(\rho)) = \sum_j \Pr(j)E(u_2^j(\rho)) \qquad (8)$$

where $j \in \{SS, SF, FS, FF\}$.

From Table 1 we can see that if banks invest in different industries, we have

$$E(u_2(0)) = \pi^2 E(u_2^{SS}) + \pi(1-\pi)E(u_2^{SF}) + (1-\pi)\pi E(u_2^{FS}) + (1-\pi)^2 E(u_2^{FF}) \qquad (9)$$

Note that from lemma 1 we know that in state *FS* liquidating the failed bank's assets to the surviving bank is the dominant strategy for the regulator, hence its expected utility in state *FS* is zero.

If the banks invest in the same industry we have:

$$E(u_2(1)) = \pi E(u_2^{SS}) + (1-\pi)E(u_2^{FF}) \qquad (10)$$

The probability of being in state *SF* or state *FS* is equal to zero if the correlation is $\rho = 1$. Using lemma 1 (see equation A.1), we can express the banks' expected payoff in state *SF* as:

$$E(u_2^{SF}) = 2E(u_2^{SS}) - p^I \qquad (11)$$

and we can use equation (6) to express the banks' expected payoff in state *FF* for $p^S > 2\pi\Delta$:

$$E(u_2^{FF}) = \frac{1}{2}(1-\beta)\left[2E(u_2^{SS}) - p^S\right] \qquad (12)$$

We then have

$$E(u_2(0)) - E(u_2(1)) = \pi(1-\pi)\left[\beta E(u_2^{SS}) - p^I + \frac{1}{2}(1-\beta)p^S\right] \qquad (13)$$

To rearrange the equation we have:

$$E(u_2(0)) - E(u_2(1)) = \pi(1-\pi)\left[\beta(E(u_2^{SS}) - p^I) - (1-\beta)(p^I - \frac{1}{2}p^S)\right] \qquad (14)$$

The first term in the square bracket is positive given lemma 1. The second term in the square bracket is also positive given that the cash-in-the-market price of the liquidated assets is lower in systemic failure than in individual failure. We can show that the sign of the difference between the expected utility of choosing correlation 0 and the expected utility of choosing correlation 1 depends on β and the price difference of the liquidated assets between systemic failure and individual failure. According to lemma 2, bailout only happens in state *FF*; if one bank is chosen to be bailed out, the expected payoff in the second period depends on β. Intuitively, the larger the fraction of the profit exploited by the regulator given bailout, the less incentive the banks have to be bailed out. The banks' profit also depends on the cash-in-the-market prices in the two states. The greater the difference between the liquidated price of the assets in systemic failure and the liquidated price of the assets in individual failure, the more incentive banks have to choose highly correlated assets in order to exploit the high profit in the systemic risk because of the relatively low liquidation price. If the difference between the liquidation price in systemic failure and the liquidation price in individual failure is not too high, banks have less incentive to herd and therefore more incentive to choose $\rho = 0$. We then have:

Lemma 3: If $p^S > 2\pi\Delta$, the regulator liquidates both banks to outside investors in state *FF*, and banks choose the lowest level of correlation $\rho = 0$. If $p^S < 2\pi\Delta$, the choice of banks' correlation level depends on the β, the expected payoff in state *SS* and the cash-in-the-market price p^S and p^I. If $\beta(E(u_2^{SS}) - p^I) \geq (1-\beta)(p^I - \frac{1}{2}p^S)$, the banks choose the lowest level of correlation $\rho = 0$, otherwise the banks choose the highest level of correlation $\rho = 1$.

The combination of the three lemmas above characterizes the unique subgame perfect equilibrium. We then have:

Proposition 1: In the unique subgame perfect equilibrium, the regulator does not intervene in state *SS*. The regulator always liquidates the assets to the surviving bank in states *SF* and *FS*. The surviving bank finds it profitable to acquire the other bank's assets at the cash-in-the-market price. In state *FF*, if $p^S < 2\pi\Delta$, the regulator has a dominant strategy: it randomly bails out just one bank and liquidates the failed bank's assets to the bailed-out bank. It is profitable for the bailed-out bank to acquire the asset at the cash-in-the-market price. If $\beta(E(u_2^{SS}) - p^I) \geq (1-\beta)(p^I - \frac{1}{2}p^S)$, the banks invest in different assets at t=0, otherwise the banks invest in the same asset, i.e. $\rho = 1$. If $p^S > 2\pi\Delta$, the regulator liquidates the two assets to outside investors. There is no "too-many-to-fail" rescue in the subgame perfect equilibrium.

5. Market Price Sensitive CAR and "Too-Many-to-Fail" Rescue

We now include market price-sensitive capital regulation in the benchmark model. We denote the asset price as p^0, if the assets are not being liquidated. Otherwise the price of the assets is determined by the cash-in-the-market pricing. The asset price p^0 can be assumed as the standard discounted value of future dividends of the assets that can be exploited by Banks *A* and *B*. Given the assumption that outside investors are inferior users of the assets and the liquidated asset price is marked to the cash-in-the-market, which decreases with the number of bank failures, we then have $p^0 > p \geq p^I > p^S$.

