Measurement of $CP$ Violation in the Phase Space of $B^\pm \to K^\pm \pi^+ \pi^-$ and $B^\pm \to K^\pm K^+ K^-$ Decays

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The charmless decays $B^\pm \to K^\pm \pi^+ \pi^-$ and $B^\pm \to K^\pm K^+ K^-$ are reconstructed using data, corresponding to an integrated luminosity of 1.0 fb$^{-1}$, collected by LHCb in 2011. The inclusive charge asymmetries of these modes are measured as $A_{CP}(B^\pm \to K^\pm \pi^+ \pi^-) = 0.032 \pm 0.008 \text{(stat)} \pm 0.004 \text{(syst)} \pm 0.007 \text{(J/ψK^±)}$ and $A_{CP}(B^\pm \to K^\pm K^+ K^-) = -0.043 \pm 0.009 \text{(stat)} \pm 0.003 \text{(syst)} \pm 0.007 \text{(J/ψK^±)}$, where the third uncertainty is due to the $CP$ asymmetry of the $B^\pm \to J/ψK^±$ reference mode. The significance of $A_{CP}(B^\pm \to K^\pm K^+ K^-)$ exceeds three standard deviations and is the first evidence of an inclusive $CP$ asymmetry in charmless three-body $B$ decays. In addition to the inclusive $CP$ asymmetries, larger asymmetries are observed in localized regions of phase space.

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Violation of the combined symmetry of charge conjugation and parity ($CP$ violation) is described in the standard model by the Cabibbo-Kobayashi-Maskawa quark-mixing matrix [1,2]. $CP$ violation is experimentally well established in the $K^0$ [3], $B^0$ [4,5], and $B^\pm$ [6] systems. One category of $CP$ violation, known as direct $CP$ violation, requires two interfering amplitudes with different weak and strong phases to be involved in the decay process [7]. Large $CP$ violation effects have been observed in charmless two-body $B$-meson decays such as $B^0 \to K^\pm \pi^\mp$ [8,9] and $B^0 \to K^\mp \pi^\pm$ [10]. However, the source of the strong phase difference in these processes is not well understood, which limits the potential to use these measurements to search for physics beyond the standard model. One possible source of the required strong phase is from final-state hadron rescattering, which can occur between two or more decay channels with the same flavor quantum numbers, such as $B^\pm \to K^\pm \pi^\mp \pi^\mp$ and $B^\pm \to K^\pm K^+ K^-$. [11–14]. This effect, referred to as “compound $CP$ violation” [15] is constrained by $CPT$ conservation so that the sum of the partial decay widths, for all channels with the same final-state quantum numbers related by the $S$ matrix, must be equal for charge-conjugated decays.

Decays of $B$ mesons to three-body hadronic charmless final states provide an interesting environment to search for $CP$ violation through the study of its signatures in the Dalitz plot [16]. Theoretical predictions are mostly based on quasi-two-body decays to intermediate states, e.g., $\rho^0 K^\pm$ and $K^{0*(892)} \pi^\mp$ for $B^\pm \to K^\pm \pi^\mp \pi^\mp$ decays and $\phi K^\pm$ for $B^\pm \to K^\pm K^+ K^-$ decays (see, e.g., Ref. [17]). These intermediate states are accessible through amplitude analyses of data, such as those performed by the Belle and BABAR Collaborations, who reported evidence of $CP$ violation in the intermediate channel $\rho^0 K^\pm$ [18,19] in $B^\pm \to K^\pm \pi^\mp \pi^\mp$ decays and more recently in the channel $\phi K^\pm$ [20] in $B^\pm \to K^\pm K^+ K^-$ decays. However, the inclusive $CP$ asymmetry of $B^\pm \to K^\pm \pi^\mp \pi^\mp$ and $B^\pm \to K^\pm K^+ K^-$ decays was found to be consistent with zero.

