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Optimal Procurement with Auditing and Bribery

A thesis submitted in partial fulfilment of

the requirements for the degree of

Doctor of Philosophy in Economics

by

Ruben Pastor Vicedo

Department of Economics

University of Warwick

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Contents

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Finally, I express my gratitude to the many relatives and friends that have shown an interest in my progress during the writing of this thesis. Any attempt at listing all of them would be unsuccessful. I will therefore not make such an attempt with the hope that they see themselves reflected in these lines. I could not however avoid explicitly expressing my heartfelt thank you to my parents, Vicent Pastor LLuch and Pilar Vicedo Galbis, to whom I dedicate this thesis.

Declaration

I submit this thesis to the University of Warwick in accordance with the requirements for the degree of Doctor of Philosophy. I declare that this thesis is my own work and has not been submitted for a degree at another university.

Ruben Pastor Vicedo

September 2012

Abstract

In this thesis I characterise an optimal procurement contract for a government that purchases a good or service from a firm that has private information about its cost of production (its type) when the government has available the reports of a corruptible internal auditor and an honest but less well informed external auditor.

In chapter 2 I assume that the government is constrained to offer the internal auditor a contract that consists of a penalty if the external auditor obtains evidence of misreporting. For the case of two cost types I show that an optimal contract exhibits a separation property: the government gives priority to achieving the first best (no private information) expected profit scheme over demanding the first best quantity scheme. For the case of a continuum of cost types I provide sufficient conditions under which this result is valid.

In chapter 3 I allow the government to offer the internal auditor a contract that consists of a transfer, a reimbursement and a penalty. For the situation in which bribery takes place after the firm makes a claim about its type I demonstrate that the government can achieve the outcome of the first best contract if the sum of the expected penalties is positive and for every type of the firm the distribution of the outcome of the audit is not the same as that of the adjacent type. For the situation in which bribery takes place before the firm makes a claim about its type I argue that the contract design problem is the same as in chapter 2 and I prove that if the sum of the expected penalties does not depend on the extent of the misreporting then in an optimal contract bribery does not take place.

Chapter 1

Overview

1.1 Motivation and research questions

It has long been recognized that governments have less accurate information about the cost of production of goods and services than the firms that supply them. Faced with this asymmetry of information, an optimal strategy for a government is to commit to a transfer scheme and a quantity scheme that induces a firm to make a truthful claim about its private information. As the interests of the government and the firm generally do not coincide, this second best contract fails to achieve the outcome of the first best contract that the government would offer if it had the same information as the firm (see Baron and Myerson (1982)).

To support the government, an audit agency (henceforth referred as an auditor) is often charged with gathering the information correlated with the cost of production of the firm that becomes available after production takes place. By requiring a reimbursement that is contingent on the report of the auditor about the outcome of the audit, the government can then alter the incentive of the firm to make a truthful claim about its private information.

This allows the government to approximate or even attain the outcome of its first best contract (see Baron and Besanko (1984) and Riordan and Sappington (1988)). As an illustration, the Defense Contract Audit Agency performs audits for the U.S. Department of Defense. In the financial year 2011 it "examined over $128 billion in defense contractor costs and issued over 7,000 audit reports. These reports recommended $11.9 billion in cost reductions" (see U.S. Department of Defense (2012, page 2)).

A major concern in the design of government contracts is that the firm, having observed the outcome of the audit, attempts to bribe the auditor to misreport it. This misreporting undermines the informativeness of auditing information, as it distorts the correlation between the private information of the firm and the report of the auditor. The government often tries to prevent it by making contracting between the firm and the auditor illegal, so that they cannot appeal to a court of law to enforce a bribery agreement. Nevertheless, when the auditor is industry specific, the repeated interaction between the auditor and the firm might make a bribery agreement enforceable through a reputation mechanism (see Tirole (1992) and Martimort (1999)).

The common institutional response to the threat of bribery is then to verify the report of the internal (industry specific) auditor with that of an external (non sectoral) auditor that is less prone to bribery due to its limited relationship with the firm. If the external auditor obtains evidence of misreporting then the government requests that the firm and the internal auditor pay a penalty. For the case of the Defense Contract Audit Agency, the external auditing is performed by the Government Accountability Office. Examples of other organisations that play the role of an external auditor include the National Audit Office in the U.K., the Bundesrechnungshof in Germany and the Cour des Comptes in France.

In this thesis I characterise an optimal procurement contract for a government that purchases a good or service from a firm that has private information about its cost of production when the government has available the reports of a corruptible internal auditor and an honest but less well informed external auditor. More precisely, I provide answers to the following questions: 1) What quantity does the government demand? 2) When does the government require a reimbursement and what amount does it request? 3) What expected profit does the firm make?

1.2 Methodology and structure of the thesis

The methodology that I employ to characterise an optimal procurement contract is due to Tirole (1986) and it consists of two steps. Applied to the contract design problem under consideration, it proceeds as follows: In the first step take as given the contract that the government offers to the firm and the internal auditor and determine the bribery agreement that the firm offers to the internal auditor. In the second step treat the optimal bribery agreement as a constraint in the contract design problem of the government and optimise to find the contract that the government offers to the firm and the internal auditor.

The fundamental assumption in the methodology that I have just described is that the bribery agreement (henceforth referred as the side contract) is enforceable even if it is illegal. The benefit of this assumption is that it allows the use of optimization theory to determine first the optimal side contract and then the optimal contract that the government offers to the firm and the internal auditor. Tirole argues that an alternative, more fundamental approach, that does not employ this assumption "traces the foundations of enforceability to repeated interaction and reputation" (see Tirole (1992, page 156)). This approach requires the use of relatively complex techniques (dynamic mechanism design with incentive constraints defined by equilibria of a repeated game). The fact that many of the lessons obtained with the enforceable approach seem to remain valid for this approach has led most research on bribery to discard it (see Martimort (1999) for an exception).

As a preliminary step to determine an optimal side contract, it is necessary to specify the contract that the government offers to the firm and the internal auditor. I assume for the entire thesis that the government offers a contract to the firm that consists of a transfer, a quantity, a reimbursement and a penalty. The transfer and the quantity depend on the claim of the firm about its type. The reimbursement depends on the claims of the firm about its type and about the outcome of the audit and on the report of the internal auditor. The firm pays the penalty if the external auditor obtains evidence of misreporting.

In chapter 2 I assume that the government is constrained to offer the internal auditor a contract that consists of a penalty if the external auditor obtains evidence of misreporting. I refer to this contract as non contingent. The benefit of restricting attention to a non contingent contract for the internal auditor is that there is no need to consider a side contract that is contingent on the claims of the firm, as the payoff of the internal auditor does not depend on them. The only relevant side contract consists of the firm requesting a report from the internal auditor and paying a bribe if the internal auditor complies with the request.

In chapter 3 I allow the government to offer the internal auditor a contract that consists of a transfer, a reimbursement and a penalty that are a function of the same contingencies as the transfer, the reimbursement and the penalty that the government offers to the firm. I refer to this contract as contingent. The consideration of a contingent contract for the internal auditor leads to several possible side contracts, depending on the contracting variables. I assume that the firm can offer the internal auditor a side contract that includes the claims of the firm from the point in time at which the offer takes place and the report of the internal auditor. I then consider two possible situations. In the first situation side contracting takes place after the firm makes a claim about its private information (so the side contract is not contingent on this claim) but before it makes a claim about the outcome of the audit. In the second situation side contracting takes place before the firm makes a claim about its private information. I refer to these two situations as ex post and ex ante side contracting respectively.

1.3 Preview of the results

1.3.1 Preview of the results in chapter 2

If the government offers the internal auditor a non contingent contract then an optimal side contract for the firm consists of paying a zero bribe if it requests the internal auditor to report truthfully and a bribe equal to the expected penalty that the internal auditor pays otherwise. The contract design problem is then as if the firm controls the report of the internal auditor at a cost of misreporting equal to the sum of the expected penalties.[1]

I first characterise an optimal contract for the case in which the private information of the firm (henceforth referred as its type) takes two possible values, with a low value denoting a lower cost of production than a high value for any given quantity. In that situation the contract design problem consists

[1] I initially assume that the firm and the internal auditor are risk neutral. In section 2.7 I discuss how the results differ if either or both of them are risk averse.

of determining how to optimally prevent the firm when its type is low from claiming that it is high.

In Proposition 1 I show that for the case of two cost types the government requires a reimbursement when the firm claims that its type is high if and only if the outcome of the audit is more likely to occur when the type of the firm is low than when it is high. The reimbursement is not greater than the sum of the expected penalties, so that misreporting does not take place. If the audit is sufficiently informative of the type of the firm or the sum of the expected penalties is sufficiently high then the government offers a contract that results in the first best (no private information) expected profit scheme and quantity scheme. Otherwise an optimal contract exhibits a separation property: the government gives priority to achieving the first best expected profit scheme over demanding the first best quantity scheme.

I then characterise an optimal contract for the case in which the type of the firm takes a continuum of possible values. This is a more complex task than the previous one, as it requires determining how to optimally prevent the firm from making a false claim for each possible type. I proceed in two steps, each ending with a proposition.

In Proposition 2 I focus on how to optimally prevent the firm from marginally exaggerating its type. I provide a sufficient condition under which the logic of Proposition 1 remains valid. More precisely, if this condition is satisfied then the government requires a reimbursement for any claim of the firm about its type if and only if the outcome of the audit is more likely to occur when the type of the firm is marginally lower than the type that the firmed claimed to have. Misreporting does not take place and the government gives priority to achieving the first best expected profit scheme over achieving the first best quantity scheme.

In Proposition 3 I then provide sufficient conditions under which preventing the firm from marginally exaggerating its type implies preventing the firm from making a false claim about its type. This ensures that the contract that I characterise in my second proposition is an optimal contract.

1.3.2 Preview of the results in chapter 3

If the government offers the internal auditor a contingent contract then the government can ensure that the internal auditor reports truthfully when it rejects an ex post side contract by requiring a reimbursement from the internal auditor that is not contingent on its report. As a result, in an optimal ex post side contract the firm pays the internal auditor a zero bribe if it requests the internal auditor to report truthfully and a bribe equal to the difference in reimbursement plus the expected penalty that the internal auditor pays otherwise. The contract design problem is then as if the firm selects its claim about the outcome of the audit and the report of the internal auditor to minimise the sum of the reimbursements plus the sum of the expected penalties.

In Proposition 4 I demonstrate that when side contracting takes place ex post the government can achieve the outcome of the first best contract if the sum of the expected penalties is positive and for every type of the firm the distribution of the outcome of the audit is not the same as that of the adjacent type.[2] The government offers the firm a reimbursement scheme that prevents the firm from making a false claim for the first best quantity scheme and expected profit scheme. The government ensures that bribery does not take place by giving the internal auditor the reimbursement that the firm pays. This is not costly for the government as for every type of the firm the government

[2]For the case of a continuum of types this proposition involves some of the conditions in Proposition 3.

pays the internal auditor a transfer equal to the expected reimbursement that the internal auditor pays.

If the government offers the internal auditor a contingent contract and side contracting takes place ex ante I assume that the internal auditor observes the type of the firm. This assumption ensures that in an optimal ex ante side contract the firm pays the internal auditor an expected bribe such that the expected payoff of the internal auditor is the same as if it rejects the side contract. The contract design problem is then as if the firm selects its claims about its type and about the outcome of the audit and the report of the internal auditor to maximise the sum of their expected payoffs.

The claims and the report that the firm selects with ex ante side contracting depend on the sum of the transfers and the sum of the reimbursements but not on how they add up. Therefore with ex ante side contracting it is optimal for the government to offer the internal auditor a non contingent contract. The contract design problem is then as in chapter 2: the firm controls the report of the internal auditor at a cost of misreporting equal to the sum of the expected penalties. An optimal contract is then given by Proposition 1 for the case of two cost types and by Propositions 2 and 3 for the case of a continuum of cost types. I employ this section to address a question that I ignored in chapter 2: whether bribery takes place in an optimal contract.

In Proposition 5 I prove that if the sum of the expected penalties that the firm and the internal auditor pay is independent of the extent of the misreporting then the government can achieve the optimal quantity scheme and expected profit scheme of a contract for which bribery takes place with a contract for which bribery does not take place. As bribery is costly for the government, it is then optimal for the government to deter it. This result is valid regardless of the the distribution of the outcome of the audit and of the belief of the government about the type of the firm.

1.4 Relationship with existing research

1.4.1 Auditing and bribery

Research on auditing and bribery began with the work of Tirole (1986) on the formation of coalitions in hierarchies. Tirole argued that "behaviour is often best predicted by the analysis of group as well as individual incentives" (see Tirole (1986, page 181)). To illustrate this idea, Tirole developed a principal, supervisor and agent model that is applicable to a wide variety of situations (including procurement). The agent (the firm) is the productive unit, with a privately known productivity (its type) that takes two possible values. The principal (the government) receives an output equal to the sum of the type of the agent and its unobservable effort. The supervisor (the auditor) observes either the type of the agent or nothing. In the former case the agent can bribe the supervisor to report that it has observed nothing.

Tirole characterised the optimal contract that the principal offers to the agent and the supervisor. He showed that this contract prevents bribery but it does not achieve the same outcome as when bribery is not an issue. His analysis is the basis of the two articles on auditing and bribery that are most closely related to the research in this thesis. I discuss each of them in turn.[3]

Kofman and Lawarree (1993) introduced the distinction between a corruptible internal auditor and an honest external auditor that I employ in this thesis. They analysed a contract design problem that features a principal,

[3]Armstrong and Sappington (2007) and Dal Bo (2006) surveyed research on auditing and bribery in the context of regulation. Mookherjee (2006) performed the same task in the context of organisational design, with a focus on whether decentralisation is an optimal response to bribery.

an agent with a privately known productivity (its type), an internal auditor and a costly external auditor. As in the model of Tirole, the type of the agent takes two possible values and the principal receives an output equal to the sum of the type of the agent and its unobservable effort. The agent and the internal auditor observe the outcome of an audit that takes two possible values. The principal pays the internal auditor a reward if the agent claims that its type is low and the internal auditor reports the outcome of the audit that is more likely to occur when the type of the agent is high. The agent can bribe the internal auditor to misreport the outcome of the audit. The external auditor observes the same outcome of the audit as the internal auditor if it exerts a positive effort and it observes nothing otherwise. If the external auditor reports a different outcome of the audit than the internal auditor then the agent and the internal auditor pay a penalty.

Kofman and Lawaree concentrated on characterising the optimal trade off between the cost of hiring the external auditor and the benefit of employing its report to eliminate the threat of bribery. By contrast, I assume that the external auditor is as costly as the internal auditor but less well informed and I focus on obtaining an optimal contract under more general assumptions than theirs. My analysis is broader than theirs in three aspects. First, I consider the case of a continuum of types and I provide sufficient conditions under which an optimal contract has the same structure as for the case of two types (Propositions 2 and 3). Second, in chapter 3 I let the government offer a contingent contract to the internal auditor and I clarify the conditions under which this contract achieves the outcome of the first best contract (Proposition 4). Third, I allow the outcome of the audit to take an arbitrary (but finite) number of possible values and I characterise when bribery does not take place in an optimal contract (Proposition 5).

Bac and Kucuksenel (2006) emphasised the distinction between ex post and ex ante side contracting. They extended the model of Tirole (1986) by assuming that the supervisor has to exert an unobservable effort to obtain its information. They then allowed the agent to offer the supervisor an ex ante side contract that consists of a bribe in exchange for the commitment of the supervisor to not monitor.

Bac and Kucuksenel were interested in how ex ante side contracting affects the optimal contract with ex post side contracting that Tirole characterised. They found that if monitoring costs are small and the probability of detection is large then the optimal contract with ex post side contracting remains optimal. My definition of ex ante side contracting (given in section 1.2) differs from theirs and as a result their conclusions do not apply. In particular, assuming that auditing is costless I find that ex ante side contracting leads to a strictly worse outcome for the government than ex post side contracting unless the audit is sufficiently informative of the type of the firm or the sum of the expected penalties is sufficiently high (Propositions 4 and 5).

1.4.2 Auditing and costly misreporting

Research on auditing with costly misreporting does not consider bribery explicitly and instead assumes that the agent controls the report of the auditor at a cost of misreporting. Several articles following this approach have more general assumptions on the distribution of the type of the agent than the research that I discussed in the previous subsection. They are then more relevant to understand my result that in an an optimal contract bribery does not take place if the expected penalties that the firm and the internal auditor pay are independent of the extent of the misreporting (Proposition 5). I now describe the two articles on auditing with costly misreporting that are most closely related to the research in this thesis.

Laffont and Tirole (1986) provided the most well known analysis on auditing with costly misreporting in the context of procurement. They studied a model in which a government contracts with a firm for the production of a good or service. The firm has a privately known efficiency parameter (its type) that takes a continuum of possible values. Before production takes place, the firm can exert an unobservable effort that reduces its expected average cost below its type. The government offers the firm a contract that consists of a transfer, a quantity and a reimbursement. The transfer and the quantity are a function of the claim of the firm about its type while the reimbursement depends on the previous claim and on the cost of production of the firm, which the government observes.

The effort that the firm exerts in the model of Laffont and Tirole can be seen as an action that leads to the misreporting of the expected average cost of the firm. The fundamental difference between effort in the model of Laffont and Tirole and bribery in my model is that effort lowers the cost of production of the firm whereas bribery does not. It is then not surprising that Laffont and Tirole found that the optimal contract induces the firm to exert a positive amount of effort whereas I find that bribery does not take place in an optimal contract. What might come as a surprise is that my result requires the condition that the expected penalties that the firm and the internal auditor pay are independent of the extent of the misreporting. To explain the need for this condition I turn to the next article.

Maggi and Rodriguez-Clare (1995) considered a procurement model with auditing and costly misreporting that it closest to mine with respect to the payoffs of the principal and the agent and the information that they observe. The agent produces a good or service for the principal. The agent has a privately known marginal cost (its type) that takes a continuum of possible values. After production takes place, an audit results in a signal that has a one to one relationship with the type of the agent. The agent can then distort it by performing a costly action before the principal observes a report about it. The principal offers the agent a contract that consists of a transfer, a quantity and a reimbursement. The transfer and the quantity are a function of the claim of the agent about its type. The principal requests a zero reimbursement if the report is the one that it anticipates given the claim of the agent about its type. Otherwise the principal requests an arbitrarily high reimbursement.

Maggi and Rodriguez-Clare demonstrated that in their model misreporting takes place in an optimal contract if there is no fixed cost of misreporting and the variable cost is convex. The reason is that by tolerating misreporting the principal makes it costlier for the agent to make a false claim about its type. This allows the principal to reduce the transfer that it pays the agent when it makes a truthful claim. I do not assume that the outcome of the audit has a one to one relationship with the type of the firm so the result of Maggi and Rodriguez-Clare does not apply to my analysis.

1.4.3 Auditing and constraints on payments

Research on auditing with constraints on payments characterises an optimal contract under the assumption that the reimbursement that the principal requests from the agent has an upper bound due to either legal restrictions or limited liability. Several articles following this approach consider the case of a continuum of types and have more general assumptions on the distribution of the outcome of the audit than those articles that I discussed in the two previous subsections. They are then more relevant to understand my characterisation of an optimal contract with a continuum of types (Propositions 2 and 3). I now describe the two articles on auditing with constraints on payments that are most closely related to the research in this thesis.

