BRAZILIAN FOREIGN TRADE:
FIXED AND TIME VARYING PARAMETER MODELS

by

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SUMMARY

In this thesis we estimate and analyse several econometric models for the Brazilian trade equations. A major attention is given to the questions of stationarity and parameter instability. We test for the presence of unit roots by using the Dickey and Fuller, and Phillips and Perron tests and the Johansen procedure, and apply an error correction mechanism to the data. To investigate the question of parameter instability we use the Kalman filter in both classical and bayesian approaches and the switching regressions technique. These tests and estimations are performed using both annual and quarterly disaggregated data. We show that, in some cases, the trade equation coefficients are indeed time varying. The changes in the trade elasticities are then related to changes in the trade policy regime and to the industrial structure of the economy.
CHAPTER I

INTRODUCTION

For the last ten years foreign trade has been a major concern of Brazilian economic policy. The debt crisis in 1982 brought about the need to obtain a large trade surplus to offset the current account deficit. As foreign loans suddenly became unavailable, the only way to keep paying interest on the foreign debt and ensure the flow of imports was through a massive increase in exports.

These severe balance of payment problems motivated several attempts to estimate trade equations for policy analysis. However all this empirical work shares the assumptions of stationarity and constant parameters. The main concern of this thesis is that these assumptions may not be appropriate in this context and, therefore, we propose to investigate this matter by using new techniques that deal with non-stationary series and time varying coefficients.

As far as the question of stability of the Brazilian trade equations coefficients is concerned the existing literature is quite inadequate. In most cases this question is mentioned but the treatment does not go further than applying structural stability tests. Regardless of the result of the structural stability test this question is not examined further.
As Goldstein and Khan (1985) have noted when discussing trade equations: "Changes in the basic relationship can either be gradual or sudden, and in either case the resulting parameters will be biased and inconsistent if allowance is not made for the shifts. There are in fact good reasons for expecting that trade relationships are subject to both types of changes. Gradual changes in the elasticities can come about as the pattern of trade changes during the process of economic development or as the result of changes in government trade policies. Sudden shocks such as changes in the exchange rate or exchange rate regime, or large oil price increases can also fundamentally alter the basic demand and supply relationships".¹ In a more general framework Lucas (1976) has also claimed that since agents have rational expectations the parameters in the econometric relationships may not remain constant after changes in the policy regime. Additionally, parameter instability may also be expected as a result of Orcutt's "quantum effect" or from aggregation problems.²

Given the large changes in the structure of Brazilian trade and sharp modifications in trade policy during the last three decades, the Brazilian trade equations might be particularly subject to parameter instability.

Brazilian exports previously relied on coffee but are now based on manufactured and semi-manufactured

²) See Orcutt (1950).
goods. The import substitution process has completely reshaped the industrial structure. Two major import substitution plans, the Plano de Metas and the II PND, were particularly important in encouraging import substitution in new industries, especially in the intermediate and capital goods sectors. Moreover, this import substitution process also contributed to alter the composition of imports, substantially reducing the share of consumer goods.

Exchange rate policy has changed from a single to a multiple rate system and back again. Direct import controls and additional costs have been imposed on imports from time to time. Substantial changes in tariff policy have also taken place and a quite generous system of export incentives has been introduced.

The main objective of this thesis is the estimation and analysis of time varying parameter models for Brazilian foreign trade. These estimations are performed mainly by using the Kalman filter in both classical and Bayesian frameworks. We then try to associate the changes in the coefficients with changes in the policy regime or the import substitution process. The thesis also provides a case study of the relative advantages and disadvantages of some of the different statistical methods that have been proposed.

The assumption of weak stationarity, that is, that the first and second moments of a time series are time-invariant, played an important part in the development of classical econometric methods.
Subsequently time series analysts drew attention to kinds of trends, in mean and variance, and Box and Jenkins (1970) popularized the approach of differencing a series to reduce it to stationarity. Such a series is referred as difference-stationary or integrated series, equivalently we can say that its autoregressive operator contains one or more unit roots. Box and Jenkins presented a number of judgemental methods for determining the required degree of differencing, that is, the order of integration or number of unit roots. It was only later that formal hypothesis tests were developed.\(^3\) Although one can make a series stationary by taking first differences, this approach implies loosing all the long run properties of the model.

Stationarity was always taken for granted in previous work on Brazilian foreign trade. No attempt was made to test for the presence of unit roots in the data. Given the results obtained by Nelson and Plosser (1982), revealing the presence of unit roots in many economic time series, it seems inadequate to treat this question by assumption. Actually, evidence of the inadequacy of assuming stationarity when dealing with economic time series had been given a long ago by Granger (1966). The "typical spectral shape", with high values at low frequencies, observed in economic time series is an indication of how important trends are in this kind of series.

\(^3\) See Fuller (1976).
Therefore, the approach adopted in this thesis consists in first testing for the presence of unit roots using a number of test statistics available, going from the simple Dickey and Fuller test to the recent and sophisticated Johansen procedure. Once the presence of unit roots is observed we test for cointegration and use an error correction approach to model the data.

The advantage of using an error correction mechanism is that economic theory is used to establish only the long run relationship between the variables, while the short run dynamics is data determined. A general to specific approach is adopted to deal with the question of dynamics. We start with a fairly general dynamic structure and test successive restrictions that are data acceptable and economically meaningful to obtain a simpler dynamics.

The question of how to deal with dynamics, especially when using quarterly data, is also a common problem far as the empirical work on Brazilian trade equations is concerned. None of the existing work employs anything more complex than the simple partial adjustment model. The use of an error correction mechanism and the general to specific approach mentioned above is also useful in overcoming this problem.

Trade equations will be estimated using quarterly and annual data. The estimation will be performed using aggregated and disaggregated data. For imports we will use total, intermediate and capital goods, while for exports we use data on total and industrial exports.
The thesis proceeds as follows. In chapter two we present the production theory and the classical supply and demand approaches for modelling trade equations. In the case of the production theory approach the question of parameter instability can be addressed by choosing the appropriate profit function. We use a translog specification as an example of a profit function that generates non-constant elasticities. Some discussion of usual econometric questions, such as the choice of variables and functional form, treatment of dynamics and stationarity and aggregation, is also included in this chapter. Finally, we present a critical survey of the relevant empirical literature on Brazilian trade equations.

The next chapter provides a general presentation of Brazilian trade policy for the period 1947 to 1988. During this period Brazil has experienced several abrupt changes in the trade policy regime. There was the creation of the tariffs system, the change from multiple to single exchange rate, the creation of an export incentive program, two major import substitution plans and several measures to control imports. In other words, this chapter contains the policy background necessary to analyse the changes in the trade equation coefficients.

In chapter four we deal with the econometric methods that will be used in the estimation work. Most of this chapter is concerned with presenting and comparing several different time varying parameters models. This discussion includes the purely random model, the
adaptative regression model, the systematic parameter variation model and the switching regressions model. Especial attention is given to the Kalman filter and its use in both classical and Bayesian framework. Some other econometric issues regarding unit roots and cointegration are also discussed, namely the Johansen and Engle and Granger procedures, seasonal cointegration and encompassing.

The fifth and sixth chapters present and discuss the empirical results for imports and exports, respectively. In these two chapters the trade models presented in chapter two are estimated using the econometric methods discussed in chapter four and the results are analysed against the background of policy change discussed in chapter three. The classical Kalman filter approach, the switching regressions model and the Bayesian Dynamic Linear model are used to estimate time varying trade elasticities. We will associate the changes in the coefficients with the changes in the trade policy regime.

Finally, in chapter seven we present general conclusions of the thesis, including some policy implications of the results obtained in chapters five and six and an evaluation of the econometric methods used.
CHAPTER II

MODELLING TRADE EQUATIONS

I) Introduction

In this chapter we will address the question of how to model trade equations. We will discuss not only the theoretical aspects but also some of the applied work using Brazilian data.

We will deal as well with some of the practical econometric problems involved such as which variables to use, how to model the dynamics, parameter instability and stationarity.

All empirical work on Brazilian trade equations to date have assumed that the parameters are constant. This may be an unrealistic assumption, if price and income elasticities in trade equations vary not only cyclically, according to movements in the business cycle, but also as a result of the implementation of different trade policies or changes in the pattern of trade due to the process of economic development. Such parameter instability might be particularly expected in the Brazilian case, given that in the last thirty years there have been several abrupt changes in trade policy and two major import substitution programs.

Following this introduction, the chapter has three sections. In the next section, we present the main theoretical aspects of trade equation modelling,
together with some empirical issues. The third section contains a discussion of the empirical work available on trade equations in Brazil and the last section presents the conclusions and remarks.

II) The Specification of Trade Equations

There are two different approaches to the modelling of trade equations. The traditional approach, surveyed by Goldstein and Khan (1985) and Magee (1975), adopts a household model treating traded goods as final goods that enter the consumer sector directly. An alternative approach, surveyed by Woodland (1982) and Kohli (1991), models trade equations from a production theory framework.

1) The Production Theory Approach

In this model the small country hypothesis is adopted, implying that only import demand and export supply have to be estimated. All imported goods are supposed to be intermediate goods, used by the production sector as inputs. On the other hand, exported and domestic goods are assumed to be separable outputs of the production sector. That is, in this model there is no consumption of traded goods, since all imports are inputs while exported goods are different from domestic production. Duality theory is used to derive econometrically convenient equations.

The model uses a short run or restricted profit function, or GNP function, to represent the technology.
Import and export decisions are made by profit maximizing firms under perfect competition conditions. The firms choose the optimum imports and output mix given a vector of domestic and international prices and a vector of domestic primary factor stocks. The technology is represented by a production possibility set from which the profit function can be derived. Therefore, given the production possibility set $T$ we can define the restricted profit function, which has the usual properties, as

$$\pi(p, v) = \max_x \{ px: (x, v) \in T \}$$

where $x$ is a vector of net output (imports enter with a negative sign), $v$ is a vector of fixed inputs and $p$ is a positive vector of domestic and international prices.

Using Hotelling's Lemma we obtain

$$x = \nabla_p \pi(p, v) \quad (1)$$

where $\nabla$ is the vector differential operator. Assuming that capital stock is fixed in the short run, the system of equations (1) represents the short run domestic output supply and the short run export supply and import demand functions. Similarly, we can obtain the system of inverse demand for domestic primary inputs (if the derivatives exist)

$$w = \nabla_v \pi(p, v) \quad (2)$$

where $w$ is the vector of factor prices.

To make the model operational we have just to specify a functional form for the profit function. Following Kohli (1978), Burgess (1974a and 1974b) and Lawrence (1987) let us use a translog function defined as
\[ \ln(z) = a_{00} + \sum_i a_{i0} \ln z_i + 1/2 \sum_i \sum_j a_{ij} \ln z_i \ln z_j \]  

(3)

Using (3) we can write the translog profit function as:

\[ \ln \pi = a + \sum_i b_i \ln p_i + 1/2 \sum_i \sum_h c_{ih} \ln p_i \ln p_h \]

\[ + \sum_j d_j \ln v_j + 1/2 \sum_j \sum_k e_{jk} \ln v_j \ln v_k \]

\[ + \sum_i \sum_j f_{ij} \ln p_i \ln v_j \]

(4)

where \( c_{ij} = c_{ji} \) and \( e_{jk} = e_{kj} \). To ensure that \( \pi \) is homogeneous of degree one in prices requires that

\[ \sum_i b_i = 1 \]  
\[ \sum_j f_{ij} = \sum_h c_{ih} = 0, \]

(5)

while to ensure that \( \pi \) is homogeneous of degree one in domestic primary inputs requires that

\[ \sum_j d_j = 1 \]  
\[ \sum_j f_{ij} = \sum_k e_{jk} = 0. \]

(6)

By logarithmic differentiation we obtain

\[ \partial \ln \pi / \partial \ln p_i = p_i (\partial \pi / \partial p_i ) / \pi = p_i x_i / \pi = S_i \]  

(7)

\[ \partial \ln \pi / \partial \ln v_j = v_j (\partial \pi / \partial v_j ) / \pi = v_j w_i / \pi = s_i \]

Applying (7) to (4) we have

\[ S_i = b_i + \sum_h c_{ih} \ln p_h + \sum_j f_{ij} \ln v_j \]

\[ s_i = d_j + \sum_i f_{ij} \ln p_i + \sum_k e_{jk} \ln v_k. \]

(8)

These equations can then be estimated econometrically to obtain the unknown parameters. Note that, since $S_i$ and $s_i$ add up to one, only $n - 1$ equations have to be estimated in each system.

One interesting feature of this method is that it allows the estimation of all cross elasticities of the domestic supply, labour demand, import demand and export supply. Moreover, since the functional form chosen for $\pi$ is not a constant elasticity one, these elasticities can be estimated for each point in time.

$$\epsilon_{ij} = \left(\frac{p_j}{x_i}\right) \frac{\partial(x_i)}{\partial p_j}$$

2) The Perfect and Imperfect Substitutes Models

In terms of the traditional approach, as shown by Goldstein and Khan (1985) and Magee (1975), there are two different ways to model trade equations, depending on whether domestic and foreign products are assumed to be perfect or imperfect substitutes. In the imperfect substitutes model, the products are supposed to be slightly different, in such a way that prices are also different.

Thus, the demand and supply curves for imported goods can be written as a function of national income, and prices of domestic and imported goods. What is relevant for the importers is not only the prices, but actually the final cost paid by the product measured in a common currency. The demand and supply of imports can then be written as
\[ M^d = f(Y_n, E, P, T), \quad (9) \]
\[ f_1 > 0, f_2 < 0, f_3 > 0, f_4 < 0 \]

\[ M^* = g(P, P_d, E^*, Y_n), \quad (10) \]
\[ g_1 > 0, g_2 < 0, g_3 > 0, g_4 > 0 \]

\[ M^d = M^*, \quad (11) \]

where \( Y_n \) is nominal income, \( P \) and \( P_d \) are the import and domestic price levels, \( E \) is the nominal exchange rate, \( S \) is the rate of export subsidies, and \( T \) stands basically for tariff, but should also include transport costs, insurance and all other factors that represent an additional cost for the importer. The asterisk is used to differentiate between the foreign and home economies. The aggregate demand function depends positively on nominal income and domestic prices, and negatively on import prices measured in local currency and the tariff rate, while in the supply function import prices, the foreign subsidy to exports, foreign income have the positive sign, and foreign domestic prices a negative sign.

In the same way, demand and supply for exports can be written as a function of effective prices, including exchange rate and export subsidies, and external income.

\[ X^d = l(P_e, P_d, Y_n, T^*), \quad (12) \]
\[ l_1 < 0, l_2 < 0, l_3 > 0, l_4 < 0 \]

\[ X^* = h(E, P_d, S, Y_n), \quad (13) \]
\[ h_1 > 0, h_2 < 0, l_3 > 0, h_4 > 0 \]

\[ X^d = X^*, \quad (14) \]

2) Note that the supply of imports for a country is also the supply of exports from the rest of the world to the country.
where $P_x$ is the export price index. The supply of exports depends positively on export prices in terms of domestic currency, export subsidies and domestic income, and negatively on domestic prices, while the demand for exports is positively related to foreign domestic prices and foreign income, and negatively with export prices and tariff rates in the rest of the world.

As is common in the literature, some additional hypotheses can be used to simplify the model presented above. First, the estimation of supply and demand curves is sometimes said not to be necessary. If one assumes the supply curves to be completely elastic, only the demand functions have to be estimated. Although it may be possible for a country to buy its imports in the world market without changing the price, the same does not seem plausible for exports. Unless there exists idle capacity, or, more generally, unless the export sector operates subject to constant or increasing returns to scale, it seems unlikely that, at least in the short run, production could be expanded without an increase in prices. In other words, although it is plausible to assume that a country is a price taker in the world market, it is less plausible to assume that the rest of the world is also a price taker with respect to the home economy. One explanation sometimes offered to justify the estimation of demand equations only is that price decisions are taken before the decision on quantities,
which "implies that within a range of variation in the data any quantity can be supplied at the given prices". 3

If that is not the case, the import and export prices cannot be treated as exogenous variables, and a simultaneous model must be estimated. The estimation of the demand equation model alone, when the supply curve is not totally elastic, would lead to an inconsistent and downward biased estimate of the price elasticity of demand. The estimated price elasticity of demand would be a weighted average of the actual demand and supply elasticities.

Another way of reducing the number of equations to be estimated is to adopt the "small country" hypothesis. If the country's share in world imports and exports is small, the supply of imports and the demand for exports will be completely price elastic or, at least have a substantially high price elasticity. Thus, the import and export volumes will depend only on the demand for imports and supply of exports functions, since the country will be able to buy its imports and sell its exports with no feedback on prices. If that is not the case, once again a simultaneous model must be estimated, otherwise the coefficients will be inconsistent and biased.

In the literature on trade equations it has been common to adopt the "small country" hypothesis for imports, estimating only a demand equation, while in the case of exports the use of a simultaneous approach has often been preferred.

A second supposition, common not only in work on trade equations but also in consumer theory as a whole, is the absence of money illusion. This assumption implies that the functions are homogeneous of degree zero in prices and nominal income. One can then normalise the variables by the domestic price level, using real income and relative prices as explanatory variables.

Another possible simplification is to combine prices and tariffs or subsidies into a single variable. Sometimes it is argued that tariff and import prices have different effect on imports. This could result from importers regarding changes in tariff as permanent, while changes in import prices are regarded as temporary, or from the fact that changes in tariff are usually announced in advance.\(^4\) The same line of argument applies to export subsidies. In the case of tariffs, empirical work has shown that import prices and tariffs have equal effect on import demand.\(^5\)

The previous model can then be rewritten as

\[
M^d = f( Y, EP_m (1+T)/ P_d ), \quad (15) \\
f_1 > 0 \quad f_2 < 0 \\
M^s = g(Y^*, P_m(1+T)/ P_d ), \quad (16) \\
g_1 > 0 \quad g_2 > 0 \\
X^d = l( Y^*, Px(1+T^*)/ P_d ), \quad (17) \\
l_1 > 0 \quad l_2 < 0 \\
X^s = h( Y, EP_x(1+S)/ P_d ). \quad (18) \\
h_1 > 0 \quad h_2 < 0
\]

\(^4\) See, for example, Balassa (1967).
Until now we have dealt with equilibrium models. The disequilibrium models applied to trade equations that are available in the literature are generally of a very simple fashion. It is normally assumed that the optimum and actual levels of imports or exports are different. A partial adjustment model is then proposed to model the disequilibrium. 

More recently, other kinds of disequilibrium models have been proposed. These models follow the work of Fair and Jaffee (1972), Fair and Kelejian (1974) and Maddala (1986). The idea is to divide the sample into points on the demand and supply curves. Although the maximum likelihood approach to the sample separation problem is more efficient, simple methods using the price variation are often used. Rios (1986) and Fachada (1990), for example, have used the methods proposed by Fair and Jaffee (1972) to estimate a disequilibrium model for the Brazilian exports.

On the other hand, if domestic and foreign goods are assumed to be perfect substitutes, having a common price given by world supply and demand, price differentials will play no role in determining the volume of trade. In this case, imports and exports could be modelled as the difference between domestic demand and supply.

Let $D$ and $S$ be the total domestic demand and supply of the traded good, $P_d$ is the domestic price, $Y$ is real

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6) See, for example, Goldstein and Khan (1978) and Khan (1974).
income and $F$ is some cost variable. Then the perfect substitutes model can be written as

\[ D = f(P_d, Y) \]  \hspace{1cm} (19)

\[ f_1 < 0 \quad f_2 > 0 \]

\[ S = g(P_d, F) \]  \hspace{1cm} (20)

\[ g_1 > 0 \quad g_2 < 0 \]

\[ M = D(P_d, Y) - S(P_d, F) \]  \hspace{1cm} (21)

\[ X = S(P_d, F) - D(P_d, Y). \]  \hspace{1cm} (22)

One should note that in this case there are no explicit demand for imports or supply for exports. The model describes only excess demand, and excess supply of domestic goods, assumed to be satisfied by imports and exports. The country's ability to influence world prices will depend basically on its share of the world market and on its own price elasticity of supply and demand.

The choice of the perfect or imperfect substitute model will depend essentially on the kind of product or the level of aggregation we are considering. For manufactured goods or aggregates such as consumer and capital goods, the imperfect substitute model seems adequate, while for homogeneous goods such as crude oil, wheat or coffee the perfect substitute model is more appropriate.

7) In this case, as Enoch (1978) shows, a possible measure of competitiveness is relative costs instead of relative prices.

8) Such models have been applied to the Brazilian imports of crude oil and wheat by Ramos (1985) and Modiano (1983).
3) Other Variables of Interest

3.1) Import Equations

It is common practice in the empirical literature on import demand to include some other variables to account for non-price restrictions, differences between the cyclical and secular income elasticities, and for long run trends in the volume of imports. 9

In the case of non-price restrictions, the basic argument is that there are extra costs that are not normally reflected in movements of prices. Depending on which phase of the business cycle the economy is in, delivery lags can change considerably, leading to an increase or decrease in imports. To deal with this problem, it is common to introduce a capacity variable such as \( Y/Y^p \), where \( Y \) and \( Y^p \) are respectively actual and potential output. Thus, when the economy is overheated, imports tend to increase, whereas in a phase of widespread spare capacity the volume of imports should decrease.

Another point, raised by Khan and Ross (1975), is that one should distinguish between cyclical and secular effects on the demand for imports. They claim basically that one should use not only actual income as an explanatory variable, but also its potential value. Their model is a kind of partial adjustment model, where

9) See Barker (1979), Khan and Ross (1975) and Barker (1987)
current imports depend on current income and the "potential demand for imports" or, in other words, the long run demand for imports, depend on the potential income. Then, by using a partial adjustment hypothesis, where deviations between the actual and potential demand for imports are a fraction of the deviation between actual and potential income, they end up with an equation in terms of price, actual real income and its potential value. In analytical terms

\[ \ln M_d = a_0 + a_1 \ln P_t + a_2 \ln y_t + u_t \]  
(23)  

\[ \ln M_{pd} = b_0 + b_1 \ln P_t + b_2 \ln y_P + v_t \]  
(24)  

and

\[ \ln M_d - \ln M_{pd} = d (\ln y_t - \ln y_P ) + w_t. \]  
(25)  

Substituting (24) into (25) we get the estimating equation

\[ \ln M_d = b_0 + b_1 \ln P_t + d \ln y_t + (b_2-d) \ln y_P + e_t \]  
(26)  

where \( e_t = v_t + w_t \). Note that the sign of the coefficient of potential income can be either positive or negative, depending on the relative sizes of \( b_2 \) and \( d \). Intuitively, one can think of this sign being positive or negative, depending on the gradual changes in the demand for imports over time. If structural changes lead to a more open economy, with an increased proportion of imports in relation to income, this coefficient should be positive, and vice versa.

Finally, a third variable normally found in import equations is the time trend. The inclusion of this variable may be justified on theoretical grounds as
accounting for long run changes in the economy and consequentially in the structure of imports. Again, the sign of its coefficient could be either positive or negative, depending on how successful is the import substitution process. A time trend has also been used as an explanatory variable in the models that adopt the production theory approach. Kohli (1978), for example, uses a time trend to account for structural changes in both import demand and export supply functions.

One should note that although these variables may improve the fit of the equation, in theoretical terms there is not much that can be said, since in all cases the expected sign of the coefficients is not clear. Even in the case of capacity utilization, it has been argued that a negative sign can be seen as appropriate. If the domestic business cycle follows the same pattern as the world economy, then it seems possible that an increase in capacity utilization will lead to a reduction in imports, since domestic producers can supply faster than foreign ones.

Moreover, as shown by Barker (1979), since potential output is usually calculated as a function of time, these three variables are interconnected. Despite the different economic interpretation that is given to justify the use of such variables, it seems that they are all capturing the same phenomenon. It is our view that variables such

10) See, for example, Weisskoff (1979).
as the potential product and the time trend may be accounting for structural changes in the demand for imports that cannot be accommodated by the assumption of fixed coefficients.

3.2) Export Equations

Exactly as in import equations, it is also common to include other variables such as capacity utilization, potential product or a time trend in export equations.

Goldstein and Khan (1978) developed a model where the potential product is one of the explanatory variables in the supply of exports. The idea is that exports should respond positively to changes in the country's capacity to produce. Their model, with some alterations, can be defined as

\[
\ln X_d = a_0 + a_1 \ln \left[ \frac{P_x (1+T^*)}{P_d} \right] + a_2 \ln Y^* \quad (27)
\]

\[
\ln X^* = b_0 + b_1 \ln \left[ \frac{EP_x (1+S)}{P_d} \right] + b_2 \ln Y^*. \quad (28)
\]

\[a_2, b_1, b_2 > 0, \quad a_1 < 0\]

This model can also be used to justify the presence of potential product, as well as other variables such as capacity utilization, in the reduced form equation. Solving (27) for \(P_x\) we obtain

\[
\ln P_x = c_0 + c_1 \ln X_d + c_2 \ln (1+T^*) + c_3 \ln P_d + c_4 \ln Y^* \quad (29)
\]

where,

\[c_0 = - \frac{a_0}{a_1}, \quad c_1 = 1/a_1, \quad c_2 = -1, \quad c_3 = 1, \quad c_4 = - \frac{a_2}{a_1}.\]

\[c_3, c_4 > 0, \quad c_1, c_2 < 0\]

Substituting (29) into (28) and assuming market clearance, we have
\[ \ln X = \frac{b_0 + b_1 c_0}{D} + \frac{b_2}{D} \ln Y^* + \frac{b_1 c_4}{D} \ln Y^* \]
\[ + \frac{b_1}{D} \ln \left[ EP_d \frac{(1+S)}{P_d(1+T^*)} \right]. \] (30)

where, \( D = 1 - b_1 c_1 \)

Since \( D \) is positive, the coefficients are all expected to be positive. Similarly, one can get the reduced form for prices by solving (28) and (29) for \( P_x \).

\[ \ln P_x = \frac{c_0 + c_1 b_0}{D} + \frac{1}{D} \ln \left[ P_d \frac{(1+S)}{(1+T^*)} \right] + \frac{c_4}{D} \ln Y^* \]
\[ + c_1 b_2 / D \ln Y^* + c_1 b_1 / D \ln \left[ E(1+S)/P_d \right]. \] (31)

As noted above, one could use this same framework to introduce capacity utilization. It is normally argued that the supply of exports depends negatively on the level of capacity utilization. The central argument is that producers always serve the home market first, because foreign markets are presumably less profitable due to more costly marketing, transport costs or greater risk. Therefore \( b_2 \) would be negative. Winters (1981) and Hotson and Gardiner (1983) adopt this approach.

A different approach to the inclusion of capacity utilization was adopted by Batchelor (1977) and Brakman and Joosten (1986). According to them, because producers prefer to serve the home market first, individual exporting firms are subject to two different regimes. When the level of capacity utilization is low, exports will be constrained by external demand, but when it is high, exports will be constrained by domestic supply. There is then a threshold point such that as overall capacity utilization rises "factors influencing demand - world trade and competition prices - became suddenly
important and those influencing supply - investment, profitability - become suddenly all important."
In analytical terms the model can be described as

\[ x = \min (x_1, x_2) \]
\[ x_1 = a_1 (Y^f, P/P^f) + b_1 q \]
\[ x_2 = a_2 (K, P/P^h) + b_2 q \]
\[ p = c_1 (P^f) + d_1 q \quad \text{if} \quad x = x_1 \]
\[ = c_2 (P^h) + d_2 q \quad \text{if} \quad x = x_2 \]

where, \( x_1 \) is the demand for exports, \( x_2 \) is the export supply, \( Y^f \) is real foreign income, \( K \) is the net stock of capital of the firm, \( P, P^f \) and \( P^h \) are the export price of the firm, price of competing goods and home market price respectively, and \( q \) is the capacity utilization rate.

Taking an individual firm, it has to be operating in the demand or supply constraint case, but for the economy as a whole there can exist both demand and supply constrained firms. Therefore for a firm there will be a threshold point (\( q' \)) where it jumps between regimes, but for the economy such jumps are clearly implausible. Observations cannot be fully characterized as belonging to one or the other regime, but rather as helpful in describing both regimes. Obviously, one observation will tell more about the supply constraint case once \( \mu_q \), the average of \( q \) for all firms, is less than \( q' \) and vice versa.

Given a distribution function for the capacity utilization \( g(q) \), which has an expected value of \( \mu_q \), the

---

volume of exports for the economy is (a similar equation could be written for export prices as well)

\[ X = \int_{-\infty}^{+\infty} x \cdot g(q) \, dq \]

\[ = \int_{q'} x_1 \cdot g(q) \, dq + \int_{q'}^{+\infty} x_2 \cdot g(q) \, dq. \]

To make the model operational we just have to add a disturbance term to the equations and specify the form of the distribution function \( g(q) \). One will then end up with a single equation that can be estimated by ordinary least squares, being the parameters in both regimes retrievable from this "reduced form".  

A time trend has also been used in export models, although its theoretical justification is sometimes not very convincing. In studies on the UK economy, the basic justification for using time is the long term reduction in the UK's share of world exports. The idea is that British exports have been subjected to a long term non price loss of competitiveness, so that the coefficient on the time trend will have negative sign.

4) Some Econometric Considerations

When estimating trade equations, as in other economic relations, one faces several econometric problems, such as choice of functional form,

13) Another approach to the estimation of this problem based in switching regressions is suggested by Goldfeld and Quandt (1973a and 1973b).
14) See, for example, Dinenis et alii (1989) or Brooks (1981).
simultaneity, dynamic specification, stability of the parameters and aggregation problems.

4.1) **Functional Form**

Almost all empirical work on trade equations is based on a log linear specification. The evidence in favour of the log linear specification in import demand was first provided by Khan and Ross (1977) and later confirmed by Boylan et alii (1980), both using the Box and Cox methodology. More recently Hitiris and Petoussis (1984), using a methodology developed by Godfrey and Wickens (1981) that allows for serial correlation, again come out in favour of the log linear specification. In terms of export equations, this problem has normally been dealt with by assumption.

4.2) **Dynamics and Stationarity**

To account for the fact that the full response of the dependent variable to changes in the explanatory variables may not be completed in the same period, different dynamic models have been used. Perhaps the two most popular ones are the partial adjustment and the polynomial distributed lag models.

Although the use of dynamic specifications seems essential when working with quarterly data, the evidence suggest that most of the adjustment occurs within a year. Goldstein and Khan (1976, 1978), for example, have estimated that the average lag of total imports is between two and four quarters, while for exports it is
between one and five quarters. If one is considering disaggregated data, the problem can be more complicated once some goods, especially capital goods, are subject to longer delivery lags.

Another noteworthy observation is that normally a "specific to general" approach has been adopted. It is common to start with simple models that are subsequently expanded to solve the problems that emerge. As shown by Hendry and Mizon (1978), this is not the most adequate way to proceed, once the test performed in the new model is conditional on the result of the tests done in the former model.\textsuperscript{15}

A more adequate and elegant way of dealing with the question of model dynamics is to adopt an error correction model (ECM). In the ECM the estimation of the long and short run coefficients is done separately. The long run coefficients, also called cointegrating vector, are obtained in a first stage by either the Engle and Granger two step method or the Johansen procedure. These coefficients are then used in the second stage when only the short run responses are estimated. This will be the approach used in this thesis.\textsuperscript{16}

In addition, the ECM is also a good answer to the question of non stationarity. All the empirical work on the Brazilian trade equations have assumed, implicitly,

\textsuperscript{15} An exception is Hitiris and Petoussis (1984), who adopt a "general to specific approach".
\textsuperscript{16} A more detailed discussion of the ECM, unit roots, cointegration and the Engle and Granger and Johansen procedures can be found in the chapter four.
that the variables of interest are stationary. If the variables are non-stationary but cointegrated then the ECM is the proper model to use.

4.3) Simultaneity

As noted above it has been more common to use simultaneous models for exports, using the "small country" hypothesis to justify the estimation of the demand equation alone in the case of imports. A better procedure, however, would be the specification of a complete model that could be tested for price exogeneity.

Although the simultaneity between prices and quantities is ruled out by the "small country" assumption, simultaneity can still be a problem in relation to imports and income. Considering an extended model, the level of imports can be seen as an explanatory variable in the income equation. This is especially true in the case of essential imports. For a country where essential goods represent a large proportion of total imports, a simultaneity problem should be expected even in the total imports equation. Most of the literature ignores this problem, failing to even test for the endogeneity of income.

17) The unit root tests carried out in chapters 5 and 6 show that all Brazilian series used in the estimation of trade equations are indeed non-stationary.
19) Exception is Moraes (1986), who estimates a simultaneous model.
4.4) **Aggregation**

The possibility of an aggregation bias in trade equations was first discussed by Orcutt (1950). He argued that goods with low price elasticity can display a larger price variation, causing a downward bias in the estimated aggregate price elasticity of demand. Actually, there can exist a specification bias as well. As Maddala (1977) points out, each aggregate coefficient is a weighted average, not only of its own individual values, but of all coefficients in the regression. In other words, unless all the individual elasticities are equal, the disaggregated price elasticity should be a weighted average of the individuals price and income elasticities.

On the other hand, disaggregated equations should also be preferred on the grounds of using all the information available. In the absence of misspecification problems or measurement errors, the use of disaggregated data should be preferred because there is no loss of information. In practice, however, it is possible that disaggregated data will have larger measurement error than aggregated data, and that the specification of demand functions is likely to be more difficult in the disaggregated equations.²⁰

If one decides to work with disaggregated equations Barker (1970), using a demand for imports, offers a simple way to obtain the aggregated price and income elasticities. When using a log linear specification, the

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²⁰ See Aigner and Goldfeld (1974). Winters (1980) has found evidence indicating that aggregated models explain total exports better than disaggregated ones.
disaggregated elasticities contribute according to the share of the individual import in total imports and the relative variation of the individual demand over that of total demand.

$$b_a = \sum_{i=1}^{n} b_i \frac{M_i}{M_a} \left[ \frac{dX_i / X_i}{dX_a / X_a} \right].$$

The ratio of relative own variation over relative total variation is the so called "distribution factor". If all goods vary in the same way as the aggregate, that is, the distribution factor is one for all goods, then the aggregated coefficient is just a simple weighted average of the individual coefficients as in the linear case.\textsuperscript{21}

4.5) Parameter Stability

A common assumption in econometrics is constancy of coefficients. In terms of trade equations, it implies that price and income elasticities are constant over time. A first challenge to this assumption was made by Orcutt (1950), arguing that parameter instability could arise due to the "quantum effect". The idea is that buyers have some kind of inertia in switching suppliers, in such a way that small and large variations of prices have different impacts on demand. If we have a sample that contains periods of both small and large variation

\textsuperscript{21} To obtain the distribution factors, one can run auxiliary regressions of $\ln x_i$ on $\ln x_a$. 
of prices, then we should estimate the equation under two different regimes. 

Other more important sources of instability are the government trade policy, changes in the pattern of trade due to the process of economic development, and sudden shocks such as changes in the exchange rate regime, large increase in oil prices, etc. Note that instability can either result in sharp changes in the coefficients, as a consequence of sudden shocks, or in smooth changes over time, resulting from the process of economic development and on the long run implementation of trade policy.

Together with these secular and sharp changes in the elasticities it is also possible to find some cyclical variation. In the case of the income elasticity, for example, changes in the composition of output over the business cycle may induce it to have a kind of cyclical behaviour.

Several authors have tested trade equations for structural change in the parameters. The evidence in most cases is in favour of structural change. In none of the cases though, the coefficient path over time was actually estimated.

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22) Learner and Stern (1970) redefine this problem in terms of speed of adjustment, that is demand will adjust faster to large variation of prices than to small ones.

23) Evidence that the relation between total import and expenditure is not stable over the business cycle was found by Giovannetti (1987).

24) See, for example, Mastropasqua (1982) for the Italian case, Joy and Stolen (1975) and Stern et alii (1979) for the United States and Abreu (1987) and Fachada (1990) using Brazilian data.
As far as Brazil is concerned, the question of parameter instability should be expected to be very important. As we will discuss in the next chapter, Brazil has had several important changes in the trade policy regime. Such changes are bound to have had some effect on the trade elasticities. Moreover, the import substitution plans carried out in the beginning of the 1950s and 1960s changed substantially the structural composition of Brazilian industry, which again could have affected the parameters stability.

Therefore, one of the central issues in this thesis will be the estimation of trade equations using time varying parameters techniques.

III) Brazilian Trade Equations

In this section, we present and discuss the empirical work available on trade equations in Brazil. We concentrate our attention on nine papers: Weisskoff (1979), Abreu and Horta (1982), Dib (1985), Abreu (1987) and Fachada (1990) on demand for imports, and Braga ad Markwald (1983) and Paula Pinto (1982), Fachada(1990) and Zini (1988) on export equations.

Two types of assessment can be undertaken. The first is concerned with econometric modelling issues and the second with the discussion and comparison of results. As far as econometric modelling is concerned, all the empirical work in question is open to the same kind of criticism of how to deal with dynamics and stationarity and residual correlation. The series are implicitly
assumed to be stationary and the treatment of dynamics does not go further than the partial adjustment model. In almost all the cases a strong residual correlation is present and is "corrected" by using the Corchrane-Orcutt technique. Moreover, the inclusion of some kind of a time trend or a trend like variable is often necessary to obtain reasonable results. Therefore the main focus in this section will not be the econometric criticism of the papers.

1) Import Demand Equations

We use the results presented by Dib (1985) to highlight the point about the use of variables such as capacity utilization, potential output and time trend to improve the fit of the equation. Table I.1 presents various experiments with different specifications done by Dib (1985). To solve the autocorrelation problem in equation 1 she tries basically to use different combinations of the capacity utilization, potential output and time variables. The theoretical questions concerning this kind of variables were discussed previously.

Note that price and income elasticities in equations 2 and 3 are almost identical, reflecting the fact that potential output is constructed just as a function of time. They are measuring, as Weisskoff would argue, the import substitution process, or, as I would argue, long run changes in the coefficients caused by the import substitution process.
On the other hand, the inclusion of the capacity utilization variable in equation 5 substantially changes the income elasticity, from 3.29 to 1.06. As argued before this variable is used to explain cyclical movements in import demand. Therefore, the reduction in the income elasticity could be associated with the separation between the cyclical and secular effects.

Table II.1

<table>
<thead>
<tr>
<th>equation</th>
<th>income</th>
<th>potential output</th>
<th>capacity utilization</th>
<th>time</th>
<th>price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.509</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>-0.751</td>
</tr>
<tr>
<td>2</td>
<td>3.293</td>
<td>-2.232</td>
<td>--</td>
<td>--</td>
<td>-1.128</td>
</tr>
<tr>
<td>3</td>
<td>3.291</td>
<td>--</td>
<td>--</td>
<td>-0.155</td>
<td>-1.129</td>
</tr>
<tr>
<td>4</td>
<td>--</td>
<td>--</td>
<td>3.292</td>
<td>0.074</td>
<td>-1.128</td>
</tr>
<tr>
<td>5</td>
<td>1.061</td>
<td>--</td>
<td>2.232</td>
<td>--</td>
<td>-1.128</td>
</tr>
<tr>
<td>6</td>
<td>--</td>
<td>1.061</td>
<td>3.293</td>
<td>--</td>
<td>-1.128</td>
</tr>
</tbody>
</table>

Source: Dib (1985).

She also tries to introduce some dynamics into the equation via a partial adjustment model, but in the end, for forecasting purposes, chooses equation 5 as the best model.

As can be gleaned from tables II.2 and II.3, the comparison of the results presented by Abreu and Horta (1982) and Abreu (1987) seems to show little change in the magnitude of price and income elasticities in the
equations estimated for the periods 1961/80 and 1960/87. However, the elasticity of imports with respect to capacity utilization changes considerably for intermediate and capital goods. Even for total imports, Abreu (1987) shows that the capacity utilization elasticity would also change substantially if we did not incorporate the dummy variable for 1974 into the equation.

Table II.2

<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>income</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>Total</td>
</tr>
<tr>
<td>intermediate goods</td>
</tr>
<tr>
<td>consumer goods</td>
</tr>
<tr>
<td>capital goods</td>
</tr>
</tbody>
</table>


According to Abreu (1987), this large change in the capacity utilization elasticity is due to measurement problems in this variable. In the construction of potential output, a constant rate of growth is built in, in his case 7.58%. However, because of the severe adjustment program in the period 1981-83, it is not plausible to suppose a continuous growth of potential output at the same rate as before. Therefore, the
constructed capacity utilization variable would overestimate the true spare capacity in the economy. Note that if capacity utilization is actually part of the true specification, this measurement problem could be overcome by a time varying parameter model. The measurement error could be compensated by allowing the capacity utilization elasticity to vary.

Table II.3

![Table](image)

A better way of approaching this problem is to use a measure of capacity utilization that allows for a time varying potential output growth rate. This question is addressed in chapter 5 where different measures of capacity utilization are calculated and compared.

As reported in table II.4, Abreu (1987) also shows that even the income and price elasticities cannot be regarded as constant over this period. Experiments using recursive least squares for the period 1969/1985, showed
that the income and price elasticities changed smoothly during this period, while the capacity utilization elasticity changed from 3.40 to 2.59.

An interesting comparison can be made between the results of Abreu (1987) and Weisskoff (1979), because disaggregated estimates are available for two substantially different periods of time. It should be said that the estimated price and income elasticities are not totally comparable, not only since the specifications are slightly different, but also because Abreu (1987) excludes crude oil and wheat from the total and intermediate goods equations.

Table II.4

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>price</td>
<td>income</td>
</tr>
<tr>
<td>1960/79 -0.844 (0.096)</td>
<td>0.981 (0.061)</td>
</tr>
<tr>
<td>1960/80 -0.852 (0.088)</td>
<td>0.974 (0.052)</td>
</tr>
<tr>
<td>1960/81 -0.864 (0.082)</td>
<td>0.961 (0.043)</td>
</tr>
<tr>
<td>1960/82 -0.864 (0.077)</td>
<td>0.961 (0.037)</td>
</tr>
<tr>
<td>1960/83 -0.734 (0.097)</td>
<td>0.032 (0.043)</td>
</tr>
<tr>
<td>1960/84 -0.701 (0.101)</td>
<td>1.058 (0.048)</td>
</tr>
<tr>
<td>1960/85 -0.685 (0.106)</td>
<td>1.074 (0.050)</td>
</tr>
</tbody>
</table>

Nevertheless, the differences in the elasticities are quite considerable. In the total imports equation Weisskoff’s income elasticity is more than twice Abreu’s reported value. For the price elasticity this relation is the other way around. The differences are particularly substantial in the capital and intermediate goods case. These changes may be associated with long run change in the elasticities, due to the import substitution process and alterations in exchange rate management. 25

Table II.5

Demand Elasticities - Weisskoff (1979) - 1953/1970

<table>
<thead>
<tr>
<th></th>
<th>income</th>
<th>time</th>
<th>price</th>
</tr>
</thead>
<tbody>
<tr>
<td>total</td>
<td>2.33</td>
<td>-0.13</td>
<td>-0.37</td>
</tr>
</tbody>
</table>
| consumer goods       | 2.19   | -0.13 | -0.27
| metallic intermediate| 2.75   | -0.13 | -0.42 |
| non metallic intermediate | 2.01   | -0.09 | -0.41 |
| capital equip. for agriculture | 6.11   | -0.34 | -1.28 |
| capital equip. for industry | 5.29   | -0.29 | -1.16 |

Note: 1) not significant at 5%.

25) This question will be addressed in more detail in chapter 5.
Finally, let us make some consideration on quarterly estimates. Some work has been done recently on import equations using quarterly data. The earlier lack of such estimates was mainly due to the absence of GDP data on a quarterly basis. Since such a GDP series became available for the period 1975-85, Abreu (1987), Moraes (1986), Fachada (1990) and Zini (1988) have fitted import demand equations using quarterly data.

Table II.6

<table>
<thead>
<tr>
<th>Demand Elasticities - Fachada (1990) - 76.4/88.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>income</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>Total</td>
</tr>
<tr>
<td>intermediate goods</td>
</tr>
<tr>
<td>capital goods</td>
</tr>
</tbody>
</table>


The quarterly data estimations are done in the same unsatisfactory fashion as the annual estimations. Although all the variables are in levels, no attempt is made to test for unit roots. The question of residual autocorrelation is dealt with by using the Cochrane-Orcutt procedure. The question of dynamics is also not properly treated. Although one should expect to have some dynamics when dealing with quarterly data, most of the estimations presented are static equations.
Whenever some dynamics is introduced it is done via the simple partial adjustment model. As a result of all this modelling problems the use of a time trend is quite frequent.

The results and problems of Abreu (1987) and Moraes (1986) are similar to those we have just presented. Therefore, given space limitations these papers will not be discussed.

It should be noticed that Zini (1988) also presents import demand estimations using quarterly data. Nevertheless, since his disaggregation of imports is not compatible with the one we intend to use in this thesis we decided not to show or discuss his estimates. The important thing to notice about Zini’s work is the improvement in the modelling aspect. Instead of simply correcting the residual autocorrelation he tries to distinguish between model misspecification and residual autocorrelation. He uses a testing procedure based on a first order autocorrelation test and the RESET test developed by Thursby (1981).

Some criticism of this test procedure by Knottnerus (1985) and Godfrey (1987) have cast doubts on the robustness of the diagnostic checks. Moreover, this procedure does not deal with the question of non stationarity.

2) Export Equations

Unlike the empirical work on import demand equations, econometric studies on Brazilian export
equations have been quite unsystematic and, therefore, comparisons between different works are more difficult. Until recently, almost all the empirical work available was based on reduced forms or, in a few cases, on supply equations only.

Studies using a reduced form equation do not allow us to retrieve the supply and demand elasticities. This is because the complete model is normally not specified, since the main objective in these works has been forecasting. On the other hand, supply elasticities that are obtained from studies that use the small country hypothesis can also be inadequate. If the price elasticity of demand is not infinite, single equation models that ignore the demand side will produce biased and inconsistent estimates.

Braga and Markwald (1983) and Paula Pinto (1982) using annual data have found some evidence to refute the hypothesis of infinite price elasticity of demand for manufactured goods. This result is somehow surprising since the Brazilian share in the world exports is quite small. For the period 1950/1985, the Brazilian share of world exports was around 1.32%. It should be noticed though that in the case of Braga and Markwald (1983) the evidence against the small country hypothesis is not very conclusive. As can be seen in table II.7 the price elasticity of demand is quite high.

