THE UNIVERSITY OF WARWICK

February 2013 No.124

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Lei Zhang, Lin Zhang and Yong Zheng

WORKING PAPER SERIES

Centre for Competitive Advantage in the Global Economy

Department of Economics

Wholesale Funding, Coordination, and Credit Risk*

Lei Zhang[a,†], Lin Zhang[b] and Yong Zheng[c]

[a] University of Warwick, Coventry CV4 7AL, UK.

[b] Southwestern University of Finance and Economics, Chengdu, 610074, P. R. China

[c] Southwestern University of Finance and Economics, Chengdu, 610074, P. R. China

November 2012

Abstract

We use the global games approach to study key factors affecting the credit risk associated with roll-over of bank debt. When creditors are heterogenous, these include the extent of short-term borrowing and capital market liquidity for repo financing. Specifically, in a model with a large institutional creditor and a continuum of small creditors independently making their roll-over decisions based on private information, we find that increasing the proportion of short-term debt and/or decreasing market liquidity reduces the willingness of creditors to roll over. This raises credit risk in equilibrium. The presence of a large creditor does not always reduce credit risk, however, unless it is better informed.

JEL classification: G01; G14; G20

Keywords: Credit Risk; Coordination; Debt Crisis; Private information; Global games

1 Introduction

Consecutive waves of deregulations and the widespread use of securitized debt instruments have led many financial institutions, particularly investment banks, to rely heavily on short-term borrowing[1] to finance their investment in long-term risky assets (Shin 2008). The

*We are grateful to Robert Akerlof, Russell Cooper, Philip Dybvig, Hugo Hopenhayn, Marcus Miller, Pengfei Wang, Jian Yang, and Shen Guo for their helpful comments. The paper has benefitted from feedbacks from the seminar/conference participants at the Hong Kong University of Science and Technology, the Tsinghua University, and the Southwestern University of Finance and Economics. We gratefully acknowledge the research support from the CAGE (Lei Zhang) and the research grant from the "211 Project" (Lin Zhang and Yong Zheng).

†Corresponding author: Lei Zhang, Department of Economics, University of Warwick, Coventry CV4 7AL, UK; Tel. +44 (0) 2476 522983; Fax: +44 (0) 2476 523032; Email address: l.zhang@warwick.ac.uk. Email address of Lin Zhang: zhanglin@swufe.edu.cn. Email address of Yong Zheng: zhengyong@swufe.edu.cn.

[1]Gorton & Metrick (2010) suggest the main reason for this reliance is the rapid growth of money under management by large creditors such as institutional investors, pension funds, mutual funds, states and municipalities, and nonfinancial firms. These institutions prefer a demand-deposit-like product.

financial crisis of 2007-2008 highlights how heavy reliance on short-term, particularly wholesale, debts can expose the financial system to credit market freeze. Such freeze, precipitated by collective actions of creditors not rolling over their existing debts, resembles the classic case of a bank run.

Recently documented evidence has revealed some interesting salient features of the banking industry leading up to the sub-prime crisis. First is that banks had become increasingly reliant on short-term instruments to finance their asset holding and these short-term debt holders were typically heterogenous in their sizes. Second is that banks had increased significantly the use of structured financial products, such as collateralized debt obligations (CDOs), to attract investors.

Several measures indicates a significant increase and fluctuation in the magnitude of short-term instruments such as repo, commercial papers and interbank deposits. King (2008) estimates that the amount of repo was \$10 trillion at the end of 2007 roughly doubled since 2000. The size of commercial papers was relatively smaller than repo, but it is as important as Treasury Bills. The total asset-backed commercial paper (ABCP) outstanding in the U.S. market grew from \$650 billion in January 2004 to \$1.3 trillion in July 2007. Prior to the Sub-prime crisis, ABCP was the largest money market instrument in the United States.[2] As the wholesale fund markets consist mostly these short-term instruments, their increasing sizes and fluctuations must have played a pivotal role in the sub-prime crisis.

There is also evidence that failures of large financial institutions may have precipitated the financial crisis. Brunnermeier (2010) pointed out that the deterioration in capital market liquidity coupled with the inability to roll over short-term wholesale debt is one of the direct causes of the failures of Bear Stearns, Lehman Brothers, Washington Mutual, and, eventually led to the collapse of a significant part of the U.S. financial system during the 2007-2008 financial crisis. In the bankruptcy filing of Lehman Brothers, nearly 26 percent of its debt was held by the 30 largest creditors among more than 100,000 creditors. This illustrates clearly that creditor composition can have substantial impact on credit markets.

The wide-spread use of structured financial products created information problem, at least to creditors of banks. As argued by Haldane (2009), structured financial products are complex objects and it is difficult to trace the risks of the structured products to their underlying assets. There may be added difficulty of objectively characterising risks because of network structure of the banking system: the risks of assets of one bank may be affected by counterparty risk associated with network structure. This indicates that private information possessed by the creditors of a bank may be crucial in assessing whether the bank is liquid or solvent.

This paper studies how these factors, such as features of wholesale funding, creditor composition and the presence of private information can affect creditor coordination and thereby influence credit risk.[3] As wholesale financiers were criticized during the financial crisis to rely heavily on information from rating agencies, we are particularly interested in two questions. The first is whether a better informed wholesale creditor can decrease credit risk. The second is whether a larger proportion of wholesale funding in finance banks'

[2]For comparison, the second largest instrument was Treasury Bills with about \$940 billion outstanding.

[3]To measure the risk in financial crisis, Gorton and Metrick (2012) constructed a weighted average of haircut of repo. From September 2007, haircut index kept rising from 5 percent and reached 45 percent at the end of 2008.

borrowing can lower credit risk.

Theoretical studies relevant to ours include those focusing on short-term debt financing (Morris & Shin 2004), wholesale funding (Calomiris 1999, Huang & Ratnovski 2011), market liquidity (Diamond & Rajan 2005, Brunnermeier & Pedersen 2009), and market freezes resulting from short-term debt rollover (Plantin 2009, Acharya, Gale & Yorulmazer 2011). However, to our knowledge, no study exists that addresses how these factors are combined to affect credit risk.

This paper provides a model to fill in this gap. The key insight that we suggest is that wholesale funding is a double-edged sword. Unlike Calomiris (1999) focusing on the "bright side" and Huang and Ratnovski (2011) stressing the "dark side" of wholesale funding, our model provides general results on the role of wholesale funding when there are interactions between private information and creditor composition.

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Formally, we consider a bank that relies on rolling over short-term debt to finance its investment in long-term risky assets. Its short-term debt is held by a large creditor (a wholesale financier) and a continuum of small creditors. When short-term debt matures, holders have to decide independently whether to roll over their loans or not. If a creditor believes that, on average, the other creditors are likely to foreclose on their loans, he/she will foreclose. As a result, creditors cannot coordinate perfectly when making their rollover decisions.

To model the role of private information, we follow the global games approach pioneered by Calsson and van Damme (1993a, 1993b), and popularised by Morris and Shin (2004). In focusing on the role of large creditor, our work is related to Corsetti, Dasgupta, Morris & Shin (2004) and Liu and Mello (2011). Corsetti et al. (2004) show that the presence of large traders makes small traders more aggressive in currency attacks. Likewise, in our model, the presence of less informed large creditors reduces the willingness of small creditors to roll over. Liu and Mello (2011) show that institutional creditors foreclose if their financial positions deteriorate through the lending channel. By focusing on the borrower's balance sheet instead, we find that the large creditor will foreclose if the bank is highly leveraged and the capital market liquidity is low.

In our model, a bank can fail because of coordination failure among creditors. This is similar in spirit to bank runs studied by Diamond and Dybvig (1983). There, perfect information produces more than one Nash equilibrium. Rochet and Vives (2004), Goldstein and Pauzner (2005), and He and Xiong (2011) use global games to obtain unique equilibrium in bank runs. The main difference between our model and theirs is that we study bank runs with heterogeneous players.

Studies on credit risk can be traced back to Merton (1974) who employed real-option method to investigate how default risk is associated with the debtor's asset quality. However, Merton (1974) considers only a single creditor's decision problem and overlook the credit risk resulting from coordination failure between the creditors and thereby underestimate the credit risk. Morris and Shin (2004; 2010) adopt a global-games framework to study how coordination failure between small creditors can increase credit risk. In our model, we study credit risk with both large and small creditors.

The main mechanism of the model is as follows. At the refinancing stage, the bank's liquidity depends on how much cash it can raise from the capital markets by pledging its assets as collateral, which, in turn, depends jointly on its asset quality and market liquidity. The bank's risky asset return is not perfectly observable. The inability to observe the risky asset return leads to imperfect coordination between short-term creditors when deciding to rollover or not their loans. The role of wholesale funding is demonstrated in the case when additional foreclosure from the wholesale financier is needed to make the bank fail. In this case, if the wholesale financier believes that the bank's financial position is not sustainable whether because there is a deterioration in capital market liquidity or because the asset quality is not good enough, he decides to foreclose. The wholesale financier's foreclosure will make the bank fail. Thus, even abstracting from modeling his financial constraint, the wholesale financier may withdraw upon a hint of negative news[4].

In general, some of the results obtained from our model are in line with existing literature. For example, we find that heavy reliance on short-term debt financing makes the bank more vulnerable to creditor runs, and thereby increases credit risk. A decrease in market liquidity raises credit risk. Other results, particularly related to the interaction between creditor composition and private information, are quite intriguing.

Given it is likely that information precision and the size could be positively correlated, our results show that a higher precision in the wholesale financier's information on the financial capacity of the bank increases the willingness of the small creditors to roll over their loans and thereby reduces credit risk. Intuitively, if the wholesale financier arbitrarily has more precise information on the fundamentals of the debtor, his own switching point is lowered. Because the switching point of the small creditors is positively related to that of the wholesale financier, it is reduced as well.

We explicitly model credit risk, which is decomposed, as in Morris and Shin (2010), into insolvency risk and illiquidity risk. In this respect, the most interesting result from this paper is that analytically the size effect of the wholesale financier is ambiguous. This result suggests that short-term wholesale funding is a double-edged sword such that only under certain conditions, an increase in the wholesale funding reduces credit risk. Comparing with the case without wholesale funding, the presence of the wholesale financier reduces the incidence of imperfect coordination among small creditors but add new imperfect coordination between

[4]As documented by Gorton and Metrick (2012), on August 9, 2007, the French bank BNP Paribas stopped withdrawals from three funds invested in mortgage-backed securities and suspended calculation of net asset values. The interest rate spread of overnight short-term asset-backed commercial paper (ABCP) over the Federal Funds rate increased from 10 basis points to 150 basis points within one day of the BNP Paribas announcement. Subsequently, the market experienced a 'bank run' that originated in shadow banking, and ABCP outstanding dropped from \$1.3 trillion in July 2007 to \$833 billion in December 2007.

the small creditors and the wholesale financier. Thus, the overall effect depends on the relative strength of these two factors. Using different combinations of parameter values, our numerical solutions reveal two conditions under which the overall size effect on credit risk can be determined. First, an increase in the size of the wholesale financier reduces credit risk provided that private information is more precise than public information. Otherwise, an increase in the size of the wholesale financier raises credit risk. This result explains why when wholesale financiers tend to rely on public but low quality information from rating agencies, wholesale funding increases credit risk. Second, an increase in the size of wholesale funding reduces credit risk if and the premium of rolling over is sufficiently high.

This paper is organized as follows. We present the model in Section 1 and solve the equilibrium in Section 2. Then, we present the equilibrium properties in Section 3, and analyze credit risk in Section 4. Finally, Section 5 provides some concluding remarks.

2 The model

To study how creditor composition can affect default risks, we focus our attention on the roll-over of short-term debts. In what follows, we first set up a roll-over game under perfect information. This is then extended to incorporate information imperfection. Finally, to set a benchmark, we outline results in a special case where all short-term creditors are small.