Baron and Besanko (1984) presented the most general characterisation of an optimal contract with auditing and constraints on payments in the context of regulation. They considered a model in which a government regulates the fixed fee and the unit price that a firm charges to its customers. The firm has private information (its type) that takes a continuum of possible values and is correlated with its cost, which the government observes if it hires an auditor. The government offers the firm a contract that consists of a fixed fee, a unit price and a reimbursement. The fixed fee and the price are a function of the claim of the firm about its type while the reimbursement depends on the previous claim and on the cost of production of the firm if the government hires an auditor. The government is restricted to request a reimbursement not greater than a legally specified amount.

Baron and Besanko proved the Maximum Punishment Principle: the government requests the highest possible reimbursement for any claim of the firm about its type if and only if the outcome of the audit is more likely to occur when the type of the firm is marginally lower than the type that the firm claimed to have. In my analysis the highest possible optimal reimbursement is a function of how well informed the external auditor is. In addition, to provide the remaining characterisation of how to optimally prevent the firm from marginally exaggerating its type (Proposition 2) I do not assume a specific distribution of the outcome of the audit whereas they assumed a normal distribution. Also, I am able to provide sufficient conditions under which preventing the firm from marginally exaggerating its type implies preventing the firm from making a false claim about its type (Proposition 3) whereas they only provided an example where this is true for some parameter configurations.

Gary-Bobo and Spiegel (2006) studied a model of procurement in which a government contracts with a firm for the production of a good or service. The firm has private information (its type) that takes a continuum of possible values and is correlated with its cost, which the government observes. The government offers the firm a contract that consists of a transfer, a quantity and a reimbursement. The transfer and the quantity are a function of the claim of the firm about its type while the reimbursement depends on the previous claim and on the cost of production of the firm. The government is restricted to request a reimbursement such that the expected profit of the firm is not lower than its liability.

Gary-Bobo and Spiegel established sufficient conditions under which the government can achieve the outcome of the first best contract in their model. Unlike Baron and Besanko, they did not restrict attention to a particular distribution of the outcome of the audit. However, they incorrectly argued that limited liability makes it optimal for the government to request the highest possible reimbursement for all the outcomes of the audit except the outcome that is most likely for the type that the firm claimed to have. More precisely, they ignored that with limited liability there is a trade off between requiring a reimbursement for as many outcomes as possible to request an amount below the limited liability of the firm and requiring a reimbursement for those outcomes of the audit that are more likely to occur for a marginally higher type to prevent the firm from marginally exaggerating its type. As a result of this omission, their characterisation of an optimal contract with a continuum of types substantially differs from mine.

Chapter 2

Optimal Procurement with a Non Contingent Contract for the Internal Auditor

2.1 Introduction

In this chapter I characterise an optimal procurement contract for a government that purchases a good or service from a firm that has private information about its cost of production when the government has available the reports of a corruptible internal auditor and an honest but less well informed external auditor.[1] I do so assuming that the government is constrained to offer the internal auditor a contract that consists of a penalty if the external auditor obtains evidence of misreporting. In that situation, an optimal side contract for the firm consists of paying a zero bribe if it requests the internal auditor to report truthfully and a bribe equal to the expected penalty that the internal auditor pays otherwise.[2] The contract design problem is then as if the firm

[1]For an example of this contract design problem see section 1.1.
[2]For a discussion of contracting and side contracting in this thesis see section 1.2.

controls the report of the internal auditor at a cost of misreporting equal to the sum of the expected penalties that the firm and the internal auditor pay if the external auditor obtains evidence of misreporting.

I first characterise an optimal contract for the case in which the cost type of the firm takes two possible values, with a low value denoting a lower cost of production than a high value for any given quantity. In Proposition 1 I show that, in order to prevent the firm when its type is low from claiming that it is high, the government requires a reimbursement when the firm claims that its type is high if and only if the outcome of the audit is more likely to occur when the type of the firm is low than when it is high. The reimbursement is not greater than the sum of the expected penalties, so that misreporting does not take place. If the audit is sufficiently informative of the type of the firm or the sum of the expected penalties is sufficiently high then the government offers a contract that results in the first best (no private information) expected profit scheme and quantity scheme. Otherwise an optimal contract exhibits a separation property: the government gives priority to achieving the first expected profit when the type of the firm is low over demanding the first best quantity when the type of the firm is high.[3]

To evaluate the robustness of the previous results I consider the case of a continuum of cost types. In order to characterise an optimal contract I follow the differential approach to contracting of Laffont and Maskin (1980): First I obtain a differential constraint that is necessary to prevent the firm from marginally exaggerating its type. Then I characterise an optimal contract in the contract design problem that results from replacing the constraint that

[3] If an audit is not available then it is well known that the government demands the first best quantity when the type of the firm is low and a second best quantity below the first best quantity when the type of the firm is high. The firm makes the first best expected profit when its type is high and a second best expected profit above the first best expected profit when its type is low (see Baron and Myerson (1982)).

the firm makes a truthful claim about its type with the differential constraint (henceforth referred as the relaxed problem). Finally I verify that an optimal contract in the relaxed problem satisfies the constraint that the firm makes a truthful claim about its type and is therefore an optimal contract.

In Proposition 2 I provide a sufficient condition under which the logic of Proposition 1 remains valid for the relaxed problem. More precisely, if this condition is satisfied then in an optimal contract the government requires a reimbursement for any claim of the firm about its type if and only if the outcome of the audit is more likely to occur when the type of the firm is marginally lower than the type that the firmed claimed to have. Misreporting does not take place and an optimal contract exhibits a separation property: for every type of the firm the government gives priority to achieving the first best marginal difference in expected profit over demanding the first best quantity.

In Proposition 3 I address a common difficulty of auditing models with more than two types: requiring that the payoff of the firm has the Spence-Mirrlees property does not suffice to verify that an optimal contract in the relaxed problem satisfies the constraint that the firm makes a truthful claim about its type.[4] I first specify a property of the distribution of the outcome of the audit that allows me to write the constraint that the firm makes a truthful claim about its type in terms of the functions that determine an optimal contract in the relaxed problem. I then find conditions on these functions that ensure that an optimal contract in the relaxed problem satisfies the resulting constraint. I illustrate the feasibility of these conditions with a simple example that has a clear economic interpretation.

[4] The Spence-Mirrlees property in this setup states that the marginal rate of substitution between the quantity that the firm produces and the money that it receives is monotonic in its type (see Laffont and Martimort (2002, Appendix 3.3)). If, as I assume, the firm has a payoff that is linear in the money that it receives then the property is satisfied if and only if the marginal cost of the firm is monotonic in its type.

The structure of the chapter is as follows: In the next section I describe the model under consideration and in section 2.3 I present the benchmark case in which the government observes the type of the firm. After this, in section 2.4 I provide the mathematical statement of the contract design problem together with two preliminary results that play a key role in the characterisation of an optimal contract. Subsequently, in section 2.5 I characterise an optimal contract for the case of two cost types and in section 2.6 I perform the same task for the case of a continuum of cost types. With the analysis completed, in section 2.7 I conclude, discuss the relationship between my results and existing research and comment on some possible extensions.[5] Finally, in the appendix I prove those results that do not follow directly from others.

2.2 The model

2.2.1 Contracting parties and information structures

A government contracts with a firm for the production of a quantity $q \geq 0$. The firm has a cost of production of $C(\theta, q)$. The firm observes the actual value of θ (henceforth referred as its type) whereas the government has a prior belief about θ given by the probability function $f(\theta)$ which is positive for all θ in Θ.

After the firm produces the quantity q, an internal auditor investigates the cost of production. The audit results in an outcome s that takes one of the n possible values in the finite set $S = \{s_1, ..., s_n\}$. The outcome s consists of data correlated with the type of the firm. It can contain for example the number of hours employed, the amount of inputs spent or the depreciation rate of assets

[5] For a preview of the relationship between the results in this thesis and existing research see section 1.4.

used per quantity produced. As auditing involves sampling procedures I allow for an imperfect correlation between this information and the type of the firm. I denote by $g(s \mid \theta)$ the probability of the outcome s conditional on the type of the firm θ.

The firm and the internal auditor observe the outcome of the audit whereas the government does not. The internal auditor presents to the government a report $\hat{s} \in S$ that might differ from the outcome of the audit ($\hat{s} \neq s$). This misreporting can take the form of including in the report hours employed and inputs spent in other projects or inflating the initial value of assets so that their perceived depreciation seems higher than it actually is. The government does not have the ability to distinguish a truthful report from a false one. However, it has available the report σ of an honest external auditor. The external auditor evaluates the veracity of the report of the internal auditor. If the internal auditor misreports then the external auditor obtains evidence against it $\sigma = e$ with probability h and no evidence $\sigma = \emptyset$ otherwise. If the internal auditor does not misreport then the external auditor never obtains evidence against it.

2.2.2 Contracts

The contract that the government offers to the firm consists of a transfer $t(\tilde{\theta})$ in exchange for a quantity $q(\tilde{\theta})$ and a reimbursement $r(\tilde{\theta}, \tilde{s}, \hat{s})$. The firm selects the claims $\tilde{\theta}$ and $\tilde{s}$ from two message spaces M_θ and M_s after observing θ and s respectively.

The side contract that the firm offers to the internal auditor consists of a bribe $b(s)$ and a report $\hat{s}(s)$. The meaning of this side contract is that, when the outcome of the audit is s, the firm pays the internal auditor the bribe $b(s)$ if the internal auditor reports $\hat{s}(s)$. The side contract $b(s) = 0$ and $\hat{s}(s) = s$

corresponds to no bribery.

If the external auditor detects misreporting then the firm and the internal auditor pay respective penalties P_F and P_A. These penalties admit several non exclusive interpretations. One interpretation is that they are the wealth of the firm and the internal auditor that a court can seize. Another interpretation is that they are the resulting loss of future rents for the firm and the internal auditor or its members as a result of legal actions. In either case I assume that the penalties are legally specified so the government has no choice over them.

2.2.3 Preferences

The payoff of the government when the firm produces a quantity q, the government pays a transfer t and it receives a reimbursement r is $U(q)-t+r$. I assume that the utility $U(q)$ is increasing, strictly concave and twice continuously differentiable and that it satisfies $U(0)=0$ and $\frac{dU(0)}{dq}=\infty$.

The payoff (profit) of the firm when it produces a quantity q, it receives a transfer t and it pays a bribe b, a reimbursement r and a penalty P_F is $\pi = t-b-r-P_F-C(\theta,q)$. I assume that $C(\theta,q)$ is increasing, convex and twice continuously differentiable in q and it satisfies that $C(\theta,0)=0$, that $C(\theta,q)$ and $\frac{dC(\theta,q)}{dq}$ are increasing in θ for $q>0$ and that $\frac{d^2C(\theta,q)}{dq^2}$ is non decreasing in θ for $q>0$.[6]

The payoff (profit) of the internal auditor when it receives a bribe b and it pays a penalty P_A is $b-P_A$. The payoff of the external auditor does not depend on the relationship with the other contracting parties.

The contracting parties are risk neutral. The firm and the internal auditor

[6] The assumption that $C(\theta,q)$ is differentiable in q at $q=0$ rules out the existence of a fixed cost of production. The purpose of this is to ensure that the government always demands a positive quantity (given that $U(0)=C(\theta,0)$ and that $\frac{dU(0)}{dq}=\infty$). It is possible to adjust the analysis to allow for a fixed cost of production by imposing additional assumptions on the utility function that guarantee that the demand is always positive.

accept a contract if their expected payoff is greater or equal than their reservation payoff, which I normalize to zero.

2.2.4 Timing

The timing of the contractual relationships is as follows:[7]

1. The firm observes θ.

2. The government offers a contract to the firm.

3. The firm makes a claim $\tilde{\theta}$, produces a quantity q and receives a transfer t.

4. The firm and the internal auditor observe s.

5. The firm offers a side contract to the internal auditor.

6. The firm makes a claim $\tilde{s}$ and the internal auditor reports $\hat{s}$. The firm pays a reimbursement r and a bribe b.

7. The external auditor observes σ and reports. If it reports evidence of misreporting then the firm and the internal auditor pay respective penalties P_F and P_A.

2.3 The benchmark case

As a benchmark case for later use, I first consider the situation in which the government observes the type of the firm. In that situation, the contract that the government offers to the firm consists of a transfer $t(\theta)$ in exchange for a

[7] In this timing I do not mention what happens when the firm rejects the contract or the internal auditor rejects the side contract as these are not optimal choices.

quantity $q(\theta)$. The payoff of the government is the utility of the quantity that it demands minus the transfers that it pays. This is given by:

$$U(q(\theta)) - t(\theta) \tag{2.1}$$

The firm accepts the contract if:

$$\pi(\theta) = t(\theta) - C(\theta, q(\theta)) \geq 0 \tag{2.2}$$

An optimal contract maximises (2.1) subject to (2.2) for all θ. The optimal transfer is equal to the cost of production so the firm makes a zero first best expected profit:

$$\pi^{FB}(\theta) = 0 \tag{2.3}$$

The government demands a first best quantity $q^{FB}(\theta)$ that maximises the surplus, defined as its utility minus the cost of production of the firm. The quantity scheme is given by the condition that the marginal surplus is zero:

$$\frac{dU(q^{FB}(\theta))}{dq} - \frac{dC(\theta, q^{FB}(\theta))}{dq} = 0 \tag{2.4}$$

2.4 The contract design problem

2.4.1 Statement of the problem

Returning to the situation in which the government does not observe the type of the firm, I first focus on the side contract. The internal auditor reports truthfully whenever it rejects the side contract to avoid paying a penalty. This results in a payoff of zero. Therefore in any optimal side contract the firm pays a zero bribe if it requests truthful reporting ($b(s) = 0$ if $\hat{s}(s) = s$, bribery does not take place) and a bribe equal to the expected penalty that the internal auditor pays otherwise ($b(s) = hP_A$ if $\hat{s}(s) \neq s$). The contract design problem is then as if the firm controls the report of the internal auditor at a cost of misreporting equal to the sum of the expected penalties. In other words, the contract design problem is a principal agent problem with hidden information and hidden action.

In this setup the Revelation Principle of Myerson (1982) applies: there is an optimal contract $\left\{q(\tilde{\theta}), t(\tilde{\theta}), r(\tilde{\theta}, \tilde{s}, \hat{s}), \hat{s}(\tilde{\theta}, \tilde{s})\right\}$ in which the message spaces are the set of types and the set of outcomes of the audit ($\tilde{\theta} \in \Theta$ and $\tilde{s} \in S$) and in which the government makes a recommendation to the firm about what report to request from the internal auditor ($\hat{s}(\tilde{\theta}, \tilde{s})$). Furthermore, this contract is truthful ($\tilde{\theta} = \theta$ and $\tilde{s} = s$) and obedient ($\hat{s} = \hat{s}(\tilde{\theta}, \tilde{s})$).

Before proceeding with the characterisation of the contract design problem, it is worth pausing to comment on the role of the recommendation in the above contract. What the recommendation captures is that for any contract the government is aware of the types of the firm and the outcomes of the audit for which bribery takes place and the resulting report. The purpose of considering it is then to facilitate the characterisation of the contract design problem.

I next note that in the model under consideration obedience does not impose any constraint: given the claims of the firm, the government can infer whether the firm followed its recommendation or not. It can then require an arbitrarily high reimbursement when the firm does not ($r(\tilde{\theta}, \tilde{s}, \hat{s}) = \infty$ if $\hat{s} \neq \hat{s}(\tilde{\theta}, \tilde{s})$). Therefore I can write the reimbursement as $r(\tilde{\theta}, \tilde{s})$, where the firm pays this reimbursement if the report of the internal auditor coincides with the recommendation of the government.

In Chapter 3 I argue that, with the information structure of the external auditor and the penalties that I have assumed, there is an optimal truthful contract for which the government recommends that the firm requests a truthful report from the internal auditor for all the types of the firm and all the outcomes of the audit ($\hat{s}(\theta, s) = s$ for all θ and all s). Using this result, I refer to a truthful claim about the outcome of the audit as bribery not taking place. I now state the contract design problem with a truthful contract for which bribery does not take place.

The payoff of the government with a truthful contract for which bribery does not take place is:

$$E_{f(\theta)}\left\{U(q(\theta)) - t\,(\theta) + \sum_{s\in S} r(\theta, s)g(s \mid \theta)\right\} \tag{2.5}$$

After the firm claims that its type is θ, when the outcome of the internal audit is s bribery does not take place if:[8]

$$r(\theta, s) \leq r(\theta, \tilde{s}) + hP_A + hP_F \quad \forall \tilde{s} \neq s \tag{2.6}$$

If this condition holds for all the types of the firm and all the outcomes of the audit then the firm prefers to make a truthful claim when its type is θ if:[9]

$$\begin{aligned} t\,(\theta) - \sum_{s\in S} r(\theta, s)g(s \mid \theta) - C(\theta, q\,(\theta)) \geq \\ t(\tilde{\theta}) - \sum_{s\in S} r(\tilde{\theta}, s)g(s \mid \theta) - C(\theta, q(\tilde{\theta})) \quad \forall \tilde{\theta} \neq \theta \end{aligned} \tag{2.7}$$

and accepts the contract if:

$$\pi\,(\theta) = t\,(\theta) - \sum_{s\in S} r(\theta, s)g(s \mid \theta) - C(\theta, q(\theta)) \geq 0 \tag{2.8}$$

[8] To simplify the presentation of my results I assume that when the firm is indifferent between bribing and not bribing the internal auditor it chooses the latter.

[9] To simplify the presentation of my results I assume that when the firm is indifferent between making a false claim about its type and making a truthful claim it chooses the latter.

An optimal contract maximises (2.5) subject to (2.6) for all θ and all s and (2.7) and (2.8) for all θ.

2.4.2 Alternative statement of the problem

I now perform a change in variables that provides more intuition into the contract design problem. I consider a contract $\left\{q(\tilde{\theta}), \pi(\tilde{\theta}), r(\tilde{\theta}, \tilde{s})\right\}$. The transfer that the firm obtains is given by (2.8).

After this change in variables I can write (2.5) as:

$$E_{f(\theta)}\left\{U(q(\theta)) - C(\theta, q(\theta)) - \pi(\theta)\right\} \tag{2.9}$$

and (2.7) becomes:

$$\begin{aligned}\pi(\theta) \geq \pi(\tilde{\theta}) + \sum_{s \in S} r(\tilde{\theta}, s)\left(g(s \mid \tilde{\theta}) - g(s \mid \theta)\right) + \\ + C(\tilde{\theta}, q(\tilde{\theta})) - C(\theta, q(\tilde{\theta})) \quad \forall \tilde{\theta} \neq \theta\end{aligned} \tag{2.10}$$

while (2.8) is simply:

$$\pi(\theta) \geq 0 \tag{2.11}$$

An optimal contract maximises (2.9) subject to (2.6) for all θ and all s and (2.10) and (2.11) for all θ. Equation (2.9) shows that the contract design problem is as if the government faces two costs: the cost of production and the expected profit of the firm. Equation (2.10) requires that the expected profit of the firm when it makes a truthful claim is greater or equal than when it does not. The right hand side shows that the expected profit of the firm when its type is θ and it claims that its type is $\tilde{\theta} \neq \theta$ is equal to its expected profit when its type is $\tilde{\theta}$ plus the difference in expected reimbursement with the reimbursement of type $\tilde{\theta}$ plus the difference in the cost of production with the quantity that the government demands for type $\tilde{\theta}$.