In this Letter, we report measurements of the inclusive $CP$-violating asymmetries in $B^\pm \to K^\pm \pi^\mp \pi^\mp$ and $B^\pm \to K^\pm K^+ K^-$ decays with unprecedented precision. (The inclusion of charge-conjugate decay modes is implied except in the asymmetry definitions.) We also study their asymmetry distributions across the phase space. The $CP$ asymmetry in $B^\pm$ decays to a final state $f^\pm$ is defined as

$$A_{CP}(B^\pm \to f^\pm) = \Phi[\Gamma(B^- \to f^-), \Gamma(B^+ \to f^+)].$$

where $\Phi[X, Y] = (X - Y)/(X + Y)$ is the asymmetry operator, $\Gamma$ is the decay width, and the final states are $f^\pm = K^\pm \pi^\mp \pi^\mp$ or $f^\pm = K^\pm K^+ K^-$. The LHCb detector [21] is a single-arm forward spectrometer covering the pseudorapidity range $2 < \eta < 5$, designed for the study of particles containing $b$ or $c$ quarks. The analysis is based on $pp$ collision data, corresponding to an integrated luminosity of 1.0 fb$^{-1}$, collected in 2011 at a center-of-mass energy of 7 TeV.

Events are selected by a trigger [22] that consists of a hardware stage, based on information from the calorimeter and muon systems, followed by a software stage, which applies a full event reconstruction. Candidate events are first required to pass the hardware trigger, which selects particles with large transverse energy. The software trigger requires a two-, three-, or four-track secondary vertex with a high sum of the transverse momenta $p_T$ of the tracks and a significant displacement from the primary $pp$ interaction vertices (PVs). At least one track should have $p_T > 1.7$ GeV/c and $\chi^2_{IP}$ with respect to any primary vertex greater than 16, where $\chi^2_{IP}$ is defined as the difference between the $\chi^2$ of a given PV reconstructed with and

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without the considered track, IP is the impact parameter. A multivariate algorithm is used for the identification of secondary vertices consistent with the decay of a $b$ hadron.

A set of off-line selection criteria is applied to reconstruct $B$ mesons and suppress the combinatorial backgrounds. The $B^\pm$ decay products are required to satisfy a set of selection criteria on their momenta, transverse momenta, the $\chi^2_p$ of the final-state tracks, and the distance of closest approach between any two tracks. The $B$ candidates are required to have $p_T > 1.7 \text{ GeV}/c$, $\chi^2_p < 10$ (defined by projecting the $B$ candidate trajectory backwards from its decay vertex) and displacement from any PV greater than 3 mm. Additional requirements are applied to variables related to the $B$-meson production and decay, such as quality of the track fits for the decay products, and the angle between the $B$ candidate momentum and the direction of flight from the primary vertex to the decay vertex. Final-state kaons and pions are further selected using particle identification information, provided by two ring-imaging Cherenkov detectors [23]. The selection is common to both decay channels, except the particle identification selection, which is specific to each final state. Charm contributions are removed by excluding the regions of $\pm 30 \text{ MeV}/c^2$ around the $D^0$ mass in the two-body invariant masses $m_{\pi\pi}$, $m_{K\pi}$, and $m_{KK}$. The contribution of the $B^\pm \to J/\psi K^\pm$ decay is also excluded from the $B^\pm \to K^\pm \pi^+ \pi^-$ sample by removing the mass region $3.05 < m_{\pi\pi} < 3.15 \text{ GeV}/c^2$.

The simulated events used in this analysis are generated using PYTHIA 6.4 [24] with a specific LHCb configuration [25]. Decays of hadronic particles are produced by EVTGEN [26], in which final-state radiation is generated using PHOTOS [27]. The interaction of the generated particles with the detector and its response are implemented using the GEANT 4 toolkit [28], as described in Ref. [29].