2.1 Timing, players, payoffs, and perfect information

There are three event dates, *ex ante* (date 0), interim (date 1), and *ex post* (date 2), and no discounting between dates. At date 0, the bank, holding equity of E, issues both long-term (maturing in two periods) and short-term (maturing in one period) debts to acquire A units of risky assets maturing at date 2 and M units of cash. The contractual values of long-term and short-term debts are L_2 and S_1 respectively. The returns of the risky assets are given exogenously, with each unit of A pays a gross amount of θ_2 at date 2. Let θ_0 and θ_1 be the expected value of θ_2 at dates 0 and 1 such that

$$\begin{aligned} \theta_1 &= \theta_0 + \varepsilon_1 \\ \theta_2 &= \theta_1 + \varepsilon_2, \end{aligned}$$

where ε_1 and ε_2 are independently distributed random variables following normal distributions with mean 0, precision σ_1 and σ_2, and cumulative functions $F_1(\cdot)$ and $F_2(\cdot)$. Here, θ_1 and θ_2 can be interpreted as public signals available at date 0 and 1, respectively.

Debt contracts considered here are incomplete that they cannot specify the legal consequences of every possible state of the world. In addition, there is a maturity mismatch between short-term debt financing and long-term asset holding. For simplicity, we assume the expected asset return at date 0 is sufficiently large so that creditors are willing to lend *ex ante* and the bank's investment decision at date 0 is exogenous. To focus our attention on the roll-over of short-term lending, we also assume that cash raised by the bank at date 1 is insufficient to cover all its short-term borrowing (details are specified below).

The roll-over game at date 1 involves a bank, a large creditor[5], and a continuum of small creditors indexed by the interval $[0, 1]$. All players are risk neutral and there is no single creditor who has enough funds to finance the total short-term debt roll-over. Short-term debt includes wholesale debt and retail debt. The wholesale component is provided by a wholesale financier, while the retail component is borrowed from small creditors. We assume that the wholesale financier has a sufficiently large amount of funds to finance the bank's short-term debt up to the limit of $p \in (0, 1)$, while the set of all small creditors together has a proportion of 1-p. The strategy of creditors is to roll over their debt or not given available information, while the bank simply bears the consequences of creditors' decisions.

When deciding to roll over at date 1, creditors take into account of two risks faced by the bank: the *insolvency risk* at date 2 and the *illiquidity risk* at date 1. Illiquidity risk is the probability that the bank will fail because of a run when it would not have been insolvent in the absence of a run, and insolvency risk is the probability that the bank will fail if there is no run. Given short-term creditors roll over their debt, the bank may face *insolvency risk* at date 2 when asset returns are low. This is illustrated by using bank's date 2 balance sheet in Table 1. Let S_2 be the total amount of rollover, i.e., the contractual value of short-term debt issued at date 1 and maturing at date 2, and E_2 the bank's equity at date 2. Thus, the bank's balance sheet at date 2 takes the following form.

Table 1: The bank's balance sheet at date 2.

Assets	Liabilities
Cash, M	Long-term debt, L_2
Risky Assets, $\theta_2 A$	Short-term debt held by the large creditor, pS_2
	Short-term debt held by the small creditors, $(1-p)S_2$
	Equity, E_2

The bank is *insolvent* at date 2 if its *ex post* equity is negative,

$$E_2 \equiv M + \theta_2 A - (L_2 + S_2) < 0,$$

The equality defines a critical value of solvency[6] in terms of the date 2 returns

$$\theta_2^* \equiv \frac{L_2 + S_2 - M}{A}.$$

If $\theta_2 < \theta_2^*$, the bank is insolvent at date 2; the recovery rate for short-term creditors is assumed to be 0 [7].

The *illiquidity risk* occurs at date 1 when the bank has difficulty in raising enough cash to repay its short-term borrowing if a sufficiently large fraction of creditors choose not to

[5]A large creditor can also refer to a group of creditors whose measure is not zero and whose actions are perfectly coordinated.

[6]For simplicity, we exclude partial liquidation in the interim stage. Introducing partial liquidation, however, does not change the qualitative nature of our results.

[7]The qualitatively nature of our results will remain the same even if a positive recovery rate is introduced.

roll over their loans. Assume the bank has an outside option of raising new financing from markets by pledging its assets as collateral, with the amount $\lambda\theta_1$ raised from each unit of its risky assets, where $\lambda \in [0, 1]$ reflecting *capital market liquidity.* The total amount of cash available at date 1 is $M+\lambda\theta_1 A$. So the bank is illiquid at date 1 if and only if $M+\lambda\theta_1 A < S_1$.

Define the liquidity ratio as

$$\pi \equiv \frac{M + \lambda\theta_1 A}{S_1}.$$

Note that the bank has no illiquidity risk when $\pi \geq 1$. We focus on the case where $\pi < 1$, in which the illiquidity risk is positive. In this case, whether the bank can continue its operation to date 2 depends on the proportion of short term creditors rolling over their loans. If this proportion is less than π, the bank fails in a run. Those who rolled over are assumed to receive a payoff normalized to zero, and those who did not receive a payoff of liquidation r^*. If this proportion is larger than π, however, the bank remains in operation until date 2. Those who rolled over received either a payoff of $r_s = S_2/S_1$ or 0 depending on whether the bank is solvent or not, and those who did not obtain r^*. Table 2 outlines payoffs to a creditor, where rows indicate its strategies and columns the states of the bank. If $r_s < r^*$, then the dominant strategy is to foreclose. For $0 < r^* < r_s$, there is no dominant strategy. In this paper, we exploit the strategic uncertainty induced by the imperfect coordination among creditors. Hence, payoffs are assumed to be ordered as $0 < r^* < r_s$.

Table 2: Payoffs to a creditor

Action/State	Continuation to date 2		Liquidation at date 1
	Solvent	Insolvent	
Roll over	r_s	0	0
Foreclose	r^*	r^*	r^*

Equilibrium under perfect information (where all short-term creditors observe θ_1) can be determined straightforwardly. Rollover decisions depend on whether the bank is liquid and its solvency probability. When the solvency probability is low, $Pr(\theta_2 \geq \theta_2^*) < r^*/r_s$, the dominant strategy is to foreclose irrespective of bank's liquidity. When the bank is liquid (i.e. $M + \lambda\theta_1 A \geq S_1$) and solvent in the next period with a sufficiently high probability (i.e. $Pr(\theta_2 \geq \theta_2^*) \geq r^*/r_s$), rollover is the dominant strategy. The coordination problem occurs when the bank is illiquid (i.e., $M + \lambda\theta_1 A < S_1$) and solvent with high probability (i.e., $Pr(\theta_2 \geq \theta_2^*) \geq r^*/r_s$). If sufficient mass of creditors rolls over their debt, a creditor will roll over; otherwise he/she will foreclose. Because of the uncertainty regarding θ_2, two types of inefficiencies can arise in equilibrium due to coordination failure. One is inefficient liquidation causing an *ex post* solvent bank to fail at the interim stage and the other is inefficient roll-over of an *ex ante* insolvent bank.

2.2 Imperfect Information

Consider a general case where creditors at date 1 receive imperfect information on θ_1, with noisy signals x_i and y, observed by small and large creditors, following

$$x_i = \theta_1 + e_i$$
$$y = \theta_1 + \upsilon$$

where e_i and υ are normally distributed with mean 0 and precision α and β, respectively. Their respective cumulative distribution functions are denoted by $G(\cdot)$ and $H(\cdot)$. Under perfect competition, creditors are reluctant to share information so that $cov(e_i, \upsilon) = 0$, and $cov(e_i, e_j) = 0$ for $i \neq j$.

The posteriors of small and large creditors' beliefs in θ_1 can be obtained through simple Baysian updating to give

$$X_i = \frac{\sigma_2 \theta_2 + \alpha x_i}{\sigma_2 + \alpha}, \tag{1}$$

$$Y = \frac{\sigma_2 \theta_2 + \beta y}{\sigma_2 + \beta}. \tag{2}$$

Following Morris & Shin (2004), we consider "sophisticated" strategies of creditors. A sophisticated strategy is a decision rule that takes into account both the private information concerning the fundamentals and high order beliefs of others. When constructing equilibrium of a game with a continuum of players, it is challenging to keep tracking each layer of each player's belief of the beliefs of others. Global games provide a simple procedure. As shown in Morris & Shin (2004), the equilibrium outcome under a sophisticated strategy is the same as that under a simple strategy in which each creditor chooses the best action for a uniform belief regarding the proportion of other creditors choosing a certain action. The equilibrium is constructed by assuming that each player adopts a switching strategy in which a creditor rolls over whenever his/her estimate of the underlying fundamentals is higher than a given threshold. Otherwise, he/she forecloses.

For given posterior thresholds X_i^* and Y^* of small and large creditors, a creditor having his/her posterior above the trigger implies his/her expected payoff from roll-over conditional on his/her signal must exceed that from not rolling over. Backward induction on the posterior gives the critical signal received by a creditor. Taking the inverse of (1) and (2), these critical signals are

$$x_i^* = \frac{\alpha + \sigma_2}{\alpha} X_i^* - \frac{\sigma_2}{\alpha} \theta_2,$$
$$y^* = \frac{\beta + \sigma_2}{\beta} Y^* - \frac{\sigma_2}{\beta} \theta_2.$$

To simplify treatment, we set $\sigma_2/\alpha \to 0$, and $\sigma_2/\beta \to 0$. This implies that either the public information is uninformative, $\sigma_2 \to 0$, for finite α and β or private information is accurate, α and/or $\beta \to \infty$, for a finite σ_2. Given such conditions, posterior and signal thresholds coincide

$$\lim_{\sigma_2/\alpha \to 0} x^* = X^*,$$
$$\lim_{\sigma_2/\beta \to 0} y^* = Y^*.$$

So a creditor will roll over whenever he/she receives a signal above his/her respective critical value. To set a benchmark, we first discuss a special case where all creditors are small.

2.3 Equilibrium with small creditors

The case with only small creditors leads to a symmetric game of Morris & Shin (2010) with the difference that, in our case, the liquidity ratio of the bank is not perfectly observable. Since all creditors are of the same type, he/she adopts the same switching strategy in which he/she forecloses if his/her signal falls below the same critical value x^*. Such equilibrium strategy implies a critical state, θ_1^*, below which the bank is liquidated at date 1.

The equilibrium is solved in two steps. The first step is to derive the liquidation threshold θ_1^* given switching trigger x^*. If the true state is θ_1^*, foreclosure of a critical mass of creditors is needed to force the liquidation of the bank. At the liquidation threshold θ_1^*, we must have

$$M + \lambda\theta_1^* A = S_1 Pr(x \leq x^* | \theta_1^*), \tag{3}$$

where $Pr(x \leq x^*|\theta_1^*)$ is the probability of foreclosure given the true state θ_1^*. Since a creditor has a uniform belief regarding the proportion of creditors that foreclose, $Pr(x \leq x^*|\theta_1^*)$ is also the fraction of creditors who do not roll over. Equation (3) simply indicates that at the liquidation threshold, the bank can raise just enough cash to cover foreclosure. Given the distribution of signal, we have

$$Pr(x \leq x^*|\theta_1^*) = G(x^* - \theta_1^*).$$

Let D be the total debt (*i.e.*, $D = S_1 + L_2$), and τ the short-term debt ratio (*i.e.*, $\tau = S_1/D$). Equation (3) can be rewritten as

$$M + \lambda\theta_1^* A = \tau D \cdot G(x^* - \theta_1^*). \tag{4}$$

Second, we derive the indifference condition between rolling over and foreclosing. This requires that the expected payoff of rollover conditional on the signal is the same as the liquidation payoff. After rollover, the conditional probability for a creditor to obtain r_s is the joint probability that the bank can continue its operation till date 2 and it is solvent given signal, i.e.,

$$\begin{aligned} Pr(\theta_1 > \theta_1^* \text{ and } \theta_2 > \theta_2^*|x) &= [1 - Pr(\theta_1 \leq \theta_1^*|x)][1 - Pr(\theta_2 \leq \theta_2^*|x)] \\ &= [1 - G(\theta_1^* - x)][1 - F_2(\theta_2^* - x)]. \end{aligned}$$

Here, conditional on the updated signal, the interim probability of insolvency is

$$N_1(x_i) = Pr(\theta_2 \leq \theta_2^* \mid x_i) = F_2(\theta_2^* - x_i).$$

This probability is derived from $Pr(\theta_2 \leq \theta_2^* \mid x_i) = Pr(\varepsilon_2 - e_i \leq \theta_2^* - x_i \mid x_i)$. Because $\sigma_2/\alpha \to 0$, it is straightforward to see that $\varepsilon_2 - e_i$ is normally distributed with precision σ_2, and its cumulative function is $F_2(\cdot)$. Hence, the indifference condition becomes

$$[1 - G(\theta_1^* - x^*)][1 - F_2(\theta_2^* - x^*)]r_s = r^*. \tag{5}$$

From (4) and (5), one can solve for unique equilibrium θ_1^* and x^* (see Morris and Shin, 2004 for the proof).