The first best contract is not an optimal contract if an audit is not available: for every type θ below the highest type the firm would claim that its type is a $\tilde{\theta} > \theta$ to make a positive profit of $C(\tilde{\theta}, q^{FB}(\tilde{\theta})) - C(\theta, q^{FB}(\tilde{\theta})) > 0$ (the right hand side of (2.10) with $g(s \mid \tilde{\theta}) - g(s \mid \theta) = 0$ for all s, $\pi(\tilde{\theta}) = 0$ and $q(\tilde{\theta}) = q^{FB}(\tilde{\theta})$).

With auditing, the government has three complementary instruments to prevent the firm when its type is a particular θ to claim that its type is a particular $\tilde{\theta}$ (equation (2.10)): 1) the profit scheme: the government can offer a higher expected profit when the type of the firm is θ than when it is $\tilde{\theta}$, 2) the quantity scheme: the government can demand a quantity below the first best quantity when the type of the firm is $\tilde{\theta}$ and 3) the reimbursement scheme: the government can require a positive reimbursement when the type of the firm is $\tilde{\theta}$ for outcomes of the audit that are more likely to occur when the type of the firm is θ than when it is $\tilde{\theta}$ ($g(s \mid \tilde{\theta}) - g(s \mid \theta) < 0$) and a negative one otherwise. Employing the profit scheme or the quantity scheme is costly for the government as it then does not achieve the same outcome as when it knows the type of the firm ($\pi(\theta) = 0$ and $q(\theta) = q^{FB}(\theta)$ for all θ). Employing the reimbursement scheme does not have a cost for the government and therefore is preferable but it is limited by the constraint that in an optimal contract bribery does not take place.

2.4.3 Simplifying results

A difficulty in the contract design problem is that for every outcome of the audit ensuring that the reimbursement does not result in bribery involves the comparison with the reimbursement for all the other outcomes of the audit (see (2.6)). Lemma 1 shows that this constraint is in fact a constraint on the range of the reimbursement:

Lemma 1. *An optimal contract satisfies that the range of the reimbursement for the outcome of the audit* $(\max_{s\in S} r(\theta,s) - \min_{s\in S} r(\theta,s))$ *is smaller or equal than the sum of the expected penalties* $(\psi = hP_A + hP_F)$ *for all the types of the firm.*

The intuition for Lemma 1 is simple: bribery does not take place if and only if it does not take place when the incentive of the firm to bribe the internal auditor is strongest. If the firm bribed the internal auditor then it would ask the internal auditor to report the outcome of the audit for which it pays the lowest reimbursement. Its incentive to bribe the internal auditor is strongest for the outcome of the audit for which it pays the highest reimbursement. Considering this case provides the bound on the range of the reimbursement for which bribery does not take place, which I denote by ψ to simplify notation. The bound is high when the external auditor is very likely to be informed and the penalties that the firm and the internal auditor receive when the external auditor detects misreporting are severe.

Lemma 1 clarifies how the threat of bribery constrains the reimbursement scheme in an optimal contract. Lemma 2 makes this constraint more tractable:

Lemma 2. *An optimal contract satisfies that:*

$$\min_{s\in S} r(\theta,s) = 0 \tag{2.12}$$

$$r\,(\theta,s) \in [0,\ \psi] \quad \forall s \tag{2.13}$$

for all the types of the firm.

The first part of Lemma 2 states that there is no point for the government to require that the firm pays a reimbursement for all the outcomes of the audit: for any type of the firm the government can alter all the reimbursements by an amount that results in a zero minimum reimbursement and alter the transfer by the same amount so that the expected profit of the firm is unchanged. This change does not affect the incentives of the firm to bribe the auditor or to make a truthful claim. Also it does not affect the payoff of the government. Therefore there is an optimal contract for which the minimum reimbursement that the firm pays is zero for all the types of the firm.

The second part of Lemma 2 follows directly from the first part together with Lemma 1: by definition, every reimbursement is not lower than the minimum reimbursement or higher than the maximum reimbursement and if the minimum reimbursement is zero then Lemma 1 implies that the maximum reimbursement for which bribery does not take place is smaller or equal than the sum of the expected penalties for all the types of the firm.

From now on I focus on characterising an optimal contract that maximises (2.9) subject to (2.10), (2.11), (2.12) and (2.13) for all θ. It follows from the previous discussion that any other contract in which for any type of the firm the government demands the same quantity and the firm makes the same expected profit is also optimal.

2.5 An optimal contract with two cost types

In this section I assume that the type of the firm is either low or high: $\Theta = \{\underline{\theta}, \overline{\theta}\}$ with $\underline{\theta} < \overline{\theta}$. This simplifies the contract design problem as the firm can only make a truthful claim or claim the other type (for every θ there is only one $\tilde{\theta} \neq \theta$).

The payoff of the government in this case is given by:

$$\Big(U(q(\underline{\theta}))-C(\underline{\theta},q(\underline{\theta}))-\pi(\underline{\theta})\Big)f(\underline{\theta})+\Big(U(q(\overline{\theta}))-C(\overline{\theta},q(\overline{\theta}))-\pi(\overline{\theta})\Big)f(\overline{\theta}) \quad (2.14)$$

The firm makes a truthful claim when its type is low if:

$$\pi(\underline{\theta}) \geq \pi(\overline{\theta}) + \sum_{s\in S} r(\overline{\theta},s)\left(g(s \mid \overline{\theta}) - g(s \mid \underline{\theta})\right) + C(\overline{\theta},q(\overline{\theta})) - C(\underline{\theta},q(\overline{\theta})) \quad (2.15)$$

and it makes a truthful claim when its type is high if:

$$\pi(\overline{\theta}) \geq \pi(\underline{\theta}) + \sum_{s\in S} r(\underline{\theta},s)\left(g(s \mid \underline{\theta}) - g(s \mid \overline{\theta})\right) + C(\underline{\theta},q(\underline{\theta})) - C(\overline{\theta},q(\underline{\theta})) \quad (2.16)$$

An optimal contract maximises (2.14) subject to (2.12), (2.13) and (2.11) for $\underline{\theta}$ and $\overline{\theta}$ and (2.15) and (2.16). To obtain it, I take the approach of conjecturing that in an optimal contract the constraint that the firm makes a truthful claim when its type is high (equation (2.16)) is not relevant and I then verify this guess. Lemma 3 presents the results that follow directly from proceeding in this way:

Lemma 3. *In an optimal contract the firm makes a zero expected profit when its type is high* ($\pi^{SB}(\overline{\theta})=0$) *and the government demands the first best quantity when its type is low* ($q(\underline{\theta})=q^{FB}(\underline{\theta})$).

Lemma 3 provides a partial characterisation of an optimal contract. To present the remaining characterisation in a format that is comparable to that of the following sections, I divide the constraint that the firm makes a truthful claim when its type is low (equation (2.15)) by the distance between types. I denote the resulting second term in the right hand side by $\phi(q(\overline{\theta}))$. The constraint is then:

$$\frac{\pi(\underline{\theta}) - 0}{\overline{\theta} - \underline{\theta}} \geq \sum_{s \in S} r(\overline{\theta}, s) \frac{g(s \mid \overline{\theta}) - g(s \mid \underline{\theta})}{\overline{\theta} - \underline{\theta}} + \phi(q(\overline{\theta})) \tag{2.17}$$

I refer to the terms in this constraint as "adjusted". The term in the left hand side is then the adjusted difference in expected profit between the low type and the high type. The second term in the right hand side is the adjusted difference in the cost of production between the high type and the low type with the quantity of the high type. It follows from the assumptions on $C(\theta, q)$ that $\phi(0) = 0$ and that $\phi(q) > 0$, $\frac{d\phi(q)}{dq} > 0$ and $\frac{d^2\phi(q)}{dq^2} \geq 0$ for $q > 0$.

As a benchmark for later use, I now consider the situation in which an audit is not available. I refer to the corresponding optimal contract as second best. Using the notation introduced above, Lemma 4 presents it:

Lemma 4. *If an audit is not available then in an optimal contract the firm makes a positive expected profit when its type is low* ($\pi^{SB}(\underline{\theta}) > 0$) *and the government demands a quantity below the first best quantity when its type is high* ($q^{SB}(\overline{\theta}) < q^{FB}(\overline{\theta})$). *These are given by:*

$$\pi^{SB}(\underline{\theta}) = (\overline{\theta} - \underline{\theta})\phi(q^{SB}(\overline{\theta})) \tag{2.18}$$

$$f(\overline{\theta}) \left(\frac{dU(q^{SB}(\overline{\theta}))}{dq} - \frac{dC(\theta, q^{SB}(\overline{\theta})))}{dq} \right) = f(\underline{\theta})(\overline{\theta} - \underline{\theta}) \frac{d\phi(q^{SB}(\overline{\theta}))}{dq} \tag{2.19}$$

Lemma 4 registers the result that when an audit is not available the government employs both the expected profit scheme and the quantity scheme to prevent the firm when its type is low from claiming that it is high ($\pi^{SB}(\underline{\theta}) > 0$ and $q^{SB}(\overline{\theta}) < q^{FB}(\overline{\theta})$). The second best expected profit follows from the fact that in an optimal contract the constraint that the firm makes a truthful claim when its type is low binds if an audit is not available. In turn, that determines the second best quantity when the type of the firm is high.

This quantity optimally trades off the loss of surplus when the type of the firm is high and the loss of expected profit when the type of the firm is low that is due to the contract for the high type. The optimum is determined considering the marginal quantity.

Returning to the situation in which an audit is available, I now focus on the first term in the right hand side of (2.17). This term is the adjusted difference in expected reimbursement between the high type and the low type with the reimbursement of the high type. To determine its role in an optimal contract I introduce a definition based on the negative of it:

Definition 1. *The value of the audit is the maximum negative adjusted difference in expected reimbursement between the high type and the low type with the reimbursement of the high type for which bribery does not take place:*

$$V = \max_{r\left(\overline{\theta},s\right)\in[0,\,\psi]} -\sum_{s\in S} r(\overline{\theta},s)\frac{g(s\mid\overline{\theta})-g(s\mid\underline{\theta})}{\overline{\theta}-\underline{\theta}} = \tag{2.20}$$

$$= -\sum_{s\in S:g(s|\overline{\theta})-g(s|\underline{\theta})<0} \psi\frac{g(s\mid\overline{\theta})-g(s\mid\underline{\theta})}{\overline{\theta}-\underline{\theta}}$$

The value of the audit captures the best use that the government can make of the audit to prevent the firm when its type is low from claiming that it is high. For those outcomes of the audit that are more likely to occur when the type of the firm is low than when the type of the firm is high ($g(s\mid\overline{\theta})$ $-g(s\mid\underline{\theta})<0$) it requests the highest reimbursement for which bribery does not take place ($r(\overline{\theta},s)=\psi$). For the other outcomes it requests no reimbursement.

Using the value of the audit I can determine the pairs of expected profit scheme and quantity scheme for which there is a reimbursement scheme such that the firm makes a truthful claim and bribery does not take place (those that satisfy (2.17) with the first term in the right hand side equal to $-V$). I can then characterise an optimal contract:

Proposition 1. *An optimal contract is given by:*

$$r(\overline{\theta}, s) = \begin{cases} \psi \min\left\{\frac{\phi(q^{FB}(\overline{\theta}))}{V},\ 1\right\} & if\ g(s \mid \overline{\theta}) - g(s \mid \underline{\theta}) < 0 \\ 0 & if\ g(s \mid \overline{\theta}) - g(s \mid \underline{\theta}) \geq 0 \end{cases} \tag{2.21}$$

$$\pi(\underline{\theta}) = (\overline{\theta} - \underline{\theta}) \max\left\{\phi(q^{SB}(\overline{\theta})) - V,\ 0\right\} \tag{2.22}$$

$$q(\overline{\theta}) = \begin{cases} q^{SB}(\overline{\theta}) & if\ V \leq \phi(q^{SB}(\overline{\theta})) \\ q^{V}(\overline{\theta}) & if\ V \in [\phi(q^{SB}(\overline{\theta})),\ \phi(q^{FB}(\overline{\theta}))] \\ q^{FB}(\overline{\theta}) & if\ V \geq \phi(q^{FB}(\overline{\theta})) \end{cases} \tag{2.23}$$

with $q^V(\overline{\theta})$ *given by* $0 = -V + \phi(q^V(\overline{\theta}))$.

The government requires a reimbursement when the type of the firm is high if and only if the outcome of the audit is more likely to occur when the type of the firm is low than when it is high. The reimbursement is uniquely defined to be the highest reimbursement for which bribery does not take place except when the value of the audit is so high that the government can achieve the first best expected profit scheme and quantity scheme with a lower reimbursement ($V > \phi(q^{FB}(\overline{\theta}))$). In that situation an optimal reimbursement is the minimum reimbursement that achieves the above outcome.[10]

If the value of the audit is not high enough for the government to achieve the same outcome as when it knows the type of the firm then there are two possible situations. For a low value of the audit ($V < \phi(q^{SB}(\overline{\theta}))$) when the type of the firm is high the government demands the second best quantity and when the type of the firm is low it offers an expected profit that is positive

[10] Any higher reimbursement up to the sum of expected penalties is also optimal. The reason to select the reimbursement in Proposition 1 is again to facilitate the comparison with the results of the following section.

but lower than the second best. For an intermediate value of the audit ($V \in [\phi(q^{SB}(\overline{\theta})),\ \phi(q^{FB}(\overline{\theta}))]$) when the type of the firm is high it demands a quantity given by the value of the audit and $\phi(q)$ and when the type of the firm is low it offers a zero expected profit. As $\frac{d\phi(q)}{dq} > 0$ it follows that this quantity is in between the second best quantity and the first best quantity.

The two previous situations can be summarised by saying that an optimal contract exhibits a separation property: when the type of the firm is high the government demands the second best quantity unless the audit allows it to offer the first best expected profit when the type of the firm is low. The intuition for this property is the following: if the value of the audit is not high enough for the government to achieve the first best outcome then the constraint that the firm makes a truthful claim when its type is low binds and the contract design problem is as if an audit is not available but with the expected profit of the firm when its type is low decreased by the value of the audit. As the trade off involving the quantity that the government demands when the type of the firm is high does not change, the optimal quantity is the same unless it results in a negative expected profit for the firm when its type is low.

2.6 An optimal contract with a continuum of cost types

2.6.1 The relaxed problem

In this section I assume that the type of the firm belongs to an interval: $\Theta = [\underline{\theta}, \overline{\theta}]$ with $\underline{\theta} < \overline{\theta}$. I denote the cumulative distribution function of θ by $F(\theta)$ and I assume that the hazard rate $\frac{F(\theta)}{f(\theta)}$ is non decreasing and differentiable.[11] I also assume that $C(\theta, q)$ and $g(s \mid \theta)$ are twice continuously differentiable in θ.

To characterise an optimal contract, I assume that it is differentiable for almost all the types of the firm and I replace the constraint that the firm makes a truthful claim about its type (equation (2.10)) with a necessary differential constraint.[12] I then obtain an optimal contract in the resulting relaxed problem and verify that it satisfies the original constraint (and is therefore an optimal contract).

The payoff of the government in this case is given by:

$$\int_{\underline{\theta}}^{\overline{\theta}} \Big(U(q(\theta)) - C(\theta, q(\theta)) - \pi(\theta) \Big) f(\theta)\, d\theta \tag{2.24}$$

To obtain the differential constraint I use the constraint that when its type is $\theta_L < \theta$ the firm does not claim that it is θ and the constraint that when it is θ the firm does not claim that it is $\theta_H > \theta$. I divide the first constraint by $\theta - \theta_L$ and the second by $\theta_H - \theta$. Taking the limit as θ_L and θ_H go to θ and denoting by $\phi(\theta, q(\theta))$ the second term in the right hand side results in:

$$-\frac{d\pi(\theta)}{d\theta} = \sum_{s \in S} r(\theta, s) \frac{dg(s \mid \theta)}{d\theta} + \phi(\theta, q(\theta)) \tag{2.25}$$

Equation (2.25) is a necessary condition to prevent the firm from marginally exaggerating its type. It is the equivalent of equation (2.17) for the case of a continuum of cost types. The term in the left hand side is the negative of the difference in expected profit between type θ and a marginally lower type. The first term in the right hand side is the difference in expected reimbursement

[11] The monotone hazard rate property is a standard condition in the literature on contracting with asymmetric information that many distributions (including normal, uniform, logistic, chi-squared and exponential) satisfy (see Bagnoli and Bergstrom (2005)).

[12] Technically I assume that $t(\theta)$, $q(\theta)$ and $r(\theta, s)$ are differentiable for almost all θ. As S is finite this means that $\pi(\theta)$ is also differentiable for almost all θ.

between type θ and a marginally lower type with the reimbursement of type θ. The second term in the right hand side is the difference in the cost of production between type θ and a marginally lower type with the quantity of type θ. It follows from the assumptions on $C(\theta, q)$ that $\phi(\theta, 0) = 0$ and that $\phi(\theta, q) > 0$, $\frac{d\phi(\theta,q)}{dq} > 0$ and $\frac{d^2\phi(\theta,q)}{dq^2} \geq 0$ for $q > 0$.

An optimal contract in the relaxed problem maximises (2.24) subject to (2.12), (2.13), (2.11) and (2.25) for all θ.[13] As before, I first consider the situation in which an audit is not available and I refer to the corresponding optimal contract as second best. Lemma 5 presents it:

Lemma 5. *If an audit is not available then in an optimal contract the firm makes a positive expected profit when its type is below the highest type ($\pi^{SB}(\theta) > 0$ for $\theta < \overline{\theta}$) and the government demands a quantity below the first best quantity when its type is above the lowest type ($q^{SB}(\theta) < q^{FB}(\theta)$ for $\theta > \underline{\theta}$). These are given by:*

$$\pi(\theta) = \int_{\theta}^{\overline{\theta}} \phi(\eta, q^{SB}(\eta))d\eta \tag{2.26}$$

$$f(\theta)\left(\frac{dU(q^{SB}(\theta)))}{dq} - \frac{dC(\theta, q^{SB}(\theta)))}{dq}\right) = F(\theta)\frac{d\phi(\theta, q^{SB}(\theta))}{dq} \tag{2.27}$$

Lemma 5 is the counterpart of Lemma 4. It states that when an audit is not available the government employs both the expected profit scheme and the quantity scheme to prevent the firm from marginally exaggerating its type. The second best expected profit for type θ follows from integrating for all types greater than θ the difference in expected profit with a marginally lower type when an audit is not available (together with $\pi(\overline{\theta}) = 0$). As for the second best quantity for type θ, it optimally trades off the loss of surplus when the

[13] I have omitted the qualifier "almost" that should accompany equation (2.25), sacrificing precision for the sake of brevity. In what follows I proceed in this way.

type of the firm is θ and the loss of expected profit when the type of the firm is below θ that is due to the contract for type θ.

