RENT-SEEKING, LEARNING AND THE DYNAMICS OF REPUTATION IN THE INTERNATIONAL CREDIT MARKET

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CONTENTS

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Whilst many have helped with this thesis, responsibility for any remaining errors is mine alone.

DECLARATION

The work in chapter six, sections 6.3 to 6.8, is based on a paper jointly written with Jonathan Thomas. The title of this paper is "An Efficiency Result in a Lobbying Model", mimeo, University of Warwick, August 1992. The idea for this paper, the basic mechanism driving the results and the form of the illustrative example used come from some of my own research, presented here as chapter seven, sections 7.1 and 7.2. This idea was formalized into a general model by Jonathan Thomas although Proposition 6.2 and its proof were a joint effort.

SUMMARY

Most reputation-based models of sovereign debt assume that a default on a loan obligation leads to the imposition of an immediate and permanent credit embargo. The first part of this thesis examines the case in which the length of exclusion is endogenously determined and may consequently be finite or infinite. In this way, lulls in activity followed by enthusiastic lending in the international credit market can be modelled. Additionally, examining the optimal exclusion strategy of the creditor allows investigation of the consequences of 'excusing' default. By not punishing a defaulter immediately, it is more likely that a complete loan embargo will be imposed in the future. The effect of excusing default on the expected value of the credit relationship to the country is also examined.

A negative externality can arise in the relationship between a sovereign borrower and a creditor due to the existence of countries which repeatedly default on their debt: a default by one country may make the creditor more cautious in lending to others. The effects of this externality are examined in a dynamic model in which the bank does not know the type of customer it faces, but can learn its identity over time. The equilibrium actions of the players then depend crucially on the borrower's reputation for creditworthiness. Even a country which is not an inherent defaulter may be tempted to repudiate its debt obligations with this type of incomplete information structure. Each successive default causes reputation to fall until a critical level is surpassed, at which point a permanent lending embargo is imposed. In this dynamic model of debtor reputation, borrowers face an additional problem as they do not always possess the funds needed to make a repayment and thus reveal their type.

In recognizing that borrowing countries can be different by nature, the final part of this thesis examines an economy which is driven to borrow externally as an endogenous outcome of a political system in which interest groups lobby political parties. The amount of borrowing is shown to depend upon the number of redistributive policies the parties can use and the attitude of the voters to external borrowing. A proposal is put forward for linking debt forgiveness in this type of lobbying economy to the level of rent-seeking carried out by the interest groups. It is demonstrated that this proposal is capable of improving the well being of the ordinary citizens of the economy who share the repayment cost but may not enjoy the benefits of external borrowing.

CHAPTER 1. INTRODUCTION

Precommitment by a sovereign borrower to repay its loan obligations is time inconsistent in most instances, so lending policies need to be designed which give these countries an incentive to repay. Chapter two examines some of the previous work in this area and suggests lines of research which have not been fully explored. It is indicated that most of these analyses deal with the incentives of a borrower which contracts sovereign loans for investment purposes. Chapters three, four and five model this type of debtor whilst the work in chapters six and seven leads to a completely different characterization in which debt is contracted as the endogenous result of competition for influence in the political system.

With the theoretically more 'traditional' debtor in mind, the following areas are highlighted in chapter two as requiring more research; these become the aims of this thesis:

i) To investigate models in which breakdowns (of variable durations) and resumptions of voluntary lending can occur as part of an equilibrium;

ii) To evaluate the extent to which default can be thought of as excusable in the sense of not precluding access to future loans and to examine the effect of excusing default on the borrower's expected value of the credit relationship;

iii) To present a model of the dynamic evolution of a debtor's reputation for creditworthiness over time;

iv) To incorporate the idea that borrowers are not a homogeneous group and that countries which often default may impose an externality upon countries which have a high propensity to repay;[1]

v) In the environment depicted in iv), assuming that country type cannot be readily observed, to allow the bank to learn the type of country to which it is lending and characterize the equilibrium lending and repayment strategies under this learning.

An additional aim, already highlighted is:

vi) To provide an alternative characterization to the traditional economic theory model of a borrowing country.

Whilst aims i) to v) are grounded in chapter two, some brief remarks are in order here. Aim i) reflects the fact that most models of reputation-based sovereign debt contracts assume an immediate and permanent credit embargo following default. This does not allow depiction of the observed phenomenon whereby lending breaks down (often for years or even decades) and can resume again voluntarily. The frequency with which these breakdowns occur and the length of time until lending

[1] Default by one borrower may make a creditor more cautious in lending to others.

resumes has obvious implications for the suitability of foreign commercial bank lending for development purposes.

The idea behind aim ii) comes from Grossman and van Huyck (1988) who present a model in which the creditor will not actually punish a default by a debtor as the borrower has an incentive to repay whatever it can, depending upon the verifiable state of nature. If the amount repaid is not the contractually specified obligation, the bank knows that this is due to the realization of a poor state of nature so that default is 'excusable'. The problem with this idea is that when breakdowns in lending occur, then a punishment appears to have taken place. It is still possible for default to be excusable in the sense of not precluding the possibility of access to future loans (weaker than Grossman and van Huyck). Lending equilibria when this can occur need to be examined. Whether excusing default is in the borrower's interest is also a question to be addressed.

Following from aims i) and ii) is the possibility that a debtor may lose its reputation for being a creditworthy borrower for some time but may regain it, so that the dynamics of reputation are important (aim iii)) as this will critically be expected to determine the path of a lending equilibrium. This question of the evolution of reputation is even more interesting in a situation where the bank knows that borrowers are of different types but cannot observe which type it has as a customer (aim iv)). Assuming that there are some types to which the bank would not lend under complete information, there is the possibility of the creditor learning its customer's type (aim v)), whilst the country can optimally reveal its private information through its debt

repayment strategy (if it has funds available to make optimal repayments). The existence of 'good' and 'bad' borrowers introduces the possibility that even good borrowers can be denied credit, having been "tarred with the same brush" as the bad creditors. In order to avoid this externality, one might reasonably expect a good borrower to signal its type through some debt repayment. We investigate a more interesting scenario in which the debtor may sometimes be prevented from making this signal due to a lack of liquidity.

Having entertained the possibility that borrowers are not alike, aim vi) considers a different model of a borrowing economy. Instead of looking at a country which invests its borrowed funds (being different can then be interpreted as having dissimilar investment technologies), aim vi) is fulfilled by picturing an economy which contracts sovereign debt as an endogenous outcome of the functioning of the political system.

After briefly surveying some of the existing literature on sovereign borrowing and indicating some areas of weakness in chapter two, aims i) to v) are fulfilled in the models of chapters three, four and five, whilst chapters six and seven work towards aim vi). The remainder of the present chapter offers a brief overview of what is to come.

Chapter three examines both why some countries are excluded from receiving external bank loans and why those which receive funds cannot expect continuous access to the international credit market. The focus here is on the type of borrower

which is usually analyzed in the literature where funds are borrowed for investment purposes. In the model presented in this chapter, it is assumed that the bank is unable to observe the investment return in the borrowing country so that there is a natural role for the debtor's willingness to repay. Whilst it is the concern about its reputation for creditworthiness which induces the borrower to make some repayments, we do not assume that default leads to an immediate and irretrievable loss of this reputation. Rather, the length of the punishment interval and the extent to which losses of creditworthiness are permanent are determined endogenously, parting company with most of the sovereign debt literature. To what extent default does not preclude access to future loans is also investigated.

To briefly pre-empt some of the results of chapter three, the international credit market depicted there can exhibit two types of equilibrium: a no lending equilibrium and multiple lending equilibria. The existence of the lending equilibria is shown to depend upon conditions relating to the degree of patience of the players, the technological capabilities of the borrower and its willingness to repay the contracted debt. Importantly, breakdowns can occur in the relationship with default leading to some (variable and not necessarily permanent) period of exclusion from international borrowing. Furthermore, the fact that a creditor does not know whether repudiation is due to insouciance or simply a bad investment return, means that the borrower can choose to default in equilibrium with a positive probability and still the bank can achieve zero profits over the longer term.

Whilst in chapter three the country is constrained to repay all of its obligation

or nothing at all, chapter four allows the borrower to decide how much of its obligation to repay when it has the funds available. In order to simplify the model, the bank is assumed to require to expect to break even each period. Due to this, the conditions for the existence of equilibrium are stricter than those of chapter three. Additionally, the country is found to repay the optimal amount when funds are available so that it must be able to achieve a sufficient surplus after repayment to keep it honest and make repayment optimal.

The lending equilibrium in chapter four indicates that the country can guarantee itself a loan by making an optimal repayment, but if a non-optimal repayment is made then a punishment interval of exclusion begins; lending breakdowns and resumptions thus characterize this lending equilibrium. The most efficient equilibrium is the one in which the country has the greatest degree of access to the international credit market. The probability of retaining this access is shown to depend on how great the chance is of the borrower receiving a loan immediately after default. The bank has two options: it can either punish the defaulter immediately and allow readmittance after a period of time, or it can moderate the immediate punishment by granting a loan following default with a relatively high probability. This latter course of action is shown to lead to an eventual credit embargo: 'excusing' default initially can actually reduce the overall expected payoff of the debtor.

Chapter five seeks to introduce the fact that borrowers are not a homogeneous group. In particular, two types of debtor country are assumed to exist: type I which invests its loans and makes optimal repayment decisions (as in chapters three and

four) and type P, a profligate country which contracts loans and never repays. A commercial bank would, under certain conditions, wish to lend to type I but never to type P. The problem is that the bank is initially unaware of the true identity of its customer and must learn its type. The actions of the 'honest' country and the bank are to be characterized here under the externality imposed by the existence of a type P debtor. Indeed, these actions are dependent upon the degree of certainty with which the bank believes it is transacting with a type I country, so that the reputation for creditworthiness of a borrowing country will condition its access to loans. As this belief changes over time, so a dynamic model of debtor reputation develops. The bank wishes to establish its customer type whilst the type I country attempts to reveal its private information optimally.

The results of chapters three and four are tempered in chapter five as lending breakdowns and resumptions can only occur up to a point. There is a critical level of reputation which imposes an endogenous limit on the tolerance of the creditor to default. Once this critical level of belief is surpassed, even the type I country is permanently excluded from receiving foreign bank loans. In optimally revealing its private information, the type I borrower is tempted to masquerade as type P and not repay through choice. As the critical level of belief is neared, this temptation falls. The fact that investment has to succeed in order for repayment to be possible lessens this temptation still further; the type I borrower sets a high repayment probability each time its investment succeeds in case funds are not available in the future to repay the loan.

Whilst most of the sovereign debt literature seeks to characterize borrowers as type I (borrowing for investment), chapters six and seven are aimed towards an alternative characterization of a sovereign borrower. Elements of the rent-seeking and political competition literature are combined in depicting external borrowing as the subject of a struggle among interest groups in an economy. In particular, lobbies spend resources trying to get their favoured political party elected and, once in office, the government borrows externally to reward the loyalty of its support lobby. The basic model is adapted from the literature on endogenous policy theory, but contains a flaw which is outlined and corrected in a general model in chapter six for the case in which voters are not directly hostile to the parties announcing redistributive policies. Chapter seven then demonstrates, in an example, how this flaw affects the outcomes when voters are hostile.

The model is then extended to two periods with borrowing in the first and repayment from taxation revenue in the second. The amount of external borrowing in this lobbying economy is shown to depend on the hostility of the voters and the number of redistributive policy instruments under the control of the parties. We therefore have a model in which rent-seeking economies contract sovereign debt as an outcome of the political process, with the differences in level of borrowing being linked to some of the characteristics underlying the political system. In such economies, the lobbies are portrayed as receiving all of the benefits from external borrowing whilst the ordinary citizens are burdened with repayment. A debt forgiveness scheme which is linked to the amount of lobbying activity is proposed in order to reduce the tax rate paid by the ordinary agent. It is shown that unconditional

forgiveness will not reduce the tax rate but by linking the amount of forgiveness (inversely) to the amount of lobbying it is possible, in principle at least, to relieve the ordinary citizens of a part of the debt repayment burden. This scheme also reduces the amount of resources spent on lobbying; if these can be channelled into productive activity, the economy can grow faster.

CHAPTER 2. THE INTERNATIONAL CREDIT MARKET

2.1 Introduction

The recent general history of lending to sovereign nations through the international credit market is well documented (see, *inter alia*, Lindert and Morten (1989) and O'Brien (1986)). Historically, there have been several episodes where initial lending surges have culminated in eventual default.[1] The latest of these began in the early 1970's and has evolved into the debt crisis in which the developing nations and international banks are still embroiled. Whereas the 1950's and 1960's witnessed most lending to Less Developed Countries (LDCs) coming from official sources, the recycling of OPEC oil surpluses by the international banks in the early 1970's led to the eruption of private, voluntary lending. Market forces coped well at first in channelling funds to parts of the world in which opportunities were numerous and returns high. The bubble finally burst in 1982 when Mexico and then Brazil suspended payments on foreign debt.

Explanations for the ensuing crisis have been many. Some attribute the blame largely to external shocks to the borrowing economies (see Allsopp and Joshi (1986)), whilst others point out the irresponsibility of the borrowing and lending policies of those involved. Massive borrowing by the LDCs can be rationalized by examining

[1] Complete accuracy would define default to be a declaration on a borrower made by the creditor in the event of repudiation. By this definition, default is a rare occurrence. In common with much of the other literature on sovereign debt, repudiation and default shall be used interchangeably.

some of the uses, both legitimate and illegitimate, for borrowed funds. These could be used, for example, for investment and development, to shelter an economy from temporary income and trade fluctuations or to reduce the costs of adjusting to a permanently lower income stream. Among the less legitimate grounds for contracting debt are unsustainable government policy support, greed on the part of the rulers (e.g. The Philippines in the Marcos era), fuel for an import or consumption boom or to increase military strength (e.g. Argentina - see Williamson (1986)).

The banks are often regarded as having loaned recklessly due to the lure of generous returns, failing to accurately assess the risk involved in lending to sovereign states.[2] Hellwig (1977) partially rationalizes continued lending in presenting a theoretical model of the (domestic) credit market which suggests that the contracting of an initial loan will necessarily entail the lender 'throwing good money after bad'. In this model, the borrower initially has zero income which is expected to jump to a permanently higher level at some unknown future date. Consumption is financed through loans which, if by the time income rises the debtor has not defaulted, are repaid up to some maximum. After a credit line has been exhausted prior to the jump in income, the debt goes into default unless more credit is forthcoming to prevent the borrower becoming bankrupt. The lender therefore has an incentive to make loans which may yield a loss themselves but retain the solvency of the borrower and hence the possibility of repayment. Realizing this, the borrower contracts debt at a rapid rate thereby raising the probability of reaching the maximum feasible repayment, at which

[2] Guttentag and Herring (1985) suggest reasons for what they term 'disaster myopia' on the part of the banks.

point the bank must cease lending. By failing to precommit to a credit ceiling, the creditor places itself in a situation in which an initial loan will necessarily lead to further loans. This situation can only be avoided by not lending in the first instance.

2.2 *The enforcement of international loan contracts*

Lending in credit markets, both domestic and international, is subject to the complications of moral hazard and adverse selection. Moral hazard can arise as the creditor often does not know the exact utilization of any extended credit. The employment of borrowed resources naturally has consequences for the prospects of servicing the debt, although, in the international context, funds are usually available to meet debt servicing requirements. This is not to suggest that moral hazard is not a potentially great problem in international lending. Eaton, Gersovitz and Stiglitz (1986) indicate three situations in which moral hazard can arise in the international arena: (i) when borrowers can affect their susceptibility to default penalties; (ii) when they can affect the likelihood of the penalty; (iii) when the total amount borrowed cannot be observed by individual lenders. Adverse selection relates to the difficulty of identifying the characteristics of the borrower which are relevant for designing an incentive compatible repayment schedule. In this case, it is possible that borrowers are attracted who know that their prospects for repayment are poor (see section 2.4 for more on information related problems).

Undoubtedly, the feature which most distinguishes the international credit market from its domestic counterpart relates to the enforceability of cross-border loan

contracts. The non-existence of a supranational legal authority to enforce such contracts apparently grants sovereign debtors the option of repudiating their loan obligations if such action is to their advantage. On this subject Keynes (1924)[3] states, "Indeed, it is probable that loans to foreign governments have turned out badly on balance... The investor has no remedy - none whatever - against default. There is, on the part of most foreign countries, a strong tendency to default on the occasion of wars and revolutions and whenever the expectation of further loans no longer exceeds in amount the interest payable on old ones." Worrall (1990) points out that, whilst it is possible in principle to obtain a default judgement through the courts[4] against a foreign government, enforcing such a judgement is more difficult. Depicting the relationship between an international bank and a sovereign debtor therefore involves designing a contract which will induce both the bank to lend and the country to repay.

At a practical level, there are three constraints on the repayment of sovereign debt - solvency, liquidity and the willingness to repay. A debtor is solvent if it has the resources to meet its agreed stream of debt servicing repayments without the need to borrow forever in order to make the interest payments. As the outstanding debt of the LDCs represents a small proportion of their national income, insolvency does not appear to be an issue of great importance in lending to foreign governments. When capital markets are imperfect, a solvent borrower may find that it is unable to raise enough reserves to meet a due repayment. Thus the liquidity situation of a country

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[3] This reference is taken from Eaton (1990).

[4] In the USA under the Foreign Sovereigns Immunities Act 1976, and in the United Kingdom under the State Immunity Act 1978.

can constrain its debt repayments. Finally, and most reflective of the unenforceable nature of sovereign debt contracts, a debtor can be thought of as performing a cost-benefit calculation when deciding on debt repayment so that its willingness to repay is an obstacle to debt servicing. This evaluation of the costs and benefits of meeting obligations need not be carried out purely in economic terms, as political considerations may also be a factor in determining the amount and timing of any debt payments.

In most of the theoretical models of the credit market, lenders are taken to operate in a competitive environment and be risk neutral so that they maximize their expected profit. If a creditor has complete information in a domestic setting without uncertainty, then collateral requirements can be used to eliminate default risk.[5] In this case, an amount will be lent at the market rate of interest up to a maximum determined by the level of collateral. The supply of credit is therefore perfectly elastic up to a point, after which it becomes perfectly inelastic. Stiglitz and Weiss (1981) demonstrate the possibility of credit rationing in equilibrium in the domestic market due to uncertainty arising from informational imperfections. In this model lenders cannot observe the risk associated with the investment project undertaken by the borrower. In seeking to charge a profit maximizing interest rate on its loans, the bank affects the riskiness of the loans through adverse selection and incentive effects. These authors prove that, for a given interest rate, a level of risk above some critical level is necessary for the borrower to wish to borrow (adverse selection).

[5] Complete information in this case means all of the characteristics of the borrower relevant to the loan contract. With this information, the creditor can accurately assess to incentives of the borrower in all situations.

Additionally, the higher the interest rate, the more attractive the borrower finds riskier projects (incentive effect). As increasing the interest rate can increase the riskiness of the loan portfolio, it cannot necessarily be used to equate the demand and supply of funds. Credit rationing in this case manifests itself in the form of restrictions on the number of loans granted.

Eaton and Gersovitz (1981) is the seminal theoretical work on the supply of loanable funds to a sovereign nation in the presence of repudiation risk. Lenders in this model are assumed to be risk-neutral, competitive and earning zero profits from one period debt contracts. Whilst retaining the option of defaulting on a debt if in its interest, a recalcitrant borrower suffers a loss of income and a permanent embargo on future loans (for more on default penalties see section 2.3). Increasing the debt servicing burden of a borrower is shown to raise the probability that the debtor reneges on its contractually specified obligations. This gives rise to a maximum credit level beyond which a debt will be repudiated with certainty. The credit ceiling in the Eaton and Gersovitz model is endogenously determined as a function of the probability of default, the obligation imposed by the loan and the banks' opportunity cost of funds. To achieve a zero profit level when default is possible, the banks charge an interest rate which rises with the size of the loan, so that the supply of funds to the sovereign borrower is upward sloping until the credit ceiling is reached, at which point it becomes infinitely inelastic.

If a sovereign borrower could make a binding pre-commitment not to default on any loan, then the functioning of the international credit market would mirror its

domestic counterpart. Such a precommitment is usually time inconsistent, however, so the bank must take default risk into account when designing an international loan contract. A broad classification of the models of sovereign debt is possible according to the incentives used to ensure cooperative behaviour and the punishments to recalcitrance, although models also exist which combine the incentives/punishments given below.

2.3 Modelling sovereign debt - carrots versus sticks

2.3.1 Direct penalties

The assumption of a direct penalty following default represents the 'stick' approach. Penalties can take several forms, for example asset sequestration, loss of production (Cooper and Sachs (1985) and Cohen and Sachs (1986)) or trade sanctions (Krugman (1985)). To demonstrate the general structure of purely penalty-based models under certainty, assume a single good economy which lasts for two periods. In period one a loan of L units of the good is granted which must be repaid in amount R(L) in the second period. A credible penalty of P units is the punishment to repudiation. The borrowing economy has a utility function which is increasing in the amount of the loan and decreasing in the amount of the repayment so that the utility from fulfilling the terms of the contract is $U_{rep} = U(L, R(L))$, whilst the default utility is $U_{def} = U(L, P)$. The debt will therefore be repaid if $U_{rep} \geq U_{def}$. Assuming that lenders are competitive and risk-neutral with i representing the opportunity cost of funds, the required repayment will be set at R(L)=(1+i)L. Substitution gives the result

that the loan will be repaid if $P \geq (1+i)L$.

Whilst undeniably simple, this model illuminates several important features of the stick approach. The borrower is credit constrained if it wishes to borrow more than $P/(1+i)$, with the size of the penalty crucially determining the credit ceiling. In this framework, the constrained borrower benefits from an increase in the penalty as this raises the ceiling, whilst the existence of the ceiling prevents default and hence the penalty is never imposed. This mirrors the working of the domestic credit market under certainty and collateral requirements described earlier.

Eaton, Gersovitz and Stiglitz (1986) introduce uncertainty into this simple model by means of a state-dependent penalty $P = P(s)$, where s is the state of nature. Utility may depend on s in other ways than through the penalty so the utilities of repayment and default become $U_{rep} = U(L, R(L), s)$ and $U_{def} = U(L, P, s)$ with $U_{rep} \geq U_{def}$ again defining the repayment criterion. Repayment states are $s \in S$ and non-repayment states are $s \in S'$. Letting f(s) denote the probability of state s occurring, the probability of repayment is $\pi = \int_S f(s)ds \leq 1$ and $\pi R(L) = (1+i)L$ is set by the bank. It follows that states may exist in which default will occur and the penalty will be imposed. Whilst increasing the amount of the penalty will generally increase the amount lent, it does not follow that the expected utility[6] of the borrower will increase as this higher punishment must be paid for $s \in S'$.

Most theoretical models which employ penalties are actually hybrids of the

[6] The expected utility of the borrower is $\int_S U_{rep} f(s)ds + \int_{S'} U_{def} f(s)ds$.

carrot and stick approach. Cohen and Sachs (1986) offer an infinite-horizon growth model under certainty of an economy with access to the international credit market. Repudiating a debt constrains the borrower to financial autarky (continued access is thus the 'carrot') and imposes a direct loss of production. Specifically, it is assumed that capital is less productive after a default, which could be thought of as reflecting the increased difficulty a recalcitrant borrower may have in conducting trade. In making optimal decisions, the social planner in the borrowing economy can choose to repudiate its repayment commitments if to its advantage, giving rise to the possibility of repudiation. Assuming sufficient linearities to achieve a closed solution, an endogenous credit ceiling can be derived in the form of a critical debt-to-capital ratio. This critical level depends upon the production technology in the borrowing economy and also the parameters in its intertemporal utility function. The Cohen-Sachs economy proceeds through two stages of growth. In the first the credit constraint is not binding and the economy grows initially at a rapid rate, declining as the debt/GDP and debt/capital ratios rise. A binding credit constraint characterizes the second stage of growth where the debt and economy grow at the same rate which is below the growth rate in stage one. Significantly, the credit ceiling reduces the productive efficiency of the borrower as the growth rate in the second phase is below that which maximizes productive wealth. During the second stage, debt repayment is less than the due amount but outright default is prevented because the lender refinances the interest payments.

The possibility of outright default in the international credit market is considered in the model of Eaton and Gersovitz (1981) mentioned earlier. The crucial

element which distinguishes between threatened and actual default is the presence of uncertainty in the borrowing economy. It was noted earlier that the supply of funds to a sovereign borrower in this model is upward sloping until the (endogenous) credit ceiling is reached. Two characterizations of the borrowing economy are considered. Income can be high or low and in the deterministic version of the model income alternates between being high in one period and then low the next. Borrowing is undertaken in periods when income is low and is repaid in the following period when income is certain to be high. Failure to meet the repayments imposes a direct penalty and financial autarky on the borrower. Default is avoided by setting the credit ceiling low enough that the debtor never finds it profitable to renege on the agreed debt servicing. In the second characterization, stochastic income determination permits an actual default to occur in the Eaton and Gersovitz model. In this case, income can again be high or low but according to an uncertain pattern. Borrowing may only occur in low income states and must be repaid in the next period regardless of income. A realization of two successive low income states may then lead to an actual default.

Instead of assuming that the default penalty is a direct one on income or production, penalties could take the form of trade sanctions. Krugman (1985) introduces the notion of uncertain future trade penalties into a model of rescheduling of debt commitments with trade sanctions. Decisions taken today therefore depend upon the expectation of the penalty in the future. Aizenman (1989) suggests that trade sanctions could be employed to move the creditor-debtor relationship towards a first best no-default precommitment equilibrium. This outcome could be achieved by increasing the openness of an economy so that trade sanctions become a more potent

punishment in the event of default. In his two period model, borrowing countries can invest in different activities with different degrees of exposure to international trade. The creditor should encourage the borrowing government to implement a tax policy which will reduce default risk by directing investment to more open sectors. Borrowing for investment purposes thus has two effects: (i) marginal borrowing raises total indebtedness so raises the probability of default; (ii) investment in exposed sectors increases the openness of an economy thereby lowering the probability of default. The optimal tax would balance these two effects.

Calvo (1989) asks why penalties are not effective enough to achieve moral hazard free equilibria. The simple theoretical analysis presented earlier suggests that a large penalty will enforce repayment whilst not imposing a welfare loss as it never needs to be implemented. He notes that theory and practice appear distanced in this respect as actual penalties are not very large and cases of repudiation do occur. In a two-period model of loan and repayment, Calvo finds two reasons for the existence of small penalties. Firstly, a penalty may impose costs on the lender so that its resolve to punish a deviant borrower is diminished. This will tend to either place an upper bound on the penalty or reduce the credibility of large penalties. Alternatively, the penalty could be related to the degree of monitoring carried out by the creditor in the borrowing country. If loans are closely monitored then a penalty could be automatically incurred on default, whereas less than perfect monitoring, with a possibility that even good loan prospects are refused refinancing of problem debts, could lead to the emergence of an upper bound on the penalty. Creditors would not wish to punish 'good' debtors but may lack the ability to distinguish between a good

and bad debtor. Additionally, Calvo indicates the potential time inconsistency of ex-post optimal penalties as lenders may be tempted to increase them after a loan contract has been agreed. This temptation is especially great if the cost of careful monitoring is high.

2.3.2 Purely reputation-based models

Sustaining sovereign debt contracts solely through penalties in effect causes the international credit market to almost mimic the market for domestic debt with bankruptcy and collateral requirements. In contrast to the hybrid direct penalty/continued access models presented in the previous section, one branch of the sovereign debt literature considers models in which continued access to the international credit market can be a great enough incentive to generate voluntary repayments on sovereign debt. Under this approach, the debtor is concerned with its reputation for creditworthiness. In a reputational equilibrium, the borrower must care enough about its future credit standing in order that the short-run gains from not servicing debt are outweighed by the long run losses associated with financial autarky. Furthermore, the withdrawal of credit must be in the interest of the lender to be a credible punishment. Although the credit embargo need not be permanent, this is often the assumption made in the literature (see, *inter alia*, Eaton and Gersovitz (1981), Kletzer (1984), Worrall (1990) and Atkeson (1991)).[7] A situation of no lending is the harshest equilibrium punishment available to the creditor in a pure

[7] Grossman and van Huyck (1988) present a reputational equilibrium in which the lender can 'forget' previous repudiation by a sovereign. Rather then endogenizing the length of memory, however, it is assumed to be random.

reputation model so provides the greatest incentive for debt repayment. However, models making this assumption miss an important class of equilibria in which lending can break down and resume again as part of the equilibrium. Chapters three and four of this thesis endogenize the length of the punishment interval and investigate the punishment structure which leads to the most efficient equilibrium. Chapter five looks at the evolution of a debtor's reputation over time in the presence of a negative externality due to the existence of inherent defaulters in the international credit market. Complementary to chapters three and four, the model of chapter five also exhibits lending breakdowns of varying durations.

Sceptics of pure reputation as a debt repayment mechanism deny the existence of reputational contracts in the international credit market. Bulow and Rogoff (1989) model a situation in which competitive creditors lend to a small developing country whose production depends upon investment and a shock, which are both observed by the lenders and borrower. There is no private information. Two types of debt contract are considered: (i) an implicit contract based on reputation; (ii) a cash-in-advance contract. This latter contract involves the country becoming a creditor itself and lending to a bank in return for a non-negative repayment in the following period. An arbitrage argument belies the proof of the non-existence of a pure reputational contract. Once the country receives a loan from bank A, it should default and therefore be barred from receiving future reputation-linked funds. This saving on debt service is then used as a deposit with bank B which yields a return next period. The country then continues to reinvest in further cash-in-advance contracts which cost it less than repaying the original loan. Realizing this, bank A will not extend a

reputation-based contract in the first instance. This non-existence result appears powerful in the confines of the model used by Bulow and Rogoff, but relies on the existence of outside investors offering enforceable cash-in-advance contracts. Worrall (1990) notes that if an enforceable contract cannot be made with bank B, then insurance purchased by the country is less than perfect as bank B may renege on the agreement if it finds such action optimal.

Rosenthal (1991) presents a proof of the non-existence of purely reputation-based sovereign debt contracts which is not based on the assumption that the borrower can become a creditor. He uses a one-sector neo-classical growth model under certainty to look at the sovereign's incentives for repaying a debt contract which specifies a time profile for capital flows to the country (loans) and repayment obligations. The result is that if the borrower is sufficiently impatient[8] then a time comes when the debtor would prefer to default on the debt contract and follow its stand alone path afterwards (as default leads to the exclusion of the borrower from the credit market). Knowing the certainty of eventual default would prevent the lender offering the initial debt contract. When the borrower is more patient, then this result need not hold. Additionally, allowing the debtor to be a lender following default produces the Bulow and Rogoff (1989) result which is independent of the discount factor.

It is possible that default may not lead to the imposition of a complete credit

[8] This is the requirement that $\beta \leq 1/(1+r)$ where β is the borrower's discount factor and r is the risk-free rate of interest.

embargo. The notion that default by a sovereign borrower may be excusable, in the sense of not precluding access to future loans, is due to Grossman and van Huyck (1988), who view sovereign debt as a claim contingent on a verifiable state of nature. The lender sets the state contingent repayment schedule in such a manner that the borrower always prefers to pay as much of the debt as it can afford. Due to the problems of writing a state contingent contract which allows for every eventuality, only the maximum repayment in the best state is specified in the contract. As this contractually specified amount can only be repaid in the best state and the sovereign is induced to pay as much as possible in other states, the lender knows that any failure to make the contractual debt repayment is due to a (verifiable) bad state of nature. Therefore when the sovereign makes only a partial repayment on its debt (or no payment at all in the worst state) its access to further loans is not affected. This result does not appear surprising in this model, for information is complete and perfect as the lender knows the identity of the borrower and the state of nature is verifiable. The extent to which default is excusable and the effect on the expected payoffs of the borrower in more complex environments are issues which are examined in this thesis. Specifically, chapters three and four investigate these questions when the bank is aware of the identity of the borrower but its investment returns are the private knowledge of the debtor and chapter five analyzes the case in which the bank is informed about neither the identity of the borrower nor its investment return.

2.4 Information and learning

In the international credit market, the borrower is likely to have some private information as it seems reasonable to assume that the debtor will know more about its own situation than will the creditor. This can lead to moral hazard and adverse selection effects which have been discussed earlier with reference to the market for domestic credit. In the international arena these problems serve to compound the non-enforceability of sovereign debt contracts.