Table II.7 shows a selection of the results presented by Paula Pinto (1982) and Braga and Markwald (1983). They both estimate simultaneous models for the
periods 1957/75 and 1959/81, respectively. Although the sample period is approximately the same, there seems to be some instability in the elasticities. The difference in the price elasticity of demand and the capacity utilization elasticity are substantial, ranging from -1.12 to -6.32, and from zero to -1.36, respectively. The income elasticity also varies, but in a less dramatic way.

Table II.7

<table>
<thead>
<tr>
<th></th>
<th>Income</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paula Pinto (1982)</td>
<td>2.19</td>
<td>-1.12</td>
</tr>
<tr>
<td>Braga and Markwald (1983)</td>
<td>3.14</td>
<td>-6.32</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Price</th>
<th>Capacity utilization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paula Pinto (1982)</td>
<td>1.69 to 3.14</td>
<td>-0.081</td>
</tr>
<tr>
<td>Braga and Markwald (1983)</td>
<td>2.20</td>
<td>-1.36</td>
</tr>
</tbody>
</table>

Note: 1) not significant at 5%.

More recently some estimations using quarterly data have become available. Both Fachada (1990) and Zini (1988) estimate a simultaneous model for the Brazilian exports of manufactured goods. As table II.8 makes clear the estimated elasticities are not similar. A possible explanation for this discrepancy is parameter
stability. Zini (1988) uses data for the period 1970/1986 while Fachada's estimation are performed using data from 1976 to 1988. To reinforce this argument note that in both cases a trend variable was used. The potential output variable present in both works was obtained by regressing the real output against the time trend.

Table II.8

<table>
<thead>
<tr>
<th>DEMAND ELASTICITIES</th>
<th>income</th>
<th>price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fachada (1990)</td>
<td>2.41</td>
<td>-1.93</td>
</tr>
<tr>
<td>Zini (1988)</td>
<td>4.92</td>
<td>-0.31</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SUPPLY ELASTICITIES</th>
<th>price</th>
<th>capacity utilization</th>
<th>potential output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fachada (1990)</td>
<td>0.33</td>
<td>-2.33</td>
<td>2.91</td>
</tr>
<tr>
<td>Zini (1988)</td>
<td>0.26</td>
<td>-1.42</td>
<td>1.66</td>
</tr>
</tbody>
</table>

Note: 1) not significant at 5%.

The same kind of criticism made in the case of the import demand can also be applied for export equations. The question of dynamics does not receive adequate treatment. Even in the case of quarterly data an instantaneous adjustment or, at most, a partial adjustment model is used. Although the variables are used in levels, no test for unit roots are done. The modelling strategy consists in starting with a simple model and try
to solve possible problems by generalising it. It is no surprise that serial autocorrelation is a problem in almost all the cases. As mentioned before, Zini (1990) is the only one to devote more attention to the question of modelling strategy. Nevertheless, he ends up using a non robust method and does not address the non stationarity problem.

IV) Conclusions

This chapter presents the main theoretical issues on the trade equations and discusses the empirical work applied to Brazil.

We have argued that parameter instability is a fundamental issue in the Brazilian case. The parameters should be expected to vary not only during the business cycle, as a result of changes in expectations and in the composition of output, but also in response to changes in trade policy.

The analysis of the available empirical work seems to offer some evidence in favour of the hypothesis of parameter instability. These evidence in the case of imports are derived from the comparison between the results presented by Abreu (1987) and Weisskoff (1979), the importance of the trend variables in the equations and the sample variation experiments carried out by Abreu (1987). In the exports equations case, there is also a considerable discrepancy between the estimated elasticities. Changes in the sample period lead to
substantial changes in some of the parameters and trend variables have been used.

Despite this there has been no serious attempt to deal with this problem. The only thing that was done in this respect was to perform some structural stability tests. Whenever such tests were performed they have rejected the hypothesis of parameter stability.

The testing and estimation of models allowing for parameter variation seems important not only because it could improve the forecast performance, but could also serve as an instrument to evaluate the impact of the different trade policy changes and import substitution programs.

Although the production theory approach offers a integrated way of dealing with imports and exports that has strong links with trade theory and a clear microeconomic foundation, we will use in this thesis the most popular imperfect substitutes model. As Kohli (1991) has shown the results the use of different functional forms has sometimes lead to conflicting results. This approach offers no proper framework to deal with dynamics and, moreover, the Brazilian data on price and quantity of primary domestic factors are practically nonexistent. A substantial amount of work will be needed to generate data set of reasonable quality.
CHAPTER III

Brazilian Trade Policies, 1947-1988

I) Introduction

The main objective of this chapter is to provide a general presentation of Brazilian trade policy during 1947-88. As shown in the following sections there have been frequent changes in the trade policy during this period. These changes were in some cases quite substantial and, therefore, can be characterized as regime changes.

In terms of the main objective of this thesis, the estimation of trade equations with time varying parameters, this policy description is important because it highlights the issue of parameter instability. In such a changing environment, coefficients can be expected to respond to the substantial changes that have occurred in the policy regime. In this sense it gives the basis for the explanation of the coefficient movements that are estimated in chapters 5 and 6.

The description and analysis of Brazilian trade policy that are available in the literature are only partial. Most of the work focuses on either imports or exports for limited periods of time. Therefore, we seek

II) From Liberalization to Direct Controls, 1946/53

By the end of the second world war, the Brazilian balance of payments position looked quite comfortable. During the war, as a result of successive trade surpluses, especially after 1942, and reduced payments on the external debt, the volume of reserves in gold and hard currency had improved substantially. In addition, the likely post war boom in consumer demand was expected to cause a large increase in the international coffee price, which was responsible for approximately 70% of total exports. As for the capital account, government expectations were also optimistic. The political and military support given to the United States during the war was expected to pay off afterwards, in terms of a substantial public capital inflow. An increase in the inflow of private capital was also expected in response to the more open external policy that was being proposed.2

2) For a more detailed description and analysis of Brazilian external policy and economic conditions in this period, see Malan et alii (1980).
In this scenario, the main focus of economic policy was to try to reduce inflation, that had increased to 14.6% in 1946. To help in this aim, the government decided to keep the nominal exchange rate constant at the 1939 level, and to liberalize exchange controls. As the differential between internal and external inflation was substantial during 1939/46, the real exchange rate in 1946 was considerably over valued compared with 1939. In the government's eyes the overvalued exchange rate would help to combat inflation by increasing the domestic supply of goods. On the other hand, a real devaluation could increase pressure on inflation. This policy was expected to have some adverse effect on the trade balance but, given the considerations mentioned above, this was not a central concern.

As the optimistic expectations about the balance of payments did not come true, Brazil experienced severe trade deficits. Although the international coffee price had increased, the fall in non coffee exports kept total exports approximately constant. The expected increase in the capital inflow did not happen, leaving the trade deficit to be financed by reductions in reserves.

The hard currency reserves ran out very rapidly, forcing the government to change policy. After some initial and unsuccessful legislation, the government introduced the Law n. 262 of 23/02/48, creating the obligation of having a licence to be able to import and

3) The exchange rate liberalization was done through the Decree-Law n. 9.025 of 27/02/1946.
export any product. As far as imports were concerned, the government would discriminate in favour of essential goods, trying to reduce total imports by reducing non essential imports. The CEXIM (Import and Export Agency), now CACEX, was put in charge of issuing the licences. The imports were divided into three categories. The total amount of foreign exchange was allocated among these categories. The first category of "most essential goods" received 75%, the second 20% and the third 5%.

Some authors argue that the licensing system was also used to control non-coffee exports, keeping the domestic market well supplied and thus helping to combat inflation. However, there seems to be no strong evidence to confirm this.

To help the non-coffee exports that were seriously affected by the overvalued exchange rate the "linked operations" (operacoes vinculadas) were created. Some exporters were allowed to sell exchange directly to the importers. The transaction was made at the official rate plus a negotiated premium. Therefore, it amounted to a devaluation of the exchange rate for some selected products.

The trade balance deterioration during 1951/52 led the government to tighten the policy. The Korean War brought about concerns of shortage and a precautionary increase in imports. The linked operations were ended and the import control was made more rigid.

4) Actually, some goods were exempted from this obligation.
5) See, for example, Leff (1967).
This policy of direct control on imports and fixed nominal exchange rate was kept in place until 1953.6 Although it allowed the government a strict control on imports, this policy had some drawbacks. Its effects on non-coffee exports were very damaging, reducing substantially their competitiveness. On the other hand, it was not agile enough to deal with short run problems as the import boom of 1951/52 made clear.

In an attempt to address these questions the exchange rate policy was changed in 1953. Instead of quantitative controls a new system of multiple exchange rates was created. By changing to price controls the government was able to encourage non-traditional exports and, at the same time, have a more efficient and flexible mechanism to manage trade flows.

III) The Multiple Exchange Rate Regime, 1953/57

The change to multiple exchange rates started in January 1953.7 On the imports side a new "free" exchange rate was created alongside the official rate. The "free" exchange rate was also controlled by the government but its value was higher than the official rate that still was fixed at the same nominal level. The official rate was used for buying essential imports and for government financial transactions while the "free" rate was used for the other imports. On the exports side a premium was added to the official rate. Initially, coffee exporters

6) For more details on the trade policy during this period see Von Doellinger at alii (1977).
7) Law n. 1807 of 7/1/53.
received an extra cr$ 5.00 per dollar on top of the official rate while the others received cr$ 10.00. These amounts were changed over time.

In October 1953, SUMOC’s Instruction 70 widened the changed in the exchange rate system in Brazil. It created a system with five different exchange rates for imports and four for exports. Actually, between 1953/54, there were still only two rates for exports. It was only in 1955 that the government extended the number of export exchange rates to four. The export products were classified according to their competitiveness in world markets. Coffee was in group I, other agricultural and mineral products such as cotton, tobacco, quartz, cocoa and iron ore were in groups II and III, while other products, including manufactures, were in group IV. The import products were also classified in different groups, according to their necessity, from group I to V.

The government had the monopoly in buying foreign currency from exporters, paying different prices in different groups. The foreign currency was then sold in auction among importers of each one of the groups. It controlled the different exchange rates by controlling the supply of currency in each auction.

---

8) The SUMOC was an embryo of the central bank created in 1945. It only became an actual central bank in 1964.
9) There were actually other three rates. The official rate, still fixed in nominal terms, was basically used by the government, the "free" rate that was used for financial transactions and the cost rate (custo de cambio), equal to the official rate plus the average premium paid to exports, used for some specific products.
### Table III.1

<table>
<thead>
<tr>
<th></th>
<th>1953¹</th>
<th>1954</th>
<th>1955</th>
<th>1956</th>
<th>1957³</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Imports</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>group I</td>
<td>31.77</td>
<td>39.55</td>
<td>80.70</td>
<td>83.05</td>
<td>60.76</td>
</tr>
<tr>
<td>group II</td>
<td>38.18</td>
<td>43.63</td>
<td>105.23</td>
<td>111.10</td>
<td>81.56</td>
</tr>
<tr>
<td>group III</td>
<td>44.21</td>
<td>57.72</td>
<td>176.00</td>
<td>149.99</td>
<td>106.34</td>
</tr>
<tr>
<td>group IV</td>
<td>52.19</td>
<td>56.70</td>
<td>223.16</td>
<td>219.58</td>
<td>151.93</td>
</tr>
<tr>
<td>group V</td>
<td>78.90</td>
<td>108.74</td>
<td>303.54</td>
<td>309.28</td>
<td>316.39</td>
</tr>
<tr>
<td><strong>Exports</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>coffee</td>
<td>23.36</td>
<td>26.25²</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>non-coffee</td>
<td>28.36</td>
<td>31.29²</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>group I</td>
<td>--</td>
<td>--</td>
<td>31.50</td>
<td>37.06</td>
<td>38.16</td>
</tr>
<tr>
<td>group II</td>
<td>--</td>
<td>--</td>
<td>37.91</td>
<td>40.10</td>
<td>43.06</td>
</tr>
<tr>
<td>group III</td>
<td>--</td>
<td>--</td>
<td>43.18</td>
<td>49.88</td>
<td>55.00</td>
</tr>
<tr>
<td>group IV</td>
<td>--</td>
<td>--</td>
<td>50.98</td>
<td>59.12</td>
<td>67.00</td>
</tr>
</tbody>
</table>


Notes:
1) October / December.
2) average rate.
3) January / August.

A major part of the currency available was allocated to groups I to III, keeping the exchange rate in the lower groups overvalued in relation to the higher groups. In this way, the government was able to control the volume of imports and encourage exports, accordingly to the needs of the trade balance. Moreover, the multiple exchange rates scheme worked as an instrument of industrial policy. The scheme was in fact a system of variable tariffs and subsidies, allowing the government to maintain its goals in terms of import substitution and to promote exports at the same time.

As far as the trade balance is concerned the multiple exchange rates seem to have had the intended
impact. Exports of non-traditional goods increased, especially after 1954. Total exports stayed relatively constant during this period because of the poor performance of the coffee exports caused by a fall in demand.

On the fiscal side Instruction 70 had also a positive impact. It became a source of extra revenue for the government. Although there were large variations in the balance for the period as a whole, the premium paid for the importers was greater than the one received by the exporters.

Another important measure was SUMOC's Instruction 113 of January 1955. It allowed the import by foreign firms in Brazil of capital goods for approved projects with external financing without exchange coverage. This scheme was intended to increase direct investment and reduce the pressure on the auctions. An increasing part of the capital goods imports was made through this scheme.

This scheme was not without its drawbacks. Although it allowed a long overdue progressive devaluation it was administratively complicated since the auctions had to be conducted simultaneously in different cities.

IV) Tariff Reform and Exchange Rate Unification, 1957/64

As noted above, the multiple exchange rate scheme started in 1953 had the same effect as a variable tariff. Before 1957, tariffs were not very effective in granting protection to domestic industry, since they were usually
specific tariffs. After some months, the high inflation would reduce substantially the effective protection.

To overcome this problem as well as the administrative drawback of the multiple exchange rates, a new tariff law was passed in August 1957. It created a new ad valorem tariff system with rates between 0% and 150%. The products were classified in the same fashion as before. The capital and intermediate goods paid lower duties, normally from 0% to 50%, while consumer goods were charged higher duties, between 50% and 150%.

The idea was to separate exchange rate and industrial policy management. The selective protection of domestic industry would be done by the tariff structure, while the exchange rate would be just an indicator of the shortage of hard currency.

The exchange rate system was also simplified. On the imports side, the number of exchange rates was reduced to two, general and special. The products considered to be essentials were imported using the lower general rate while consumer goods and others with a domestic substitute used the higher special rate. On the export side, the reduction in the number of exchange rates was done gradually, but by 1959 there were only two rates. Other rates such as the "free" rate and the cost rate were not abolished. In fact, many of the export products were gradually transferred to the higher "free" rate.

10) A new agency, the Customs Policy Council (CPA), was created to set and revise the tariffs.
11) Products like wheat and crude oil were still imported using the cost rate.
A substantial change in exchange rate policy happened in 1961. The multiple exchange rate system was ended and a more aggressive approach in terms of exchange rate management was adopted. All exports, including coffee, were unified under the "free" rate. A similar movement happened in imports. Apart from a very few products all imports started using the "free" rate. The import of crude oil and wheat was still done using the cost rate. As a result of a more realistic exchange rate management this rate was devalued by 100%.

Table III.2

<table>
<thead>
<tr>
<th>Year</th>
<th>General</th>
<th>Special</th>
</tr>
</thead>
<tbody>
<tr>
<td>1957¹</td>
<td>80.29</td>
<td>179.67</td>
</tr>
<tr>
<td>1958</td>
<td>149.35</td>
<td>300.36</td>
</tr>
<tr>
<td>1959</td>
<td>201.75</td>
<td>365.88</td>
</tr>
<tr>
<td>1960</td>
<td>222.79</td>
<td>527.37</td>
</tr>
<tr>
<td>1961²</td>
<td>208.86</td>
<td>638.76</td>
</tr>
</tbody>
</table>

Notes: 1) September / December.
2) January / March.

Although there was a clear movement towards a single exchange rate, unification was not complete. The creation of a compulsory deposit implied some exchange rate differentials. Importers were expected to deposit in local currency the equivalent of 100% of the imports' value for a period of 150 days. As the rate of interest of 6% per year paid on the deposit certificates was below
the market value the compulsory deposit represented an extra cost of buying foreign currency. Moreover, as some products were exempt from the compulsory deposit there was an actual difference in the implied exchange rates.\(^{12}\)

Apart from these changes in economic policy it was also during this period that an ambitious import substitution plan, the Plano de Metas, was carried out. Most of the import substitution process had taken place in the consumer goods industry. The objective of the Plano de Metas was to extend this process to the intermediate and capital goods industry.

As it would be impossible for Brazil to finance a large-scale import substitution program using only its own resources, a substantial increase in the foreign capital inflow became essential. This was especially important since exports were falling because of the fall in coffee prices. In 1957 the inflow of direct investment was 60% greater than in the previous year. Moreover, after this increase the level of direct foreign investment stayed high until the end of the plan in 1961.

Several criticisms can be made of the way the Plano de Metas was implemented, mainly in terms of increasing inflation and foreign debt. Nevertheless, it seems to be widely acceptable that it had an important impact in changing the Brazilian industrial structure.

\(^{12}\) Crude oil, wheat, goods imported from countries within the Latin America Free Trade Alliance (ALALC), government imports and goods that were going to be used as inputs in the exports production were exempt from the compulsory deposit.
V) The "Crawling Peg" System and Incentives to Exports, 1964/74

The main characteristic of this period is the gradual change from import substitution to export promotion as the central objective of trade policy. Throughout this period the government adopted several measures towards liberalizing imports and boosting exports.

The final unification of the exchange rate occurred in 1964 with the end of the compulsory deposits. A more important change in exchange rate policy happened in 1968 with the introduction of the crawling peg system. Between 1957 and 1968, the government used to correct the nominal exchange rate by the domestic price index only after a long delay. This procedure had the inconvenience of creating substantial variations in the real exchange rate since the overvaluation of the national currency was then corrected all at once.

In 1968, the government created a crawling peg system in which small devaluations of the nominal exchange rate occurred at short intervals, keeping the real exchange rate approximately constant. The real exchange rate was not actually kept constant, having changed according to market conditions and policy objectives. The important feature of this system was to achieve a much smoother path for the real exchange rate and to end the chronic overvaluation problem that had been damaging to export growth.
In this period, all the remaining controls on imports were lifted. The tariffs that had been increasing since 1957 were reduced after 1967. The average tariff for manufactured goods fell from 99% in 1966 to 57% in 1973\textsuperscript{13}, while for the total imports it came down from 16.5% to 10.7% over the same period. It was still compulsory to obtain a licence to import, but the licences were awarded without any special selective criteria. As a result of this more open policy and the continuous economic growth after 1967, imports grew steadily. During the period 1965/1973 the total imports grew from 0.941 to 6.199 billions of dollars, which represents an increase of 559%.

It was also during this period that the government started to give a wide range of fiscal and financial incentives to exports. Brazilian exports had been stagnating for a long period, between 1950 and 1963 increasing only by 3.5%. Such stagnation was seen as the result of the import substitution policies that had discriminated against exports.

At first, between 1964/68, the government removed all obstacles to the increase of exports. Although the rebating of indirect tax is common practice in foreign trade, such rebates were not given in Brazil. Exports had to pay the two existing indirect taxes, the IPI and ICM.\textsuperscript{14}

\textsuperscript{13} See Braga and Tyler (1992).
\textsuperscript{14} Both IPI and ICM are value added taxes but the IPI is a federal tax while the ICM is a state tax. Other exemptions were also granted for some specific taxes such as the tax on minerals (imposto unico sobre minerais), on electricity (imposto unico sobre energia eletrica), and
In 1964 exports were exempt from IPI and in 1968 this exemption was extended to the ICM as well.\textsuperscript{15} The "drawback system" was created, allowing inputs to be used in export production to be imported free of import tax.\textsuperscript{16}

Several other measures, mainly taken after 1968, were intended to create export subsidies. Firstly, the government granted full exemption of income tax on export profits.\textsuperscript{17} It also granted financial subsidies via loans for working capital with interest rates lower than the market rates.\textsuperscript{18} Finally, a special scheme called IPI Credit was created to give exporters rebates on other taxes according to the exemption received. The exporter was not only exempt from paying indirect taxes but also received a credit related to the tax exemption that could be used to pay for other taxes.\textsuperscript{19}

These were just the main subsidies. Several other less important or comprehensive measures were also taken. For firms with a long-term export commitment the BEFIEX program was created.\textsuperscript{20} This program allowed the import of

on oil products (imposto unico sobre combustiveis e lubrificantes).
\textsuperscript{15) Decree-Law n. 61514 of October 1967 and Decree-Law n. 406 of December of 1968, respectively.}
\textsuperscript{16) Law n. 53967 of June 1964. The imports done through the "drawback system" were exempt not only from the import tax itself but also from IPI, ICM and other imported related taxes.}
\textsuperscript{17) Law n. 4663 of October 1966.}
\textsuperscript{18) Central Bank Resolution n. 71 of November 1967. Although the rules governing this loans changed over time the interest rates were normally negative since inflation was not taken into account.}
\textsuperscript{19) Decree-Law n. 491 of March of 1969 for the IPI and Convenio CONFAZ AE01/70 of January 1970 for the ICM. The IPI and ICM exemptions could be credit up to a limit of 15%.}
\textsuperscript{20) The BEFIEX was created by the Decree-Law n. 1219 of May 1972.}
machinery and inputs to increase the domestic production without paying import tax or IPI. The difference between the BEFIEX and the "drawback system" was that in the former the exemption of duties was granted even for goods that would not be used in export production. Moreover, the imports done through the BEFIEX were not subjected to the Similarity Law. A special fund, the FINEX, to finance exports of capital and consumer goods was created. Incentives were given to the creation of trading companies and special sectoral programs were also used to give incentives to exports. 21

It should be noticed that the conceding of subsidies was not general. They were mainly targeted to manufactured goods. However, the definition of manufactured products given by the government was rather loose, including some minerals, processed foods, such as sugar and meat, and some agricultural products, such as soybean.

In short, this period represents a substantial change from an import substitution policy that discriminated against exports to a policy of more realistic exchange rate management, reduction of import controls and creation of export subsidies. The figures in table III.3 give a good idea of the importance of the export subsidies. They grew from 5% of the exports of manufactured goods in 1966 to 54.1% in 1974.

---

21) A detailed analysis of the export financing system in Brazil can be found in Baumann and Braga (1986).
Table III.3

<table>
<thead>
<tr>
<th>Year</th>
<th>Exports Incentives (%)</th>
<th>Tariff (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1966</td>
<td>5.0</td>
<td>16.5</td>
</tr>
<tr>
<td>1967</td>
<td>21.3</td>
<td>11.0</td>
</tr>
<tr>
<td>1968</td>
<td>26.2</td>
<td>14.5</td>
</tr>
<tr>
<td>1969</td>
<td>34.0</td>
<td>14.3</td>
</tr>
<tr>
<td>1970</td>
<td>43.8</td>
<td>11.8</td>
</tr>
<tr>
<td>1971</td>
<td>47.5</td>
<td>10.8</td>
</tr>
<tr>
<td>1972</td>
<td>48.5</td>
<td>10.8</td>
</tr>
<tr>
<td>1973</td>
<td>50.5</td>
<td>10.7</td>
</tr>
<tr>
<td>1974</td>
<td>54.1</td>
<td>9.5</td>
</tr>
</tbody>
</table>

Source: Musalem (1981) and Comercio Exterior do Brasil, several issues.

Note: 1) Calculated as the ratio between total imports and the import tax.

Brazilian exports grew from 1.6 to 8.0 billions of dollars between 1966 and 1974. Even considering the growth in world trade and in the export prices that happened during this period, the growth in Brazilian exports is quite remarkable. The imports show a similar pattern growing 375% between 1966 and 1973. Despite the steady growth in exports, the import liberalization process, mainly after 1967, contributed to growing deficits in the balance of payments current account. Such deficits could only be sustained thanks to huge capital inflows. Long term loans increased from 0.51 billion dollars in 1966 to 6.89 billions in 1974.
VI) The Oil and Debt Crises, 1974/83

After a period of more liberal import policies and an emphasis on export promotion, the oil crisis in 1974 marked a return to the inward looking policies. The subsequent increase in oil prices and interest rates in 1979 and the debt crisis in 1983 also ensured that import restrictions remained high on the agenda. Therefore, the main characteristics of this period are the reintroduction of import-restrictive policies and a more aggressive exchange rate policy, especially in 1983, while the system of export incentives was not altered in any major way.

Given the fall in international demand associated with the oil crisis, the best hope of reducing the trade deficit seemed to be import reduction. In the second half of 1974 the government introduced the first restrictive measures. Tariffs on 900 non-essential products were increased by 100%.22 The issuing of import licences for goods subject to tariffs of greater than 55% was conditional on cash payment for the foreign exchange.23 The state owned enterprises were forbidden from importing consumer goods.24

In 1975 it became clear that the restrictive measures that had been adopted were not enough to close the trade gap. Therefore, new restrictive measures were introduced. In the beginning of 1975 the obligation of

22) Decree-Law n. 1334 of June 1974. Later in the year another 1000 products also had their tariff increased in 100% by the Decree-Law n. 1364 of November 1974.
cash payment to buy foreign exchange was extended to products with a tariff rate greater than 37%.\textsuperscript{25} A compulsory deposit was created for goods paying tariffs of more than 37%. Importers had to deposit the equivalent of 100% of the imports' value in national currency in the Central Bank. These deposits were held for 180 days and were not entitled to receive any interest or compensation for inflation.\textsuperscript{26} Another 680 products were also subject to the payment of the compulsory deposit despite their tariff.\textsuperscript{27} This measure represented in practice the return to a multiple exchange rate system for imports.

By the end of 1975 the government decided to tighten the restrictions even more. Tariffs on approximately 2000 products including some intermediate goods were increased.\textsuperscript{28} The issuing of licences for goods considered superfluous was suspended. New limits were imposed on government imports including the prohibition of imported goods when a similar domestic product was available.

As part of the return to the inward looking policies the government started a new import substitution plan, namely the Second National Development Plan (II PND). The idea was to promote import substitution mainly in the capital goods sector. This Plan was seen as part of a

\textsuperscript{25} Central Bank Resolution n. 319 of February 1975. 26) Central Bank Resolution n. 331 of July 1975. The period was later extended to 360 Days by the Decree-Law n. 1427 of December 1975. 27) As it happened in 1961 some products were exempt of paying the compulsory deposit. Imports of crude oil, wheat, coal, goods imported from countries within the Latin America Free Trade Alliance (ALALC) or associated with the BEFIEX or Drawback systems were exempt. 28) Decree-Law n. 1427 of December 1975.
long term adjustment process as opposed to the short term adjustment represented by import restrictions. Although it would lead to more imports and the need for foreign capital the II PND was seen as fundamental to allow long term growth. Castro and Souza (1985) argue that this goal was actually achieved. According to them the II PND led to import substitutions that were important in explaining the period of growth with relatively stable imports from 1983 onwards.

The import restriction policies seem to have had a substantial impact. Despite the continuous economic growth during this period the level of imports was kept relatively constant. Between 1974 and 1978 average GDP growth was 6.7% while the level of imports was relatively stable around 12.2 billion dollars. In 1978 it rises to 13.7 billions because of a sharp increase in import prices. Actually, the success in containing imports is even greater if we take into account the fact that import prices were increasing. The quantum of imports excluding oil and wheat was reduced by 24.2% during this period.

Total exports also increased from 8.0 to 12.7 billion dollars between 1974 and 1978. In this case though the main contributor to this good performance was the increase in export prices. Nevertheless, the quantum of exports managed to growth on average 5.3% per year despite the adverse international background.

In 1977/78 the balance of payments seemed under control. In 1977, for the first time since the oil crisis, a trade surplus was achieved. Although in 1978
there was again a trade deficit it was not very large. On the other hand the capital account had also improved with an increase in direct investment and long term loans. The government actually started relaxing some of the import controls. In particular, several products became gradually exempt from the compulsory deposit.

Unfortunately the events of 1979 would lead the government back to the import restrictive policies, although now with greater reliance on the exchange rate. The second large increase in the oil price again put pressure on the government. In just one year imports increased from 13.7 to 15.2 billion dollars. On the other hand, the jump in interest rates also had a negative impact on the Brazilian balance of payments. The USA economic policy combining fiscal expansion and monetary austerity caused an substantial increase in the worlds' interest rates. Since Brazil had accumulated a large foreign debt after 1974 it meant a large increase in interest payments. In 1978 the foreign debt was 2.5 times greater than it was in 1974. Therefore between 1978 and 1979 interest payment on the external debt almost doubled, going from 2.7 to 4.2 billion dollars.

In the beginning of 1979 the government decided to attempt a gradual adjustment policy based on exchange rate management. The compulsory deposit and IPI credit were to be gradually removed but on the other hand the real exchange rate was to be devalued by 6% in 1979 and
at least 1.1% each quarter thereafter until 1983. By the end of the year, as this gradual approach failed to deliver, a more aggressive policy was adopted. In December 1979 the government ended the compulsory deposit and IPI credit at once and devalued the exchange rate by 30%. This represented a major change in relation to the crawling peg system, but according to the government it was necessary to correct the balance of payments problem.

The inflationary consequences of the "maxi devaluation" were expected to be very large. Moreover, there were expectations that another "maxi devaluation" would follow soon. Therefore, frightened that inflation would get completely out of control during 1980 the government changed course once again and abandoned the policy of aggressive exchange rate devaluation. Trying to contain inflationary expectations the government then commit itself to increase the nominal exchange rate by only 40% during 1980. By the end of the year although the increase in the nominal exchange rate reached 54% the inflation rate of 100% meant a substantial overvaluation of the real exchange rate.

This overvaluation of the real exchange rate would last until 1982 when the "mini devaluations" were accelerated. During this time the government had to rely on other aspects of trade policy. In December 1979 some import limits were imposed on state owned enterprises.30

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29) The end of the IPI credit was also a demand from the GATT that considered it an instrument of unfair competition.
30) Decree 82268 of December 1979. Some specific companies such as Petrobras, Siderbras and Eletrobras
In the second quarter of 1980 a tax on financial operations (IOF) was introduced. When buying foreign exchange for imports IOF of 15% was due. On the export side the main change was the reintroduction of the IPI credit in April 1981. Some other adjustments were also made in export incentives. The resources available for export financing were increased and a fixed interest rate lower than the rate of inflation for financing working capital for exporters was introduced.

Together with these measures the stabilization program started in the second quarter of 1980 was also expected to have positive effects on the trade balance. The basic idea was to reduce domestic absorption in order to reduce imports and at the same time increase exports via the reduction in the level of capacity utilization.

In 1981/82 Brazil experienced its worse recession since records began in 1947.

Brazil managed to restore the trade surplus in 1981/82 thanks to increased exports in 1981 and reduced imports in 1982. The quantum of imports did not change in 1980 but was in a constant and deep decline thereafter. Between 1981/83 the quantum of imports excluding crude oil and wheat was reduced on average by 18.7% per year.

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31) Central Bank Resolution n. 610 of April 1980. The rate was later in the year increased to 25% by the Central Bank Resolution n. 672 of December 1980.
32) In 1979 the government started charging a fixed interest rate plus a compensation for the inflation ("monetary Correction"). In 1981 the interest rate on these loans was fixed at 40% while the inflation in this year was 95.2%.
Because of the high interest rates the interest payments during this period were quite substantial. Between 1979 and 1982 Brazil paid 31.0 billion dollars in interest. Since the resources necessary to cover this payment could not come from the trade balance new loans were contracted to pay for the old ones. Such a situation is obviously unsustainable in the long run as the debt crisis at the end of 1982 proved.

After the Mexican interruption of debt payments it became virtually impossible for Brazil to obtain new loans. The IMF then stepped in and a new stabilization program was agreed. The basic feature of the agreement was that the IMF and the private banks would keep a minimum inflow of loans while Brazil would undertake an adjustment program to be able to generate future trade surpluses. No reduction on the debt stock or interest rates was contemplated in the agreement. Brazil, as well as other developing countries, was to bear all the costs of avoiding the collapse of the international financial market.

As for the trade balance, the main features of the stabilization program were the reduction of demand and an aggressive exchange rate policy. In February 1983 the exchange rate was devalued by 30%. To guarantee that this "maxi devaluation" would mean a real devaluation the "mini devaluations" were accelerated to account for the inevitable increase in inflation. Contrary to what
happened in 1980 inflation control was not the main aim any more.\textsuperscript{33}

Although the inflation rate more than doubled from 99.7\% to 211.0\% and GDP fell by 3.4\% the stabilization program has to be considered quite successful. The main aim of obtaining a trade surplus was accomplished. In 1983 Brazil had a 6.5 billions trade surplus. This surplus was mainly achieved by import reduction. The total imports were reduced by 20.4\% while exports increased only slightly.

VII) The Relaxation of Controls, 1984/88

The Brazilian balance of trade position that had shown a substantial improvement in 1983 was even more comfortable in the next two years. Helped by the increase in world demand exports during 1984/85 were around 26.3 billion dollars.

The exchange rate policy also deserves some credit for the good export performance. The government implemented an exchange rate policy aimed at maintaining the level of the real exchange rate. The "mini devaluations" were such that the real devaluation that had happened in 1983 was not lost. A change in the exchange rate rules in the beginning of 1985 actually led to a further, though temporary, real devaluation. From March 1985 the nominal exchange rate was corrected by the average inflation in the last three months. As the rate

\textsuperscript{33) For a description of the this stabilization plan and comparisons with the previous one see Carneiro (1986).}
of inflation came down during the following months this rule resulted in a further real devaluation.\textsuperscript{34}

Imports were reduced even further, from 15.4 to around 13.5 billion dollars between 1983 and 1984/85. This reduction was mainly due to the fall in oil prices. Crude oil imports fell from 7.6 to 5.4 billion dollars between 1983 and 1985.

The good performance of the trade balance allowed government to adopt a more liberal approach towards the trade policy. Several imported goods were, for a temporary period, exempted from IOF. On the export side the measures were focused on reducing the pressure on the public sector deficit. The interest rate conditions for working capital financing were changed, reducing the difference to the market interest rate. From August 1984 onwards the financial operations were done by private banks at a rate 10\% lower than the normal market rate. The difference between the two rates was covered by the government.\textsuperscript{35} Finally, the IPI credit was ended in April 1985.

The increase in inflation in the first months of 1986 and the relatively comfortable balance of payments position put domestic macroeconomic policy back at the top of the agenda. On February 28th 1986 the government froze all prices, including the exchange rate and wages, and ended the widespread formal indexation system. During the next few months the real exchange rate was stable.

\textsuperscript{34} This new rule was introduced by the Central Bank Resolution n. 1001 of March 1985.
\textsuperscript{35} Central Bank Resolution n. 950 of August of 1984.
since the new stabilization plan was succeeding in controlling prices. After June, though, the increase in domestic prices led to a progressive deterioration in the real exchange rate.

Exports fell substantially, from 25.6 to 22.3 billion dollars between 1985 and 1986. Export prices actually increased but the fall in the quantum was much greater. Other factors contributed to the reduction in exports, such as the large reduction in spare capacity and the large increase in wages. On the other hand, there was a growing expectation that the government would have to make another "maxi devaluation" to correct the exchange rate. The dollar premium in the black market rose to 100% of the official rate. The expectations of a new "maxi devaluation" led to exports been delayed and imports anticipated. Trying to break such expectations the government reinstated the crawling peg policy in October and allowed exporters and importers to hold deposits in dollars in the Central Bank. 36 It was too late though to prevent the fall in exports. 37

The trade balance deterioration would have been worse had not the huge increase in non-oil imports been partially offset by the drop in crude oil imports. Non-oil imports increased from 7.7 to 11.2 billion dollars between 1985 and 1986. The success in increasing the domestic oil production and the lower international

36) Central Bank Resolutions n. 1208 and 1209 of October 1986. This deposits in dollars were later extinct by the Central Bank Resolution n. 1492 of June 1988.
37) For a analysis of the stabilization plan see Modiano (1987) and Franco (1986).
prices for this product meant a reduction in oil imports from 5.5 to 2.8 billions between 1985 and 1986. This increase in imports should be explained not only by the increase in GDP and wages but also by the government policy of keeping the domestic market well supplied. After some months of price controls several goods were in short supply and a large black market for such goods was developing.

As a result the trade surplus was reduced from 12.5 to 8.3 billion dollars. The overall balance of payments deficit also got worse leading to a fast reduction in reserves. This process ultimately led to the suspension of debt payments in February 1987.

In the beginning of 1987 the focus of the economic policy changed again to the balance of payments. The daily "mini devaluations" were accelerated and in two days in May and June a more significant devaluation, of 8.5% and 9.5% respectively, was undertaken. Because of the increasing inflation this more aggressive exchange rate policy failed to restore the real exchange rate to its previous level.

The trade balance position nevertheless was improved significantly as a result of the increase in exports. The reduction in capacity utilization caused by the GDP slow down and the increase in international demand are the main factors explaining the increase in exports. In 1987 exports went back to the level of 1984/85 and in 1988 it reached the all time record of 33.8 billions of dollars.
VIII) Conclusions

We have shown in this chapter that Brazilian trade policy between 1947 and 1988 experienced several regime changes. During this time we have had multiple and single exchange rates systems, the introduction of the crawling peg rule and its temporary break by the "maxi devaluations". The partial end of the import substitution policies in 1964 led to the creation of a new structure of export incentives. Imports, on the other hand, have been subject to all kinds of controls. The main changes as far as imports are concerned were the introduction of the ad valorem tariffs in 1957, the compulsory deposits and the quantitative controls.

Such changes in economic policy can be expected to have some impact on the stability of the trade equation coefficients. As suggested by Lucas (1976) changes in the policy regime will probably have some impact on the stability of economic relations.

On the other hand two major import substitution plans were carried out in 1957/60 and 1974/78. Although several criticisms can be made of the conduct and timeliness of these plans, it is generally accepted that they had an important impact on Brazilian industrial structure. These plans were responsible for the development of new industries, especially in the capital and intermediate goods sector.

These large changes in the industrial structure of the economy may have affected the trade pattern and, therefore, had an impact on the stability of the trade
elasticities. Moreover, as discussed in chapter 2 section 4.5, there remains Orcutt's "quantum effect" and the aggregation question that may also lead to some parameter instability.
**APPENDIX III.1**

Main Economic Indicators

<table>
<thead>
<tr>
<th>Balance of Payments and External Debt (US$ billions)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Years</strong></td>
</tr>
<tr>
<td>Trade Balance</td>
</tr>
<tr>
<td>Exports</td>
</tr>
<tr>
<td>Imports</td>
</tr>
<tr>
<td>Services</td>
</tr>
<tr>
<td>Interest</td>
</tr>
<tr>
<td>Current Account</td>
</tr>
<tr>
<td>Capital Account</td>
</tr>
<tr>
<td>Direct Investment</td>
</tr>
<tr>
<td>Long Term Loans</td>
</tr>
<tr>
<td>Short Term Capital</td>
</tr>
<tr>
<td>Amortizations</td>
</tr>
<tr>
<td>Others</td>
</tr>
<tr>
<td>Total</td>
</tr>
<tr>
<td>Foreign Debt</td>
</tr>
<tr>
<td>Debt/exports</td>
</tr>
</tbody>
</table>

**Inflation and Growth Indicators**

<table>
<thead>
<tr>
<th><strong>Years</strong></th>
<th><strong>1947</strong></th>
<th><strong>1948</strong></th>
<th><strong>1949</strong></th>
<th><strong>1950</strong></th>
<th><strong>1951</strong></th>
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<tbody>
<tr>
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<td>7.70</td>
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<td>11.00</td>
<td>12.70</td>
<td>5.30</td>
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</table>
### Main Economic Indicators

#### Balance of Payments and External Debt (US$ billions)

<table>
<thead>
<tr>
<th>Years</th>
<th>1952</th>
<th>1953</th>
<th>1954</th>
<th>1955</th>
<th>1956</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trade Balance</td>
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<td>0.424</td>
<td>0.148</td>
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<td>0.437</td>
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<tr>
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<td>1.540</td>
<td>1.558</td>
<td>1.419</td>
<td>1.483</td>
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<tr>
<td>Imports</td>
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<td>1.116</td>
<td>1.410</td>
<td>1.099</td>
<td>1.046</td>
</tr>
<tr>
<td>Services</td>
<td>-0.336</td>
<td>-0.355</td>
<td>-0.338</td>
<td>-0.308</td>
<td>-0.369</td>
</tr>
<tr>
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<td>-0.034</td>
<td>-0.048</td>
<td>-0.035</td>
<td>-0.067</td>
</tr>
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<td>Current Account</td>
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<td>0.055</td>
<td>-0.195</td>
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<td>0.057</td>
</tr>
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<td>0.059</td>
<td>-0.018</td>
<td>0.003</td>
<td>0.151</td>
</tr>
<tr>
<td>Direct Investment</td>
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<td>0.022</td>
<td>0.011</td>
<td>0.043</td>
<td>0.089</td>
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<td>Long Term Loans</td>
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<td>0.044</td>
<td>0.109</td>
<td>0.084</td>
<td>0.231</td>
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<tr>
<td>Short Term Capital</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Amortizations</td>
<td>-0.033</td>
<td>-0.046</td>
<td>-0.134</td>
<td>-0.140</td>
<td>-0.187</td>
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<td>0.039</td>
<td>-0.004</td>
<td>0.016</td>
<td>0.018</td>
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<td>Total</td>
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<td>0.016</td>
<td>-0.203</td>
<td>0.017</td>
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<td>1.317</td>
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<td>Debt/exports</td>
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<td>0.753</td>
<td>0.845</td>
<td>1.018</td>
<td>1.817</td>
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#### Inflation and Growth Indicators

<table>
<thead>
<tr>
<th>Years</th>
<th>1952</th>
<th>1953</th>
<th>1954</th>
<th>1955</th>
<th>1956</th>
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<tbody>
<tr>
<td>Inflation</td>
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<td>20.55</td>
<td>25.84</td>
<td>12.15</td>
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<td>GDP Growth</td>
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<td>9.30</td>
<td>11.10</td>
<td>5.50</td>
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## Main Economic Indicators

### Balance of Payments and External Debt (US$ billions)

<table>
<thead>
<tr>
<th>Years</th>
<th>1957</th>
<th>1958</th>
<th>1959</th>
<th>1960</th>
<th>1961</th>
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</thead>
<tbody>
<tr>
<td>Trade Balance</td>
<td>0.107</td>
<td>0.065</td>
<td>0.072</td>
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<tr>
<td>Exports</td>
<td>1.392</td>
<td>1.244</td>
<td>1.282</td>
<td>1.270</td>
<td>1.404</td>
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<tr>
<td>Imports</td>
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<td>1.179</td>
<td>1.210</td>
<td>1.293</td>
<td>1.292</td>
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<td>Services</td>
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<td>-0.373</td>
<td>-0.459</td>
<td>-0.350</td>
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<td>--</td>
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<tr>
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### Inflation and Growth Indicators

<table>
<thead>
<tr>
<th>Years</th>
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<th>1958</th>
<th>1959</th>
<th>1960</th>
<th>1961</th>
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<tr>
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Main Economic Indicators

Balance of Payments and External Debt (US$ billions)

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<th>1963</th>
<th>1964</th>
<th>1965</th>
<th>1966</th>
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<td>1.294</td>
<td>1.086</td>
<td>0.941</td>
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<td>Services</td>
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<td>-0.259</td>
<td>-0.362</td>
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<td>-0.155</td>
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<td>0.070</td>
<td>0.074</td>
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<tr>
<td>Short Term Capital</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
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Inflation and Growth Indicators

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## Main Economic Indicators

### Balance of Payments and External Debt (US$ billions)

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<td>0.545</td>
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### Inflation and Growth Indicators

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<td>25.45</td>
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<td>14.20</td>
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<td>11.81</td>
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### Main Economic Indicators

#### Balance of Payments and External Debt (US$ billions)

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#### Inflation and Growth Indicators

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### Main Economic Indicators

#### Balance of Payments and External Debt (US$ billions)

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#### Inflation and Growth Indicators

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## Main Economic Indicators

### Balance of Payments and External Debt (US$ billions)

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### Inflation and Growth Indicators

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### Main Economic Indicators

#### Balance of Payments and External Debt (US$ billions)

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#### Inflation and Growth Indicators

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Source: All data in this appendix is from IBGE (1990) and Boletin do Banco Central, several issues.
CHAPTER IV

TIME VARYING PARAMETER MODELS,
UNIT ROOTS AND COINTEGRATION

I) Introduction

For a long time now people have been studying models that allow parameter variation across the sample period. The first attempts at building varying coefficient models were made in the 1950s and 1960s by Rubin (1950), Klein (1953), Kuh (1959), Rao (1965) and others.

Different stories can be told about why one should allow estimated coefficients to change across the sample. The most obvious and reasonable one is that true coefficients themselves are the result of a stochastic process. In terms of a time series model the coefficient may be assumed autocorrelated, depending on some economic variable, or just random with a given mean and variance.

On the other hand, even when the true coefficient is stable, the varying coefficient model can be seen as a way to tackle specification errors such as excluded variables. Thus, a parameter variation not explained by some theoretical consideration can be seen as an evidence of misspecification.

A great encouragement to the use time varying parameter model came from the Lucas critique. According to Lucas when the economic agents have rational expectations the parameters in econometric models will no
longer be constant. Agents will change their behaviour in response to changes in the policy regime. In Lucas words: "To this point, I have argued simply that the standard stable parameter view of econometric theory and quantitative policy appears not to match several important characteristics of economic practice, while an alternative general structure, embodying stochastic parameter drift, matches these characteristics more closely".

Note that if the parameter changes in some parametric way, there will be other "deep parameters" that are still constant, namely the mean and variance of the underlying stochastic process.

On the other hand, it has been noticed that most of the economic time series have a trend pattern. The existence of unit roots in economic series has been confirmed by the work of Granger (1966) and Nelson and Plosser (1982). To retain the stationarity assumption it has been proposed that the variables should be tested for cointegration. If they cointegrate, the estimation of an error correction model seems to be the appropriate course of action.

In this chapter we present the econometric methods that will be used in chapters 5 and 6.

The main objective of this chapter is to discuss different time varying parameter models developed during the last decades, giving special attention to the Kalman filter. The comparison with earlier models seems to point

1) Lucas (1976).
out the superiority of the Kalman filter in dealing with time varying parameter models. Additionally, we also present some discussion on how to deal with non-stationary series.

