Sequential Decision-Making and Asymmetric Equilibria: An Application to Takeovers*

David Gill and Daniel Sgroi

Abstract

With indivisible shareholdings and simultaneous shareholder decision-making, the existing takeover literature provides a reasonable profit only in asymmetric equilibria. We allow the raider to approach shareholders sequentially and thereby find a unique equilibrium that produces the same outcome.

KEYWORDS: takeovers, sequential, pivotal

*David Gill can be contacted at: Trinity College, Oxford, OX1 3BH, UK. Daniel Sgroi can be contacted at: Faculty of Economics, Austin Robinson Building, Sidgwick Avenue, Cambridge, CB3 9DE, UK.
1. Introduction

Consider an unconditional offer to buy shares made by a raider, motivated by the raider’s correct belief that he can improve the value of the firm, say from 0 to 1 per share. In the standard model decisions are made simultaneously by an infinite number of atomistic shareholders. No bid can succeed at a price below 1, so the raider cannot extract any value from the takeover. If the bid were going to succeed, then each atomistic shareholder (who can have no influence on the outcome), would refuse to sell in order to wait for the takeover to raise the value of the share to 1, leading to a contradiction. This provides the basic setup in Grossman and Hart (1980). Bagnoli and Lipman (1988) and Holmstrom and Nalebuff (1992) consider a variation in which shareholders are finite in number, and so can exercise some influence over the probability of success. They find a unique symmetric mixed strategy equilibrium which gives very slight profits, tending to zero as the number of shareholders becomes large. They also find numerous asymmetric pure strategy equilibria which give greater profits to the raider, but which are indistinguishable but for the choice of who sells and who does not. It is hard to motivate why some would sell and others not in the simultaneous case.

The method used in this paper is a specific application of the well-understood fact that an asymmetric pure strategy equilibrium in a simultaneous game can often be reproduced in a sequential game. In particular, we can reproduce the better profits for a raider under the asymmetric equilibria, by allowing sequential decision-making by shareholders. This completely pins down what each shareholder in the sequence will do and so restores a unique single equilibrium.

The vast majority of takeover codes for publicly listed companies insist on the identical treatment of shareholders, and this legally restricts any attempt
at a sequential approach. However, the method is perfectly valid for attempted takeovers of non-listed companies, most obviously private medical, legal, consultancy and accounting partnerships or employee-owned businesses. It would be incorrect to assume that such partnerships are invariably economically insignificant or not subject to takeover activity. To give just one example: Deloitte Touche Tohmatsu, one of the largest auditing and consulting firms in the world with revenues of over $15 billion in 2003, has grown through numerous mergers and takeovers in its history including 50 mergers in the 1960s, the merger of Touche Ross and Tohmatsu Awoki and Co. in 1975, and the merger with Deloitte Haskins & Sells to form Deloitte Touche Tohmatsu in 1990.

Given the legal restrictions, we suggest that the sequential approach in this paper is best applied to non-listed firms. For this reason, though we stick with convention and describe decision-makers as shareholders, in most cases it may be sensible to think of them as partners or employee-owners. We might further argue that where takeover codes covering listed companies prohibit sequential decision-making, they might be revised in the light of our findings to improve the incentives for raiders to take over under-performing firms, though of course shareholders would still need to be adequately protected against expropriation and dilution.

2. The Model

The scenario is simple, and is based on Bagnoli and Lipman (1988). A firm is viewed by a raider as badly managed and undervalued. The raider believes there is a gain to be made through takeover, rationalization and the exploitation of synergies. This gain per share we normalize to 1 while the current value of the firm per share we normalize to 0. Let there be $N \geq 3$ shareholders all holding an identical and indivisible share of the firm. The raider makes

---

1 Most takeover codes are very clear that shareholders should be treated equally, and sequential offers would certainly break such codes. Some are a little more complex and may indeed allow differential treatment: e.g., in China the regulatory authorities can grant a waiver releasing the raider from the need to make a mandatory offer to all shareholders.

2 Recall that in most cases we are thinking of shareholders as partners or employee-owners in a non-listed firm, and therefore it is more reasonable to consider one share per shareholder.
an offer $a \in [0, 1]$ to these shareholders. The set of actions available to each shareholder is $A_i \in \{\text{Sell, Refuse}\}$.

We assume that offers are unconditional on whether the takeover succeeds or fails, and that success requires at least an integer number of shares $K \in \left(\frac{N}{2}, N\right)$ to be sold to the raider. Payoffs for shareholder $i$ are $\pi_i(\text{Sell}|\text{Takeover succeeds}) = \pi_i(\text{Sell}|\text{Takeover fails}) = a; \pi_i(\text{Refuse}|\text{Takeover succeeds}) = 1; \text{and } \pi_i(\text{Refuse}|\text{Takeover fails}) = 0$. A shareholder would like to sell if the takeover fails, but if it succeeds the shareholder would rather hang on to the share and see it rise in value to 1.

