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Essays on Economic Fluctuations

by

Michail Rousakis

Thesis

Submitted to the University of Warwick

for the degree of

Doctor of Philosophy

Principal advisor: Professor Herakles Polemarchakis

Department of Economics

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THE UNIVERSITY OF WARWICK

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for my mother's mother

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Declarations

The thesis is the candidate's own work and has not been submitted for a degree at another university.

Abstract

This thesis consists of two essays on economic fluctuations.

The first essay (Chapter 2) explores the role of expectations in economic fluctuations. It does so within a cashless, monetary, and competitive economy featuring producers and consumers/workers with *asymmetric* information. Only workers observe current productivity and hence they perfectly anticipate prices, whereas all agents observe a noisy signal about long-run productivity. Information asymmetries imply that monetary policy and consumers' expectations have real effects. Non-fundamental, purely expectational shocks are conventionally thought of as demand shocks. While this remains a possibility, expectational shocks can also have the characteristics of supply shocks: if positive, they increase output and employment, and lower inflation. Whether expectational shocks manifest themselves as demand or supply shocks depends on the monetary policy pursued. Forward-looking policies generate multiple equilibria in which the role of consumers' expectations is arbitrary. Optimal policies restore the complete information equilibrium. They do so by manipulating prices so that producers correctly anticipate their revenue despite their uncertainty about current productivity. I design targets for forward-looking interest-rate rules which restore the complete information equilibrium for any policy parameters. Inflation stabilization per se is typically suboptimal as it can at best eliminate uncertainty arising through prices. This offers a motivation for the Dual Mandate of central banks.

The second essay (Chapter 3) shows that implementation cycles, introduced in Shleifer (1986) , are possible in the presence of capital and the absence of borrowing constraints. In a two-sector economy, patents on cost-saving ideas which take the form of investment-specific technological change arrive exogenously at a sequential, perfectly smooth rate: in odd-numbered periods, they

reach a firm producing capital of type 1 and, in the even-numbered ones, a firm producing capital of type 2 . Firms can make profits out of these once. While the immediate appropriation (henceforth, "implementation") of patents is always a possibility, for accordingly formed expectations, firms can alternatively implement their patents simultaneously. This is because investment-specific technological change naturally introduces a one-period discrepancy between the time firms implement their patents and the time they receive revenue out of them. The implementation of a patent implies a sharp fall in investment which, in turn, causes a boom in current consumption. As a result, the consumption boom takes place before the wealth boom. This not only eliminates the need to smooth consumption away from the wealth boom to the period before it as conjectured, but, further, it implies that the interest rate paid when revenue is realized -and wealth expands- falls. Consequently, present discounted profits rise and implementation cycles can become a possibility. In a policy extension, I show that prolonging patent rights to two periods rules out "implementation cycles" and may lead to a welfare improvement.

Chapter 1

Introduction

Economic fluctuations are ubiquitous. This thesis consists of two essays exploring them from two entirely different perspectives.

The first essay (Chapter 2), motivated by recent empirical studies[1] illustrating the major contribution of shocks to expectations in business cycle fluctuations, explores how the economy responds to non-fundamental shocks to expectations. Understanding the role of "sentiments" in economic fluctuations can lead to policies more successful in containing them.

The conventional wisdom is that purely expectational shocks exhibit features associated with demand shocks (think, for instance, of shocks to government expenditure):[2] when positive, that is when agents overstate the capacity of the economy, they increase output, employment and inflation. Inflation stabilization emerges then as a natural policy recommendation.[3] Nevertheless, the US economy was characterized by high cyclical employment and relatively

[1] See for example Beaudry and Portier (2006), Schmitt-Grohe and Uribe (2008), Blanchard et al. (2009)), Beaudry and Lucke (2010) and Barsky and Sims (2011a,b).

[2] See for example Blanchard (2009), Angeletos and La'O (2009), and especially Lorenzoni (2009, 2011).

[3] See for example the baseline case in Lorenzoni (2009).

low inflation in the mid-80s and the second half of the 90s, which are recalled as periods of exuberant optimism. More strikingly, data suggest that consumer sentiment and inflation are negatively correlated. An interpretation of expectational shocks as demands shocks does not seem to fit.

I reconsider the nature of expectational shocks within a monetary, cashless, and competitive economy where producers and consumers/workers have asymmetric information about current fundamentals and inflation (prices). I show that expectational shocks can indeed have implications for the business cycle actually associated with supply shocks (think, for instance, of oil shocks): when positive, they increase output and employment, and they lower inflation, implications at odds with the Phillips curve. Nonetheless, the possibility that expectational shocks manifest themselves as demand shocks remains. The underlying forces are producers' expectations about current productivity which push toward a supply-shock interpretation and consumers' expectations about long-run productivity which push toward a demand-shock interpretation; which one (supply or demand) prevails depends on the monetary policy pursued.

The above result was for the monetary authority following a "contemporaneous" (Taylor-type) rule. My second result concerns forward-looking policies of the type the literature traditionally considers (for instance, see Clarida et al. (2000)) . I show that forward-looking policies can generate a continuum of equilibria for any choice of policy parameters. Interestingly, what distinguishes equilibria is the role of consumers' expectations about the long-run prospects of the economy which is arbitrarily specified. This is because the equilibrium real interest rate is independent of them. A policy implication of this result, which can contribute to the discussion about the desirability of forward-looking policies, is that the short-run volatility of output due to expectational shocks can be substantially higher under forward-looking policies than under "contemporaneous" ones.

A question that emerges naturally concerns the role of monetary policy and a monetary authority's optimal response to shocks. Monetary policy has real effects by virtue of asymmetric, as opposed to incomplete yet symmetric, information about inflation (prices), an insight also offered in Weiss (1980) and King (1982). With flexible prices, producers' incomplete information is the only source of inefficiency. Optimal policies must restore then the complete information equilibrium. I show that inflation stabilization per se typically fails to do so -even in the case of "demand" shocks- as it at best eliminates uncertainty arising through inflation. Optimal policies manipulate inflation so that producers correctly anticipate their revenue despite their uncertainty about current productivity. Bearing this in mind, I design targets for forward-looking policies which restore the complete information equilibrium for any chosen policy parameters. These results offer a motivation for the Dual Mandate of central banks.

The second essay (Chapter 3) shows that implementation cycles, introduced in Shleifer (1986), are possible in the presence of capital or any storable commodity and the absence of borrowing constraints.

More analytically, the economy consists of a representative agent, a competitive final-good firm and two capital-good sectors each comprising a high number of capital-good firms. Capital makers use foregone consumption (investment) to produce a capital good they specialize in used in the production of the final good. In odd-numbered periods, a patent reaches randomly a firm in sector 1 and, in the even-numbered ones, a firm in sector 2. Patents are on cost-reducing technologies which imply that a unit of capital requires less resources in order to be produced. Initially, as in Shleifer (1986), I let firms make profits out of a patent only for one period; once a patent is utilized, the innovating firms' competitors costlessly copy the idea the patent was on and drive sector profits to zero -until a new patent arrives to the sector. As competitors cannot reverse-engineer an idea a patent is on before it is actually implemented, I use the terms patent and idea interchangeably.

Firms need to decide when they will implement their patents. I show that, if they share expectations about future and have perfect foresight, multiple "sunspot" equilibria can arise: firms can either implement their patents as soon as they receive them, which implies that patents are in place at the same -perfectly smooth- rate as that of their arrival, or they may instead coordinate their implementation, in which case "implementation cycles" with capital are generated.

To see this, note that imperfect competition invites demand externalities among capital-good sectors. Since a capital maker can postpone implementation of a patent, for instance, to the following period, when with certainty no improved technology will arrive, it needs to decide whether to implement it immediately or in the following period. As Matsuyama (1995) notes, it is precisely the intertemporal decision that firms face in combination with the presence of intratemporal demand externalities that can result in multiple Pareto-rankable equilibria.

Nevertheless, and despite this intuition, implementation cycles in the presence of capital and the absence of borrowing constraints (or constrained investment volatility) is something that Shleifer (1986) conjectured against:

anticipating future profits, agents would attempt to reduce their current savings in order to smooth out consumption. In turn, that would lower production and hence profits in a hypothetical implementation boom. For consumption smoothing to be mitigated, higher real interest rates would be necessary, which would in turn imply that firms discount future profits more. Both effects could rule out implementation cycles.

Why is this not so here? The reason is that patents are on investment-specific technological change, in the spirit of Greenwood et al. (1997, 2000). Investment-specific technological change introduces a one-period discrepancy between the consumption boom and the wealth boom. To see this, note that the implementation of a patent in the technology of a capital good reduces its current production cost, whereas the revenue out of it becomes realized in the following period. The latter implies that the wealth boom occurs one period *after* the coordinated implementation of patents takes place. The former implies that investment is substantially reduced in the implementation periods -in fact, it can even undershoot- and drives consumption above trend. As a result, the interest rate paid then is higher than the interest rate paid in the following, "wealth-boom," period. This increases investment in implementation booms, smoothing out consumption in the opposite direction from the conjectured one -without overturning the result on consumption which is a general equilibrium one-, and implies that more capital is installed in the following period which, given the elastic demand for it, leads to greater profits. Taking all into account, discounted profits after a conjectured coordinated implementation of patents become greater and, therefore, implementation cycles with capital possible.

Turning to policy considerations, it is important to note that the abstractions this essay makes allow it to concentrate solely on the effects of patent rights on the implementation -rather than the generation- of patents and, thereby, offer a clean argument from a different perspective to the ongoing discussion about the length of patent rights.

In particular, I show that letting firms appropriate a patent for two periods renders implementation cycles impossible and may lead to a welfare improvement. This suggests that a prolongation of patent rights is potentially desirable.

Chapter 2

Expectations and Fluctuations: The Role of Monetary Policy

2.1 Introduction

Recent empirical work suggests that shocks to expectations contribute significantly to economic fluctuations.[1] But how so? This is a recurrent question for academics, practitioners, and op-ed columnists. There is a growing consensus that if, for instance, consumers overstate the economy's fundamentals, the economy booms at the cost of inflation. A recent literature has formalized this idea:[2] non-fundamental, purely expectational shocks behave like demand shocks. When positive, they increase output and employment, and are inflationary. Stabilizing inflation emerges then as a natural policy recommenda-

[1] Empirical studies on the contribution of changes in expectations to business cycle fluctuations include Beaudry and Portier (2006), Schmitt-Grohe and Uribe (2008), Blanchard et al. (2009)), Beaudry and Lucke (2010) and Barsky and Sims (2011a,b).

[2] See for example Blanchard (2009), Angeletos and La'O (2009), and especially Lorenzoni (2009, 2011).

tion.[3]

Nevertheless, Figures 2.1 - 2.4 show that the US economy was characterized by high cyclical employment and relatively low inflation in the mid-80s and the second half of the 90s, which are recalled as periods of exuberant optimism. Notably, Figures 2.3 and 2.4 reveal that consumer sentiment and inflation are negatively correlated.[4] An interpretation of expectational shocks as demands shocks does not seem to fit.

This essay reconsiders the nature of purely expectational shocks within a competitive, monetary, cashless economy where producers and consumers/workers have asymmetric information about fundamentals and inflation (prices). I show that expectational shocks can have implications for the business cycle associated with supply shocks: when positive, they increase output and employment, and they lower inflation, which is incompatible with the Phillips curve.[5] Nonetheless, the possibility that expectational shocks manifest themselves as demand shocks remains. The underlying forces are producers' expectations which push toward a supply-shock interpretation and consumers' expectations which push toward a demand-shock interpretation; which one (demand or supply) prevails depends on the monetary policy pursued.

A natural question that emerges concerns the role of the monetary authority and its optimal response to shocks. With flexible prices, producers' incomplete information is the only source of inefficiency. Asymmetric, as opposed to incomplete but symmetric, information about inflation (prices)

[3]See the baseline case in Lorenzoni (2009).

[4] At a quarterly basis (Figure 2.3), the correlation of consumer sentiment and inflation is -0.53. Data are described in Appendix 2.9.

[5] Gali (1992) considers the textbook IS-LM model coupled with a Phillips curve and explores the effects of demand and supply shocks on the US business cycle. A discussion of the Phillips curve can be found in Mankiw (2001).

implies that monetary policy has real effects. Optimal policies restore the complete information equilibrium. Inflation stabilization per se is typically suboptimal as it at best eliminates uncertainty arising through inflation without removing producers' incomplete information. Optimal policies manipulate inflation so that producers correctly anticipate their revenue despite their uncertainty about productivity. Bearing this in mind, I design targets for forward-looking policies which restore the complete information equilibrium for any chosen policy parameters.

A competitive (neoclassical) economy features two representative agents, a consumer/worker and a producer, and a monetary authority. The worker supplies labor to a firm, managed by the producer, which produces a single commodity. Productivity consists of a permanent and a temporary component. There is asymmetric information about its current realization: it is specific and known to the worker, while the producer faces uncertainty about it. The monetary authority sets the riskless short-term nominal interest rate. I consider two interest-rate rules: a "contemporaneous" one and a forward-looking one.

Each period is split into two stages: In the first stage, the worker realizes his current productivity -not its individual components- , both agents observe a noisy public signal about the permanent (equivalently, long-run) productivity component, and the labor market opens (and closes). In the second stage, with production pre-determined from stage 1 , the commodity and the nominal bond markets open (and close) and all payments materialize. Prices are flexible in all markets and agents are price-takers.

The nominal wage, announced in stage 1, reflects the producer's expectations about productivity as well as stage 2 inflation (or prices). With constant returns to scale, the scale of production is pinned down by labor supply. The worker has complete information, so his labor decision and, consequently, production depend on the nominal wage and the inflation he knows will prevail in stage 2.

Inflation, in turn, depends on current productivity, on the producer's expectations about it, and the consumer's expectations about long-run productivity in a way decided by monetary policy. Asymmetric information about current productivity leads agents to form heterogeneous expectations about the inflation to prevail; this opens the door to monetary policy. Further, to the extent that inflation depends on the consumer's expectations about long-run productivity, the producer needs to second-guess the consumer. Then the consumer's expectations also have real effects, indirectly, through inflation. Therefore, that inflation is realized after the labor market has cleared not only prevents productivity from being revealed, but, in combination with asymmetric information, it implies that monetary policy and the consumer's expectations have real effects.

Purely expectational shocks affect both agents' expectations. The consumer's expectations about long-run productivity push toward a demand-shock interpretation. A consumption smoothing motive underlies this. Consider, for instance, positive purely expectational shocks. A consumer overly optimistic about the long-run prospects of the economy raises his current demand. If the producer had complete information about current productivity, flexible prices would increase and wages would proportionally adjust leaving the real wage intact. However, under incomplete information, the producer overestimates the inflationary pressure caused due to the consumer's expectations. As a result, the nominal wage increases more than proportionally and a higher real wage prevails. This induces the worker to increase his labor supply and production to expand.

The producer's expectations about current productivity per se point toward a supply-shock interpretation. A higher real wage reflects the producer's overly optimistic expectations; employment increases, production expands and, for a certain demand level, prices need to fall for the commodity market to clear.

It should not perhaps come as a surprise that the producer's incomplete information manifests itself as a distortion in the labor wedge originating from the labor demand side. The labor wedge is defined as the ratio of the marginal product of labor to the marginal rate of substitution of leisure for consumption.[6] Chari et al. (2007) find that it is countercyclical and accounts for more than half of the US output variance. When the real wage exceeds the marginal product of labor, the labor wedge falls. Positive expectational shocks, then, induce a countercyclical labor wedge.[7]

Whether expectational shocks cause an inflationary or a deflationary pressure depends on the monetary policy pursued. Taking into account that employment and output both increase (positive co-movement), it follows that it is up to the monetary authority whether a demand- or a supply-shock interpretation best fits expectational shocks.

In particular, the policy weight on the current output gap is central to which interpretation prevails. To see this, fix the real interest rate and note that, for a "contemporaneous" rule, expected inflation is zero, which implies

[6] See for example Hall (1997), Chari et al. (2007) and Shimer (2009).

[7] Related papers generating a countercyclical labor wedge in response to expectational shocks include Angeletos and La'O (2009), La'O (2010) and Venkateswaran (2011). Unlike these papers, the present essay emphasizes the connection of monetary policy with the labor wedge.

that the real interest rate coincides with the nominal one. The nominal interest rate targets inflation and the output gap. A positive expectational shock results in a positive output gap. A higher weight on the output gap implies less inflationary pressure which, in fact, may turn to a deflationary one.

Turning to productivity shocks, agents' expectations underreact in response to positive productivity shocks. As a result, a lower real wage prevails which induces employment to fall,[8] whereas output increases, however by less than under complete information. Following the same line of thought as above, the policy weight on current output gap determines whether productivity shocks are inflationary or disinflationary. Of course, agents learn over time and their expectations eventually converge to the underlying productivity level.

Considering forward-looking policies, the main difference with "contemporaneous" ones is that forward-looking policies generate a continuum of equilibria for any choice of policy parameters.[9] Importantly, what distinguishes equilibria is the role of the consumer's expectations which is arbitrarily specified. Furthermore, the short-run volatility of output due to expectational shocks is considerably higher under forward-looking policies than under "contemporaneous" ones. These results can contribute to the discussion about the desirability of forward-looking policies.[10]

[8] In the business cycle literature, Gali (1999) and Basu et al. (2006) also argue that positive technology shocks cause a temporary fall in employment.

[9] As my focus is on the effects of purely expectational shocks as well as those of productivity shocks, I do not discuss determinacy in the sense, for example, of Clarida et al. (2000) or Bullard and Mitra (2002) (although that discussion has been recently revived with Cochrane (2011)). Nevertheless, it is important to mention that there is no real indeterminacy here, since possible "sunspot" shocks lie outside the information sets of both agents thereby having no real effects, a point on which I elaborate below. I resume this discussion in fn. 60.

[10] Clarida et al. (1999, 2000) and Giannoni and Woodford (2003) also consider forward-looking policies, however in different settings. The Bank of England is suggested to follow

The nominal implications for forward-looking rules also differ, even after controlling for the consumer's expectations. This is because "contemporaneous" interest-rate rules pin down inflation, whereas forward-looking ones pin down price levels. To see this, consider a positive purely expectational shock and let prices depend positively on the producer's expectations, which is true for "active" policies, i.e. policies in which the monetary authority responds to inflation more than one-to-one. Price levels exhibit a non-monotonic pattern in response to expectational shocks: they increase on impact, however as agents update their beliefs over time, they gradually return to their long-run level. Thus, positive expectational shocks cause an inflationary pressure on impact and a deflationary one from the following period onwards. By the same logic, positive permanent productivity shocks are inflationary, until prices reach their higher steady-state level.

The producer's incomplete information is the only source of inefficiency. Optimal monetary policies restore then the complete information equilibrium. To do so they manipulate inflation (prices) so that the producer correctly anticipates his stage-2 revenue, even though still uncertain about current productivity. Inflation stabilization per se is typically suboptimal as it at best eliminates the indirect, inflation, channel of expectations without removing the producer's uncertainty about current fundamentals. I design forward-looking interest-rate rules which restore the complete information equilibrium. The rules "punish" deviations of expected inflation and expected growth from targets which adjust to their complete information levels.

In an extension, I consider a forward-looking monetary authority with superior information and let it communicate its information with noise. The

a forward-looking policy (Nelson (2000) provides an account of the period 1992 - 1997).

noise could be thought of as a measurement error or a monetary policy shock. The nominal interest rate serves then as an endogenous public signal. To the extent that prices depend positively on productivity, I show that positive measurement errors and monetary policy shocks raise the producer's expectations about the following period's productivity which results in higher prices and output.

Related literature. The idea that changes in expectations affect the business cycle has its origins at least in Pigou (1926) and has recently been revived by Beaudry and Portier (2004).[11] Christiano et al. (2010) show that expectational shocks are disinflationary in a New-Keynesian framework.[12] However, this strand of literature distinguishes between shocks to current and future productivity, whereas I emphasize the distinction between fundamental and non-fundamental shocks to expectations.

This essay lies in the literature following Phelps (1970) and Lucas (1972) which has formalized the idea that incomplete information can open the door to non-neutralities of non-fundamental factors.[13] The closest paper is Lorenzoni (2009). Lorenzoni (2009) restricts attention to the consumer side within a New-Keynesian framework and suggests that purely expectational shocks cause effects associated with demand shocks. Instead, I consider both the producer and the consumer side in a competitive economy with flexible prices[14]

[11]See also Beaudry and Portier (2006, 2007) and Jaimovich and Rebelo (2009).

[12]It has also been suggested in the empirical work of Barsky and Sims (2011b).

[13] Polemarchakis and Weiss (1977), Weiss (1980), King (1982), Bulow and Polemarchakis (1983) and, especially, Grossman and Weiss (1982) are related papers of the early literature. The literature has been revived with Woodford (2001), Morris and Shin (2002), Mankiw and Reis (2002) and Sims (2003). Hellwig (2008), Mankiw and Reis (2010), Lorenzoni (2011) and Chapter 9 in Veldkamp (2011) offer excellent surveys of the literature.

[14] A strand of literature, which for instance includes Angeletos and La'O (2009), Angeletos and La'O (2011a) and La'O (2010), also considers both sides however within non-monetary "Lucas-islands" frameworks featuring Dixit-Stiglitz monopolistic competition. This strand

and suggest that purely expectational shocks can behave like supply or demand shocks depending on the monetary policy pursued. To the best of my knowledge, this essay is the first to suggest so.

This essay shares with Weiss (1980), King (1982) and Lorenzoni (2010) the idea that monetary policy is non-neutral when there is asymmetric information about variables the monetary authority will respond to.[15] Crucially, it is asymmetric, rather than incomplete but symmetric, information that breaks the policy irrelevance, proposed in Sargent and Wallace (1975, 1976). Furthermore, the proposed optimal policies here differ from the one in Weiss (1980). In Weiss (1980), prices perfectly reveal the unknown fundamentals, while here prices are observed with a delay, so, by construction, this possibility is nonexistent.

The structure of the essay is as follows. Sections 2.2 and 2.3 present the model. Section 2.4 considers a "contemporaneous" interest-rate rule and shows that purely expectational shocks can have the features of demand or supply shocks for different policy specifications. Section 2.5 presents and analyzes the equilibria when a forward-looking interest-rate rule is followed and, in an extension, endows the monetary authority with superior information. Section 2.6 discusses the role of monetary policy and proposes optimal policies. Section 2.7 concludes.

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of literature emphasizes the link between dispersed information and strategic complementarities across islands which I abstract from.

[15] Recent papers studying monetary policy in environments with informational frictions include Adam (2007), Paciello and Wiederholt (2011) and Angeletos and La'O (2011b).

2.2 Environment

The competitive economy features two agents: a representative consumer/worker supplying labor to a representative firm he owns and a producer managing the firm. The firm produces a non-storable commodity. The economy is cashless and the only relevant financial market is a nominal bond market; a monetary authority sets the price of a riskless short-term nominal bond according to a "Taylor-type" rule.[16] Agents are price-takers in all markets. Time is discrete and infinite commencing in period 0. Each period comprises two stages: in stage 1 only the labor market opens, whereas in stage 2 the commodity and the nominal bond markets open.

The consumer's preferences are given by

$$E_0^c \sum_{t=0}^{\infty} \beta^t \, U(C_t, N_t) \,, \tag{2.1}$$

with period-t utility

$$U(C_t, N_t) \;=\; \log C_t \;-\; \frac{1}{1+\zeta}\, N_t^{1+\zeta} \,. \tag{2.2}$$

C_t and N_t denote consumption and employment in period t, respectively, and $\zeta \,>\, 0$ denotes the inverse of the constant marginal utility of wealth ("Frisch") elasticity of labor supply. The consumer's time preference is parametrized by $\beta \,\in (0,1)$.

The consumer faces a sequence of budget constraints given by

$$P_t\, C_t \;+\; Q_t\, B_{t+1} \;=\; B_t \;+\; W_t\, N_t \;+\; \Pi_t \,, \tag{2.3}$$

[16]Chapter in Woodford (2003) provides a treatment of cashless monetary economies.

where Q_t and B_{t+1} denote the price and holdings of nominal bonds maturing in $t+1$, respectively, P_t and W_t the commodity price and the nominal wage in t, respectively, and Π_t the firm's profits that accrue to the consumer.

The firm's technology is

$$Y_t \,=\, A_t\, N_t\,, \tag{2.4}$$

where A_t denotes the worker's productivity.

Productivity consists of a permanent and a temporary component (henceforth lowercase letters will denote natural logarithms),

$$a_t \,=\, x_t \,+\, u_t\,, \tag{2.5}$$

where x and u denote the permanent and temporary productivity components, respectively. Productivity -not its components- is specific and known to the worker, whereas the producer faces uncertainty about it.[17]

The permanent component x_t follows a random walk stochastic process

$$x_t \,=\, x_{t-1} \,+\, \epsilon_t\,, \tag{2.6}$$

where ϵ_t is an i.i.d shock and $\epsilon \sim N(0, \sigma_\epsilon^2)$. The temporary component u_t is i.i.d. and $u \sim N(0, \sigma_u^2)$.

All agents have costless access to a public signal about the permanent

[17] It may be argued that it is in the worker's best interest to reveal his type as he is the firm's owner. This is only an abstraction. Although I have not explored this possibility, an economy with many islands and complete financial markets which preserves the asymmetry of information *within* an island would presumably generate similar implications. See also fn. 20.

productivity component

$$s_t = x_t + e_t\,, \tag{2.7}$$

where e_t is i.i.d. and $e \sim N(0, \sigma_e^2)$. Shocks u , ϵ , and e are mutually independent. Hereafter, I will call e a purely expectational shock.

The distinction between permanent and temporary productivity introduces persistence in the shock effects.

2.2.1 Timing

Each period is divided into two stages. In the first stage, the consumer/worker realizes his period productivity a_t , both agents and the monetary authority realize the public signal about the permanent productivity component s_t , and the labor market opens (and closes). In the second stage, the commodity market and the nominal bond market open. All payments materialize in stage 2 and are perfectly enforceable.

Stage 1 is in turn divided into two sub-stages: in sub-stage 1 , new information is realized and the nominal wage is announced, whereas, in sub-stage 2 , the worker decides on his labor supply. This can be so by virtue of the firm's technology given by (2.4) : constant returns to scale imply that the nominal wage is independent of the amount of labor to be submitted in sub-stage 2 of stage 1 .[18]

In stage 2 , the monetary authority -whose role I specify below- steps in to set the nominal interest rate according to an interest-rate rule and the

[18]I have introduced a lag in the labor supply decision to prevent it from fully revealing the worker's productivity. An alternative would be to let labor supply be subject to a preference shock. Under such a specification, labor supply would (generically) be partially revealing about the worker's productivity. In the limit case in which the preference shock's variance tended to infinity, the producer would dismiss the informational content of labor supply and his information set would coincide with the one here.

commodity market opens. The consumer decides on his bond holdings and consumption at the prevailing prices. With nominal bonds in zero net supply, the nominal bond price adjusts to clear the nominal bond market. With production pre-determined from stage 1 , the commodity price adjusts to clear the commodity market.

Since output (alternatively, the commodity price) perfectly reveals productivity, in stage 2 both agents and the monetary authority have identical information.

2.3 Rational Expectations Equilibrium

The state of the economy as of period t coincides with the entire history $\Psi_t = \{(a_\tau)_{\tau=0}^{t}, (s_\tau)_{\tau=0}^{t}\}$. This is so due to the agents' formation of expectations on which I elaborate below. Turning to the agents and the monetary authority's information sets, it follows from the above that $I_{t,1}^{p} = \{(a_\tau)_{\tau=0}^{t-1}, (s_\tau)_{\tau=0}^{t}\} = \Psi_t \setminus \{a_t\}$ and $I_{t,2}^{m} = I_{t,2}^{p} = I_t^c = \Psi_t$.

Definition 1 (Equilibrium)**.** *A rational expectations equilibrium* under an interest-rate rule $Q_t(\Psi_t)$ *consists of prices* $\{P_t(\Psi_t), W_t(\Psi_t \setminus \{a_t\})\}$*, an allocation* $\{N_t^d(\Psi_t \setminus \{a_t\}), Y_t(\Psi_t)\}$ *for the producer and an allocation* $\{C_t(\Psi_t), N_t^s(\Psi_t), B_{t+1}(\Psi_t)\}$ *for the consumer such that:*

1. $\{N_t^d(\Psi_t \setminus \{a_t\}), Y_t(\Psi_t)\}$ *solves the producer's problem, laid out below, at prices* $\{P_t(\Psi_t), W_t(\Psi_t \setminus \{a_t\}), Q_t(\Psi_t)\}$.

2. $\{C_t(\Psi_t), N_t^s(\Psi_t), B_{t+1}(\Psi_t)\}$ *solves the consumer's problem, laid out below, at prices* $\{P_t(\Psi_t), W_t(\Psi_t \setminus \{a_t\}), Q_t(\Psi_t)\}$.

3. *All markets clear:* $Y_t = C_t$, $N_t^d = N_t^s$, $B_{t+1} = 0$ *for all* t *with* $B_0 = 0$.

The producer's labor demand in stage 1 maximizes the firm's expected evaluated profits, $E_t^p[\lambda_t \Pi_t \,|\, I_{t,1}^p]$. Expectations are conditional on the producer's information set in stage 1, $I_{t,1}^p$.[19] Using (2.4), profits are given by $\Pi_t = (P_t A_t - W_t) N_t$ and are evaluated using the consumer/owner's Lagrange multiplier, λ_t.[20] Constant returns to scale imply that the producer accommodates any labor supply at[21,22]

$$W_t = \frac{E_t^p [\lambda_t P_t A_t]}{E_t^p [\lambda_t]} . \tag{2.8}$$

The consumer has complete information about the state of the economy. Effectively, he makes all decisions in stage 1. Given $B_0 = 0$, the consumer chooses consumption, labor supply, and nominal bond holdings to maximize his expected utility (2.1) - (2.2) subject to his sequence of budget constraints (3.3) and a no-Ponzi-scheme constraint (for instance, requiring that $B_{t+1} > -\Gamma$

[19]Henceforth, the producer's expectations will always refer to his expectations as of stage 1 (sub-stage 1) unless otherwise stated.

[20]One may correctly point out that the consumer's Lagrange multiplier would perfectly reveals (his) productivity. Implicitly I have assumed that, at the beginning of the each period, the consumer and the producer physically separate. This assumption allows me to abstract from the "Lucas-Phelps" islands framework and to consider only one "island" in its stead. Given this, by maximizing the firm's evaluated profits, the producer operates the firm in the way the consumer/owner would want him to (see also Chapter 6 in Magill and Quinzii (1996)).

[21] It is central to this essay that the nominal wage in stage 1 be such that the producer's *expected* evaluated profits are zero. Given the linear technology (2.4), the producer is willing to hire any labor supplied at that nominal wage given by (2.8). This will typically result in a production level not ex-post desirable: once the state of the economy is realized, the real wage will typically be higher or lower than productivity, yielding losses or profits, respectively, with profits (losses) added (subtracted) in a lump-sum fashion to (from) the consumer/owner's income. Even though the nominal wage is flexible and competitive, the current setting could very roughly be thought to imply a form of nominal wage stickiness.

[22]On another note, since production takes place after the nominal wage is announced and depends on the consumer/worker's productivity, Y_t in the definition of equilibrium above is a function of the state Ψ_t rather than the producer's information set in stage 1, $\Psi_t \setminus \{a_t\}$.

for any $\Gamma > 0$ at all t is fine). The consumer's optimality conditions are

$$N_t^{\zeta} = \frac{W_t}{P_t C_t} \tag{2.9}$$

$$Q_t = \beta E_t^c \left[\frac{P_t}{P_{t+1}} \frac{C_t}{C_{t+1}} \right], \tag{2.10}$$

where $E_t^c [\cdot]$ refers to the consumer's expectations conditional on his information set I_t^c.[23]

In addition, the no-Ponzi-scheme condition and the fact that nominal bonds are in zero net supply imply $B_{t+1} = 0$ for all t in equilibrium. Suppressing then bond holdings from the state of the economy is harmless.

In what follows, I restrict attention to linear equilibria.[24] This simplifies considerably the agents' information extraction problems and allows me to use the Kalman filter algorithm in order to study the evolution of agents' beliefs.

For simplicity, I will work with log-linear approximations of (2.8)-(2.10). This is with no loss of generality: Appendix 2.10 shows that second-order terms are constants. Log-linear approximations will be around the stochastic steady state: since permanent productivity follows a random walk, output and consumption (as well as price levels) are non-stationary and the steady state stochastic.[25] To restore stationarity one needs to normalize the non-stationary variables with the permanent productivity component (see also King et al. (1988)). For instance, in the case of consumption, one could instead write $C_t^s = \frac{C_t}{e^{x_t}}$ (in logs, $c_t^s = c_t - x_t$).[26] I find it more convenient

[23] I have made no distinction between stages 1 and 2 for the consumer's expectations because the consumer's information set does not change within a period.

[24] I ignore whether non-linear equilibria exist.

[25] See also fn. 41.

[26] As in Lorenzoni (2005), when it comes to the expectation of a normalized variable, one

though to work with the non-normalized variables throughout.

2.3.1 Linear equilibria

I focus on linear rational expectations equilibria. In log-linear form the optimality equations are[27]

$$w_t \;=\; E_t^p\left[a_t\right] \;+\; E_t^p\left[p_t\right] \tag{2.11}$$

$$\zeta n_t \;=\; w_t \;-\; p_t \;-\; c_t \tag{2.12}$$

$$c_t \;=\; -\log\beta + \log Q_t \;+ E_t^c\left[c_{t+1} \;+\; \pi_{t+1}\right]. \tag{2.13}$$

Combining (2.11) and (2.12) results in

$$\zeta n_t \;=\; E_t^p\left[a_t\right] \;+\; E_t^p\left[p_t\right] \;-\; p_t \;-\; c_t\,. \tag{2.14}$$

I use the optimality conditions (2.13) and (2.14) in the rest of the analysis.

The existence of a monetary policy rule can get round the equilibrium indeteterminacy, nominal or real depending on whether agents have complete information or not, that would have prevailed in its absence. However, as Section 2.5 illustrates, the presence of a monetary authority per se need not

needs to additionally consider the expectation of permanent productivity. For instance, in the case of consumption, $E_t\left[c_t^s\right] \;=\; c_t \;-\; E_t\left[x_t\right]$.

[27]Where applicable, approximations are first-order around the stochastic steady state. For the log-linear approximations see Appendix 2.8.1 , whereas for the characterization of the steady state see Section 2.4.4 . A version without log-linear approximations of the equilibrium under rule 1 (see below for this) can be found in Appendix 2.10 . It should be straightforward to do the same with equilibrium under rule 2 .

be enough.

Monetary authority. The monetary authority sets the gross nominal interest rate (equivalently, the inverse of the logarithm of the nominal bond price), $i_t = -\log Q_t$, according to an interest-rate rule. Two commonly used rules will be considered in sequence, a contemporaneously-looking one (henceforth, rule 1) and a forward-looking one (henceforth, rule 2):[28]

$$i_t = -\log\beta + \phi_\pi \pi_t + \phi_y (y_t - a_t) \qquad \text{(Rule 1)}$$

$$i_t = -\log\beta + \phi_\pi E_t^m [\pi_{t+1}], \qquad \text{(Rule 2)}$$

where i_t denotes the nominal interest rate and π_t denotes inflation in period t, defined as $\pi_t := p_t - p_{t-1}$. In the case of rule 1, the monetary authority targets the output gap defined as the deviation of output from its complete information counterpart a_t. I restrict attention to non-negative values of the policy weights, ϕ_π and ϕ_y.

The monetary authority's information is solely based on the sequence of public signals as well as information extraction from prices and quantities. In Section 2.5.5, I let it be endowed with superior information when it follows rule 2 and subsequently study the information extraction problem of the agents. I consider more rules in Section 2.6 which explicitly studies the optimal monetary policies in the current framework.

[28] Rule 1 has been suggested by Taylor (1993, 1999) to capture adequately the Fed's policy during the period 1987 - 1992. Among other papers, rule 2 is considered in Clarida et al. (1999, 2000). Nelson (2000) proposes that a forward-looking rule fits well the Bank of England's policy in the period 1992 - 1997.

2.3.2 Expectations and the state of the economy

The state of the economy as of period t coincides with the the entire history $\Psi_t = \{(a_\tau)_{\tau=0}^{t}, (s_\tau)_{\tau=0}^{t}\}$. Past realizations of productivity and the public signal are part of the current state due to the agents' formation of expectations. In particular, I assume that the evolution of the agents' expectations about permanent productivity is given by the Kalman filter algorithm.[29] This is because inflation (prices) and/or quantities perfectly reveal productivity in stage 2 of each period. For this reason, the monetary authority's information set when it steps in, I_t^m, coincides with the state. It follows then that $I_t^m = I_{t,2}^p = I_t^c = \Psi_t$. The producer's expectation about current productivity as of stage 1 coincides with his expectation about its permanent component which follows from (2.5) and the fact that his information set in stage 1, which I show in the next section, is $I_{t,1}^p = \{(a_\tau)_{\tau=0}^{t-1}, (s_\tau)_{\tau=0}^{t}\}$. More analytically and bearing (2.5) in mind, agents and the monetary authority's expectations evolve as

$$E_t^p[a_t] = E_{t,1}^p[x_t] = (1-\mu)\, E_{t-1,2}^p[x_{t-1}] + \mu\, s_t \tag{2.15}$$

$$E_{t,2}^p[x_t] = E_t^c[x_t] = E_t^m[x_t] = (1-k)\, E_{t-1}^c[x_{t-1}] + k\,[\theta\, s_t + (1-\theta)\, a_t], \tag{2.16}$$

where μ, k, θ depend on the variances $\sigma_\epsilon^2, \sigma_e^2, \sigma_u^2$ and are in $(0,1)$. Appendix 2.8.2 offers an explicit treatment of the formation of expectations.

[29]Throughout this essay, I assume that agents learn/form expectations in a Bayesian way. I have not explored alternative learning/expectation-formation specifications, a subject beyond the scope of this essay. Further, agents could possibly update in other Bayesian ways which I ignore.

2.4 Equilibrium under Rule 1: Demand or Supply?

2.4.1 Complete information benchmark

Consider the case in which the state of the economy is common knowledge. Then, the real side of the economy is determined irrespectively of the public signal and the pursued monetary policy; we can confirm that $n_t^* = 0$ and $y_t^* = a_t$. On the nominal side, conjecture that $\pi_t = \vartheta_1 E_t^c[x_t] + \vartheta_2 a_t$ and then confirm that $\pi_t^* = \frac{1}{\phi_\pi}(E_t^c[x_t] - a_t)$. The consumer's expectations about permanent (long-run) productivity have only nominal effects: a consumption smoothing motive leads to changes in the consumer's current demand depending on his expectations about permanent productivity; however, flexible prices appropriately adjust in stage 2 and the nominal wage proportionally adjusts in stage 1 leaving the real wage intact and preventing the consumer's expectations from having real effects.