This chapter has four other sections. In the next section we present the different non-bayesian models that were introduced during the 1970s, exploring also the relations between them. The third section is dedicated to the Kalman filter. It contains not only the derivation of the recursive equations for filtering and smoothing, but also a discussion of some other topics such as different schemes of parameter variation and the Bayesian approach to time varying parameters. In the fourth section we deal with the question of unit roots and cointegration. The fifth section presents the conclusions and remarks.

II) Different Models Proposed

1) Purely Random Model

The purely random coefficient model was one of the first models to tackle the time varying problem. The basic idea is to create a model where the estimated fixed coefficient represents the average of the time varying coefficient distribution. Thus, although it incorporates explicitly the possibility that coefficients are not fixed over the sample period, it does not allow the

2) See, for example, Rao (1965), Hildreth and Houck (1968) and Swamy (1970).
estimation of different coefficients for each point in time.

The basic model can be written as

\[ y_t = X'_t \beta_t + \nu_t \quad (1) \]

\[ \beta_t = \mu + \epsilon_t \quad (2) \]

where,

\[ \nu_t \sim N(0, \sigma^2_v) \] and \[ \epsilon_t \sim N(0, \sigma^2_e). \]

Additionally, both error terms are assumed to be independent and non-autocorrelated. Combining (1) and (2)

\[ y_t = X'_t \mu + u_t \quad (3) \]

where

\[ u_t = X'_t \epsilon_t + \nu_t. \]

One should note that after this substitution the model becomes a fixed coefficient model, with an heteroscedastic error term. Equation (3) can then be estimated by generalized least squares (GLS), and the fixed coefficient \( \mu \) is the estimated mean of \( \beta_t \). In other words, it shows the average change in the dependent variable due to a one unit change in the exogenous variables. The GLS estimator is

\[ b = (X'\Omega^{-1}X)^{-1} X'\Omega^{-1} y. \quad (4) \]

The problem in using this is that normally we do not know the error covariance matrix \( \Omega \), and an estimate of this matrix should be used in (4).

The procedure proposed to obtain the estimated \( \Omega \) is based on the residuals of the ordinary least squares (OLS) estimation of (3)
\[ e = y - X'\beta \]
\[ = Mu \]

where, \( M = I - X(X'X)^{-1}X' \) is a known matrix. Then,
\[ E(ee') = E(Muu'M') \]
\[ = M \Omega M \]

from which,
\[ E(e^*) = M^* \text{Var}(u) \]

where, \( e^* = e^2 \) and \( m^* = m^2 \)

Finally, define \( w = e^* - E(e^*) \) and \( G = M^*X^* \), then:
\[ e^* = E(e^*) + w \]
\[ = M^* \text{Var}(u) + w \]
\[ = M^* (X^*\sigma_{e^2} + \sigma_{v^2}) + w \]
\[ e^* = M^*X^*\sigma_{e^2} + M^*\sigma_{v^2} + w \quad (5) \]

Since \( M \) and \( X \) are known matrices we can estimate (5) by OLS to get estimates of the variances, and hence of \( \Omega \).

As shown by Swamy (1970) and Hildreth and Houck (1968), this procedure leads to an unbiased and consistent estimator of the variance-covariance matrix and a consistent and asymptotically efficient estimator of the mean.

2) The Adaptive Regression Model

This model, developed initially by Cooley and Prescott (1973a), is basically a standard regression model where the intercept term is allowed to vary. It

3) A similar procedure is derived in Theil (1971) p.622-625. The difference is that in Theil's case the random term \( w \) is again heteroscedastic with a known pattern of heteroscedasticity. Thus, he uses GLS to obtain the estimated variances.
assumes that the additive error term is a sum of "not only a transitory element that has effect in the current period, but also a permanent component whose effect persists into the future". 4

One example of this permanent component could be a new variable that, at some point in time, becomes relevant to the explanation of the dependent variable. 5 In this model, the role of the time varying intercept is precisely to take into account this permanent component of the error term.

Normally, when dealing with this kind of problem, especially when the presence of omitted variables is acknowledged, the common procedure is to postulate an autoregressive error term. It is also very common to "adjust" the constant term whenever the ex-post forecasts do not perform very well.

Instead of using these ad hoc procedures Cooley and Prescott propose to define the model allowing the constant term to vary. The model is:

\[ y_t = \alpha_t + X_t' \beta + \epsilon_t \]  \hspace{1cm} (6)
\[ \alpha_t = \alpha_{t-1} + u_t \]  \hspace{1cm} (7)

where

\[ \epsilon_t \sim N(0, (1-\delta)\sigma^2) \]
\[ u_t \sim N(0, \delta \cdot \sigma^2) \]

and \( 0 \leq \delta \leq 1 \).

5) Other possible sources of this permanent component are changes in tastes and technological development.
The parameter $\delta$ is used to measure the relative importance of the permanent component. Note that if there is no such permanent component in the additive error term, that is $\delta = 0$, then $\alpha_t = \alpha_{t-1}$ and we are back to the fixed coefficient model. To obtain the estimating equation, we solve (6) recursively and substitute the result in (7).

$$\alpha_t = \alpha_{t-1} + u_t$$
$$= \alpha_{t-2} + u_t + u_{t-1}$$
$$= \alpha_{t-3} + u_t + u_{t-1} + u_{t-2}$$
$$\vdots$$
$$\vdots$$
$$\alpha_0 + \sum_{i=1}^{t-1} u_{t-i}$$

Then,

$$y_t = \alpha_0 + X_t' \beta + \varepsilon_t + \sum_{i=0}^{t-1} u_{t-i}$$

Actually, because they are concerned about forecasting, Cooley and Prescott solve the recursive calculation the other way around, obtaining $\alpha_t$ in terms of $\alpha_{t+1}$ and the error sum. Instead of adjusting the constant, they propose to estimate the most recent value of it.

$$y_t = \alpha_{t+1} + X_t' \beta_t + \varepsilon_t + \sum_{s=t}^{T} u_s \quad (8)$$

In matrix notation$^6$,

$$y = X' \beta + v \quad (9)$$

where,

---

$^6$) Now $X'$ includes a column of ones as usual.
\[ \beta^* = [\alpha_{t+1}, \beta_1, \beta_2, \ldots, \beta_k] \]

\[ \nu_t = \epsilon_t + \sum_{s=t}^{T} u_s \]

Equation (9) is identical to the standard fixed coefficient regression model. The only problem is the error term, that is now not independent. If \( \delta \) is known then the GLS estimator of \( \beta^* \) and \( \sigma^2 \) is given as

\[ b_\delta = (X' \Omega_\delta X)^{-1} X' \Omega_\delta y \]

\[ S_\delta = T^{-1} (y - X'b_\delta)' \Omega_\delta^{-1} (y - X'b_\delta) \]

where

\[ \Omega_\delta = (1-\delta)I + \delta R \]

and \( R \) is a \( T \times T \) matrix with elements \( r_{ij} \) such that

\[ r_{ij} = \min [t-i+1, T-j+1] \]

If we have no a priori information about the value of \( \delta \), we have to rely on some numerical search technique. Given the likelihood function of the observations

\[ L(y; \beta^*, \sigma^2, \delta) = -\frac{T}{2} \ln 2\pi - \frac{T}{2} \ln \sigma^2 - \frac{1}{2} \ln |\Omega_\delta| \]

\[-\frac{1}{2}\sigma^2 [(y - X \beta^*)' \Omega_\delta^{-1} (y - X \beta^*)] \]

we can substitute the estimators of \( \sigma^2 \) and \( \beta^* \) to get the concentrated likelihood function

\[ L(y, \beta^*, \sigma^2; \delta) = -\frac{T}{2} (\ln 2\pi + 1) - \frac{T}{2} \ln S_\delta - \frac{1}{2} \ln |\Omega_\delta| \]

and then search between zero and one to get a value \( \delta \) that maximizes the concentrated likelihood.

---

7) Note that \( R \) is constructed in this fashion because of the cumulative sums in (8) that make \( u_t \) autocorrelated.
Note that computational cost is a problem for this routine once the variance-covariance matrix has to be inverted several times, given different values of $\delta$. Cooley and Prescott (1973a) solve this problem by transforming the variables in such a way that $\Omega\delta$ is diagonal, thereby making calculation easier. Cooley and Prescott (1973b and 1973c) also show that $b_\delta$ is consistent, asymptotically efficient and robust to specification errors causing structural change over time, relative to least squares methods with and without first order autoregressive errors.

This model, like the purely random one, does not allow the estimation of a path of coefficients over time, as its main focus is forecasting. A rather inconvenient generalization of this model, where slope and intercept are allowed to vary and be estimated in different points in time, is presented by Cooley and Prescott (1973c and 1976). The generalized model can be written as

$$y_t = X' \beta_t + \epsilon_t \quad (10)$$
$$\beta_t = \beta_{t-1} + u_t \quad (11)$$

where

$$\epsilon_t \sim N[0, (1-\delta)\Sigma_\epsilon]$$
$$u_t \sim N[0, \delta \Sigma_u]$$

and $\Sigma_\epsilon$ and $\Sigma_u$ are known up to a scale factor. Following the same steps as before one can solve it recursively to obtain

$$y = X'\beta + v$$

where,
\[ \beta = \beta_{t+1} \]

\[ v_t = \varepsilon_t - X_t' \sum_{s=t}^{T} u_s \]

The composition of the matrix \( R \) used to form \( \Omega_s \) will also change slightly. Its elements are now

\[ r_{ij} = \min \left[ T-i+1, T-j+1 \right] X_i' \Sigma_u X_j \]

or, in the most general case, if we want to estimate the coefficient vector in some period \( t \)

\[ r_{ij} = \min \left[ |t-1|, |t-j| \right] X_i' \Sigma_u X_j \]

Then, given \( \Omega_s \) we can proceed as before.

3) **Systematic Parameter Variation**

Instead of postulating sequentially varying coefficients, this model, due to Belsley (1973a b and c), assumes that coefficients will vary according to a set of economic variables.

\[ y_t = X_t' \beta_t + \varepsilon_t \quad (12) \]

\[ \beta_t = \delta Z_t + u_t \quad (13) \]

\[ \varepsilon_t \sim N(0, \sigma^2) \]

\[ u_t \sim N(0, \sigma_u Q) \]

where \( X_t \) and \( Z_t \) are \( k \) and \( r \) vectors, respectively, and \( \beta \) is a \( k \) vector. Substituting (13) into (12) we get

\[ y_t = X_t' (\delta Z_t + u_t) + \varepsilon_t \]

\[ = (X_t' \otimes Z_t') \Gamma + v_t \]
\[ y = W\Gamma + v \]  
\[ y = W\Gamma + v \]  
where, 
\[ v_t = X'\beta_t + \epsilon_t \]  
\[ w_t = X' \Theta Z' \]  
\[ E(v_t) = E(X'\beta_t + \epsilon_t) = 0 \]  
\[ \text{Var}(v_t) = \text{Var}(X'\beta_t + \epsilon_t) = \sigma^2_e + \sigma^2_u X'Q X \]  
Again, if we stack the observations in (14) to get a more suitable matrix form 
\[ y = W\Gamma + v \]  
where \( y \) and \( v \) are \( T \times 1 \) vectors, \( W \) is a \( T \times k \) matrix and \( \Gamma \) is a \( k \times 1 \) vector (\( \Gamma = \text{vec}(\delta) \)). 
The estimation procedure will depend on the assumptions one makes about \( Z_t \) and \( \beta_t \). In the simplest case where \( Z_t \) is known, one could apply OLS or GLS depending on the variance of \( \beta_t \). If the model is assumed systematic, that is \( \text{Var}(\beta_t) = 0 \), then \( v_t \) is no longer heteroscedastic and we can apply OLS to (15). On the other hand, if we have a random component in \( \beta_t \), we should apply GLS. A solution in this case, when \( Q \) is known, is to follow the same steps proposed for the purely random model, that is use the OLS residuals to get an estimate of the error variances and use these to correct the heteroscedasticity. 
The model actually proposed in Belsley (1973a) is one where \( Z_t \) is not known but \( \text{Var}(\beta_t) \) is equal to zero.
Therefore the central problem is not how to estimate \((15)\), but how to identify the vector \(Z_t\).\(^8\)

Belsley (1973a) proposes a fairly complicated way to get \(Z_t\). The choice of \(Z_t\) could be based on trying different \(Z\)'s in the regression

\[
bt = \delta Z_t + ht \tag{16}
\]

where \(bt\) is an estimator of \(\beta_t\) and \(ht\) is a white noise error, such that

\[
bt = \beta_t + ht \tag{17}
\]

This corresponds to \((13)\), in the case in which \(\text{Var } (ut)\) is zero. The problem is how to get \(bt\). Belsley (1973a) provides an algorithm to generate a series of \(bt\). The basic idea is to find \(bt\) by running regressions of \(y\) on \(X\) for different periods of time, starting with the period \(t = 1\) to \(t = k + \tau\), to get \(b_1\), and keep changing the period up to \(t = T - k - \tau\) to \(t = T\), to get \(bt\). Hence, this "moving-window" technique can provide a series for \(bt\).\(^9\)

This procedure does not completely solve the problem, because as shown by Belsley (1973a) \(bt\) is a biased estimator of \(\beta_t\). This bias will basically depend on the rate of change of \(Z_t\). Whenever \(Z_t\) moves slowly over time the bias would be small.

---

8) A simpler, practical way would be run an OLS regression assuming fixed parameters and then plot the residuals against several possible variables. Although a possible non linearity between \(Z_t\) and the residuals might complicate the conclusions, a simple inspection of the plot may provide good insights.

9) Belsley (1973b) also provides an F-test for constancy of parameters when \(Z\) is not known.
Finally, one should note that Belsley actually does not provide a new way to estimate time varying parameters, but a way to choose the set of explanatory variables $Z_t$. Note also that the proposed method is by no means theoretically superior to simply trying different $Z$s directly in (14). Once $Z_t$ is known the estimation of (14) is trivial.

4) **Switching Regressions**

The switching regression model was initially developed in the 1970s by Goldfeld and Quandt, and various approaches were proposed to estimate and test the switching regression hypothesis. This basically claims that sample observations can be divided into a, usually small, number of different regimes. These are distinguished by the fact that, although the form of the equation is the same throughout the whole period, parameter values may change between regimes.

Thus, supposing there are only two regimes, the switching regression model can be expressed as

$$y_t = X' \beta_1 + u_{1t} \quad \text{if } t \text{ belongs to regime 1} \quad (18)$$

$$y_t = X' \beta_2 + u_{2t} \quad \text{if } t \text{ belongs to regime 2}. \quad (19)$$

In general, we can have two different sets of parameters $(\beta_1, \sigma_1)$ and $(\beta_2, \sigma_2)$.

Note that the central point is that dividing the sample using some kind of a priori knowledge is not

---

10) One could argue in favor of Belsley procedure on the basis of computational costs, since his algorithm involves only inversion of $2x2$ matrices.
possible. If the observations can be classified as belonging to a specific regime, the problem of estimating and performing stability tests is trivial.

The first and simplest approach to this model is due to Quandt (1958), who assumes that, in a sample of size \( n = n_1 + n_2 \), there exist only two regimes such that the first \( n_1 \) observations are associated with the first regime and the last \( n_2 \) observations with the second one. Therefore his problem consists of finding the optimal break point \( (t') \) and then estimating the coefficients in the two periods. Let us define the likelihood function of the observations \( y_t \), given \( t' \), as

\[
L(y_t/t') = (1/2\pi)^{n/2} \sigma_1^{t'} \sigma_2^{(n-t')} \exp \left[ \frac{1}{2} \sigma_1^2 \sum_{t=1}^{t'} (y_t - X_t' \beta_1)^2 \right] \exp \left[ -\frac{1}{2} \sigma_2^2 \sum_{t=t'+1}^{n} (y_t - X_t' \beta_2)^2 \right].
\]  

(20)

Then, one can use a numerical technique to maximize this function for different choices of \( t' \).

Two aspects of this procedure should be highlighted. First, it allows only one jump, at time \( t' \), and secondly, this break point \( t' \) cannot be close to either end of the sample, otherwise the likelihood function may be unbounded.

This model was then extended by Goldfeld and Quandt (1972) and Quandt (1972) to allow for several changes between regimes. Goldfeld and Quandt (1972) suppose that at each point in time nature chooses between the two regimes according to some variables \( Z_t \). They assume that
if \( \sum_{i=1}^{P} \delta_i Z_{it} \leq 0 \) then nature chooses regime 1,

if \( \sum_{i=1}^{P} \delta_i Z_{it} > 0 \) then nature chooses regime 2,

where the \( \delta_s \) are unknown coefficients.\(^{11}\)

Now define a diagonal matrix \( D \) with elements \( d(Z_t) \) such that

\[
d(Z_t) = \begin{cases} 
1 & \text{if } \sum_{i=1}^{P} \delta_i Z_{it} \leq 0 \\
0 & \text{if } \sum_{i=1}^{P} \delta_i Z_{it} > 0.
\end{cases}
\]

Stacking the observations in (18) and (19) and using the matrix \( D \) to combine them we have

\[
y = (I-D)X \beta_1 + DX \beta_2 + w \quad (21)
\]

where \( W = (I-D)u_1 + Du_2 \).

The log-likelihood function can then be written as, (excluding a constant)

\[
L = - (1/2) |\Omega| - (1/2) [(y - (I-D)X \beta_1 + DX \beta_2)'] \Omega^{-1} 
\[
(y - (I-D)X \beta_1 + DX \beta_2)]. \quad (22)
\]

The problem in maximizing (22) is that it implies searching over several different combinations of the diagonal matrix \( D \). To solve this problem Goldfeld and Quandt suggest approximating the unit step function \( d(Z_t) \) by a continuous function. They choose the normal cumulative density function,

\(11\) It is possible that \( Z_t \) includes some or even all the regressors.
\[
d(Z_t) = \sum_{i=1}^{p} \delta_i Z_{it} \int_{-\infty}^{\infty} \frac{1}{\sigma \sqrt{2\pi}} \exp \left[ -\frac{\Phi^2}{2\sigma^2} \right] d\Phi \quad (23)
\]

where \(\sigma\) measures the goodness of fit of the discrimination between the two regimes. For small \(\sigma\) the expression (23) should give a good approximation, but when \(\sigma\) is not small, that is when the two regimes cannot be easily separated, some values of \(d(Z_t)\) will not be close to zero or one.

Thus, one can substitute (23) into (22) and then maximize it to get \(\beta's, \delta's\) and \(\sigma\).\(^{12}\)

Another possibility is to suppose that nature's choice of regime is random. Quandt (1972) develops such a model with stochastic choice of regime. At each point in time there is a probability \(\alpha\) that the observation comes from regime 1 and a probability \(1-\alpha\) that it comes from regime 2.

Given the normality assumption the conditional density of \(y_t\) can be written as a weighted average of the probability density functions.

\[
g(y_t/X_t) = \alpha f_1(y_t/X_t) + (1-\alpha) f_2(y_t/X_t)
\]

\[
= \left( \alpha/\sigma_1 \sqrt{2\pi} \right) \exp\left[ -(1/2\sigma_1^2) \left( y_t - X_t^\prime \beta_1 \right)^2 \right] \\
+ \left( 1-\alpha/\sigma_2 \sqrt{2\pi} \right) \exp\left[ -(1/2\sigma_2^2) \left( y_t - X_t^\prime \beta_2 \right)^2 \right] \quad (24)
\]

\(^{12}\) A more intuitive approach would be to use the estimated \(\delta's\), or the \(d(Z_t)\)s they imply, to separate the sample and then use OLS, but inference carried out using OLS procedures would then be conditional on this sample separation.
from which the likelihood function follows,

\[
L = [g_1 \times g_2 \times g_3 \times \ldots \times g_n].
\]  

(25)

Note that again we face the same potential problem as in (20), that is (25) may be unbounded. One way to overcome this possible problem is to impose a priori that \( \sigma_1 = k \sigma_2 \), where \( k \) is any positive constant, to make (25) bounded. Even if this a priori supposition is not made, Kiefer (1978)\(^{13}\) has shown that there would exist a consistent solution.\(^{14}\)

The generalization for the case of more than two regimes is quite straightforward. If there are \( f \) different regimes one just needs to redefine the log-likelihood function as

\[
L = \sum_{i=1}^{f} \alpha_i f_i
\]

where, \( \sum_{i=1}^{f} \alpha_i = 1 \).

Then the system may switch several times, not only between two specific regimes, but also to newer ones.

A final generalization may allow for the possibility of making a switch from one regime to another depending on which regime is in effect. Goldfeld and Quandt (1973a) provide this generalization, putting the model just described into a Markov chain framework.

13) See also Goldfeld and Quandt (1973b).
14) The problem is more complicated because, as shown in Kiefer (1978), the starting value should itself be consistent. One alternative would be to use a moment generating function, instead of the likelihood function. This procedure is suggested by Quandt and Ramsey (1978), but there is no evidence that it would be better than the likelihood estimation.
Define a transition matrix $\Gamma$ where the $rsth$ element $(r,s = 1, 2)$ is the probability of switching to regime $s$ given that regime $r$ was in effect. Given the probabilities at time zero one can use the matrix $\Gamma$ to obtain the probabilities in all the subsequent periods.

$$
\Gamma = \begin{bmatrix}
\tau_1 & 1-\tau_1 \\
1-\tau_2 & \tau_2
\end{bmatrix}
$$

Now, the probability vector can be written as $\alpha = (\alpha_1 t \ \alpha_2 t)$ where $\alpha_1 t = 1 - \alpha_2 t$. Then, the combined density function can be rewritten as

$$
g = \alpha_1 t f_1(y_t/X_t) + (1-\alpha_1 t) f_2(y_t/X_t)
$$

from which we can write down the likelihood function and maximize it to get $\beta_s, \sigma_s, \tau_s$ and $\alpha_{10}$.  

Finally, one should note that the use of the likelihood function in all these different models gives a natural test for the constancy of parameters. The likelihood ratio test can be used under the null hypothesis that parameters are restricted to be constant.

5) **Some Relations Between the Models**

In some sense we can divide the different models presented here into two categories. The first two models, the purely random (PRM) and the adaptive regression (ARM), can be classified as random models, while the other two, the systematic parameter variation (SPV) and switching regressions (SRM), can be described as systematic models. The basic point is that in the PRM and

15) One just needs to estimate $\alpha_{10}$ and $\tau$ because $\alpha_t$ can be found recursively as $\alpha_t = \alpha_{t-1} \Gamma$, then $\alpha_t = \alpha_0 \Gamma^t$.  

ARM the parameter vector is itself a random variable, while in the SPR and SRM the parameters are deterministic functions of some specific economic variables.\(^{16}\)

Let us first compare the random models. Here the superiority of the ARM over the PRM is clear, not only because it allows the coefficient path to be estimated, but also because the PRM assumes a constant mean for \(\beta_t\) while in the ARM the conditional mean is allowed to vary.

The links between the SPV and the SRM are also easy to establish. In the SPV model with only two regimes \((R=2)\), we can define \(Z_t\) as

\[
Z_t = \begin{cases} 
(1 & 0) & 1 \leq t \leq t' \\
(0 & 1) & t' \leq t \leq T 
\end{cases}
\]

to get the simplest SRM, and as

\[
Z_t = \begin{cases} 
(1 & 0) & \text{with probability } \alpha, \\
(0 & 1) & \text{with probability } 1-\alpha 
\end{cases}
\]

to get the \(\alpha\)-method in the SRM. In both cases the \(\text{Var}(u_t)\) in (13) is supposed to be zero.

The upshot of all this is that the SPV is not a method for estimating time varying parameters, but rather a way of choosing the vector \(Z_t\). In this sense it could be seen as a previous step to the SRM.

Another interesting comparison can be made between the ARM and the SVP. If \(u_t\) is assumed to be distributed with independent increments (13) can be rewritten as

\[
\beta_t = \beta_{t-1} + \delta \Delta Z_t + v_t \quad (26)
\]

where, \(v_t = \Delta u_t\).

---

\(^{16}\) Note that even when nature chooses stochastically between regimes this can be true. Substituting \(\alpha\) by \(d(Z_t)\) in (24), the choice of regime would depend on \(Z_t\).
Note that for small $\Delta Z_t$, (26) can be approximated by (11). But small $\Delta Z_t$, that is $Z_t$ moves slowly over time, is exactly the requirement for the SVP work well.

III) The Kalman Filter

The Kalman filter was originally developed in the engineering literature to deal with random signals.\textsuperscript{17} Its applications to econometrics was probably made difficult because it approaches the standard regression problem in a very different manner. In the way it is usually derived the estimator $b_t$ is a result of regressing $b_t$ on the dependent variable $y_t$, instead of regressing $y_t$ on the exogenous variable $X_t$. That is the conditional mean of $\beta_t$ given $y_t, y_{t-1}, \ldots, y_1$.

As Duncan and Horn (1972) have observed, in comparing the Kalman filter with the standard regression analysis: "Specifically, the central objective in both cases is to estimate $\beta$, the entire vector of regression coefficients involved. Speaking in terms of vector space projections, as Kalman does, the conventional regressionist approaches this problem by projecting an observation $y$ vector onto a linear vector spanned by the columns of an $X$ matrix, that is by 'fitting a regression $y^*$ of $y$ on $X'$. The optimal estimates $b$ of $\beta$ are then found as the coefficients of the orthogonal projection $y^*$ so obtained, that is the coefficient of $X$ in the equation for $y^*$. The Kalman approach, on the other hand, makes use of the fact that $\beta_t$ is now a random variable. By a

\textsuperscript{17} See Kalman (1960).
suitable redefinition of the vector space terms involved, the optimal estimate itself is found directly as an orthogonal projection $bt/\varepsilon$ of $\beta_t$ on a vector space spanned by the $\mu_1, y_1, \ldots, y_s$ of the data involved. In regression terms, this is 'fitting a regression of $\beta_t$ on $\mu_1, y_1, \ldots, y_s'".18

The derivation of the Kalman filter equations presented in this section will be carried out in a regression context since our main objective is to point out its use in estimating time varying parameter regressions. It should be clear though that the Kalman filter may have many other uses. Different kinds of models can be cast in the state space framework, that is as in equations (27) and (28), and, therefore, estimated by the Kalman filter. Examples of this are ARIMA and unobserved components models or, as it is sometimes called, structural time series models.19 We shall explore the state space representation of ARIMA and unobserved components models later in this section.

The basic idea behind the Kalman filter is that estimation is performed recursively in two steps. In the first step the "best" estimate at time $t$, using all the information available up to time $t-1$, is found. Then, this estimate is updated using the new information that becomes available at time $t$. This estimate is the best in

18) See Duncan and Horn (1972) p.816. Note that $\mu_1$ represents the prior mean of $\beta_t$ and $y^*$ the estimated $y$.
the sense that it minimizes the mean square error (MMSE).\textsuperscript{20}

The discrete time Kalman filter model is composed by a measurement and a transition equations, that can be written as

\begin{equation}
Y_t = X_t' \beta_t + \epsilon_t \quad (27)
\end{equation}

\begin{equation}
\beta_t = M_t \beta_{t-1} + S_t c_t + R_t u_t \quad (28)
\end{equation}

where, $\beta_t$ = state vector;

$Y_t$ = vector of actual measurements;

c_t = input vector;

$X_t$, $M_t$, $S_t$, $R_t$ = known fixed matrices with appropriate dimensions.

It is also assumed that

\begin{align*}
E (\epsilon_i, \epsilon_j) & = \delta_{ij} \sigma^2 H_t \quad i, j = 1, \ldots, T \\
E (u_i, u_j) & = \delta_{ij} \sigma^2 Q_t \quad i, j = 1, \ldots, T \\
E (\epsilon_t, u_t) & = 0 \quad t = 1, \ldots, T \\
E (\beta_0, \epsilon') & = 0 \quad t = 1, \ldots, T \\
E (\beta_0, u') & = 0 \quad t = 1, \ldots, T
\end{align*}

where $\delta_{ij}$ is the Kronecker delta. These conditions just state that $\epsilon_t$ and $u_t$ have zero mean and covariance matrix $\sigma^2 H_t$ and $\sigma^2 Q_t$, respectively, are non-autocorrelated, non-correlated between themselves and non-correlated with the state vector at time zero. It is also assumed that $H_t$ and $Q_t$ are known and that both error terms are normally distributed, although normality is not crucial.\textsuperscript{21}

\textsuperscript{20} It is usual to refer to it as the MMSE estimate, instead of minimum variance unbiased estimate, because of the stochastic nature of the parameter.

\textsuperscript{21} Occasionally people prefer to work with a wide sense distribution. In this case the Kalman filter estimator
1) Finding the Prediction Equations

At any point in time we know $b_{t-1}$ and its covariance matrix, which is

$$\sigma^2 \Sigma_{t-1} = E [(bt/t-1 - \beta_t)(bt/t-1 - \beta_t)'] \quad (29)$$

To get the best estimator of $\beta_t$ we should use the transition equation (28), which shows how the state vector changes over time.

$$bt/t-1 = Mt bt-1 + St ct \quad (30)$$

To find the equation to predict the covariance matrix at time $t$, given its value at time $t-1$ let us define the forecast error in (29) as

$$bt/t-1 - \beta_t = Mt (bt-1 - \beta_t) + Rt Ut \quad (31)$$

Substituting (31) in (29)

$$\sigma^2 \Sigma_{t-1} = E \left[ \left[ Mt (bt-1 - \beta_t) + Rt Ut \right] \right]$$

$$= E \left[ Mt (bt-1 - \beta_t) (bt-1 - \beta_t)'M' + Mt (bt-1 - \beta_t) u'R' + Rt Ut (bt-1 - \beta_t)'M' \right]$$

$$+ Rt Ut u'R'$$

$$= E \left[ Mt (bt-1 - \beta_t) (bt-1 - \beta_t)'M' + Rt Ut u'R' \right] + 2 Mt (bt-1 - \beta_t) u'R'$$

$$= Mt E \left[ (bt-1 - \beta_t)(bt-1 - \beta_t) \right] M' +$$

$$+ Rt E (utu') R' - 2 Mt E \left[ (bt-1 - \beta_t)u' \right] R'.$$

would be the MMSE among the linear estimators. Athans (1974) for example uses normality while Harvey and Phillips (1982) work with a wide sense distribution.
It is shown in Jazwinski (1970) that according to the Orthogonal Projection Lemma\textsuperscript{22}

\[
E [(b_{t-1} - \beta_{t-1})u'] = 0
\]

then,

\[
\sigma^2 \Sigma_{t-1} = M_t \sigma^2 \Sigma_{t-1} M' + R_t \sigma^2 Q_t R'
\]

\[
\Sigma_{t-1} = M_t \Sigma_{t-1} M' + R_t Q_t R'.
\] \hspace{1cm} (32)

Thus, (30) and (32) can then be used to predict the state vector $\beta$ and the covariance matrix $\Sigma$ at time $t$, given their estimates at time $t-1$.

2) Finding the Updating Equations

Let us first define the residual $v_t$ as

\[
v_t = Y_t - \hat{Y}_{t-1} = X' \beta_t + \epsilon_t - X' b_{t-1-1}
\]

\[
\hat{v}_t = X' (\beta_t - b_{t-1-1}) + \epsilon_t \hspace{1cm} (33)
\]

Hence,

\[
E (v_t) = X' E (\beta_t - b_{t-1-1}) + E(\epsilon_t) = 0
\]

since $b_t$ is unbiased and,

\[
\text{Var} (v_t) = E (v_t^2)
\]

\[
= X' E [(\beta_t - b_{t-1-1})(\beta_t - b_{t-1-1})'] X_t + E (\epsilon_t^2)
\]

\[
= X' \sigma^2 \Sigma_{t-1} X_t + \sigma^2 H_t
\]

\[
= \sigma^2 [X' \Sigma_{t-1} X_t + H_t]
\]

\[
\text{Var} (v_t) = \sigma^2 F_t \hspace{1cm} (34)
\]

where

\[
F_t = X' \Sigma_{t-1} X_t + H_t \hspace{1cm} (35)
\]

\textsuperscript{22) See Jazwinski (1970) p.202-203.}
The derivation of the updating equations for $b$ and $\Sigma$ can be found as the result of applying GLS to a model with varying parameters.\textsuperscript{23} The trick in making the proof simple is to redefine the model as

$$
\begin{bmatrix}
  b_{t/t-1} \\
  Y_t
\end{bmatrix} =
\begin{bmatrix}
  I \\
  X_t
\end{bmatrix}
\beta_t +
\begin{bmatrix}
  b_{t/t-1} - \beta_t \\
  \epsilon_t
\end{bmatrix}
$$

or in more compact notation

$$
Y^*_t = X^* \beta^*_t + \epsilon^*_t
$$

where the covariance matrix of $\epsilon^*_t$ is

$$
E(\epsilon^*_t, \epsilon^*_t') = \sigma^2 \Omega =
\begin{bmatrix}
  \Sigma_{t/t-1} & 0 \\
  0 & H_t
\end{bmatrix}.
$$

Applying GLS to (36a) we obtain

$$
b_{t/t} = (X^*, \Omega^{-1} X^*)^{-1} X^* \Omega^{-1} Y^*. \quad (38)
$$

To obtain a more suitable formula we can substitute (36) and (37) into (38).

$$
b_{t/t} = (\Sigma_{t/t-1} + X_t H^{-1} X_t)^{-1} (\Sigma_{t/t-1} b_{t/t-1} + X_t H^{-1} Y_t) \quad (39)
$$

Using some well known lemmas of matrix inversion, equation (39) can be rewritten as

$$
b_{t/t} = b_{t/t-1} + K_t (Y_t - X^* b_{t/t-1}) \quad (40)
$$

where the Kalman gain $K_t$ is

$$
K_t = \Sigma_{t/t-1} X_t F^{-1}.
$$

Note that the filter gain is used to improve the state vector forecasts. Every time the state vector is such that predicted and actual $Y_t$ are different, the

\textsuperscript{23} Proof of the updating equations using the GLS principle can be found in Duncan and Horn (1972), Sant (1977), Harvey (1981), and Diderrich (1985).
error is incorporated into the new state vector estimate to make it more accurate. In other words, (40) says that
the expected value of the state vector, given all the information up to time \( t \), can be divided into two components: the expected value of this vector given the information up to \( t-1 \), plus a coefficient times the forecast error of \( Y_t \) given the information available up to \( t-1 \). The filter gain is exactly this coefficient which discounts the forecast error. Following Chow (1983) we can write (40) as

\[
E(\beta_t/Y_t, I_{t-1}) = E(\beta_t/I_{t-1}) + K_t[Y_t - E(Y_t/I_{t-1})]
\]

where, \( Y_t \) stands for the actual observation to time \( t \) and \( I_{t-1} \) for \( (Y_1 Y_2 ... Y_{t-1}) \).

Note that if the forecast error in (40) is zero, then \( b_t = b_{t-1} \) and we are back to the fixed coefficient case. In this sense, the fixed coefficient model can be seen as a special case of the time varying model when we do not use the forecast error at time \( t-1 \) to improve the forecast for the period \( t \).

To get the updating equation for the covariance matrix we can multiply (40) by \(-1\), add \( \beta_t \) and use (27) to obtain

\[
\beta_t - b_t/t = \beta_t - b_{t-1}/t - K_t [X'(\beta_t - b_{t-1}/t) + \epsilon_t].
\]

(41)

Taking the expected value of the cross product of (41) we have,

\[
\sigma^2 \Sigma_t/t = \sigma^2 \Sigma_t/t-1 - K_t [X' \sigma^2 \Sigma_t/t-1 X_t + \sigma^2 H_t] K_t'
\]

using the definitions of \( K_t \) and \( F_t \)

\[
\Sigma_t/t = \Sigma_t/t-1 - \Sigma_t/t-1 X_t F^{-1} X' \Sigma_t/t-1 .
\]

(42)
As we said before, the time varying parameter model is just one of the possible applications of the Kalman filter. In this case $y_t$ is a vector with observations on the endogenous variable, $\beta_t$ a vector of parameters to be estimated, $X_t$ a fixed matrix of exogenous variable, $c_t = 0$ and $H_t = M_t = R_t = I$.

To end the derivation the only thing we need is an estimator for $\sigma^2$. This can be obtained using a maximum likelihood approach. Since $v_t$ in (33) is normal and serially uncorrelated we can write the log likelihood function as

$$
\ln L = -\frac{(T/2)}{2} \ln 2\pi \left[ \frac{1}{2} \sum_{t=1}^{T} \ln \sigma^2 (X_t' \Sigma_{t/t-1} X_t + H_t) + \frac{1}{2} \sum_{t=1}^{T} \frac{v_t^2}{F_t} \right].
$$

Doing some manipulations and using (33) and (35) we get

$$
\ln L = -\frac{(T/2)}{2} \ln 2\pi - \frac{(T/2)}{2} \ln \sigma^2 - \frac{1}{2} \sum_{t=1}^{T} \ln F_t
$$

$$
- \frac{1}{2\sigma^2} \sum_{t=1}^{T} \frac{v_t^2}{F_t}.
$$

Equation (43) can then be maximized with respect to $\sigma^2$ which gives

$$
-(T/2) \left( \frac{1}{\sigma^2} \right) + \frac{(1/2\sigma^4)}{\sum_{t=1}^{T} v_t^2 / F_t} = 0.
$$

Multiplying both sides by $2\sigma^2$ and rearranging we get

---

24) The variances of the state variables is sometimes called hyperparameters, see Harvey (1987).
\[ S^2 = \frac{1}{T} \sum_{t=1}^{T} \frac{v_t^2}{F_t} \quad (44) \]

which is the maximum likelihood estimator of \( \sigma^2 \) conditional on \( M_t, H_t, S_t, X_t, R_t, \) and, \( Q_t. \)

Note also that if \( Q_t \) was not known it could be estimated by maximizing the concentrated log-likelihood. Substituting (44) into (43) we get

\[ \ln L_c = - (T/2) (\ln 2\pi + 1) - (T/2) \ln S^2 - \frac{1}{2} \sum_{t=1}^{T} \ln F_t. \]

Note that we could use the maximization procedure described above to find other unknowns in the model. It sometimes happens that \( M_t \) is not known. That is, for example, the case when we are dealing with ARIMA models.

The problem though is still the same, that is, find \( \sigma^2 \) and \( M_t \) that maximizes the likelihood function given \( H_t, X_t, R_t \) and \( Q_t. \) Once the model is in the state space form and the initial values are determined, the Kalman filter is used to generate the prediction error \( v_t \) that is used as input to the log-likelihood function which is then maximized by numerical optimization.

3) The Recursive Estimation

In short, the Kalman filter can be summarized by the forecast equations (30) and (32) and updating equations (40) and (42). The estimation is done recursively. Given \( b_0 \) and \( \Sigma_0 \) we can use (30) and (32) to obtain an initial estimate and then update this estimate using (40) and (42).
In practice, we can use as starting values a covariance matrix \( \Sigma_0 = kI \) where \( k \) is a large number. The value for \( b_0 \) can be assumed to be zero. Note that \( b_0 \) does not matter very much because as \( k \) is large \( \Sigma_0 \) will dominate \( b_0 \). The initial estimates will probably be not very precise, but as the Kalman filter is time invariant one can smooth the coefficients after filtering it.

One can also, as suggested by Sant (1977), use the equivalence between GLS and the Kalman filter to obtain the initial conditions. If one has enough degrees of freedom one can apply GLS to the first \( m \) observations to get \( b_m \) and \( \Sigma_m \) and then use them as starting values to filter the other \( T - m \) observations.

If on the other hand, we know the model is stationary we could use the mean and the covariance matrix of the prior distribution of \( b_0 \). Since the model (27) and (28) has no constant term we should set \( b_0 = 0 \), and \( \Sigma_0 \) could be found by solving equation (32) as

\[
\Sigma_0 = M_t \Sigma_0 M'_t + R_t Q_t R'_t.
\]

Thus, the actual estimation consists in, given \( b_0 \) and \( \Sigma_0 \), find \( b_{t-1}/t \) and \( \Sigma_{t-1}/t \) and after this \( v_t \) and \( F_t \). Once the new observation \( Y_t \) is available, one can then get the updated estimates \( b_t/t \) and \( \Sigma_t/t \). In this procedure the key equations are (30), (32), (33), (34), (40) and (42).

4) The Stochastically Convergent Coefficient Model

The stochastically convergent coefficient model, also called "return to normality model", was first used by Rosenberg (1973) and more recently by Harvey and Phillips (1982). In this model the parameter in each point in time is a linear combination of what it was last period and some fixed value

\[ \alpha_t = \phi \alpha_{t-1} + (1 - \phi) \mu + u_t \]  \hspace{1cm} (45)

or, rearranging the terms

\[ \alpha_t - \mu = \phi (\alpha_{t-1} - \mu) + u_t \]  \hspace{1cm} (46)

where

\[ u_t \sim N(0, \sigma^2 \Sigma_t) \]

and \( \phi \) is a diagonal matrix that satisfies stationarity condition, that is, it has eigenvalues less than one in absolute value.

In this model the coefficient \( \alpha_t \) is variable but changes slowly over time around a fixed mean \( \mu \). Note that in the usual transition equation, where \( \alpha \) follows a random walk process, its value at the end of the sample can be substantially different from the beginning of the sample. Thus, in cases where a substantial structural change is expected one would prefer to use the random walk approximation, whereas in the case of smooth changes over time the stochastically convergent coefficient model seems to be more appropriate.

One should also note that the purely random model, as well as the random walk specification of \( \beta_t \), can be regarded as special cases of the stochastically convergent coefficient model. If \( \phi = 0 \) we are back to the
\[ P(\beta_t / I_t) = P(\beta_t / Y_t, I_{t-1}) = P(\beta_t / V_t, I_{t-1}) \]

\[ P(Y_t / \beta_t, I_{t-1}) = P(V_t / \beta_t, I_{t-1}). \]

This is true because, since \( M_t, S_t, c_t \) and \( X_t \) are known, observing \( V_t \) is the same as observing \( Y_t \). Then (49) can be rewritten as

\[ P(\beta_t / V_t, I_{t-1}) = P(V_t / \beta_t, I_{t-1}) \cdot P(\beta_t / I_{t-1}). \quad (51) \]

The distribution of the likelihood \( P(V_t / \beta_t, I_{t-1}) \) is normal and its mean and variance are as defined before.

To finally obtain the posterior distribution from (51) one can use some standard results in multivariate statistics to get \(^{29}\)

\[ (\beta_t / V_t, I_{t-1}) = N \left[ \frac{\mu_{t-1} + K_{t}(Y_t - X'b_{t-1})}{\Sigma_{t-1} + K_{t}X't^{-1}X'} \right]. \quad (52) \]

A simple comparison between (52) and (40) and (42) makes clear that the updating equations can be seen as the mean and variance of the posterior distribution. The Kalman filter can then be seen as an update procedure from which beliefs about the state vector are updated depending on how good was the prior guess.

Finally one should note that normality is not crucial to prove the equivalence between the GLS and bayesian approaches. In deriving (50) and (52) normal distribution was used to simplify matters. \(^{30}\)

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\(^{29}\) See, for example, Chow (1983) p.8-14.

\(^{30}\) The general proof of the updating equations via Bayes' Theorem can be found in West and Harrison (1989).
purely random model whereas if $\phi = 1$ we get (28), when $M_t = I$ and $c_t = 0$.

To formulate this model in the Kalman filter framework we may define

$$\beta_t = (\mu', \delta')'$$

where

$$\mu_t = \mu$$ and $$\delta_t = \alpha_t - \mu$$, then

$$Y_t = X^* ' \beta_t$$  \hspace{1cm} (47)$$

$$\begin{bmatrix} \mu_t \\ \delta_t \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & \phi \end{bmatrix} \begin{bmatrix} \mu_{t-1} \\ \delta_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} u_t$$  \hspace{1cm} (48)$$

where $X^* ' = (X'_t \ X'_t)$.

5) The Bayesian Approach

Until now we have remained in the classical statistics framework. Nevertheless, the time varying parameters model of equations (27) and (28) can also be approached from a bayesian view point. We will try to show in this section that as far as the prediction and updating equations are concerned the classical and bayesian approaches are equivalent. The difference is in the treatment of the variances.

5.1) A Bayesian Interpretation of the Prediction and Updating Equations

The Kalman filter updating equations were derived above using the GLS principle. What is shown in the literature is that these equations can also be derived from a bayesian point of view.\textsuperscript{26}

\textsuperscript{26} See for example Meinhold and Singpurwalla (1983).
The Bayesian approach to statistics is mainly concerned with how information changes the beliefs one has about hypotheses and parameter values. In Lindley's words: "The main subject matter of statistics is the study of how data (events) change degrees of belief; from prior, by observation of A, to posterior". This is the idea behind Bayes' Theorem on conditional probability

\[ P(H_n/A) \propto P(A/H_n) P(H_n) \]

where \( n = 1, 2, \ldots \), and \( A \) is fixed.

Put simply, it says that a prior belief or probability of a set of hypotheses \( P(H_n) \), can be transformed in a posterior belief or probability \( P(H_n/A) \), given the likelihood of \( H_n \) on \( A \) \( P(A/H_n) \). This sequential approach to estimation also lies at the heart of the Kalman filter.

Let us take equation (27) and (28) and rewrite Bayes' Theorem as

\[ P(\beta_t/It) = P(Y_t/\beta_t, It-1) P(\beta_t/It-1) \]

where, as before \( It-1 \) stands for \( (Y_1, Y_2, \ldots, Y_{t-1}) \). At the beginning one knows the prior distribution \( (\beta_t/It-1) \)

\[ (\beta_t/It-1) \sim N(M_t b_{t-1}, M_t \Sigma_{t-1} M_t' + R_t Q_t R_t') \]

where \( b_{t-1} \) and \( \Sigma_{t-1} \) are the mean and covariance matrix of \( (\beta_{t-1}/It-1) \). To get (50) we just used (27), (28) and some basic results of the normal distribution.28

Note also that using the residual \( v_t \), defined in (33), we can rewrite the posterior probability and the likelihood of \( Y_t \) as

5.2) **Variance Learning and Discount Factors**

The basic difference between the classical and bayesian approaches to the time varying parameters models is the estimation of the observational and evolution variances. In the classical framework the hyperparameters are estimated by a maximum likelihood procedure, as described in equations (43) and (44). On the other hand, the bayesian approach uses variance learning and discount factors when dealing with the variances.

