Measuring Mathematical Resilience: An application of the construct of resilience to the study of mathematics

Objectives

To meet the challenge of accelerating demands for quantitative literacy in the work force, improvements are needed in mathematics education. Student skill must be increased at all ability levels while also reducing the achievement gap across gender, racial and ethnic groups to increase their participation in advanced mathematics coursework and representation in mathematics related careers (National Mathematics Advisory Panel, 2008). Research has shown that affective traits such as motivation and attitude are linked to increased likelihood of taking advanced mathematics courses (Ma, 2006) and are significant predictors of improved cognitive activity and achievement (Buff, Reusser, Rakoczy,& Pauli, 2011; Ethington & Wolfe, 1986). In addition, males generally score more favorably than females on affective variables related to mathematics achievement and persistence (McGraw, Lubienski, & Strutchens, 2006; Sherman & Fennema, 1977; Wilkins and Ma, 2003). Although psychological resilience has been researched extensively (Luthar, Cicchetti, & Becker, 2000; Luthar, 2007) the study of mathematical resilience, defined as a positive adaptive stance to mathematics which allows students to continue learning despite adversity, represents a new approach (Johnston-Wilder & Lee, 2010; Rivera & Waxman, 2011). Math anxiety looks at maladaptive response to learning mathematics and is well-studied (Hembree, 1990; Richardson & Suinn, 1977; Tobias, 1978). In contrast, resilience incorporates factors associated with optimal functioning. Although mathematical resilience has been identified as important for success (Johnston-Wilder & Lee, 2010; Rivera & Waxman, 2011), little consensus exists around its definition and no measures of resilience have been rigorously developed and/or validated. Rivera & Waxman (2011) identified the use of teacher nomination of resilient students as a limitation of their study, further motivating development of an instrument. This presentation will report on efforts to develop and validate an instrument measuring mathematical resilience. Ultimately, the measure will aid in developing and testing models that gauge the role of mathematical resilience in student achievement and persistence in advanced coursework. These models can be used to develop interventions to improve mathematical resilience, achievement, and quantitative literacy (Johnston-Wilder & Lee, 2010).

Theoretical Framework

The construct of mathematical resilience stems from the concept of psychological resilience which represents a positive response to negative stimuli. “Resilience is a multidimensional construct regulating optimal human functioning and locates itself in a positive psychology which addresses mental wellness rather than mental illness,” (Karairmak, 2010, p. 350). It involves “a dynamic process encompassing positive adaptation despite experiences of significant adversity or trauma,” (Luthar, 2006, p 747). Protective factors help to reduce the effects of negative stimuli by helping an individual to persist through the challenges they face. Luthar (2007) argued that since resilience is complex, it needs to be examined in the context of a particular domain, encouraging researchers to study many types of resilience, including educational resilience.

Based upon this research on psychological resilience as well as theory proposed by Johnston-Wilder & Lee (2010), we hypothesized that mathematical resilience is multidimensional, with four correlated factors:

1. Value: Belief that math is a valuable subject and is worth studying,

2. Struggle: Recognition that struggle with math is universal even with people who have a high level of mathematical skill,

3. Growth: Confidence that all people can develop mathematical skill and disbelief that some are born with or without the ability to learn, and

4. Resilience: An orientation towards negative situations or difficulties in the study of mathematics that results in a positive response.

The importance of each factor is discussed next.

The first factor, value, is based on expectancy-value theory. In the context of academic achievement, this theory posits that students will be more interested and more motivated to study mathematics if they believe it is valuable (Chouinard, 2007). In this context, value is determined by the student's perception of the importance of mathematics to their life and the world. The more valuable the student perceives math to be, the greater the motivation to study it, and the more likely they are to persist in the face of difficulties.

The second factor, struggle, is based upon Bandura's (1989) theory of personal agency as "the capacity to exercise control over one's own thought processes, motivation, and action" (p. 175). Human agency is also exercised through the collective experiences and culture of a group. Bandura (2000) found that the higher the perceived collective efficacy, the higher the groups' motivational investment in their undertakings, the stronger their staying power in the face of impediments and setbacks, and the greater their performance accomplishments" (p. 78). A student who believes that struggle in mathematics is common to their peer group or to all who study mathematics will have tolerance and stronger staying power in the face of setbacks.

Growth, factor three, refers to the belief that knowledge of math is not fixed and that growth is possible. According to Dweck's growth theory of learning (Dweck, 2000), students who attribute their success to internal factors have a mastery goal orientation, in which they seek challenges and develop strategies in response to setbacks. In contrast, having a fixed theory of intelligence orients students to a concern over performance and avoidance of activities that would result in difficulties (Dweck, 2000).

