Ownership structure, Voting and Risk

September 16, 2011

Abstract

We analyze the determinants of ownership structure in firms when conflicts of interest on risk arise endogenously via different ownership stakes and firm decisions are made through majority voting. A large block is chosen to incentivize monitoring. Because a large blockholder holds a large share of the firm, he is averse to risky investing. This generates a conflict of interest with dispersed shareholders. Mid-size blockholders, voting in favor of middle of the road projects, mitigate the conflict of interest. Depending on monitoring costs, voting institutions and the nature of the industry, three types of ownership structures arise: one large shareholder with a fringe of dispersed owners, multiple large shareholders and fully dispersed shareholders.

1 Introduction

Much of the literature on corporate governance has focused on the role of large shareholders in resolving free riding problems that arise when there is dispersed ownership.[1] There is, however, a vast empirical literature documenting that ownership structure takes very diverse forms, ranging from one large shareholder, to multiple intermediate sized shareholders and fully dispersed structures. In the US, 67% of public firms has more than one blockholder with a participation larger than 5%, while only 13% are widely held and 20% has only one blockholder (using the database in Dlugosz, Fahlenbrach, Gompers, and Metrick (2006)). In eight out of nine largest stock markets of the European Union the median size of the second largest voting block in large publicly listed companies exceeds five percent (data from the European Corporate Governance Network).[2] In this paper we provide a novel theory why blockholders may emerge. A larger block implies increased voting rights (and hence an increased ability to affect important firm decisions) but a less diversified portfolio. We investigate how potential conflicts of interest between shareholders on the risk profile of a firm affect the ownership structure of a firm when majority voting is the mechanism for aggregating shareholder preferences. We show that depending on model parameters a variety of ownership structures may emerge, including the often observed multiple large blockholders. Several novel empirical predictions are derived. Our theory links firm characteristics (such as investment size, industry characteristics and minority shareholders participation) to its ownership structure. Moreover, the model makes predictions about how a firm's ownership structure affects its choice of project risk.

Empirically, the literature on ownership structure and risk is sparse. To the best of our knowledge, the issue of how potential conflicts of interest can affect a firms

[1] See e.g. Grossman and Hart (1980), Stiglitz (1985), Shleifer and Vishny (1986), Holmstrom and Tirole (1993), Admati, Pfleiderer, and Zechner (1994), Burkart, Gromb, and Panunzi (1997), Pagano and Röell (1998), Bolton and Von Thadden (1998), Maug (1998)).

[2] La Porta, Lopez-De-Silanes, and Shleifer (1999) find that 25% of the firms in various countries have at least two blockholders while Laeven and Levine (2008b) find that 34% (12%) of listed Western European firms have more than one (two) large owners where large owners are considered shareholders with more than a 10% stake.

ownership structure has not been studied empirically. The question of how ownership structure affects risk taking has been studied by Laeven and Levine (2008a) who analyze data for more than 250 privately owned banks across 48 countries, and show that banks that have well diversified shareholders, are likely to take more risks. In the case of non-financial firms, similar patterns are observed between ownership structure and risk decisions, e.g. (Carlin and Mayer (2003) and John (2008)). While the exact direction in which ownership structures affect risk changes across regulatory environments, the main lesson to be drawn from the empirical literature is that there is a close link between important risk decisions of the firm and its ownership structure. In this literature several questions remain unanswered: what is the mechanism which determines the link between ownership structure and important decisions of the firm? In the presence of multiple blockholders, whose preferences do decisions reflect? Given the apparent cost of block ownership, what exactly is the governance role of a second third etc. blockholder? Our paper is an attempt to answer these questions.

Formally, suppose there is an initial owner or founder of a new firm who needs to raise capital to finance a project. The initial owner of the firm cannot commit to take a value enhancing action, e.g. monitoring the manager, unless he has a sufficiently large stake (e.g. Maug (1998)). The higher the cost of monitoring, the higher the minimum threshold of shares to which the initial owner must commit.[3]

The initial owner raises capital through issuing shares and/or debt. Outside investors who are ex-ante identical decide their share ownership conditional on the offer price set by the initial owner. Both the owner as well as other investors are risk averse, but their preferences on risk/return depend only on the size of their stakes in the firm: the larger the stakes the lower the risk/return trade-off they prefer. Once the ownership structure is established, shareholders vote on the riskiness of the investment projects that the firm subsequently undertakes. At the time of buying shares therefore, investors

[3] In general the initial owner may hold a larger stake because of other reasons, e.g. signaling his faith in the firm to outside investors, or because of complementarities between the the entrepreneur and the value of the firm (e.g. the case of Sun Microsystems and Apple). This would not change the main results of the paper.

face a trade off between holding a diversified portfolio and having little influence on the firm's decisions, or buying more shares, maybe holding a suboptimal portfolio, but influencing the firm's decisions. This trade-off matters because the initial owner prefers less risky and lower return projects due to his large initial stake, while outside investors would ideally want the highest risk/return, creating an endogenous conflict of interest.[4]

We show that some investors buy mid-sized blocks, so as to guarantee (by their votes) that the risk/return profile of the firm is higher than what would be chosen by the initial large shareholder alone. Paradoxically, of course, when they do buy a larger fraction of shares, their preferences move closer to those of the initial large shareholder.

Our main results are that for low values of the monitoring cost, which correlates with low potential conflicts of interest on risk, we should observe a dispersed ownership structure, while for very high values of monitoring costs (high potential conflicts) we should observe one large shareholder with a fringe of dispersed owners. Finally, for intermediate values of the initial owner's monitoring costs (conflicts on risk profile exist but are not too big), multiple blockholders should be observed. Other important determinants of ownership structure are voting institutions, the size of the firm and the degree of minority protection.

We show that in turn, ownership structure affects the decisions on risk: a single large shareholder is associated with lower risk (as the large shareholder makes all decisions) while multiple large shareholders are associated with higher risk and higher firm value. Mid-sized blockholders play a role in mitigating the conflicts of interests between the largest shareholder, who prefers to reduce risk at the expense of value and liquidity shareholders who are value maximizing. Moreover, our model predicts that when there are multiple blockholders, it is not necessarily the largest shareholder who is pivotal for the risk taking decision.

[4]The effects on the risk choices when a controlling shareholder is less diversified can be seen in the choices of the Swedish bank Skandinaviska Enskilda Banken (SEB). The controlling shareholder, the Wallenberg family, has a big part of its wealth invested in the bank. The bank's approach is to be prudent as "sometimes life can turn sour". For this reason it faced the financial crises with a lot of cash which helped it to perform better than its' peers and in general than the stock market (The Economist, 2009). For a more systematic study see Faccio, Marchica, and Mura (2009).

This is broadly consistent with what the empirical literature has found on the relationship between ownership structure, firm risk and value : Carlin and Mayer (2000; 2003) show that firms with more dispersed ownership tend to invest in higher risk projects, like R&D and skill intensive activities. Laeven and Levine (2008b) find that firms with several blockholders have a higher Tobin's Q than firms with only one big shareholder. On the other hand, Lehmann and Weigand (2000), Volpin (2002), (Faccio, Lang, and Young, 2001), Maury and Pajuste (2005), Gutierrez and Tribo (2004) find that multiple blockholders lead to higher firm value.

Other important determinants of ownership structure in our model are the nature of the industry -innovative (high potential returns and risk) or mature (low risk, low return, less potential for conflicts of interest) as well as the legal framework around voting institutions and the way they function. Anecdotal evidence suggests that in more mature sectors it is more common to see families in control of firms. Becht, Franks, Mayer, and Rossi (2009) suggest that activism by large investors is much more common in the UK than in US, because of the legal and regulatory structure as well as different voting institutions in the two countries.

The literature on explaining ownership structure is sparse. Zwiebel (1995) was the first to explain the existence of multiple intermediate sized blocks. He argues that when wealth constraints limit block ownership, multiple blockholders may form a controlling coalition. Investors can get partial benefits of control based on their initial endowments. We depart from Zwiebel (1995) in endogenizing (i) the amount of wealth each investor puts in the firm and (ii) endogenize control benefits through the firm's decision on risk. This allows us to generate new empirical implications regarding firm characteristics and ownership structure.

In a more normative vein, Bennedsen and Wolfenzon (2000) show that multiple large shareholders can reduce expropriation of minority shareholders and ownership structure can thus commit the initial owner to more efficient decisions. Their work applies to closely held corporations characterized by an absent resale market for shares. In contrast our paper analyzes ownership structures in publicly held corporations where trading is allowed. Gomes and Novaes (2001) consider the role of veto power by blockholders instead of considering majority rule as voting mechanisms as we do. The shared control among a few large shareholders may be an efficient way to protect minority shareholders reducing the extraction of private benefit of control. Their emphasis is however on the hold up problems induced by the veto power of blockholders and they derive empirical implications on the effects of veto power.

Noe (2002) and Edmans and Manso (2008) look at the link between multiple blockholders and financial markets. In Noe (2002) blockholders can trade and profit from private information but because price movements have no disciplining role, 'exit' threat by selling shares does not have any effect on managerial effort. In Edmans and Manso (2008) instead, share price movements have a disciplining role and the threat of exit induces the manager to exert more effort. They conclude that share price movements are more effective than direct governance tools. Our paper focuses instead on how decision are taken in relation to the role of activist blockholders. The assumption that blockholders are activist on policy decisions of the firm is confirmed e.g. by Helwege, Intintoli, and Zhang (2011) who find that institutional investors prefer to affect firm policy through activism and voting rather than acting through the threat of exit. Yermack (2010) provides a survey of evidence on how shareholder voting has led to significant changes in corporate governance and strategy.

Admati, Pfleiderer, and Zechner (1994) and DeMarzo and Urosevic (2006) analyze the trade off between risk sharing and monitoring: a larger block improves the incentives for monitoring but comes at the cost of greater exposure to risk. In our model too, the entrepreneur may need to hold a larger block to commit to monitoring and this comes at the risk of greater exposure to firm risk. However, the focus of our paper is on how conflicts of interests between a large (monitoring) blockholder and dispersed shareholders lead to the emergence of additional smaller blockholders.

An additional prediction from our model is that ownership structure affects underpricing in IPOs. In an equilibrium where blockholders are present, share prices are lower than the willingness to pay by well diversified liquidity shareholders. Hence IPO underpricing is correlated with multiple blocks sharing control of the firm. It does not occur with other ownership structures. This is consistent with some empirical studies (Brennan and Franks (1997), Fernando, Krishnamurthy, and Spindt (2004), Goergen and Renneboog (2002) and Nagata and Rhee (2009)).[5]

Finally, the paper contributes to the literature on voting. Typically, the models on voting do not endogenize individual preferences, the price of votes and hence the voting power of an agent (Dhillon, 2005). When applying voting theories to corporate governance issues, on the other hand, the firm value (and hence share prices) and shareholders decisions are closely related. An investor can decide how many shares to buy and their voting decision changes depending on the block he chooses to buy. The price, being set by the initial owner, becomes an endogenous variable that affects and is affected by the existence of a second blockholder and the voting outcome.

The paper is organized as follows. Section 2 outlines the model. Section 3 provides sufficient conditions for different ownership structures to be equilibria. In section 4 we derive the empirical implications of the model. Finally, section 5 concludes. All the proofs are in the Appendix.

2 The Model

An initial owner of a firm seeks to raise a minimum amount of capital K through equity finance for the firm.[6] He is endowed with 1 unit of wealth which he allocates between the firm (a fraction w_E) and a risk free asset. Potential investors are similarly endowed with 1 unit of wealth of which they invest a fraction w_i in the firm. In return for their investment they receive shares $\alpha_i = \frac{w_i}{K-w_E}(1-\alpha_E)$ each, where α_E denotes the shares retained by the initial owner. The set of (potential) outside investors is partitioned into two types of investors; a fraction λ are *active* shareholders and the remaining who are

[5] Stoughton and Zechner (1998) and DeMarzo and Urosevic (2006) show that IPO underpricing can serve to ensure the participation of large investors who can monitor and hence be value enhancing. However, in these papers control considerations are absent and the role of multiple large investors is not analyzed. Empirically this implies that there is no difference between one or multiple blockholders.

[6] Initially we assume the initial owner raises just K; later in section 3.2 we show that this assumption is without loss of generality as the initial owner always prefers to raise the minimum amount of capital.

passive. Active shareholders are assumed to vote anticipating that their vote is going to have an impact on the decisions of the firm.[7] Passive shareholders act competitively and take the firm's decisions as given, ignoring their own potential influence: for technical simplicity we assume they do not vote. The parameter λ allows us to capture the effects of minority protection on firm ownership structures: the higher is λ the higher is the participation of active minority shareholders.

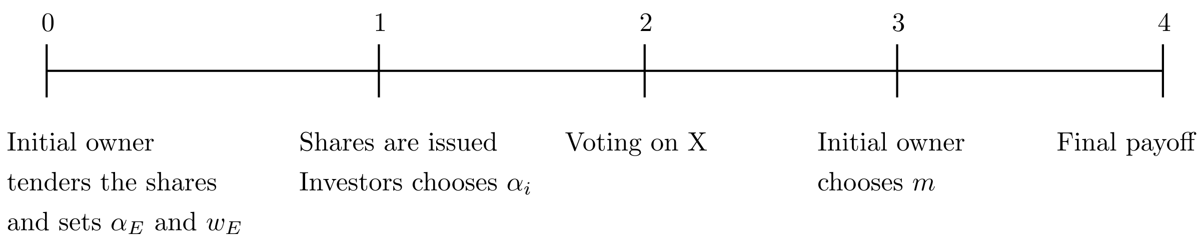

Figure 1: The Time Structure

Figure 1 shows the timeline of the game: in period 0, the initial owner decides w_E and the fraction of the shares to retain, α_E. This is equivalent to announcing the fraction of shares retained together with the share price, $\frac{K-w_E}{1-\alpha_E}$. In period 1, investor i decides the fraction of shares of the firm to buy, α_i. There are sufficiently many investors in the market so that there is never a problem of excess supply of shares. If there is under-subscription, then the project cannot go ahead. If there is oversubscription, we assume that this is a stable situation only when no investor who gets shares is willing to sell them at a price lower than the maximum price that an excluded investor is willing to pay.[8]

In period 2, shareholders have to take a decision by voting. At this stage, the ownership structure is common knowledge. The voting decision is about the risk profile of the firm. For example, we may think of this as a decision about the projects that a firm

[7] Institutional investors such as hedge funds can be interpreted as active investors. It is well known that the value creation effect of hedge funds is highly significant both in the short and in the long run. For more evidence on the role of active investors see Smith (1996), Brav, Jiang, Partnoy, and Thomas (2008) and Becht, Franks, Mayer, and Rossi (2009).

[8] We do not explicitly model the secondary market in shares but we capture some of the spirit of the secondary market by imposing this particular refinement of the Nash equilibrium concept.

will invest in or the CEO's nomination. The project values are affected by a variable X as follows: it is normally distributed with mean $\bar{R}X + f(m)$ and and standard deviation σX, where $f(m)$ is the extra expected cash flow from monitoring (which is chosen in period 3) and $\bar{R} > 0, \sigma > 0$ are parameters. Shareholders thus choose a project profile $X \in [0, \bar{X}]$: the higher is X the higher is the risk and return of the firm. To ensure that there is an interesting conflict of interest, we assume that $\bar{X} > \frac{\bar{R}}{\gamma\sigma^2}$. Hence, conflicts of interest between shareholders can be captured in a simple way through the uni-dimensional linear efficiency frontier of possible projects. The decision X is taken through majority voting. We show later that once the ownership structure is fixed, preferences on X are single peaked (Lemma 1) so that the median shareholders ideal point on $[0, \bar{X}]$ is the Condorcet Winner.[9] Voting is costless. However, since only a subset of investors are assumed to vote the Condorcet winner must be chosen from among the ideal points X_j of the voting subset only.

In period 3 the initial owner decides whether to take a value enhancing action, which we refer to as monitoring. His choice variable is $m \in \{0, 1\}$. If the initial owner monitors ($m = 1$), the expected firm value increases by $f(1) = K$ at a monetary cost of $c(1) = \bar{m}K$, with $\bar{m} \leq 1$. If the initial owner does not monitor, $f(0) = 0$ and $c(0) = 0$.

Finally at period 4 the payoffs are realized.

Investors and initial owner have identical preferences represented by the following utility function:

$$u_j = -\frac{1}{\gamma} e^{-\gamma Y_j} \tag{1}$$

where $j = \{i, E\}$, i refers to an outside investor, E refers to the initial owner, γ is the parameter of risk aversion and $Y_j = g(w_j)$ is the final wealth when a fraction w_j of the wealth is invested in the project.

The certainty equivalent representation of the utility function (1) with X fixed is:

$$U_i = \alpha_i \left(\bar{R}X + f(m)\right) - \frac{K - w_E}{1 - \alpha_E}\alpha_i - \frac{\gamma}{2}\sigma^2 X^2 \alpha_i^2 + 1 \tag{2}$$

[9] The Condorcet winner is the project that wins against every other project in pair-wise majority voting.

The first part is the expected wealth from investing in the firm, the second part is the price paid and the third is the dis-utility from investing in a risky asset. Investor i will maximize (2) by choice of α_i, given $\alpha_E, K - w_E$ and the beliefs on X, $f(m)$. Since the initial owner is the residual claimant, his exponential utility function can be written in terms of certainty equivalent as:

$$U_E = \alpha_E(\bar{R}X + f(m)) - \frac{\gamma}{2}\sigma^2 X^2 \alpha_E^2 + 1 - w_E - c(m) \tag{3}$$

The initial owner chooses α_E, w_E in period 1 and m in period 3 to maximize (3) subject to the constraint that he needs to raise the capital, i.e. $K - w_E \leq \sum_i w_i$, or equivalently $1 - \alpha_E \leq \sum_{i=1}^{N} \alpha_i$ where $\sum_i \alpha_i$ is the sum of the total shares demanded. Furthermore $\alpha_E + \sum_i \alpha_i = 1$.