Let us consider the simplest dynamic linear model represented by equations (53) and (56). The observational variance $V$ is estimated sequentially as new observations become available.

\[
Y_t = \theta_t + \nu_t \\
\theta_t = \theta_{t-1} + \omega_t \\
\nu_t \sim \mathcal{N}(0, V) \\
\omega_t \sim \mathcal{N}(0, W_t)
\]

The point estimate of $V$ is $S_{t-1}$ defined by equations (57) to (59). The updating equations now include the updating equation for $S_{t-1}$. As more observations become available we correct our previous estimate of $V$. Note that $n_t$ represent the degrees of freedom at each time, but $d_t$ is not exactly the residual sum of squares. If $S_{t-1}/Q_t = 1$, then $d_t$ is the residual sum of squares.

\[
S_t = d_t/n_t \quad (57) \\
n_t = n_{t-1} + 1 \quad (58) \\
d_t = d_{t-1} + e_t^2 S_{t-1}/Q_t \quad (59) \\
e_t = \text{one step forecast errors (} Y_t - \hat{Y}_t \text{)} \\
Q_t = \text{variance of the one step forecast}
\]
A correction is made every time the squared residuals is different from its expected value, since $E(\epsilon_t) = \nu_t$. Note that since $\delta_t = n_t \sigma_t$, equation (59) can be rewritten as

$$n_t \sigma_t^2 = n_{t-1} \sigma_{t-1}^2 + \sigma_{t-1}^2 \frac{\epsilon_t}{\nu_t}$$

$$n_t \sigma_t^2 = (n_{t-1}) \sigma_{t-1}^2 + \sigma_{t-1}^2 \frac{\epsilon_t}{\nu_t}$$

$$\sigma_t^2 = \sigma_{t-1}^2 + \sigma_{t-1}^2/n_t \left[ (\frac{\epsilon_t}{\nu_t}) - 1 \right]. \quad (60)$$

Every time $\epsilon_t = \nu_t$ there will be no correction in the estimated variance. If the square residuals is greater then its expected value the expression in brackets in equation (60) will be positive and then estimated variance will be corrected upwards and vice versa. To start the estimations some value for $\sigma_0$ must be available.

The evolution variance $W_t$ is not directly estimated. Ameen and Harrison (1984, 1985a, 1985b) show how the estimation of $W_t$ can be replaced by choosing a discount factor $0 < \delta \leq 1$. The discount factor can be regarded as describing how fast the system is changing. It tells us how informative an observation is as it ages. If $\delta = 1$, or equivalently $W = 0$, we have the fixed parameters model.

6) Autoregressive Residuals

It may be the case that after the estimation by the Kalman filter evidence of residual autocorrelation is found. The model should then be augmented in order to
account for the presence of serial correlation. Consider the following augmented model:

\[ Y_t = X_t' \beta_t + \epsilon_t \quad (61) \]

\[ \beta_t = \beta_{t-1} + u_t \quad (62) \]

\[ \epsilon_t = \rho \epsilon_{t-1} + \eta_t \quad (63) \]

where \( \eta_t \) is white noise error.

This is a simplified version of the state space model of equations (27) and (28) that includes first order autocorrelated residuals. Apart from the residual autocorrelation, all other hypotheses related to the state space model are maintained. In addition we also assume that

\[ E (\eta_i, u_j) = 0 \quad \text{for } i, j = 1, \ldots, T \]

\[ E (\eta_i, \beta_j) = 0 \quad \text{for } i, j = 1, \ldots, T \]

\[ E (\beta_0, \eta') = 0 \quad \text{for } t = 1, \ldots, T. \]

Using (61) and (63) we can then write the quasi first differences as

\[ Y_t = \rho Y_{t-1} + X_t' \beta_t - X_t' \rho \beta_{t-1} + \eta_t. \quad (64) \]

Let us define

\[ \delta_t = \begin{bmatrix} \rho \\ \beta_t \\ -\rho \beta_{t-1} \end{bmatrix}, \quad Z_t' = \begin{bmatrix} Y_{t-1} \\ X_t' \\ X_{t-1}' \end{bmatrix} \quad \text{and} \quad \xi = \begin{bmatrix} 0 \\ u_t \\ -\rho u_{t-1} \end{bmatrix}. \]

The state space model can then be written as

---

31) For an application of this kind of model, see Wolff (1985).
\[ Y_t = Z_t' \delta_t + \eta_t \quad (65) \]

\[ \delta_t = \delta_{t-1} + \xi_t. \quad (66) \]

Equations (65) and (66) can then be estimated using the Kalman filter. In the model above \( \rho \) is supposed to be fixed. It can be estimated by a recursive procedure from the filter residuals in a Cochrane-Orcutt fashion.

7) Recursive Least Squares

The recursive least squares estimation can also be performed by the Kalman filter once the relevant model is cast in the state space framework. Note that in this case the true parameter is not time varying. Therefore, one should expect to see the state vector settling down to a constant as more data are used. The Kalman filter is in this case simply an algorithm for repeating the OLS estimation as the sample size increases.

Consider the simplified state space model

\[ Y_t = X_t' \beta_t + \epsilon_t \quad (67) \]

\[ \beta_t = \beta_{t-1} \quad (68) \]

where \( \epsilon_t \sim N(0, \sigma^2) \) and \( \beta_0 = \beta_{t-m} \sim N(\beta_{t-m}, \Sigma_{t-m}) \).

Therefore, after using the first \( m \) observations to obtain the initial conditions the filter can then be started. The updating equations are given by

\[ b_{t-1} = b_{t-1} = (X'X)_{t-1}^{-1} (X'Y)_{t-1} \]

\[ \Sigma_{t-1} = \Sigma_{t-1} = \sigma^2 (X'X)_{t-1}^{-1} \]

while the prediction equations are
\[ b_t = b_{t-1} + k_t [Y_t - X'_t b_{t-1}] \]
\[ \Sigma_t = \Sigma_{t/1-t-1} - \Sigma_{t/1-t-1} X_t f_t^{-1} X'_t \Sigma_{t/1-t-1} \]

where,
\[ k_t = (X'X)^{-1}_{t-1} X'_t f_t^{-1} \]
\[ f_t = \sigma^2 [1 + X'_t (X'X)^{-1}_{t-1} X_t]. \]

8) **ARIMA Models in State Space Form**

As we have said before, ARIMA models can also be easily cast in the state space framework and, therefore, estimated by the Kalman filter.\(^{32}\) Consider the AR(2) model
\[ y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \eta_t. \]

Its state space representation can be written as
\[ y_t = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \beta_{1t} \\ \beta_{2t} \end{bmatrix} \begin{bmatrix} X'_t \\ \beta_t \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \eta_t. \]

In this case the \( M_t \) matrix is not known and, therefore, will have to be estimated using the maximum likelihood approach described before.

Unfortunately the state space representation is not unique. Consider the ARMA (2,1) model
\[ y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \epsilon_t \theta_1 \epsilon_{t-1}. \]

The model above can be represented as

\(^{32}\) For more details of how ARIMA model can be in state space form see Priestley (1981) and Akaike (1976).
Let us now consider the general case of an ARMA (p, q) model:

\[ y_t = \phi_1 y_{t-1} + \ldots + \phi_p y_{t-m} + \epsilon_t + \theta_1 \epsilon_{t-1} + \ldots + \theta_q \epsilon_{t-q+1} \]

where \( m = \max(p, q+1) \). The state space representation for the general ARMA model can be written as

\[
\begin{bmatrix}
\beta_{1t} \\
\beta_{2t} \\
\ldots \\
\beta_{mt}
\end{bmatrix} = \begin{bmatrix} 0 & 1 \\
\phi_2 & \phi_1 \\
\vdots & \vdots \\
\end{bmatrix} \begin{bmatrix}
\beta_{1t-1} \\
\beta_{2t-1} \\
\ldots \\
\beta_{mt-1}
\end{bmatrix} + \begin{bmatrix} 0 \\
1 \\
\vdots \\
\end{bmatrix} \epsilon_t.
\]

Let us now consider the general case of a ARMA (p, q) model:

\[ y_t = \phi_1 y_{t-1} + \ldots + \phi_p y_{t-m} + \epsilon_t + \theta_1 \epsilon_{t-1} + \ldots + \theta_q \epsilon_{t-q+1} \]

where \( m = \max(p, q+1) \). The state space representation for the general ARMA model can be written as

\[
\begin{bmatrix}
\beta_{1t} \\
\beta_{2t} \\
\ldots \\
\beta_{mt}
\end{bmatrix} = \begin{bmatrix} 0 & 1 \\
\phi_2 & \phi_1 \\
\vdots & \vdots \\
\end{bmatrix} \begin{bmatrix}
\beta_{1t-1} \\
\beta_{2t-1} \\
\ldots \\
\beta_{mt-1}
\end{bmatrix} + \begin{bmatrix} 0 \\
1 \\
\vdots \\
\end{bmatrix} \epsilon_t.
\]

33) The ARIMA representation can be obtained by substituting \( y_t \) by \( z_t \), where \( z_t = \Delta y_t \).
9) Unobserved Components Models

In this framework a time series is modeled by decomposing it into its basic forming elements. That is, it is decomposed into a tendency ($\mu_t$), cycle ($\psi_t$), seasonal ($\gamma_t$) and irregular ($\epsilon_t$) components.

$$y_t = \mu_t + \psi_t + \gamma_t + \epsilon_t$$

Note that the components themselves are time varying. That is, the model allows for a changeable tendency, cycle and seasonal pattern.

The complete model should specify also the behaviour of each one of the individual components. A complete treatment of unobserved component models is beyond the scope of this thesis. Therefore, we will concentrate on the case of the "basic structural time series model".\textsuperscript{34}

In "basic structural time series model" only three components are used since the cycle component is omitted. The tendency is modelled as a random walk with a time varying drift while the drift itself is a random walk. The seasonal component is modelled by a combination of sine and cosine waves. The model can be express as

\[\begin{bmatrix}
\beta_{1t} \\
\vdots \\
\beta_{mt}
\end{bmatrix} = \begin{bmatrix}
\phi_1 & 1 \\
\phi_2 & 0 \\
\vdots & \vdots \\
\phi_m & 0
\end{bmatrix} \begin{bmatrix}
\beta_{1t-1} \\
\vdots \\
\beta_{mt-1}
\end{bmatrix} + \begin{bmatrix}
1 \\
\vdots \\
\theta_{m-1}
\end{bmatrix} \epsilon_t.\]

\textsuperscript{34) The trend plus cycle model is discussed in Portugal (1991). A complete discussion of unobserved components models is found in Harvey (1989).}
\[
\gamma_t = \mu_t + \gamma_t + \epsilon_t \\
\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t \\
\beta_t = \beta_{t-1} + \zeta_t \\
(69)
\]

\[
\gamma_t = \sum_{j=1}^{s/2} \gamma'_{jt}
\]

where \( \gamma'_{tj} \) is formed by

\[
\begin{bmatrix}
\gamma_{jt} \\
\gamma_{jt}^*
\end{bmatrix}
= \begin{bmatrix}
\cos \lambda_j & \sin \lambda_j \\
-sin \lambda_j & \cos \lambda_j
\end{bmatrix}
\begin{bmatrix}
\gamma_{jt-1} \\
\gamma_{jt-1}^*
\end{bmatrix} + \begin{bmatrix}
\omega_{jt} \\
\omega_{jt}^*
\end{bmatrix}
\]

(70)

\[
\gamma_{jt} = \gamma_{jt-1} + \omega_{jt}
\]

\[
\gamma_{jt}^* = -\gamma_{jt-1} + \omega_{jt}^*
\]

\[
\gamma_{2t} = -\gamma_{2t-1} + \omega_{2t}
\]

\[
\gamma_{2t}^* = -\gamma_{2t-1} + \omega_{2t}^*
\]

(71)

The state space representation of the equations (69) and (71) is

\[
\gamma_t = \begin{bmatrix}
1 & 0 & 1 & 0 & 0
\end{bmatrix} \alpha_t + \epsilon_t
\]
10) **Smoothing**

Until now we have been dealing with filtering. In this section we shall discuss the opposite problem, that is smoothing. The Kalman filter is a forward looking algorithm that produces estimates of the state vector at time $t$ using all the information available until time $t-1$. The smoothed estimates, on the other hand, use all the information in the sample. It starts at time $T$ and goes backwards giving the best estimations for time $T-1$, $T-2$ and so on. The smoothing algorithm is started by $b_{T/T}$ and $\Sigma_{T/T}$ given by the Kalman filter and works backwards until $t=1$.

Let $b_{t/T}$ and $\Sigma_{t/T}$ denote the state vector and covariance matrix smoothed estimators. Then, the smoothing equations can be written as

\[
b_{t/T} = b_t + \Sigma^* (b_{t+1/T} - M_{t+1} b_t)
\]

and

\[
\Sigma_{t/T} = \Sigma_t + \Sigma^* (\Sigma_{t+1/T} - \Sigma_{t+1/t}) \Sigma^*
\]

where

\[
\Sigma^* = \Sigma_t M_{t+1}^{-1} \Sigma_{t+1/t}^{-1}, \quad t = T-1, \ldots, 1
\]

This is the optimal fixed interval smoother. There are other smoothing algorithms that can be used but the
fixed point smoother seems to be the most popular in econometrics.\textsuperscript{35}

IV) Trending Variables, Unit Roots and Cointegration

Until recently it was quite common to assume at least weak stationarity when estimating regression equations. Nevertheless as, Granger pointed out some time ago, most economic time series have a "typical spectral shape", with most power concentrated at low frequencies. That is, they exhibit a strong trend and, consequently, are non-stationary.\textsuperscript{36} In terms of import demand equations, several authors have acknowledged the presence and importance of trends. Following Khan and Ross (1975) they have tried to model separately the "secular" and cyclical features of import demand.

Differencing the data to remove the trend in a Box and Jenkins fashion does not seem to be the solution. By differencing all the data prior to estimation the long run properties of the model are lost. A better approach is to tackle this problem within the model and not in the context of the variables themselves.

The theory concerning integrated and cointegrated series and their relation with the error correction mechanism is now well known and, therefore, will not be

\textsuperscript{35) See Anderson and Moore (1979) for a description of other smoothing algorithms.}
\textsuperscript{36) See, Granger (1966) and also Granger and Newbild (1974). A more recent discussion of this problem, with tests for unit roots for many macroeconomic time series, can be found in Nelson and Plosser (1982).}
reviewed in detail here.\textsuperscript{37} Rather, we shall concentrate only on some points related to the estimation of the error correction vector and recent developments that extend the cointegration ideas to seasonal series.

1) \textbf{The Engle-Granger Procedure}

Let us start by defining a integrated series of order $d$, say $I(d)$, as a series that needs to be differenced not more than $d$ times to become stationary.

$$x_t - I(d)$$

The primary problem of dealing with $I(d)$ series is that the usual statistical properties of the first and second moments do not hold. Thus the usual distributional theory cannot be used. A different distributional theory is needed for non-stationary series.

Note that as a linear combination of $I(d)$ series is also $I(d)$, the error term in a regression involving $I(d)$ variables is going to be $I(d)$. Nevertheless, there may be a linear combination of $I(d)$ series that is $I(d-b)$ for $b > 0$. In that case the series are said to be cointegrated. Let us assume that $d = b = 1$, then

$$x_t - I(1) \text{ and } y_t - I(1)$$

but

$$z_t = y_t - a x_{t-1} - I(0)$$

where $a$ is the cointegrating vector.

The relation between cointegration and the error correction mechanism was established by Engle and Granger.

\textsuperscript{37} For a discussion on this questions see Engle and Granger (1987) and Hendry (1986).
Engle and Granger (1987) suggested a two step procedure when dealing with an error correction mechanism. In the first step, the cointegrating vector, containing the long run coefficients, is estimated from a static regression in levels. In the second step, the error correction term, that is the residuals from the first step, is used in a equation in differences to obtain the impact coefficients.

The interesting feature of this procedure is that both steps only involve single equation estimation by least squares. In the first step, using $I(1)$ series, all the long run information is extracted and considerations about dynamics are ignored. Moreover, as a by-product of this first step one obtains the cointegrated regression Durbin-Watson that is used as one of the tests for cointegration.\(^{38}\)

The dynamic structure is incorporated only in the second step when the variables appear in differences, while the restrictions on the levels from the first step are incorporated through the error correction term.

Stock (1987) has shown that the coefficients from the first step are not only consistent but in fact "super

---

38) Since unit root tests are relatively well known and also because of space restrictions we will not discuss such tests.
consistent", converging faster to the true parameter. This is an important result because it allows us to regard the estimated error correction vector from the first step as the true one when performing the second step. This result justifies the omission of the dynamics in the first step and the incorporation of the cross equation restrictions through the error correction term only in the second step.

Although there seems to be no problem in large samples, Stock (1987) also finds evidence of a sizeable small sample bias. This bias is especially relevant in cointegrated regressions with a low $R^2$.

Nevertheless, there are some points that are overlooked by the two-step procedure. Firstly, if one has more than one explanatory variable, as is often the case, there may be multiple cointegrating vectors. Therefore the Engle-Granger procedure will result in the estimation of a linear combination of different cointegrating vectors. Only in the special case when all cointegrating vectors are equal will the two-step procedure lead to a sensible result.

Secondly, many economic time series exhibit a seasonal pattern. In frequency domain terms an I(1) series has a peak at low frequencies, that is the behaviour of the series, especially its infinite variance, comes from the contribution of the low frequencies, or long run part of the series. An analogous case happens when there is a peak in the seasonal frequencies. Therefore one should ask not only if two series are
cointegrated in the usual sense, but also if they are seasonally cointegrated.

These two aspects concerning multiple cointegrating vectors and seasonal cointegration are discussed below.

2) The Johansen Procedure

Recently, a new procedure has been proposed to find the cointegrating vector allowing for the possibility of more than one vector. The Johansen procedure was first put forward by Johansen (1988). The critical values for the test of multiple cointegrating vectors, as well as a practical application to the demand for money in Denmark and Finland were presented by Johansen and Juselius (1990).

The starting point is a $p$ dimensional vector autoregressive representation

$$A(L) X_t = \varepsilon_t$$

where $\varepsilon_t \sim N(0, \Omega)$.

Let us rewrite the VAR as

$$X_t = A_1 X_{t-1} + A_2 X_{t-2} + \ldots + A_k X_{t-k} + \varepsilon_t$$

or

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \ldots + \Gamma_{k-1} \Delta X_{t-k-1} + \Gamma_k X_{t-k} + \varepsilon_t$$

where $\Gamma_i = -I + A_1 + \ldots + A_i$ for $i = 1, 2, \ldots, k$

so that $-\Gamma_k = A(1) = -I + A_1 + A_2 + \ldots + A_k$.

When $\Gamma_k$ has either zero or full rank, we are in the usual cases of a VAR in differences or a stationary $X_t$.

39) Dickey and Pantula (1987) also provide a test for multiples cointegrating vectors. Because of space limitations and as it is based in the well known Dickey and Fuller framework of estimating auxiliary regressions, we will not discuss it here.
given a VAR in levels. The interesting case is when the rank($\Gamma_k$) = $r$ for $0 < r < p$, that is $\Gamma_k$ is less than full rank. So let us assume that $|A(L)|$ has unit roots, so that the cointegrating matrix $A(1)$ is less than full rank.

So far there is no substantial difference from the Engle-Granger approach. As they pointed out, if a vector is cointegrated the VAR either in levels or in differences will not be an appropriate representation of the data generation process. A VAR in levels will not take into account the fact that variables are integrated, therefore omitting important constraints, while a VAR in differences will be misspecified, given the omission of the term in levels, that is the error correction term.

The existence of multiple cointegrating vectors can be formulated as the hypotheses that

$$\Gamma_k = \alpha \beta'$$

where $\beta$ is a $p \times r$ matrix that has the cointegrating vectors in its columns and $\alpha$ is also a $p \times r$ matrix containing the feedback effect or the loading vector as Johansen calls it.

Johansen suggests starting by isolating the $\Gamma_k$ influence using partitioned regressions and then using the result to make inference on $\Gamma_k$ by likelihood methods. Two auxiliary regressions can be estimated. First regress $\Delta x_t$ on $\Delta x_{t-1}$, $\Delta x_{t-2}$, ..., $\Delta x_{t-k-1}$, and then regress $x_{t-k}$ on $\Delta x_{t-1}$, $\Delta x_{t-2}$, ..., $\Delta x_{t-k-1}$, to obtain the residuals $R_{\Delta t}$ and $R_{kt}$, respectively. These two quantities are $\Delta x_t$ and $x_t$ free of the influence of the other
variables involved. An eventual regression of Rot on Rkt would be actually the same as regressing ΔXt on Xt without the influence of the other variables in differences.

The problem is then to maximize the likelihood function

\[ L(\alpha, \beta, \Omega) = \left| \Omega \right|^{-T/2} \exp \left\{ -\frac{1}{2} \sum_{t=1}^{T} (\text{Rot} + \alpha \beta'Rkt)' \Omega^{-1} (\text{Rot} + \alpha \beta'Rkt) \right\} . \]

The likelihood function can then be concentrated in terms of the parameters of interest, that is the cointegrating vector matrix \( \beta \). Taking the first derivative of the likelihood function with respect to \( \Omega \) and \( \alpha \) one obtains the well known results

\[ \hat{\Omega}(\beta) = S_{oo} - S_{ok} \beta (\beta'S_{kk} \beta)^{-1} \beta'S_{ko} \]

\[ \hat{\alpha}(\beta) = - S_{ok} \beta (\beta'S_{kk} \beta)^{-1} \]

where \( T^{-1} \sum_{t=1}^{T} R_{it}R_{jt} \) for \( i,j = 0, k \).

That is, \( S_{ij} \) is the product moment matrix of the residuals.

Using the expressions for \( \hat{\Omega}(\beta) \) and \( \hat{\alpha}(\beta) \) one can then concentrate the likelihood which is now proportional to \( \left| \hat{\Omega}(\beta) \right|^{-t/2} \).

\[ L_c(\beta) = \left| S_{oo} - S_{ok} \beta (\beta'S_{kk} \beta)^{-1} \beta'S_{ko} \right| \]

Because of the proportionality, maximizing the concentrated likelihood is the same as minimizing \( \left| \hat{\Omega}(\beta) \right| \). The estimation of \( \hat{\beta} \) can be more easily achieved by the
minimisation problem. Given some matrix relations the
above expression can be rewritten as
\[ |(\beta' S_{kk} \beta - \beta' S_{ko} S_{oo} S_{ok} \beta)| / |\beta' S_{kk} \beta| \]
which has to be minimized with respect to \( \beta \).

As shown by Johansen (1988) this entails the
calculation of the eigenvalues of \( S_{ko} S_{oo} S_{ok} \) with
respect to \( S_{kk} \), that is
\[ |\lambda S_{kk} - S_{ko} S_{oo} S_{ok}| = 0. \]

The problem with the formulation above is that it
is not a standard eigenvalue problem. Johansen and
Juselius (1990) have shown that by using the Cholesky
decomposition of \( S_{kk} = C'C \) for some non singular pxp
matrix \( C \), the same eigenvalues can be found by solving
\[ |\lambda I - C^{-1} S_{ko} S_{oo} S_{ok} C^{-1}| = 0. \]
Although the eigenvalues are the same, this expression
will give different eigenvectors \( e_1, e_2, \ldots, e_p \). Thus
they will have to be transformed as \( C^{-1} e \).

Putting the problem in these terms is quite useful
since most econometric programs will be able to solve the
above equation.

Let us then put the estimated eigenvalues
\( \hat{\lambda}_1 > \hat{\lambda}_2 > \hat{\lambda}_p \) in this decreasing order into a diagonal
matrix \( D \), and the corresponding eigenvectors in a matrix
\( E \), then
\[ S_{kk} ED = S_{ko} S_{oo} S_{ok} E \]
where \( E \) is normalised such that \( E'S_{kk}E=I \).

Then the estimator \( \hat{\beta} \) is given by the first \( r \) columns
of \( E \), that is the first \( r \) eigenvectors of \( S_{ko} S_{oo} S_{ok} \nwith \) respect to \( S_{kk} \). As the eigenvectors are normalised
by the condition $\beta'S_{kk} \beta = I$, the solution for $\alpha$, $\Gamma_k$ and $\Omega$ is

$$\hat{\alpha} = -S_{0k} \beta (\hat{\beta}'S_{kk} \hat{\beta})^{-1} = -S_{0k} \beta$$

$$\hat{\Gamma}_k = \hat{\alpha} \beta' = -S_{0k} \beta \beta'$$

$$\hat{\Omega} = S_{00} - S_{0k} \beta \beta' S_{k0} = S_{00} - \hat{\alpha} \hat{\alpha}'$$.

The final point is how to find $r$, the number of cointegrating vectors. This is done by a likelihood ratio test. The test that there exists at most $r$ cointegrating vectors is applied sequentially for decreasing values of $r$. Therefore, if there are four explanatory variables one should start by testing for at most four cointegrating vectors. If it is accepted then test for at most three cointegrating vectors and so on.

Johansen and Juselius (1990) and Johansen (1991) provide two tests, as well as their critical values, for the number of cointegrating vectors. They are the trace and $\lambda_{max}$ tests. The trace test is defined as

$$-2 \ln(q) = -T \sum_{i=r+1}^{p} (1-\lambda_i).$$

The $\lambda_{max}$ test is simply the difference between successive trace statistics.

3) Seasonal Cointegration

So far we have been dealing with cointegration applied to trended variables. In other words, how to deal
with variables that are trended but have a specific linear combination that is not trended. The same kind of question can be addressed to seasonal series.

In frequency domain terms, an I(d) series has a peak at low frequencies. Similarly, a seasonally integrated series, say SI(d_s), has a peak at seasonal frequencies. As before, if two variables are seasonally integrated of order d but there exists a linear combination of them that is seasonally integrated of order d_s - b_s for b_s > 0, these series are said to be seasonally cointegrated. Taking again the case where d_s = b_s = 1 one can write

\[ x_t - SI(1) \text{ and } y_t - SI(1) \]

but

\[ z_t = y_t - a x_t - SI(0) \]

where a is the seasonal cointegrating vector.

This question has been analysed by Dickey, Hasza and Fuller (1984), Engle Granger and Hallman (1989) and Hylleberg et alii (1990).

In terms of testing for seasonal unit roots the problem can be seen as testing for a polynomial \((1-L^4)\), as the usual test for unit roots involve testing for \((1-L)\). Dickey, Hasza and Fuller (1984) have offered tables with the critical values for such a test. As the usual DF test it is based on the auxiliary regression

\[ \Delta y_t = \beta y_{t-4} + \epsilon_t \]

The test for seasonal unit roots is then a "t-test" on \(\beta\). If the residuals are not white noise the test can be augmented by adding extra lags of \(\Delta y_t\).
The problem with this approach is that it only tests for seasonal unit roots at all frequencies. The test is actually a joint test. The possibility of having seasonal unit roots at only some frequencies is not contemplated. Such a test is provided by Hylleberg et alii (1990).

Let us start by defining a series as being integrated of order $d$ at frequency $\theta$ if it has a spectrum $f(\omega)$ which takes the form

$$f(\omega) = c (\omega - \theta)^{-2d}$$

for $\omega$ near $\theta$. In their notation this is denoted $I_{\theta}(d)$. In the usual case of a peak at the zero frequency we write $I_{0}(d)$.

Consider now a series with peaks at seasonal frequencies $\omega_s = 2\pi j/s$ for $j = 1, 2, \ldots, s/2$. Concentrating on the quarterly case we have $\pi/2$ (one cycle per year) and $\pi$ (two cycles per year). A vector $x_t$ which has all its components $I_{\theta}(d)$ is said to be cointegrated at that frequency if there exists a vector $b_\theta$ such that

$$z^\theta = x_t b_\theta$$

is integrated of order lower than $\theta$. Finally, to unify the notation, a series that is $I_{0}(d)$ and $S_{1}(d_s)$ can be denoted by $S_{1}(d_0,d_s)$.

To derive the test for unit roots let us consider an $x_t$ series generated by

$$(1-L^4) x_t = \epsilon_t.$$  

The polynomial $(1-L^4)$ can be factorized as

$$(1-L^4) x_t = (1-L)(1+L+L^2+L^3) x_t$$

$$= (1-L)(1+L)(1-L^2) x_t$$
\[ = (1-L) S(L) x_t. \]

Normally, inference is based on \((1-L)\) to see if \(x_t\) is integrated. Hylleberg et alii (1990) suggest to do inference on \(S(L)\) also to look for seasonal unit roots. So the objective is to test for the roots \(+1\), \(-1\), \(+i\) and \(-i\), which solve the above polynomial.

To perform such a test it is suggested to estimate

\[ x_{4t} = \pi_1 x_{1t-1} + \pi_2 x_{2t-1} + \pi_3 x_{3t-2} + \pi_4 x_{3t-1} + \epsilon_t \]

where

\[
x_{1t} = (1 + L + L^2 + L^3) x_t
\]
\[
x_{2t} = -(1 - L + L^2 - L^3) x_t
\]
\[
x_{3t} = -(1 - L^2) x_t
\]
\[
x_{4t} = (1 - L^4) x_t.
\]

The estimation can be done by least squares, using extra lags of \(x_{4t}\) to ensure that \(\epsilon_t\) is white noise.

The test for the roots \(+1\) and \(-1\) is done by testing whether \(\pi_1 = 0\) and \(\pi_2 = 0\), respectively. For the complex roots \(+i\) and \(-i\) we need to test jointly \(\pi_3 = 0\) and \(\pi_4 = 0\). Therefore, if \(\pi_2\) and either \(\pi_3\) or \(\pi_4\) are different from zero there will be no seasonal unit root. On the other hand if \(\pi_1\) is different from zero there will be no unit root associated with the trend. Therefore, to establish if a series has no unit roots at either seasonal or zero frequency, being then stationary, one must test that each of the \(\pi_s\), except for either \(\pi_3\) or \(\pi_4\), is different from zero. The tables with critical values to test the \(\pi_s\) is provided by Hylleberg et alii (1990).
Given these considerations about seasonal cointegration changes should be made in the Engle-Granger procedure. As Engle, Granger and Hallman (1989) have shown if there exist seasonal unit roots the coefficients in the cointegrated regression are no longer super consistent. The superconsistency result will hold only under the restriction that both cointegrating vectors, at zero and seasonal frequency, are equal.

An alternative would be to test for seasonal unit roots first and then, if any are found, use a filter to remove their effect before applying the Engle-Granger procedure. A possible choice of filter, if seasonal unit roots are found in all seasonal frequencies, is \((1-L^4)/(1-L)\). Therefore, one would generate new variables

\[
\begin{align*}
\hat{x}_t &= [(1-L^4)/(1-L)] x_t \\
\hat{y}_t &= [(1-L^4)/(1-L)] y_t
\end{align*}
\]

and then apply the two step procedure on \(\hat{x}_t\) and \(\hat{y}_t\). If there is a seasonal unit root only in some specific frequency the appropriate filter, suggested by the test, should be used. In the quarterly data case either \((1+L)/(1-L)\) or \((1+L^2)/(1-L)\). The problem with this approach is that the consequences of pre-testing for seasonal unit roots on the Engle-Granger procedure are not known.

Another problem is that although one can test each series for seasonal unit roots, there is no available test for seasonal cointegration. Testing the residuals of the cointegrated regression is obviously not satisfactory since the superconsistency result is lost when seasonal
unit roots are present. The test proposed by Hylleberg et alii (1990) would possibly have to be changed, or at least the critical values would be different.

A possible test for seasonal cointegration would be to estimate a "seasonal cointegrated regression", after removing a possible zero frequency unit root, and then to test the residuals. Thus one could estimate

\[ \Delta x_t = \sum_{j=0}^{s-2} \alpha_j \Delta y_{t-j} + \epsilon_t \]

and then test \( \epsilon_t \) for seasonal unit roots in the same approach proposed above. The problem is that in this case the critical values provided by Hylleberg et alii (1990) are no longer applicable. Changes would have to be made to allow for the estimated \( \alpha_s \).

Finally, it should be noted that seasonal cointegration tests, as in the case with tests for cointegration at zero frequency\(^40\), are troubled by structural breaks. Hall and Scott (1990) show that changes in the seasonal pattern may lead to a S(1,1) result when the series is actually S(1,0).

4) **Encompassing**

Encompassing tries, basically, to provide a framework for model comparison. The idea that new models should account for previous models findings was first put

\(^{40}\) Perron (1989) has shown that an I(0) series with a shift in the mean may come out as an I(1). He proposed some unit root tests that account for structural breaks, based on a graphical inspection of the series. More recently, Zivot and Andrews(1990) have developed a variant of this test that endogenises the break point.
forward by Davidson et alii (1978) and later formalised by Mizon (1984) and Mizon and Richard (1986).

Let us follow Hendry (1988) and put the problem in terms of comparing two non nested models $M_1$ and $M_2$. These models have parameter vectors $\alpha$ and $\beta$ while the data generation process actual parameter vector is $\theta$. Both $\alpha$ and $\beta$ are functions of $\theta$ since they are simple tentative ways of approximating $\theta$. Let us denote these functions by $\alpha(\theta)$ and $\beta(\theta)$. Suppose, for the sake of argument, that $M_1$ is the true representation of the data generation process. One can then obtain $\beta(\alpha)$, which represents how $M_1$ would anticipate $M_2$ when $M_1$ is the true representation of the data generation process.

Let $\psi = \beta(\theta) - \beta(\alpha(\theta))$. The first term tells us how $\beta$ is anticipated given $\theta$ while the second term indicates how $\beta$ is anticipated by $\alpha$, which is a function of $\theta$. If this difference, called the population encompassing difference, is zero then $M_1$ encompasses $M_2$, or $M_1 \in M_2$. On the other hand, if $M_1 \not\in M_2$ then there are some features of the data generation process reflected by $M_2$ that are not accounted for by $M_1$. If $M_1 \not\in M_2$ then $M_2$ is a redundant model, it provides no extra information in relation to $M_1$.

Although $M_1$ and $M_2$ are non nested it is possible to find another model $M_{\min}$, the minimal model, in which $M_1$ and $M_2$ are nested. The obvious choice in the regression model is a combination that accommodates, without repetition, all variables present in $M_1$ and $M_2$. 
Obviously $M_m \subset M_1$ and $M_m \subset M_2$ since $M_m$ contains both models. To account for the degrees of freedom correction one can use the concept of parsimonious encompassing. Then, if $M_i \subset M_m$ it must be true that $M_i \subset M_2$ since the only extra source of information present in $M_m$, apart from $M_i$, is that of $M_2$. Actually, in linear models $M_i \subset M_2$ if and only if $M_i \subset M_m$.

Several encompassing tests have been proposed in the literature. We are going to highlight five of them.41

i) The Cox Test

This is a variance based test. The central idea is that if $M_i$ is true then $M_2$ should give a poor fit. The test statistic is the difference between the maximised log likelihood under both hypotheses, less a estimate of how much this difference is expected to be under the hypotheses that $M_i$ is true.42

$$T_0 = [\text{L}_{i \text{max}} - \text{L}_{2 \text{max}}] - \hat{\sigma}^2 [\text{L}_{i \text{max}} - \text{L}_{2 \text{max}}]$$

A significant negative value for the statistic implies that $M_2$ fits much better than would be expected if $M_i$ were true. In the linear regression case the test statistic $D_0 = T_0 / \text{SE}(T_0)$ is asymptotically a $N(0,1)$.

$$T_0 = n/2 \log \left( \frac{\sigma_i^2}{\sigma_1^2} \right)$$

where $\sigma_i^2$ is the actual error variance in $M_2$ and $\sigma_1^2$ is the error variance that $M_2$ would have if $M_i$ were the correct model.

41) Two references that present different encompassing tests are Mackinnon (1983) and Ericsson (1983).
42) The peculiar notation used here follows Ericsson (1983).
ii) The J Test

Now consider the two models

\[ M_1: \ y = X_1 \alpha_1 + u_1 \quad u_1 \sim N(0, \sigma_1^2 I) \]

\[ M_2: \ y = X_2 \alpha_2 + u_2 \quad u_2 \sim N(0, \sigma_2^2 I). \]

For simplicity let us assume that there are no common elements in \( X_1 \) and \( X_2 \). A combined model can be written as

\[ y = (1-\delta) (X_1 \alpha_1) + \delta (X_2 \alpha_2) + \epsilon \]

and the null hypothesis that \( \delta = 0 \) corresponds to saying that \( M_1 \) is true or \( M_1 \in M_2 \). In finite samples there will be of course more than just two values (0 or 1) for \( \delta \), that is the discrimination will not be perfect.

The suggestion is then to use the fitted value of \( M_2 \) in the composite regression. Under \( M_2 \), \( X_2 \alpha_2 \) may be approximated by its estimated value. The combined regression is then

\[ y = (1-\delta) (X_1 \alpha_1) + \delta \hat{y} + v \]

A t test can then be performed to test the null hypothesis that \( \delta = 0 \), that is the forecast given by \( M_2 \) does not offer any new information about \( y \). If we accept that \( \delta = 0 \) then \( M_1 \in M_2 \), if the null is rejected then \( M_1 \not\in M_2 \).

iii) The F Test

An alternative test is to perform a F test on the combined model, to test the joint hypotheses that \( \delta \alpha_2 = 0 \). Since both models are nested with the combined
model this is just a usual F test. If the models have any common variable repetition should obviously be avoided.

iv) The Sargan IV Test

Sargan has proposed a test for over identified restrictions after estimation by instrumental variables that can be used for encompassing test purposes. Given a set of instruments Z with m elements, the test statistic, which is distributed as a $X^2$ with $m - k_1$ degrees of freedom, can be defined as

$$C_1 = y' (N - Q_1) y / \sigma^2_1$$

where

$$N = Z (Z'Z)^{-1} Z'$$
$$P_1 = X_i (X'NX_i)^{-1} X'_i$$
$$Q_1 = N P_1$$

and $S^2_1$ is the instrumental variable estimator of the variance in $M_1$.

In this context $C_1$ can be seen as testing the non nested hypotheses that $M_1$ encompasses the unrestricted reduced form equation for $y$.

v) The Ericsson IV Test

This test is a variance based test like the Cox test but is derived using the instrumental variables framework in the same fashion as the Sargan test. The test statistic is

$$C_6 = y' (Q_2 - P'_1 Q_2 P_1) y / \sigma^2_1 n^{1/2}.$$
IV) Conclusions

In this chapter we have presented different procedures to estimate time varying parameter models. In terms of the random models, the superiority of the Kalman filter is quite obvious. Not only the computational costs are small, because there is no need for sequential matrix inversion, but also it provides a natural way of estimating the parameter covariance matrix and to carry out the decomposition of the prediction error.

The comparison with the systematic parameter variation models is not so straightforward, but the Kalman filter seems also to be superior. A systematic parameter variation could be accommodated in the Kalman filter framework using the input vector $c_t$. It also avoids problems such as the unboundedness of the likelihood function.

The Kalman filter seems to offer a complete and flexible framework for dealing with the problem of parameter variation. It can be used in both classical and bayesian frameworks. The main difference between these two approaches is the way the variances of the random walk processes are estimated. In the classical framework these hyperparameters are estimated using a maximum likelihood method, while in the bayesian case the observational variance is estimated by a learning process and the question of estimating the evolution variance is dealt with by choosing discount factors for the different parts of the model.
On the other hand, we have also presented a discussion of some aspects concerning unit roots, cointegration and the error correction mechanism. Since many economic variables are I(1), the error correction mechanism seems to offer a simple and efficient way to estimate the short and long run elasticities.

The Kalman filter and the error correction mechanism will play a crucial role on the estimations performed in the next two chapters.
CHAPTER V

ESTIMATIONS OF IMPORT DEMAND EQUATIONS

I) Introduction

In this chapter we present the empirical results for different models of imports demand. We concentrate on single equation models. Since Brazil can be considered a small country only the demand equation needs to be estimated. Prices in international markets are supposed to be unaffected by the Brazilian actions. Therefore, the supply curves are taken to be completely elastic. The imperfect substitute hypothesis is adopted and, additionally, we also assume absence of money illusion and homogeneity between price and tariffs.

We start with a fixed coefficient model. Most of the work that has been done on import demand has implicitly or explicitly assumed stationarity. In the fourth chapter we have included some discussion of how to deal with non-stationary series and in this chapter the estimation of an error correction mechanism for Brazilian import demand is presented.

The advantage of using an error correction mechanism is that economic theory is used to establish only the long run relationship between the variables, while the short run dynamics is data determined. We start with a

1) For a more complete discussion of different trade models see chapter two.
general model and after successive restrictions that are data acceptable end up with a simpler model.

After presenting and discussing the results of the fixed coefficient models, we then turn to the time varying parameter model. The econometric techniques to handle this kind of model was discussed in chapter four. Basically, the coefficients are allowed to follow a random walk and the Kalman filter is used to obtain the time path of the coefficients. We try to keep the same structure that we had for the fixed coefficients model, especially with respect to the error correction approach.

We use not only data on total imports but also on intermediate and capital goods imports. For each of these categories quarterly and annual estimations are presented.

The chapter has another three sections. In the next section we discuss the results of the fixed coefficient models while in the third section the empirical results of the time varying models are presented. In both cases there are two sub-sections, one presenting the annual estimations and another with the quarterly results. In each of these sub-sections the imports of total, capital and intermediate goods is treated separately. The last section contains the conclusions.

II) Fixed Parameter Models

In most of the current econometric literature, the cointegration techniques, briefly outlined in chapter four, have been used to deal with the modelling of long
run equilibrium and short run dynamics. In this section we present the results obtained from applying this technique to Brazilian import demand for total, intermediate and capital goods.

We first test the series for the order of integration using the cointegrated regression Durbin-Watson (CRDW), Dickey-Fuller (DF) and augmented Dickey-Fuller (ADF) tests and the Phillips and Perron test (PP). The tests were performed in all three versions, that is, without constant, with constant and with constant and trend. To save space though, the DF and ADF tests are presented in the version without constant or time trend while the PP test presented is calculated from an auxiliary regression with a constant. The time trend was not included because graphical inspection of the series pointed to a random walk without a drift process.

The cointegrated regression is estimated to obtain the long run elasticities and tests for cointegration are presented. The error correction equation is then estimated in a general to specific framework to obtain the short run dynamics. When possible encompassing tests are performed. We started with four lags for the quarterly data estimations and two lags for the annual

2) A application of the Johansen Procedure to import demand equations can be found in Urbain (1989).
3) The critical values for these tests can be found in Sargan and Bhargava (1983) and Fuller (1976).
4) A description of these tests can be found in Fuller (1976), Dickey and Fuller (1979,1981), Phillips (1987) and Phillips and Perron (1988).
data estimations and tested down to find the more adequate model.\footnote{All estimations were performed using RATS 3.11 and PC-Give 6.01.}

The Johansen procedure is also used and the results are presented in appendix V.\footnote{See Stock (1987).} No evidence of multiple cointegrating vectors is found and therefore we decided to retain the Engle-Granger methodology. It should be said though, that some discrepancy in the estimated cointegrated vector exists between the two methods. It is sometimes claimed that the Johansen procedure gives better results since it is a maximum likelihood process and also because it avoids a possible small sample bias that may be present in the Engle and Granger estimate. In our case though, as we intend to use cointegration in a varying parameters framework the Engle and Granger two step procedure is the logical option.

II. 1) \textit{Quarterly Data Estimations}

As noted in the fourth chapter the question of unit roots is not confined to trended variables. The same type of analysis can also be applied to seasonality. Therefore, before testing for unit roots at the zero frequency one should test for the presence of seasonal unit roots.

Tests for seasonal integration have shown the presence of seasonal unit roots in the output and capacity utilisation series, while the remaining series have shown no signs of it. As the theory for dealing with
seasonally integrated series is not well developed yet we choose to ignore such result. It should be noticed that in all series \( \pi_1 \) indicates presence of unit roots at the zero frequency.

Table V.1

<table>
<thead>
<tr>
<th>( \pi_1 )</th>
<th>( \pi_2 )</th>
<th>( \pi_3 )</th>
<th>( \pi_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>lqm1q</td>
<td>-0.667</td>
<td>-4.083</td>
<td>-3.016</td>
</tr>
<tr>
<td>lrer6q</td>
<td>0.439</td>
<td>-1.456</td>
<td>-3.025</td>
</tr>
<tr>
<td>ly4q</td>
<td>0.742</td>
<td>-3.157</td>
<td>-0.497</td>
</tr>
<tr>
<td>lu1q</td>
<td>-0.634</td>
<td>-0.829</td>
<td>-2.849</td>
</tr>
<tr>
<td>lqm2q</td>
<td>-0.106</td>
<td>-4.173</td>
<td>-3.032</td>
</tr>
<tr>
<td>lrer7q</td>
<td>0.783</td>
<td>-4.402</td>
<td>-3.241</td>
</tr>
<tr>
<td>ly1q</td>
<td>0.963</td>
<td>-0.620</td>
<td>-0.188</td>
</tr>
<tr>
<td>lu2q</td>
<td>-0.532</td>
<td>-0.898</td>
<td>-2.940</td>
</tr>
<tr>
<td>ldkgq</td>
<td>-0.545</td>
<td>-5.274</td>
<td>-2.930</td>
</tr>
</tbody>
</table>

The data used consist of seasonally unadjusted quarterly series from 1975 to 1988, except for total imports where sample period starts in 1976. A full description of the data and sources can be found in appendix V.2

1) Import Demand for Intermediate Goods

The table V.2 shows the unit root tests for the variables used in the intermediate goods import demand equation. The variables in levels are all I(1) and their first differences are I(0) as expected.

The cointegrating regression is presented in equation 1. The values for the price and income elasticities close to one and the large value for the elasticity with respect to capacity utilisation are in
line with findings of other authors. Fachada (1990) reports -0.87, 1.16 and 2.88 for the period 1976-1988, while Abreu (1986) using data from 1976 to 1985 obtained -0.74, 1.13 and 1.87.