2.1 Simultaneous Decisions by Shareholders

Where shareholders all decide simultaneously whether to accept an offer, we can consider two possible symmetric equilibrium candidates in which the takeover succeeds (all sell and all mix with the same probability). First consider the “all sell” equilibrium. If all shareholders were selling at $a < 1$, then each shareholder would have an incentive to deviate by refusing to sell, thus guaranteeing 1 as the takeover would still succeed. Thus, there can be no “all sell” equilibrium in which the raider can extract positive surplus.

Bagnoli and Lipman show that the raider can always make a strictly positive profit in a symmetric mixed strategy equilibrium. However, the profit relies on the small difference between the probability of success relevant to a shareholder, namely the probability of success if he does not sell but the other shareholders mix, and the probability of success relevant to the raider, namely the probability of success if all shareholders mix. As $N \to \infty$, the difference between these probabilities tends to zero, so the expected profit to the raider vanishes.

The payoff can be improved by considering asymmetric pure strategy equilibria. The trick is to make shareholders pivotal. In an asymmetric equilibrium where precisely $K$ shareholders sell and precisely $N - K$ do not, all those who sell are pivotal and hence accept the offer of $a$ instead of forcing the takeover to fail and getting 0, while all those who do not sell are not pivotal and hence prefer $1 > a$, since the takeover succeeds. In such an equilibrium, with $a$ close
to zero, the raider will get a payoff close to $\frac{K}{N}$ per share. With the number of shares required for control defined as a proportion of the total number, the payoff per share will be constant in $N$, in contrast to the result for the unique symmetric equilibrium.

There are many equilibria with this property since for each permutation of shareholders such that precisely $K$ accept and $N - K$ decline there exists an asymmetric equilibrium, so it is hard to motivate how shareholders would be able to coordinate on any particular one, or indeed why they would play an asymmetric equilibrium at all given the small profits to the raider in the unique symmetric equilibrium. We see in the next section that using this idea of pivotal shareholders gives us another way of inducing the same outcome as in the asymmetric type of equilibrium, but where the outcome is uniquely defined by the arbitrary sequential ordering imposed by the raider.

### 2.2 Sequential Decisions by Shareholders

As shown we can have asymmetric equilibria that render takeovers possible and profitable. However, there are multiple equilibria which begs the question: why do some shareholders sell and others don’t? We resolve this by moving away from simultaneous decision-making by shareholders to sequential decision-making. There are two major impacts of moving to a sequential framework: shareholders are now heterogeneous as they are all indexed by a unique number in the ordering; and each shareholder must consider the impact of his decision on others, and the impact of others’ decisions on him. We now have a more strategic problem from the shareholders’ perspective.

We denote the history of shareholder actions up to but not including $j$ as $H_j = \{A_1, A_2, \ldots, A_{j-1}\}$, and the number of sales and refusals in this history as $\#H_j(\text{Sell})$ and $\#H_j(\text{Refuse})$ respectively. Before going on to the main
proposition, we will define a concept which goes to the heart of the sequential model: refuse-pivotalness, and then state and prove a crucial lemma.

**Definition.** Shareholder \( j \) is refuse-pivotal if \( \#H_j(\text{Refuse}) = N - K \). Thus, if \( j \) refuses to sell, the takeover is certain to fail, while if \( j \) sells, the takeover may yet succeed.

**Lemma.** For any offer \( a > 0 \), if shareholder \( j \) is refuse-pivotal he will sell and so too will all shareholders \( i > j \). The takeover must then succeed.

**Proof.** If \( j \) is refuse-pivotal then \( \pi_j(\text{Sell}) = a > \pi_j(\text{Refuse} | \text{Takeover fails}) = 0 \), so \( j \) sells. Now \( A_j = \text{Sell} \Rightarrow \#H_j(\text{Refuse}) = \#H_{j+1}(\text{Refuse}) = N - K \), so \( j + 1 \) is also refuse-pivotal and sells. By induction this must also be the case for all \( i > j \). The takeover succeeds as there were exactly \( N - K \) refusals before \( j \) and so there must be \( K \) sales in total given \( j \) and all later shareholders sell. \( \square \)

**Proposition.** Under sequential decision-making, for any offer \( a \in (0,1) \) the unique equilibrium involves a successful takeover in which the first \( N - K \) shareholders refuse to sell, gaining a payoff of 1, and the remaining \( K \) shareholders all sell, gaining a payoff of \( a \).

**Proof.** Take any shareholder in the sequence. There are four possibilities: (i) Shareholder \( j \) is refuse pivotal, i.e., \( \#H_j(\text{Refuse}) = N - K \). (ii) Shareholder \( j \) is not refuse pivotal and \( \#H_j(\text{Sell}) \geq K \). (iii) Shareholder \( j \) is not refuse pivotal and \( \#H_j(\text{Sell}) < K \); \( \#H_j(\text{Refuse}) > N - K \). (iv) Shareholder \( j \) is not refuse pivotal and \( \#H_j(\text{Sell}) < K \); \( \#H_j(\text{Refuse}) < N - K \).