The market price sensitive capital regulation implies that the capital requirement increases if the asset price decreases. We then have $k'(p) < 0$ and $k''(p) > 0$. The increasing margin of capital requirement is in line with Basel 2's regulation on asset credit rating and CAR as shown in Figure 1.[48]

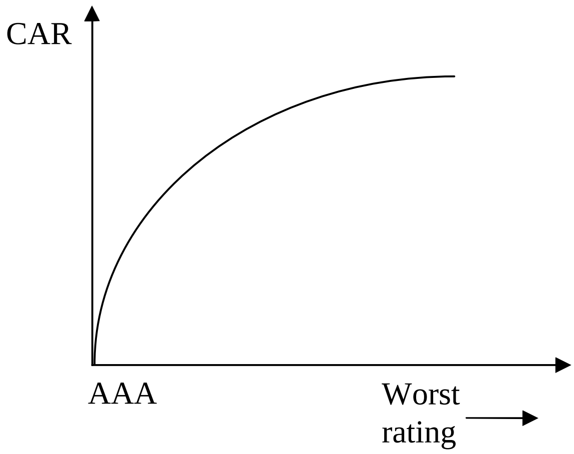

Figure 1

We denote the cost of capital for individual banks as $\theta(k)$, and $\theta'(k) > 0$ and $\theta''(k) \geq 0$.[49] There is a conflict of interest between social welfare and individual banks' interest in capital-raising effort, as banking capital generates a high benefit for social welfare relative to its individual cost.[50] Here we assume that the social cost of the bank's capital adequacy requirement is equal to zero.

Note that the fundamental reason for capital regulation is that it can curb individual banks' risk-shifting incentive. We can assume there are two levels of risky portfolio

[48] Note that the credit rating is also based on VaR for risk measurement. In general, the decrease in the asset price increases the VaR measure and the required rate of return for the underlying asset, which increases its probability of default and thus decreases its credit rating (see Brunnermeier and Pedersen, 2008 and Brunnermeier et al., 2009).

[49] Hellwig (2008) and Kashyap, Rajan and Stein (2008) argue that banks compete in lowering the equity/debt ratio. A high equity/debt ratio can send a negative signal to the market. The greater the equity issuance, the less competitive a bank is, in turn making it more difficult to attract finance.

[50] Bernanke (2008) argues that the capital-raising effort is crucial for the stability of the financial system. Failing to do so will pose negative externalities for the broader economy, losing new profit opportunities and deepening the economic recession. Miles et al. (2011) also argue that the social benefit greatly outweighs individual banks' cost of capital. The current capital regulation imposed on banks is much less than the socially desirable one.

that a bank can choose in each industry – high risk and low risk – denoted $i \in \{H, L\}$, with a mean-preserving spread. In the benchmark model, since there is no capital regulation, bank tends to choose a high-risk asset portfolio due to the risk-shifting problem. When capital regulation is imposed on banks, banks are likely to choose an investment portfolio with lower risk. We then have $R(H) > R(L)$ and $\pi(H) < \pi(L)$. Since this chapter focuses on the regulator's bailout decision under systemic risk, we can take the effect of capital requirement on the individual risk-shifting problem as given, and just focus on banks' investment correlation. Without loss of any generality, we then assign the same notation to asset returns and the probability of success as in the benchmark model. The reader should bear in mind that the individual risk levels can be different in the two cases.[51]

5.1 Analysis

We start with our analysis of four states at *t*=1, that is, $j \in \{SS, SF, FS, FF\}$.

1. Both banks survive (SS)

As before, the deposit insurance in the first period implies $r_0 = 1$. In the second period, the deposit rate is denoted as r_1. We then have $R > r_1 \geq r_0$. In this state, both banks have enough consumption goods to invest again. The bank's expected payoff in state *SS*, denoted $E(u_2^{SS})$, is:

[51] Also note that we do not consider the case in which two banks play a game on the choice of risk level. There may be an equilibrium in which banks choose a low risk level at *t*=0, in order to purchase liquidated assets at a low price at *t*=1. However, since our paper focuses on the effect of capital regulation on the strategic choice of banks' correlation, this equilibrium does not affect the main result of the paper. We will leave this matter for further research. We can impose the condition that the decision on the risk level is only made at *t*=0 and a high-risk portfolio can generate a high enough return at *t*=1, which is greater than the sum of the returns at *t*=1 and *t*=2 if the banks choose a low-risk portfolio.

$$E(u_2^{SS}) = \pi(R - r_1(1 - k(p^0))) - k(p^0) - \theta(k(p^0)) \\ = \pi R - 1 - \theta(k(p^0)) \tag{15}$$

where, in the first equation, the first term is the expected return net of the promised return to the depositors, which is r_1 times the total amount of the deposit $(1-k(p^0))$, the second term is the banks' capital investment at asset price p^0 and the last term refers to the banks' cost of capital at p^0. We can simplify the first equation as the second expression.

***2. Only one bank fails* (*SF* or *FS*)**

The regulator's objective is to maximize the total expected output of the banking sector, subtracting any cost of bailout or liquidation. Again, these two states happen if and only if the correlation is $\rho = 0$, and we only consider the state in which *A* has a high return and *B* has zero return due to the state symmetry. Bank *B* cannot operate the assets for one more period because it cannot pay back the depositor r_0. Hence, its assets will be liquidated at the cash-in-the-market price, unless there is a bailout. We can show that:

Lemma 4: The profitability for Bank *A* to purchase the liquidated assets depends on the cost of capital at the cash-in-the-market price:

1. If $(\pi R - 1 - p^I) > \theta(k(p^I))$, Bank *A* is willing to purchase the assets at the liquidated price, and it is optimal for the regulator to liquidate *B*'s assets to *A*.