EQ(1) Modelling lqm2q by OLS
The sample is 1975(1) to 1987(4)

<table>
<thead>
<tr>
<th>variable</th>
<th>coeff</th>
<th>S.E.</th>
<th>H.C.S.E.</th>
<th>t-value</th>
<th>partial r²</th>
</tr>
</thead>
<tbody>
<tr>
<td>lrer7qt</td>
<td>-0.908</td>
<td>.16730</td>
<td>.19691</td>
<td>-5.424</td>
<td>.3800</td>
</tr>
<tr>
<td>ly1qt</td>
<td>.972</td>
<td>.19747</td>
<td>.29316</td>
<td>4.924</td>
<td>.3356</td>
</tr>
<tr>
<td>lu2qt</td>
<td>3.672</td>
<td>.41065</td>
<td>.45170</td>
<td>8.943</td>
<td>.6249</td>
</tr>
<tr>
<td>const</td>
<td>-11.722</td>
<td>2.06999</td>
<td>2.22185</td>
<td>-5.663</td>
<td>.4005</td>
</tr>
</tbody>
</table>

R² = 0.770  σ = 0.1349  F(3,48) = 53.51  DW = 0.961
DF = -4.537 ADF = -4.048 PP = -4.63

Table V.2

Unit Root Tests - 1975/1988

<table>
<thead>
<tr>
<th></th>
<th>CRDW</th>
<th>DF</th>
<th>ADF</th>
<th>PP</th>
</tr>
</thead>
<tbody>
<tr>
<td>lqm2q</td>
<td>0.462</td>
<td>-0.167</td>
<td>-0.046</td>
<td>-2.64</td>
</tr>
<tr>
<td>lrer7q</td>
<td>0.283</td>
<td>0.360</td>
<td>0.382</td>
<td>-1.74</td>
</tr>
<tr>
<td>ly1q</td>
<td>0.121</td>
<td>1.362</td>
<td>0.828</td>
<td>-2.32</td>
</tr>
<tr>
<td>lu2q</td>
<td>0.212</td>
<td>-0.158</td>
<td>-0.017</td>
<td>-1.91</td>
</tr>
<tr>
<td>Δlqm2q</td>
<td>2.171</td>
<td>-7.949</td>
<td>-7.989</td>
<td>-8.83</td>
</tr>
<tr>
<td>Δlrer7q</td>
<td>2.137</td>
<td>-7.771</td>
<td>-4.629</td>
<td>-8.34</td>
</tr>
<tr>
<td>Δly1q</td>
<td>2.120</td>
<td>-7.952</td>
<td>-2.111</td>
<td>-9.52</td>
</tr>
<tr>
<td>Δlu2q</td>
<td>1.927</td>
<td>-7.156</td>
<td>-4.995</td>
<td>-7.22</td>
</tr>
</tbody>
</table>

Given the error correction vector from the above equation the error correction mechanism is presented in equation 2. The results below seem to be quite reasonable. All the variables are significant and have the expected sign. The Lagrange Multiplier, White and Reset tests indicate no serial correlation, heterocedasticity or misspecification. The Chow forecasting test points to coefficient stability while
the ARCH test shows no indication of autoregressive conditional heterocedasticity.

EQ(2) Modelling $\Delta lqm2q$ by OLS
The sample is 1975(3) to 1987(4)

<table>
<thead>
<tr>
<th>variable</th>
<th>coeff</th>
<th>S.E.</th>
<th>H.C.S.E.</th>
<th>t-value</th>
<th>partial $r^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta rer7qt$</td>
<td>-0.889</td>
<td>0.16818</td>
<td>0.18163</td>
<td>-5.286</td>
<td>0.3884</td>
</tr>
<tr>
<td>$\Delta ly1qt$</td>
<td>0.700</td>
<td>0.33103</td>
<td>0.26758</td>
<td>2.115</td>
<td>0.0923</td>
</tr>
<tr>
<td>$\Delta ly1qt$</td>
<td>0.741</td>
<td>0.35106</td>
<td>0.42984</td>
<td>2.112</td>
<td>0.0920</td>
</tr>
<tr>
<td>$\Delta lu2qt$</td>
<td>2.346</td>
<td>0.71531</td>
<td>1.51116</td>
<td>3.279</td>
<td>0.1964</td>
</tr>
<tr>
<td>cvt-1</td>
<td>-0.539</td>
<td>0.12837</td>
<td>0.16563</td>
<td>-4.201</td>
<td>0.2862</td>
</tr>
<tr>
<td>const</td>
<td>-0.014</td>
<td>0.01590</td>
<td>0.01709</td>
<td>-0.863</td>
<td>0.0166</td>
</tr>
</tbody>
</table>

$R^2 = 0.708$  $\sigma = 0.1055$  $F(5,44) = 21.32$

$DW = 2.010$  $RSS = 0.4898$

Information Criteria: SC = -4.156  HQ = -4.298  FPE = 0.012

<table>
<thead>
<tr>
<th>date</th>
<th>actual</th>
<th>forecast</th>
<th>Y - Yhat</th>
<th>forecast SE</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1988 1</td>
<td>-0.2483</td>
<td>-0.1925</td>
<td>-0.0559</td>
<td>0.1086</td>
<td>-5.141</td>
</tr>
<tr>
<td>1988 2</td>
<td>0.0451</td>
<td>0.1271</td>
<td>-0.0820</td>
<td>0.1116</td>
<td>-7.347</td>
</tr>
<tr>
<td>1988 3</td>
<td>0.1889</td>
<td>0.1667</td>
<td>-0.0223</td>
<td>0.1103</td>
<td>-2.015</td>
</tr>
<tr>
<td>1988 4</td>
<td>0.0366</td>
<td>-0.0512</td>
<td>0.0878</td>
<td>0.1100</td>
<td>0.7984</td>
</tr>
</tbody>
</table>

Chow (4,44) = 0.38  LM(4) = 2.569  LM(8) = 8.809

ARCH(4) = 1.878  WHITE(10,33) = 1.170  RESET(1,43) = 0.420.

From equation 2 we can see that the impact of the real exchange rate and real income is well concentrated in the first quarter. In the real exchange rate case the impact elasticity is -0.889, leaving only -0.019 to be accounted for later on. A similar but less strong pattern is followed by the income elasticity. The impact elasticity is 0.700 rising to 0.741 after one quarter. Therefore, 76% of the total adjustment happens in the first two quarters, leaving the remaining 24% for later on. For capacity utilisation, the impact elasticity accounts for only 64% of the total adjustment, leaving more than a third of it to be accomplished subsequently.
Finally, although we are using seasonally unadjusted data the inclusion of seasonal dummy variables does not appear necessary. It seems that most of the seasonal behaviour in the import series is accounted for by the seasonality in the income variable. The inclusion of seasonal dummy variables invariably leads to the income variables becoming not significant. The same can be said for the other equations below. 7

2) Import Demand for Capital Goods

We start by attempting to estimate an import demand for capital goods in the same fashion as above. Nevertheless, given the drastic reduction in the imports of capital goods during the period under consideration, the income elasticity always came out with the wrong sign for different definitions of income. This is a problem recognised in other attempts to estimate an import equation for capital goods using quarterly data for the period post 1975. 8 In 1988 the quantity of capital goods imports was 66% of the level in 1975.

Some authors have suggested that this reduction is basically related to an import substitution process related to the Second National Plan of Development (II PND) that was taking place. The idea is that the creation of some new industries in the capital goods sector had the impact of reducing the income elasticity

7) This comment is also valid for the total and capital goods imports.
for this type of imports. Therefore, all econometric work on the import demand for capital goods has included some kind of trend variable to account for this fact.

EQ(3) Modelling lm3q by OLS
The sample is 1975(3) to 1987(4)

<table>
<thead>
<tr>
<th>variable</th>
<th>coeff</th>
<th>S.E.</th>
<th>H.C.S.E.</th>
<th>t-value</th>
<th>partial r^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>lrer8qt</td>
<td>-0.477</td>
<td>0.1109</td>
<td>0.11235</td>
<td>-4.300</td>
<td>0.2908</td>
</tr>
<tr>
<td>ly4qt</td>
<td>1.254</td>
<td>0.2542</td>
<td>0.22354</td>
<td>4.932</td>
<td>0.3509</td>
</tr>
<tr>
<td>lu3qt-2</td>
<td>0.965</td>
<td>0.19136</td>
<td>0.22532</td>
<td>5.043</td>
<td>0.3610</td>
</tr>
<tr>
<td>trend</td>
<td>-0.021</td>
<td>0.00278</td>
<td>0.00268</td>
<td>-7.635</td>
<td>0.5643</td>
</tr>
<tr>
<td>const</td>
<td>-2.099</td>
<td>1.04566</td>
<td>1.20072</td>
<td>-2.007</td>
<td>0.0822</td>
</tr>
</tbody>
</table>

R^2 = 0.914 σ = 0.1320 F(4,45) = 118.94

DW = 1.986 RSS = 0.7839

Information Criteria: SC = -3.764 HQ = -3.883 FPE = 0.019

<table>
<thead>
<tr>
<th>date</th>
<th>actual</th>
<th>forecast</th>
<th>Y - Yhat</th>
<th>forecast SE</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1988 1</td>
<td>4.7459</td>
<td>4.6517</td>
<td>0.0945</td>
<td>0.1436</td>
<td>0.6571</td>
</tr>
<tr>
<td>1988 2</td>
<td>4.9207</td>
<td>4.7809</td>
<td>0.1399</td>
<td>0.1467</td>
<td>0.9529</td>
</tr>
<tr>
<td>1988 3</td>
<td>4.8737</td>
<td>4.7973</td>
<td>0.0764</td>
<td>0.1456</td>
<td>0.5249</td>
</tr>
<tr>
<td>1988 4</td>
<td>4.8811</td>
<td>4.7010</td>
<td>0.1800</td>
<td>0.1484</td>
<td>1.2131</td>
</tr>
</tbody>
</table>

Chow (4,45) = 0.55 LM(4) = 0.468 LM(8) = 5.472

ARCH(4) = 0.944 WHITE(8,36) = 0.972 RESET(1,44) = 1.172.

As a first approximation we follow the same route. Instead of using an error correction mechanism, we first estimate an equation in levels adding a time trend. We adopt a general to specific approach, starting with an equation with four lags of all variables and then searching for a more parsimonious model using successive tests. The result is presented in equation 3.

The main difference between the above equation and the ones available in the literature is the capacity utilisation variable that here appears lagged two

9) See, for example, Castro and Souza (1985).
periods, whereas it normally is a contemporaneous variable. Some encompassing tests can be done since in this case the dependent variable is the same.

The results are overwhelming in favour of our model (M1). According to each and every one of the tests our model is accepted, that is, it encompasses previous models.

Table V.3

Encompassing Test Statistics

M1 is: lqm3qt on 1rer8qt ly4qt lu3qt-2 trend const
M2 is: lqm3qt on 1rer8qt ly4qt lu3qt trend const

Instruments used:
1rer8qt ly4qt lu3qt-2 trend const lu3qt
\[ \sigma_1 = 0.1320 \quad \sigma_2 = 0.1430 \quad \sigma_{[\text{Joint}]} = 0.1330 \]

<table>
<thead>
<tr>
<th>Form M1 v M2</th>
<th>Test Form M2 v M1</th>
</tr>
</thead>
<tbody>
<tr>
<td>-.754 N(0,1) Cox N(0,1)</td>
<td>-4.216</td>
</tr>
<tr>
<td>.693 N(0,1) Ericsson IV N(0,1)</td>
<td>3.459</td>
</tr>
<tr>
<td>.429 ( \chi^2(1) ) Sargan ( \chi^2(1) )</td>
<td>7.020</td>
</tr>
<tr>
<td>.424 F(1, 44) Joint Model F(1, 44)</td>
<td>8.133</td>
</tr>
<tr>
<td>[0.519] Probability</td>
<td>[0.007]</td>
</tr>
</tbody>
</table>

The problem with equation 3 is that the variables involved are all I(1). Moreover, if we assume that, given the import substitution process and the existing legislation protecting the domestic production of capital goods, there exists a "preference" to buy domestically produced capital goods, then the relevant equation to be estimated is the demand for capital goods. The import of capital goods would then be just a residual, with only the excess capital goods demand going to imports. Thus we
decided to focus on the estimation of a capital goods demand equation.\textsuperscript{10}

The problems in estimating the demand for capital goods for Brazil are tremendous given the total inadequacy of the data. Therefore, the results presented here should be regarded as a tentative approximation and taken with extreme care.

The demand for capital goods was constructed using the series of production ($y_{3q}$) and imports of capital goods ($qm_{3q}$). To create the weights we used the value in Cr\$ of the imports of capital goods and the production value of the capital goods industry.

Table V.4

<table>
<thead>
<tr>
<th>CRDW</th>
<th>DF</th>
<th>ADF</th>
<th>PP</th>
</tr>
</thead>
<tbody>
<tr>
<td>lqm3q</td>
<td>0.217</td>
<td>-0.882</td>
<td>-1.338</td>
</tr>
<tr>
<td>lrer8q</td>
<td>0.292</td>
<td>0.380</td>
<td>0.345</td>
</tr>
<tr>
<td>ly4q</td>
<td>0.426</td>
<td>0.530</td>
<td>0.483</td>
</tr>
<tr>
<td>ld1qg</td>
<td>0.159</td>
<td>-0.442</td>
<td>-0.704</td>
</tr>
<tr>
<td>lu3q</td>
<td>0.221</td>
<td>-0.288</td>
<td>-0.389</td>
</tr>
<tr>
<td>A1qm3q</td>
<td>3.032</td>
<td>-12.662</td>
<td>-6.649</td>
</tr>
<tr>
<td>A1rer8q</td>
<td>2.166</td>
<td>-7.938</td>
<td>-5.206</td>
</tr>
<tr>
<td>A1y4q</td>
<td>1.908</td>
<td>-6.760</td>
<td>-2.259</td>
</tr>
<tr>
<td>A1u3q</td>
<td>2.651</td>
<td>-10.200</td>
<td>-5.407</td>
</tr>
<tr>
<td>A1dkgq</td>
<td>1.885</td>
<td>-6.895</td>
<td>-6.930</td>
</tr>
</tbody>
</table>

The key variables to explain the demand for capital goods are real GDP, the ratio of capital and labour price and level of capacity utilisation in the economy.\textsuperscript{11} The

10) There is another method of dealing with this problem. One could try to approach this question by using a model that allows for a fall in the income elasticity. This strategy will be used later in this chapter.

11) For a discussion of the different models of investment for the UK economy see Wallis et alii (1987).
positive relationship between the demand for capital goods and real GDP comes from the accelerator model, while the negative relationship with the capital-labour price ratio comes from the neoclassical model of investment.

As far as capacity utilisation is concerned the idea is that the firm has a planned optimum level of capacity, so that it increases its investment whenever it is near this optimum level and vice versa. Alternatively, since the capacity utilization term can be seen as reflecting the intensity with which capital is used, it serves as a proxy for depreciation. In both cases the expected sign of this variable is positive.

The model we estimate here is accelerator based, including also capacity utilisation. The inclusion of the factor price ratio was not possible since such series does not exist.

EQ(4) Modelling \( \Delta qg \) by OLS
The sample is 1975(1) to 1987(4)

<table>
<thead>
<tr>
<th>variable</th>
<th>coeff</th>
<th>S.E.</th>
<th>H.C.S.E.</th>
<th>t-value</th>
<th>partial r²</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ly_{4t} )</td>
<td>0.192</td>
<td>0.09290</td>
<td>0.09993</td>
<td>2.070</td>
<td>0.0804</td>
</tr>
<tr>
<td>( lu_{1q} )</td>
<td>3.031</td>
<td>0.19657</td>
<td>0.21187</td>
<td>15.422</td>
<td>0.8292</td>
</tr>
<tr>
<td>const</td>
<td>-9.309</td>
<td>1.02442</td>
<td>1.18580</td>
<td>-9.087</td>
<td>0.6276</td>
</tr>
</tbody>
</table>

\( R^2 = 0.829 \quad \sigma = 0.0879 \quad F(2,49) = 119.02 \quad DW = 0.769 \quad DF = -3.460 \quad ADF = -2.413 \quad PP = -3.50 \)

The unit root tests applied to the residuals of the cointegrated regression show that the variables

12) Other variables also have been used when estimating a demand for capital goods. Leff and Sato (1987) for example, use variables such as expected rate of inflation and changes in the credit supply, while Lomax (1990) uses a profit share variable.
cointegrate. The estimated error correction mechanism gives reasonable results, showing no sign of forecast failure, serial correlation, heteroscedasticity, or misspecification.

EQ(5) Modelling $\Delta \text{ldkgq}$ by OLS
The sample is 1975(2) to 1987(4)

<table>
<thead>
<tr>
<th>variable</th>
<th>coeff</th>
<th>S.E.</th>
<th>H.C.S.E.</th>
<th>t-value</th>
<th>partial $r^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta y4qt$</td>
<td>.816</td>
<td>.07415</td>
<td>.08594</td>
<td>11.006</td>
<td>.7205</td>
</tr>
<tr>
<td>$\Delta u1qt$</td>
<td>1.075</td>
<td>.22070</td>
<td>.20524</td>
<td>4.873</td>
<td>.3356</td>
</tr>
<tr>
<td>ecvt-1</td>
<td>-.180</td>
<td>.06391</td>
<td>.07206</td>
<td>-2.819</td>
<td>.1447</td>
</tr>
<tr>
<td>const</td>
<td>-.010</td>
<td>.00535</td>
<td>.00545</td>
<td>-1.933</td>
<td>.0737</td>
</tr>
</tbody>
</table>

$R^2 = 0.809 \quad \sigma = 0.0376 \quad F(3,47) = 66.33$

$DW = 2.131 \quad RSS = 0.0664$

Information Criteria: SC = -6.336 HQ = -6.429 FPE = 0.002

analysis of 1-step forecasts

date | actual | forecast $Y - Y\hat{q}$ | forecast SE | t-value
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1988 1</td>
<td>-.0480</td>
<td>-.1099</td>
<td>.0618</td>
<td>.038835</td>
</tr>
<tr>
<td>1988 2</td>
<td>.0472</td>
<td>.0455</td>
<td>.0017</td>
<td>.038157</td>
</tr>
<tr>
<td>1988 3</td>
<td>.0109</td>
<td>.0597</td>
<td>-.0487</td>
<td>.038819</td>
</tr>
<tr>
<td>1988 4</td>
<td>-.0611</td>
<td>-.1044</td>
<td>.0432</td>
<td>.039432</td>
</tr>
</tbody>
</table>

Chow (4,47) = 1.24 \quad LM(4) = 2.959 \quad LM(8) = 8.133

ARCH(4) = 1.112 \quad WHITE(6,40) = 1.532 \quad RESET(1,44) = 0.194.

The most striking feature is the low value for the feedback coefficient, only $-0.180$, indicating that only a small part of the short run disequilibrium is corrected each period. This result contrasts with the one previously obtained for intermediate goods. In that case the feedback coefficient is $-0.539$. As we are now dealing with capital investment this result is not surprising, since a slow adjustment process to the long run equilibrium may be expected in this case.\textsuperscript{13}

\textsuperscript{13} Lomax (1990), for example, finds a value of $-0.285$ for the feedback coefficient for British economy.
In the short run the effects of changes in output and capacity utilization are roughly the same. Both impact elasticities are around one. In the long run though, the main determinant of the demand for capital goods is capacity utilization, having a long run elasticity of 3.031.

3) Total Imports Demand

We tried also to estimate an error correction model for total import demand. The unit root tests show that all the variables under consideration are I(1) and also that they cointegrate.

Table V.5

<table>
<thead>
<tr>
<th>CRDW</th>
<th>DF</th>
<th>ADF</th>
<th>PP</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>lqm1q 0.226</td>
<td>-0.332</td>
<td>-0.637</td>
<td>-1.76</td>
<td>I(1)</td>
</tr>
<tr>
<td>lrer6q 0.732</td>
<td>0.092</td>
<td>-0.056</td>
<td>-3.48</td>
<td>I(1)</td>
</tr>
<tr>
<td>ly4q 0.426</td>
<td>0.530</td>
<td>0.483</td>
<td>-2.76</td>
<td>I(1)</td>
</tr>
<tr>
<td>lu1q 0.181</td>
<td>-0.483</td>
<td>-0.684</td>
<td>-1.88</td>
<td>I(1)</td>
</tr>
<tr>
<td>△lqm1q 2.144</td>
<td>-7.516</td>
<td>-6.942</td>
<td>-7.76</td>
<td>I(0)</td>
</tr>
<tr>
<td>△lrer6q 2.426</td>
<td>-9.022</td>
<td>-4.785</td>
<td>-9.57</td>
<td>I(0)</td>
</tr>
<tr>
<td>△ly4q 1.908</td>
<td>-6.760</td>
<td>-2.259</td>
<td>-7.50</td>
<td>I(0)</td>
</tr>
<tr>
<td>△lu1q 2.250</td>
<td>-8.032</td>
<td>-3.958</td>
<td>-8.12</td>
<td>I(0)</td>
</tr>
</tbody>
</table>

The problem here is a possible bias in the income elasticity. As we have said above, given the wide reduction in the capital goods imports, all attempts to estimate a capital goods import demand lead to a negative income elasticity. Therefore, although the share of

14) Again, as noted in the previous section, a time varying coefficient model may be the answer to this problem.
capital goods imports has been reducing in the past years, it is still possible that a large negative income elasticity for these goods would bias downwards the value for the total income elasticity. This is like a reduced form effect. The "structural" model has a demand for capital goods with a positive income elasticity and an import substitution policy that leads to a declining share of imported goods in the total demand.

In fact the cointegrated regression shows just that. The values for the price and capacity utilization coefficients are in line with results reported by other authors. The income elasticity though is substantially smaller than what seems to be the consensus.\textsuperscript{15} Most authors, using a time trend, report an income elasticity around one whereas we obtained 0.344.

EQ(6) Modelling $\log(m_{t-1})$ by OLS
The sample is 1976(1) to 1987(4)

\begin{tabular}{lcccccc}
variable & coeff & S. E. & H.C.S.E. & t-value & partial r\textsuperscript{2} \\
1rer6qt & -.910 & .22579 & .21545 & -4.032 & .2698 \\
1y4qt & .344 & .19366 & .16898 & 1.776 & .0669 \\
1u1q & 3.865 & .36061 & .34058 & 10.718 & .7231 \\
const & -9.338 & 2.08402 & 2.22181 & -4.481 & .3133 \\
\end{tabular}

\[ R^2 = 0.797 \quad \sigma = 0.1467 \quad F(3,44) = 57.58 \quad DW = 0.835 \]

\[ DF = -3.517 \quad ADF = -2.880 \quad PP = -3.57 \]

In theory it should always be preferred to work with disaggregated data since it increases the amount of information available. In practice, however, it has been argued that disaggregated data usually have larger measurement error and also that it increases the risk of

\textsuperscript{15) For these comparisons see Abreu (1986), Zini Jr. (1988) and Fachada (1990).}
misspecification. The results presented here seem, at least as far as the fixed parameter model using quarterly data is concerned, to point in favour of using disaggregated data.

Therefore, in terms of policy analysis it seems clear that the use of an aggregated imports demand equation should be avoided. The bias will lead to an under estimation of the total imports in the long run.

EQ(7) Modelling Δ1qm1q by OLS
The Sample is 1977(1) to 1987(4)

<table>
<thead>
<tr>
<th>variable</th>
<th>coeff</th>
<th>S. E.</th>
<th>H.C.S.E.</th>
<th>t-value</th>
<th>partial r²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ1qm1qt-1</td>
<td>-0.301</td>
<td>0.09622</td>
<td>0.13659</td>
<td>-3.132</td>
<td>0.2190</td>
</tr>
<tr>
<td>Δ1qm1qt-2</td>
<td>-0.261</td>
<td>0.09387</td>
<td>0.09389</td>
<td>-2.776</td>
<td>0.1804</td>
</tr>
<tr>
<td>Δ1qm1qt-3</td>
<td>0.217</td>
<td>0.10254</td>
<td>0.09782</td>
<td>2.111</td>
<td>0.1130</td>
</tr>
<tr>
<td>Δ1rer6qt</td>
<td>-0.476</td>
<td>0.14529</td>
<td>0.14043</td>
<td>-3.274</td>
<td>0.2345</td>
</tr>
<tr>
<td>Δ1y4qt</td>
<td>0.301</td>
<td>0.18831</td>
<td>0.24874</td>
<td>1.598</td>
<td>0.0680</td>
</tr>
<tr>
<td>Δ1y4qt-1</td>
<td>0.932</td>
<td>0.19923</td>
<td>0.24496</td>
<td>4.677</td>
<td>0.3847</td>
</tr>
<tr>
<td>Δ1u1qt</td>
<td>2.603</td>
<td>0.52257</td>
<td>0.54723</td>
<td>4.981</td>
<td>0.4149</td>
</tr>
<tr>
<td>ecvt-1</td>
<td>-0.182</td>
<td>0.09315</td>
<td>0.08421</td>
<td>-1.956</td>
<td>0.0986</td>
</tr>
<tr>
<td>const</td>
<td>-0.013</td>
<td>0.01155</td>
<td>0.01298</td>
<td>-1.117</td>
<td>0.0344</td>
</tr>
</tbody>
</table>

R² = 0.799 ρ = 0.0741 F(8,35) = 17.43

DW = 2.068 RSS = 0.1925

Information Criteria: SC = -4.658 HQ = -4.888 FPE = 0.007

<table>
<thead>
<tr>
<th>date</th>
<th>actual</th>
<th>forecast</th>
<th>Y - Yhat</th>
<th>forecast SE</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1988 1</td>
<td>-0.2461</td>
<td>-0.1418</td>
<td>0.1043</td>
<td>0.0786</td>
<td>-1.3260</td>
</tr>
<tr>
<td>1988 2</td>
<td>0.0686</td>
<td>0.0675</td>
<td>0.0011</td>
<td>0.0827</td>
<td>0.0128</td>
</tr>
<tr>
<td>1988 3</td>
<td>0.1612</td>
<td>0.1599</td>
<td>0.0013</td>
<td>0.0813</td>
<td>0.0160</td>
</tr>
<tr>
<td>1988 4</td>
<td>0.0483</td>
<td>-0.0478</td>
<td>0.0961</td>
<td>0.0832</td>
<td>1.1550</td>
</tr>
</tbody>
</table>

Chow (4,35) = 0.85 LM(4) = 4.156 LM(8) = 16.236

ARCH(4) = 1.945 WHITE(16,18) = 0.616 RESET(1,34) = 0.346.

As far as the error correction equation goes the main feature is the complicated short run dynamics and small value for the feedback coefficient. The presence of
terns in ΔIQM imply a complex adjustment pattern.\textsuperscript{16} That is especially true for the income elasticity. The impact coefficient is 0.300 but it overshoots to 0.841 after one quarter. For the other coefficients the profile is smoother but still with some bouncing.

This overshoot of the income elasticity may help to explain why other authors have come up with a much higher value for this coefficient. Abreu (1986) and Fachada (1990) both use a time trend to account for long run changes in the import demand. Therefore, it can be argued that either the time trend solves the aggregation bias problem or that their coefficients actually represent or are influenced by the short run elasticity, or both. In the case of Zini Jr (1988) such comparisons are more difficult since the sample period is substantially different. In any case, as proposed above, the best way to tackle the problem is to use disaggregated data.

II.2) Annual Data Estimations

The annual data estimations were performed using data from 1947 to 1988. The first problem in using annual data is that homogeneous series are difficult to obtain for the whole period in question. It took some work to be able to extend the data until 1947. The details on how the series were constructed can be found in appendix V.2.

\textsuperscript{16} The actual profile of coefficients is given by the polynomials ratio. If the equation is written as \( \gamma(L)y_t = a(L)x_{1t} + \beta(L)x_{2t} \), the profile is given by \( a(L)/\gamma(L) \) and \( \beta(L)/\gamma(L) \).
The main problem is associated with the capacity utilization series. This series is only available from 1970 onwards. To overcome this problem we created alternative measures of capacity utilization and evaluate which one would be the most appropriate to use. Therefore, before presenting the results for the annual equations we shall, in the next section, first discuss, present and compare different measures of capacity utilization.

1) Different Measures of Capacity Utilization

Measuring the level of capacity utilization in the economy is important not only in itself but also because this variable is expected to be relevant in explaining the behaviour of other economic aggregates. Specifically, in the case of empirical work on trade equations the use of such variable has been widespread.¹⁷

The different measures of capacity utilization available in the literature can be divided into two major groups according to the way they handle the potential output. It is the potential output that is actually estimated since the capacity utilization or output gap are obtained from, respectively, the ratio or difference between actual and potential output. The potential output can be assumed to increase at a constant or time varying rate.

¹⁷) See, for example, Abreu (1987), Fachada (1990), Barker (1987), Braga and Markwald (1983), Kahn and Ross (1975), and Goldstein and Khan (1985).
1.1) Measures with a Fixed Potential Output Growth Rate

A first measure of capacity utilization can be obtained by regressing the logarithm of output on a time trend. That is,

\[ \ln y_t = \alpha + \beta \text{trend} + \epsilon_t \]

where \( \ln y_t \) is the logarithm of real GDP. In this case, the residual \( \epsilon_t \) can be taken as a measure of the output gap in the economy.

In this context the key idea is one of normal output. There exists a kind of natural or normal path which is represented by the estimated output in the regression above. Therefore, the potential output is not the maximum output and the actual output can be above as well as below it. If the residual \( \epsilon_t \) is positive this implies that the economy is overheated and vice versa. When the actual output is equal to potential output the residual will be zero, which means that output is at its normal or natural level.

Using Brazilian GDP data for the period 1920/1988, we estimated the regression above by least squares. The data was obtained from IBGE (1990) and Zerkowiski and Veloso (1982). The results show the potential output growing at around 6.0% per year.

\[ \ln y_t = 0.7471 + 0.0599 \text{trend} \]

The residuals, named \( u_t \), representing the output gap are shown in figure V.1.

The variable seems to behave in accordance with expectations. It shows positive values for the 20s while the beginning of the recession after 1929 leads to a
reduction in the output gap that reaches its lowest negative level in 1932. The economy overheats again in 1937 and then slumps until the end of the second world war to its all time low. The post war prosperity is acknowledged by the reduction in the negative gap until the economy overheats again in 1961. With the recession of the 60s the output gap goes down to a new trough in 1967 to then recover again until 1980 when the recession leads to its reduction.

A slightly different approach is to determine a priori the year of maximum capacity utilization, that is years where the actual and potential output are equal, and then calculate the potential output series in between these years. Suppose we know that at some specific times, say t and t+s, the capacity was at its maximum. The fixed rate of potential output growth ($\theta$) can then be calculated from the relation

$$y_{pt+s} = y_{pt} (1 + \theta)^s$$

Once the potential output $y_p$ is obtained the level of capacity utilization can be calculated as the ratio $y/y_p$. As before the potential output is supposed to grow at a constant rate $\theta$ but in this case we have a more intuitive measure of capacity utilization bounded at 100%.

As we had various years of probable maximum capacity utilization between 1920 and 1988 we decided to do the potential output estimation in steps. The years designated as maximum capacity ones were 1928, 1961, 1974, 1980 and 1986. We have then four different growth
rates for potential output. Therefore, although potential output grows at a constant rate within each period, this rate changes from one period to another.

Table V.6

<table>
<thead>
<tr>
<th>Rate of growth (β)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1920/1961</td>
</tr>
<tr>
<td>1961/1974</td>
</tr>
<tr>
<td>1974/1980</td>
</tr>
<tr>
<td>1980/1986</td>
</tr>
</tbody>
</table>

The capacity utilization index calculated in this fashion, named $u_{14a}$, is shown in figure V.2. The behaviour of $u_{14a}$ is again quite reasonable, meeting what is intuitively known about the level of capacity utilization of the economy.

1.2) Measures with a Varying Potential Output Growth Rate

Some time varying methods of estimating the potential product have been put forward by Moreira (1985) and Pereira (1986). They use a moving average process and a structural time series, respectively.

The structural time series methodology tries to decompose a series into its different unobserved components. In this case we use the trend plus cycle model where the actual GDP is decomposed into trend, cycle and irregular components.

$$y_{1a} = \mu_t + \psi_t + \epsilon_t$$

18) For details on the this approach see Harvey (1989).
The cycle component is obtained as a combination of sine and cosine waves. Given the frequency $\lambda_c$ and a damping factor $\rho$ the cycle component can be written as

$$\begin{bmatrix} \psi_t \\ \psi^*_t \end{bmatrix} = \rho \begin{bmatrix} \cos \lambda_c & \sin \lambda_c \\ -\sin \lambda_c & \cos \lambda_c \end{bmatrix} \begin{bmatrix} \psi_{t-1} \\ \psi^*_{t-1} \end{bmatrix} + \begin{bmatrix} \kappa_t \\ \kappa^*_t \end{bmatrix}$$

where $\kappa$ and $\kappa^*$ are white noise errors with a common constant variance $\sigma^2$. The complete model can then be written as

$$\begin{align*}
ly1_{at} &= \mu_t + \psi_t + \epsilon_t \\
\mu_t &= \mu_{t-1} + \beta_{t-1} + \eta_t \\
\beta_t &= \beta_{t-1} + \zeta_t \\
\psi_t &= \frac{(1-\rho \cos \lambda_c L)\kappa_t + (\rho \sin \lambda_c L)\kappa^*_t}{1 - 2\rho \cos \lambda_c L + \rho^2 L^2}
\end{align*}$$

where $\eta$ and $\zeta$ are also white noise with variances $\sigma^2$ and $\sigma^2$.

The estimation of this model is then performed using the Kalman filter. The state space representation of the model can be written as

$$\begin{align*}
ly1_{at} &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \alpha_t + \epsilon_t \\
\alpha_t &= \begin{bmatrix} \mu_t \\ \beta_t \\ \psi_t \\ \psi^*_{t} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \rho \cos \lambda_c & \rho \sin \lambda_c \\ 0 & 0 & -\rho \sin \lambda_c & \rho \cos \lambda_c \end{bmatrix} \begin{bmatrix} \mu_{t-1} \\ \beta_{t-1} \\ \psi_{t-1} \\ \psi^*_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_t \\ \zeta_t \\ \kappa_t \\ \kappa^*_t \end{bmatrix}.
\end{align*}$$

In this case we again calculate the output gap by taking the difference between the actual and potential output but now the potential output represented by the trend $\mu_t$ does not have a constant rate of growth.
The estimated hyperparameters $\sigma_n$, $\sigma_\xi$, $\sigma_\kappa$ and $\sigma_\epsilon$, the state vector at the end of the sample $x_{1988}$ and the frequency and damping factor are shown below.

### Time Domain Estimation

Dependent variable is $\text{LOG}(Y_{1A})$

Sample period 1920 to 1988 69 Observations

<table>
<thead>
<tr>
<th>Estimate</th>
<th>Parameter</th>
<th>Standard Error</th>
<th>t-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0000013</td>
<td>$\sigma(\text{Level})$</td>
<td>0.0028546</td>
<td>0.004649</td>
</tr>
<tr>
<td>0.000288</td>
<td>$\sigma(\text{Trend})$</td>
<td>0.000315</td>
<td>9.125</td>
</tr>
<tr>
<td>0.0011365</td>
<td>$\sigma(\text{Cycle})$</td>
<td>0.0021098</td>
<td>5.387</td>
</tr>
<tr>
<td>0.8648</td>
<td>Damping Factor</td>
<td>0.0966</td>
<td>8.9557</td>
</tr>
<tr>
<td>0.3487</td>
<td>Frequency</td>
<td>0.1662</td>
<td>2.0985</td>
</tr>
<tr>
<td>18.0165</td>
<td>Period</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0000000</td>
<td>$\sigma(\text{Irregular})$</td>
<td>0.002704</td>
<td>0.000000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Estimate</th>
<th>State</th>
<th>RMSE</th>
<th>t-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.8505</td>
<td>Level</td>
<td>0.0539</td>
<td>89.9692</td>
</tr>
<tr>
<td>0.0393</td>
<td>Trend</td>
<td>0.0131</td>
<td>2.9897</td>
</tr>
<tr>
<td>-0.0583</td>
<td>Cycle</td>
<td>0.0539</td>
<td>-1.0809</td>
</tr>
<tr>
<td>-0.0004129</td>
<td>Cycle</td>
<td>0.0551</td>
<td>-0.0074982</td>
</tr>
</tbody>
</table>

Skewness $X^2 (1) = 2.4231$

Kurtosis $X^2 (1) = 0.1935$

Normality $X^2 (2) = 2.6166$

$Q(1) = 0.3271$ $Q(3) = 1.643$ $Q(5) = 8.364$ $Q(10) = 11.47$

Heteroscedasticity test $F(22, 22) = 0.5613$

Log-likelihood kernel 176.2228

Prediction error variance 0.0018620

Sum of squares 0.1309

$R^2 = 0.9987$

$RD^2 = 0.0230$

The output gap calculated using the structural time series model, named u10a, is shown in figure V.3 and, as in previous examples, it also looks like what would be intuitively expected. The output gap is calculated in this case as the difference between the actual output and
the trend component. The average potential output rate of growth is this case is 5.8%. It reaches a maximum of 7.7% in the 1970s and falls back to a average of only 4.4% during the 1980s.

An alternative way of allowing for a varying rate of growth for the potential output was proposed by Moreira (1985). The idea is to construct the potential product using a two period moving average of the actual output.

Suppose that the actual output expected rate of growth in each period, say $g^*$, is formed as an average of the rates of growth in the last two periods and that investment is done accordingly to this expectations trying to keep constant the level of capacity utilization. Formally,

$$g_t^* = \frac{g_{t-1} + g_{t-2}}{2}$$

$$u_t^* = u_{t-1} \quad \text{or} \quad \frac{y_{pt}}{y_{pt-1}} = \frac{y_t}{y_{t-1}}$$

as $1+g_t = \frac{y_t}{y_{t-1}}$ we then have

$$\frac{y_{pt}}{y_{pt-1}} = 1+g_t$$

As $g_t$ is not known its expected value $g^*$ is used instead. Therefore,

$$\frac{y_{pt}}{y_{pt-1}} = 1 + \frac{g_{t-1} + g_{t-2}}{2}$$

$$\frac{y_{pt}-y_{pt-1}}{y_{pt-1}} = \frac{y_{t-1}/y_{t-2} + y_{t-2}/y_{t-3}}{2} = \theta_t$$

The potential output is then calculated recursively as

$$y_{pt} = \theta_t \cdot y_{pt-1}$$

and

$$y_{po} = y_0.$$
We chose 1928 as the year to start the recursions since, as we noted above, the level of capacity utilization in this year is probably near the maximum. The resulting variable is presented in figure V.4 under the name of $u_{12a}$. Experiments where $\theta_t$ is a two period geometric mean lead to very similar results. The average potential output rate of growth obtained in this case, 6.0%, is close to the figure from the structural time series model. However, it shows a more erratic behaviour. As before, this variable also seems to provide a reasonable approximation of what is intuitively known about the level of capacity utilization.

The final measure of capacity utilization we calculate, named $u_{13a}$, combines the information available
in quarterly data for the period post 1970 with the variable u12a. Therefore, for the period post 1970 we use the annual average whereas for the period before 1970 the series is calculated using the rate of change in u12a.

There exists a further method of generating the capacity utilization series by using the stock of capital and the capital/output ratio. If a series of average capital stock is available and the capital/output ratio is known for a year of maximum capacity utilization, the potential output series can be created by multiplying the capital stock in each year by this capital/output ratio. No prior information is actually needed about the year of maximum capacity utilization since it should correspond to the lowest capital output ratio. This method is not used here since it is impossible to obtain reliable data on average stock of capital for the period under consideration.

1.3) **Comparing the Different Measures**

The main thing to note about these different measures of capacity utilization is that variables that allow for a time varying rate of growth for the potential product (u10a and u12a) have a smaller standard deviation when compared with those which have a fixed potential product growth rate (u11a and u14a). It can be seen from figures V.6 and V.7 that u11a and u14a are normally below

19) See Conjuntura Economica, several issues.  
20) See Bonelli and Malan (1976) for an application of this method.
u10a and u12a during the recession years and above it during the years of boom.

Table V.7

<table>
<thead>
<tr>
<th></th>
<th>u10a</th>
<th>u11a</th>
<th>u12a</th>
<th>u13a</th>
<th>u14a</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0009</td>
<td>0.0000</td>
<td>90.2105</td>
<td>81.6612</td>
<td>89.0488</td>
</tr>
<tr>
<td>Std.Devn.</td>
<td>0.0588</td>
<td>0.1234</td>
<td>4.7452</td>
<td>4.3140</td>
<td>7.9186</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.0928</td>
<td>-0.4285</td>
<td>-0.4110</td>
<td>-0.3416</td>
<td>-0.0777</td>
</tr>
<tr>
<td>Exc. Kurtosis</td>
<td>-0.7194</td>
<td>-1.0490</td>
<td>-0.2457</td>
<td>-0.3739</td>
<td>-1.1787</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.1225</td>
<td>-0.2091</td>
<td>79.1901</td>
<td>71.2996</td>
<td>73.3718</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.1243</td>
<td>0.2294</td>
<td>100.0000</td>
<td>90.0360</td>
<td>100.0000</td>
</tr>
</tbody>
</table>

This behaviour is in line with what would be expected since in the fixed potential output growth rate case, investment decisions are not influenced by short run movements in GDP. When the potential output rate of growth is allowed to vary it seems that during recession periods it falls below the constant rate and vice versa.

Figure V.6

u10a and u11a
Although the hypothesis of a time varying potential output growth rate is much more appealing, the structural time series model presented in the previous section does not allow us to accept it. The t ratio test on $\sigma_r^2$ cannot reject the hypothesis of a fixed rate of growth.

The estimation of a structural time series model with a fixed potential product growth rate, that is using $\beta$ instead of $\beta_t$, leads to a constant growth rate of 5.3%. The comparison of these two models using the AIC and the BIC also favours the fixed growth rate model. Therefore, although it makes more economic sense to think in terms of a varying rate of growth, at least in the structural time series framework, there seems to be no statistical confirmation of it.
As a way of testing the different measures of capacity utilization provided here we tried to compare them with the available quarterly data on industrial capacity utilization for the post 1970 period, denoted by $u_{13a}$. Obviously the level of capacity utilization for the economy as a whole does not have to follow exactly the same path of the capacity utilization in the industrial sector. Nevertheless, especially for the recent period when the industrial sector has became more and more important, a high correlation is expected.

The evidence in the correlation matrix shown above is strongly in favour of $u_{12a}$, which has a correlation coefficient of 0.8803. The second higher correlation coefficient, 0.6405, was obtained by $u_{10a}$. Therefore, this test seems to give backing to the idea that measures allowing a time varying rate of growth for potential output are closer to reality.

In conclusion, although the statistical evidence from the structural time series model cannot confirm the time varying growth rate hypothesis, this alternative still seems more appropriate not only in economic terms
but also on the grounds of the information available for the industrial sector.

From this analysis of the five different series created it seems that $u_{12a}$ is the most appropriate measure to use. This choice is based not only on the comparison with $u_{13a}$ but also on the fact that $u_{12a}$, being bounded between zero and one, is a more intuitive measure. It should also be said that in preliminary test using the import demand equations, the variable $u_{12a}$ has shown a slightly better performance.

2) Import Demand for Intermediate Goods

As before we start by calculating the unit root tests for all the series involved in the estimation to see if they are stationary. For the variables in levels the presence of a unit root cannot be rejected, while after taking first differences the series are reduced to stationarity.

Table V.9

<table>
<thead>
<tr>
<th>CRDW</th>
<th>DF</th>
<th>ADF</th>
<th>PP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_{qm2a}$</td>
<td>0.141</td>
<td>0.602</td>
<td>-1.064</td>
</tr>
<tr>
<td>$l_{y1a}$</td>
<td>0.008</td>
<td>8.544</td>
<td>2.183</td>
</tr>
<tr>
<td>$l_{u12a}$</td>
<td>0.549</td>
<td>-0.049</td>
<td>-0.077</td>
</tr>
<tr>
<td>$1_{rer2a}$</td>
<td>0.355</td>
<td>0.203</td>
<td>0.213</td>
</tr>
<tr>
<td>$\Delta l_{qm2a}$</td>
<td>2.286</td>
<td>-7.326</td>
<td>-4.687</td>
</tr>
<tr>
<td>$\Delta l_{y1a}$</td>
<td>1.059</td>
<td>-1.993</td>
<td>-1.467</td>
</tr>
<tr>
<td>$\Delta l_{rer2a}$</td>
<td>1.909</td>
<td>-5.956</td>
<td>-6.195</td>
</tr>
<tr>
<td>$\Delta l_{u12a}$</td>
<td>1.747</td>
<td>-5.243</td>
<td>-4.247</td>
</tr>
</tbody>
</table>
It is only in the case of GDP (ly1a) that there might be some doubt since the ADF test does not reject the unit root hypothesis. In the case of this variable though, since it exhibits a very strong trend, the most appropriate unit root test includes a time trend.\textsuperscript{21} The value of the ADF and PP tests in the version including the time trend and a constant are, respectively, -2.72 and -3.98. The contradiction between the tests does not vanish. Strictly speaking the ADF and PP tests are not necessary in this case, since no autocorrelation was found in the residuals of the auxiliary regression used to calculate the DF test. Therefore, we accept the hypothesis of no unit root in $\Delta$ly1a.

The results for the cointegrated regression are shown in equation 8. the income and price elasticities are both significantly different from zero at the 5\% level and have the expected sign. The coefficient of capacity utilization, although with the right sign is only significant at 10 \%.

\begin{equation}
\text{EQ(8) Modelling lqm2a by OLS} \\
The Sample is 1947 to 1985
\end{equation}

\begin{center}
\begin{tabular}{lcccccc}
variable & coeff & S. E. & H.C.S.E. & t-value & partial r\textsuperscript{2} \\
\hline
lrer2at & -.363 & .09166 & .10381 & -3.961 & .3096 \\
ly1at & .639 & .04997 & .04923 & 12.797 & .8239 \\
lul2at & 1.184 & .69938 & .65573 & 1.693 & .0757 \\
const & -1.713 & 3.22776 & 3.10038 & -.531 & .0080 \\
\hline
\end{tabular}
\end{center}

\begin{itemize}
\item $R^2 = 0.830$ 
\item $\sigma = 0.2048$ 
\item $F(3,35) = 56.86$ 
\item $DW = 0.872$
\end{itemize}

\begin{itemize}
\item $DF = -3.507$ 
\item $ADF = -2.460$ 
\item $PP = -3.66$
\end{itemize}

\textsuperscript{21} For a discussion of the different versions of the unit root tests see Perron(1988).
Comparing the coefficients from the cointegrated regression with other empirical work is somewhat difficult since the sample and, mainly, the method we adopt are different from other authors. The nearest possible comparison is with Abreu (1987). He uses data for the period 1960/1985 and his equations are estimated by ols with correction for residual autocorrelation. The values of the price and capacity utilization elasticities are more or less in line with his results while the income elasticity is smaller.\footnote{See Abreu (1987).}

All the unit root tests indicate that the variables cointegrate. Therefore we try to estimate an error correction mechanism. The results for the ECM were not very good, with very large coefficients and standard errors. The problem is that there exists a very strong correlation between $\Delta ly_{1a}$ and $\Delta lu_{12a}$. The use of alternative measures of capacity utilization did not solve the problem. All the measures of capacity utilization proposed early in this chapter led to similarly bad results. The correlation coefficient between $\Delta \log y_t$ and the different measures of $\Delta \log ut$ is around 0.7.