The wealth invested, w_j, is not bounded either for the initial owner or for the outside investors. Hence, investors can short the shares ($w_i < 0$). This never occurs in equilibrium as the initial owner always lowers price to the point where investors want to own a positive fraction of the shares. For the initial owner $w_E < 0$ means that he receives wealth instead of investing in the project. This can be interpreted as initial owner's rent for the entrepreneurial idea. Finally when $w_j > 1$ with $j \in \{E, i\}$ it means that the investors borrow money (at the risk free rate) in order to invest in the firm. Hence, for the initial owner the decision is not only on the ownership structure but also the composition of his portfolio between (personal) debt and equity. The higher the debt, the higher is the risk exposure but the higher the control he has.

We now derive the ideal point $X_j(\alpha_j)$ shown in Figure 2, for an investor j. We then determine the payoff functions of players given the ownership structure defined as the vector of shares owned by investors: $\vec{\alpha} = (\alpha_E, \alpha_1, ..., \alpha_k)$ where k is the number of active investors who hold shares.

Lemma 1 *The preferred choice of X given α_j, for any shareholder $j \in \{i, E\}$, denoted X_j is uniquely defined by:*

$$X_j = \min\left[\frac{\bar{R}}{\gamma\sigma^2\alpha_j}, \bar{X}\right] \tag{4}$$

The choice of X depends only on the investor's shareholdings α_j as $\frac{\bar{R}}{\gamma\sigma^2\alpha_j}$ is a one-to one function of α_j. We define $\bar{\alpha} \equiv \frac{\bar{R}}{\gamma\sigma^2\bar{X}}$ as the fraction of shares $\bar{\alpha}$ such that $X_j(\bar{\alpha}) = \bar{X}$. It follows from Lemma 1 above that once $\vec{\alpha}$ is fixed, preferences of investors and the initial owner on X are single peaked. Hence, by the median voter theorem (Black (1948)), there exists a Condorcet Winner on the set $[0, \bar{X}]$ and it coincides with the preferred point of the median shareholder. Denote $X_{med}(\vec{\alpha})$ as the median X when the ownership structure is $\vec{\alpha}$. To save on notation, we suppress the argument $\vec{\alpha}$. For convenience we denote the median shareholdings as α_{med}.[10]

In the first best situation, i.e. no monitoring issues, both the initial owner and the investors would like to have as few shares as possible and choose maximum return (i.e. $\bar{X}$), even though it comes with high risk (Lemma 2). However, the incentive problems associated with monitoring imply that the initial owner faces a trade off between increasing the value of the firm through monitoring and his portfolio diversification.[11]

Notice from equations (2)-(4) that α_E determines both the price paid, $\frac{K-w_E}{1-\alpha_E}$, and (potentially) X_{med} through the share ownership structure. Hence the indirect utility function for active investors depends on α_E, as well as the anticipated m and $\vec{\alpha}$ (given α_E and $K - w_E$). Passive investors' indirect utility depends also on α_E, but X is taken as given. Pure strategies of the owner are 3-tuples $(w_E, \alpha_E, m(\alpha_E, X_{med}))$ together with a function from α_E to a voting decision over X. Pure strategies of investors are functions from $(\alpha_E, K - w_E)$ to a shareholding α_i and a voting decision over X.[12] This describes an extensive form game, where the set of players are the initial owner and other active investors, the pure strategies and payoffs are as above. We look for subgame perfect equilibria of the game described in Fig. 1. See the appendix for a formal definition of equilibrium.

[10]Consider the frequency distribution of shares of *initial owner and active investors only* on the set X. The median X is the unique X_j such that exactly half the shares are on either side of it. Since it is common knowledge that passive investors never vote, α_{med} is defined only on the basis of shares of initial owner and active investors.

[11]We use the term 'diversify' loosely to capture the notion that an agent may benefit from reducing his exposure to the project risk.

[12]Since there is pairwise voting, voting is assumed to be sincere and we rule out strategic agenda setting issues.

3 Equilibria

We solve the game by backward induction. The last stage is the monitoring decision. The initial owner can commit to monitoring only when he owns enough shares, $\alpha_E \geq \bar{m}$ (see Lemma 7 in Appendix). Observe that the decision to monitor is independent of the voting outcome.

The second last stage is the voting game. This is trivial given the share ownership. Each voter votes for his ideal point given his shares (Lemma 1) and the median shareholder's preferred point X_{med} is chosen. Finally we come to what we call the *ownership subgame*. In this subgame, investors buy their shares given (α_E, w_E), anticipating the effects of their share ownership on the voting outcome and the monitoring decision. Since the monitoring outcome is independent of ownership structure except through α_E, we can therefore partition the subgames at this stage into those where $\alpha_E \geq \bar{m}$ and those where it is less. There is a continuum of such subgames. We define the *Equilibrium Ownership Structure* (EOS) as the Nash equilibrium of the subgame for each pair $(\alpha_E, w_E) \in S \equiv [0, 1] \times (-\infty, +\infty)$.

We define outside investors who buy shares as *liquidity shareholders*, or *blockholders* depending on whether they hold an optimally diversified portfolio or not. Liquidity shareholders' shares are denoted by $\alpha_{l,j}$,which represents the optimal portfolio when the voting outcome is assumed to be X_j. Given the investors' beliefs on $X_{med} = X_j$ a straightforward maximization of the utility function for outside investors yields the demand for shares by liquidity investors of (see Lemma 8 in Appendix):

$$\alpha_{l,j} = \frac{X_j \bar{R} + f(m) - \frac{K - w_E}{1 - \alpha_E}}{\gamma X_j^2 \sigma^2} \tag{5}$$

The fraction of shares chosen by the liquidity investors depends on their beliefs on the voting outcome X_j: $\alpha_{l,j}$ is decreasing in X_j. In general, investors can short their shares so $U_{l,j} \geq 1$ even if $\alpha_{l,j} < 0$. However, the constraint on full subscription by the initial owner implies that in equilibrium $\alpha_{l,j} > 0$.

Observe that the value function of liquidity investors, $U_{l,j}$ is always bigger than

1 since there is no restriction on $\alpha_{l,j}$: if the utility from holding the firms' shares is less than the risk free asset, then $\alpha_{l,j}$ could be negative, i.e. investors can go short on the shares. We show later that the constraint on full subscription by the initial owner implies that in equilibrium $\alpha_{l,j} > 0$. In the particular case there $X_j = \bar{X}$, we define $\bar{\alpha}_l$ the optimal shareholding of the liquidity investors.

A *blockholder* is an outside investor who in equilibrium holds a suboptimal portfolio, i.e. $\alpha_i > \alpha_{l,j}$ where α_i is the shareholding of blockholder i. Let U_j^{nBH} denote the value function of a representative blockholder, who owns α_i fraction of shares, in an ownership structure which admits n blockholders and has $X_{med} = X_j$ (j being the pivotal blockholder).

We can now define the *first best* choice for investor i formally as the optimal α_i assuming that in the second stage agent i acts as a dictator in the choice of X.

Lemma 2 *Assume* $\frac{K-w_E}{1-\alpha_E} > f(m)$*, the first best choice of the outside investors is* $X = \bar{X}$ *and* $\alpha_i = \bar{\alpha}_l$.

Definition 1 *An Equilibrium Ownership Structure (EOS) corresponding to a pair (*α_E*,* w_E*) is an equilibrium of the subgame beginning at the information set (*α_E*,* w_E*). In particular the following must be satisfied in equilibrium: 1.* $U_j^{nBH} \geq 1$ *(if there are any blockholders in equilibrium) and* $U_{l,j} \geq 1$ *(if there are any liquidity shareholders in equilibrium). We call this the Participation Constraint, 2. Capital is raised, i.e.* $\sum_i = \alpha_i \geq 1 - \alpha_E$*. 3. No active investor wants to unilaterally increase or decrease his shares, given* α_E*,* w_E*. We call this the Incentive Constraint. 4. Passive investors maximize their utility conditional on the anticipated* $X = X_{med}$*. No investor who receives shares is willing to sell them at a price lower than the maximum price that any excluded investor is willing to pay.*

The equilibrium concept is standard– that of Nash between active investors, and passive investors act to maximize their utility given their beliefs on X_j, which must be the right beliefs so $X_{med} = X_j$. Refinement 4. is imposed because in case of oversubscription, we would like a rationing rule that allocated shares in a “stable” way.

In other words, we want to ensure that no excluded investor can do better by deviating unilaterally.

The ownership structure can be of three types based on who is the median shareholder: (A) (i) *No Conflicts EOS* where $X_{med} = X_E = \bar{X}$, hence there are no conflicts of interest; between the initial owner and shareholders (who hold a perfectly diversified portfolio). (ii) *Liquidity Shareholder EOS* where active liquidity investors are in control of the firm, $X_{med} = \bar{X}$. (B) An *n-Blockholder EOS*, where $X_{med} = X_j$ and n active investors plus the initial owner hold a non-perfectly diversified portfolio. For simplicity, we focus first on blockholder equilibria where all investors hold the same shares α_1. (C) The *Initial Owner EOS*, where $X_{med} = X_E < \bar{X}$ and all outside investors are liquidity shareholders.

Definition 2 *A symmetric n-Blockholder ownership structure is one where there are $n > 0$ active investors (blockholders) with shares α_1 each and $N_A \geq 0$ active liquidity investors with shares $\alpha_{l,1}$ such that $\alpha_1 > \alpha_{l,1} > 0$, $N_A \alpha_{l,1} = \lambda(1 - \alpha_E - n\alpha_1)$, and $X_{med} = X_1$.*

The participation constraint of liquidity investors, $U_{l,j} \geq 1$, also implies that the maximum price they are willing to pay, $\frac{K - w_E}{1 - \alpha_E}$, is higher, the higher is the anticipated risk/return profile of the firm, i.e. the higher is X_j.

Lemma 3 *Liquidity shareholders are better off than blockholders regardless of the monitoring decision.*

This is true by definition: Liquidity investors hold the fraction of shares that provides optimal diversification given the anticipated voting outcome, X_j. Any other shareholding gives lower utility.

The next lemma shows that if the share price is sufficiently high, the pivotal investor holds more shares than the liquidity investors.[13]

[13]If, to the contrary, the share price is too low, so that $f(m) \geq \frac{K - w_E}{1 - \alpha_E}$, investors will demand all the shares tendered. We show later that this cannot be an equilibrium as the initial owner will always maximize the share price.

Lemma 4 *Assume* $\frac{K-w_E}{1-\alpha_E} > f(m)$*, then* $\alpha_{l,j} \leq \alpha_j$*.*

We now identify how many shares an investor buys when she decides to hold a block.

Lemma 5 *Assume full subscription of shares. If an n-Blockholder EOS exists with* $\alpha_E > 0$ *and* $\frac{K-w_E}{1-\alpha_E} > f(m)$*, each blockholder holds*

$$\alpha_1 = \frac{\alpha_E (1+\lambda) - \lambda}{(1-\lambda) n} \tag{6}$$

To illustrate the intuition behind this lemma, consider a situation where the initial owner retains $\alpha_E = 30\%$ of the shares, and liquidity investors are anticipated to buy 50%. Say that $\lambda = \frac{1}{5}$, this means that 10%of the liquidity investors are active. Clearly, given that only 10% of the liquidity investors vote, at least 20% of the remaining shares must be held by blockholders in order to have $X_{med} = X_1$ rather than X_E. Suppose $n =$ 5 this means that each blockholder must own 4% of the shares in a symmetric situation. The essence of Lemma 5 is that under the stated conditions, no blockholder wants to own *more* than the minimal fraction of shares needed to guarantee that $X_{med} = X_1$ as that would expose them to higher risk without gaining anything in terms of control. If they hold any less, they do not affect the voting outcome.

From Lemmas 4 and 5, it follows that blockholders, if they arise, are pivotal and hold more shares than liquidity shareholders, but less than the initial owner, $\alpha_{l,j} < \alpha_j < \alpha_E$. The intuition behind this uses Lemma 2: the underlying ex-ante preferences of outside investors are such that they would prefer to move the anticipated voting decision closer to their ex-ante ideal point $\bar{X}$. However, as soon as they buy the shares, their ex-post preferences change (Lemma 1). Now, the whole purpose of buying more shares was to move the decision away from X_E towards $\bar{X}$: if they buy more shares than α_E they end up with a decision that is worse for them (ex-ante) than X_E! Hence, X_1 will lie between the preferred choice of the liquidity shareholders and the initial owner. This is the sense in which blockholders mitigate the conflicts of interest between the initial owner and the liquidity shareholders.

Finally, we solve the first stage maximization for the initial owner given what he anticipates will happen for each (α_E, w_E). This depends on the EOS that is anticipated for each pair (α_E, w_E). These will be used in the derivations of the subgame perfect equilibria: the method of proof is to solve for the Initial Owner's maximization problem, assuming a common belief about the EOS following every information set (α_E, w_E). Since the analysis of the EOS is quite technical with limited economic insight, it is done in the Appendix (Lemmas 9- 11 and Corollaries 3-4).

Before moving onto describing the subgame perfect equilibria we need some additional notation. In order to get full subscription, the initial owner must invest enough of his own wealth in the firm to satisfy the participation constraint of the outside investors (full subscription implies that the shares demanded must be strictly positive). Recall that liquidity shares demanded depend on the particular X anticipated. The following equations indicate the minimum w_E to guarantee that the demand for shares for the different ownership structures: $\underline{w}_E^k$ with $k = \{E, n, LS\}$ is the minimum w_E in case of respectively Initial Owner Ownership Structure, n Blockholders one and Liquidity Shareholders one. This last one is equivalent also to the No conflicts one.

$$\underline{w}_E^E(\alpha_E) \equiv K - \left(\frac{\bar{R}^2}{\gamma\sigma^2\alpha_E} + f(m)\right)(1-\alpha_E) + \epsilon_E \tag{7}$$

$$\underline{w}_E^n(\alpha_E) \equiv K - \left(\bar{R}X_1 + f(m) - \frac{\gamma}{2}X_1^2\sigma^2\alpha_1\right)(1-\alpha_E) \tag{8}$$

$$\underline{w}_E^{LS}(\alpha_E) \equiv K - \left(\bar{R}\bar{X} + f(m)\right)(1-\alpha_E) + \epsilon_{LS} \tag{9}$$

Defining $\eta > 0$ is the fraction of shares corresponding to one share, $\epsilon_j = \gamma X_j^2\sigma^2\eta$ the extra capital the initial owner needs to invest to guarantee that the investors buy one share (relative to the 0 condition).XX

We also need a participation constraint for the initial owner: he can always choose not to monitor the manager, and either not raise any capital or simply sell the firm. Suppose the initial owner decides not to monitor in the last stage (i.e. $\alpha_E < \bar{m}$). Proposition 1 shows that then he always chooses $\alpha_E = 0$ and the unique ownership structure that emerges is a Liquidity Shareholder Equilibrium Ownership Structure: the initial owner either does not raise capital or sells the firm letting the liquidity shareholders be in control of a non-monitored firm.

Proposition 1 *Suppose $m = 0$ in period 3, in period 0 the initial owner sells the firm when the firm has a positive NPV setting ($w_E = \underline{w}_E^{LS}$), otherwise he does not raise the capital and invests in the risk free asset ($w_E = 0$). In both cases, $\alpha_E = 0 < \bar{m}$. His value function is given by:*

$$V_E^{NM} = \max(\bar{R}\bar{X} - K + 1, 1) \tag{10}$$

When the firm is sold, the unique EOS is a Liquidity Shareholder one with $X = \bar{X}$.

where $\underline{w}_E^{LS}$ is the minimum wealth that the initial owner must pledge to get full subscription. This proposition illustrates the trade-offs faced by the initial owner. Since he acts as a monopolist in the pricing of shares, when he has no constraint on his shareholdings from monitoring, he can extract the full value of the firm without incurring any risk, by simply selling the firm (i.e. $\alpha_E = 0$, $w_E < 0$). However, investors are willing to buy shares only if the expected NPV is positive. Otherwise he prefers not to raise capital ($w_E = \alpha_E = 0$) .

Proposition 1 shows that the choice of $\alpha_E > 0$, $m = 0$ by the initial owner is dominated by the choice of either selling the firm or not raising capital. Hence, in what follows it is sufficient to show that the participation constraint and the non-selling constraints are satisfied, to ensure that the initial owner prefers to hold a block and take the value enhancing action ($\alpha_E \geq \bar{m}$, $m = 1$) to any non-value enhancing monitoring equilibrium.

3.1 Monitoring Equilibria

As discussed above the equilibrium ownership structure is of three types: in section 3.1.1 we analyze the first type of diversified ownership structure: this is the case where monitoring costs are so low ($\bar{m} \leq \bar{\alpha}$) that the final choice is $X_{med} = X_E = \bar{X}$. Since this is the first best point for outside investors (Lemma 2), there is no conflict of interest and no incentive to hold blocks. This is the benchmark case A(i) in that it gives the first best outcome for all investors. We discuss the case of multiple blockholders (case B) in Section 3.1.2, and ownership structures with one large block and a fringe of small shareholders (case C) is discussed in Section (3.1.3). Finally case A(ii) is the other possibility when there is a dispersed ownership structure, but in this case $X_{med} = \bar{X} > X_E$. This is discussed in Section 3.1.4.

Before moving to the equilibria we show that investors are willing to receive less shares than what they proportionally contribute. Conditional on monitoring, investing in the firm increases the utility of the investors because it widens the possible portfolios they can choose among. Since the initial owner is a monopolist he can push share prices up to the point where the investors' participation constraint is satisfied with equality. Put another way, the initial owner contributes to capital proportionally less than what he receives in cash flow rights: $\alpha_E > \frac{w_E}{K}$ when $m = 1$. This is shown in the next Lemma:

Lemma 6 *Assume that $m = 1$. In any subgame perfect equilibrium, $\frac{K - w_E}{1 - \alpha_E} > K$.*

In all monitoring equilibria, the initial owner sets the price per share as high as possible to avoid dilution of his shareholdings. If the initial owner monitors, the expected firm value is above the return on the risk-free asset. Hence, the minimum possible price that guarantees the participation of the investors is above the price of the risk-free asset.