Finally, factor 4, resilience, is based on the literature on psychological resilience which includes exposure to significant threats followed by a positive response where, in this case, each is related to learning mathematics. According to Bandura (1989), "ordinary social realities are strewn with difficulties" and "the acquisition of knowledge and competencies usually requires sustained effort in the face of difficulties and setbacks; it is resiliency of self-belief that counts" (p. 1176). Therefore this factor incorporates two components, (1) the experience of some setback, followed by (2) a positive response.

Method

After completion of the literature review and development of the theoretical model for mathematical resilience, items were written for each of the four factors with a seven point Likert scale response structure, a score of "1" indicating "Completely Disagree" to "7" indicating "Completely Agree." Items then underwent content validation to determine their degree of fit to the constructs of interest, using McKenzie's (1999) guidelines. Eleven subject matter experts were selected using the criteria of knowledge of mathematics, mathematics education, and the actuarial profession, of which two did not reply. Respondents included professors of mathematics, mathematics education, and assessment. Two educational psychology graduate students and one actuary also participated.

The instrument was pilot tested in two stages. First, a convenience sample of 262 was collected and underwent Exploratory Factor Analysis (EFA) using SPSS (2009). The results of EFA1 served to identify the factor structure and to reduce the number of items on each scale. A second convenience sample of 603 participants was later collected and randomly divided into two sets. Half the sample contributed to a second EFA, EFA2, to determine if the different sample and item revision affected the factor structure. Finally, a Confirmatory Factor Analysis (CFA) was completed. The CFA tested the fit of the data to the theoretical structure using Amos 18.0.0 (Arbuckle, 2009; Thompson, 2010).

Both EFAs used oblique rotation to identify the structure of the instrument and to allow for correlated factors. Although eight different estimates were considered to identify the number of factors, preference was given to the PCA estimates and MAP procedures based upon recommendations of Pett et al (2003). A principal axis factoring extraction, again using oblique rotation (direct oblimin with $\delta = 0$) provided pattern and structure coefficients along with factor correlations. Following recommendations from the literature (Netemeyer, Beardon & Sharma, 2003) items were retained with loadings of .40 on one factor and loadings no greater than .25 on a second factor.

Using the factor structure identified by the EFA2, a CFA was completed on the second half of the sample, with factor loadings estimated using maximum likelihood estimation. Following recommendations by Netemeyer et al. (2003), three fit statistics were considered in evaluating goodness of fit. These were the Chi Square, the comparative fit index (CFI >.90), and the root mean square error of approximation (RMSEA<.08).

Data Sources and Materials

For EFA1, a convenience sample of 262 was collected, comprised mostly of insurance actuaries from across the United States and community college mathematics students living in the northeast. After listwise deletion of missing values, 253 respondents remained. Actuaries were hypothesized to have "mathematical resilience" because they are required to pass a series of mathematics-related exams for which the pass rate is quite low. In fact, most actuaries fail at least one exam in their career. To provide variation, students in community college were also sampled. We hypothesized that these students are likely to have struggled with mathematics and perhaps have not developed the qualities related to mathematical resilience. The original sampling plan provided for overrepresentation of community college students, but due to a low response rate, the sample was not well balanced.

For EFA2 and for the CFA, a second convenience sample of 603 was collected from university undergraduate students attending a research-intensive state university located in the northeast. These students were enrolled in large lecture hall courses, including Philosophy, Art History, and Introductory Statistics. The sample was split in two (N=293 and N=310) using a random number generator. The first sample was drawn for EFA2 and the second for the CFA, with 280 and 290 remaining after listwise deletion respectively.

Results

Because three analyses were conducted, we summarize findings for each analysis separately.

EFA1. The first EFA supported a four factor model, as estimated by MAP and PCA statistics. Table 1 presents the items and their loadings on each factor. Total amount of variance explained by this factor structure was 45%. While many items clearly loaded as hypothesized on the value, struggle, and growth factors, the results were mixed for resilience items. Resilience items did not load solely on one factor and many were multidimensional. Theoretically, we concluded that the factor of resilience included dimensions of the three other factors. Therefore, we revised the instrument to include only value, struggle, and growth. In addition, items G1, S4, and G6 loaded on more than one factor and were reworded. Two items were added struggle to address how respondents react to error.

EFA 2. The second EFA was run on the revised instrument and supported a three factor model, as indicated by MAP and PCA statistics. Total amount of variance explained by this factor structure was 42%. Table 2 presents the items and their loadings on each factor. All items loaded on factors as hypothesized, providing further support for three factors. Again, items were selected for the final factor structure using criteria set forth in Netemeyer et al (2003). V6, S9, and S1 were eliminated; G1 was also eliminated based upon the EFA1 results.