The interim illiquidity risk is the probability that the bank will fail in a run but would have been solvent if no run occurs. With small creditors alone, the interim illiquidity risk is

$$L_1(\theta_1) = \begin{cases} 1 - F_2(\theta_2^* - \theta_1), & \theta_1 \leq \theta_1^* \\ 0, & \theta_1 > \theta_1^* \end{cases}.$$

The feature of the interim credit risk is shown in Figure 1, where the horizontal axis represents θ_1 and the vertical the interim credit risk. The interim insolvency risk is shown by the broken line. As θ_1 is the unconditional expectation of θ_2, the interim insolvency risk is decreasing in θ_1. The interim illiquidity risk is represented by the distance between the horizontal continuous line and the broken line in the shaded area. When $\theta_1 > \theta_1^*$, the bank has enough cash to meet its short-term claims, the interim illiquidity risk is zero. The insolvency risk on the right side of the critical point θ_1^* represents the probability that the bank will fail even after a successful rollover. Thus, the decomposition of interim credit risk depends on θ_1: for θ_1 to the left of the liquidation threshold, θ_1^*, credit risk contains both illiquidity and insolvency risk; to the right of θ_1^*, it only contains insolvency risk.

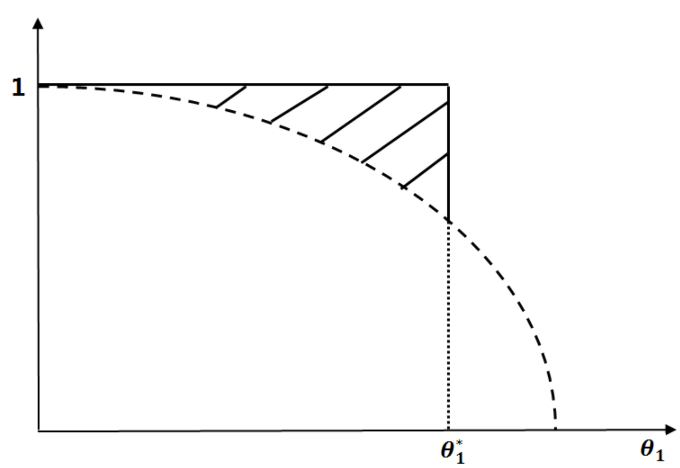

Figure 1: Interim credit risk with small creditors only.

From the point of view of a long-term creditor, knowing the *ex ante* credit risk at date 0 is of central importance. The *ex ante* insolvency risk is

$$N_0(\theta_0) = \int_{-\infty}^{+\infty} F_2(\theta_2^* - \theta_1) f(\theta_1 - \theta_0) d\theta_1, \tag{6}$$

where f is the density function of F_1. This is given by the expectation of the area under the broken line indicated in Figure 1. The *ex ante* illiquidity risk is

$$L_0(\theta_0) = \int_{-\infty}^{\theta_1^*} [1 - F_2(\theta_2^* - \theta_1)] f(\theta_1 - \theta_0) d\theta_1, \tag{7}$$

which is given by the expectation of the shaded area indicated in Figure 1.

3 Equilibrium with heterogenous creditors

We now turn to the case with both small and large creditors. As shown in Corsetti *et al.* (2004), there is unique equilibrium characterized by both types of creditors using their respective switching strategies around their trigger point x^* and y^*. These are the only equilibrium strategies that can survive the iterative elimination of strictly dominated strategies. To determine the equilibrium, we consider two situations under which the bank fails at the interim stage. The first is when foreclosures by small creditors alone are sufficient to make the bank fail. The second is when the additional foreclosure by the large creditor is needed.

First, consider the liquidation brought by small creditors alone. Let $\underline{\theta}_1$ be the liquidation threshold with small creditors only. The discussion in section 2.3 suggests that this threshold must satisfy

$$M + \lambda A \underline{\theta}_1 = \tau D(1 - p)G(x^* - \underline{\theta}_1), \tag{8}$$

where $1 - p$ is the total measure of small creditors. For $\theta_1 < \underline{\theta}_1$, the bank will be liquidated at date 1 due to foreclosures by small creditors regardless of the large creditor's action. For $\theta_1 \geq \underline{\theta}_1$, whether the bank remains in operation depends on whether the large creditor foreclosures or not.

Next, consider the liquidation brought by the additional foreclosure by the large creditor. The total amount of foreclosure includes two components: foreclosure by the large creditor τDp and foreclosures by small creditors $\tau D(1 - p)G(x^* - \theta_1)$. Thus, the bank will fail whenever

$$\tau D[p + (1 - p)G(x^* - \theta_1)] > M + \lambda \theta_1 A.$$

Define the liquidation threshold of both types of creditors as $\overline{\theta_1}$ such that the above holds as an equality

$$M + \lambda A \overline{\theta}_1 = \tau D[p + (1 - p)G(x^* - \overline{\theta}_1)]. \tag{9}$$

It is clear that $\overline{\theta_1} \geq \underline{\theta_1}$. For $\overline{\theta_1} \geq \theta_1 > \underline{\theta_1}$, the bank fails if the large creditor forecloses. For $\theta_1 > \overline{\theta_1}$, bank can continue its operation till date 2 irrespective of actions by both creditors.

How $\underline{\theta}_1$ and $\overline{\theta}_1$ are determined is illustrated in Figure 2 where the horizontal axis represents θ_1 and the vertical the probabilities of foreclosures. Line 1 is $\lambda A\theta_1/\tau D + M/\tau D$. Line 2 is $(1 - p)G(x^* - \theta_1)$, representing the probability of foreclosure by small creditors. Line 3 is $p + (1 - p)G(x^* - \theta_1)$, representing the probability of foreclosure by both creditors. Line 1 intersects Line 2 and Line 3, respectively, at $\underline{\theta}_1$ and $\overline{\theta}_1$. Note that both $\overline{\theta}_1$ and $\underline{\theta}_1$ are functions of the switching point x^*, which, in turn, depends on the large creditor's switching point y^* because each creditor's payoff depends on the others' actions.

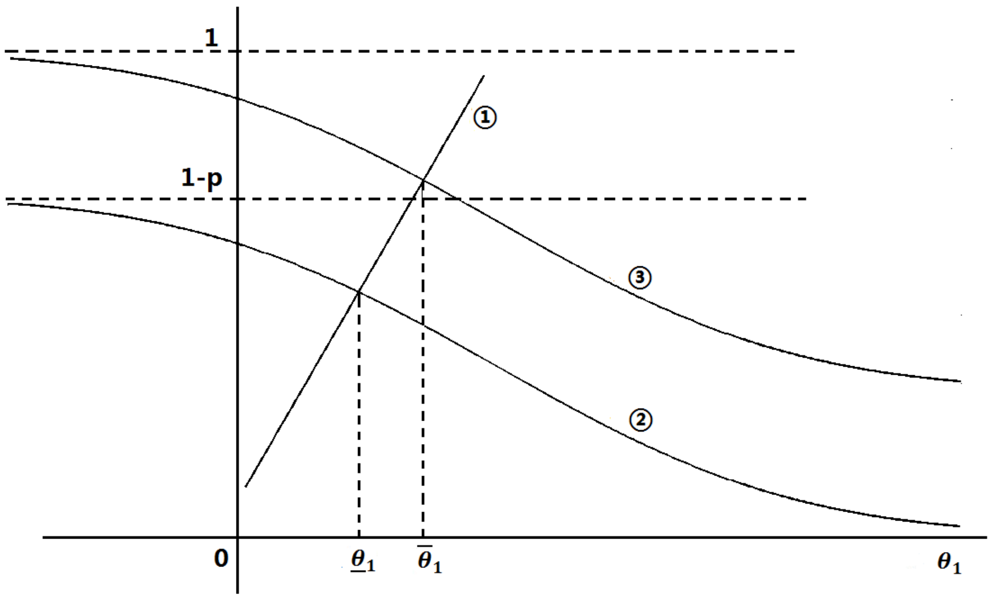

Figure 2: Liquidation thresholds with heterogenous creditors.

To solve for these two switching points, two additional equations in terms of $\overline{\theta}_1$, $\underline{\theta}_1$, x^*, and y^* are needed. We appeal to the fact that both types of creditors are indifferent between foreclosing and rolling over at their own switching point, x^* or y^*. The large creditor, based on the signal he/she receives, assigns probability $H(\underline{\theta}_1 - y)$ to the event that $\theta_1 \leq \underline{\theta}_1$. Only when $\theta_1 > \underline{\theta}_1$ can the large creditor's rollover save the bank at date 1. The insolvency risk that the large creditor assigns is $F_2(\theta_2^* - y)$. Thus, the indifference condition for the large creditor is

$$[1 - H(\underline{\theta}_1 - y^*)][1 - F_2(\theta_2^* - y^*)]r_s = r^* \tag{10}$$

where y^* is the large creditor's switching point. The large creditor will roll over if and only if his/her signal surpasses this switching point.

A small creditor's problem is a bit more complicated. In the region $(-\infty, \underline{\theta}_1]$, a small creditor receiving a signal x assigns probability $\int_{-\infty}^{\underline{\theta}_1} g(\theta_1 - x)d\theta_1$ to the event that the bank fails regardless of the actions of the large creditor, where $g(\cdot)$ is the density function of $G(\cdot)$. In the region of $(\underline{\theta}_1, \overline{\theta}_1]$, the bank fails if the large creditor forecloses. The probability that the large creditor forecloses at θ_1, given his trigger strategy around y^*, is $H(y^* - \theta_1)$. Hence, the indifference condition is

$$\left[1 - \left(G\left(\underline{\theta}_1 - x^*\right) + \int_{\underline{\theta}_1}^{\overline{\theta}_1} g\left(\theta_1 - x^*\right) H\left(y^* - \theta_1\right) d\theta_1\right)\right]\left(1 - F_2\left(\theta_2^* - x^*\right)\right) r_s = r^* \tag{11}$$

where $F_2(\theta_2^* - x^*)$ is the insolvency risk assigned by a small creditor based on his/her signal. With these four equations, the unique equilibrium defined by $\{x^*, y^*, \underline{\theta}_1, \overline{\theta}_1\}$ can be obtained. The existence of such equilibrium is given by the following proposition.

Proposition 1 *There exists a unique dominance solvable equilibrium in the game in which the large creditor uses the switching strategy around y^* and the small creditors use the switching strategy around x^*.*

Proof: See Appendix.

4 Equilibrium properties

We can now address the question of how short-term debt financing, capital market liquidity and the presence of the large creditor affect the bank's vulnerability to a run. The equilibrium effects in the presence of the large creditor consist of an information and a size effect, which leads to two natural questions. Does the involvement of a better informed large creditor increase the willingness of the small creditors to roll over? Does an increase in the size of the larger creditor make the small creditors more willing to roll over? To provide answers to these questions, we present comparative statics of equilibrium effects by means of propositions. All proofs are relegated to the Appendix.

What is the effect of having a larger proportion of short-term debt financing in the bank's capital structure? The following proposition summarizes this result.