3.1.1 A(i): No Conflicts Equilibrium

Proposition 2 *Suppose*

$$\bar{m} \in \left(0, \min\left[\bar{\alpha}, \bar{m}_{NC}^{RC}(K), \bar{m}_{NC}^{S}(K)\right]\right] \tag{11}$$

then there exists a No Conflicts (NC) equilibrium where the initial owner monitors, $m = 1$, $\alpha_E = \bar{m}$, $X_{med} = X_E = \bar{X}$ and $w_E = \underline{w}_E^{LS}(K)$.
(The exact values of $\bar{m}_{NC}^{RC}$ and $\bar{m}_{NC}^{S}$ as functions of the parameters are given in the proof in the Appendix.)

In this equilibrium, $\bar{m}$ is so low ($\bar{m} \leq \bar{\alpha}$) that the Initial Owner's first best is the same as other investors ($X_E = \bar{X}$). The initial owner retains just enough shares to have the incentive to monitor, i.e. $\alpha_E = \bar{m}$. A higher α_E implies more risk exposure without any gain in terms of monitoring. Since $X_{med} = \bar{X}$ there is no incentive for any investor to hold a block. However observe that the initial owner's shareholdings are determined by the monitoring costs, $\bar{m}$, and depending on the monitoring costs, the initial owner can hold more or less than the liquidity shareholders. This equilibrium ownership structure shows a benchmark situation when all shareholders interests are aligned so conflict of interests are absent in this case.

Liquidity shareholders are willing to buy shares only if the returns are high enough to compensate for the risk, $w_E \geq \underline{w}_E^{LS}$, otherwise they are better off just investing all their wealth in the risk-free asset. The initial owner then sets the highest price to extract all the rents: $w_E = \underline{w}_E^{LS}$.

The monitoring requirement sets a maximum threshold on the shares that can be distributed to the liquidity shareholders. Given this threshold there is a maximum fraction of wealth that liquidity shareholders are willing to invest. The rest of the capital (if needed) must be pledged by the initial owner ($w_E = \underline{w}_E^{LS}$). The higher the amount of capital the initial owner needs for the project, K, the higher the wealth he needs to pledge, i.e. $\underline{w}_E^{LS}$ is increasing in K.

Note that given that there are no financial constraints, the initial owner could finance the project completely on his own through borrowing money. However, he prefers to rely on outside equity rather than issuing debt in order to limit his risk exposure. If the value created is very high the initial owner does not need to invest *any* money; indeed, he can be compensated by the investors for the monitoring exerted and the entrepreneurial idea ($\underline{w}_E^{LS} < 0$). These characteristics of the capital invested by the initial owner are common to all equilibria we find.

Alternatively the initial owner could choose not to raise the capital or to sell the firm. To be a viable project for the initial owner, the value remaining to the initial owner after compensating the investors, must be high enough to compensate him for the money invested and the risk (i.e. $\bar{m} \leq \bar{m}^{RC}_{NC}$).

At the same time to be willing to remain a shareholder of the firm, rather than sell it outright, the value created by monitoring ($f(m) = K$) has to be high enough. The extra value due to monitoring compensates the initial owner for the direct cost of monitoring as well as the indirect costs related to holding a sub-optimal portfolio. If the extra utility created by monitoring can be achieved by a dispersed ownership structure without monitoring then he would prefer to sell the firm (i.e. $\bar{m} \geq \bar{m}^{S}_{NC}$).

Figure 2 offers a graphic representation of the No Conflicts Equilibrium. The equilibrium exists when the area in the graph that satisfies both the no selling and raising capital constraints is positive. Since $\bar{m}^{RC}_{NC}, \bar{m}^{S}_{NC}, \bar{\alpha} > 0$, we can conclude that this is the case.

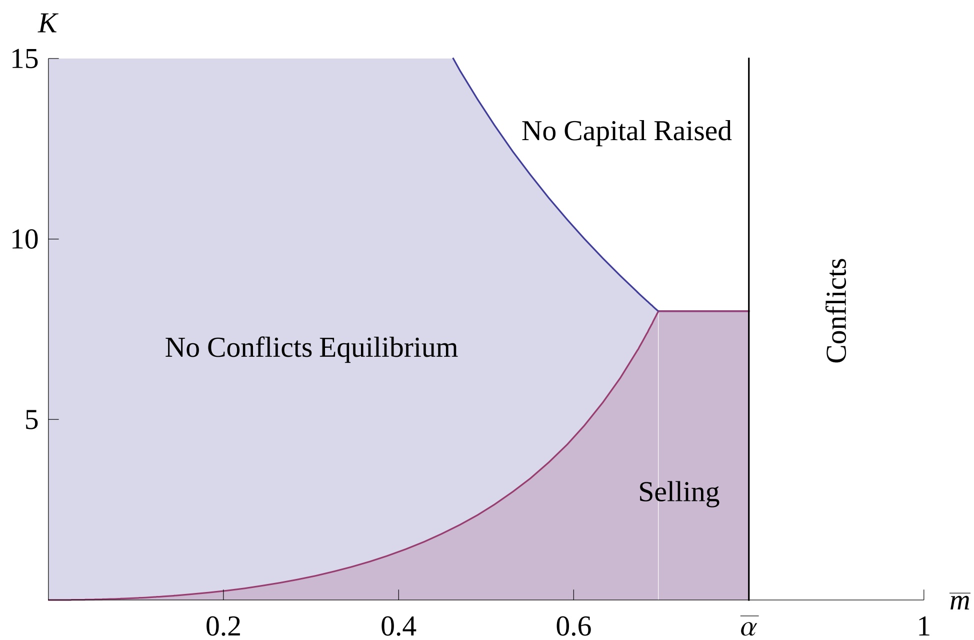

Figure 2: No Conflicts Equilibrium ($\bar{R} = 0.8$, $\gamma = 10$, $\sigma = 0.1$, $\bar{X} = 10$).

3.1.2 B: Blockholder equilibria

The existence of blockholder equilibria depends on three necessary conditions (1) a conflict of interest between investors and initial owner that is generated endogenously by the fact that they have different shares in the firm and hence different ex-ante preferences on the risk/return profile of the firm; (2) shareholders are able to influence the voting decision by buying more shares and (3) in equilibrium the initial owner's shareholding is large enough that active liquidity investors in the firm cannot jointly ensure that $X_{med} = \bar{X}$. If any of these three requirements is not met, then we do not have a blockholder equilibrium.

This section shows our main result: the existence of blockholder equilibria for intermediate monitoring costs. In the following, we discuss two different cases of blockholder equilibria. For one range of parameters, it is possible that n-Blockholder equilibria exist (Proposition 3), but we cannot exclude other equilibria.[14] However, we have a stronger result: for some parameter values of monitoring costs, there exist *only* blockholder equilibria. So, even though there are multiple equilibria in terms of the number of blockholders, we know that there will be no non-blockholder equilibria (Corollary 1).

Proposition 3 *Suppose:*

$$\begin{aligned}\bar{m} \quad \in \quad & \left(\max\left[\bar{\alpha}, \tilde{\alpha}(n,\lambda), \bar{m}_1^E(n,\lambda), \bar{m}_{1,n}^{RC}(n,K), \bar{m}_{1,n}^S(n,K)\right],\right. \\ & \left.\min\left[\frac{1}{2}, \hat{\alpha}(n,\lambda), \bar{m}_2^E(n,\lambda), \bar{m}_{2,n}^{RC}(n,K), \bar{m}_{2,n}^S(n,K)\right]\right] \end{aligned} \tag{12}$$

then there exists an n-Blockholder-Equilibrium where $m = 1$, $\alpha_E = \bar{m}$, $X_{med} = X_1$ and $w_E = \underline{w}_E^n$.
(The full expressions for $\tilde{\alpha}$, $\hat{\alpha}$, $\bar{m}_{1,n}^{RC}, \bar{m}_{2,n}^{RC}, \bar{m}_{1,n}^S, \bar{m}_{2,n}^S, \bar{m}_1^E, \bar{m}_2^E$ are given in the Appendix)

where $\underline{w}_E^n$ denotes the minimum w_E to guarantee that the participation constraint of blockholders is satisfied when $X_{med} = X_1$. Proposition 3 demonstrates that there exist blockholder equilibria where n investors prefer to hold a large block of shares (and a sub-optimally diversified portfolio) in order to shift the decision to a higher level of risk/return. The presence of blocks mitigates the conflicts of interests between the initial

[14] In Proposition 5 we show that an Initial Owner equilibrium is possible for this parameter range.

owner and the investors ($\bar{X} > X_1 > X_E$). There can be multiple EOS, parametrized by n depending on the beliefs on the EOS.

Note that the initial owner cannot prevent the entry of blockholders by setting a different w_E. If he sets a higher w_E he loses rent while if he sets a lower w_E (a higher share price) drives out the participation of the blockholders. In such a case the participation constraint of the liquidity shareholders is also not satisfied. So the initial owner would not be able to raise capital. The condition $\alpha_E < \hat{\alpha}(n)$ guarantees that the incentive compatibility condition to become a blockholder is satisfied: the utility of an investor is higher when he is a blockholder with decision X_1 than a liquidity shareholder with decision $X_E < X_1$.

The reader may find it puzzling that we start with ex-ante identical outside (active) investors, yet only some of them decide to become blockholders. This is because there are multiple Nash equilibria for every n, and the identity of the blockholders could be different in each of these equilibria and the rest of the investors (liquidity shareholders) free ride on these. Like in a (discrete) public goods provision problem, the blockholders contribute to the public good provision (i.e. moving the decision on the project closer to the most preferred point of all outside investors) because given the other shareholders contributions, it is a Nash equilibrium for them to contribute as long as the value of the public good to them is sufficiently high. XX DISCUSS Allowing for collusion is not going to change this result since all of the EOS are in the Pareto efficient setXX This translates into the condition that α_E is sufficiently low: as α_E decreases, the incentives to hold larger blocks increases. This is because, in the first place, as α_E decreases, fewer shares are needed in order to gain control over X and hence the cost of holding a block is lower. Second, because of the convexity of X_j with respect to α_j (equation (4)), it follows that the smaller α_E is, the larger is the shift in X (for the same $\alpha_E - \alpha_1$), i.e. $X_E - X_1$ is greater. This implies a higher increase in the expected return of becoming a blockholder.

Hence there exists a threshold, $\hat{\alpha}(n)$, such that when $\alpha_E \leq \hat{\alpha}(n)$ and $w_E = \underline{w}_E^n$, the utility of being a blockholder is higher than being a liquidity shareholder with the initial owner in control. If he decreases α_1 the outcome is $X_{med} = X_E$, while if he increases his shares he is still the median shareholder but he is moving further away from his first best. The initial owner could get full control on the voting decision if he chose $\alpha_E \geq \frac{1}{2}$ but this comes at the expense of a higher share price and lower risk exposure. Finally, for the liquidity shareholders, given that there are n blockholders there are no gains to be had from increasing α_i in equilibrium.

Unlike the usual public goods contribution game, however, when blockholders buy a larger block of shares, their preferences over X are closer to those of the initial owner. This is why the presence of blockholders mitigates, but does not remove, the conflict of interest between the initial owner and the outside investors.

Lemma 3 implies that in the blockholder equilibrium, liquidity shareholders free ride on blockholders as they hold the optimal portfolio. The price which satisfies the participation constraints of the blockholders is lower than the maximum price that liquidity shareholders are willing to pay. Therefore the existence of blockholders allows the liquidity shareholders to extract some of the rent, that in all other equilibria goes entirely to the initial owner. In all the other equilibria the initial owner sets the price low enough to satisfy the participation constraint of the liquidity shareholders with equality, and hence he extracts all the rent (see next Propositions).

To guarantee that the initial owner prefers to monitor rather than not, we have the condition $\bar{m} \in [max\left(\bar{m}_{1,n}^{RC}(n), \bar{m}_{1,n}^{S}(n)\right), min\left(\bar{m}_{2,n}^{RC}(n), \bar{m}_{2,n}^{S}(n)\right)]$ (See Fig. 3). If the monitoring costs are too high the residual value for the initial owner after compensating the blockholder for the extra risk they bear due to the undiversified portfolio, is not high enough to compensate for holding an undiversified portfolio and the monitoring costs. When the monitoring costs are too low, there are blockholders with small blocks or liquidity shareholders determining X. In such a case the risk/return outcome is very high compared to the initial owner's preferred point. So when the monitoring costs are too high or too low the risk the initial owner bears is so high and the gain from monitoring is so low that he prefers to sell the firm or not to raise capital.

Now we explain the remaining conditions that need to be satisfied to have an n-

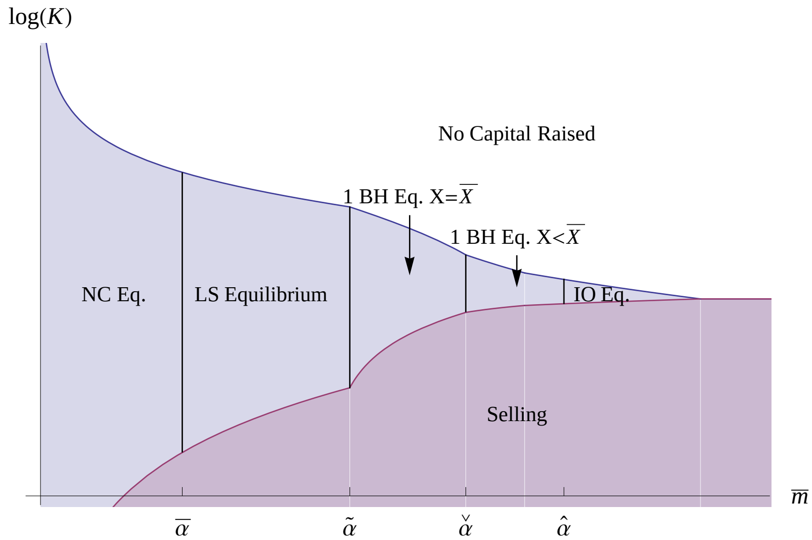

Figure 3: 1 Blockholder equilibrium ($\bar{R} = 1$, $\gamma = 12$, $\sigma = 0.2$, $\bar{X} = 50$, $\lambda = 0.1$). $\bar{\alpha}$ and $\bar{m}_2^E$ are not binding.

Blockholder Equilibrium (expression 12). First note that the initial owner can always choose to set an $\alpha_E \geq \frac{1}{2} \geq \bar{m}$ to get full control and hence it is not obvious that an n-Blockholder Equilibrium exists. The trade off the initial owner faces is between setting a high share price and retaining control. When $\bar{m} \in [\bar{m}_{1,n}^E, \bar{m}_{2,n}^E]$, the initial owner *prefers* to be in a n-Blockholder Equilibrium. There is a lower bound, $\alpha_E < \bar{m}_{1,n}^E$, then the cost to the initial owner from relinquishing control and ending up with an X that is very far from X_E, is too high. If $\alpha_E > \bar{m}_{2,n}^E$, then the cost of getting control is low. In such cases there does not exist an $n-$Blockholder equilibrium - there is only an Initial Owner equilibrium (see Corollary 2). Any α_E lower than $\frac{1}{2}$ does not guarantee control, so the equilibrium α_E is superior, given the beliefs on EOS off the equilibrium path.

If $\bar{m} < \bar{\alpha}$ there are no conflicts (see Proposition 2). Alternatively when $\bar{m} \leq \tilde{\alpha}(n)$, outside investors can affect the vote outcome even without holding blocks, each active investor is pivotal, even while holding liquidity stocks and $X_{med} = \bar{X}$ (see Proposition 6 for a formal analysis of the case).

A possible objection to our main result is that there are multiple equilibria for this configuration of monitoring costs. Corollary 1 shows that for some values of the monitoring costs only n Blockholder equilibria are possible.

Corollary 1 *Suppose:*

$$\bar{m} \in \Big(\ \max\left[\bar{\alpha}, \tilde{\alpha}(1,\lambda), \bar{m}_1^E(1,\lambda), \bar{m}_1^{RC}(1)(K), \bar{m}_1^S(1,K)\right], \\ \min\left[\frac{1}{2}, \hat{\alpha}(1,\lambda), \bar{m}_2^E(1,\lambda), \bar{m}_2^{RC}(1,K), \bar{m}_2^S(1,K)\right] \ \Big] \tag{13}$$

then there exist n Blockholder equilibria with $m = 1$, $\alpha_E = \bar{m}$, $X_{med} = X_1 < \bar{X}$, $w_E = \underline{w}_E^1$. No other types of equilibria with positive trade exist for these parameter values.

This corollary is a special case of Proposition 3. The intuition behind this result is that when monitoring costs are low enough, the cost of losing diversification for a single blockholder to deviate and demand a block is low enough (equilibrium α_E is lower than $\hat{\alpha}(1,\lambda)$). Hence, because she needs a smaller block to both win the vote (using the vote of the active liquidity investors) and to have a large shift in X, an Initial Owner equilibrium is ruled out. Among the possible n-Blockholder equilibria we might want to consider the one which is most preferred by the initial owner. Of course, in our model the equilibrium depends on the anticipated EOS and this need not be the one that the initial owner most prefers. However, the initial owner *can* choose prices to rule out equilibria with $n < n^*$.