The stochastic version of the model in Eaton and Gersovitz (1981) is extended by Kletzer (1984) to highlight the role played by the informational structure. Debtors make optimal repudiation decisions but face financial autarky forever after non-repayment. Prior to default, lenders offer a loan contract (principal and interest rate) every period. Loans may be contracted from many banks and two cases are examined according to the information an individual bank has on the total indebtedness of the borrowing economy. If lenders can observe this total amount of indebtedness, then the loan contract specifies the rate of interest to be repaid as a function of total borrowing. An equilibrium always exists at the tangency of the set of loans giving a non-negative expected profit and a country indifference curve. Typically, the equilibrium will entail credit rationing. Assuming an informational asymmetry, so that each individual bank does not know the total borrowing undertaken in the country, means that contracts conditioned on this magnitude are no longer possible. Indeed, as the probability of repayment depends upon the total amount of indebtedness, it can only be observed when the debtor refuses loans at the market rate of interest. An

equilibrium in this case will occur along the demand curve for loans. Non-existence of a lending equilibrium is possible if, along the demand curve for loanable funds, only loans are possible which yield negative returns to the bank. Whilst the case with observability of total indebtedness gives rise to a constrained equilibrium, unobservability leads to more debt being contracted at a higher rate of interest.

Atkeson (1991) uses a dynamic game of international borrowing in order to investigate the optimal pattern of capital flows between a lender and a borrower when optimal default decisions are made and moral hazard exists. In this model, moral hazard arises due to the lenders' inability to observe and monitor the proportion of a loan which is used for investment and that which is consumed. With the assumed punishment to default being financial autarky, the optimal contract overcomes the problems of repudiation and moral hazard. A noticeable feature of the optimal contract is that it prescribes a capital outflow and fall in consumption and investment after the lowest realizations of output if this low output is a strong enough indicator of low past investment.

When a model includes some incomplete or imperfect information as part of its structure, learning of the informational asymmetry becomes a possibility. Lang and Nakamura (1989) develop a model of the credit market with two types of loans - riskless and risky in the sense that default is a possibility. Supply and demand for the two kinds of loans depends upon the expected returns to the borrowers and lenders, which in turn depend upon the level of information possessed by these agents. Returns on risky loans are comprised of a permanent and transitory component and

lenders do not initially know the expected returns of the risky loans but use a Kalman filter to learn the permanent element over time. The larger the number of loans at any time, the greater the amount of information there is available to aid inference of future returns. Increasing the amount of risk/uncertainty associated with a risky loan is shown to lead to a lower number of loans so that less information is released which leads to a further increase in risk/uncertainty and so on in a vicious circle. Similarly, the effects of an adverse shock to the credit market can be magnified by the learning process due to lower information levels. Multiple long-run equilibria are a feature of the Lang and Nakamura learning model so that an adverse shock can move the economy away from one equilibrium and towards another. Consequently there are two channels through which a shock can affect the credit market.

In a credit market, borrowers may not be a homogeneous group. Cole, Dow and English (1989) consider a sovereign borrower with two different types of government - stable and unstable - neither of which can be observed by the lenders. An unstable government is more myopic than the stable one and is consequently more prone to default on a debt obligation. Furthermore, the transition of these governments evolves over time according to a first-order Markov process. A large number of risk-neutral lenders make loans according to their belief about the type of government currently in power. Two classes of Bayesian Perfect Equilibria are investigated: (i) those enforced by a <u>fixed</u> punishment interval; (ii) a signalling equilibrium.

When default is followed by a fixed period of exclusion from the credit

markets, both separating and pooling equilibria are found to exist. In a separating equilibrium, the stable government is induced by the punishment to repay its loan whereas the unstable is not. Therefore, default is assumed by the bank to indicate an unstable type and the punishment interval begins. During the punishment interval, the type of government is changing so that the lenders belief gradually reverts to the population mean. When the belief in a stable government is sufficiently strong, loans will again be forthcoming. In the model, the default decision is such that if a stable type defaults, then so does an unstable type. As a result, two types of pooling equilibria can be supported by a fixed punishment interval: a no lending equilibrium and a no default equilibrium where the punishment induces the unstable government (and hence the stable one) to repay.

According to the fixed punishment interval equilibria, a country cannot receive a loan until the end of the punishment even if a stable government has gained power. A signalling equilibrium allows the stable government to indicate that this change has occurred and regain access to the credit market before the exclusion period ends. The signal used is a partial repayment of a loan in default. Furthermore, the equilibrium signal size is just greater than a signal which the unstable type would be willing to make. Therefore an unstable government will never signal so that partial repayment indicates a stable government. In the signalling equilibrium, partial repayment is the only way that readmittance to the credit market can be gained, for reputation does not evolve during the exclusion period in this type of equilibrium. Using the Cho-Kreps (1987) intuitive criterion, the existence of a signalling equilibrium allows the fixed punishment equilibria to be ruled out. The stable government will always prefer to

signal its type and end the period of punishment, whereas the unstable type will never make a repayment. Therefore an observed signal indicates to the lenders that the stable government is in power and the punishment interval should cease. Equilibria supported by a fixed punishment interval can therefore be ruled out as the punishment will end as soon as a stable government evolves. This is used to explain why defaulters can borrow again after a (variable) period of exclusion.

In the model of Cole, Dow and English (1989), a problem arises with the suggested signalling equilibrium if the stable government does not have sufficient funds to make the signal. The model of chapter five of this thesis presents a model of the international credit market in which a bank attempts to distinguish over time between two borrowing countries who have different inherent attitudes to debt repayment. In particular, one country always defaults (unstable) and the other makes optimal default decisions but may be constrained in its attempts to reveal its type through illiquidity (stable). This model analyzes the manner in which the presence of the unstable inherent defaulter and this liquidity constraint affect the dynamics of the reputation of the stable debtor. As in Cole, Dow and English (1989) lending resumptions are found to follow default but, in contrast to their paper, the model here suggests that this can only happen up until a critical level of reputation below which no loans are forthcoming. Illiquidity may cause the stable borrower to fail to make the signal in time and so fall below the reputation threshold even though this may not have been its intention. Additionally, chapters three and four investigate lending breakdowns and resumptions when the type of borrower is known. Here, the punishment to default is found to be a <u>variable</u> length of exclusion from the

international credit market.

2.5 Conclusion

This chapter has looked at some of the literature on sovereign debt and indicated several of the problems which have been encountered and overcome in modelling this phenomenon. Despite the volume of work, there are still areas open for research. Chapters three and four examine the pattern of sovereign loans in a reputation-based model when the type of borrower is known whilst chapter five explores a reputational model in which borrowers can be heterogeneous and the bank does not know the type of borrower it faces. Customer type can be learned over time in this framework. A common feature of these three models is that a default is not assumed to lead to an immediate and permanent credit embargo. Rather, the length of the punishment interval following default is determined endogenously in equilibrium. Following from this, the type of punishment which maximizes the expected payoffs of the players can be characterized and compared to the assumption usually made in the sovereign debt literature. Furthermore, the phenomenon of lending breaking down for varying durations can be depicted.

Whilst most of the literature deals with a country which borrows for investment purposes, chapters six and seven work towards a different characterization where sovereign borrowing is seen as an endogenous outcome of the interaction of the economic and political systems where interest groups lobby the political parties to borrow in order to fund government transfers. The amount of borrowing undertaken

in the equilibrium of a two-period model is determined by both of these systems. Debt forgiveness linked to the level of lobbying is considered as a means of improving the welfare of the ordinary citizens and increasing growth in this rent-seeking economy.

CHAPTER 3. LENDING TO A KNOWN SOVEREIGN BORROWER

Chapter two suggests the lack of a theoretical model depicting the discontinuous nature of loans to a sovereign borrower through time. Phases of enthusiastic lending are observed, followed by eventual default and the drying up of loans for a (variable) period of time, after which credit is again extended. Eichengreen (1989) states: " After 1970 a period of inactivity first gave way to a surge of bank lending, followed by the development of debt servicing difficulties and finally the curtailment of foreign lending. To a surprising extent, the recent rise and retreat of foreign lending resembles previous historical episodes in which surges of foreign lending were abruptly terminated by waves of default, only to start up again after a lull of several decades". Much of the existing theoretical work has neglected to model this series of events. This chapter presents a reputational model of sovereign borrowing in an attempt to fill this apparent gap in the literature.

Presented here is a model involving the interaction of one competitive bank and one sovereign borrower through time. The bank must decide whether to lend to the borrower, given the inherently unenforceable nature of sovereign debt contracts. If the country receives a loan, then it invests in its investment technology which produces uncertain returns. Should a high enough return be generated, then the country has the option of repudiating or repaying the loan, whichever is optimal. A failed investment leads to non-fulfillment of the terms of the debt contract. Both players have the same information, except that the bank cannot observe the realized investment return. This is an important feature of this model of the international

credit market, for it allows a role for the country's *willingness* to repay a debt as the bank cannot discern whether default is due to country insouciance or, more legitimately, failed investment. In the presence of this informational imperfection, the bank cannot control the repayment action of the borrower. Therefore it must lend in such a manner that the borrower is given the correct incentives to repay when able.

Cole, Dow and English (1989) state that, empirically, the most recent default will have the greatest effect on the bank's lending policy. This observation is used to justify concentration on Markov strategies and Markov Perfect Equilibrium (MPE). With Markov strategies, the past has an effect on current actions only in as much as it affects the current state variable which summarizes the past history of the economic system. Given this definition of the state variable, if one player uses a Markov strategy, then the opponent can do no better than to use a Markov strategy also (Maskin and Tirole (1988)). Essentially, the model presented below is a stochastic game in which the state of the system follows a first-order Markov process. In this case, the state of the system in the next period depends solely upon the current state and current actions.

As the bank is competitive, solutions are examined in which it achieves an expected return of zero over the long run. Two types of MPE can be distinguished - those in which there is lending and those in which there is not. Naturally, the former is the more interesting and necessary conditions are given for this type of equilibrium to occur. In order for a country to get a loan, it must pass some very strong criteria. Specifically, the investment technology in the borrowing nation must be capable of

producing high enough returns to facilitate repayment. Additionally, given that the bank cannot observe realized returns, the country must be able to avoid being tempted to fool the bank by claiming that investment has failed when, in reality, it did not. In short, the bank will only loan to a country which it considers will be able and willing to repay the contracted debt with a high probability. Even then, a lending equilibrium requires the bank to be sufficiently far sighted not to be panicked into closing credit lines completely after an isolated case of default.[1]

There are several interesting implications of the lending equilibrium if it exists. For any set of permissible parameter values, there are multiple equilibria. Whilst equilibria do exist in which default is punished by a permanent credit embargo, this is not a feature imposed on the solution in contrast to much of the reputational sovereign debt literature examined in chapter two. Furthermore, equilibria are found to exist in which a repudiation of an obligation may not lead to an irreversible loss of the borrower's reputation for creditworthiness. Specifically, it is possible in equilibrium that default could be followed by a loan in the very next period or, failing this, after a finite period of exclusion.[2] This finding is in a similar vein to the 'excusable default' result of Grossman and van Huyck (1988) in that default does not necessarily preclude access to loans. Excusable default in the strict sense of no punishment to non-repayment is not a feature of the equilibria in this model, however.

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[1] A weaker form of 'excusable default' than Grossman and van Huyck (1988).

[2] The expected length of the embargo is determined endogenously as a function of the model parameters.

Some numerical analysis suggests that non-repayment of debt is punished very strongly with a long lasting, but possibly not infinite, credit embargo likely to follow default. In some periods therefore, the sovereign is credit constrained whilst in others it is not. This idea of credit rationing contrasts with most existing models in which a sovereign faces a credit ceiling in each period. Allsopp and Joshi (1986) ask whether private lending to LDCs is suitable for development purposes. The volatile time path of credit rationing found in this model would tend to suggest that it is not.

We now turn to details of the model.

3.1 The environment

The credit relationship to be modelled here is between a competitive, risk-neutral bank and a risk-neutral sovereign borrower who both operate in a stationary economic environment. We shall restrict attention to situations in which the bank has a zero expected level of profit over the infinite time horizon. The borrower is taken to be risk-neutral as we are concerned with incentive rather than risk-sharing issues. There is assumed to be one (perishable) good in existence which is best thought of as the output of the borrowing country. For simplicity, all loans are normalized to be one unit of the good for the duration of one period at an exogenously fixed rate of interest r. Most of the existing work in this area focusses on one period debt contracts (see, *inter alia*, Eaton and Gersovitz (1981), Kletzer (1984), Cole, Dow and English (1989)). O'Brien (1986) notes that short-run maturities are often perceived as carrying lower risk than longer term ones, for it gives

the creditor an earlier option of non-renewal than do long-term contracts. This option is found to be of particular significance for the incentive structure designed by the bank in the present model (see section 3.5 for details).

In order to generate the funds necessary to make required repayments on any granted loan, we assume that the borrowing nation has at its disposal a non-divisible investment technology in which it always invests the whole loan. Stochastic returns are produced by this investment technology in that with fixed probability $0 < \gamma < 1$ the return is 'high' and with probability $1-\gamma$ 'low'. In particular, we assume that in a period of high returns the country regains its invested unit of the good plus some positive constant $x > 0$, but should a low return be realized, this shall be taken to indicate the loss of the borrowed unit of the good.[3] Both the country and the bank are taken to know the value of γ. The repayment options of the borrower are simplified by requiring either all obligations or none at all to be repaid upon realization of the investment return.

At this point it is necessary to introduce an informational asymmetry into the model, for if the bank were able to observe the investment return achieved by the borrowing country, then it could immediately discern whether any non-repayment of debts was due to borrower insouciance or simply bad luck. The former would be punished and the latter possibly not. The task of the bank is made more difficult here as it is assumed that the creditor has imperfect information about the actual realized

[3] The role of γ can be interpreted more widely as anything which affects the value of the output of the borrowing country, e.g. a terms of trade shock.

investment return in as much as the return is not directly observable by the bank. In this way, we build into the model a role for the borrower's willingness to repay.[4] Providing the borrower with the correct incentives to prevent insouciance can only be achieved through the rationing of loans over time.

A final assumption is that the borrowing country cannot become a depositor with any financial institutions. This is necessary due to the point by Bulow and Rogoff (1989) demonstrating that the possibility of making such deposits can lead, in some settings, to the non-existence of reputational sovereign debt contracts.

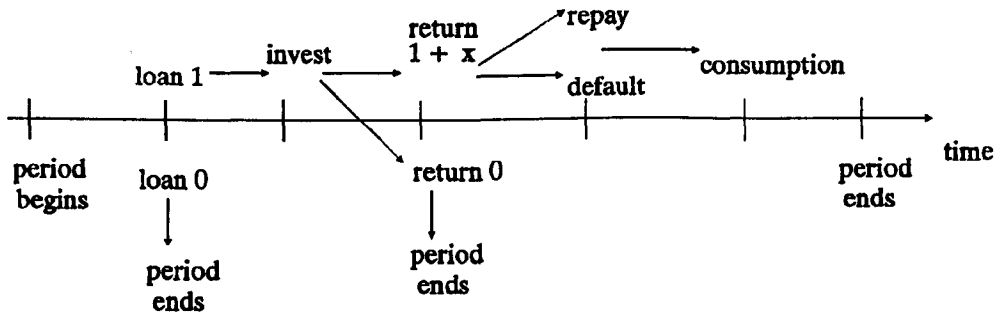
3.2 The play

The play of the game in any one period is described in Figure 3.1. At the beginning of the period, the bank makes a decision on whether to extend a loan of one or zero units of the good. If no loan is granted, the period ends with a current period payoff to both parties of zero. Should the bank decide to grant a loan, it purchases one unit of the good at a price of unity and the country invests this unit in its investment technology. Next the return is realized and revealed to the country only. If the investment has failed, then no repayment can be made so the bank loses its loaned unit and the country achieves a zero payoff in the current period. When the investment succeeds, the debtor chooses whether to repay its obligations to the creditor. Repayment yields a current period payoff of x-r to the country and r to the

[4] The willingness of the borrower to repay is taken as an economic decision in this model. Of course in practice, political considerations may be an important influence, for example the willingness or ability of the government to raise taxation revenue to repay a debt.

bank, whereas default in this case generates 1+x and -1 respectively. Naturally, it is assumed that x > r in order to facilitate any gains from trade.[5] After repayment or repudiation of the obligation, the period ends.[6]

Figure 3.1



The unique solution to the one period game is easily verified by backwards induction to be a situation where no lending and no repayment takes place. Extending the model to any finite number of periods under this setup generates the same result. To avoid such problems of backwards unravelling, we assume that the game is repeated infinitely many times.

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[5] In fact, we would expect an equilibrium with lending to have $\gamma(1+x) > 1+r$.

[6] Notice that all of the action takes place inside one period. There are no explicit connections between periods which makes the repeated game easier to solve.

3.3 Strategies

In the model it is assumed that the Markov property holds so that, given the present state of the system, all information concerning the past of the system is irrelevant for its future behaviour. It is possible to identify four states depending upon actions taken in the previous period:

1) no loan made last period;

2) loan granted which was not repaid through choice;

3) loan granted which was repaid;

4) loan granted which was not repaid due to low investment return.

The strategies which we shall consider here are Markov strategies where the repayment/lending decisions in the current period depend only on the payoff relevant history. In this model, the payoff relevant history is summarized by the state of the system so that if one player uses a Markov strategy then the opponent can do no better than to also employ a strategy of this type. This leads to the following definition:

Definition 3.1 Let p represent the probability that a loan is granted by the bank and s_i (i = 1, 2, 3) denote the possible states of the system.[7] A Markov strategy for the bank is a state dependent lending rule $p(s_i) \in [0,1]$ which selects a loan probability for each of the three states. Similarly, a Markov strategy for the country is a state dependent repayment probability $a(s_i) \in [0,1]$ which is defined when investment succeeds.

Markov strategies can be used in situations where the most recent actions have a strong bearing on current and future payoffs (see Maskin and Tirole (1988)). Cole, Dow and English (1989) argue that this is the case for the international credit market. In using this type of strategy, players' actions are still rational and we may expect the solutions obtained to be simple and thus easier to study and more likely to arise in practice than complex equilibria.

3.4 Expected payoffs

The assumptions about the economic environment and the play of the game lead to the following expression for the present value of the expected payoffs for the bank in each state (denoted V_i).[8] To ease notation, we represent the state dependence of the players' strategies simply by p_i and a_i.

[7] Notice that we do not need the fourth state in what follows. This becomes subsumed into state two as the bank observes repudiation without knowing the cause.

[8] Time subscripts are omitted as the equilibrium path is state and not time dependent.

$$V_i = (1 - p_i)\,\delta V_1 + p_i\,\{(1 - \pi_i)(-1 + \delta V_2) + \pi_i(r + \delta V_3)\} \qquad (3.1)$$

Here, $0 < \delta < 1$ is a discount factor assumed common to both players and $\pi_i = \gamma a_i$ indicates the total probability that a loan will be repaid in state i. Notice that this equation expresses the loaning preferences of the bank, for δV_1 is the expected return from not extending funds whilst the term in braces indicates the expected return from lending. The relative magnitudes of these two terms determines the bank's lending policy.

We can similarly state an expression for the present value of the expected payoffs for the borrower in this stationary environment. These are given as U_i in equation (3.2).

$$U_i = (1 - p_i)\,\delta U_1 + p_i\,\{\pi_i(x - r + \delta U_3) + \gamma(1 - a_i)(1 + x + \delta U_2) + (1 - \gamma)\delta U_2\} \qquad (3.2)$$

The repayment preferences of the country can be seen from (3.2) to depend upon the relative magnitudes of the return from repudiating $(1 + x + \delta U_2)$ and from repaying $(x - r + \delta U_3)$. The term $(1 - \gamma)\delta U_2$ reflects the expected continuation payoff achieved if investment fails.

3.5 Equilibrium

We are now in a position to state the conditions which a Markov Perfect Equilibrium (MPE) must satisfy.

<u>Definition 3.2</u> Assuming that both players are rational, a stationary MPE consists of a pair of strategies p_i^* and a_i^* such that p_i^* maximizes equation (3.1) given a_i^* and the current state and a_i^* maximizes equation (3.2) given p_i^* and the current state. Additionally, we require that $p_3^* > p_2^*$ and $p_3^* > p_1^*$ in a lending equilibrium.

This definition ensures that, for each state, the strategy employed by one player is an optimal response to the strategy of the opponent. The restriction $p_3^* > p_2^*$ prevents the bank from rewarding a country more (in terms of a higher loan probability) for non-repayment than for fulfilling the terms of the debt contract. Additionally, $p_3^* > p_1^*$ is needed in order that the country does not find it optimal to default when receiving a loan in order to miss out on a loan next period and then receive a loan with a high probability in the following period.

The solution of the model is stated in the following proposition:

PROPOSITION 3.1 There are two possible types of MPE in which the bank has a zero expected return:

(I) if i) $\gamma \geq 1/(1+r)$

and ii) $\gamma(1+x) > 1+r$

and iii) $\delta \geq (1+r)/\gamma(1+x)$

then $a_i^* = 1/[\gamma(1+r)]$ $\forall$ i, and the p_i^* satisfy

$$\delta(1 + r)\, p_1^* + \delta[\gamma(1 + x) - (1 + r)]\, p_2^* - \delta\gamma(1 + x)\, p_3^* + (1 + r) = 0 \tag{3.3}$$

such that $p_3^* > p_2^*$ and $p_3^* > p_1^*$.

(II) If any of i), ii), iii) above do not hold then

$p_i^* = a_i^* = 0 \quad \forall$ i

Proof Let the term in braces in equation (3.1) be represented by M. Then it is clearly the case that: (A) $\delta V_1 > M$ implies $p_i = 0$; (B) when $\delta V_1 < M$ then $p_i = 1$; (C) $\delta V_1 = M$ gives bank randomization so that $p_i \in [0,1]$. The repayment preferences of the country are given from (3.2) by:

(X) $x - r + \delta U_3 = 1 + x + \delta U_2$ implies $a_i \in [0,1]$;

(Y) $x - r + \delta U_3 > 1 + x + \delta U_2$ indicates $a_i = 1$;

(Z) $x - r + \delta U_3 < 1 + x + \delta U_2$ yields $a_i = 0$.

I. Assume that conditions (C) and (X) hold so that $p_i \in [0,1]$ and $a_i \in [0,1]$. From condition (C) and equation (3.1), bank randomization indicates that $V_i^* = 0$ $\forall$ i. Substitution of this fact back into equation (3.1) gives a_i^* as in equilibrium I of the proposition. Condition i) is needed to ensure $1 \geq a_i^*$.

Now use condition (X) to eliminate U_3 from the three equations given by (3.2), viz

$$[\delta(1 - p_1) - 1]\, U_1 + [\delta p_1]\, U_2 = - p_1\gamma(1 + x) \tag{3.2a}$$

$$[\delta(1 - p_2)]\, U_1 + [\delta p_2 - 1]\, U_2 = - p_2\gamma(1 + x) \tag{3.2b}$$

$$[\delta(1 - p_3)]\, U_1 + [\delta p_3 - 1]\, U_2 = \{(1 + r)/\delta\} - p_3\gamma(1 + x) \tag{3.2c}$$

The three equations (3.2a), (3.2b) and (3.2c) form a system in two unknowns (U_1 and U_2) so can be solved only if one of the equations is redundant i.e. iff

$$\begin{vmatrix} \delta(1-p_1)-1 & \delta p_1 & -\gamma(1+x)p_1 \\ \delta(1-p_2) & \delta p_2-1 & -\gamma(1+x)p_2 \\ \delta(1-p_3) & \delta p_3-1 & \dfrac{1+r-\delta p_3\gamma(1+x)}{\delta} \end{vmatrix} = 0$$

This yields

$$(1-\delta)\left[p_1(1 + r) - p_2(1 + r - \gamma(1 + x)) + \{1 + r - p_3\gamma\delta(1 + x)\}/\delta \right] = 0$$

which implies (3.3) since $\delta \neq 1$. From (3.3) it is the case that if $1 + r > \gamma(1 + x)$ then $p_3^* > 1$, so condition ii) is needed (although this condition is actually implied by iii)).

Next let $p_1^* = p_3^* - \alpha$ and $p_2^* = p_3^* - \beta$, where $1 \geq \alpha > 0$ and $1 \geq \beta > 0$ by Definition 3.2 and the fact that the p_i^* are probabilities.[9] Substituting this into (3.3) delivers

$$\alpha(1 + r) + \beta[\gamma(1 + x) - (1 + r)] = (1 + r)/\delta \tag{3.4}$$

Rearranging (3.4) gives an expression for β:

$$\beta = \frac{(1 + r)(1 - \delta\alpha)}{\delta[\gamma(1 + x) - (1 + r)]}$$

[9] In fact, it can readily be established from (3.3) that $0 > \beta$ is impossible in a lending equilibrium so that the condition $p_3^* > p_2^*$ is derived endogenously in equilibrium.

In order that $1 \geq \beta$, the following restriction on δ is necessary

$$\delta \geq \frac{1 + r}{\gamma (1 + x) - (1 + r)(1 - \alpha)} \tag{3.5}$$

The minimum δ thus consistent with equilibrium (I) occurs when $\alpha = 1$ in (3.5) as indicated by condition iii).

II. Firstly we assert that $p_i^* = a_i^* = 0$ is an equilibrium and then verify this fact. Notice that this strategy combination implies that conditions (A) and (Z) above must hold. Substituting $p_i^* = a_i^* = 0$ into payoff equations (3.1) and (3.2) yields $V_i^* = U_i^* = 0 \ \forall$ i. Clearly then only conditions (A) and (Z) can obtain so this is indeed an equilibrium.

Finally, we must check that no other combinations of conditions (A), (B), (C) and (X), (Y), (Z) is a candidate for equilibrium. This is easily done by contradiction. Take, for example, the combination of conditions (A) and (Y) where we assume $p_i = 0$ and $a_i = 1$ is an equilibrium. From equation (3.2), the expected payoff for the country is $U_i = 0 \ \forall$ i. Using this, we have $1 + \delta U_3 < 1 + x + \delta U_2$ which gives $a_i = 0$, a contradiction. Contradictions can similarly be derived for all other condition pairs. ■

3.6 Remarks

Equilibrium II is one in which there is no lending and no repayment. If the bank knows that its loans will not be repaid, then clearly it will never loan. Similarly, if the bank will not loan regardless of country actions, then there is no reason for the country to set a positive repayment probability in any state. In this case both the bank and the country have an expected return of zero.

Equilibrium I is more interesting as it depicts some potential activity in the international credit market. The intertemporal properties of this equilibrium indicate that there may be periods of lending and repayment, lending with default or no lending activity. In other words, it is quite possible that breakdowns in the relationship can occur for an endogenously determined length of time. The lending equilibrium only exists, however, if three conditions are met. Condition i) is required in order that $1 \geq a_i^*$. Table 3.1 gives an indication of the interest rate conditions prevailing in the period 1974-89 in the international credit market, indicating the highest and lowest interest rates and the period average.

Table 3.1:

Average annual interest rate terms of new commitments by private creditors 1974-89 (%).

	HIGH r	LOW r	PERIOD AVE. r
ALL DEVELOPING COUNTRIES[10]	11.1	6.4	7.98
BRAZIL	16.2	7.8	10.63

Source: World Bank World Debt Tables, various editions.

Given the values of r in Table 3.1, Table 3.2 then gives the minimum value of γ which satisfies condition i).

Table 3.2: Minimum γ values given r from Table 3.1.

	γ (high r)	γ (low r)	γ (ave. r)
ALL DEVELOPING COUNTRIES	0.9000	0.9398	0.9261
BRAZIL	0.8606	0.9276	0.9309

[10] As defined by the World Bank in the World Bank Debt Tables.

For the interest rate conditions which occurred over the period 1974-89, the probability of an investment succeeding in the borrowing country would have to be very high in order for a loan to be granted.

Condition ii) states that the expected investment return should be greater than the actual debt to be repaid. In other words, the investment technology in the developing country must be potentially capable of producing the returns necessary to repay the debt. This too is a difficult criterion for a borrower to meet. Take, for example, the period average interest rate from Table 3.1 and $\gamma = .95$: condition ii) then requires $x > 13.66\%$ for the all country sample and $x > 16.45\%$ for Brazil.

The third condition which must be fulfilled in order for there to be a lending equilibrium in the international credit market implies that both parties must have sufficient regard for the future. This restriction derives from the equilibrium lending policy of the bank so that the creditor must be able to overlook current default to some extent in anticipation of future fulfillment of loan obligations. Additionally, the debtor must value the future enough that the short run gains to default do not outweigh the benefits of continued access. From equation (3.3), the intuitively appealing and common result obtains that the larger the discount factor, the greater the number of equilibria which can be supported.

Turning to the equilibrium actions themselves, the current mixed strategy of the bank makes the country indifferent in the current period between repaying and defaulting whilst the current mixed strategy of the country renders the bank indifferent

in the current period between loaning and withholding funds. From equation (3.1) it can be seen that the bank is only indifferent in all states if a_i is the same in all states. This is reflected in equilibrium I. Furthermore, the form of the repayment probability is intuitively appealing, for the higher the interest obligation, the less willing is the country to repay. Similarly, an increase in the probability of a successful investment requires a low willingness to repay, a_i, to preserve bank indifference. Notice that the lender in this environment cannot make loan contracts completely free of the risk of repudiation caused by insouciance. Whilst some other models of the debtor/creditor relationship may permit default only due to a poor state of nature, $a_i^* < 1$ in this model indicates that default may be chosen <u>in equilibrium</u> in the good state. The bank can still achieve zero profits in the long-run, however.

Table 3.3 gives the equilibrium a_i^* given the interest rate conditions in Table 3.1, taking $\gamma = 0.95$. Tables 3.2 and 3.3 suggest that a lending equilibrium will occur only if the country has an 'adequate' investment technology and is extremely willing to use returns generated by investment to finance debt repayment.

Table 3.3: Equilibrium repayment action (a_i^*), given γ = .95 and the interest rate conditions from Table 3.1.

	a_i^* (high r)	a_i^* (low r)	a_i^* (ave. r)
ALL DEVELOPING COUNTRIES	0.9475	0.9893	0.9748
BRAZIL	0.9059	0.9765	0.9515

For the bank, any $p_i^* \in [0,1]$ represent an equilibrium lending strategy if they satisfy equation (3.3) and the additional constraints from Definition 3.2. An appealing feature of equilibrium I is that current lending decisions are linked to previous repayment performance. It follows from $p_3^* > p_1^*$ and $p_3^* > p_2^*$ that all default is punished to some extent in this setup because the bank cannot discern whether non-repayment is due to country insouciance or failed investment. The equilibrium lending strategy in this case is to punish all default through a reduced loan probability. This conflicts with Grossman and Van Huyck's (1988) notion of excusable default which, in the context of this model, would require $p_2^* = p_3^*$ (> 0). To see why this is an impossible outcome for this game, notice from the proof of Proposition 3.1 that, in equilibrium, condition (X) is satisfied so that $x - r + \delta U_3 = 1 + x + \delta U_2$. Given that a_i^* is constant across states in equilibrium, $p_2^* = p_3^*$ implies $U_2 = U_3$. From condition (X) again, this equilibrium is only valid if r = -1, a contradiction. In this model it is impossible, in equilibrium, to make the country willing to repay in all states. This fact

precludes the Grossman and van Huyck (1988) result in a strict sense.

Whilst all default is punished through a lower loan probability, a weaker form of the Grossman and van Huyck (1988) result can arise in this model. Specifically, although the loan probability is reduced after non-repayment, it is still possible to observe a loan being granted in the period directly following default if this is the outcome of the bank's randomization process. Naturally, this turn of events is most likely when the immediate punishment is weakest. To examine the minimum punishment to default in a lending equilibrium, the following definition is adopted.

<u>Definition 3.3</u> The 'most excusable default' lending equilibrium satisfies Definition 3.2 and has the least distance between p_2^* and p_3^* (i.e. the minimum value of β).

In other words, the most excusable default lending equilibrium has the highest possible loan probability following a default.

<u>PROPOSITION 3.2</u> The most excusable default lending equilibrium has a lending strategy for the bank

$$p_1^* = 0 \; ; \qquad p_2^* = \frac{\delta\gamma(1 + x) - (1 + r)}{\delta\,[\gamma(1 + x) - (1 + r)]} \; ; \qquad p_3^* = 1$$

and is valid if conditions i), ii) and iii) in Proposition 3.1 are satisfied. The equilibrium repayment action for the country is as in equilibrium I of Proposition 3.1.