2. If $(\pi R - 1 - p^I) < \theta(k(p^I))$, Bank *A* is unwilling to purchase the assets at the liquidated price. The regulator bails out the failed bank if $p^I \leq \pi\Delta$, and the regulator liquidates the assets to outside investors if $p^I > \pi\Delta$.

Proof: See appendix.

The argument is similar to lemma 1: the regulator is always willing to liquidate the assets to the surviving bank because it avoids the asset misallocation cost and decreases the cost of deposit insurance. However, Bank *A* could find it unprofitable to purchase the liquidated asset at the cash-in-the-market price if there is a high cost of capital holding. Bank *A*'s decision on the purchase of the liquidated assets depends on the cost of capital holding at p^I.

3. Both banks fail (FF)

Without loss of any generality, we denote $E(U_2^{FF})$ as the regulator's objective function in *FF*. Note that the regulator's available strategies depend on the bank's willingness to acquire the liquidated asset. There are four possible strategies for the regulator:

a) *The regulator bails out both banks*

The objective function takes the value:

$$E(U_2^{FF}) = 2(\pi R - 1) - 2(1 - k(p^0)) \tag{16}$$

In this strategy, the only cost is to pay off the depositors, which is $2(1 - k(p^0))$.

b) *The regulator liquidates both banks' assets to outside investors*

The regulator's objective function takes the value:

$$E(U_2^{FF}) = 2(\pi R - 1) + 2p^S - 2(1 - k(p^0)) - 2\pi\Delta \tag{17}$$

The regulator pays off the depositors $2(1-k(p^0))$, receives the proceeds from the liquidation of the assets to outside investors $2p^S$ and incurs the cost of misallocation for two liquidated assets $2\pi\Delta$.

c) *The regulator randomly bails out one bank and liquidates the other bank's assets to outside investors*

The regulator's objective function in this case is:

$$E(U_2^{FF}) = 2(\pi R-1) + p^S - 2(1-k(p^0)) - \pi\Delta \qquad (18)$$

The regulator faces the social cost of asset misallocation of just one liquidated asset and receives p^S amount of funds to cover the cost of the deposit insurance $2(1-k(p^0))$.

d) *The regulator randomly bails out one bank and lets the bailed-out bank acquire the failed bank's assets at the cash-in-the-market price?*

The feasibility of this strategy depends on the cost of capital holding at the cash-in-the-market price p^S. The expected payoff for the bailed out bank purchasing the liquidated assets is:

$$\begin{aligned} E(u_2^{FF}) &= (1-\beta)\left[2\pi(R-r_1) - p^S - \theta(k(p^0)) - \theta(k(p^S))\right] \\ &= (1-\beta)\left[E(u_2^{SS}) + \pi R - 1 - p^S - \theta(k(p^S))\right] \end{aligned} \qquad (19)$$

The profitability for the bailed out bank to purchase the assets depends on the cost of capital holding $\theta(k(p^S))$. If $(\pi R - 1 - p^S) < \theta(k(p^S))$, the banks are reluctant to purchase the liquidated assets at the cash-in-the-market price, which in turn rules out the strategy for the regulator to bail out one bank randomly and let the bailed out bank acquire the other bank's assets at the cash-in-the-market price.

However, for $(\pi R - 1 - p^S) > \theta(k(p^S))$, the regulator can still implement this strategy. The payoff for the regulator is:

$$E(U_2^{FF}) = 2(\pi R - 1) + p^S - 2(1 - k(p^0)) \tag{20}$$

There is no misallocation cost in this case.

Comparing these expected payoffs, we can obtain the following bailout policy for the regulator in state *FF*:

Lemma 5: In state *FF*, the regulator's strategy is as follows:

1. If $(\pi R - 1 - p^S) > \theta(k(p^S))$, for $p^S > 2\pi\Delta$ the regulator liquidates both banks' assets to outside investors. If $p^S < 2\pi\Delta$, the regulator always randomly bails out one bank and lets this bailed-out bank acquire the other failed bank's assets at the cash-in-the-market price.

2. If $(\pi R - 1 - p^S) < \theta(k(p^S))$, for $p^S > \pi\Delta$, the regulator liquidates both banks' assets to outside investors; for $p^S < \pi\Delta$, the regulator bails out both banks.

Given lemma 5, we can then derive the expected payoff for individual banks in state *FF*:

1. If $(\pi R - 1 - p^S) > \theta(k(p^S))$, we then have:

$$E(u_2^{FF}) = \begin{cases} 0 & \text{if} \quad p^S > 2\pi\Delta \\ \frac{1}{2}(1-\beta)\left[2\pi(R - r_1) - p^S - \theta(k(p^0)) - \theta(k(p^S))\right] & \text{if} \quad p^S < 2\pi\Delta \end{cases} \tag{21}$$

2. If $(\pi R - 1 - p^S) < \theta(k(p^S))$, the expected payoff for banks is:

$$E(u_2^{FF}) = \begin{cases} 0 & \text{if} \quad p^S > \pi\Delta \\ (1-\beta)\left[\pi R - 1 - \theta(k(p^0))\right] & \text{if} \quad p^S < \pi\Delta \end{cases} \tag{22}$$

First-period (t=0) investment correlation

We now characterize banks' strategy for choosing their correlation level at t=0. In this section, we only focus on the interesting case in which there is no "too big to fail", thus $(\pi R - 1 - p^I) < \theta(k(p^I))$ and $p^I > \pi\Delta$, and consider the case of a "too many to fail" rescue $(\pi R - 1 - p^S) < \theta(k(p^S))$ and $p^S < \pi\Delta$. The full analysis of banks' strategic choices of correlation and the regulator's bailout policy under different conditions is shown in the appendix.