CFA. A CFA was run on the third sample using the hypothesized factor structure described above. A three factor model adequately fit the data, χ^2 (227, N=290)=512.0, p<.001, RMSEA = .066, and CFI=.91. Pattern and structure coefficients along with coefficient alpha, presented in Table 3, confirmed three intercorrelated factors. The path diagram is presented in Figure 1.

Scholarly Significance

The results of this study confirm the items tested measure three affective dimensions of studying mathematics, Value, Struggle, and Growth, which together form the construct of Mathematical Resilience. As an important response to foundational discussions on mathematical resilience (Johnston-Wilder & Lee, 2010, Rivera & Waxman, 2011), this study represents the first time it has been rigorously defined and factor analyzed. This instrument will be useful in future studies including research on whether mathematical resilience is a significant predictor of mathematical achievement, what interventions can increase mathematical resilience, and whether mathematical resilience can be enhanced in students at risk for failure in mathematics. According to Rivera et al. (2010) the study of resilience will benefit all students, “but, most important, those at risk of academic failure” (p. 186). This topic is particularly salient this year in that the study of resilience offers promise towards improving mathematical education for all levels of achievement.

Table 1.
Pattern matrix for the four factor principal-axis factor analysis of the mathematical resilience scale, EFA1.

		Factor			
Item No.	Item	Value	Struggle	Growth	Resilience
V1	Math is essential for my future.	.793	-.027	.041	-.089
V2	Math will be useful to me in my life's work.	.860	.020	-.097	-.027
V3	Math courses are very helpful no matter what I decide to study.	.625	.048	.134	-.024
V4	Knowing math contributes greatly to achieving my goals.	.905	.017	-.115	-.035
V5	Having a solid knowledge of math helps me understand more complex topics in my field of study.	.855	.075	-.026	.084
V6	People who are good at math have more opportunities than those who aren't good at math.	.398	.064	-.125	.220
V7	Thinking mathematically can help me with things that matter to me.	.774	.054	.006	.162
V8	It would be difficult to succeed in life without math.	.450	-.066	.100	.065
V9	Math develops good thinking skills that are necessary to succeed in any career.	.579	.007	.186	.119
S1	Everyone struggles with math at some point.	-.095	.685	.021	-.022
S3	Good mathematicians experience difficulties when solving problems.	.050	.654	.037	.162
S4	Successful people who work in math related fields struggle when working on hard math problems.**	.103	.626	.079	.218
S5	Everyone makes mistakes at times when doing math.	.131	.504	.049	-.093
S6	Struggle is a normal part of working on math.	-.067	.494	-.138	-.110
S7	People in my peer group struggle sometimes with math.	-.066	.481	.067	-.144
S8	People who are good at math may fail a hard math test.	.138	.479	.097	.189
S9	Math teachers are sometimes stumped by a math problem.	.151	.449	-.065	.210
S10	When someone struggles in math, it doesn't mean they have done something wrong.	*	*	*	*

S11	Making mistakes is necessary to get good at math.	*	*	*	*
G1	Everyone can get better at math if they try.**	.302	.058	.557	.050
G2	Math can be learned by anyone.	-.028	.007	.668	-.016
G3	If someone is not a math person, they won't be able to learn much math.	.195	-.059	.706	.155
G4	If someone is not good at math, there is nothing that can be done to change that.	.140	.115	.511	-.042
G5	People are either good at math or they aren't.	.103	.001	.553	.068
G6	I believe a person's math ability is determined at birth.**	-.252	.019	.623	-.092
G7	Some people cannot learn math.	-.081	.039	.762	-.072
G8	Only smart people can do math.	*	*	*	*
G9	I believe I can grown in my knowledge of math	.408	.095	.104	-.159
R1	When I have done poorly on something related to math, I know how to adapt.	.551	.076	.080	-.274
R2	I sometimes get discouraged by difficulties in mathematics, but I bounce back.	.035	.590	.131	-.123
R3	I have strategies to use when I get stuck trying to solve math problems.	.582	-.037	.075	-.148
R4	When I fail or do poorly on a math test, I know I have to work harder.	.262	.061	.117	-.377
R5	When I struggle with math, I return to it until I get it.	.603	.052	.082	-.427
R6	When I experience a setback in something related to math, I seek encouragement from others.	.035	.082	.026	-.427
R7	I sometimes find math confusing, but I stick with it.	-.151	.603	-.012	-.274
R8	When I don't do as well as I hoped on a math task or test, I keep trying until I can do it.	.496	.034	.072	-.454