Proof

From (3.4), the minimum value of β (= β^{min}) occurs when α is at its largest (= 1) so that

$$\beta^{min} = \frac{(1 + r)(1 - \delta)}{\delta\,[\gamma(1 + x) - (1 + r)]}$$

As, by definition, $p_1^* = p_3^* - \alpha$, p_3^* must equal one and $p_1^* = 0$. Also $p_2^* = p_3^* - \beta = 1 - \beta^{min}$ in the most excusable default lending equilibrium which gives p_2^* as in the proposition. The action of the country is given in Proposition 3.1. ■

Essentially, Proposition 3.2 gives the minimum equilibrium punishment immediately following a default. In this case, repayment of a loan is rewarded by a loan for certain next period. Non-repayment is punished according to p_2^*, and if there was no loan last period, then lending ceases altogether.[11] In other words, lending is terminated with probability β^{min}. Assuming r = .08, δ = .99, γ = .95 and x = .15 gives β^{min} = .873. Therefore, whilst the most excusable default lending equilibrium gives the case in which a loan is most likely to follow a non-repayment, a defaulter runs a great risk of losing its access permanently, for once reputation is lost in this equilibrium, it cannot be regained. This result is endogenously derived in this model and not assumed as in, for example, Eaton and Gersovitz (1981), Kletzer (1984) and Atkeson (1991).

[11] Due to this feature, the most excusable default equilibrium is unlikely to be the most efficient equilibrium. This is demonstrated in the numerical example of the next section.

PROPOSITION 3.3 If constraints i), ii) and iii) are satisfied, then the harshest punishment in a lending equilibrium has $p_1^* = p_2^* = 0$ and $p_3^* = (1+r)/\delta\gamma(1+x)$.

Proof The harshest punishment available to the bank is to withhold funds i.e. $p_1^* = p_2^* = 0$. By Definition 3.2, it must be the case that $p_3^* > 0$ in a lending equilibrium - substituting $p_1^* = p_2^* = 0$ into (3.3) delivers p_3^* as stated. ■

If this harshest punishment lending equilibrium is played then, lending is terminated with a high probability, $1 - p_3^*$, and repayment does not necessarily guarantee a loan next period. This is the lending equilibrium in which the country most easily loses its access to the international credit market and consequently is the least efficient. Indeed, it is this equilibrium which imposes complete financial autarky following default, which corresponds to those investigated in most of the sovereign debt literature. This is the only equilibrium of the current model in which at least one breakdown and resumption of lending cannot occur.

It appears that the equilibria which could arise from this simple model are capable of capturing a wide range of situations. On the one hand, when $p_1^* > 0$ and $p_2^* > 0$ it is possible that a defaulter will receive a loan in the next period or after a finite period of exclusion depending on the outcome of the bank's randomization process. When $p_1^* = p_2^* = 0$, not only does a defaulter lose its creditworthiness irretrievably, but this can also happen to a country which has repaid its debt. As debt repayment does not guarantee a loan for certain in this case (Proposition 3.3), even if a country meets its obligation it may fail to obtain a loan immediately afterwards.

The system then reverts to state 1 in which no loan is ever granted as $p_1^* = 0$. In addition to the most excusable and harshest punishment lending equilibria, situations are depicted in which there are periods of enthusiastic lending followed by periods of inactivity. Moreover, the transition between these different phases of lending activity is governed in equilibrium by the repayment actions of the country. Additionally, the expected length of the punishment period is determined as part of the equilibrium without recourse to *a priori* restrictions.

3.7 Numerical analysis

In order to examine possible lending equilibria more closely, equation (3.3) has been evaluated numerically using $\delta = .99$, $\gamma = .95$, $x = .15$ and $r = .08$. These parameters satisfy the conditions necessary for a lending equilibrium to exist. In this equilibrium, the country repays with probability $a_i^* = 0.9747$. The lending actions of the bank are presented in Table 3.4, where the cells of the table represent p_3^*. In deriving these values, p_1^* and p_2^* were allowed to range between zero and one in steps of 0.001 for p_1^* and 0.01 for p_2^*.

For this set of parameters, it is apparent that the equilibrium lending policy of the bank will involve p_1^* being no more than 0.001 and p_2^* assuming its largest value at 0.12. Giving the country enough of an incentive to repay any sovereign debt which it incurs leads to very high equilibrium values for p_3^*, the equilibrium probability of a loan following repayment.

Table 3.4: Equilibrium values of p_3^*, given $\delta = .99$, $\gamma = .95$, $x = .15$ and $r = .08$ and values for p_1^* and p_2^*

p_1^*	p_2^*						
	0	0.02	0.04	0.06	0.08	0.10	0.12
0	.9985	.9988	.9990	.9992	.9995	.9997	.9999
0.001	.9995	.9998	1.000	+	+	+	+

Note: A '+' indicates a value of $p_3^* > 1$.

The most excusable default lending equilibrium has $p_1^* = 0$, $p_2^* = .12$ and $p_3^* = .9999$, [12] with a high probability that the debtor loses its reputation permanently. Indeed when $p_1^* = 0$, the debtor should expect at some stage to be permanently excluded from the credit market. This mirrors existing reputational models of sovereign debt. However, there are equilibria in which $p_1^* > 0$ implying that reputation is never lost permanently but can be regained after a period of punishment. Periods of renewed lending after <u>variable</u> exclusion intervals are thus captured by these equilibria.

Corresponding to the equilibria in Table 3.4, Tables 3.5, 3.6 and 3.7 indicate that the expected payoff of the country (U_i^*) is low in states one and two but

[12] This does not exactly equal one due to the coarseness of the grid used to search for equilibria.

comparatively high in state three. The expected return of the bank is zero in all cases.

Table 3.5: U_1^* for the equilibria in Table 3.4.

p_1^*	p_2^*						
	0	0.02	0.04	0.06	0.08	0.10	0.12
0	0	0	0	0	0	0	0
0.001	.1091	.1113	.1136	+	+	+	+

Table 3.6: U_2^* for the equilibria in Table 3.4.

p_1^*	p_2^*						
	0	0.02	0.04	0.06	0.08	0.10	0.12
0	0	.0223	.0455	.0697	.0949	.1213	.1488
0.001	.1081	.1325	.1580	+	+	+	+

Table 3.7: U_3^* for the equilibria in Table 3.4.

p_1^*	p_2^*						
	0	0.02	0.04	0.06	0.08	0.10	0.12
0	1.0909	1.1132	1.1364	1.1606	1.1858	1.2122	1.2397
.001	1.1990	1.2234	1.2489	+	+	+	+

The steady state probabilities for these equilibria are $\mu_1 = .627$, $\mu_2 = .0276$ and $\mu_3 = .346$ for the case of most excusable default and $\mu_1 = .6672$, $\mu_2 = .02463$ and $\mu_3 = .3082$ for the harshest punishment. Multiplying the payoffs in the preceding three tables by their steady state probabilities leads to Table 3.8 in which the total payoffs of the country in each equilibrium are represented.

From this Table 3.8, it is clear and unsurprising that the harshest punishment lending equilibrium is the situation in which the country achieves the lowest expected payoff. This is due to the high probability of a total exclusion from receiving external funds. It is exactly this equilibrium which is often presented in the sovereign debt literature.

Table 3.8: Expected payoffs for the equilibria in Table 3.4.

p_1^*	p_2^*						
	0	0.02	0.04	0.06	0.08	0.10	0.12
0	.3362	.3507	.3656	.3812	.3976	.4147	.4330
.001	.4456	.4623	.4795	+	+	+	+

Although the most excusable default lending equilibrium is the one in which the country faces the least immediate punishment for default, this is not the most efficient outcome. Indeed, it is dominated by all of the equilibria in which reputation is never lost permanently following default (in other words where $p_1^* > 0$). The most efficient equilibrium sets $p_1^* = .001$, $p_2^* = .04$ and $p_3^* = 1$ so that repayment is rewarded with a loan for certain whilst p_2^* is at the highest level consistent with a non- zero p_1^*. In other words, the most efficient equilibrium is the one in which the borrower has the most access to the international credit market.

An efficiency argument may be applied in order to reduce the multiple equilibria of this model to a unique equilibrium.[13] Before the lending game is played, imagine that a number of banks compete with each other over the right to lend to the sovereign. Each bank offers a contract and the sovereign selects the one its prefers

[13] I am grateful to Norman Ireland for pointing this out.

(that is the one which offers the highest expected payoffs). If it is possible to offer loan contracts which specify a loan probability in each state, then the competition among the banks will ensure that the contract is offered which provides the sovereign with the greatest possible expected payoff. In terms of Tables 3.4 and 3.8, the bank which offers the contract specifying loan probabilities of .001, .04 and 1 in states one, two and three respectively will gain the right to lend to the sovereign. Assuming that this contract legally binds the bank, the lending game will then possess a unique equilibrium.

3.8 Comparative statics

If binding state-dependent contracts are not possible, then it is interesting to look at how the size of the equilibrium set varies in response to changes in the model parameters. The comparative statics of the lending side of the model are easily evaluated to be[14]

$$\frac{\partial p_1^*}{\partial r} < 0 \qquad \frac{\partial p_2^*}{\partial r} < 0 \qquad \frac{\partial p_3^*}{\partial r} > 0$$

$$\frac{\partial p_1^*}{\partial x} > 0 \qquad \frac{\partial p_2^*}{\partial x} > 0 \qquad \frac{\partial p_3^*}{\partial x} < 0$$

$$\frac{\partial p_1^*}{\partial \gamma} > 0 \qquad \frac{\partial p_2^*}{\partial \gamma} > 0 \qquad \frac{\partial p_3^*}{\partial \gamma} < 0$$

[14] Economic interpretation of these comparative statics is not meaningful due to the existence of multiple equilibria.

with the interaction of the p_i^* having the following signs:

$$\frac{\partial p_1^*}{\partial p_2^*} < 0 \qquad \frac{\partial p_2^*}{\partial p_1^*} < 0$$

$$\frac{\partial p_1^*}{\partial p_3^*} > 0 \qquad \frac{\partial p_2^*}{\partial p_3^*} > 0$$

$$\frac{\partial p_3^*}{\partial p_1^*} > 0 \qquad \frac{\partial p_3^*}{\partial p_2^*} > 0$$

This interaction of the lending probabilities seems to suggest that, in a lending equilibrium, the bank must set an appropriate distance between the loan probability in the reward state (state three) and the loan probabilities in the other two states. Also, if r increases or γ or x falls, equilibria with p_3^* close to one may see this probability increase above one and hence will cease to be equilibria. The same applies to p_1^* or p_2^* close to zero. The intuition here is straightforward: as r increases, the country becomes less inclined to repay unless the bank provides the correct incentive by increasing the distance between the reward (state three) and punishment (states one and two) loan probabilities. For equilibria at the edge of the original equilibrium set this is not possible without driving the loan probabilities above one or below zero. Similarly, as the technology in the borrowing country deteriorates (x or γ fall), the larger the required repayment becomes as a proportion of the expected investment return so the less willing the country is to repay. Making the country want to repay by adjusting the gap between punishment and reward again shrinks the set of possible equilibria.

3.9 Conclusion

In the type of international credit market modelled here, many countries will be refused sovereign loans. Even those which pass the criteria and receive loans cannot expect continuous access to external funds. The special feature of the model of this chapter is that it does not assume that default necessarily leads to a complete future lending embargo. An endogenously determined length of exclusion results as the punishment to default in equilibrium. Depending on the exact equilibrium played, this punishment can be either infinite or finite. Models assuming a permanent loss of reputation after default necessarily rule out the latter class of equilibria. Specifically, equilibria have been demonstrated to exist in which reputation may not be lost irreversibly in that a default may be followed by a loan immediately or after a period of exclusion. The lulls in activity in the market for sovereign debt suggested earlier in this chapter by Eichengreen (1989) can thus be captured, at least in principle, by this model. Whether the international credit market is in a phase of lending or non-lending was shown to depend critically on the action of the borrowing country. An interesting feature of the model is that some defaults are due to country insouciance and not just caused by a bad state of nature.

The most efficient lending equilibria are achieved when the country retains the possibility of access to external funds. The least efficient lending equilibrium was found to be the one which constrains the borrower to permanent financial autarky following default. A numerical example was presented, however, which suggested that sovereign borrowers should expect default to lead to extremely long, possibly

permanent, periods of financial isolation. Even the most efficient equilibrium exhibits this characteristic, implying that commercial bank lending is not suitable for the purposes of development - the flow of funds is likely to be much too volatile.

Although the model presented above is quite rich in implication, it is very simple and shall be extended in the next two chapters. Chapter four investigates the relationship of the punishment interval and access to the international credit market in determining the payoff of the borrower when the country has the flexibility of choosing the amount of its obligation it wishes to repay. The benefits of excusing default are also examined. Chapter five introduces heterogeneous borrowers into the international credit market so that the bank is not aware of the inherent characteristics of the borrower but must learn them over time.

CHAPTER 4. EXCUSABLE DEFAULT AND ACCESS TO THE INTERNATIONAL CREDIT MARKET

In the lending equilibria of chapter three, the country sometimes defaults even when it has funds available to repay its debt. Also in some of the equilibria, it is not certain that repayment of an obligation will generate a loan for certain in the next period. This chapter uses the concept of self-generation of payoffs introduced by Abreu, Pearce and Stacchetti (1986) to solve a model similar to that of chapter three when the country can decide how much of its obligation to repay and the bank must break even in each period. This gives the country a greater freedom of choice than in the previous chapter, for its action set is expanded. Using this model, the precise correlation between the size of the expected country payoff and the level of access to the international credit market is investigated. We shall demonstrate that the outcomes of this model imply that the country always repays the optimal amount when able and the bank always rewards this repayment with a further loan immediately. That the bank can design an incentive scheme which ties the borrower to a policy of always being honest means that the debtor loses its incentive to cheat so that it never repudiates when it has funds to repay the debt. This is comparable to the incentive compatible state-dependent repayment schedule in Grossman and van Huyck (1988) where the borrower pays what it can.

Furthermore, it is demonstrated that the equilibria of this model can be achieved using Markov strategies. The conditions which guarantee the existence of a lending equilibrium are stricter than those of chapter three. In particular, the country

will always repay more than the gross rate of interest in order to compensate the (myopic) bank for the possibility of default. For a lending equilibrium to exist, the investment technology of the country must be able to generate sufficient returns for the borrower to achieve a great enough surplus after repayment that the honest policy is always dominant.

A numerical example is presented to illustrate the findings for an economy which has potentially greater investment returns than that of chapter three. The country analyzed previously is found not to pass the criteria required for receiving loans in this new framework. Using the definition from chapter three, the most excusable default lending equilibrium is examined and found to imply that a borrower will lose its access to the international credit market eventually, making it the least efficient equilibrium in this model. This contrasts with the result in the previous chapter that the most excusable default lending equilibrium was relatively efficient. There, each equilibrium prescribed different expected payoffs in each state so that the most excusable default equilibrium could still be quite efficient in spite of its predicted credit embargo, for the benefits of continued access in other equilibria were smaller than the potential benefits of having default 'overlooked' in the most excusable default lending equilibrium. In the model of the present chapter, the different MPEs are found to give the same payoff following repayment and the same payoff following default, so the most efficient lending equilibrium is the one in which the country has the greatest degree of access to external funds.

The results on the efficiency of the most excusable default lending equilibrium

stand in contrast to the excusable default equilibrium of Grossman and van Huyck (1988). It appears that the crucial difference between the two models is that borrowers repay what they can in their model and when they can in the current chapter, but the bank cannot observe whether default is attributable to insouciance or investment failure. Designing an incentive compatible repayment scheme in the presence of this constraint on funds available for debt service gives the bank a series of options in its lending policy. At one extreme it may excuse default as much as possible in a lending equilibrium but at the same time increase the chance of a complete credit embargo, or at the other extreme it can implement the harshest equilibrium punishment after default whilst potentially maintaining credit lines open. The latter case - the least excusable default lending equilibrium - is shown to be more efficient for the country. Indeed, excusing default will actually harm the borrower in the long run.

4.1 The model

The economic environment is assumed to be that described in chapter three, section 3.1 and the play conforms to section 3.2 with the exception that the country decides how much of its obligation to repay after it has obtained a loan and investment has succeeded. Specifically, we assume that the action of the country at time t is to repay $R_t \in [0, 1+x]$. The time horizon is infinite. Denote the probability that an amount $R_t' \in [0, 1+x]$ is repaid as $a(R_t') \in [0,1]$ so that $\Sigma_{R_t} a(R_t) = 1$ and $\gamma \Sigma_{R_t} a(R_t) R_t$ is the expected repayment made. The value a(0) is then the probability that no repayment is made at all.

A strategy for the bank selects a loan probability at each time period depending upon the history up to that point. Similarly, a strategy for the country, defined only when investment succeeds, selects a repayment probability 'a' for each $R_t \in [0, 1+x]$ at time t depending upon history.

4.2 Payoff self-generation

The concept of self-generation of payoffs is due to Abreu, Pearce and Stacchetti (1986, 1990) and has previously been applied to a game involving sovereign debt by Atkeson (1991). Let $E(\delta)$ be the set of equilibrium payoffs for a game, given discount factor δ, and let W be the set of continuation payoffs. Further, denote by $B(\delta,W)$ the set of payoffs generated by δ and W.

<u>Definition 4.1</u> W is self-generating if $W \subseteq B(\delta,W)$.

Definition 4.1 states that the set of continuation payoffs is self-generating if the set of payoffs which can be achieved through continuation payoffs from the set W includes all of W. The following theorem relates W and $E(\delta)$.

THEOREM 4.1 (Abreu, Pearce and Stacchetti 1986, 1990):

If W is self-generating, then $W \subseteq E(\delta)$.

If the set of continuation payoffs is self-generating, then the set of continuation payoffs is contained in the set of equilibrium payoffs. We shall use this to solve the

model of this chapter.

Let e^m be the maximum expected equilibrium payoff achievable by the debtor nation and let $e_t \in [0, e^m]$ be a particular payoff which we seek to achieve. The following lemma demonstrates that any $e_t \in [0, e^m]$ can be supported as an equilibrium.

LEMMA 4.1 $e_t \in [0, e^m]$ can be supported as an equilibrium.

<u>Proof</u> $e_t \in [0, e^m]$ can be supported as an equilibrium iff there exists a lending action for the bank, $p_t \in [0, 1]$, and an action for the borrower, $a(R_t) \in [0, 1]$, and future country utilities e^c_{NL} if there is no loan in period t and $e^c(R_t)$ if R_t is repaid, such that the following three conditions, (4.1), (4.2) and (4.3), hold

$$e_t = (1 - p_t)\, \delta e^c_{NL} + p_t [(1 - \gamma)\, \delta e^c(0) + \gamma \{\textstyle\sum_{Rt} a(R_t)\, (1 + x - R_t + \delta e^c(R_t))\}] \tag{4.1}$$

where e^c_{NL}, $e^c(0)$, $e^c(R_t) \in [0, e^m]$.

$$\gamma \textstyle\sum_{Rt} a(R_t) R_t = 1+r \quad \text{for } p_t > 0$$
$$\gamma \textstyle\sum_{Rt} a(R_t) R_t \leq 1+r \quad \text{for } p_t = 0 \tag{4.2}$$

$$1 + x - R_t + \delta e^c(R_t) = 1 + x - R_t' + \delta e^c(R_t') \geq 1 + x + \delta e^c(0)$$
$$\forall\, R_t, R_t' \text{ in the support of } a(.) \tag{4.3}$$

We must show that $[0, e^m]$ is an interval. Set $e^c_{NL} = 0$ for simplicity and assume that e^m requires $p_t = 1$. Let $e_t^+ = \lambda e^m$ for $0 \leq \lambda \leq 1$ be a particular payoff which we seek to achieve. From (4.1), e_t^+ is achieved by setting $p_t = \lambda$ implying that lending is terminated with probability $(1-\lambda)$. Repeating for all permissible λ establishes that $[0, e^m]$ is indeed an interval. ■

Essentially, Lemma 4.1 demonstrates that the set of payoffs $[0, e^m]$ self-generates and hence belongs to the set of equilibrium payoffs by Theorem 4.1. As there are no other country payoffs in this model by definition, this is the entire set of equilibrium payoffs i.e. $E(\delta) = [0, e^m]$. In solving the model at time t, both a current and continuation payoff for the borrower is selected from the set $E(\delta)$. In other words, the set of equilibrium payoffs $E(\delta)$ does not vary between periods. The time subscript is therefore dropped for the remainder of this chapter.

Equation (4.1) is the expected payoff of the borrower and is comparable to (3.2), although is more general as the level of repayment can be chosen by the debtor. The per-period zero profit condition for the bank is (4.2) which states that if a loan is granted with positive probability, the expected repayment must be equal to the gross interest rate. A loan will not be granted if the expected repayment is below this level.[1] Condition (4.3) reflects the fact that for all R played with positive probability, i.e. for all R in the support of a(R), $1 + x - R + \delta e^c(R)$ must attain the maximum. If the country sets a positive probability on two different repayment levels, then it must

[1] Indeed, (4.2) demonstrates that it is possible for a loan not to be granted even if the expected repayment is high enough. As shown below, a loan needs to be granted with certainty to achieve e^m.

get the same expected payoff from each of these actions. If not, one of the repayments would strictly be preferred.

4.3 Characterization of the equilibrium payoff set $E(\delta)$

We have demonstrated that the equilibrium payoff set for this game is $E(\delta) = [0, e^m]$, so now we attempt to characterize this set. This is carried out by establishing several claims, the first of which demonstrates that a country will never renege on its debt contract if it is able to repay.

<u>Claim 4.1</u> There is a unique $R^* \in [0, 1+x]$ for which $a(R^*) > 0$ and $e^c(R^*) = e^m$. In fact, for $R^* = (1+r)/\gamma$, $a(R^*) = 1$.

<u>Proof</u> The expected country payoff in (4.1) is increasing in the choice of continuation payoffs $e^c(R)$ so, in order to achieve e^m, the country must be rewarded as much as possible for some repayment. From Lemma 4.1, there must exist some R^* with $a(R^*) > 0$ and $e^c(R^*) = e^m$. To see that this R^* is unique, consider the following argument: assume that there exists $R' \neq R^*$ with $a(R') > 0$ and $e^c(R') = e^m = e^c(R^*)$. Equation (4.3) is immediately violated indicating that R^* is unique and hence $a(R^*) = 1$. With $a(R^*) = 1$, (4.2) implies that for the bank to make zero profits, the average value of R^* (conditional on a loan being made and investment succeeding) must be $R^* = (1+r)/\gamma$. ■

Claim 4.1 states that in order to achieve the maximum payoff, the country must

repay R^* when able. Notice that we require $\gamma(1 + x) > 1 + r$, for this repayment to be feasible and to ensure that the bank does not capture all of the country's surplus.[2] Furthermore, an amount greater than the gross interest rate is repaid in order to compensate the bank for repudiation due to investment failure. Such compensation is necessary in this model as the bank must expect to break even each period, in contrast to chapter three where the repayments were $1 + r$ as it could take a longer term view.

Achieving e^m requires that the bank loan with probability $p^* = 1$ as long as

$$(1 - \gamma)\, \delta e^c(0) + \gamma\, [1 + x - \{(1 + r)/\gamma\} + \delta e^m] > \delta e^c_{NL} \qquad (4.4)$$

which simply states that the country should expect, in the best equilibrium, to achieve more by obtaining a loan in the current period than by not receiving credit. Indeed, this must hold at least for the equilibrium delivering e^m if there are to be non-trivial solutions to this model (equilibria with lending). As (4.2) is clearly satisfied in the best equilibrium, the bank is indifferent between lending to the sovereign borrower and not. We can therefore set $p^* = 1$ as required to achieve the highest equilibrium payoff for the borrower. In the numerical example of chapter three, the most efficient equilibrium also had repayment being rewarded with a loan for certain in the following period.

[2] This condition was one of the pre-requisites in chapter three for a lending equilibrium to occur.

The following two claims help to characterize $E(\delta)$, the equilibrium payoff set for the borrower.

Claim 4.2 $\quad e^c(0) = e^m - (1+r)/\delta\gamma$.

Proof With $e^c(R^*) = e^m$, it must be the case that the return from repaying this amount is at least as good as that from making no repayment at all, viz.

$$1 + x - R^* + \delta e^m \geq 1 + x + \delta e^c(0) \tag{4.5}$$

From (4.1), the expected payoff of the borrower is increasing in $e^c(0)$ so achieving the best country payoff involves (4.5) holding with equality. This establishes the claim using R^* from Claim 4.1. ■

Claim 4.3

$$e^m = \frac{1}{1-\delta}\left[\gamma(1+x) - \frac{(1+r)}{\gamma}\right] \tag{4.6}$$

Proof In (4.1) set $p^* = 1$, $R^* = (1+r)/\gamma$, $a(R^*) = 1$ and $e^c(0)$ from Claim 4.2 to obtain

$$e^m = (1-\gamma)\delta\left[e^m - \frac{(1+r)}{\delta\gamma}\right] + \gamma\left[1 + x - \frac{(1+r)}{\gamma} + \delta e^m\right]$$

Rearranging this yields (4.6) as claimed. ■

Notice that for the existence of a lending equilibrium, we require that $e^c(0) \geq 0$ [3] so that borrowers with small γ or x will be denied loans. The fact that the least technically advanced countries will not receive commercial bank loans was a feature of the model of chapter three. The condition which ensures the existence of a lending equilibrium can be derived from Claims 4.2 and 4.3 as $\delta\gamma(1 + x) \geq R^*$.[4] The discounted expected value of the country's investment must be at least as large as the repayment made in the best equilibrium. In other words, the country will only want to repay anything (and hence be rewarded with a further loan) if the discounted expected return next period from receiving and investing a loan is at least as great as the loss incurred this period due to repayment. With (4.4), the best expected country payoff is attainable only if a loan is made with certainty. Furthermore, e^m is the normalized expected difference between investment return and the optimal repayment.

4.4 Lending equilibrium in Markov strategies

In restricting attention to Markov strategies, we require that the bank selects its current loan probability depending upon what has happened in the previous period. Similarly, a Markov strategy for the country, defined only when investment succeeds, selects a probability for each feasible repayment depending upon actions in the previous period. In fact we have already established in Claim 4.1 that the country will

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[3] If $e^c(0) < 0$ then Claim 4.2 implies $\delta e^m < R^*$ so that the country would maximize its payoff by not repaying. Consequently the bank would not lend.

[4] Earlier, it was noted that $\gamma(1+x) > 1+r$ is needed for a lending equilibrium to exist. As $R^* = (1+r)/\gamma$, the condition $\delta\gamma(1+x) \geq R^*$ is the strictest condition which must be met for the existence of a lending equilibrium.

always repay R^* when able.

The following lemma shows that a lending equilibrium of this model exists in Markov strategies.

LEMMA 4.2 Payoff $e \in [0, e^m]$ can be achieved using Markov strategies.

Proof Notice that expected payoff $e^+ = \lambda e^m$ with $0 \leq \lambda \leq 1$ can be attained by setting $p = \lambda$ and $e^c_{NL} = 0$. In this case, lending is terminated with probability $(1-\lambda)$. This strategy is Markovian and works in the following way: if R^* is repaid, then a loan is extended next period with probability one; if R^* is not repaid then a loan is extended next period with probability λ and if no loan was granted last period, no loan shall be forthcoming this period. For the country $a(R^*) = 1$ is a weakly dominant strategy in all states by construction. ■

Given that, under certain conditions, a lending equilibrium in Markov strategies exists, the set of equilibrium actions is straightforward to characterize. The country sets $a(R^*) = 1$, $R^* = (1+r)/\gamma$ when investment succeeds, conditional on a loan being made. This is a weakly dominant strategy irrespective of actions in the previous period.[5] Using Claims 4.2 and 4.3 yields

[5] In chapter three, it was weakly dominant in all states to set a constant probability of repayment. Here, a constant repayment amount is set.

$$e^c(0) = \frac{\delta\gamma^2(1 + x) - (1 + r)}{\delta\gamma\ (1 - \delta)}$$

(4.7)

From equation (4.1), the bank action, p, which gives this value of $e^c(0)$ can be found to be

$$p^*_{NR} = \frac{\delta\gamma\ [\gamma(1 + x) - (1 - \delta)\delta e^c_{NL}\ ] - (1 + r)}{\delta\gamma\ [\gamma(1 + x) - (1 - \delta)\delta e^c_{NL}\ ] - \delta(1 + r)}$$

(4.8)

where p^*_{NR} indicates the probability of a loan following a non-optimal repayment of debt.

In order to induce the country to want to repay R^*, the bank must reward repayment as much as possible. Setting $p^* = 1$ is only a reward for the country if it attains a higher expected payoff from obtaining a loan than from being refused credit. In other words, we require that $e^m > e^c_{NL}$. Furthermore, an upper bound can be set on e^c_{NL} so that $p^*_{NR} \geq 0$ in (4.8). A lower bound on e^c_{NL} occurs when $p = 0$ in (4.1) giving $e^c_{NL} = 0$. Therefore, it must be the case that

$$e^m > \frac{1}{\delta\ (1 - \delta)}\ [\ \gamma(1 + x) - \frac{(1 + r)}{\delta\gamma}] \geq e^c_{NL} \geq 0$$

(4.9)

In order to achieve some e^c_{NL} in this region requires

$$p^*_{NL} = \frac{e^c_{NL}\,(1 - \delta)}{\delta e^c(0) + \gamma(1 + x) - \delta e^c_{NL}}$$

(4.10)

When $\delta\gamma(1+x) > (1+r)/\gamma$ and (4.9) is met, a multiplicity of lending MPEs exist. This mirrors the findings in chapter three where the country again had a weakly dominant strategy and the bank had many ways of achieving zero profit. Notice that $\delta\gamma(1+x) > (1+r)/\gamma$ is a stronger condition than ii) and iii) in chapter three. In the present model, not only must the country have the potential for generating returns in order for the bank to be induced to lend, but the technology must be able to compensate the borrower for making debt repayments which exceed the gross interest rate. Recall that repayments were equal to the gross rate of interest in chapter three. That the requirements for the existence of equilibrium are stricter than in the previous chapter should not be surprising, for the bank is taking a shorter term view here, breaking even each period. Additionally, the equilibria are such that the country must have a great enough surplus after repayment that it is not tempted to renege on the grounds of insouciance. These facts combine to prevent more countries obtaining loans in this framework than the one presented in chapter three.

The Markov strategy for the bank works in the following way: for some e^c_{NL} in the region given by (4.9),

if R^* is repaid then $p^*_{+1} = 1$;

if $R \neq R^*$ is repaid then $p^*_{+1} = p^*_{NR}$;

if no loan this period, then $p^*_{+1} = p^*_{NL}$.

where p^*_{+1} is the equilibrium loan probability in the next period. There are a number of MPEs corresponding to different levels of punishment for a non-optimal repayment of debt. The 'most excusable default' lending equilibrium as defined in chapter three occurs when there is least distance between the loan probability following a repayment and the loan probability following default. This distance is increasing in e^c_{NL} so that the most excusable default lending equilibrium sets $e^c_{NL} = 0$. Notice that if $e^c_{NL} = 0$, then there is always a positive probability of the credit relationship terminating as $p^*_{NL} = 0$ so that the bank terminates lending in this case with probability $1-p^*_{NR}$. Possible termination of credit was found to be a part of the most excusable default lending equilibrium in the previous chapter. The 'least excusable default' lending equilibrium sets the loan probability at its lowest level following a default i.e. e^c_{NL} is at its maximum level in (4.8).

The economy depicted in the numerical analysis of chapter two does not pass the criteria set in this model to obtain loans. If $\gamma = .95$, $\delta = .99$ and $r = .08$ as in chapter three, it must be the case that $x \geq .2088$ in order for a lending MPE to exist.[6] To illustrate the extreme points of the model of this chapter, consider a potentially more productive economy than has been used so far with the parameter values

[6] Alternatively, if $\gamma = .95$, $\delta = .99$ and $x = .15$, a lending equilibrium would only exist if $0.0275 > r$.