Returning to the situation in which an audit is available, I now adapt Definition 1 to the case of a continuum of cost types:

Definition 2. *The value of the audit for type θ is the maximum negative difference in expected reimbursement between type θ and a marginally lower type with the reimbursement of type θ for which bribery does not take place:*

$$V(\theta) = \max_{r(\theta,s)\in[0,\,\psi]} -\sum_{s\in S} r(\theta,s)\,\frac{dg(s\mid\theta)}{d\theta} = -\sum_{s\in S:\,\frac{dg(s|\theta)}{d\theta}<0} \psi\frac{dg(s\mid\theta)}{d\theta} \tag{2.28}$$

Using the value of the audit for type θ, I can determine the pairs of expected profit scheme and quantity scheme for which there is a reimbursement scheme such that the firm does not have an incentive to marginally exaggerate its type and bribery does not take place (those that satisfy (2.25) for all θ with the first term in the right hand side equal to $-V(\theta)$). I can then obtain the following proposition:

Proposition 2. *An optimal contract in the relaxed problem is given by:*

$$r(\theta,s) = \begin{cases} \psi\min\left\{\frac{\phi(\theta,q^{FB}(\theta))}{V(\theta)},\ 1\right\} & if\ \frac{dg(s|\theta)}{d\theta}<0 \\ 0 & if\ \frac{dg(s|\theta)}{d\theta}\geq 0 \end{cases} \tag{2.29}$$

$$\pi(\theta) = \int_{\theta}^{\bar{\theta}} \max\left\{\phi(\eta,q^{SB}(\eta)) - V(\eta)\,,\ 0\right\} d\eta \tag{2.30}$$

$$q(\theta) = \begin{cases} q^{SB}(\theta) & if\ V(\theta)\leq\phi(\theta,q^{SB}(\theta)) \\ q^{V}(\theta) & if\ V(\theta)\in[\phi(\theta,q^{SB}(\theta)),\ \phi(\theta,q^{FB}(\theta))] \\ q^{FB}(\theta) & if\ V(\theta)\geq\phi(\theta,q^{FB}(\theta)) \end{cases} \tag{2.31}$$

with $q^V(\theta)$ *given by* $0 = -V(\theta) + \phi(\theta, q^V(\theta))$ *if for any* θ *such that* $V(\theta) \in [\phi(\theta, q^{SB}(\theta)),\ \phi(\theta, q^{FB}(\theta))]$ *then:*

$$\frac{d}{d\theta}\left(\frac{\frac{dU(q^V(\theta))}{dq} - \frac{dC(\theta, q^V(\theta))}{dq}}{\frac{d\phi(\theta, q^V(\theta))}{dq}}\right) \leq 0 \qquad (2.32)$$

Proposition 2 provides a sufficient condition under which an optimal contract in the relaxed problem has the same structure as the contract in Proposition 1. For any type θ with a relatively low value of the audit ($V(\theta) < \phi(q^{SB}(\theta))$) the government demands the second best quantity and it offers an expected profit that is marginally decreasing in type but less so than the second best. For any type θ with a relatively intermediate value of the audit ($V(\theta) \in [\phi(\theta, q^{SB}(\theta)),\ \phi(\theta, q^{FB}(\theta))]$) it demands a quantity given by the value of the audit and $\phi(\theta, q)$ and it offers an expected profit that is marginally constant in type. As $\frac{d\phi(\theta,q)}{dq} > 0$ it follows that this quantity is in between the second best quantity and the first best quantity.[14]

To understand the role of the condition in Proposition 2 consider first a type θ with a relatively intermediate value of the audit and suppose that it is optimal for the government to demand $q^V(\theta)$.[15] By the definition of $q^V(\theta)$, the government prevents the firm from marginally exaggerating its type without generating a loss of expected profit. However there is a loss of surplus as $q^V(\theta) \leq q^{FB}(\theta)$. Now consider a marginally higher type. This results in a change in $q^V(\theta)$. Equation (2.32) ensures that the comparative importance of the loss of surplus at $q^V(\theta)$ is non increasing. As the hazard rate is non decreasing this implies that it is also optimal to eliminate the loss of expected profit.

[14]The qualifier relatively is due to the fact that $\phi(\theta, q^{SB}(\theta))$ and $\phi(\theta, q^{FB}(\theta))$ depend on the type of the firm.

[15]If there exists a type θ for which $V(\theta) = \phi(\theta, q^{SB}(\theta))$ it is optimal for the government to demand $q^V(\theta) = q^{SB}(\theta)$.

It is possible to write (2.32) in terms of the value of the audit for type θ (rather than in terms of $q^V(\theta)$). I did not do so to facilitate its interpretation. I now look at the implication of (2.32) on the value of the audit for type θ in an example:

Example 1. Suppose that the cost function is given by $C(\theta, q) = \theta q$. This results in $\phi(q) = q$ so $q^V(\theta) = V(\theta)$. Also $\frac{dC(\theta,q)}{dq} = \theta$ and $\frac{d\phi(\theta,q)}{dq} = 1$. Equation (2.32) becomes:

$$\frac{d}{d\theta}\left(\frac{\frac{dU(V(\theta)))}{dq} - \theta}{1}\right) = \frac{d^2U(V(\theta)))}{dq^2}\frac{dV(\theta)}{d\theta} - 1 \leq 0 \tag{2.33}$$

The utility function is concave so equation (2.33) is satisfied if the value of the audit for type θ does not decrease too fast.

The general lesson of Example 1 is that equation (2.32) implies a bound on the rate of decrease of $q^V(\theta)$, which in turn implies a bound on the rate of decrease of the value of the audit for type θ. As an optimal contract in the relaxed problem depends on the value of the audit for type θ, this raises the question of how an optimal contract in the relaxed problem varies across Θ. Lemma 6 provides the result that answers this question:

Lemma 6. *The condition in Proposition 2 implies that if* $V(\theta) \geq \phi(\theta, q^{SB}(\theta))$ *for any type of the firm then this also holds for any higher type and the same is true for* $V(\theta) \geq \phi(\theta, q^{FB}(\theta))$.

Lemma 6 states that the condition in Proposition 2 implies that in relative terms the value of the audit is non decreasing: if it switches it goes from low to intermediate or from intermediate to high. This result follows from the fact equation (2.32) implies that $q^V(\theta)$ does not decrease faster than $q^{SB}(\theta)$ whenever they are equal and the same is true for $q^{FB}(\theta)$ (see equations (2.4) and (2.27)). This in turn implies that $V(\theta)$ does not decrease faster than $\phi(\theta, q^{SB}(\theta))$ whenever they are equal and the same is true for $\phi(\theta, q^{FB}(\theta))$.

Using Lemma 6 I can be more specific about an optimal contract in the relaxed problem. My first additional result concerns the expected profit scheme:

Corollary 1. *The expected profit scheme in Proposition 2 is positive and decreasing if the type of the firm is below a threshold θ_{SB} and zero otherwise. It is given by:*

$$\pi(\theta) = \begin{cases} \int_{\theta}^{\theta_{SB}} \phi(\eta, q^{SB}(\eta)) - V(\eta)\, d\eta & if\ \theta \in [\underline{\theta}, \theta_{SB}] \\ 0 & if\ \theta \in [\theta_{SB}, \overline{\theta}] \end{cases} \tag{2.34}$$

where $\theta_{SB} = \min \theta \in \Theta : V(\theta) \geq \phi(\theta, q^{SB}(\theta))$ if $V(\theta) \geq \phi(\theta, q^{SB}(\theta))$ for some θ and $\theta_{SB} = \overline{\theta}$ otherwise.

Corollary 1 conveys two pieces of additional information on the expected profit scheme. First, it clarifies for which types the firm makes a positive expected profit: only those below the type for which the value of the audit switches from low to intermediate ($\theta \in [\underline{\theta}, \theta_{SB})$). Second, it rules out the possibility that the firm makes an expected profit that is positive and constant for some types. In other words, if the expected profit of the firm is positive then it is decreasing in the type of the firm.

My second additional result focuses on the quantity scheme:

Corollary 2. *The quantity scheme in Proposition 2 consists of at most three intervals. It is given by:*

$$q(\theta) = \begin{cases} q^{SB}(\theta) & if\ \theta \in [\underline{\theta}, \theta_{SB}]\ and\ \theta_{SB} > \underline{\theta} \\ q^{V}(\theta) & if\ \theta \in [\theta_{SB}, \theta_{FB}],\ \theta_{SB} > \underline{\theta}\ and\ \theta_{SB} < \overline{\theta} \\ q^{FB}(\theta) & if\ \theta \in [\theta_{FB}, \overline{\theta}]\ and\ \theta_{FB} < \overline{\theta} \end{cases} \tag{2.35}$$

where $\theta_{FB} = \min \theta \in \Theta : V(\theta) \geq \phi(\theta, q^{FB}(\theta))$ *if* $V(\theta) \geq \phi(\theta, q^{FB}(\theta))$ *for some* θ *and* $\theta_{FB} = \overline{\theta}$ *otherwise and* θ_{SB} *is as in Corollary 1 (so* $\theta_{SB} < \theta_{FB}$ *if* $\theta_{SB} \in (\underline{\theta}, \overline{\theta})$ *and* $\theta_{SB} = \theta_{FB}$ *if* $\theta_{SB} \in \{\underline{\theta}, \overline{\theta}\}$*).*

Figure 2.1 presents a situation in which the three intervals in Corollary 2 exist ($\theta_{SB} > \underline{\theta}$ and $\theta_{FB} < \overline{\theta}$) together with the resulting quantity scheme. The perhaps surprising feature is that the government demands the first best quantity when the type of the firm is at or above the type for which the value of the audit switches from intermediate to high $\theta \in [\theta_{FB}, \overline{\theta}]$. This is in contrast with the situation in which an audit is not available, where the government demands a second best quantity that is closest to the first best quantity when the type of the firm is close to the lowest type.

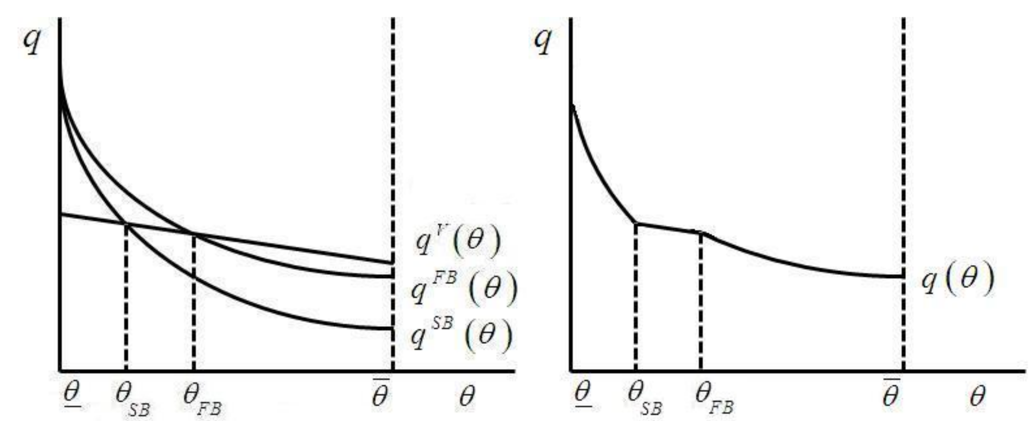

Figure 2.1: The case of three contract intervals (left) and the resulting quantity scheme (right) with a continuum of cost types.

Closest inspection of Corollary 2 reveals that there are three other possible situations: 1) the government demands the second best quantity for all the types of the firm ($\theta_{SB} = \theta_{FB} = \overline{\theta}$), 2) the government demands the first best quantity for all types of the firm ($\theta_{SB} = \theta_{FB} = \underline{\theta}$), 3) the government demands the second best quantity when the type of the firm is in the interval $[\underline{\theta}, \theta_{SB}]$ and it demands $q^V(\theta)$ when the type of the firm is in the interval $[\theta_{SB}, \overline{\theta}]$.

2.6.2 An optimal contract

I now consider under what conditions on the distribution of the outcome of the audit and on the cost of production the contract in Proposition 2 satisfies the constraint that the firm makes a truthful claim about its type (equation (2.10)) and is therefore an optimal contract. Using the reimbursement scheme (equation (2.29)) shows that when its type is θ the firm makes a truthful claim if:

$$\pi(\theta) \geq \pi(\tilde{\theta}) + \sum_{s \in S: \frac{dg(s|\tilde{\theta})}{d\theta} < 0} \min \left\{ \frac{\phi(\tilde{\theta}, q^{FB}(\tilde{\theta}))}{V(\tilde{\theta})},\ 1 \right\} \psi(g(s \mid \tilde{\theta}) - g(s \mid \theta)) + \tag{2.36}$$

$$+C(\tilde{\theta}, q(\tilde{\theta})) - C(\theta, q(\tilde{\theta})) \quad \forall \tilde{\theta} \neq \theta$$

For any type θ this constraint involves all the other types and their respective distribution of the outcome of the audit. I first find a condition on the latter that ensures that it plays no role in the constraint beyond the determination of the value of the audit for the corresponding type. Afterwards I obtain conditions on the value of the audit and on the cost of production that guarantee that the contract in Proposition 2 satisfies the resulting constraint. I start by writing the second and the third term in the right hand side of equation (2.36) in differential form:

$$\pi(\theta) \geq \pi(\tilde{\theta}) - \min \left\{ \frac{\phi(\tilde{\theta}, q^{FB}(\tilde{\theta}))}{V(\tilde{\theta})},\ 1 \right\} \int_{\theta}^{\tilde{\theta}} - \sum_{s \in S: \frac{dg(s|\tilde{\theta})}{d\theta} < 0} \psi \frac{dg(s \mid \eta)}{d\eta} d\eta + \tag{2.37}$$

$$+ \int_{\theta}^{\tilde{\theta}} \phi(\eta, q(\tilde{\theta})) d\eta \quad \forall \tilde{\theta} \neq \theta$$

The term inside the first integral of equation (2.37) differs from the value of the audit for type η (any type between θ and $\tilde{\theta}$) only in that it considers those outcomes of the audit that are more likely to occur when the type of the firm is marginally lower than $\tilde{\theta}$ than when it is $\tilde{\theta}$ ($\frac{dg(s|\tilde{\theta})}{d\theta} < 0$) instead of the corresponding outcomes for type η. Nevertheless those outcomes are the same if the audit satisfies the following property:

Definition 3. *The audit has the monotonicity property if the probability of every outcome of the audit is either decreasing for all the types of the firm ($\frac{dg(s|\theta)}{d\theta} < 0$ for all θ) or non decreasing ($\frac{dg(s|\theta)}{d\theta} \geq 0$ for all θ).*

If the audit has the monotonicity property then the outcomes of the audit for which the government requires a reimbursement are the same for all the types of the firm and equation (2.37) becomes:

$$\pi(\theta) \geq \pi(\tilde{\theta}) - \min\left\{\frac{\phi(\tilde{\theta}, q^{FB}(\tilde{\theta}))}{V(\tilde{\theta})},\ 1\right\} \int_{\theta}^{\tilde{\theta}} V(\eta) d\eta + \tag{2.38}$$

$$+ \int_{\theta}^{\tilde{\theta}} \phi(\eta, q(\tilde{\theta})) d\eta \quad \forall \tilde{\theta} \neq \theta$$

My main result for an audit that has the monotonicity property is this:

Proposition 3. *If the audit has the monotonicity property, the value of the audit is non increasing and both the cost of production and the marginal cost of production are convex in θ then the contract in Proposition 2 is an optimal contract.*

Proposition 3 provides sufficient conditions under which the contract in Proposition 2 is an optimal contract. The assumption that the marginal cost of production is convex in θ ensures that the second best quantity is decreasing in θ. When the value of the audit is low for all the types of the firm ($\theta_{SB} = \overline{\theta}$) this result together with the monotonicity property suffice to verify that equation (2.38) is satisfied for all θ. The reason is that in this situation for any type θ

if the firm claims that its type is any $\tilde{\theta} \neq \theta$ the difference in expected reimbursement with the reimbursement of type $\tilde{\theta}$ is equal to the difference in expected profit that is attributable to the value of the audit. As a result the incentive of the firm to make a truthful claim about its type does not depend on the value of the audit.

The assumption that the cost of production is convex in θ means that $\phi(\theta, q)$ is increasing in θ. Together with the assumption that the value of the audit is non increasing this assumption has two important implications. First, it ensures that the firm prefers to make a truthful claim that results in a zero expected profit ($\theta \in [\theta_{SB}, \overline{\theta}]$) than to claim that its type is a $\tilde{\theta}$ for which the expected profit is positive ($\tilde{\theta} < \theta_{SB}$). Second, it guarantees that the firm makes a non positive expected profit when it claims that its type is a $\tilde{\theta}$ for which the expected profit is zero ($\tilde{\theta} \geq \theta_{SB}$) for all θ. These two observations together with the previous one prove that equation (2.38) is satisfied for all θ.

I conclude this section with a simple example that satisfies the conditions in both Propositions 2 and 3 and has a clear economic interpretation:

Example 2. The cost of production is $C(\theta, q) = \theta q$ (as in Example 1). The set of outcomes of the audit is $S = \{s_1, ..., s_F, s_{F+1}, ...s_n\}$. The distribution of the outcome of the audit satisfies that $\frac{dg(s|\theta)}{d\theta} < 0$ and $\frac{d^2 g(s|\theta)}{d\theta^2} = 0$ for all θ and all $s \in \{s_1, ..., s_F\}$ and that $\frac{dg(s|\theta)}{d\theta} \geq 0$ for all θ and all $s \in \{s_{F+1}, ..., s_n\}$.

Both the cost of production and the marginal cost of production are convex in θ. Any outcome of the audit between s_1 and s_F has the natural interpretation of the firm failing the audit as it results in the firm paying a reimbursement ($\frac{dg(s|\theta)}{d\theta} < 0$). Analogously, any outcome of the audit above s_F has the interpretation of the firm passing the audit. The first condition on the distribution of the outcome of the audit requires that the probability of any fail is decreasing in the type of the firm at a constant rate. The second condition on the distribution of the audit requires that the probability of any pass is non decreasing in the type of the firm. The value of the audit for type θ is given by $V(\theta) = \sum_{s \leq s_F} -\psi \frac{dg(s|\theta)}{d\theta}$. It is constant for all θ ($\frac{dV(\theta)}{d\theta} = \sum_{s<s^+} -\psi \frac{d^2 g(s|\theta)}{d\theta^2} = 0$) so the conditions in both Propositions 2 (as given in (2.33)) and 3 are satisfied.

2.7 Conclusions and extensions

In this chapter I began by arguing that in an optimal procurement contract the threat of bribery constrains the reimbursement that the government requests from the firm to be smaller or equal than the sum of the expected penalties that the firm and the internal auditor pay. For the case of two cost types I then constructed a measure (the value of the audit) that captures the best use that the government can make of the audit to prevent the firm from making a false claim about its type. I employed this measure to characterise an optimal contract and I showed that it exhibits a separation property: the government gives priority to achieving the first best expected profit scheme over demanding the first best quantity scheme. I explained the intuition behind this result and I provided sufficient conditions under which it extends to the case of a continuum of cost types.

Baron and Besanko (1984) presented the most general characterisation of an optimal contract with auditing in the context of regulation. They assumed that the regulator is restricted to request a reimbursement from the firm not greater than a legally specified amount. My analysis differed from theirs in three aspects. First, the highest possible optimal reimbursement in my model was a function of how well informed the external auditor is. Second, to characterise an optimal contract in the relaxed problem I did not assume a specific distribution of the outcome of the audit whereas they assumed a normal distribution with a mean non decreasing and convex in the type of the firm. Third, I was able to provide sufficient conditions under which preventing the firm from marginally exaggerating its type implies preventing the firm from making a false claim about its type whereas they only provided an example where this is true for some parameter configurations.