* New Item in EFA2
** Reworded in EFA2

Table 2.
P*attern matrix for the three factor principal-axis factor analysis of the mathematical resilience scale, EFA2.*

Item No.	Item	Factor Value (α=920)	 Struggle (α=.790)	 Growth (α=.791)
V1	Math is essential for my future.	.763	.082	-.135
V2	Math will be useful to me in my life's work.	.851	-.011	-.036
V3	Math courses are very helpful no matter what I decide to study.	.697	.059	-.170
V4	Knowing math contributes greatly to achieving my goals.	.867	-.055	-.100
V5	Having a solid knowledge of math helps me understand more complex topics in my field of study.	.732	-.009	.031
V6	People who are good at math have more opportunities than those who aren't good at math.	.421	.043	.259
V7	Thinking mathematically can help me with things that matter to me.	.627	.025	-.118
V8	It would be difficult to succeed in life without math.	.557	.071	-.233
V9	Math develops good thinking skills that are necessary to succeed in any career.	.571	.187	-.155
S1	Everyone struggles with math at some point.	-.113	.521	.050
S3	Good mathematicians experience difficulties when solving problems.	.135	.561	-.048
S4	People who work in math related fields sometimes find math challenging.**	.184	.556	.025
S5	Everyone makes mistakes at times when doing math.	-.001	.471	-.026
S6	Struggle is a normal part of working on math.	.089	.575	.068
S7	People in my peer group struggle sometimes with math.	.070	.612	.111

S8	People who are good at math may fail a hard math test.	-.097	.592	-.080
S9	Math teachers are sometimes stumped by a math problem.	.018	.395	-.016
S10	When someone struggles in math, it doesn't mean they have done something wrong.	.008	.309	-.036
S11	Making mistakes is necessary to get good at math.	.103	.533	-.186
G1	Everyone can get better at math.**	.195	.228	-.523
G2	Math can be learned by anyone.	-.171	-.207	.469
G3	If someone is not a math person, they won't be able to learn much math.	-.020	-.011	.618
G4	If someone is not good at math, there is nothing that can be done to change that.	-.121	-.089	.681
G5	People are either good at math or they aren't.	-.134	.232	.563
G6	Everyone's math ability is determined at birth.**	.053	-.099	.561
G7	Some people cannot learn math.	-.126	.176	.538
G8	Only smart people can do math.	.097	-.163	.604

* New Item in EFA2
** Reworded in EFA2

Table 3. *Pattern and structure matrix for the three factor confirmatory factor analysis of the mathematical resilience scale.*

		Pattern Matrix			Structure Matrix		
Item No.	Item	Value (α=.942)	Struggle (α=.706)	Growth (α=.829)	Value (α=.942)	Struggle (α=.706)	Growth (α=.829)
V1	Math is essential for my future.	0.914	0	0	0.914	0.388	0.398
V2	Math will be useful to me in my life's work.	0.932	0	0	0.932	0.395	0.405
V3	Math courses are very helpful no matter what I decide to study.	0.807	0	0	0.807	0.342	0.351
V4	Knowing math contributes greatly to achieving my goals.	0.914	0	0	0.914	0.388	0.398
V6	Having a solid knowledge of math helps me understand more complex topics in my field of study.	0.811	0	0	0.811	0.344	0.353
V7	Thinking mathematically can help me with things that matter to me.	0.765	0	0	0.765	0.324	0.333
V8	It would be difficult to succeed in life without math.	0.685	0	0	0.685	0.290	0.298
V9	Math develops good thinking skills that are necessary to succeed in any career.	0.697	0	0	0.697	0.296	0.303
S1	Everyone struggles with math at some point.	0	0.481	0	0.204	0.481	0.137
S3	Good mathematicians experience difficulties when solving problems.	0	0.554	0	0.235	0.554	0.157
S4	People who work in math related fields sometimes find math challenging.**	0	0.483	0	0.205	0.483	0.137
S5	Everyone makes mistakes at times when doing math.	0	0.422	0	0.179	0.422	0.120
S6	Struggle is a normal part of working on math.	0	0.638	0	0.271	0.638	0.181
S7	People in my peer group struggle sometimes with math.	0	0.392	0	0.166	0.392	0.111
S8	People who are good at math may fail a hard math test.	0	0.287	0	0.122	0.287	0.082
S11	Making mistakes is necessary to get good at math.	0	0.606	0	0.257	0.606	0.172
G2	Math can be learned by anyone.	0	0	0.613	0.267	0.174	0.613
G3	If someone is not a math person, they won't be able to	0	0	0.746	0.325	0.212	0.746

	learn much math.						
G4	If someone is not good at math, there is nothing that can be done to change that.	0	0	0.720	0.313	0.204	0.720
G5	People are either good at math or they aren’t.	0	0	0.608	0.264	0.173	0.608
G6	Everyone's math ability is determined at birth.**	0	0	0.552	0.240	0.157	0.552
G7	Some people cannot learn math.	0	0	0.670	0.291	0.190	0.670
G8	Only smart people can do math.	0	0	0.589	0.256	0.167	0.589