In case (iv), from \( j \)'s perspective, there are only two possibilities. Either the takeover succeeds without anybody ever becoming refuse pivotal, or there are enough later refusals that a later shareholder \( k \) becomes refuse pivotal, in which case by the Lemma the takeover succeeds. Thus \( j \) knows that whatever he does, the takeover will succeed, so he refuses to sell to get \( \pi_j(\text{Refuse} | \text{Takeover succeeds}) = 1 > \pi_j(\text{Sell}) = a \).

\( \#H_1(\text{Sell}) = 0 < K \) and \( \#H_1(\text{Refuse}) = 0 < N - K \). Thus, the first shareholder is in situation (iv) and so refuses to sell. Thus the second is in
the same situation and also refuses, so long as $\#H_2(\text{Refuse}) = 1 < N - K$. The same argument applies by induction until the $(N - K + 1)$th shareholder, for whom $\#H_{N-K+1}(\text{Refuse}) = N - K$. This shareholder is therefore refuse-pivotal, so by the Lemma $j = N - K + 1$ and all later shareholders sell, and the takeover succeeds. $\Box$

Thus, the raider can offer any $a \in (0, 1)$, and buy $K$ shares from the last $K$ shareholders approached, earning $(1 - a)$ per share purchased. From the raider’s perspective the sequential outcome produces an identical profit to the asymmetric outcome in Bagnoli and Lipman (1988), and a better outcome than the symmetric one where profits are small, vanishing to zero as the number of shareholders becomes large.

3. Applications

We require that the raider be able to commit to the order in which he approaches the shareholders; otherwise the later shareholders might try to hold out hoping that the raider will be forced to approach some of the earlier ones once more. However, a raider involved in multiple raids, e.g., a private equity firm, may well be able to build up a suitable reputation through repeated takeover activity. To give one example, Oasis Healthcare PLC, through its wholly-owned subsidiary, Oasis Dental Care Ltd., has taken over 67 independent dental partnerships in the UK since 2002. Oasis note that the UK dental market (worth over $5 billion, and growing at 10% annually) is highly fragmented and under-developed, with the individual practice partnership, of which there are 3,000 with three or more surgeries, representing the principal business unit. The scope for continuous and ongoing takeover activity in this industry is considerable, and provides an excellent opportunity for a raider to establish a reputation.

Another example application is to employee-ownership in the transport industry. In the 1990s, UK municipal bus services were privatized, leading to
a proliferation of small employee-owned city-based bus companies. There followed a period, to some extent still ongoing, of takeovers leading to the creation of Stagecoach Group, First Travel and other large transport companies which operate at a national or even international level. Spear (1999) suggests that the UK experience with employee-owned bus companies raises theoretical and strategic issues concerning their ability, as non-listed companies, to resist raiders. This is in stark contrast to the Grossman-Hart paradox which focuses on the ability of inefficient firms to resist takeover. One rationale for the boom in takeovers in this sector might well be the ability of raiders to avoid takeover codes, which are not applicable to non-listed companies, and approach the shareholders (in this case employee-owners) without the need to make simultaneous mandatory offers.

Any attempt to approach shareholders sequentially, whether partners or employees-owners, will be much more likely to be practical where shareholdings are not too diffuse, as was certainly the case with dental practices and bus companies in the 1990s, and continues to be the case in much of the medical, legal, accounting and consulting sectors.

4. Conclusion

We have modified the basic framework of a takeover model to allow a raider to approach shareholders sequentially. As stressed in the introduction, where the shareholders are partners in a non-listed company, such as most private consulting, accounting or medical firms, there are no legal restrictions on a sequential approach. We then have a unique equilibrium providing profits for the raider in contrast to the situation in Grossman and Hart (1980). The sequential decision-making framework closely duplicates the outcome in the multiple asymmetric equilibria under simultaneous decision-making in Bagnoli and Lipman (1988). Since in our model partners are distinguished by their place in the ordering, we can eliminate all but one equilibrium and pin down which partners will gain most from the takeover. The intuition is quite clear: under sequential decision-making those towards the end of the sequence will have to sell to obtain a strictly positive payoff. This allows the raider the
freedom to make a much lower offer, and enables partners approached earlier to earn a higher payoff than later decision-makers by holding onto their shares. Our model applies best to targets whose shareholdings are indivisible and not too diffuse, giving another reason for considering the case of non-listed partnerships.

We also need credibility. Recalling the efforts of Oasis Healthcare PLC to consolidate the UK dental industry, which have resulted in a long sequence of ongoing acquisitions of small partnerships with no end in sight, we might hypothesize that the reputation of the raider can work to ensure credibility.

Though the application to takeovers is particularly clear, in principle, the general argument might help with any related simultaneous decision-making problem with freeriding. It might be that considering sequential decision-making can provide for a unique solution with reasonable payoffs, though in each application, as with takeovers, the feasibility of a sequential ordering of decision-making would need to be considered.

References