Banks' objective function is to maximize $E(u_1) + E(u_2(\rho))$. Again, since the first-period expected payoff does not depend on the correlation level, we only focus on the second-period profits when banks choose ρ.

The expected second-period return has to take account of different states. We have:

$$E(u_2(\rho)) = \sum_j \Pr(j) E(u_2^j(\rho))$$

where $j \in \{SS, SF, FS, FF\}$.

From Table 1 we can see that if banks invest in different industries, we have

$$E(u_2(0)) = \pi^2 E(u_2^{SS}) + \pi(1-\pi)E(u_2^{SF}) + \pi(1-\pi)E(u_2^{FS}) + (1-\pi)^2 E(u_2^{FF}) \qquad (23)$$

whereas if banks invest in the same industry we have:

$$E(u_2(1)) = \pi E(u_2^{SS}) + (1-\pi)E(u_2^{FF}) \qquad (24)$$

The probability of being in state SF is equal to zero, if the correlation is $\rho = 1$.

For $(\pi R - 1 - p^I) < \theta(k(p^I))$, the banks' expected payoff in state FS is:

$$E(u_2^{FS}) = \begin{cases} 0 & if \quad p^I > \pi\Delta \\ (1-\beta)\left[\pi R - 1 - \theta(k(p^0))\right] & if \quad p^I < \pi\Delta \end{cases} \qquad (25)$$

In the case where there is no "too big to fail" rescue, we then have $p^I > \pi\Delta$; in addition, if the cash-in-the-market price of the liquidated assets is small in systemic failure, we then have $p^S < \pi\Delta$. In this case, the banks' expected payoff under correlation zero is:

$$E(u_2(0)) = \pi^2 E(u_2^{SS}) + \pi(1-\pi)E(u_2^{SF}) + (1-\pi)^2 E(u_2^{FF})$$

From lemma 4 we have that banks' expected payoff in state *SF* is:

$$E(u_2^{SF}) = E(u_2^{SS})$$

Since the surviving bank finds it unprofitable to purchase the liquidated assets at p^I, the bank's expected payoff in state *FS* is zero, since the regulator finds it less costly to liquidate the assets to an outside investor.

Recall lemma 5, we can show:

$$E(u_2^{FF}) = (1-\beta)E(u_2^{SS}) \tag{26}$$

The bank's expected payoff under correlation 1 is:

$$E(u_2(1)) = \pi E(u_2^{SS}) + (1-\pi)(1-\beta)E(u_2^{SS}) \tag{27}$$

We then have

$$E(u_2(0)) - E(u_2(1)) = -\pi(1-\pi)(1-\beta)E(u_2^{SS}) \tag{28}$$

In this case, the banks choose correlation 1 as long as β is less than 1. Compared with equation (14), in which the banks' strategy regarding correlation depends on the value of four variables, we could have an increase in systemic risk due to the market price sensitive capital requirement and the "too many to fail" bailout policy. We can show that:

Proposition 2: Consider $(\pi R - 1 - p^I) < \theta(k(p^I))$ and $(\pi R - 1 - p^S) < \theta(k(p^S))$, in the unique subgame perfect equilibrium: the regulator does not intervene in the state *SS*. If $p^I > \pi\Delta$ and $p^S < \pi\Delta$, in state *SF* and *FS*, the regulator liquidates the assets to outside investors. In state *FF*, the regulator bails out both banks. The banks choose the highest correlation, $\rho = 1$, as long as $\beta < 1$. If $p^I > \pi\Delta$ and $p^S > \pi\Delta$, in states *SF*, *FS* and *FF*, the regulator liquidates the failed bank(s) assets to outside investors. The banks are indifferent between choosing correlation 1 and choosing correlation 0. If $p^I < \pi\Delta$ and $p^S < \pi\Delta$, in states *SF*, *FS* and *FF*, the regulator bails out the bank(s) as long as there is bank failure. The banks are indifferent between choosing correlation 1 and choosing correlation 0.

Proof: See Appendix.

6. Discussion

6.1. Social Losses

We first discuss the social losses under the benchmark model and the model with a market sensitive capital requirement. Suppose that equation 14 is positive, the banks then choose different industries to invest in the benchmark model. In this case, the regulator's expected loss at *t*=0 is:

$$2\pi(H)(1-\pi(H))(1-p^I)+(1-\pi(H))^2(2-p^S)$$

Given equation 27, we then have the expected social cost in the economy with a price-sensitive capital requirement:

$$2(1-\pi(L))(1-k(p^0))$$

We assume that the banks choose different risky portfolios; intuitively we have $\pi(H) \le \pi(L))$. The social cost is not necessarily higher in the benchmark model than in the model with market price sensitive capital requirement. The social cost is greater with the market price-sensitive capital requirement if:

$$2\pi(H)(1-\pi(H))(1-p^I)+(1-\pi(H))^2(2-p^S) < 2(1-\pi(L))(1-k(p^0)) \qquad (29)$$