* New Item in EFA2
** Reworded in EFA2

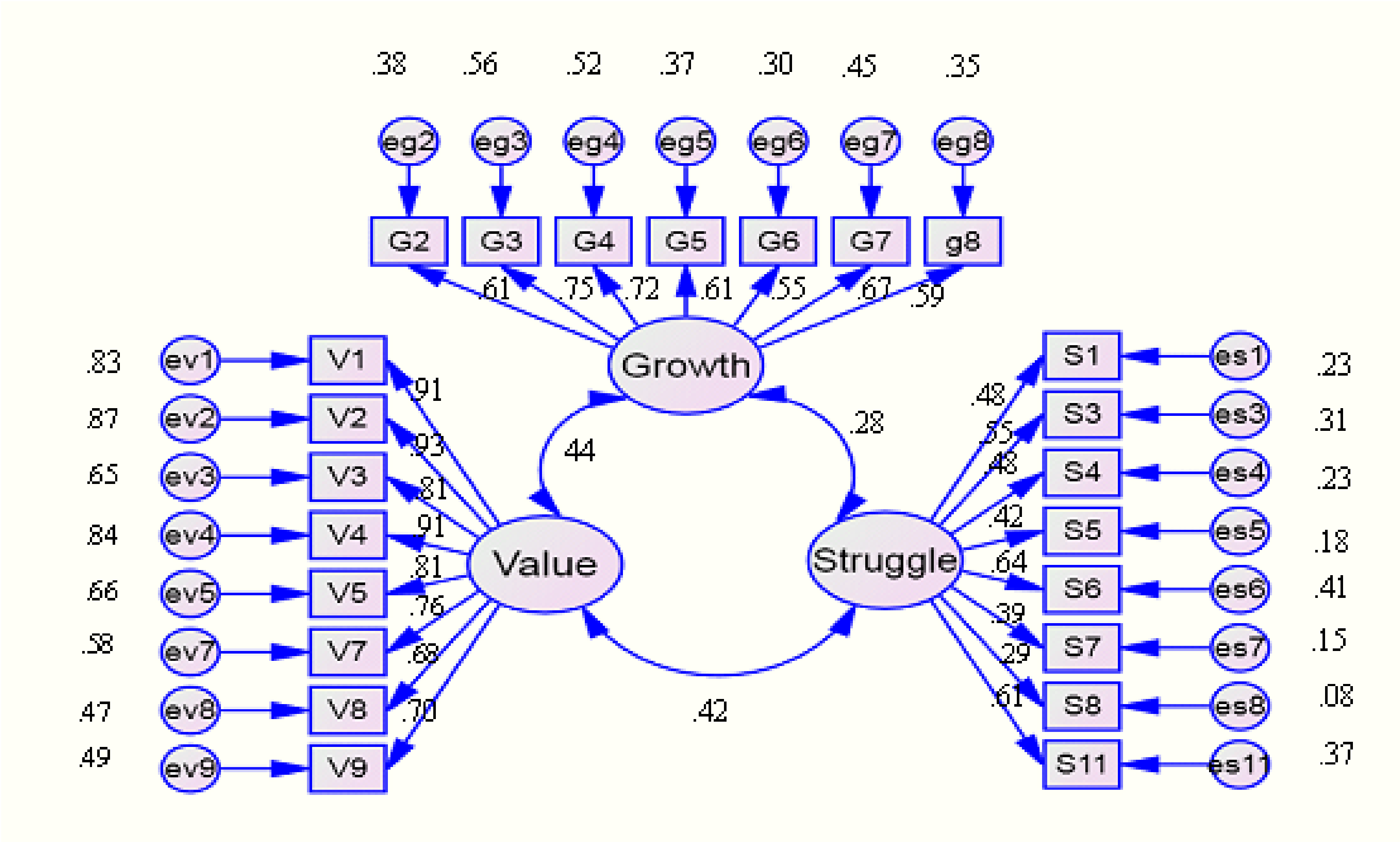

Figure 1. *The path diagram of the factor structure of the mathematical resilience scale.*

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