Kofman and Lawarree (1993) introduced the distinction between a corruptible internal auditor and an honest external auditor that I employ in this thesis. They analysed a contract design problem that features a principal, an agent with a privately known productivity (its type), an internal auditor and a costly external auditor. They obtained a separation property assuming that both the type of the agent and the outcome of the audit take two possible values. I provided sufficient conditions under which an optimal contract has a separation property for the case of a continuum of cost types. I then showed that it implies that the government demands the first best quantity when the type of the firm is either the lowest type or a type at or above the type for which the value of the audit switches from intermediate to high.[16]

Gary-Bobo and Spiegel (2006) studied a model of procurement with auditing in which the type of the firm takes a continuum of possible values. They assumed that the government is restricted to request a reimbursement such that the expected profit of the firm is not lower than its liability. They then established sufficient conditions under which the government can achieve the outcome of the first best contract. Unlike Baron and Besanko, they did not restrict attention to a particular distribution of the outcome of the audit. However, they incorrectly argued that limited liability makes it optimal for the

[16]Kofman and Lawarree assumed that the principal pays the internal auditor a reward if the agent claims that its type is low and the internal auditor reports the outcome of the audit that is more likely to occur when the type of the agent is high. In chapter 3 I allow the government to offer the internal auditor a contract that is a function of the same contingencies as the contract that the government offers to the firm.This generalises their analysis in terms of contracting and side contracting.

government to request the highest possible reimbursement for all the outcomes of the audit except for the outcome that is most likely to occur for the type that the firm claimed to have.[17] As a result, their characterisation of an optimal contract with a continuum of types substantially differs from mine.

My analysis is robust to several extensions: First, I can let the internal auditor be the one that proposes the side contract. In that situation my results do not change as in an optimal side contract the internal auditor chooses its report to minimise the reimbursement that the firm pays plus the sum of the expected penalties. Second, I can consider the payoff of the government to be a weighted sum of its utility and the profit of the firm (as in Baron and Myerson (1982) and Laffont and Tirole (1986)). In that circumstance the closer the weight on the profit of the firm is to the weight on the utility of the government the closer the second best quantity scheme is to the first best quantity scheme. Third, I can assume that the internal auditor is risk averse. In that scenario the expected disutility of the penalty that the internal auditor pays replaces the expected penalty that the internal auditor pays in the characterisation of both an optimal side contract and an optimal contract.[18, 19]

More interesting is the question of how my results vary if I allow the government to offer the internal auditor a contract that is a function of the same contingencies as the contract that the government offers to the firm. I devote chapter 3 to answering this question. In the answer I also prove and

[17] More precisely, they ignored that with limited liability there is a trade off between requiring a reimbursement for as many outcomes as possible to request an amount below the limited liability of the firm and requiring a reimbursement for those outcomes of the audit that are more likely to occur for a marginally higher type to prevent the firm from marginally exaggerating its type.

[18] If the firm can offer a side contract that is contingent on the report of the external auditor then in an optimal side contract the firm pays a bribe equal to the penalty that the internal auditor pays if the external auditor detects misreporting and a zero bribe otherwise. The risk aversion of the internal auditor has then no effect on my results.

[19] If the firm is risk averse then the government cannot achieve the outcome of the first best contract for any value of the audit as an audit involves risk. Lemma 2 is not valid when the firm is risk averse.

explain a result that I employed in this chapter: that, with the information structure of the external auditor and the penalties that I had assumed, there is an optimal truthful contract for which bribery does not take place for any type of the firm and any outcome of the audit.

Appendix

Proof. (Lemma 1) For any θ equation (2.6) is satisfied for all s if and only if:

$$\max_{s \in S} r(\theta, s) \leq \min_{s \in S} r(\theta, s) + hP_A + hP_F \tag{2.39}$$

Rearranging (2.39) gives the condition on the range of the reimbursement in the lemma. This completes the proof. □

Proof. (Lemma 2) Suppose that there is an optimal contract $\left\{q_0(\tilde{\theta}), \pi_0(\tilde{\theta}), r_0(\tilde{\theta}, \tilde{s})\right\}$ that satisfies (2.10), (2.11) and (2.39) for all θ and $\min_{s \in S} r_0(\theta, s) \neq 0$ for some θ. I can construct the alternative contract $\left\{q_A(\tilde{\theta}), \pi_A(\tilde{\theta}), r_A(\tilde{\theta}, \tilde{s})\right\}$ as follows: $q_A(\tilde{\theta}) = q_0(\tilde{\theta})$ and $\pi_A(\tilde{\theta}) = \pi_0(\tilde{\theta})$ for all $\tilde{\theta}$ and $r_A(\tilde{\theta}, \tilde{s}) = r_0(\tilde{\theta}, \tilde{s}) - \min_{\tilde{s} \in S} r_0(\tilde{\theta}, \tilde{s})$ for all $\tilde{\theta}$ and all $\tilde{s}$.

Applying the minimum function to $r_A(\theta, s)$ gives $\min_{s \in S} r_A(\theta, s) = \min_{s \in S} r_0(\theta, s) - \min_{s \in S} r_0(\theta, s) = 0$ for all θ. Equation (2.39) for all θ then gives $\max_{s \in S} r_A(\theta, s) \leq \psi = hP_A + hP_F$ for all θ. Therefore $r_A(\theta, s) \in [0, \psi]$ for all θ and all s.

Also, as $\sum_{s \in S} r_A(\tilde{\theta}, s)\left(g(s \mid \tilde{\theta}) - g(s \mid \theta)\right) = \sum_{s \in S} r_0(\tilde{\theta}, s)\left(g(s \mid \tilde{\theta}) - g(s \mid \theta)\right)$ for all $\tilde{\theta}$, replacing $q_A(\tilde{\theta})$ and $\pi_A(\tilde{\theta})$ in (2.10) gives (2.10) for the initial contract for all θ. Therefore the alternative contract satisfies (2.10) for all θ.

By construction, the alternative contract satisfies (2.11) for all θ. Furthermore, replacing $q_A(\tilde{\theta})$ and $\pi_A(\tilde{\theta})$ in (2.9) gives (2.9) for the initial contract. Therefore the alternative contract is also optimal. This completes the proof. □

Proof. (Proposition 1) An optimal contract maximises (2.14) subject to (2.12), (2.13) and (2.11) for $\underline{\theta}$ and $\overline{\theta}$ and (2.15) and (2.16). If (2.16) is not a constraint then any $r(\underline{\theta}, s)$ that satisfies (2.12) and (2.13) for $\underline{\theta}$ is optimal. Also, in an optimal contract $\pi(\overline{\theta}) = 0$ and $q(\underline{\theta}) = q^{FB}(\underline{\theta})$. Equation (2.15) then becomes (2.17). The Lagrangian for this problem when (2.12) and (2.13) are satisfied for $\overline{\theta}$ is:

$$L = (-\pi(\underline{\theta}))f(\underline{\theta}) + (U(q(\overline{\theta}) - C(\overline{\theta}, q(\overline{\theta})))f(\overline{\theta}) + \tag{2.40}$$

$$+\gamma\left(\frac{\pi(\underline{\theta})}{\overline{\theta}-\underline{\theta}} - \sum_{s\in S} r\left(\theta, s\right)\frac{g(s\mid\overline{\theta}) - g(s\mid\underline{\theta})}{\overline{\theta}-\underline{\theta}} - \phi(q(\overline{\theta}))\right) + \lambda\pi(\underline{\theta})$$

where γ and λ are the Lagrange multipliers of (2.17) and (2.11).

The Kuhn-Tucker necessary and sufficient conditions for an optimal contract when (2.12) is satisfied for $\overline{\theta}$ are:

$$\left(\frac{dU(q(\overline{\theta}))}{dq} - \frac{dC(\overline{\theta}, q(\overline{\theta}))}{dq}\right)f(\overline{\theta}) = \gamma\frac{d\phi(q(\overline{\theta}))}{dq} \tag{2.41}$$

$$r(\overline{\theta}, s) = \begin{cases} \psi & if\ \gamma(g(s\mid\overline{\theta}) - g(s\mid\underline{\theta})) < 0 \\ [0,\ \psi] & if\ \gamma(g(s\mid\overline{\theta}) - g(s\mid\underline{\theta})) = 0 \\ 0 & if\ \gamma(g(s\mid\overline{\theta}) - g(s\mid\underline{\theta})) > 0 \end{cases} \tag{2.42}$$

$$-f(\underline{\theta}) + \frac{\gamma}{\overline{\theta}-\underline{\theta}} + \lambda \leq 0,\ \pi(\underline{\theta}) \geq 0,\ (-f(\underline{\theta}) + \frac{\gamma}{\overline{\theta}-\underline{\theta}} + \lambda)\pi(\underline{\theta}) = 0 \tag{2.43}$$

$$\gamma \geq 0,\ \frac{\pi(\underline{\theta})}{\overline{\theta}-\underline{\theta}} - \sum_{s\in S} r\left(\theta, s\right)\frac{g(s\mid\overline{\theta}) - g(s\mid\underline{\theta})}{\overline{\theta}-\underline{\theta}} - \phi(q(\overline{\theta})) \geq 0, \tag{2.44}$$

$$\gamma\left(\frac{\pi(\underline{\theta})}{\overline{\theta}-\underline{\theta}} - \sum_{s\in S} r\left(\theta, s\right)\frac{g(s\mid\overline{\theta}) - g(s\mid\underline{\theta})}{\overline{\theta}-\underline{\theta}} - \phi(q(\overline{\theta}))\right) = 0$$

$$\lambda \geq 0,\ \pi(\underline{\theta}) \geq 0,\ \lambda \pi(\underline{\theta}) = 0 \tag{2.45}$$

Suppose that $\gamma > 0$. Equation (2.42) gives $-\sum_{s\in S} r(\theta, s)\frac{g(s|\overline{\theta})-g(s|\underline{\theta})}{\overline{\theta}-\underline{\theta}} = V$. Equation (2.44) then gives $\frac{\pi(\underline{\theta})}{\overline{\theta}-\underline{\theta}} + V - \phi(q(\overline{\theta})) = 0$. Equations (2.43) and (2.45) give $\gamma \leq f(\underline{\theta})(\overline{\theta}-\underline{\theta})$. There are two possible situations. First, suppose that $\gamma = f(\underline{\theta})(\overline{\theta}-\underline{\theta})$ so $\lambda = 0$. Equation (2.41) then gives $q(\overline{\theta}) = q^{SB}(\overline{\theta})$. Equation (2.44) then gives $\pi(\underline{\theta}) = (\overline{\theta}-\underline{\theta})(\phi(q^{SB}(\overline{\theta})) - V)$. As equations (2.43) and (2.45) require $\pi(\underline{\theta}) \geq 0$ this then requires $V \leq \phi(q^{SB}(\overline{\theta}))$. Second, suppose that $\gamma \in (0,\ f(\underline{\theta})(\overline{\theta}-\underline{\theta}))$. If $\lambda = 0$ then equation (2.43) gives $\pi(\underline{\theta}) = 0$. If $\lambda > 0$ then equation (2.45) gives $\pi(\underline{\theta}) = 0$. Equation (2.44) then gives $0 = -V + (\phi(q^{V}(\overline{\theta}))$. The left hand side of equation (2.41) is decreasing in $q(\overline{\theta})$ whereas the right hand side is non decreasing. This together with $\gamma \in (0,\ f(\underline{\theta})(\overline{\theta}-\underline{\theta}))$ then require $q^{V}(\overline{\theta}) \in (q^{SB}(\overline{\theta}),\ q^{FB}(\overline{\theta}))$. As $\frac{d\phi(q)}{dq} > 0$ this then requires $V \in (\phi(q^{SB}(\overline{\theta})),\ \phi(q^{FB}(\overline{\theta}))$.

Suppose that $\gamma = 0$. Equation (2.41) gives $q(\overline{\theta}) = q^{FB}(\overline{\theta})$. If $\lambda = 0$ then (2.43) gives $\pi(\underline{\theta}) = 0$. If $\lambda > 0$ then (2.45) gives $\pi(\underline{\theta}) = 0$. Replacing in (2.44) gives $-\sum_{s\in S} r(\theta, s)\frac{g(s|\overline{\theta})-g(s|\underline{\theta})}{\overline{\theta}-\underline{\theta}} - \phi(q^{FB}(\overline{\theta})) \geq 0$. As $-\sum_{s\in S} r(\theta, s)\frac{g(s|\overline{\theta})-g(s|\underline{\theta})}{\overline{\theta}-\underline{\theta}} \leq V$ this then requires $V \geq \phi(q^{FB}(\overline{\theta}))$. From (2.42) any $r(\overline{\theta}, s)$ that satisfies $V \geq \phi(q^{FB}(\overline{\theta}))$ is optimal. The reimbursement scheme in (2.21) gives $-\sum_{s\in S} r(\theta, s)\frac{g(s|\overline{\theta})-g(s|\underline{\theta})}{\overline{\theta}-\underline{\theta}} = \phi(q^{FB}(\overline{\theta}))$ so it is optimal.

From the fact that $g(s \mid \theta)$ is a probability function there exists at least one outcome s such that $g(s \mid \overline{\theta}) - g(s \mid \underline{\theta}) \geq 0$. Therefore (2.12) is satisfied for $\overline{\theta}$. To verify that (2.16) is satisfied I consider $r(\underline{\theta}, s) = 0$ for all s. Equation (2.16) then requires $0 \geq \pi(\underline{\theta}) - (\overline{\theta}-\underline{\theta})\phi(q^{FB}(\underline{\theta}))$. This is satisfied if $0 \geq (\overline{\theta}-\underline{\theta})\phi(q^{SB}(\overline{\theta})) - (\overline{\theta}-\underline{\theta})\phi(q^{FB}(\underline{\theta}))$. As $q^{SB}(\overline{\theta}) < q^{FB}(\overline{\theta}) < q^{FB}(\underline{\theta})$ and $\frac{d\phi(q)}{dq} > 0$ this is satisfied. This completes the proof. □

Proof. (Proposition 2) An optimal contract in the relaxed problem maximises

(2.24) subject to (2.12), (2.13), (2.11) and (2.25) for all θ. The Hamiltonian for this problem when (2.12) and (2.13) are satisfied for all θ is:

$$H = (U(q(\theta)) - C(\theta, q(\theta)) - \pi(\theta))f(\theta) + \tag{2.46}$$

$$+\mu(\theta)\left(-\sum_{s\in S} r(\theta,s)\frac{dg(s\mid\theta)}{d\theta} - \phi(\theta, q(\theta))\right) + \tau(\theta)\,\pi(\theta)$$

where $\mu(\theta)$ is the costate variable and $\tau(\theta)$ is the Lagrange multiplier of (2.11).

The necessary and sufficient conditions for an optimal contract when (2.12) is satisfied for all θ are:[20]

$$f(\theta)\left(\frac{dU(q(\theta))}{dq} - \frac{dC(\theta, q(\theta))}{dq}\right) = \mu(\theta)\frac{d\phi(\theta, q(\theta))}{dq} \tag{2.47}$$

$$r(\theta, s) = \begin{cases} \psi & if\ \mu(\theta)\frac{dg(s|\theta)}{d\theta} < 0 \\ [0,\ \psi] & if\ \mu(\theta)\frac{dg(s|\theta)}{d\theta} = 0 \\ 0 & if\ \mu(\theta)\frac{dg(s|\theta)}{d\theta} > 0 \end{cases} \tag{2.48}$$

$$\frac{d\pi(\theta)}{d\theta} = -\sum_{s\in S} r(\theta,s)\frac{dg(s\mid\theta)}{d\theta} - \phi(\theta, q(\theta)) \tag{2.49}$$

$$\frac{d\mu(\theta)}{d\theta} = -\frac{dH}{d\pi(\theta)} = f(\theta) - \tau(\theta) \tag{2.50}$$

$$\tau(\theta)\pi(\theta) = 0,\ \tau(\theta) \geq 0,\ \pi(\theta) \geq 0 \tag{2.51}$$

$$\mu(\underline{\theta}) \leq 0,\ \mu(\underline{\theta})\pi(\underline{\theta}) = 0 \tag{2.52}$$

$$\mu(\overline{\theta}) \geq 0,\ \mu(\overline{\theta})\pi(\overline{\theta}) = 0 \tag{2.53}$$

The costate variable determines the quantity scheme and the reimbursement scheme (equations (2.47) and (2.48)), which together determine

[20]For a proof of the necessity and sufficiency of these conditions see Seierstad and Sydsaeter (1987).

the marginal decrease in the expected profit scheme (equation (2.49)). To characterise the costate variable for which an optimal contract in the relaxed problem has the separation property, I first define $\mu^V(\theta)$ as the value of the costate variable that results in $q(\theta) = q^V(\theta)$:

$$\left(\frac{dU(q^V(\theta))}{dq} - \frac{dC(\theta, q^V(\theta))}{dq}\right) f(\theta) = \mu^V(\theta)\frac{d\phi(\theta, q^V(\theta))}{dq} \tag{2.54}$$

The left hand side of (2.54) is decreasing in $q^V(\theta)$ whereas the right hand side is non decreasing. The comparison with (2.4) gives $\mu^V(\theta) \geq 0$ if $q^V(\theta) \leq q^{FB}(\theta)$. The comparison with (2.27) gives $\mu^V(\theta) \leq F(\theta)$ if $q^V(\theta) \leq q^{FB}(\theta)$. As $\frac{d\phi(\theta,q)}{dq} > 0$ this is equivalent to $\mu^V(\theta) \in [0,\ F(\theta)]$ if $V(\theta) \in [\phi(\theta, q^{SB}(\theta)),\ \phi(\theta, q^{FB}(\theta))]$.

I now consider the following costate variable:

$$\mu(\theta) = \begin{cases} F(\theta) & if\ V(\theta) \leq \phi(\theta, q^{SB}(\theta)) \\ \mu^V(\theta) & if\ V(\theta) \in [\phi(\theta, q^{SB}(\theta)),\ \phi(\theta, q^{FB}(\theta))] \\ 0 & if\ V(\theta) \geq \phi(\theta, q^{FB}(\theta)) \end{cases} \tag{2.55}$$

Replacing (2.55) in (2.47) results in the quantity scheme in (2.31). Also, the reimbursement scheme in (2.29) satisfies (2.48) as (2.55) gives $\mu(\theta) > 0$ if $V(\theta) < \phi(\theta, q^{FB}(\theta))$. Using (2.29) and (2.31) I can then write (2.49) as $\frac{d\pi(\theta)}{d\theta}$ $=\min\left\{V(\theta) - \phi(\theta, q^{SB}(\theta)),\ 0\right\} = -\max\left\{\phi(\theta, q^{SB}(\theta)) - V(\theta),\ 0\right\}$. Integrating the previous expression from θ to $\overline{\theta}$ with $\pi(\overline{\theta}) = 0$ results in the expected profit scheme in (2.30).