Proposition 2 *All thresholds $(\underline{\theta}_1, \overline{\theta}_1, x^*, y^*)$ are increasing in the short-term debt ratio.*

A number of studies, such as Bulow & Shoven (1978), White (1989), Morris and Shin (2001) and Detragiache & Garella (1994), find that a larger number of creditors makes debt renegotiation more difficult because of coordination failure. With an increase in the short-term debt financing, the bank is more likely to encounter liquidity problem at date 1, making coordination of debt rollover even more difficult. Consistent with such intuition, this proposition suggests that heavy reliance on short-term debt financing makes the bank more vulnerable to creditor runs.

Are creditors more willing to roll over if the capital markets are more liquid? The following proposition provides the answer.

Proposition 3 *All thresholds $(\underline{\theta}_1, \overline{\theta}_1, x^*, y^*)$ are decreasing with respect to market liquidity λ.*

Proposition 3 implies that, when the capital markets are more liquid, creditors are more willing to roll over. This is intuitive because high capital market liquidity makes it easy for the bank to access an alternative source of short-term financing at the interim stage to ease its liquidity problem, and thereby making coordination problem less severe. Conversely, a deterioration in capital market liquidity reduces the bank's liquidity, and raises all thresholds. Here, we focus on the borrower's balance sheet by implicitly assuming that the creditors' balance sheets are not affected. Thus, a creditor does not foreclose because his/her financial position deteriorates. Considering creditors' balance sheets would amplify this effect.

Does it matter whether the large creditor has greater precision in its information on θ_1? This question raises a central issue in the analysis regarding the equilibrium effect, if any, of improving the quality of the large creditor's information. The following proposition synthesizes the result.

Proposition 4 *All thresholds* $(\underline{\theta}_1, \overline{\theta}_1, x^*, y^*)$ *decrease with the precision of the large creditor's information.*

This proposition suggests, *ceteris paribus*, a higher precision in the information for the large creditor increases the willingness of the small creditors to roll over their loans. Here high precision of information on θ_1 means creditors have more precise information on the bank's liquidity situation at date 1. Intuitively, if the large creditor has arbitrarily more precise information on the bank's liquidity, his switching point is reduced. This reduction, in turn, lowers the switching point of the small creditors because of the interaction between their high order beliefs. Given such interaction, if the value of continuation is *ex post* higher than the value of liquidation, a small creditor relying on precise information from the large creditor can minimize the error of foreclosing and losing the opportunity of receiving higher payoffs. By taking this into account, the large creditor is more likely to roll over. The iteration of such interaction reenforce itself, so in equilibrium, increasing the accuracy of the large creditor's information makes small creditors more willing to roll over.

Does an increase in the size of the larger creditor make the small creditors more willing to roll over? Unfortunately, it is not always possible to analytically provide a definitive answer to this question (i.e., whether x^* is decreasing with the size of the large creditor p). Generally speaking, we know that the size effects on thresholds x^*, y^* and $\underline{\theta}_1$ are the same. They are negative (or positive otherwise) if and only if

$$\frac{b_2 b_5 (1 - G(x^* - \overline{\theta}_1))}{g(x^* - \overline{\theta}_1)} < \frac{b_1 (b_3 b_6 + b_4) G(x^* - \underline{\theta}_1)}{g(x^* - \underline{\theta}_1)},$$

where parameters b_1, b_2, b_3, b_4, b_5, and b_6 are as defined in Appendix A. The liquidation threshold $\overline{\theta}_1$ is decreasing in p if

$$\frac{dx^*}{dp} > \frac{1 - G(x^* - \overline{\theta}_1)}{(1-p) g(x^* - \overline{\theta}_1)}.$$

To further explore the size effect, we proceed in two ways: analytically and numerically. Analytically, we focus on the limiting case where $\alpha \to \infty$, $\beta \to \infty$, and $\sigma_2 \to 0$. In other words, creditors have precise information, but the public information is uninformative.

To assess analytically the size effect in the limiting case, we first summarise some interim technical results in the following lemmas.

Lemma 1 *For* $\alpha \to \infty$, $\beta \to \infty$, *and* $\sigma_2 \to 0$,

$$x^* = y^* = \underline{\theta}_1 = \overline{\theta}_1, \quad \textit{if} \quad \overline{\theta}_1 \le t,$$

and

$$x^* = y^* = \underline{\theta}_1 < \overline{\theta}_1, \quad \textit{if} \quad \overline{\theta}_1 > t,$$

where $t \equiv [\tau D(1-p) - M]/\lambda A$.

This suggests that we can focus on $\underline{\theta}_1$ and $\overline{\theta}_1$ for the size effects in equilibrium. To proceed further, we show how $\underline{\theta}_1$ can be determined in the limiting case.

Lemma 2 *For $\alpha \to \infty$, $\beta \to \infty$, and $\sigma_2 \to 0$, the liquidation threshold $\underline{\theta}_1$ is*

$$\underline{\theta}_1 = [\tau D(1-p)G(-\underline{\delta}) - M]/\lambda A,$$

where $\underline{\delta} \equiv \sqrt{\alpha}(\underline{\theta}_1 - x^*)$.

For $\overline{\theta}_1 > t$, $\underline{\delta}$ is the unique solution to

$$1 - G(\underline{\delta}) - \int_{\underline{\delta}}^{+\infty} g(k) H\left[\sqrt{\frac{\alpha}{\beta}}(\underline{\delta} - k) - H^{-1}\left(1 - \frac{2r^*}{r_s}\right)\right] dk = \frac{2r^*}{r_s}. \tag{12}$$

For $\overline{\theta}_1 \leq t$, $\underline{\delta}$ is the unique solution to

$$1 - G(\underline{\delta}) - \int_{\underline{\delta}}^{\Gamma} g(k) H\left[\sqrt{\frac{\alpha}{\beta}}(\underline{\delta} - k) - H^{-1}\left(1 - \frac{2r^*}{r_s}\right)\right] dk = \frac{2r^*}{r_s}, \tag{13}$$

where $\Gamma = G^{-1}(G[\underline{\delta}) + p/(1-p)]$.

From the lemma above, the size effect on $\underline{\theta}_1$ in the limiting case is given by

$$\frac{d\underline{\theta}_1}{dp} = \frac{\tau D}{\lambda A}\left[-G(-\underline{\delta}) - (1-p)g(-\underline{\delta})\right]\frac{d\underline{\delta}}{dp}.$$

We can only determine the sign of the size effect on $\underline{\theta}_1$ unambiguously if $\overline{\theta}_1 > t$. This result is summarised as follows.

Proposition 5 *In the limit as $\alpha \to \infty$, $\beta \to \infty$, and $\sigma_2 \to 0$, the liquidation threshold $\underline{\theta}_1$ is decreasing in p provided that $\overline{\theta}_1 > t$.*

Hence, increasing the size of the large creditor raises the willingness of the small creditors to roll over in the limiting case when $\underline{\theta}_1 < \overline{\theta}_1$.[8]

We need to emphasize that in the limiting case even when everyone has arbitrarily precise information, the interval of inefficient liquidation or roll-over persists due to strategic uncertainty. This is because in either the case where $\underline{\theta}_1 = \overline{\theta}_1 = x^* = y^*$ or $\underline{\theta}_1 = x^* = y^* < \overline{\theta}_1$, these thresholds are still above 0. The positive thresholds imply that in equilibrium there is always inefficient liquidation or inefficient rollover.

To further explore the size effect we resort to numerical examples. We calibrate parameters under two conditions. First, the payoff to foreclosing is chosen to be quite low relative to that of roll-over so that the insolvency risk is high and/or the capital markets are quite

[8] The size effect in our model is similar to that in Corsetti *et al.* (2004), i.e., it can be generally ambiguous. They obtained unambiguous size effect in two limiting cases: when $\alpha/\beta \to 0$ or when $\alpha/\beta \to \infty$. In our model, we can show that, as $\alpha/\beta \to \infty$, the liquidation threshold $\underline{\theta}_1$ is decreasing in p when $\overline{\theta}_1 \leq t$. However, when $\alpha/\beta \to 0$, the left hand side of (12) and (13) is zero, which means that solvency risk is 1 and $r^*/r_s = 1$. Therefore, we cannot use this method to prove that the size effect is globally positive because that it lowers $\underline{\theta}_1$.

Table 3: Parameter value for numerical solutions.

Cash, M	10	Assets, A	121
Haircut, $1-\lambda$	0.25	Payoff ratio, r^*/r_s	0.45
Long-term debt, L_2	21	Short-term debt, S_1	100
Private information precision, α, β	1	Public information precision, σ_2	1/3

illiquid.[9] Second, to be consistent with our model, the liquidity ratio is chosen to be strictly smaller than 1, or $\pi < 1$. This leads to $\overline{\theta}_1 < (S_1 - M)/\lambda A$. Values of these parameters are given in Table 3. Setting values for the parameters, such as M, S_1, L_2, A, affects only the size of the bank's balance sheet, and thereby does not affect the qualitative nature of our primary results. Because insolvency risk is high, the payoff ratio is set to 0.45, and the haircut is 25 percent so that the capital markets are quite illiquid. Both types of creditors' information precision is set to 1, while the public information precision is 1/3.

Then, we solve a system of four nonlinear equations. We plotted all thresholds as a function of p in Figure 3. Figure 3 shows that all thresholds, except $\overline{\theta}_1$, are decreasing in p. Recall that $\overline{\theta}_1$ is defined as the critical state where additional foreclosure by the large creditor is needed to make the bank fail. As p increases, this additional foreclosure becomes larger. However, both the large and small creditors' switching points are decreasing in p. Thus, an increase in p not only makes the large creditor more willing to roll over, but also raises the willingness of the small creditors to roll over. To verify the robustness of the results, we perform the same computation using different values for $\{\lambda, \alpha, \beta, \sigma_2\}$. The results are proven to be robust.

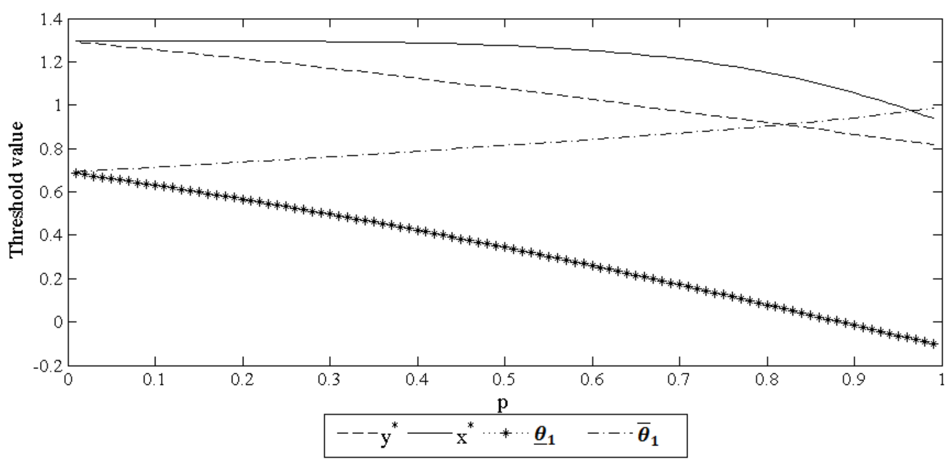

Figure 3: Thresholds as functions of the size of the large creditor p.

[9]We make this choice to avoid distribution functions being too close to step functions. This requires that the variances of distributions are not too small. As the variance of asset returns represents risk, we adjust the ratio of the liquidation value to the continuation return proportionally with the variances.