2.4.2 Incomplete information

Conjecture that

$$c_t = \xi_1 E_t^p[a_t] + \xi_2 a_t \tag{C1}$$

$$\pi_t = \kappa_1 E_t^p[a_t] + \kappa_2 E_t^c[x_t] + \kappa_3 a_t . \tag{C2}$$

Conjectures (C1) and (C2) imply the state of the economy can be summarized as $\Psi_t = \{E_t^p[a_t], E_t^c[x_t], a_t\}$. This is a direct consequence of the way agents form their expectations, described in Section 2.3.2, which disciplines the treatment of public signals and productivities within the state. The monetary authority can fully extract the current state by observing the public signal in stage 1 and inflation in stage 2 (alternatively, production or employment) which by conjecture (C2) (respectively, (C1)) perfectly reveals productivity a_t. In other words, when the monetary authority steps in at the beginning of stage 2, it shares the same information set with the consumer. This applies to the producer in stage 2 as well; that is $I_t^m = I_{t,2}^p = I_t^c = \Psi_t$.

Adding and subtracting p_{t-1} in the labor market optimality condition (2.14) and combining the Euler equation (2.13) with rule 1 implies

$$\zeta n_t = E_t^p [a_t] + E_t^p [\pi_t] - \pi_t - c_t \tag{2.17}$$

$$c_t = -[\phi_\pi \pi_t + \phi_y (y_t - a_t)] + E_t^c [c_{t+1} + \pi_{t+1}], \tag{2.18}$$

respectively.

Combining conjectures (C1) and (C2) with the optimality conditions, (2.17) and (2.18), and market clearing (Appendix 2.8.3 collects the derivations) yields

$$y_t = \xi_1 E_t^p [a_t] + (1 - \xi_1) a_t \tag{2.19}$$

$$\pi_t = \frac{1}{\phi_\pi} [-(1 + \phi_y) \xi_1 E_t^p [a_t] + E_t^c [x_t] + [(1 + \phi_y) \xi_1 - 1] a_t] \tag{2.20}$$

$$\xi_1 = \frac{\phi_\pi - 1 + k (1 - \theta)}{\phi_\pi (1 + \zeta) - (1 + \phi_y)}, \tag{2.21}$$

where k, θ are parameters associated with the consumer's learning problem introduced previously and derived in Appendix 2.8.2 .[30]

Equation (2.19) shows that output is a weighted average[31] of productivity and the producer's expectations about it. The respective weights depend on the Frisch elasticity of labor supply, parametrized by ζ, and the monetary policy parameters ϕ_π , ϕ_y .

The presence of ϕ_π , ϕ_y in (2.19) leads to the *first* key remark: monetary policy is non-neutral. This is attributed to the *heterogeneity* of the agents' expectations in stage 1 about inflation in stage 2 as we can see from (2.17) . Of course, heterogenous expectations are attributed to the agents' asymmetric information about current productivity. Crucially, incomplete yet symmetric information would imply a neutral monetary policy.

A *second* key remark is that the consumer's expectations have real effects despite prices being flexible. Once again, this is a direct consequence of asymmetric information. To the extent that inflation depends on the consumer's expectations, the producer needs to second-guess the consumer when forming expectations about inflation.[32] In particular, as (2.91) in Appendix 2.8.3 shows,

$$E_t^p\left[\,E_t^c\left[x_t\right]\,\right] \;=\; E_t^c\left[x_t\right] \;+\; k\left(1-\theta\right)\left(E_t^p\left[a_t\right] \;-\; a_t\right).$$

[30]There is no solution for $\phi_\pi = 0$ and there seems to be no well-defined solution for $\phi_\pi = 1$ (yet the latter case I need to explore further) . Therefore, I will assume both of these values of ϕ_π away.

[31]This is a direct consequence of preferences logarithmic in consumption.

[32]One could conjecture that consumption in (C1) also depends directly on the consumer's expectations only to verify that, in fact, the consumer's expectations do not enter equilibrium output directly. This happens because what matters for the labor decision in stage 1 , and hence the real side of the economy, is productivity and the producer's -not the consumer's- expectations about it as well as about inflation, as (2.17) attests.

What matters for the labor decision and hence production -through the inflation channel- is the *wedge* between the producer's and the consumer's expectations about inflation. Given conjecture (C2) and the fact that $E_t^c[\pi_t] = \pi_t$, it follows that

$$E_t^p[\pi_t] - E_t^c[\pi_t] = E_t^p[\pi_t] - \pi_t = [\kappa_2 \, k \,(1-\theta) + \kappa_3]\,(E_t^p[a_t] - a_t)\,. \quad (2.22)$$

The presence of the parameter κ_2 in (2.22) attests that the consumer's expectations have real effects.

Importantly, what lies in the intersection of the agents' information sets (for example, the producer's expectations) and what lies outside the union of the agents' information sets (possibly, non-fundamental shocks - see fn. 60) has *no* real effects through the inflation channel.

I will first discuss purely expectational shocks, which operate only through agents' expectations. Insulating the analysis from productivity shocks will allow me to focus solely on the "mechanics" of agents' expectations. Subsequently I discuss productivity shocks which operate both directly and through agents' expectations. Before continuing, let me point out that

$$\kappa_1 + \kappa_2 + \kappa_3 = 0 \quad (2.23)$$

$$\kappa_1 + \kappa_3 = -\frac{1}{\phi_\pi}\,. \quad (2.24)$$

Combining (2.23) and (2.24) implies $\kappa_2 = \frac{1}{\phi_\pi} > 0$, which we can see in (2.20); the consumer's expectations are positively related to inflation, and, consequently, indirectly through inflation positively related to output. The logic underlying this is a permanent income hypothesis one: if, for instance, a purely expectational shock leads the consumer to overstate the long-run prospects of the economy, consumption smoothing results in an increase in current demand which in turn causes an inflationary pressure. If the producer had complete information, prices would fully absorb the increased demand in stage 2 and nominal wages would proportionally adjust in stage 1; both would imply an unaffected real wage and, as a result, the absence of real effects. However, this is not the case under incomplete information: an overly optimistic producer -the public signal coordinates agents- overestimates the inflationary pressure. This implies the nominal wage increases more than proportionally compared to inflation, which results in a higher real wage. The latter causes labor to increase and production to expand, therefore partly accommodating the increased demand. Purely expectational shocks via the consumer's expectations push then toward a demand shock interpretation.

Turning to the producer, we can see from (2.19) - (2.21) that his expectations cause output and inflation to move in opposite directions. In other words, they point toward a supply-shock interpretation.[33] A sufficient condition for the producer's expectations to be positively related to output and negatively related to inflation is $\phi_\pi > \max\{\frac{1+\phi_y}{1+\zeta}, 1\}$.[34] That is for suffi-

[33] As already argued, since the producer second-guesses the consumer when forming expectations about inflation, the consumer's expectations matter indirectly as the term $\kappa_2\, k\,(1-\theta)$ in (2.22) shows. In fact, since $\kappa_2 > 0$, this term only accentuates the supply shock interpretation as (2.93) in Appendix 2.8.3 shows.

[34] For $\min\{\frac{1+\phi_y}{1+\zeta}, 1 - k\,(1-\theta)\} < \phi_\pi < \max\{\frac{1+\phi_y}{1+\zeta}, 1 - k\,(1-\theta)\}$, positive expectational shocks behave like negative supply shocks: they lower output and raise inflation. For appropriate policy parameters (ϕ_π, ϕ_y), inflation depends negatively on productivity. Then the indirect, inflation, channel of expectations lowers the total effect of the producer's expectations: an overly optimistic producer expects the worker to be more productive than he is, while inflation lower than it will actually be. For the suggested parameter values, the negative indirect effect outweighs the positive direct one. As a result, the nominal wage increases by less than inflation, hence, the real wage falls compared to its complete

ciently "active" policies, expectational shocks via the producer's expectations push toward a co-monotone supply shock interpretation. As I have already implied, the inefficiency caused due to the producer's incomplete information manifests itself as a distortion in the labor optimality condition. In particular, it causes a shift in labor demand: the overly optimistic, for instance, expectations of the producer will result in a higher real wage. This induces the worker to increase his labor supply and, as a result, production to expand. For a given demand level, this causes a deflationary pressure; prices need to fall for the commodity market to clear.

Will a demand- or a supply-shock interpretation prevail for purely expectational shocks? Suppose that the expectational shock affects the agents' expectations in the same way.[35] Then it follows that a positive expectational shock lowers inflation as long as $\kappa_1 + \kappa_2 < 0$. By (2.23) and (2.24), this is equivalent to requiring $\kappa_3 > 0$. Inspecting (2.20), we can see that the term κ_2 does not respond to changes in the monetary policy weight on the output gap, ϕ_y, whereas κ_3 increases in it;[36] the sign of κ_3 depends on how the policy weight on the output gap relates to the Frisch elasticity of labor supply $1/\zeta$. In particular, a value of ϕ_y greater than or equal to the inverse Frisch elasticity of labor supply ζ is a sufficient condition for κ_3 to be positive and, consequently, expectational shocks to be negatively related to inflation.[37]

The picture that emerges is that expectational shocks can exhibit features associated with supply or demand shocks depending on the monetary

information counterpart, which induces the worker to decrease his labor supply.

[35] That is I assume $k\,\theta = \mu$ in the learning problems of the agents. This is a good approximation if the temporary productivity shock has a high variance relative to the expectational shock, which is consistent with the parametrization in Table 2.1 below.

[36] A sufficient condition for this is that $\phi_\pi > 1$.

[37] The term κ_3 exhibits discontinuity at $\phi_\pi (1+\zeta) - 1$. As a result, this is true as long as $\phi_y < \phi_\pi (1+\zeta) - 1$.

policy pursued. The policy weight on the current output gap is central to how expectational shocks manifest themselves. In particular, as long as $\phi_\pi > \max\{\frac{1+\phi_y}{1+\zeta}, 1\}$, a higher weight on the output gap pushes toward a supply-shock interpretation. To provide an intuition for this, first note that (2.20) implies expected inflation is zero, that is the nominal and the real interest rate coincide:

$$r_t = i_t = \phi_\pi \pi_t + \phi_y (y_t - a_t). \tag{2.25}$$

Fix for a moment the real interest rate and consider the case of overly optimistic expectations which implies a positive output gap. Controlling for general equilibrium effects, the higher the policy weight on the output gap, the lower the inflationary pressure has to be for the real rate to remain constant.[38]

However, the real -and, hence, the nominal- interest rate increases in response to a positive purely expectational shock. This is a consequence of the overreaction of expectations: expected future output increases by more than current output since the latter is in part disciplined by current productivity, whose long-run component agents overstate. To what extent or whether this increase will be translated into higher inflation depends on the weight put on the (positive) output gap.

Turning to productivity shocks, $\phi_\pi > (1 + \phi_y) \max\{\frac{1}{1+\zeta}, \frac{k(1-\theta)}{\zeta-\phi_y}\}$ is a sufficient condition for them to be positively related to output. On the nominal side, maintaining the assumption that expectational shocks affect the agents' expectations in the same way, a direct implication of (2.23) and (2.24) is that productivity and expectational shocks cannot both increase or lower inflation.

[38]One may wonder what happens when the policy weight on inflation, ϕ_π, changes. In fact, general equilibrium effects complicate things considerably as both κ_2 and κ_3 (alternatively, κ_1) depend on ϕ_π. As a result, a similar reasoning applies only locally and it becomes hard to generalize. Hence, I will abstract from this consideration.

To connect the results with the previous analysis, consider a positive productivity shock. Under complete information, inflation would depend positively on the wedge between the consumer's expectations and productivity. Following a positive productivity shock the consumer's expectations underreact; as a result, demand underreacts as well which implies that prices must fall for the market to clear. However, under incomplete information, prices will not fall as much as they would under complete information, whereas they can even increase. The reason is that the producer's expectations also underreact, hence supply underreacts as well.

Along the lines of the above analysis, the weight on the output gap proves to be key as to how supply responds. Revisiting the real side, the underreaction of the producer's expectations implies that the increase in output falls short of the increase in productivity, therefore the output gap is negative and employment falls. Holding the real and, since they coincide, the nominal interest rate constant, the higher the weight on the output gap, the lower the nominal interest rate will be, hence the less the required fall in prices (see also (2.25)). However, both the nominal and the real interest rate fall after a positive productivity shock, a consequence of the underreaction of expectations.

Last, note that for $E_t^p [a_t] = a_t$ the complete information equilibrium prevails.

2.4.3 Labor wedge

Formalizing the intuition above, the producer's incomplete information has an impact on his labor demand and, consequently, distorts the labor optimality condition. This causes fluctuations in the labor wedge. Following Chari et al. (2007) and Shimer (2009), the labor wedge is defined as the ratio of the marginal product of labor to the marginal rate of substitution of leisure for consumption by construction equal to $\frac{1}{1-\tau_{n,t}}$ in the expression below:

$$-\frac{U_{n,t}}{U_{c,t}} = (1 - \tau_{n,t})\, MP_{n,t}\,.$$

$U_{n,t}$ and $U_{c,t}$ denote the marginal disutility of labor and marginal utility of consumption, respectively, and $MP_{n,t}$ denotes the marginal product of labor in period t. The above expression becomes in this case

$$N_t^{-(1+\zeta)} = \frac{1}{1 - \tau_{n,t}}\,.$$

Under complete information, $N_t^* = 1$ and the labor wedge is equal to 1. Under incomplete information this will generally not be the case; switching to logs and using $n_t = y_t - a_t$ from the firm's technology implies

$$n_t = \frac{\phi_\pi - 1 + k\,(1-\theta)}{\phi_\pi\,(1+\zeta) - (1+\phi_y)} \left(E_t^p\,[a_t] - a_t\right). \tag{2.26}$$

For $\phi_\pi > \max\{\frac{1+\phi_y}{1+\zeta}\,, 1\}$, employment depends positively on the distance of the producer's expectations from the underlying productivity. The labor wedge in logs is given by the LHS below:

$$-\log\,(1-\tau_{n,t}) = -\frac{[\phi_\pi - 1 + k\,(1-\theta)]\,(1+\zeta)}{\phi_\pi\,(1+\zeta) - (1+\phi_y)} \left(E_t^p\,[a_t] - a_t\right). \tag{2.27}$$

Maintaining that $\phi_\pi > \max\{\frac{1+\phi_y}{1+\zeta}\,, 1\}$, in case $E_t^p\,[a_t] > a_t$, the log-labor wedge is negative, and positive, otherwise. Then purely expectational shocks induce a countercyclical labor wedge. This is easy to see: a positive, for instance, purely expectational shock raises output whereas it lowers the labor wedge. This is in line with the documented countercyclicality of the labor wedge (see for example Chari et al. (2007) and Shimer (2009)) and suggests that purely expectational shocks can possibly account for it. Interestingly, fluctuations in the labor wedge depend on the monetary policy pursued. Section 2.6 elaborates on this.

2.4.4 Equilibrium dynamics

I deal with this case numerically, even though a closed-form representation of the dynamics can be obtained along the lines of Section 2.5.3 below. The baseline parametrization is in Table 2.1. In that I follow Lorenzoni (2009) and one may check the references therein. The parametrization implies the Kalman gain terms, μ and k, are 0.22 and 0.23, respectively, whereas the relative weight the consumer places on the public signal, θ, is 0.96. In addition, I initially set the response to the output gap $\phi_y = 0.5$.[39]

Before continuing with the impulse response functions,[40] let me repeat that the *stochastic* steady state is pinned down by the permanent productivity component x, which by (2.6) evolves as a random walk. In particular, at the

[39]The monetary policy parameters are based on Taylor (1993).

[40]"An impulse response function is [...] the dynamic path of some variables [...] in response to some shock [...] (Mankiw, 2001, page C54)." More precisely, letting rows represent the three shocks, ϵ, u, and e, respectively, and columns represent periods t, $t+1$, ..., in the case, for instance, of a one-standard-deviation ϵ shock, I explore the dynamic response of the economy to

$$\begin{bmatrix} \sigma_\epsilon & \mathbf{0} \\ \mathbf{0} & \\ \mathbf{0} & \end{bmatrix}.$$

Table 2.1: Calibrated parameters

Inverse Frisch elasticity of labor supply	ζ	0.5
Monetary policy weight on inflation	ϕ_π	1.5
Standard deviation of permanent productivity shock	σ_ϵ	0.0077
Standard deviation of temporary productivity shock	σ_u	0.15
Standard deviation of expectational shock	σ_e	0.03

steady state $y = c = x$, $n = 0$, $\pi = 0$, and $r = i = -\log\beta$.[41,42] In all figures, impulse response functions are for one-standard-deviation shocks. Periods, appearing on the horizontal axis of the figures, are interpreted as quarters. Further, with no loss of generality, I assume that $x = 0$ before any shocks hit.

Figure 2.5 shows the impulse responses to positive purely expectational shocks. As already argued, as expectations increase, output and employment increase, the labor wedge falls and the interest rates increase. For the considered parametrization, inflation falls. With no change in the underlying productivity, all effects die out in the long run and variables return to their steady-state values.

Figure 2.6 shows that after a positive permanent productivity shock

[41]The steady state level of an economy, pinned down by the permanent (long-run) productivity component x, is conceptually different from the efficient, complete information level of the economy which is pinned down by temporal aggregate productivity a_t. To see how the two connect with each other, when the economy is at its steady state, it is also at its complete information level but not vice versa. More precisely, at the steady state $a = x$ and $E^p[a] = E^c[x] = x$, whereas at the complete information level of the economy, we only need $E_t^p[a_t] = a_t$. In the impulse response functions analysis, I assume that, before any shocks hit, the economy has already reached its steady state which, based on the above, coincides with its complete information level. The two will remain coincidental after a permanent productivity or a purely expectational shock and they will differ only on impact after a temporary productivity shock.

[42] For ease of exposition, I have suppressed constants, hence in all figures the nominal and the real interest rate equal zero at the steady state.

agents expectations underreact. This causes an increase in output, however by less than under complete information, which in turn causes a fall in employment. As employment falls, the labor wedge increases and is therefore procyclical. For the considered parametrization, inflation increases, whereas, since expectations underreact, the nominal and the real interest rate fall. As expectations converge to the underlying higher productivity level, all variables converge to their steady-state levels.

The impulse responses to a temporary productivity shock (Figure 2.7) are initially similar to the ones of a permanent productivity shock and, subsequently, to the ones of an expectational shock. As argued above, this is because they affect productivity only on impact, whereas from the following period onwards they serve as purely expectational shocks.

As I have already pointed out, the impulse responses when rule 1 is followed are generally sensitive to the specification of the monetary policy rule and, in particular, to the policy weight on the output gap, ϕ_y. Consider now the case in which the authority does not respond to the output gap, that is $\phi_y = 0$, with all other parameters as in Table 2.1. Figures 2.8 - 2.10 show the impulse responses to one-standard-deviation positive purely expectational, permanent productivity, and temporary productivity shocks, respectively. While everything else remains unchanged, the implications for inflation are reversed. In particular, positive permanent productivity shocks lower inflation whereas positive expectational shocks increase inflation. The last results are in line with Lorenzoni (2009). Notably, unlike in Lorenzoni (2009), they are generated in a perfectly competitive environment where prices are flexible and the real interest rate can freely adjust.

Juxtaposing figures 2.5 and 2.8 illustrates the first main result of the essay: purely expectational shocks can behave like supply or demand shocks. A natural question is why the current framework can accommodate both cases. The reasons are, first, the explicit role assigned to the producer's expectations and, second, the presence of asymmetric information between consumers and producers. Crucially, the latter pushes monetary policy and the consumer's expectations through the door.[43] The producer's expectations point toward a supply-shock interpretation, whereas the consumer's expectations, as in Lorenzoni (2009), point toward a demand-shock interpretation. The monetary authority decides which one will prevail.

2.5 Equilibria under Rule 2: Beyond Demand and Supply

2.5.1 Complete information benchmark

Like before, under incomplete information $y_t^* = a_t$ and $n_t^* = 0$. Conjecture for prices that $p_t = \vartheta_3 \, E_t^c\,[x_t] + \vartheta_4 \, a_t$. The Euler equation (2.13) becomes

$$E_t^c[a_{t+1}] - a_t = (\phi_\pi - 1)\,(E_t^c\,[p_{t+1}] - p_t)\,. \tag{2.28}$$

A family of solutions is given by $p_t^* = \frac{1}{\phi_\pi - 1}\, a_t + \vartheta_3 \, E_t^c\,[x_t]$; price levels depend arbitrarily on the consumer's expectations.

[43]It is key that the consumer has complete information about the current state; this enables me to abstract from wealth effects which are the subject of Beaudry and Portier (2004) and Jaimovich and Rebelo (2009) among other papers.

2.5.2 Incomplete information

Consider the conjectures:[44]

$$c_t \;=\; \xi_3\, E_t^p\,[a_t] \;+\; \xi_4\, a_t \tag{C3}$$

$$p_t \;=\; \kappa_4\, E_t^p\,[a_t] \;+\; \kappa_5\, E_t^c\,[x_t] \;+\; \kappa_6\, a_t\,. \tag{C4}$$

Conjectures (C3) and (C4) imply the state can sufficiently be described by $\Psi_t \;=\; \{E_t^p[a_t],\, E_t^c[x_t],\, a_t\}$. The information sets of the agents and the monetary authority are like before. Since $I_t^m \;=\; I_t^c$, the Euler equation (2.13) becomes

$$E_t^c[c_{t+1}] \;-\; c_t \;=\; (\phi_\pi - 1)\,(E_t^c\,[p_{t+1}] \;-\; p_t)\,. \tag{2.29}$$

Taking familiar steps (see Appendix 2.8.4) yields

$$\xi_3 \;=\; \frac{1 \;+\; \kappa_6 \;+\; k\,(1-\theta)\,\kappa_5}{1+\zeta} \tag{2.30}$$

$$\xi_4 \;=\; 1 \;-\; \xi_3 \tag{2.31}$$

$$\xi_3 \;=\; (\phi_\pi - 1)\,\kappa_4 \tag{2.32}$$

$$\kappa_4 \;+\; \kappa_6 \;=\; \frac{1}{\phi_\pi - 1}\;. \tag{2.33}$$

There are 4 equations and 5 unknowns. Equations (2.30) - (2.31) follow from

[44] See also fn. 32 for why I do not include $E_t^c\,[x_t]$ in (C3).

the labor market optimality condition (2.14), whereas (2.32) - (2.33) follow from the Euler equation (2.29). When matching coefficients, an equation is missing from the latter because the real interest rate is determined irrespectively of the consumer's expectations.

Combining (2.30) - (2.33) yields

$$\xi_3 = \frac{\phi_\pi}{\phi_\pi + \zeta(\phi_\pi - 1)} + \frac{(\phi_\pi - 1)\, k\, (1 - \theta)}{\phi_\pi + \zeta(\phi_\pi - 1)}\, \kappa_5 \,. \tag{2.34}$$

Prices can be expressed as

$$p_t = \frac{1}{\phi_\pi - 1}\, y_t + \kappa_5\, E_t^c\, [x_t] \,. \tag{2.35}$$

As in the case of rule 1, asymmetric information implies monetary policy and the consumer's expectations have real effects as the presence of ϕ_π and κ_5, respectively, in (2.34) attests.

However, a crucial difference with the case of rule 1 is the existence of multiple equilibria each corresponding to a different value of κ_5. An immediate monetary policy implication is that targeting expected inflation invites multiple (linear) equilibria, notably for any value ϕ_π in the interest-rate rule. Interestingly, the role of the consumer's expectations is arbitrarily specified across equilibria, which is the second main finding of the essay. As already pointed out, this is because the real interest rate is independent of the consumer's expectations. Additionally, rule 2 specifies price levels as opposed to inflation in the case of rule 1.

As expected, depending on κ_5, expectational and productivity shocks can raise or lower employment and price levels. Further, equation (2.34) suggests that short-run output volatility caused by expectational shocks increases in the absolute value of κ_5 .

2.5.3 A baseline equilibrium

To explore the dynamics of the producer's expectations in the equilibrium under rule 2, I will suppress the role of the consumer's expectations. This corresponds to setting $\kappa_5 = 0$ in (2.34) and (2.35) . It is straightforward to extend the results to equilibria with $\kappa_5 \neq 0$.

Complete information benchmark

Like before, on the real side complete information implies $y_t^* = a_t$ and $n_t^* = 0$. A solution for price levels is $p_t^* = \frac{1}{\phi_\pi - 1} a_t$.[45]

Incomplete information

Setting $\kappa_5 = 0$ in (2.34) and (2.35) pins down the equilibrium given by equations (2.36) - (2.37) below:[46]

[45] Since $E_t^c[a_{t+1}] = E_t^c[x_t] \neq a_t$ (see also Section 2.3.2) , the possibility of prices being fixed in equilibrium appears only as a limit case for $\phi_\pi \to \infty$. It is also a possibility in the special case where $\sigma_e^2 = \sigma_u^2 = 0$. Constant prices could also have prevailed (as a unique non-explosive path) if either productivity a_t evolved as a random walk, or if the economy was a static one.

[46] Conjectures (C3) and (C4) for $\kappa_5 = 0$ combined with (2.11) imply $w_t = (1 + \kappa_4 + \kappa_6) E_t^p[a_t]$; the nominal wage perfectly reveals $E_t^p[a_t]$ to the consumer and the monetary authority in stage 1 . Hence, if the signal s_t , instead of publicly observed, was privately observed by the producer, the nominal wage would generally perfectly communicate it to the consumer and the monetary authority.

$$y_t = \frac{1}{\phi_\pi + \zeta(\phi_\pi - 1)} \left(\phi_\pi \, E_t^p\,[a_t] + \zeta(\phi_\pi - 1)\, a_t\right) \tag{2.36}$$

$$p_t = \frac{1}{\phi_\pi - 1}\, y_t \,. \tag{2.37}$$

Like before, equation (2.36) shows that output is a weighted average (also fn. 31) of productivity and the producer's expectations about it. The respective weights depend on the Frisch elasticity of labor supply, parametrized by ζ , and the monetary policy parameter ϕ_π . By (2.37) , prices are a monotone transformation of output.[47] For an "active" monetary policy ($\phi_\pi > 1$) , output and prices depend positively on the producer's expectations about productivity and productivity itself.[48]

It follows from (2.36) and (2.37) that

$$\pi_t = \frac{1}{(\phi_\pi - 1)\,[\phi_\pi + \zeta(\phi_\pi - 1)]} \big(\phi_\pi \left(E_t^p\,[a_t] - E_{t-1}^p\,[a_{t-1}]\right) + \zeta\,(\phi_\pi - 1)\,(a_t - a_{t-1})\big) \,. \tag{2.38}$$

Inflation, by equation (2.38) , is a weighted average of the change in producer's expectations and the change in productivity in the last two periods.

Let me make some remarks. First, each value of ϕ_π is associated with a unique equilibrium; the equilibrium with constant prices is obtained in the

[47] Output and prices are non-stationary. See also the analysis in Section 2.3 .

[48] For $\frac{1}{1+\zeta} < \phi_\pi < 1$ output depends positively on the producer's expectations about productivity and negatively on productivity, whereas for $0 < \phi_\pi < \frac{1}{1+\zeta}$ it depends negatively on the producer's expectations and positively on productivity. The opposite relations are true for price levels. Employment has the same sign as the weight of expectations in output as (2.39) below shows.

limit as $\phi_\pi \to \infty$. Second, observe that the optimal monetary policy in the baseline equilibrium under rule 2 is a zero-response to expected inflation policy, $\phi_\pi = 0$. In this case, all variables are at their complete information (efficient) level. I elaborate on this in Section 2.6 where I further consider an enriched version of rule 2. Last, note that for $E_t^p[a_t] = a_t$ the complete information equilibrium prevails.

Labor wedge

It follows from (2.36) and the firm's technology (2.4) that

$$n_t = \frac{\phi_\pi}{\phi_\pi + \zeta(\phi_\pi - 1)} \left(E_t^p[a_t] - a_t\right) . \tag{2.39}$$

Equation (2.39) shows that employment depends proportionally on the wedge between the producer's expectations about productivity and productivity itself. Taking the same steps as in the case of rule 1, the labor wedge in logs is given by

$$-\log\left(1 - \tau_{n,t}\right) = -\frac{\phi_\pi(1+\zeta)}{\phi_\pi + \zeta(\phi_\pi - 1)} \left(E_t^p[a_t] - a_t\right) . \tag{2.40}$$

For $\phi_\pi > \frac{1}{1+\zeta}$, in case $E_t^p[a_t] > a_t$, the log-labor wedge is negative, and positive, otherwise. In addition, it is decreasing in the monetary policy parameter, ϕ_π,[49] and becomes zero for $\phi_\pi = 0$. We can once again observe that purely expectational shocks induce a countercyclical labor wedge, which is in line with the documented countercyclicality of the labor wedge.

[49]Note that there is a discontinuity for $\phi_\pi = \frac{1}{1+\zeta}$.

Equilibrium dynamics

Turning to the impulse response functions, the signs I report below refer to $\phi_\pi > 1$; that is the monetary authority follows an "active" policy, along the lines of Taylor (1999).[50] Figures 2.11 - 2.16 show the impulse response functions to one-standard-deviation shocks for the parametrization in Table 2.1. Periods are interpreted as quarters.

If a unit purely expectational shock, e_t, arises, the consumer's expectations in period $t+s$ increase by $(1-k)^s\, k\,\theta$. The producer's expectations increase on impact by μ and in period $t+s$ for $s \geq 1$ by $(1-k)^{s-1}\,(1-\mu)\,k\,\theta$. The impulse response functions are

$$\frac{dy_t}{de_t} = \frac{dn_t}{de_t} = \frac{\phi_\pi\,\mu}{\phi_\pi + \zeta(\phi_\pi - 1)} > 0 \tag{2.41}$$

$$\frac{dy_{t+s}}{de_t} = \frac{dn_{t+s}}{de_t} = (1-k)^{s-1}\,\frac{\phi_\pi\,(1-\mu)\,k\,\theta}{\phi_\pi + \zeta(\phi_\pi - 1)} > 0\,, \quad \text{for } s \geq 1 \tag{2.42}$$

(2.43)

[50]See fn. 48 for the dynamics when $\phi_\pi < 1$.

$$\frac{d\pi_t}{de_t} = \frac{\phi_\pi \, \mu}{(\phi_\pi - 1)\,[\phi_\pi + \zeta(\phi_\pi - 1)]} > 0 \qquad (2.44)$$

$$\frac{d\pi_{t+1}}{de_t} = -\frac{\phi_\pi \,[\mu - (1-\mu)\, k\, \theta]}{(\phi_\pi - 1)\,[\phi + \zeta(\phi_\pi - 1)]} \qquad (2.45)$$

$$\frac{d\pi_{t+s}}{de_t} = -\,(1-k)^{s-2}\,\frac{\phi_\pi\,(1-\mu)\,k^2\,\theta}{(\phi_\pi - 1)\,[\phi + \zeta(\phi_\pi - 1)]} < 0\,, \quad \text{for } s \geq 2\,. \qquad (2.46)$$

Equations (2.41) and (2.42) (also Figure 2.11) demonstrate the positive co-movement result, already discussed: output and employment increase in response to a positive expectational shock. The result is due to the producer overstating the worker's productivity. In the limit as $s \to \infty$, expectations converge to the true level of productivity implying both output and employment return to their steady-state levels.

A key difference between the equilibrium under rule 1 and the equilibrium under rule 2 is that the former specifies inflation whereas the latter price levels. In the baseline case considered here, prices are positively related to the producer's expectations. Hence, a positive expectational shock causes an increase in price levels (Figure 2.12). However, as agents update their beliefs over time, their expectations become more aligned with fundamentals and, hence prices return monotonically to their steady-state level, $p = \frac{1}{\phi_\pi - 1}\, x$, generating thereby a deflationary pressure as (2.46) shows from the following period onwards.[51] Put differently, price levels respond non-monotonically to positive expectational shocks. They are higher compared to their complete information level, yet inflation, by definition, measures changes in price levels

[51] Whether there is inflation or disinflation in period $t+1$ depends on the variances of the shocks. The parametrization here implies the latter.

between periods.[52] All effects vanish as $s \to \infty$.

To pin down the impulse responses of the nominal and the real interest rate (Figure 2.12) I need to specify the impulse response of expected inflation:

$$\frac{dE_t^c\,[\pi_{t+1}]}{de_t} = \frac{\zeta(\phi_\pi - 1)\,k\,\theta \,-\, \phi_\pi\,(\mu - k\,\theta)}{(\phi_\pi - 1)\,[\phi_\pi \,+\, \zeta(\phi_\pi - 1)]} \tag{2.47}$$

$$\frac{dE_{t+s}^c\,[\pi_{t+s+1}]}{de_t} = (1-k)^{s-1}\frac{\phi_\pi\,(\mu - k) \,+\, [\zeta(\phi_\pi - 1)\,(1-k)]\,k\,\theta}{(\phi_\pi - 1)\,[\phi_\pi \,+\, \zeta(\phi_\pi - 1)]}\,, \quad \text{for } s \geq 1\,. \tag{2.48}$$

The nominal interest rate is $i_{t+s} = \phi_\pi\, E_{t+s}^c\,[\pi_{t+s+1}]$ and the real interest rate is $r_{t+s} = (\phi_\pi - 1)\, E_{t+s}^c\,[\pi_{t+s+1}]$.[53]

Inflation expectations increase,[54] given the parametrization, resulting in higher nominal and real interest rates. In the limit $s \to \infty$, inflation expectations, the nominal, and the real interest rate all return to their steady-state values.

If a shock to the permanent productivity component $\epsilon_t = 1$ arises, the consumer's expectations about productivity in period $t+s$ increase by $1 - (1-k)^{s+1}$ as (2.16) implies, whereas the producer's expectations increase by $1 - (1-\mu)\,[1-(1-k)^s]$ as (2.15) implies. The impulse response functions

[52] A similar result is obtained in Lorenzoni (2005), though not associated with disinflation. The increase in prices can become less severe and prices can even fall for reasonable values of ϕ_π under an extended forward-looking rule targeting expected growth in addition to expected inflation. The logic is similar to the rule 1 case: for a given real interest rate, the greater the weight placed on expected growth, the lower expected inflation will be, controlling for general equilibrium effects.

[53] In addition, notice that $E_t^c\,[y_{t+s}] = E_t^c\,[x_t]$ for $s \geq 1$ and $E_t^c\,[\pi_{t+s}] = 0$ for $s > 1$. These results follow from (2.36) and (2.38) combined with (2.16).

[54] This is unlike the case of rule 1. The difference between the two lies in that rule 1 specifies inflation rather than price levels.

are

$$\frac{dy_{t+s}}{d\epsilon_t} = 1 - (1-k)^s \frac{\phi_\pi (1 - \mu)}{\phi_\pi + \zeta(\phi_\pi - 1)} \in (0,1) \tag{2.49}$$

$$\frac{dn_{t+s}}{d\epsilon_t} = -(1-k)^s \frac{\phi_\pi (1 - \mu)}{\phi_\pi + \zeta(\phi_\pi - 1)} < 0 \tag{2.50}$$

$$\frac{d\pi_t}{d\epsilon_t} = \frac{1}{\phi_\pi - 1}\left(1 - \frac{\phi_\pi (1 - \mu)}{\phi_\pi + \zeta(\phi_\pi - 1)}\right) > 0 \tag{2.51}$$

$$\frac{d\pi_{t+s}}{d\epsilon_t} = (1-k)^{s-1} \frac{\phi_\pi (1 - \mu)\, k}{(\phi_\pi - 1)\,[\phi_\pi + \zeta(\phi_\pi - 1)]} > 0\,, \quad \text{for } s \geq 1\,. \tag{2.52}$$

A unit increase in the permanent productivity shock causes an equivalent change in steady-state output and no change in steady-state employment. We can see from (2.49) and (2.50) (see also Figure 2.13) that a positive permanent productivity shock causes output to increase by less than one and employment to fall temporarily. By (2.40), the labor wedge increases temporarily. This happens because expectations underreact after a positive permanent productivity shock. As a result, labor demand shifts inwards and the real wage falls relative to its efficient level. Equation (2.52) suggests productivity shocks are inflationary (see also Figure 2.14). The positive dependence of prices on expectations for $\phi_\pi > 1$, as (2.36) and (2.37) imply, underlies this result. Hence, as expectations converge to the new permanent productivity level, prices get closer to their steady-state level, implying inflation along the way.

The impulse response of the consumer's inflation expectations (also Figure 2.14) is

$$\frac{dE^c_{t+s}\left[\pi_{t+s+1}\right]}{d\epsilon_t} = (1-k)^s \, \frac{\phi_\pi\,(k-\mu) \, - \, \zeta(\phi_\pi - 1)\,(1-k)}{(\phi_\pi - 1)\left[\phi_\pi \, + \, \zeta(\phi_\pi - 1)\right]} \, . \tag{2.53}$$

Figure 2.14 shows that following a permanent productivity shock inflation expectations fall and so do the nominal and the real interest rate. In the limit as $s \rightarrow \infty$, expectations become aligned with the new productivity level, output and prices converge to their new steady-state levels, whereas the remaining variables return to their pre-shock levels.