One should expect to find that changes in GDP and capacity utilization were close together, especially since the potential output normally grows slowly over time. In analytical terms

\[
\Delta \log ut = \log \left( \frac{y}{yp} \right)_t - \log \left( \frac{y}{yp} \right)_{t-1} = \Delta \log yt - \Delta \log ypt.
\]
Therefore, if $\Delta \log y_{pt}$ is approximately constant the change in the level of capacity utilization will follow the change in the GDP. The fact that the capacity utilization series we are using was constructed from the GDP series only makes matters worse.

Therefore the short term effects of income and capacity utilization cannot be identified separately. The presence of multicollinearity between these two variables creates an identification problem. The way out seems to be to include both variables in the cointegrating regression but to drop one of them in the ECM. We decided to drop capacity utilization. Equation 9 presents the ECM estimated in this way.

EQ(9) Modelling $\Delta qm2a$ by OLS
The Sample is 1948 to 1985

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coef</th>
<th>S.E.</th>
<th>H.C.S.E.</th>
<th>t-value</th>
<th>Partial $r^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta r_{2at}$</td>
<td>-.741</td>
<td>.39109</td>
<td>.38625</td>
<td>-1.896</td>
<td>.1234</td>
</tr>
<tr>
<td>$\Delta y_{lat}$</td>
<td>1.383</td>
<td>.65982</td>
<td>.55139</td>
<td>2.096</td>
<td>.1144</td>
</tr>
<tr>
<td>$ecv_{t-1}$</td>
<td>-.493</td>
<td>.12451</td>
<td>.12839</td>
<td>-3.961</td>
<td>.3158</td>
</tr>
<tr>
<td>const</td>
<td>-.063</td>
<td>.04814</td>
<td>.04629</td>
<td>-1.300</td>
<td>.0473</td>
</tr>
</tbody>
</table>

$R^2 = 0.439$ $\sigma = 0.1441$ $F(3,34) = 8.86$

$DW = 2.176$ $RSS = .7055$

Information Criteria: SC = -3.604 HQ = -3.715 FPE = 0.023

<table>
<thead>
<tr>
<th>Date</th>
<th>Actual</th>
<th>Forecast Y</th>
<th>Yhat</th>
<th>Forecast SE</th>
<th>T-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1986</td>
<td>.1941</td>
<td>.1214</td>
<td>.0727</td>
<td>.1474</td>
<td>.4930</td>
</tr>
<tr>
<td>1987</td>
<td>-.0833</td>
<td>.0011</td>
<td>-.0844</td>
<td>.1474</td>
<td>-.5726</td>
</tr>
<tr>
<td>1988</td>
<td>-.0958</td>
<td>-.0211</td>
<td>-.0746</td>
<td>.1530</td>
<td>-.4875</td>
</tr>
</tbody>
</table>

Chow (3,34) = 0.27 $LM(1) = 0.747$ $LM(3) = 2.681$

ARCH(3) = 0.824 WHITE(6,27) = 0.579 RESET(1,33) = 3.019
The coefficients have the expected sign and are significant. The estimation of the feedback coefficient is relatively high indicating that half of the short run disequilibrium is corrected in each period. Since we are dealing with annual data this is an expected result. Actually, in the case of intermediate goods imports a high value for the feedback coefficient was obtained even for the quarterly data equations. Comparison of short and long run coefficients shows that both price and "income elasticity" overshoot. The overshoot in the "income elasticity" is probably due to stock adjustments. As the economy picks up, firms build up stocks while in the recession they try to do the opposite to reduce costs.  

Although the equation passes most of the diagnostic tests, the results for the ECM should be taken with some scepticism, since the $R^2$ is very low and the equation fails the RESET test. These problems are certainly a consequence of the omission of the capacity utilization variable.

As we shall see later in this chapter the problem of multicolinearity is solved by the use of time varying parameters. In such a model the coefficients of the ECM do not present the same sort of abnormal behaviour as in the fixed coefficients case.

23) The use of the term income elasticity is not totally appropriated in this context. The multicolinearity problem does not allow us to identify this coefficient as such.
3) Import Demand for Capital Goods

The unit root tests in the case of imports of capital goods do not show any contradiction. For this category of imports, instead of using GDP it seems more appropriate to use a measure of the real gross capital formation. The use of such variable can be criticised on the grounds that the capital goods imports is one of the components used to calculate the gross capital formation. Nevertheless, we decided to retain this variable not only because the imports of capital goods is expected to respond to a investment type of variable but also because it yields better statistical results.

Table V.10

<table>
<thead>
<tr>
<th>CRDW</th>
<th>DF</th>
<th>ADF</th>
<th>PP</th>
</tr>
</thead>
<tbody>
<tr>
<td>lqm3a</td>
<td>0.231</td>
<td>0.119</td>
<td>-0.001</td>
</tr>
<tr>
<td>ly3a</td>
<td>0.016</td>
<td>3.922</td>
<td>2.396</td>
</tr>
<tr>
<td>lu12a</td>
<td>0.549</td>
<td>-0.049</td>
<td>-0.077</td>
</tr>
<tr>
<td>lrer6a</td>
<td>0.369</td>
<td>0.477</td>
<td>0.927</td>
</tr>
<tr>
<td>Δlqm2a</td>
<td>2.286</td>
<td>-7.326</td>
<td>-4.687</td>
</tr>
<tr>
<td>Δly3a</td>
<td>1.505</td>
<td>-3.669</td>
<td>-3.062</td>
</tr>
<tr>
<td>Δlrer6a</td>
<td>2.023</td>
<td>-6.107</td>
<td>-7.080</td>
</tr>
<tr>
<td>Δlu12a</td>
<td>1.747</td>
<td>-5.243</td>
<td>-4.247</td>
</tr>
</tbody>
</table>

The cointegrated regression, equation 10, shows a reasonably good fit. Again the coefficients have the expected sign and are significantly different from zero. All the unit root tests indicate that the variables cointegrate.

The capacity utilization elasticity is higher than in the cases of total or intermediate goods imports. This
confirms the intuitive idea that non-price costs are more important as far as capital goods are concerned. On the other hand it also has a higher price elasticity in comparison with the other aggregates. That is because in the case of capital goods there is the possibility of changing the relative intensity with which capital and labour are used. The price elasticity is more important in this case because firms can use labour more or less intensively depending on the price of capital.

EQ(10) Modelling \( lq_{m3a} \) by OLS
The Sample is 1947 to 1985

<table>
<thead>
<tr>
<th>variable</th>
<th>coeff</th>
<th>S. E.</th>
<th>H.C.S.E.</th>
<th>t-value</th>
<th>partial r²</th>
</tr>
</thead>
<tbody>
<tr>
<td>lrer6at</td>
<td>-.855</td>
<td>.14398</td>
<td>.21303</td>
<td>-5.941</td>
<td>.5021</td>
</tr>
<tr>
<td>ly3at</td>
<td>.647</td>
<td>.06518</td>
<td>.04992</td>
<td>9.922</td>
<td>.7377</td>
</tr>
<tr>
<td>lu12at</td>
<td>3.590</td>
<td>1.09224</td>
<td>.83783</td>
<td>3.287</td>
<td>.2359</td>
</tr>
<tr>
<td>const</td>
<td>-9.953</td>
<td>5.03120</td>
<td>4.36336</td>
<td>-1.978</td>
<td>.1006</td>
</tr>
</tbody>
</table>

R² = 0.749 \( \sigma = 0.3226 \) F(3,35) = 34.84 DW = 1.029

DF = -3.586 ADF = -3.231 PP = -3.75

As before the values we obtain for the price and capacity utilization elasticities are compatible with the results presented by Abreu (1987), while our value for income elasticity is below his.²⁴

When estimating the ECM, we encountered the same problem as before. The multicollinearity between \( \Delta u_{12a} \) and \( \Delta y_{3a} \) leads to completely unreasonable estimates of the coefficient values and their standard errors. Therefore, we had to drop the capacity utilization variable. The results for the ECM are presented in equation 11.

EQ(11) Modelling Δlqm3a by OLS
The Sample is 1948 to 1985

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{variable} & \text{coeff} & \text{S. E.} & \text{H.C.S.E.} & \text{t-VALUE} & \text{PARTIAL r^2} \\
\hline
\Delta \text{rer6at} & -.229 & .14629 & .15337 & -1.565 & .0672 \\
\Delta \text{lq3at} & 1.077 & .37437 & .27662 & 2.877 & .1958 \\
\text{ecvt}-1 & -.483 & .12820 & .13682 & -3.763 & .2941 \\
\text{const} & -.062 & .04502 & .03822 & -1.366 & .0520 \\
\hline
\end{array}
\]

R2 = 0.487 \sigma = 0.2249 F(3,34) = 10.76

DW = 1.993 RSS = 1.7191

Information Criteria: SC = -2.713 HQ = -2.824 FPE = 0.056

Analysis of 1-step forecasts

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{data} & \text{actual} & \text{forecast Y - Yhat} & \text{forecast SE} & \text{t-value} \\
\hline
1986 & .1935 & .3385 & -.1450 & .2359 & -.6148 \\
1987 & -.1237 & .2741 & -.3978 & .2331 & -1.7062 \\
1988 & .1069 & .1426 & -.0357 & .2396 & -.1489 \\
\hline
\end{array}
\]

Chow (3,34) = 1.05 \ LM(1) = 0.400 \ LM(3) = 0.001
ARCH(3) = 0.183 \ WHITE(6,27) = 2.224 \ RESET(1,33) = 0.111

Although it does not fail the RESET test, the ECM has a very low R^2 as a result of the exclusion of Δlu12a. It is also noteworthy that the one step ahead forecasts almost break down in 1987.

The coefficients are significant at the 5% level, apart from the constant and price elasticity. The price elasticity is only significant at the 10% level. The short term coefficient for the income response overshoots but not as much as in the case of intermediate goods. Finally, the feedback coefficient is also high, around -0.5. That is a feature of all three categories of imports considered. When dealing with annual data one should expect to get a substantial part of the short run disequilibrium corrected each period.
4) **Total Imports Demand**

Apart from the problem with Δly1a which we have discussed before, the unit root tests indicate that the variables are I(1).

The results of the cointegrated regression, equation 12, are very much as before. The elasticities are significant and have the expected sign and the unit root tests show that the variables cointegrate.

In terms of comparing the elasticities with other estimates available, the same pattern is repeated. Our price and capacity utilization elasticities are in line with the other authors findings but the income elasticity is below what seems to be the consensus.25 The reason for this discrepancy may be found in the method used. In most of the previous work the equations are corrected for residual autocorrelation. As the GDP series has a strong trend component it is more affected than the others by such correction.

### Table V.11

<table>
<thead>
<tr>
<th>CRDW</th>
<th>DF</th>
<th>ADF</th>
<th>PP</th>
</tr>
</thead>
<tbody>
<tr>
<td>lqm1a</td>
<td>0.122</td>
<td>0.608</td>
<td>0.741</td>
</tr>
<tr>
<td>ly1a</td>
<td>0.008</td>
<td>8.544</td>
<td>2.183</td>
</tr>
<tr>
<td>l1u12a</td>
<td>0.549</td>
<td>-0.049</td>
<td>-0.077</td>
</tr>
<tr>
<td>lrer1a</td>
<td>0.403</td>
<td>-0.069</td>
<td>-0.097</td>
</tr>
<tr>
<td>Δlqm1a</td>
<td>1.953</td>
<td>-6.179</td>
<td>-4.224</td>
</tr>
<tr>
<td>Δly1a</td>
<td>1.059</td>
<td>-1.993</td>
<td>-1.467</td>
</tr>
<tr>
<td>Δlrer1a</td>
<td>1.859</td>
<td>-5.816</td>
<td>-5.993</td>
</tr>
<tr>
<td>Δl1u12a</td>
<td>1.747</td>
<td>-5.243</td>
<td>-4.247</td>
</tr>
</tbody>
</table>

25) For these comparisons, see Abreu (1987) and Dib (1985).
EQ(12) Modelling lqm1a by OLS
The Sample is 1947 to 1985

<table>
<thead>
<tr>
<th>variable</th>
<th>coeff</th>
<th>S. E.</th>
<th>H.C.S.E.</th>
<th>t-value</th>
<th>partial r²</th>
</tr>
</thead>
<tbody>
<tr>
<td>lrrelat</td>
<td>-0.648</td>
<td>0.133</td>
<td>0.120</td>
<td>-4.860</td>
<td>0.4030</td>
</tr>
<tr>
<td>ly1at</td>
<td>0.675</td>
<td>0.060</td>
<td>0.059</td>
<td>11.225</td>
<td>0.7826</td>
</tr>
<tr>
<td>lul2at</td>
<td>2.307</td>
<td>0.947</td>
<td>0.755</td>
<td>2.437</td>
<td>0.1451</td>
</tr>
<tr>
<td>const</td>
<td>-5.405</td>
<td>4.407</td>
<td>3.523</td>
<td>-1.226</td>
<td>0.0412</td>
</tr>
</tbody>
</table>

R² = 0.796  σ = 0.2770  F(3,35) = 45.53  DW = 0.495
DF = -2.334  ADF = -2.055  PP = -2.51

The ECM for the total imports is presented in equation 13. The non inclusion of Δlu12a lead again to a low R² and, therefore, any conclusions should interpreted with care.

EQ(13) Modelling Δlqm1a by OLS
The Sample is 1948 to 1985

<table>
<thead>
<tr>
<th>variable</th>
<th>coeff</th>
<th>S. E.</th>
<th>H.C.S.E.</th>
<th>t-value</th>
<th>partial r²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δlrrelat</td>
<td>-0.332</td>
<td>0.117</td>
<td>0.14140</td>
<td>-2.830</td>
<td>0.1906</td>
</tr>
<tr>
<td>Δly1at</td>
<td>2.149</td>
<td>0.694</td>
<td>0.40837</td>
<td>3.097</td>
<td>0.2200</td>
</tr>
<tr>
<td>ecvt-1</td>
<td>-0.337</td>
<td>0.098</td>
<td>0.08693</td>
<td>-3.427</td>
<td>0.2567</td>
</tr>
<tr>
<td>const</td>
<td>-0.111</td>
<td>0.050</td>
<td>0.04230</td>
<td>-2.188</td>
<td>0.1234</td>
</tr>
</tbody>
</table>

R² = 0.520  σ = 0.1514  F(3,34) = 12.27
DW = 2.252  RSS = 0.7789

Information Criteria: SC = -3.505  HQ = -3.616  FPE = 0.025

### Analysis of 1-step forecast

<table>
<thead>
<tr>
<th>data</th>
<th>actual</th>
<th>forecast</th>
<th>Y - Yhat</th>
<th>forecast SE</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1986</td>
<td>0.3894</td>
<td>0.2004</td>
<td>0.1890</td>
<td>0.1569</td>
<td>1.2047</td>
</tr>
<tr>
<td>1987</td>
<td>-0.1368</td>
<td>-0.0221</td>
<td>-0.1147</td>
<td>0.1555</td>
<td>-0.7377</td>
</tr>
<tr>
<td>1988</td>
<td>-0.0420</td>
<td>-0.0309</td>
<td>-0.0111</td>
<td>0.1605</td>
<td>-0.0692</td>
</tr>
</tbody>
</table>

Chow (3,34) = 0.69  LM(1) = 1.335  LM(3) = 4.201
ARCH(3) = 3.631  WHITE(6,27) = 1.095  RESET(1,33) = 0.592

The coefficients have the expected sign and are all significantly different from zero at 5%. The income elasticity shows a substantial overshoot. Although the
feedback coefficient is higher than in the quarterly data equation it is the lower among the annual data estimates.

III) **Time-Varying Parameter Models**

In this section we present the results of the Kalman filter estimation. The same kind of structure used for the fixed parameter estimation is kept. We first estimate the cointegrated regression and use the Kalman filter residuals in the error correction mechanism.

The structural stability test performed confirm the existence of structural break justifying, therefore, the use of a time varying parameter model. We have used the CUSUM, CUSUMSQ and different sequential Chow tests. The tests are not presented here to save space.

We shall try to relate the movements in the coefficients with the different economic policies and import substitution plans that were implemented during the period under consideration. As we have said in previous chapters the basic idea is that because of changes in the policy regime and in the industrial structure of the economy the coefficients of the import demand function are expected not to be stable.

As we shall see, in some cases the time varying parameters hypothesis does not hold. In other cases some of the coefficients of the import demand are fixed while others are time varying.

26) A similar application, though not using the error correction mechanism, for the Chilean import demand can be found in Meller and Cabezas (1989).

27) For a description of the sequential Chow tests see Hendry (1989).
We expect to get long run coefficients moving more smoothly in response to long term changes in policies or import substitution plans. On the other hand, the short term coefficients are expected to jump much more abruptly in response to changes, or expected changes, in the policy regime.

The estimation follows the procedure discussed in the fourth chapter. The coefficients variances or hyperparameters are calculated using a maximum likelihood approach. The filter is started by a diffuse prior with a large covariance matrix and a unity state vector at time zero.28

The reader should be aware that much of the discussion in this section will rely on the trade policy issues presented in the chapter three. When analysing the changes in the trade elasticities we shall refer only briefly to those trade policies.

III.1) **Annual Data Estimation**

The estimations are preformed for the period 1947 to 1988. The graphs presented in this section do not cover the full sample since for the initial observations the filter gives results highly dependent on the initial conditions. In general we discard the first eight observations.

In the case of the ECM there is an additional loss of observations. That is because the initial errors from

28) The estimations in this section were performed using the program REG-X.
the cointegrated regression were particularly big and, therefore, were not included in the ECM. The sample used starts in 1955.

1) Total Imports Demand

The results for the estimated hyperparameters of the cointegrated regression are presented in equation 14. It can be seen that all coefficients, except the constant, can be assumed as time varying at 5%. The figures V.8 to V.11 show the coefficient paths.

EQ(14) Modelling lqm1a by Kalman Filter

<table>
<thead>
<tr>
<th>hyperparameter</th>
<th>standard error</th>
<th>t statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>12.0284</td>
<td>1.2336</td>
</tr>
<tr>
<td>lrer1a</td>
<td>0.0559</td>
<td>2.4735</td>
</tr>
<tr>
<td>lya2a</td>
<td>0.0158</td>
<td>3.8536</td>
</tr>
<tr>
<td>lu12a</td>
<td>2.1223</td>
<td>2.0289</td>
</tr>
</tbody>
</table>

In the case of income elasticity four distinct periods can be found. For the period 1955/65 there is a steady reduction in the income elasticity. It goes from 0.72 in 1955 to 0.51 in 1965. This reduction is certainly related to the import substitution process occurred in the Brazilian economy since 1945. Special credit may be given to the Plano de Metas. As we have said in the third chapter this plan aimed to promote import substitution, especially in the capital goods sectors. Therefore, the change in the industrial structure of the economy was affecting the stability of the income elasticity.

It can also be argued that the reduction of GDP growth that was brought about in the beginning of the sixties also helped this movement of reduction in the
income elasticity. As we shall see in many cases it is possible to find a cyclical pattern in the income elasticity movements. As we pointed out in chapter two, several reasons can be put forward to explain such cyclical behaviour. These movements can, for example, be credited to changes in the composition of the output during the business cycle. Another possibility is the existence of a "quality effect". As people are well off during the periods of fast GDP growth they switch to better quality products. Because of the high level of protection experienced by Brazilian industry the domestic production sometimes is of lower quality. Therefore, during the boom period people switch to imported goods. A final argument to explain the cyclical movement of the income elasticity is the behaviour of the government. During the recession the government may try to reduce the level of imports, to try to help the domestic industry, by imposing direct controls.

For the period 1965/74 the income elasticity shows a rapid rise. In 1974 the income elasticity of 0.98 is at its highest point. This can be explained by the continued and rapid GDP growth after 1968 and by the removal of import controls.

After the oil price shock and the reduction in GDP growth after 1974, the income elasticity starts falling again. As part of the stabilisation policy the government introduced new quantitative controls. These controls included prohibition of imports of a considerable number of luxury goods and a more strict interpretation of the
Similarity Law. The income elasticity falls from 0.98 in 1974 to 0.81 in 1982. It might be argued that some credit for this reduction should be given to the II PND. This import substitution plan, as we shall see later in the present chapter, had the impact of reducing the income elasticity for capital goods imports. Therefore, it probably also had some indirect impact on the total imports income elasticity. Finally, for the period after 1983 the income elasticity stays relatively stable.

\[ \text{Figure V.8} \]

Total Imports (Coint Reg)

Note that the intuitive idea put forward by Castro and Souza (1985) that the II PND lead to a reduction in the income elasticity, allowing the economy to start growing again without a substantial increase in imports seems to find some support. Nevertheless, if we are
correct in inferring that the income elasticity has a cyclical pattern and that it was also influenced by import controls, the importance of the II PND must be diminished. Actually, as we shall see later in this chapter the effectiveness of the II PND in reducing the income elasticity of the capital goods imports was quite limited.

**Figure V.9**

Total Imports (Coint Reg)

In the case of the price elasticity there exists an almost continuous increase during the whole period. The reason for this is found in the long term process of liberalizing the exchange rate regime. Although Brazil has experienced periods of adoption of more restrictive policies, in the long run there has been a gradual movement towards a more realistic exchange rate regime.
Therefore, prices are becoming more important in determining the volume of imports. More recently this process has been confirmed by the introduction of a "free" exchange market where the central bank does not set the exchange rate.29

![Figure V.10](image)

Figure V.10
Total Imports (Coint Reg)

As far as the capacity utilization elasticity is concerned, the main feature is the sharp reduction in 1973/74. For the period after and before the coefficient is relatively stable. We shall argue that this fall is related to the fact that capacity utilization is a bounded variable. As it is bounded between zero and one whenever it gets closer to the bound there is no room for further movements in the same direction. Therefore, as

29) The exchange rate is at present determined in the market were only importers and exporters have access.
the economy gets closer to full capacity this variable becomes less important. In the limit the capacity utilization elasticity should go to zero as the economy goes to full capacity. If the demand increases as a result of people borrowing to consume or the government producing an unexpected monetary expansion, then this excess of demand will go to imports since the domestic economy can not increase production. That is what was happening in Brazil in the mid seventies.

In other words the relationship between imports and capacity utilization is non linear. The regression coefficient depends on the level of capacity utilization. The boundedness problem can be seen as trying to estimate a linear relationship when the true one is non linear. To
check this explanation of the sharp fall in 1973/74, we
decided to use a transformation of the variable in
question to get rid of the problem. The most obvious
function to use is the logistic transformation \( g(u) \).

\[
g(u) = \log \left( \frac{u}{1 - u} \right)
\]

The logistic function transform a variable bounded
between zero and one in an unbounded variable over the
real line. After performing this transformation we
reestimate the cointegrated regression and the results
seem to confirm our hypothesis. The capacity utilization
elasticity does not show the sharp fall in 1973/74 as
before. Actually, in this case we cannot reject the
hypothesis that this coefficient is fixed. As far as the
other variables are concerned there is no substantial
change.

We also tried two different functions that, although
are still bounded between zero and one, give
transformations that, specially close to the boundary,
are non linear. This are the OGIVE function \( f(u) \)\(^{30}\) and the
cubic \( h(u) \).

\[
h(u) = -2u^3 + 3u^2 \quad f(u) = \frac{1}{1 + e^{a+bu}}
\]

The cubic function gives a almost linear
transformation expect for values near the boundary. The
OGIVE function was used in the case were \( b = -2a \) so that
it is symmetric around 0.5. The only question remaining
is the choice of \( a \). By choosing a large \( a \), say \( a = 50 \),
the OGIVE becomes a step function. If we reduce the value
of \( a \) it becomes s shaped. In the estimations we actually

\(^{30}\) See Hendry and Ericsson (1990).
use $a = 5$. Although the results are also good in the sense that it removes the sharp jump in 1973/74, the OGIVE function lead to more significant changes in the other coefficients path. The cubic transformation is the less effective one, since it is almost linear over the relevant range.

Equation 15 show the results for the ECM using the residuals from the time varying cointegrated regression. It should be noted that in this case, and also for capital and intermediate goods imports, multicolinearity does not seem to be a problem. Experiments using both income and capacity utilization lead to sensible results and, therefore, both variables were included in the time varying ECM. Equation 15 presents not only the hyperparameters for the time varying coefficients but also the estimate of the fixed parameters.

In this case we fixed all coefficients but the constant and price elasticity. The path of the time varying coefficients are shown in figures V.12 and V.13. As we said before, we expect the short run coefficients to be more responsive to changes in the policy regime. Therefore, the coefficient most expected to be affected is the price elasticity. As discussed in chapter three, there have been changes in the exchange rate regime over the period in consideration.

For the period 1960/1967 the short run price response increased from -0.42 to -0.56. This increase is related to a more realistic exchange rate management implemented after 1961. The exchange rate was devalued by
100%, the auction system was ended and a process towards a single exchange rate began.

**EQ(15) Modelling Δlqm1a by Kalman Filter**

<table>
<thead>
<tr>
<th>hyperparameter</th>
<th>standard error</th>
<th>t statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
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<td>0.0061</td>
</tr>
<tr>
<td>Δlrer1at</td>
<td>0.0184</td>
<td>0.0056</td>
</tr>
</tbody>
</table>

**fixed parameters**

| Δly1at         | 1.2309         | 0.2580     | 4.7710     |
| Δlu12at        | 1.6897         | 0.7719     | 2.1889     |
| ecvt-1         | -0.5364        | 0.1566     | -3.4254    |

The crawling peg system introduced in 1968 did not have any destabilizing effects on the short run price elasticity. On the contrary, it kept the price elasticity very much stable. Since one of the aims of this new system was to reduce instability in the foreign exchange management, it can be said that it was successful in this respect.

**Figure V.12**

TotalImports(ECM)
The only break in the coefficient stability over this period occurs in 1974/75. The oil price crisis led the government to impose changes in the exchange rate regime. The crawling peg was kept in place but a compulsory deposit was created and cash payment for imports with tariff rates superior to 55% became obligatory. For six months an amount equal to 100% of the FOB value of the imports had to be deposited in the Central Bank. No interest, and worst no inflation correction, was paid on this deposits.

There seems to be a over reaction to this new policy measures. After a 10% reduction in the short run price elasticity in 1974/75 it goes back to a new level slightly above as it was before.
After 1983 the stability in the short run price elasticity seems to be broken again. It increases continuously until 1988. As a result of IMF pressures the government introduces a much more aggressive exchange rate management, starting with a maxi devaluation of 30% in 1983. The foreign debt crisis imposed the need for large trade surplus to be able to meet the interest payments.

2) Import Demand for Intermediate Goods

In the case of intermediate goods imports the time varying parameters models does not appear to be appropriate. As it can be seen in equation 16, none of the coefficients in the cointegrated regression are time varying. The t test on the hyperparameters do not allow us to reject the hypothesis of fixed coefficients. The only case of the constant that there may be some doubt, since it just fails the test. The figures V.14 to V.17 show the coefficient path.

To allow the constant term to be time varying would create a conflict with the result we obtained in the fixed coefficient model. In that case we have concluded that the variables involved were cointegrated, that is there was no unit root in the residuals of the cointegrated regression. If the constant term is truly time varying this is not possible. The residuals in the fixed coefficient model would have incorporated this and, therefore, would not be stationary.
Figure V.14
Intermediate Goods Imports (Coint Reg)

Figure V.15
Intermediate Goods Imports (Coint Reg)
The income elasticity is fixed since none of the reasons we mention in the previous section to justify the time variation of total imports are present in this case. On the one hand, the import substitution plans have never particularly targeted the intermediate goods industry. On the other hand, the quality effect is probably inoperative since these are mainly homogeneous goods and, given that these goods are normally essential to keep the economy working, there have been no direct controls imposed by the government.

As far as the price elasticity is concerned, the long term process of exchange rate liberalization has happened on a much more reduced scale. Across all the changes in regime the government has held the intermediate goods, and actually also the capital goods, imports as special cases. During the auction system they were in a category where the supply of foreign exchange was such to keep the exchange rate relatively over valued. This imports also manage to pay lower compulsory deposits or avoid them altogether whenever they were introduced.

EQ(16) Modelling lqm2a by Kalman Filter

<table>
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<th>hyperparameter</th>
<th>standard error</th>
<th>t statistic</th>
</tr>
</thead>
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<tr>
<td>lrer2at</td>
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<td>0.0746</td>
</tr>
<tr>
<td>lylat</td>
<td>0.0071</td>
<td>0.0075</td>
</tr>
<tr>
<td>l1u2at</td>
<td>0.2636</td>
<td>0.2575</td>
</tr>
</tbody>
</table>

Since the cointegrated regression is not time varying we used the same cointegrating vector we had before in the fixed coefficient model. Because of this we
have some extra observations to be used to estimate the ECM. The estimation runs from 1947 to 1988 and the coefficients paths, from 1955 onwards, are shown in figures V.18 and V.19. For the reasons discussed before we have only allowed time variation in the price and constant coefficients.

The behaviour of the short term price elasticity follows very much the same pattern as before. It is stable, around -0.52, before 1960. As a result of the policy changes outlined above it increases to -0.89 in 1965. It becomes more or less stable again until 1974/75 and, as it happen with the price elasticity for total imports, overshoots and find a new stable level at around -0.65. After 1983 it starts to increase again reaching -0.79 in 1988.

EQ(17) Modelling Δ1qm2a by Kalman Filter

<table>
<thead>
<tr>
<th>hyperparameter</th>
<th>standard error</th>
<th>t statistic</th>
</tr>
</thead>
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<td>const</td>
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<td>0.0113</td>
</tr>
<tr>
<td>Δ1rer2at</td>
<td>0.3886</td>
<td>0.1792</td>
</tr>
</tbody>
</table>

fixed parameters

<table>
<thead>
<tr>
<th>variable</th>
<th>coefficient</th>
<th>standard error</th>
<th>t statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δly1at</td>
<td>1.0685</td>
<td>0.2412</td>
<td>4.4299</td>
</tr>
<tr>
<td>Δlu12at</td>
<td>0.9135</td>
<td>0.4169</td>
<td>2.1912</td>
</tr>
<tr>
<td>ecvt-1</td>
<td>-0.6228</td>
<td>0.1998</td>
<td>-3.1171</td>
</tr>
</tbody>
</table>

3) Import Demand for Capital Goods

The cointegrated time varying regression for capital goods imports is presented in equation 18. Initial experiments lead us to the conclusion that the price elasticity should be treated as fixed. The same argument used for intermediate goods imports also apply here. The
continuous increase in the price elasticity for total imports was due to a succession of policies aimed at a more realistic exchange rate management. That was not the case for capital goods imports. These imports were normally protected, having a more favourable exchange rate.

Figure V.20

Figures V.20 to V.22 show the coefficient path for the constant, income and capacity utilization elasticity. The income elasticity falls continuously from 1.1 to 0.44 between 1957 and 1963. As it was mentioned before this fall is the result of the Plano de Metas. As a new segment of the capital goods industry is implanted the effect was to reduce the need of imports for a given GDP growth. From 1963 to 1974 the income elasticity begins to increase
again. It reaches 0.75 in 1974, well below its value before the Plano de Metas.

Figure V.21

From 1975 onwards it declines constantly to 0.68 in 1988. The main thing to notice here is that the II PND did not work as well as the Plano de Metas in terms of reducing the income elasticity. Therefore, as the effect of the II PND was limited in this respect the fall in the income elasticity for total imports must be mainly attributed to cyclical factors and government direct controls.

The capacity utilization elasticity shows a sharp fall in 1975/76. As we have discussed before this happen because the level of capacity utilization is close to its maximum.
EQ(18) Modelling lqm3a by Kalman Filter

<table>
<thead>
<tr>
<th>Hyperparameter</th>
<th>Standard Error</th>
<th>t Statistic</th>
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<tbody>
<tr>
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<td>2.0321</td>
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<tr>
<td>lY3at</td>
<td>0.0216</td>
<td>3.1304</td>
</tr>
<tr>
<td>lU12at</td>
<td>1.2826</td>
<td>2.9863</td>
</tr>
</tbody>
</table>

Fixed Parameters
lReretat = -0.8213 0.1941 -4.2313

Figure V.22
Capital Goods Imports (Coint Reg)

The ECM in equation 19 is estimated as before fixing the income, capacity utilization and feedback coefficients. The path for the constant and price elasticity is shown in figures V.23 and V.24.

The behaviour of the short run price response in this case is different from the pattern observed for total and intermediate goods imports. Although there is a jump downwards in 1961, from -0.48 to -0.63, the short term price elasticity becomes stable until 1974. Then it
Figure V.23
Capital Goods Imports (ECM)

Figure V.24
Capital Goods Imports (ECM)
jumps downwards again from -0.70 in 1975 to -0.91 in 1976. It is easy to understand why it does not jump upwards in 1974/75 as in the cases discussed before, since the capital goods imports were in general exempt from paying the compulsory deposit. Only products with tariff rate of more than 37% were liable to pay the compulsory deposit. Nevertheless this does not explain why it jumped downwards. After 1983 its behaviour is unconventional again since it stays stable for awhile, increasing only after 1986.

EQ(19) Modelling $\Delta \text{qm3a}$ by Kalman Filter

<table>
<thead>
<tr>
<th>hyperparameter</th>
<th>standard error</th>
<th>t statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>0.0013</td>
<td>0.0012</td>
</tr>
<tr>
<td>$\Delta \text{r6at}$</td>
<td>0.1905</td>
<td>0.0617</td>
</tr>
</tbody>
</table>

fixed parameters

| $\Delta \text{y3at}$ | 0.9658 | 0.3359 | 2.8753 |
| $\Delta \text{u12at}$ | 2.0354 | 0.4437 | 4.5874 |
| $\text{ecvt-1}$     | -0.5065 | 0.1576 | -3.2138 |

III.2) Quarterly Data Estimations

In the quarterly data estimations we tried to follow the same approach used in the annual equations. We were expecting to get a more smooth path for the long run elasticities and a jumpy behaviour for the short run responses. Although this worked well before, in the quarterly data equations the behaviour is mixed. The long run coefficients also present sharp jumps up and down in response to policy changes. In general the coefficient path is more erratic than before.

This mixed pattern may be due to the use of quarterly data itself. Within a year there is more time...
for an eventual adjustment process to take place. The point here is similar to what is normally argued in terms of dynamic adjustment. Using quarterly data one is much more likely to find dynamic relationships than when using annual data.

The estimations were performed over the period 1975 to 1988 for capital and intermediate goods imports and from 1976 to 1988 for total imports. As before the graphs in this section do not cover the full sample period since some allowance had to be made for the filter to reach a more informative prior. All the graphics actually start in the first quarter of 1979.

1) **Total Imports Demand**

In the case of total imports, only the price and income elasticities were found to be time varying. The results for the cointegrated regression are presented in equation 20 and the coefficients path are shown in figures V.25 to V.27.

The main feature in the price elasticity case is the significant jumps downwards that occur in 80.1, 83.1 and 86.3. The first two jumps are probably related to the maxi devaluations of the exchange rate that took place in December 1979 and February 1983. The increase in oil prices and interest rates significantly worsen the Brazilian balance of payments. As an answer to the balance of payment problems the government decided to change the rules of the crawling peg system and promote a realignment of the exchange rate. On both occasions the
exchange rate was devalued by 30%. It is interesting to note that the maxi devaluation of 1983 seems to have had a greater impact on the price elasticity. This happens because during 1980 the government, trying to contain the inflationary consequences of the maxi devaluation, decided kept the increase in the exchange rate below the rate of inflation.

EQ(20) Modelling \( lq_{1t} \) by Kalman Filter

<table>
<thead>
<tr>
<th>hyperparameter</th>
<th>standard error</th>
<th>t statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_{t+1} )</td>
<td>7.0670</td>
<td>29.9703</td>
</tr>
<tr>
<td>( l_{t+1}rert )</td>
<td>0.0607</td>
<td>0.0125</td>
</tr>
<tr>
<td>( l_{t+1}yt )</td>
<td>0.1871</td>
<td>0.0923</td>
</tr>
</tbody>
</table>

In the case of 86.3, the jump is associated with an expected maxi devaluation. As part of the anti inflationary program started in February 1986, the Cruzado Plan, the nominal exchange rate was fixed. When, by the middle of the year, the program started to be perceived as failing there was some expectation that the exchange rate was going to be devalued. The long run price elasticity increased from -0.81 to -0.97 between the third quarter of 1986 and the first quarter of 1987.

Therefore, although there was no actual maxi devaluation during 1986 for a period of time the long run price elasticity behaved as if it had happened. As people expected an increase in the imports costs in the future the actual exchange rate became more important to explain the level of imports. The maxi devaluation did not happen and after 87.1 the price elasticity begins to reduce. It
Figure V.25
Total Imports (Coint Reg)

Figure V.26
Total Imports (Coint Reg)
was only in the middle of 1987, under a new finance minister, that the government started correcting the exchange during 1987.

The income elasticity is driven by the factors we discussed before when dealing with the annual equations. Between 79.1 and 81.2 the income elasticity goes from 0.75 to 0.60 as a result of the reduction in the growth rate and also the import substitution process started by the II PND. The increase in the income elasticity after 81.3 is short lived since the recession of 1983 brings it down again. It increases again after 83.4 to a new level around 0.68.

Figure V.27
Total Imports (Coint Reg)

The main thing to note here is that the long run income elasticity seems to be free of the possible bias
noticed in the fixed coefficient model. In the fixed coefficient model the income elasticity was just 0.34, well below the other estimates available in the literature. The previous estimations normally used a time trend to account for structural changes in the parameters. By using time varying parameter techniques we can accomplish the same goal in a more elegant and informative way.

The ECM in equation 21 allow for a time varying short run price elasticity and constant. The coefficients path are shown in figures V.28 and V.29.

The main features of the price elasticity path are, as in the case of the long run elasticity, associated with the two maxi devaluations in 1979 and 1983 and the expected maxi devaluation in 1986. The difference is in terms of the magnitude of the changes. In this case the coefficients change much more extensively. After the maxi devaluations the short run price elasticity goes from around -0.5 to -1.0. In 1986 the change is even more substantial as it goes from -0.54 to -1.4. Therefore, although the short and long run coefficients both jump in response of large unexpected changes in policy the former seems to be more responsive.

EQ(21) Modelling Δlm1q by Kalman Filter

<table>
<thead>
<tr>
<th>hyperparameter</th>
<th>standard error</th>
<th>t statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>0.0685</td>
<td>0.1205</td>
</tr>
<tr>
<td>Δlrer6qt</td>
<td>0.2985</td>
<td>0.1042</td>
</tr>
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fixed parameters

<table>
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<tr>
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<th>t statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δly4qt</td>
<td>0.2856</td>
<td>0.0781</td>
</tr>
<tr>
<td>Δlu1qt</td>
<td>1.0325</td>
<td>0.2878</td>
</tr>
<tr>
<td>ecvt-1</td>
<td>-0.2564</td>
<td>0.0574</td>
</tr>
</tbody>
</table>
Figure V.28
Total Imports (ECM)

Figure V.29
Total Imports (ECM)
A comparison between equations 21 and 7 shows that the time varying coefficient model did not require the use of extra lags for the dependent and income variables. In equation 7 we had a quite complicated short run dynamics. Attempts to use extra lags in the time varying parameters model were not successful. The extra lags were always not significantly different from zero.

2) Import Demand for Intermediate Goods

As it can be seen by the cointegrated regression, equation 22, none of the parameters in the demand function can be assumed to be time varying. This result is consonant with what we obtained in the annual equations. In that case as well, the coefficients of the cointegrated regression were found to be constant. The evidence seems to point in favour of constant long run elasticities for intermediate goods imports.

EQ(22) Modelling $lqm2q$ by Kalman Filter

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<th>standard error</th>
<th>t statistic</th>
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</thead>
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<td>34.3172</td>
</tr>
<tr>
<td>$lrer7qt$</td>
<td>0.0236</td>
<td>0.0453</td>
</tr>
<tr>
<td>$ly1qt$</td>
<td>0.0556</td>
<td>0.2365</td>
</tr>
<tr>
<td>$lu2qt$</td>
<td>0.2836</td>
<td>0.7753</td>
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</table>

EQ(23) Modelling $Δlqm2q$ by Kalman Filter

<table>
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<th>t statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
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<td>0.0162</td>
</tr>
<tr>
<td>$Δlrer7qt$</td>
<td>0.0309</td>
<td>0.0147</td>
</tr>
</tbody>
</table>

fixed parameters

| $Δly1qt$ | 0.7123 | 0.2186 | 3.2584 |
| $Δlu2qt$ | 2.5481 | 0.6400 | 3.9814 |
| $ecvt-1$ | -0.5690 | 0.2199 | -2.5875 |
Figure V.32
Intermediate Goods Imports (Coint Reg)

Figure V.33
Intermediate Goods Imports (Coint Reg)
Figure V.34
Intermediate Goods Imports (ECM)

Figure V.35
Intermediate Goods Imports (ECM)
Equation 23 presents the results for the ECM and figures V.34 and V.35 show the coefficients path. The main features are again the responses of the short term price elasticity to the maxi devaluations of 1979 and 1983 and the expected maxi devaluation in 1986.

3) Import Demand for Capital Goods

Contrary to what happened in the fixed parameters model we were able to obtain a reasonable equation for capital goods imports. Before we were getting a negative income elasticity. We said then that one of the possible explanations for this unusual result was that the income elasticity was actually declining over time. The impact of the II PND and government policies put together could be used to explain such behaviour.

Equation 24 presents the results for the cointegrated regression. It can be seen the hyperparameter of $1y_{4q}$ is significantly different from zero. Moreover, as can be seen in figure V.38, the income elasticity shows a declining pattern from 1979 to 1983. It fell from 0.65 in 1979.1 to 0.43 in 1983.4.

The price elasticity can only be assumed different from zero at 10%. Its behaviour does not quite match the one we had for the total imports. Although the maxi devaluations seem to have had same impact, the main feature in figure V.37 in the continuous increase during 84.3 and 85.4. During this period the price elasticity goes from -0.49 to -0.61.
Figure V.38
Capital Goods Imports (Coint Reg)

Figure V.39
Capital Goods Imports (ECM)
As far as the ECM is concerned, the short run price elasticity also do not behave very much as in the case of total and intermediate goods imports. Although it increases in response to the maxi devaluations there are some other particular features. After 1980 it decreases substantially getting close to zero over from 82.2 to 83.1. It not only increases as a result of the maxi
devaluation in 1983, but actually keeps increasing until 86.3.

**EQ(25) Modelling Δlqm3q by Kalman Filter**

<table>
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<th>t statistic</th>
</tr>
</thead>
<tbody>
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<td>0.0028</td>
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<tr>
<td>Δlrer8qt</td>
<td>0.4523</td>
<td>0.1267</td>
</tr>
</tbody>
</table>

**fixed parameters**

| Δly4qt         | 0.7658         | 0.3914     | 1.9567     |
| Δlu3qt         | 1.3659         | 0.5062     | 2.6984     |
| ecvt-1         | -0.2013        | 0.0519     | -3.8785    |

**IV) Conclusions and Remarks**

In this chapter we tried to offer some fresh insights about the behaviour of Brazilian trade equations. A new approach based in the theory of cointegration was applied to the import demand case. This new procedure allows a simple distinction between short and long run properties of the model escaping from the simple partial adjustment hypothesis that has been used so widely in empirical works on this subject.

We use the Engle and Granger procedure to estimate models with fixed and time varying coefficients.

As far as the quarterly fixed coefficient models are concerned, the main point that stands out from the analysis is the complex adjustment mechanism that seems to exist. The exception seems to be intermediate goods, where most of the adjustment happens in the first quarter and the short run disequilibrium correction works quickly. For the capital goods and total imports the results indicate a complex and slow path of adjustment as well as a small disequilibrium correction in each period.
The fixed coefficient model estimated with annual data does not give good results. That is because there is some multicollinearity between the first difference of the income and capacity utilization variables. Their short run impact can not be estimated separately. We were, therefore, were forced to drop the capacity utilization from the ECM.

We also address the question of time varying parameters. As we discuss in the previous chapter, given the substantial changes in the industrial structure of the economy, stimulated by import substitution plans, and in economic policy regimes the parameters of the import demand function are expected not to be fixed. The Kalman filter was used to estimate a time varying parameters model.

Most of the previous empirical work on import demand have used a time trend to account for the instability in the coefficients due to structural changes. By using the Kalman filter we can accomplish the same goal in a more elegant and informative way.

It is difficult to find a global pattern in terms of how and why the coefficients change but some general remarks can be made. The long run income elasticity was found to have a cyclical pattern and to respond to the import substitution plans. In this respect the Plano de Metas was found to have had a greater impact when compared to the II PND. In the short run it was found that the income elasticity and the capacity utilization elasticity were fixed.
The price elasticity, both short and long run, seem to be affected by the changes in the exchange rate regime. Whenever the government adopts a more realistic exchange rate management the price variable becomes more important to explain the volume of imports or, in other words, the price elasticity increases.

The results show that in the case of the ECM, the short run price elasticity changes abruptly in response to the maxi devaluations. They also show that in the case of annual data estimations, the changes in the coefficients of the cointegrated regression is slow over time, whereas in the case of the quarterly data estimations the coefficients of both ECM and cointegrated regression changes abruptly.
APPENDIX V.1

In this appendix we present the results for the Johansen procedure. It was estimated with a constant but no trend or seasonal dummies. The lag length $k$ was chosen to make the residuals white noise.