Unbinned extended maximum likelihood fits to the mass spectra of the selected $B^\pm$ candidates are performed. The $B^\pm \to K^\pm \pi^+ \pi^-$ and $B^\pm \to K^+K^-K^-$ signal components are parametrized by so-called Cruijff functions [30] to account for the asymmetric effect of final-state radiation on the signal shape. The combinatorial background is described by an exponential function, and the background due to partially reconstructed four-body $B$ decays is parametrized by an ARGUS function [31] convolved with a Gaussian resolution function. Peaking backgrounds occur due to decay modes with one misidentified particle and consist of the channels $B^\pm \to K^+K^-\pi$, $B^\pm \to \pi^+\pi^-\pi^\pm$, and $B^\pm \to \eta(\rho^0\gamma)K^\pm$ for the $B^\pm \to K^+K^-\pi^\pm$ mode, and $B^\pm \to K^+K^-K^\pm$ for the $B^\pm \to K^+K^-K^-$ mode. The shapes of the peaking backgrounds are obtained from simulation. The peaking background yields are obtained from simulation to be $N_{\eta/K} = 2140 \pm 154$ (most of which lie at masses lower than the signal), $N_{\pi\pi\pi} = 528 \pm 58$, and $N_{KK\pi} = 219 \pm 25$ for $B^\pm \to K^+K^-\pi^\pm\pi^\mp$, and $N_{KK\pi} = 192 \pm 20$ for $B^\pm \to K^+K^-K^\pm$ decays. The invariant mass spectra of the $B^\pm \to K^+K^-\pi^\pm\pi^\mp$ and $B^\pm \to K^+K^-K^-$ candidates are shown in Fig. 1.

The mass fits of the two samples are used to obtain the signal yields $N(K\pi\pi) = 35901 \pm 327$ and $N(KKK) = 22119 \pm 164$, and the raw asymmetries, $A_{\text{raw}}(K\pi\pi) = 0.020 \pm 0.007$ and $A_{\text{raw}}(KKK) = -0.060 \pm 0.007$, where the uncertainties are statistical. In order to determine the $CP$ asymmetries, the measured raw asymmetries are corrected for effects induced by the detector acceptance and interactions of final-state particles with matter, as well as for a possible $B$-meson production asymmetry. The decay products are regarded as a pair of charge-conjugate hadrons $h^+h^- = \pi^+\pi^-$, $K^+K^-$, and a kaon with the same charge as the $B^\pm$ meson. The $CP$ asymmetry is expressed in terms of the raw asymmetry and a correction $A_D$.

$$A_{CP} = A_{\text{raw}} - A_D, \quad A_D = A_D(K^\pm) + A_R(B^\pm).$$  \hspace{1cm} (2)

Here, $A_D(K^\pm)$ is the kaon detection asymmetry, given in terms of the charge-conjugate kaon detection efficiencies $e_D(K^\pm)$ by $A_D(K^\pm) = \Phi[e_D(K^-), e_D(K^+)]$, and $A_R(B^\pm)$ is the production asymmetry, defined from the $B^\pm$ production rates $R(B^\pm)$ as $A_R(B^\pm) = \Phi[R(B^-), R(B^+)]$.

![Fig. 1 (color online). Invariant mass spectra of (a) $B^\pm \to K^\pm \pi^+ \pi^-$ decays and (b) $B^\pm \to K^+K^-K^-$ decays. The left panel in each figure shows the $B^-$ modes, and the right panel in each shows the $B^+$ modes. The results of the unbinned maximum likelihood fits are overlaid. The main components of the fit are also shown.](101801-2)
The correction term $A_\Delta$ is measured from data using a sample of approximately $6.3 \times 10^4$ $B^+ \rightarrow J/\psi K^\pm K^\mp$ decays. The $B^\pm \rightarrow J/\psi K^\mp$ sample passes the same trigger, kinematic, and kaon particle identification selections as the signal samples, and it has a similar event topology. The kaons from $B^\pm \rightarrow J/\psi K^\mp$ decay also have similar kinematics in the laboratory frame to those from the $B^\pm \rightarrow K^\pm \pi^+ \pi^-$ and $B^\pm \rightarrow K^\pm K^+ K^-$ modes. The correction is obtained from the raw asymmetry of the $B^\pm \rightarrow J/\psi K^\mp$ mode as

$$A_\Delta = A_{\text{raw}}(J/\psi K) - A_{\text{CP}}(J/\psi K),$$

using the world average of the CP asymmetry $A_{\text{CP}}(J/\psi K) = (0.1 \pm 0.7)\%$ [32]. The CP asymmetries of the $B^\pm \rightarrow K^\pm \pi^+ \pi^-$ and $B^\pm \rightarrow K^\pm K^+ K^-$ channels are then determined using Eqs. (2) and (3).