A temporary productivity shock causes on impact responses similar to those in the permanent productivity shock case; from the following period onwards, it only affects the agents' expectations, hence the responses resemble the ones in the expectational shock case. The consumer's expectations in period $t+s$ increase by $(1-k)^s\,k\,(1-\theta)$, whereas the producer's expectations are unchanged on impact, as changes in the temporary productivity component affect their expectations with one-period lag, and increase by $(1-k)^{s-1}\,(1-\mu)\,k\,(1-\theta)$ in period $t+s$ for $s \geq 1$. In particular, in period t the responses are

$$\frac{dy_t}{du_t} = \frac{\zeta(\phi_\pi - 1)}{\phi_\pi \, + \, \zeta(\phi_\pi - 1)} \in (0,1) \tag{2.54}$$

$$\frac{dn_t}{du_t} = -\frac{\phi_\pi}{\phi_\pi \, + \, \zeta(\phi_\pi - 1)} < 0 \tag{2.55}$$

$$\frac{d\pi_t}{du_t} = \frac{\zeta}{\phi_\pi \, + \, \zeta(\phi_\pi - 1)} > 0\,. \tag{2.56}$$

In the subsequent periods the responses are

$$\frac{dy_{t+s}}{du_t} = \frac{dn_{t+s}}{du+_t} = (1-k)^{s-1} \frac{\phi_\pi (1-\mu)\, k\, (1-\theta)}{\phi_\pi + \zeta(\phi_\pi - 1)} > 0\,, \quad \text{for } s \geq 1 \tag{2.57}$$

$$\frac{d\pi_{t+1}}{du_t} = -\frac{\zeta\,(\phi_\pi - 1) - \phi_\pi\,(1-\mu)\,k(1-\theta)}{(\phi_\pi - 1)\,[\phi_\pi + \zeta(\phi_\pi - 1)]} \tag{2.58}$$

$$\frac{d\pi_{t+s}}{du_t} = -\,(1-k)^{s-2} \frac{\phi_\pi\,(1-\mu)\,k^2\,(1-\theta)}{(\phi_\pi - 1)\,[\phi_\pi + \zeta(\phi_\pi - 1)]} < 0\,, \quad \text{for } s \geq 2\,. \tag{2.59}$$

The response of inflation expectations is given by

$$\frac{dE_t^c\,[\pi_{t+1}]}{du_t} = \frac{\phi_\pi\, k\,(1-\theta) - \zeta\,(\phi_\pi - 1)\,[1 - k\,(1-\theta)]}{(\phi_\pi - 1)[\phi_\pi + \zeta(\phi_\pi - 1)]} \tag{2.60}$$

$$\frac{dE_{t+s}^c\,[\pi_{t+s+1}]}{du_t} = (1-k)^{s-1} \frac{[\phi_\pi\,(\mu - k) + \zeta(\phi - 1)\,(1-k)]\,k\,(1-\theta)}{(\phi_\pi - 1)\,[\phi_\pi + \zeta(\phi_\pi - 1)]}\,, \quad \text{for } s \geq 1\,. \tag{2.61}$$

Figures 2.15 and 2.16 display the impulse response functions.

2.5.4 Short-run volatility

In what is a separate exercise, I compare the short-run (one-period) output volatility caused by purely expectational shocks, e_t, among the equilibria for the considered interest-rate rules.[55] The parametrization is the one in Table 2.1 . I normalize to one the short-run output volatility generated by rule 1 for $\phi_y = 0$ to make comparisons easier. Table 2.2 reports the results.

Table 2.2: Short-run volatility

Rule 1 ($\phi_y = 0$)	1
Rule 1 ($\phi_y = 0.5$)	2.78
Rule 2 (baseline)	4.43

We can see that the baseline case of rule 2 generates considerably higher short-run output volatility than the considered cases of rule 1 . This can be further increased by assigning a role to the consumer's expectations (see also (2.34)) . Considering the analyzed equilibria for rule 1 , "supply" shocks ($\phi_y = 0.5$) generate considerably higher volatility than "demand" shocks ($\phi_y = 0$) . Indeed, for ϕ_π high enough so that $\phi_\pi (1 + \zeta) - (1 + \phi_y) > 0$, the short-run output volatility due to expectational shocks increases in ϕ_y .

2.5.5 Monetary authority with superior information

In this section I lift the assumption that the monetary authority has no superior information compared to the agents. Instead, I assume that the monetary authority has information about the following period's state. To prevent the

[55] Short-run output volatility in the cases of rule 1 and 2, respectively, is

$$\left(\frac{\phi_\pi - 1 + k(1-\theta)}{\phi_\pi (1+\zeta) - (1+\phi_y)} \mu \right)^2 \sigma_e^2$$

$$\left(\frac{\phi_\pi}{\phi_\pi + \zeta(\phi_\pi - 1)} \mu \right)^2 \sigma_e^2 .$$

forward-looking[56] nominal interest rate from being fully revealing about the following period's state, I require that the monetary authority either reports the following period's price with a measurement error or transmits "surprise" monetary policy shocks. In both cases the nominal interest rate serves as a public signal about the following period's productivity. However, in the former case the monetary authority misreports the following period's prices unintentionally, as opposed to intentionally in the latter. The aim of this section is twofold: first, to analyze the informational implications per se when the monetary authority communicates its superior information with noise; second, to equip the monetary authority with an additional monetary policy tool, the monetary policy shocks, and pin down its equilibrium effects. I further explore monetary policy shocks in Section 2.6. The focus throughout this section will be on the baseline case of rule 2, which corresponds to setting $\kappa_5 = 0$ in (2.34) and (2.35). Extending the results to the other equilibria should be straightforward although this is something I have not explored yet.

When the monetary authority reports the following period's price with a measurement error, the prevailing nominal interest rate in $t-1$ is

$$i_{t-1} = \phi_\pi \, \tilde{\pi}_t \,, \tag{2.62}$$

where $\tilde{\pi}_t \equiv \tilde{p}_t - p_{t-1}$, with

$$\tilde{p}_t = p_t + w_t \,. \tag{2.63}$$

[56] Since agents have complete information about the current state when the monetary authority steps in, there can only be information extraction from the nominal interest rate in -what seems to me- a meaningful way if the monetary authority is forward-looking. Therefore, I restrict attention only to rule 2.

The error term is i.i.d with $w_t \sim N(0, \sigma_w^2)$ and is independent of the shocks ϵ_t, e_t, and u_t .

In terms of observables as of stage 2 of period $t-1$, this can be expressed as

$$\tilde{p}_t \;=\; \frac{1}{\phi_\pi}(i_{t-1} \;+\; \phi_\pi \, p_{t-1})\,.$$

In the case of monetary policy shocks the nominal interest rate is

$$i_{t-1} \;=\; \phi_\pi \, \pi_t \;+\; \omega_t\,, \tag{2.64}$$

where ω is i.i.d. with $\omega_t \sim N(0, \sigma_\omega^2)$ and is independent of the shocks ϵ_t, e_t, u_t, and w_t .

Agents now extract

$$\hat{p}_t \;=\; \phi_\pi \, p_t + \omega_t\,, \tag{2.65}$$

which in terms of observables as of stage 2 of period $t-1$ can be expressed as

$$\hat{p}_t \;=\; i_{t-1} \;+\; \phi_\pi \, p_{t-1}\,.$$

2.5.6 Linear equilibria

Equilibrium is given by equations (2.36) - (2.38) . The state of the economy is now augmented by the public signal about period t's productivity which the monetary authority transmits. I denote this by z_t in the case of a measurement error and $\hat{z}_t$ in the case of a monetary policy shock. The state can sufficiently be described then by $\Omega_t \;=\; (\{a_\tau\}_{\tau=0}^t\,,\; \{s_\tau\}_{\tau=0}^t\,,\; z_t)$, replacing Ω_t with $\hat{\Omega}_t$ and z_t with $\hat{z}_t$ in the case of a monetary policy shock. What distinguishes the two cases is the information set of the monetary authority in $t-1:$ in the case of measurement errors it is $I^m_{t-1} \,=\, \Omega_t \setminus \{z_t\}$, whereas in the case of monetary policy shocks it is $I^m_{t-1} \,=\, \hat{\Omega}_t$. That is, in the latter case, the monetary authority takes into account the effects of the signal it transmits. I assume it is common knowledge what the case is each time a shock hits. As I show in Appendix 2.8.4, the endogenous public signals associated with the two cases are

$$z_t \,=\, a_t \,+\, \frac{\phi_\pi + \zeta(\phi_\pi - 1)}{\zeta}\, w_t \tag{2.66}$$

$$\hat{z}_t \,=\, a_t \,+\, \frac{\phi_\pi + \zeta(\phi_\pi - 1)}{\phi_\pi\, \zeta}\, \omega_t\,. \tag{2.67}$$

Agents (perfectly) disentangle the endogenous public signals upon the realization of the public signal s_t in stage 1 of period t.[57] The producer's information set then becomes $I^p_{t,1} \,=\, \Omega_t \setminus \{a_t\}$, whereas the consumer's $I^c_t \,=\, \Omega_t$. As I show in Appendix 2.8.4, the producer's expectations about productivity are

$$E^p_t \left[a_t \,|\, I^p_{t,1}\right] \,=\, \delta\, E^p_t \,\left[x_t \,|\, I^p_{t,1} \setminus \{z_t\}\right] \,+\, (1-\delta)\, z_t\,, \tag{2.68}$$

where δ is a coefficient in $(0,1)$ (respectively use $\hat{\delta}$, $\hat{z}_t$ and $\hat{\Omega}_t$ in the case of a monetary policy shock). Importantly, δ and $\hat{\delta}$ depend on the monetary policy parameter ϕ_π.[58]

It is apparent from (2.66) and (2.67) that the economy's response to measurement errors and monetary policy shocks is very similar. In particular,

[57] This happens because they know the stochastic process of prices given by (2.37).

[58] The case analyzed in Section 2.3 corresponds to $\delta \,=\, 1$ which would prevail if the conditional variance of the endogenous signals was infinite.

for $\phi_\pi > 1$ positive interest rate shocks raise the producer's expectations about productivity in the following period. This happens because for $\phi_\pi > 1$ prices are positively related to productivity. Therefore, a higher nominal interest rate overstates the following period's price and leads the producer to partially attribute it to an increase in productivity.[59]

2.5.7 Equilibrium dynamics

The dynamics when shocks ϵ_t, e_t, and u_t are realized are very similar to the ones in Section 2.5.3 .

Unlike there, the effects of a measurement error or a monetary policy shock last only one period. This is because it generates a signal about a_t, which consumers learn and producers realize once the labor decision is made . If a shock $w_t = 1$ arises, the impact responses are

$$\frac{dy_t}{dw_t} = \frac{dn_t}{dw_t} = \frac{\phi_\pi (1-\delta)}{\zeta} > 0 \tag{2.69}$$

$$\frac{dp_t}{dw_t} = \frac{d\pi_t}{dw_t} = -\frac{d\pi_{t+1}}{dw_t} = \frac{\phi_\pi (1-\delta)}{\zeta (\phi_\pi - 1)} > 0 \, . \tag{2.70}$$

It can be seen from (2.69) and (2.70) that interest rate shocks boost

[59] In case the monetary authority has no superior information and this is common knowledge, monetary policy shocks have no real effects because they are unanticipated by both agents, hence they have no effect on the labor decision in stage 1 . They can immediately be extracted by the agents which implies they have no effect on the consumer's inflation and output expectations for the following period. As a result, they only affect the current price, in a co-monotone way for $\phi_\pi > 1$. On the contrary, in the superior information case agents extract monetary policy shocks with a one-period lag, hence their (nominal and real) effects are realized in the following period. They have real effects because they are not simultaneously fully extracted by both agents.

output and prices. These responses are in the same direction as the ones after a shock to the public signal s_t . This is because measurement errors and monetary policy shocks serve as purely expectational shocks: when positive, they increase the producer's expectations about productivity without any change in the underlying fundamentals.

The impact responses to a policy shock $\omega_t = 1$ are scaled down by ϕ_π as (2.67) suggests:

$$\frac{dy_t}{d\omega_t} = \frac{dn_t}{d\omega_t} = \frac{(1-\hat{\delta})}{\zeta} > 0 \tag{2.71}$$

$$\frac{dp_t}{d\omega_t} = \frac{d\pi_t}{d\omega_t} = -\frac{d\pi_{t+1}}{d\omega_t} = \frac{(1-\hat{\delta})}{\zeta(\phi_\pi - 1)} > 0\,. \tag{2.72}$$

The previous comments apply. However, in the next section I show that the two cases generate partly different monetary policy implications.

2.6 Monetary Policy

The equilibrium nominal wage in stage 1 is given by

$$w_t = E_t^p[a_t] + E_t^p[p_t]\,.$$

Consequently, through the nominal wage, the real side of the economy reflects the producer's expectations about productivity. The producer's expectations enter the nominal wage both directly and indirectly through inflation in the case of rule 1 and prices in the case of rule 2 . Monetary policy can have real effects through the indirect inflation (price) channel. To see this, observe that the labor market optimality condition (2.17) can more generally be written

$$\zeta n_t = E_t^p[a_t] + E_t^p[\pi_t] - E_t^c[\pi_t] - E_t^c[c_t]. \tag{2.73}$$

Taking the producer's uncertainty as given, monetary policy has real effects as long as agents form *heterogeneous* expectations about the inflation to prevail in stage 2, that is $E_t^p[\pi_t] \neq E_t^c[\pi_t]$. By construction, this is the case here. Crucially, what matters for labor decision and, hence the real side, is the wedge in the agents's expectations about inflation, $E_t^p[\pi_t] - E_t^c[\pi_t]$. Anything common in the agents' information sets and anything lying outside both agents' information sets (for instance, non-fundamental shocks - see fn. 60 below) has no real effects through the inflation channel. Then, it should not perhaps come as a surprise that incomplete yet symmetric information about current productivity would imply a neutral monetary policy.

The producer's incomplete information is the only source of inefficiency. Optimal monetary policy restores then the complete information equilibrium. An infinitely aggressive policy on inflation policy implies $\pi_t = 0$ and only removes the indirect, inflation (price) channel of expectations. As a result, it is typically suboptimal.

By direct implication of (2.14), the complete information equilibrium is restored if and only if

$$E_t^p[a_t] + E_t^p[\pi_t] - (\pi_t + a_t) = 0 \tag{2.74}$$

$$E_t^p[a_t] + E_t^p[p_t] - (p_t + a_t) = 0 , \tag{2.75}$$

for rules 1 and 2 , respectively.

It follows that monetary policy succeeds, not by removing the producer's uncertainty, but rather by making it irrelevant. To see this, note that inflation (prices) depends on productivity and agents' expectations in a way decided by monetary policy. Optimal monetary policy manipulates inflation in such a way that the producer correctly anticipates his stage-2 revenue, which is all he is interested in.

One would argue that the inefficiency here arises exactly because of agents's asymmetric information; if agents had incomplete yet symmetric information, then the complete information equilibrium would prevail. However, this is true only because of logarithmic preferences in consumption; in more general environments, the producer's incomplete information would suffice. Nevertheless, it is asymmetric, rather than incomplete but symmetric, information in combination with the existence of a nominal bond market that enables the monetary authority to drive the economy closer to the complete information equilibrium. If a real bond market was in the place of the nominal bond market, then the inflation (price) channel would be absent, and there would be no way to drive the economy to the first best.

Optimal policy here has different implications from the one in Weiss (1980) which implies that prices perfectly communicate fundamentals. By construction, this is a nonexistent possibility here. However, this essay shares with Weiss (1980), King (1982) and Lorenzoni (2010) the insight that, at the time the labor decision is made, it is asymmetric, as opposed to incomplete but symmetric, information about variables the monetary policy will be based on that breaks the policy irrelevance proposed in Sargent and Wallace (1975, 1976) . Implicit in this is that the monetary authority is more informed when it steps in than the least informed agent (here, the producer) at the time the labor decision is made. This is true here since the time advantage of the monetary authority is essentially an informational advantage; in fact, the monetary authority perfectly observes or extracts the variables in question (inflation, output, current productivity) when it steps in.

Crucially, that inflation stabilization is suboptimal is in contrast with the baseline case in Lorenzoni (2009) in which the limit $\phi_\pi \to \infty$ restores the efficient equilibrium. In Lorenzoni (2009) , producers have complete information, however nominal rigidities prevent prices from fully absorbing the consumer's expectations about long-run productivity. Stabilizing inflation resolves this. In contrast, here prices flexibly adjust, however producers have incomplete information about productivity and, consequently, their (stage-2) revenue; stabilizing inflation only eliminates their uncertainty about inflation but not about their revenue.

Below I consider both interest-rate rules and explore how the monetary authority can mitigate the effects of incomplete information and drive the economy closer to its complete information counterpart in each case. In the context of rule 2 , I design policy targets which restore the complete information equilibrium for any choice of policy parameters.[60]

[60]My focus so far has been on the effects of purely expectational shocks as well on those of productivity shocks. However, a fair question, especially when it comes to monetary policy, is about the possible presence of indeterminacies. The answer is that here there is no *real* indeterminacy. To see this, first note that the labor decision is intratemporal and is anyway made before inflation prevails. Turning to inflation, even if it is indeterminate, i.e. susceptible to possibly non-fundamental ("sunspot") shocks, since the "sunspot" shocks lie outside the information sets of both agents, they cannot affect the labor decision in stage 1 , i.e. they cannot have real effects (see (2.73) and the analysis that comes with it) . Nevertheless, there may well be nominal indeterminacy. To rule out "sunspot" shocks in the case, for instance, of rule 1, on which the positive part of this essay is based , we would need $\phi_\pi > 1$. But I should repeat here, that I abstract from such considerations.

2.6.1 Rule 1

If interest-rate rule 1 is followed, setting $\phi_\pi = 1 - k\,(1-\theta)$ is optimal; however, this policy is unrealistic as it requires the monetary authority to be fully aware of the agents' learning problems which is hardly realistic. Crucially, in the limit as $\phi_\pi \to \infty$, $\pi_t \to 0$;[61] inflation is constant and the indirect (inflation/price) channel of expectations, through which the consumer's expectations also operate, is muted. However, even in this limit case, the producer's expectations continue to matter via the direct channel. Hence, inflation stabilization can at best eliminate the uncertainty arising through the inflation channel and, as such, is suboptimal.

An implication of Section 2.5.4 is that, for moderate values of ϕ_y, short-run output volatility due to purely expectational shocks increases in ϕ_y. However, perhaps not surprisingly, in the limit $\phi_y \to \infty$, the economy is at its complete information counterpart; a policy infinitely responsive to deviations of output from its complete information level is therefore optimal.

2.6.2 Rule 2

A first policy implication generated by the equilibrium analysis (see Section 2.5) is that a forward-looking rule, like rule 2, invites multiple equilibria in which the consumer's expectations is arbitrarily specified, which is not the case when a contemporaneously-looking rule is followed.[62] Second, as I showed in Section 2.5.4, the short-run volatility of output due to expectational shocks

[61] As ϕ_π increases, the sign of the change in the weight of the producer's expectations, ξ_1, is given by the sign of $-(1+\phi_y) + [1 - k\,(1-\theta)]\,(1+\zeta)$ (see also (2.19) and (2.21)). It is negative for a high enough ϕ_y relative to the inverse Frisch elasticity ζ, while there is a discontinuity at $\frac{1+\phi_y}{1+\zeta}$.

[62] At least, I have failed to find other linear equilibria for rule 1.

is substantially higher for forward-looking rules than for contemporaneously-looking ones, for the parametrization in Table 2.1.

I initially consider the baseline equilibrium in which the consumer's expectations have no role.[63] This corresponds to setting $\kappa_5 = 0$ in (2.34). Observe in (2.36) that the weight of output placed on producer's expectations is $\xi_3 = \frac{\phi_\pi}{\phi_\pi + \zeta\,(\phi_\pi - 1)}$, whereas the weight placed on productivity is $1 - \xi_3$. The former decreases in ϕ_π (see also fn. 49); the greater ϕ_π, the weaker the indirect (price) channel of expectations will be. In the limit as $\phi_\pi \to \infty$, prices are constant. As in the previous case, only the indirect channel of expectations is muted, therefore inflation stabilization is suboptimal.

The focus so far has been on active policies, which correspond to the monetary authority setting $\phi_\pi > 1$. However, setting $\phi_\pi = 0$ in (2.36) and (2.37) returns $y_t = a_t$ and $p_t = -a_t$; a Friedman-rule policy completely eliminates the role of expectations and keeps the economy at its complete information level. To provide an intuition for this, observe that for $\phi_\pi < \frac{1}{1+\zeta}$ the price effect becomes negative, which implies the indirect channel effect mitigates the direct one. For $\phi_\pi = 0$, the two effects precisely offset each other, rendering, therefore, incomplete information irrelevant in equilibrium. Notably, such a policy implies that the producer's revenue is constant across states: high prices prevail for low productivities and vice versa. Summarizing the above, the Friedman rule, for different from the usual reasons, emerges as an optimal policy; however, if an active policy is to be pursued, then it should be as aggressive on inflation as possible.

Next, I analyze monetary policy when the monetary authority has superior information. As we saw earlier, the monetary authority can either, un-

[63] See fn. 66 below for the general case.

intentionally, report prices with a measurement error or, intentionally, fuel the economy with "surprise" monetary policy shocks. A straightforward option for a "benevolent" monetary authority in the latter case is to use monetary policy shocks to eliminate the producer's expectational errors. However, I will focus on the monetary policy parameters that can insulate the economy against measurement errors and can serve as a commitment device against monetary policy shocks.

One can see from (2.69) and (2.71) and Appendix 2.8.4 that the monetary policy parameter ϕ_π affects the equilibrium not only directly, but also indirectly by affecting the precision of the endogenous public signal, z_t or $\hat{z}_t$, it generates. The precision of the public signal is inversely related to δ ($\hat{\delta}$ for the monetary policy shock).

Considering the case where the authority reports prices with a measurement error, in the limit as $\phi_\pi \to \infty$, the precision of the endogenous public signal z_t becomes zero and $\delta \to 1$; hence, agents ignore the public signal which then has no real effects. Alternatively, a Friedman-rule policy ensures immunity to measurement errors as well, for the reasons outlined above. Hence, both extreme policies imply measurement errors have no real effects.

In the case of monetary policy shocks, ϕ_π matters only through the parameter $\hat{\delta}$ as we can see from (2.71). Appendix 2.8.4 shows that the variance of the signal $\hat{z}_t$ tends to infinity only when a Friedman-rule policy is pursued, which is the unique optimal policy in this case allowing the monetary authority to commit against "surprise" shocks. Even though, for $\phi_\pi > 1$ (a sufficient condition), the variance of the signal increases in ϕ_π, in the limit $\phi_\pi \to \infty$ the public signal's variance is still finite, hence $\hat{\delta} \neq 1$. This implies that a policy infinitely aggressive on inflation cannot serve as a commitment device against monetary policy shocks.

Optimal monetary policies

In this section I design targets for forward-looking interest-rate rules which restore the complete information equilibrium for any choice of policy parameters $(\phi_\pi\,,\,\phi_y)$. I start with the baseline case of rule 2 and subsequently deal with the general form that equilibria can have when a forward-looking policy is followed, given by (2.30) - (2.33).

Baseline equilibrium. I will follow a reverse engineering process. The optimal policy suggested above requires setting $\phi_\pi = 0$. It is straightforward to check that this implies $y_t = a_t$ and $p_t = -a_t$ (see also fn. 66 below).

Consider the rule

$$i_t = -\log\beta + \phi_\pi\, E_t^m\,[\pi_{t+1} - \hat{\pi}_{t+1}] + \phi_y\, E_t^m\,\Delta[y_{t+1} - \hat{y}_{t+1}]\,, \tag{2.76}$$

where the target levels of prices and output are set equal to their above specified levels: $\hat{\pi}_{t+1} = -E_t^m\,[\Delta a_{t+1}]$ and $\hat{y}_t = a_t$. This rule involves the monetary authority "punishing" deviations from the efficient inflation and growth rates.[64,65]

Taking the same steps as in the derivations of (2.36) - (2.37) shows that *any* chosen coefficients $(\phi_\pi\,,\,\phi_y)$ can drive the economy to its efficient level. The Friedman rule is a special case obtained for $\phi_\pi = \phi_y = 0$.

[64] As already emphasized, the authority has complete information when it sets the nominal interest rate.
[65] Orphanides (2003) discusses the benefits of targeting output growth.

Multiple equilibria. Consider the interest-rate rule given by (2.76) . Make the following modification:

$$\hat{\pi}_{t+1} \;=\; \kappa_7 \, E_t^m[\Delta a_{t+1}] \tag{2.77}$$

$$\kappa_7 \;=\; -\,[1 \,+\, \frac{\phi_\pi - 1}{\phi_\pi} \, k\,(1-\theta)\,\kappa_5]\,. \tag{2.78}$$

This rule drives the economy to its complete information counterpart. Once again, observe that the inflation and growth targets are related to their complete information levels.[66] The proposed rule is invariant to changes in ϕ_y , whereas, as (2.78) shows, it adjusts to the chosen value of ϕ_π .[67] To get an intuition for the latter, first use the former result and set $\phi_y \;=\; 0$ in order to bring the equilibrium closer to the equilibrium given by (2.34) - (2.35) . Then observe in (2.34) that the real effects of the consumer's expectations depend on the monetary authority's response to expected inflation, ϕ_π . Hence, the targets in the suggested policy (2.76) - (2.78) also need to adjust accordingly

[66] To get an intuition for this, recall that the efficient equilibrium requires $\xi_3 \;=\; 0$; this implies $\kappa_4 \;=\; 0$ by (2.32) and $\kappa_6 \;=\; \frac{1}{\phi_\pi - 1}$ by (2.33) . Given these, we can see in (2.34) that $\xi_3 \;=\; 0$ prevails for ϕ_π such that $\kappa_5 \;=\; -\,\frac{1}{k\,(1-\theta)}\,\frac{\phi_\pi}{\phi_\pi - 1}$. Then, the equilibrium is

$$y_t \;=\; a_t \tag{2.79}$$

$$p_t \;=\; \frac{1}{\phi_\pi - 1}\, a_t \;-\; \frac{1}{k\,(1-\theta)}\,\frac{\phi_\pi}{\phi_\pi - 1}\, E_t^c\,[x_t]\,. \tag{2.80}$$

The rule given by (2.76) - (2.78) and $\hat{y}_t \;=\; a_t$ yields

$$y_t \;=\; a_t \tag{2.81}$$

$$p_t \;=\; -\,[1 \,+\, k\,(1-\theta)\,\kappa_5]\, a_t \;+\; \kappa_5\, E_t^c[x_t]\,. \tag{2.82}$$

Setting ϕ_π such that $\kappa_5 \;=\; -\,\frac{1}{k\,(1-\theta)}\,\frac{\phi_\pi}{\phi_\pi - 1}$ returns (2.79) - (2.80) .

[67] The rule will not adjust to changes in ϕ_π for $\kappa_5 \;=\; 0$, as already shown.

to changes in ϕ_π. All derivations are collected in Appendix 2.8.5.

The monetary authority can extract the role of the consumer's expectations, parametrized by κ_5, and productivity a_t by observing output and prices (see also (2.35)) when it steps in; subsequently, it can invoke the rule given by (2.76) - (2.78) and $\hat{y}_t = a_t$ and restore the complete information equilibrium for any choice of policy parameters (ϕ_π, ϕ_y).

Last, observe that setting $\kappa_5 = 0$ returns $\kappa_6 = -1$ and $\kappa_7 = -1$, which correspond to the baseline rule (2.76).

2.7 Conclusion

This essay has reconsidered the nature of purely expectational shocks within a competitive, cashless, monetary economy. Asymmetric information about current fundamentals is the driving force in the model. Informational asymmetries lead agents to form heterogeneous expectations about inflation; as a result, monetary policy and consumers' expectations have real effects through inflation. Traditionally, expectational shocks are viewed as Keynesian demand shocks: when positive, they increase output, employment and inflation. I have shown that this interpretation remains a possibility but is not the only one; expectational shocks can cause business cycle patterns associated with supply shocks: when positive, they increase output and employment and they lower inflation. Such an interpretation seems in line with the low inflation and the high cyclical employment in the mid-80s and the second half of the 90s, which are recalled as periods of exuberant optimism. Whether expectational shocks manifest themselves as demand or supply shocks reflects the monetary policy pursued.

I have considered different interest-rate rules and shown that forward-looking rules generate multiple equilibria in which consumers' expectations have an arbitrary role. Further to this, expectational shocks cause substantially higher short-run output volatility under forward-looking policies than under "contemporaneous" ones. Inflation stabilization per se is typically suboptimal, as it can at best eliminate uncertainty arising through prices. Optimal monetary policies manipulate inflation so that the producer correctly anticipates his revenue. In this way, producers' incomplete information about productivity becomes irrelevant. I have designed targets for forward-looking interest-rate rules which restore the complete information equilibrium for any chosen policy parameters.

Recovering purely expectational shocks from the data will shed light on their seemingly shifting nature. Of course, the literature on the identification of expectational shocks remains far from settled (for example, Beaudry and Portier (2006), Blanchard et al. (2009) and Barsky and Sims (2011a,b)). On the policy front, introducing capital, investment and credit constraints is a rather natural extension with potentially promising monetary policy implications.

2.8 Appendix: Omitted Derivations

2.8.1 Log-linear approximations

First, with the exception of the Lagrange multiplier λ, let X denote the steady-state value of X_t, x_t denote $\log X_t$, and $\hat{x}_t$ denote $\log \frac{X_t}{X}$. In the case of $\log \frac{\lambda_t}{\lambda}$, I will use $\hat{\lambda}_t$.

Starting with (2.8) , note that at the steady state it implies that $W = P\,A$.

Multiplying and dividing variables on both sides of (2.8) by their steady state values yields

$$W\,\lambda\,\frac{W_t}{W}\,E_t^p\left[\frac{\lambda_t}{\lambda}\right] = \lambda\,P\,A\,E_t^p\left[\frac{\lambda_t\,P_t\,A_t}{\lambda\,P\,A}\right]. \tag{2.83}$$

Simplifying and rewriting (2.83) using natural logarithms results in

$$E_t^p\,[e^{\hat{\lambda}_t + \hat{w}_t}] = E_t^p\,[e^{\hat{\lambda}_t + \hat{p}_t + \hat{a}_t}]. \tag{2.84}$$

Taking a first-order approximation around the steady state of both sides of (2.84) (see also Appendix A in Campbell (1994) for a similar exercise) results in

$$E_t^p\,[1 + \hat{\lambda}_t + \hat{w}_t] = E_t^p\,[1 + \hat{\lambda}_t + \hat{p}_t + \hat{a}_t]. \tag{2.85}$$

Simplifying (2.85) by taking into account that $W = P\,A$ and yields (2.11) .

For (2.10) one can check Chapter 3 in Gali (2008) .

2.8.2 Kalman filter

Let me start with the consumer's case which is easier to handle. Suppose that the consumer's prior for period-t permanent productivity, x_t , is

$$x_t\,|\,I_{t-1}^c \sim N(x_{t|t-1}\,,\,\sigma_{x,t-1}^2)\,,$$

where $\sigma_{x,t-1}^2 \equiv Var_{t-1}^c\,[x_t]$.

Upon the arrival of new information, $\{s_t, a_t\}$, the consumer's information set becomes $I_t^c = I_{t-1}^c \cup \{s_t, a_t\}$. Taking into account that all shocks are serially uncorrelated, mutually independent, and normally distributed and using Bayes' Law implies that the consumer's posterior distribution is

$$x_t \,|\, I_t^c \sim N\left((1-k_t)\, x_{t|t-1} + k_t \left[\theta\, s_t + (1-\theta)\, a_t\right],\, \left(\frac{1}{\sigma_{x,t-1}^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_u^2}\right)^{-1}\right),$$

where $k_t \equiv \frac{\frac{1}{\sigma_e^2}+\frac{1}{\sigma_u^2}}{\frac{1}{\sigma_{x,t-1}^2}+\frac{1}{\sigma_e^2}+\frac{1}{\sigma_u^2}}$ and $\theta \equiv \frac{\frac{1}{\sigma_e^2}}{\frac{1}{\sigma_e^2}+\frac{1}{\sigma_u^2}}$. The former denotes the Kalman gain term, that is the precision of new information, $\{s_t, a_t\}$, relative to the total precision of the consumer's information; the latter denotes the precision of the signal s_t relative to that of the consumer's new information.

Letting $\sigma_{x,t}^2 \equiv Var_t^c\left[x_{t+1}\right]$, the following period's prior is

$$x_{t+1} \,|\, I_t^c \sim N\left(x_{t+1|t}\,,\, \sigma_{x,t}^2\right),$$

where $x_{t+1|t} = (1-k_t)\, x_{t|t-1} + k_t\left[\theta\, s_t + (1-\theta)\, a_t\right]$ and

$$\sigma_{x,t}^2 = \left(\frac{1}{\sigma_{x,t-1}^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_u^2}\right)^{-1} + \sigma_\epsilon^2\,. \tag{2.86}$$

Let σ_x^2 denote the solution (a fixed point) to the Riccati equation (2.86) (simply set $\sigma_x^2 \equiv \sigma_{x,t-1}^2 = \sigma_{x,t}^2$). A solution does not exist in the limit case where $\sigma_e^2 \to \infty$ and $\sigma_u^2 \to \infty$ which I therefore dismiss.

I assume that both agents' prior in period 0 is $x_{0\,|-1} \sim N(0\,,\,\sigma_x^2)$, which implies that their learning problems (see below for the producer) are at their steady state when time commences. As a result, the Kalman gain term in (2.16) is time invariant and given by

$$k \equiv \frac{\frac{1}{\sigma_e^2} + \frac{1}{\sigma_u^2}}{\frac{1}{\sigma_x^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_u^2}} .$$

Turning to the producer's learning problem, recall from the main text analysis that, by the end of each period, both agents have received the same new information. Given their (assumed to be) common prior in period 0, this implies that, at the end of each period, $I^p_{t-1,2} = I^c_{t-1}$. As a result, agents have the same prior distribution over the following period's x. That is

$$x_t \,|\, I^p_{t-1,2} \sim N(x_{t|t-1} \,,\, \sigma^2_{x,t-1}) \,,$$

where $\sigma^2_{x,t-1} \equiv Var^c_{t-1}[x_t] = Var^p_{t-1,2}[x_t]$.

Using (2.5), the producer's prior distribution of period-t productivity, a_t, is

$$a_t \,|\, I^p_{t-1,2} \sim N\left(x_{t|t-1} \,,\, \sigma^2_{x,t-1} + \sigma^2_u\right) . \tag{2.87}$$

The consumer and the producer's information sets differ though in stage 1 of each period. In particular, the producer's information set is $I^p_{t,1} = I^p_{t-1,2} \cup \{s_t\}$. Then,

$$x_t \,|\, I^p_{t,1} \sim N\left((1-\mu_t)\, x_{t|t-1} + \mu_t\, s_t \,,\, \left(\frac{1}{\sigma^2_{x,t-1}} + \frac{1}{\sigma^2_e}\right)^{-1}\right) ,$$

where $\mu_t \equiv \frac{\frac{1}{\sigma_e^2}}{\frac{1}{\sigma^2_{x,t-1}} + \frac{1}{\sigma_e^2}}$.

Along the same lines, the producer's stage-1 distribution of a_t is

$$a_t \,|\, I^p_{t,1} \sim N\left((1-\mu_t)\, x_{t|t-1} + \mu_t\, s_t \,,\, \left(\frac{1}{\sigma^2_{x,t-1}} + \frac{1}{\sigma^2_e}\right)^{-1} + \sigma^2_u\right) . \tag{2.88}$$

In stage 2 of t, the producer extracts a_t. Since the producer and the consumer have the same information, the producer's stage-2 distribution of x_t coincides with that of the consumer and is given by

$$x_t \mid I^p_{t,2} \sim N\left((1-k_t)\, x_{t|t-1} + k_t \left[\theta\, s_t + (1-\theta)\, a_t\right], \left(\frac{1}{\sigma^2_{x,t-1}} + \frac{1}{\sigma^2_e} + \frac{1}{\sigma^2_u} \right)^{-1} \right).$$

Like before, $k_t \equiv \frac{\frac{1}{\sigma_e^2} + \frac{1}{\sigma_u^2}}{\frac{1}{\sigma^2_{x,t-1}} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_u^2}}$ and $\theta \equiv \frac{\frac{1}{\sigma_e^2}}{\frac{1}{\sigma_e^2} + \frac{1}{\sigma_u^2}}$.

Let $\sigma^2_{x,t} \equiv Var^c_t\left[x_{t+1}\right] = Var^p_{t,2}\left[x_{t+1}\right]$. The producer's prior distributions for x_{t+1} and a_{t+1}, respectively, are

$$x_{t+1} \mid I^p_{t,2} \sim N\left(x_{t+1|t}\,,\, \sigma^2_{x,t}\right)$$

$$a_{t+1} \mid I^p_{t,2} \sim N\left(x_{t+1|t}\,,\, \sigma^2_{x,t} + \sigma^2_u\right), \tag{2.89}$$

where $x_{t+1|t} = (1-k_t)\, x_{t|t-1} + k_t\left[\theta\, s_t + (1-\theta)\, a_t\right]$ and

$$\sigma^2_{x,t} = \left(\frac{1}{\sigma^2_{x,t-1}} + \frac{1}{\sigma^2_e} + \frac{1}{\sigma^2_u} \right)^{-1} + \sigma^2_\epsilon .$$

Observe that the last expression is the same as the Riccati equation (2.86). Letting σ^2_x denote its solution and assuming that the agents' prior in period 0 is $x_{0\,|\,-1} \sim N(0\,,\,\sigma^2_x)$, implies that the producer's learning problem is also at its steady state when time commences. As a result, the Kalman gain term in (2.16) is time invariant -see above for this. This is also the case for μ, the coefficient in (2.15), which is given by

$$\mu \equiv \frac{\frac{1}{\sigma^2_e}}{\frac{1}{\sigma^2_x} + \frac{1}{\sigma^2_e}}.$$

Last, as long as σ_x^2 solves (2.86), the variance of the producer's (prior or posterior) distribution for a_t is also time invariant. To see this in the case of the producer's prior distribution, juxtapose (2.87) and (2.89). In the case of the producer's posterior (stage-1) distribution, observe that the only possibly time-varying term in (2.88) is the prior's variance; if this is time invariant, so is the variance of the producer's posterior.

A thorough demonstration of the Kalman filter can be found in Anderson and Moore (1979), Harvey (1989), and Technical Appendix B in Ljungqvist and Sargent (2004).

2.8.3 Equilibrium under Rule 1

Let me elaborate first on the filtering problems of the agents. The producer's and the consumer's expectations, respectively, are (see also (2.15) and (2.16)):

$$E_t^p\,[a_t] \;=\; E_{t,1}^p\,[x_t] \;=\; (1-\mu)\,E_{t-1,2}^p\,[x_{t-1}] \;+\; \mu\, s_t$$

$$E_{t,2}^p\,[x_t] \;=\; E_t^c\,[x_t] \;=\; (1-k)\,E_{t-1}^c\,[x_{t-1}] \;+\; k\,[\theta\, s_t \;+\; (1-\theta)\,a_t]\,.$$

Then, the consumer's expectations in period t of the producer's expectations in $t+1$ are given by

$$E_t^c\,[\,E_{t+1}^p\,[a_{t+1}]\,] \;=\; E_t^c\,[x_t] \tag{2.90}$$

and the producer's expectations in period t of the consumer's expectations in t are given by

$$E^p_t\,[\,E^c_t\,[x_t]\,] \;=\; (1{-}k)\,E^c_{t-1}\,[x_{t-1}] + k\,[\theta\,s_t + (1{-}\theta)\,E^p_t\,[a_t]] \;=\; E^c_t\,[x_t] + k\,(1{-}\theta)\,(E^p_t\,[a_t] - a_t)\,. \tag{2.91}$$

Substituting conjectures (C1), (C2), and (2.91) in (2.17) implies

$$\zeta n_t \;=\; (1 + \kappa_3 - \xi_1)\,E^p_t\,[a_t] + \kappa_2\,k\,(1{-}\theta)\,(E^p_t\,[a_t] - a_t) - (\kappa_3 + \xi_2)\,a_t\,. \tag{2.92}$$

Substituting (2.92) in the firm's technology, $y_t \;=\; a_t \,+\, n_t\,,$ using market clearing, $y_t \;=\; c_t\,,$ and, subsequently, matching coefficients with conjecture (C1) yields

$$\xi_1 \;=\; \frac{1\,+\,\kappa_3\,+\,\kappa_2\,k\,(1-\theta)}{1+\zeta} \tag{2.93}$$

$$\xi_2 \;=\; \frac{\zeta\,-\,\kappa_3\,-\,\kappa_2\,k\,(1-\theta)}{1+\zeta}\,. \tag{2.94}$$

Observe that

$$\xi_1\,+\,\xi_2 \;=\; 1\,, \tag{2.95}$$

a direct consequence of preferences logarithmic in consumption.