I next check that (2.55) satisfies (2.50). Equation (2.51) gives $\tau(\theta) \geq 0$ so (2.50) is satisfied if $\frac{d\mu(\theta)}{d\theta} \leq f(\theta)$. From (2.55) this requires $\frac{d\mu^V(\theta)}{d\theta} \leq f(\theta)$ whenever $V(\theta) \in [\phi(\theta, q^{SB}(\theta)),\ \phi(\theta, q^{FB}(\theta))]$. Applying the implicit function theorem in (2.54) I can write this inequality as:

$$\frac{d\mu^V(\theta)}{d\theta} = f(\theta)\left(\frac{d}{d\theta}\left(\frac{\frac{dU(q^V(\theta)))}{dq} - \frac{dC(\theta,q^V(\theta)))}{dq}}{\frac{d\phi(\theta,q^V(\theta))}{dq}}\right) + \frac{\mu^V(\theta)}{f(\theta)^2}\frac{df(\theta)}{d\theta}\right) \leq f(\theta) \quad (2.56)$$

The assumption that the hazard rate is non decreasing gives $\frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right) = = 1 - \frac{F(\theta)}{f(\theta)^2}\frac{df(\theta)}{d\theta} \geq 0$. As $\mu^V(\theta) \in [0,\ F(\theta)]$ this gives $1 - \frac{\mu^V(\theta)}{f(\theta)^2}\frac{df(\theta)}{d\theta} \geq 0$. As a result $\frac{d\mu^V(\theta)}{d\theta} \leq f(\theta)$ is satisfied if the first term in the parenthesis is non positive. This is the condition in Proposition 2.

I now verify that (2.55) and the expected profit scheme in (2.30) satisfy (2.51). If $V(\theta) \leq \phi(\theta, q^{SB}(\theta))$ then (2.55) and (2.50) give $\tau(\theta) = 0$ so (2.51) is satisfied. If $V(\theta) \in [\phi(\theta, q^{SB}(\theta)),\ \phi(\theta, q^{FB}(\theta))]$ and $\frac{d\mu^V(\theta)}{d\theta} < f(\theta)$ then (2.55) and (2.50) give $\tau(\theta) > 0$. If $V(\theta) \geq \phi(\theta, q^{FB}(\theta))$ then (2.55) and (2.50) give $\tau(\theta) = f(\theta)$. In both situations (2.51) gives $\pi(\theta) = 0$. The expected profit scheme in (2.30) satisfies $\pi(\theta) = 0$ for $V(\theta) \geq \phi(\theta, q^{SB}(\theta))$ if whenever this holds for any type θ it also holds for any higher type. The proof of Lemma 6 shows that (2.32) implies this.

Equation (2.55) and the expected profit scheme in (2.30) satisfy (2.52) and (2.53) as $\mu(\underline{\theta}) = 0$ and $\pi(\overline{\theta})$. From the fact that $g(s \mid \theta)$ is a probability function there exists at least one outcome s such that $\frac{dg(s|\theta)}{d\theta} \geq 0$ for all θ. Therefore (2.12) is satisfied for all θ. This completes the proof. □

Proof. (Lemma 6) Suppose that $V(\theta) \geq \phi((\theta, q^{SB}(\theta))$ for some type θ and this does not hold for some type higher than θ. There is then a type θ such that $V(\theta) = \phi(\theta, q^{SB}(\theta))$ and $\frac{dV(\theta)}{d\theta} < \frac{d\phi(\theta,q^{SB}(\theta))}{d\theta}$. As $V(\theta) = \phi(\theta, q^V(\theta))$ and $\frac{d\phi(\theta,q)}{dq} > 0$ the first equation gives $q^V(\theta) = q^{SB}(\theta)$. As $\frac{dV(\theta)}{d\theta} = \frac{d\phi(\theta,q^V(\theta))}{d\theta}$ and $\frac{d\phi(\theta,q)}{dq} > 0$ the second equation gives $\frac{dq^V(\theta)}{d\theta} < \frac{dq^{SB}(\theta)}{d\theta}$.

I now show that (2.32) gives $\frac{dq^{SB}(\theta)}{d\theta} \leq \frac{dq^V(\theta)}{d\theta}$. Equation (2.27) and the assumption that $\frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right) \geq 0$ gives:

$$\frac{d}{d\theta}\left(\frac{\frac{dU(q^{SB}(\theta)))}{dq} - \frac{dC(\theta,q^{SB}(\theta)))}{dq}}{\frac{d\phi(\theta,q^{SB}(\theta))}{dq}}\right) \geq 0 \tag{2.57}$$

Comparing (2.57) with (2.32) when $q^V(\theta) = q^{SB}(\theta)$ gives:

$$\frac{d}{dq}\left(\frac{\frac{dU(q^{SB}(\theta)))}{dq} - \frac{dC(\theta,q^{SB}(\theta)))}{dq}}{\frac{d\phi(\theta,q^{SB}(\theta))}{dq}}\right)\frac{dq^{SB}(\theta)}{d\theta} \geq \tag{2.58}$$

$$\frac{d}{dq}\left(\frac{\frac{dU(q^{SB}(\theta)))}{dq} - \frac{dC(\theta,q^{SB}(\theta)))}{dq}}{\frac{d\phi(\theta,q^{SB}(\theta))}{dq}}\right)\frac{dq^{V}(\theta)}{d\theta}$$

The common derivative in (2.58) is negative as $\frac{dU(q^{SB}(\theta)))}{dq} - \frac{dC(\theta,q^{SB}(\theta)))}{dq} > 0$, $\frac{d\phi(\theta,q)}{dq} > 0$ and $\frac{d^2\phi(\theta,q)}{dq^2} \geq 0$. Equation (2.58) then gives $\frac{dq^{SB}(\theta)}{d\theta} \leq \frac{dq^{V}(\theta)}{d\theta}$.

The proof for $V(\theta) \geq \phi((\theta, q^{FB}(\theta))$ follows the same steps dividing (2.4) by $\frac{d\phi(\theta,q^{FB}(\theta))}{dq}$. This completes the proof. $\square$

Proof. (Proposition 3) If the audit has the monotonicity property then the contract in Proposition 2 is an optimal contract if it satisfies (2.38) for all θ. I show that if the value is non increasing and both the cost of production and the marginal cost of production are convex in θ then the contract in Proposition 2 satisfies (2.38) for all θ. If the cost of production is convex in θ then $\frac{d\phi(\theta,q)}{d\theta} \geq 0$. If the marginal cost of production is convex in θ then $\frac{d^2\phi(\theta,q)}{d\theta dq} \geq 0$. Equation (2.27) and the assumption that $\frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right) \geq 0$ then gives $\frac{dq^{SB}(\theta)}{d\theta} < 0$.

I consider the situation in which the three intervals in Corollary 2 exist. The proof applies a fortiori if not all of them do:

Suppose that $\tilde{\theta} \in [\underline{\theta}, \theta_{SB}]$ and $\theta_{SB} > \underline{\theta}$. There are two possible situations. First, if $\theta \in [\underline{\theta}, \theta_{SB}]$ then (2.38) reduces to $0 \geq \int_{\theta}^{\tilde{\theta}} \phi(\eta, q^{SB}(\tilde{\theta})) - \phi(\eta, q^{SB}(\eta))d\eta$ for all $\theta \in [\underline{\theta}, \theta_{SB}]$ and $\tilde{\theta} \neq \theta$. As $\frac{d\phi(\theta,q)}{dq} > 0$ and $\frac{dq^{SB}(\theta)}{d\theta} < 0$ then

$\phi(\eta, q^{SB}(\tilde{\theta})) < \phi(\eta, q^{SB}(\eta))$ if $\tilde{\theta} > \theta$ and $\phi(\eta, q^{SB}(\tilde{\theta})) > \phi(\eta, q^{SB}(\eta))$ if $\tilde{\theta} < \theta$ so the integral is negative. Second, if $\theta \in [\theta_{SB}, \overline{\theta}]$ then (2.38) reduces to $0 \geq \int_{\tilde{\theta}}^{\theta_{SB}} \phi(\eta, q^{SB}(\eta)) - \phi(\eta, q^{SB}(\tilde{\theta}))d\eta + \int_{\theta_{SB}}^{\theta} V(\eta) - \phi(\eta, q^{SB}(\tilde{\theta}))d\eta$ for all $\theta \in [\theta_{SB}, \overline{\theta}]$ and $\tilde{\theta} \neq \theta$. As $\frac{d\phi(\theta,q)}{dq} > 0$ and $\frac{dq^{SB}(\theta)}{d\theta} < 0$ the first integral is non positive. As $\frac{dV(\theta)}{d\theta} \leq 0$, $\frac{d\phi(\theta,q)}{d\theta} \geq 0$, $\frac{d\phi(\theta,q)}{dq} > 0$ and $\frac{dq^{SB}(\theta)}{d\theta} < 0$ then $V(\eta) \leq V(\theta_{SB}) = \phi(\theta_{SB}, q^{SB}(\theta_{SB})) \leq \phi(\eta, q^{SB}(\theta_{SB})) < \phi(\eta, q^{SB}(\eta))$ so the second integral is non positive.

Suppose that $\tilde{\theta} \in [\theta_{SB}, \theta_{FB}]$, $\theta_{SB} > \underline{\theta}$ and $\theta_{SB} < \overline{\theta}$. Equation (2.38) reduces to $\pi(\theta) \geq 0 + \int_{\theta}^{\tilde{\theta}} \phi(\eta, q^V(\tilde{\theta})) - V(\eta)d\eta$ for all $\tilde{\theta} \neq \theta$. As $\frac{dV(\theta)}{d\theta} \leq 0$ and $\frac{d\phi(\theta,q)}{d\theta} \geq 0$ then $V(\eta) \geq V(\tilde{\theta}) = \phi(\tilde{\theta}, q^V(\tilde{\theta})) \geq \phi(\eta, q^{SB}(\tilde{\theta}))$ if $\tilde{\theta} > \theta$ and $V(\eta) \leq V(\tilde{\theta}) = \phi(\tilde{\theta}, q^V(\tilde{\theta})) \leq \phi(\eta, q^{SB}(\tilde{\theta}))$ if $\tilde{\theta} < \theta$ so the integral is non positive for all $\tilde{\theta} \neq \theta$.

Suppose that $\tilde{\theta} \in [\theta_{FB}, \overline{\theta}]$ and $\theta_{FB} < \overline{\theta}$. Equation (2.38) reduces to $\pi(\theta) \geq 0 + \int_{\theta}^{\tilde{\theta}} \phi(\eta, q^{FB}(\tilde{\theta})) - \frac{\phi(\tilde{\theta}, q^{FB}(\tilde{\theta}))}{V(\tilde{\theta})} V(\eta)d\eta$ for all $\tilde{\theta} \neq \theta$. As $\frac{dV(\theta)}{d\theta} \leq 0$ and $\frac{d\phi(\theta,q)}{d\theta} \geq 0$ then $V(\eta) \geq V(\tilde{\theta})$ and $\phi(\tilde{\theta}, q^{FB}(\tilde{\theta})) \leq \phi(\eta, q^{FB}(\tilde{\theta}))$ if $\tilde{\theta} > \theta$ and $V(\eta) \leq V(\tilde{\theta})$ and $\phi(\tilde{\theta}, q^{FB}(\tilde{\theta})) \geq \phi(\eta, q^{FB}(\tilde{\theta}))$ if $\tilde{\theta} < \theta$ so the integral is non positive for all $\tilde{\theta} \neq \theta$. This completes the proof. $\square$

Chapter 3

Optimal Procurement with a Contingent Contract for the Internal Auditor

3.1 Introduction

In this chapter I characterise an optimal procurement contract for a government that purchases a good or service from a firm that has private information about its cost of production when the government has available the reports of a corruptible internal auditor and an honest but less well informed external auditor. In contrast with chapter 2, I do so allowing the government to offer the internal auditor a contract that consists of a transfer, a reimbursement and a penalty that are a function of the same contingencies as the transfer, the reimbursement and the penalty that the government offers to the firm.[1] This contract leads to several possible side contracts, depending on the contracting variables. I assume that the firm can offer the internal auditor a side contract

[1]This chapter makes substantial use of the terminology, notation, definitions and results in chapter 2. I therefore strongly recommend reading that chapter before proceeding further.

that includes the claims of the firm from the point in time at which the offer takes place and the report of the internal auditor. I then consider two possible situations. In the first situation side contracting takes place after the firm makes a claim about its type but before it makes a claim about the outcome of the audit. In the second situation side contracting takes place before the firm makes a claim about its type. I refer to these two situations as ex post and ex ante side contracting respectively.

I first characterise an optimal contract with ex post side contracting. In that situation the government can ensure that the internal auditor reports truthfully when it rejects an ex post side contract by requiring a reimbursement from the internal auditor that is not contingent on its report. As a result, in an optimal ex post side contract the firm pays the internal auditor a zero bribe if it requests the internal auditor to report truthfully and a bribe equal to the difference in reimbursement plus the expected penalty that the internal auditor pays otherwise. The contract design problem is then as if the firm selects its claim about the outcome of the audit and the report of the internal auditor to minimise the sum of the reimbursements plus the sum of the expected penalties.

In Proposition 4 I demonstrate that when side contracting takes place ex post the government can achieve the outcome of the first best contract both for the case of two cost types and for the case of a continuum of cost types in the relaxed problem if the value of the audit is positive (for all the types of the firm in the latter case). The government offers the firm a reimbursement scheme that prevents the firm from making a false claim for the first best quantity scheme and expected profit scheme. The government ensures that bribery does not take place by giving the internal auditor the reimbursement that the firm pays. This is not costly for the government as for every type of the firm the government pays the internal auditor a transfer equal to the expected reimbursement that the internal auditor pays.

I next characterise an optimal contract with ex ante side contracting assuming that the internal auditor observes the type of the firm. This assumption ensures that in an optimal ex ante side contract the firm pays the internal auditor an expected bribe such that the expected payoff of the internal auditor is the same as if it rejects the side contract. The contract design problem is then as if the firm selects its claims about its type and about the outcome of the audit and the report of the internal auditor to maximise the sum of their expected payoffs.

The claims and the report that the firm selects with ex ante side contracting depend on the sum of the transfers and the sum of the reimbursements but not on how they add up. Therefore with ex ante side contracting it is optimal for the government to offer the internal auditor a non contingent contract. The contract design problem is then as in chapter 2: the firm controls the report of the internal auditor at a cost of misreporting equal to the sum of the expected penalties. An optimal contract is then given by Proposition 1 for the case of two cost types and by Propositions 2 and 3 for the case of a continuum of cost types. I employ this section to address the question of why bribery does not take place in an optimal contract.

In Proposition 5 I prove that if the sum of the expected penalties that the firm and the internal auditor pay is independent of the extent of the misreporting then the government can achieve the optimal quantity scheme and expected profit scheme of a contract for which bribery takes place with a contract for which bribery does not take place. As bribery is costly for the government, it is then optimal for the government to deter it. This result is valid regardless of the the distribution of the outcome of the audit and of the belief of the government about the type of the firm.

The structure of the chapter is as follows: In the next section I describe the model under consideration and in section 3.3 I present the benchmark case in which the government observes the type of the firm. After this, in section 3.4 I state the contract design problem with ex post side contracting and characterise an optimal contract and in section 3.5 I perform the same tasks with ex ante side contracting. With the analysis completed, in section 3.6 I conclude, discuss the relationship between my results and existing research and comment on some possible extensions. Finally, in the appendix I prove those results that do not follow directly from others.

3.2 The model

3.2.1 Contracting parties and information structures

The contracting parties are the same as in subsection 2.2.1. The information structures of the government, the firm and the external auditor are also the same. I now assume that the internal auditor observes both the outcome of the audit and the type of the firm.[2] For the case of a continuum of cost types I also assume that $g(s \mid \theta)$ is twice continuously differentiable in θ, as I did in subsection 2.6.1.

3.2.2 Contracts

The contract that the government offers to the firm consists of a transfer $t_F(\tilde{\theta})$ in exchange for a quantity $q(\tilde{\theta})$ and a reimbursement $r_F(\tilde{\theta}, \tilde{s}, \hat{s})$. The contract that the government offers to the internal auditor consists of a transfer $t_A(\tilde{\theta})$

[2] This assumption is not relevant in section 3.4 but I use it to simplify the exposition of my results.

and a reimbursement $r_A(\tilde{\theta}, \tilde{s}, \hat{s})$. The firm selects the claims $\tilde{\theta}$ and $\tilde{s}$ from two message spaces M_θ and M_s after observing θ and s respectively.

The ex post side contract consists of a bribe $b(s, \tilde{s})$ and a report $\hat{s}(s, \tilde{s})$. The meaning of this side contract is that, when the outcome of the audit is s, the firm makes the claim $\tilde{s}$ and pays the internal auditor the bribe $b(s, \tilde{s})$ if the internal auditor reports $\hat{s} = \hat{s}(s, \tilde{s})$. The side contract $b(s, s) = 0$ and $\hat{s}(s, s) = s$ corresponds to no ex post bribery.

The ex ante side contract consists of a bribe $b(\theta, \tilde{\theta}, s, \tilde{s})$ and a report $\hat{s}(\theta, \tilde{\theta}, s, \tilde{s})$ for all s. The meaning of this side contract is that, when the type of the firm is θ, the firm makes the claim $\tilde{\theta}$ and after observing s makes the claim $\tilde{s}$ and pays the internal auditor the bribe $b(\theta, \tilde{\theta}, s, \tilde{s})$ if the internal auditor reports $\hat{s} = \hat{s}(\theta, \tilde{\theta}, s, \tilde{s})$. The side contract $b(\theta, \theta, s, s) = 0$ and $\hat{s}(\theta, \theta, s, s) = s$ for all s corresponds to no ex ante bribery.

If the external auditor detects misreporting then the firm and the internal auditor pay respective penalties P_F and P_A. I assume that the penalties are legally specified so the government has no choice over them.

3.2.3 Preferences

The payoff of the government when the firm produces a quantity q, the government pays transfers t_F and t_A and it receives reimbursements r_F and r_A is $U(q) - t_F - t_A + r_F + r_A$. I make the assumptions on $U(q)$ that I made in subsection 2.2.3.

The payoff (profit) of the firm when it produces a quantity q, it receives a transfer t_F and it pays a reimbursement r_F, a bribe b and a penalty P_F is $\pi_F = t_F - r_F - b - P_F - C(\theta, q)$. I make the assumptions on $C(\theta, q)$ that I made in subsection 2.2.3. For the case of a continuum of cost types I also assume that $C(\theta, q)$ is twice continuously differentiable in θ, as I did in subsection 2.6.1.

The payoff (profit) of the internal auditor when it receives a transfer t_A and a bribe b and it pays a reimbursement r_A and a penalty P_A is $\pi_A = t_A + b - r_A - P_A$. The payoff of the external auditor does not depend on the relationship with the other contracting parties.

The contracting parties are risk neutral. The firm and the internal auditor accept a contract if their expected payoff is greater or equal than their reservation payoff, which I normalize to zero.

3.2.4 Timing

The timing of the contractual relationships is as follows:[3]

1. The firm and the internal auditor observe θ.
2. The government offers a contract to the firm and the internal auditor.
3. With ex ante side contracting the firm offers a side contract to the internal auditor.
4. The firm makes a claim $\tilde{\theta}$, produces a quantity q and receives a transfer t_F. The internal auditor receives a transfer t_A.
5. The firm and the internal auditor observe s.
6. With ex post side contracting the firm offers a side contract to the internal auditor.
7. The firm makes a claim $\tilde{s}$ and the internal auditor reports $\hat{s}$. The firm pays a reimbursement r_F and a bribe b. The internal auditor pays a reimbursement r_A.

[3]In this timing I do not mention what happens when the firm or the internal reject the contract or the internal auditor rejects the side contract as these are not optimal choices.

8. The external auditor observes σ and reports. If it reports evidence of misreporting then the firm and the internal auditor pay respective penalties P_F and P_A.