Proposition 4 *The optimal number of blockholders for the initial owner is:*

$$n^* = \left[\frac{(1+\bar{m})(\bar{m} - \lambda + \bar{m}\lambda)}{2\bar{m}^2(1-\lambda)}\right] > 0 \tag{14}$$

Moreover

$$\frac{\partial n^*}{\partial \bar{m}} = -\frac{\bar{m} - 2\lambda}{2\bar{m}^3(1-\lambda)} \tag{15}$$

and

$$\frac{\partial n^*}{\partial \lambda} = -\frac{1 - \bar{m} - 2\bar{m}^2}{2\bar{m}^2(1-\lambda)^2} \tag{16}$$

The optimal number of blockholders for the initial owner is a trade-off between the benefits from having few blockholders and a vote outcome closer to his optimal outcome or having many blockholders and being able to sell the shares at a high price, i.e. low w_E. When the monitoring costs are high ($\bar{m} \geq 2\lambda$) a further increase of the monitoring costs would induce the initial owner to prefer less blockholders. In such a case blockholders are already holding a highly undiversified portfolio and the initial owner needs to set a very low price in order to induce them to buy the shares. Decreasing the number of blockholders further does not reduce the share price by much but it decreases the costs of holding a suboptimal portfolio.

On the other hand, when the monitoring costs are low, i.e. $\bar{m} < 2\lambda$, an increase in the monitoring costs induces the initial owner to prefer more blockholders. In this case the discrepancy between the preferences of the blockholders and the initial owner is not that high so that increasing the number of blockholders allows the initial owner to raise the price at which the shares are tendered.

Finally, the effect of the proportion of active investors among the liquidity shareholders, λ, has a negative effect on the optimal number of blockholders preferred by the initial owner. $\frac{\partial n^*}{\partial \lambda}$ is negative for $\bar{m} \leq \frac{1}{2}$. When more liquidity investors vote, it becomes cheaper for blockholders to hold sufficiently large blocks so that they are jointly pivotal in the voting. In this sense, stronger investor protection (proxied by higher λ) is complementary to shared control ownership structures. However, this is very costly for the initial owner in terms of risk exposure: when λ is higher the initial owner would prefer to set a lower price to have the vote outcome closer to his preferred point through the expected equilibrium ownership structure. Hence the higher the participation of the liquidity shareholders to the vote, the lower the number of blockholders the initial owner would like to have. The overall effect is ambiguous.

3.1.3 C: Initial Owner Equilibrium

This equilibrium exists in 3 cases. First, when the monitoring costs are so high that the initial owner has to hold more than 50% of the shares and hence no outside investor can influence the decision. The second case occurs when the initial owner holds less than 50% but the equilibrium α_E (i.e. $\bar{m}$) is high enough such that 1 blockholder would not have a unilateral incentive to deviate from holding liquidity shares. The third case occurs when the Initial Owner chooses to hold more shares than what the monitoring threshold requires in order to avoid the loss of full control. In the first and third cases the Initial Owner equilibrium is unique. Case 1 and 2 is analyzed in Proposition 5, case 3 in Corollary 2.

Proposition 5 *Suppose that*

$$\bar{m} \in \left(\max\left[\bar{\alpha}, \min\left[\frac{1}{2}, \max\left[\hat{\alpha}(1,\lambda), \check{\alpha}(1,\lambda)\right]\right]\right], \min\left[\bar{m}_E^{RC}(K), \bar{m}_E^{S}(K), 1\right]\right] \tag{17}$$

then there exists an Initial Owner equilibrium where the initial owner is the only blockholder, $m = 1$*,* $\alpha_E = \bar{m}$*,* $X_{med} = X_E = \frac{\bar{R}}{\gamma\sigma^2\bar{m}}$ *and* $w_E = \underline{w}_E^E$*.*

When $\bar{m} \in \left(\max\left[\bar{\alpha}, \frac{1}{2}\right], \min\left[\bar{m}_E^{RC}, \bar{m}_E^{S}, 1\right]\right]$ *the Initial Owner equilibrium is unique. (The exact values of* $\hat{\alpha}(1,\lambda)$*,* $\check{\alpha}(1,\lambda)$*,* $\bar{m}_E^{RC}$ *and* $\bar{m}_E^{S}$ *as functions of the parameters are given in the appendix.)*

where $\underline{w}_E^E$ denotes the minimum w_E to guarantee that the demand for shares is at least 1 share when the outcome is X_E.

Consider first the case, when the monitoring costs are very high ($\bar{m} \geq \frac{1}{2}$), such that the initial owner is willing to monitor only if he holds more than 50% of the shares. This implies that he is highly exposed to firm risk and because he is in control of the vote outcome he chooses a low risk/return project. As usual, the participation constraint of the initial owner requires $\bar{m} \leq \min[\bar{m}_E^{RC}, \bar{m}_E^{S}]$ (Figure 4).

A less obvious result is that an Initial Owner equilibrium also exists if the monitoring costs are *smaller* than $\frac{1}{2}$, although in this case it is not unique. As we saw in such a case there may be blockholder equilibria as well, but we cannot rule out an Initial Owner equilibrium. An Initial Owner equilibrium can exist when $\bar{m} < \frac{1}{2}$, when no single investor has a unilateral incentive to deviate to become a blockholder.Condition $\bar{m} > \hat{\alpha}(1,\lambda)$ ensures that then an investor is not willing to unilaterally hold more shares

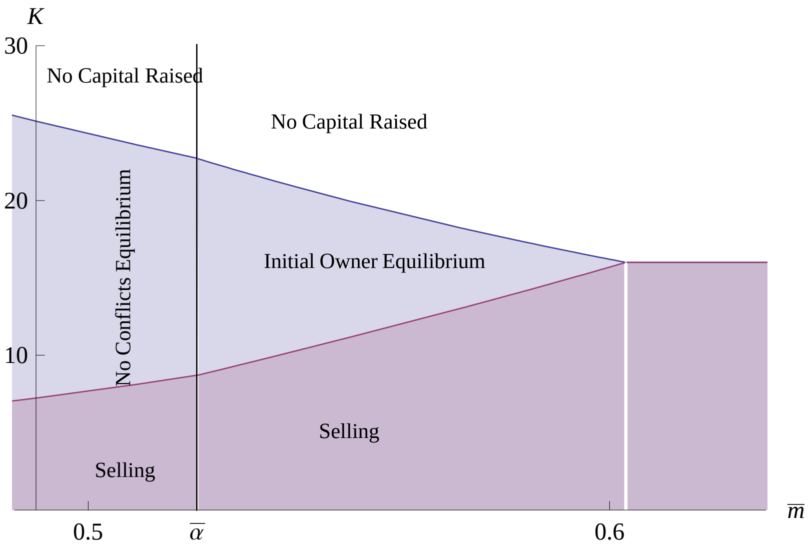

Figure 4: Initial Owner Equilibrium ($\bar{R} = 2$, $\gamma = 12$, $\sigma = 0.2$, $\bar{X} = 10$, $\lambda = 0.05$).

in order to influence the voting decision, given $w_E = \underline{w}_E^E$. The condition $\bar{m} > \breve{\alpha}(1, \lambda)$ ensures that no excluded outside (active) investor is willing to buy shares from an included passive investor at a higher price than the equilibrium price. Finally, in order to guarantee conflicts of interests between investors and initial owner, we set $\bar{m} > \bar{\alpha}$. As in the previous cases the participation constraint of the liquidity shareholders has to be satisfied and hence $w_E = \underline{w}_E^E$.

We now analyze the case where an Initial Owner equilibrium arises because the initial owner chooses to retain full control ($\alpha_E = \frac{1}{2} + \eta_E$) in order to avoid an n-Blockholder equilibrium.

Corollary 2 *Suppose*

$$\bar{m} \in \left(\max\left[\bar{\alpha}, \tilde{\alpha}(n, \lambda),\right], \min\left[\bar{m}_E^{RC}(K), \bar{m}_E^S(K)\right]\right) \tag{18}$$

and either $\bar{m} \leq \bar{m}_{1,n}^E(n, \lambda)$*, or* $\bar{m} > \bar{m}_{2,n}^E(n, \lambda)$ *then an Initial Owner Equilibrium exists and is unique where* $m = 1$*,* $\alpha_E = \frac{1}{2} + \eta_E$*,* $X_{med} = X_E$ *and* $w_E = \underline{w}_E^E$*.*

The initial owner is willing to hold $\alpha_E > \frac{1}{2} > \bar{m}$ when the monitoring costs are either very high, $\bar{m} > \bar{m}^E_{2,n}$ or very low $\bar{m} < \bar{m}^E_{1,n}$. The intuition behind this result is that when the monitoring costs are very high, $\bar{m} > \bar{m}^E_{2,n}$, the cost of increasing α_E a little bit and reducing diversification is very small relative to the gain from controlling the vote outcome and choosing his preferred risk/return combination. When the monitoring costs are low, on the other hand, the initial owner does not hold many shares and so blockholders have a low cost from owning blocks. Hence, the initial owner faces a large cost arising from the conflict of interests on the risk/return vote outcome. For this reason he prefers to increase α_E to a point where he can control the vote outcome, $\alpha_E > \frac{1}{2}$. The other conditions have the same rationale as previous Propositions so we do not repeat them here.

3.1.4 A(ii): Liquidity Shareholders equilibrium

We now consider the second type of equilibria with a dispersed ownership structure. Intuitively this happens when there are sufficiently many active liquidity shareholders and $\bar{m}$ is not too high, so that the first best X can be achieved even without blockholders.

Proposition 6 *Suppose*

$$\bar{m} \in \left(\bar{\alpha}, \min\left[\frac{1}{2}, \tilde{\alpha}(n,\lambda), \bar{m}_1^{NC}(K), \bar{m}_2^{NC}(K)\right]\right] \tag{19}$$

then there exists a Liquidity Shareholders equilibrium with n (in addition to the fraction λ) active investors where $m = 1$, $\alpha_E = \bar{m}$, $X_{med} = \bar{X}$ and $w_E = \underline{w}_E^{LS}(K)$.
(The exact values of $m_1^{NC}(K), \bar{m}_2^{NC}(K)$ as functions of the parameters are given in the proof in the appendix.)

A Liquidity Shareholder equilibrium exists when the monitoring costs are relatively low, but not low enough that there are no conflicts (i.e. $\bar{m} > \bar{\alpha}$). In this equilibrium, the initial owner finds it optimal to choose $\alpha_E = \bar{m}$, but $\bar{m}$ is sufficiently small so that there are enough active liquidity shareholders to be pivotal. In particular we can have two cases: one where the fraction λ of the liquidity shareholders is sufficient to change the

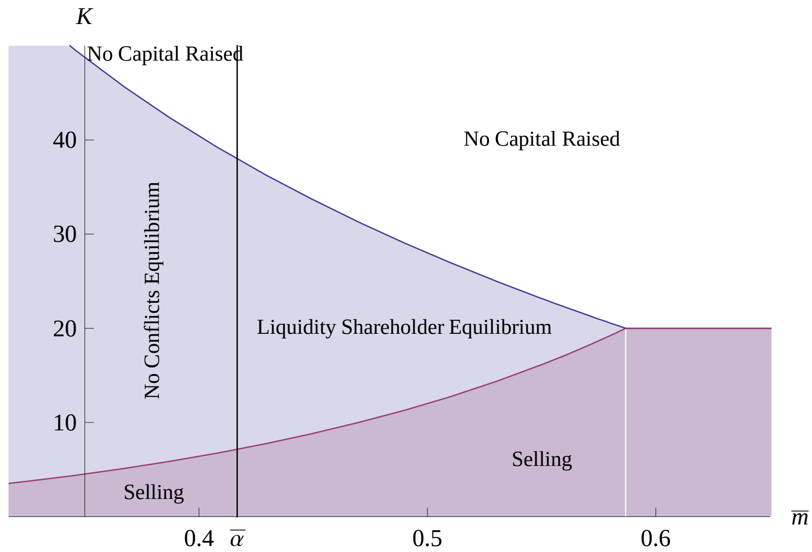

Figure 5: Liquidity Shareholder Equilibrium ($\bar{R} = 2$, $\gamma = 12$, $\sigma = 0.2$, $\bar{X} = 10$, $\lambda = 0.05$).

vote outcome (i.e. $n = 0$) and the other where there are n active investors in addition to the fraction λ active liquidity investors who vote and ensure that the outcome is $\bar{X}$. If monitoring costs are higher ($\bar{m} > \tilde{\alpha}(n, \lambda)$), then there is an n blockholder EOS and if $\bar{m} > \frac{1}{2}$ then there is an Initial Owner EOS. Finally, as in the case of the No Conflicts and the Initial Owner equilibrium, when the monitoring costs are higher $\bar{m}_1^{NC}$ than the initial owner prefers to raise no capital and if monitoring costs are higher than $\bar{m}_2^{NC}$, he prefers to sell the firm.

Again, the participation constraint of liquidity shareholders is satisfied, at $w_E = \underline{w}_E^{LS}(K)$.

Note that the initial owner could always choose to retain a strictly higher fraction of shares ($\alpha_E > \bar{m}$) to induce a blockholder EOS or even to hold α_E bigger than half, to induce an Initial Owner EOS. In either of these cases, the vote outcome is closer to his own preferred point. However the higher control comes at the expense of both a lower price paid by the investors, more dilution and a less diversified portfolio for the initial owner.

3.2 Optimal amount of capital raised

In this section we relax the assumption that the initial owner raises just enough capital needed to implement the project, K. We allow the initial owner to invest more capital $I \geq K$ and use the difference, $I - K$ to buy the risk free asset. This would offer him the possibility to achieve the preferred degree of diversification through the firm's investment in the risk free asset: hence the conflict of interest between the initial owner and outside investors may disappear. We show in the proposition below that our results are robust to relaxing this assumption.

Proposition 7 *The initial owner always strictly prefers to raise the minimum amount of capital, i.e.* $I = K$.

Observe that the initial owner acts as a monopolist when setting the share price. Hence, if he increases I, this lowers the price per share in equilibrium and it lowers the risk of the project for the same X. The decrease in price decreases the initial owner's utility in such a way that it more than offsets the increase in utility due to a lower risk of the project. This conclusion follows because a higher amount raised implies a lower utility for outside investors since they get profits lower than their contribution (Lemma 6). Moreover equilibrium X does not change. To satisfy the participation constraint therefore the initial owner must increase his own participation for the same α_E.

4 Comparative Statics and Empirical Implications

In what follows we look more in depth the the important predictions of our model and discuss how they relate to the empirical literature on ownership structure (See Lemmas 16 –19 in the Appendix for formal results supporting our claims in this section.).

4.1 What does Ownership Structure depend on?

We interpret higher λ as higher participation by small investors in voting decisions, which could be due to higher legal protection to small shareholders or voting institutions that encourage participation in shareholder meetings.[15]

Suppose we start out in an n blockholder equilibrium with number of blockholders sufficiently small ($n \leq \frac{1}{2\eta}$) and monitoring costs sufficiently small ($\bar{m} < \frac{1}{2}$). The effect of higher λ in our model is ambiguous: on the one hand when λ increases, $\hat{\alpha}(n, \lambda)$ also increases which means that it becomes less costly to hold blocks relative to the benefits. On the other hand when λ increases, $\tilde{\alpha}(n, \lambda)$ increases: outside investors may be able to shift the decision on X without holding blocks and hence it is easier to obtain a liquidity shareholders equilibrium. In both cases, the size of the median block decreases and hence more risky projects are chosen, making it worse for the initial owner when price is fixed. The initial owner may respond by changing the price. Hence, if the starting point is a blockholder equilibrium and $\bar{m}$ is close to $\frac{1}{2}$ the initial owner may switch to holding majority control (see point (d) Lemma 16 in the Appendix). This higher concentration implies a lower risk outcome for the firm. If the starting point is a blockholder equilibrium and $\bar{m}$ is sufficiently far from $\frac{1}{2}$, then there may be no change in the ownership structure in response to a change in λ.

In contrast, if we start out in an Initial Owner equilibrium (high $\bar{m}$), the increase in λ may have no effect on the ownership structure or on the size of the median share. Similarly, starting out in a No Conflicts equilibrium or in a Liquidity Shareholders equilibrium, the ownership structure does not change.

Our predictions for the effect of higher minority participation via say shareholder protection laws depends on the nature of the industry. Industries characterized by a high or very low degree of agency problems are less likely to respond with a more dispersed ownership structure, while industries with intermediate levels of agency problems should switch to more dispersed ownership structure when facing an increase in minority participation.

As Becht, Bolton and Roell (2002) conclude in their overview of corporate governance, there is a trade off between small investor protection and monitoring of man-

[15]For example until a few years ago in Germany only shareholders with more than 5% participation could vote. Alternatively, shareholders' vote participation can be related to information disclosure and hence the ability to make informed decisions.

agerial discretion. Our results are consistent with this view: higher λ may lead to decisions closer to small shareholder's preferred points without compromising on monitoring only when monitoring is not very costly in a firm. When monitoring is costly, in our model shareholder protection reduces the willingness of the initial owner to raise capital, or to monitor. *So, excessive minority protection can be detrimental for firm value. Counter-intuitively, the higher is λ, the higher may be the initial owner's incentives not to participate or to keep control of the voting decision.*

Second, $\bar{X}$ captures the potential for conflict in a firm. For example, high risk and high return firms are likely to be more innovative industries while mature sectors would be likely to have a low $\bar{X}$. In our model, as $\bar{X}$ increases, $\bar{\alpha}$ decreases so that the probability of observing a dispersed ownership structure with no conflicts is lower. Increasing the conflict of interest in general reduces the willingness of the initial owner to monitor or even to raise capital ($\frac{\partial V_E^{NM}}{\partial \bar{X}} \geq 0$). Assuming that the participation constraints of the initial owner are satisfied, the range of $\bar{m}$ such that an n blockholder equilibrium exists becomes smaller ($\frac{\partial \hat{\alpha}}{\partial \bar{X}} \geq 0$) Ceteris paribus, in more mature sectors having a choice only of low risk/low return projects (low $\bar{X}$), ownership structures with multiple blockholders are less likely to be observed. Anecdotal evidence suggests that in more mature sectors it is more common to see families in control of firms. In very innovative industries (high $\bar{X}$) , on the other hand, we should observe multiple blockholder ownership structures. These blockholders are usually represented by institutional investors, e.g. venture capitalists, who professionally look for firms with a high risk/return profile. *Ceteris paribus, the higher the potential for conflict in a firm the more likely are multiple blockholder ownership structures.*

The results of Carlin and Mayer (2000) note a positive relationship between the size of the largest blockholder and the second and third blockholders. The size of the second blockholder is positively related to the size of the largest blockholder. Our model provides a nice explanation of this phenomenon: when the initial large shareholder has a bigger stake, the size of other blocks must be sufficiently big to counterbalance the voting power of the first. Moreover, Carlin and Mayer (2000) find that when a third blockholder is present, the size of the second is much smaller and similar to the third one: our core mechanism suggests that this is because when there are more blockholders, each one can be smaller as long as they counterbalance the initial large shareholder. An interesting prediction of our theory is that regardless of the number and size of blockholders (even in asymmetric equilibria), there is going to be a size difference between the largest block and the second largest block.