5 Credit risk

Having established the equilibrium effects, we can now address the primary question of how short-term financing, capital market liquidity and the presence of the large creditor affects credit risk. In the interim period, insolvency risk is

$$N_1(\theta_1) = F_2(\theta_2^* - \theta_1),$$

and illiquidity risk is

$$L_1(\theta_1) = \begin{cases} 1 - F_2(\theta_2^* - \theta_1) & \theta_1 \leq \underline{\theta}_1 \\ H(y^* - \theta_1)(1 - F_2(\theta_2^* - \theta_1)) & \underline{\theta}_1 < \theta_1 \leq \overline{\theta}_1 \\ 0 & \theta_1 > \overline{\theta}_1 \end{cases}$$

For θ_1 in the region $(\underline{\theta}_1, \overline{\theta}_1]$, $H(y^* - \theta_1)$ is the probability that the large creditor forecloses at θ_1, given his trigger strategy around y^*.

The interim credit risk is $C_1(\theta_1) = N_1(\theta_1) + L_1(\theta_1)$ such that

$$C_1(\theta_1) = \begin{cases} 1 & \theta_1 \leq \underline{\theta}_1 \\ H(y^* - \theta_1)(1 - F_2(\theta_2^* - \theta_1)) + F_2(\theta_2^* - \theta_1) & \underline{\theta}_1 < \theta_1 \leq \overline{\theta}_1 \\ F_2(\theta_2^* - \theta_1) & \theta_1 > \overline{\theta}_1 \end{cases}$$

The *ex ante* insolvency risk is

$$N_0(\theta_0) = \int_{-\infty}^{+\infty} f_1(\theta_1 - \theta_0) F_2(\theta_2^* - \theta_1)\, d\theta_1,$$

and the *ex ante* illiquidity risk is

$$\begin{aligned} L_0(\theta_0) &= \int_{-\infty}^{\underline{\theta}_1} (1 - F_2(\theta_2^* - \theta_1)) f_1(\theta_1 - \theta_0)\, d\theta_1 \\ &\quad + \int_{\underline{\theta}_1}^{\overline{\theta}_1} H(y^* - \theta_1)(1 - F_2(\theta_2^* - \theta_1)) f_1(\theta_1 - \theta_0)\, d\theta_1. \end{aligned} \tag{14}$$

The *ex ante* credit risk is

$$C_0(\theta_0) = N_0(\theta_0) + L_0(\theta_0).$$

Note that the changes in thresholds affect the *ex ante* credit risk only through the *ex ante* illiquidity risk. Thus, we focus on how the *ex ante* illiquidity risk is affected. We study first how the credit risk is affected by short-term financing, capital market liquidity and a better informed large creditor. The following proposition provides the answer.

Proposition 6 *The ex ante illiquidity risk is increasing in the short term debt ratio, but it is decreasing in market liquidity and in the precision of the large creditor's information.*

First, greater reliance on short-term debt financing increases the probability of creditor runs and credit risk. Second, an increase in market liquidity reduces credit risk. Conversely, a deterioration in capital market liquidity raises credit risk. Finally, a higher precision in the large creditor's information concerning the bank's liquidity at date 1 decreases credit risk.

To study the size effect on credit risk, we differentiate (14) with respect to p. Interestingly, it depends on how $\underline{\theta}_1$, y^* and $\overline{\theta}_1$ vary with respect to p (See Appendix A). Because $\underline{\theta}_1$ and y^* are decreasing in p, while $\overline{\theta}_1$ is increasing in p, the sign of $dL_0(\theta_0)/dp$ cannot be determined unambiguously. To explain this result, we use the liquidation threshold θ_1^*, when all creditors are small, as a benchmark. The interim illiquidity risk with two types of creditors is displayed in Figure 4.

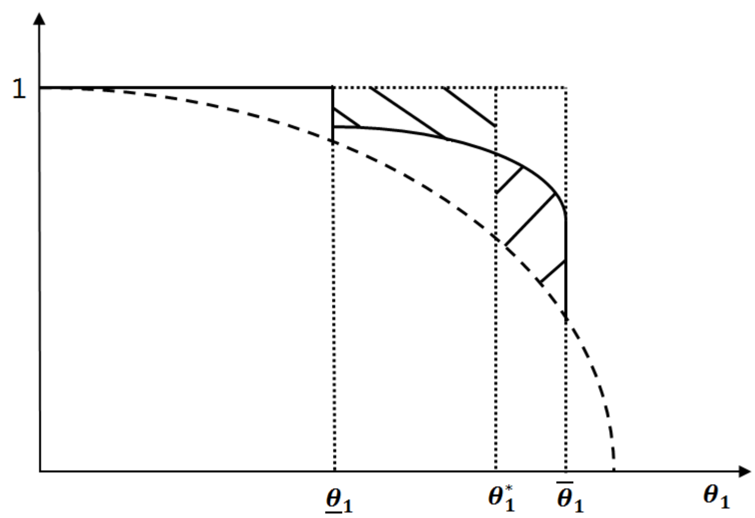

Figure 4: Interim credit risk with two types of creditors.

Figure 4 depicts the interim credit risk as a function of the expected asset return θ_1 with two types of creditors. The broken line represents the insolvency risk. The distance between the horizontal solid lines and the broken line represents the interim illiquidity risk. The shaded area to the left of θ_1^* represents the portion of the illiquidity risk that is decreased due to the presence of the large creditor, while the shaded area to the right of θ_1^* represents the portion of the illiquidity risk that is increased due to the presence of the large creditor.

Suppose, initially, that the short-term debt is all held by the small creditors and that the critical sate without the large creditor is θ_1^*. Now, the short-term debt is held by both the large and the small creditors. The presence of the large creditor will lower $\underline{\theta}_1$ but raise $\overline{\theta}_1$. Since $\underline{\theta}_1 \leq \theta_1^* \leq \overline{\theta}_1$), θ_1^* separates the $[\underline{\theta}_1, \overline{\theta}_1]$ into two regions. Without the large creditor, the interim illiquidity risk is the distance between the horizontal continuous line and the broken line in the area to the left of θ_1^*. With the large creditor, because the large creditor rolls over with a positive probability $1 - H(y^* - \theta_1)$ for θ_1 in the region $(\underline{\theta}_1, \overline{\theta}_1]$, the shaded area to the left of θ_1^* represents part of the illiquidity risk that is decreased due to the presence of the large creditor. This is primarily because the presence of the large creditor reduces the incidence of imperfect coordination between small creditors and thereby decreases illiquidity risk. At the same time, the presence of the large creditor pushes $\overline{\theta}_1$ to the right of θ_1^*. The shaded area to the right of θ_1^* represents the part of the illiquidity risk that is increased due to the presence of the large creditor. This is because the presence of the large creditor introduces new imperfect coordination between small creditors and the large one and thereby increases illiquidity risk. The overall impact of the presence of the large creditor on the credit risk depends on which of the aforementioned effects dominates.

Analytically, considering the expectation of the shaded area indicated in Figure 4, if the decreased part is larger than the increased part, then the *ex ante* illiquidity risk will decrease in p. In this case, an increase in the size of the large creditor reduces the credit risk. However, if the decreased part is smaller than the increased part in terms of credit risk, then the *ex ante* illiquidity risk will increase in p. Finally, if these opposite effects are equal, then the *ex ante* illiquidity risk is constant in p.

To further explore the size effect on credit risk, we numerically solve the model and compute the *ex ante* illiquidity risk as a function of p using the same parameter setting in Table 3. We plotted the *ex ante* illiquidity risk as a function of p in Figure 5. Figure 5 shows that an increase in the size of the large creditor lowers credit risk. This result holds provided that public information is less precise than private information and the payoff ratio r^*/r_s is quite low.

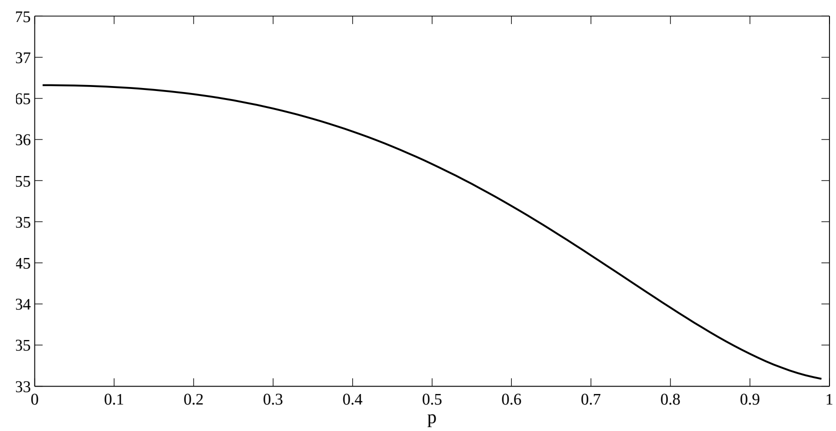

Figure 5: Ex ante illiquidity risk as a function of the size of the large creditor p.

What is the size effect if private information is less precise than public information? The question is relevant, as pointed out by Huang & Ratnovski (2011), that creditors invest less on improving their private information and rely on costly public information provided by rating agencies. We set private and public information precision to 1 and 2 respectively in keeping other parameters unchanged. Interestingly, as Figure 6 shows, an increase in the size of the large creditor raises credit risk. When private information is less precise than public information, a larger proportion of wholesale funding raises credit risk.

What is the size effect of the large creditor if liquidation value is only slightly lower than continuation value? We set the payoff ratio r^*/r_s to 0.8 keeping other parameters unchanged. As Figure 7 shows, an increase in the size of the large creditor raises credit risk if the premium of rolling over is small.

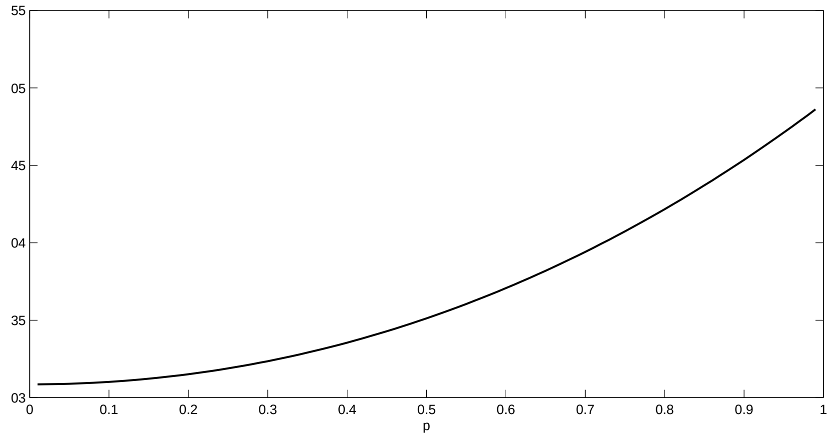

Figure 6: Ex ante illiquidity risk as a function of the size of the large creditor p with $\alpha/\sigma_2 = \beta/\sigma_2 = 1/2$.

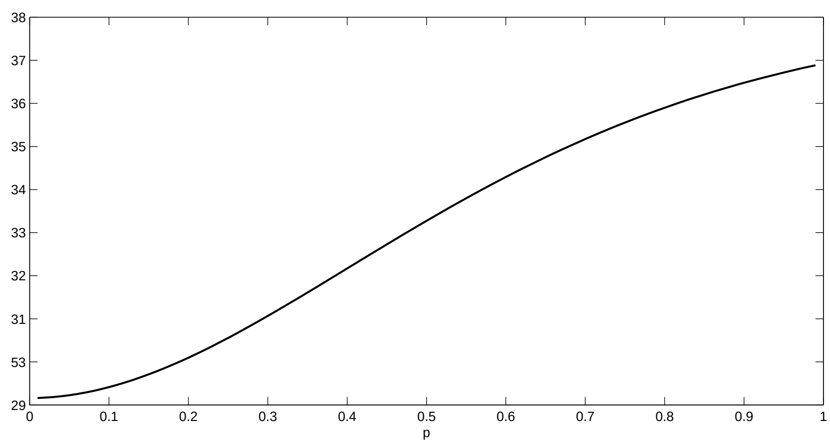

Figure 7: Ex ante illiquidity risk as a function of the size of the large creditor p with $r^*/r_s = 0.8$.