Even if the bank chooses a high correlation in the benchmark model, the regulator can still use a market solution upon bailout, because the bailed-out bank still finds it profitable to purchase the liquidated assets by borrowing from the outside investors; this then decreases the regulator's cost on bailout. In the case of a price-sensitive capital requirement, however, the market solution is ruled out because of the high cost of capital holding, which may increase the cost of bailout for the regulator. Suppose that in both cases the banks choose high correlation, equation 5 indicates that the regulator's expected cost of bailout at t=0 is then $(1-\pi(H))(2-p^S)$, whereas the expected bailout cost in the case of a market price-sensitive capital requirement is $2(1-\pi(L))(1-k(p^0))$, as in equation 16. It can be the case that:

$$(1-\pi(H))(2-p^S) < 2(1-\pi(L))(1-k(p^0)) \qquad (30)$$

In particular, if a bank can create off-balance-sheet "conduits", with loans placed in "conduits", the bank just needs to maintain relatively low capital against them. However, to make it easier for the "conduits" to obtain funds, the bank provides guarantees of the underlying credit, essentially bringing the risk back onto itself, even if it is not shown on the balance sheet. This "regulation arbitrage" can then let the bank invest in high-risk assets with low capital holding in a normal time. In the extreme case, if we have $\pi(H)=\pi(L)$ and $k(p^0)$ is small enough, conditions 29 and 30 hold.

6.2. Market Liquidity

Our results rely on the assumption that the market is illiquid. The notion that the market value of the asset can be below the expected present value of future cash flows seems to contradict the traditional theory of asset pricing under an information-efficient market. However, this theory is based on the assumption that the market is completely frictionless and the market prices fully reflect the fundamental values of all the assets and liabilities. The efficiency properties of competitive equilibrium can then be derived. However, theoretical analyses have shown if market is incomplete (see Gale, 2012), borrowers have agency problem (see Kiyotaki and Moore, 1997), or market participants have the "lemons" concern (see Brunnermeier 2008), the market reactions to shocks can be quite extreme. In times of financial crisis, the interaction of institutions and markets leads to situations in which the prices in illiquid markets do not reflect the future payoffs but rather reflect the amount of cash available to the buyers in the market. If the financial market is illiquid, imposing a market price-sensitive capital requirement on financial institutions may not be beneficial. The relevant analogy here is with the theory of the second best from welfare economics. When there is more than one imperfection in a competitive economy, removing just one of these imperfections, like an individual bank's risk-shifting problem, is not necessarily welfare-improving. It is possible that the removal of one of the imperfections will magnify the likelihood of systemic failure, which imposes a high cost on the overall welfare, as we have discussed in section 6.1. Thus, simply moving to a market price sensitive capital regulation without addressing the other imperfections in the financial system need not guarantee a welfare improvement, as pointed out by Plantin, Haresh and Shin (2008).

6.3. Macro-Prudential Policy

To tackle the problem, financial authorities need to focus on regulating the financial system as a whole. However, the current capital regulation is excessively focused on seeking to improve the behaviour and risk management practices of individual banks, which is too micro-prudential. At the same time, wider systemic issues have been ignored, meaning an insufficient macro-prudential policy. More importantly, we have shown that an inadequate *ex ante* micro-prudential policy could lead to a high probability of systemic risk *ex post*. We know that monetary policy can have a role in the leaning against the wind approach. Bank regulation should also have this property. Our analysis supports the idea of state-dependent or countercyclical capital regulation, suggested by, for instance, Alessandri and Haldane (2009), Barrell and Davis (2011), Korinek (2011), Kashyap and Stein (2004) and Kashyap, Rajan and Stein (2008), which argue that the regulator should commit to relaxing the capital ratio in a bad state of the world. Kashyap and Stein (2004) argue that time-varying capital requirements emerge as an optimal scheme in a model in which the social planner maximizes a welfare function that weights both the micro-prudential objective of protecting the deposit insurance fund and the macro-prudential objective of maintaining credit creation during recessions. At a time when the market stops functioning, when bank capital is scarce and the credit supply is tight, the regulator concerned with both objectives should be willing to tolerate a higher probability of bank failure than in good times. Alessandri and Haldane (2009) and Kashyap, Rajan and Stein (2008) additionally suggest the idea of capital insurance, which involves a bank purchasing an insurance policy that pays off in a bad state of the world. To address concerns about the insurer defaulting, the policy would be fully collateralized, that is, the insurer would put the full amount of the policy into a locked box up front. In both cases, the idea is to make capital less costly in a bad state relative to a good state. Our model suggests that if the regulator can identify that the asset price volatility is due to market malfunctioning, then this macro-prudential policy could not only alleviate the procyclical effect of a fire sale as they suggested, but it can also decrease the likelihood of systemic risk *ex ante*, since banks may find it profitable to purchase liquidated assets if the margin requirement is not marking to the market. The time-varying capital requirement can then regulate the individual risk-shifting problem *ex ante* without causing the problem for banks to generate high systemic risk *ex post*.

7. Conclusion

This chapter argues that a "too many to fail" rescue only arises if a bank is unwilling to purchase the liquidated assets. We show that this can happen if the margin requirement of capital holding is marked to the market price, and the market price is subject to the cash-in-the-market constraint. We then show that in the presence of a "too many to fail" rescue, banks can have a higher incentive to invest in highly correlated assets *ex ante*, in order to increase the probability of systemic risk, thus trigger the regulator's bailout policy. We then compare the social welfare losses in which there is no capital requirement at all and in which there is a market sensitive capital requirement. We argue that, given the incentive for regulatory arbitrage, the social loss can be greater under the market price-sensitive capital requirement.