$\gamma = .95$, $\delta = .99$, $r = .08$ and $x = .25$. This yields $e^m = 5.0658$ and $e^c(0) = 3.9175$. The most and least excusable default equilibria for these parameter values are given below (p^*_R indicates the loan probability following repayment of R^* last period and μ_j are the steady state probabilities of being in state j in the two equilibria, where j = R, NR, NL, indicating the states 'loan and optimal repayment', 'loan and sub-optimal repayment' and 'no loan'):

<u>Most excusable default lending equilibrium:</u>

$e^c_{NL} = 0$;	$p^*_R = 1$;	$p^*_{NR} = .7733$;	$p^*_{NL} = 0$.
	$\mu_R = .5615$	$\mu_{NR} = .0296$	$\mu_{NL} = .4089$

<u>Least excusable default lending equilibrium:</u>

$e^c_{NL} = 3.957$;	$p^*_R = 1$;	$p^*_{NR} = .00003$;	$p^*_{NL} = .0345$.
	$\mu_R = .3276$	$\mu_{NR} = .0172$	$\mu_{NL} = .6552$

Although these equilibria lie at opposite extremes of the set of lending options open to the bank, they both deliver e^m if optimal repayment was made in the previous period and $e^c(0)$ following a non-optimal repayment. The crucial difference between the two equilibria is that the most excusable default lending equilibrium precludes further loans if ever the bank withholds credit for one period, whereas the other equilibrium maintains the possibility of retaining access to external funds indefinitely. It should not be surprising then that the expected value of the game to the country in the most excusable default lending equilibrium (2.9604) is smaller than in the least excusable default lending equilibrium (4.3196). This apparent paradox is caused by

the definition of the most (least) excusable default equilibrium to be the smallest (largest) distance between the loan probability following repayment and that following default. In other words, the degree to which default is excusable is measured only in the state directly following the default with no regard for the future. Although the most excusable default lending equilibrium has the largest loan probability following repudiation, when seen as a whole this equilibrium will be highly inefficient as it prescribes the eventual termination of credit. By punishing a default strongly, on the other hand, there is no need for the bank to threaten a total embargo in order to keep the country honest. Overlooking default actually harms the country in the longer term.

As suggested in chapter three, the most efficient equilibrium will be the one in which access to the international credit market is greatest and this will be the unique equilibrium if the pre-play contracting process outlined in chapter three is carried out among the banks. In the present model, the equilibria are distinguished by the value of e^c_{NL} chosen. Recall that e^c_{NL} is the continuation payoff if no loan was granted in the previous period so that the most efficient lending equilibrium occurs when this value is greatest. In contrast, when $e^c_{NL} = 0$, access to the international credit market can be lost permanently so this is the least efficient lending equilibrium.

4.5 Summary

The model of this chapter has relaxed the assumption made in chapter three that debt repayment is all or nothing, whilst the bank must expect to break even each period. The conditions which guarantee the existence of lending MPEs in this case

are stronger than for the case analyzed in chapter three. Indeed, the economy analyzed numerically previously would not receive loans in the current framework. In equilibrium, the country has no incentive to renege on its debt obligation unless it has experienced a bad state of nature. The investment technology must therefore be capable of generating returns which leave enough of a surplus after debt repayment that the country is not tempted to cheat. Complete repudiation of obligations when the country can choose how much to repay is due to a bad state of nature in the borrowing country. Knowing this, the bank rewards loan repayments with another loan for certain in the next period and punishes non-optimal repayment in the form of a reduced loan probability.

The equilibrium payoff set of this game is easily characterized using the concept of self-generation and we have shown that these payoffs can be achieved as Markov Perfect Equilibria. As in the previous chapter, some parameters were used in order to examine the MPEs more closely. In the worst lending equilibrium for the country - paradoxically the most excusable default equilibrium - there is likely to be a complete loss of the country's reputation for creditworthiness and hence a permanent credit embargo will be imposed. The best outcome - the least excusable default lending equilibrium - indicates that losses of reputation are only temporary. Indeed, several periods can pass in which no loan is made and then lending can resume. This was also found in chapter three. The apparent paradox in the relative efficiency of the most and least excusable default lending equilibria was indicated to be caused by the definitions adopted for these two equilibria.

The MPEs found in this chapter resemble the outcomes in Grossman and van Huyck (1988) as the country has no incentive to cheat. In the current setting, when the country may be constrained in the frequency of repayments it is able to make and the bank cannot observe the realized investment return, the country is most likely to face a permanent credit embargo when default is excused most. Indeed the borrower does better in terms of its total expected payoff when the bank implements a policy of harsh punishment of default whilst retaining the possibility of further loans in the future. The difference between the results here and those of Grossman and van Huyck is attributable to the fact that debt repayments in the current model can only be made if investment succeeds but this is not observed by the bank. In this setup, the bank must design its lending policy to make the country repay even though its investment return is private knowledge. As the payoffs to repayment of the optimal amount (e^m) and complete repudiation ($e^c(0)$) are the same in all equilibria, it is the payoff following a no-loan state (e^c_{NL}) which determines the overall expected payoff. By not punishing default strongly initially, the bank must compensate by reducing e^c_{NL} in order to induce the country to follow the honest policy. This reduces the overall expected payoff of the borrower so that excusing default is actually the action which harms the country the most.

Reinforcing the findings of chapter three is the important result that lending to a sovereign nation can temporarily break down and then resume as part of the equilibrium. Previous work has tended to assume that policies need to be implemented to ensure the resumption of commercial bank lending to developing countries (for example, Aizenman and Borensztein (1989)). The results of this and

the previous chapter suggest that, in some cases at least, this is a false presumption. Whilst it is clearly not the case that lending to all sovereign borrowers will resume automatically as it were, it is also not true that all breakdowns in lending are permanent.

CHAPTER 5. LEARNING AND THE DYNAMICS OF REPUTATION IN THE INTERNATIONAL CREDIT MARKET

5.1 Heterogeneous borrowers

The previous two chapters have characterized the relationship between a bank and a known sovereign borrower. The task of the bank is straightforward in this case, for it loans only to countries which are willing and technically capable of generating funds to service the debt. In this chapter, we introduce two 'types' of borrower and examine the outcomes when the bank does not know which type of borrower it faces. A type P country is assumed to be profligate as a received loan is never invested but is always consumed immediately.[1] Thus this debtor is forced into always defaulting on its obligations assuming that the one good in existence cannot be stored. No loans would be granted to type P if its type were known. The second type of borrower, type I, has identical characteristics to the one dealt with in chapters three and four. This debtor always invests its loans in the same stochastic technology described previously and makes optimal repayment decisions.[2]

At the outset of the game, a creditor identical to the one dealt with in chapter three is matched with a debtor of unknown type with whom it transacts for the

[1] Williamson (1986) documents that Argentina, for example, used her excursions into the international credit market not for productive purposes, but to purchase arms.

[2] These two types of debtor are comparable with the assumption by Calvo (1989) that a country can invest in 'legitimate' or 'illegitimate' activities. In the present framework, type P invests in the illegitimate activity (consumption) and type I invests legitimately.

remainder of the (potentially infinite) game. Again, the realized investment return in the type I debtor country is unobservable by the bank. The creditor thus has incomplete and imperfect information in this model as compared with complete but imperfect information in chapters three and four. Repayment is again dichotomized to be all or nothing. We are interested in capturing both the behaviour of the bank in the face of this uncertainty and of the type I borrower who wishes optimally to reveal its private information.

The play of the game in each period is as described in chapter three, section 3.2. The difference in this scenario is that actions will depend upon the belief of the creditor that it is transacting with a type I debtor. This belief can be interpreted as a borrower's reputation for creditworthiness. As the belief changes through time, we develop a dynamic theory of debtor reputation. Naturally, the bank would not wish to lend if it knew that its customer were type P. If there has been a loan this period which the debtor repays, the game becomes the one of complete information in chapter three, for repayment is only made by type I. If the loan is not repaid at the end of the period, the bank will revise downward its assessment that the borrower is type I. If there is no loan in the present period, the bank has no new information so cannot revise its beliefs. The task of the bank is to establish its borrower's type, whilst the type I country wishes to optimally reveal this information. The type P country merely accepts loans and never repays its obligations.[3]

[3] In the exposition and solution of the model, it should be clear that, unless otherwise stated, a type I country is being referred to. A type P borrower makes no decisions.

The results suggest that lending breakdowns and the resumption of voluntary lending can occur in equilibrium. This feature is common with the previous two chapters. At least up to a point, default does not preclude access to future loans. This appears to extend a weaker form of the excusable default result of Grossman and van Huyck (1988) to an incomplete information structure with potential country insouciance. In contrast to chapters three and four, there is an endogenous limit on the tolerance of default by the bank, for once the reputation of a country sinks too low, a complete and permanent loan embargo is imposed.

A numerical example, taking the economy from chapter three, suggests that parameters satisfying the criteria for the existence of a lending equilibrium are likely to deliver a high critical reputation threshold. Consequently, only a small number of loans are made until debtor type is learned by the bank. Whilst it is possible in principle that a creditworthy country can masquerade as an inherent defaulter, the example shows that this is likely to happen only with a small probability due to the threat of a loan embargo. Nevertheless, some default occurs due to country insouciance and not a realization of a poor state of nature. The temptation to repudiate through choice decreases, however, as the critical level of reputation, and hence the complete embargo, is neared.

This chapter can be regarded as complementing the work on learning in the international credit market by Cole, Dow and English (1989). They also examine the possibility that borrowers may be inherently different, although in their case it is the government in a single country which is capable of being two different types, stable

and unstable (more prone to default). These governments evolve continuously in their model according to a first-order Markov process so that the 'learning' is carried out even in a period of non-lending. In common with this chapter, loans are made according to the belief that the borrower is stable. In the Cole, Dow and English model, the only reasonable equilibrium is of the signalling type where a stable government indicates that it has taken power by means of a partial repayment of outstanding debt.[4]

Naturally, in order to make the signal, a stable type must have the funds available. In the context of a two borrower model, the present chapter examines the case in which the stable type of borrower (type I) may be prevented from making this signal by lack of funds. In the Cole, Dow and English model lending resumes following default after the stable type has signalled that it is in power. In the model of this chapter, the presence of a constraint on repayment means that lending resumptions only follow default up until a critical level of belief. Once this level of belief has been surpassed, even the stable borrower will permanently lose its access to external funds. The uncertainty caused by the presence of the unstable borrower can have a profound effect on the borrowing capabilities of the more stable type.

[4] See chapter two for details.

5.2 Beliefs and strategies

Denote by $q_t \in [0,1]$ the creditor's assessment that it is dealing with a type I country at time t (the time horizon is potentially infinite).[5] This assessment is the state of the system containing all of the payoff relevant history so that attention can again be focussed on Markov strategies.[6] Let $z(q_t)$ be the probability that a loan is granted at time t and $y(q_t)$ be the repayment probability by a type I country which is defined when investment succeeds.

<u>Definition 5.1</u> A Markov strategy for the bank selects a loan probability $z \in [0,1]$ for each level of belief $q_t \in [0,1]$ i.e. $\{z(q_t)\}^{\infty}_{t=1}$. A Markov repayment strategy for the type I country is defined when investment succeeds and selects a repayment probability $y \in [0,1]$ for each level of belief i.e. $\{y(q_t)\}^{\infty}_{t=1}$. A type P country never repays its debt irrespective of time or state.

At the end of each period, the bank updates its belief according to the action which the country has taken on receiving a loan. If no loan is granted, no revision of the prior belief can occur. Following a loan, repayment of a debt immediately reveals the borrower to be type I and the game becomes the one of complete information in chapter three. If a loan is granted and not repaid, the bank rationally updates its belief downwards according to Bayes' rule. This gives rise to the following rule for the updating of prior beliefs:

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[5] In effect, q_t is the borrower's reputation for creditworthiness.

[6] In the setup of the model, history matters only to the extent that it affects beliefs.

$$q_{t+1} = q_t \qquad \textit{if no loan}$$

$$q_{t+1} = \frac{q_t\,[\,1 - \gamma\, y(q_t)\,]}{1 - q_t \gamma\, y(q_t)} \qquad \textit{if loan + no repayment}$$

$$q_{t+1} = 1 \qquad \textit{if loan + repayment}$$

(5.1)

From this updating rule, it is clear that the sequence of beliefs will depend upon the sequence of country actions and the prior belief with which the bank begins the game. For any sequence of $\{y(q_t)\}^{\infty}_{t=1}$, conditional on a loan being made, investment succeeding and default being observed, there will be J+1 revisions of the belief, where J depends on the prior at the beginning of the game and the sequence of type I country actions. Assume for now that there exists a level of belief q^J which is so low that if $q < q^J$ the country will face a complete lending embargo as the bank believes it to be type P. That such a q^J exists is shown later. Denoting the belief at the beginning of the game by q^0, the sequence of beliefs generated by a sequence of country actions <u>with default observed</u> will be $Q = \{q^0, q^1, \ldots, q^J, q^{J+1}\}$ where $q^j > q^{j+1}$. If repayment is made at level of belief q^j, then $q^{j+1} = 1$ and the game of complete information from chapter three is played.

Although the sequence of q is determined by the country's action, the length of the time interval between jumps in the sequence is determined by the action of the bank, for only when a loan is granted is the belief updated. Let the belief of the bank that it faces a type I debtor at the beginning of the game (time t = 0) be denoted by q^0_0. This prior is held for as many time periods as it takes for the first loan to be

issued. Assume that a loan is granted in the s(0)'th period[7] so that the prior is updated to q^1 in period s(0)+1. Assuming that default is observed, this q^1 is then held for a further s(1) periods until a loan is again forthcoming. The j'th revision of the belief, delivering q^j in the sequence (contingent on no prior repayment), occurs after S(j-1)+1 time periods[8] where $S(j-1) = 1 + \sum^{j-1}_{k=0} s(k)$. This belief is then held for s(j) periods until a loan is granted.

The belief q^J is arrived at in time period S(J-1)+1 and is held for s(J) periods up until a loan is made. When this particular loan is made, either the country repays its loan, revealing it to be type I, or it repudiates. This latter action causes the bank to impose a complete loan embargo by the definition of q^J. The game of incomplete information therefore lasts a total of S(J)+1 periods, which depends upon the lending strategy of the bank.[9] The total number of loans which will be made until the belief falls below the critical level will be J+1.

We shall denote the j'th revision of the prior belief to q^j, occurring at time S(j-1)+1 and lasting for s(j) time periods, by $q^j_{t(j)}$ so that t(j) represents any time period between S(j-1)+1 and S(j).

It is apparent from the updating rule in (5.1) that the distance between each q

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[7] s(0) + 1 thus denotes the length of time for which q^0 is held.

[8] S(j-1) is thus the total time which elapses before the j'th revision of the belief.

[9] S(J)+1 has the potential to be infinite.

in the sequence Q will be determined by Bayes' rule.[10]

<u>Definition 5.2</u> Let $q^{j-1}_{t(j-1)}$ and $q^{j}_{t(j)}$ be two beliefs in the sequence Q such that $q^{j}_{t(j)}$ directly follows $q^{j-1}_{t(j-1)}$. The j'th step locus gives combinations of $q^{j-1}_{t(j-1)}$ and $y(q^{j-1}_{t(j-1)})$ from (5.1) which deliver $q^{j}_{t(j)}$.

Rearranging Bayes' rule gives the equation of the j'th step locus as

$$y(q^{j-1}_{t(j-1)}) = \frac{q^{j-1}_{t(j-1)} - q^{j}_{t(j)}}{\gamma\, q^{j-1}_{t(j-1)}\,[\,1 - q^{j}_{t(j)}\,]} \tag{5.2}$$

The reason for the construction of these step loci is as follows. Assume that Q^* represents an equilibrium sequence of beliefs. Naturally, any action taken by the type I country must be consistent with this path of beliefs being followed. Equation (5.2) gives exactly these actions.

5.3 Expected payoffs

Assume that the level of belief for any single time period $t \in t(j)$ is q^{j}_{t}. If a loan is granted and not repaid, then this belief is revised downwards to q^{j+1}_{t+1} where $t+1 = S(j)+1$. If the loan is repaid, then the bank sets $q_{t+1} = 1$ and the country gets a continuation payoff of $\delta U(1)$, the discounted value of playing the game of complete

[10] Recall that the sequence Q is defined when default is observed.

information from chapter three. The probability that a loan is granted is $z(q^j_t)$ and the country wishes to repay with probability $y(q^j_t)$. As γ is the probability that funds are generated for repayment, $\gamma y(q^j_t)$ is the total probability of repayment.[11]

As in chapter three, δ is a discount factor and r a fixed interest rate earned by the bank in the event of a repayment. The present value of the type I country's payoff at time $t \in t(j)$ with level of belief q^j_t is

$$U(q^j_t) = (1 - z(q^j_t))\, \delta U(q^j_t) + z(q^j_t) \{\gamma y(q^j_t)\, (x - r + \delta U(1)) + \gamma(1 - y(q^j_t))\, (1 + x + \delta U(q^{j+1}_{t+1})) + (1 - \gamma)\, \delta U(q^{j+1}_{t+1})\} \qquad (5.3)$$

If no loan is granted in period $t \in t(j)$, then beliefs remain at q^j_t and the expected payoff of the country gets discounted one period. If a loan is made and investment fails, as it does with probability $(1 - \gamma)$, then beliefs are updated and the country receives nothing in the current period and a continuation payoff of $\delta U(q^{j+1}_{t+1})$. If investment succeeds, then the type I country obtains $x - r + \delta U(1)$ if it repays and $1 + x + \delta U(q^{j+1}_{t+1})$ if it repudiates. The action of the country in period t will depend upon the relative magnitudes of its payoff from repudiating and repaying.

Letting V(1) be the payoff which the bank expects from the complete information game, the present expected value of the bank's payoff at time $t \in t(j)$ can

[11] Instead of characterizing the types of country by their underlying willingness to repay, a type I country could be thought of as having a high γ, whereas a type P country has γ=0.

be written as[12]

$$V(q^j_t) = (1 - z(q^j_t))\, \delta V(q^j_t) + z(q^j_t) \left\{ (1 - q^j_t \gamma y(q^j_t))\, (\delta V(q^{j+1}_{t+1}) - 1) + q^j_t \gamma y(q^j_t)\, (r + \delta V(1)) \right\} \tag{5.4}$$

If no loan is granted in period $t \in t(j)$ then beliefs remain at q^j_t and the expected payoff of the bank gets discounted one period. If a loan is made and is not repaid, then the bank receives a payoff of $\delta V(q^{j+1}_{t+1}) - 1$ due to the loss of the loaned unit and consequent updating of beliefs. The probability that this return is achieved is $(1-q^j_t) + q^j_t(1-\gamma y(q^j_t))$ which is the probability that the bank assigns to the customer being type P plus the probability that a type I country does not repay (through either choice or bad luck). The bank achieves a payoff of $r + \delta V(1)$ if the loan is repaid as this indicates that the customer is type I. The probability of repayment is $q^j_t \gamma y(q^j_t)$, the probability that the country is type I and is able and willing to repay.

Notice that (5.4) reflects the loaning preferences of the bank as the payoffs from extending and withholding a loan are compared when it makes its decision. The following result proves useful:

[12] Chapter three suggests that $V(1) = 0$, but is retained here as a constant for generality.

Result 5.1 The bank is indifferent between loaning and not at time $t \in t(j)$ (i.e. $z(q^j_t) \in [0,1]$), $j = 0,\ldots, J$, if

$$y(q^j_t) = \frac{1}{\gamma q^j_t (1 + r + \delta V(1))} \tag{5.5}$$

Proof Bank indifference requires that the expected return from lending be equal to the expected return from withholding funds. This requires in (5.4) that the term in braces equals $\delta V(q^j_t)$ which implies $V(q^j_t) = 0$, $\forall\, j = 0,\ldots, J$, $t \in t(j)$. Therefore

$$0 = z(q^j_t) \{ (1 - q^j_t \gamma y(q^j_t)) (-1) + q^j_t \gamma y(q^j_t) (r + \delta V(1)) \}$$

which rearranges to give (5.5). ■

Corollary The bank strictly prefers to lend at time $t \in t(j)$ i.e. $z(q^j_t) = 1$, if $y(q^j_t)$ is greater than that in (5.5), whilst a repayment probability below this level implies a preference for withholding funds ($z(q^j_t) = 0$).

Equation (5.5) permits a simple intuitive interpretation. As the level of belief that the country is type I falls, the country must be extremely willing to repay its debt when able in order to induce the bank to lend. A low value of γ implies that funds will be generated infrequently to facilitate repayment. Again the bank is only motivated to lend if there is a high likelihood that any return from the country's investment will be used to repay a debt obligation. Similarly, as the return to lending

falls, the bank requires compensation in the form of a low repudiation probability.

Let $q^{j-1}_{t(j-1)}(X)$ denote the level of belief at which the j'th step locus crosses the line of bank indifference given by (5.5). It is easily determined that, for j = 1,..., J

$$q^{j-1}_{t(j-1)}(X) = \frac{1 + q^{j}_{t(j)}\,[r + \delta V(1)]}{1 + r + \delta V(1)} \tag{5.6}$$

Letting $q^j(W)$ denote the level of belief at which the j'th step locus reaches the point y = 1, the j'th step locus begins at the point $q^j(X)$, $y(q^j(X)) = 0$ and ends at $q^j(W)$, $y(q^j(W)) = 1$. This locus crosses the bank indifference line at $q^{j-1}(X)$. [13]

Notice that (5.6) does not define $q^J(X)$, for which we adopt the following notation. Let q^J_t for $t \in t(J)$ denote the lowest level of belief at which a loan will still be forthcoming with positive probability.[14]

<u>Result 5.2</u>

$$q^{J}_{t \in t(J)} = \frac{1}{\gamma\,[\,1 + r + \delta V(1)\,]} \tag{5.7}$$

[13] The level of belief $q^j(W)$ is held in periods t(j-1), as the j'th revision of the belief will be constructed to take the belief to $q^j(X)$ for time periods t(j).

[14] In fact it should be clear that $q^J = q^J(X)$, which shall simply be referred to as q^J_t.

Proof By the definition of q^J, this is the lowest level of belief at which the country's action can still induce the bank to be indifferent between lending and not. Result 5.1 and its corollary indicate that a loan is most likely to be granted when the country repays with certainty if able. Setting $y(q^J_t) = 1$, for $t \in t(J)$, into (5.5) delivers (5.7). At any level of belief $q < q^J$, the bank requires $y(q) > 1$ in order to be induced to lend with positive probability. ■

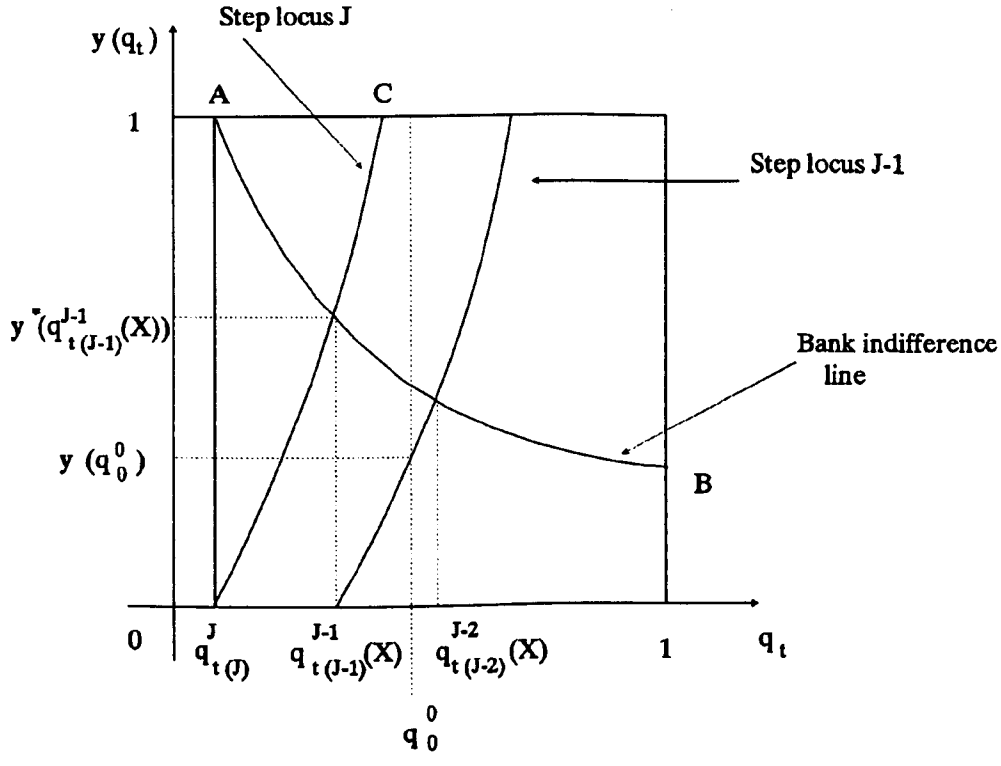
Result 5.2 is important, for it is the lowest level of belief in the sequence Q at which a lending equilibrium to the game of incomplete information can exist. Given that the sequence Q is defined for observed default, any $q^{J+1} < q^J$ is too low for lending to occur. The level of belief q^J is thus critical. Immediately, we have an explanation for why some countries cannot obtain bank credit. If $q^0 < q^J$ then no equilibrium exists in which lending takes place. Furthermore, q^J is likely to be different for each country so that we have the potential for explaining, through a reputation argument, why some countries enjoy short-lived credit relationships with commercial banks whilst others may last longer. Notice also that q^J depends upon γ and that a lending equilibrium requires $\gamma > 1/[1+r+\delta V(1)]$ in order that q^J is bounded below one.[15]

Figure 5.1 draws together findings so far. The line AB represents the locus along which the bank is indifferent between extending and withholding credit. Above and to the right of this curve is the region in which the bank strictly prefers to lend, whereas a loan will never be forthcoming in the area below and to the left of the bank

[15] This also ensures that Bayes' rule revises beliefs downwards.

indifference locus.

Figure 5.1



The step locus J, indicated in the figure by $Cq^J_{t(J)}$, gives the combinations of the belief preceding $q^J_{t(J)}$, i.e. $q_{t(J-1)}$, and corresponding country action $y(q_{t(J-1)})$ which are consistent with a default causing the prior to be revised to exactly $q^J_{t(J)}$. The curvature of the step loci reflects the fact that the bank learns more about its customer when the willingness of the type I country to repay is relatively high. A type I country which chooses $y(q^j_{t(j)}) = 0$ is indistinguishable from a type P debtor so that an observed default imparts no new information to the bank so that beliefs remain at $q^j_{t(j)}$. The largest downward revision of the belief occurs when type I sets $y(q^j_{t(j)}) = 1$ as this

is the situation in which the bank can be most certain that default indicates a type P borrower.

5.4 Existence of a Markov Perfect Equilibrium with lending

<u>Definition 5.3</u> A Markov Perfect Equilibrium of this game of incomplete information consists of a sequence of beliefs $Q^* = \{q^{(0)*}_{t(0)},.....,q^{(J+1)*}_{t(J+1)}\}$, conditional on default being observed, and bank actions $Z^* = \{z(q^{(0)*}_{t(0)}),.....,z(q^{(J+1)*}_{t(J+1)})\}$ and type I country actions $Y^* = \{y(q^{(0)*}_{t(0)}),.....,y(q^{(J+1)*}_{t(J+1)})\}$ such that Z^* maximizes (5.4) given Y^* and Q^*, and Y^* maximizes (5.3) given Z^* and Q^*. The equilibrium sequence of beliefs is determined by (5.1) and is such that Q^* is the equilibrium belief path until repayment is observed. An observed repayment ends the game of incomplete information, so that the level of belief becomes one for the duration of the complete information game in chapter three.

This definition of an MPE ensures that each player's action at each stage of the game is an optimal response to the action played by the opponent. Furthermore, the equilibrium sequence of beliefs is determined by the rational updating procedure of the bank. Given an equilibrium sequence of beliefs, the equilibrium actions must be consistent with this sequence being followed. This means that if, in Figure 5.1, $q^{J}_{t(J)}$ and $q^{J-1}_{t(J-1)}(X)$ are in the equilibrium belief sequence, the equilibrium action of the type I country at time $t \in t(J-1)$ must be $y^*(q^{J-1}_{t(J-1)}(X))$ with the bank randomizing between loaning and not.

Before proceeding to a full solution of this game, several observations are useful. Notice, from Result 5.1, that for time period $t \in t(j)$ and belief q^j_t, it is the action of the type I borrower at time t which makes the bank indifferent between loaning and not at time t. From (5.3), however, the country randomizes between repaying and repudiating its debt only if $x - r + \delta U(1) = 1 + x + \delta U(q^{j+1}_{t+1})$, $t+1 = S(j)+1$. Clearly, $U(q^{j+1}_{t+1})$ depends on the action of the bank at time t+1, $z(q^{j+1}_{t+1})$. Therefore, the bank can only induce the borrower to be indifferent between repaying and repudiating in period $t \in t(j)$ through its action in periods t(j+1).

An intuitive explanation of the construction of an MPE for this game can be gained by examining Figure 5.1. The level of belief $q^J_{t(J)}$ is critical, so we can begin at this point and work backwards through time and the belief sequence. Assume that $q^J_{t(J)}$ forms part of the equilibrium sequence of beliefs and that default causes a revision to the next (arbitrary) equilibrium belief in the sequence $q^{(J+1)*} < q^J_{t(J)}$. At $q^J_{t(J)}$, the only country action which can secure a loan with positive probability is $y(q^J_{t(J)}) = 1$, rendering the bank indifferent between lending and not. To form part of an equilibrium, $y(q^J_{t(J)}) = 1$ must be an optimal action for the country at this level of belief. Indeed, this action weakly dominates all others if $\delta U(1) \geq 1+r$. To see this, notice that the expected return from $y(q^J_{t(J)}) = 1$ is $\gamma(x-r+\delta U(1))$, whereas any other action $y' < 1$ yields $\gamma y'(x-r+\delta U(1)) + \gamma(1-y')(1+x)$. For $y(q^J_{t(J)}) = 1$ to be optimal requires that the country cannot unilaterally switch from this action in order to achieve a better payoff. Clearly, the return to always repaying when able at level of belief

$q^J_{t(J)}$ is always at least as good as that from any other action as long as $\delta U(1) \geq 1+r$.[16]

Assuming that this condition is met, then $q^J_{t(J)}$ and $y(q^J_{t(J)}) = 1$ can form a part of the lending equilibrium. If $q^J_{t(J)}$ is a part of the equilibrium, Definition 5.3 states that this level of belief must be reached following revision of the prior $q_{t(J-1)}$. In other words, the country action at level of belief $q_{t(J-1)}$ must lie on the J'th step locus. We can choose $z(q^J_{t(J)})$ in order to ensure that the type I borrower has the correct incentive to play on the J'th step locus.[17]

Imagine now that the level of belief $q^{J-1}_{t(J-1)}(X)$ in Figure 5.1 is part of the equilibrium sequence Q^*. In order for the pair $q^J_{t(J)}$, $y(q^J_{t(J)}) = 1$ to be a part of the equilibrium, we must ensure that beliefs are revised from $q^{J-1}_{t(J-1)}(X)$ to $q^J_{t(J)}$. Clearly this involves the country selecting action $0 < y^*(q^{J-1}_{t(J-1)}(X)) < 1$ in the figure. Such an action is only optimal if the country is indifferent at this level of belief between repaying and repudiating. Country indifference requires that $x-r+\delta U(1) = 1+x+\delta U(q^J_{t(J)})$ which can be ensured by appropriate setting of $z(q^J_{t(J)})$ which determines $U(q^J_{t(J)})$. Again, $y^*(q^{J-1}_{t(J-1)}(X))$ is at least as good as any other action as long as $\delta U(1) \geq 1+r$. When the country plays $y^*(q^{J-1}_{t(J-1)}(X))$, the bank is indifferent between extending and withholding credit when it holds belief $q^{J-1}_{t(J-1)}(X)$. This implies that $z(q^{J-1}_{t(J-1)}(X))$ can be set to make the country indifferent at the level of belief immediately preceding $q^{J-1}_{t(J-1)}(X)$ in the equilibrium sequence of beliefs.

---

[16] If this were not the case, then even the type I country would never wish to repay so that a lending equilibrium could not exist.