Turning to the Euler equation (2.18), conjectures (C1) and (C2) combined with (2.90) imply

$$E_t^c\,[c_{t+1}] \,-\, c_t \,=\, -\,\xi_1\,E_t^p\,[a_t] \,+\, (\xi_1 \,+\, \xi_2)\,E_t^c\,[x_t] \,-\, \xi_2\,a_t \tag{2.96}$$

$$i_t \,-\, E_t^c\,[\pi_{t+1}] \,=\, (\phi_y\,\xi_1 \,+\, \phi_\pi\,\kappa_1)\,E_t^p\,[a_t] \,+\, [-\kappa_1 \,+\, (\phi_\pi - 1)\kappa_2 \,-\, \kappa_3]\,E_t^c\,[x_t] \,+$$

$$+\,[\phi_y\,\xi_2 \,+\, \phi_\pi\,\kappa_3 \,-\, \phi_y]\,a_t\,. \tag{2.97}$$

Matching coefficients in (2.96) and (2.97) yields

$$-\,\xi_1 \,= \phi_y\,\xi_1 \,+\, \phi_\pi\,\kappa_1 \tag{2.98}$$

$$\xi_1 \,+\, \xi_2 \,=\, -\,\kappa_1 \,+\, (\phi_\pi - 1)\,\kappa_2 \,-\, \kappa_3 \tag{2.99}$$

$$-\,\xi_2 \,= \phi_y\,\xi_2 \,+\, \phi_\pi\,\kappa_3 \,-\, \phi_y\,. \tag{2.100}$$

Summing (2.98) - (2.100) across sides and using (2.95) yields

$$\kappa_1 \,+\, \kappa_2 \,+\, \kappa_3 \,=\, 0\,, \tag{2.101}$$

whereas summing across (2.98) and (2.100) and again using (2.95) yields

$$\kappa_1 \,+\, \kappa_3 \,=\, -\,\frac{1}{\phi_\pi}\,, \tag{2.102}$$

which are equations (2.23) and (2.24) , respectively, in the main text.

Solving (2.94), (2.95) and (2.98) - (2.100) for ξ_1 , ξ_2 , κ_1 , κ_2 , κ_3 returns

(2.19) - (2.21) in the main text.

2.8.4 Equilibria under Rule 2

Equations (2.30) - (2.31) can be obtained by combining the equilibrium labor decision (2.14) with the firm's technology and market clearing. They coincide with (2.93) - (2.95), derived in Appendix 2.8.3 above (one only needs to replace ξ_1 with ξ_3 , ξ_2 with ξ_4 , κ_1 with κ_4 , κ_2 with κ_5 , and κ_3 with κ_6) .

Turning to the Euler equation, conjectures (C3) - (C4) imply $E_t^c\,[c_{t+1}] = (\xi_3 + \xi_4)\,E_t^c\,[x_t]$ and $E_t^c\,[p_{t+1}] = (\kappa_4 + \kappa_5 + \kappa_6)\,E_t^c\,[x_t]$. Then the LHS and the RHS of the Euler equation (2.13) after taking into account that $I_t^m = I_t^c$ (equation (2.29) in the main text) become, respectively,

$$E_t^c\,[c_{t+1}] - c_t = (\xi_3 + \xi_4)\,E_t^c\,[x_t] - \xi_3\,E_t^p\,[a_t] - \xi_4\,a_t \tag{2.103}$$

$$i_t - E_t^c\,[\pi_{t+1}] = (\phi_\pi - 1)\,E_t^c\,[\pi_{t+1}] = (\phi_\pi - 1)\,[(\kappa_4 + \kappa_6)\,E_t^c\,[x_t] - \kappa_4\,E_t^p\,[a_t] - \kappa_6\,a_t]\,. \tag{2.104}$$

Matching coefficients in (2.103) - (2.104) and using (2.31) yields (2.32) - (2.33) .

Omitted derivations in Section 2.5.5

First, I deal with the case in which the monetary authority reports the following period's prices with a measurement error. Next, I follow the same process in the case of a monetary policy shock. Recall that what distinguishes the two cases is the information set of the monetary authority.

Measurement error. Suppose at the end of period $t-1$ the nominal interest rate serves as a noisy signal about the price in t, as in (2.62). Agents extract

$$\tilde{p}_t \,=\, p_t \,+\, w_t \,, \tag{2.105}$$

where $\tilde{p}_t \,\equiv\, \frac{i_{t-1} + \phi_\pi \, p_{t-1}}{\phi_\pi}$. The monetary authority's information set is $I^m_{t-1} \,=\, \Omega_t \setminus \{z_t\}$, where z_t is the public signal about period-t productivity which I derive below and Ω_t denotes the state of the economy in t. The latter is $\Omega_t \,=\, (\{a_\tau\}^t_{\tau=0}\,,\, \{s_\tau\}^t_{\tau=0}\,,\, z_t)$. Using (2.37), (2.105) becomes

$$\frac{\tilde{p}_t \,-\, \kappa_4\, E^p_t\left[a_t \,|\, I^p_{t,1} \setminus \{z_t\}\right]}{\kappa_6} \,=\, a_t \,+\, \frac{1}{\kappa_6}\, w_t\,, \tag{2.106}$$

where κ_4, κ_6 are coefficients given by (2.30) - (2.33) for $\kappa_5 \,=\, 0$. The producer's information set in stage 1 is $I^p_{t,1} \,=\, \Omega_t \setminus \{a_t\}$. The LHS in (2.106) is the endogenous public signal in stage 1 of t denoted by z_t. It follows then that

$$z_t \,\equiv\, \frac{\left[\phi_\pi \,+\, \zeta\,(\phi_\pi - 1)\right] \tilde{p}_t \,-\, \frac{\phi_\pi}{\phi_\pi - 1}\, E^p_t\left[a_t \,|\, I^p_{t,1} \setminus \{z_t\}\right]}{\zeta} \,=\, a_t + \frac{\phi_\pi \,+\, \zeta\,(\phi_\pi - 1)}{\zeta}\, w_t\,. \tag{2.107}$$

The conditional variance of productivity is then $\sigma^2_z \,\equiv\, Var\left[a_t \,|\, z_t\right] \,=\, \left(\frac{\phi_\pi \,+\, \zeta\,(\phi_\pi - 1)}{\zeta}\right)^2 \sigma^2_w$.

Turning back to the producer, suppose for a moment that z_t is not part of his information set. Then, the producer's posterior distribution of a_t is

$$a_t \,|\, I^p_t \setminus \{z_t\} \,\sim\, N\left(E^p_t\,\left[x_t \,|\, I^p_{t,1} \setminus \{z_t\}\right]\,,\, \left(\frac{1}{\sigma^2_x} \,+\, \frac{1}{\sigma^2_e}\right)^{-1} \,+\, \sigma^2_u\right)\,,$$

where $E^p_t\,\left[x_t \,|\, I^p_{t,1} \setminus \{z_t\}\right]$ is given by (2.15) and σ^2_x is the fixed point in (2.86).

Taking z_t into account, the producer's posterior becomes

$$a_t \,|\, I^p_{t,1} \sim \; N \; \left(\delta \, E^p_t \; \left[x_t \,|\, I^p_{t,1} \setminus \{z_t\}\right] \; + \; (1-\delta)\, z_t \; , \; \sigma^2_a\right) \; , \tag{2.108}$$

where $\delta \;=\; \left(\left(\frac{1}{\sigma^2_x} \,+\, \frac{1}{\sigma^2_e}\right)^{-1} \,+\, \sigma^2_u\right)^{-1} \sigma^2_a$ and $\sigma^2_a \;=\; \left[\left(\left(\frac{1}{\sigma^2_x} \,+\, \frac{1}{\sigma^2_e}\right)^{-1} \,+\, \sigma^2_u\right)^{-1} \,+\, \frac{1}{\sigma^2_z}\right]^{-1}$.

Monetary policy shock. In case the monetary authority transmits monetary policy shocks, its information set additionally includes $\hat{z}_t$, that is $I^m_{t-1} \;=\; \hat{\Omega}_t$. Taking the same steps as before, agents observe $\hat{p}_t \;=\; \phi_\pi \, p_t \,+\, \omega_t$, where $\hat{p}_t \;\equiv\; i_{t-1} \,+\, \phi_\pi \, p_{t-1}$. The monetary authority transmits the public signal

$$\hat{z}_t \;=\; a_t \;+\; \frac{\phi_\pi \,+\, \zeta(\phi_\pi - 1)}{\phi_\pi \, \zeta} \, \omega_t \, ,$$

where

$$\hat{z}_t \;\equiv\; \frac{\left[\phi_\pi \,+\, \zeta(\phi_\pi - 1)\right] \hat{p}_t \;-\; \frac{\phi^2_\pi}{\phi_\pi - 1} \, E^p_t[a_t \,|\, I^p_{t,1}]}{\phi_\pi \, \zeta} \, . \tag{2.109}$$

The conditional variance of productivity is $\sigma^2_{\hat{z}} \;\equiv\; Var[a_t \,|\, \hat{z}_t] \;=\; \left(\frac{\phi_\pi \,+\, \zeta(\phi_\pi - 1)}{\phi_\pi \, \zeta}\right)^2 \sigma^2_\omega$. The producer's posterior is

$$a_t \,|\, I^p_{t,1} \sim \; N \; \left(\hat{\delta} \, E^p_t \; \left[x_t \,|\, I^p_{t,1} \setminus \{\hat{z}_t\}\right] \; + \; (1-\hat{\delta})\, \hat{z}_t \; , \; \sigma^2_{\hat{a}}\right) \; , \tag{2.110}$$

where $\hat{\delta} \;=\; \left(\left(\frac{1}{\sigma^2_x} \,+\, \frac{1}{\sigma^2_e}\right)^{-1} \,+\, \sigma^2_u\right)^{-1} \sigma^2_{\hat{a}}$ and $\sigma^2_{\hat{a}} \;=\; \left[\left(\left(\frac{1}{\sigma^2_x} \,+\, \frac{1}{\sigma^2_e}\right)^{-1} \,+\, \sigma^2_u\right)^{-1} \,+\, \frac{1}{\sigma^2_{\hat{z}}}\right]^{-1}$.

Observe that unlike in (2.107), the producer's expectations in (2.109) are conditional on the entire information set of the producer. To fully extract

$\hat{z}_t$ use (2.110) to get

$$\hat{z}_t \equiv [\phi_\pi (1-\hat{\delta}) + \zeta(\phi_\pi - 1)]^{-1} \left\{ \frac{(\phi_\pi - 1)[\phi_\pi + \zeta(\phi_\pi - 1)]}{\phi_\pi} \hat{p}_t - \phi_\pi \hat{\delta} E_t^p \left[x_t \mid I_{t,1}^p \setminus \{\hat{z}_t\} \right] \right\}.$$

2.8.5 Derivations in Section 2.6.2

Consider the interest-rate rule

$$i_t = -\log\beta + \phi_\pi E_t^m [\pi_{t+1} - \hat{\pi}_{t+1}] + \phi_y E_t^m \Delta[y_{t+1} - \hat{y}_{t+1}],$$

where $\hat{\pi}_{t+1} = \kappa_7 E_t^m (a_{t+1} - a_t)$ and $\hat{y}_t = a_t$. A reverse engineering process will pin down κ_7.

The labor market optimality condition implies

$$\xi_3 = \frac{1 + \kappa_6 + k(1-\theta)\kappa_5}{1+\zeta}$$

$$\xi_4 = 1 - \xi_3,$$

which correspond to equations (2.30) - (2.31) in the main text. Taking familiar steps, the Euler equation implies

$$1 - \phi_y = (\phi_\pi - 1)(\kappa_4 + \kappa_6) - \phi_\pi \kappa_7 - \phi_y \tag{2.111}$$

$$(1 - \phi_y)\xi_3 = (\phi_\pi - 1)\kappa_4 \tag{2.112}$$

$$(1 - \phi_y)\xi_4 = (\phi_\pi - 1)\kappa_6 - \phi_\pi \kappa_7 - \phi_y. \tag{2.113}$$

Setting

$$\kappa_7 = \frac{(\phi_\pi - 1)\,\kappa_6 - 1}{\phi_\pi} \tag{2.114}$$

implies $\xi_3 = 0$ and $\xi_4 = 1$ as required, $\kappa_4 = 0$ and $\kappa_6 = -\left[1 + k\,(1-\theta)\,\kappa_5\right]$. Combining the latter with (2.114) yields (2.78) in text.

2.9 Appendix: Data

Data in Figures 2.1 - 2.4 are collected from the St. Louis Fed and refer to the US economy for the period 1965 : 1 - 2010 : 1 . Data in Figures 2.1 and 2.3 are quarterly, whereas in Figures 2.2 and 2.4 they are annual. Employment refers to "All Employees: Total Nonfarm Employees (Thousands of Persons)" (series PAYEMS) and is seasonally adjusted. It is logged and HP-filtered with penalty 1600 for quarterly and 100 for annual data, respectively. Figures 2.1 - 2.4 show its cyclical component scaled up by 50 for expositional clarity. Inflation in Figures 2.1 and 2.3 refers to percent changes in the "Gross Domestic Product: Implicit Price Deflator" (series GDPDEF) and is seasonally adjusted. Inflation in Figures 2.2 and 2.4 refers to percent changes in the "Consumer Price Index for All Urban Consumers: All Items" (series CPIAUCSL) and is seasonally adjusted. Consumer Sentiment refers to "University of Michigan: Consumer Sentiment" (series UMCSENT1, UMCSENT) and is not seasonally adjusted. It is scaled down by 25 in Figure 2.3 and by 10 in Figure 2.4 .[68]

[68] In the figures, GDP is an abbreviation for Gross Domestic Product and CPI is an abbreviation for Consumer Price Index.

2.10 Appendix: Solving for the Equilibrium under Rule 1 Without Approximations

Plug the producer's labor demand condition, (2.8) , into the consumer's labor supply condition, (2.9) , replace for λ_t by taking into account that $\lambda_t \,=\, \frac{1}{P_t\,C_t}$, multiply and divide the RHS of the generated expression by P_{t-1} , and confirm that

$$N_t^{\zeta} \;=\; \frac{1}{\Pi_t\,C_t}\,\frac{E_t^p\,[\frac{A_t}{C_t}]}{E_t^p\,[\frac{1}{\Pi_t\,C_t}]}\,. \tag{2.115}$$

Turning to the Euler equation, (2.10) , suppose that the monetary authority sets the nominal bond price according to the following interest-rate rule:

$$Q_t \;=\; \beta\,\Pi_t^{-\,\phi_\pi}\,\left(\frac{Y_t}{A_t}\right)^{-\,\phi_y}\,,$$

where $\Pi_t \;\equiv\; \frac{P_t}{P_{t-1}}$.

Plugging this into (2.10) yields

$$\Pi_t^{-\,\phi_\pi}\,\left(\frac{Y_t}{A_t}\right)^{-\,\phi_y} \;=\; E_t^c\,\left[\frac{1}{\Pi_{t+1}}\,\frac{C_t}{C_{t+1}}\right]\,. \tag{2.116}$$

Let me now post the following conjectures for log-consumption and log-inflation:

$$c_t \;=\; \xi_0 \,+\, \xi_1\,E_t^p[a_t] \,+\, \xi_2\,a_t \tag{C1'}$$

$$\pi_t \;=\; \kappa_0 \,+\, \kappa_1\,E_t^p[a_t] \,+\, \kappa_2\,E_t^c[x_t] \,+\, \kappa_3\,a_t\,. \tag{C2'}$$

Given that all shocks are normally distributed, I will show below that, conditional on the agents' information sets, conjectures (C1') and (C2') imply that C_t and Π_t are log-normally distributed.

Let me start with the labor optimality condition, (2.115). Take technology ($y_t = a_t + n_t$) into account to express it as follows:

$$e^{\zeta\,(y_t - a_t)} = e^{-(\pi_t + c_t)} \frac{E_t^p\left[e^{a_t - c_t}\right]}{E_t^p\left[e^{-(\pi_t + c_t)}\right]}. \tag{2.117}$$

Next, use market clearing and rearrange terms in (2.117) to get

$$e^{(1+\zeta)\,c_t - \zeta\,a_t + \pi_t} = \frac{E_t^p\left[e^{a_t - c_t}\right]}{E_t^p\left[e^{-(\pi_t + c_t)}\right]}. \tag{2.118}$$

Take conjectures (C1') and (C2') into account and express the LHS of (2.118) as

$$e^{(1+\zeta)\,\xi_0 + \kappa_0 + [(1+\zeta)\,\xi_1 + \kappa_1]\,E_t^p[a_t] + \kappa_2\,E_t^c[x_t] + [(1+\zeta)\,\xi_2 - \zeta + \kappa_3]\,a_t}. \tag{2.119}$$

The RHS of (2.118) is equal to

$$\frac{E_t^p\left[e^{-\xi_0 - \xi_1\,E_t^p[a_t] + (1-\xi_2)\,a_t}\right]}{E_t^p\left[e^{-\{\kappa_0 + \xi_0 + (\kappa_1 + \xi_1)\,E_t^p[a_t] + \kappa_2\,E_t^c[x_t] + (\kappa_3 + \xi_2)\,a_t\}}\right]}. \tag{2.120}$$

Conditional on the producer's information set, $I_{t,1}^p = \Psi_t \setminus \{a_t\}$, the exponent in the nominator of (2.120) is normally distributed with mean $-\xi_0 + (1 - \xi_1 - \xi_2)\,E_t^p[a_t]$ and variance $(1-\xi_2)^2\,\sigma_{p,a}^2$, where $\sigma_{p,a}^2 \equiv Var\left[a_t \,|\, I_{t,1}^p\right] = \left(\frac{1}{\sigma_x^2} + \frac{1}{\sigma_e^2}\right)^{-1} + \sigma_u^2$ and σ_x^2 solves the Riccati equation (2.86) (see also the analysis in 2.8.2). Then, the nominator of (2.120) is equal to

$$e^{-\xi_0 + (1-\xi_1-\xi_2)\,E_t^p[a_t] + \frac{1}{2}\,(1-\xi_2)^2\,\sigma_{p,a}^2} \tag{2.121}$$

Let me now turn to the exponent in the denominator of (2.118). It is also normally distributed with mean

$$E_t^p\left[-\left(\pi_t + c_t\right)\right] =$$

$$-\,\left\{\kappa_0 + \xi_0 + \left[\kappa_1 + \kappa_2\, k\,(1-\theta) + \kappa_3 + \xi_1 + \xi_2\right] E_t^p\left[a_t\right] + \kappa_2\, E_t^c\left[x_t\right] - \kappa_2\, k\,(1-\theta)\, a_t\right\},$$

where I have used (2.91).

To find its variance, $Var_t^p\left[-\left(\pi_t + c_t\right)\right]$, first bring $\pi_t + c_t$ into the following form:

$$\pi_t + c_t = \kappa_0 + \xi_0 + (\kappa_1 + \xi_1)\, E_t^p[a_t] + \kappa_2\left[(1-k)\, E_{t-1}^c[x_{t-1}] + k\,\theta\, s_t\right] + \left[\kappa_2\, k\,(1-\theta) + \kappa_3 + \xi_2\right] a_t\,.$$

It then follows that

$$Var_t^p\left[-\left(\pi_t + c_t\right)\right] = \left[\kappa_2\, k\,(1-\theta) + \kappa_3 + \xi_2\right]^2\, \sigma_{p,a}^2\,.$$

The denominator on the RHS of (2.120) is then equal to

$$E_t^p\left[e^{-\left(\pi_t + c_t\right)}\right] = e^{-\left\{\kappa_0 + \xi_0 + \left[\kappa_1 + \kappa_2\, k\,(1-\theta) + \kappa_3 + \xi_1 + \xi_2\right] E_t^p\left[a_t\right] + \kappa_2\, E_t^c\left[x_t\right] - \kappa_2\, k\,(1-\theta)\, a_t\right\}} \times$$

$$\times\, e^{\frac{1}{2}\left[\kappa_2\, k\,(1-\theta) + \kappa_3 + \xi_2\right]^2\, \sigma_{p,a}^2}\,. \qquad (2.122)$$

Therefore, the RHS of of (2.120) is equal to (simply divide (2.121) by

(2.122))

$$e^{\kappa_0 + \frac{1}{2}\{(1-\xi_2)^2 - [\kappa_2\, k\,(1-\theta) + \kappa_3 + \xi_2]^2\}\,\sigma_{p,a}^2 + [1+\kappa_1+\kappa_2\, k\,(1-\theta)+\kappa_3]\, E_t^p\,[a_t] + \kappa_2\, E_t^c\,[x_t] - \kappa_2\, k\,(1-\theta)\, a_t}\,. \tag{2.123}$$

Matching coefficients in (2.119) and (2.123) yields (2.93), (2.94), and

$$\xi_0 \;=\; \frac{1}{2\,(1+\zeta)}\,\{(1-\xi_2)^2 \,-\, [\kappa_2\, k\,(1-\theta) \,+\, \kappa_3 \,+\, \xi_2]^2\}\,\sigma_{p,a}^2\,. \tag{2.124}$$

Turning to the Euler equation, (2.116), it can be expressed as follows:

$$e^{-\,[\phi_\pi\, \pi_t + \phi_y\,(y_t - a_t)]} \;=\; e^{c_t}\, E_t^c\,[e^{-\,(c_{t+1} + \pi_{t+1})}]\,. \tag{2.125}$$

Let me start with the LHS of (2.125). Market clearing and conjectures (C1') and (C2') imply that it can be expressed as

$$e^{-\,[\phi_\pi\, \pi_t + \phi_y\,(y_t - a_t)]} \;=\; e^{-\,\{\phi_\pi\, \kappa_0 + \phi_y\, \xi_0 + (\phi_\pi\, \kappa_1 + \phi_y\, \xi_1)\, E_t^p\,[a_t] + \phi_\pi\, \kappa_2\, E_t^c\,[x_t] + [\phi_\pi\, \kappa_3 + \phi_y\,(\xi_2 - 1)]\, a_t\}}\,. \tag{2.126}$$

Turning to the RHS of (2.125), $c_{t+1} \,+\, \pi_{t+1}$ conditional on the consumer's information set $I_t^c \;=\; \Psi_t$ is normally distributed with mean

$$E_t^c\,[c_{t+1} \,+\, \pi_{t+1}] \;=\; \xi_0 \,+\, \kappa_0 \,+\, (\xi_1 \,+\, \xi_2 \,+\, \kappa_1 \,+\, \kappa_2 \,+\, \kappa_3)\, E_t^c\,[x_t]\,,$$

where once again I have used conjectures (C1') - (C2').

To find the variance, $Var^c_{t+1}\,[c_{t+1}\,+\,\pi_{t+1}]$, express, first, $c_{t+1}\,+\,\pi_{t+1}$ as

$$c_{t+1} + \pi_{t+1} =$$

$$= \xi_0 + \kappa_0 + [(\xi_1+\kappa_1)(1-\mu) + \kappa_2(1-k)]\,E^c_t\,[x_t] + [(\xi_1+\kappa_1)\mu + \kappa_2 k\theta]\,s_{t+1}+$$

$$+ [\xi_2 + \kappa_3 + \kappa_2\,k(1-\theta)]\,a_{t+1}$$

$$= G + [(\xi_1 + \kappa_1)\,\mu + \kappa_2\,k + \xi_2 + \kappa_3]\,x_{t+1} + [(\xi_1 + \kappa_1)\,\mu + \kappa_2\,k\,\theta]\,e_{t+1}$$

$$+ [\xi_2 + \kappa_3 + \kappa_2\,k\,(1-\theta)]\,u_{t+1}\,,$$

where $G \equiv \xi_0 + \kappa_0 + [(\xi_1 + \kappa_1)\,(1-\mu) + \kappa_2\,(1-k)]\,E^c_t\,[x_t]$ is a known term to the consumer in period t. Given that shocks are mutually independent, it follows that

$$Var^c_{t+1}\,[c_{t+1} + \pi_{t+1}] = [(\xi_1 + \kappa_1)\,\mu + \kappa_2\,k + \xi_2 + \kappa_3]^2\,\sigma^2_x + [(\xi_1 + \kappa_1)\,\mu + \kappa_2\,k\,\theta]^2\,\sigma^2_e +$$

$$+ [\xi_2 + \kappa_3 + \kappa_2\,k\,(1-\theta)]^2\,\sigma^2_u\,.$$

Hence,

$$E^c_t\,[e^{-\,(c_{t+1}\,+\,\pi_{t+1})}] = e^{-\,[\,\xi_0 + \kappa_0 + (\xi_1 + \xi_2 + \kappa_1 + \kappa_2 + \kappa_3)\,E^c_t\,[x_t]\,]} \times$$

$$\times\; e^{\frac{1}{2}\,\{[(\xi_1 + \kappa_1)\,\mu + \kappa_2\,k\, + \xi_2 + \kappa_3]^2\,\sigma^2_x + [(\xi_1 + \kappa_1)\,\mu + \kappa_2\,k\,\theta]^2\,\sigma^2_e + [\xi_2 + \kappa_3 + \kappa_2\,k\,(1-\theta)]^2\,\sigma^2_u\}}\,.$$

Consequently, the RHS of (2.125) becomes

$$e^{c_t}\, E_t^c\,[e^{-\,(c_{t+1}\,+\,\pi_{t+1})}] = e^{-\,\kappa_0+\frac{1}{2}\{[(\xi_1+\kappa_1)\mu+\kappa_2 k+\xi_2+\kappa_3]^2\,\sigma_x^2+[(\xi_1+\kappa_1)\mu+\kappa_2 k\theta]^2\,\sigma_e^2+[\xi_2+\kappa_3+\kappa_2 k(1-\theta)]^2\,\sigma_u^2\}}\times$$

$$\times\; e^{\xi_1\,E_t^p\,[a_t]\,-\,(\xi_1\,+\,\xi_2\,+\,\kappa_1\,+\,\kappa_2\,+\,\kappa_3)\,E_t^c\,[x_t]\,+\,\xi_2\,a_t}\,. \tag{2.127}$$

Matching coefficients in (2.126) and (2.127) yields (2.98) - (2.100) and

$$\kappa_0 \;=\; -\frac{\phi_y\,\xi_0}{\phi_\pi-1} - \tag{2.128}$$

$$-\;\frac{\frac{1}{2}\,\{[(\xi_1+\kappa_1)\mu+\kappa_2 k+\xi_2+\kappa_3]^2\,\sigma_x^2+[(\xi_1+\kappa_1)\mu+\kappa_2 k\theta]^2\,\sigma_e^2+[\xi_2+\kappa_3+\kappa_2 k(1-\theta)]^2\,\sigma_u^2\}}{\phi_{\pi-1}},$$

where ξ_0 is given by (2.124) and ξ_1 , ξ_2 , κ_1 , κ_2 , κ_3 solve (2.94), (2.95) and (2.98) - (2.100) (see also the analysis in 2.8.3) .

Figure 2.1: Changes in GDP Deflator and Cyclical Employment: 1965 - 2010 (quarterly)

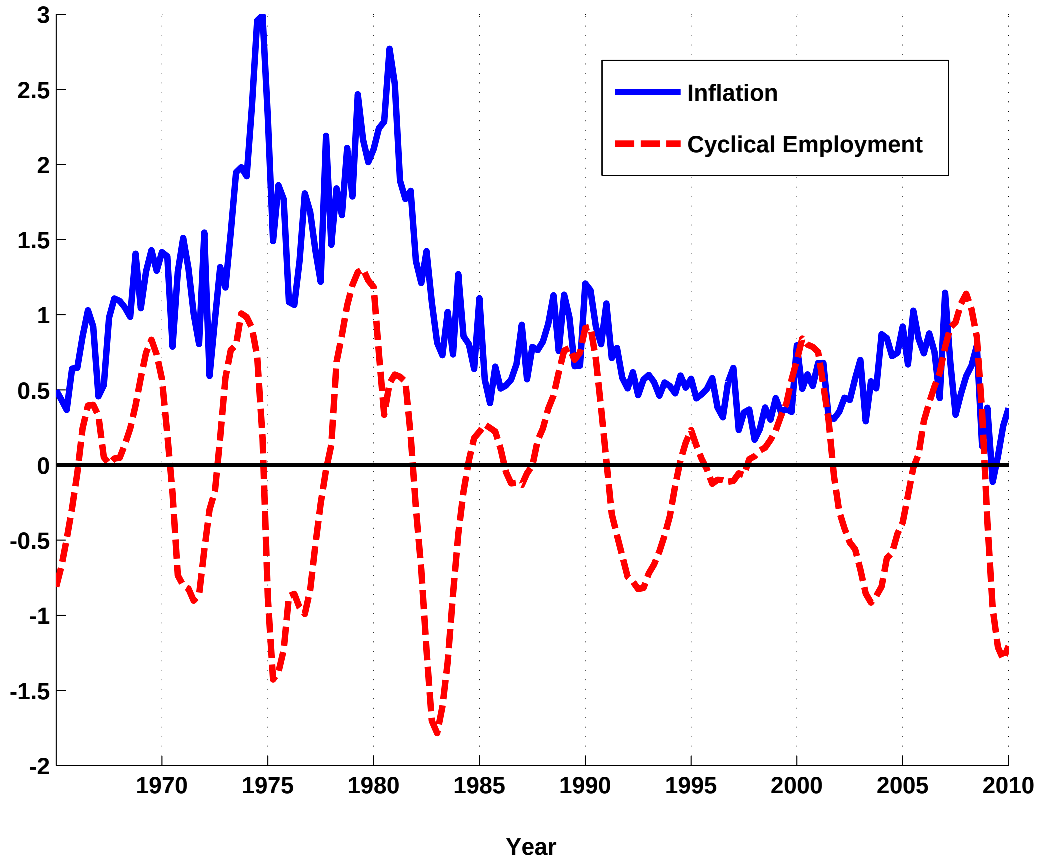


Figure 2.2: CPI Inflation and Cyclical Employment: 1965 - 2010 (annual)

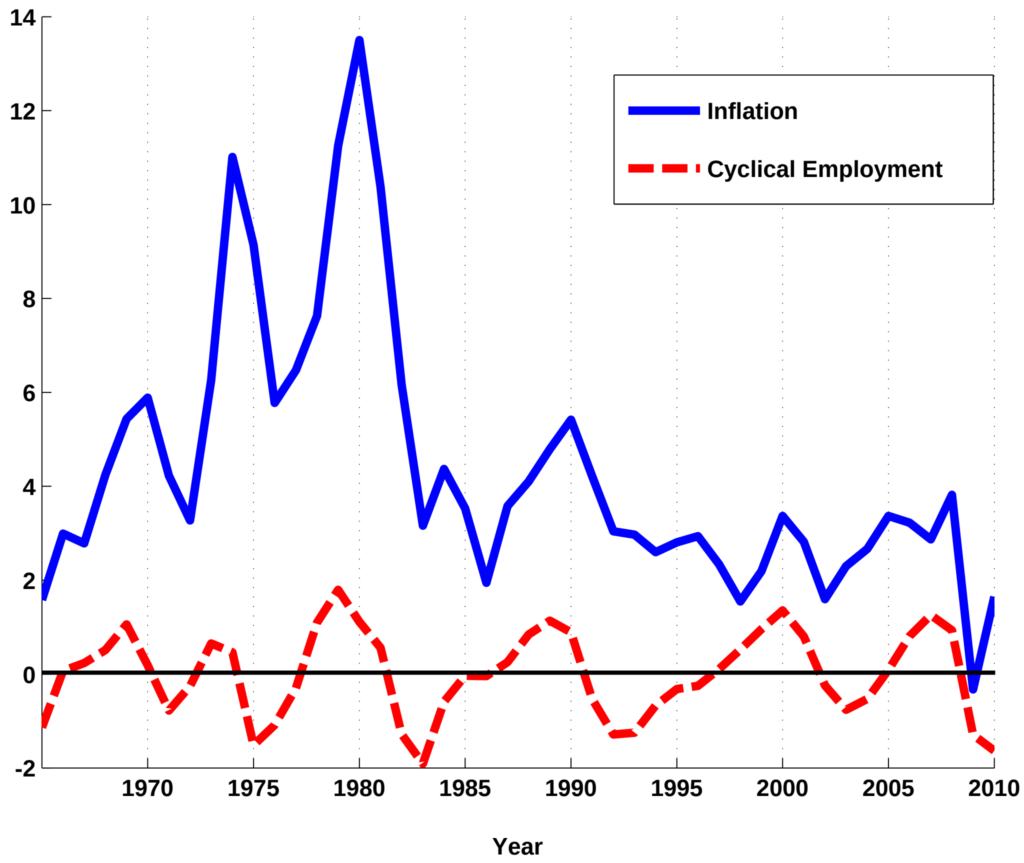


Figure 2.3: Changes in GDP Deflator, Cyclical Employment and Consumer Sentiment: 1965 - 2010 (quarterly)

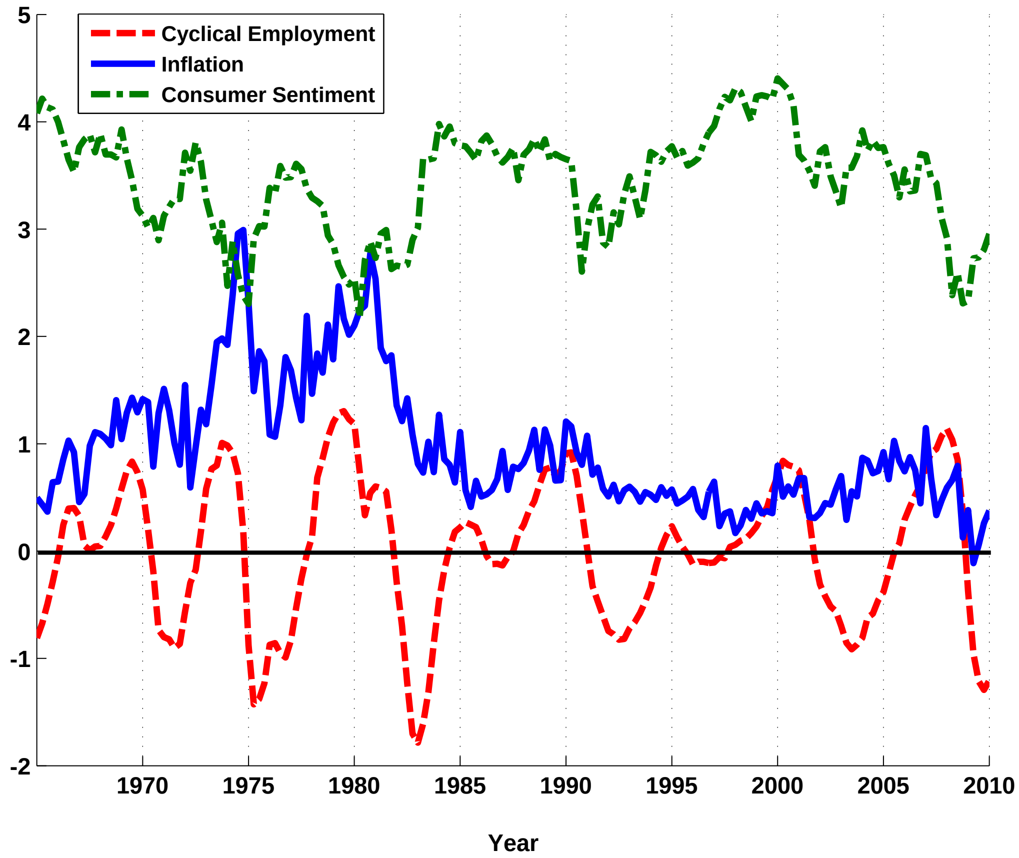


Figure 2.4: CPI Inflation, Cyclical Employment and Consumer Sentiment: 1965 - 2010 (annual)

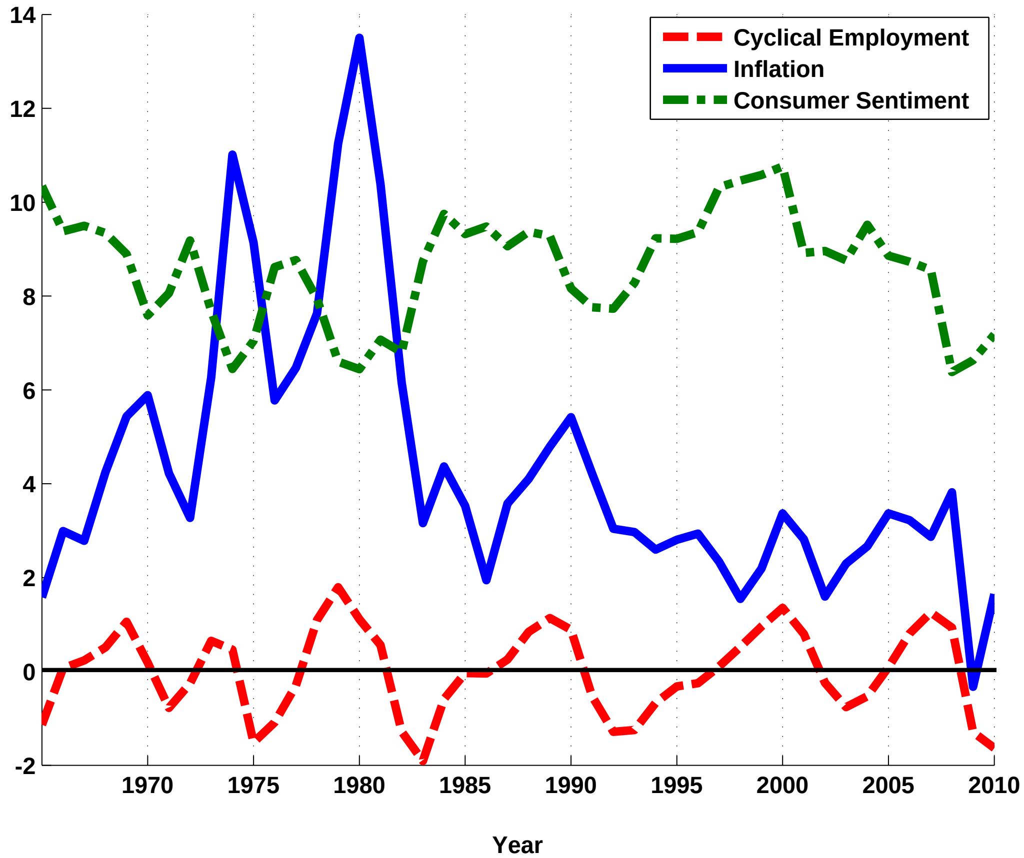


Figure 2.5: Impulse responses to a positive expectational shock for rule 1 with $\phi_y = 0.5$