Since the detector efficiencies for the signal modes are not flat in the corners of the Dalitz plot and the raw asymmetries are also not uniformly distributed, an acceptance correction is applied to the integrated raw asymmetries. It is determined by the ratio between the $B^+$ and $B^-$ average efficiencies in simulated events, reweighted to reproduce the population in the Dalitz plot of signal data. Furthermore, the detector acceptance and reconstruction efficiency depend on the trigger selection. The efficiency of the hadronic hardware trigger is found from calibration data to have a small charge asymmetry for final-state kaons. Therefore, the data are divided into two samples with respect to the hadronic hardware trigger decision: events with candidates selected by the hadronic trigger and events selected by other triggers independently of the signal candidate. In order to apply Eq. (3) to $B^\pm \rightarrow K^\pm K^+ K^-$ events selected by the hadronic hardware trigger, the difference in trigger efficiencies caused by the presence of three kaons compared to one kaon is taken into account. The acceptance correction and subtraction of $A_\Delta$ are performed separately for each trigger configuration. The trigger-averaged value of the asymmetry correction is $A_\Delta = -0.014 \pm 0.04$, which is consistent with other LHCb analyses [6,33,34]. The integrated CP asymmetries are then the weighted averages of the CP asymmetries for the two trigger samples.

The systematic uncertainties on the asymmetries are related to the mass fit models, possible trigger asymmetry, and phase-space acceptance correction. In order to estimate the uncertainty due to the choice of the signal mass shape, the initial model is replaced with the sum of a Gaussian and a crystal ball function [35]. The uncertainty associated with the combinatorial background model is estimated by repeating the fit with a first-order polynomial. We evaluate three uncertainties related to the peaking backgrounds: one due to the uncertainty on their yields, another due to the difference in mass resolution between simulation and data, and a third due to their possible non-zero asymmetries. The deviations from the nominal results are accounted for as systematic uncertainties. The systematic uncertainties related to the possible asymmetry induced by the trigger selection are of two kinds: one due to an asymmetric response of the hadronic hardware trigger to kaons and a second due to the choice of sample division by trigger decision. The former is evaluated by reweighting the $B^\pm \rightarrow J/\psi K^\mp$ mode with the charge-separated kaon efficiencies from calibration data. The latter is determined by varying the trigger composition of the samples in order to estimate the systematic differences in trigger admixture between the signal channels and the $B^\pm \rightarrow J/\psi K^\mp$ mode. Two distinct uncertainties are attributed to the phase-space acceptance corrections: one is obtained from the uncertainty on the detection efficiency given by the simulation, and the other, due to the choice of binning, is evaluated by varying the binning of the acceptance map. The systematic uncertainties for the measurements of $A_{\text{CP}}(B^\pm \rightarrow K^\pm \pi^+ \pi^-)$ and $A_{\text{CP}}(B^\pm \rightarrow K^\pm K^+ K^-)$ are summarized in Table I.