In pulling together our discussion, the overall conclusion that we draw from our analysis is that short-term financing, capital market liquidity and the presence of the large creditor are important determinants of credit risk. These conclusions are the most clear cut regarding the effects of short-term financing, capital market liquidity and a better informed large creditor. Analytically the size effect of the large creditor is ambiguous. Our numerical calculations reveal that an increase in the size of the large creditor lowers credit risk provided that private information is more precise than public information and the payoff ratio r^*/r_s is quite low. However, an increase in the size of the large creditor raises credit risk if public information is more precise than private information or the payoff ratio r^*/r_s is quite high.

6 Concluding remarks

Recent documented evidence has indicated that heavy reliance on short-term debt financing had a significant impact on the severity of the 2007/8 financial crisis. This may be partly rationalised in terms of our model. The size effect discussed earlier suggests that if private information on the bank's future profitability is very coarse, an increase in the size of the whole-sale financier will raise the credit risk. The wide-spread use of structured financial products and their complex nature make it difficult for individual banks to gather accurate information about the underlying risks associated with these products. In this case, the presence of the large whole-sale financier makes it more difficult for creditors to coordinate their action to roll over the existing debt. The situation is exacerbated when the market liquidity declines, resulting in a credit market freeze.

These results are obtained when the focus is placed on the balance sheet effect of banks (borrowers). However, when the market condition deteriorates, creditors financial positions may be adversely affected. This may generate fire-sales and make creditors less likely to roll over their existing debt. Incorporating the balance sheet effect of creditors, one can endogenize the market liquidity. The interaction between the balance sheet effects of banks and creditors may lead to a downward spiral on debt roll-overs. So, taking into account of creditor's financial positions can generally amplify the effects on credit risks discussed in this paper.

Given the main assumptions adopted in the paper, our results need to be interpreted with caution. The first issue is to do with the simultaneous nature of roll-over decisions. In practice, creditors can make sequential decisions based on, for example, debt seniority. Our results will be retained if actions are not observable and/or signals are "cheap talks". The second issue is associated with the assumed maturity structure of debt ex ante. Given a bank has substantial short-term debt, it is more likely that the bank will face a liquidity problem in the interim period. This is the main driving force of the credit risk (in addition to the coordination problems among heterogenous creditors) in the interim period. To provide a proper foundation, one needs to tackle the problem of how such maturity structure can emerge when creditors can select the type of debt contract ex ante.

Appendix A

Proof of Proposition 1 In what follows, we show first that there exists a unique x^* that solves equation (11). We then show that this unique switching equilibrium is dominance solvable.

Differentiating (8) and (9) with respect to x^*, respectively, yields

$$\frac{d\underline{\theta}_1}{dx^*} = \frac{\tau D(1-p)g(x^* - \underline{\theta}_1)}{\lambda A + \tau D(1-p)g(x^* - \underline{\theta}_1)} \in (0,1),$$
$$\frac{d\overline{\theta}_1}{dx^*} = \frac{\tau D(1-p)g(x^* - \overline{\theta}_1)}{\lambda A + \tau D(1-p)g(x^* - \overline{\theta}_1)} \in (0,1).$$

Let $\underline{\delta} = \underline{\theta}_1 - x^*$ and $\overline{\delta} = \overline{\theta}_1 - x^*$. Both $\underline{\delta}$ and $\overline{\delta}$ are monotonically decreasing in x^* as

$$d\underline{\delta}/dx^* = d\underline{\theta}_1/dx^* - 1 < 0$$
$$d\overline{\delta}/dx^* = d\overline{\theta}_1/dx^* - 1 < 0.$$

Note that (10) gives the mapping from small creditors' trigger x^* to the large creditor's trigger y^*. Differentiating (10) with respect to x^*, one can show that this mapping is monotonic, i.e.,

$$dy^*/dx^* = b_3 d\underline{\theta}_1/dx^* \in (0,1), \tag{A.1}$$

where

$$b_3 = \frac{h(\underline{\theta}_1 - y^*)[1 - F_2(\theta_2^* - y^*)]}{f_2(\theta_2^* - y^*)[1 - H(\underline{\theta}_1 - y^*)] + h(\underline{\theta}_1 - y^*)[1 - F_2(\theta_2^* - y^*)]} \in (0,1).$$

Rewriting (11) as

$$\Psi[x^*, y^*(x^*)] \equiv \left[1 - G\left(\underline{\delta}\right) - \int_{\underline{\delta}}^{\overline{\delta}} g\left(k\right) H\left(y^* - x^* - k\right) dk\right] \left[1 - F_2\left(\theta_2^* - x^*\right)\right] = r^*/r_s. \tag{A.2}$$

Differentiating the left hand side of (A.2) with respect to x^* yields

$$\begin{aligned}\frac{d\Psi[x^*, y^*(x^*)]}{dx^*} = & -g(\underline{\delta})[1 - H(y^* - x^* - \underline{\delta})]\frac{d\underline{\delta}}{dx^*} - g(\overline{\delta})H(y^* - x^* - \overline{\delta})\frac{d\overline{\delta}}{dx^*} \\ & - \int_{\underline{\delta}}^{\overline{\delta}} g(k)h(y^* - x^* - k)\left(\frac{dy^*}{dx^*} - 1\right) dk \\ & + \frac{f_2(\theta_2^* - x^*)}{1 - F_2(\theta_2^* - x^*)}\left[1 - G\left(\underline{\delta}\right) - \int_{\underline{\delta}}^{\overline{\delta}} g\left(k\right) H\left(y^* - x^* - k\right) dk\right]\end{aligned}$$

Substitution of (A.1) into the above expression, one can show that $\Psi[x^*, y^*(x^*)]$ is strictly increasing in x^*. For sufficiently small x^*, $\Psi[x^*, y^*(x^*)]$ approaches 0, while for sufficiently large x^*, it approaches 1. As $\Psi[x^*, y^*(x^*)]$ is continuous in x^*, there exists a unique solution to (11). With x^* determined in this way, (10) is then used to solve for the large creditor's switching point y^*.

We can now proceed to show that this unique switching equilibrium is the only equilibrium surviving the iterative elimination of strictly dominated strategies. Consider the expected payoff to roll-over for a small creditor conditional on signal x when all other small creditors follow the switching strategy around $\widehat{x}$, and when the large creditor plays his best response $y(\widehat{x})$ to this switching strategy, obtained from (10). Denote this expected payoff by $u(x, \widehat{x})$. It is given by

$$u(x,\widehat{x}) = \left[1 - \left(G\left(\underline{\theta}_1(\widehat{x}) - x\right) + \int_{\underline{\theta}_1(\widehat{x})}^{\overline{\theta}_1(\widehat{x})} g\left(\theta_1 - x\right) H\left(y(\widehat{x}) - \theta_1\right) d\theta_1\right)\right]\left[1 - F_2\left(\theta_2^* - x\right)\right] r_s,$$

where $\underline{\theta}(\widehat{x})$ and $\overline{\theta}(\widehat{x})$ indicate the value of $\underline{\theta}$ and $\overline{\theta}$ when small creditors follow the switching strategy around $\widehat{x}$. We allow $\widehat{x} \in \mathbb{R} \cup \{-\infty, \infty\}$ take the values $-\infty$ and ∞, by which the small creditors respectively never and always foreclose. As shown above, $u(.,.)$ is increasing in its first argument and decreasing in its second.

For sufficiently high values of x, rolling over is a dominant action for a small creditor, regardless of the actions of others, small or large. Denote by $\overline{x}^1$ the threshold value of x above which it is a dominant action to roll over for a small creditor. Since all creditors realize this, any strategy to foreclose above $\overline{x}^1$ is dominated by rolling over. Then, it cannot be rational for a small creditor to foreclose whenever his signal is higher than $\overline{x}^2$, where $\overline{x}^2$ solves

$$u(\overline{x}^2, \overline{x}^1) = r^*$$

It is so, since the switching strategy around $\overline{x}^2$ is the best reply to the switching strategy around $\overline{x}^1$ played by other small creditors and to that of the wholesale financier $y(\overline{x}^1)$, and since even the small creditor that assumes the lowest possibility of the continuation of the project believes that the incidence of continuation is higher than that implied by the switching strategy around $\overline{x}^1$and $y(\overline{x}^1)$. Since the payoff to rolling over is increasing in the incidence of continuation by the other creditors, any strategy that refrains from rolling over for signals higher than $\overline{x}^2$ is strictly dominated. Since

$$u\left(\overline{x}^1, \infty\right) = u\left(\overline{x}^2, \overline{x}^1\right) = r^*$$

monotonicity of u implies $\overline{x}^1 > \overline{x}^2$. Thus, suppose $\overline{x}^{k-1} > \overline{x}^k$, monotonicity implies that $\overline{x}^k > \overline{x}^{k+1}$. We can generate a decreasing sequence

$$\overline{x}^1 > \overline{x}^2 > \overline{x}^3 ... > \overline{x}^k > ...$$

where any strategy that refrains from rolling over for signal $x > \overline{x}^k$ does not survive k rounds of deletion of dominated strategies. Since the sequence is bounded, assuming $\overline{x}$ is the largest solution to $u(x, x) = r^*$, then monotonicity of u implies that

$$\overline{x} = \lim_{k \to \infty} \overline{x}^k$$

Any strategy that refrains from rolling over for signal higher than $\overline{x}$ does not survive iterated dominance.

Conversely, if $\underline{x}$ is the smallest solution to $u(x,x) = r^*$, any strategy that refrains from foreclosing for a signal below $\underline{x}$ does not survive iterative elimination. If there is a unique solution to $u(x,x) = \overline{L}$, then the smallest solution is the largest solution. Therefore, there is only one strategy that remains after eliminating all iteratively dominated strategies. This strategy is the only equilibrium strategy. This completes the argument.

Proof of Proposition 2 Differentiating (8) and (9) with respect to τ, respectively, provides

$$\begin{aligned}\frac{dx^*}{d\tau} &= \frac{1}{b_1}\frac{d\underline{\theta}_1}{d\tau} - \frac{G\left(x^* - \underline{\theta}_1\right)}{\tau g\left(x^* - \underline{\theta}_1\right)}, \\ \frac{dx^*}{d\tau} &= \frac{1}{b_2}\frac{d\overline{\theta}_1}{d\tau} - \frac{p + (1-p)\, G\left(x^* - \overline{\theta}_1\right)}{\tau\left(1-p\right) g\left(x^* - \overline{\theta}_1\right)},\end{aligned}$$

where

$$\begin{aligned}b_1 &= [1 + \lambda A/\tau D(1-p)g(x^* - \underline{\theta}_1)]^{-1} < 1, \\ b_2 &= [1 + \lambda A/\tau D(1-p)g(x^* - \overline{\theta}_1)]^{-1} < 1.\end{aligned}$$

Let $\underline{\delta} = \underline{\theta}_1 - x^*$ and $\overline{\delta} = \overline{\theta}_1 - x^*$. Then we obtain

$$\begin{aligned}\frac{d\underline{\delta}}{d\tau} &= (b_1 - 1)\frac{dx^*}{d\tau} + \frac{b_1 G\left(x^* - \underline{\theta}_1\right)}{\tau g\left(x^* - \underline{\theta}_1\right)} \\ \frac{d\overline{\delta}}{d\tau} &= (b_2 - 1)\frac{dx^*}{d\tau} + \frac{b_2\left[p + (1-p)\, G\left(x^* - \overline{\theta}_1\right)\right]}{\tau\left(1-p\right) g\left(x^* - \overline{\theta}_1\right)}\end{aligned}$$

Differentiating (10) with respect to τ, we obtain

$$\frac{dy^*}{d\tau} = b_3\frac{d\underline{\theta}_1}{d\tau},$$

where

$$b_3 = \frac{h\left(\underline{\theta}_1 - y^*\right)\left[1 - F_2(\theta_2^* - y^*)\right]}{f_2\left(\theta_2^* - y^*\right)\left(1 - H\left(\underline{\theta}_1 - y^*\right)\right) + h\left(\underline{\theta}_1 - y^*\right)\left[1 - F_2(\theta_2^* - y^*)\right]} \in (0,1).$$