Perhaps the most supportive empirical evidence for our model is the U.S. Treasury arranged a bailout of $700 billion funding for the Troubled Asset Relief Program (TARP), inter alia, in 2008. The U.S. government argued that TARP was necessary because many banks held Mortgage Backed Securities (MBS) which were in trouble and became illiquid, which could have led to sector-wide banking difficulties in the absent of intervention. Although it is optimal for the government to intervene *ex post*, this chapter argues that it can distort banks' incentive *ex ante* and generate a "too correlated to fail" problem. The TARP is intended to purchase the troubled assets so that it could stabilize the asset price hence release banks from the pressure of high margin calls on capital ratio. We argue that if capital regulation can be made state-dependent - so that the authorities commit to relax the capital ratio in a bad state of the world, then it could not only alleviate the problem of high margin on capital ratio *ex post*, but also give banks the incentive *ex ante* to invest in less correlated assets. This will lower the probability of systemic failure.

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Appendices

1. ***Lemma 1***: It is optimal for the regulator to liquidate Bank *B*'s assets to Bank *A*, and it is also profitable for Bank *A* to purchase the assets at the cash-in-the-market price by accessing Bank *B*'s depositors and borrowing from outside investors.

Proof: We first show that the regulator is willing to liquidate the assets to the surviving bank. The regulator has three options: 1) the regulator can bail out the failed bank, which will cost $r_0 = 1$ to pay off the depositors; 2) the regulator can liquidate the assets to outside investors. The cost of the deposit insurance is $r_0 - p^I$. However, there is a misallocation cost of $\pi\Delta$; 3) if the regulator liquidates the assets to Bank *A* at the cash-in-the-market price, there is no misallocation cost, and since the bank can borrow from outside investors to purchase the assets, the cost of providing the deposit insurance is again $r_0 - p^I$.

Next we show that it is profitable for Bank *A* to purchase the liquidated assets. Bank *A*'s expected profit after the purchase is:

$$E(u_2^{SF}) = \pi\left(R - r_1\right) + \pi\left(R - r_1(1 + p^I)\right) = 2(\pi R - 1) - p^I \qquad \text{(A.1)}$$

where the first term in the first equality is the expected payoff from the existing assets, and the second term is the expected payoff from purchasing the assets by borrowing from Bank *B*'s depositor and outside investors. It is profitable for Bank *A* to acquire the asset if $(\pi R - 1) > p^I$. Note that the maximum willingness to pay for outside investors is:

$$p = \pi(R - \Delta) - 1 \qquad \text{(A.2)}$$

where outside investors can borrow 1 unit of endowment from depositors. Given the cash-in-the-market price $p \geq p^I$, we then find that the minimum profit for Bank *A* from acquiring the asset is $\pi\Delta$.

2. Lemma 4: The profitability for Bank *A* to purchase the liquidated assets depends on the cost of capital at the cash-in-the-market price:

1. If $(\pi R - 1 - p^I) > \theta(k(p^I))$, Bank *A* is willing to purchase the assets at the liquidated price, and it is optimal for the regulator to liquidate *B*'s assets to *A*.

2. If $(\pi R - 1 - p^I) < \theta(k(p^I))$, Bank *A* is unwilling to purchase the assets at the liquidated price. The regulator bails out the failed bank if $p^I < \pi\Delta$, and the regulator liquidates the assets to outside investors otherwise.

Proof: We first show the profitability condition for Bank *A* to acquire the liquidated assets. Bank *A*'s expected profit after the purchase is:

$$E(u_2^{SF}) = \left[\pi R - 1 - \theta(k(p^0))\right] + \left[\pi R - 1 - p^I - \theta(k(p^I))\right] \qquad \text{(A.2)}$$

where the first term is the expected payoff from the existing assets net of the cost of capital and the second term is the expected payoff from purchasing the liquidated assets net of the cost of capital at the cash-in-the-market price p^I. It is profitable for Bank *A* to acquire the assets if and only if $(\pi R - 1 - p^I) > \theta(k(p^I))$, otherwise Bank *A* will make a loss.

We then show that the regulator is willing to liquidate the assets to the surviving bank. The regulator faces three options: 1) the regulator can bail out the failed bank, which will cost $(1 - k(p))$ to pay off the depositor; 2) the regulator can liquidate the assets to an outside investor. The cost of the deposit insurance is $(1 - k(p)) - p^I$.

However, there is a misallocation cost of $\pi\Delta$; 3) if the regulator liquidates the assets to Bank *A* at the cash-in-the-market price, there is no misallocation cost, and since the bank can borrow from outside investors to purchase the assets, the cost of providing the deposit insurance is again $(1-k(p))-\overline{p}$. However, if $(\pi R-1-p^I)<\theta(k(p^I))$, the regulator can only choose either to liquidate the assets to an outside investor, which induces a cost of misallocation $\pi\Delta$, or to bail out Bank *A*, which gives up p^I of its funds to make up the cost of the deposit insurance cover. We then have that if $p^I>\pi\Delta$ the regulator will liquidate the funds to an outside investor, and it will bail out Bank *A* otherwise.