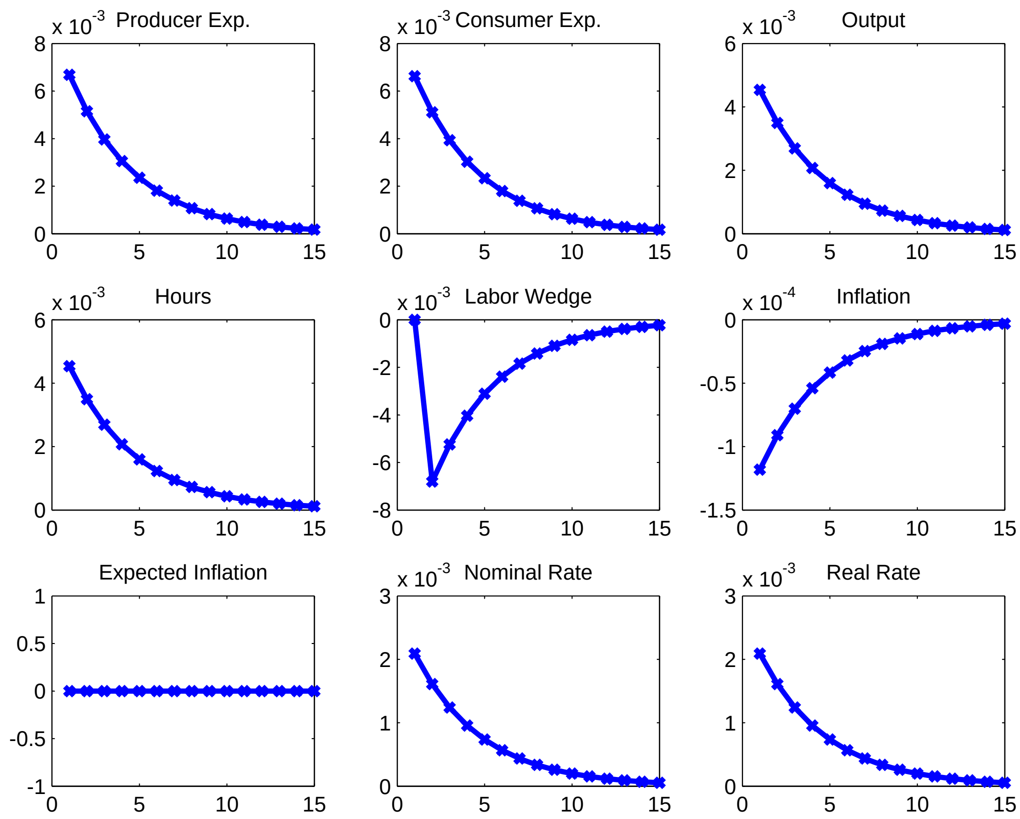


Figure 2.6: Impulse responses to a positive permanent productivity shock for rule 1 with $\phi_y = 0.5$

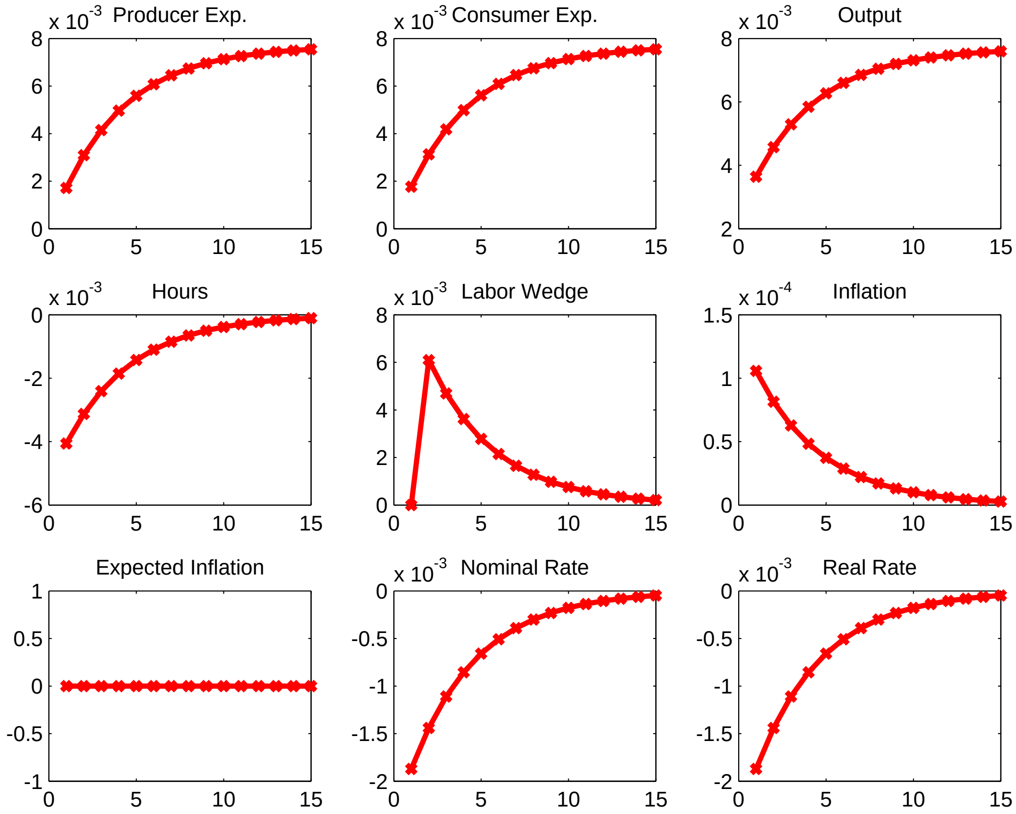


Figure 2.7: Impulse responses to a positive temporary productivity shock for rule 1 with $\phi_y = 0.5$

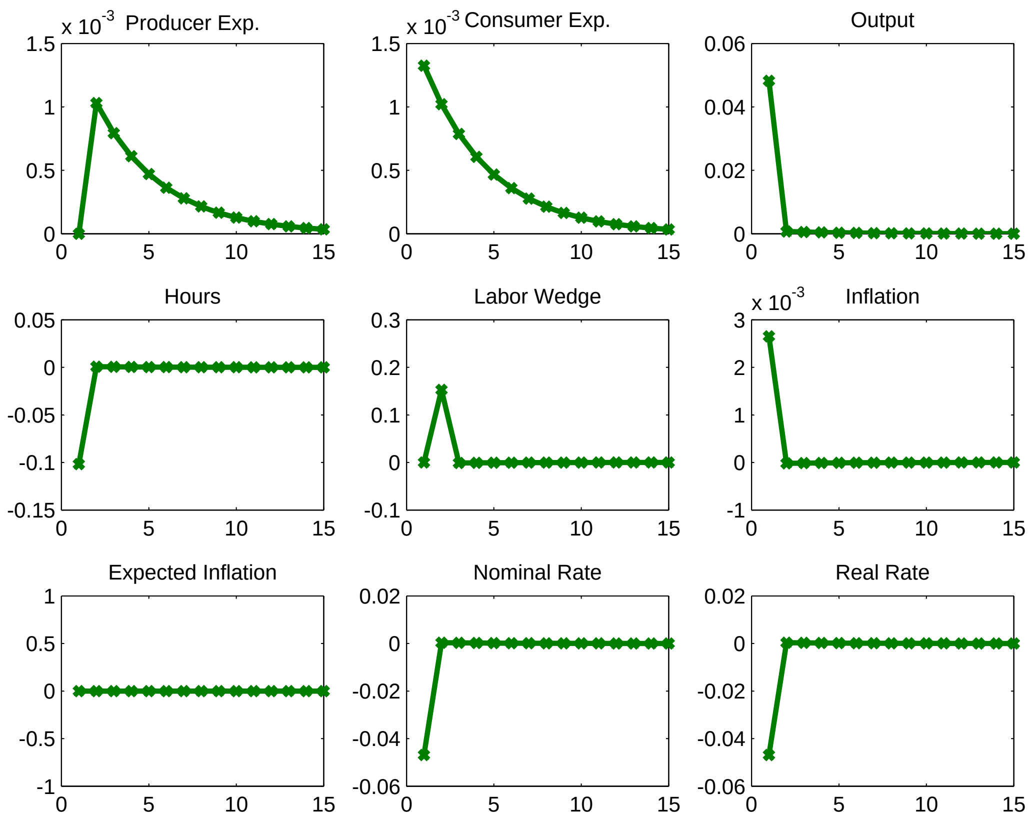


Figure 2.8: Impulse responses to a positive expectational shock for rule 1 with $\phi_y = 0$

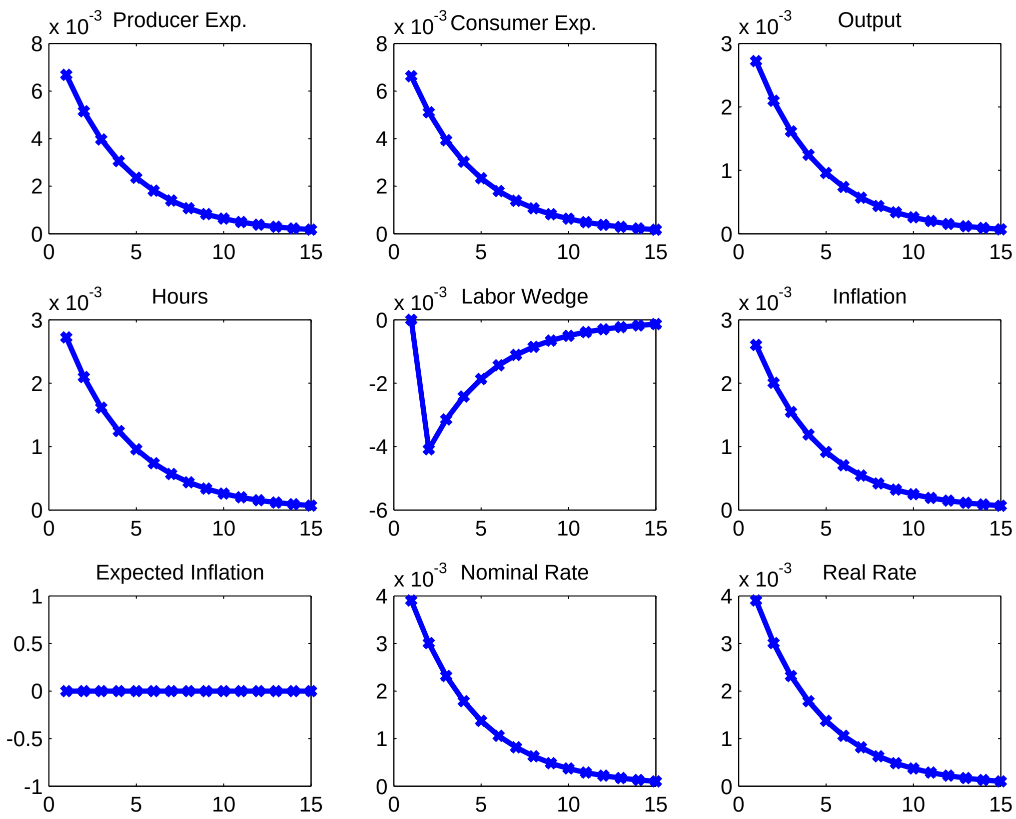


Figure 2.9: Impulse responses to a positive permanent productivity shock for rule 1 with $\phi_y = 0$

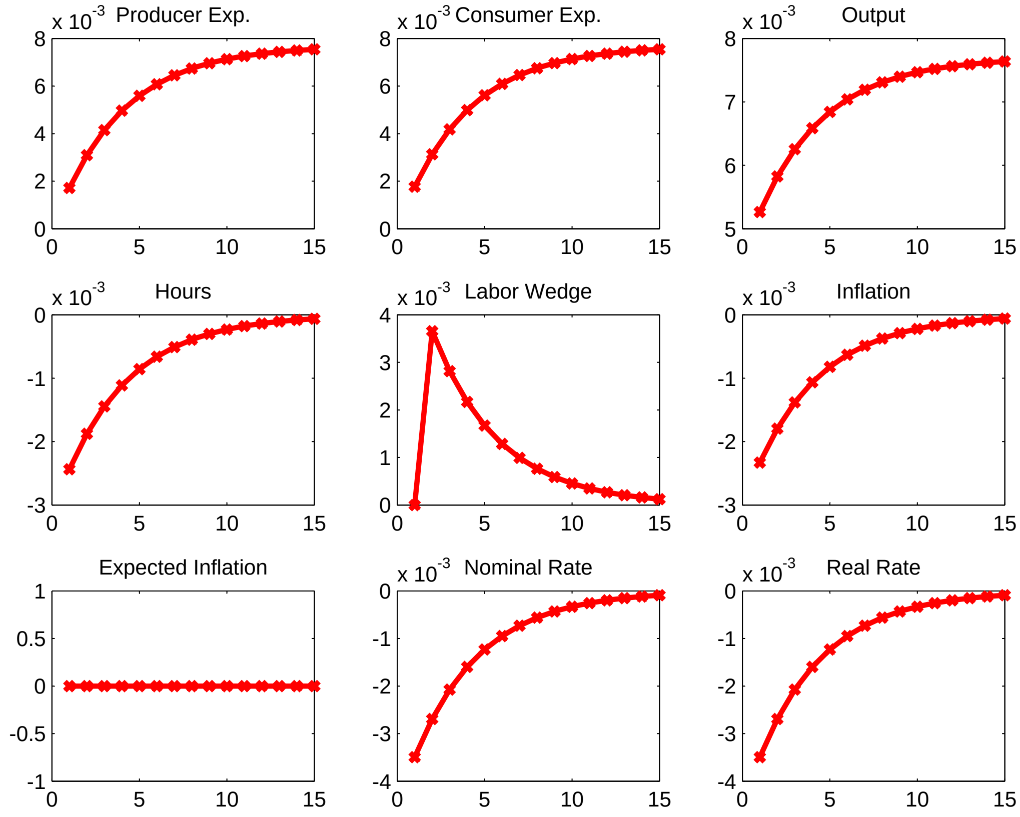


Figure 2.10: Impulse responses to a positive temporary productivity shock for rule 1 with $\phi_y = 0$

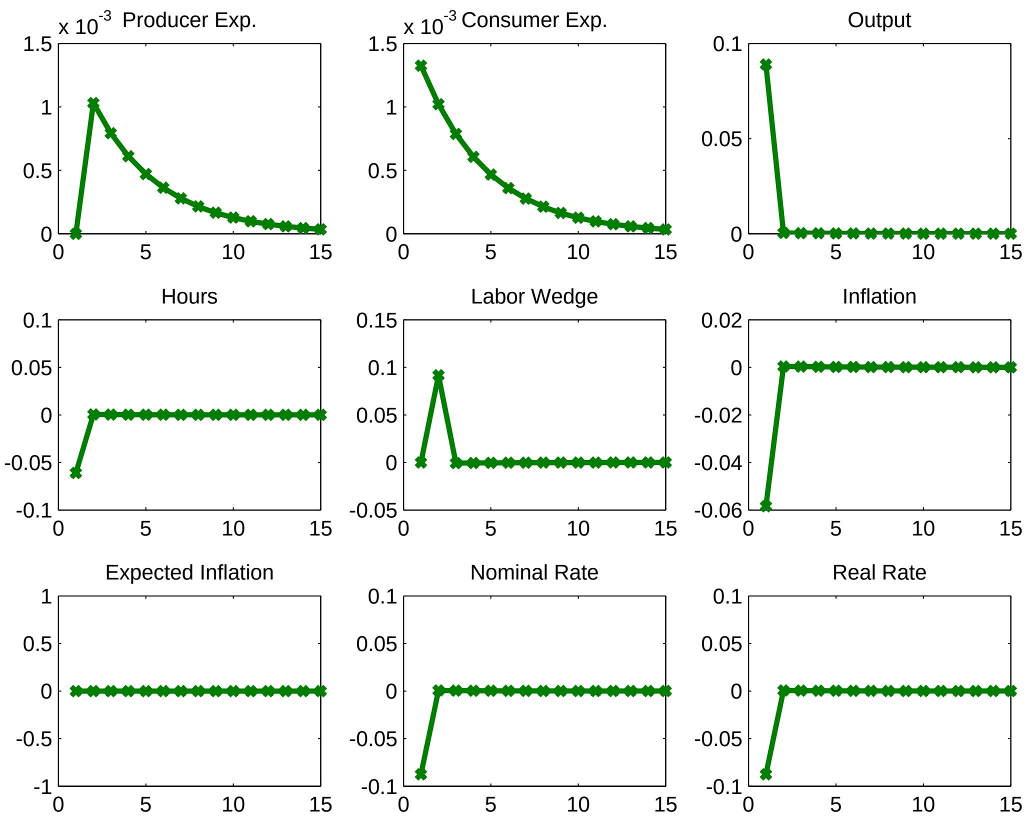


Figure 2.11: Impulse responses to a positive expectational shock for baseline rule 2 (1)

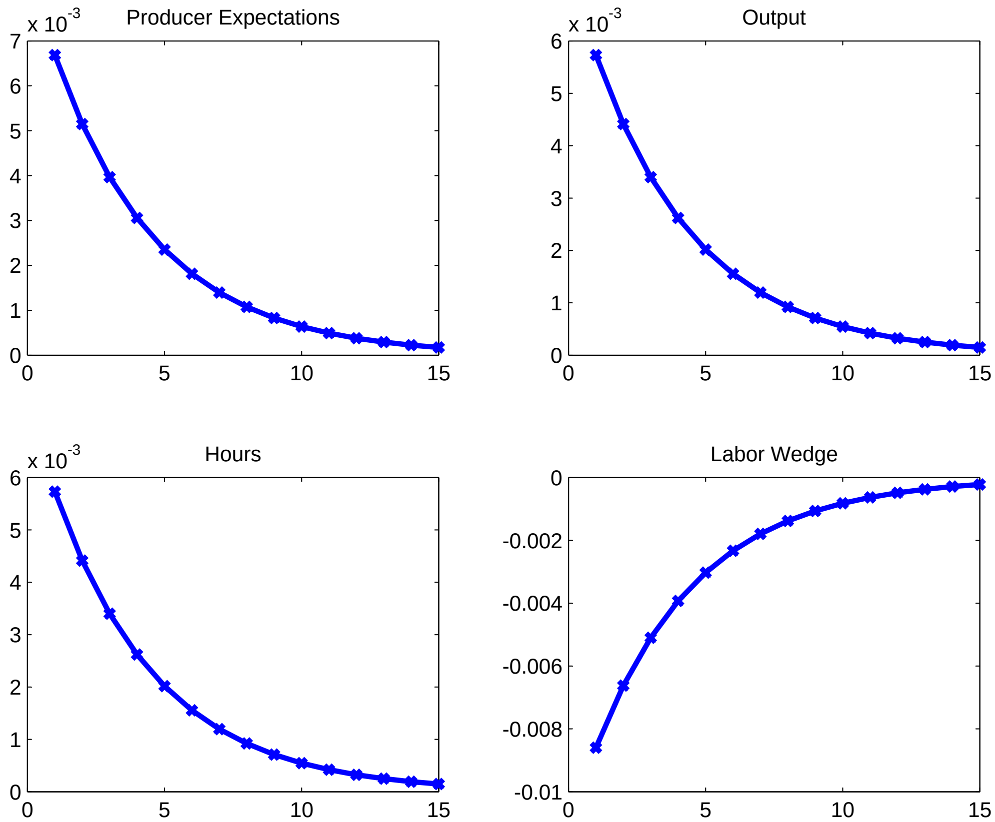


Figure 2.12: Impulse responses to a positive expectational shock for baseline rule 2 (2)

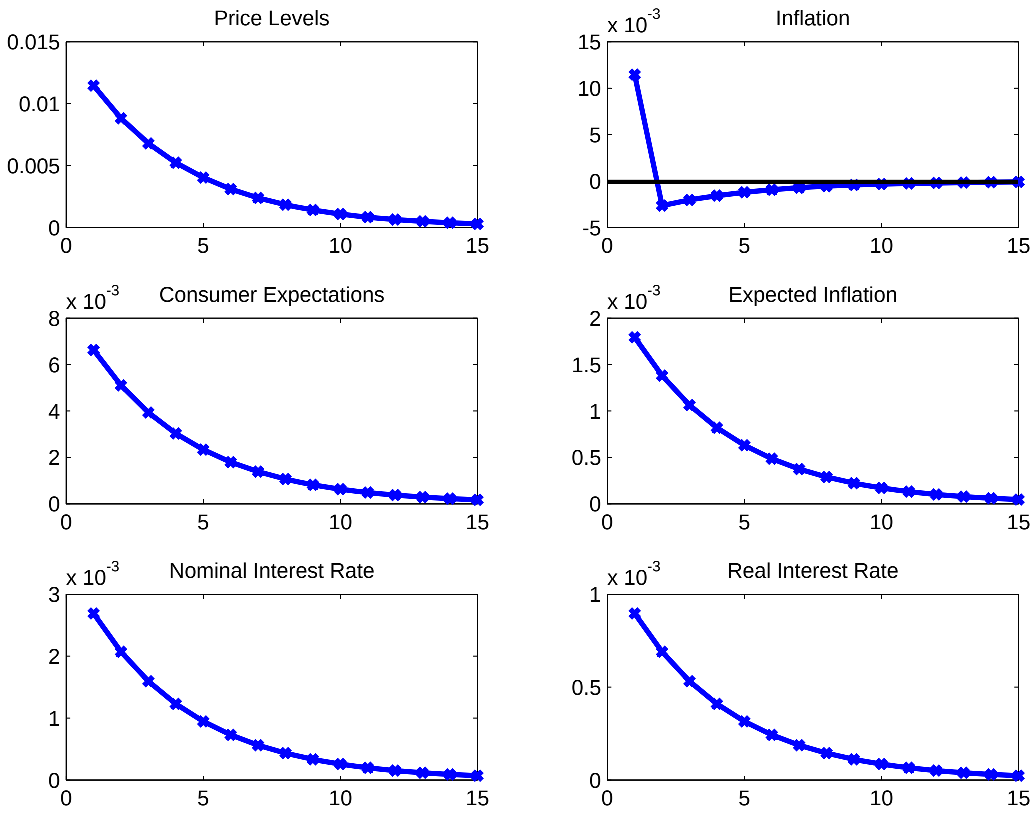


Figure 2.13: Impulse responses to a positive permanent productivity shock for baseline rule 2 (1)

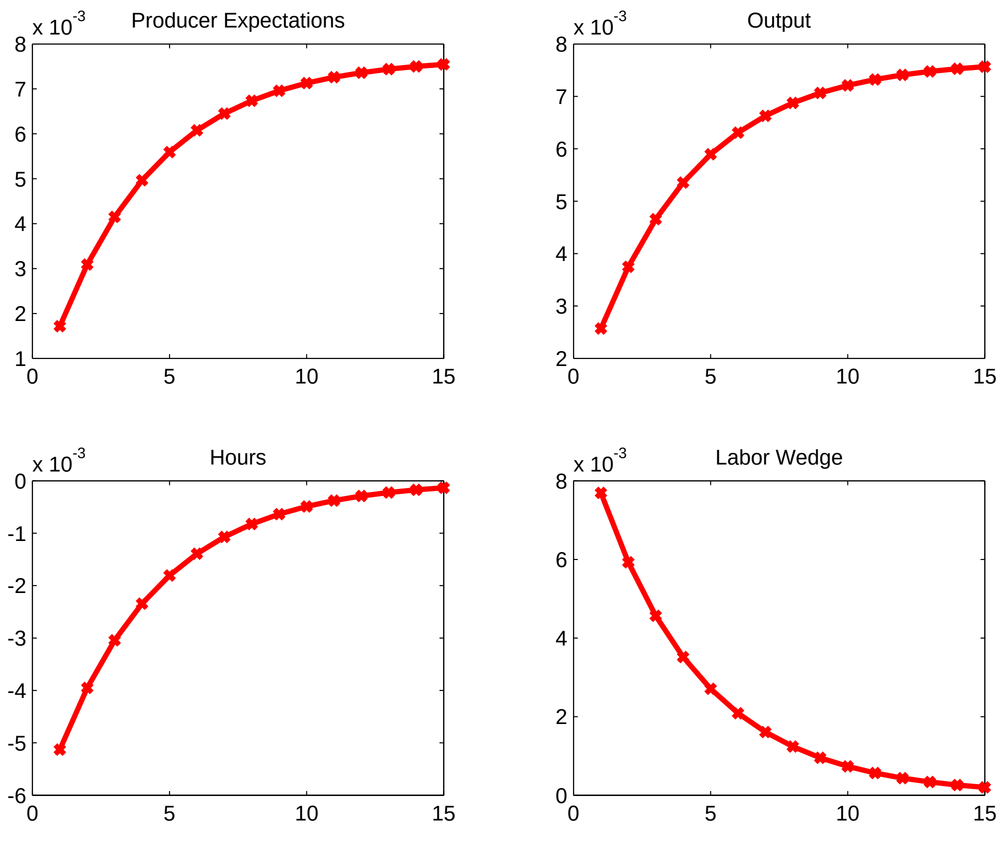


Figure 2.14: Impulse responses to a positive permanent productivity shock for baseline rule 2 (2)

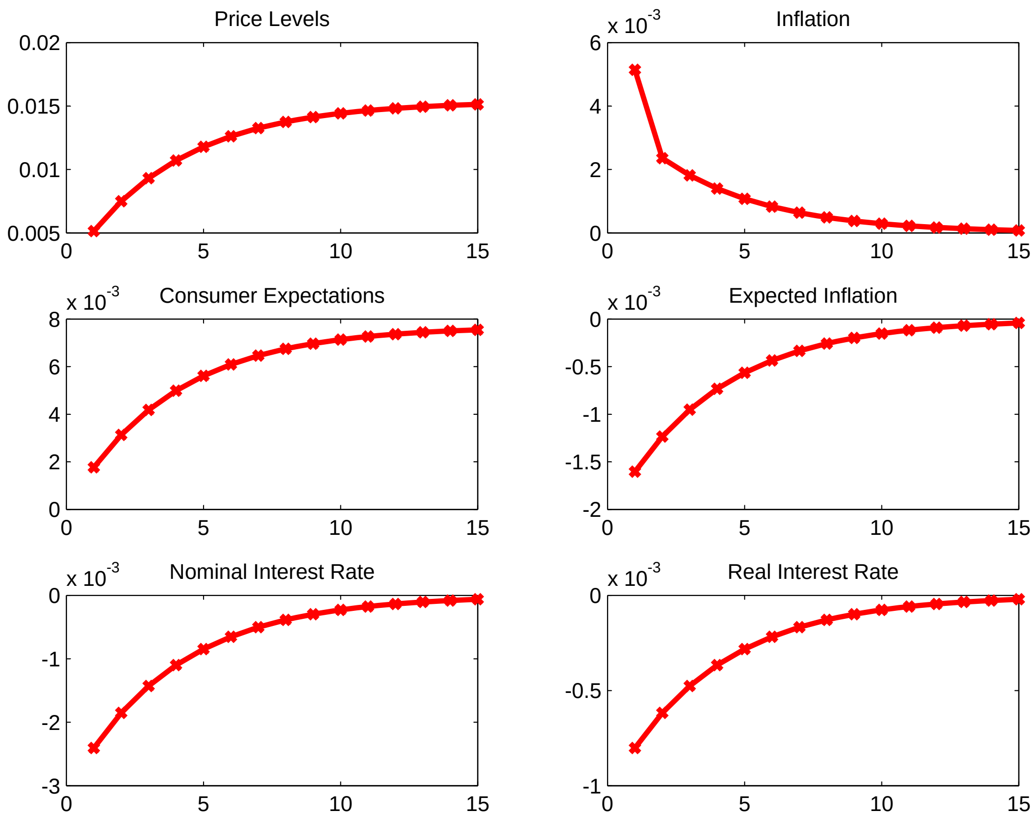


Figure 2.15: Impulse responses to a positive temporary productivity shock for baseline rule 2 (1)

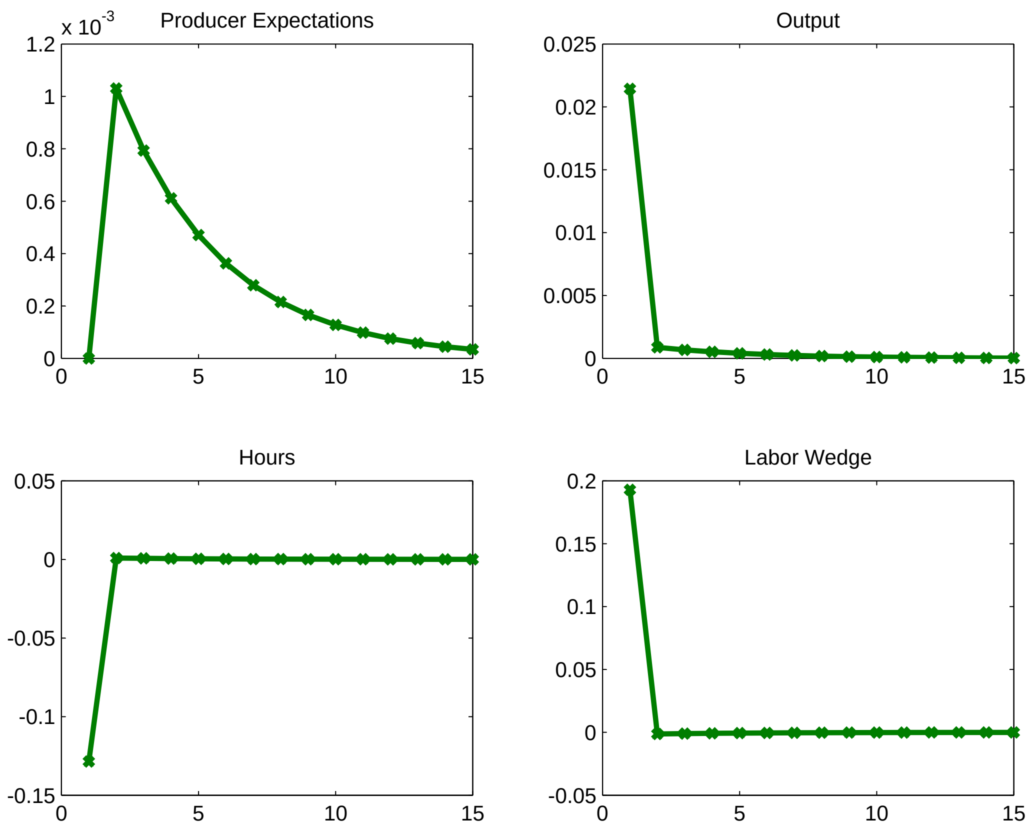
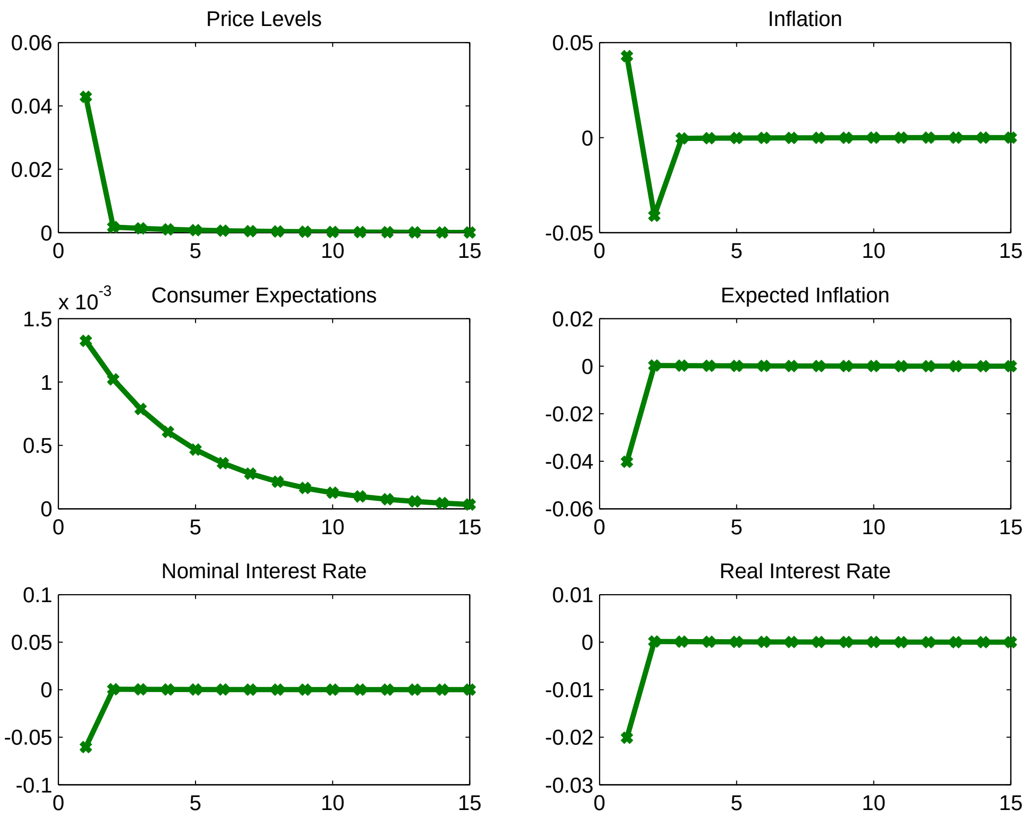


Figure 2.16: Impulse responses to a positive temporary productivity shock for baseline rule 2 (2)



Chapter 3

Implementation Cycles: Investment-Specific Technological Change and the Length of Patents

3.1 Introduction

When it comes to the release of new products, companies, especially technology ones, are particularly concerned about two things: its timing and being secretive. The latter suggests that potential gains are short-lived; the former suggests that timing affects them.

This essay attempts to address three questions, related to the above remarks. First, can companies coordinate the launch of new (improved) products even though they may develop them time-separately in the presence of capital (or any storable commodity) and the absence of borrowing constraints?

Second, can regulators affect this possibility by extending rights over improved technologies? And if so, will this necessarily lead to welfare improvements? Yes, yes, and perhaps.

A natural starting point in this attempt is Shleifer (1986) which introduced "implementation cycles." However, it did so in an economy where storable commodities are absent and argued that their absence is indeed indispensable.[1] This essay builds on it, allows for capital and savings and shows that, in sharp contrast with Shleifer's conjecture, implementation cycles are still possible.

More analytically, the economy consists of a representative agent who consumes a final good produced by a respective firm which he owns and to which he supplies his labor. The final-good firm is competitive and, besides labor, uses two different types of capital. For each type of capital, there is a respective sector comprising a number of Bertrand-competing capital makers. Capital makers use foregone consumption (investment) to produce the capital good they specialize in.

Suppose that in odd-numbered periods, a patent reaches randomly a firm (capital maker) in sector 1 and, in the even-numbered ones, a firm in sector 2 . Patents are on cost-reducing technologies which imply that a unit of capital requires less resources in order to be produced. Initially, as in Shleifer (1986) , I let firms make profits out of a patent only for one period; once a patent is utilized, the innovating firms' competitors costlessly copy the idea the patent was on and drive sector profits to zero -until a new patent arrives to the sector. As competitors cannot reverse-engineer an idea a patent is on before it is actually implemented, I will henceforth use the terms patent and

[1] See (Shleifer, 1986, page 1183) .

idea interchangeably.

Firms need to decide when they will implement their patents. I show that, if they share expectations about future and have perfect foresight, multiple "sunspot" equilibria can arise: firms can either implement their patents as soon as they receive them, which implies that patents are in place at the same -perfectly smooth- rate as that of their arrival, or they may instead coordinate their implementation, in which case "implementation cycles" with capital are generated.

To see this, note that imperfect competition invites demand externalities among capital-good sectors. Since a capital maker can postpone implementation of a patent, for instance, to the following period, when with certainty no improved technology will arrive, it needs to decide whether to implement it immediately or in the following period. As Matsuyama (1995) notes, it is precisely the intertemporal decision that firms face in combination with the presence of intratemporal demand externalities that can result in multiple equilibria which can be Pareto-ranked.

Nevertheless, and despite this intuition, implementation cycles in the presence of capital and the absence of borrowing constraints (or constrained investment volatility) is something that Shleifer (1986) conjectured against: anticipating future profits, agents would attempt to reduce their current savings in order to smooth out consumption. In turn, that would lower production and hence profits in a hypothetical implementation boom. For consumption smoothing to be mitigated, higher real interest rates would be necessary, which would in turn imply that firms discount future profits more. Both effects could rule out implementation cycles.

Why is this not so here? The reason is that patents are on investment-specific technological change, in the spirit of Greenwood et al. (1997, 2000). Investment-specific technological change introduces a one-period discrepancy between the consumption boom and the wealth boom. To see this, note that the implementation of a patent in the technology of a capital good reduces its current production cost, whereas the revenue out of it becomes realized in the following period. The latter implies that the wealth boom occurs one period *after* the coordinated implementation of patents takes place. The former implies that investment is substantially reduced in the implementation periods -in fact, it can even undershoot- and drives consumption above trend. As a result, the interest rate paid then is higher than the interest rate paid in the following, "wealth-boom," period. This increases investment in implementation booms, smoothing out consumption in the opposite direction from the conjectured one -without overturning the result on consumption which is a general equilibrium one-, and implies that more capital is installed in the following period which, given the elastic demand for it, leads to greater profits. Taking all into account, discounted profits after a conjectured coordinated implementation of patents become greater and, therefore, implementation cycles with capital possible.

In a policy extension, I let firms appropriate a patent for two periods. It turns out that implementation cycles become impossible. To see this, note that, in that case, postponing implementation to the following period is equivalent to postponing implementation to two periods afterwards in the one-period monopoly case. In a stationary equilibrium, every other period is the same. As a result, as long as firms discount future at a positive rate, which turns out to be always the case, the possibility of implementation cycles is ruled out.