Then

$$\frac{dy^*}{d\tau} = b_3 b_1\frac{dx^*}{d\tau} + \frac{b_1 b_3 G\left(x^* - \underline{\theta}_1\right)}{\tau g\left(x^* - \underline{\theta}_1\right)}.$$

We can rewrite (7) as

$$\left[1 - G\left(\underline{\delta}\right) - \int_{\underline{\delta}}^{\overline{\delta}} g\left(k\right) H\left(y^* - x^* - k\right) dk\right]\left[1 - F_2\left(\theta_2^* - x^*\right)\right] r_s = r^* \tag{A.3}$$

Differentiating (A.3) with respect to τ, we obtain

$$
\begin{aligned}
& -g\left(\underline{\delta}\right)\left(1-H\left(y^{*}-x^{*}-\underline{\delta}\right)\right)\frac{d\underline{\delta}}{d\tau}-g\left(\overline{\delta}\right)H\left(y^{*}-x^{*}-\overline{\delta}\right)\frac{d\overline{\delta}}{d\tau} \\
& -\int_{\underline{\delta}}^{\overline{\delta}} g\left(k\right)h\left(y^{*}-x^{*}-k\right)\left(\frac{dy^{*}}{d\tau}-\frac{dx^{*}}{d\tau}\right)dk \\
& +\frac{f_2\left(\theta_2^{*}-x^{*}\right)}{1-F_2\left(\theta_2^{*}-x^{*}\right)}\left[1-G\left(\underline{\delta}\right)-\int_{\underline{\delta}}^{\overline{\delta}} g\left(k\right)H\left(y^{*}-x^{*}-k\right)dk\right]\frac{dx^{*}}{d\tau}=0
\end{aligned}
$$

Let $w=r^{*}/[1-F_2(\theta_2^{*}-y^{*})]r_s>0$. Then, $1-H(y^{*}-x^{*}-\underline{\delta})=w$. By substitution, we obtain

$$
\begin{aligned}
& b_4\left(1-b_1\right)\frac{dx^{*}}{d\tau}+b_5\left(1-b_2\right)\frac{dx^{*}}{d\tau}+b_6(1-b_1b_3)\frac{dx^{*}}{d\tau}+b_7\frac{dx^{*}}{d\tau} \\
= \; & \frac{b_1(b_3b_6+b_4)G\left(x^{*}-\underline{\theta}_1\right)}{\tau g\left(x^{*}-\underline{\theta}_1\right)}+\frac{b_2b_5\left[p+(1-p)G\left(x^{*}-\overline{\theta}_1\right)\right]}{\tau\left(1-p\right)g\left(x^{*}-\overline{\theta}_1\right)}
\end{aligned}
$$

where

$$
\begin{aligned}
b_4 &= wg\left(\underline{\delta}\right)>0 \\
b_5 &= g\left(\overline{\delta}\right)H\left(y^{*}-x^{*}-\overline{\delta}\right)>0, \\
b_6 &= \int_{\underline{\delta}}^{\overline{\delta}} g\left(k\right)h\left(y^{*}-x^{*}-k\right)dk>0, \\
b_7 &= \frac{f_2\left(\theta_2^{*}-x^{*}\right)}{1-F_2\left(\theta_2^{*}-x^{*}\right)}\left[1-G\left(\underline{\delta}\right)-\int_{\underline{\delta}}^{\overline{\delta}} g\left(k\right)H\left(y^{*}-x^{*}-k\right)dk\right]>0.
\end{aligned}
$$

Because b_1 and b_2 are smaller than one, all of the coefficients on the left hand side of the above equation are positive. Thus, we have $dx^{*}/d\tau>0$, then $dy^{*}/d\tau>0$, $d\underline{\theta}_1/d\tau>0$, and $d\overline{\theta}_1/d\tau>0$. This completes the proof.

Proof of Proposition 3. Differentiating (8) and (9) with respect to λ, respectively, yields

$$
\frac{dx^{*}}{d\lambda}=\frac{1}{b_1}\frac{d\underline{\theta}_1}{d\lambda}+\frac{\underline{\theta}_1A}{\tau D\left(1-p\right)g\left(x^{*}-\underline{\theta}_1\right)}
$$

$$
\frac{dx^{*}}{d\lambda}=\frac{1}{b_2}\frac{d\overline{\theta}_1}{d\lambda}+\frac{\overline{\theta}_1A}{\tau D\left(1-p\right)g\left(x^{*}-\overline{\theta}_1\right)}.
$$

We have

$$
\frac{d\underline{\delta}}{d\lambda}=\left(b_1-1\right)\frac{dx^{*}}{d\lambda}-\frac{b_1\underline{\theta}_1A}{\tau D\left(1-p\right)g\left(x^{*}-\underline{\theta}_1\right)}
$$

$$\frac{d\overline{\delta}}{d\lambda} = (b_2 - 1)\frac{dx^*}{d\lambda} - \frac{b_2\overline{\theta}_1 A}{\tau D(1-p)\, g\left(x^* - \overline{\theta}_1\right)}$$

Differentiating (10) with respect to λ, we obtain

$$\frac{dy^*}{d\lambda} = b_1 b_3 \frac{dx^*}{d\lambda} - \frac{b_1 b_3 \underline{\theta}_1 A}{\tau D(1-p)\, g\left(x^* - \underline{\theta}_1\right)}.$$

Differentiating (A.3) with respect to λ, we obtain

$$\begin{aligned} & b_4(1-b_1)\frac{dx^*}{d\lambda} + b_5(1-b_2)\frac{dx^*}{d\lambda} + b_6(1-b_1b_3)\frac{dx^*}{d\lambda} + b_7\frac{dx^*}{d\lambda} \\ = \; & -\frac{b_1(b_3b_6+b_4)\underline{\theta}_1 A}{\tau D(1-p)\, g\left(x^*-\underline{\theta}_1\right)} - \frac{b_2 b_5 \overline{\theta}_1 A}{\tau D(1-p)\, g\left(x^* - \overline{\theta}_1\right)} \end{aligned}$$

Thus, we have $dx^*/d\lambda < 0$, $dy^*/d\lambda < 0$, $d\underline{\theta}_1/d\lambda < 0$, and $d\overline{\theta}_1/d\lambda < 0$. This completes the proof.

Proof of Proposition 4. Differentiating (8) and (9) with respect to the precision of the large lender's information β, we obtain

$$\begin{aligned} \frac{dx^*}{d\beta} &= \frac{1}{b_1}\frac{d\underline{\theta}_1}{d\beta}, \\ \frac{dx^*}{d\beta} &= \frac{1}{b_2}\frac{d\overline{\theta}_1}{d\beta}, \end{aligned}$$

Then, we have

$$\begin{aligned} \frac{d\underline{\delta}}{d\beta} &= (b_1 - 1)\frac{dx^*}{d\beta} \\ \frac{d\overline{\delta}}{d\beta} &= (b_2 - 1)\frac{dx^*}{d\beta}. \end{aligned}$$

Moreover, we can write (10) in standard normal and differentiate it with respect to β such that

$$\begin{aligned} & \left(1 - F_2\left(\theta_2^* - y^*\right)\right)\left[\phi\left(\sqrt{\beta}\left(\underline{\theta}_1 - y^*\right)\right)\left(\sqrt{\beta}\left(\frac{d\underline{\theta}_1}{d\beta} - \frac{dy^*}{d\beta}\right) + \frac{1}{2\sqrt{\beta}}\left(\underline{\theta}_1 - y^*\right)\right)\right] \\ = \; & \left[1 - H\left(\underline{\theta}_1 - y^*\right)\right] f_2\left(\theta_2^* - y^*\right)\frac{dy^*}{d\beta}, \end{aligned}$$

which yields

$$\frac{dy^*}{d\beta} = b_1 c_1 \frac{dx^*}{d\beta} + \frac{c_1}{2\beta}\left(\underline{\theta}_1 - y^*\right),$$

where

$$c_1 = \frac{\left(1 - F_2\left(\theta_2^* - y^*\right)\right)\phi\left(\sqrt{\beta}\left(\underline{\theta}_1 - y^*\right)\right)\sqrt{\beta}}{\left(1 - F_2\left(\theta_2^* - y^*\right)\right)\phi\left(\sqrt{\beta}\left(\underline{\theta}_1 - y^*\right)\right)\sqrt{\beta} + \left(1 - H\left(\underline{\theta}_1 - y^*\right)\right) f_2\left(\theta_2^* - y^*\right)} \in (0,1).$$

Differentiating (A.3) with respect to β, we obtain

$$
\begin{aligned}
&-g\left(\underline{\delta}\right)\left(1-H\left(y^*-x^*-\underline{\delta}\right)\right)\frac{d\underline{\delta}}{d\beta}-g\left(\overline{\delta}\right)H\left(y^*-x^*-\overline{\delta}\right)\frac{d\overline{\delta}}{d\beta} \\
&-\int_{\underline{\delta}}^{\overline{\delta}} h_x\left(k\right)\phi\left(\sqrt{\beta}\left(y^*-x^*-k\right)\right)\left[\sqrt{\beta}\left(\frac{dy^*}{d\beta}-\frac{dx^*}{d\beta}\right)+\frac{y^*-x^*-k}{2\sqrt{\beta}}\right]dk \\
&+\frac{f_2\left(\theta_2^*-x^*\right)}{1-F_2\left(\theta_2^*-x^*\right)}\left[1-G\left(\underline{\delta}\right)-\int_{\underline{\delta}}^{\overline{\delta}} g\left(k\right)H\left(y^*-x^*-k\right)dk\right]\frac{dx^*}{d\beta}=0,
\end{aligned}
$$

which can be rearranged as

$$
\begin{aligned}
&-g\left(\underline{\delta}\right)\left(1-H\left(y^*-x^*-\underline{\delta}\right)\right)\left(b_1-1\right)\frac{dx^*}{d\beta}-g\left(\overline{\delta}\right)H\left(y^*-x^*-\overline{\delta}\right)\left(b_2-1\right)\frac{dx^*}{d\beta} \\
&-\int_{\underline{\delta}}^{\overline{\delta}} g\left(k\right)\phi\left(\sqrt{\beta}\left(y^*-x^*-k\right)\right)\left[\sqrt{\beta}\left(b_1c-1\right)\frac{dx^*}{d\beta}+\frac{c_1(\underline{\theta}_1-y^*)+y^*-\underline{\theta}_1+\underline{\delta}-k}{2\sqrt{\beta}}\right]dk \\
&+\frac{f_2\left(\theta_2^*-x^*\right)}{1-F_2\left(\theta_2^*-x^*\right)}\left[1-G\left(\underline{\delta}\right)-\int_{\underline{\delta}}^{\overline{\delta}} g\left(k\right)H\left(y^*-x^*-k\right)dk\right]\frac{dx^*}{d\beta}=0.
\end{aligned}
$$

Then, we obtain

$$
\begin{aligned}
& b_4(1-b_1)\frac{dx^*}{d\beta}+b_5\left(1-b_2\right)\frac{dx^*}{d\beta}+b_6\sqrt{\beta}\left(1-b_1c_1\right)\frac{dx^*}{d\beta}+b_7\frac{dx^*}{d\beta} \\
= & \int_{\underline{\delta}}^{\overline{\delta}} g\left(k\right)\phi\left(\sqrt{\beta}\left(y^*-x^*-k\right)\right)\frac{c_1(\underline{\theta}_1-y^*)+y^*-\underline{\theta}_1+\underline{\delta}-k}{2\sqrt{\beta}}dk,
\end{aligned}
$$

Note that $\underline{\theta}_1-y^*<0$. Because $\underline{\theta}_1<\overline{\theta}_1$, the integrand $y^*-\underline{\theta}_1+\underline{\delta}-k$ evaluated between $\underline{\delta}$ and $\overline{\delta}$ is strictly negative. Hence, that $\frac{dx^*}{d\beta}<0$, then $dy^*/d\beta<0$, $d\underline{\theta}_1/d\beta<0$, and $d\overline{\theta}_1/d\beta<0$ is straightforward. This completes the proof.