3.3 The benchmark case

As a benchmark case for later use, I first consider the situation in which the government observes the type of the firm. In that situation, the contract that the government offers to the firm consists of a transfer $t_F(\theta)$ in exchange for a quantity $q(\theta)$. The contract that the government offers to the internal auditor consists of a transfer $t_A(\theta)$. The payoff of the government is the utility of the quantity that it demands minus the transfers that it pays. This is given by:

$$U(q(\theta)) - t_F(\theta) - t_A(\theta) \tag{3.1}$$

The firm accepts the contract if:

$$\pi_F(\theta) = t_F(\theta) - C(\theta, q(\theta)) \geq 0 \tag{3.2}$$

whereas the internal auditor accepts the contract if:

$$\pi_A(\theta) = t_A(\theta) \geq 0 \tag{3.3}$$

An optimal contract maximises (3.1) subject to (3.2) and (3.3) for all θ. The optimal transfer that the government pays to the firm is equal to the cost of production so the firm makes a zero first best expected profit:

$$\pi_F^{FB}(\theta) = 0 \tag{3.4}$$

The optimal transfer that the government pays to the internal auditor is equal to zero so the internal auditor makes a zero first best expected profit:

$$\pi_A^{FB}(\theta) = 0 \tag{3.5}$$

The government demands the first best quantity $q^{FB}(\theta)$ defined in (2.4).

3.4 An optimal contract with ex post side contracting

3.4.1 Statement of the problem

I state the contract design problem with ex post side contracting under two restrictions on the contract that the government offers to the firm and the internal auditor. The purpose of these restrictions is to facilitate the characterisation of an optimal contract by making the contract design problem comparable to that in chapter 2. I will later show that there is a contract that satisfies these restrictions and results in the first best quantity scheme and expected profit scheme.

First, I assume that the government offers a contract in which the message spaces are the set of types and the set of outcomes of the audit ($\tilde{\theta} \in \Theta$ and $\tilde{s} \in S$). Second, I assume that the government offers a contract that is truthful ($\tilde{\theta} = \theta$ and $\tilde{s} = s$) and for which bribery does not take place ($b(s, s) = 0$ and $\hat{s}(s, s) = s$). I then consider a contract in which the government requests an arbitrarily high reimbursement from the firm when its claim about the outcome of the audit does not coincide with the report of the internal auditor ($r_F(\tilde{\theta}, \tilde{s}, \hat{s}) = \infty$ if $\hat{s} \neq \tilde{s}$). I also restrict attention to a contract in which the government requests a reimbursement from the internal auditor that is not contingent on its report. With these simplifications I can write the contract that the government offers to the firm and the internal auditor as $\left\{q(\tilde{\theta}), t_F(\tilde{\theta}), r_F(\tilde{\theta}, \tilde{s}), t_A(\tilde{\theta}), r_A(\tilde{\theta}, \tilde{s})\right\}$, where the firm pays the reimbursement $r_F(\tilde{\theta}, \tilde{s})$ if its claim about the outcome of the audit coincides with the report of the internal auditor.

The payoff of the government with a truthful contract for which bribery does not take place is:

$$E_{f(\theta)}\left\{Uq((\theta)) - t_F(\theta) + \sum_{s \in S} r_F(\theta, s) g(s \mid \theta) \right. \tag{3.6}$$
$$\left. - t_A(\theta) + \sum_{s \in S} r_A(\theta, s) g(s \mid \theta)\right\}$$

I now focus on the side contract. The internal auditor reports truthfully whenever it rejects the side contract to avoid paying a penalty. The firm then makes a truthful claim about the outcome of the audit to avoid paying an arbitrarily high reimbursement. Therefore, after the firm claims that its type is $\tilde{\theta}$ and the outcome of the internal audit is s, the payoff of the internal auditor if it rejects the side contract is $t_A(\tilde{\theta}) - r_A(\tilde{\theta}, s)$.

In any optimal side contract the firm requests a report equal to its claim about the outcome of the audit ($\hat{s}(s, \tilde{s}) = \tilde{s}$) as otherwise it pays an arbitrarily high reimbursement. As a result, in any optimal side contract the internal auditor pays an expected penalty hP_A if the claim of the firm about the outcome of the audit differs from the truth ($\tilde{s} \neq s$). Therefore, after the firm claims that its type is $\tilde{\theta}$, the outcome of the internal audit is s and the firm claims that it is $\tilde{s}$, the payoff of the internal auditor is $t_A(\tilde{\theta}) + b(s, s) - r_A(\tilde{\theta}, s)$ if $\tilde{s} = s$ and $t_A(\tilde{\theta}) + b(s, \tilde{s}) - r_A(\tilde{\theta}, \tilde{s}) - hP_A$ otherwise. In consequence, in any optimal side contract the firm pays a zero bribe if it makes a truthful claim about the outcome of the audit ($b(s,s) = 0$ if $\tilde{s} = s$, bribery does not take place) and a bribe equal to the difference in the reimbursement plus the expected penalty that the internal auditor pays otherwise ($b(s,\tilde{s}) = r_A(\tilde{\theta},\tilde{s}) - r_A(\tilde{\theta},s) + hP_A$ if $\tilde{s} \neq s$).

I refer to a truthful claim about the outcome of the audit as bribery not taking place. After the firm claims that its type is θ, when the outcome of the internal audit is s bribery does not take place if:

$$r_F(\theta,s) \leq r_F(\theta,\tilde{s}) + r_A(\theta,\tilde{s}) - r_A(\theta,s) + hP_A + hP_F \quad \forall \tilde{s} \neq s \tag{3.7}$$

If this condition holds for all the types of the firm and all the outcomes of the audit then the firm prefers to make a truthful claim when its type is θ if:

$$\begin{gathered} t_F(\theta) - \sum_{s \in S} r_F(\theta,s) g(s \mid \theta) - C(\theta,q(\theta)) \geq \\ t_F(\tilde{\theta}) - \sum_{s \in S} r_F(\tilde{\theta},s) g(s \mid \theta) - C(\theta,q(\tilde{\theta})) \quad \forall \tilde{\theta} \neq \theta \end{gathered} \tag{3.8}$$

The firm and the internal auditor accept the contract if:

$$\pi_F(\theta) = t_F(\theta) - \sum_{s \in S} r_F(\theta,s) g(s \mid \theta) - C(\theta,q(\theta)) \geq 0 \tag{3.9}$$

$$\pi_A(\theta) = t_A(\theta) - \sum_{s \in S} r_A(\theta,s) g(s \mid \theta) \geq 0 \tag{3.10}$$

An optimal truthful contract for which bribery does not take place maximises (3.6) subject to (3.7) for all θ and all s and (3.8), (3.9) and (3.10) for all θ.

3.4.2 Alternative statement of the problem

I next perform a change in variables that provides more intuition into the contract design problem and facilitates its comparison with that in chapter 2.

I consider a contract $\left\{q(\tilde{\theta}), \pi_F(\tilde{\theta}), r_F(\tilde{\theta}, \tilde{s}), \pi_A(\tilde{\theta}), r_A(\tilde{\theta}, \tilde{s})\right\}$. The transfers that the firm and the internal auditor obtain are given by (3.9) and (3.10) respectively.

After this change in variables I can write (3.6) as:

$$E_{f(\theta)}\left\{U(q(\theta)) - C(\theta, q(\theta)) - \pi_F(\theta) - \pi_A(\theta)\right\} \tag{3.11}$$

and (3.8) becomes:

$$\pi_F(\theta) \geq \pi_F(\tilde{\theta}) + \sum_{s \in S} r_F(\tilde{\theta}, s)\left(g(s \mid \tilde{\theta}) - g(s \mid \theta)\right) + \tag{3.12}$$

$$+C(\tilde{\theta}, q(\tilde{\theta})) - C(\theta, q(\tilde{\theta})) \quad \forall \tilde{\theta} \neq \theta$$

while (3.9) and (3.10) are simply:

$$\pi_F(\theta) \geq 0 \tag{3.13}$$

$$\pi_A(\theta) \geq 0 \tag{3.14}$$

An optimal truthful contract for which bribery does not take place maximises (3.11) subject to (3.7) for all θ and all s and (3.12), (3.13) and (3.14) for all θ. The contract design problem presents two differences with respect to that in chapter 2. First, it is as if the government faces an additional cost: the expected profit of the internal auditor. Second, the constraint that guarantees that bribery does not take place (equation (3.7)) differs from the equivalent constraint in chapter 2 (equation (2.6)) in that it contains the difference in the reimbursement that the internal auditor pays.

3.4.3 An optimal contract

The crucial observation to characterise an optimal contract with ex post side contracting is that the constraint that bribery does not take place is in fact a constraint on the range of the sum of the reimbursements. To see this, it suffices to write (3.7) as:

$$r_F(\theta, s) + r_A(\theta, s) \leq r_F(\theta, \tilde{s}) + r_A(\theta, \tilde{s}) + hP_A + hP_F \quad \forall \tilde{s} \neq s \tag{3.15}$$

and follow the same logic as in Lemma 1.

Making use of this observation, Proposition 4 characterises an optimal contract both for the case of two cost types and for the case of a continuum of cost types:

Proposition 4. *A reimbursement scheme of the internal auditor for which bribery does not take place is given by:*

$$r_A(\theta, s) = -r_F(\theta, s) + \rho(\theta, s) \tag{3.16}$$

where the range of $\rho(\theta, s)$ *for the outcome of the audit* $(\max\limits_{s \in S} \rho(\theta, s) - \min\limits_{s \in S} \rho(\theta, s))$ *is smaller or equal than the sum of the expected penalties for all the types of the firm.*

For the case of two cost types if the value of the audit is positive then in an optimal contract the firm and the internal auditor make a zero expected profit ($\pi_F(\theta) = 0$ *and* $\pi_A(\theta) = 0$) *and the government demands the first best quantity* ($q(\theta) = q^{FB}(\theta)$) *for both types of the firm. The reimbursement scheme of the firm is given by:*

$$r_F(\overline{\theta}, s) = \begin{cases} \psi \frac{\phi(q^{FB}(\overline{\theta}))}{V} & if \ g(s \mid \overline{\theta}) - g(s \mid \underline{\theta}) < 0 \\ 0 & if \ g(s \mid \overline{\theta}) - g(s \mid \underline{\theta}) \geq 0 \end{cases} \tag{3.17}$$

For the case of a continuum of cost types if the value of the audit is positive for all the types of the firm then in an optimal contract in the relaxed *problem the firm and the internal auditor make a zero expected profit and the government demands the first best quantity for all the types of the firm. The reimbursement scheme of the firm is given by:*

$$r_F(\theta, s) = \begin{cases} \psi \frac{\phi(\theta, q^{FB}(\theta))}{V(\theta)} & if \ \frac{dg(s|\theta)}{d\theta} < 0 \\ 0 & if \ \frac{dg(s|\theta)}{d\theta} \geq 0 \end{cases} \tag{3.18}$$

If the audit has the monotonicity property, the value of the audit is non increasing and the cost of production is convex in θ then this is an optimal contract.

In the reimbursement scheme of the internal auditor in (3.16) the government gives the internal auditor the reimbursement that the firm pays $(-r_F(\theta, s))$ and it requests an amount $\rho(\theta, s)$. The sum of the reimbursements is then $\rho(\theta, s)$. As this has a range for the outcome of the audit smaller or equal than the sum of the expected penalties it then follows from Lemma 1 that bribery does not take place.

The construction of the reimbursement scheme of the internal auditor in (3.16) is possible for any reimbursement scheme of the firm. In an optimal contract for every type of the firm the government pays the internal auditor a transfer equal to the expected reimbursement that the internal auditor pays so its expected payoff is zero. The contract design problem is then as in chapter 2 but with no constraint on the range of the reimbursement of the firm. For the case of two cost types the reimbursement scheme of the firm in (3.17) achieves the first best quantity scheme and expected profit scheme if the value of the audit is positive. For the case of a continuum of cost types the reimbursement scheme of the firm in (3.18) achieves the first best quantity scheme and expected profit scheme in the relaxed problem if the value of the audit is positive for all θ.[4] It then follows from Proposition 3 that an optimal contract in the relaxed problem is an optimal contract if the remaining assumptions in Proposition 4 are satisfied.[5]

It is worth pointing out that Proposition 4 holds if the internal auditor does not observe the type of the firm. In that situation the internal auditor has a belief about the type of the firm that it uses to determine its expected payoff from accepting the contract that the government offers. For the contract in Proposition 4 this belief does not play a role in the decision of accepting the contract as for any type of the firm the internal auditor makes a zero expected profit.

3.5 An optimal contract with ex ante side contracting

3.5.1 Ex post versus ex ante side contracting

I begin this section by showing that with ex ante side contracting the contract in Proposition 4 might result in bribery and as a result fail to achieve the first best quantity scheme and expected profit scheme. I consider a side contract with a bribe $b(\theta, \tilde{\theta}, s, s) = r_A(\tilde{\theta}, s) - t_A(\tilde{\theta})$ and a report $\hat{s}(\theta, \tilde{\theta}, s, s) = s$ for all s. That is, when its type is θ, the firm claims that its type is $\tilde{\theta}$ and for all the outcomes of the audit it makes a truthful claim ($\tilde{s} = s$), it requests the

[4]Proposition 4 is valid if the value of the audit is zero because the sum of the expected penalties is zero. Unlike the situation in which the sum of the expected penalties is positive, this is however a knife edge result that is due to my assumption that when the firm is indifferent between bribing an not bribing the internal auditor it chooses the latter.

[5]The condition in Proposition 2 given in (2.32) is not necessary as the government achieves the outcome of the first best contract. The assumption in subsection 2.6.1 that the hazard rate is non decreasing and the assumption in Proposition 3 that the marginal cost of production is convex in θ are also not necessary as its unique role is to ensure that the second best quantity is non increasing.

internal auditor to report truthfully and it pays the internal auditor a bribe equal to the reimbursement that the internal auditor pays minus the transfer that it receives. The payoff of the internal auditor is then zero for all the outcomes of the audit. Therefore its expected payoff is zero so it accepts the side contract.[6]

The expected payoff of the firm when its type is θ and it offers the above side contract is given by:

$$t_F(\tilde{\theta}) - \sum_{s \in S} r_F(\tilde{\theta}, s) g(s \mid \theta) - \sum_{s \in S} r_A(\tilde{\theta}, s) g(s \mid \theta) + t_A(\tilde{\theta}) - C(\theta, q^{FB}(\tilde{\theta})) \quad (3.19)$$

For the contract in Proposition 4 the transfers $t_F(\tilde{\theta})$ and $t_A(\tilde{\theta})$ are given by (3.9) and (3.10) with $\theta = \tilde{\theta}$, $\pi_F(\tilde{\theta}) = 0$ and $\pi_A(\tilde{\theta}) = 0$. The expected payoff of the firm when its type is θ is then:

$$\begin{aligned} &\sum_{s \in S} r_F(\tilde{\theta}, s) g(s \mid \tilde{\theta}) + C(\tilde{\theta}, q^{FB}(\tilde{\theta})) - \sum_{s \in S} r_F(\tilde{\theta}, s) g(s \mid \theta) \\ &- \sum_{s \in S} r_A(\tilde{\theta}, s) g(s \mid \theta) + \sum_{s \in S} r_A(\tilde{\theta}, s) g(s \mid \tilde{\theta}) - C(\theta, q^{FB}(\tilde{\theta})) \end{aligned} \quad (3.20)$$

which using (3.16) is simply:

$$\sum_{s \in S} \rho(\tilde{\theta}, s) \left(g(s \mid \tilde{\theta}) - g(s \mid \theta) \right) + C(\tilde{\theta}, q^{FB}(\tilde{\theta})) - C(\theta, q^{FB}(\tilde{\theta})) \quad (3.21)$$

For the contract in Proposition 4 if bribery does not take place then the expected profit of the firm is zero. Therefore the firm does not offer the side contract under consideration when its type is θ if:

$$\begin{aligned} 0 \geq \sum_{s \in S} \rho(\tilde{\theta}, s) \left(g(s \mid \tilde{\theta}) - g(s \mid \theta) \right) + \\ + C(\tilde{\theta}, q^{FB}(\tilde{\theta})) - C(\theta, q^{FB}(\tilde{\theta})) \quad \forall \tilde{\theta} \neq \theta \end{aligned} \quad (3.22)$$

[6] This is true for any type of the firm and any claim of the firm about it. Therefore the assumption that the internal auditor observes the type of the firm does not play a role here.

Equation (3.22) is the same as (2.10) for the first best quantity scheme and expected profit scheme replacing $\rho(\theta, s)$ with $r(\theta, s)$. Also, the constraint on the range of $\rho(\theta, s)$ in Proposition 4 is the same as that on the range of $r(\theta, s)$ in Lemma 1. It then follows from Proposition 1 that for the case of two cost types the firm does not offer the side contract under consideration only if the value of the audit is sufficiently high ($V \geq \phi(q^{FB}(\overline{\theta}))$). For the said reason it follows from Proposition 2 that the same is true only if the value of the audit for type θ is sufficiently high for all the types of the firm ($V(\theta) \geq \phi(\theta, q^{FB}(\theta))$ for all θ).

3.5.2 Statement of the problem

I now start the characterisation of the contract design problem with ex ante side contracting considering the side contract. The assumption that the internal auditor observes the type of the firm makes it unnecessary to obtain the expect payoff of the internal auditor if it rejects the side contract: whatever this expected payoff is in an optimal ex ante side contract the firm pays an expected bribe such that the expected payoff of the internal auditor is the same as if it rejects the side contract.[7] The contract design problem is then as if the firm selects its claims and the report of the internal auditor to maximise the sum of their expected payoffs.

The claims and the report that the firm selects with ex ante side contracting depend on the sum of the transfers and the sum of the reimbursements but not on how they add up. Therefore for any optimal contract there is an alternative optimal contract in which the firm receives the transfer of the internal auditor and pays its reimbursement. In other words, with ex ante side contracting it

[7] If the internal auditor did not observe the type of the firm then the firm and the internal auditor would be in an informed principal, uninformed agent situation. It is well known that this situation might result in a multiplicity of optimal side contracts, some of which are coalitionally inefficient (see Maskin and Tirole (1992)).

is optimal for the government to offer the internal auditor a non contingent contract.

In an optimal side contract of a non contingent contract for the internal auditor the firm pays the internal auditor an expected bribe equal to the expected penalty that the internal auditor pays.[8] The contract design problem is then as in chapter 2: for every outcome of the audit it is as if the firm controls the report of the internal auditor at a cost of misreporting equal to the sum of the expected penalties. Therefore, it follows from subsection 2.4.1 that there is an optimal contract $\left\{q(\tilde{\theta}), t_F(\tilde{\theta}), r_F(\tilde{\theta},\tilde{s}), \hat{s}(\tilde{\theta},\tilde{s})\right\}$ in which the message spaces are the set of types and the set of outcomes of the audit ($\tilde{\theta} \in \Theta$ and $\tilde{s} \in S$) and in which the government makes a recommendation to the firm about what report to request from the internal auditor ($\hat{s}(\tilde{\theta},\tilde{s})$). Furthermore, this contract is truthful ($\tilde{\theta} = \theta$ and $\tilde{s} = s$) and obedient ($\hat{s} = \hat{s}(\tilde{\theta},\tilde{s})$).