Third, consider the effect of $\bar{m}$ on the possible equilibria. One interpretation of high $\bar{m}$ is the degree of agency problems. The higher is the degree of agency problems, the higher is the monitoring required from the initial owner. Empirically, one can distinguish firms on the basis of measures of information asymmetry based on a firms growth opportunities, the market microstructure of the firms stock and analyst forecasts of a firm's earnings per share (Clark and Shastri (2000)). High monitoring costs are more common in firms dedicated to innovation or R&D where moral hazard issues are much more pervasive. The higher are the agency problems, the higher is the participation of the largest blockholder (through higher monitoring costs). This implies that in a blockholder equilibrium, if the number of blockholders is fixed, each of them would hold a higher stake. *Hence, according to our model, the higher is $\bar{m}$ the higher is the stake of the largest blockholder.* In firms with very high measures of information asymmetry we should see more concentrated ownership and less risky project choices.

A higher voting threshold on the other hand increases the cost of full control to the owner, so *countries with higher voting thresholds should have less concentrated ownership structures according to our theory.*

When the project size K increases the initial owner has to contribute the extra amount needed to finance the project. Thus, the share price does not change when more capital is needed. However because the relative value added by the monitoring decreases, it becomes more attractive for the initial owner to sell the idea and have a totally dispersed ownership structure or not raise capital at all (see Lemma 19 in the Appendix). This can best be understood in Figure 3: when K increases the area showing the various equilibria shrinks.

Finally, we should mention that one of the significant patterns that emerges from empirical studies is how hedge funds are much more activist and lead to higher returns for firms than pension funds (Yermack (2010)): this is partly explained by the greater freedom that hedge funds have to choose their stakes strategically relative to pension funds. In our theory, this difference is not analyzed.

4.2 The implications of the Ownership Structure

Let us now look at the predictions we make on how ownership structure influences firm choices. In our model, the risk/return decision depends on the size of the median shareholder (we assume a 50% majority threshold, but the generalization to any threshold is the pivotal voter). *Ceteris paribus, the smaller the size of the median block the higher is the predicted firm value and firm risk.* Of course, in our model, the smaller the median block, the more diversified is the shareholder. The general prediction is that it is the level of diversification of the median shareholder that affects the value and risk profile of a firm, and the median shareholder need not be the largest shareholder.

To the best of our knowledge there is no detailed study of ownership structure and firm risk. However, we can find some indirect evidence of our mechanism. Carlin and Mayer (2003) and John (2008) find a correlation between firm risk and ownership structures. In the first paper, blockholders are present in high risk firms, while a single blockholder is common in low risk firms, while in the second there is a negative relationship between concentration and risk. Laeven and Levine (2008b) focus on banks ownership structure and show that risk taking behavior of banks does depend on ownership structure: banks with at least one blockholder are more conservative than firms with dispersed ownership. Our results, while broadly consistent with these findings, suggest that the story is more nuanced as we can distinguish on the basis of the size of the median block. Moreover our model shows how primitives like the potential for conflict over risk drive the ownership structure: the causation therefore runs both ways.

Our predictions on how ownership structure affects firm value are consistent with the empirical literature which shows that the effect of blockholders on the value of the firm is usually positive (Barclay and Holderness (1989) and Kirchmaier and Grant (2005)). Most studies find too that a second blockholder or third blockholder increases firm value. This evidence holds across countries and across publicly listed or private firms.Lehmann and Weigand (2000) find that a second large shareholder improves the profitability of listed companies in Germany. According to Volpin (2002) in Italy, when blockholders form syndicates the firm market value is higher than when there is a single blockholder. In Europe and Asia, higher dividends are positively related to the number of blockholders (Faccio, Lang, and Young, 2001). Maury and Pajuste (2005) have shown that when there are 2 blockholders with similar interests, the existence of a third blockholder increases firm value. In Spain the number of blockholders is positively related to a better performance of private firms (Gutierrez and Tribo, 2004). Roosenboom and Schramade (2006) studying French IPOs find that when the owner is powerful, the firm is less valued; when the initial owner shares control with other blockholders the value increases.

Our paper offers an alternative explanation for the underpricing observed in IPOs. (see Brennan and Franks (1997), Boulton, Smart, and Zutter (forthcoming), Nagata and Rhee (2009) and Yeh and Shu (2004)) In particular Brennan and Franks (1997) argue that underpricing can be more severe when the initial owner wants to avoid blocks. However, they note that this is not a stable outcome and over time blocks are formed anyway. The findings of Brennan and Franks (1997) are in line with our predictions. If the initial owner could choose the share allocation and retain control, he would be willing to do so even though this implies a lower price. However, if share trade is allowed this outcome cannot be stable. Our paper implicitly takes into account the possibilities of re-trading. In our blockholder equilibria moreover, we find that liquidity shareholders free ride on blockholders in that they are willing to pay a higher price for the shares than what they actually pay (this is despite the the stability against re-trading that we impose). In this sense we show how there can be underpricing even in this more general setting for an IPO. Our theory predicts therefore that underpricing occurs when the size of the initial block (initial owner) is not too large (in particular less than the relevant voting threshold) and not too small. In such a case the predictions of our model are similar to those of Stoughton and Zechner (1998) and DeMarzo and Urosevic (2006): underpricing occurs when blockholders are present and it is higher, the higher the size of the block.

We can differentiate between our results and those of Stoughton and Zechner (1998) and DeMarzo and Urosevic (2006). Underpricing in their model occurs to guarantee the participation of a single large shareholder who must undertake costly monitoring and so underpricing is associated with one large shareholder. In our model underpricing occurs not when there is a single large shareholder but rather when there is shared control among multiple blockholders. The larger is the differential between the size of the blocks and the size of minority stakes, the higher is the rent that liquidity shareholders obtain, and it is in this sense that underpricing is more severe when the differential is bigger.

5 Conclusions

This paper analyzes the determinants of ownership structure and its effect on the risk profile of a firm when decisions are taken through shareholders' vote. Agency problems and the need for monitoring lead to a less diversified initial large shareholder. Because of his stakes he is more conservative than outside shareholders on risk, leading to a conflict of interests on this decision. This gives incentives for mid-sized blockholders who mitigate the conflicts of interests between the largest blockholder and minority investors. We use the model to explain the three different ownership structures that can arise: one large shareholder with a fringe of minority shareholders, mutiple mid-sized blocks and fully dispersed structures.

The model provides the framework to explain a variety of phenomena reported in empirical studies such as the positive relationship between the presence of blockholders and firm value, ownership concentration and risk, the role of ownership in IPO underpricing. An important take away message of our paper is that there is a clear distinction between ownership structures with one large shareholder and those with multiple intermediate sized blockholders, both in terms of the conditions under which they are observed as well as in the implications for firm policy on risk.

Although our paper assumes that all outside investors are identical, the paper could be easily extended to the case of heterogenous agents. Indeed this would help to reduce the problem of multiple equilibria. In this case less risk averse (or wealthier) investors would be the natural blockholders and we would expect that if this occurs, the risk/return choice is even more stark. The other important extensions to the paper are to allow different voting mechanisms (e.g.dual class shares, cumulative voting, agenda choic, plurality vs majority rules etc) and allowing more than one firm in the economy.

References

Admati, A. R., P. Pfleiderer, and J. Zechner (1994): "Large Shareholder Activism, Risk Sharing, and Financial Market Equilibrium," Journal of Political Economy, 102(6), 1097–1130.

Barclay, M. J., and C. G. Holderness (1989): "Private Benefit from Control of Public Corporations," Journal of Financial Economics, 25(2), 371–395.

Becht, M., J. Franks, C. Mayer, and S. Rossi (2009): "Returns to Shareholder Activism: Evidence from a Clinical Study of the Hermes UK Focus Fund," Rev. Financ. Stud., 22(8), 3093–3129.

Bennedsen, M., and D. Wolfenzon (2000): "The balance of power in closely held corporations," Journal of Financial Economics, 58(1–2), 113–139.

Black, D. (1948): "On the rationale of group decision-making," Journal of Political Economy, 56(1), 23–34.

Bolton, P., and E. L. Von Thadden (1998): "Blocks, Liquidity, and Corporate Control," The Journal of Finance, 53(1), 1–25.

Boulton, T. J., S. B. Smart, and C. J. Zutter (forthcoming): "IPO Underpricing and the Market for Corporate Control," Journal of International Business Studies.

Brav, A., W. Jiang, F. Partnoy, and R. S. Thomas (2008): "Hedge Fund Activism, Corporate Governance, and Firm Performance," The Journal of Finance, 63(4), 1729–1775.

Brennan, M., and J. Franks (1997): "Underpricing, ownership and control in initial public offerings of equity securities in the UK," Journal of Financial Economics, 45(3), 391–413.

Burkart, M., D. Gromb, and F. Panunzi (1997): "Large Shareholders, Monitoring, and the Value of the Firm," Quarterly Journal of Economics, 112(3), 693–728.

Carlin, W., and C. Mayer (2000): "How Do Financial Systems Affect Economic Performance?," in Corporate Governance: Theoretical and Empirical Perspectives, ed. by X. Vives. Cambridge University Press, Cambridge, UK.

——— (2003): "Finance, investment, and growth," Journal of Financial Economics, 69(1), 191–226.

Clark, J., and K. Shastri (2000): "On Information Asymmetry Indices," Unpublished manuscript.

DeMarzo, P. M., and B. Urosevic (2006): "Ownership Dynamics and Asset Pricing with a Large Shareholder," Journal of Political Economy, 114(4), 774–815.

Dhillon, A. (2005): "Poliltical Parties and Coalition Formation," in Group Formation in Economics: Networks, Clubs and Coalitions, ed. by G. Demange, and M. Wooders. Cambridge University Press, Cambridge, UK.

Dlugosz, J., R. Fahlenbrach, P. Gompers, and A. Metrick (2006): "Large Blocks of stock: Prevalence, size and measurement," .

Edmans, A., and G. Manso (2008): "Governance Through Exit and Voice: A Theory of Multiple Blockholders," Discussion Paper 225/2006, ECGI.

Faccio, M., L. H. P. Lang, and L. Young (2001): "Who Disciplines Management in Poorly Performing Companies?," American Economic Review, 91(1), 54–78.

Faccio, M., M.-T. Marchica, and R. Mura (2009): "Large Shareholder Diversification and Corporate Risk-Taking," Unpublished manuscript.

Fernando, C. S., S. Krishnamurthy, and P. A. Spindt (2004): "Are Share Price Levels Informative? Evidence from the Ownership, Pricing, Turnover, and Performance of IPO Firms," Journal of Financial Markets, 7(4), 377–403.

Goergen, M., and L. Renneboog (2002): "Prediction of Ownership and Control Concentration in German and UK Initial Public Offerings," in Convergence and Diversity of Corporate Governance Regimes and Capital Markets, ed. by J. McCahery, P. Moerland, T. Raaijmakers, and L. Renneboog, pp. 251–267. Oxford University Press, Oxford, UK.

GOMES, A., AND W. NOVAES (2001): "Sharing of Control as a Corporate Governance Mechanism," Discussion Paper PIER Working paper 01-029, University of Pennsylvania Law School.

GROSSMAN, S. J., AND O. D. HART (1980): "Takeover Bids, the Free-Rider Problem: and the Theory of Corporation," Bell Journal of Economics, 11(1), 42–64.

GUTIERREZ, M., AND J. A. TRIBO (2004): "Private Benefits Extraction In Closely-Held Corporations: The Case For Multiple Large Shareholders," Discussion Paper 53/2004, ECGI.

HELWEGE, J., V. INTINTOLI, AND A. J. ZHANG (2011): "Voting with their feet or activism? Institutional Investors impact on CEO turnover," Unpublished manuscript.

HOLMSTROM, B., AND J. TIROLE (1993): "Market Liquidity and Performance Monitoring," Journal of Political Economy, 101(4), 678–709.

JOHN, KOSE, L. L. P. Y. B. Y. (2008): "Corporate Governance and Risk taking: Theory and Evidence," Journal of Finance, 63(4), 1607–1728.

KIRCHMAIER, T., AND J. GRANT (2005): "Corporate Ownership Structure and Performance in Europe," European Management Review, 2(3), 231–245.

LA PORTA, R., F. LOPEZ-DE-SILANES, AND A. SHLEIFER (1999): "Corporate Ownership Around the World," The Journal of Finance, 54(2), 471–517.

LAEVEN, L., AND R. LEVINE (2008a): "Bank Governance, Regulation and Risk taking," .

——— (2008b): "Complex Ownership Structures and Corporate Valuations," Review of Financial Studies, 21(2), 579–604.

LEHMANN, E., AND J. WEIGAND (2000): "Does the Governed Corporation Perform Better? Governance Structures and Corporate Performance in Germany," European Finance Review, 4(2), 157–195.

MAUG, E. (1998): "Large Shareholders as Monitors: Is there a tradeoff between liquidity and control?," The Journal of Finance, 53(1), 65–98.

MAURY, B., AND A. PAJUSTE (2005): "Multiple Large Shareholders and Firm Value," Journal of Banking and Finance, 29(7), 1813–1834.

NAGATA, K., AND S. RHEE (2009): "Ownership, Control and Underpricing in Initial Public Offerings: Evidence from Japan," Unpublished manuscript.

NOE, T. H. (2002): "Investor Activism and Financial Market Structure," Review of Financial Studies, 15(1), 289–318.

PAGANO, M., AND A. RÖELL (1998): "The Choice of Stock Ownership Structure: Agency Costs, Monitoring, and the Decision to Go Public," Quarterly Journal of Economics, 113(1), 187–226.

ROOSENBOOM, P., AND W. SCHRAMADE (2006): "The price of power: Valuing the controlling position of owner-managers in French IPO firms," Journal of Corporate Finance, 12(2), 270–295.

SHLEIFER, A., AND R. W. VISHNY (1986): "Large Shareholders and Corporate Control," Journal of Political Economy, 94(3), 461–488.

SMITH, M. P. (1996): "Shareholder Activism by Institutional Investors: Evidence for CalPERS," Journal of Finance, 51(1), 227–52.

STIGLITZ, J. E. (1985): "Credit Markets and the Control of Capital," Journal of Money, Credit and Banking, 12(2), 133–152.

STOUGHTON, N., AND J. ZECHNER (1998): "IPO-Mechanisms, Monitoring and Ownership Structure," Journal of Financial Economics, 49(1), 45–77.

THE ECONOMIST (2009): "Business: The ties that bind; The Wallenbergs," 390(8615), 68.

VOLPIN, P. (2002): "Governance with poor investor protection: evidence from top executive turnover in Italy," Journal of Financial Economics, 64(1), 61–90.

YEH, Y. H., AND P.-G. SHU (2004): "Corporate Governance and Valuation of IPOs," Unpublished manuscript.

YERMACK, D. (2010): "Shareholder Voting and Corporate Governance," Annual Review of Financial Economics, 2, 103–125.

ZWIEBEL, J. (1995): "Block Investment and Partial Benefits of Corporate Control," Review of Economic Studies, 62(2), 161–185.

A Appendices

A.1 The Model

A.1.1 Lemma 1:

Proof. The proof is obvious. We just maximize the objective functions (over X) of the outside investors, equation (2), and of the initial owner, equation (3), given the fraction of shares held, α_j. Concavity ensures uniqueness of the solution. ■

A.1.2 Equilibrium definition

Our notion of equilibrium is subgame perfect equilibrium of the game described in Fig. 1. Note that because in many potential equilibria more than one investor needs to buy shares for the initial owner to find it worthwhile to start the firm, there is always a No Trade equilibrium. In this equilibrium, no investor buys any shares anticipating that no other investor will buy shares. Below we provide a definition for equilibria with positive trade.

Equilibrium is a monitoring level, $m \in \{0,1\}$, a fraction α_E^* of shares held by the initial owner, a fraction of wealth invested w_E^*, a decision X_{med} and an allocation of shares among investors, $\vec{\alpha}^*$ such that: (i) α_E^* and w_E^* maximize the utility of the initial owner given the anticipated demand, the anticipated monitoring level, m, and the anticipated ownership structure $\vec{\alpha}(K-w_E, \alpha_E)$. (ii) Each active investor chooses α_i to maximize her utility given $K - w_E^*$, α_E^*, the anticipated m and the anticipated shares of all other active investors denoted α_{-i}. (iii) Each passive investor chooses α_i to maximize her utility given $K - w_E^*$, α_E^* and the anticipated m and X_{med}. (iv) In equilibrium there must be full subscription. There can be excess demand in equilibrium as long as no investor who owns shares is willing to sell them at a price lower than the maximum willingness to pay of the excluded investors. (v) The value enhancing decision must be optimal for the initial owner given his stake and the vote outcome. (vi) Expectations are rational.

A.2 The equilibria.