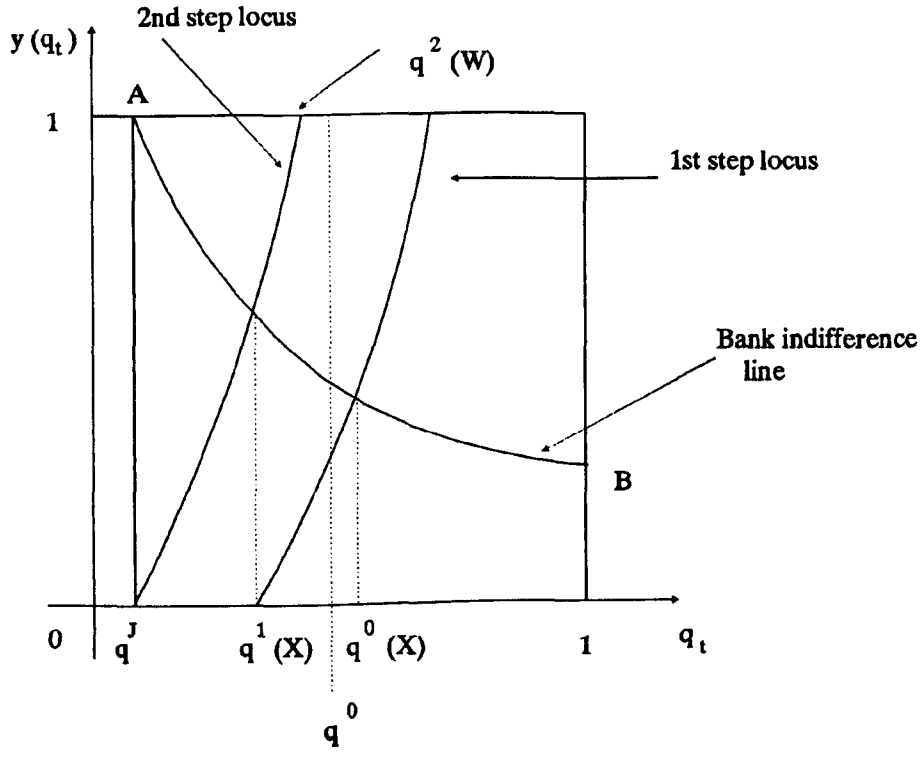
[17] Recall that $z(q^J_{t(J)})$ affects the repayment preferences of the country at level of belief $q_{t(J-1)}$. As we have constructed bank indifference at $q^J_{t(J)}$, we can choose $z(q^J_{t(J)})$ as required.

We must next examine what will happen in Figure 5.1 if the belief immediately preceding $q^J_{t(J)}$ falls in the region between $q^J_{t(J)}$ and $q^{J-1}_{t(J-1)}(X)$. Clearly, attention should be restricted to points on and above the line of bank indifference as we wish the equilibrium to exhibit lending. In this region, it is easily verified that the optimal country I action is to repay with certainty when able as long as $\delta U(1) \geq 1+r$. Notice that if the initial prior belief in time period 0 falls in the region between $q^J_{t(J)}$ and $q^{J-1}_{t(J-1)}(X)$, then $q^J_{t(J)}$ will not form a part of the equilibrium sequence of beliefs. For all other starting values of q, the equilibrium can be constructed so that $q^J_{t(J)}$ is a part of the equilibrium belief sequence.

The recursive method outlined above can be used to determine actions and beliefs which satisfy Definition 5.3. We have seen that unless $\delta U(1) \geq 1+r$, there will be no MPE which exhibits lending. One further problem with the existence of this type of equilibrium also occurs for some initial levels of belief q^0_0. From the previous argument, the pair $q^{J-1}_{t(J-1)}(X)$, $y^*(q^{J-1}_{t(J-1)}(X))$ will form a part of the equilibrium, but only if this level of belief can be reached from all levels of belief which immediately precede $q^{J-1}_{t(J-1)}(X)$. In Figure 5.1, it is possible that the initial level of belief is q^0_0. In this case, it is not possible to reach $q^{J-1}_{t(J-1)}(X)$ as so doing requires a country action of $y(q^0_0)$ which is below the bank indifference line. This action is inconsistent with a loan being granted, so that any lending equilibrium would break down at this point. This existence problem occurs as q^0_0 falls in the region between the point where the J'th step locus reaches y = 1 and the point where the J-1'st step locus crosses the line depicting bank indifference. In general, this problem of non-existence of a lending equilibrium occurs for $q^0(X) > q^0 > q^2(W) \geq q^J(W)$, as in Figure 5.2, where $q^j(W)$

indicates the level of belief at which the j'th step locus reaches y = 1.

Figure 5.2

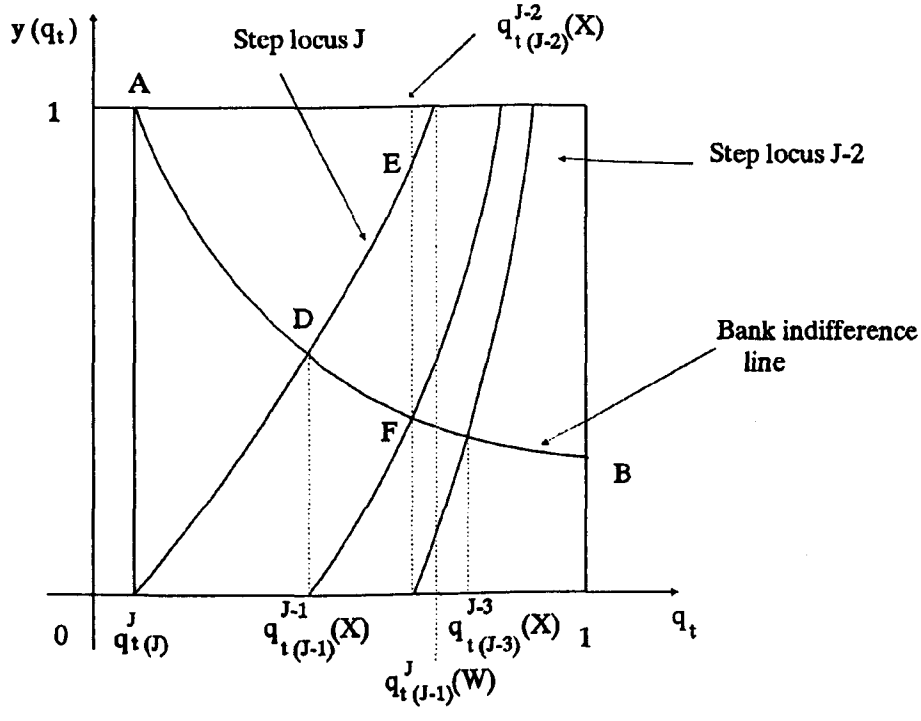


Solving this problem requires imposing a parameter restriction which effectively 'bends' the step loci according to Figure 5.3.

Assume again that $q^{J-1}_{t(J-1)}(X)$ and $q^{J}_{t(J)}$ form a part of the equilibrium sequence of beliefs and that the prior belief at the very beginning of the game q^0_0 does not fall between these points. For any initial level of belief between $q^{J-3}_{t(J-3)}(X)$ and $q^{J-2}_{t(J-2)}(X)$, the equilibrium sequence of $q^{J-1}_{t(J-1)}(X)$ and $q^{J}_{t(J)}$ can be continued by the type I country selecting the action on step locus J-1 which corresponds to the current level of belief.

An observed default would then take the game along the equilibrium path via points D and then A if default is observed again

Figure 5.3



Assume now that the level of belief is $q^{J-2}_{t(J-2)}(X)$. The equilibrium then follows points F, D and A if default is observed after each loan. In the situation depicted in Figure 5.3, a lending equilibrium can exist which will not completely break down until the critical level of belief is passed. Apparently, the crucial difference between Figures 5.1 and 5.3 lies with the points where the step loci cross the line of bank indifference. Solving the problem means that the first step locus (j = 1) crosses the

bank indifference line at a lower level of belief than where the second step locus (j = 2) reaches the point y = 1. In Figure 5.3, $q^{J-2}_{t(J-2)}(X) < q^{J}_{t(J-1)}(W)$. [18] Therefore, the parameter restriction which gives rise to Figure 5.3 restricts the size of the jumps between the members of the belief sequence.[19] The restriction needed for this situation to obtain is given in Lemma 5.1

LEMMA 5.1 For $q^{J}_{t(J)} \leq q^{0}_{0} \leq q^{J}_{t(J-1)}(W)$ or $q^{1}_{t(0)}(W) \geq q^{0}_{0} \geq q^{0}_{t(0)}(X)$, a MPE with lending exists if a lending equilibrium exists with a type I country in the game of complete information and

(1) $\delta U(1) \geq 1+r$ and

(2) $\gamma > 1/[1+r+\delta V(1)]$.

For all other values of q^{0}_{0}, i.e. $q^{0}_{t(0)}(X) > q^{0}_{0} > q^{2}_{t(1)}(W) \geq q^{J}_{t(J-1)}(W)$, the existence of a MPE with lending requires additionally that

$$\gamma \geq \frac{[\,2 + r + \delta V(1)\,]\,[\,1 + 2(r + \delta V(1))\,] - [\,1 + r + \delta V(1)\,]^2}{[\,1 + r + \delta V(1)\,]\,[\,1 + 2(r + \delta V(1)\,]} \tag{5.8}$$

<u>Proof</u> See Appendix 5.1.

[18] The reason that $q^{J}(W)$ has a t(J-1) time subscript is that the J'th revision takes beliefs to q^{J} at time S(J-1)+1. When beliefs are at $q^{J}(W)$, the J'th revision has not yet occurred.

[19] The weakest requirement is that $q^{0}_{0} = q^{2}_{t(1)}(W)$.

We are now in a position to state the main result of this chapter.

PROPOSITION 5.1 Let q^{J+1} be the equilibrium level of belief which follows $q^J_{t(J)}$, with $q^J_{t(J)}$ defined as in Result 5.2. Assuming that the existence conditions from Lemma 5.1 are met, a MPE with lending and learning has the following bank and type I country actions and expected payoffs at each level of belief.

i) $q = q^J_{t(J)}$; for $t \in t(J)$

$$z(q_t^J) = \frac{[1 - \delta]\,[\delta U(1) - (1 + r)]}{\delta\,[\,1 + x - (1 - \gamma)\,(x - r + \delta U(1))\,]}$$

$$y(q_t^J) = 1$$

$$V(q_t^J) = 0$$

$$U(q_t^J) = U(1) - \frac{(1 + r)}{\delta}$$

Beliefs revised to q^{J+1} on default (game ends).

ii) $q^{J}_{t(J)} < q_t < q^{J-1}_{t(J-1)}(X)$; for $t \in t(J-1)$

$$z(q_t) = 1$$

$$y(q_t) = 1$$

$$V(q_t) = q_t \gamma \ (1 + r + \delta V(1)) - 1$$

$$U(q_t) = \gamma \ (x - r + \delta U(1))$$

Beliefs revised to q^{J+1} on default (game ends).

iii) $q = q^{j}_{t(j)}(X)$, $j = 1,\ldots, J-1$; for $t \in t(j)$

$$z(q^j_t(X)) = \frac{[1 - \delta] \ [\delta U(1) - (1 + r)]}{\delta \gamma \ (1 + x)}$$

$$y(q^j_t(X)) = \frac{1}{\gamma q^j_t(X) \ (1 + r + \delta V(1))}$$

$$V(q^j_t(X)) = 0$$

$$U(q^j_t(X)) = U(1) - \frac{(1 + r)}{\delta}$$

Beliefs revised to q^{j+1} on default (if $q^{j+1} = q^J$ then (i), if not then $q^{j+1} = q^{j+1}(X)$ so (iii))

iv) $q^{j-1}_{t(j-1)}(X) < q_t < q^{j}_{t(j)}(X)$, $j = 1,\ldots, J-1$; for $t \in t(j)$

$$z(q_t) = 1$$

$$y(q_t) = \frac{q_t - q^{j+1}_{t(j+1)}(X)}{\gamma q_t \, [1 - q^{j+1}_{t(j+1)}(X)]}$$

$$V(q_t) = q_t \, \gamma \, y(q_t) \, (1 + r + \delta V(1)) - 1$$

$$U(q_t) = \gamma(1 + x) - (1 + r) + \delta U(1)$$

Beliefs revised to q^{j+1} on default (if $q^{j+1} = q^{J}$ then (i), if not then $q^{j+1} = q^{j+1}(X)$ so (iii)).

<u>Proof</u> See Appendix 5.2.

Let us examine how these strategies work. The action and belief sequences depend crucially on the level of belief at the beginning of the game, q^0_0 (held for s(0)+1 periods until a loan is granted). Assume for example, that q^0_0 coincides with $q^{J-2}(X)$. The equilibrium belief sequence, assuming that each loan is repudiated, is $Q^* = \{ q^{J-2}(X), q^{J-1}(X), q^{J}, q^{J+1} \}$. Initially, the players choose their action according to iii) in the proposition. The bank randomizes between loaning and not and a loan is made after s(0)+1 periods. The type I country repays with the probability given in iii), and the belief is revised to $q^{J-1}(X)$ where iii) is again used as part of the equilibrium strategy. The next revision of the belief is to $q^{J}(X)$ in period S(J-1)+1 so that actions are chosen according to i).

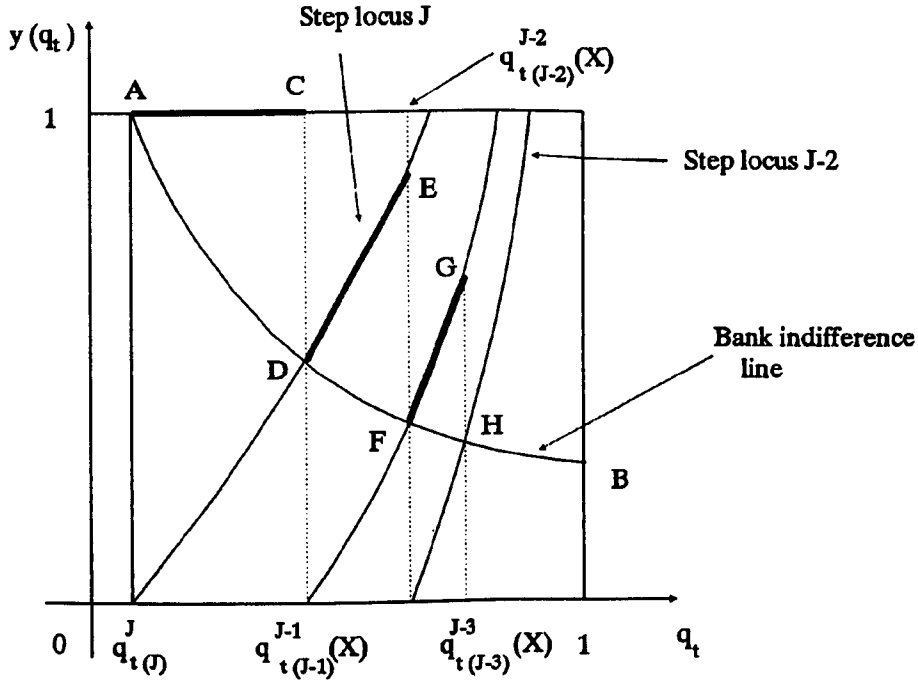
Assume now that $q^{J-2}(X) < q^0_0 < q^{J-3}(X)$. The play accords initially to iv) so that the first revision of the belief leads to $q^{J-1}(X)$ and the players play according to

iii). The equilibrium belief sequence is thus $Q^* = \{ q^0, q^{J-1}(X), q^J, q^{J+1} \}$. The strategies work then by making the equilibrium belief sequence correspond to those levels of belief at which the bank indifference line is crossed by the step loci. The exception to this is the case where $q^J < q^0 < q^{J-1}(X)$ which gives rise to the sequence of beliefs $Q^* = \{ q^0, q^{J+1} \}$, and play according to ii). In this case only one loan will be made until the game of private information ends.

In Figure 5.4, heavy lines depict the equilibrium country I actions for all values of q up to $q^{J-3}(X)$. The equilibrium country I actions which conform to the equilibrium belief sequence are marked by the intervals AC, DE, and FG for values of q below $q^{J-3}(X)$. These intervals are closed on the left and open on the right. The country prefers F to E as this gives it access to at least three loans whereas E gives only two until the incomplete information structure ends.

At each stage of the game, except possibly the first period if $q^0_0 \neq q^0(X)$, the bank is indifferent between loaning and withholding funds. Each successive default causes the reputation of the borrower to fall. As this reputation falls, so the country seeks to repay with a higher probability. The temptation to masquerade as a type P country is greatest at high levels of belief, and falls progressively as the critical level of belief and the complete credit embargo are neared. Breakdowns and resumptions of voluntary lending can occur until the critical belief is passed, in which case the country faces financial autarky.

Figure 5.4



5.6 An example of a lending MPE with learning

In order to see exactly what this model predicts, it is useful to carry out some numerical analysis. This section takes some parameters for the type I borrower and examines the MPE which arises for different levels of initial belief q^0_0.

Example 5.1: $V(1) = 0$, $U(1) = 5$, $\gamma = .99$, $x = .25$, $r = .15$, $\delta = .99$

The conditions in Lemma 5.1 are satisfied so a lending equilibrium exists. These parameters yield $q^J = .8783$, $q^{J-1}(X) = .9841$, $q^{J-2}(X) = .9979$ and $q^{J-3}(X) = .9997$. The equilibrium path depends upon the initial belief held by the bank. We take three cases here, corresponding to q^0 in the region defined by ii), iii) and iv) in Proposition 5.1.

<u>Case 5.1.1:</u> $q^0 = .98$

This initial value corresponds to ii) in Proposition 5.1 as $q^J < q^0 < q^{J-1}(X)$. The equilibrium sequence of beliefs, bank and type I country actions are (for any $q^{J+1} < .8783$):

$Q^* = \{ .98_{t(0)}, q^{J+1}{}_{t(1)} \}$ (following default)

$Z^* = \{ 1_{t(0)}, 0_{t(1)} \}$

$Y^* = \{ 1_{t(0)}, + \}$ (+ indicates undefined)

where $t(0) = \{ 0 \}$, $t(1) = \{ 1,.., \infty \}$. In this case $J = 0$ as beliefs fall immediately below the critical level on the first revision. The expected payoffs are $V(.98) = .1157$, $U(.98) = 4.9995$.

The initial level of belief in this case corresponds to a point somewhere between A and C in Figure 5.4. In the first period, the bank expects a positive profit so grants a loan with certainty and the country repays if it is able. Default, caused

only by investment failure in this equilibrium, leads to an irreversible breakdown of lending from period two onwards. This leads to the fact that U(.98) < U(1).

<u>Case 5.1.2:</u> $q^0 = .9997$

This initial belief corresponds to the point where the J-2nd step locus crosses the bank indifference line. Therefore iii) from Proposition 5.1 is used until the belief is revised as far downwards as q^J when i) is then used. This gives rise to the following equilibrium:

$$Q^* = \{ .9997_{t(0)}, .9979_{t(1)}, .9841_{t(2)}, .8783_{t(3)}, q^{J+1}_{t(4)} \}$$

$$Z^* = \{ .031_{t(0)}, .031_{t(1)}, .031_{t(2)}, .032_{t(4)}, 0_{t(4)} \}$$

$$Y^* = \{ .8786_{t(0)}, .8802_{t(1)}, .8925_{t(2)}, 1_{t(3)}, + \}$$

The number of belief revisions until the critical level of belief is reached is J = 3 in this case, indicating that a maximum of four loans will be made until the game of incomplete information ends. A loan is made initially with a small probability so that $t(0) = \{ 0,..., S(0) \}$, $t(1) = \{ S(0)+1,..., S(1) \}$ and so on until $t(4) = \{ S(3)+1,..., \infty \}$.

In terms of Figure 5.4, the game begins at H and proceeds through F, D and A assuming default is observed. From the outset, the bank is indifferent between loaning and not. Indeed a loan is forthcoming with a small probability so that the time which elapses between loans may be long. Bank indifference throughout the

game gives the bank an expected total payoff of zero. Notice that at the critical belief, the loan probability increases slightly. This is due to the fact that, with a permanent credit embargo imminent, the bank can only induce good behaviour in the previous period by a higher setting of the loan probability at the critical level of the belief.

Knowing that at most four loans will be made until borrower type is revealed, the type I country increases its willingness to repay with each successive loan. It does this in order that it will not lose access to the credit market by attempting to reveal its type before the level of belief is below the critical level. Notice that country insouciance may be partly responsible if any of the first three loans are repudiated. The total expected country payoff in this case is 3.834 which is below U(.98) and U(1) as long periods may elapse between loans.

<u>Case 5.1.3:</u> $q^0 = .9985$

This initial level of belief lies between $q^{J-2}(X)$ and $q^{J-3}(X)$ so that the initial actions are prescribed by iv) in Proposition 5.1. After an initial loan, the type I country plays an action on the J-1st step locus taking the game to $q^{J-1}(X)$ and iii) in the proposition. The next revision of the belief is to $q^J(X)$ so that the actions then correspond to i).

With this initial belief, the equilibrium is as follows:

$$Q^* = \{ .9985_{t(0)}, .9841_{t(1)}, .8783_{t(2)}, q^{J+1}{}_{t(3)} \}$$

$$Z^* = \{ 1_{t(0)}, .031_{t(1)}, .032_{t(2)}, 0_{t(3)} \}$$

$$Y^* = \{ .9162_{t(0)}, .8925_{t(1)}, 1_{t(2)}, + \}$$

where $t(0) = \{ 0 \}$, $t(1) = \{ 1,..., S(1) \}$, $t(2) = \{ S(1)+1,..., S(2) \}$, $t(3) = \{ S(2)+1,..., \infty \}$, $J = 2$.

The starting point for this equilibrium is above the bank indifference line so that it strictly prefers to lend in period one. At level of belief q^0, the bank expects a payoff of .0415. The first loan is repaid with a high probability in order to conform to the equilibrium belief path. The bank makes two further loans as outcomes of its randomization process. The country sets a high probability of repayment in order to avoid an embargo, with a small chance of it masquerading as a type P borrower.

5.7 Remarks

In order that a lending equilibrium can exist when the bank has incomplete information about its type of debtor, the conditions in Lemma 5.1 must be satisfied. This cannot be achieved unless $U(1) > 0$. In other words, the existence of a lending equilibrium to the game of complete information in chapter three is a pre-requisite for a lending equilibrium under incomplete information. Indeed, when the bank randomizes, corresponding to levels of belief at i) or iii) in Proposition 5.1, the loan probability is increasing in $U(1)$. As there are likely to be multiple equilibria in the game of complete information, the most efficient outcome in the complete information

game yields the best outcome in the incomplete information structure if the equilibrium proceeds through i) and/or iii).[20]

The example presented indicates that parameters satisfying all of the conditions are likely to give a high critical belief which is reached after a small number of loans, so that there is an endogenous limit placed on the tolerance of the bank to default by a debtor. The equilibrium is dependent upon the level of belief with which the bank begins the game. Indeed, for each initial belief there is only one lending equilibrium which contrasts with the multiple equilibria found in the learning model of Lang and Nakamura (1989). The overall expected payoffs are thus dependent on the initial belief, with the bank being able to expect to earn a small profit when the initial belief is above the line of bank indifference. With such a prior, the country achieves a relatively high expected payoff as the bank loans with certainty in the first period. When the prior lies on the bank indifference line, long lengths of time may elapse between loans which reduces the expected payoff of the country due to discounting.

Alongside the initial belief, the number of loans preceding an embargo depends on the distance of the jumps in the belief sequence. From (5.6) it is easily seen that the distance between the intersections of the step and bank loci is increasing in the interest rate. As these intersections crucially determine the path of the equilibrium, beliefs are revised down quicker the larger is the interest rate. This appears to suggest a reaction to adverse selection on the part of the bank. As the interest rate

[20] Other comparative statics are identical to chapter three.

increases,[21] the less willing is the type I country to contract debt so that it is most likely that the bank is lending to a type P customer who does not care about the interest rate as it never repays. Thus the belief is revised down quickly.

The equilibrium sequence of bank actions indicates that lending breakdowns and resumptions can occur, at least up until the critical level of belief is surpassed. This is important as it is a feature which will certainly be missed by models of sovereign lending which assume that default calls forth an immediate credit embargo. Indeed, the fact that, at least up to a point, default does not preclude further loans, suggests that this model has extended a weaker version of Grossman and van Huyck's (1988) excusable default to a scenario with incomplete information and possible country insouciance. A country can fail to repay through choice and not be totally excluded from future loans until the critical level of belief. Default is always punished, however, in the form of a small loan probability in the following period.

As the example predicts that only a small number of loans will be made, a type I country sets a high repayment probability in order to reveal its type before the critical belief is surpassed. In the complete information case of chapter three, the country examined repays with a high constant probability in every period. The model presented in this chapter looks at the dynamics of the country's reputation for creditworthiness so that the repayment probability changes through time. It is still possible, in principle, that a type I country will wish to masquerade as a type P debtor.

[21] At some stage the interest rate will be so high that a lending equilibrium will cease to exist altogether.

This temptation falls over time as the critical level of belief, and permanent credit embargo, is neared and the type I country becomes more concerned with its reputation for creditworthiness.

5.8 Conclusion

This chapter has extended the model of chapter three to allow for the possibility that the bank can lend to a sovereign nation without knowing its inherent 'type'. Type can be interpreted in a variety of ways. Here, a type P country was assumed not to invest its loan whereas type I responsibly invested its funds obtained from the international credit market. Alternatively, one could think of type as relating to political stability so that debt repudiation by a type P country is due to political instability. A further possibility is that a type P country is inefficient and characterized by unproductive redistributional activity carried out by interest groups. This idea is explored in the next two chapters.

We have shown that in order for a lending MPE to exist when the bank has incomplete information, a lending MPE must exist when the bank knows its customer type. The bank, in other words, must want to learn its borrower's type. An additional existence condition, which is stronger than those in the model of chapter three, must be met in order to generate a lending MPE with incomplete information from some levels of initial belief.

When lending takes place, the bank has an opportunity to learn its type of

customer. Therefore, we have been able to examine the dynamics of a country's reputation for creditworthiness over time when an externality is imposed by the existence of inherent defaulters. A feature of the MPE is that lending can break down and resume as part of the equilibrium, at least up to some critical level of belief. Models which assume the imposition of a credit embargo immediately following a single default will miss this important point. Indeed, it is possible that a period of non-lending can be followed by voluntary commercial bank loans. Thus, not all lending following a temporary lapse is defensive in the sense of Krugman (1985). Additionally, 'renewed' lending may not just be observed after a lending equilibrium has broken down as in Aizenman and Borensztein (1989). The model presented in this chapter has as its equilibrium the feature of 'renewed' lending after a period where funds have been withheld. This has also been found by Cole, Dow and English (1989) in a different context.

The existence of uncertainty regarding the type of borrower makes it possible, in principle at least, for a type I country to masquerade as an inherent defaulter in order to save on its debt repayments. Indeed, some default may be through choice rather than necessity. As the critical level of belief is neared and the bank believes to be close to learning its type of customer, the type I country will wish to reveal its type in order to avoid a credit embargo. The possibility that even a type I borrower is excluded from future loans is not ruled out as it may be prevented from revealing its type in time due to investment failures, contrasting with Cole, Dow and English (1989) where the stable borrower always has funds to reveal its type.

APPENDIX 5.1 - Proof of Lemma 5.1

We shall suppress time subscripts for ease of notation. That $\delta U(1) \geq 1+r$ is required for a lending equilibrium to exist is explained in the text. That this is necessary for a lending equilibrium to exist if $q^J \leq q^0_0 \leq q^J(W)$ is also explained in the text. For $q^1(W) \geq q^0 \geq q^0(X)$, the game proceeds along the intersections of the step loci and the bank indifference line after the first action which may lie on the first step locus. Therefore, the game eventually arrives at point A in Figure 5.3. At this point, the type I country can only persuade the bank to loan if $y = 1$ which, as explained in the text, is an optimal move if $\delta U(1) \geq 1+r$.

We now derive equation (5.8). Let $q^j(W)$ denote the level of belief at which the j'th step locus attains the value $y(q^j(W)) = 1$. For the existence of a lending equilibrium we must rule out the case $q^0(X) > q^0 > q^2(W) \geq q^J(W)$. We can do this by ensuring $q^0(X) \leq q^2(W)$.

From $q^0(X)$, the next belief in the sequence in a lending equilibrium will be constructed to be $q^1(X)$. From Bayes' rule

$$q^1(X) = \frac{q^0(X)\,[1 - \gamma y(q^0(X))]}{1 - q^0(X)\gamma y(q^0(X))} \tag{A5.1}$$

and by bank indifference at $q^0(X)$ we obtain $y(q^0(X))$ which can be substituted into (A5.1) to yield

$$q^1(X) = \frac{q^0(X)(1 + r + \delta V(1)) - 1}{r + \delta V(1)}$$

(A5.2)

From $q^1(X)$, the next belief in the equilibrium sequence will be $q^2(X)$ so that

$$q^2(X) = \frac{q^2(W)(1 - \gamma)}{1 - \gamma q^2(W)} = \frac{q^1(X)(1 + r + \delta V(1)) - 1}{r + \delta V(1)}$$

(A5.3)

Substituting (A5.2) into (A5.3) yields

$$q^0(X) = \frac{[r + \delta V(1)][1 + q^2(W)[(1 - \gamma)(r + \delta V(1)) - \gamma]]}{(1 - \gamma q^2(W))(1 + r + \delta V(1))^2} + \frac{1}{(1 + r + \delta V(1))}$$

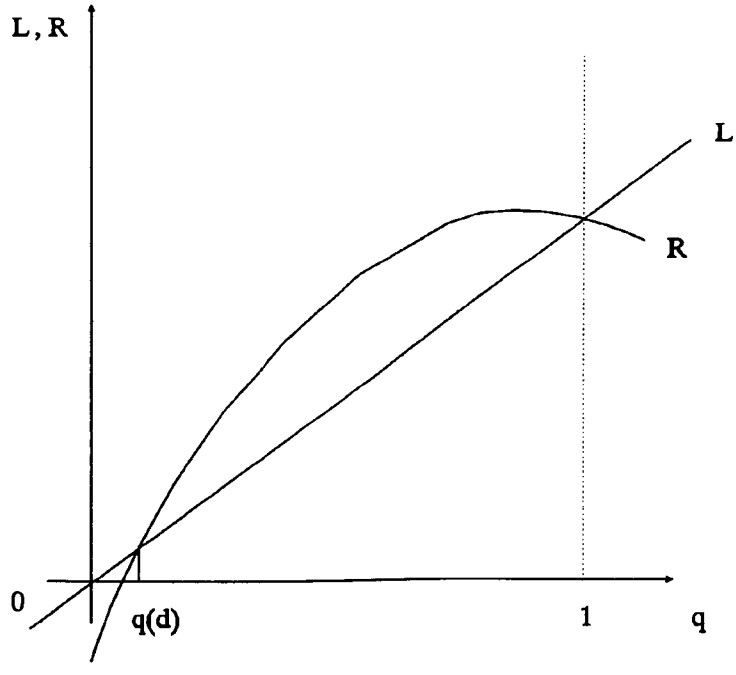
Clearly $q^0(X) \leq q^2(W)$ if

$$q^2(W)(1 - \gamma)(r + \delta V(1))^2$$
$$\leq (1 - \gamma q^2(W))[q^2(W)(1 + r + \delta V(1))^2 - 2(r + \delta V(1)) - 1]$$

(A5.4)

Let (A5.4) be written as $L \leq R$. For $q^2(W) = 1$, $L = R$. The following can be shown to be the case.

Figure A5.1



There is an interval from 0 to q(d) where L > R. As we can construct a lending equilibrium for $q^J \leq q^0 \leq q^J(W)$ by the discussion in the text, the fact that L > R for the interval 0 to q(d) is of no consequence as long as $q^J(W) \geq q(d)$. The value of q(d) can easily be determined in (A5.4) using $q^2(W) = 1$ as one root, so that

$$q(d) = \frac{1 + 2(r + \delta V(1))}{\gamma\ (1 + r + \delta V(1))^2} \tag{A5.5}$$

Using Result 5.2

$$q^J(W) = \frac{1}{\gamma\ [(1 - \gamma)\ (1 + r + \delta V(1)) + 1]}$$

A necessary condition for a lending equilibrium to exist if $q^0(X) > q^0 > q^2(W) \geq q^J(W)$ is that $q^J(W) \geq q(d)$ which is given as equation (5.8). This ensures that if $q^0 > q^J(W)$, it cannot be the case that $q^0(X) > q^0 > q^2(W)$ because equation (5.8) restricts $q^0(X)$ and $q^2(W)$ so that $q^0(X) \leq q^2(W)$. ■

APPENDIX 5.2 - Proof of Proposition 5.1

i) $q = q^J_{t(J)}$ reached at time S(J-1)+1.