3. Full analysis of the bank's strategy and the regulator's bailout under market price-sensitive CAR

3.1 Consider the case in which $(\pi R-1-p^I)<\theta(k(p^I))$ and $(\pi R-1-p^S)<\theta(k(p^S))$; the banks' expected payoff in state *FS* is:

$$E(u_2^{FS})=\begin{cases}0 & if \quad p^I>\pi\Delta \\ (1-\beta)\left[\pi R-1-\theta(k(p^0))\right] & if \quad p^I<\pi\Delta\end{cases} \tag{A.3}$$

and the banks' payoff in state *FF* is:

$$E(u_2^{FF})=\begin{cases}0 & if \quad p^S>\pi\Delta \\ (1-\beta)\left[\pi R-1-\theta(k(p^0))\right] & if \quad p^S<\pi\Delta\end{cases} \tag{A.4}$$

We have shown the case in which $p^I>\pi\Delta$ and $p^S<\pi\Delta$ in section 5. We consider two other cases in which $p^I>\pi\Delta$ and $p^S>\pi\Delta$, and $p^I<\pi\Delta$ and $p^S<\pi\Delta$. Note that since we have $p^I>p^S$, we can rule out the possibility that $p^I<\pi\Delta$ and $p^S>\pi\Delta$.

3.1.1 $p^I > \pi\Delta$ and $p^S > \pi\Delta$

The regulator has full commitment to not bailing out banks in any state. A bank's expected payoff in state *SF* is thus the same as in state *SS*:

$$E(u_2^{SF}) = E(u_2^{SS})$$

The bank's expected payoff in states *FS* and *FF* is zero since the regulator finds it optimal to liquidate the assets to outside investors:

$$E(u_2^{FS}) = 0$$

$$E(u_2^{FF}) = 0$$

The second-period expected payoff when the banks invest in different industries is:

$$E(u_2(0)) = \pi^2 E(u_2^{SS}) + \pi(1-\pi)E(u_2^{SS})$$

The second-period expected payoff when the banks invest in the same industries is:

$$E(u_2(1)) = \pi E(u_2^{SS})$$

We then have:

$$E(u_2(0)) - E(u_2(1)) = 0$$

The banks are indifferent between choosing correlation level one and choosing correlation level zero.

3.1.2 $p^I < \pi\Delta$ and $p^S < \pi\Delta$

The regulator suffers from both a "too big to fail" and a "too many to fail" rescue. Given equation A.3 and equation 15, we then have the bank's expected payoff in state *FF*:

$$E(u_2^{FF}) = (1-\beta)E(u_2^{SS})$$

Equations A.4 and 15 give the bank's expected payoff in state *FS*:

$$E(u_2^{FS}) = (1-\beta)E(u_2^{SS})$$

The bank's expected payoff in state *SF* is the same as in *SS*:

$$E(u_2^{SF}) = E(u_2^{SS})$$

The second-period expected payoff when banks invest in different industries is then:

$$E(u_2(0)) = \pi^2 E(u_2^{SS}) + \pi(1-\pi)E(u_2^{SS}) + \pi(1-\pi)(1-\beta)E(u_2^{SS}) + (1-\pi)^2(1-\beta)E(u_2^{SS})$$

The second-period expected payoff when banks invest in the same industries is:

$$E(u_2(1)) = \pi E(u_2^{SS}) + (1-\pi)(1-\beta)E(u_2^{SS})$$

We then have:

$$E(u_2(0)) - E(u_2(1)) = 0$$

Given $p^I < \pi\Delta$ and $p^S < \pi\Delta$, the banks are indifferent between choosing the correlation level of one and choosing the correlation level of zero.

3.2 Consider the case in which $(\pi R - 1 - p^I) > \theta(k(p^I))$ and $(\pi R - 1 - p^S) < \theta(k(p^S))$; the banks' payoff in state *FS* is:

$$E(u_2^{FS}) = 0$$

Because in this case the surviving bank finds it profitable to purchase the liquidated assets at the cash-in-the-market price, it is also optimal for the regulator to liquidate the assets to the surviving bank as in lemma 4.

The bank's expected payoff in state *FF* depends on the cash-in-the-market price of the assets and the misallocation cost; we then have equation 22:

$$E(u_2^{FF}) = \begin{cases} 0 & if \quad p^S > \pi\Delta \\ (1-\beta)\left[\pi R - 1 - \theta(k(p^0))\right] & if \quad p^S < \pi\Delta \end{cases}$$

3.2.1 $p^S > \pi\Delta$

The bank's expected payoff in state *SF* is given by equation A.2; combining equation A.2 and 15, we then have:

$$E(u_2^{SF}) = E(u_2^{SS}) + \left[\pi R - 1 - p^I - \theta(k(p^I))\right]$$

The bank's payoff in state *FF* is zero if $p^S > \pi\Delta$, as in equation 22.

$$E(u_2^{FF}) = 0$$

The second-period expected payoff when the banks invest in different industries is then:

$$E(u_2(0)) = \pi^2 E(u_2^{SS}) + \pi(1-\pi)[E(u_2^{SS}) + \pi R - 1 - p^I - \theta(k(p^I))]$$

and the second-period expected payoff when the banks invest in the same industries is:

$$E(u_2(1)) = \pi E(u_2^{SS})$$

We then have:

$$E(u_2(0)) - E(u_2(1)) = \pi(1-\pi)[\pi R - 1 - p^I - \theta(k(p^I))]$$

In this case, the bank's dominant strategy is to choose the correlation level of 1.