A natural question is whether the immediate implementation of patents when patent rights last two periods is welfare-improving over the equilibria that can prevail when they last one period. Relative to the immediate implementation equilibrium of the latter case the answer is negative: patents diffuse faster to the implementing firms' competitors which implies that the economy reaches a certain consumption level faster. Nevertheless, relative to the cyclical equilibrium the answer can be positive. There are three effects which push in opposite directions. First is the one already described: patents (only of sector 2 though) diffuse faster in the cyclical equilibrium. Second, in the "two-period-patent" equilibrium, sector-1 patents are implemented one period ahead. Both effects result in lower investment -that is less resources to be directed towards the following period's capital- in the immediate implementation, two-period-patent equilibrium. Nevertheless, general equilibrium effects favor output in the "one-period-patent," synchronized implementation equilibrium. It turns out that for patents being on not too drastic (cost-reducing) ideas, the "investment" effects outweigh the "output" effect, rendering, thereby, a prolongation of patent rights potentially desirable.

Related literature. The closest paper to this essay is Shleifer (1986) , the main differences with which I highlighted above. In the remaining parts of the essay, I frequently refer to how the two relate to each other in greater detail.[2,3]

[2] A simplified version of Shleifer (1986) which could serve as an intermediate step between Shleifer's model and mine can be found on Lawrence Christiano's teaching webpage. The link to this is `http://faculty.wcas.northwestern.edu/~lchrist/d11/d1101/implement.pdf` .

[3] With expectations arbitrarily supporting one of the possible multiple equilibria, this essay relates to the "sunspots" literature, which, includes, among other articles, Azariadis (1981) , Cass and Shell (1983) and Grandmont (1985) . Benhabib and Farmer (1999) offers an overview of this literature. A complementary earlier survey with an emphasis on endogenous cycles can be found in Boldrin and Woodford (1990) .

Francois and Lloyd-Ellis (2008) also generates implementation cycles with capital which, further, can be sustained as a unique equilibrium outcome. Although, their model is quite different from the one in this essay, I will restrict attention to two key differences. A central assumption the authors make is that patents arrive after firms have incurred an endogenous search cost. A consequence of this assumption is that patents arrive simultaneously in all sectors, which is in sharp contrast with the perfectly smooth rate of their arrival in Shleifer (1986) and here: if patents arrive in cycles, they are more easily implemented in cycles as well. A second key difference is that, in Francois and Lloyd-Ellis (2008) , patents do not affect the technology of capital but they instead affect the technology of intermediate goods which requires only labor. Consequently, the wealth and the consumption booms coincide. Here, patents affect the investment technology which naturally introduces a one-period discrepancy between the consumption and the wealth boom.[4]

Turning to the literature related to the essay's result on patent policy, this essay differs from a recent and growing literature on patent protection and intellectual property rights which includes, among other articles, Boldrin and Levine (2002, 2008b) and Henry and Ponce (2011) .[5,6] This literature focuses on the incentives to innovate and analyzes whether markets for patents can substitute for the absence of patent rights. Here, innovation is exogenous, whereas allowing for a market for patents would leave the results intact.

[4]Certainly this essay shares features, which I will not attempt to review here, with the growth literature, prominent contributions to which include Romer (1990) , Grossman and Helpman (1991) and Aghion and Howitt (1992) , the literature on strategic delay (for instance, Chamley and Gale (1994)) , as well as Matsuyama (1999) and, more recently, Jovanovic (2009) . Needless to say, this list is by no means exhaustive.

[5]See also the discussion in Boldrin and Levine (2008a) .

[6]This literature dates back to Schumpeter (1942) and Arrow (1962) . Chapter 12 in Acemoglu (2009) offers an excellent overview of the early literature. Holmes and Schmitz (2010) offers a more general discussion on how competition affects productivity.

Hopenhayn and Squintani (2010) also considers the effects of patent rights on the timing of patent releases. However it does so in a model of sequential innovation which allows for preemptive entry by an innovating firm's competitors. Consequently, the timing of patents balances the possibility of preemption and the generation of future patents. I abstract from such considerations here: firms can utilize a patent immediately or with delay without the fear of preemption and without the fear that the implementation of a patent affects their future generation of patents, which happens independently over time, randomly and costlessly.

These abstractions allow the essay to concentrate solely on the effects of patent rights on the implementation -rather than the generation- of patents and, thereby, offer a clean argument from a different -and, to the best of my knowledge, new- perspective to the ongoing -and lively- discussion about the length of patent rights.

The rest of the essay is organized as follows. Section 3.2 presents the model. Section 3.3 analyzes the equilibria when patent rights last one period. Section 3.4 discusses welfare. Section 3.5 extends patent rights to two periods and performs welfare comparisons with the equilibria which can prevail when patent rights last one period. Section 3.6 concludes.

3.2 The Model

The economy is populated by an infinitely-lived (representative) agent. The agent consumes a single storable commodity (final good) produced by a (representative) final-good firm to which he supplies labor. Further, the final-good firm uses two storable (composite) capital goods in an additively-separable way. For each capital good, there is a respective sector which comprises at least two capital-good firms Bertrand-competing for its production. It takes one period to produce a capital good and foregone consumption (investment) is used as input in its production. Capital-good firms become the recipients of patents on cost-saving ideas (henceforth, simply "patents") and, once they implement these, they can make temporary monopoly profits before being imitated by their competitors. I elaborate more on this last -and central- feature of the economy below.

There is no uncertainty; agents and firms share expectations about the future and have perfect foresight. Agents (firms) can perfectly borrow against their future profits (revenue). Time is discrete and infinite and commences in period 1.

3.2.1 Representative agent

The preferences of the agent are given by

$$\sum_{t=1}^{\infty} \beta^{t-1}\, U(x_t\,,\, l_t) \tag{3.1}$$

with

$$U(x_t\,,\, l_t) \;=\; \log x_t \,+\, \chi\, \log l_t\,, \tag{3.2}$$

where x_t denotes consumption of the final good, l_t denotes leisure, $\beta \in (0,1)$ parametrizes the agent's time preference, and $\chi > 0$ parametrizes the relative weight on leisure within the period utility of the agent.[7]

[7] The intertemporal elasticity of substitution is 1. However, the implementation cycles that I specify below are more easily generated the greater the intertemporal elasticity of substitution is.

The agent is endowed with one unit of time, owns all firms in the economy and can freely borrow against his perfectly foreseen future profits. This last assumption is essentially an assumption of perfect capital markets and allows me to use the agent's intertemporal budget constraint given by

$$\sum_{t=1}^{\infty} m_t \, x_t \;\leq\; \sum_{t=1}^{\infty} m_t \left[w_t \,(1 \,-\, l_t) \,+\, \Pi_t^f \,+\, \sum_{i=1}^{2} \Pi_{t,i} \,\right], \tag{3.3}$$

where $m_t \;=\; \frac{1}{R_1 \,...\, R_{t-1}}$ for $t \;>\; 1$, with $m_1 \;=\; 1$; R_t denotes the gross real interest rate paid in period $t+1$, w_t denotes the real wage paid by the final-good firm, and Π_t^f and $\Pi_{t,i}$ denote the profits that accrue to the agent by the final-good firm and capital-good firms in sector i for $i \;=\; 1, 2$, respectively. All prices and the real interest rate are expressed in units of the final good.

The agent chooses $\{(x_t\,,\, l_t)_{t=1}^{\infty}\}$, where $x_t \;>\; 0$ and $l_t \;\in\; (0, 1]$ to maximize his lifetime utility given by (3.1) - (3.2) subject to his intertemporal budget constraint given by (3.3). The first order conditions with respect to x_t and l_t imply the following relations:

$$\frac{x_{t+1}}{x_t} \;=\; \beta \, R_t \tag{3.4}$$

$$\frac{x_{t+1}}{x_{t-1}} \;=\; \beta^2 \, R_t \, R_{t-1} \tag{3.5}$$

$$\frac{l_{t+1}}{l_t} = \beta\, R_t\, \frac{w_t}{w_{t+1}} \tag{3.6}$$

$$\frac{l_{t+1}}{l_{t-1}} = \beta^2\, R_t\, R_{t-1}\, \frac{w_{t-1}}{w_{t+1}}\,. \tag{3.7}$$

3.2.2 Final-good firm

The final-good firm is competitive in both the final-good and the input markets. Its (neoclassical) technology is given by

$$F\,(n_t\,,\, k_{t,1}\,,\, k_{t,2}) \;=\; n_t^{\alpha}\,(k_{t,1}^{1-\alpha} \;+\; k_{t,2}^{1-\alpha})\,, \tag{3.8}$$

where n_t denotes employed labor, $k_{t,1}$ and $k_{t,2}$ denote capital of types 1 and 2, respectively, rented[8] in period t and $\alpha \in (0,1)$ measures the labor share and the inverse elasticity of substitution between the two types of capital.[9] Note that the marginal products of the two capital types are conditionally -on labor- independent of each other as in Romer (1990). I assume that capital depreciates fully within a period.[10]

In each period t, the final-good firm chooses $\{n_t\,,\, k_{t,1}\,,\, k_{t,2}\,,\, y_t\}$ (all in non-negative quantities) to maximize its temporal profits given by

$$\Pi_t^f \;=\; y_t \;-\; w_t\, n_t \;-\; q_{t,1}\, k_{t,1} \;-\; q_{t,2}\, k_{t,2}\,,$$

[8]I assume that final-good firms rent rather than buy capital in order to avoid possible problems with the final-good firm buying capital in advance in order to use it in the future.

[9]The technology of the final-good firm can be alternatively expressed as $F\,(n_t, K_t) = n_t^{\alpha}\, K_t^{1-\alpha}$, where $K_t \equiv (k_{t,1}^{\frac{\epsilon_\alpha - 1}{\epsilon_\alpha}} + k_{t,2}^{\frac{\epsilon_\alpha - 1}{\epsilon_\alpha}})^{\frac{\epsilon_\alpha}{\epsilon_\alpha - 1}}$ with $\epsilon_\alpha = \frac{1}{\alpha}$.

[10] Since capital depreciates fully within a period and with demand for it being positive in equilibrium, it may be pointed out that this is essentially an irreversibility constraint since it effectively rules out disinvestment. That may be true at face value, nevertheless, at the same time, it implies that investment has the highest possible volatility as each period the economy needs new capital to be produced. See also fn. 33.

where $y_t \leq F(n_t, k_{t,1}, k_{t,2})$ with $F(\cdot)$ given by (3.8); $q_{t,i}$ denotes the real rental price of capital type i for $i = 1, 2$. In period 1, the firm is endowed with quantities of capital $k_{1,1}$ and $k_{1,2}$, on which I elaborate below and in Section 3.4.1.

The firm's maximization problem yields the following inverse demand functions:

$$w_t = \alpha n_t^{\alpha-1} (k_{t,1}^{1-\alpha} + k_{t,2}^{1-\alpha}) \tag{3.9}$$

$$q_{t,i} = (1-\alpha) n_t^{\alpha} k_{t,i}^{-\alpha} \;, \text{ for i} = 1, 2. \tag{3.10}$$

Observe that both demand functions are elastic ($\epsilon_{w,n} = -\frac{1}{1-\alpha}$ and $\epsilon_{q,k_i} = -\frac{1}{\alpha}$, respectively) and that profits are zero in each period, an implication of constant returns to scale.

3.2.3 Capital-good firms

I consider two types of capital goods.[11] For each capital good, there is a number of capital makers which Bertrand-compete for its production with no capacity constraints. They are indexed by j and j' for firms 1 and 2, respectively, where $j = 1, 2, \ldots, J$ and $j' = 1, 2, \ldots, J'$ with $J, J' \geq 2$.

[11] This is an abstraction. In fact, I need two *baskets* of capital goods with each basket including a sufficiently high number of capital goods so that each capital-good firm is small enough within its "basket." The desired implication of this assumption is that capital-good firms take aggregate outcomes as given; put differently, any strategic interactions among firms within and across baskets are ruled out. For simplicity, let each basket contain a continuum of unit measure of capital goods. That is $k_{t,i} = \int_{i' \in [0,1]} k_{t,i,i'} \, di'$, where $k_{t,i,i'}$ denotes capital good i' in basket i in t. Assuming a symmetric treatment of capital goods within a composite capital good allows me henceforth, for expositional clarity, to mention capital-good type i and actually refer to the representative capital good (i, i') in basket i.

Capital-good firms operate a constant-returns-to-scale technology. For instance, the technology of capital maker j manufacturing capital of type 1 is given by

$$k_{t,1,j} \;=\; \psi_{t-1,1,j}\, i_{t-1,1,j} \;, \tag{3.11}$$

where $k_{t,1,j}$ denotes capital of type 1 produced by capital maker j in period t and $i_{t-1,1,j}$ denotes capital maker j's investment, which is foregone consumption, in period $t-1$ used as input in the production of capital type 1. Likewise for capital good of type 2 (see also fn. 10).

Aggregate investment in t is

$$i_t \;=\; \sum_{j=1}^{J} i_{t,1,j} \;+\; \sum_{j'=1}^{J'} i_{t,2,j'} \,. \tag{3.12}$$

Supposing that, before the economy starts, firms j and j' in sectors 1 and 2, respectively, have no inferior technology relative to that of their competitors, the initial level of investment required is given by

$$i_0 \;=\; \frac{k_{1,1,j}}{\psi_{0,1,-j}} \;+\; \frac{k_{1,2,j'}}{\psi_{0,2,-j'}} \,, \tag{3.13}$$

where $\psi_{0,1,-j}$ and $\psi_{0,2,-j'}$ denote the technology level of their competitors before the economy starts.

Since I focus on the balanced growth path (steady state) of the economy, I treat time as if had commenced in $-\infty$. This implies that capital in period 1 summarizes the state of an economy which started in $-\infty$. Equivalently, an economy which starts in period 1 must do so with the "right" levels of capital. I resume this discussion in Section $3.4.1$.

Pattern of patents

I make two assumptions on the arrival of patents which are as in Shleifer (1986).

Assumption 1. Patents on improved technologies arrive exogenously. They reach sectors sequentially, at a perfectly smooth rate. With no loss of generality, a patent reaches randomly a firm in capital sector 1 in odd periods and a firm in capital sector 2 in the even ones. Patents are on ideas that affect the technology of capital of type i in the following way:

$$\frac{\psi_{t+1,i,j}}{\psi_{t-1,i,j''}} = \mu\,,$$

where $\mu > 1$ and $\psi_{\cdot,i,j}$ denotes the state-of-the-art technology (inverse marginal cost of capital) in sector i possessed by firm j.[12] As time commences in period 1, an odd period, imposing $\psi_{0,1,j} = 1$ and $\psi_{0,2,j'} = \mu^{\frac{1}{2}}$ removes the first-mover advantage of sector 1, thereby ensuring the symmetric treatment of capital-good sectors: the lead in the technology race alternates between sectors with their relative technology "distance" remaining fixed at $\mu^{\frac{1}{2}}$. Put differently, it is as if time commenced in $-\infty$.[13]

[12]Observe that the economy encompasses only investment-specific technological change as in Greenwood et al. (1997). For expositional reasons, I completely abstract from total factor productivity (TFP).

[13]Although I have not explored this case, presumably the assumption that patents arrive periodically could be partially relaxed; what really matters is that the probability with which a patent reaches a particular sector before the indicated time is sufficiently low. The fact that patents reach the economy at a perfectly smooth rate renders the generation of cycles harder and crystallizes the forces which underpin them. If patents arrived in a cyclical fashion, then their cyclical implementation would perhaps come as no surprise. In fact, this is what underpins the result in Francois and Lloyd-Ellis (2008), in which, though, the timing of the patent arrival is endogenous.

Assumption 2. A firm can appropriate an idea a patent is on only for *one* period: in the period following its implementation, this idea becomes publicly disclosed and imitators enter driving prices down to marginal cost and profits to zero.[14]

Profits

Profits of capital maker j [15] which produces capital good i are given by

$$\Pi_{t,i,j} = q_{t,i,j}\, k_{t,i,j} - R_{t-1}\, i_{t-1,i,j}\,.$$

Capital maker j of capital good i chooses $\{k_{t,i,j}\,,\, i_{t-1,i,j}\}$ for each t to maximize its profits subject to the technology given by (3.11). Since revenue is realized one period after investment is made, I allow capital-good firms to be able to perfectly borrow against their future revenue.

Below I distinguish between two cases: in the first case, all firms within sector i operate the same technology in which case a firm, say j, is randomly selected to produce capital of type i. In the second case, firm j has a technological advantage over its competitors which allows it to enjoy monopoly profits.

[14]The fact that firms make temporary monopoly profits is an assumption in line with Shleifer (1986) as well as the Schumpeterian growth literature originating in Aghion and Howitt (1992), which also focuses on quality improvements ("process innovations"); it is in contrast with Romer (1990) in which firms' rights over the *use* of an idea last forever. Further, unlike the endogenous growth literature, this essay entirely abstracts from issues concerning the generation of patents. This will prove useful in Section 3.5 where I extend the horizon of patent rights to two periods, as it will enable me to focus on the "implementation" effects of patent rights and to entirely abstract from their effect on the generation of patents, an issue which the literature traditionally studies.

[15]With a slight abuse of notation, henceforth I will make no distinction between j and j'.

Perfect competition. When $\psi_{t,i,j} = \psi_{t,i,-j}$, where $\psi_{\cdot,\cdot,-j}$ denotes the inverse marginal cost of the competitors of capital maker j, there is perfect competition which, given the constant returns to scale, implies zero profits.

Firm j in sector i supplies the capital good i at price

$$q^*_{t,i,j} = \frac{R_{t-1}}{\psi_{t-1,i,-j}}\,. \tag{3.14}$$

Inverse demand for capital from the final-good firm (3.10) pins down the competitive quantity given by

$$k^*_{t,i,j} = \left(\frac{(1-\alpha)\,\psi_{t-1,i,-j}}{R_{t-1}}\right)^{\frac{1}{\alpha}} n_t\,. \tag{3.15}$$

Monopoly in the presence of a competitive fringe. Capital maker j implementing a patent in the production of capital type i, for instance, in period $t-1$ enjoys monopoly profits in the following period, t, as it takes one period to build capital. Capital maker j chooses $k_{t,i,j}$ to maximize its profits given by

$$\Pi_{t,i,j} = q_{t,i,j}\,k_{t,i,j} - R_{t-1}\frac{k_{t,i,j}}{\mu\,\psi_{t-1,i,-j}}\,, \tag{3.16}$$

subject to the (inverse) demand for capital given by (3.10). Since demand for capital is elastic, the solution is well defined:

$$q^m_{t,i,j} = \frac{R_{t-1}}{(1-\alpha)\,\mu\,\psi_{t-1,i,-j}} \quad \text{and} \quad k^m_{t,i,j} = \left(\frac{(1-\alpha)^2\,\mu\,\psi_{t-1,i,-j}}{R_{t-1}}\right)^{\frac{1}{\alpha}} n_t\,.$$

Henceforth, I restrict attention to the case in which $q^m_{t,i,j} \geq q^*_{t,i,j}$. In this case, the "limit" price $q^*_{t,i,j}$ is set: a capital maker which implements a patent cannot charge more than the price its competitors would set, $q^*_{t,i,j}$, else its competitors would undercut it and capture the whole market. Limit pricing takes place (i.e. $q^m_{t,i,j} \geq q^*_{t,i,j}$) when

$$\mu \, (1 - \alpha) \leq 1 \, . \tag{3.17}$$

The lower the innovation rate, μ , and the less elastic the demand for capital is, the more easily (3.17) is satisfied. For a certain level of the elasticity of the demand for capital, condition (3.17) imposes an upper bound on the innovation rate. Likewise, for a certain innovation rate, (3.17) imposes an upper bound on the elasticity of capital demand.[16]

Then, a monopolist sells a quantity given by (3.15) , which corresponds to the technology level of its competitors, at the price its competitors would set, given by (3.14) , and makes profits because of its lower -by μ relative to its competitors- marginal cost of producing a unit of capital. Combining (3.16) with (3.14) and (3.15) implies that the monopolist's profits are given by

$$\Pi_{t,i,j} = (1 - \alpha)^{\frac{1}{\alpha}} \, \psi_{t-1,i,-j}^{\frac{1}{\alpha}-1} \left(\frac{\mu - 1}{\mu} \right) \frac{n_t}{R_{t-1}^{\frac{1}{\alpha}-1}} \, . \tag{3.18}$$

Profits depend negatively on the interest rate paid in the period they are made. This is because demand for capital is elastic and a higher real interest rate implies that capital becomes more costly. That the demand for capital is elastic also explains the increase of profits in the technology level of a monopolist's competitors. In addition, profits depend proportionally on contemporaneous

[16]For $\alpha = 2/3$, this implies that $\mu \leq 3$. In R&D theory, the complementary case in which $q^m_{t,i,j} < q^*_{t,i,j}$ refers to "drastic innovations." See also Chapter 12 in Acemoglu (2009) .

employment by virtue of the technology of the final-good firm.

The implementation decision

As I have already argued, the implementation of a patent results in profits in the following period. In the presence of two capital-good sectors, suppose that the recipient of a patent needs to decide whether to implement it immediately or in the following period.

A capital maker j in sector i receiving a patent in, say, period $t-1$ will implement it immediately rather than in the following period as long as its present discounted period t profits exceed its present discounted $t+1$ profits. That is (superscripts denote the date a patent arrives and time in subscripts refers to the date profits are made), it must be that

$$\frac{\Pi_{t,i,j}^{t-1}}{R_{t-1}} \geq \frac{\Pi_{t+1,i,j}^{t-1}}{R_{t-1}\,R_t}\,.$$

By (3.18), this boils down to

$$\frac{n_t}{n_{t+1}}\left(\frac{R_t}{R_{t-1}}\right)^{\frac{1}{\alpha}-1} \geq \frac{1}{R_t}\,. \tag{3.19}$$

In the complementary case in which

$$\frac{n_t}{n_{t+1}}\left(\frac{R_t}{R_{t-1}}\right)^{\frac{1}{\alpha}-1} < \frac{1}{R_t}\,, \tag{3.20}$$

capital maker j prefers to postpone implementation to the following period.

Implicit in these is that a capital maker cannot affect the real interest rate by its actions alone (see also fn. 11).

Last, note that, even though profits depend on the competitors' technology level at the period of implementation, the implementation decision is independent of it.

3.3 Balanced Growth Path Equilibria

I restrict attention to perfect-foresight, balanced growth path equilibria which involve cycles of constant periodicity.

On the balanced growth path (BGP) , consumption, production, and investment grow all at the same long-run rate. However, since, by construction, a patent reaches a sector every other period, the growth rate of the economy's variables may differ between odd and even periods within a cycle. Consumption needs then to satisfy the following stationarity conditions on the balanced growth path :

$$\frac{x_{\tau+1}}{x_{\tau}} = v \tag{3.21}$$

$$\frac{x_{\tau+1}}{x_{\tau-1}} = \lambda \, , \tag{3.22}$$

where τ denotes an even period, with no loss of generality (see also fn. 19) . Combining (3.21) and (3.22) with (3.4) and (3.5) implies

$$R_t = \begin{cases} \frac{v}{\beta} & \text{if } t = \tau \\ \frac{\lambda}{v\,\beta} & \text{if } t = \tau + 1 \end{cases} . \tag{3.23}$$

Combining (3.22) with (3.5) leads to the following remark:

Remark 1. $R_\tau \, R_{\tau+1} \; = \; \frac{\lambda}{\beta^2}$.

Next, I provide the equilibrium definition:

Definition 2 (BGP Equilibrium)**.** *A perfect-foresight (periodic) balanced growth path equilibrium is a set of interest rates* $\{R_t\}_{t=1}^{\infty}$ *and prices* $\{w_t\, , \, \{q_{t,i}\}_{i=1}^{2}\}_{t=1}^{\infty}$ *, an allocation* $\{l_t, \, x_t\}_{t=1}^{\infty}$ *for the representative agent, an allocation* $\{n_t\, , \, \{k_{t,i}^d\}_{i=1}^{2}\, , \, y_t\}_{t=1}^{\infty}$ *for the final-good firm, and an allocation* $(\{k_{t,i}^s\, , \, i_{t,i}\}_{t=1}^{\infty})_{i=1,2}$ *for the technology-leading firms in the capital-good sectors,*[17] *such that*

1. *(Optimality) The allocations of the agent, the final-good firm and the leaders in the capital-good sectors solve their problems, laid out in Section 3.2 , at the stated prices.*

2. *(Market clearing)* $k_{t,i}^d \; = \; k_{t,i}^s \; \equiv \; k_{t,i}$ *for all* i *,* $n_t \, + \, l_t \; = \; 1$ *and* $y_t \; = \; x_t \, + \, i_t$ *where* $i_t \; = \; i_{t,1} \, + \, i_{t,2}$ *, for all* t *.*

3. *(Consistency) For expectations arbitrarily centered around an equilibrium ("sunspots") , capital-good firms must find it optimal to implement their patents as conjectured.*

4. *(No storage) No storage takes place.*

I start with the no-storage condition (requirement (4) in Definition 2) :[18]

Condition 1 (No storage)**.** $R_t \; > \; 1$ *for all* t *rules out storage in equilibrium.*

Proof. See the Appendix. □

[17] $k_{t,i}$ refers to capital produced by *sector* i . Furthermore, as I have already mentioned, if all firms within a sector have the same technology level, a capital-good firm is randomly chosen.

[18] The storage technology I assume is one-to-one.

As my focus is on stationary equilibria, I will restrict attention to just period τ, an even period, and the periods before and after it. Hence, following the analysis in Section 3.2.3 , in period $\tau - 1$, an odd period, a patent reaches a firm in sector 1 and the state-of-the-art technology, irrespectively of whether the patent is implemented or not, becomes $\mu\,\psi$, greater by by μ compared to its assumed previous level ψ , whereas in sector 2 it remains $\mu^{1/2}\,\psi$; in period τ a patent reaches a firm in sector 2 in which the state-of-the-art technology becomes $\mu^{3/2}\,\psi$, whereas in sector 1 it remains $\mu\,\psi$. As I have already pointed out, the ratio of leading technologies in the two sectors equals $\mu^{1/2}$ in odd periods and $\frac{1}{\mu^{1/2}}$ in even ones; that is, the lead of the patent race alternates between sectors ad infinitum.

I analyze two perfect-foresight equilibria: an acyclical, immediate implementation equilibrium and a cyclical, synchronized implementation one. In the former, capital makers implement a patent as soon as they receive it. In the latter, the capital maker receiving a patent first (henceforth, "firm 1") waits and implements it together with the capital maker receiving a patent second (henceforth, "firm 2") ; that is patents are implemented in even periods.[19]

For each equilibrium, I first ensure that requirements (1) - (2) in Definition 2 are satisfied. Next, I specify the conditions under which the conjectured timing of the patents' implementation is optimal for the capital-good firms (requirement (3) in Definition 2) . Last, I confirm that the no-storage condition is met and pin down the transversality condition.

[19] Although I have not explored this possibility explicitly, I find no reason for why there cannot be a symmetric equilibrium in which patents are implemented in odd periods, which, in fact, could well be the case under the premise that period 1 summarizes the state of an economy starting in $-\infty$. This would require simply letting τ denote an odd period. Since time however starts in an odd period, welfare in the two cyclical equilibria will be different. See my conjecture on that in fn. 42 .

3.3.1 Immediate implementation equilibrium

In the immediate (acyclical) implementation equilibrium, firms expect each other to implement their patents immediately.

Period $\tau - 1$. Firm 1 receives a patent which it immediately implements. Since I assume throughout that condition (3.17) holds, firm 1 sets the same price as its competitors would, given by (3.14), and produce the quantity which the technology level of their competitors justifies, given by (3.15). In line with the analysis above, the technology levels in the two sectors are $\psi_{\tau-1,1,-j} = \psi_{\tau-2,1} = \psi < \psi_{\tau-1,1,j} = \mu\,\psi$, since $\mu > 1$, and $\psi_{\tau-1,2} = \mu^{\frac{1}{2}}\,\psi$, respectively.[20] Then,

$$k_{\tau,1} = \left(\frac{(1-\alpha)\,\psi}{R_{\tau-1}}\right)^{\frac{1}{\alpha}} (1 - l_\tau) \tag{3.24}$$

$$k_{\tau,2} = \left(\frac{(1-\alpha)\,\mu^{\frac{1}{2}}\,\psi}{R_{\tau-1}}\right)^{\frac{1}{\alpha}} (1 - l_\tau)\,. \tag{3.25}$$

Investment by (3.11) and (3.12) is

$$i_{\tau-1} = \frac{k_{\tau,1}}{\mu\,\psi} + \frac{k_{\tau,2}}{\mu^{\frac{1}{2}}\,\psi}\,. \tag{3.26}$$

Combined with (3.24) and (3.25), (3.26) becomes

$$i_{\tau-1} = \left[\mu^{-1} + \mu^{\frac{1}{2}\left(\frac{1}{\alpha}-1\right)}\right] \psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\,\frac{(1-l_\tau)}{R_{\tau-1}^{\frac{1}{\alpha}}}\,. \tag{3.27}$$

[20]Whenever I omit j or $-j$ from the technology subscript, I refer to all firms within a sector.

Observe that investment depends negatively on the interest rate paid in the following period, whereas it is proportionally related to the following period's employment, an implication of the constant-returns-to-scale technology of the final good.

Similarly, since $\psi_{\tau-2,1,-j} = \psi$ and $\psi_{\tau-2,2,-j} = \mu^{-\frac{1}{2}}\,\psi$,

$$k_{\tau-1,1} = \left(\frac{(1-\alpha)\,\psi}{R_{\tau-2}}\right)^{\frac{1}{\alpha}} (1-l_{\tau-1}) \tag{3.28}$$

$$k_{\tau-1,2} = \left(\frac{(1-\alpha)\,\mu^{-\frac{1}{2}}\,\psi}{R_{\tau-2}}\right)^{\frac{1}{\alpha}} (1-l_{\tau-1})\,. \tag{3.29}$$

Combining (3.8) with (3.28) and (3.29) yields

$$y_{\tau-1} = \left[1+\mu^{-\frac{1}{2}(\frac{1}{\alpha}-1)}\right]\,\psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}-1}\,\frac{(1-l_{\tau-1})}{R_{\tau-2}^{\frac{1}{\alpha}-1}}\,. \tag{3.30}$$

Output depends negatively on the interest rate paid currently: the higher the current interest rate the lower the investment in capital and, thus, the lower current production is.

Market clearing in the final-good market implies that consumption is given by

$$x_{\tau-1} = \psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\,\left[\frac{\left(1+\mu^{-\frac{1}{2}(\frac{1}{\alpha}-1)}\right)(1-l_{\tau-1})}{(1-\alpha)\,R_{\tau-2}^{\frac{1}{\alpha}-1}} - \frac{\left(\mu^{-1}+\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right)(1-l_{\tau})}{R_{\tau-1}^{\frac{1}{\alpha}}}\right]. \tag{3.31}$$

Period τ. Firm 2 receives and immediately implements a patent. In period τ, technology in the two sectors is $\psi_{\tau,1,} = \mu\,\psi$ and $\psi_{\tau,2,-j} = \mu^{\frac{1}{2}}\,\psi < \psi_{\tau,2,j} =$

$\mu^{\frac{3}{2}}\,\psi$. Capital in the following period is given by

$$k_{\tau+1,1} \;=\; \left(\frac{(1-\alpha)\,\mu\,\psi}{R_\tau}\right)^{\frac{1}{\alpha}} (1-l_{\tau+1}) \tag{3.32}$$

$$k_{\tau+1,2} \;=\; \left(\frac{(1-\alpha)\,\mu^{\frac{1}{2}}\,\psi}{R_\tau}\right)^{\frac{1}{\alpha}} (1-l_{\tau+1})\,. \tag{3.33}$$

By (3.11) and (3.12), investment is

$$i_\tau \;=\; \frac{k_{\tau+1,1}}{\mu\,\psi} \;+\; \frac{k_{\tau+1,2}}{\mu^{\frac{3}{2}}\,\psi}\,,$$

which combined with (3.32) and (3.33) becomes

$$i_\tau \;=\; \left[\mu^{\frac{1}{\alpha}-1} + \mu^{\frac{1}{2\alpha}-\frac{3}{2}}\right]\,\psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\,\frac{(1-l_{\tau+1})}{R_\tau^{\frac{1}{\alpha}}}\,. \tag{3.34}$$

Substituting (3.24) and (3.25) in (3.8) yields

$$y_\tau \;=\; \left[1 + \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right]\,\psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}-1}\,\frac{(1-l_\tau)}{R_{\tau-1}^{\frac{1}{\alpha}-1}}\,.$$

Market clearing in the final-good market implies that consumption is

$$x_\tau \;=\; \psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\,\left[\frac{\left(1+\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right)\,(1-l_\tau)}{(1-\alpha)\,R_{\tau-1}^{\frac{1}{\alpha}-1}} \;-\; \frac{\left(\mu^{\frac{1}{\alpha}-1} \;+\; \mu^{\frac{1}{2\alpha}-\frac{3}{2}}\right)\,(1-l_{\tau+1})}{R_\tau^{\frac{1}{\alpha}}}\right]\,. \tag{3.35}$$

Period $\tau+1$ **.** Firm 1 receives and implements a patent. Technology in the two sectors is $\psi_{\tau+1,1,-j} = \mu\,\psi \,<\, \psi_{\tau+1,1,j} \,=\, \mu^2\,\psi$ and $\psi_{\tau+1,2} \,=\, \mu^{\frac{3}{2}}\,\psi$. Then,

$$k_{\tau+2,1} \;=\; \left(\frac{(1-\alpha)\,\mu\,\psi}{R_{\tau+1}}\right)^{\frac{1}{\alpha}} (1-l_{\tau+1}) \tag{3.36}$$

$$k_{\tau+2,2} \;=\; \left(\frac{(1-\alpha)\,\mu^{\frac{3}{2}}\,\psi}{R_{\tau+1}}\right)^{\frac{1}{\alpha}} (1-l_{\tau+2})\,. \tag{3.37}$$

Proceeding in the same way as before, investment is given by

$$i_{\tau+1} \;=\; \frac{k_{\tau+2,1}}{\mu^2\,\psi} \;+\; \frac{k_{\tau+2,2}}{\mu^{\frac{3}{2}}\,\psi}\,,$$

which combined with (3.36) and (3.37) becomes

$$i_{\tau+1} \;=\; \left[\mu^{\frac{1}{\alpha}-2} + \mu^{\frac{3}{2}(\frac{1}{\alpha}-1)}\right]\,\psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\,\frac{(1-l_{\tau+2})}{R_{\tau+1}^{\frac{1}{\alpha}}}\,.$$

Substituting (3.32) and (3.33) into (3.8) yields

$$y_{\tau+1} \;=\; \left[\mu^{\frac{1}{\alpha}-1} + \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right]\,\psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}-1}\,\frac{(1-l_{\tau+1})}{R_{\tau}^{\frac{1}{\alpha}-1}}\,.$$

Market clearing in the final-good market implies that consumption is

$$x_{\tau+1} \;=\; \psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\,\left[\frac{\left(\mu^{\frac{1}{\alpha}-1} + \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right)\,(1-l_{\tau+1})}{(1-\alpha)\,R_{\tau}^{\frac{1}{\alpha}-1}} \;-\; \frac{\left(\mu^{\frac{1}{\alpha}-2} + \mu^{\frac{3}{2}(\frac{1}{\alpha}-1)}\right)\,(1-l_{\tau+2})}{R_{\tau+1}^{\frac{1}{\alpha}}}\right]\,. \tag{3.38}$$

The above satisfy optimality and market clearing, that is requirements (1) - (2) of the equilibrium definition. Next, I exploit stationarity and check for consistency and no storage in turn.

For accordingly formed expectations, an immediate implementation (acyclical) equilibrium is sustained as long as each firm which receives a patent finds it optimal to implement it immediately. I split this into two steps, as in Shleifer (1986), which I label "Profit Condition 1 (IPC1)" and "Profit Condition 2 (IPC2)."

Profit condition 1 . In the acyclical equilibrium the economy grows at a constant rate. By symmetry, employment and the real interest rate remain constant across periods, that is $l_\tau = l_{\tau+1} \equiv l$ and $R_{\tau-1} = R_\tau \equiv R$ (see also fn. 21).

A firm prefers to implement a patent immediately rather than in the following period if and only if condition (3.19) is satisfied. Given that employment and the real interest rate are constant, condition (3.19) simplifies to

$$R \geq 1 . \tag{IPC1}$$

Profit condition 2 . I look for the condition under which no firm receiving a patent has an incentive to wait for two periods irrespectively of the fact that a new patent will arrive in its sector rendering the one in question obsolete. That is interest rates must be such that no firm has an incentive to wait "too much." It must be then that

$$\frac{\Pi_{t,i}^{t-1}}{R_{t-1}} \geq \frac{\Pi_{t+2,i}^{t-1}}{R_{t-1}\, R_t\, R_{t+1}} .$$

Since $R_{t-1} = R_t \equiv R$, the above condition boils down once again to

$$R \geq 1\,. \tag{IPC2}$$

Profit condition 2 ensures that firm 1 does not postpone implementation to the next odd period and firm 2 to the next even one.

Combining profit conditions (IPC1) and (IPC2) implies that no firm postpones implementation to any period after the next odd (for firm 1) or even (for firm 2) one either. To see this, note that profit condition (IPC2) implies that, for instance, firm 1 does not wait until the next odd period, whereas by profit condition (IPC1), it does not wait until the even period following this, and so forth, and likewise for firm 2.

Balanced growth path

As I have already noted, by symmetry, $l_{odd} = l_{even} \equiv l$ and $R_{\tau-1} = R_\tau \equiv R$.[21] Then, it follows from (3.23) that $v = \lambda^{\frac{1}{2}}$ and from (3.22), (3.31) and (3.38) together that $\lambda = \mu^{\frac{1}{\alpha}-1}$. Then, by (3.23), the real interest rate along the balanced growth path is $R = \frac{\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}}{\beta}$. Since $\mu > 1$, $\alpha \in (0\,,1)$ and $\beta \in (0\,,1)$, it follows that $R > 1$, which satisfies profit conditions (IPC1) and (IPC2) as well as the no-storage condition (Condition 1).[22]

These lead to the following corollary:

Corollary 1 (Steady growth). *An acyclical (steady-growth) equilibrium is always possible for accordingly formed expectations.*

Further, the endogenous variables evolve as follows on the balanced

[21] One can confirm that this is in fact a unique stationary solution by taking the same steps as in the synchronized implementation equilibrium. The steps in the case of the synchronized implementation equilibrium are collected in the Appendix.