Lemma 7 *The initial owner chooses $m = 1$ $\underline{iff}$ $\alpha_E \geq \bar{m}$.*

Proof. At date 3, the ownership structure, $\vec{\alpha}$, and thus X_{med} are already fixed. Given the initial owner's objective function (3), he chooses $m = 1$ iff the utility from $m = 1$ is greater than from $m = 0$, that is when:

$$\alpha_E \left(X_{med}\bar{R} + K\right) - \frac{\gamma}{2}\alpha_E^2 X_{med}^2 \sigma^2 - \bar{m}K \geq \alpha_E X_{med}\bar{R} - \frac{\gamma}{2}\alpha_E^2 X_{med}^2 \sigma^2 \tag{20}$$

Rearranging, we get the condition $\alpha_E \geq \bar{m}$. ■

Lemma 8 *Let X_j be the belief on the voting outcome and $K - w_E$ the capital demanded. Then liquidity shareholders demand $\alpha_{l,j}$ as defined in equation (5).*

Proof. Investor l chooses $\alpha_{l,j}$ to maximize equation (2) where $X_j = X_{med}$. The first order condition implies equation (5). The second order condition is satisfied as long as $X_j > 0$, so this is a maximum. ■

A.2.1 Lemma 3

Proof. As the liquidity shareholding $\alpha_{l,j}$ maximizes the utility of an investor given a vote outcome, any shareholding $\alpha_1 \neq \alpha_{l,j}$ gives less utility. ■

A.2.2 Lemma 4

Proof. Consider first the case where $\alpha_j \geq \bar{\alpha}$. Hence by Lemma 1, $X_j = \frac{\bar{R}}{\gamma\sigma^2\alpha_j}$ and $\alpha_{l,j} = \alpha_j + \frac{f(m)-\frac{K-w_E}{1-\alpha_E}}{\frac{\bar{R}^2}{\gamma\sigma^2\alpha_j^2}}$. By assumption, $f(m) - \frac{K-w_E}{1-\alpha_E} < 0$ and thus $\alpha_{l,j} < \alpha_j$.

Now let $\alpha_j < \bar{\alpha}$. By definition, j is the median shareholder, hence $X_{med} = X_j = \bar{X}$. As $f(m) - \frac{K-w_E}{1-\alpha_E} < 0$ a liquidity investor always chooses $\bar{\alpha}_l < \bar{\alpha}$ and hence it is sufficient to show that no active investors hold $\alpha_j < \bar{\alpha}_l$. Assume to the contrary investors j holds $\alpha_j < \bar{\alpha}_l$. (Active) Investor j can improve his utility by choosing $\bar{\alpha}_l$ and voting for $X = \bar{X}$ without changing X_{med}. Contradiction to the equilibrium definition where shareholders are maximizing their utility when X is fixed. This proves that in equilibrium $\alpha_j \geq \alpha_{l,j}$. ■

A.2.3 Lemma 2:

Proof. By Lemma 1, the first best X for investor i is given $X_i = \min\left[\frac{\bar{R}}{\gamma\sigma^2\alpha_i}, \bar{X}\right]$. Plugging the function $X_i = X(\alpha_i)$ into equation (2), investor i' utility function is decreasing in α_i. Investors ideal point is $X = \bar{X}$ and $\bar{\alpha}_l$. ■

A.2.4 Lemma 5:

Proof. The proof follows these steps: (A) compute the utility of the median shareholder (a) when $X_{med} = X_1 < \bar{X}$ and (b) when $X_{med} = X_1 = \bar{X}$; (B) Show that (a) $\alpha_{-1} > 0$; (b) $\alpha_1 + \alpha_{-1} \geq \alpha_E > \alpha_1$; (c) $\alpha_E \geq \lambda(1 - \alpha_E - n\alpha_1)$; (C) Derive equation (6).

(A.a) Since $X_{med} = X_1 < \bar{X}$, by Definition 2, shareholder 1 is the median shareholder and $\alpha_1 > \bar{\alpha}$. Using equation (2), the utility of the median shareholder is:

$$U_1^{nBH} = 1 + \alpha_1 \left(X_1 \bar{R} + f(m) - \frac{K - w_E}{1 - \alpha_E} \right) - \frac{\gamma}{2}\alpha_1^2 X_1^2 \sigma^2$$

By Lemma 1 and the fact that $X_{med} = X_1 < \bar{X}$, this is equivalent to

$$\alpha_1(f(m) - \frac{K - w_E}{1 - \alpha_E}) + 1 + \frac{\bar{R}^2}{2\gamma\sigma^2} \tag{21}$$

By assumption, $f(m) - \frac{K-w_E}{1-\alpha_E} < 0$, hence U_1^{nBH} is decreasing in α_1.

(A.b) Suppose instead that $X_{med} = X_1 = \bar{X}$. By definition $\bar{\alpha}_l < \alpha_1$ in an n-Blockholder EOS and $\bar{\alpha}_l = \text{argmax } U_i(\alpha_i)|_{X=\bar{X}}$. Hence, given the shape of the utility function (2), U_1^{nBH} is decreasing in α_1.

(B.a) First note that α_{-1} can only be non negative as it is defined as the sum of the anticipated shares of the active investors so those that vote. Hence it is sufficient to show that $\alpha_{-1} \neq 0$. Suppose to the contrary that $\alpha_{-1} = 0$ in an n-Blockholder EOS. In order to have $X_{med} = X_1$ we must have $\alpha_1 \geq \alpha_E$, since investor 1 is the only outside investor who votes. From part (a), we know that U_1^{nBH} above is decreasing in α_1 and hence investor 1 is better off setting $\alpha_1 + \alpha_{-i} = \alpha_E$, otherwise the initial owner becomes the median shareholder. But in this case, $X_{med} = X_1 = X_E$ and *de facto* investor 1 does not affect the vote outcome. Hence he prefers to be a liquidity shareholder and his optimal shareholding is given by $\bar{\alpha}_l$. Contradiction to the fact that this is an n-Blockholder EOS (since investor 1 wants to deviate unilaterally). Hence, $\alpha_{-1} > 0$ in any n-Blockholder EOS.

(B.b) $\alpha_E > \alpha_1$ follows from the proof of part (1) above. We need to prove that $\alpha_1 + \alpha_{-1} \geq \alpha_E$. Suppose to the contrary that $\alpha_1 + \alpha_{-1} < \alpha_E$. Then $\alpha_E = \alpha_{med}$ and so $X_{med} = X_E$. This is not an n-Blockholder EOS by Definition 2. Contradiction.

(B.c) $\alpha_E \geq \lambda(1-\alpha_E-n\alpha_1)$. Suppose to the contrary that $\alpha_E < \lambda(1-\alpha_E-n\alpha_1)$. Then it is optimal for an investor holding α_1 to deviate and hold $\alpha_{l,1}$. This contradicts the Definition 2 of a n-Blockholder EOS.

(C) As U_1^{nBH} is decreasing in α_1 and $n\alpha_1 \geq \alpha_E - \lambda(1 - \alpha_E - n\alpha_1)$. Equation (6) follows. ■

A.2.5 Proposition 1:

Proof. We apply backward induction. Suppose the initial owner raises capital he maximizes the following objective function:

$$\max_{\alpha_E, w_E} U\,(m=0) = \bar{R}X_{med}(\alpha_E)\alpha_E - \frac{\gamma}{2}X_{med}(\alpha_E)^2\sigma^2\alpha_E^2 + 1 - w_E \tag{22}$$

subject to the relevant participation constraint, $w_E \geq w_E^j(\alpha_E)$ where $w_E^j(\alpha_E) = \{\underline{w}_E^E, \underline{w}_E^n, \underline{w}_E^{LS}\}$.

Substituting for w_E^j in the objective function, it can be checked that $U\,(m=0)$ is decreasing in α_E for all w_E^j, for $X_{med} \leq \bar{X}$. Therefore $\alpha_E^* = 0$ is the optimal choice of the initial owner for *any* ownership structure.

When $\alpha_E^* = 0$, and there is at least one active investor ($\lambda > 0$), all active investors vote for $\bar{X}$ and hence $X_{med} = \bar{X}$. To satisfy the participation constraint of outside investors $w_E = \underline{w}_E*^{LS}$ and the Liquidity Shareholder EOS is the unique EOS. Also it is trivial to see that no investor is willing to sell his shares at a price lower than the maximum that an excluded investor is willing to pay.

The initial owner's utility is given by $\bar{R}\bar{X} - K + 1 - \epsilon_{LS}$ If he invests in the riskfree asset his utility is 1. Hence when the project has a positive NPV he sells the firm, otherwise he does not raise capital. The initial owner's value function is then given by equation (10). ■

A.3 Monitoring Equilibria

First we provide full expressions for $\underline{w}_E^E, \underline{w}_E^n, \underline{w}_E^{LS}$ of the minimum wealth that the initial owner needs to pledge to guarantee that the participation constraints are satisfied:

Let $\epsilon_j = \gamma X_j^2 \sigma^2 \eta$ where $\eta > 0$ is the fraction of shares corresponding to one share.

$$\underline{w}_E^E(\alpha_E) \equiv K - \left(\frac{\bar{R}^2}{\gamma\sigma^2\alpha_E} + f(m)\right)(1-\alpha_E) + \epsilon_E \tag{23}$$

$$\underline{w}_E^n(\alpha_E) \equiv K - \left(\bar{R}X_1 + f(m) - \frac{\gamma}{2}X_1^2\sigma^2\alpha_1\right)(1-\alpha_E) \tag{24}$$

$$\underline{w}_E^{LS}(\alpha_E) \equiv K - \left(\bar{R}\bar{X} + f(m)\right)(1-\alpha_E) + \epsilon_{LS} \tag{25}$$

A.3.1 The EOS

In this section, we provide sufficient conditions under which various EOS exist for different pair of $(\alpha_E, w_E) \in S \equiv [0,1] \times (-\infty, \infty)$. This is needed, since in order to show that a putative equilibrium is subgame perfect, we must specify what happens off the equilibrium path.

First, observe that for certain combinations of (α_E, w_E) there always exists a no trade equilibrium where no investors participate in the share issue. Define $\underline{w}_E^{NT} = \begin{cases} \underline{w}_E^E & \text{if } \alpha_E > \max\left[\frac{1}{2}, \bar{\alpha}\right] \\ \underline{w}_E^1 & \text{if } \bar{\alpha} \le \alpha_E \le \frac{1}{2} \\ \underline{w}_E^{LS} & \text{if } \alpha_E \le \bar{\alpha} \end{cases}$.

This is the minimum amount the initial owner needs to invest in order to guarantee that the participation constraint of investors is satisfied (across all X) for different levels of α. If $w_E < \underline{w}_E^{NT}$ then there is no trade, i.e. the project does not go ahead.

Define:

$$\underline{\underline{w}}_E(\alpha_E) \equiv K - f(m)(1-\alpha_E) \tag{26}$$

$$\hat{\alpha}(n) \equiv \max[\hat{\alpha}_1(n), \hat{\alpha}_2(n)] \tag{27}$$

$$\hat{\alpha}_1(n) \equiv \frac{2\lambda}{2(1+\lambda) - n(1-\lambda)} \tag{28}$$

$$\hat{\alpha}_2(n) \equiv \frac{2\bar{\alpha}(1-\lambda)n + \lambda - \sqrt{\lambda^2 + 4\bar{\alpha}(1-\lambda)n\left(\bar{\alpha}(1-\lambda)n - 1 - 2\bar{\alpha}(1+\lambda)\right)}}{2(1+\lambda)} \tag{29}$$

$$\breve{\alpha}(n) \equiv \frac{n(1-\lambda)\bar{\alpha} + \lambda}{1+\lambda} \tag{30}$$

$$\tilde{\alpha}(n) \equiv \frac{\lambda}{1+\lambda} + \frac{n\eta(1-\lambda)}{1+\lambda} \tag{31}$$

Suppose α_E is fixed, then $\underline{\underline{w}}_E$ is the maximum w_E that guarantees that $f(m) \le \frac{K-w_E}{1-\alpha_E}$. $\hat{\alpha}(n)$ is the value of α_E such that if $\alpha_E \le \hat{\alpha}(n)$ then $\underline{w}_E^E \le \underline{w}_E^n$. This may differ depending on X: In particular if $\alpha_E \le \hat{\alpha}_1(n)$, then $\underline{w}_E^E \le \underline{w}_E^n$ where $X_1 < \bar{X}$, and if $\alpha_E \le \hat{\alpha}_2(n)$, then $\underline{w}_E^E \le \underline{w}_E^n$ where $X_1 = \bar{X}$. $\breve{\alpha}(n)$ is the value of α_E such that when $\alpha_E \le \breve{\alpha}(n)$, $\alpha_1 = \bar{\alpha}$. Finally $\tilde{\alpha}(n)$ is the value of α_E such that when

$\alpha_E \leq \tilde{\alpha}(n)$ we have $\alpha_E \leq n\bar{\alpha}_l + \lambda(1-\alpha_E - n\bar{\alpha}_l)$ and $\bar{\alpha}_l$ is evaluated at $w_E = \underline{w}_E^{LS}$ so that $\bar{\alpha}_l = \eta$.

Lemma 9 *There exists an Initial Owner EOS, with $X_{med} = X_E < \bar{X}$ for any pair (α_E, w_E), satisfying the following conditions:*

$$\alpha_E \in \left(\max\left(\bar{\alpha}, \min\left[\frac{1}{2}, \max\left[\hat{\alpha}(1), \check{\alpha}(1)\right]\right]\right), 1\right] \tag{32}$$

$$w_E \in \left[\underline{w}_E^E(\alpha_E), \min[\underline{w}_E^1(\alpha_E), \underline{\underline{w}}_E(\alpha_E)\right) \tag{33}$$

When $m = 1$, the set of α_E such that conditions (32) and (33) are satisfied is non-empty.

Proof. Note that as $w_E \leq \underline{\underline{w}}_E$, $f(m) \leq \frac{K-w_E}{1-\alpha_E}$.

From Definition 1: An Initial Owner EOS exists for any combination of (α_E, w_E) iff

1. $\sum_i \alpha_i \geq 1-\alpha_E$ and $U_{l,E} \geq 1$. This implies that $U_{l,E} > 1$, i.e. $\alpha_{l,E} > 0$. This holds iff iff $w_E \geq \underline{w}_E^E$. This is the first part of condition (33).
2. The incentive constraint of active investors is satisfied; i.e. no liquidity investor has an incentive to switch to becoming a blockholder and get a higher utility when the decision changes to X_1 from X_E. Since $\alpha_E > \hat{\alpha}(1)$, $\underline{w}_E^E < \underline{w}_E^1$. Hence condition (33) implies that $U_{l,E} > 1 > U_1^{1BH}$.
3. $\alpha_E > \bar{\alpha}$ ensures that $X_E < \bar{X}$.
4. No shareholder is willing to sell his participation at a price lower than the price at which the excluded investors are willing to buy. Any active investor who is excluded can do better by buying liquidity shares $\bar{\alpha}_l$ from a *passive* investor (who does not vote) at a price slightly higher than the initial owner's price, if by voting he is able to become pivotal and change the outcome to $\bar{X}$. To avoid this possibility we impose the following condition:

$$\lambda(1-\alpha_E - \bar{\alpha}_l) + \bar{\alpha}_l < \alpha_E \tag{34}$$

   A sufficient condition for this is that $\alpha_E > \check{\alpha} > \tilde{\alpha}(1)$.

   Therefore, the above condition plus condition arising in 2. $\alpha_E > \max[\hat{\alpha}(1), \check{\alpha}(1)]$. However these conditions become irrelevant when the initial owner has the majority of the shares since then, no active investors can change the vote outcome. This implies that $\alpha_E > \min\left[\frac{1}{2}, \max[\hat{\alpha}(1), \tilde{\alpha}(1)]\right]$.

Putting together the constraints of point 3. and 4., the lower bound of (32) is given. ■

Lemma 10 *There exists an n-Blockholder EOS, with $X_{med} = X_1 < \bar{X}$, for any pair (α_E, w_E), satisfying the following conditions:*

$$\alpha_E \in \left(\check{\alpha}(n), \min\left[\hat{\alpha}_1(n), \frac{1}{2}\right]\right] \tag{35}$$

$$w_E \in [\underline{w}_E^n(\alpha_E), \underline{\underline{w}}_E(\alpha_E)) \tag{36}$$

Proof. Observe that $X_1 < \bar{X}$ iff $\alpha_1 > \bar{\alpha}$ (Lemma 1). Using Lemma 5 this condition is equivalent to $\alpha_E > \breve{\alpha}(n)$, the lower boundary of condition (35).

An n-Blockholder EOS with $X_1 < \bar{X}$ exists iff the conditions of Definition 1 are satisfied:

1. The participation constraint of blockholders is satisfied iff

$$U_1^{nBH} = \left(\bar{R}X_1 + f(m)\right)\alpha_1 - \frac{K - w_E}{1 - \alpha_E}\alpha_1 - \frac{\gamma}{2}X_1^2\sigma^2\alpha_1^2 + 1 \geq 1 \tag{37}$$

   Rearranging, this is equivalent to $w_E \geq \underline{w}_E^n(\alpha_E)$, the lower boundary of condition (36). By Lemma 3, liquidity shareholders participation constraint is always satisfied whenever the blockholders' is, hence $U_{l,1} \geq 1$.

2. From Lemma 5, since $w_E \leq \underline{\underline{w}}_E$ by condition (36) his utility U_1^{nBH} is decreasing in α_1 so the blockholder holds just enough to be pivotal. Hence, no blockholder wants to increase his shares from α_1.

   Lemma 5 shows that if the blockholder decreases his shareholding, the highest possible utility a blockholder investor can achieve, is given by being a liquidity shareholder. Hence, the incentive compatibility constraint is $U_1^{nBH} \geq U_{l,E}$. Because $\alpha_E \geq \hat{\alpha}_1(n)$ we know from point 2. of the proof of Lemma 9 that since $w_E \geq \underline{w}_E^n$, $U_1^{nBH} \geq U_{l,E}$. This is the first upper bound in condition (35).