I) Quarterly Data Estimations

1) Intermediate Goods Imports

**EIGENVALUES $\mu_i$ are**

<table>
<thead>
<tr>
<th>$\mu_1$</th>
<th>$\mu_2$</th>
<th>$\mu_3$</th>
<th>$\mu_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.023979</td>
<td>.206959</td>
<td>.296711</td>
<td>.453370</td>
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</table>

There are 4 valid eigenvalues $\mu_i$ out of 4

<table>
<thead>
<tr>
<th>$\lambda_{max}$</th>
<th>Trace</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-T.\log(1-\mu_i)$</td>
<td>$-T.\sum \log(1-\mu_i)$</td>
</tr>
<tr>
<td>1.140769</td>
<td>1.140769</td>
</tr>
<tr>
<td>10.898384</td>
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<td>16.543394</td>
<td>28.582548</td>
</tr>
<tr>
<td>28.387183</td>
<td>56.969731</td>
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</tbody>
</table>

**$\beta'$ EIGENMATRIX [largest $\mu_i$ first]**

<table>
<thead>
<tr>
<th>Variable</th>
<th>$lq_{m2q}$</th>
<th>$lr_{er7q}$</th>
<th>$l_{y1q}$</th>
<th>$lu_{2q}$</th>
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</thead>
<tbody>
<tr>
<td>$lq_{m2q}$</td>
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<td>-.98938</td>
<td>7.15443</td>
<td>73.16358</td>
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<td>$lr_{er7q}$</td>
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<td>-2.43370</td>
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**STANDARDIZED $\beta'$ EIGENVECTORS**

<table>
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<tr>
<th>Variable</th>
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<th>$lr_{er7q}$</th>
<th>$l_{y1q}$</th>
<th>$lu_{2q}$</th>
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**STANDARDIZED $\alpha$ Coefficients**

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<td>.09442</td>
<td>.05542</td>
<td>.04751</td>
<td>-.00171</td>
</tr>
</tbody>
</table>
2) Capital Goods Imports

**EIGENVALUES $\mu_i$ are**

\[ .076638 \quad .268261 \quad .331932 \]

There are 3 valid eigenvalues $\mu_i$ out of 3

\[
\begin{array}{ccc}
\lambda_{\text{max}} & \text{Trace} \\
-\text{T.} \log(1-\mu_i) & -\text{T.} \Sigma \log(1-\mu_i) \\
3.906939 & 3.906939 \\
15.304248 & 19.211188 \\
19.764867 & 38.976055 \\
\end{array}
\]

**$\beta'$ EIGENMATRIX [largest $\mu_i$ first]**

<table>
<thead>
<tr>
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<th>ly4q</th>
<th>lulq</th>
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<td>lulq</td>
<td>9.70534</td>
<td>5.03994</td>
<td>-21.68857</td>
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**STANDARDIZED $\beta'$ EIGENVECTORS**

<table>
<thead>
<tr>
<th>Variable</th>
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**STANDARDIZED $\alpha$ Coefficients**

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</table>

3) Total Imports

**EIGENVALUES $\mu_i$ are**

\[ .060527 \quad .085384 \quad .396343 \quad .423554 \]

There are 4 valid eigenvalues $\mu_i$ out of 4

\[
\begin{array}{ccc}
\lambda_{\text{max}} & \text{Trace} \\
-\text{T.} \log(1-\mu_i) & -\text{T.} \Sigma \log(1-\mu_i) \\
2.809617 & 2.809617 \\
4.016297 & 6.825914 \\
22.713679 & 29.539593 \\
24.789330 & 54.328923 \\
\end{array}
\]
\[ \beta' \text{ EIGENMATRIX [largest } \mu_i \text{ first]} \]

<table>
<thead>
<tr>
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<th>ler6q</th>
<th>ly4q</th>
<th>lu1q</th>
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\[ \text{STANDARDIZED } \beta' \text{ EIGENVECTORS} \]

<table>
<thead>
<tr>
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<th>ly4q</th>
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\[ \text{STANDARDIZED } \alpha \text{ Coefficients} \]

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<td>0.00818</td>
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</table>

II) Annual Estimations

1) Intermediate Goods Imports

\[ \text{EIGENVALUES } \mu_i \text{ are} \]

\[ .008496 \quad .107763 \quad .239845 \quad .256162 \]

There are 4 valid eigenvalues \( \mu_i \) out of 4

\[ \lambda_{max} \quad \text{Trace} \]

\[ -T. \log(1-\mu_i) : -T. \sum \log(1-\mu_i) \]

\[ .307147 \quad .307147 \]

\[ 4.104829 \quad 4.411976 \]

\[ 9.872398 \quad 14.284375 \]

\[ 40.624198 \quad 54.908573 \]

\[ \beta' \text{ EIGENMATRIX [largest } \mu_i \text{ first]} \]

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\[ \text{STANDARDIZED } \beta' \text{ EIGENVECTORS} \]

<table>
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<th>ler2a</th>
<th>ly1a</th>
<th>lu12a</th>
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<td>lu12a</td>
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</table>
STANDARDIZED α Coefficients

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<th>ly1a</th>
<th>lu12a</th>
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<td>.01384</td>
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<td>-.10898</td>
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<td>-.00095</td>
<td>-.00141</td>
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<tr>
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<td>-.09004</td>
<td>-.00072</td>
<td>-.00144</td>
<td>-0.44870</td>
</tr>
</tbody>
</table>

2) Capital Goods Imports

EIGENVALUES $\mu_i$ are

<table>
<thead>
<tr>
<th></th>
<th>.057630</th>
<th>.206386</th>
<th>.346655</th>
<th>.429150</th>
</tr>
</thead>
</table>

There are 4 valid eigenvalues $\mu_i$ out of 4

$\lambda_{\text{max}}$ Trace

$$-T \log(1-\mu_i) = -T \sum \log(1-\mu_i)$$

<table>
<thead>
<tr>
<th></th>
<th>2.196237</th>
<th>2.196237</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>8.552847</td>
<td>10.749084</td>
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<td></td>
<td>15.749063</td>
<td>26.498147</td>
</tr>
<tr>
<td></td>
<td>31.357655</td>
<td>57.855802</td>
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</table>

$\beta'$ EIGENMATRIX [largest $\mu_i$ first]

<table>
<thead>
<tr>
<th>Variable</th>
<th>lgm3a</th>
<th>ler6a</th>
<th>ly3a</th>
<th>lu12a</th>
</tr>
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<tbody>
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STANDARDIZED $\beta'$ EIGENVECTORS

<table>
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<th>lgm3a</th>
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<th>ly3a</th>
<th>lu12a</th>
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STANDARDIZED α Coefficients

<table>
<thead>
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<th>ler6a</th>
<th>ly3a</th>
<th>lu12a</th>
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</thead>
<tbody>
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3) Total Imports

EIGENVALUES $\mu_i$ are

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<tr>
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<th>.024383</th>
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<th>.414774</th>
</tr>
</thead>
</table>

There are 4 valid eigenvalues out of 4
### $\lambda_{\text{max}}$ Trace

\[-T. \log(1-\mu_i) : -T. \Sigma \log(1-\mu_i)\]

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>0.888683</td>
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</tr>
<tr>
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### $\beta'$ EIGENMATRIX [largest $\mu_i$ first]

<table>
<thead>
<tr>
<th>Variable</th>
<th>lqm1a</th>
<th>1rer1a</th>
<th>lyl1a</th>
<th>lu12a</th>
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</table>

### STANDARDIZED $\beta'$ EIGENVECTORS

<table>
<thead>
<tr>
<th>Variable</th>
<th>lqm1a</th>
<th>1rer1a</th>
<th>lyl1a</th>
<th>lu12a</th>
</tr>
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<tbody>
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</table>

### STANDARDIZED $\alpha$ Coefficients

<table>
<thead>
<tr>
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<th>lqm1a</th>
<th>1rer1a</th>
<th>lyl1a</th>
<th>lu12a</th>
</tr>
</thead>
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</tr>
</tbody>
</table>
I) Quarterly Data

All data are from 1975 to 1988 and correspond to seasonally unadjusted indexes 1984=100 unless stated otherwise.

1) Foreign Trade Indexes

The price indexes are Paasche while the indexes of quantum are Laspeyres.

- qm1q = Total imports except crude oil and wheat.
Source: for 1976-1986, unpublished data from Fundacao Getulio Vargas; for 1987-1988, the series was extended by Fachada (1990) using the same methodology.

- qm2q = Imports of intermediate goods except crude oil and wheat.

- qm3q = Imports of capital goods.

- pm1q = Total imports except crude oil and wheat.

- pm2q = Imports of intermediate goods except crude oil and wheat.
- \( pm3q \): Imports of capital goods.


2) Activity Variables

- \( y1q \): GDP.


- \( y3q \): Industrial production in the capital goods sector.

Source: IBGE, Indicadores de Conjuntura, several issues.

- \( y4q \): Industrial production (manufacturing industry).

Source: Same as above.

- \( u1q \): Capacity utilization in the industrial sector.

Source: Comjuntura Economica, several issues.

- \( u2q \): Capacity utilization in the intermediate goods industry.

Source: same as above.

- \( u3q \): Capacity utilization in the capital goods industry.

Source: same as above.

- \( dgkq \): Demand for capital goods. Constructed using the \( y3q \) and \( qm3q \) series. The weights were obtained from the annual value of the capital goods imports in cr$, converted by \( er3q \), and the production value of the domestic capital goods industry (metallurgy and mechanic industries).

3) **Domestic Price Variables**

- $pd_{2q}$ = Intermediate goods wholesale price.
  Source: Conjuntura Economica, column 9, several issues.
- $pd_{3q}$ = Capital goods wholesale price.
  Source: Conjuntura Economica, column 13, several issues.
- $pd_{4q}$ = Industrial goods wholesale price.
  Source: Conjuntura Economica, column 27, several issues.
- $er_{1q}$ = Index of exchange rate for total imports (cr$/US$). This data is actually the effective cost of buying foreign exchange. Therefore, as discussed in the main text, it includes not only the exchange rate but also the extra costs imposed from time to time.
  Source: Boletin do Banco Central, several issues; Rosa et alii (1979).
- $er_{2q}$ = Index of exchange rate for intermediate goods imports (cr$/US$).
  Source: same as above.
- $er_{3q}$ = Index of exchange rate for capital goods imports (cr$/US$).
  Source: same as above.

4) **Variables Names**

- $lqm_{1q} = \ln (qm_{1q})$
- $lqm_{2q} = \ln (qm_{2q})$
- $lqm_{3q} = \ln (qm_{3q})$
- $ldkg_{q} = \ln (dkg_{q})$
- $lrer_{6q} = \ln (er_{1q} \times pm_{1q} / pd_{4q})$
- $lrer_{7q} = \ln (er_{2q} \times pm_{1q} / pd_{2q})$
- $lrer_{8q} = \ln (er_{3q} \times pm_{3q} / pd_{3q})$
- $lu_{1q} = \ln (u_{1q})$
- $lu_{2q} = \ln (u_{2q})$
- $lu_{3q} = \ln (u_{3q})$
- $ldkg = \ln (dkg)$
- $ly_{1q} = \ln (y_{1q})$
- $ly_{4q} = \ln (y_{4q})$
II) **Annual Data**

All data are from 1947 to 1988 and correspond to index 1984=100 unless stated otherwise.

1) **Foreign Trade Indexes**

The price indexes are Paasche while the indexes of quantum are Laspeyres.

- \( qm1a \) = Total imports except crude oil and wheat. Before 1960 the series was constructed by deflating the values in dollars by the US wholesale price index. The series in dollars was obtained using the share of oil and wheat in the total imports in cruzeiros.


- \( qm2a \) = Imports of intermediate goods except crude oil and wheat. Data before 1960 had to be constructed as above.

Source: same as above.

- \( qm3a \) = Imports of capital goods. Before 1959 the series was calculated using the rate of change of a Fisher index.

Source: for 1947-1958, Fundacao Getulio Vargas (1968); for 1975-1986, Comjuntura Economica, column 28, several issues; for 1987-1988, the series was extended by Fachada (1990) using the same methodology.
- \( p_{1a} \) = Total imports except crude oil and wheat. Before 1960 the series was obtained using the USA wholesale price index.
Source: for 1947-1959, IFS/IMF, several issues; for 1975-1986, unpublished data from Fundacao Getulio Vargas; for 1987-1988, the series was extended by Fachada (1990) using the same methodology.

- \( p_{2a} \) = Imports of intermediate goods except crude oil and wheat. Data before 1960 had to be constructed as above.
Source: same as above.

- \( p_{3a} \) = Imports of capital goods. Before 1959 the series was calculated using the rate of change of a Fisher index.
Source: for 1947-1958, Fundacao Getulio Vargas (1968); for 1975-1986, Conjuntura Economica, column 42, several issues; for 1987-1988, the series was extended by Fachada (1990) using the same methodology.

2) **Activity Variables**

- \( y_{1a} \) = GDP.

- \( y_{3a} \) = Real gross capital formation. The series in real terms was obtained using the GDP deflator.
Source: same as above.

- \( u_{10a} \) = output gap. It was calculated by decomposing the GDP in unobservable components and taking the GDP gap as the observed value minus the tendency.
- \( u_{11a} \) = GDP gap from regression residuals. 
  \((\ln y_{1a} = a + b \text{time})\)

Source: same as above.

- \( u_{12a} \) = GDP gap from a two period moving average process with 1928 as the base year. 
  \((b_t = [(y_{1at-1}/y_{1at-2}) + (y_{1at-2}/y_{1at-3})]/2) \) and \( y_{pt} = b_t \ y_{pt-1} \)

Source: same as above.

- \( u_{13a} \) = GDP gap. For the period 1970/1988 annual average of \( u_{1q} \). Before 1970 the rate of change in \( u_{12a} \) was used.

Source: same as above.

- \( u_{14a} \) = GDP gap. Calculated by adjusting a exponential to the years of maximum capacity. The years chosen as maximum capacity were 1928, 1961, 1974, 1980.

Source: same as above.

3) **Domestic Price Variables**

- \( p_{d1a} \) = Total wholesale price.

Source: Conjuntura Economica, column 4, several issues.

- \( p_{d2a} \) = Intermediate goods wholesale price.

Source: Conjuntura Economica, column 8, several issues.

- \( p_{d3a} \) = Capital goods wholesale price. The series was extended before 1970 using the total wholesale price index.

Source: Conjuntura Economica, column 13, several issues.

- \( p_{d4a} \) = Industrial goods wholesale price.

Source: Conjuntura Economica, column 27, several issues.

- \( e_{r1a} \) = Index of exchange rate cr$/US$ for the total imports. This data is actually the effective cost of buying foreign exchange. Therefore, as discussed in the
main text, it includes not only the exchange rate but also the extra costs imposed from time to time.


- er2a = Index of exchange rate cr$/US$ for imports of intermediate goods. As before it includes all extra costs.

Source: same as above, but for 1953 1957, weighted average of categories I and II.

- er3a = Index of exchange rate cr$/US$ for imports of capital goods. As before it includes all extra costs.

Source: same as above, but for 1953 1957, weighted average of categories II and II.

- ta = Rate of tariff, calculated as the ratio of import tax plus tax on financial operations (IOF) revenues over total imports except crude oil and wheat in cr$.

Source: for the import tax and total imports in cr$, IBGE (1990); and for the tax on financial operations (IOF), Arrecadacao de Tributos Federais, several issues.

4) Variables Names

- \( lqm1a = \ln(qm1a) \)
- \( lq2m = \ln(qm2a) \)
- \( lqm3a = \ln(qm3a) \)
- \( lym1a = \ln(y1a) \)
- \( lym2a = \ln(y3a) \)
- \( lym3a = \ln(y3a) \)
- \( lut12a = \ln(u12a) \)
- $\ln(r_{er1a} = \ln\left(\frac{er1a \cdot pm1a \cdot (1 + ta)}{pd1a}\right)$

- $\ln(r_{er2a} = \ln\left(\frac{er2a \cdot pm2a \cdot (1 + ta)}{pd2a}\right)$

- $\ln(r_{er6a} = \ln\left(\frac{er3a \cdot pm3a \cdot (1 + ta)}{pd1a}\right)$

III) Graphs

[Graphs showing time series data with labels Igm1a, Igm1q, Igm2a, Igm2q, Igm3a, Igm3q]
CHAPTER VI

ESTIMATIONS OF EXPORT EQUATIONS

I) Introduction

In this chapter we present the empirical results for different models of export behaviour. We consider a supply and demand model of the kind represented by equations (27) and (28) in chapter two. Although the Brazilian share in world exports is not large it has became usual in the literature to estimate a simultaneous equation model and therefore we follow this path.

We follow the same approach adopted in the previous chapter. We start by estimating an error correction model with fixed coefficients. The next step is to allow the coefficients to be time varying and to try to associate their variation with changes in trade policy.

Three different approaches, all discussed in chapter four, are used to model the parameter change. The Kalman filter estimations do not lead to good results and therefore the switching regressions and bayesian approaches are also used.

The estimations are performed for total non-coffee exports as well as for industrial exports. In both cases annual and quarterly data are used. Coffee was excluded not only because the imperfect substitute model seems inappropriate in this case but mainly because the coffee
market is quite regulated by the major producers through the World Coffee Organization.

The chapter has another three sections. The following section deals with the fixed parameter estimation, while in the third section we present the results for the time varying parameter model. In some cases industrial and total non-coffee exports are discussed in separate subsections and in others cases they are discussed together. Finally, the last section presents the conclusions and remarks.

II) Fixed Parameter Models

In this section we present the fixed parameter estimates for the supply and demand equations of industrial and total non-coffee exports. The estimations are performed using two stage least squares (2SLS). Full information methods such as three stage least squares (3SLS) and full information maximum likelihood (FIML) are normally preferred since they yield asymptotically efficient estimators under the assumption that the model is specified correctly. If the model is not correct these methods may lead to inconsistent estimations that can be transferred from one equation to the remainder of the system. Since there is no firm consensus about the exact model specification we opted to use 2SLS to restrict the effect of misspecification to the equation under consideration.

As in the previous chapter we start by testing the order of integration of each of the variables to be used.
The CRDW, DF, ADF and PP tests are performed in all three versions. To save space the DF and ADF are presented only in the version including the constant while the PP test presented is calculated from an auxiliary regression with a constant and time trend. With the quarterly data we also performed the seasonal integration tests proposed by Hyllerberg et alii (1990).

We followed the Engle and Granger method, estimating first the cointegrated regression to get the long run elasticities and then using the residuals in the error correction mechanism. As in the previous chapter we adopted a general to specific approach. We started by estimating the ECM with a fairly general dynamic structure that was then restricted to obtain a more parsimonious specification.

II.1) Annual Data Estimations

In the estimations presented below we have used annual data for the period 1947 to 1988 for total non-coffee exports and from 1950 to 1988 for industrial exports. The problems in obtaining such a long data series were not as complicated as in the case of imports. Nevertheless, the data series are still not completely satisfactory. The details on how the series were constructed, their sources and names can be found in appendix VI.1.

1) The estimations in this section were performed using Microfit 3.0, RATS 3.11 and PC-Give 6.01.
The unit root tests for the annual data series are presented in table VI.1. The critical values for the tests are roughly -2.93 for the DF and ADF tests and -3.50 for the PP test. The tests all point in the same direction. The variables in levels are I(1) while the differences are I(0).

Table VI.1

<table>
<thead>
<tr>
<th>Unit Root Tests</th>
</tr>
</thead>
<tbody>
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</tr>
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</tbody>
</table>

1) Total Non-Coffee Exports

Equations (1s) and (1d) are respectively the cointegrated supply and demand equations for total non-coffee exports. The coefficients have all the right sign and are significant at 5%. Moreover the unit root tests show that the variables cointegrate since the residuals are I(0).

The demand price elasticity is quite high, partially confirming the hypothesis that Brazilian exports do not
affect international prices. Therefore, a simultaneous model is not really necessary as far as the estimation of the supply equation is concerned. The estimation of the supply equation using OLS yield coefficients that are quite close to the 2SLS estimations. The OLS estimations for the price and capacity utilization were respectively 2.610 and -5.354. The major advantage in the simultaneous model approach is therefore to obtain an estimation for the demand income elasticity.\(^2\)

**EQ(1s) Modelling \(lq_{x1a}\) by 2SLS**  
**The Sample is 1947 to 1985**

<table>
<thead>
<tr>
<th>variable</th>
<th>coeff</th>
<th>S. E.</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
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</tr>
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<td>.1998</td>
<td>13.6982</td>
</tr>
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<td>(lu_{12a})</td>
<td>-5.2374</td>
<td>1.4309</td>
<td>-3.6603</td>
</tr>
</tbody>
</table>

\[R^2 = 0.8454\] \[\sigma = 0.43096\] \[F(2,36) = 98.4330\] \[DW = 0.8681\]

\[DF = -2.685\] \[ADF = -2.496\] \[PP = -2.95\]

**EQ(1d) Modelling \(lq_{x1a}\) by 2SLS**  
**The Sample is 1947 to 1985**

<table>
<thead>
<tr>
<th>variable</th>
<th>coeff</th>
<th>S. E.</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
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<td>(ly_{w3a})</td>
<td>1.6313</td>
<td>.1186</td>
<td>13.7500</td>
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</tbody>
</table>

\[R^2 = 0.9256\] \[\sigma = 0.29907\] \[F(2,36) = 223.7755\] \[DW = 1.2759\]

\[DF = -4.164\] \[ADF = -4.178\] \[PP = -5.47\]

The income elasticity of 1.63 is quite low when compared with the values available in the literature. The consensus figure seems to be around 2.5. It should be

---

\(^2\) As instruments we used all the exogenous variables including the constant. The export price was deleted from variables such as \(lr_{er8a}\) and \(lr_{p3a}\) when they were used as instruments to ensure that only the exogenous variables were being used.
noticed though that most of the estimations available deals with manufactured goods only. As we will see in the next section the income elasticity for industrial goods is much closer to the consensus value.

EQ(2s) Modelling Δlq1a by 2SLS
The Sample is 1948 to 1985

<table>
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<tr>
<th>variable</th>
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<th>S. E.</th>
<th>t-value</th>
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</thead>
<tbody>
<tr>
<td>const</td>
<td>.0562</td>
<td>.0268</td>
<td>2.0931</td>
</tr>
<tr>
<td>Δlr8at</td>
<td>1.5226</td>
<td>.5770</td>
<td>2.6386</td>
</tr>
<tr>
<td>Δlu12at</td>
<td>-1.2094</td>
<td>.7935</td>
<td>-1.5241</td>
</tr>
<tr>
<td>ecvt-1</td>
<td>-.5962</td>
<td>.1994</td>
<td>-2.9899</td>
</tr>
</tbody>
</table>

R² = 0.5988 σ = 0.16374 F(3,34) = 8.8750 DW = 2.0138

analysis of 1-step forecasts

<table>
<thead>
<tr>
<th>date</th>
<th>actual</th>
<th>prediction</th>
<th>error</th>
<th>S.D. of error</th>
<th>t-value</th>
</tr>
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<tbody>
<tr>
<td>1986</td>
<td>-.23302</td>
<td>-.07965</td>
<td>-.15337</td>
<td>.17965</td>
<td>-0.85372</td>
</tr>
<tr>
<td>1987</td>
<td>.19008</td>
<td>-.12834</td>
<td>.31842</td>
<td>.17599</td>
<td>1.80093</td>
</tr>
<tr>
<td>1988</td>
<td>.14161</td>
<td>-.17507</td>
<td>.31668</td>
<td>.18554</td>
<td>1.70680</td>
</tr>
</tbody>
</table>

LM(1) = 0.5136 LM(3) = 1.8726 WHITE(1) = 0.4350
RESET(1) = 0.8047 RMSE = 0.2740

EQ(2d) Modelling Δlq1a by 2SLS
The Sample is 1948 to 1985

<table>
<thead>
<tr>
<th>variable</th>
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<th>S. E.</th>
<th>t-value</th>
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</thead>
<tbody>
<tr>
<td>const</td>
<td>.0511</td>
<td>.0337</td>
<td>1.5152</td>
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<td>Δlr3at</td>
<td>-1.8733</td>
<td>.9820</td>
<td>-1.9077</td>
</tr>
<tr>
<td>Δly3at</td>
<td>1.1349</td>
<td>.9191</td>
<td>1.2348</td>
</tr>
<tr>
<td>ecvt-1</td>
<td>-.62609</td>
<td>.2886</td>
<td>-2.1691</td>
</tr>
</tbody>
</table>

R² = 0.6524 σ = 0.11834 F(3,34) = 8.1615 DW = 1.8456

analysis of 1-step forecasts

<table>
<thead>
<tr>
<th>date</th>
<th>actual</th>
<th>prediction</th>
<th>error</th>
<th>S.D. of error</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1986</td>
<td>-.23302</td>
<td>.10208</td>
<td>-.33510</td>
<td>.19153</td>
<td>-1.74959</td>
</tr>
<tr>
<td>1987</td>
<td>.19008</td>
<td>.47329</td>
<td>-.28322</td>
<td>.28378</td>
<td>-0.99803</td>
</tr>
<tr>
<td>1988</td>
<td>.14161</td>
<td>.39563</td>
<td>-.25402</td>
<td>.23381</td>
<td>-1.08644</td>
</tr>
</tbody>
</table>

LM(1) = 1.5155 LM(3) = 2.3657 WHITE(1) = 1.3590
RESET(1) = 0.25248 RMSE = 0.2927
As far as the supply equation is concerned, the main thing to notice is the high value for the capacity utilization elasticity. It indicates a substantial flexibility of switching between the foreign and domestic markets. This characteristic is quite useful during periods of recession since firms can avoid a worse scenario by increasing sales abroad.

Equations (2s) and (2d) above are the error correction mechanisms, estimated using the error correction vector given by the cointegrated regressions. The results seem to be reasonable. All the variables have the expected sign although some of them are only significant at 10% or 15%. The Lagrange Multiplier, White and Reset tests performed for both supply and demand equations indicate no serial correlation, heterocedasticity or misspecification problems. The forecast performance of the model is not exceptional but in none of the cases, as the t tests for the forecasts show, the equations break down.

The fit in both equations as measured by the $R^2$ is relatively low. On one hand this is not surprising since we are dealing with I(0) variables. On the other hand, as the share of non-coffee exports in the total exports was not very high in the beginning of the sample. Therefore, it is possible that other factors not considered in the equations were of relevance. Moreover, the usual interpretation of the $R^2$ as the proportion of total

3) In the early 1950s coffee exports represented approximately 60% of the total Brazilian exports.
variation in the dependent variable explained by the exogenous variables do not apply in this context since there are two endogenous variables. The 2SLS may even produce a negative $R^2$ since the residual sum of squares may exceed the total sum of squares.

In most of the cases the impact elasticity is quite high, indicating that a major part of the total adjustment of the non-coffee exports to changes in the exogenous variables is done in the current year. In the case of the supply price elasticity and the demand price and income elasticity at least 56% of the total impact happens in the first year.

It is only in the case of capacity utilization that such a thing does not happen. The short run elasticity is -1.21 compared with a the long run elasticity of -5.24. This means only 23% of the total impact of a change in the level of capacity utilization on the total non-coffee exports happens in the current year. This result cast some doubt on the short run flexibility between the foreign and domestic markets we have mentioned before. Nevertheless, as the impact elasticity is greater than one it is possible to talk of some flexibility even in the short run.

Finally, as the coefficients of the error correction vectors are also high it becomes clear that the adjustment process is on the whole very fast. Not only a major part of the total adjustment happens in the first year but also some 60% of the disequilibrium is corrected each year.
2) **Industrial Exports**

The results presented in equations (3s) and (3d) show that in the case of industrial exports the variable in question also cointegrate. The estimated coefficients have the expected sign and are significant at 5%.

As before the high demand price elasticity indicates that a single equation model estimated by OLS is quite adequate if one is just interested in the supply equation. The coefficients of the OLS estimation of the supply equation, 2.35 for the price elasticity and -5.19 for the capacity utilization elasticity, are very close to the 2SLS estimates.

**EQ(3s) Modelling $lq_x2a$ by 2SLS**
The Sample is 1950 to 1985

<table>
<thead>
<tr>
<th>variable</th>
<th>coeff</th>
<th>S. E.</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>14.5953</td>
<td>8.4899</td>
<td>1.7191</td>
</tr>
<tr>
<td>lrer5at</td>
<td>2.4849</td>
<td>.2228</td>
<td>11.1602</td>
</tr>
<tr>
<td>lu12at</td>
<td>-5.1638</td>
<td>1.8650</td>
<td>-2.7689</td>
</tr>
</tbody>
</table>

$R^2 = 0.7914$ $\sigma = 0.55873$ $F(2,33) = 62.6061$ $DW = 0.6554$

$DF = -2.657$ $ADF = -2.801$ $PP = -2.87$

**EQ(3d) Modelling $lq_x2a$ by 2SLS**
The Sample is 1950 to 1985

<table>
<thead>
<tr>
<th>variable</th>
<th>coeff</th>
<th>S. E.</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>11.1520</td>
<td>4.0841</td>
<td>2.7306</td>
</tr>
<tr>
<td>lrp4at</td>
<td>-3.8869</td>
<td>.9102</td>
<td>-4.2702</td>
</tr>
<tr>
<td>lyw3at</td>
<td>2.4827</td>
<td>.1695</td>
<td>14.6471</td>
</tr>
</tbody>
</table>

$R^2 = 0.8590$ $\sigma = 0.45933$ $F(2,33) = 100.5474$ $DW = 1.0073$

$DF = -4.260$ $ADF = -2.833$ $PP = -4.40$

4) Actually, the PP test for the supply equation is marginally below the critical value of -2.93.
The results are in general similar to those obtained in the previous section. The main difference is the demand income elasticity. For the industrial exports this coefficient is 2.48 compared with only 1.63 for the total non-coffee exports. This is an expected result since industrial goods are in general more income elastic than agricultural goods and raw materials.

The value of the elasticities are in range of the figures available in the literature. Although most of the results produce a smaller capacity utilization elasticity a high value is not without precedent. 5

The error correction mechanisms presented in equations (4s) and (4d). The tests performed in both equations show no evidence of serial correlation, heterocedasticity or misspecification. The forecast performance of the supply equation is quite good, having the smallest root mean square error (RMSE) so far. The same good results can not be obtained from the demand equation. Although the t-test in the individual forecasts are not significant at 5% the errors are substantial generating a large RMSE. As the prediction errors are all negative a adjustment in the constant seems appropriate.6

As far as the speed of adjustment is concerned, the results in this section confirm the fact that exports

5) Braga and Markwald (1983) in one of their experiments obtain a value of 5.2 for the capacity utilization elasticity.
6) Actually, the demand equation for industrial exports also have forecast errors all of the same sign. This may be taken as an indication that the constant is drifting off at the end of the sample. The results shown later in this chapter confirm that some of the coefficients are time varying.
seen to have an overall fast adjustment. Although the impact coefficients are in general not proportionally as large as in the case of non-coffee exports, they still imply that a substantial part of the total impact is accounted for in the first year. Moreover, as the feedback coefficients are high any disequilibrium error is greatly corrected each year.

EQ(4s) Modelling \( \Delta \log x_2a \) by 2SLS
The Sample is 1951 to 1985

<table>
<thead>
<tr>
<th>variable</th>
<th>coeff</th>
<th>S. E.</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
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<td>0.0365</td>
<td>1.8743</td>
</tr>
<tr>
<td>( \Delta \text{lrer5at} )</td>
<td>1.2266</td>
<td>0.6462</td>
<td>1.8982</td>
</tr>
<tr>
<td>( \Delta \text{alu12at} )</td>
<td>-1.5797</td>
<td>1.0164</td>
<td>-1.5542</td>
</tr>
<tr>
<td>ecvt-1</td>
<td>-0.6202</td>
<td>0.2151</td>
<td>-2.8837</td>
</tr>
</tbody>
</table>

\( R^2 = 0.5881 \)  \( \sigma = 0.11129 \)  \( F(3,31) = 8.6068 \)  \( DW = 1.9179 \)

Analysis of 1-step forecasts

<table>
<thead>
<tr>
<th>date</th>
<th>actual</th>
<th>prediction</th>
<th>error</th>
<th>S.D. of error</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1986</td>
<td>-0.08516</td>
<td>-0.089993</td>
<td>.00483</td>
<td>.22147</td>
<td>0.02181</td>
</tr>
<tr>
<td>1987</td>
<td>0.06052</td>
<td>-0.025743</td>
<td>.08626</td>
<td>.22157</td>
<td>0.38931</td>
</tr>
<tr>
<td>1988</td>
<td>0.23044</td>
<td>-0.053120</td>
<td>.28356</td>
<td>.22458</td>
<td>1.26262</td>
</tr>
</tbody>
</table>

\( \text{LM(1)} = 1.7088 \)  \( \text{LM(3)} = 2.6547 \)  \( \text{WHITE(1)} = 0.10140 \)

RESET(1) = 0.25369  \( \text{RMSE} = 0.1712 \)

EQ(4d) Modelling \( \Delta \log x_2a \) by 2SLS
The Sample is 1951 to 1985

<table>
<thead>
<tr>
<th>variable</th>
<th>coeff</th>
<th>S. E.</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>0.0620</td>
<td>0.0460</td>
<td>1.3480</td>
</tr>
<tr>
<td>( \Delta \text{lp4at} )</td>
<td>-1.5722</td>
<td>.7837</td>
<td>-2.0061</td>
</tr>
<tr>
<td>( \Delta \text{lyw3at} )</td>
<td>1.0835</td>
<td>.7483</td>
<td>1.4480</td>
</tr>
<tr>
<td>ecvt-1</td>
<td>-0.5135</td>
<td>.2190</td>
<td>-2.3444</td>
</tr>
</tbody>
</table>

\( R^2 = 0.6753 \)  \( \sigma = 0.20951 \)  \( F(3,31) = 9.2358 \)  \( DW = 1.9525 \)

Analysis of 1-step forecasts

<table>
<thead>
<tr>
<th>date</th>
<th>actual</th>
<th>prediction</th>
<th>error</th>
<th>S.D. of error</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
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<td>-1.74681</td>
</tr>
<tr>
<td>1987</td>
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<td>0.57000</td>
<td>-0.50948</td>
<td>0.32641</td>
<td>-1.56086</td>
</tr>
<tr>
<td>1988</td>
<td>0.23044</td>
<td>0.58848</td>
<td>-0.35804</td>
<td>0.33148</td>
<td>-1.08013</td>
</tr>
</tbody>
</table>

\( \text{LM(1)} = 2.7508 \)  \( \text{LM(3)} = 3.2564 \)  \( \text{WHITE(1)} = 0.04258 \)

RESET(1) = 0.35379  \( \text{RMSE} = 0.4666 \)
II.2) Quarterly Data Estimations

The quarterly data estimations were performed using data from the first quarter of 1975 to the last quarter of 1988. All the series are seasonally unadjusted and the details about the sources and variables names can be found in appendix VI.1.

Table VI.2

<table>
<thead>
<tr>
<th>Seasonal Integration</th>
<th>$\pi_1$</th>
<th>$\pi_2$</th>
<th>$\pi_3$</th>
<th>$\pi_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>lqx1q</td>
<td>2.331</td>
<td>-4.564</td>
<td>-1.925</td>
<td>-2.788</td>
</tr>
<tr>
<td>lqx2q</td>
<td>2.144</td>
<td>-4.776</td>
<td>-2.128</td>
<td>-3.484</td>
</tr>
<tr>
<td>ler4q</td>
<td>1.021</td>
<td>-4.015</td>
<td>-4.038</td>
<td>-4.548</td>
</tr>
<tr>
<td>ler5q</td>
<td>-1.118</td>
<td>-3.285</td>
<td>-4.859</td>
<td>-3.637</td>
</tr>
<tr>
<td>lusq</td>
<td>-0.634</td>
<td>-0.829</td>
<td>-2.849</td>
<td>-5.472</td>
</tr>
<tr>
<td>lrp3q</td>
<td>-1.489</td>
<td>-4.208</td>
<td>-3.399</td>
<td>-4.951</td>
</tr>
<tr>
<td>lrp4q</td>
<td>-1.297</td>
<td>-3.394</td>
<td>-4.849</td>
<td>-4.806</td>
</tr>
<tr>
<td>lqw2q</td>
<td>4.125</td>
<td>-1.127</td>
<td>-4.003</td>
<td>-4.215</td>
</tr>
</tbody>
</table>

To address the question of unit roots properly we have to test for their presence not only at the zero frequency but also at the seasonal ones. The seasonal unit root tests are reported in table VI.2. The tests on $\pi_1$ indicate strongly a unit root at the zero frequency for all the variables. Moreover, the tests on $\pi_2$, $\pi_3$ and $\pi_4$ indicate the presence of seasonal unit roots at the biannual and annual cycle for most of the variables. Only in the case of lusq and lqw2q does there seem to be no seasonal unit root at the biannual cycle.7

7) Using the notation developed in chapter four all the variables are $I_0(1)$ and $I_{1/2}(1)$, and apart from lusq and lqw2q also $I_{1/4}(1)$. The critical values for the tests
Table VI.3

Unit Root Tests

<table>
<thead>
<tr>
<th>CRDW</th>
<th>DF</th>
<th>ADF</th>
<th>PP</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>lx1q</td>
<td>0.169</td>
<td>-1.913</td>
<td>-1.144</td>
<td>-1.74</td>
</tr>
<tr>
<td>lx2q</td>
<td>0.087</td>
<td>-1.582</td>
<td>-1.415</td>
<td>-1.53</td>
</tr>
<tr>
<td>lrx4q</td>
<td>0.068</td>
<td>-2.270</td>
<td>-1.194</td>
<td>-2.30</td>
</tr>
<tr>
<td>lrx5q</td>
<td>0.251</td>
<td>-0.821</td>
<td>0.558</td>
<td>-0.75</td>
</tr>
<tr>
<td>lu1q</td>
<td>0.160</td>
<td>-1.749</td>
<td>-1.748</td>
<td>-1.80</td>
</tr>
<tr>
<td>lpr3q</td>
<td>0.061</td>
<td>-1.073</td>
<td>-0.724</td>
<td>-1.15</td>
</tr>
<tr>
<td>lpr4q</td>
<td>0.113</td>
<td>-1.946</td>
<td>-1.008</td>
<td>-1.99</td>
</tr>
<tr>
<td>lyw2q</td>
<td>0.103</td>
<td>-0.986</td>
<td>0.025</td>
<td>-0.49</td>
</tr>
<tr>
<td>Δlx1q</td>
<td>1.952</td>
<td>-7.264</td>
<td>-4.730</td>
<td>-8.41</td>
</tr>
<tr>
<td>Δlx2q</td>
<td>1.872</td>
<td>-6.893</td>
<td>-5.017</td>
<td>-7.30</td>
</tr>
<tr>
<td>Δlrx4q</td>
<td>1.666</td>
<td>-6.767</td>
<td>-4.034</td>
<td>-7.10</td>
</tr>
<tr>
<td>Δlrx5q</td>
<td>2.116</td>
<td>-9.001</td>
<td>-4.133</td>
<td>-10.06</td>
</tr>
<tr>
<td>Δlu1q</td>
<td>2.265</td>
<td>-8.247</td>
<td>-3.181</td>
<td>-8.35</td>
</tr>
<tr>
<td>Δlpr3q</td>
<td>1.595</td>
<td>-6.748</td>
<td>-3.131</td>
<td>-6.88</td>
</tr>
<tr>
<td>Δlpr4q</td>
<td>1.866</td>
<td>-8.278</td>
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<td>-8.58</td>
</tr>
<tr>
<td>Δlyw2q</td>
<td>2.395</td>
<td>-8.076</td>
<td>-3.101</td>
<td>-17.69</td>
</tr>
</tbody>
</table>

Although we recognise the presence of seasonal unit roots and the problems associated with that we pursue this path no further. Apart from some special cases where the cointegrating vector is known there is not a well developed framework to deal with this question.

The traditional unit root tests are presented in table VI.3. They confirm the test on $\pi_1$ in indicating the presence of a unit root (1-L) in all the variables.

1) Total Non-Coffee Exports

Attempts to estimate a quarterly data model in the same fashion as before did not produce good results. The main reason for this poor performance is the way one defines the relative price or real exchange rate given by Hylleberg et alii (1990) are -2.96, -1.95, -1.90 and -1.72 for respectively $\pi_1$, $\pi_2$, $\pi_3$ and $\pi_4$. 
variable. Until now this variable has been defined as the ratio between the export prices in dollars, including nominal exchange rates and exports subsidies, and the domestic prices. The claim made here is that the domestic price index is not the appropriate variable to be used in these circumstances.

The slow-down in GDP growth after 1974 and especially the recession of the 1980s put pressure on firms to export. As domestic demand was depressed, firms were driven to foreign markets as a way of escaping business failure. Therefore, the export decision was made not on the basis of comparing foreign and domestic prices but by contrasting foreign prices and domestic export costs.

A second similar line of argument, that leads to the same kind of conclusion, is also possible. Since the level of capacity utilization was quite below the maximum, profitability became the relevant variable. Therefore, firms were making export decisions based on comparing foreign prices and domestic export costs.

Unfortunately, there exists no variable measuring the evolution of export costs. As the construction of such a variable is out of our reach we decided to use the wage costs instead. The importance of the ratio between exchange rate and wages has been discussed in the literature.\(^8\) We therefore use the evolution of the domestic wages instead of wholesale prices in constructing the relative price variable to be used in

\(^{8}\) See, for example, Paula Pinto (1982).
the supply equation. The use of such variable increased greatly the quality of the results.

The cointegrated regressions are presented in equations (5s) and (5d). Since there exist only two other attempts to estimate a supply and demand model for the Brazilian exports, comparison between the values of the long run elasticities obtained here and a possible consensus value is difficult. Moreover, these two previous attempts did not produce similar results. It is interesting to note though that in the case of Fachada (1990) and Zini (1988), the estimated supply price elasticity was not significant at 5%.

\[
\text{EQ(5s) Modelling } lq \times lq \text{ by 2SLS} \\
\text{The Sample is 1975(1) to 1987(4)}
\]

<table>
<thead>
<tr>
<th>variable</th>
<th>coeff</th>
<th>S. E.</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>10.1623</td>
<td>2.6707</td>
<td>3.8051</td>
</tr>
<tr>
<td>lrer4qt</td>
<td>1.4721</td>
<td>.1783</td>
<td>8.2554</td>
</tr>
<tr>
<td>l1lqt</td>
<td>-3.0647</td>
<td>.4972</td>
<td>-6.1641</td>
</tr>
</tbody>
</table>

\[
R^2 = 0.7586 \quad \sigma = 0.2077 \quad F(2,49) = 76.9714 \quad DW = 1.2522
\]

\[
\text{DF} = -4.994 \quad \text{ADF} = -5.021 \quad \text{PP} = -3.21
\]

\[
\text{EQ(5d) Modelling } lq \times lq \text{ by 2SLS} \\
\text{The Sample is 1975(1) to 1987(4)}
\]

<table>
<thead>
<tr>
<th>variable</th>
<th>coeff</th>
<th>S. E.</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
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<td>3.7765</td>
<td>.14032</td>
</tr>
<tr>
<td>lrp3qt</td>
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<td>.8997</td>
<td>-1.9170</td>
</tr>
<tr>
<td>l1y2qt</td>
<td>1.5579</td>
<td>.4709</td>
<td>3.3087</td>
</tr>
</tbody>
</table>

\[
R^2 = 0.7616 \quad \sigma = 0.20639 \quad F(2,49) = 78.2659 \quad DW = 0.9137
\]

\[
\text{DF} = -3.770 \quad \text{ADF} = -4.515 \quad \text{PP} = -3.53
\]

The unit root test indicates that the variables cointegrate. The demand price elasticity seems to be low.

9) See table I.8 in chapter one.
showing that the small country hypothesis is not appropriate in this case. This contrasts with the findings in the annual data estimation. The reason for this apparent contradiction may lie in the fast growth of the Brazilian exports since the mid sixties. It seems that the small country hypothesis has recently become inadequate. A time varying parameter model estimated using annual data may be used to give support to such idea.

The error correction mechanisms are presented in equations (6s) and (6d). As it happened with the annual data estimations most of the total impact is accounted for in the first period. At least 50% of the total impact of a change in the exogenous variable happens in the first quarter. The feedback coefficients are also quite high considering we are using quarterly data. Some 26% to 30% of the disequilibrium is corrected each quarter.

The equations not only pass all the tests for serial correlation, heterocedasticity and misspecification but also show a very good predictive power. The RMSE in both equations are remarkably small, showing a great improvement in relation to the annual equations. Another noteworthy feature is that in all the equations the dynamics is quite simple.

The seasonality question was addressed by adding seasonal dummies for the first three quarters. In both supply and demand equations the seasonal dummies contributed substantially to improve the results. An F
test for the joint significance of the seasonal dummies indicated that they are jointly significant at 5%.