[22] Since $R > 1$, the transversality condition, which I show for the synchronized implementation equilibrium in the Appendix, always holds.

growth path:[23]

$$\frac{y_{t+1}}{y_t} = \frac{i_{t+1}}{i_t} = \frac{x_{t+1}}{x_t} = \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)} \ , \text{ for all } t \tag{3.39}$$

$$l = \left[\frac{\alpha}{\chi} \frac{\mu\,(1 + \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)})}{\mu - \beta(1-\alpha) + (1-\beta(1-\alpha))\mu^{1+\frac{1}{2}(\frac{1}{\alpha}-1)}} + 1 \right]^{-1} . \tag{3.40}$$

That is, in the acyclical (baseline) equilibrium, output, consumption, and investment grow at the same constant rate, whereas employment remains constant across time.

A higher innovation rate, μ, sets the economy onto a steeper growth path and results in a higher real interest rate, while a lower subjective discount factor β also calls for a higher interest rate. Turning to leisure, it increases in μ and the relative taste parameter for leisure χ and decreases in β.

3.3.2 Synchronized implementation equilibrium

I focus on the synchronized (*cyclical*) implementation equilibrium at which, firm 1, which receives a patent in an odd period, finds it optimal to save it and implement it in the following even period together with firm 2 which receives a patent then (see also fn. 19).

Period $\tau - 1$. A patent reaches firm 1 and is not implemented but is instead stored and implemented in τ. Effectively, from the viewpoint of $\tau - 1$, a not implemented patent is as if it had never arrived. As (3.17) holds, firm 1 (likewise, firm 2) sets the same price and produces the same quantity as its

[23] To find leisure, I use the intratemporal optimality condition $\chi \frac{x_t}{l_t} = w_t$, where, from the final-good firm's problem, $w_t = \alpha \frac{y_t}{1-l_t}$.

competitors would, given by (3.14) and (3.15), respectively, and it produces at the same marginal cost as they would. The technology with which the following period's capital is produced is $\psi_{\tau-1,1,-j} = \psi$ and $\psi_{\tau-1,2} = \mu^{\frac{1}{2}}\,\psi$. Then, capital in the following period is given by

$$k_{\tau,1} = \left(\frac{(1-\alpha)\,\psi}{R_{\tau-1}}\right)^{\frac{1}{\alpha}} (1-l_\tau) \tag{3.41}$$

$$k_{\tau,2} = \left(\frac{(1-\alpha)\,\mu^{\frac{1}{2}}\,\psi}{R_{\tau-1}}\right)^{\frac{1}{\alpha}} (1-l_\tau). \tag{3.42}$$

Since

$$i_{\tau-1} = \frac{k_{\tau,1}}{\psi} + \frac{k_{\tau,2}}{\mu^{\frac{1}{2}}\,\psi}, \tag{3.43}$$

investment in period $\tau-1$ is

$$i_{\tau-1} = \left[1+\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right]\psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\,\frac{(1-l_\tau)}{R_{\tau-1}^{\frac{1}{\alpha}}}. \tag{3.44}$$

Likewise, as the technology of firm j's competitors in $\tau-2$ is given by $\psi_{t-2,1,-j} = \mu^{-1}\,\psi$ and $\psi_{t-2,2,-j} = \mu^{-\frac{1}{2}}\,\psi$, capital in $\tau-1$ is given by

$$k_{\tau-1,1} = \left(\frac{(1-\alpha)\,\mu^{-1}\,\psi}{R_{\tau-2}}\right)^{\frac{1}{\alpha}} (1-l_{\tau-1}) \tag{3.45}$$

$$k_{\tau-1,2} = \left(\frac{(1-\alpha)\,\mu^{-\frac{1}{2}}\,\psi}{R_{\tau-2}}\right)^{\frac{1}{\alpha}} (1-l_{\tau-1}). \tag{3.46}$$

The above imply

$$y_{\tau-1} \;=\; \left[\mu^{-\left(\frac{1}{\alpha}-1\right)} \;+\; \mu^{-\frac{1}{2}\left(\frac{1}{\alpha}-1\right)}\right] \psi^{\frac{1}{\alpha}-1} \, (1-\alpha)^{\frac{1}{\alpha}-1} \, \frac{(1-l_{\tau-1})}{R_{\tau-2}^{\frac{1}{\alpha}-1}} \,. \tag{3.47}$$

Market clearing in the final-good market implies that consumption in $\tau - 1$ is

$$x_{\tau-1} = \left[1 \;+\; \mu^{\frac{1}{2}\left(\frac{1}{\alpha}-1\right)}\right] \psi^{\frac{1}{\alpha}-1} \, (1{-}\alpha)^{\frac{1}{\alpha}} \, \left[\frac{1-l_{\tau-1}}{\mu^{\frac{1}{\alpha}-1}(1-\alpha)\, R_{t-2}^{\frac{1}{\alpha}-1}} \;-\; \frac{1-l_{\tau}}{R_{t-1}^{\frac{1}{\alpha}}}\right] . \tag{3.48}$$

Period τ. Implementation takes place in both sectors. Technology across capital makers in the two sectors is $\psi_{\tau,1,-j} \;=\; \psi \;<\; \psi_{\tau,1,j} \;=\; \mu\,\psi$ and $\psi_{\tau,2,-j} \;=\; \mu^{\frac{1}{2}}\,\psi \;<\; \psi_{\tau,2,j} \;=\; \mu^{\frac{3}{2}}\,\psi$, which implies that in period $\tau + 1$ capital is

$$k_{\tau+1,1} \;=\; \left(\frac{(1-\alpha)\,\psi}{R_{\tau}}\right)^{\frac{1}{\alpha}} \, (1-l_{\tau+1}) \tag{3.49}$$

$$k_{\tau+1,2} \;=\; \left(\frac{(1-\alpha)\,\mu^{\frac{1}{2}}\,\psi}{R_{\tau}}\right)^{\frac{1}{\alpha}} \, (1-l_{\tau+1})\,. \tag{3.50}$$

Since

$$i_{\tau} \;=\; \frac{k_{\tau+1,1}}{\mu\,\psi} \;+\; \frac{k_{\tau+1,2}}{\mu^{\frac{3}{2}}\,\psi}\,,$$

investment in period τ is

$$i_{\tau} \;=\; \left[\frac{1+\mu^{\frac{1}{2}\left(\frac{1}{\alpha}-1\right)}}{\mu}\right] \psi^{\frac{1}{\alpha}-1} \, (1-\alpha)^{\frac{1}{\alpha}} \, \frac{(1-l_{\tau+1})}{R_{t}^{\frac{1}{\alpha}}}\,. \tag{3.51}$$

The production function (3.8) combined with (3.41) and (3.42) yields

$$y_\tau = \left[1 + \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right] \psi^{\frac{1}{\alpha}-1}(1-\alpha)^{\frac{1}{\alpha}-1} \frac{(1-l_\tau)}{R_{t-1}^{\frac{1}{\alpha}-1}} . \tag{3.52}$$

Market clearing in the final-good market then implies that

$$x_\tau = \left[1 + \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right] \psi^{\frac{1}{\alpha}-1} (1-\alpha)^{\frac{1}{\alpha}} \left[\frac{1-l_\tau}{(1-\alpha)\, R_{\tau-1}^{\frac{1}{\alpha}-1}} - \frac{1-l_{\tau+1}}{\mu\, R_\tau^{\frac{1}{\alpha}}}\right] . \tag{3.53}$$

Period $\tau+1$ **.** A patent reaches firm 1 but it is kept stored until period $\tau+2$, when the next implementation boom takes place. Since it is not implemented, effectively it is as if it had never arrived. Effective technology in the two sectors is $\psi_{\tau+1,1,-j} = \mu\,\psi$ and $\psi_{\tau+1,2} = \mu^{\frac{3}{2}}\,\psi$, which implies that capital in $\tau+2$ is

$$k_{\tau+2,1} = \left(\frac{(1-\alpha)\,\mu\,\psi}{R_{\tau+1}}\right)^{\frac{1}{\alpha}} (1-l_{\tau+2})$$

$$k_{\tau+2,2} = \left(\frac{(1-\alpha)\,\mu^{\frac{3}{2}}\,\psi}{R_{\tau+1}}\right)^{\frac{1}{\alpha}} (1-l_{\tau+2}) .$$

Taking familiar steps, since

$$i_{\tau+1} = \frac{k_{\tau+2,1}}{\mu\,\psi} + \frac{k_{\tau+2,2}}{\mu^{\frac{3}{2}}\,\psi} ,$$

investment in period $\tau+1$ is given by

$$i_{\tau+1} = \mu^{\frac{1}{\alpha}-1} \left[1 + \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right] \psi^{\frac{1}{\alpha}-1} (1-\alpha)^{\frac{1}{\alpha}} \frac{(1-l_{\tau+2})}{R_{\tau+1}^{\frac{1}{\alpha}}} . \tag{3.54}$$

Combining the production function (3.8) with (3.49) and (3.50) implies

$$y_{\tau+1} = \left[1+\mu^{\frac{1}{2}\left(\frac{1}{\alpha}-1\right)}\right] \psi^{\frac{1}{\alpha}-1} (1-\alpha)^{\frac{1}{\alpha}-1} \frac{(1-l_{\tau+1})}{R_{\tau}^{\frac{1}{\alpha}-1}}. \tag{3.55}$$

Market clearing then implies

$$x_{\tau+1} = \left[1+\mu^{\frac{1}{2}\left(\frac{1}{\alpha}-1\right)}\right] \psi^{\frac{1}{\alpha}-1} (1-\alpha)^{\frac{1}{\alpha}} \left[\frac{1-l_{\tau+1}}{(1-\alpha) R_{\tau}^{\frac{1}{\alpha}-1}} - \frac{\mu^{\frac{1}{\alpha}-1}(1-l_{\tau+2})}{R_{\tau+1}^{\frac{1}{\alpha}}}\right]. \tag{3.56}$$

A two-period implementation cycle -of the type that I focus on- requires firm 1 , which receives a patent in an odd period to find it optimal to wait exactly *one* period before implementing it, and firm 2, which receives a patent in the following even period to find it optimal to implement it immediately. I will take the same steps as in the case of the acyclical equilibrium. Starting with consistency and taking stationarity into account, I first derive the condition under which firm 1 prefers to postpone implementation to the following period rather than implement immediately, which I label "Profit Condition 1 (SPC1) ;" subsequently, I explore the condition under which firm 1 prefers not to postpone implementation from the following implementation boom to the one after that, which I label "Profit Condition 2 (SPC2) ." The two conditions combined imply that firm 1 prefers to wait exactly one period to implement and firm 2 prefers to implement immediately than in any future period. Omitted derivations in what follows are collected in the Appendix.

Profit condition 1 . Firm 1 prefers to implement in the following period rather than immediately as long as its present discounted profits in the former case exceed its present discounted profits in the latter (recall profits are realized one period after implementation) .[24] Given stationarity and τ being an even period, the implementation decision, made in $\tau - 1$, must then satisfy

$$\frac{\Pi_{\tau+1,1}^{\tau-1}}{R_{\tau-1}\, R_{\tau}} > \frac{\Pi_{\tau,1}^{\tau-1}}{R_{\tau-1}} . \tag{SPC1}$$

Given (3.18) , (SPC1) becomes

$$\frac{1 - l_{\tau+1}}{1 - l_{\tau}} > \frac{R_{\tau}^{\frac{1}{\alpha}}}{R_{\tau-1}^{\frac{1}{\alpha}-1}} , \tag{3.57}$$

which as I show in the Appendix boils down to

$$\frac{\mu\,[1 + \beta\,(1-\alpha)]}{\mu + \beta\,(1-\alpha)} > \frac{1}{\beta} . \tag{3.58}$$

We can see that (3.58) is more easily satisfied as μ and β increase and as α decreases. A higher innovation rate implies higher profits, hence a greater incentive for firms to coordinate in the presence of demand externalities. A higher β implies that firm 1 is more likely to wait for a certain level of profits; in the limit as $\beta \to 1$, (3.58) is always satisfied. Turning to α , it parametrizes both the capital share $(1-\alpha)$ as well as the elasticity of substitution between the two types of capital $(\frac{1}{\alpha})$. Greater values of the former and lower of the latter imply that implementation cycles are more easily sustained. As α falls, both increase. However, the former effect dominates, hence (3.58) is more

[24]Of course, expectations are centered around the synchronized implementation equilibrium in both cases. On the lack of strategic interactions among firms see fn. 11 .

easily met. Last, observe that (3.58) is independent of the relative weight of leisure χ in the agent's preferences which is due to the separability of leisure and consumption within the flow utility and, given this, to preferences being logarithmic in consumption.[25]

Profit condition 2 . For a synchronized implementation (cyclical) equilibrium to be sustained, no firm must find it optimal to postpone implementation past the two-period cycle. For this to be the case, it suffices to show that no firm has an incentive to wait until the next implementation period, i.e. period $\tau + 2$, an argument also appearing in Shleifer (1986) .

To see this in the case of firm 1 , note that, given the stationary structure of the economy, if firm 1 prefers to implement in τ rather than in $\tau - 1$, i.e. when condition (SPC1) holds, then it also finds it optimal to postpone implementation from $\tau + 1$ to $\tau + 2$. In other words, condition (SPC1) effectively implies that implementation can only take place in even periods. Thus, showing that firm 1 opts to implement in τ as opposed to doing so in $\tau + 2$ or, by the same token, any future even period, is what I need to complete the consistency requirement of Definition 2 . Note that this argument is independent of the fact that a new patent will reach sector 1 in $\tau + 1$ rendering the patent received in $\tau - 1$ obsolete. An analogous reasoning applies to firm 2 .

Then, in the case of firm 1 , the following condition must be satisfied:

$$\frac{\Pi^{\tau-1}_{\tau+1,1}}{R_{\tau-1}\, R_{\tau}} \geq \frac{\Pi^{\tau-1}_{\tau+3,1}}{R_{\tau-1}\, R_{\tau}\, R_{\tau+1}\, R_{\tau+2}} \,. \tag{SPC2}$$

[25]For $\alpha = 2/3$, which by (3.17) requires $\mu \leq 3$, $\chi = 1.5$ and $\beta = 0.97$, (3.58) requires approximately that $\mu > 1.14$.

Given that $R_{\tau-1} = R_{\tau+1}$ and $R_\tau = R_{\tau+2}$, condition (SPC2) simplifies to

$$R_\tau \, R_{\tau+1} \geq 1 \,. \tag{3.59}$$

Condition (SPC2) implies that profits are discounted at an on average positive net real interest rate and we can think of it as a weak version of the transversality condition which I analyze below.

One can confirm that conditions (3.57) and (3.59) combined imply that firm 2 finds it optimal to implement immediately.[26]

Combining (3.22) and (3.23) with (3.48) and (3.56) implies that

$$\lambda = \mu^{\frac{1}{\alpha}-1} > 1 \;\; \text{, since } \mu > 1 \,. \tag{3.60}$$

Lemma 1. *Remark 1 and (3.60) imply that (3.59) always holds.*

Balanced growth path

Leisure. On the balanced growth path of the synchronized implementation equilibrium, leisure takes values which alternate between odd and even periods and remain constant every other period, that is $l_{\tau-1} = l_{\tau+1} = l_{odd}$ and

[26] Starting with profit condition 2, it would be like (SPC2) but without $R_{\tau-1}$ in the denominators, which plays no role anyway. Then, (3.59) is what we need.

Turning to profit condition 1, firm 2 implements immediately if

$$\frac{\Pi^{\tau}_{\tau+1,i}}{R_\tau} \geq \frac{\Pi^{\tau}_{\tau+2,i}}{R_\tau \, R_{\tau+1}} \,.$$

After substituting for profits, given by (3.18), and exploiting stationarity, which implies $l_\tau = l_{\tau+2}$, the above condition boils down to

$$R_{\tau+1} \left(\frac{R_{\tau+1}}{R_\tau} \right)^{\frac{1}{\alpha}-1} \geq \frac{1 - l_\tau}{1 - l_{\tau+1}} \,.$$

This holds if conditions (3.57) and (3.59) hold together which we can confirm by multiplying the LHS of (3.57) by $R_\tau \, R_{\tau+1}$.

$l_\tau = l_{\tau+2} = l_{even}$.[27] As I show in the Appendix, leisure is given by

$$l_{odd} = \left[\frac{\alpha}{\chi}\,\frac{\mu\,(1+\beta\,(1-\alpha))}{\mu-\beta^2(1-\alpha)^2}+1\right]^{-1} \tag{3.61}$$

$$l_{even} = \left[\frac{\alpha}{\chi}\,\frac{\mu+\beta\,(1-\alpha)}{\mu-\beta^2(1-\alpha)^2}+1\right]^{-1}. \tag{3.62}$$

Remark 2. *It is* $l_{odd} < l_{even}$.

Remark 2 implies that employment falls when implementation takes place.

Interest rates. Equilibrium interest rates are given by (3.23), with λ given by (3.60) and v given by

$$v = \left[\mu^{\left(\frac{1}{\alpha}-1\right)^2}\frac{\alpha\,(\mu+\beta\,(1-\alpha))+\chi\,(\mu-\beta^2\,(1-\alpha)^2)}{\alpha\,\mu\,(1+\beta\,(1-\alpha))+\chi\,(\mu-\beta^2\,(1-\alpha)^2)}\right]^{\frac{\alpha}{2-\alpha}}. \tag{3.63}$$

Claim 1. *It is* $v < \mu^{\frac{1}{2}\left(\frac{1}{\alpha}-1\right)}$.

Proof. See the Appendix. □

It follows from (3.60) that the geometric average growth rate is $\mu^{\frac{1}{2}\left(\frac{1}{\alpha}-1\right)}$, which is the one that prevails every period in the immediate implementation (steady-growth) equilibrium. Controlling for deviations from it and using Claim 1 implies that consumption booms when implementation takes place.

[27] I use interchangeably throughout $l_{\tau-1}$, $l_{\tau+1}$ and l_{odd} for leisure in odd periods and l_τ, $l_{\tau+2}$ and l_{even} for leisure in the even ones.

Letting R_{even} and R_{odd} denote the real interest rate paid in odd and even periods in the synchronized equilibrium, respectively,[28] and, with R denoting the real interest rate in the immediate implementation equilibrium, Claim 1 leads to the following remark:

Remark 3. *It is* $R_{even} < R < R_{odd}$ *.*

Controlling for variations in employment, Remark 3 bears witness to the presence of demand externalities between the two capital-good sectors: since $R_{even} < R$, profits (both discounted and current-valued) of the following firm (firm 2), which implements a patent immediately in both equilibria, are greater in the synchronized than in the immediate implementation one (see also (3.18)). Allowing for variations in employment does not overturn this observation for all the parametrizations that I have considered.

Transversality condition. In what is the last step, I check the transversality condition. The transversality condition requires the present discounted value of the agent's lifetime wealth to converge. In other words, the present discounted value of the labor income and the capital-good firms' profits needs to converge. In the Appendix I show that the transversality condition is always satisfied.[29]

With (SPC2) and the transversality condition always satisfied, the following proposition can be generated:

[28]Throughout I use interchangeably R_{even}, R_τ and $R_{\tau+2}$ for the interest rate paid in odd periods and R_{odd}, $R_{\tau-1}$ and $R_{\tau+1}$ for the interest rate paid in the even ones.

[29]This result is due to preferences being logarithmic in consumption. For general CRRA preferences, the transversality condition is more easily satisfied the lower the intertemporal elasticity of substitution is.

Proposition 1. *A two-period synchronized implementation equilibrium prevails as a perfect-foresight equilibrium as long as condition (SPC1) and the no-storage condition (1) are satisfied.*

This proposition is central to the essay. In sharp contrast with the conjecture in (Shleifer, 1986, page 1183) , implementation cycles with capital can be generated and they do so for plausible values of the parameters. Importantly, this happens in the presence of storable commodities and in the absence of borrowing constraints and investment irreversibilities (on the latter see also fn. 33). But how so? Following Shleifer's line of thought, one would expect that, in the prospect of future profits, agents would reduce current savings and, thereby, future capital stock in order to smooth out their consumption across periods. At the same time, a (real) interest rate increase would be necessary to prevent agents from borrowing in the period before the wealth expansion. Both effects combined imply that firms' present discounted profits in an implementation boom would fall which could eventually eliminate their incentives to postpone implementation until then.

This intuition does not apply here. This is because, in sharp contrast with Shleifer (1986) in which innovations are sector-neutral, that is they enhance total factor productivity (TFP), innovations here are investment-specific as in Greenwood et al. (1997, 2000) . This difference in modeling technological change is important. Unlike changes in TFP, investment-specific technological change introduces a one-period discrepancy between the date firms invest and the date they receive their revenue. As a result, a coordinated implementation of patents implies a concurrent considerable fall in savings/investment -in fact, investment can even undershoot- due to the reduced cost of producing capital and a considerable increase in the wealth of agents in the period following it.[30] The former implies that consumption grows considerably in an implementation boom even after taking into account the effects on output.[31] The latter implies that the consumption boom takes place before the wealth boom. This not only eliminates the need to smooth consumption away from the wealth boom to the period before it, but, additionally, it implies that the interest rate linking the implementation period to the one when revenue is realized and wealth expands actually falls (Remark 3) . As a result, firms discount future profits less rather than more. The fall in the real interest rate, in turn, causes two, tied to each other, effects. First, it increases investment in an implementation period which thereby smooths out consumption in the opposite direction from the conjectured one, though (without overturning results; Claim 1 is a general equilibrium result) . Second, it leads to an increase of the capital stock in the period revenue is realized relative to that in the implementation period.[32] With the demand for capital being elastic, this implies that the profits capital-good firms make following an implementation boom grow rather than fall (see also (3.18)) relative to the profits they would have made had they implemented alone. Taking everything into account, present discounted profits increase which eventually makes implementation cycles possible (Proposition 1) .[33]

[30]In fact, output and, consequently, labor income also boosts with a one-period lag (see the analysis below) . This need not be always true. For instance, in the extreme case in which labor is inelastically supplied, labor income booms when implementation takes place. Nevertheless, for all the parametrizations that I have considered, variations in labor income prove insufficient to make the wealth boom happen simultaneously with the consumption boom and, hence, potentially overturn the intuition in the main text.

[31]More precisely, consumption grows above trend as Claim 1 attests.

[32]Simply compare equations (3.41) - (3.42) with (3.49) - (3.50) . The result follows from Remarks 2 and 3 .

[33]As I noted in fn. 10 , that capital depreciates fully effectively rules out disinvestment and, therefore, it could be argued that I impose investment irreversibilities which according

Below I discuss the role of parameters in the generation of cycles and the balanced growth path. I set $\alpha = \frac{2}{3}$, which by (3.17) implies that $\mu \leq 3$ and, based on Greenwood et al. (2000), $\chi = 1.5$ and consider values of β close to 1.[34] A consequence of setting $\alpha = \frac{2}{3}$ is that v, given by (3.63), is greater than one. In turn, this implies that R_τ is greater than one, whereas by Claim 1, $R_{\tau+1}$ is also greater than one. As a result, the no-storage condition is always satisfied.

Generation of cycles. A greater innovation rate sets the economy onto a steeper growth path which is accompanied with higher interest rates, R_τ and $R_{\tau+1}$. However, $R_{\tau+1}$ increases sufficiently more than the interest rate paid after an implementation boom, R_τ, does so that the RHS of (3.57) falls. In other words, controlling for changes in employment which as I argue next actually reinforce this effect, as the innovation rate increases, discounted profits in an implementation boom become greater relative to profits that would be realized if firm 1 instead opted to implement alone. This is because a greater innovation rate results in considerably reduced savings/investment in an implementation period. In turn, this implies a substantial increase in contemporaneous consumption both in absolute and, crucially, in relative to trend terms, where I define as trend the geometric average growth rate $\lambda^{\frac{1}{2}} = \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}$ which char-

to (Shleifer, 1986, page 1183) could render implementation cycles possible. The way I interpret his argument, even though such an interpretation may be susceptible to criticism, is that he expects that too volatile investment would rule implementation cycles with capital out. By allowing for full capital depreciation, I indeed maximize the volatility of investment as new capital needs to be produced every period. In fact, as it may have already become apparent, it is the excessive investment volatility -actually investment can even undershoot- that renders implementation cycles with capital possible here.

[34]The numerical values that I report below correspond to $\beta = 0.97$, which is again based on Greenwood et al. (2000). In fact, many results below hold for a much wider range of parametrizations, however I abstract from such considerations.

acterizes the immediate implementation (steady-growth) equilibrium.[35]

Turning to leisure/employment, a greater innovation rate, μ, results in an increase in leisure in both periods; the leisure ratio $\frac{l_{\tau+1}}{l_\tau}$ falls, whereas the employment ratio $\frac{1-l_{\tau+1}}{1-l_\tau}$, which is on the LHS of (3.57) increases. Taking both effects into account, the greater the innovation rate μ, the more attractive an implementation boom is to firm 1.

A greater β lowers v, that is consumption in the implementation periods becomes higher both in absolute and relative to trend terms, and both interest rates. This is because agents become more patient. However, $R_{\tau+1}$ falls less relative to R_τ and the RHS of (3.57) decreases. Parallel to this, a greater discount factor β decreases leisure in all periods as well as the leisure ratio $\frac{l_{\tau+1}}{l_\tau}$, whereas it increases the employment ratio $\frac{1-l_{\tau+1}}{1-l_\tau}$, which is on the LHS of (3.57). Once again both effects imply that a higher β leads more easily to implementation cycles.

Next, I analyze the balanced growth path.

Balanced growth path. Output, consumption and investment grow by $\mu^{\frac{1}{\alpha}-1}$ every two periods:

$$\frac{y_{t+1}}{y_{t-1}} = \frac{x_{t+1}}{x_{t-1}} = \frac{i_{t+1}}{i_{t-1}} = \mu^{\frac{1}{\alpha}-1}. \tag{3.64}$$

Within a cycle, we know that $\frac{x_{\tau+1}}{x_\tau} = v$, whereas output and invest-

[35]The trend of output, consumption and investment is $\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}$, that of capital is $\mu^{\frac{1}{2\alpha}}$ and that of employment is 1.

ment's evolution is given by

$$\frac{y_{\tau+1}}{y_\tau} = \left(\frac{1-l_{\tau+1}}{1-l_\tau}\right)\left(\frac{R_{\tau+1}}{R_\tau}\right)^{\frac{1}{\alpha}-1}$$
$$\frac{i_{\tau+1}}{i_\tau} = \left(\frac{1-l_\tau}{1-l_{\tau+1}}\right)\left(\frac{\mu\, R_\tau}{R_{\tau+1}}\right)^{\frac{1}{\alpha}},$$

which follow from (3.52) and (3.55), and (3.51) and (3.54), respectively. After substituting for the interest rates, given by (3.23), and using eq. (3.97) in the Appendix, the above expressions become

$$\frac{y_{\tau+1}}{y_\tau} = v\left(\frac{\mu\,(1+\beta(1-\alpha))}{\mu+\beta(1-\alpha)}\right) \tag{3.65}$$
$$\frac{i_{\tau+1}}{i_\tau} = v\left(\frac{\mu+\beta(1-\alpha)}{1+\beta(1-\alpha)}\right), \tag{3.66}$$

where v is given by (3.63).

For the considered parametrization, output grows above trend in the period following the implementation of patents. Investment is procyclical and undershoots: it falls when implementation takes place, as fewer resources need to be directed towards the production of capital goods, and rises sharply in the period following implementation. Further, by Remark 2 employment is procyclical, whereas, most notably, by Claim 1 consumption is countercyclical. As I argued above, it is necessary for the generation of cycles that consumption booms in the implementation periods.

The above and (3.65) and (3.66) imply that investment is more volatile than output since $\frac{i_{\tau+1}}{i_\tau} > \frac{y_{\tau+1}}{y_\tau}$. In turn, output is more volatile than employment. Notably (for the considered parametrization), consumption is also more volatile than output[36] but less volatile than investment. The interpretation for this is simple: with output relatively stable, a very volatile investment implies a very volatile consumption.

As the innovation rate increases, investment, consumption and employment become more volatile, whereas the volatility of output responds non-monotonically increasing at low values of μ and falling at higher ones.[37]

3.4 Welfare

From a planner's viewpoint, both equilibria are suboptimal which is due to the (periodic) presence of monopolies in the capital-good markets.[38] Nevertheless, the equilibria can be Pareto ranked.

In the immediate implementation equilibrium, the lifetime utility of the

[36]As consumption is countercyclical, I compare $\frac{y_{\tau+1}}{y_\tau}$ given by (3.65) with $\frac{x_\tau}{x_{\tau-1}}$, where $\frac{x_\tau}{x_{\tau-1}} = \frac{\mu^{\frac{1}{\alpha}-1}}{v}$.

[37]In addition to these, note that, for the considered parametrization, capital is countercyclical. To see this, recall that, because of "limit-pricing," implementing firms produce the quantity of capital that their competitors would produce. This implies that, controlling for variations in interest rates and leisure, patents will affect the quantity of capital installed after two periods. Further, capital is more volatile than output, which should not come as a surprise given that it depreciates fully within a period, is less volatile than investment and its volatility increases in μ.

[38]Since the implementation of a patent improves the technology of an implementing firm's competitors, one would suggest that externalities is an additional source of inefficiency. My argument for why this is not indeed an issue is the same as the one in Shleifer (1986) (in particular, see (Shleifer, 1986, page 1178) and fn. 10 there). In principle, this potential problem could be corrected by allowing for decentralized markets on a one-to-one basis between the firm endowed with a patent and one of its competitors with the former setting the price in exchange for sharing the rights to its patent (we can equivalently think in terms of contracts on an individual, "take it or leave it" basis). The presence of constant returns to scale in the capital-good's technology implies that competitors, which behave symmetrically, will demand zero at any positive price since, afterwards, they will Bertrand-compete at least with the firm owning the patent which, in turn, is not willing to suggest a zero price. Hence, markets would clear at positive prices small enough so that demand and supply are equal to zero in each period. Therefore, externalities is not an additional source of inefficiency.

representative agent, given by (3.1) - (3.2) , is equal to

$$U_i = \log x_1^i + \chi \log l + \beta \left(\log \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)} x_1^i + \chi \log l \right) + \beta^2 \left(\log \mu^{\frac{1}{\alpha}-1} x_1^i + \chi \log l \right) + \ldots ,$$

where I have taken into account that leisure is constant across time and that consumption grows each period by $\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}$ (see (3.39) - (3.40)) . This expression boils down to

$$U_i = \frac{1}{1-\beta} \left(\frac{1}{2} \frac{\beta}{1-\beta} \log \mu^{\frac{1}{\alpha}-1} + \log x_1^i + \chi \log l \right) , \tag{3.67}$$

where l is given by (3.40) .

In the synchronized implementation equilibrium, the lifetime utility of the agent is

$$U_s = \log x_1^s + \chi \log l_{odd} + \beta \left(\log \frac{\mu^{\frac{1}{\alpha}-1}}{v} x_1^s + \chi \log l_{even} \right) + \beta^2 \left(\log \mu^{\frac{1}{\alpha}-1} x_1^s + \chi \log l_{odd} \right) + \ldots ,$$

where I have taken into account that leisure is constant controlling for the period being odd or even and that consumption grows as (3.21) and (3.22) prescribe. The above expression simplifies to

$$U_s = \frac{1}{1-\beta} \left[\frac{\beta}{1-\beta^2} \log \mu^{\frac{1}{\alpha}-1} - \frac{\beta}{1+\beta} \log v + \log x_1^s + \frac{\chi}{1+\beta} \left(\log l_{odd} + \beta \log l_{even} \right) \right] , \tag{3.68}$$

where v is given by (3.63) and l_{odd} , l_{even} are given by (3.61) and (3.62) , respectively.

To make welfare comparisons, simply subtract (3.68) from (3.67) to get

$$U_i - U_s = \frac{1}{1-\beta}\left[\frac{\beta}{1+\beta}\left(\log v - \log \mu^{\frac{1}{2}\left(\frac{1}{\alpha}-1\right)}\right) + \log x_1^i - \log x_1^s\right] +$$

$$+ \frac{\chi}{1-\beta}\left(\log l - \frac{\log l_{odd} + \beta \log l_{even}}{1+\beta}\right). \tag{3.69}$$

For the considered parametrization, welfare is greater in the immediate implementation equilibrium than in the synchronized implementation one.[39] To analyze this result, I will start with the last terms in (3.69), which reflect differences in welfare due to differences in leisure levels. For the considered values of α, χ and β and sufficiently high values of μ, the leisure component of lifetime utility is greater in the synchronized implementation equilibrium. However, the effect of leisure in welfare comparisons is typically negligible.

What is crucial is differences in lifetime consumption. To analyze these, I will draw a distinction between the difference in the initial consumption levels, captured by the third and the fourth term in (3.69) combined, and the difference in the consumption growth rates between the two equilibria, captured by the first two terms. The latter takes a negative value by Claim 1. To see this, recall that the growth rate every two periods is the same across equilibria. Hence, the difference in consumption growth is due to the difference in the growth rate within a cycle, captured by the first two terms in (3.69). Since consumption grows faster in the synchronized implementation equilibrium (Claim 1), the first two terms combined take a negative value.

Turning to the initial level of consumption, it is higher in the immediate

[39] In fact, I have failed to obtain the opposite result for a wide range of parametrizations.

implementation equilibrium. To see this compare (3.31) and (3.48).[40] It turns out that in the immediate implementation equilibrium output is greater and investment lower compared to the synchronized implementation equilibrium.[41] Controlling for the interest rates which do not overturn the result for the considered parametrization, investment is lower in the immediate implementation equilibrium since one firm (firm 1) implements a patent as opposed to none doing so in the synchronized implementation one (compare (3.26 with (3.43)). As for output, once again controlling for the interest rates, it is greater in the immediate implementation equilibrium because patents in sector 1 are implemented faster which leads to a greater level of capital of type 1 (compare (3.28) with (3.45)) and, hence, a greater level of output.[42]

3.4.1 Initial levels of capital/investment

Section 3.3 analyzed the two equilibria independently of each other. By this I mean that, in each equilibrium, the economy starts with the "right" quantity of capital-goods.

Below, I find the initial (period-0) investment required in each equilibrium. I suppose that the level of initial investment is the one that would have prevailed if time had started in $-\infty$. Therefore, I set $\psi = 1$ and divide the

[40] Since what matters is relative consumption, any odd-period consumption would do for the comparison between equilibria.

[41] The result for investment is true for the standard parametrization but not in all the ones that I have considered, whereas the results for output and, crucially, consumption hold for all the parametrizations that I have considered.

[42] I have not explored welfare in the case of a synchronized implementation equilibrium in which implementation booms take place in odd periods (see also fn. 19). Nevertheless, since that cyclical equilibrium is symmetric with the one considered here and given that time here commences in an odd period, my conjecture is that results would possibly be overturned. Such an argument could apply to the welfare considerations in Section 3.5 as well.

RHS of both (3.34) and (3.51) by $\mu^{\frac{1}{\alpha}-1}$.[43] This yields, respectively,

$$i_0^i = \left[1 + \mu^{-\frac{1}{2}(\frac{1}{\alpha}+1)}\right] (1-\alpha)^{\frac{1}{\alpha}} \frac{1-l}{R^{\frac{1}{\alpha}}} \tag{3.70}$$

$$i_0^s = \left[\mu^{-\frac{1}{\alpha}} + \mu^{-\frac{1}{2}(\frac{1}{\alpha}+1)}\right] (1-\alpha)^{\frac{1}{\alpha}} \frac{1-l_{\tau+1}}{R_\tau^{\frac{1}{\alpha}}}, \tag{3.71}$$

where l is given by (3.40), $R = \frac{\mu^{\frac{1}{\alpha}-1}}{\beta}$, $l_{\tau+1}$ is given by (3.61) and $R_\tau = \frac{v}{\beta}$ with v given by (3.63).

For the considered parametrization,[44] initial investment is greater in the immediate implementation equilibrium. As a result, the two cases can be Pareto ranked only conditional on this difference in initial investment.

3.5 The Desirability of Extending Patent Rights

In this section I explore whether extending the patent horizon is potentially welfare-improving. Therefore, the only assumption that I relax concerns the duration of patent rights. In particular, I let firms make monopoly profits out of a patent for *two* periods instead of one. Everything else remains unchanged.

[43] I opted for initial investment rather than initial levels of capital in order to facilitate comparisons between equilibria. To find initial levels of capital one needs to set $\psi = 1$ in $k_{\tau-1,1}$ and $k_{\tau-1,2}$.

[44] In particular, this is true for all the parametrizations that I have considered.