The results obtained for the inclusive CP asymmetries of the $B^\pm \rightarrow K^\pm \pi^+ \pi^-$ and $B^\pm \rightarrow K^\pm K^+ K^-$ decays are

$$A_{\text{CP}}(B^\pm \rightarrow K^\pm \pi^+ \pi^-) = 0.032 \pm 0.008 \pm 0.004 \pm 0.007,$$

$$A_{\text{CP}}(B^\pm \rightarrow K^\pm K^+ K^-) = -0.043 \pm 0.009 \pm 0.003 \pm 0.007,$$

where the first uncertainty is statistical, the second is the experimental systematic, and the third is due to the CP asymmetry of the $B^\pm \rightarrow J/\psi K^\mp$ reference mode [32]. The significances of the inclusive charge asymmetries, calculated by dividing the central values by the sum in quadrature of the statistical and both systematic uncertainties, are 2.8 standard deviations ($\sigma$) for $B^\pm \rightarrow K^\pm \pi^+ \pi^-$ and 3.7$\sigma$ for $B^\pm \rightarrow K^\pm K^+ K^-$ decays.

In addition to the inclusive charge asymmetries, we also study the asymmetry distributions in the two-dimensional phase space of two-body invariant masses. The background-subtracted Dalitz plot distributions of the signal region, defined as the mass region within three Gaussian widths from the signal peak, are divided into bins with equal numbers of events in the combined $B^+$ and $B^-$ samples. The background under the signal is estimated from the sideband distributions. A raw asymmetry variable $A^0_{\text{raw}} = \Phi[N(B^-), N(B^+)]$ is computed from

<table>
<thead>
<tr>
<th>Systematic uncertainty</th>
<th>$A_{\text{CP}}(K \pi \pi)$</th>
<th>$A_{\text{CP}}(KKK)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal model</td>
<td>0.0010</td>
<td>0.0002</td>
</tr>
<tr>
<td>Combinatorial background</td>
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<td>&lt;0.0001</td>
</tr>
<tr>
<td>Peaking background</td>
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<td>0.0001</td>
</tr>
<tr>
<td>Trigger asymmetry</td>
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<td>Acceptance correction</td>
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<td>0.0019</td>
</tr>
<tr>
<td>Total</td>
<td>0.0040</td>
<td>0.0027</td>
</tr>
</tbody>
</table>
are defined as trized and its two-body invariant mass squared variables $m_B$ of $B$ in the invariant mass region, around the $\rho(770)$ resonance, as seen by Belle [18] and BABAR [19], and above the $f_0(980)$ resonance. This can also be seen in the inset figure of the $\pi^+\pi^-$ invariant mass projection, where there is an excess of $B$ candidates. No significant asymmetry is present in the low-mass region of the $K^{\pm}\pi^\mp$ invariant mass projection. The $A^{\text{raw}}_\text{reg}$ distribution of the $B^\pm \rightarrow K^{\pm}\pi^\mp\pi^\mp$ mode reveals an asymmetry concentrated at low values of $m_{K^\pm\pi^\mp}^2$ in the Dalitz plot. The distribution of the projection of the number of events onto the $m_{K^+K^-}^2$ invariant mass (inset in the right plot of Fig. 2) shows that this asymmetry is not related to the $\phi(1020)$ resonance but is instead located in the region $1.2 < m_{K^+K^-}^2 < 2.0 \text{ GeV}^2/c^4$.

The CP asymmetry in each of the channels is further studied in the region where the raw asymmetry is observed to be large. The $B^\pm \rightarrow K^{\pm}\pi^\mp\pi^\mp$ mode, we measure the CP asymmetry of the region $m_{K^{\pm}\pi^\mp}^2 < 1.5 \text{ GeV}^2/c^4$ and $0.08 < m_{\pi^+\pi^-}^2 < 0.66 \text{ GeV}^2/c^4$, which spans the lowest $\pi^+\pi^-$ masses, including the $\rho(770)$ resonance. Unbinned extended maximum likelihood fits are performed to the mass spectra of the candidates in the two regions, using the same models as the global fits. The spectra are shown in Fig. 3. The resulting signal yields and raw asymmetries for the two regions are $N^{\text{reg}}(K\pi\pi) = 552 \pm 47$ and $A^{\text{raw}}_{\text{reg}}(K\pi\pi) = 0.687 \pm 0.078$ for the $B^\pm \rightarrow K^{\pm}\pi^\mp\pi^\mp$ mode, and $N^{\text{reg}}(KKK) = 2581 \pm 55$ and $A^{\text{reg}}_{\text{raw}}(KKK) = -0.239 \pm 0.020$ for the