In subsection 2.4.1 I discussed the interpretation of the recommendation. I then claimed that, with the information structure of the external auditor that I have assumed, there is an optimal truthful contract for which the government recommends that the firm requests a truthful report from the internal auditor for all the types of the firm and all the outcomes of the audit ($\hat{s}(\theta, s) = s$ for all θ and all s). I next stated the contract design problem using this result and referring to a truthful claim about the outcome of the audit as bribery not taking place. I now state the contract design problem without this result, reserving the qualifier "bribery does not take place" for when the government recommends that the firm requests a truthful report from the internal auditor.

The payoff of the government in a truthful contract is given by:

$$E_{f(\theta)}\left\{Uq((\theta)) - t_F(\theta) + \sum_{s\in S} r_F(\theta,s)g(s \mid \theta)\right\} \tag{3.23}$$

[8] In this sentence the expected penalty is both over the report of the external auditor and over the outcome of the audit.

After the firm claims that its type is θ, when the outcome of the internal audit is s the firm prefers to make a truthful claim about it if:

$$r_F(\theta, s) + 1_{\hat{s}(\theta,s)\neq s}\psi \leq r_F(\theta, \tilde{s}) + 1_{\hat{s}(\theta,\tilde{s})\neq s}\psi \quad \forall \tilde{s} \neq s \tag{3.24}$$

If this condition holds for all the types of the firm and all the outcomes of the audit then the firm prefers to make a truthful claim about its type when its type is θ if:

$$\begin{gathered} t_F(\theta) - \sum_{s\in S}\Big(r_F(\theta, s) + 1_{\hat{s}(\theta,s)\neq s}\psi\Big)g(s \mid \theta) - C(\theta, q(\theta)) \geq \\ t_F(\tilde{\theta}) - \sum_{s\in S}\Big(r_F(\tilde{\theta}, s) + 1_{\hat{s}(\tilde{\theta},s)\neq s}\psi\Big)g(s \mid \theta) - C(\theta, q(\tilde{\theta})) \quad \forall \tilde{\theta} \neq \theta \end{gathered} \tag{3.25}$$

and accepts the contract if:

$$\pi_F(\theta) = t_F(\theta) - \sum_{s\in S}\Big(r_F(\theta, s) + 1_{\hat{s}(\theta,s)\neq s}\psi\Big)g(s \mid \theta) - C(\theta, q(\theta)) \geq 0 \tag{3.26}$$

An optimal contract maximises (3.23) subject to (3.24) for all θ and all s and (3.25) and (3.26) for all θ.

3.5.3 Alternative statement of the problem

I next perform a change in variables that provides more intuition into the contract design problem and facilitates its comparison with that in chapter 2. I consider a contract $\Big\{q(\tilde{\theta}), \pi_F(\tilde{\theta}), r_F(\tilde{\theta}, \tilde{s}), \hat{s}(\tilde{\theta}, \tilde{s})\Big\}$. The transfer that the firm obtains is given by (3.26).

After this change in variables I can write (3.23) as:

$$E_{f(\theta)}\left\{Uq((\theta)) - C(\theta, q(\theta)) - \pi_F(\theta) - \sum_{s\in S} 1_{\hat{s}(\theta,s)\neq s}\psi g(s \mid \theta)\right\} \tag{3.27}$$

and (3.25) becomes:

$$\pi_F(\theta) \geq \pi_F(\tilde{\theta}) + \sum_{s \in S} \Big(r_F(\tilde{\theta}, s) + 1_{\hat{s}(\tilde{\theta}, s) \neq s} \psi \Big) \Big(g(s \mid \tilde{\theta}) - g(s \mid \theta) \Big) + \tag{3.28}$$

$$+ C(\tilde{\theta}, q(\tilde{\theta})) - C(\theta, q(\tilde{\theta})) \quad \forall \tilde{\theta} \neq \theta$$

while (3.26) is simply:

$$\pi_F(\theta) \geq 0 \tag{3.29}$$

An optimal contract maximises (3.27) subject to (3.24) for all θ and all s and (3.28) and (3.29) for all θ. The contract design problem presents three differences with respect to that in chapter 2. First, from (3.28) it follows that the government has an additional instrument, the "bribery scheme", to prevent the firm when its type is a particular θ to claim that its type is a particular $\tilde{\theta}$: the government can offer a contract for which bribery takes place when the type of the firm is $\tilde{\theta}$ for outcomes of the audit that are more likely to occur when the type of the firm is θ than when it is $\tilde{\theta}$ ($\hat{s}(\tilde{\theta}, s) \neq s$ if $g(s \mid \tilde{\theta}) - g(s \mid \theta) < 0$). Second, (3.24) shows that the use that the government makes of this instrument affects the use that it can make of the reimbursement scheme: the firm makes a truthful claim about the outcome of the audit if and only if the reimbursement that it pays plus the sum of the expected penalties when bribery takes place is not greater than when it makes a false claim. Third, (3.27) makes it clear that this instrument is costly for the government: for any type of the firm, given the expected profit, quantity and expected reimbursement, the higher is the expected (over the outcome of the audit) sum of the expected penalties the higher is the transfer that the government pays (see (3.26)).

3.5.4 An optimal contract

I finally address the question of why bribery does not take place in an optimal contract. The approach that I take consists of comparing the optimal quantity scheme and expected profit scheme that the government can achieve in a contract for which bribery takes place with the corresponding schemes in a contract for which bribery does not take place. Proposition 5 presents the result of this comparison:

Proposition 5. *If the sum of the expected penalties does not depend on the extent of the misreporting then the government can achieve the optimal quantity scheme and expected profit scheme of a contract for which bribery takes place with a contract for which bribery does not take place. An optimal contract is then given by Proposition 1 for the case of two cost types and by Propositions 2 and 3 for the case of a continuum of cost types.*

Proposition 5 states that in the model under consideration the government does not benefit from having an additional instrument to prevent the firm from making a false claim about its type. In other words, any quantity scheme and expected profit scheme that the government can achieve through the joint use of the reimbursement scheme and the bribery scheme the government can also achieve them through the use of the reimbursement scheme alone. As bribery is costly for the government, it is then optimal for the government to deter it.

To understand the logic behind Proposition 5, consider a reimbursement scheme in a contract for which bribery does not take place that consists of the reimbursement scheme in the optimal contract for which bribery takes place plus the sum of the expected penalties when bribery takes place. By construction, the firm pays the same expected amount in the two contracts when it makes a truthful claim about the outcome of the audit. Therefore, conditional on the firm making a truthful claim about the outcome of the audit, the quantity scheme and the expected profit scheme that the government can achieve are the same in the two contracts. The question is then whether the firm makes a truthful claim about the outcome of the audit in the contract for which bribery does not take place if it does so in the optimal contract for which bribery takes place. The answer is positive due to the fact that in the contract for which bribery does not take place if the firm makes a false claim about the outcome of the audit then it pays the reimbursement in the contract for which bribery takes place plus at least the sum of the expected penalties.

It is worth mentioning that Proposition 5 holds regardless of the the distribution of the outcome of the audit and of the belief of the government about the type of the firm. The constraints that the firm makes a truthful claim about about the outcome of the audit and about its type (equations (3.24) and (3.28)) do not depend on these functions and as a result neither does Proposition 5.

3.6 Conclusions and extensions

In this chapter I began by arguing that when side contracting takes place ex post the government prevents bribery by giving the internal auditor the reimbursement that the firm pays. This is not costly for the government as in an optimal contract for every type of the firm the government pays the internal auditor a transfer equal to the expected reimbursement that the internal auditor pays. The government can then achieve the outcome of the first best contract both for the case of two cost types and for the case of a continuum of cost types in the relaxed problem if the value of the audit is positive (for all the types of the firm in the latter case).

I also argued in this chapter that when side contracting take place ex ante and the internal auditor observes the type of the firm it is optimal for the government to offer the internal auditor a non contingent contract. I then proved that if the sum of the expected penalties that the firm and the internal auditor pay is independent of the extent of the misreporting then the government can achieve the optimal quantity scheme and expected profit scheme of a contract for which bribery takes place with a contract for which bribery does not take place. In that situation it is then optimal for the government to deter bribery regardless of the distribution of the outcome of the audit and of the belief of the government about the type of the firm.

Maggi and Rodriguez-Clare (1995) considered a principal and agent procurement model with auditing in which the agent has a privately known marginal cost of production (its type) that takes a continuum of possible values. They assumed that the audit results in a signal that has a one to one relationship with the type of the agent. They allowed the agent to distort the signal by performing a costly action before the principal observes a report about it. They then demonstrated that misreporting takes place in an optimal contract if there is no fixed cost of misreporting and the variable cost is convex. I did not assume that the outcome of the audit has a one to one relationship with the type of the firm. Therefore, unlike them, I could not simplify the characterisation of an optimal contract with ex ante side contracting by restricting attention to a reimbursement that is arbitrarily high when the claims of the firm about its type and about the outcome of the audit do not coincide. I was nevertheless able to prove that if the cost of misreporting (the sum of the expected penalties) is fixed then misreporting (bribery) does not take place in an optimal contract.

Bac and Kucuksenel (2006) emphasised the distinction between ex post and ex ante side contracting. They considered a principal, supervisor and agent model in which the agent has a privately known productivity (its type) that takes two possible values. They assumed that the supervisor has a positive monitoring cost and defined the ex ante side contract as a bribe in exchange for the commitment of the supervisor to not monitor. They found that if monitoring costs are small and the probability of detection is large then the optimal contract with ex post side contracting is optimal with ex ante side contracting. With a different definition of ex ante side contracting and assuming that auditing is costless I found that ex ante side contracting leads to a strictly worse outcome for the government than ex post side contracting unless the audit is sufficiently informative of the type of the firm or the sum of the expected penalties is sufficiently high.

The assumption that the internal auditor observes the type of the firm helped me to characterise an optimal contract with ex ante side contracting by making it optimal for the government to offer the internal auditor a non contingent contract. There is research on auditing and bribery which does not make this assumption (see Faure-Grimaud, Laffont, and Martimort (2003) and Celik (2009)). The focus of this research is on characterising how the government can exploit the transaction costs that the firm and the internal auditor have due to the asymmetry of information between them. The issue of bribery is dealt with by assuming that the outcome of the audit takes two possible types, a situation in which the sum of the expected penalties trivially does not depend on the extent of the misreporting as there is only one possible misreport.[9]

In the characterisation of an optimal contract with ex ante side contracting I also used the assumption that the sum of the expected penalties does not depend on the extent of the misreporting. This assumption is not related to whether the government receives some or all of the penalties that the firm and

[9]Che and Kim (2006) study optimal contracting with multiple colluding agents that are asymmetrically informed. They provide conditions under which collusion imposes no cost for the principal without assuming that the information of any of the contracting parties take two possible values. Those conditions involve more than two colluding parties so they are not valid for the analysis of side contracting between a firm and an internal auditor.

the internal auditor pay (which seems realistic when penalties are monetary fines) so my results do not change in that situation. My analysis can also easily accommodate an extension in which I assume that the sum of the expected penalties depends on the outcome of the audit, an assumption which might reflect that the external auditor finds it easier to detect misreporting for some outcomes of the audit than for others. In that scenario bribery is not optimal and my results are identical with a suitably redefined value of the audit. How my results vary with more general assumptions on the information structure of the external auditor and on the penalties that the firm and the internal auditor remains an open question.

Appendix

Proof. (Proposition 4) Equations (3.11) and (3.14) give that in an optimal contract $\pi_A(\theta) = 0$ for all θ. Equations (3.11) and (3.13) give that if (3.15) and (3.12) are not constraints then in an optimal contract $\pi_F(\theta) = 0$ and $q(\theta) = q^{FB}(\theta)$ for all θ.

The reimbursement scheme of the firm in (3.17) satisfies (3.12) with $\pi_F(\theta) = 0$ and $q(\theta) = q^{FB}(\theta)$ for all $\theta \in \left\{\underline{\theta}, \overline{\theta}\right\}$ if $g(s \mid \overline{\theta}) - g(s \mid \underline{\theta}) \neq 0$ for some s. The reimbursement scheme of the firm in (3.18) satisfies (2.25) with $\pi_F(\theta) = 0$ and $q(\theta) = q^{FB}(\theta)$ for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$ if $\frac{dg(s|\theta)}{d\theta} \neq 0$ for some s and all θ. From the proof of Proposition 3 the reimbursement scheme of the firm in (3.18) satisfies (3.12) for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$ if the audit has the monotonicity property, the value of the audit is non increasing and the cost of production is convex in θ (see $\theta_{FB} = \underline{\theta}$).

Replacing (3.16) in (3.15) and using Lemma 1 and the definition of $\rho(\theta, s)$ in Proposition 4 gives that any $r_F(\theta, s)$ satisfies (3.15) for all θ and all s. This completes the proof. $\square$

Proof. (Proposition 5) Suppose that there is an optimal contract $\left\{q_0(\tilde{\theta}), \pi_0(\tilde{\theta}), r_0(\tilde{\theta}, \tilde{s}), \hat{s}_0(\tilde{\theta}, \tilde{s})\right\}$ that satisfies (3.24) for all θ and all s, (3.28) and (3.29) for all θ and $\hat{s}_0(\theta, s) \neq s$ for some θ and some s such that $g(s \mid \theta) > 0$.[10] I can construct the alternative contract $\left\{q_A(\tilde{\theta}), \pi_A(\tilde{\theta}), r_A(\tilde{\theta}, \tilde{s}), \hat{s}_A(\tilde{\theta}, \tilde{s})\right\}$ as follows: $q_A(\tilde{\theta}) = q_0(\tilde{\theta})$ and $\pi_A(\tilde{\theta}) = \pi_0(\tilde{\theta})$ for all $\tilde{\theta}$ and $r_A(\tilde{\theta}, \tilde{s}) = r_0(\tilde{\theta}, \tilde{s}) + 1_{\hat{s}_0(\tilde{\theta}, \tilde{s}) \neq \tilde{s}}\psi$ and $\hat{s}_A(\tilde{\theta}, \tilde{s}) = \tilde{s}$ for all $\tilde{\theta}$ and all $\tilde{s}$.

Replacing $r_A(\tilde{\theta}, \tilde{s})$ and $\hat{s}_A(\tilde{\theta}, \tilde{s})$ in (3.24) gives:

$$r_0(\theta, s) + 1_{\hat{s}_0(\theta,s) \neq s}\psi \leq r_0\left(\theta, \tilde{s}\right) + 1_{\hat{s}_0(\theta,\tilde{s}) \neq \tilde{s}}\psi + \psi \quad \forall \tilde{s} \neq s \tag{3.30}$$

The left hand side of (3.30) is the same as that of (3.24) for the initial contract whereas the right hand side is at least as high for all θ and all s. Therefore the alternative contract satisfies (3.24) for all θ and all s.

Also, as $r_A(\tilde{\theta}, s) + 1_{\hat{s}_A(\tilde{\theta},s) \neq s}\psi = r_0(\tilde{\theta}, s) + 1_{\hat{s}_0(\tilde{\theta},s) \neq s}\psi$ for all $\tilde{\theta}$ and all s, replacing $q_A(\tilde{\theta})$ and $\pi_A(\tilde{\theta})$ in (3.28) gives (3.28) for the initial contract for all θ. Therefore the alternative contract satisfies (3.28) for all θ.

By construction, the alternative contract satisfies (3.29) for all θ. Furthermore, replacing $q_A(\tilde{\theta})$, $\pi_A(\tilde{\theta})$ and $\hat{s}_A(\theta, s)$ in (3.27) gives a strictly higher amount than (3.27) for the initial contract. Therefore the initial contract is not optimal. This completes the proof. $\square$

[10] If $g(s \mid \theta) = 0$ then the issue of bribery is irrelevant with a truthful contract.

Bibliography

Armstrong, M., and D. E. Sappington (2007): "Recent Developments in the Theory of Regulation," *Handbook of Industrial Organization volume 3*, Elsevier.

Bac, M., and S. Kucuksenel (2006): "Two Types of Collusion in a Model of Hierarchical Agency," *Journal of Institutional and Theoretical Economics*, 162(2), 262–276.

Bagnoli, M., and T. Bergstrom (2005): "Log-Concave Probability and its Applications," *Economic Theory*, 26(2), 445–469.

Baron, D. P., and D. Besanko (1984): "Regulation, Asymmetric Information, and Auditing," *RAND Journal of Economics*, 15(4), 447–470.

Baron, D. P., and R. B. Myerson (1982): "Regulating a Monopolist with Unknown Costs," *Econometrica*, 50(4), 911–30.

Celik, G. (2009): "Mechanism design with collusive supervision," *Journal of Economic Theory*, 144(1), 69–95.

Che, Y.-K., and J. Kim (2006): "Robustly Collusion-Proof Implementation," *Econometrica*, 74(4), 1063–1107.

Dal Bo, E. (2006): "Regulatory Capture: A Review," *Oxford Review of Economic Policy*, 22(2), 203–225.

Faure-Grimaud, A., J.-J. Laffont, and D. Martimort (2003): "Collusion, Delegation and Supervision with Soft Information," *Review of Economic Studies*, 70(2), 253–279.

Gary-Bobo, R., and Y. Spiegel (2006): "Optimal State-Contingent Regulation Under Limited Liability," *RAND Journal of Economics*, 37(2), 431–448.

Kofman, F., and J. Lawarree (1993): "Collusion in Hierarchical Agency," *Econometrica*, 61(3), 629–56.

Laffont, J.-J., and D. Martimort (2002): "The Theory of Incentives: The Principal-Agent Model," Princeton University Press.

Laffont, J.-J., and E. Maskin (1980): "A Differential Approach to Dominant Strategy Mechanisms," *Econometrica*, 48(6), pp. 1507–1520.

Laffont, J.-J., and J. Tirole (1986): "Using Cost Observation to Regulate Firms," *Journal of Political Economy*, 94(3), 614–41.

Maggi, G., and A. Rodriguez-Clare (1995): "Costly Distortion of Information in Agency Problems," *The RAND Journal of Economics*, 26(4), 675–689.

Martimort, D. (1999): "The Life Cycle of Regulatory Agencies: Dynamic Capture and Transaction Costs," *Review of Economic Studies*, 66(4), 929–47.

Maskin, E., and J. Tirole (1992): "The Principal-Agent Relationship with an Informed Principal, II: Common Values," *Econometrica*, 60(1), 1–42.

Mookherjee, D. (2006): "Decentralization, Hierarchies, and Incentives: A

Mechanism Design Perspective," *Journal of Economic Literature*, 44(2), 367–390.

Myerson, R. B. (1982): "Optimal Coordination Mechanisms in Generalized Principal-Agent Problems," *Journal of Mathematical Economics*, 10(1), 67–81.

Riordan, M. H., and D. E. M. Sappington (1988): "Optimal Contracts with Public ex post Information," *Journal of Economic Theory*, 45(1), 189–199.

Seierstad, A., and K. Sydsaeter (1987): "Optimal Control Theory with Economic Applications," North-Holland.

Tirole, J. (1986): "Hierarchies and Bureaucracies: On the Role of Collusion in Organizations," *Journal of Law, Economics and Organization*, 2(2), 181–214.

——— (1992): "Collusion and the Theory of Organizations," *Advances in Economic Theory, Proceedings of the Sixth World Congress of the Econometric Society Vol. II*, Cambridge University Press.

U.S. Department of Defense (2012): "Report to Congress on FY 2011 Activities at the Defense Contract Audit Agency," .