3.5.1 Immediate implementation equilibrium

As in Section 3.3.1 , leisure and the real interest rate remain constant across periods with values given by $\tilde{l}$ and R , respectively, which I find subsequently.[45]

Period $\tau - 1$ **.** Firm 1 receives a patent which it immediately implements, whereas firm 2 , which implemented its patent in the previous period, enters its second and last period as a monopolist. The technology levels in the two sectors are $\psi_{\tau-1,1,-j} = \psi < \psi_{\tau-1,1,j} = \mu\,\psi$ and $\psi_{\tau-1,2,-j} = \mu^{-\frac{1}{2}}\,\psi < \psi_{\tau-1,2,j} = \mu^{\frac{1}{2}}\,\psi$, respectively. Then,

$$k_{\tau,1} = \left(\frac{(1-\alpha)\,\psi}{R}\right)^{\frac{1}{\alpha}} (1-\tilde{l}) \tag{3.72}$$

$$k_{\tau,2} = \left(\frac{(1-\alpha)\,\mu^{-\frac{1}{2}}\,\psi}{R}\right)^{\frac{1}{\alpha}} (1-\tilde{l})\,. \tag{3.73}$$

Investment is given by

$$i_{\tau-1} = \frac{k_{\tau,1}}{\mu\,\psi} + \frac{k_{\tau,2}}{\mu^{\frac{1}{2}}\,\psi}\,. \tag{3.74}$$

Combining the above expression with (3.72) and (3.73) yields

$$i_{\tau-1} = \left[\mu^{-1} + \mu^{-\frac{1}{2}(\frac{1}{\alpha}+1)}\right] \psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\,\frac{(1-\tilde{l})}{R^{\frac{1}{\alpha}}}\,. \tag{3.75}$$

Similarly, since $\psi_{\tau-2,1,-j} = \mu^{-1}\,\psi$ and $\psi_{\tau-2,2,-j} = \mu^{-\frac{1}{2}}\,\psi$,

[45] As it will become evident below, the real interest rate in an immediate implementation (steady-growth) equilibrium is the same irrespectively of the duration of patent rights, hence the use of R as opposed to, for instance, $\tilde{R}$.

$$k_{\tau-1,1} \;=\; \left(\frac{(1-\alpha)\,\mu^{-1}\,\psi}{R}\right)^{\frac{1}{\alpha}}(1-\tilde{l})$$

$$k_{\tau-1,2} \;=\; \left(\frac{(1-\alpha)\,\mu^{-\frac{1}{2}}\,\psi}{R}\right)^{\frac{1}{\alpha}}(1-\tilde{l})\,.$$

I proceed in a familiar way to find output and consumption:

$$y_{\tau-1} \;=\; \left[\mu^{-\left(\frac{1}{\alpha}-1\right)} + \mu^{-\frac{1}{2}\left(\frac{1}{\alpha}-1\right)}\right]\,\psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}-1}\,\frac{(1-\tilde{l})}{R^{\frac{1}{\alpha}-1}} \tag{3.76}$$

$$x_{\tau-1} \;=\; \psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\,(1-\tilde{l})\,R^{-\frac{1}{\alpha}}\left[\frac{R\left(\mu^{-\left(\frac{1}{\alpha}-1\right)} + \mu^{-\frac{1}{2}\left(\frac{1}{\alpha}-1\right)}\right)}{1-\alpha} \;-\; \left(\mu^{-1} \;+\; \mu^{-\frac{1}{2}\left(\frac{1}{\alpha}+1\right)}\right)\right]. \tag{3.77}$$

Period τ . Firm 2 receives and immediately implements a patent. In period τ , technology in sector 1 remains as in $\tau-1$, that is $\psi_{\tau,1,-j} \;=\; \psi \;<\; \psi_{\tau,1,j} \;=\; \mu\,\psi$, whereas in sector 2 it becomes $\psi_{\tau,2,-j} \;=\; \mu^{\frac{1}{2}}\,\psi \;<\; \psi_{\tau,2,j} \;=\; \mu^{\frac{3}{2}}\,\psi$. Capital in the following period is given by

$$k_{\tau+1,1} \;=\; \left(\frac{(1-\alpha)\,\psi}{R}\right)^{\frac{1}{\alpha}}(1-\tilde{l}) \tag{3.78}$$

$$k_{\tau+1,2} \;=\; \left(\frac{(1-\alpha)\,\mu^{\frac{1}{2}}\,\psi}{R}\right)^{\frac{1}{\alpha}}(1-\tilde{l})\,. \tag{3.79}$$

Investment is given by

$$i_\tau = \frac{k_{\tau,1}}{\mu\,\psi} + \frac{k_{\tau,2}}{\mu^{\frac{3}{2}}\,\psi}\,, \tag{3.80}$$

which combined with (3.78) and (3.79) yields

$$i_\tau = \left[\mu^{-1} + \mu^{\frac{1}{2\alpha}-\frac{3}{2}}\right]\,\psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\,\frac{(1-\tilde{l})}{R^{\frac{1}{\alpha}}}\,. \tag{3.81}$$

Output and consumption are given by

$$y_\tau = \left[1 + \mu^{-\frac{1}{2}(\frac{1}{\alpha}-1)}\right]\,\psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}-1}\,\frac{(1-\tilde{l})}{R^{\frac{1}{\alpha}-1}}$$

$$x_\tau = \psi^{\frac{1}{\alpha}-1}\,(1-\alpha)^{\frac{1}{\alpha}}\,(1-\tilde{l})\,R^{-\frac{1}{\alpha}}\,\left[\frac{R\left(1+\mu^{-\frac{1}{2}(\frac{1}{\alpha}-1)}\right)}{1-\alpha} - \left(\mu^{-1} + \mu^{\frac{1}{2\alpha}-\frac{3}{2}}\right)\right]\,.$$

Period $\tau + 1$**.** Firm 1 receives and implements a patent. Technology in the two sectors is $\psi_{\tau+1,1,-j} = \mu\,\psi < \psi_{\tau+1,1,j} = \mu^2\,\psi$ and $\psi_{\tau+1,2,-j} = \mu^{\frac{1}{2}}\,\psi < \psi_{\tau+1,2,j} = \mu^{\frac{3}{2}}\,\psi$. Then,

$$k_{\tau+2,1} = \left(\frac{(1-\alpha)\,\mu\,\psi}{R}\right)^{\frac{1}{\alpha}}\,(1-\tilde{l})$$

$$k_{\tau+2,2} = \left(\frac{(1-\alpha)\,\mu^{\frac{1}{2}}\,\psi}{R}\right)^{\frac{1}{\alpha}}\,(1-\tilde{l})\,.$$

Investment is given by

$$i_{\tau+1} = \frac{k_{\tau,1}}{\mu^2\,\psi} + \frac{k_{\tau,2}}{\mu^{\frac{3}{2}}\,\psi}\,,$$

which is equal to

$$i_{\tau+1} \;=\; \left[\mu^{\frac{1}{\alpha}-2} + \mu^{\frac{1}{2\alpha}-\frac{3}{2}}\right] \psi^{\frac{1}{\alpha}-1} (1-\alpha)^{\frac{1}{\alpha}} \frac{(1-\tilde{l})}{R^{\frac{1}{\alpha}}} \,.$$

Output and consumption are given by

$$y_{\tau+1} \;=\; \left[1 + \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right] \psi^{\frac{1}{\alpha}-1} (1-\alpha)^{\frac{1}{\alpha}-1} \frac{(1-\tilde{l})}{R^{\frac{1}{\alpha}-1}}$$

$$x_{\tau+1} \;=\; \psi^{\frac{1}{\alpha}-1} (1-\alpha)^{\frac{1}{\alpha}} (1-\tilde{l})\, R^{-\frac{1}{\alpha}} \left[\frac{R\left(1 + \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\right)}{1-\alpha} \;-\; \left(\mu^{\frac{1}{\alpha}-2} + \mu^{\frac{1}{2\alpha}-\frac{3}{2}}\right)\right] .$$

Profit conditions.

As each firm can make profits for two periods out of a patent, profit condition 1 in the case of a firm receiving a patent in period $t-1$ becomes

$$\frac{\Pi_t}{R_{t-1}} + \frac{\Pi_{t+1}}{R_{t-1}\,R_t} \geq \frac{\Pi_{t+1}}{R_{t-1}\,R_t} + \frac{\Pi_{t+2}}{R_{t-1}\,R_t\,R_{t+1}} \,.$$

The LHS refers to discounted profits as of $t-1$ made when a patent is immediately implemented, whereas the RHS refers to discounted profits made when a patent is implemented in the following period. The above condition does not take into account that, in the latter case, after one period a new patent will render the one in question obsolete, which would imply that the condition would be more easily satisfied.

Likewise, profit condition 2 becomes

$$\frac{\Pi_t}{R_{t-1}} + \frac{\Pi_{t+1}}{R_{t-1}\, R_t} \geq \frac{\Pi_{t+2}}{R_{t-1}\, R_t\, R_{t+1}} + \frac{\Pi_{t+3}}{R_{t-1}\, R_t\, R_{t+1}\, R_{t+2}}\,.$$

Since in the steady-growth equilibrium, employment and interest rates are constant, so are temporal profits. Then, both profit conditions boil down to $R \geq 1$.[46]

Balanced growth path

Along the balanced growth path, output, consumption and investment grow all by $\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}$ and the real interest rate is $R = \frac{\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}}{\beta}$, which implies that both profit conditions are always met. Leisure is given by (see also fn. 23)

$$\tilde{l} = \left[\frac{\alpha\,\mu}{\chi\,(\mu - \beta(1-\alpha))} + 1\right]^{-1}. \tag{3.82}$$

3.5.2 Synchronized implementation equilibrium

In the synchronized implementation equilibrium (of the type I considered in Section 3.3), firm 1 postpones implementation until the following even period when it implements together with firm 2. Let me start with the profit conditions and, subsequently, show a synchronized implementation equilibrium is not possible when rights over a patent last two periods.

For firm 1 which receives a patent, say, in $\tau - 1$ to prefer to implement in period τ rather than immediately, it must be that

[46] As in Section 3.3, the no-storage condition requires that $R > 1$ and the TVC always holds.

$$\frac{\Pi_{\tau+1}}{R_{\tau-1}\,R_\tau} + \frac{\Pi_{\tau+2}}{R_{\tau-1}\,R_\tau\,R_{\tau+1}} > \frac{\Pi_\tau}{R_{\tau-1}} + \frac{\Pi_{\tau+1}}{R_{\tau-1}\,R_\tau}\,. \tag{3.83}$$

The first term on the LHS and the second on the RHS cancel out so that (3.83) becomes

$$\frac{\Pi_{\tau+2}}{R_{\tau-1}\,R_\tau\,R_{\tau+1}} > \frac{\Pi_\tau}{R_{\tau-1}}\,. \tag{3.84}$$

By juxtaposing (3.84) with (SPC1) , we can see that postponing implementation to the following period when the monopoly horizon is two periods is equivalent to postponing implementation to two periods afterwards in the context of the synchronized equilibrium in the one-period monopoly case.

Since in a stationary equilibrium interest rates and employment remain constant controlling for the period being odd or even, it follows that $\Pi_\tau = \Pi_{\tau+2}$. This implies that (3.84) becomes

$$R_\tau\,R_{\tau+1} < 1\,. \tag{3.85}$$

Turning to profit condition 2 ,[47] this is

$$\frac{\Pi_{\tau+1}}{R_{\tau-1}\,R_\tau} + \frac{\Pi_{\tau+2}}{R_{\tau-1}\,R_\tau\,R_{\tau+1}} \geq \frac{\Pi_{\tau+3}}{R_{\tau-1}\,R_\tau\,R_{\tau+1}\,R_{\tau+2}} + \frac{\Pi_{\tau+4}}{R_{\tau-1}\,R_\tau\,R_{\tau+1}\,R_{\tau+2}\,R_{\tau+3}}\,. \tag{3.86}$$

By stationarity, one can confirm that (3.86) simplifies to

$$\left(1 - \frac{1}{R_{\tau+1}\,R_{\tau+2}}\right)\left(\Pi_{\tau+1} + \frac{\Pi_{\tau+2}}{R_{\tau+1}}\right) \geq 0.$$

Given that it is not possible that firms make negative profits in equilibrium

[47]As it has been so far in this essay, whenever it comes to profit condition 2 , I ignore the possibility that new patents can render the ones in question obsolete, which would imply that profit condition 2 is more easily met.

since in that case they would rather not implement their patents, this expression becomes (recall that, by stationarity, $R_\tau = R_{\tau+2}$)

$$R_\tau \, R_{\tau+1} \geq 1 \,. \tag{3.87}$$

Juxtaposing (3.85) and (3.87) implies that synchronized implementation is *not* possible when patent rights last two periods.

Below I explore whether extending patent rights to two periods can lead to a welfare improvement.

3.5.3 Welfare

I showed above that only the immediate implementation equilibrium is possible when patent rights last two periods.[48] Then the agent's lifetime utility is

$$\tilde{U}_i = \frac{1}{1-\beta} \left(\frac{1}{2} \frac{\beta}{1-\beta} \log \mu^{\frac{1}{\alpha}-1} + \log \tilde{x}_1^i + \chi \log \tilde{l} \right) , \tag{3.88}$$

where $\tilde{l}$ is given by (3.82). I will compare (3.88) with the lifetime utility in the two equilibria which can prevail when firms profit out of a patent once. That is I will compare (3.88) with (3.68) and (3.67) in that order.

Welfare comparison with the synchronized implementation equilibrium

The difference in lifetime utilities is given by

[48] To be precise, I have failed to find an equilibrium besides this one.

$$U_s - \tilde{U}_i = \frac{1}{1-\beta}\left[\frac{\beta}{1+\beta}\left(\log \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)} - \log v\right) + \log x_1^s - \log \tilde{x}_1^i\right] +$$

$$+ \frac{\chi}{1-\beta}\left(\frac{\log l_{odd} + \beta \log l_{even}}{1+\beta} - \log \tilde{l}\right) . \tag{3.89}$$

I will take the same steps as in Section 3.4 . Starting with the leisure terms, one can confirm that $\tilde{l} > l_{even} > l_{odd}$. The first inequality follows by comparing (3.82) with (3.62) , whereas the second follows from Remark 2 . Then, although its role is non-pivotal in welfare comparisons, leisure utility is greater in the two-period-patent equilibrium.

Turning to consumption, by Claim 1 and as explained in Section 3.4 the growth terms together take a negative value. However, what is once again crucial for the welfare comparison outcome is the distance between the initial levels of consumption in the two equilibria given by (3.48) and (3.77) . It turns out that for sufficiently low values of the innovation rate, μ , initial consumption is greater in the two-period-patent equilibrium (see also Figure 3.1) . To show this, I will proceed into two steps.

Starting with initial output levels, we can see from (3.47) and (3.76) for $\psi = 1$ that

$$\Delta y \equiv y_1^s - \tilde{y}_1^i = \left[\mu^{-(\frac{1}{\alpha}-1)} + \mu^{-\frac{1}{2}(\frac{1}{\alpha}-1)}\right] (1-\alpha)^{\frac{1}{\alpha}-1} \left(\frac{1-l_{odd}}{R_{even}^{\frac{1}{\alpha}-1}} - \frac{1-\tilde{l}}{R^{\frac{1}{\alpha}-1}}\right) . \tag{3.90}$$

Taking into account that $\tilde{l} > l_{odd}$ and that, by Remark 3 , $R_{even} < R$ implies that the last term is positive, hence initial output is greater in the synchronized implementation, one-period-patent equilibrium than in the two-period-patent one. Furthermore, for the considered parametrization, the initial output difference increases in μ.

Turning to initial investment levels, we can see from (3.44) and (3.75) for $\psi = 1$ that

$$\Delta i \equiv i_1^s - \tilde{i}_1^i = \left[\frac{1}{\mu} + \mu^{-\frac{1}{2}(\frac{1}{\alpha}+1)}\right](1-\alpha)^{\frac{1}{\alpha}} \left[\mu^{\frac{1}{2}(\frac{1}{\alpha}+1)} \frac{1-l_{even}}{R_{odd}^{\frac{1}{\alpha}}} - \frac{1-\tilde{l}}{R^{\frac{1}{\alpha}}}\right]. \tag{3.91}$$

For the considered parametrization, the last term is positive and the initial (positive) investment difference grows in μ.[49]

It follows from the above that both initial output and investment are greater in the synchronized implementation (one-period-patent) equilibrium relative to the two-period-patent one. To inspect things a bit more, control for the differences in interest rates and employment, and observe that output is the same in the two equilibria, whereas investment in greater in the synchronized implementation one. The former is due to firms using the same level of capital in odd periods -again, controlling for interest rates and employment; to see this, observe that, in even periods, the implementing firms competitors' technology level is the same in both equilibria. In turn, this, of course, means that initial output differences are entirely attributed to differences in interest rates (general equilibrium effects) and employment rather than directly to technology/implementation reasons. It is the presence of volatility then that favors initial output in the synchronized implementation one-period-patent equilibrium.

Initial investment differences can be accounted for by cyclical as well

[49]In fact, the initial output and investment differences are positive for all the parametrizations that I have considered.

as -importantly- by technology/implementation differences directly. Starting with the latter, observe that, first, patents in sector 2 diffuse to the economy faster in the synchronized implementation one-period-patent equilibrium; second, patents in sector 1 are implemented earlier in the two-period immediate implementation equilibrium. Both remarks point to lower investment in the two-period-patent equilibrium. To see the first remark, simply juxtapose (3.42) and (3.73) and observe that -again after controlling for interest rates and leisure- more type-2 capital needs to be installed in the synchronized implementation one-period-patent equilibrium. The reason is that, even though sector-2 patents are implemented simultaneously in the two equilibria, firms appropriate them for one additional period in the two-period one. As a result, they become available to their competitors with a one-period delay. To see the second remark, simply recall that sector-1 patents are implemented immediately in the two-period-patent equilibrium, whereas they do so with a one-period delay in the synchronized implementation one-period-patent one (confirm this by juxtaposing (3.43) and (3.74)). Turning to interest rate and employment differences (cyclical/general equilibrium effects), they push in the opposite direction,[50] however without overturning the implications of the technology/implementation ones.

Is there a dominant effect? Not for the considered parametrization. At sufficiently low values of μ the "investment" effect dominates the "output" effect $(\Delta y < \Delta i)$, hence initial consumption is greater in the two-period-patent equilibrium. However, $\Delta y \,/\, \Delta i$ increases in μ and becomes greater

[50]That is, controlling for technology differences, investment is lower in the synchronized implementation one-period-patent equilibrium. To see this, note that, for all considered parametrizations, the expression $\left(\frac{1-l_{even}}{R_{odd}^{\frac{1}{\alpha}}} - \frac{1-\tilde{l}}{R^{\frac{1}{\alpha}}}\right)$ is negative. Furthermore, it decreases in μ.

than one for high enough values of it. The threshold value is $\mu^* \simeq 2.14$.

Since the role of initial consumption level is pivotal in welfare comparisons, they, in turn, will exhibit a nearly identical pattern: for sufficiently low values of μ, the two-period-patent equilibrium is Pareto-superior to the synchronized implementation one-period-patent equilibrium.[51] The threshold value in the welfare comparison is $\mu^{**} \simeq 1.95$ (see also Figure 3.1), which is lower than μ^* since the combination of the growth and the leisure effects favors the synchronized implementation equilibrium.

Welfare comparison with the immediate implementation equilibrium

The difference in lifetime utilities is given by

$$U_i - \tilde{U}_i = \frac{1}{1-\beta}\left[\log x_1^i - \log \tilde{x}_1^i + \chi\,(\log l - \log \tilde{l})\right]. \tag{3.92}$$

Starting with the leisure terms, we can confirm by comparing (3.40) and (3.82) that $\tilde{l} > l$. However, the effect of leisure in the welfare comparison is negligible.

Turning to consumption, I show in the Appendix that, for all parameter values, $x_1^i > \tilde{x}_1^i$. Noting that the sequence of interest rates is the same in the two equilibria, technology/implementation related reasons underlie this result. More precisely, patents might be (first) implemented at the same time in the two equilibria, however, in the two-period-patent equilibrium, firms can profit out of these for one additional period. This implies that -in the two-period-patent equilibrium- patents become available to the implementing firms' competitors and, hence, to the whole economy with a one-period delay.

[51] Of course, one needs to make sure first that condition (SPC1) is met so that a synchronized implementation equilibrium is possible.

As a result, the economy reaches a certain consumption level slower.

To see this, control for differences in employment and confirm that, in each period t, one type of capital, with the type alternating between odd and even periods, is greater by $\mu^{\frac{1}{\alpha}}$ in the one-period-patent equilibrium. On the one hand, this of course implies that output is greater in the one-period-patent equilibrium. Confirm, for instance, from (3.30) and (3.76) that $y_t = \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\,\tilde{y}_t$ at all t.[52] On the other hand, the one-period delay with which patents diffuse in the two-period-patent equilibrium results in lower investment. This is because less capital of some type needs to be installed in the following period. Once again controlling for differences in employment, we can see (for instance, from (3.27) and (3.75)) that $i_t > \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\,\tilde{i}_t$, for all t. That is extending patent rights to two periods implies that investment falls proportionally more relative to output. Nevertheless, these are in relative terms; in absolute terms the "output" effect always dominates the "investment" effect and consumption falls. Furthermore, the initial consumption difference grows in μ.[53]

With initial consumption's role being pivotal in the welfare comparison, welfare is greater in the one-period-patent immediate implementation equilibrium.

Combining the results above leads to the central policy conclusion of the essay:

Proposition 2. *For a sufficiently low innovation rate, extending the length of patent rights to two periods can lead to a welfare improvement.*

[52]Taking into account differences in employment would imply that $y_t > \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\,\tilde{y}_t$ since $1-l > 1-\tilde{l}$.

[53] This result holds for all the parametrizations that I have considered.

Initial level of investment

Proceeding as in Section 3.4.1 , to find the initial level of investment in the two-period-patent equilibrium, I set $\psi = 1$ and divide the RHS of (3.81) by $\mu^{\frac{1}{\alpha}-1}$. This yields

$$\tilde{i}_0^i = \left[\mu^{-\frac{1}{\alpha}} + \mu^{-\frac{1}{2}(\frac{1}{\alpha}+1)}\right] (1-\alpha)^{\frac{1}{\alpha}} \frac{1-\tilde{l}}{R^{\frac{1}{\alpha}}} . \tag{3.93}$$

For all considered parametrizations, initial investment in the two-period-patent equilibrium is lower than in both one-period-patent equilibria. Hence, the welfare improvement that Proposition 2 considers is unconditional on the initial level of investment.

3.6 Conclusion

This essay showed that implementation cycles in the presence of capital and the absence of borrowing constraints or constraints on investment volatility are possible. The reason is that patents are on investment-specific technological change which introduces a one-period discrepancy between the time a new patent is implemented and the time revenue out of it is realized.

Furthermore, the exogenous generation of patents permitted it to view patent rights from a different perspective. The "default" one concerns the incentives of innovators which, ultimately, affect the generation of patents. This essay abstracted from this, otherwise important and heated, debate and, instead, focused on the effects that the length of patent rights can have on the implementation of patents. In particular, it showed that a prolongation of patent rights eliminates implementation cycles and may lead to a welfare improvement.

The model, arguably highly stylized, certainly has its limitations. From a macro-perspective, consumption booms before output and is too volatile. From a more theoretical one, the analysis was silent about transitional dynamics, whereas, what is a related issue, the initial conditions were assumed to be the "right" ones. Resolving these issues could be part of future research.

3.7 Appendix: Proofs

Proof of Condition 1 (No storage) : There are two kinds of storable commodities in the economy: the capital goods and the final good. I deal with these in turn. The storage technology I assume in both cases is one-to-one.

Capital goods. Suppose that capital-good firm j produces an additional unit of capital good i in period $t-1$ and, instead of selling it to the final-good firm in period t, it instead stores and sells it in period $t+1$ (I assume that capital depreciates only if used). The cost of producing it as of $t-1$ is, say, $\frac{1}{\psi}$, whereas the revenue generated out of it as of $t-1$ is $\frac{q_{t+1}}{R_t\,R_{t-1}}$, where q is the competitive price offered by the final-good firm given by (3.14). No storage takes place if

$$\frac{R_t}{\psi\,R_t\,R_{t-1}} < \frac{1}{\psi}\,,$$

which is equivalent to

$$R_{t-1} > 1\,. \tag{3.94}$$

If, instead, the capital-good firm considers selling the additional unit of capital in period $t + 2$, then, ignoring the possibility that a new idea will render the one in question obsolete, the no-storage condition becomes

$$R_t \, R_{t-1} \, < \, 1 \, .$$

It follows then that (3.94) suffices to rule out storage in this case as well. Of course, if a new patent renders the one in question obsolete and supposing that the firm in question receives the new patent (see also the last paragraph in this proof), discounted revenue will be even lower and the no-storage condition will hold even more easily.

Proceeding in this way, (3.94) suffices to rule out storage and sale of a capital good in any period after period $t + 2$. Therefore, positive net interest rates rule out storage in equilibrium.

Let me underline that, in the above argument, I have implicitly assumed that a capital-good firm sells at least an infinitesimally small quantity of the capital good it specializes in in t. This implies that in case it possesses and makes use of a superior technology (patent), that becomes publicly available in t so that its competitors copy it and the competitive price prevails in $t{+}1$. And, of course, I rule out the possibility that a firm uses two different technologies at the same time.

I deal with the possibility that a capital-good firm possesses a superior technology and does not implement it in later sections of the main text and I label the respective conditions profit conditions 1 and 2. In other words, the profit conditions and the no-storage condition act in a somewhat complementary way. The former specify that a capital-good firm implements a patent and, since it maximizes profits, meets the whole demand for the type of capital it specializes in *when* it is conjectured to do so, whereas the latter rules out the possibility that it produces an additional amount of capital which it stores in order to sell it in a future period.

But still there is the possibility that a firm with a superior technology prefers to implement it in the following period, but considers using it immediately in secrecy aiming to sell the capital it produces using it in two periods. Given that it faces a given demand from the final-good firm and acts as a profit maximizer -that is it sells a certain profit-maximizing quantity-, for positive net real interest rates the discounted cost of producing it tomorrow is lower than today. Given that the price per unit is constant and common under both scenarios, once again positive net real interest rates rule this possibility out. As for the case in which that firm considers selling capital in any other future period (from the third period following the considered one onwards), see my analysis above.

Last, note that in some of the above arguments I assumed that the capital-good firm deciding whether to store capital or not will with certainty have the chance to produce capital in the future which, of course, need not be the case and which would render some of my above arguments irrelevant.

Final good. It is easy to confirm that condition (3.94) rules out storage of the final good as well.

Derivations in Section 3.3.2 :

Profit condition (3.58). Combining (3.53) and (3.56) and given that by stationarity $\frac{x_{\tau+1}}{x_\tau} = v$, $R_{\tau-1} = R_{\tau+1}$, and $l_\tau = l_{\tau+2}$ one can get

$$\frac{1-l_{\tau+1}}{(1-\alpha)\, R_\tau^{\frac{1}{\alpha}-1}} - \frac{\mu^{\frac{1}{\alpha}-1}(1-l_\tau)}{R_{\tau-1}^{\frac{1}{\alpha}}} = v\left(\frac{1-l_\tau}{(1-\alpha)R_{\tau-1}^{\frac{1}{\alpha}-1}} - \frac{1-l_{\tau+1}}{\mu\, R_\tau^{\frac{1}{\alpha}}}\right). \tag{3.95}$$

Rearranging terms in (3.95) yields

$$\left(\frac{1-l_{\tau+1}}{1-l_\tau}\right)\left(\frac{1}{1-\alpha} + \frac{v}{\mu R_\tau}\right) = \left(\frac{R_\tau}{R_{\tau-1}}\right)^{\frac{1}{\alpha}-1}\left(\frac{\mu^{\frac{1}{\alpha}-1}}{R_{\tau-1}} + \frac{v}{1-\alpha}\right). \tag{3.96}$$

Combining (3.96) with (3.23) implies that

$$\frac{1-l_{\tau+1}}{1-l_\tau} = \left(\frac{R_\tau}{R_{\tau-1}}\right)^{\frac{1}{\alpha}-1} R_\tau \left[\frac{\beta\,\mu\,(1+\beta(1-\alpha))}{\mu+\beta(1-\alpha)}\right]. \tag{3.97}$$

The profit condition (3.57) requires that

$$\frac{1-l_{\tau+1}}{1-l_\tau} > R_\tau \left(\frac{R_\tau}{R_{\tau-1}}\right)^{\frac{1}{\alpha}-1}.$$

Substituting in the LHS of (3.57) the RHS of (3.97), taking into account that R_τ and $R_{\tau+1}$ are both positive (since λ and v, given by (3.60) and (3.63), respectively, are positive) and rearranging yields the profit condition (3.58).

Leisure equations (3.61) and (3.62). Combining (3.6) with (3.9) and the production function (3.8), we get

$$\beta = \frac{y_{\tau+1}}{y_\tau}\,\frac{1-l_\tau}{1-l_{\tau+1}}\,\frac{l_{\tau+1}}{l_\tau}\,\frac{1}{R_\tau}. \tag{3.98}$$

Equations (3.52) and (3.55) together imply that

$$\frac{y_{\tau+1}}{y_\tau} = \frac{1-l_{\tau+1}}{1-l_\tau} \left(\frac{R_{\tau-1}}{R_\tau}\right)^{\frac{1}{\alpha}-1} . \tag{3.99}$$

Substituting (3.99) into (3.98) yields

$$\frac{l_{\tau+1}}{l_\tau} = \beta\, R_\tau \left(\frac{R_\tau}{R_{\tau-1}}\right)^{\frac{1}{\alpha}-1} . \tag{3.100}$$

Combining (3.97) and (3.100) results in

$$\frac{1-l_{\tau+1}}{l_{\tau+1}} = \frac{1-l_\tau}{l_\tau} \left[\frac{\mu\,(1+\beta(1-\alpha))}{\mu+\beta(1-\alpha)}\right] . \tag{3.101}$$

The intratemporal optimality condition of the household in period τ is

$$\chi\,\frac{x_\tau}{l_\tau} = w_\tau\,, \tag{3.102}$$

where x_τ is given by (3.53).

Labor demand from the final-good firm (see (3.9)) is $w_\tau = \frac{\alpha\, y_\tau}{1-l_\tau}$, with y_τ given by (3.52). Combining these and substituting on the RHS of (3.102) yields

$$\frac{1}{l_\tau} \left[\frac{1-l_\tau}{(1-\alpha)R_{\tau-1}^{\frac{1}{\alpha}-1}} - \frac{1-l_{\tau+1}}{\mu R_\tau^{\frac{1}{\alpha}}}\right] = \frac{\alpha}{\chi\,(1-\alpha)}\,\frac{1}{R_{\tau-1}^{\frac{1}{\alpha}-1}} . \tag{3.103}$$

Multiplying both sides of (3.103) by $R_{\tau-1}^{\frac{1}{\alpha}-1}$ and using (3.100) yields

$$\frac{1-l_{\tau}}{l_{\tau}}\,\frac{1}{1-\alpha}-\frac{1-l_{\tau+1}}{l_{\tau+1}}\frac{\beta}{\mu}=\frac{\alpha}{\chi\,(1-\alpha)}\,. \tag{3.104}$$

Using (3.101) to substitute for $\frac{1-l_{\tau+1}}{l_{\tau+1}}$ in (3.104) results in (3.62). In turn, inserting (3.62) into (3.101) yields (3.61).

Derivation of (3.63). Substituting for the interest rates given by (3.23) into (3.100) and rearranging implies

$$v=\left(\mu^{(\frac{1}{\alpha}-1)^{2}}\,\frac{l_{\tau+1}}{l_{\tau}}\right)^{\frac{\alpha}{2-\alpha}}\,. \tag{3.105}$$

To find $\frac{l_{\tau+1}}{l_{\tau}}$, divide (3.61) by (3.62) across sides which yields

$$\frac{l_{\tau+1}}{l_{\tau}}=\frac{\frac{\alpha}{\chi}\,\frac{\mu+\beta(1-\alpha)}{\mu-\beta^{2}(1-\alpha)^{2}}+1}{\frac{\alpha}{\chi}\,\frac{\mu\,(1+\beta(1-\alpha))}{\mu-\beta^{2}(1-\alpha)^{2}}+1}\,. \tag{3.106}$$

Inserting (3.106) into (3.105) results in (3.63).

Proof of Claim 1: As the fraction term in (3.63) is lower than one, it suffices to show that

$$\mu^{(\frac{1}{\alpha}-1)^{2}\,\frac{\alpha}{2-\alpha}}<\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}\,.$$

It is straightforward to confirm that this is true.

Transversality Condition: The transversality condition (TVC) requires the agent's present discounted lifetime wealth to converge. I explore it for the synchronized implementation equilibrium.

The first step is to find the present discounted lifetime wealth of the agent. The agent's wealth consists of his labor income (DLI) and profits ($D\Pi$) out of the ownership of capital-good firms. I check both in turn.

The present discounted lifetime labor income is

$$DLI = w_1 n_1 + \frac{w_2 n_2}{R_1} + \frac{w_3 n_3}{R_1 R_2} + \dots \quad .$$

Using the fact that, from the final-good firm's problem, $w_t = \alpha \frac{y_t}{n_t}$ and that, by stationarity, $n_1 = n_3 = \dots$, $n_2 = n_4 = \dots$, $R_1 = R_3 = \dots$ and $R_2 = R_4 = \dots$ the above expression simplifies to

$$DLI = n_1 \left[\frac{\alpha y_1}{n_1} + \frac{\alpha y_3}{n_1 R_1 R_2} + \frac{\alpha y_5}{n_1 (R_1 R_2)^2} \dots \right] +$$

$$+ \frac{n_2}{R_1} \left[\frac{\alpha y_2}{n_2} + \frac{\alpha y_4}{n_2 R_1 R_2} + \frac{\alpha y_6}{n_2 (R_1 R_2)^2} \dots \right] .$$

Since output grows by $\mu^{\frac{1}{\alpha}-1}$ every two periods, the above expression boils down to

$$DLI = \alpha (y_1 + \frac{y_2}{R_1}) \left[1 + \frac{\mu^{\frac{1}{\alpha}-1}}{R_1 R_2} + \left(\frac{\mu^{\frac{1}{\alpha}-1}}{R_1 R_2} \right)^2 + \dots \right] .$$

Turning to present discounted profits and noting that they grow by $\mu^{\frac{1}{\alpha}-1}$ every two periods,

$$D\Pi = \frac{\Pi_{3,1} + \Pi_{3,2}}{R_1 R_2} \left[1 + \frac{\mu^{\frac{1}{\alpha}-1}}{R_1 R_2} + \left(\frac{\mu^{\frac{1}{\alpha}-1}}{R_1 R_2} \right)^2 + \dots \right] .$$

Given that $R_1 R_2 = \frac{\mu^{\frac{1}{\alpha}-1}}{\beta^2}$, the above expressions converge, hence, the TVC is always satisfied.

Proceeding in the same way, it is straightforward to check that the TVC is also always satisfied in the case of the immediate implementation equilibrium in which $R = \frac{\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)}}{\beta}$.

Proof in Section 3.5.3: Subtracting (3.77) from (3.31) -having set $\psi = 1$ in both- and taking into account that $1-l > 1-\tilde{l}$ implies that

$$x_1^i - \tilde{x}_1^i > (1-\alpha)^{\frac{1}{\alpha}} R^{-\frac{1}{\alpha}} (1-\tilde{l}) \left(\frac{R}{1-\alpha} \left[1 - \mu^{-(\frac{1}{\alpha}-1)} \right] - \left[\mu^{\frac{1}{2}(\frac{1}{\alpha}-1)} - \mu^{-\frac{1}{2}(\frac{1}{\alpha}+1)} \right] \right)$$

$$= (1-\alpha)^{\frac{1}{\alpha}-1} R^{-\frac{1}{\alpha}} (1-\tilde{l}) \left(R \left[1 - \mu^{-(\frac{1}{\alpha}-1)} \right] - (1-\alpha)\, \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)} \left(1 - \mu^{-\frac{1}{\alpha}} \right) \right)$$

$$> (1-\alpha)^{\frac{1}{\alpha}-1} R^{-\frac{1}{\alpha}} (1-\tilde{l})\, \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)} \left[1 - \mu^{-(\frac{1}{\alpha}-1)} - (1-\alpha) \left(1 - \mu^{-\frac{1}{\alpha}} \right) \right]$$

$$= (1-\alpha)^{\frac{1}{\alpha}-1} R^{-\frac{1}{\alpha}} (1-\tilde{l})\, \mu^{\frac{1}{2}(\frac{1}{\alpha}-1)} \left[\alpha - \mu^{-(\frac{1}{\alpha}-1)} + (1-\alpha)\, \mu^{-\frac{1}{\alpha}} \right].$$

In the third line, I use the fact that $\beta \in (0,1)$ taking into account that $\mu > 1$ and $\alpha \in (0,1)$, which imply that $1 - \mu^{-(\frac{1}{\alpha}-1)} > 0$.

The next step is to show that

$$\alpha - \mu^{-(\frac{1}{\alpha}-1)} + (1-\alpha)\, \mu^{-\frac{1}{\alpha}} \geq 0,$$

or equivalently that

$$\alpha\, \mu^{\frac{1}{\alpha}} + 1 - \alpha \geq \mu. \tag{3.107}$$

With $\mu > 1$ and $\alpha \in (0,1)$, one can confirm that eq. (3.107) is always true. Hence, $x_1^i > \tilde{x}_1^i$, as desired.

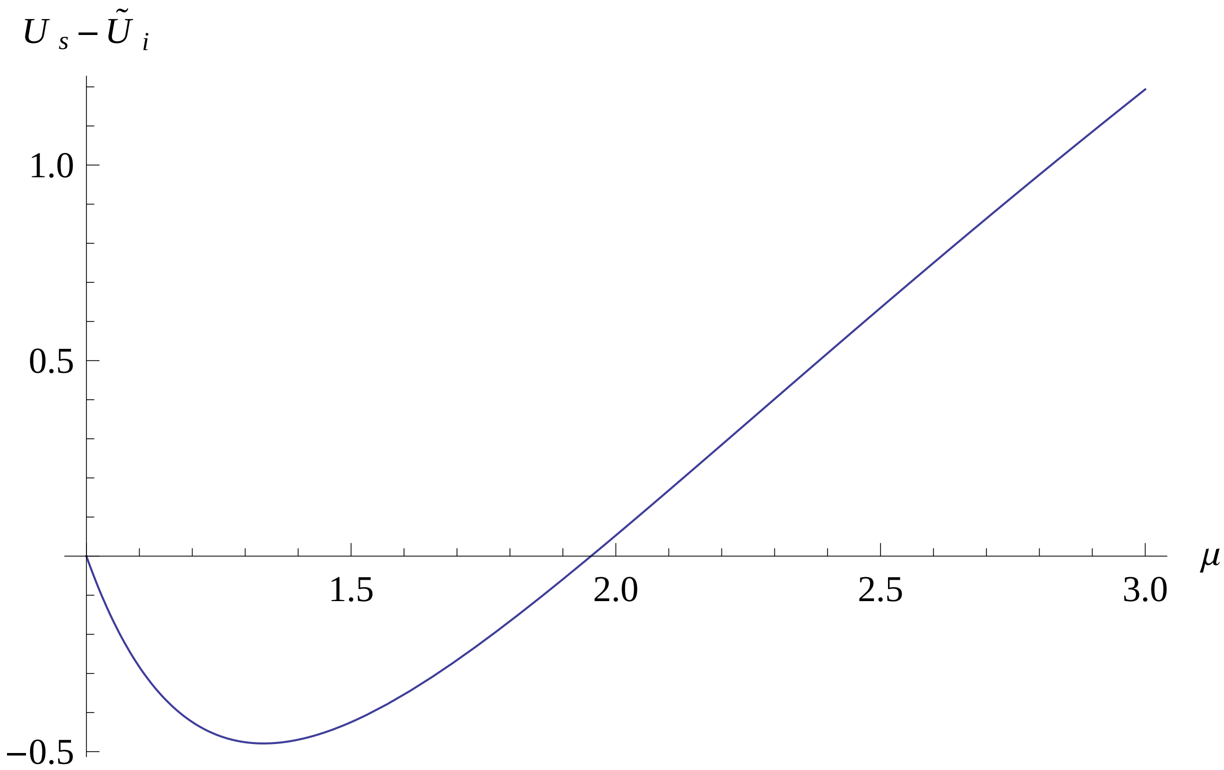


Figure 3.1: Welfare comparison between the synchronized implementation equilibrium and the two-period patent equilibrium

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