   Finally because an investor wants to hold a block only if he is pivotal in the vote outcome, we need to impose that $\alpha_E \leq \frac{1}{2}$. Suppose $\alpha_E > \frac{1}{2}$ then $X_{med} = X_E$ always, so there is no Blockholder EOS. This gives the second upper bound in condition (35).

   Hence, no blockholder wants to change his shares unilaterally from α_1.

   Finally we check that liquidity shareholders cannot gain from unilateral deviation. No (active) liquidity shareholder has an incentive to hold shares bigger than α_1 in order to change $X_{med} < X_1$, as this would reduce their utility (which is increasing in X and decreasing in shareholdings when $\alpha_i > \alpha_{l,j}$).

   If the investors choose any $\alpha_i < \alpha_1$ they do not change the outcome, hence $\alpha_l(X_1)$ maximizes their utility.

3. If an included liquidity investor sold his shares to an an excluded investor, the excluded investor would have to buy more than the block α_1 to change the outcome and get a bigger rent than included liquidity shareholders. However blockholders do not make any rents as their participation constraint is binding, so there is no incentive to re-trade.

■

Corollary 3 *There exists an n-Blockholder EOS, with* $X_{med} = X_1 = \bar{X}$*, for any pair* (α_E, w_E)*, satisfying the following conditions:*

$$\alpha_E \in \left(\max\left[\bar{\alpha}, \tilde{\alpha}(n)\right], \min\left[\check{\alpha}(n), \hat{\alpha}_2(n), \frac{1}{2}\right]\right] \quad (38)$$

$$w_E \in [\underline{w}_E^n(\alpha_E), \underline{\underline{w}}_E(\alpha_E)) \quad (39)$$

where $1 \leq n \leq M_A$.

Proof.

The conditions follow from Lemma 10. However to ensure that the blockholders always hold a non diversified portfolio such that $X_{med} = \bar{X}$, we impose the conditions $\alpha_E > \tilde{\alpha}(n)$ and $\alpha_E \leq \check{\alpha}(n)$. Of course $\tilde{\alpha}_n$ depends on w_E, α_E. Notice that $\tilde{\alpha}(n) < \alpha_E < \check{\alpha}_E(n)$, since $\bar{\alpha}_l < \alpha_1 < \bar{\alpha}$. ■

Lemma 11 *For any pair* (α_E, w_E) *there exists a Liquidity Shareholder EOS, with* $X_{med} = \bar{X}$ *with* $n + \frac{\lambda(1-\alpha_E - n\bar{\alpha}_l)}{\bar{\alpha}_l}$ *active investors holding* $\bar{\alpha}_l$ *if:*

$$\alpha_E \in \left(\bar{\alpha}, \min\left(\tilde{\alpha}(n), \frac{1}{2}\right)\right] \quad (40)$$

$$w_E(\alpha_E) \geq \underline{w}_E^{LS}(\alpha_E) \quad (41)$$

Proof.

Let $U_{l,\bar{\alpha}}$ denote the value function of a liquidity shareholder when $X = \bar{X}$, and he holds the optimal shareholdings. A liquidity shareholder EOS exists iff the conditions of Definition 1 are satisfied:

1. By the proof of Lemma 8, point 1. and 2. Definition 1 are satisfied iff $w_E \geq \underline{w}_E^{LS}(\alpha_E)$.
2. As long as $\alpha_E \leq \tilde{\alpha}_E(n)$, $X_{med} = \bar{X}$. No active liquidity investor wants to increase or decrease his shareholdings since this is the most preferred point (see Lemma 2). So point 3. of Definition 1 is satisfied.
3. Passive investors hold $\bar{\alpha}_l$ which maximizes their utility.
4. No investor is willing to sell his shares to any excluded investors as the maximum price at which excluded investors are willing to buy the shares is the minimum price at which the liquidity shareholders are willing to sell since X is already at its first best for outside investors.

■

Corollary 4 *There exists a No Conflicts EOS, with* $X_{med} = X_E = \bar{X}$ *if*

$$\alpha_E \in (0, \bar{\alpha}] \quad (42)$$

$$w_E(\alpha_E) \in \left[\underline{w}_E^{LS}(\alpha_E), \underline{\underline{w}}_E\right) \quad (43)$$

Proof. When $\alpha_E \in (0, \bar{\alpha}]$ there are no conflicts of interests between outside investors and the initial owner, $X_{med} = X_E = \bar{X}$. Investors maximize their utility holding $\bar{\alpha}_l$. Their participation constraint is satisfied when $w_E \geq \underline{w}_E^{LS}$. Active investors hold $\bar{\alpha}_l$, and are at their most preferred X. Hence they have no incentive to deviate. No investor is willing to sell shares at a price lower than the maximum that an excluded investor will pay. ■

A.3.2 Lemma 6:

Proof. From Section A.3.1 it results that all the possible EOS are characterized by outside investors who are either liquidity shareholders or blockholders.

Consider an EOS where no blockholders exist. Suppose to the contrary, that there is an equilibrium with $\frac{K-w_E}{1-\alpha_E} < K$, $\alpha_{l,j} > 0$ and $U_{l,j} > 1$. By assumption there are sufficiently many investors in the market, so there always exist passive shareholders who have a strictly positive demand for shares for any $0 < X_j \leq \bar{X}$. Hence, the initial owner can increase his utility by decreasing w_E, for any α_E and still ensure that there is a (smaller) positive demand by passive investors, ensuring full subscription. As the demand of the liquidity shareholders is given by equation (5), the initial owner will do this until $\frac{K-w_E}{1-\alpha_E} > K$. Contradiction.

Consider an EOS where blockholders and liquidity shareholders exist. In such a case either blockholders or liquidity shareholders will have the most binding constraint. We already showed above that when the liquidity investors participation is more binding then $\frac{K-w_E}{1-\alpha_E} > K$. So it is sufficient to show that this is true when the binding constraint is that of blockholders.

When the blockholders's participation constraint is more binding given $m = 1$ the value function for blockholders given $X_{med} = X_j$ is given by :

$$U_1^{nBH} = \bar{R}X_j\alpha_1 + \left(K - \frac{K-w_E}{1-\alpha_E}\right)\alpha_1 - \frac{\gamma}{2}X_j^2\sigma^2\alpha_1^2 + 1 \tag{44}$$

By the same logic as for the first part, suppose that there is an equilibrium with $\frac{K-w_E}{1-\alpha_E} < K$. Because $\alpha_1 \leq \max[\alpha_j, \bar{\alpha}]$ when $\frac{K-w_E}{1-\alpha_E} < K$, then the participation constraint of the blockholders is satisfied with strict inequality, i.e. $U_1^{nBH} > 1$. The initial owner can decrease w_E and still satisfy the constraints and ensure full subscription. Contradiction to the fact that it is an equilibrium. ■

A.3.3 Proposition 2:

Proof. Before we prove the next proposition, we need a few lemmas which provide expressions for the value function of the initial owner under the alternative ownership structures that could be obtained. By Lemma 6, we can drop the requirement for the EOS, that $w_E < \underline{\underline{w}}_E$.

Lemma 12 *Suppose the conditions for the No Conflicts EOS (Corollary 4) are satisfied, and the equilibrium of the game is the No Conflicts equilibrium, then the initial owner sets* $\alpha_E = \bar{m}$, $X_{med} = X_E = \bar{X}$, $w_E = \underline{w}_E^{LS}$ *and the value function of the Initial Owner is given by:*

$$V_E^{NC} = \bar{R}\bar{X} + 1 - \frac{\gamma}{2}\bar{X}^2\bar{m}^2\sigma^2 - \bar{m}K - \epsilon_{LS} \tag{45}$$

Proof. By Lemma 7, in any monitoring equilibrium, $\alpha_E \geq \bar{m}$. By Corollary 4, the No Conflicts EOS with $X_{med} = X_E = \bar{X}$ exists if $\alpha_E \in (0, \bar{\alpha}]$ and $w_E \geq \underline{w}_E^{LS}(\alpha_E)$. Therefore the maximization problem of the Initial Owner in the No Conflicts equilibrium is:

$$\max_{\alpha_E, w_E} U_E = (\bar{R}\bar{X} + K)\alpha_E - \frac{\gamma}{2}\bar{X}^2\alpha_E^2\sigma^2 + 1 - w_E - \bar{m}K \tag{46}$$

$$\text{s.t} \quad w_E \geq \underline{w}_E^{LS}(\alpha_E) \tag{47}$$

$$\alpha_E \in [\bar{m}, \bar{\alpha}] \tag{48}$$

The initial owner's utility is decreasing in the wealth invested, w_E. Hence he chooses w_E such that it satisfies the participation constraint of the liquidity investors, (47), at equality. Inserting it in the initial owner's objective function we obtain:

$$\bar{R}\bar{X} + 1 - \frac{\gamma}{2}\bar{X}^2\alpha_E^2\sigma^2 - \bar{m}K - \epsilon_{LS} \tag{49}$$

This expression is decreasing in α_E. Hence the initial owner will retain just enough shares to satisfy the monitoring constraint with equality: $\alpha_E = \bar{m}$. Inserting $\alpha_E = \bar{m}$ in the initial owners utility function, we have expression (45). ■

Let $\underline{b} = \left(\max\left[\bar{\alpha}, \min\left[\frac{1}{2}, \max\left[\hat{\alpha}(1), \tilde{\alpha}(1)\right]\right]\right]\right) + \eta$.

Lemma 13 *Suppose the conditions of the Initial Owner EOS are satisfied (Lemma 9), and the equilibrium of the game is an Initial Owner equilibrium, then* $X_{med} = X_E < \bar{X}$, $w_E = \underline{w}_E^E$, $\alpha_E = \max[\bar{m}, \underline{b}]$, *and the value function of the Initial Owner is given by:*

$$V_E^E = \bar{R}X_E + 1 - \frac{\gamma}{2}X_E^2 \max[\bar{m}, \underline{b}]^2\sigma^2 - \bar{m}K - \epsilon_E = \frac{\bar{R}^2}{\gamma\sigma^2}\left(\frac{1}{\max[\bar{m}, \underline{b}]} - \frac{1}{2}\right) + 1 - \bar{m}K - \epsilon_E \tag{50}$$

Proof. The proof follows the same steps as for the proof of Lemma 12 using Lemmas 7 and 9. Detailed proof is available upon request. ■

Let $\underline{c} = \max[\breve{\alpha}(n) + \eta, \bar{m}]$.

Lemma 14 *Suppose the conditions of the n-Blockholder EOS are satisfied (Lemma 10 and Corollary 3) and the equilibrium of the game is an n Blockholder equilibrium with* $X_{med} = X_1$, *the initial owner* *sets $\alpha_E = \underline{c}$, $w_E = \underline{w}_E^n$ and the value function of the Initial Owner is given by:*

$$V_E^n = \bar{R}X_1 + 1 - \bar{m}K - \frac{\gamma}{2}X_1^2\sigma^2(\underline{c}^2 + \alpha_1 - \alpha_1\underline{c}) \tag{51}$$

Proof. The proof follows the same steps as for the proof of Lemma 12 and it applies Lemmas 7 and 10. Detailed proof is available upon request. ■

Lemma 15 *Suppose the conditions for the Liquidity Shareholder EOS are satisfied and there exists a Liquidity Shareholder equilibrium with monitoring. Then, $\alpha_E = \max(\bar{\alpha} + \bar{\eta}, \bar{m})$, $w_E = \underline{w}_E^{LS}$ and the value function of the Initial Owner is given by:*

$$V_E^{LS} \equiv \bar{R}\bar{X} + 1 - \bar{m}K - \frac{\gamma}{2}\bar{X}^2\sigma^2(\max[\bar{m}, \bar{\alpha} + \bar{\eta}])^2 - \epsilon_{LS} \tag{52}$$

Proof. The proof follows the same steps as for the proof of Lemma 12 and it applies Lemmas 7 and 11. Detailed proof is available upon request. ■

Define $\bar{m}_{NC}^{RC}$ and $\bar{m}_{NC}^{S}$ the values of $\bar{m}$ such that when $\bar{m}$ is smaller than $\bar{m}_{NC}^{RC}$ the initial owner prefers monitoring to not raising capital and $\bar{m}_{NC}^{S}$ the value of $\bar{m}$ such that the initial owner prefers monitoring rather than selling the firm (i.e. $V_E^{NC} \geq V_E^{NM}$ whenever $\bar{m} \leq \min\left(\bar{m}_{NC}^{RC}, \bar{m}_{NC}^{S}\right)$).

$$\bar{m}_{NC}^{RC} \equiv \frac{\sqrt{K\left(K + 2\bar{X}^2\gamma\sigma^2\right)} - K}{\bar{X}^2\gamma\sigma^2} \tag{53}$$

$$\bar{m}_{NC}^{S} \equiv \frac{\sqrt{2\bar{R}\gamma\sigma^2\bar{X}^3 + K^2} - K}{\bar{X}^2\gamma\sigma^2} \tag{54}$$

We are now ready to prove the proposition. We solve the game by backward induction. The initial owner chooses α_E and w_E, anticipating the EOS. His problem can be broken into the following: (1) $\alpha_E \in [\bar{m}, \bar{\alpha}]$, (2) $\alpha_E \in (\bar{\alpha}, 1]$, (3) $\alpha_E \in [0, \bar{m}]$.

We first describe the beliefs on the EOS and the corresponding value functions in each interval.
Case (1). By Corollary 4 there exists a No Conflicts EOS whenever $w_E \geq \underline{w}_E^{LS}$. Hence in this interval the beliefs of all players on the EOS are the No Conflicts EOS. Now, if $w_E < \underline{w}_E^{LS}$, for any possible X there exists a No Trade EOS where all investors believe that no one else will participate, and we assume that the belief is on the No Trade EOS. The initial owner's value function is given by equation (45) when $w_E \geq \underline{w}_E^{LS}$ and by the no trade value function, $V_E^{NT} = 1$ in case $w_E < \underline{w}_E^{LS}$.
Case (2). By Lemmas 9, 10 and 11 the possible EOS in this interval are the Initial Owner, the n Blockholder or the Liquidity Shareholder ones if $w_E \geq \underline{w}_E^j$ where $j = \{IO, n, LS\}$ depending on the sub-interval within $[\bar{\alpha}, 1]$. It is easy to see that if $\bar{\alpha} < \underline{b}$, and as long as $\check{\alpha}(1) \geq \frac{1}{2}$, the interval $[\bar{\alpha}, 1]$ can be partitioned into the three types of EOS, and we will assume that the IO EOS is in the partition $[\frac{1}{2}, 1]$ regardless of $\underline{b}$. If $\check{\alpha}(1) < \frac{1}{2}$, then there always exists an n sufficiently large such that $\check{\alpha}(n) \geq \frac{1}{2}$ so that we always have a blockholder equilibrium when $\alpha_E \leq \frac{1}{2}$. On the other hand if $\bar{\alpha} \geq \frac{1}{2}$ then

there is only the IO EOS in the interval $[\bar{\alpha}, 1]$. Again if $w_E < \underline{w}_E^j$ for the corresponding sub interval, we assume a No Trade EOS and hence $V_E^{NT} = 1$. Thus, e.g. if an Initial Owner EOS exists the initial owner sets $\alpha_E = \max[\bar{m}, \frac{1}{2}] = \frac{1}{2}$ as $\bar{m} \leq \bar{\alpha}$ and his value function, V_E^E is given by equation (50) setting $\underline{b} = \frac{1}{2}$. If an n Blockholder EOS exists, Lemma 14 shows that the initial owner's utility is decreasing in α_E. Observe that for an n Blockholder equilibrium to exist $\tilde{\alpha}(n) > \bar{\alpha}$. Hence $\underline{c} = \tilde{\alpha}(n) + \eta$ and the value function V_E^n is given by (51). If a Liquidity Shareholders EOS arises, Lemma 15 shows that $\alpha_E = \max[\bar{\alpha} + \bar{\eta}, \bar{m}] = \bar{\alpha} + \bar{\eta}$. Hence the value function V_E^{LS} is given by equation (52).

Case (3). In this interval Proposition 1 applies. The initial owner's value function is $V_E^{NM} = \max\left[1, \bar{R}\bar{X} - K + 1\right]$.

The initial owner will choose α_E to maximize his value function across the intervals (1)–(3) above.

First consider Case (2), ignoring V_E^{NT} for the moment: It is easy to see from (50) that $V_E^E|_{\alpha_E = \underline{b}} < V_E^E|_{\alpha_E = \bar{m}} = V_E^{NC}$. Hence the initial owner is better off in a No Conflicts equilibrium than in an Initial Owner equilibrium. If an n-Blockholder EOS exists, it is easy to see from equation (51) that $V_E^n|_{\alpha_1 < \bar{m}} < V_E^n|_{\alpha_1 = \bar{m}}$. $V_E^n|_{\alpha_1 = \bar{m}} < V_E^{NC}$ iff

$$\frac{1}{\alpha_1}\left(1 + \alpha_E - \frac{\alpha_E^2}{\alpha_1}\right) < \frac{2}{\bar{\alpha}} - \frac{\bar{m}^2}{\bar{\alpha}^2}$$

which is always true as $\bar{m} \leq \bar{\alpha} \leq \alpha_E$. Hence the initial owner is better off in a No Conflicts equilibrium than in an n-Blockholder equilibrium. If a Liquidity Shareholder EOS exists then by Lemma 15, $V_E^{LS}|_{\alpha_E > \bar{\alpha}} < V_E^E|_{\alpha_E = \bar{m}} = V_E^{NC}$. Hence the initial owner is better off in a No Conflicts equilibrium than in an Liquidity Shareholders equilibrium.

Now consider Case (3): The initial owner's value function is $V_E^{NM} = \max\left(1, \bar{R}\bar{X} - K + 1\right)$. Hence he prefers to monitor iff $V_E^{NC} \geq V_E^{NM}$, i.e. iff $V_E^{NC} \geq \bar{R}\bar{X} - K + 1$ and $V_E^{NC} \geq 1$.