Proof of Lemma 1. Note that (10) can be rewritten as

$$
H\left(\sqrt{\beta}(\underline{\theta}_1-y^*)\right)=1-\frac{2r^*}{r_s}, \tag{A.4}
$$

in which as $\sigma_2\to 0$, $1-F_2\left(\theta_2^*-y^*\right)=1/2$ is employed. Because $H\left(\sqrt{\beta}(\underline{\theta}_1-y^*)\right)\geq 0$, we have $r^*/r_s\leq 1/2$. To make the analysis tractable, we assume $r^*/r_s<1/2$. As $\beta\to\infty$, we must have $y^*\to\underline{\theta}_1$, or else $H\left(\sqrt{\beta}(\underline{\theta}_1-y^*)\right)$ will be either zero or one. Hence, the large creditor will roll over at states to the right of $\underline{\theta}_1$. When the small creditors have very precise information, they will also roll over at sates to the right of $\underline{\theta}_1$. Thus, in the limit case, we have

$$x^* = y^* = \underline{\theta}_1.$$

The bank fails if and only if $\theta_1 < \underline{\theta}_1$. The question of whether a larger creditor raises the willingness of the small creditors to roll over hinges on the behavior at the critical state $\underline{\theta}_1$.

In solving for the critical state $\underline{\theta}_1$ in the limiting case, we need to distinguish two cases. In the limit, from (8) and (9), we have

$$\begin{aligned} \underline{\theta}_1 &\in \left[-\frac{M}{\lambda A}, \frac{\tau D(1-p) - M}{\lambda A}\right] \\ \overline{\theta}_1 &\in \left[\frac{\tau D p - M}{\lambda A}, \frac{\tau D - M}{\lambda A}\right]. \end{aligned}$$

Thus, we can distinguish the case when $\overline{\theta}_1 \leq t$ from the case when $\overline{\theta}_1 > t$, where $t \equiv [\tau D(1-p) - M]/\lambda A$. In the former case, $\underline{\theta}_1 = \overline{\theta}_1$. However, in the latter case, $\underline{\theta}_1 < \overline{\theta}_1$.

Proof of Lemma 2. First, suppose that $\lim \underline{\theta}_1 < \lim \overline{\theta}_1$ so that $\lim \overline{\theta}_1 \geq [\tau D(1-p) - M]/\lambda A$. Because $x^* \to \underline{\theta}_1$, we must have $\overline{\delta} = \sqrt{\alpha}(\overline{\theta}_1 - x^*) \to +\infty$. Then (11) in this case is

$$1 - G(\underline{\delta}) - \int_{\underline{\delta}}^{+\infty} g(k) H\left(\sqrt{\frac{\alpha}{\beta}}(\underline{\delta} - k) - H^{-1}\left(1 - \frac{2r^*}{r_s}\right)\right) dk = \frac{2r^*}{r_s}$$

where $y^* = \underline{\theta}_1 - H^{-1}(1 - 2r^*/r_s)$ is used.

Second, consider the case where $\lim \underline{\theta}_1 = \lim \overline{\theta}_1$ so that $\overline{\delta}$ is finite and

$$(1-p)(1 - G(\underline{\delta})) = p + (1-p)\left(1 - G\left(\overline{\delta}\right)\right),$$

which yields

$$\overline{\delta} = G^{-1}\left(G(\underline{\delta}) + \frac{p}{1-p}\right).$$

Hence, in this case, (11) is

$$1 - G(\underline{\delta}) - \int_{\underline{\delta}}^{\Gamma} g(k) H\left(\sqrt{\frac{\alpha}{\beta}}(\underline{\delta} - k) - H^{-1}\left(1 - \frac{2r^*}{r_s}\right)\right) dk = \frac{2r^*}{r_s},$$

where $\Gamma = G^{-1}[G(\underline{\delta}) + p/(1-p)]$. This completes the proof.

Proof of Proposition 5. Differentiating (8) and (9) with respect to p provides

$$\begin{aligned} \frac{dx^*}{dp} &= \frac{1}{b_1}\frac{d\underline{\theta}_1}{dp} + \frac{G(x^* - \underline{\theta}_1)}{(1-p)\, g(x^* - \underline{\theta}_1)}, \\ \frac{dx^*}{dp} &= \frac{1}{b_2}\frac{d\overline{\theta}_1}{dp} - \frac{1 - G\left(x^* - \overline{\theta}_1\right)}{(1-p)\, g\left(x^* - \overline{\theta}_1\right)}, \end{aligned}$$

and

$$\begin{aligned}
\frac{d\underline{\delta}}{dp} &= (b_1 - 1)\frac{dx^*}{dp} - \frac{b_1 G\left(x^* - \underline{\theta}_1\right)}{(1-p)\, g\left(x^* - \underline{\theta}_1\right)} \\
\frac{d\overline{\delta}}{dp} &= (b_2 - 1)\frac{dx^*}{dp} + \frac{b_2\left(1 - G\left(x^* - \overline{\theta}_1\right)\right)}{(1-p)\, g\left(x^* - \overline{\theta}_1\right)}.
\end{aligned}$$

Differentiating (10) with respect to p provides

$$\frac{dy^*}{dp} = b_1 b_3 \frac{dx^*}{dp} - \frac{b_1 b_3 G\left(x^* - \underline{\theta}_1\right)}{(1-p)\, g\left(x^* - \underline{\theta}_1\right)}.$$

Differentiating (A.3) with respect to p provides

$$\begin{aligned}
& b_4 (1 - b_1)\frac{dx^*}{dp} + b_5 (1 - b_2)\frac{dx^*}{dp} + b_6(1 - b_1 b_3)\frac{dx^*}{dp} + b_7 \frac{dx^*}{dp} \\
= & -\frac{b_1(b_3 b_6 + b_4) G\left(x^* - \underline{\theta}_1\right)}{(1-p)\, g\left(x^* - \underline{\theta}_1\right)} + \frac{b_2 b_5\left(1 - G\left(x^* - \overline{\theta}_1\right)\right)}{(1-p)\, g\left(x^* - \overline{\theta}_1\right)}
\end{aligned}$$

Thus, only when

$$\frac{b_2 b_5\left(1 - G\left(x^* - \overline{\theta}_1\right)\right)}{g\left(x^* - \overline{\theta}_1\right)} < \frac{b_1(b_3 b_6 + b_4) G\left(x^* - \underline{\theta}_1\right)}{g\left(x^* - \underline{\theta}_1\right)},$$

we have $dx^*/dp < 0$. Because, analytically, we cannot prove whether this condition holds or not, we solve the model numerically. We find that $dx^*/dp < 0$, $dy^*/dp < 0$, and $d\underline{\theta}_1/dp < 0$. However, $d\overline{\theta}_1/dp > 0$.

Proof of Proposition 6. Differentiating(14) with respect to the short-term debt ratio provides

$$\begin{aligned}
\frac{dL_0\left(\theta_0\right)}{d\tau} &= \left(1 - H\left(y^* - \underline{\theta}_1\right)\right)\left(1 - F_2\left(\theta_2^* - \underline{\theta}_1\right)\right) f_1\left(\underline{\theta}_1 - \theta_0\right)\frac{d\underline{\theta}_1}{d\tau} \\
&\quad + H\left(y^* - \overline{\theta}_1\right)\left(1 - F_2\left(\theta_2^* - \overline{\theta}_1\right)\right) f_1\left(\overline{\theta}_1 - \theta_0\right)\frac{d\overline{\theta}_1}{d\tau} \\
&\quad + \int_{\underline{\theta}_1}^{\overline{\theta}_1} \left[h\left(y^* - \theta_1\right)\left(1 - F_2(\theta_2^* - \theta_1)\right)\right] f_1\left(\theta_1 - \theta_0\right) d\theta_1 \frac{dy^*}{d\tau}
\end{aligned}$$

Because we have $d\overline{\theta}_1/d\tau > 0$, $d\underline{\theta}_1/d\tau > 0$ *and* $dy^*/d\tau > 0$, we obtain

$$\frac{dL_0\left(\theta_0\right)}{d\tau} > 0.$$

Differentiating (14) with respect to market liquidity λ provides

$$\begin{aligned}\frac{dL_0(\theta_0)}{d\lambda} &= (1-H(y^*-\underline{\theta}_1))(1-F_2(\theta_2^*-\underline{\theta}_1))f_1(\underline{\theta}_1-\theta_0)\frac{d\underline{\theta}_1}{d\lambda} \\ &+H\left(y^*-\overline{\theta}_1\right)\left(1-F_2\left(\theta_2^*-\overline{\theta}_1\right)\right)f_1\left(\overline{\theta}_1-\theta_0\right)\frac{d\overline{\theta}_1}{d\lambda} \\ &+\int_{\underline{\theta}_1}^{\overline{\theta}_1}\left[h(y^*-\theta_1)(1-F_2(\theta_2^*-\theta_1))\right]f_1(\theta_1-\theta_0)\,d\theta_1\frac{dy^*}{d\lambda}\end{aligned}$$

Because we have $d\overline{\theta}_1/d\lambda<0$, $d\underline{\theta}_1/d\lambda<0$ and $dy^*/d\lambda<0$, we have

$$\frac{dL_0(\theta_0)}{d\lambda}<0.$$

Differentiating (14) with respect to the precision of the large creditor's information gives

$$\begin{aligned}\frac{dL_0(\theta_0)}{d\beta} &= (1-H(y^*-\underline{\theta}_1))(1-F_2(\theta_2^*-\underline{\theta}_1))f_1(\underline{\theta}_1-\theta_0)\frac{d\underline{\theta}_1}{d\beta} \\ &+H\left(y^*-\overline{\theta}_1\right)\left(1-F_2\left(\theta_2^*-\overline{\theta}_1\right)\right)f_1\left(\overline{\theta}_1-\theta_0\right)\frac{d\overline{\theta}_1}{d\beta} \\ &+\int_{\underline{\theta}_1}^{\overline{\theta}_1}\left[\phi\left(\sqrt{\beta}(y^*-\theta_1)\right)(1-F_2(\theta_2^*-\theta_1))\right]f_1(\theta_1-\theta_0)\left[\sqrt{\beta}\frac{dy^*}{d\beta}+\frac{y^*-\theta_1}{2\sqrt{\beta}}\right]d\theta_1\end{aligned}$$

Because $\underline{\theta}_1<\overline{\theta}_1$, the integrand $y^*-\theta_1$ evaluated between $\underline{\theta}_1$ and $\overline{\theta}_1$, is strictly negative. Furthermore, we have proven that $d\underline{\theta}_1/d\beta<0$, $dy^*/d\beta<0$, and $d\overline{\theta}_1/d\beta<0$. Hence,

$$\frac{dL_0(\theta_0)}{d\beta}<0.$$

Differentiating (14) with respect to the size of the large creditor provides

$$\begin{aligned}\frac{dL_0(\theta_0)}{dp} &= (1-H(y^*-\underline{\theta}_1))(1-F_2(\theta_2^*-\underline{\theta}_1))f_1(\underline{\theta}_1-\theta_0)\frac{d\underline{\theta}_1}{dp} \\ &+H\left(y^*-\overline{\theta}_1\right)\left(1-F_2\left(\theta_2^*-\overline{\theta}_1\right)\right)f_1\left(\overline{\theta}_1-\theta_0\right)\frac{d\overline{\theta}_1}{dp} \\ &+\int_{\underline{\theta}_1}^{\overline{\theta}_1}\left[h(y^*-\theta_1)(1-F_2(\theta_2^*-\theta_1))\right]f_1(\theta_1-\theta_0)\,d\theta_1\frac{dy^*}{dp}\end{aligned}$$

Because we have $d\overline{\theta}_1/dp>0$, while $d\underline{\theta}_1/dp<0$ and $dy^*/dp<0$, the sign of $dL_0(\theta_0)/dp$ is ambiguous. Together, these complete the proof.

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