![Graph and Diagrams]
$B^\pm \rightarrow K^\pm K^+ K^-$ mode. The $CP$ asymmetries are obtained from the raw asymmetries by applying an acceptance correction and subtracting the detection and production asymmetry correction $A_D$ obtained from $B^\pm \rightarrow J/\psi K^\pm$ decays. The validity of the global $A_D$ from $B^\pm \rightarrow J/\psi K^\pm$ decays for the results in the regions was tested by comparing the kinematic distributions of their decay products. Systematic uncertainties are estimated due to the signal models, trigger asymmetry, acceptance correction for the region, and the limited validity of Eq. (2) for large asymmetries. The local charge asymmetries for the two regions are measured to be

$$A_{CP}^{reg}(K\pi\pi) = 0.678 \pm 0.078 \pm 0.032 \pm 0.007,$$

$$A_{CP}^{reg}(KKK) = -0.226 \pm 0.020 \pm 0.004 \pm 0.007,$$

where the first uncertainty is statistical, the second is the experimental systematic, and the third is due to the $CP$ asymmetry of the $B^\pm \rightarrow J/\psi K^\pm$ reference mode.

In conclusion, we have measured the inclusive $CP$ asymmetries of the $B^\pm \rightarrow K^\pm \pi^+ \pi^-$ and $B^\pm \rightarrow K^\pm K^+ K^-$ modes with significances of $2.8\sigma$ and $3.7\sigma$, respectively. The latter represents the first evidence of an inclusive $CP$ asymmetry in charmless three-body $B$ decays. These charge asymmetries are not uniformly distributed in the phase space. For $B^\pm \rightarrow K^\pm \pi^+ \pi^-$ decays, we observe positive asymmetries at low $\pi^+ \pi^-$ masses, around the $\rho(770)^0$ resonance, as indicated by Belle [18] and BABAR [19], and also above the $f_0(980)$ resonance, where it is not clearly associated to resonances. The asymmetry appears only at low $K^\pm \pi^-$ mass around the $\rho(770)^0$ invariant mass. A signature of $CP$ violation is present in the $B^\pm \rightarrow K^\pm K^+ K^-$ Dalitz plot, mostly concentrated in the region of low $m_{KK}^2$ and low $m_{KK}^2$. A similar pattern of the $CP$ asymmetry was shown in the preliminary results of the $B^\pm \rightarrow K^+ K^- \pi^+ \pi^-$ and $B^\pm \rightarrow \pi^+ \pi^- \pi^0$ decay modes by LHCb [36], in which the positive asymmetries are at low $\pi^+ \pi^-$ masses and the negative at low $K^+ K^-$ masses, both not clearly associated with intermediate resonant states.

Moreover, the excess of $B^\rightarrow K^- \pi^+ \pi^-$ decays with respect to $B^\rightarrow K^+ \pi^+ \pi^-$ is comparable to the excess of $B^\rightarrow K^+ K^+ K^-$ decays with respect to $B^- \rightarrow K^- K^+ K^-$. This apparent correlation, together with the inhomogeneous $CP$ asymmetry distribution in the Dalitz plot, could be related to compound $CP$ violation. Since the $B^\rightarrow K^\pm \pi^+ \pi^-$ and $B^\rightarrow K^\pm K^+ K^-$ modes have the same flavor quantum numbers (as do the pair $B^\rightarrow K^+ K^- \pi^\pm$ and $B^\rightarrow \pi^+ \pi^- \pi^0$), $CP$ violation induced by hadron rescattering could play an important role in these charmless three-body $B$ decays. In order to quantify a possible compound $CP$ asymmetry, the introduction of new amplitude analysis techniques, which would take into account the presence of hadron rescattering in three-body $B$ decays, is necessary.

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