This first condition is satisfied when $\bar{m} \in [c, \bar{m}_{NC}^S]$, where $c < 0$. Hence $V_E^{NC} \geq \bar{R}\bar{X} - K + 1$, whenever $\bar{m} < \bar{m}_{NC}^S$.[16] The second condition, $V_E^{NC} \geq 1$, is satisfied when $\bar{m} \in [d, \bar{m}_{NC}^{RC}]$, where $d < 0$.

Finally, the No Trade equilibrium gives the same value to the initial owner as not raising capital, so under the conditions of the proposition, the No Conflicts equilibrium is preferred by the initial owner.

■

A.3.4 Proposition 3:

Proof.

Define as $\bar{m}_{1,n}^{RC}(n)$ and $\bar{m}_{2,n}^{RC}(n)$ the first two biggest solutions of the equation $V_E^n = 1$ and $\bar{m}_{1,n}^S(n)$

[16] Note that as ϵ is a very small number we just consider a strict inequality.

and $\bar{m}^S_{2,n}(n)$ the two biggest solutions of the equation $V^n_E = \bar{R}\bar{X} + 1 - K$. Let:

$$\bar{m}^E_1(n) \equiv \frac{\lambda(1+\lambda) + n(1-\lambda)\left(1 - \sqrt{8(3n-1)\lambda^2 + (4-24n)\lambda}\right)}{11n^2(1-\lambda)^2 + (n(1-\lambda) - (1+\lambda))^2} \tag{55}$$

$$\bar{m}^E_2(n) \equiv \frac{\lambda(1+\lambda) + n(1-\lambda)\left(1 + \sqrt{8(3n-1)\lambda^2 + (4-24n)\lambda}\right)}{11n^2(1-\lambda)^2 + (n(1-\lambda) - (1+\lambda))^2} \tag{56}$$

$\bar{m}^E_1(n)$ and $\bar{m}^E_2(n)$ are the two monitoring costs for which the initial owner is indifferent between being in a n Blockholder equilibrium or in an Initial Owner equilibrium holding the majority of the shares, i.e. $V^n_E = V^E_E\left(\alpha_E = \frac{1}{2}\right)$. For monitoring cost values within this interval $[\bar{m}^E_1(n), \bar{m}^E_2(n)]$ the initial owner prefers to be in an n Blockholder equilibrium.

Following the steps of the proof of Proposition 2, we break up the initial owner's maximization problem into the following intervals of α_E : (1) $\alpha_E \in \left[\bar{m}, \frac{1}{2}\right]$; (2) $\alpha_E \in (\frac{1}{2}, 1]$; (3) $\alpha_E \in [0, \bar{m}]$. We first describe the beliefs on the EOS and the corresponding value functions in each of these intervals.

Case (1). By Lemma 7, $m = 1$. Then all investors anticipate monitoring in the last stage. We assume the following beliefs about the EOS at date 1: if $w_E \geq \underline{w}^n_E$ then the anticipated EOS is the n Blockholder equilibrium which exists by Lemma 10. By Lemma 14 in such a case the initial owner's value function, V^n_E, is given by equation (51). If $w_E < \underline{w}^n_E$ then the EOS is the No Trade EOS with corresponding value function V^{NT}_E.

Case (2). By Lemma 7, $m = 1$. In this interval whenever $w_E \geq \underline{w}^E_E$ there exists an Initial Owner EOS (Lemma 9). By Lemma 13, he minimizes α_E, i.e. $\alpha_E = \frac{1}{2}$ and his value function becomes:

$$V^E_E = \frac{3}{2}\frac{\bar{R}^2}{\gamma\sigma^2} + 1 - \bar{m}K - \epsilon_E \tag{57}$$

Case (3). This is the same as Proposition 2, Case 3 and generates a value of V^{NM}_E.

Now we show that the conditions under which the initial owner chooses $\alpha_E = \bar{m}$, i.e. Case (1).

We first check that $V^n_E \geq V^E_E\left(\alpha_E = \frac{1}{2}\right)$. This occurs when:

$$\frac{1}{\alpha_1}\left(1 - \frac{\bar{m}^2}{\alpha_1} + \bar{m}\right) \geq 3 \tag{58}$$

This condition is satisfied iff:

$$\frac{1 + \bar{m} - \sqrt{1 + 2\bar{m} - 11\bar{m}^2}}{6} \leq \alpha_1 \leq \frac{1 + \bar{m} + \sqrt{1 + 2\bar{m} - 11\bar{m}^2}}{6} \tag{59}$$

Substituting α_1 we obtain that $V^E_n \geq V^E_E$ iff $\bar{m} \in [\bar{m}^E_{1,n}, \bar{m}^E_{2,n}]$.

Second we check that $V^E_n \geq V^E_{NM}$:

(i) $V^E_n \geq 1$ iff:

$$n(1-\lambda)\lambda\bar{R}^2 + \bar{m}\left(\bar{m}n^2(1-\lambda)^2\bar{R}^2 - n(1-\lambda)(\lambda\bar{m} + \bar{m} + 1)\bar{R}^2 + 2K\gamma(\lambda\bar{m} + \bar{m} - \lambda)^2\sigma^2\right) < 0 \tag{60}$$

The left hand side is a third degree inequality which goes from $-\infty$ to ∞, and it is positive at $\bar{m} = \frac{\lambda}{1+\lambda}$. Note also that when $\bar{m} = \frac{1}{2}$, the left hand side can be either positive or negative. Hence of the 3 potential roots for which the left hand side is equal to 0, we are interested for the two biggest ones which are defined as $\bar{m}_{1,n}^{RC}$ and $\bar{m}_{2,n}^{RC}$ and the negative values are between these two values, that is $\bar{m}_{1,n}^{RC} < \bar{m} < \bar{m}_{2,n}^{RC}$.

(ii) $V_n^E \geq \bar{R}\bar{X} - K + 1$ iff:

$$2\bar{\alpha}K\gamma(1+\lambda)^2\sigma^2\bar{m}^3 + \left(\bar{R}^2\left(2(1+\lambda)^2 - \bar{\alpha}n(1-\lambda)(\lambda+1)\right) - n(1-\lambda) - 2\bar{\alpha}K\gamma\left(3\lambda^2+4\lambda+1\right)\sigma^2\right)\bar{m}^2$$
$$+\left(\bar{\alpha}\left(2K\gamma\lambda(3\lambda+2)\sigma^2 - n\bar{R}^2(1-\lambda)\right) - 4\bar{R}^2\lambda(1+\lambda)\right)\bar{m} + \lambda\left(2\lambda\bar{R}^2 + \bar{\alpha}\left(n\bar{R}^2(1-\lambda) - 2K\gamma\lambda\sigma^2\right)\right) < 0 \quad (61)$$

The left hand side has the same features of the left hand side of condition (60). Hence this condition is satisfied when $\bar{m}_{1,n}^S < \bar{m} < \bar{m}_{2,n}^S$ where $\bar{m}_1^S$ and $\bar{m}_2^S$ are the biggest solutions of the left hand side set equal to zero. ■

A.3.5 Corollary 1:

Proof. This is special case of Proposition 3. ■

A.3.6 Proposition 4:

Proof. The initial owner's value function has a maximum for $n = n^*$. As $\bar{m} > \frac{\lambda}{1+\lambda}$, $n > 0$. ■

A.3.7 Proposition 5:

Proof. The proof follows the same steps as the proof of Proposition 2. Let

$$\bar{m}_E^{RC} \equiv \frac{1}{2}\left(1 - \frac{\bar{R}\bar{X}}{K}\right) - \frac{\bar{R}^2}{4K\gamma\sigma^2} + \frac{\sqrt{16K\bar{R}^2\gamma\sigma^2 + \left(\bar{R}^2 + 2\gamma\sigma^2\left(\bar{R}\bar{X} - K\right)\right)^2}}{4K\gamma\sigma^2} \quad (62)$$

$$\bar{m}_E^S \equiv -\frac{\bar{R}^2}{4K\gamma\sigma^2} + \frac{\bar{R}\sqrt{\bar{R}^2 + 16K\gamma\sigma^2}}{4K\gamma\sigma^2} \quad (63)$$

We break up the maximization problem of the initial owner into the following cases: (1) $\alpha_E \in [\max[\underline{b}+\eta, \bar{m}], 1] = [\bar{m}, 1]$ (2) $\alpha_E \in [0, \bar{m}]$. We first describe the beliefs on the EOS and the corresponding value functions in each interval.

Case (1). By Lemma 7, $m = 1$. Then all investors anticipate monitoring in the last stage. We assume that in this interval the initial owner EOS is anticipated as long as w_E satisfies condition (33) (Lemma 9). Lemma 13 implies then that: $\alpha_E = \bar{m}$, $w_E = \underline{w}_E^E$ and the initial owner's value function, V_E^E, is given by equation (50). Otherwise when $w_E < \underline{w}_E^E$ we assume there is no trade and $V_E^{NT} = 1$.

Case (2). This case is the same as in Proposition 2, Case (3). The initial owner's value function is $V_E^{NM} = \max[\bar{R}\bar{X} - K + 1, 1]$.

Maximizing across intervals of Cases (1) and (2) the initial owner will choose $\alpha_E = \bar{m}$ as long as $V_E^E \geq \max(V_E^{NM}, V_E^{NT}) = V_E^{NM}$. This occurs when $\bar{m} \leq \min\left[\bar{m}_E^{RC}, \bar{m}_E^S\right]$.

When $\bar{m} > \left[\frac{1}{2}, \bar{\alpha}\right]$ this is the unique equilibrium (under the conditions of the proposition), induced by the uniqueness of the Initial Owner EOS. ■

A.3.8 Corollary 2:

Proof. It follows directly from Propositions 5 and 3. ■

A.3.9 Proposition 6:

Proof. Following the same steps as in the proof of Proposition 2, we break up the maximization problem into the following intervals of α_E : (1) $\alpha_E \in \left[\bar{m}, \min\left(\tilde{\alpha}(n), \frac{1}{2}\right)\right)$; (2) $\alpha_E \in \left[\tilde{\alpha}_E(n), \min\left[\hat{\alpha}(n), \frac{1}{2}\right]\right]$; (3) $\alpha_E \in \left(\min\left[\hat{\alpha}(n), \frac{1}{2}\right], 1\right]$; (4) $\alpha_E \in [0, \bar{m})$. As before, we first describe the beliefs on the EOS in each interval and the corresponding value functions.

Case (1). By Lemma 7, $m = 1$. All investors anticipate monitoring in the last stage. We will assume the following beliefs about the EOS in date 1: if $w_E \geq \underline{w}_E^{LS}$ then the anticipated EOS is the Liquidity Shareholder EOS which exists by Lemma 11. If $w_E < \underline{w}_E^{LS}$ the project does not go ahead and the initial owner gets value V_E^{NT}. By Lemma 15, if a Liquidity Shareholder equilibrium exists the initial owner's value function, V_E^{LS}, is given by equation (52).

Case (2). If $w_E \geq \underline{w}_E^n$ there exists an n-Blockholder EOS by Lemma 14 and Corollary 3. The proof of Lemma 14 shows that the initial owner's minimizes α_E and the initial owner's utility function is continuous between these two intervals of α_E and is given by expression (51). Hence he prefers to minimize α_E, i.e. $\alpha_E = \tilde{\alpha}(n)$. The value function is therefore given by V_E^n, expression (51) with $\underline{c} = \tilde{\alpha}(n) + \eta$. If $w_E < \underline{w}_E^n$ then the belief on the EOS is the No Trade EOS, with value function V_E^{NT}.[17]

Case (3). In this case the unique EOS is the Initial Owner EOS for $w_E \geq \underline{w}_E^E$. Using the proof of Lemma 13 we know that the initial owner minimizes α_E, i.e. $\alpha_E = \underline{d} \equiv \min[\hat{\alpha}(n), \frac{1}{2}]$ and the value function is given by V_E^E. If $w_E < \underline{w}_E^E$ the belief on the EOS is the No Trade EOS, with value function V_E^{NT}.

Case (4). See Proposition 2, Case 3. The initial owner's value function is given by V_E^{NM}.

We now show that the initial owner chooses $\alpha_E = \bar{m}$, i.e. a Liquidity Shareholder EOS. This is true whenever $V_E^{LS} \geq \max(V_E^E, V_E^{NM}, V_E^n, V_E^{NT})$. Because the initial owner's value function is decreasing in α_E, $V_E^E|_{\alpha_E = \underline{d}} < V_E^E|_{\alpha_E = \bar{\alpha}} = V_E^{LS}$. Hence the liquidity shareholder ownership structure of Case (1) is preferred over the Case (3) one. Second, as in the proof of Proposition 2, $V_E^n|_{\alpha_E = \tilde{\alpha}(n)} < V_E^n|_{\alpha_E = \bar{m}} <$

[17] In this interval there can be also an Initial Owner EOS if $n > 1$ and $w_E \geq w_E^E$. In such a case the proof that shows that the initial owner prefers the Liquidity Shareholder EOS follow the same steps as Case (3).

V_E^{LS}. Hence Case (1) is preferred over Case (2). Moreover $V_E^{NM} \geq V_E^{NT}$. Hence we only need to check that $V_E^{LS} \geq V_E^{NM}$ and this is true iff $\bar{m} \leq \min\left(\bar{m}_{NC}^{RC}, \bar{m}_{NC}^{S}\right)$. ■

A.3.10 Proposition 7

Proof. When the initial owner can raise an amount of capital $I \geq K$ and invest the remaining amount in the risk free asset his objective function becomes $\alpha_E(X\bar{R} + K + I - K) - \frac{\gamma}{2}\alpha_E^2 X^2\sigma^2 - w_E - \bar{m}$. The objective function of the investors is instead:

$$(1 - \alpha_i \frac{I - w_E}{\alpha_I}) + \alpha_i \left[X\bar{R} + K + I - K\right] - \frac{\gamma}{2}\alpha_i^2 X^2 \sigma^2 \tag{64}$$

Repeating the same steps of Propositions 2, 5, 6, 3, we obtain the optimal w_E. Inserting it in the initial owner objective function, we obtain that the initial owner's objective function is decreasing in I. ■

A.3.11 Comparitive Statics

Lemma 16 *Suppose* $\alpha_E \leq \frac{1}{2}$ *and* $n \leq \frac{1}{2\eta}$ *(a)* $\frac{\partial \bar{\alpha}}{\partial \lambda} = \frac{1-2n\eta}{(1+\lambda)^2} > 0$. *(b)* $\frac{\partial \hat{\alpha}}{\partial \lambda} \geq 0$. *(c)* $\frac{\partial \breve{\alpha}}{\partial \lambda} = \frac{1-2\bar{\alpha}n}{(1+\lambda)^2} > 0$. *(d)* $\frac{\partial V_E^n}{\partial \lambda} < 0$, *(e)* $\frac{\partial \alpha_1}{\partial \lambda} = -\frac{1-2\alpha_E}{n(1-\lambda)^2} < 0$

Proof. The proof of parts (a) and (d) follow directly from the study of the derivative

(b) $\hat{\alpha}(n) \equiv \max[\hat{\alpha}_1(n), \hat{\alpha}_2(n)]$. $\frac{\partial \hat{\alpha}_1(n)}{\partial \lambda} = -\frac{2(n-2)}{(n(1-\lambda)-2(1+\lambda))^2}$. This is positive if $n < 2$, that is when $\hat{\alpha}_1(n) < 1/2$. $\hat{\alpha}_2(n)$ is defined as the threshold such that if $\alpha_E \leq \hat{\alpha}(n)$ then $\underline{w}_E^E \leq \underline{w}_E^n$ when $X_1 = \bar{X}$. This condition can be rewritten as:

$$\bar{R}X_E \geq \bar{R}\bar{X} - \frac{\gamma}{2}\bar{X}^2\sigma^2\alpha_1 \tag{65}$$

$\frac{\partial \alpha_1}{\partial \lambda} < 0$. The above condition is less binding for higher λ and hence $\frac{\partial \hat{\alpha}_2}{\partial \lambda} > 0$.

(c) The sign of $\frac{\partial V_E^n}{\partial \lambda}$ is the same of the expression $\frac{1}{\alpha_1}\left(1 + \bar{m} - \frac{\bar{m}}{\alpha_1}\right)$. This is always negative as $m\left(1 - 2n\left(1 - \lambda\right)\right) - \lambda + m^2\left(1 + \lambda\right) < 0$. ■

Lemma 17 *(a)* $\frac{\partial \bar{\alpha}}{\partial X} < 0$; *(b)* $\frac{\partial \hat{\alpha}}{\partial X} \geq 0$; *(c)* $\frac{\partial \breve{\alpha}}{\partial X} = \frac{1-2\bar{\alpha}n}{(1+\lambda)^2} < 0$; *(d)* $\frac{\partial V_E^n}{\partial X} = 0$, $\frac{\partial V_E^{NM}}{\partial X} \geq 0$, $\frac{\partial V_E^{NC}}{\partial X} = \frac{\partial V_E^{LS}}{\partial X} < 0$.

Lemma 18 *Suppose* $\alpha_E \leq \frac{1}{2}$*: (a)* $\frac{\partial \bar{\alpha}}{\partial n} > 0$; *(b)* $\frac{\partial \hat{\alpha}}{\partial n} > 0$; *(c)* $\frac{\partial \breve{\alpha}}{\partial n} > 0$; *(d)* $\frac{\partial V_E^n}{\partial n} < 0$; *(e)* $\frac{\partial \alpha_1}{\partial n} < 0$; *(f)* $\frac{\partial w_E^n}{\partial n} > 0$.

Lemma 19 *(a)* $\frac{\partial V_E^{NM}}{\partial K} \geq 0$; *(b)* $\frac{\partial V_E^i}{\partial K} < 0$ *with* $i = \{NC, n, E, LS\}$; *(c)* $\frac{\partial w_E^j}{\partial K} > 0$ *with* $j = \{NC, n, E, LS\}$;

Proof. Proof of Lemmas 17-19 follows from the study of the sign of the derivatives. ■