Fix an action for the bank $z(q^{J+1})$ and country $y(q^{J+1})$ at level of belief $q^{J+1} < q^J$, reached in time period t = S(J)+1. By the definition of $q^J_{t(J)}$, it must be that $z(q^{J+1}) = 0$ and hence $U(q^{J+1}) = 0$.

It was established in the text that $y(q^J_{t(J)}) = 1$ is optimal for the type I country if $\delta U(1) \geq 1+r$. From Result 5.1, this makes the bank indifferent between lending and not at level of belief $q^J_{t(J)}$. Therefore, as explained in the text, $z(q^J_{t(J)})$ is chosen to ensure that the country randomizes between repaying and repudiating at the previous level of belief. This requires that $x - r + \delta U(1) = 1 + x + \delta U(q^J_{t(J)})$ which can be written as

$$1 + x + \frac{\delta z(q^J_{t(J)})\ [\ \gamma\ (x - r + \delta U(1))\ ]}{1 - \delta[\ 1 - z(q^J_{t(J)})\ ]} = x - r + \delta U(1)$$

which delivers $z(q^J_{t(J)})$ as stated. The expected payoffs are a consequence of bank and type I country indifference.

ii) $q^J_{t(J)} < q_t < q^{J-1}_{t(J-1)}(X)$; $t \in t(J-1)$

For this q, it is the case that the revised belief on default is q^{J+1} so that $z(q^{J+1}) = V(q^{J+1}) = U(q^{J+1}) = 0$ on updating of the belief. Again repayment is optimal i.e. $y = 1$ as long as $\delta U(1) \geq 1+r$. The corollary to Result 5.1 and the fact that $V(q^{J+1}) = 0$ imply that the bank's optimal response is $z(q) = 1$. Payoffs are obtained by substitution.

iii) $q = q^j_{t(j)}(X)$, $j = 1,.., J-1$; $t \in t(j)$

This states that the belief is at one of the points at which the step locus crosses the bank indifference line. We prove the case for $j = J-1$ with other cases being analogous. From $q^{J-1}_{t(J-1)}(X)$, the next belief in the equilibrium sequence is $q^J_{t(J)}$. We have already set $z(q^J_{t(J)})$ so that the country is indifferent between repaying and not at any level of belief which directly precedes $q^J_{t(J)}$. Given this indifference, the type I country can do no better than to play $y(q^{J-1}_{t(J-1)}(X))$ at the intersection between the bank indifference line and the J'th step locus. This makes the bank indifferent between lending and not at level of belief $q^{J-1}_{t(J-1)}(X)$, so $z(q^{J-1}_{t(J-1)}(X))$ is set in order to make

the type I country indifferent between defaulting and repaying at the preceding level of belief $q_{t(J-2)}$. This gives the action stated in the proposition. Payoffs are again due to player indifference.

iv) $q^{j-1}_{t(j-1)}(X) < q_t < q^{j}_{t(j)}(X)$, $j = 1,..., J-1$; $t \in t(j)$.

Again we prove the case $j = J-1$ so that $q^{J-2}_{t(J-2)}(X) < q_t < q^{J-1}_{t(J-1)}(X)$. The next belief in the equilibrium sequence for such a q is again $q^{J}_{t(J)}$. By the proof to i), the country is thus indifferent between repaying and defaulting at this level of belief, so that $y(q_t)$ is chosen to ensure that $q^{J}_{t(J)}$ is reached upon default. The type I country action is thus on the J'th step locus which induces the bank to lend with certainty as this action is above the bank indifference locus. Payoffs are obtained by substitution. ■

CHAPTER 6. EFFICIENT ENDOGENOUS POLICY IN A LOBBYING MODEL

As indicated briefly in the previous chapter, the type P country can be thought of as an economy characterized by 'rent-seeking' activity.[1] This and the next chapter seek to depict a rent-seeking economy in political equilibrium. In particular, the approach adopted here combines some of the elements of rent-seeking theory with some ideas from the literature on political competition to produce a model of endogenous policy based on Brock and Magee (1978).[2] To this end, some of the central results from the rent-seeking literature are firstly discussed, followed by a brief account of the theory of political competition. Combining these two elements is the probabilistic voting model of Brock and Magee (1978) and Magee and Brock (1983) which forms the basis for the work undertaken in the remainder of this thesis. A general representation of this model is presented and a major problem is indicated in its underlying framework. In adjusting the model, some of those authors' conclusions are demonstrated not to hold.

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[1] I am grateful to Professor William A. Brock, University of Wisconsin-Madison, for this interpretation and for arousing my subsequent interest in this topic.

[2] An excellent summary of endogenous policy theory is Magee, Brock and Young (1989).

6.1 Rent-seeking and DUP activities

The term 'rent-seeking' originates in Krueger (1974). In a survey article, Tollison (1982) indicates that 'rents' can emanate from two sources: i) naturally in the price system by, for instance, shifts in demand and supply; ii) rents can be artificially contrived through, for example, government action. The theory of rent-seeking studies the competition among agents for these contrived rents.

Agents can act individually or be organized into interest groups or lobbies who collectively fight for political influence and a share in the rent created. Terrones (1990) analyzes a one-good neo-classical growth model in which individual agents divide up their labour time between work, human-capital investment and engaging in activity to capture a government transfer. The amount of the transfer obtained depends positively on the agent's own level of 'influence' activity and negatively on the total emanating from all other agents. He demonstrates that both the level and rate of growth of output in the economy are related inversely to the amount of redistributive activity undertaken, so that cross-country differences in growth can be partially explained by the presence/absence of rent-seeking activity. Instead of acting individually, agents may get together into interest groups or lobbies in order to achieve political pressure. A proof of the existence of a lobbying economy in competitive general equilibrium has recently been provided by Coggins, Graham-Tomasi and Roe (1991) in which agents are not price takers but rather a central authority sets prices on the basis of lobbying pressure from two groups with opposed interests.

The welfare implications of agents attempting to capture an artificially contrived rent have been extensively examined. Tollison (1982) illustrates the negative effect on welfare in the case of a government granting a monopoly right to one of a number of firms. Traditionally, the welfare loss was taken to be small - the Harberger (1954) triangle at less than 1% of U.S. GNP in the U.S. manufacturing sector in 1929. Tullock (1967) suggested a further welfare loss which would be caused due to the resources spent by the firms on trying to capture the monopoly right. Indeed, Krueger (1974) shows that if there were perfect competition for this rent, then the total amount spent on rent-seeking will equal the total value of the monopoly right. She estimates that these rents may be high: 7.3% of national income in the Indian public sector in 1964, with the rents for Turkish import licences being worth 15% of GNP in 1968. Posner (1975) finds that approximately 3% of GNP must be added to the Harberger triangle in order to get a true picture of the welfare losses due to monopoly. It is further possible that consumers could join together in an anti-monopoly lobby in which case these costs too must be added to the Harberger and Posner type losses.

Krueger (1974) presents a model of competitive rent-seeking among firms for a fixed number of import licences in a small country which produces and exports food and imports consumption goods subject to the quantitative restriction.[3] In her model, labour can be used to produce food, distribute goods or it can be employed in attempting to capture an import licence. Under free trade, the economy operates efficiently on the consumption possibility frontier, whilst an exogenously imposed

[3] Bhagwati (1982) labels this 'premium seeking'.

tariff or import restriction also yields an equilibrium on this frontier, albeit at a lower level of imports. The case of agents competing to obtain an exogenously fixed number of import licences is then considered under the assumption that there is free entry into rent-seeking[4]; labour is devoted to rent-seeking activity and the economy operates inside its feasible region in equilibrium. Under the free entry assumption, such competitive rent-seeking is shown by Krueger to lead to a welfare loss equal to the total amount of the rent.

There has been much debate in the literature about the welfare effects of rent seeking activity; Olson (1982) speaks of the welfare reducing effects of political competition over redistribution, but Pryor (1984) fails to find empirical evidence to support this and Brock and Magee (1984) examine three theoretical models in which the Olson conjecture fails to hold. Indeed, Bhagwati (1980) reverses the welfare results of Krueger (1974), creating the apparent paradox that lobbying actually *increases* welfare. The problem is that Krueger compares the equilibrium with an exogenous import restriction and lobbying to the (first best) case of free trade, whereas Bhagwati (1980) notes that the lobbying solution should be compared to the (second best) situation in which the distortion already exists; the effect on welfare of adding another distortion to an already distorted outcome is ambiguous. By simply allowing lobbying for an exogenous rent, Krueger also misses an important point made by Bhagwati (1980) in which he uses a Heckscher-Ohlin trade model to show that an <u>endogenous</u> tariff which has been lobbied for can improve on an <u>exogenously</u> imposed

[4] Findlay and Wellisz (1984) regard this free entry assumption as the most serious weakness in this model as the right to rent-seek is often a jealously guarded privilege.

tariff in some cases.[5]

Bhagwati (1982) broadens the base of rent-seeking activities by proposing Directly Unproductive Profit-seeking (DUP - pronounced 'dupe') activities as an umbrella term for activities which yield gain without being productive in themselves. He proposes four categories of DUP and looks at the welfare consequences in each case. Category one captures initially distorted and finally distorted situations such as the premium-seeking example of Krueger (1974). As suggested earlier, second best analysis applies so that the paradox of a beneficial outcome is possible.[6] The second category deals with situations which are initially distorted but finally distortion-free, for example, a tariff destroying lobby. Again the beneficial outcome paradox is possible although it may also be the case that the resources used to eliminate the distortion are socially more valuable than the distortion-free outcome. Category three includes those situations which were initially distortion-free but finally distorted, such as the monopoly creating case discussed earlier; the total outcome is necessarily immiserizing. Lastly, lobbying in situations which are initially and finally distortion-free is immiserizing due to the resources spent.

A further classification is provided by Bhagwati, Brecher and Srinivasan (1984) who divide DUP activities into (i) situations in which the policy is exogenous while the DUP activity is endogenous to that policy and (ii) situations in which the policy

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[5] For a sceptical view of Bhagwati's results, see Tullock (1980).

[6] Except in the case of pure quantity distortions as shown by Bhagwati and Srinivasan (1982) and Anam (1982).

is the endogenous result of the interaction of the DUP activity with the economic system, for example the tariff seeking model of Brock and Magee (1978) and Young and Magee (1986). It is this latter category which shall be the focus of the new model presented in section 6.5.

6.2 Political competition

Models of competition among political parties can be distinguished by the assumption used regarding their preferences: following Downs (1957), parties could be office-motivated in that they care solely about gaining power, or alternatively they could have preferences concerning the intrinsic properties of economic policy, for example by representing the interests of different groups in society.[7] The latter, 'partisan' approach was first formalized by Wittman (1977) and has been applied to macroeconomics, and in particular economic cycles, by Alesina (1987, 1988) who represents each party as having different attitudes to inflation and the growth of output. Alesina and Tabellini (1989) use the partisan approach in order to explain capital flight in developing countries in terms of political uncertainty; the idea here is that a group of 'capitalists' will flight its capital in order to avoid high taxation should the party representing the 'workers' be elected into government. Alesina and Tabellini (1987) and Aghion and Bolton (1989) present models in which the parties can attempt to reduce the extent of partisan policies followed by opponent governments in the future through the use of fiscal deficits and government domestic debt; leaving a successor government with a deficit and debt to repay reduces its

[7] Alesina and Cukierman (1987) combine both of these motivations.

policy options.

Models which are based upon the assumption that the political parties' sole objective is to win an election yield the median voter theorem (Black (1958)) in which the policies of each party converge fully to the preferred policy of the median voter (assuming that voters' preferences are known). By moving closer to the position of the median voter, the parties hope to (probabilistically) capture the votes of those near the median. One of the better known models which assumes that parties maximize their chances of election is the political business cycle model of Nordhaus (1975) in which a government stimulates the economy in the period directly preceding an election, with a post-election recession occurring; this can occur before and after each election, as voters are myopic and irrational in the sense that they forget that they are fooled each election time. Correcting the failings of the Nordhaus paper, Rogoff (1990) develops a model of the political budget cycle in which voters are rational and the leader scores 'ego rents' for being in office.

Brock and Magee (1978), Magee and Brock (1983) and Young and Magee (1986) combine DUP actors in the form of lobbies with Downsian political parties in their 'probabilistic voting model' which shall form the basis for the remainder of this thesis.

6.3 A 'reaction' to the probabilistic voting model (Brock and Magee (1978))

Brock and Magee (1978) combine the rent-seeking sphere with a model of political competition in modelling the redistributive conflicts between two interest groups or lobbies, one of which prefers a tariff whilst the other has a preference for an export subsidy.[8] The endogenous policies of the two political parties clear the political market in much the same way that prices act as an equilibriating device in economic markets. In the partial equilibrium version of this model, there are assumed to be two political parties, two lobbies and a group of voters.[9] The voters are imperfectly informed and rationally ignorant in the sense of Downs (1957) so that they do not spend resources on information collection. It is assumed that these voters' choice of party can be described by a probability of election function.

Prior to the election, each of the political parties commits itself to the policy platform which it considers will maximize its chance of being elected. Any active policy is assumed to be disliked by the voters so that an active policy enters negatively in the probability of election function. A positive influence on the chance of election is the level of political contributions which a party receives from the lobbies. In maximizing their expected income, the lobbies, who stand to lose or gain depending upon which party gets elected, may give political contributions to their preferred party in order to increase the chances of that party's election. The parties,

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[8] Magee (1984) surveys the literature on endogenous tariff models.

[9] Magee and Brock (1983) and Young and Magee (1986) add to this two goods and two factors in a general equilibrium Heckscher-Ohlin framework.

who commit to their policy platform before these contributions are made, thus have an incentive to quote an active policy in order to elicit contributions from the lobbies. The problem of the parties is therefore to trade off the directly negative effect with the indirectly positive effect of active policy, whilst the lobbies must undertake that level of lobbying which maximizes their expected income.

Brock and Magee (1978) have modelled this as a game in which both parties simultaneously choose policies under the following hypothesis about the reaction of the lobbies. First they prove that, in a two party - two lobby world, each lobby will only make contributions to one party - this is their campaign contribution specialization theorem (Magee and Brock (1976)). When the lobbies observe the two policies, they simultaneously choose their lobby contributions to their own party so as to maximize their expected utility. They therefore play a straightforward simultaneous move game for which the Nash equilibrium is the appropriate solution concept. Now each political party, when considering changing its policy, only takes into account the reaction of its own lobby in this Nash equilibrium, and assumes that the rival lobby's contributions remain fixed. Magee and Brock (1983) denote this the 'limited information solution'. One justification offered for this reaction hypothesis is, reasonably, that the task for the party of computing the rival lobby's reaction may be too complicated.[10]

Brock and Magee (1978) and Young and Magee (1986) find that positive

[10] Although, it turns out in our model that this calculation is unnecessary: they need only be aware of Proposition 6.1.

lobbying activity leads the parties to adopt active redistribution policies, generating Pareto inefficient allocations. By only taking into account the favourable reaction of the own lobby, these solutions are likely to overstate the extent of active policy as in a full information structure, each party would also consider the adverse reaction of the opponent lobby in committing to a policy platform. Naturally, this would be expected to dampen the policies found in the limited information setup. This is demonstrated in chapter seven, sections 7.1 and 7.2.[11]

Consider the case in which each of the parties' policies has the same adverse effect on their probability of election (so that policies of equal magnitude cancel out in the probability of election function) and assume symmetric lobbies so that neither is able to contribute more than the other and equal contributions are equally effective in the voters' reaction function. If party 1 proposes a policy which redistributes X amount of revenue from lobby 1 to lobby 2, then lobby 2 has the same incentive as lobby 1 to give its own party political contributions. In other words, the lobbying contributions cancel out in aggregate [12] and the party could only reduce its chances of election by proposing an active policy.

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[11] For the symmetric case of the Young and Magee (1986) model, Appendix 6.1 demonstrates that the full information solution (where the reactions of both lobbies are taken into account in policy formation) yields an equilibrium in which there is no distortionary policy. For their limited information solution, however, distortion is possible in this symmetric case.

[12] This is a feature of the lobbying in Becker (1983).

6.4 An efficiency result in a lobbying model

In Clark and Thomas (1992), the information structure in the probabilistic voting model is adapted so that the reaction hypothesis takes into account the full response of both lobbies to any change in policy. The remainder of this chapter is taken from that paper. We use a standard two-stage game in which, at the first stage, both parties simultaneously select policies, and at the second stage both lobbies simultaneously choose contribution levels. Payoffs are then determined by the resulting election probabilities and the policies. At a sub-game perfect equilibrium, parties are forced to take into account the reaction of both lobbies at the second stage. This change in the reaction hypothesis leads to completely different results.

What we find is that provided the model satisfies certain symmetry conditions and with a quasi-linear specification of preferences over policies and money for the lobbies, then the equilibrium of the game will involve both parties choosing a policy which is efficient in the sense of maximizing the sum of the utilities of the two lobbies. If such a policy is unique then there will be no lobbying taking place in equilibrium since both parties choose the same policy: there is nothing to be gained by contributing to one of the parties. This result is in considerable contrast to some of the results obtained in the limited information structure. If the party additionally takes into account the adverse reaction of the rival lobby then, under general conditions, we obtain the following result: if the policy is inefficient (relative to the policy of the other party) in the sense that the utility gain to the own lobby is smaller than the loss to the other lobby then the adverse reaction of the rival lobby will

always at least offset any reaction of the own lobby. It follows that parties will not choose such inefficient policies since they can only reduce the probability of election success.

Thus in a world where redistribution of income between interest groups can only be achieved by means of distortionary policies, such as trade tariffs/subsidies, our results suggest that the political game will not lead to distortionary policies. This is true however only when the economy can be considered as a conflict between just two interest groups, so that a distortionary policy involves a smaller gain to the gainers than the loss to the losers. It may be the case that there are other groups who do not actively lobby a political party who can be "squeezed" to the benefit of the two represented groups, so that a distortionary policy does lead to an increase in the joint utility of the two lobby groups. Our results show that this is all that matters for electoral success, and in this case distortionary policies will be followed. (Hence our narrow definition of efficiency need not imply efficiency taking into account all groups in society). Section 6.7 presents an example of the general model in which a group can be squeezed; it is shown there that in the case where two extreme distributionary policies lead to the same maximum utility sum, positive lobbying may be observed in equilibrium.

6.5 The political game

Assume there are two players ("lobbies"), $i = 1, 2$, whose payoffs $u_i(\alpha)$ depend upon a policy parameter α belonging to some policy set A. The players can make

contributions c_i to political parties: a quasi-linear form for the final payoff is assumed:[13]

$$u_i(\alpha) - c_i \qquad\qquad i = 1, 2.$$

There are two political parties, j = 1, 2, who commit to a policy and attempt to get elected. Let α_j be the policy of party j. Given the policies of the two parties, the two lobbies decide upon their lobby contributions, with lobby 1 contributing to party 1, and lobby 2 to party 2. The probability that party 1 gets elected depends upon the lobby contributions and the policies: $\pi(c_1, c_2; \alpha_1, \alpha_2)$.

<u>Assumption 6.1:</u> π is strictly increasing in c_1 and strictly decreasing in c_2.

This corresponds to the idea that lobby 1 supports party 1 and likewise for lobby 2.[14] Magee and Brock (1983) give an interpretation of Assumption 6.1 based on the level of information possessed by a voter; the potential cost to the voter of choosing the 'wrong' party may be high so the parties use their received contributions to unearth and distribute unfavourable (but socially valuable) information about the opposing parties. The more contributions a party elicits, the more adverse information the voters are given about the opposing party. Moreover π may depend directly on the policies, α_1 and α_2 : it is assumed that the more 'efficient' α_i is in terms of the

[13] Our results also hold if utility is separable and concave in c_i i.e. $u_i(\alpha) - v(c_i)$.

[14] It is not necessary to make this restriction: our results go through if either lobby can choose which party to support.

sum of utilities, the greater the chance of party i being elected. More formally,

Assumption 6.2:

If $u_1(\alpha_1') + u_2(\alpha_1') > u_1(\alpha_1) + u_2(\alpha_1)$ then $\pi(c_1, c_2; \alpha_1', \alpha_2) \geq \pi(c_1, c_2; \alpha_1, \alpha_2)$ with equality in the utility sums implying equality in the probabilities. Likewise for party 2's policy.

The idea here is that some voters vote according to the total utility generated by each policy and not just according to the amount of lobbying. We can imagine that certain voters are not sure ex ante which group, 1 or 2, they belong to. Consequently they should rationally vote for the party whose policy yields the highest average utility $0.5\ u_1(\alpha) + 0.5\ u_2(\alpha)$. Alternatively there might be voters not belonging to either interest group and whose utility is positively correlated with the sum of lobby utility. (Our formulation also allows for the case where the policies do not have any effect directly on the probability of election).

Assumption 6.2 reflects two of the results in Becker (1983): that the political effectiveness of a group is mainly determined not by its absolute efficiency, but by its efficiency relative to other groups (corollary to his proposition 1); political policies that raise efficiency are more likely to be adopted than policies that lower efficiency (corollary to his proposition 2).

More formally, the game is a two-stage game. At the first stage, both parties j simultaneously choose $\alpha_j \in A$. The two parties attempt to maximize their probability

of election, hence the payoff to party 1 is $\pi(c_1, c_2; \alpha_1, \alpha_2)$ and that to party 2 is $1 - \pi(c_1, c_2; \alpha_1, \alpha_2)$. The lobbies maximize, by choosing $c_i \in [0, c^{max}]$, their expected payoff, which for lobby i is

$$\pi(c_1, c_2; \alpha_1, \alpha_2)\, u_i(\alpha_1) + (1 - \pi(c_1, c_2; \alpha_1, \alpha_2))\, u_i(\alpha_2) - c_i \qquad (6.1)$$

We look for pure-strategy sub-game perfect equilibria of this two stage game in order to determine endogenous economic policies and endogenous lobbying levels. A key assumption is the following.

<u>Assumption 6.3:</u> $\pi(c, c; \alpha_1, \alpha_2)$ is independent of c for all $c \in [0, c^{max}]$

Assumption 6.3 states that, given the policies, the election probability is always the same when the two lobbies contribute the same amount: the scale of their lobbying does not affect the probability. This is slightly weaker than requiring homogeneity of degree zero in contributions.

6.5.1 The lobby sub-game

To solve for the sub-game perfect equilibria of the model it is necessary to solve backwards, starting with the lobby sub-game in the second stage, <u>given</u> a choice of policies (α_1, α_2). Our first step is to show that if a party has a more efficient policy in the sense of a larger utility sum, then it must receive at least the lobby contribution which the other party receives.

LEMMA 6.1 Suppose that $\{u_1(\alpha_i) + u_2(\alpha_i)\} > \{u_1(\alpha_j) + u_2(\alpha_j)\}$ for $i \neq j$, and that a Nash equilibrium (c_i, c_j) of the lobby sub-game exists. Then $c_i \geq c_j$.

<u>Proof</u> We give the proof for $i = 1$; the argument for $i = 2$ is symmetric. From (6.1) the expected payoff to lobby i can be written as

$$u_i(\alpha_2) + \pi(c_1, c_2; \alpha_1, \alpha_2)\,(u_i(\alpha_1) - u_i(\alpha_2)) - c_i \tag{6.2}$$

Suppose that $c_2 > c_1$. We want to prove a contradiction. By the definition of a Nash equilibrium, lobby 1 cannot gain by increasing its contribution from c_1 to c_2 :

$$(\pi(c_2, c_2; \alpha_1, \alpha_2) - \pi(c_1, c_2; \alpha_1, \alpha_2))\,(u_1(\alpha_1) - u_1(\alpha_2)) - (c_2 - c_1) \leq 0 \tag{6.3}$$

Likewise lobby 2 cannot gain by reducing its contribution from c_2 to c_1 :

$$(\pi(c_1, c_1; \alpha_1, \alpha_2) - \pi(c_1, c_2; \alpha_1, \alpha_2))\,(u_2(\alpha_1) - u_2(\alpha_2)) + (c_2 - c_1) \leq 0 \tag{6.4}$$

Using $\pi(c_1, c_1; \alpha_1, \alpha_2) = \pi(c_2, c_2; \alpha_1, \alpha_2)$ and adding (6.3) and (6.4):

$$(\pi(c_1, c_1; \alpha_1, \alpha_2) - \pi(c_1, c_2; \alpha_1, \alpha_2))\,(u_1(\alpha_1) + u_2(\alpha_1) - u_1(\alpha_2) - u_2(\alpha_2)) \leq 0$$

but the first factor is positive by the assumption that π is strictly decreasing in c_2,

while the second factor is also positive by the supposition of the lemma, a contradiction. ■

The intuition behind this result is straightforward. Suppose that party 1's policy yields a higher sum of utilities than that of party 2. If party 2 has a policy more favourable to lobby 2 than party 1 then it hopes to elicit contributions from its lobby; however because party 1 offers a higher utility sum, it must be the case that not only does lobby 1 prefer party 1, but by more (in terms of utility difference) than lobby 2 prefers party 2: lobby 1 has a larger incentive to lobby. Together with Assumption 6.3, which essentially implies that contributions to either lobby are equally effective, this means that lobby 1 will lobby at least as much as lobby 2.

The next lemma will prove useful.

LEMMA 6.2 Let α^* be a policy which maximizes the utility sum $\{u_1(\alpha) + u_2(\alpha)\}$. Then if party 1 plays α^* it is guaranteed a payoff of at least $\pi(0, 0; \alpha^*, \alpha^*)$. Likewise, party 2 by playing α^* must receive at least $(1 - \pi(0, 0; \alpha^*, \alpha^*))$.

<u>Proof</u> Suppose party 1 plays α^* against α_2 and let (c_1, c_2) be the contributions in the lobby sub-game after (α^*, α_2), so party 1's payoff is $\pi(c_1, c_2; \alpha^*, \alpha_2)$. We have $c_1 \geq c_2$ by Lemma 6.1, so

$$\pi(c_1, c_2; \alpha^*, \alpha_2) \geq \pi(c_2, c_2; \alpha^*, \alpha_2) = \pi(0, 0; \alpha^*, \alpha_2) \geq \pi(0, 0; \alpha^*, \alpha^*) \tag{6.5}$$

where the inequalities and equalities follow respectively from Assumptions 6.1, 6.3 and 6.2. Symmetrically, party 2 can guarantee itself $(1 - \pi(0, 0; \alpha^*, \alpha^*))$. ■

We shall initially also need the following.

Assumption 6.4: There exists an α^* which is the unique policy which maximizes $\{u_1(\alpha) + u_2(\alpha)\}$ over the policy set A.

Assumption 6.4 implies that α^* is the "efficient" policy in the sense of maximizing the sum of utilities. Such a policy will exist under standard assumptions: for example continuity of u_i and compactness of A together with strict concavity of the utility sum.

6.5.2 The political equilibrium

Given this assumption, we can now show that both parties choosing α^* is a Nash equilibrium.

PROPOSITION 6.1 Assume that for each (α_1, α_2) a pure strategy Nash equilibrium of the lobby game exists. Then $\alpha_1 = \alpha_2 = \alpha^*$, $c_1 = c_2 = 0$, is the outcome of a sub-game perfect equilibrium.

Proof Given that both parties are choosing the same policy α^* the lobbies must set $c_1 = c_2 = 0$ (there is no gain from lobbying, only cost), so the election probability of party 1 at the proposed equilibrium is $\pi(0, 0; \alpha^*, \alpha^*)$. By Lemma 6.2, if party 2 plays

α^*, this probability can be no more than $\pi(0, 0; \alpha^*, \alpha^*)$, hence party 1 cannot gain by deviating. Likewise for party 2. ■

In the equilibrium with a unique joint utility maximizer, both parties choose the efficient policy and there is no lobbying. By deviating from α^*, it may be possible for a party to elicit contributions from its lobby, however these will always be at least offset by contributions from the other lobby.

While the proposition states that there will be a sub-game perfect Nash equilibrium in which both parties choose the efficient policy, it does not rule out the existence of other equilibria in which policies may be different. The reason for this is that in the case in which the policies have no direct effect on π, policies close to α^* may lead to no lobbying by either lobby and are consequently as good for the parties as α^*. If however the policies do have a direct effect, so π is strictly increasing in the utility sum offered by α_1 and strictly decreasing in that of α_2, then the equilibrium must be unique.

<u>PROPOSITION 6.2</u> If π is strictly increasing in the utility sum offered by policy α_1, $\{u_1(\alpha_1) + u_2(\alpha_1)\}$, and strictly decreasing in that offered by policy α_2, then the outcome path identified in Proposition 6.1 is the only possible equilibrium outcome path.

<u>Proof</u> First notice that the equilibrium probability must be $\pi(0, 0; \alpha^* \alpha^*)$. This follows immediately from Lemma 6.2 as both parties can guarantee this probability

and they are playing a constant-sum game. Now suppose that the equilibrium involves at least one of the parties choosing $\alpha_i \neq \alpha^*$. If $\alpha_2 \neq \alpha^*$, then by the assumptions of the proposition, the last inequality in (6.5) is strict, so party 1 by playing α^* against α_2 would achieve a payoff greater than $\pi(0, 0; \alpha^*, \alpha^*)$. This means that party 2 gets a payoff less than it can guarantee itself by playing α^* , hence this cannot be an equilibrium. The argument is symmetric for $\alpha_1 \neq \alpha^*$. Hence the unique equilibrium has $\alpha_1 = \alpha_2 = \alpha^*$. ■

When the probability of election function depends directly upon the policy, we obtain the standard median voter type result of complete policy convergence, whereas it is possible that there exist equilibria which do not exhibit this feature if the election probability is policy independent.

6.6 Non-uniqueness of efficient policies

The results of section 6.5 have shown that under certain conditions the outcome of the lobbying game will involve both parties choosing the efficient policy and neither lobby making contributions. The critical assumptions were firstly the existence of a unique efficient policy and secondly a substantial degree of symmetry in the model: in particular both lobbies' contributions were equally effective in the sense that any amount of lobbying by one lobby could be 'nullified' by an equal amount of lobbying by the other lobby, <u>and</u> lobbying is equally costly in terms of payoffs to both lobbies. As relaxing these assumptions may lead to different results, we dispense with the first of these in this section and with the second in the example

of section 6.7.

PROPOSITION 6.3 Suppose that there exists more than one policy which maximizes the utility sum, with the distribution of utilities varying between these policies. Then possible equilibria will involve any combination of such policies.

Proof Lemma 6.2 establishes that playing α^*, where α^* is one of the efficient policies, guarantees party 1 $\pi(0, 0; \alpha^*, \alpha^*)$ and likewise guarantees party 2 $(1 - \pi(0, 0; \alpha^*, \alpha^*))$. If party 1 plays α_1^* and party 2 plays α_2^*, where both policies are efficient, neither can gain by changing policy since this argument proves that against α_2^*, party 1 cannot do better than $\pi(0, 0; \alpha_2^*, \alpha_2^*)$ and by playing α_1^* gets $\pi(c_1, c_2; \alpha_1^*, \alpha_2^*) = \pi(0, 0; \alpha_2^*, \alpha_2^*)$ since $c_1 = c_2$ (both α_1^* and α_2^* offer the same utility sum). Likewise party 2 cannot do better than α_2^*. This establishes that any combination of two efficient policies leads to an equilibrium. ■

Again, it is possible to observe the median voter result of complete policy convergence, or alternatively policies could be opposed. If the policies have a direct effect on π as assumed in Proposition 6.2, then again there can be no other equilibria than the efficient ones in Proposition 6.3: from Lemma 6.2 the equilibrium payoff to party 1 must be $\pi(0, 0; \alpha^*, \alpha^*)$, and the argument of the proof of Proposition 6.2 establishes that playing a policy other than an efficient one would lead to a lower payoff. If α_1^* and α_2^* offer different utility distributions, it is quite possible that lobbying is positive in equilibrium. It might, however, seem unlikely that multiple efficient policies would arise. In the example below we show how this might

nevertheless happen in an interesting fashion.