EQ(6s) Modelling $\Delta lq x1q$ by 2SLS
The Sample is 1975(2) to 1987(4)

<table>
<thead>
<tr>
<th>variable</th>
<th>coeff</th>
<th>S. E.</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>-.0278</td>
<td>.0338</td>
<td>-8.244</td>
</tr>
<tr>
<td>$\Delta$rer4qt</td>
<td>.9256</td>
<td>.5005</td>
<td>1.8495</td>
</tr>
<tr>
<td>$\Delta$l1u4qt</td>
<td>-2.9069</td>
<td>.6562</td>
<td>-4.4298</td>
</tr>
<tr>
<td>ecvt-1</td>
<td>-.3016</td>
<td>.1057</td>
<td>-2.8531</td>
</tr>
<tr>
<td>s1</td>
<td>-.1124</td>
<td>.0510</td>
<td>-2.2029</td>
</tr>
<tr>
<td>s2</td>
<td>.1870</td>
<td>.0558</td>
<td>3.3488</td>
</tr>
<tr>
<td>s3</td>
<td>.1044</td>
<td>.0458</td>
<td>2.2804</td>
</tr>
</tbody>
</table>

$R^2 = 0.6959 \quad \sigma = 0.10596 \quad F(6,44) = 16.7809 \quad DW = 1.9541$

analysis of 1-step forecasts

<table>
<thead>
<tr>
<th>date</th>
<th>actual</th>
<th>prediction</th>
<th>error</th>
<th>S.D. of error</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>88Q1</td>
<td>-.14474</td>
<td>-.20572</td>
<td>.06098</td>
<td>.11908</td>
<td>.51290</td>
</tr>
<tr>
<td>88Q2</td>
<td>.24728</td>
<td>.08125</td>
<td>.16603</td>
<td>.11571</td>
<td>1.43490</td>
</tr>
<tr>
<td>88Q3</td>
<td>.06710</td>
<td>-.00941</td>
<td>.07651</td>
<td>.11678</td>
<td>.65510</td>
</tr>
<tr>
<td>88Q4</td>
<td>-.16926</td>
<td>-.19420</td>
<td>.02494</td>
<td>.11469</td>
<td>.21746</td>
</tr>
</tbody>
</table>

LM(1) = 1.2358 \quad LM(4) = 4.3367 \quad WHITE(1) = 3.0321
RESET(1) = 0.33249 \quad RMSE = .00944

EQ(6d) Modelling $\Delta lq x1q$ by 2SLS
The Sample is 1975(2) to 1987(4)

<table>
<thead>
<tr>
<th>variable</th>
<th>coeff</th>
<th>S. E.</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>-1.1524</td>
<td>.0717</td>
<td>-2.1245</td>
</tr>
<tr>
<td>$\Delta$rp3qt</td>
<td>-1.6499</td>
<td>.5003</td>
<td>-3.2981</td>
</tr>
<tr>
<td>$\Delta$yw2qt</td>
<td>.8190</td>
<td>.4905</td>
<td>1.6698</td>
</tr>
<tr>
<td>ecvt-1</td>
<td>-.2576</td>
<td>.1012</td>
<td>-2.5470</td>
</tr>
<tr>
<td>s1</td>
<td>.0372</td>
<td>.1015</td>
<td>.3666</td>
</tr>
<tr>
<td>s2</td>
<td>.2851</td>
<td>.0647</td>
<td>4.4056</td>
</tr>
<tr>
<td>s3</td>
<td>.2651</td>
<td>.0942</td>
<td>2.8156</td>
</tr>
</tbody>
</table>

$R^2 = 0.6302 \quad \sigma = 0.1162 \quad F(6,43) = 12.2111 \quad DW = 1.486$

analysis of 1-step forecasts

<table>
<thead>
<tr>
<th>date</th>
<th>actual</th>
<th>prediction</th>
<th>error</th>
<th>S.D. of error</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>88Q1</td>
<td>-.14474</td>
<td>-.14141</td>
<td>-.00333</td>
<td>.12351</td>
<td>-.02696</td>
</tr>
<tr>
<td>88Q2</td>
<td>.24728</td>
<td>.20373</td>
<td>.04355</td>
<td>.12182</td>
<td>.35749</td>
</tr>
<tr>
<td>88Q3</td>
<td>.06713</td>
<td>-.02184</td>
<td>.08896</td>
<td>.12806</td>
<td>.69467</td>
</tr>
<tr>
<td>88Q4</td>
<td>-.16926</td>
<td>-.10878</td>
<td>-.06048</td>
<td>.12086</td>
<td>-.50041</td>
</tr>
</tbody>
</table>

LM(1) = 3.0325 \quad LM(4) = 10.2040 \quad WHITE(1) = 0.31536
RESET(1) = 0.41612 \quad RMSE = .05804
We also tried to estimate the ECM in terms of annual changes, that is using $\Delta_4$ instead of $\Delta$. It did not improve the results. The only noteworthy aspect is the increase in the value of the feedback coefficient. It got closer to the high values we obtained in the annual estimations, reinforcing the idea of a fast adjustment process.

2) Industrial Exports

The results the industrial exports are in general quite similar to those of total non-coffee exports. Actually, a closer examination of the series involved, especially the dependent variables, shows a substantially close evolution during the period considered.

EQ(7s) Modelling $lq x 2q$ by 2SLS
The Sample is 1975(1) to 1987(4)

<table>
<thead>
<tr>
<th>variable</th>
<th>coeff</th>
<th>S. E.</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>12.0790</td>
<td>2.8560</td>
<td>4.2294</td>
</tr>
<tr>
<td>$lrq5q_t$</td>
<td>1.7941</td>
<td>.1907</td>
<td>9.4087</td>
</tr>
<tr>
<td>$lu1qt$</td>
<td>-3.8968</td>
<td>.5317</td>
<td>-7.3293</td>
</tr>
</tbody>
</table>

$R^2 = 0.8098$  $\sigma = 0.22211$  $F(2,49) = 104.314$  $DW = 1.1691$

$DF = -4.757$  $ADF = -4.409$  $PP = -3.50$

EQ(7d) Modelling $lq x 2q$ by 2SLS
The Sample is 1975(1) to 1987(4)

<table>
<thead>
<tr>
<th>variable</th>
<th>coeff</th>
<th>S. E.</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>-2.0838</td>
<td>4.7556</td>
<td>-.43819</td>
</tr>
<tr>
<td>$lrp4q_t$</td>
<td>-1.7533</td>
<td>1.1378</td>
<td>-1.5409</td>
</tr>
<tr>
<td>$lyw2qt$</td>
<td>2.1460</td>
<td>.5742</td>
<td>3.7373</td>
</tr>
</tbody>
</table>

$R^2 = 0.81371$  $\sigma = 0.21982$  $F(2,49) = 107.018$  $DW = 0.6735$

$DF = -3.066$  $ADF = -2.803$  $PP = -2.98$
EQ(8s) Modelling $\Delta lq x 2q$ by 2SLS
The Sample is 1975(2) to 1987(4)

<table>
<thead>
<tr>
<th>variable</th>
<th>coeff</th>
<th>S. E.</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>0.0303</td>
<td>0.0312</td>
<td>0.9705</td>
</tr>
<tr>
<td>$\Delta lr e r 5 q t$</td>
<td>1.0422</td>
<td>0.5890</td>
<td>1.7693</td>
</tr>
<tr>
<td>$\Delta l u 1 q t$</td>
<td>-3.0428</td>
<td>0.6405</td>
<td>-4.7509</td>
</tr>
<tr>
<td>$ec v t - 1$</td>
<td>-0.2288</td>
<td>0.0917</td>
<td>-2.4955</td>
</tr>
<tr>
<td>s1</td>
<td>-1.543</td>
<td>0.0473</td>
<td>-3.2636</td>
</tr>
<tr>
<td>s2</td>
<td>0.0895</td>
<td>0.0490</td>
<td>1.8253</td>
</tr>
<tr>
<td>s3</td>
<td>0.0328</td>
<td>0.0443</td>
<td>0.7395</td>
</tr>
</tbody>
</table>

$R^2 = 0.60233$  $\sigma = 0.10362$  $F(6,44) = 11.1075$  $DW = 1.8591$

<table>
<thead>
<tr>
<th>analysis of 1-step forecasts</th>
</tr>
</thead>
<tbody>
<tr>
<td>date</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>88Q1</td>
</tr>
<tr>
<td>88Q2</td>
</tr>
<tr>
<td>88Q3</td>
</tr>
<tr>
<td>88Q4</td>
</tr>
</tbody>
</table>

LM(1) = 2.3256  LM(4) = 5.2019  WHITE(1) = 0.44181
RESET(1) = 1.0307  RMSE = 0.08124

EQ(8d) Modelling $\Delta lq x 2q$ by 2SLS
The Sample is 1975(2) to 1987(4)

<table>
<thead>
<tr>
<th>variable</th>
<th>coeff</th>
<th>S. E.</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>-1.401</td>
<td>.0846</td>
<td>-1.6585</td>
</tr>
<tr>
<td>$\Delta lr p 4 q t$</td>
<td>-1.4723</td>
<td>.5960</td>
<td>-2.4700</td>
</tr>
<tr>
<td>$\Delta l y w 2 q t$</td>
<td>1.4858</td>
<td>.8992</td>
<td>1.6523</td>
</tr>
<tr>
<td>$ec v t - 1$</td>
<td>-.2013</td>
<td>.0989</td>
<td>-2.0350</td>
</tr>
<tr>
<td>s1</td>
<td>.0806</td>
<td>.1209</td>
<td>.6670</td>
</tr>
<tr>
<td>s2</td>
<td>.1805</td>
<td>.0684</td>
<td>2.6384</td>
</tr>
<tr>
<td>s3</td>
<td>.2627</td>
<td>.1110</td>
<td>2.3665</td>
</tr>
</tbody>
</table>

$R^2 = 0.6417$  $\sigma = 0.13332$  $F(6,44) = 8.8068$  $DW = 1.7316$

<table>
<thead>
<tr>
<th>analysis of 1-step forecasts</th>
</tr>
</thead>
<tbody>
<tr>
<td>date</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>88Q1</td>
</tr>
<tr>
<td>88Q2</td>
</tr>
<tr>
<td>88Q3</td>
</tr>
<tr>
<td>88Q4</td>
</tr>
</tbody>
</table>

LM(1) = 1.2689  LM(4) = 1.9488  WHITE(1) = 0.00011
RESET(1) = 0.80561  RMSE = 0.059735
As far as the cointegrated regressions are concerned the main differences are the long-run income and capacity utilization elasticities. As argued above, the income elasticity is normally expected to be higher in the case of industrial goods. The greater capacity utilization elasticity is also expected since the industrial sector is in general more agile in changing sales between the domestic and foreign markets.

The error correction mechanisms are shown in equations (8s) and (8d). Once more the similarity with the previous results is evident. The adjustment process is very fast not only in the sense that a substantial part of the disequilibrium is corrected each period but also that most of the total impact happens in the first quarter.

It seems generally true that the adjustment process for the exports is quite fast in both annual and quarterly data. This is especially so when compared with the results for imports presented in the previous chapter. Apart from the case of intermediate goods the overall pattern seems to suggest that imports have a more slow and complicated adjustment process.

III) Time Varying Parameter Models

In this section we present the results for the time varying parameters models. In contrast to chapter five we do not rely only on the Kalman filter estimation, but use three different methods all of which were discussed in chapter four.
As a first approximation the Kalman filter is applied to the supply and demand equations obtained above. As the results were not at all satisfactory we also considered the bayesian and switching regressions approaches.\textsuperscript{10}

The objective of these estimations is to determine whether or not the changes in Brazilian trade policies or other international events had an impact on the stability of the coefficients. As we will see below the results are much less conclusive or clear than in the case of imports.

1) Kalman Filter Estimation

The Kalman filter estimations were performed in the same fashion as in chapter five. The hyperparameters were calculated by a maximum likelihood approach and then used in the filter recursions. A diffuse prior and a unit state vector were used to start the filter.

As we have a simultaneous model the Kalman filter cannot be applied to each individual equation. In such cases the extended Kalman filter is the appropriate algorithm to use.\textsuperscript{11} Unfortunately, we did not have the software facilities to perform the extended Kalman filter. Therefore, we had to simply apply the Kalman filter to each one of the equations. We did not consider the reduced form since in this case errors in one equation are carried out to the other ones.

\textsuperscript{10} The estimations in this section were performed using the programs REG-X, GAUUS 2.0 and BATS 1.4.
\textsuperscript{11} See Chow (1983).
As far as the supply equations are concerned this shortcoming should not distort the results very much since the demand price elasticity is high. Even in the case of the quarterly data estimations where the demand price elasticity was not quite so high it is still around 1.7 to 1.8.

The results of the Kalman filter estimations are shown below. Equations (9s), (9d), (10s) and (10d) are, respectively, the cointegrated regressions and error correction mechanism for the total non-coffee exports while equations (11s), (11d), (12s) and (12d) are concerned with the industrial exports.

As it can be seen by the t tests on the hyperparameters, none of the coefficients in any of the equations can be assumed to be time varying. This result can be interpreted in different ways. The first conclusion one can draw from these results is that the coefficients are indeed not time varying and, therefore, the models in the previous sections are the appropriate ones.

In the case of the quarterly data estimations this explanation is quite plausible. Since the main change in the export policy happened in the mid sixties, one should not be surprised that equations estimated using data from 1975 to 1988 have fixed coefficients. Even the "maxi devaluations" of 1979 and 1983 that had an impact on the coefficients of the import equations should be expected to have a smaller effect on the export supply coefficients. Exporters traditionally had access to the
black market and, therefore, the "maxi devaluation" may have affected mainly the amount of the foreign currency sold for the central bank, leaving the coefficients relatively stable. On the other hand, it is more difficult to accept that the comprehensive exports incentives policy of the mid sixties did not have an impact on the coefficients of the annual equations, at least as far as the long run elasticities are concerned.

A second possible explanation for the failure of the t test in pointing out parameter instability is that we are not using the appropriate algorithm. The use of the extended Kalman filter may change the results in favour of the time varying parameters model. Moreover, in some of the equations above the t statistics are not reliable due to singularities in the Hessian matrixes when estimating the hyperparameters.

Finally, it is also possible that the change in the parameters is not being picked up because it is concentrated in a short period of time. If the parameters have a step change it is possible that the maximum likelihood estimation of the hyperparameters misses such a change. The maximum likelihood approach may be not powerful enough to pick up a step change. As the introduction of export incentives is concentrated in the mid sixties a step change can not be ruled out.

EQ(9s) Modelling $\text{lqxl}_1$ by Kalman Filter

<table>
<thead>
<tr>
<th>Hyperparameter</th>
<th>standard error</th>
<th>t statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>216.31</td>
<td>2713.3</td>
</tr>
<tr>
<td>$\text{lre8r8}_t$</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>$\text{lu12}_t$</td>
<td>10.266</td>
<td>187.11</td>
</tr>
</tbody>
</table>
**EQ(9d) Modelling $lq_{1a}$ by Kalman Filter**

<table>
<thead>
<tr>
<th>Hyperparameter</th>
<th>Standard Error</th>
<th>$t$ statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>54.176</td>
<td>59.580</td>
</tr>
<tr>
<td>$lr_{p3a}$</td>
<td>688.80</td>
<td>1122.5</td>
</tr>
<tr>
<td>$ly_{w3a}$</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
</tbody>
</table>

**EQ(10s) Modelling $\Delta lq_{1a}$ by Kalman Filter**

<table>
<thead>
<tr>
<th>Hyperparameter</th>
<th>Standard Error</th>
<th>$t$ statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>0.01674</td>
<td>0.01787</td>
</tr>
<tr>
<td>$\Delta lr_{r8a}$</td>
<td>0.01179</td>
<td>0.09186</td>
</tr>
<tr>
<td>$\Delta lu_{12a}$</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>ecvt-1</td>
<td>0.05603</td>
<td>0.09876</td>
</tr>
</tbody>
</table>

**EQ(10d) Modelling $\Delta lq_{1a}$ by Kalman Filter**

<table>
<thead>
<tr>
<th>Hyperparameter</th>
<th>Standard Error</th>
<th>$t$ statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>$\Delta lr_{p3a}$</td>
<td>348.24</td>
<td>199.32</td>
</tr>
<tr>
<td>$\Delta ly_{w3a}$</td>
<td>1.4083</td>
<td>3.3410</td>
</tr>
<tr>
<td>ecvt-1</td>
<td>0.14596</td>
<td>0.18676</td>
</tr>
</tbody>
</table>

**EQ(11s) Modelling $lq_{2a}$ by Kalman Filter**

<table>
<thead>
<tr>
<th>Hyperparameter</th>
<th>Standard Error</th>
<th>$t$ statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>25794.0</td>
<td>49236.0</td>
</tr>
<tr>
<td>$lr_{r5a}$</td>
<td>116.92</td>
<td>152.93</td>
</tr>
<tr>
<td>$lu_{12a}$</td>
<td>19714.0</td>
<td>30096.0</td>
</tr>
</tbody>
</table>

**EQ(11d) Modelling $lq_{2a}$ by Kalman Filter**

<table>
<thead>
<tr>
<th>Hyperparameter</th>
<th>Standard Error</th>
<th>$t$ statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>97.492</td>
<td>56.358</td>
</tr>
<tr>
<td>$lr_{p4a}$</td>
<td>1287.7</td>
<td>1830.7</td>
</tr>
<tr>
<td>$ly_{w3a}$</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
</tbody>
</table>

**EQ(12d) Modelling $\Delta lq_{2a}$ by Kalman Filter**

<table>
<thead>
<tr>
<th>Hyperparameter</th>
<th>Standard Error</th>
<th>$t$ statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>0.15208</td>
<td>0.14110</td>
</tr>
<tr>
<td>$\Delta lr_{r5a}$</td>
<td>0.50255</td>
<td>0.74880</td>
</tr>
<tr>
<td>$\Delta lu_{12a}$</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>ecvt-1</td>
<td>0.01194</td>
<td>0.12902</td>
</tr>
</tbody>
</table>
To take into account this idea that there is a possible pitfall in the hyperparameters estimation, we use two other approaches to time varying parameters that avoid this problem.

2) The Switching Regressions Approach

The switching regressions approach was discussed in chapter four. Here we use the simplest version of the switching regressions model where there is only one change in regime. The procedure used is a variant of the one discussed in chapter four since we are dealing with simultaneous equations. We have used the adaptation of the 2SLS method proposed by Goldfeld and Quandt (1976a). Although two other more efficient full information methods exist, the 2SLS is of much simpler implementation. It only involves performing successive OLS estimations. For the reasons briefly discussed in the previous section we will restrict the analysis thereafter to the long run elasticities.

The results of the switching regressions estimations is presented in tables VI.4 and VI.5. They show the maximum of the likelihood function for a selected period around the switching date as well as the posterior odds ratio (POR).
\[ \text{POR} = \exp (\log L_t - \log L_{t'}) \]

where \( t' \) is the switching point.

The POR gives a measure of the degree of confidence one can have in the estimated switching point. It indicates the likelihood that the switch actually occurred around the switching date. It, therefore, gives a region where the switch is likely to have occurred. In other words it indicates how flat is the likelihood function around the maximum. If it is flat the POR's around the switching point will be close to one, indicating that a number of dates can be picked as the possible switching point.

### Table VI.4

<table>
<thead>
<tr>
<th>Year</th>
<th>Total Exports</th>
<th>Industrial Exports</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(-\log L)</td>
<td>POR</td>
</tr>
<tr>
<td>1958</td>
<td>15.688</td>
<td>0.0017</td>
</tr>
<tr>
<td>1959</td>
<td>14.893</td>
<td>0.0037</td>
</tr>
<tr>
<td>1960</td>
<td>13.920</td>
<td>0.0098</td>
</tr>
<tr>
<td>1961</td>
<td>12.520</td>
<td>0.0396</td>
</tr>
<tr>
<td>1962</td>
<td>11.102</td>
<td>0.1632</td>
</tr>
<tr>
<td>1963</td>
<td>12.049</td>
<td>0.0633</td>
</tr>
<tr>
<td>1964</td>
<td>9.290</td>
<td>1.0000</td>
</tr>
<tr>
<td>1965</td>
<td>9.551</td>
<td>0.7698</td>
</tr>
<tr>
<td>1966</td>
<td>11.665</td>
<td>0.0930</td>
</tr>
<tr>
<td>1967</td>
<td>12.056</td>
<td>0.0629</td>
</tr>
<tr>
<td>1968</td>
<td>14.621</td>
<td>0.0048</td>
</tr>
<tr>
<td>1969</td>
<td>13.448</td>
<td>0.0156</td>
</tr>
<tr>
<td>1970</td>
<td>11.584</td>
<td>0.1008</td>
</tr>
</tbody>
</table>

The results indicate that the switching point for the supply equations is around 1964/1966 while for the demand equations it is around 1974/1976. We have also
tried to confirm the switching point by performing a likelihood ratio test. The results are in table VI.6 and the critical values are given for a \( \chi^2 \) with three degrees of freedom.\(^\text{12}\) The tests confirm 1964 and 1965 for the supply equation and 1975 and 1976 for the demand equation as, respectively, the switching dates of total non-coffee and industrial exports.

Table VI.5

<table>
<thead>
<tr>
<th>Year</th>
<th>(-\log L)</th>
<th>POR</th>
<th>(-\log L)</th>
<th>POR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1969</td>
<td>15.016</td>
<td>0.0021</td>
<td>10.736</td>
<td>0.0075</td>
</tr>
<tr>
<td>1970</td>
<td>13.503</td>
<td>0.0095</td>
<td>8.682</td>
<td>0.0588</td>
</tr>
<tr>
<td>1971</td>
<td>13.543</td>
<td>0.0091</td>
<td>7.302</td>
<td>0.2339</td>
</tr>
<tr>
<td>1972</td>
<td>11.975</td>
<td>0.0437</td>
<td>9.006</td>
<td>0.0426</td>
</tr>
<tr>
<td>1973</td>
<td>10.618</td>
<td>0.1699</td>
<td>10.370</td>
<td>0.0109</td>
</tr>
<tr>
<td>1974</td>
<td>9.747</td>
<td>0.4059</td>
<td>9.085</td>
<td>0.0393</td>
</tr>
<tr>
<td>1975</td>
<td>8.845</td>
<td>1.0000</td>
<td>6.813</td>
<td>0.3813</td>
</tr>
<tr>
<td>1976</td>
<td>9.278</td>
<td>0.6486</td>
<td>5.849</td>
<td>1.0000</td>
</tr>
<tr>
<td>1977</td>
<td>10.150</td>
<td>0.2712</td>
<td>7.381</td>
<td>0.2161</td>
</tr>
<tr>
<td>1978</td>
<td>10.649</td>
<td>0.1647</td>
<td>8.536</td>
<td>0.0681</td>
</tr>
<tr>
<td>1979</td>
<td>11.781</td>
<td>0.0531</td>
<td>10.034</td>
<td>0.0152</td>
</tr>
<tr>
<td>1980</td>
<td>11.760</td>
<td>0.0542</td>
<td>9.116</td>
<td>0.0381</td>
</tr>
<tr>
<td>1981</td>
<td>10.395</td>
<td>0.2122</td>
<td>9.273</td>
<td>0.0326</td>
</tr>
<tr>
<td>1982</td>
<td>18.352</td>
<td>0.0001</td>
<td>7.456</td>
<td>0.2005</td>
</tr>
</tbody>
</table>

The change in regime in the mid sixties coincides with the beginning of the export incentive policy. On the other hand, the regime change in the demand equations may be related to the increase in protectionism in the

\(^{12}\) Goldfeld and Quandt (1976b) point out that although the likelihood ratio test for the switching regressions do not have an "appropriate" asymptotic \( \chi^2 \) distribution, it seems that, "for reasons that are not entirely clear", a \( \chi^2 \) with three degrees of freedom "may give an acceptable approximation".
developed countries, the dollar crisis in the early 1970's or to an increase of the Brazilian share in the world exports. The recession in the developed countries since the end of the sixties and the oil crises lead to protectionist policies that may account for the change in the parameters of the exports demand equations. This situation was not helped by the financial crisis caused by the run on the dollar in the early seventies. Another possible complementary explanation of the change in regime is that since the world share of the Brazilian export was increasing Brazil was moving away from its traditional position of small country. Therefore, one might expect a possible increase in the supply price elasticity and a reduction in the demand income and price elasticities.

<table>
<thead>
<tr>
<th>Table VI.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Likelihood Ratio Tests</td>
</tr>
<tr>
<td>total exports</td>
</tr>
<tr>
<td>supply</td>
</tr>
<tr>
<td>18.344</td>
</tr>
</tbody>
</table>

To examine the possible source of the change in regimes we reestimate the cointegrated regressions allowing both the slope and intercept coefficients to change between regimes. The use of dummy variables was then restricted to those coefficients that were significantly different from zero.
The results for the demand of total non-coffee exports are in contradiction with the early findings since none of the coefficients showed a significant step change. Equations (13s) and (14s) are the supply equations for total non-coffee and industrial exports respectively. In both cases there seems to be a step change in the relative price variable that may account for the change in regime. The step change happened as result of the announcement and introduction of export subsidies.

As could be expected, the change in the long-run supply price elasticity of industrial goods is substantially bigger than in the case of total non-coffee exports, since the export incentives were mainly directed to manufactured goods. The supply price elasticity of industrial exports increased by 15.5% while for total non-coffee exports the increase was only 5.6%.

Equation (14d) is the cointegrated regression for the demand for industrial exports. It shows that both price and income elasticities change after 1976. Both elasticities are reduced, which is in line with the argument of increases both in protectionism and in the Brazilian share of world industrial export markets. The demand price elasticity for industrial exports is reduced by 42.4% while the income elasticity falls by 65.2%.

Although the changes happen at the expected dates and in the right direction in both supply and demand cases they are not quite substantial. This is especially so when comparing with some of the changes in the import
demand elasticities. The demand elasticities of total and capital goods imports, for example, show changes of over 100% during the estimation period.

EQ(13s) Modelling $\log x_{1a}$ by 2SLS
The Sample is 1947 to 1988

<table>
<thead>
<tr>
<th>variable</th>
<th>coeff</th>
<th>S. E.</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>17.2984</td>
<td>6.5293</td>
<td>2.6494</td>
</tr>
<tr>
<td>$1_{\text{rer}8a}_t$</td>
<td>2.1823</td>
<td>.3462</td>
<td>6.3044</td>
</tr>
<tr>
<td>$1_{\text{lul}2a}_t$</td>
<td>-5.5298</td>
<td>1.4541</td>
<td>-3.8027</td>
</tr>
<tr>
<td>$1_{\text{rer}8a}_D_t$</td>
<td>.1229</td>
<td>.0506</td>
<td>2.4299</td>
</tr>
</tbody>
</table>

$R^2 = 0.86067$ $\sigma = 0.44336$ $F(3,38) = 78.246$ $DW = 0.76195$

DF = -2.732 ADF = -2.268 PP = -2.81

EQ(14s) Modelling $\log x_{2a}$ by 2SLS
The Sample is 1950 to 1988

<table>
<thead>
<tr>
<th>variable</th>
<th>coeff</th>
<th>S. E.</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>18.8945</td>
<td>6.8768</td>
<td>2.7476</td>
</tr>
<tr>
<td>$1_{\text{rer}5a}_t$</td>
<td>1.5372</td>
<td>.2996</td>
<td>5.1303</td>
</tr>
<tr>
<td>$1_{\text{lul}2a}_t$</td>
<td>-5.2928</td>
<td>1.5196</td>
<td>-3.4829</td>
</tr>
<tr>
<td>$1_{\text{rer}5a}_D_t$</td>
<td>.23771</td>
<td>.0527</td>
<td>4.5168</td>
</tr>
</tbody>
</table>

$R^2 = 0.87353$ $\sigma = 0.46902$ $F(3,35) = 80.583$ $DW = 0.43833$

DF = -2.191 ADF = -2.922 PP = -2.57

EQ(14d) Modelling $\log x_{2a}$ by 2SLS
The Sample is 1950 to 1988

<table>
<thead>
<tr>
<th>variable</th>
<th>coeff</th>
<th>S. E.</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>10.1548</td>
<td>4.0454</td>
<td>2.5102</td>
</tr>
<tr>
<td>$1_{\text{lwp}4a}_t$</td>
<td>-3.9235</td>
<td>1.0166</td>
<td>-3.8595</td>
</tr>
<tr>
<td>$1_{\text{lyw}3a}_t$</td>
<td>2.8148</td>
<td>.3437</td>
<td>8.1890</td>
</tr>
<tr>
<td>$1_{\text{lwp}4a}_D_t$</td>
<td>1.6654</td>
<td>.6709</td>
<td>2.4824</td>
</tr>
<tr>
<td>$1_{\text{lyw}3a}_D_t$</td>
<td>-1.8345</td>
<td>.7321</td>
<td>-2.5059</td>
</tr>
</tbody>
</table>

$R^2 = 0.89767$ $\sigma = 0.46902$ $F(4,34) = 74.563$ $DW = 1.1222$

DF = -5.072 ADF = -3.456 PP = -5.17

In conclusion, then, unlike Kalman filter, the switching regression approach shows some evidence of parameter variation, although this variation is less
substantial than that observed in the results obtained for imports.

3) The Bayesian Approach

A second way to by-pass the hyperparameters estimation is to use the Dynamic Linear Model (DLM) proposed by West and Harrison (1989). This model was discussed in chapter four for the single equation case. As we are dealing with a simultaneous equation system we follow Rothenberg (1975) and instrument the price variable in both the supply and demand equations. In other words, we perform a kind of 2SLS where the correlation between the right hand side endogenous variable and the error is eliminated in the first stage and the DLM is applied in the second stage.

Consider the DLM described below where $Y_t$ is the dependent variable and $X_{1t}$ and $X_{2t}$ are the exogenous variables. The model is quite similar to the Kalman filter model used above in which the coefficients follow a random walk. The only difference is in the intercept or level term which now is of second order.

\[
Y_t = \alpha_t + X_{1t} \delta_{1t} + X_{2t} \delta_{2t} + \nu_t
\]

\[
\alpha_t = \alpha_{t-1} + \beta_{t-1} + \epsilon_{1t}
\]

\[
\beta_t = \beta_{t-1} + \epsilon_{2t}
\]

\[
\delta_{1t} = \delta_{1t-1} + \xi_{1t}
\]

\[
\delta_{2t} = \delta_{2t-1} + \xi_{2t}
\]
In terms of West and Harrison (1989) notation the model above can be rewritten in a more compact form as

\[ Y_t = F'_t \theta_t + \nu_t \quad \nu_t \sim N(0,V) \]
\[ \theta_t = G \theta_{t-1} + \omega_t \quad \omega_t \sim N(0,W_t) \]

where

\[ G = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad F_t = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \omega_t = \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \]
\[ W_t = \begin{bmatrix} W_{1t} & 0 \\ 0 & W_{2t} \end{bmatrix} \]

The state vector \( \theta_t \) is estimated in the same way as in the Kalman filter, but the "hyperparameters" \( V \) and \( W_t \) are obtained in a different way. As discussed in chapter four the variance \( V \) is obtained by "variance learning", while the problem of estimating the covariance matrix \( W_t \) is substituted by the setting of discount factors, one for each block of the model.

The DLM is constructed by putting together separate building blocks. In our case we have two of such blocks, the linear trend and regression components. That is why \( W_t \) is decomposed into \( W_{1t} \) and \( W_{2t} \). Therefore, we will also need two discount factors.

A reference prior created from a initial uninformative prior and the first five observations was
used to start the estimations. Once the first results were obtained a more informative prior was considered but the results were not much different. The discount factors used were 0.98 for the linear trend component and 0.90 to 0.95 for the regression component.

The coefficient paths for the supply and demand equations of total non-coffee exports are presented in figures VI.1 to VI.8. The supply price elasticity shown in figure VI.1 is quite stable. The only recognisable change happens between 1968 and 1974 when the supply price elasticity increases smoothly, from 0.76 in 1968 to 0.98 in 1974. The other supply coefficients in figures VI.2 to VI.4 remain relatively constant.

This change in the supply price elasticity is partly in line with the findings of the previous section. The supply equation estimated using dummy variables also pointed to a small increase in the price elasticity. Nevertheless, the timing and shape of the change do not match. Previously we had the coefficient having a step change in 1964 while now the change happens smoothly between 1968 and 1974. Therefore, the two models give different results for the impact of the export incentive policies of the mid sixties. In the first case the change is concentrated in the beginning of the implementation of the export incentive policies, while in the second case it is distributed along the period over which such policies were implemented.

As far as the demand equation is concerned the results are also not quite the same as the ones in the
Figure VI.1
ON-LINE COEFFICIENT OF 1rer6a

Figure IV.2
ON-LINE COEFFICIENT OF 1u12a
Figure IV.3
ON-LINE LEVEL

Figure IV.4
ON-LINE GROWTH
Figure IV.5
ON-LINE COEFFICIENT OF lrp3a

Figure IV.6
ON-LINE COEFFICIENT OF lwu3a
previous section. Both the demand price elasticity in figure VI.5 and the level term in figure VI.7 show some small variation. The price elasticity decreases from -2.5 in 1965 to -2.1 in 1982, while the level shows a constant decline.

Note that again the change in the price elasticity is quite smooth, suggesting that a step change may be inappropriate. A further indication that a logistic switching may be more appropriate is given by the fact that the break point in the switching regressions is included in the period over which the change in the price elasticity happens. Possible reasons for this change in the demand price elasticity can again be found in the increase in protectionism and in the Brazilian share of world export markets.

Finally, it must be said that although the income elasticity in figure IV.6 appears to be very much constant there exists a substantial increase in uncertainty over this coefficient after 1972. As the standard errors fastly increase the confidence that this coefficient is actually constant may be challenged. In other words one is not so sure about the fixed point estimate in the end of the sample as one was in the beginning.

Figures VI.19 to VI.12 and VI.13 to VI.16 show the coefficient path for the supply and demand for industrial exports equations respectively. The results are in general less satisfactory than those for total non-coffee exports. The supply price elasticity does not show an.
Figure IV.9
ON-LINE COEFFICIENT OF ler5a

Figure IV.10
ON-LINE COEFFICIENT OF lu12a
Figure IV. 13
ON-LINE COEFFICIENT OF $l_{rp4a}$

Figure IV. 14
ON-LINE COEFFICIENT OF $l_{ywGa}$
significant variation pattern. That is quite surprising since the export incentive policies were directed targeted to this kind of exports. In the light of this result the supposed impact that this policy had on the supply price elasticity of total exports must be reconsidered.

The capacity utilization elasticity shows a step change in 1983, jumping from -6.25 to -5.10. This jump may be explained by the magnitude of the recession in 1983. This was the deepest recession since records began in 1947. Firms that were already switching sales to foreign markets since the recession of 1981 found it increasingly difficult. It should be noted, however, that the standard error of this coefficient is quite large throughout the sample period, hence not too much importance should be attached to this change.

The level term is the only coefficient to show a clear change. As can be seen in figure VI.11 the level term increases continuously during the sample period.

The demand equation coefficients also exhibit little change. Although the demand price elasticity and the level term in figures VI.13 and VI.15 show clear and smooth changes they are both very small. The income elasticity seems to be constant but, as in the case of total exports, there is an increase in uncertainty since the early seventies.
IV) Conclusions and Remarks

In this chapter we have adopted the same approach as in the previous one, which deals with import demand. We started by estimating a fixed coefficient error correction model and then considered the possibility of time varying parameters by using the Kalman filter. In this case, however, the Kalman filter failed to produce time varying coefficients. Since this result may have been caused by the way the hyperparameters are estimated we decided to investigate this matter further through the use of Bayesian and switching regression techniques. In other words the maximum likelihood estimation of the hyperparameters seen not to be the appropriate approach in this case.

As far as the fixed parameter models are concerned, the main feature is the simple and fast adjustment process. Not only most of the total impact of changes in the exogenous on the dependent variables happen in the first period, but also the high feedback coefficient indicates a fast correction of towards the equilibrium. This result is in direct contrast to the one we have obtained for imports.

This contrast is also present in the time varying parameters models. In the case of exports the results in this chapter seem to suggest that the parameter instability is not as substantial as in the case of imports. Although the three different approaches give some different indications the general impression is that of possible small change in coefficients. This change
seems to be concentrated in the second half of the sixties for the supply equation and in the mid seventies for the demand equation.

There is no doubt that further research is needed to clarify the results. In the switching regressions approach the use of logistic switching may give some interesting results. As the bayesian approach have shown, some of the coefficients seem to present a logistic change between the mid sixties and early seventies. The logistic change made explained by a learning process or for costs associated to a fast adjustment to the new regime. If it takes time for the economic agents to fully realise the consequences of the new policy or if the information set is asymmetric there will exist the need for a learning period. On the other hand, if there is a possibility that the policy will be reverted in the near future agents may choose to adjust slowly to the new regime.

In the Kalman filter approach the use of the extended Kalman filter may also bring some more light to the discussion.

More generally the different results given by the Kalman filter and bayesian estimation seem to point out the need for a more extensive research comparing this two approaches. Future theoretical and empirical research should concentrate on the effects of the way the variances or hyperparameters are obtained on the final results. As far as the results in chapter are concerned the bayesian approach seems to be more versatile than the
Kalman filter. It seems to be able to accommodate step changes as well as steady coefficient changes.
APPENDIX VI.1

I) Quarterly Data

All data are from 1975 to 1988 and correspond to seasonally unadjusted indexes 1984=100 unless stated otherwise.

1) Foreign Trade Indexes

The price indexes are Paasche while the indexes of quantum are Laspeyres.
- qx1q = Total exports except coffee.
  Source: for 1975-1986, Comjuntura Economica, column 2, several issues; for 1987-1988, the series was extended using the rate of growth of the same index calculated by the Central Bank.
- qx2q = Exports of industrial goods.
- px1q = Total exports except coffee.
  Source: for 1975-1986, Comjuntura Economica, column 14, several issues; for 1987-1988, the series was extended using the rate of growth of the same index calculated by the Central Bank.
- px2q = Exports of industrial goods.
2) **Activity Variables**
- $y_{w2q}$ = World import volume, calculated as the ratio between value of world import and world import unit value index.
  Source: IFS/IMF, several issues.
- $u_{1q}$ = capacity utilization in the industrial sector.
  Source: Comjuntura Economica, several issues.

3) **Price Variables**
- $p_{d5q}$ = exports cost index. Constructed using wage data from IBGE and FIESP.
  Source: IBGE (1990) and Comjuntura Economica, several issues.
- $e_{r6q}$ = Index of exchange rate for total exports (basket of currencies/cr$). In the basket of we have included 7 currencies: dollar, pound, French franc, Italian lira, florin, Japonese yen, German mark.
  Source: Boletin do Banco Central, several issues.
- $p_{xw2q}$ = Industrial countries export unit value index.
  Source: IFS/IMF, several issues.
- $s_{q}$ = rate of export subsidies for manufactured goods.
  The quarterly data was obtained from the annual estimates by assuming a linear growth.
  Source: for 1975-1985, Baumann and Moreira (1987); for 1986 we assumed a reduction of 10%; and for 1987-1988 we assumed the subsidies were kept constant at the 1986 level.
4) **Variable Names**

- \( lq1q = \ln(qx1q) \)
- \( lq2q = \ln(qx2q) \)
- \( lre4q = \ln((er6q*px1q*(1+sq))/pd5q) \)
- \( lre5q = \ln((er6q*px2q*(1+sq))/pd5q) \)
- \( lrp3q = \ln((px1q/pxw2q)*100) \)
- \( lrp4q = \ln((px2q/pxw2q)*100) \)
- \( lyw2q = \ln(yw2q) \)
- \( lu1q = \ln(u1q) \)

II) **Annual Data**

All data are from 1947 to 1988 and correspond to index 1984=100 unless stated otherwise.

1) **Foreign Trade Indexes**

The price indexes are Paasche while the indexes of quantum are Laspeyres.

- \( qx1a = \) Total exports except coffee.

Source: for 1947-1986, Comjuntura Economica, column 2, several issues; for 1987-1988, the series was extended using the rate of growth of the same index calculated by the Central Bank.

- \( qx2a = \) Exports of industrial goods.


- \( px1a = \) Total exports except coffee.

Source: for 1947-1986, Comjuntura Economica, column 14, several issues; for 1987-1988, the series was extended
using the rate of growth of the foreign trade indexes calculated by the Central Bank.
- px2a = Exports of industrial goods.

2) Activity Variables
- yw3a = World import volume index. Before 1950 the real world imports is used
- u12a = GDP gap from a two period moving average process with 1928 as the base year. \((bt = \frac{(y1at-1/y1at-2) + (y1at-2/y1at-3)}{2})\) and \(ypt = bt ypt-1\)

3) Price Variables
- pd4a = Industrial goods wholesale price.
Source: Conjuntura Economica, column 27, several issues.
- er4a = Index of exchange rate cr$/US$ for total exports except coffee. As before this export rate represents the amount effectively receive when selling foreign exchange.
- er5a = Index of exchange rate cr$/US$ for exports of industrial goods. As before this includes all extra revenues of selling foreign exchange.
Source: same as above but for 1953-1957, category IV only.
- sa = rate of export subsidies for manufactured goods. 
  Source: for 1969-1985, Baumann and Moreira (1987); for 1986 we assumed a reduction of 10%; and for 1987-1988; we assumed the subsidies were kept constant at 1986's level.
- pxw2a = Industrial countries export unit value index. 
  Again, for 1947/48 the series was obtained using USA data.
  Source: IFS/IMF, several issues.

4) Variable Names
- lqx1a = ln(qx1a)
- lqx2a = ln(qx2a)
- lrer5a = ln((er5a*px2a*(1+sa))/pd4a)
- lrer8a = ln((er4a*px1a*(1+sa))/pd4a)
- lrp3a = ln((px1a/pxw2a)*100)
- lrp4a = ln((px2a/pxw2a)*100)
- lyw3a = ln(yw3a)
- lul2a = ln(u12a)

III) Graphs
CHAPTER VII

CONCLUSIONS

The main objective of this thesis was to increase the understanding of Brazilian foreign trade by estimating and analysing the results of time varying parameter models for the Brazilian trade equations.

The question of parameter variation rises, on the one hand, as result of the several changes in the trade policy regime Brazil has experienced over the last four decades, as described in chapter three. During this time we have seen multiple and single exchange rate systems, the introduction of the crawling peg rule and its temporary break by the "maxi devaluations", the introduction of ad valorem tariffs, the partial end of the import substitution policies in 1964 associated with the creation of a new and extensive structure of export incentives, and the imposition of all kinds of direct controls on imports. On the other hand, the two major import substitution plans, carried out in 1957/60 and 1974/78, aimed at developing new industries, especially in the capital and intermediate goods sector, had an important impact on Brazilian industrial structure and, consequently, on the trade pattern.

As Lucas (1976) and Goldstein and Khan (1985) have pointed out, these facts should have had some impact in terms of creating instability in the trade equation
coefficients. The results presented in chapters five and six generally confirm the existence of parameter instability, especially in the case of the import demand.

In the case of the fixed parameter models the values we obtained for the long run trade elasticities are in general not very different from the ones available in the literature. The main difference from previous work is in the short run dynamics. The use of the error correction mechanism allowed the dynamics to be data determined and, therefore, it overcomes the limitations of the simple partial adjustment models. The results show a sharp contrast between imports and exports. While imports present in general a complicated and slow adjustment process, in the case of exports most of the adjustment happens in the first period and a major part of any eventual disequilibrium is corrected each period. Therefore, if the government is in need of an increase in the trade surplus in the short run the best way of achieving this is to use polices aimed at boosting exports rather than reducing imports.

As far as the import elasticities are concerned it is difficult to find a global pattern in terms of how and why the coefficients change but some general remarks can be made. The long run income elasticity was found to have a cyclical pattern and to respond to the import substitution plans. In this respect the Plano de Metas was found to have had a greater impact when compared to the II PND. The short run income elasticity and capacity utilization elasticity seem to be fixed. Both short and
long run price elasticity, seem to be affected by the changes in the exchange rate regime. Whenever the government adopts a more realistic exchange rate policy the price variable becomes more important in explaining the volume of imports, that is the price elasticity increases.

The export equation coefficients show much less instability than the import demand ones. Attempts to model the export equation coefficients by using the Kalman filter did not show evidence of parameter variation. The use of the Bayesian dynamic linear model and switching regressions indicate that in the case of exports we are not dealing with steady changes over time. In this case a step change seems to be more appropriate. In the case of the export supply equation the change occurred in the second half of the sixties in response to the export incentive polices, while for export demand it happened in the mid seventies as a result of the oil and dollar crises and the increase in the Brazilian share of world markets.

The results in this thesis show that, since the trade elasticities are not constant, any quantitative evaluation of possible trade policies may give biased results if the question of time variation is not considered. Although the actual value of the parameter will not be known at the time the different policies are being considered, our results might give some indication about the direction and magnitude of the change. Policies aiming to liberalise exchange rate controls, for example,
will lead to an increase in the import demand price elasticity and to a possibly smaller increase in the export supply price elasticity. On the other hand, since the import income demand elasticity exhibits a cyclical behaviour the timing of the policy change is also important. A liberalization of exchange rate controls implemented during a recession would see not only an increase in the import demand price elasticity but also a reduction in the import demand income elasticity.

Secondly, they also show that the hopes of some authors that by deepening the import substitution process we would solve or at least postpone the chronic Brazilian balance of payments problems is not justified. Their idea is that in the context of a two gap model, where the economy hits a inflationary or balance of payments constraint when it grows too fast, the import substitution process, by reducing the import income elasticity, would ease the foreign resources problem allowing faster growth. Even though the import substitution plans have had an impact in reducing the import demand income elasticity, our results showed that this reduction is by no means permanent and, therefore, the relief in terms of the trade balance is also temporary. Although the import substitution process played an important part in creating and diversifying Brazilian industry it seems that a change of strategy is now overdue. Abandonment of the import substitution policy in favour of a more open economy will improve
resource allocation and make the Brazilian industry more efficient and competitive in the world market.

Moreover, our results also show that the export incentive policy of the sixties did not have a substantial impact on the export supply price elasticity. Although the rebates on indirect taxes and the "drawback" and BEFIEX systems are sound policies, there seems to be a case to stop trying to increase exports through indiscriminate subsidy policies such as exemption of income tax, negative interest rates and the IPI Credit. It is not clear the actual specific contribution of these later measures to the growth of export and, on the other hand, they place an extra burden on the already affected public sector finances. The export performance after 1984 seems to show that a more realistic exchange rate policy could be the appropriate alternative to export subsidies.

In this thesis we have followed most of the empirical literature on time varying parameters and started by using the Kalman filter to deal with the question of parameter variation. The results in chapter six, though, cast some doubt on the capability of the Kalman filter to deal with step changes. The filter seems to be more appropriate in situations where the coefficients change continuously over time. The Bayesian dynamic linear model seems to be more versatile in this respect, as it is able to pick up the step change in the supply and demand of exports. More theoretical and empirical research is needed on the comparison of the Bayesian and classical approaches to parameter variation.
As far as the theoretical comparison goes, the key question seems to be the estimation of the variances or hyperparameters. The results in chapter six also show the importance of the simple step change models and the need to develop them further.

In terms of future research there are at least four lines that can be pursued. First there is the production theory approach for estimation of the import demand and export supply. This approach seems to suit the Brazilian case quite well since a major part of the Brazilian imports are intermediate goods. The problem in using this approach is the arduous job involved in generating reliable data on quantity and prices of domestic primary resources, capital and labour.

Secondly, an extension can be made to the work developed in this thesis on export equations by using more sophisticated step switching models. The extension could be made in two directions. One can make the model more general by allowing for more than one switching point or can keep a single switching date but make the transition between regimes smooth. In the later case a logistic switching using the versatile OGIVE function presented in chapter five may be a good approximation.

The logistic switching may happen because it takes time for the economic agents to learn or because there may be costs attached to a fast adjustment. If information is costly or if there is asymmetric information it may take time for all the economic agents to recognise the change. On the other hand, if for
example the policy that induced the change in the coefficients is reversed one may be better off by adjusting slowly. In other words, if the policy is not fully credible the adjustment may be sluggish.

Thirdly, there is the question of increasing the level of disaggregation in the export equations. Instead of using data for the industrial sector as a whole we could disaggregate it by industries, such as transport equipment, chemical products, textiles fibres, electronic equipment, etc. It is possible that by using more disaggregated data the results in terms of parameter variation will be more apparent.

Finally, one may try to use the equations estimated in this thesis as part of a macroeconomic model of the whole economy. This would allow a better understanding of the relationship between the trade sector and the rest of the economy, especially in what concerns the effects of parameter variation in the trade equations on other economic relations.
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