3.2.2 $p^S < \pi\Delta$

In this case the bank's expected payoffs in states *SS*, *SF* and *FS* are the same as in section 3.2.1. The only difference is that in state FF the bank has a positive expected payoff; given equations 22 and 15 we have:

$$E(u_2^{FF}) = (1-\beta)E(u_2^{SS})$$

The second-period expected payoff when the banks invest in different industries is then:

$$E(u_2(0)) = \pi^2 E(u_2^{SS}) + \pi(1-\pi)[E(u_2^{SS}) + \pi R - 1 - p^I - \theta(k(p^I))] + (1-\pi)^2(1-\beta)E(u_2^{SS})$$

and the second-period expected payoff when the banks invest in the same industries is:

$$E(u_2(1)) = \pi E(u_2^{SS}) + (1-\pi)(1-\beta)E(u_2^{SS})$$

We then have:

$$E(u_2(0)) - E(u_2(1)) = \pi(1-\pi)\{\beta[\pi R - 1 - p^I - \theta(k(p^I))] - (1-\beta)[p^I + \theta(k(p^I)) - \theta(k(p^0))]\}$$

The first term in the equation is positive; the second term is also positive since the capital requirement at p^I is greater than that at p^0. The bank's decision regarding the correlation level depends on β and $\theta(k(p^I))$. Holding β constant, the greater the cost of capital at p^I, the less incentive the bank has to choose correlation zero.

3.3 Consider the case in which $(\pi R - 1 - p^I) > \theta(k(p^I))$ and $(\pi R - 1 - p^S) > \theta(k(p^S))$.

In this case we are back to the benchmark model.

The bank's expected payoff in *FF* is given by equation 22:

$$E(u_2^{FF}) = \begin{cases} 0 & if \quad p^S > 2\pi\Delta \\ \frac{1}{2}(1-\beta)\left[2\pi(R-r_1) - p^S - \theta(k(p^0)) - \theta(k(p^S))\right] & if \quad p^S < 2\pi\Delta \end{cases}$$

3.3.1 $p^S > 2\pi\Delta$

The bank's expected payoff in state *FS* is zero, because both the surviving bank and the regulator find it optimal to liquidate the assets to the surviving bank.

$$E(u_2^{FS}) = 0$$

The bank's expected payoff in state *SF* is again given by equations A.2 and 15.

$$E(u_2^{SF}) = E(u_2^{SS}) + \left[\pi R - 1 - p^I - \theta(k(p^I))\right]$$

Equation 21 shows that the bank's expected payoff in state *FF* is zero, since the regulator finds it optimal to liquidate the assets to outside investors.

$$E(u_2^{FF}) = 0$$

The second-period expected payoff when the banks invest in different industries is then:

$$E(u_2(0)) = \pi^2 E(u_2^{SS}) + \pi(1-\pi)[E(u_2^{SS}) + \pi R - 1 - p^I - \theta(k(p^I))]$$

and the second-period expected payoff when the banks invest in the same industries is:

$$E(u_2(1)) = \pi E(u_2^{SS})$$

We then have

$$E(u_2(0)) - E(u_2(1)) = \pi(1-\pi)[\pi R - 1 - p^I - \theta(k(p^I))]$$

Since the regulator finds it optimal to liquidate the assets to outside investors in state FF and banks find it profitable to purchase the liquidated assets in state *SF*, it is optimal for the bank to choose the low correlation level.

3.3.2 $p^S < 2\pi\Delta$

In this case, the bank's expected payoffs in states *SS*, *SF* and *FS* are the same as in 3.3.1. The bank's expected payoff in state *FF* is given by equation 21. We have:

$$E(u_2^{FF}) = \frac{1}{2}(1-\beta)\left[2\pi(R-r_1) - p^S - \theta(k(p^0)) - \theta(k(p^S))\right]$$

The second-period expected payoff when the banks invest in different industries is then:

$$\begin{aligned} E(u_2(0)) = {} & \pi^2 E(u_2^{SS}) + \pi(1-\pi)[E(u_2^{SS}) + \pi R - 1 - p^I - \theta(k(p^I))] \\ & + (1-\pi)^2 \frac{1}{2}(1-\beta)\left[2\pi(R-r_1) - p^S - \theta(k(p^0)) - \theta(k(p^S))\right] \end{aligned}$$

and the second-period expected payoff when the banks invest in the same industries is:

$$E(u_2(1)) = \pi E(u_2^{SS}) + (1-\pi)\frac{1}{2}(1-\beta)\left[2\pi(R-r_1) - p^S - \theta(k(p^0)) - \theta(k(p^S))\right]$$

We then have:

$$\begin{aligned} E(u_2(0)) - E(u_2(1)) = \pi(1-\pi)\{&\beta[\pi R - 1 - p^I - \theta(k(p^I))] \\ &- (1-\beta)[p^I + \theta(k(p^I)) - \frac{1}{2}(p^S + \theta(k(p^S)) + \theta(k(p^0)))]\} \end{aligned}$$

The first term in the equation is positive, and the sign of the second term depends on the cost of capital in systemic failure; if $\theta(k(p^S))$ is large enough, banks have a low incentive to choose a high correlation, and also if $\theta(k(p^I)$ low. In general, for a given β, the sign of the equation really depends on the expected profit that the surviving bank can obtain in SF relative to the expected payoff in state FF, which faces a high cost of capital.