6.7 Example: A simple redistribution game

In this section we give an example to illustrate the results so far. Depending upon the parameters of the model there may be a unique maximizer of the utility sum where no interference takes place, or there may be multiple maximizers involving a third group of agents losing out to the benefit of one of the lobby groups. The example also allows us to consider what might happen when the symmetry assumptions are dropped.

It is supposed that the agents in the economy are composed of three groups: lobbies 1 and 2 together with a third group of politically non-active agents. The numbers in each group are N, N and n respectively, and each agent has an endowment of income of one unit. The government has a single instrument of redistribution, namely to impose a uniform lump sum tax τ, where $0 \leq \tau \leq \tau^{max} < 1$, and to redistribute the money raised to one of the lobby groups. There is however a deadweight loss from this policy in the form of a fraction $(1 - \lambda)$, where $1 \geq (1 - \lambda) \geq 0$, of the tax revenue which simply gets "lost". There are two political parties and party 1's probability of election success is

$$\pi = \frac{K + c_1}{2K + c_1 + c_2} \qquad c_1 + c_2 > 0$$

$$\pi = 1/2 \qquad c_1 = c_2 = 0$$

where $K \geq 0$ is a constant; π does not depend directly on the policy chosen. We consider policies of the form: levy a tax τ_i and redistribute the tax revenue $\lambda\tau_i(2 + n/N)$ to lobby i, with $\tau_i \in [0, \tau^{max}]$. (The subscript on τ refers not to the party whose policy this is, but to the lobby which benefits; so $\alpha_1 = \tau_2$ will mean that party 1 chooses a policy to benefit lobby 2 for example). Lobby i's utility is simply net per capita income.

Given a policy τ_1 which distributes tax revenue to lobby 1, the utility of lobby 1 becomes

$$(1 - \tau_1) + \lambda\tau_1(2 + n/N)$$

and that of lobby 2 is $(1 - \tau_1)$. The utility sum is then

$$\tau_1 (\lambda(2 + n/N) - 2) + 2 \tag{6.6}$$

which is increasing in τ_1 if

$$2\lambda + \lambda n/N - 2 > 0 \tag{6.7a}$$

The maximum is reached in this case at $\tau_1 = \tau^{max}$, and by symmetry also at $\tau_2 = \tau^{max}$ (the policy which distributes all revenue to lobby 2). The utility sum is decreasing in τ_1 if

$$-1 < 2\lambda + \lambda n/N - 2 < 0 \tag{6.7b}$$

The maximum is reached in this case at $\tau_1 = 0$, or equivalently $\tau_2 = 0$. (If the expression is less than -1 any positive policy would reduce both lobbies' welfare, so we do not consider this case).

We consider first the lobby sub-game after each party has chosen a policy favourable to its own lobby: $\alpha_1 = \tau_1$ and $\alpha_2 = \tau_2$. The expected utility of lobby 1 is

$$\pi\,(1 - \tau_1 + \lambda\tau_1(2 + n/N)) + (1 - \pi)\,(1 - \tau_2) - c_1$$
$$= (1 - \tau_2) + \pi(\tau_1 + \tau_2 + \tau_1(2\lambda + \lambda n/N - 2)) - c_1$$

Lobby 1 chooses c_1 to maximize this expression given c_2, which leads to the first-order conditions

$$\frac{(K + c_2)\,(\tau_1 + \tau_2 + \tau_1(2\lambda + \lambda n/N - 2))}{(2K + c_1 + c_2)^2} \leq 1 \;;\; c_1 \geq 0 \tag{6.8}$$

with complementary slackness. Given the symmetric condition for lobby 2, and assuming an interior solution c_1, $c_2 > 0$, we get

$$\frac{(K + c_1)}{(K + c_2)} = \frac{(\tau_1 + \tau_2 + \tau_1(2\lambda + \lambda n/N - 2))}{(\tau_1 + \tau_2 + \tau_2(2\lambda + \lambda n/N - 2))} \tag{6.9}$$

Notice that if the utility sum is increasing in τ_1 (or τ_2), that is (6.7a) holds, then we have $c_1 > (<) c_2$ as $\tau_1 > (<) \tau_2$; if the utility sum is decreasing in τ_1 (or τ_2) then $c_1 < (>) c_2$ as $\tau_1 > (<) \tau_2$. Taking into account the complementary slackness conditions we get the same relations except the strict inequality between c_1 and c_2 becomes an equality when $c_1 = c_2 = 0$. (This corresponds to Lemma 6.1).

We have dealt with the lobby sub-game after each party has chosen a policy favourable to its own lobby. We need also to consider the case where both parties choose policies favourable to the same lobby, say lobby 1. Let party 1 choose policy τ_1, and party 2 policy τ_1^+, with $\tau_1 > \tau_1^+$. Then the corresponding first-order conditions which hold with complementary slackness are:

$$\frac{(K + c_2)(\tau_1 - \tau_1^+)(2\lambda + \lambda n/N - 1)}{(2K + c_1 + c_2)^2} \leq 1 \ , \quad c_1 \geq 0$$

$$\frac{(K + c_1)(\tau_1 - \tau_1^+)}{(2K + c_1 + c_2)^2} \leq 1 \ , \quad c_2 \geq 0$$

At an interior solution with $c_1, c_2 > 0$, we have

$$\frac{K + c_1}{K + c_2} = 2\lambda + \frac{\lambda n}{N} - 1$$

and taking account of the possibility that $c_1 = c_2 = 0$, this implies that when (6.7a) holds, $c_1 \geq c_2$, and vice versa when (6.7b) holds; again this corresponds to Lemma 6.1.

Finally there is the possibility that party 1 chooses a policy more favourable to lobby 2 than does party 2. In this case clearly $c_1 = c_2 = 0$. We are now in a position to analyze the equilibria of the model. There are two cases to consider.

6.7.1 Unique efficient policy

First we take the case where (6.7b) holds, corresponding to the basic idea of this chapter where Assumption 6.4 is satisfied. The sum of utilities is maximized only when $\tau_1 = \tau_2 = 0$; the deadweight loss of redistribution exceeds the amount which can be raised from the third group. In this case, from Proposition 6.1, each party having a non-active policy followed by zero lobbying is a sub-game perfect equilibrium outcome. This is easily verified: at the proposed equilibrium the probability of election is one half and from the above derivations any deviation by party 1 will lead to a probability no greater than one half and likewise for party 2. This equilibrium is Pareto efficient and not simply efficient in the sense of maximizing the joint utility of the lobby groups.

6.7.2 Multiple efficient policies and the possibility of positive lobbying

Secondly, when (6.7a) holds, so that Assumption 6.4 fails, active policy pays because the two lobby groups gain at the expense of the third group by more than the deadweight loss. Here, by the discussion of section 6.6, both parties playing either

of the maximum redistribution policies, which are both efficient, is an equilibrium.[15] To see if positive lobbying is possible in equilibrium, consider the equilibrium in which party i plays $\tau_i = \tau^{max}$ (both parties act as favourably to their lobbies as possible). For an interior solution, from (6.8),

$$c = \tau^{max}\lambda(2 + n/N)/4 - K \tag{6.10}$$

needs to be positive, otherwise $c_1 = c_2 = 0$. Clearly if $K = 0$, then provided only that τ^{max}, λ, $n > 0$, we have $c > 0$. When K is low, starting from a situation where $c_1 = c_2 = 0$, the effect on the probability of a small increase in c_i is very large, and provided this multiplied by the utility difference is larger than unity, some lobbying must occur. If on the other hand the expression given by (6.10) is non-positive, then $c_1 = c_2 = 0$ is the solution: the gain from lobbying is less than the cost.

So we have the result that positive lobbying may occur in equilibrium once we dispense with Assumption 6.4. In the other three equilibrium configurations, namely i) both play $\tau_1 = \tau^{max}$, ii) both play $\tau_2 = \tau^{max}$, and iii) party 1 plays $\tau_2 = \tau^{max}$, party 2 plays $\tau_1 = \tau^{max}$, there is clearly no lobbying. [16] However, in all cases an "efficient" policy is chosen by both parties (recall we have defined efficiency only in terms of the utility sum of the two lobbies, ignoring any third parties). The equilibrium with

[15] Recall that we defined "efficiency" only from the narrow point of view of the two lobbies. Clearly maximum redistribution is not a Pareto efficient allocation.

[16] This is because we restrict lobby i to contribute only to party i: we could easily dispense with this restriction in which case the latter equilibrium would be essentially identical to the one in which party i plays $\tau_i = \tau^{max}$.

positive lobbying is Pareto inferior to the other symmetric equilibrium.

An interesting feature of such multiple equilibria is that it is possible to obtain both the median voter result where the parties choose identical policies and the polar case in which "extremist" policies are adopted.

An objection to this model is that it seems unlikely that the utility sum should have multiple maximizers. A positive lobbying equilibrium could nevertheless be obtained even in the absence of this feature if we restrict each party to choose only policies favouring their own lobbies. Suppose the constraints on redistribution are $\tau_1 \leq \tau_1^{max}$, $\tau_2 \leq \tau_2^{max}$ with $\tau_1^{max} > \tau_2^{max}$ so $\tau_1 = \tau_1^{max}$ is the unique maximizer of the utility sum - we are assuming that (6.7a) holds. Then from (6.9) and the definition of π, π is increasing in τ_1 and decreasing in τ_2 (taking into account the complementary slackness conditions this may be weakly increasing or decreasing), hence $\tau_1 = \tau_1^{max}$, $\tau_2 = \tau_2^{max}$ are in fact dominant strategies and so constitute an equilibrium, and this might involve positive lobbying as before.

When the joint utility of the lobby groups has a unique maximizer, then the efficient policy - zero lobbying result is based on the idea that each party can adopt the position of the opponent in order to guarantee an election probability of one half. If it is not possible to mimic the opposing party, a positive lobbying result is possible.

6.7.3 *Asymmetric lobbying effectiveness*

It is interesting to ask how the results vary if we dispense with the symmetry between the lobbying costs and benefits which has played an important role so far. For example suppose that the marginal cost of lobbying for lobby 1 is reduced to $\mu < 1$: so μc_1 is now the utility loss. (Alternatively we could have changed the form of π, with similar effects). Let us return to the world of Assumption 6.4, remaining within the example, so assume that (6.7b) holds, with $\tau_1 = \tau_2 = 0$ maximizing the utility sum.

First we show that $\tau_1 = \tau_2 = 0$ may no longer be an equilibrium. Suppose that party 2 plays $\tau_2 = 0$, and party 1 plays $\tau_1 > 0$. The first-order conditions are

$$\frac{(K + c_2)(\tau_1 + \tau_1(2\lambda + \lambda n/N - 2))}{(2K + c_1 + c_2)^2} \leq \mu \quad , \quad c_1 \geq 0 \tag{6.11}$$

$$\frac{(K + c_1)\,\tau_1}{(2K + c_1 + c_2)^2} \leq 1 \quad , \quad c_2 \geq 0 \tag{6.12}$$

with complementary slackness. Suppose that μ is sufficiently low that the following holds:

$$1 + (2\lambda + \lambda n/N - 2) > \mu \tag{6.13}$$

This implies that whenever $c_1 = c_2$, the first inequality in (6.12) is more likely to hold with strict inequality than (6.11). Then provided lobby 1 has an incentive to lobby when $c_2 = 0$, that is if

$$\frac{\tau_1 + \tau_1 \left(2\lambda + \frac{\lambda n}{N} - 2\right)}{4K} > \mu \tag{6.14}$$

(which in view of the assumption in (6.13), certainly follows if additionally $\tau_1 \geq 4K$), the lobby sub-game has an equilibrium with $c_1 > c_2$.[17] Hence by playing $\tau_1 > 0$ such that (6.14) is satisfied (always possible if $\tau^{max} > 4K$) party 1 pushes π above 1/2, so both parties playing efficient policies is not an equilibrium.

If an equilibrium exists when (6.13) and (6.14) hold, it must be the case that $\pi = 1/2$ since any other equilibrium probably could be improved upon by one or other of the parties changing its policy to that of its rival to guarantee itself $\pi = 1/2$. In fact both parties playing $\tau_1 = \tau_1^{max}$ is an equilibrium. Clearly party 1 cannot gain by changing its policy since there will be no lobbying and π will remain at 1/2. If party 2 plays $\tau_1 \geq 0$ or $\tau_2 > 0$ then (6.13) can easily be seen to imply $c_1 \geq c_2$ so it cannot

[17] This is easily seen. First, the reaction functions for both lobbies are continuous and bounded above with $dc_i/dc_j > (<) \; 0$ as $c_i > (<) \; c_j$; hence there exists a unique equilibrium of the lobby sub-game. Second, $c_1 = c_2 = 0$ cannot be a solution as (6.14) would imply that (6.11) is violated; it cannot be that $c_2 > c_1 = 0$ since the first inequality in (6.12) then holds with equality, and (6.14) then implies that (6.11) is violated. Finally, $c_2 > c_1 > 0$ is impossible since $c_1 > c_2$ at an interior solution by (6.13).

gain.[18]

Hence we have the result that when one lobby is more effective at lobbying, the equilibrium policy may necessarily be biased in favour of that lobby at the expense of efficiency. The party of the other lobby has no choice but to acquiesce in this situation and also favour the effective lobby: it cannot afford a fight. In equilibrium there is no lobbying. This result is in considerable contrast to those of Brock and Magee (1978) and Young and Magee (1986) where asymmetric lobbies do actually give political contributions in equilibrium and the parties set partisan policies. The difference between their and the current results is due once again to the contrasting specification of the information structure.

The fact that asymmetric lobbying effectiveness can lead to active policy in the absence of any actual lobbying in equilibrium tends to suggest the lobbies in a punishment rather than rewarding role. In the Brock and Magee framework, the reward to setting active policy is the promise of lobbying contributions, whereas in the current model, with asymmetric lobbies, it is the threat of being punished through contributions to the opponent which leads to active policy. That there is no lobbying in equilibrium suggests an even more fundamental role for lobbies in policy formation than in Brock and Magee's work; the mere existence of asymmetric lobbies and the threat that these lobbies will dent the election chances of the parties is enough to create an active, and still endogenous, policy.

[18] Again this is easily checked by showing that when $c_1 = c_2$, the ratio of the marginal benefit from lobbying to its cost is greater for lobby 1 than lobby 2.

6.8 Summary

This chapter has presented a general model of the political process reflecting the influence of interest groups. Economic policy in this setup is determined as the result of a political equilibrium. Young and Magee (1986) present a model where the endogenous policy to be determined is a tariff. Their model, however, restricts the reaction of the lobbies in an irrational manner. The model of the present paper allows agents to use all of the information available to them when making decisions and consequently, policy is not as distorted as Young and Magee would suggest - indeed, according to the narrow definition of efficiency adopted in the present paper, the political equilibrium often involves the parties choosing efficient policies whilst the interest groups do not lobby.

The conditions under which these results hold are fairly general. Underlying the results is a simple mechanism: when the lobbies are symmetric and there is a unique maximizer of their utility sum, any attempt by a party to pursue a policy other than the one guaranteeing this maximum joint utility will harm the opposing lobby more than it benefits the own one. Consequently the party will suffer a net loss of lobby contributions which we have defined as important for the outcome of the election. Even if the sum of the lobbies' utility does not have a unique maximum, we have demonstrated in the general model that an 'efficient' equilibrium will arise, in which lobbying may or may not be present.

A simple redistribution game was used to illustrate our results. The structure

of the general model was enhanced by assuming the existence of a group of politically non-active agents who could be 'squeezed' by the parties to court favour with the interest groups. Working within this example, we have been able to examine the role of our key assumptions. If the lobbies' utility sum has a unique maximizer then our general result (no lobbying and efficient policy) was shown to hold. In the case of multiple efficient policies, an equilibrium with active distortionary policy and lobbying is possible. This crucially depends upon the existence of the non-politically active agents who lose out to the interest groups in a lobbying equilibrium. Finally, when one lobby has the most power, both parties favour the most effective lobby so that policy is active even though there is no lobbying in equilibrium. Both parties acquiesce to the wishes of the strong lobby as neither party can afford to do otherwise. Although the main intention of this chapter is to demonstrate that efficient policies are possible in the presence of interest groups, this latter result suggests a much more fundamental role for lobbies in the political process than Young and Magee. In their model, it is the actual lobbying which leads to distortionary policy, but our final result suggests that the mere threat of a strong interest group using its influence may be sufficient to lead to distortion.

APPENDIX 6.1

This appendix takes the symmetric case of the Young and Magee (1986) model and shows that if the reaction of both lobbies is taken into account when setting policy, then there will be no active redistribution policy. When only the limited information solution is considered, distortion is still possible in this symmetric case.

There are two political parties and two lobbies. Each lobby has an endowment of E and lobby i makes c_i in contributions to party i (i = 1, 2) in response to the announcement of a distortionary policy α_i. The probability that party 1 is elected is

$$\pi = \frac{c_1 \alpha_2}{c_1 \alpha_2 + c_2 \alpha_1} \tag{A6.1}$$

Young and Magee calculate the following solution to the lobby sub-game at an interior solution $c_1 > 0$, $c_2 > 0$:

$$c_1 \alpha_2 = c_2 \alpha_1 [\{(1 - (\alpha_1 \alpha_2)^{-m}) (1 + \frac{E \alpha_2}{c_2 \alpha_1})\}^{1/2} - 1] \tag{A6.2}$$

$$c_2 \alpha_1 = c_1 \alpha_2 [\{(1 - (\alpha_1 \alpha_2)^{-m}) (1 + \frac{E \alpha_1}{c_1 \alpha_2})\}^{1/2} - 1] \tag{A6.3}$$

where m is a constant. The following lemma indicates that if party 1 sets a more

redistributive policy than party 2, then the outcome of the lobby sub-game is such that the probability of election of party 1 is below one-half (this being the probability it can achieve by setting a policy of equal magnitude to the other party).

LEMMA A6.1 If $\alpha_1 > \alpha_2$ then $\pi < 1/2$.

<u>Proof</u> Rearrange (A6.2) and (A6.3) to get

$$c_1\ \alpha_2 + c_2\ \alpha_1 = c_2\ \alpha_1\ \{(1 - (\alpha_1\ \alpha_2)^{-m})\ (1 + \frac{E\ \alpha_2}{c_2\ \alpha_1})\}^{1/2} \tag{A6.4}$$

$$c_1\ \alpha_2 + c_2\ \alpha_1 = c_1\ \alpha_2\ \{(1 - (\alpha_1\ \alpha_2)^{-m})\ (1 + \frac{E\ \alpha_1}{c_1\ \alpha_2})\}^{1/2} \tag{A6.5}$$

Dividing (A6.4) by (A6.5) and rearranging yields

$$\frac{c_1\ \alpha_2}{c_2\ \alpha_1} = \frac{c_2\ \alpha_1 + E\ \alpha_2}{c_1\ \alpha_2 + E\ \alpha_1} \tag{A6.6}$$

By (A6.1), showing that $\alpha_1 > \alpha_2$ implies $\pi < 1/2$ requires that $\alpha_1 > \alpha_2$ yields $c_2\ \alpha_1 > c_1\ \alpha_2$ from the lobby sub-game. We can prove this by contradiction. Suppose that $\alpha_1 > \alpha_2$ and $c_1\ \alpha_2 \geq c_2\ \alpha_1$, then (A6.6) implies

$$c_2\ \alpha_1 + E\ \alpha_2 \geq c_1\ \alpha_2 + E\ \alpha_1 \tag{A6.7}$$

but $E\ \alpha_1 > E\ \alpha_2$ and $c_1\ \alpha_2 \geq c_2\ \alpha_1$ so that (A6.7) is violated, a contradiction. ■

This lemma demonstrates that if the parties were to take into account the reactions of both of the lobbies from the lobby sub-game then both parties would set policies of equal magnitudes. In fact the implication is stronger so that the following occurs:

<u>PROPOSITION A6.1</u> Given the solutions for lobbying contributions at an interior outcome of the lobby sub-game represented by (A6.2) and (A6.3) and the probability of election function in (A6.1), then the equilibrium policies of each party are $\alpha_1 = \alpha_2 = 1$ if the parties take into account the reaction of each of the lobbies when forming their policies. ($\alpha = 1$ is a non-active policy in this model).

<u>Proof</u> By Lemma A6.1 it must be the case that $\alpha_1 = \alpha_2$. Now if $\alpha_1 = \alpha_2 > 1$ then either of the parties can reduce its policy below this level and increase its probability of election by Lemma A6.1. Worthwhile deviations are not possible only if $\alpha_1 = \alpha_2 = 1$, so this is an equilibrium. ■

CHAPTER 7. VOTER MYOPIA AND EXTERNAL DEBT IN A LOBBYING ECONOMY

In the previous chapter, the mechanism which drives the results works through the joint utility of the lobbies; the policies had an effect on lobby contributions and the probability of election only through the utility sum. The original formulation of Brock and Magee (1978) reflects voter hostility to redistributive policies in that any active policy is assumed to directly reduce that party's probability of election. This is not a feature of the model in chapter six due to Assumption 6.2. This chapter recasts the specific example used previously in a setting where the voters dislike being taxed. In determining their optimal policies, political parties therefore face a trade-off between pleasing the lobbies and pacifying the voters. The work in this chapter thus approaches that of Brock and Magee but with the important difference that each party takes the reaction of both of the lobbies into account when deciding its optimal policy.

The simple taxation game is firstly solved for the full-information case and this solution is compared to those from the limited-information framework (Magee and Brock (1983)).[1] Next we assume that the lobbying economy lasts for two periods: in the first it is possible for the political party which forms the government to borrow funds externally to channel to its support lobby but repayment has to be made out of taxation revenue in the final period.[2] The amount of borrowing and rent-seeking is shown to depend upon how far-sighted the voters are and the number of redistributive

[1] A great simplification is achieved by focussing only on symmetric equilibria.

[2] Creditors are assumed able to impose a penalty which enforces repayment.

instruments the parties have at their disposal.

Finally, a proposal is made which is designed to decrease the amount of rent-seeking in the borrowing economy with a view to decreasing the tax rate and improving the well-being of the ordinary citizens. The idea is that when the debt is in place, the lender or an organization like the International Monetary Fund announces a debt forgiveness schedule which is decreasing in the amount of lobbying contributions. This forgiveness reduces the amount of tax revenue which flows out of the country so increases the size of the transfer, *ceteris paribus*. In this case, the marginal cost of lobbying increases as the lobby loses each contributed unit but additionally, each extra unit reduces the amount of forgiveness and hence the available transfer. The design of the optimal contribution-linked forgiveness schedule indicates a critical condition which the schedule must satisfy if it is to have the desired effect of reducing the amount of lobbying <u>and</u> the size of the tax rate.

7.1 Lobbying with direct voter hostility: The full-information case

As in the previous chapter, we depict a two-stage game with two parties and two lobbies: in the first stage the parties commit to a tax rate τ_j ($j = 1, 2$) and in the second stage the lobbies contribute to their favoured party in order to aid its election campaign. We simplify by assuming that if party 1 sets an active policy then it is to the advantage of lobby 1 and likewise for the second party and lobby. In this case, reminiscent of the Brock and Magee formulation, each lobby will only contribute to its own party. In setting an active redistribution policy, the party displeases the n

voters in the economy who must also pay tax out of their one unit of endowment. The problem of the representative lobby member is examined so that with $c_1 \geq 0$ as the contribution of the representative lobby 1 member and $c_2 \geq 0$ for the representative lobby 2 member, the probability that party 1 is elected, π, is assumed to be of the logit form so that

$$\ln (\pi/1-\pi) = a \ln (\delta K+c_1) + b \ln (1-\tau_1) - d \ln (\delta K+c_2) - e \ln (1-\tau_2) \tag{7.1}$$

where $K > 0$ is a constant and δ is a Kronecker delta where $\delta = 1$ if $c_1 = 0$ or $c_2 = 0$, and $\delta = 0$ if $c_1 > 0$ and $c_2 > 0$.[3] The magnitude $\pi/1-\pi$ is the odds ratio and 'a' is the elasticity of the odds with respect to lobby 1 contributions (b, d and e are defined similarly). The odds ratio is increasing (decreasing) in the contributions of lobby 1 (2) and the amount of income left after tax paid to party 1 (2). To avoid problems of indeterminacy in (7.1), we assume $0 \leq \tau_j \leq \tau^{max} < 1$, $j = 1, 2$. Notice that any positive tax rate will directly harm that party's chances of election: this is the hostile voter effect. Rearranging (7.1) and assuming the (symmetric) case in which ab = de = 1 for simplicity, the probability that party 1 is elected becomes:

$$\pi = \frac{(\delta K + c_1)(1 - \tau_1)}{(\delta K + c_1)(1 - \tau_1) + (\delta K + c_2)(1 - \tau_2)} \tag{7.2}$$

This formulation for π ensures that if the contributions are identical and the policies

[3] This δK term is used to ensure that (7.1) is defined for all $c_i \geq 0$, $i = 1, 2$.

are equal, then the probability of election is one half for each party.

Assume that party 1 sets τ_1 and redistributes the taxation revenue to its own lobby (lobby 1)[4]; with the fact that each of the lobby members has one unit of endowment, the transfer to each of the N lobby members is $\tau_1(2+n/N)$ so that lobby 1's expected payoff, taken to be per capita income can be written as

$$\pi\,[(1-\tau_1) + \tau_1(2+n/N)] + (1 - \pi)\,(1-\tau_2) - c_1 \tag{7.3}$$

Lobby 1 maximizes (7.3) by its choice of c_1, taking both policies and the contributions of the other lobby as fixed. The first-order conditions are

$$\frac{(\delta K+c_2)\,(1-\tau_1)\,(1-\tau_2)\,[\tau_2 + \tau_1(1 + n/N)]}{[(\delta K+c_1)\,(1-\tau_1) + (\delta K+c_2)\,(1-\tau_2)]^2} \leq 1\,, \quad c_1 \geq 0$$

with complementary slackness. Solving a similar problem for lobby 2 yields the best response function for lobby i as

$$c_i = Max\ \left\{ \frac{[A_i(\delta K+c_j)(1-\tau_1)(1-\tau_2)]^{1/2} - (1-\tau_j)(\delta K+c_j)}{1-\tau_i}\,,\ 0 \right\} \tag{7.4}$$

where $A_i = \tau_j + \tau_i(1+n/N)$.

[4] The deadweight loss term of chapter six is ignored in this model.

At an interior solution $c_1 > 0$, $c_2 > 0$ we get

$$\frac{c_1}{c_2} = \frac{(\tau_2 + \tau_1(1 + n/N))}{(\tau_1 + \tau_2(1 + n/N))} = \frac{A_1}{A_2} \tag{7.5}$$

Again, as in the previous chapter, Lemma 6.1 holds. The difference here is that even if $\tau_1 > \tau_2$ does imply that $c_1 > c_2$ it still might be the case that τ_2 is the preferable policy due to the directly negative effect which active policy has on the election probability.

At an interior solution of the lobby problem it is also the case that

$$\frac{\partial \pi}{\partial c_1} = \frac{c_2\,(1-\tau_1)\,(1-\tau_2)}{[c_1\,(1-\tau_1) + c_2\,(1-\tau_2)]^2} = \frac{1}{\tau_2 + \tau_1(1 + n/N)}$$

which rearranges to give an optimal interior contribution for lobby 2 as[5]

$$c_2 = \frac{(1 - \tau_1)\,(1 - \tau_2)\,A_1\,A_2^2}{[(1 - \tau_1)\,A_1 + (1 - \tau_2)\,A_2]^2} \tag{7.6}$$

By symmetry, the optimal interior contribution for lobby 1 is

[5] Only the positive square root emanating from the previous equation need be considered as contributions cannot be negative.

$$c_1 = \frac{(1 - \tau_1)(1 - \tau_2) A_1^2 A_2}{[(1 - \tau_1) A_1 + (1 - \tau_2) A_2]^2} \tag{7.7}$$

This interior solution to the lobby sub-game is valid as long as c_2 and c_1 given by (7.6) and (7.7) are greater than zero which requires $A_1 > 0$ and $A_2 > 0$, or in other words

$$\tau_2 + \tau_1 (1 + n/N) > 0$$

$$\tau_1 + \tau_2 (1 + n/N) > 0$$

so that $\tau_1 > 0$ <u>or</u> $\tau_2 > 0$ is required. The interpretation here is as follows: if $\tau_1 = 0$ and $\tau_2 > 0$ then lobby 2 has an incentive to contribute as the promised transfer from its own party if elected is positive; additionally, lobby 1 has an incentive to contribute as it prefers party 1 to party 2 (party 1 being elected offers no positive benefit but party 2 holding office means that lobby 1 loses some of its endowment to taxation). If, on the other hand, $\tau_1 = \tau_2 = 0$ then $A_i = 0$, $i = 1, 2$ so that $c_i = 0$ from (7.4).[6]

Notice that in a symmetric situation when each party chooses the same tax rate τ, the

[6] If we had considered a deadweight loss in transferring taxation revenue then we would have $A_i = \tau_j + \tau_i (2\lambda + \lambda n/N - 1)$ so that $A_i > 0$, $i = 1, 2$, additionally requires $\lambda > N/(2N+n)$ so that the deadweight loss must be sufficiently small to induce an interior solution of the lobby sub-game.

contributions of the lobbies are identical:[7]

$$c = \frac{\tau(2N + n)}{4N} \tag{7.8}$$

If lobbying takes place in a symmetric equilibrium, then the level of lobby contributions to each party by each member of the own lobby is equal to one quarter of the prospective per capita transfer.[8] The optimal contributions reflect the fact that if the tax rates (and hence transfers) are both zero then there will be no lobbying. When both of the parties set a zero tax rate to please the voters, there is no lobbying.

Having found expressions for the lobbies' optimal contributions at an interior solution, we now proceed to examine the choice of actions by the parties who take the reactions of <u>both</u> of the lobbies into account when making their decisions. The goal of each party is assumed to be the maximization of its own probability of election taking the action of the other party as fixed. The first-order condition for party 1's problem under this full information setup is

$$\frac{\partial \pi}{\partial c_1}\frac{\partial c_1}{\partial \tau_1} + \frac{\partial \pi}{\partial c_2}\frac{\partial c_2}{\partial \tau_1} + \frac{\partial \pi}{\partial \tau_1} \leq 0 \qquad \tau_1 \geq 0 \tag{7.9}$$

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[7] In all of the following analysis we shall assume that N is large enough to ensure that the optimal contribution is less or equal to the amount of the endowment.

[8] The proportion one quarter comes from assuming ab = de = 1 in (7.1). More generally, this proportion is $1/(ab + de)^2$.

with complementary slackness, which, using $\partial\pi/\partial c_1 = - \partial\pi/\partial c_2 \ (c_2/c_1)$ at an interior solution from (7.2), rearranges to

$$\frac{\partial\pi}{\partial c_1} \left[\frac{\partial c_1}{\partial \tau_1} - \frac{c_1}{c_2} \frac{\partial c_2}{\partial \tau_1} \right] + \frac{\partial\pi}{\partial \tau_1} = 0 \tag{7.10}$$

From (7.5) we can get an expression for $\partial c_1/\partial\tau_1$ as

$$\frac{\partial c_1}{\partial \tau_1} = \frac{A_2 \left[\frac{\partial c_2}{\partial \tau_1} A_1 + c_2 (1 + n/N) \right] - A_1 c_2}{A_2^2} \tag{7.11}$$

Substituting (7.11) into (7.10) and evaluating at a symmetric situation $c_1 = c_2$, $\tau_1 = \tau_2$ and using (7.8) gives the following result:

<u>Result 7.1</u> An interior symmetric equilibrium sets $c_1 = c_2 = c^f$ and $\tau_1 = \tau_2 = \tau^f$ where

$$c^f = \frac{n\,(2N + n)}{8N\,(N + n)} \tag{7.12}$$

$$\tau^f = \frac{n}{2\,(N + n)} \tag{7.13}$$

The comparative statics of the interior solution are intuitively appealing. An increase in either τ or n increases the size of the transfer and hence the lobby contribution. The larger the contribution, the greater the tax rate so that τ^f is increasing in n. As the number of lobby members, N, increases, the smaller will be the per capita transfer and consequently lobbying is reduced which in turn lowers the tax rate. Result 7.1 and the ensuing comparative statics reflect the fact that the parties, in setting their policies, trade off between the interests of the lobbies and those of the hostile group of voters.

To highlight the role which the possibility of squeezing of the voters plays in this model, consider the case when $n = 0$. This gives $\tau^f = 0$ and $c^f = 0$ (which is consistent with the discussion of the best response function in (7.4)). This is due to the fact that in this special case, the lobbies have exactly symmetric incentives; there is no income to be gained from the third group so that the tax rate merely redistributes income among the lobbies. Any gain to lobby 1 is exactly the loss to lobby 2 so that they would contribute equal amounts to their preferred party, with lobbying canceling out in aggregate. This fact precludes an interior lobbying solution under full information but not within the limited information framework as the next section demonstrates.

When each party only takes into account the reaction of its support lobby when choosing its policy then the first order condition for party 1 can be written as

$$\frac{\partial \pi}{\partial c_1} \frac{\partial c_1}{\partial \tau_1} + \frac{\partial \pi}{\partial \tau_1} \leq 0 \qquad \tau_1 \geq 0 \tag{7.14}$$

with complementary slackness which is simply (7.9) without the adverse reaction of the opposing lobby. The lobby reaction functions for this problem are still given in (7.4) so that at least one of the tax rates must be positive in order for the interior solution to the lobby sub-game to be valid. To solve the model under limited information for an <u>interior symmetric equilibrium</u> involves differentiating the expression for the optimal interior contribution of lobby 1 given by (7.7) and substituting this expression into (7.14) which is then evaluated at a symmetric situation. This yields the following result:

<u>Result 7.2</u> An interior symmetric equilibrium of this game with limited information sets $c_1 = c_2 = c^I$ and $\tau_1 = \tau_2 = \tau^I$ where

$$c^I = \frac{(2N + n)\,(N + n)}{4N\,(3N + 2n)} \tag{7.15}$$

$$\tau^{l} = \frac{N + n}{3N + 2n} \tag{7.16}$$

Whilst Results 7.1 and 7.2 yield the same intuitive comparative static implications, there are important differences which are stated as three corollaries.

COROLLARY 7.1: When the lobbies have symmetric incentives to contribute, that is n = 0, an interior symmetric lobbying equilibrium still exists under limited information with $c^{l} = 1/6$ and $\tau^{l} = 1/3$. Under full information, an interior symmetric lobbying equilibrium will not exist in this case.

The existence of this lobbying equilibrium under limited information is due to the fact that only the positive response by the support lobby is taken into account by the parties when setting policy; although lobbying contributions cancel out in aggregate, this is not realized by the parties. This solution is therefore unreasonable but is akin to those provided by Young and Magee (1986).

The following statement is obvious from Results 7.1 and 7.2

COROLLARY 7.2: Active policy is overstated in the interior symmetric equilibrium in the limited information case, i.e. $\tau^{l} > \tau^{f}$.

This arises again as the direct result of only considering the positive lobbying impulse.

The limited information solution therefore overstates the 'true' redistributive policy arising due to interest group pressure. Moreover, this implies the following:

COROLLARY 7.3: At an interior symmetric equilibrium, $c^l > c^f$.

Using Results 7.1 and 7.2 it is easy to verify that the lobbies have a higher expected income in the limited information solution which is directly at the expense of the taxpayers who must meet a larger taxation bill in this scenario. Conversely, a move towards the full information equilibrium reduces the welfare of the lobbies but increases that of the taxpayers.

To summarize then, we have a model which is strategically equivalent to those used by Brock and Magee (1978), Magee and Brock (1983) and Young and Magee (1986) in which policy receives positive impulses in response to interest group pressure but is tempered by hostile voters. By only considering the favourable reaction of the support lobby, the solutions these authors find are biased. In particular, the level of lobbying activity is overstated as is the resulting level of the redistributive policy and amount of transfers. One justification for using the limited information framework is that these equilibria are less complex to calculate. Whilst this is undoubtedly true, the model of chapter six transforms the lobbying problem to a more manageable (and more general) setup in which the full information solutions are easily obtained. This general framework does not incorporate the direct voter hostility which

is present in the current chapter however.[9]

7.3 A two-period problem with external debt

This section extends the model to two periods; in the first period the parties can borrow from an external source if elected in order to reward the loyalty of the support lobby, with the debt being repaid in the second period. The idea here is to examine whether and to what extent the inherent characteristics of a lobbying economy lead it to contract large amounts of external debt. The results naturally depend upon the exact specification of the economy; in particular, the fore- (or short-) sightedness of the voters[10] is shown to play a crucial role as is the number of redistributive instruments at the disposal of the parties once elected. The reason that borrowing may arise from this lobbying situation is that one lobby receives the funds (the lobby whose party has won the election) whilst the debt must be shared equally by all agents in the economy. Lobbying the parties to borrow externally can therefore be to the advantage of the interest groups. This mechanism also underlies the partisan political model of Alesina and Tabellini (1989) in which the parties maximize the utility of their own interest groups through external borrowing.

At the beginning of the first period, each party announces its policies which it will follow if elected. In section 7.3.2 and 7.3.3 it is assumed that the only

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[9] Voter hostility is incorporated up to a point in chapter six: if the sum of the lobbies utility is decreasing in the policy, then the voters react negatively to active policy.

[10] This can alternatively be interpreted as the hostile or indifferent attitude of the voters to external borrowing.

redistributive instrument in the first period is the level of external borrowing whereas section 7.3.4 allows for both redistributive borrowing and taxation in the first period. After these announcements are made, the lobbies contribute to the political campaigns of the parties and the voters determine the election result probabilistically. The elected party then carries out its proposed level of borrowing and the period ends. At the beginning of period two, the parties announce tax rates which are compatible with repaying the debt and making a non-negative transfer to one of the lobbies. We assume that the debt repayment is enforceable by a fixed penalty.[11] Upon lobby contributions, the election is decided and repayment is made, ending the game. Only full-information symmetric solutions will be considered.

7.3.1 The second period

In order to solve for the sub-game perfect equilibria of this model, it is necessary to begin in the second period assuming that an amount B^* of external debt has been contracted which imposes a second period obligation of $R(B^*)$ on the elected party.[12] If party 1 sets tax rate τ_1, then the gross amount of transfer which it can channel to each lobby 1 member upon election is $\tau_1(2+n/N) - R(B^*)/N$, where B^* is first period borrowing. In order to be compatible with repaying the debt, this transfer must be at least zero. The expected income of a lobby 1 member is

[11] Let P be a fixed penalty associated with default and let R(B) be the repayment to be made on borrowing level B. Then a rational lender will set a credit ceiling B^{max} where $R(B^{max}) = P$.

[12] We assume $R' > 0$.

$$\pi\,[1 - \tau_1 + \tau_1(2+n/N) - R(B^*)/N] + (1 - \pi)\,(1 - \tau_2) - c_1$$

Using (7.2) as the reaction function of the voters, then the lobbies best response function is given by (7.4) with A_i replaced by $G_i = \tau_j + \tau_i(1+n/N) - R(B^*)/N$. Solving as before for an interior solution to the lobby sub-game yields equations of the form of (7.6) and (7.7) for the optimal interior contributions.

$$c_1 = \frac{(1 - \tau_1)\,(1 - \tau_2)\,G_1^2\,G_2}{[(1 - \tau_1)\,G_1 + (1 - \tau_2)\,G_2]^2} \tag{7.17}$$

$$c_2 = \frac{(1 - \tau_1)\,(1 - \tau_2)\,G_1\,G_2^2}{[\,(1 - \tau_1)\,G_1 + (1 - \tau_2)\,G_2\,]^2} \tag{7.18}$$

For the interior solution of the lobby sub-game to be valid requires $G_i > 0$ $i = 1, 2$. As the debt is enforceable, the announced tax rates must be compatible with repaying the debt obligation which requires $\tau_i \geq R(B^*)/(2N+n)$. In fact it is easily seen that $G_i > 0$, $i = 1, 2$, as long as one of the tax rates is strictly greater than $R(B^*)/(2N+n)$. This is parallel to the existence condition in section 7.1 where it was found that if both tax rates are zero then neither lobby can expect to receive a transfer so that lobby contributions were equal to zero. Here if $\tau_i = R(B^*)/(2N+n)$ $i = 1, 2$ then just enough tax revenue is taken to repay the debt leaving nothing as a transfer. The best response on the part of the lobbies is $c_i = 0$, $i = 1, 2$.

Using (7.17) and (7.18), at a symmetric interior solution with $\tau_1 = \tau_2 = \tau^+$ the optimal interior contribution is given by

$$c^+ = \frac{\tau^+ (2N + n) - R(B^*)}{4N} \tag{7.19}$$

Solving as in section 7.1 for the tax rate at an interior symmetric equilibrium under full information yields

$$\tau^+ = \frac{n + R(B^*)}{2 (N + n)} \tag{7.20}$$

so that the amount of lobbying decreases with the size of the obligation as this lessens the second period transfer, whilst the amount to be repaid must increase the tax rate set. Comparison of (7.19) and (7.8) and (7.20) and (7.13) highlights the way in which the debt obligation changes the results of section 7.1. For the tax rate to be compatible with repaying the loan requires $\tau^+ (2N+n) \geq R(B^*)$: using (7.20) this just reduces to $(2N+n) \geq R(B^*)$ which simply says that national income must be large enough to repay the debt. It would seem reasonable to accept this condition as satisfied as the amount of borrowing in most countries is only a small proportion of national income. Furthermore, no rational lender would loan more than the country has the resources to repay in this two-period model. Moreover, we must check that with tax rate τ^+, the conditions for the existence of an interior solution of the lobby sub-game are satisfied. In the symmetric case this requires that $\tau^+ (2N+n) > R(B^*)$.

Substituting for τ^+ from (7.20) reduces this condition to $2N+n > R(B^*)$ so that the interior symmetric equilibrium is valid if national income is greater than the debt to be repaid.

The interior lobbying solution gives the parties a payoff of 1/2 in the second period whilst substituting the lobbying and policy outcomes into the lobbies expected income equations yields that each lobby can expect $X + Z\,R(B^*)$ where

$$X = 1/2 + \frac{n^2}{8N\,(N + n)} \quad ; \quad Z = -\frac{(2N + n)}{8N\,(N + n)} < 0$$

so that the expected payoff of the lobbies is decreasing in the amount to be repaid.

In the following sections, four cases will be considered: i) borrowing is the only first period instrument and voters are myopic (relevant variable indexed by a bm superscript); ii) rational voters and borrowing only (br); iii) myopic voters and the possibility of taxation and borrowing (tbm); and finally iv) rational voters with the two policy instruments (tbr). In all cases, the lobbies are pictured as rational players who realize that borrowing in the first period imposes an obligation on them via taxation in the second period.[13]

[13] Discussion of the results is deferred to section 7.4.

7.3.2 *Borrowing with myopic voters*

When voters[14] are myopic in the current model, it is taken to mean that they do not realize that borrowing undertaken in the first period imposes an increased taxation burden in the second. When borrowing is the only instrument of redistribution in the first period, the voters reaction function at this time takes the form

$$\pi^{bm} = \frac{\delta K + c_1^{bm}}{2\delta K + c_1^{bm} + c_2^{bm}}$$

which is similar to the example used in chapter six so that the methods introduced there are appropriate here. In this case, the voters are not hostile to the first period policy of the parties. Myopic voters can be rationalized in this setup by considering that their utility is positively correlated with national income; as borrowing increases national income, this may increase the voters' utility. Assume that party 1 chooses to borrow B_1^{bm} and redistribute these funds to lobby 1; with α as the common discount factor, the two period expected lobby utility sum in this case is

$$2 + B_1^{bm}/N + 2\alpha(X + Z\,R(B_1^{bm}))$$

which increases in B_1^{bm} if

[14] Voters here refer to the n non-lobby members, as the votes of the lobbies cancel out in aggregate.

$$1/N + 2\alpha Z\, R'(B_1^{bm}) > 0 \tag{7.21}$$

The lobbies are pictured as rational players who realize the second period implication of first period borrowing. In this case, the optimal policy for the parties is the one which trades off the first period gains of the lobbies with the second period losses. Let β be the level of borrowing which makes (7.21) hold with equality; in fact this is the optimal level of borrowing at an interior lobbying solution. To see this, assume that party 1 proposes to borrow $B_1^{bm} > \beta$ whilst party 2 sets $B_2^{bm} = \beta$. By the definition of β it must be the case that the utility sum is decreasing in the amount borrowed after β so that, by Lemma 6.1 $c_2^{bm} \geq c_1^{bm}$ so that party 1 could gain by setting $B_1^{bm} = \beta$. If party 1 sets $B_1^{bm} < \beta$ on the other hand, the utility sum is increasing in the amount borrowed up to β so that party 2 playing β would still get more contributions. Both parties playing $B_1^{bm} = \beta$ is therefore the equilibrium outcome. It is exactly this level of borrowing which balances the gains to the lobbies of borrowing in the first period with the losses incurred due to second period repayment.

If the country is constrained in its amount of borrowing, that is $B^{max} < \beta$ then it is optimal to borrow B^{max} as the utility sum is increasing at this amount of borrowing. If the country is unconstrained then it will borrow β unless $\beta \leq 0$ in which case no borrowing will be undertaken.

The members of lobby 1 choose their amount of contributions in order to maximize expected income which is given by

$$\pi^{bm} [1 + B_1^{bm}/N] + (1 - \pi^{bm}) - c_1^{bm} + \alpha[X + Z \{\pi^{bm} R(B_1^{bm}) + (1 - \pi^{bm}) R(B_2^{bm})\}] \tag{7.22}$$

where π^{bm} is the first period probability of election of party 1 (the second period probability having been set equal to its equilibrium value of 1/2). This equation reflects the fact that if party 1 wins the election each lobby 1 member will receive a transfer of B_1^{bm}/N with no transfer being made to lobby 1 if party 2 forms the government. The second period payoff depends upon the amount of borrowing in the first period which depends upon which party wins that election. The first-order conditions for a maximum are given by

$$\frac{\partial \pi^{bm}}{\partial c_1^{bm}} H_1 \leq 1 \quad ; \qquad c_1^{bm} \geq 0 \tag{7.23}$$

with complementary slackness and a symmetric condition for lobby 2 members replacing H_1 by H_2 where

$$H_1 = B_1^{bm}/N + \alpha Z [R(B_1^{bm}) - R(B_2^{bm})]$$

$$H_2 = B_2^{bm}/N - \alpha Z [R(B_1^{bm}) - R(B_2^{bm})]$$

These first-order conditions yield a best response function for lobby i of the form:

$$c_i^{bm} = Max \{ (\delta K + c_j^{bm})^{1/2} H_i^{1/2} - 2\delta K - c_j^{bm} , 0 \} \tag{7.24}$$

Again, the interior optimum contribution of lobby i can readily be determined to be

$$c_i^{bm} = \frac{H_i^2 \, H_j}{[H_1 + H_2]^2} \tag{7.25}$$

so that the interior solution to the lobby sub-game is valid if $H_i > 0$ i = 1, 2. This is certainly true if the parties choose a <u>symmetric</u> level of borrowing $B^{bm} > 0$. If the parties both choose a zero level of borrowing then the best response by the lobbies is to set $c_i = 0$, i = 1, 2 as neither party is pursuing a policy which benefits either of the lobbies. On the other hand, if both parties choose $B^{bm} > 0$ then contributions by both lobbies will be positive as both lobbies attempt to get their own party elected in order to benefit from the foreign borrowing. In fact, it was shown earlier that as long as $\beta > 0$ we have $B^{bm} > 0$ so the interior solution of the lobby sub-game is valid.

Setting B^{bm} as the common policy of both parties from the discussion earlier and using (7.25), an interior symmetric lobbying equilibrium gives $c^{bm} = B^{bm}/4N$ so that the contribution of each lobby member is one quarter of the potential transfer.

7.3.3 Borrowing with rational voters

If the voters are rational, then they realize that any borrowing to benefit the lobbies will lead to a higher second period tax rate. Assuming that voters react in a hostile manner to this obligation and using a logit probability of election function, the probability that party 1 is elected is

$$\pi^{br} = \frac{(\delta K + c_1^{br})\ (\delta^R M + R(B_2^{br}))}{(\delta K + c_1^{br})\ (\delta^R M + R(B_2^{br})) + (\delta K + c_2^{br})\ (\delta^R M + R(B_1^{br}))}$$

(7.26)

where $M > 0$ is a constant and δ^R is a Kronecker delta equal to one if $R(B_1) = 0$ or $R(B_2) = 0$ and zero if both of these magnitudes are positive. Notice that the larger the obligation imposed by party i's borrowing (the higher the borrowing) the lower the chance that this party is elected. Lobby 1's expected income in this case is identical to (7.22) with the bm superscript replaced by br. Likewise, the first-order conditions for the lobby problem are given in (7.23) with the change of superscript and replacing H_i by J_i where $J_1 = B_1^{br}/N + \alpha Z\ [R(B_1^{br}) - R(B_2^{br})]$, $J_2 = B_2^{br}/N - \alpha Z\ [R(B_1^{br}) - R(B_2^{br})]$. These first-order conditions yield a best response function for lobby i as

$$c_i^{br} = Max\ \{\ \frac{[(\delta K + c_j^{br})\ \psi\ J_i]^{1/2} - (\delta K + c_j^{br})\ (\delta^R M + R(B_i^{br}))}{\delta^R M + R(B_j^{br})}\ ,\ 0\ \}$$

(7.27)

where $\psi = (\delta^R M + R(B_1^{br}))(\delta^R M + R(B_2^{br}))$.

At an interior solution of the lobby sub-game, $c_i^{br} > 0$, $i = 1, 2$ we have

$$c_i^{br} = \frac{\psi\ J_j\ J_i^2}{[J_1\ (\delta^R M + R(B_2^{br})) + J_2\ (\delta^R M + R(B_1^{br}))]^2}$$

(7.28)

For this interior solution to be valid requires $J_1 > 0$ and $J_2 > 0$. Whilst in general this may not hold, the interior lobbying solution is certainly valid in the symmetric case as long as the symmetric level of borrowing is strictly positive. If both parties set their level of borrowing at zero then (7.27) indicates that the best response is for neither lobby to give any contributions as there is no transfer to be gained by lobbying, only cost to be incurred.

In a full information symmetric interior equilibrium, the amount of borrowing per lobby member can easily be shown to satisfy

$$\frac{B^{br}}{N} = \frac{R(B^{br})\,[\frac{1}{N} + 2\alpha Z\, R'(B^{br})]}{R'(B^{br})} \tag{7.29}$$

with lobbying given by $c^{br} = B^{br}/4N$.

7.3.4 A choice of redistributive policies

In this section we consider the possibility that the parties can not only borrow, but may also use taxation policy to channel funds to their support groups after an election victory. The parties thus announce both a level of borrowing and a tax rate prior to the lobbies making their contributions. When the voters are myopic (or simply not hostile to borrowing) then the voters' reaction function is given by (7.2) (adding a tbm superscript). Lobby 1 now not only stands to gain a potential transfer from the external borrowing, but also from the other agents in the economy through

taxation revenue so that each member chooses the level of contribution to maximize

$$\pi^{tbm} [1 - \tau_1^{tbm} + B_1^{tbm}/N + \tau_1^{tbm}(2+n/N)] + (1 - \pi^{tbm}) [1 - \tau_2^{tbm}]$$
$$+ \alpha\{X + Z [\pi^{tbm}R(B_1^{tbm}) + (1 - \pi^{tbm})R(B_2^{tbm})]\} - c_1^{tbm} \qquad (7.30)$$

where the tbm superscripts denote that both taxation and borrowing are possible and that the voters are myopic.

Letting $P_1 = \tau_2^{tbm} + \tau_1^{tbm} (1+n/N) + B_1^{tbm}/N + \alpha Z [R(B_1^{tbm}) - R(B_2^{tbm})]$ and $P_2 = \tau_1^{tbm} + \tau_2^{tbm} (1+n/N) + B_2^{tbm}/N - \alpha Z [R(B_1^{tbm}) - R(B_2^{tbm})]$, then at an interior solution of the lobby sub-game $c_i^{tbm} > 0$, i = 1, 2, the optimal contribution of lobby i is

$$c_i^{tbm} = \frac{(1 - \tau_1^{tbm}) (1 - \tau_2^{tbm}) P_j P_i^2}{[(1 - \tau_1^{tbm}) P_1 + (1 - \tau_2^{tbm}) P_2]^2} \qquad (7.31)$$

which is valid for $P_1 > 0$ and $P_2 > 0$. Again, in a symmetric equilibrium this is satisfied as long as the symmetric tax rate or the symmetric level of borrowing is strictly positive. If both of these policies are at zero then no lobbying occurs as no transfer will be granted. As long as one of the policies is positive, implying a positive transfer, the lobbies become active. At the interior symmetric equilibrium with the tax rate set at τ^{tbm} and the level of borrowing at B^{tbm}, the amount of lobbying by each lobby member is

$$c^{tbm} = \frac{\tau^{tbm}(2N + n) + B^{tbm}}{4N} \tag{7.32}$$

so that each lobby member spends one quarter of the prospective transfer on lobbying; the larger the transfer, the more the amount of lobbying.

Turning to the choices of the parties, as the voters do not care about first period borrowing in this case, the optimal interior B is the one which trades off the short-term gains of the lobbies with the longer-term losses; this is precisely $B^{tbm} = \beta$ from section 7.3.2, assuming that this level of borrowing is feasible. The interior equilibrium symmetric tax rate in response to lobby contributions is easily found to be $\tau^{tbm} = (n - B^{tbm})/2(N + n)$. Notice that the equilibrium tax rate is decreasing in the amount of borrowing. The voters do not care about the level of borrowing but are concerned that a low tax rate should be set, therefore the parties use their external borrowing to placate the lobbies and adjust the tax rate downwards to please the voters.

Taking the case where voters rationally take into account the future burden of current borrowing leads to a probability that party 1 is elected into office in the first period as

$$\pi^{tbr} = \frac{(\delta K + c_1^{tbr})\ \phi}{(\delta K + c_1^{tbr})\ \phi + (\delta K + c_2^{tbr})\ \eta}$$

where $\phi = (\delta^R M + R(B_2^{tbr}))\ (1-\tau_1^{tbr})$ and $\eta = (\delta^R M + R(B_1^{tbr}))\ (1-\tau_2^{tbr})$. Lobby 1 maximizes (7.30) (with tbr replacing the tbm superscripts), with interior lobbying contributions of lobby i being derived as

$$c_i^{tbr} = \frac{\phi\ \eta\ Q_j\ Q_i^2}{[Q_1\ \phi + Q_2\ \eta]^2}$$

again valid for $Q_i > 0$ i = 1, 2 where

$$Q_1 = \tau_2^{tbr} + \tau_1^{tbr}\ (1+n/N) + B_1^{tbr}/N + \alpha Z\ [R(B_1^{tbr}) - R(B_2^{tbr})]$$

$$Q_2 = \tau_1^{tbr} + \tau_2^{tbr}\ (1+n/N) + B_2^{tbr}/N - \alpha Z\ [R(B_1^{tbr}) - R(B_2^{tbr})]$$

As in case tbm, this is valid at a symmetric situation as long as at least one of the policies is strictly positive. At a symmetric interior situation (assuming both policies are positive), we find an expression similar to section 7.3.2 for the symmetric contribution as

$$c^{tbr} = \frac{\tau^{tbr}\ (2N + n) + B^{tbr}}{4N} \tag{7.33}$$

In choosing its policies, party 1 requires

$$\frac{\partial \pi^{tbr}}{\partial c_1^{tbr}} \frac{\partial c_1^{tbr}}{\partial \tau_1^{tbr}} + \frac{\partial \pi^{tbr}}{\partial c_2^{tbr}} \frac{\partial c_2^{tbr}}{\partial \tau_1^{tbr}} + \frac{\partial \pi^{tbr}}{\partial \tau_1^{tbr}} \leq 0 \qquad \tau_1^{tbr} \geq 0 \tag{7.34}$$

with complementary slackness and

$$\frac{\partial \pi^{tbr}}{\partial c_1^{tbr}} \frac{\partial c_1^{tbr}}{\partial B_1^{tbr}} + \frac{\partial \pi^{tbr}}{\partial c_2^{tbr}} \frac{\partial c_2^{tbr}}{\partial B_1^{tbr}} + \frac{\partial \pi^{tbr}}{\partial B_1^{tbr}} \leq 0 \qquad B_1^{tbr} \geq 0 \tag{7.35}$$

with complementary slackness, to be satisfied simultaneously. Tedious calculation reduces this to the requirement that the following two equations are satisfied simultaneously in an interior symmetric equilibrium

$$\frac{\tau^{tbr}\,(2N + n) + B^{tbr}}{N} = \frac{R(B^{tbr})\,[\frac{1}{N} + 2\alpha Z\, R'(B^{tbr})]}{R'(B^{tbr})} \tag{7.36}$$

$$\tau^{tbr} = (n - B^{tbr})/2(N + n) \tag{7.37}$$

At this interior, the parties are using both of their redistributive instruments to trade off the interests of the lobbies and the voters. Notice that the parties use their borrowing to reduce the tax rate.

7.4 Implications

Now that we have characterized the symmetric interior equilibria of the different 'regimes', we are in a position to carry out some comparative analysis. As we are concerned with the extent to which a lobbying economy would carry out borrowing if able, we examine the case where the country's access to funds is unconstrained in this two period model.

Result 7.3 Assuming that $B^{max} \geq \beta > 0$, so that borrowing is unconstrained, at an interior symmetric lobbying equilibrium it is the case that

$$\beta = B^{bm} = B^{tbm} > B^{br} \geq B^{tbr} > 0$$

Proof That $\beta = B^{bm} = B^{tbm}$ is proved earlier in the text. To show $\beta > B^{br}$, first notice that $\beta = B^{br}$ is not a solution to (7.29) so that $\beta \neq B^{br}$. Given positive lobbying, it must be the case that $B^{br} > 0$ so that the left-hand side of (7.29) is positive. As $Z < 0$, $R'(.) > 0$ and given the definition of β, the right-hand side of (7.29) can only be positive if $\beta > B^{br}$ as claimed. Next we show $B^{br} \geq B^{tbr}$. When $\tau^{tbr} = 0$ we get $B^{br} = B^{tbr}$ from (7.29) and (7.36) - although this can only be an equilibrium outcome if $B = n$ solves these two equations. From (7.37), as τ^{tbr} increases so B^{tbr} falls, indicating that if $\tau^{tbr} > 0$ then $B^{tbr} < B^{br}$. Putting these results together gives $B^{br} \geq B^{tbr}$. Finally, assume $B^{tbr} = 0$. Then the right-hand side of (7.36) equals zero which implies $\tau^{tbr} = 0$. But $\tau^{tbr} = 0$ only if $B^{tbr} = n > 0$, a contradiction. ■

Some interesting observations emanate from Result 7.3. Irrespective of the number of redistributive instruments under the control of the government, an economy which has myopic voters[15] will borrow more than one in which the voters rationally include the future when deciding the current election.[16] The reason for this is obvious: when voters do not realize (or care about) the future burden imposed by present borrowing, the parties promise to contract debt solely as a means of eliciting pre-election campaign funds from the lobbies - there is no hostile voter effect of borrowing. The optimal level of borrowing in this case is independent of the availability other redistributive instruments and is set at the level which balances lobby gains in period one with lobby losses in period two. If the lobbies themselves were myopic (or perceived that the repayment obligation cannot be imposed upon them), then the parties, in the absence of the hostile voter effect, would borrow up to the maximum level permitted by the credit market. This overborrowing would naturally reduce the overall efficiency of the outcome if the debt indeed had to be repaid.

Instead of regarding voters as myopic, the above remarks also hold in (undemocratic) countries where the policy preferences of the voters hold little relevance for policy formulation. In this case, the size of the policies determines who forms the government only through the amount of induced lobby contributions; the

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[15] Or, as suggested earlier, voters whose utility is positively correlated with the size of national income. The important point is that voters do not react in a hostile manner to borrowing.

[16] Guttentag and Herring (1985) indicate that large amounts of external debt could result as a consequence of the banks being myopic - disaster myopia in their terms. Result 7.3 suggests that if the myopia were on the part of the citizens (voters) of an economy then external debts would be large.

party with the most lobbying support will gain power. In this case, the parties would maximize the utility of the support lobby (akin to a partisan model) so that borrowing would optimally take account of first period gains and second period losses, but the tax rate (and hence redistribution) would be maximal.

When voters are far-sighted and so react in a hostile manner to borrowing, then the number of policy instruments available to the government can potentially affect the size of the economy's external borrowing. If there are two channels through which income can be redistributed in the first period and the parties choose not to avail themselves of the taxation option, then borrowing is the same as the case where this is the <u>only</u> option. On the other hand, equation (7.37) shows that taxation and borrowing move in different directions so that redistributing income via taxation will lessen the need to borrow externally. The fact that $B^{tbm} > B^{tbr}$ implies that $\tau^{tbm} < \tau^{tbr}$ so that the composition of redistributed revenue in myopic voter economies has a larger borrowing and smaller taxation component than in an economy characterized by rational voters.

The amount of lobby contributions in the different regimes are related by the following result:

<u>Result 7.4</u> At an interior symmetric equilibrium, with borrowing unconstrained:

$$c^{tbm} \geq c^{bm} > c^{br} \geq c^{tbr} > 0$$

Proof That $c^{tbm} \geq c^{bm}$ follows from the fact that

$$c^{tbm} = \frac{\tau^{tbm}(2N + n) + \beta}{4N} \geq \frac{\beta}{4N} = c^{bm}$$

When the level of borrowing is the only instrument, the level of contributions is equal to one quarter of the amount of borrowing per lobby member. The fact that $B^{bm} > B^{br}$ gives that $c^{bm} > c^{br}$. Finally, notice that when $\tau^{tbr} = 0$ we have the level of transfers (T), $T^{tbr} = T^{br}$. When $\tau^{tbr} > 0$ we have from (7.36) and (7.37)

$$T^{tbr} = \frac{n(2N + n) + nB^{tbr}}{2(N + n)} \quad ; \quad \frac{\partial T^{tbr}}{\partial B^{tbr}} > 0$$

An increase in τ^{tbr} causes B^{tbr} to fall which in turn reduces T^{tbr} below T^{br}. Therefore $T^{br} \geq T^{tbr}$ which implies $c^{br} \geq c^{tbr}$ as $c^{w} = (T^{w}/4N)$, w = tbr, br. ■

In the myopic voter economies, where borrowing incurs no hostile reaction, the greatest amount of transfers are granted and hence the amount of resources spent on capturing these contrived rents is large. Additionally, the availability of a redistributive tax rate will tend to (weakly) increase the level of redistributive activity. The reason for this is clear: when borrowing incurs no hostile voter reaction, the parties take on credit to please the lobbies but additionally set a tax rate in order to trade off the interests of the lobbies and the voters as in the model of section 7.1 (or the second period of this model in section 7.3.1).

Where voters are rational, the seemingly perverse result obtains that transfers,

and rent-seeking contributions are largest when only borrowing is possible; if a tax rate is available too, then this (weakly) decreases the transfer size. It was noted earlier that the tax rate and level of borrowing move in opposite directions in order to balance the opposed interests of the voters and lobbies. The reason that the economy with both instruments available exhibits least rent-seeking is that voters react negatively both to a positive tax rate and to a positive level of borrowing. Equilibriating these two negative forces with the benefit of (net) lobbying contributions leads the economy towards less rent-seeking. Even when setting a tax rate is not an option in the rational economy, the level of redistributive activity is lower than in the myopic economy due to the adverse reaction of the voters.

If the debt obligation is perceived as non-binding (and the penalty incredible), then the parties can borrow with no fear of the hostile voter effect. Borrowing up to the maximum permitted would occur if the lobbies too regarded costless default as an alternative to repayment. This links in with the profligate economy of chapter five which borrows and never repays. Such an economy, it was suggested earlier, can obtain loans due to the lender having incomplete information about its customer type.

7.5 Debt forgiveness and lobbying economies

The previous section has highlighted the types of lobbying economies which are most likely to contract a large amount of external debt and hence be likely candidates for repayment difficulties. In particular, borrowing countries in which the voters are myopic or have very little influence may accumulate debt quickly in order

to purchase the loyalty of the lobby groups but it is the average citizen who must bear the burden of the repayment through taxation. One possible solution to these difficulties would be to forgive a proportion of the debt in order to reduce the tax rate. Whilst the creditor may not wish to carry out such a policy, an organization such as the International Monetary Fund may see this tax reduction as a reasonable objective. The IMF could then agree to repay some of the debt itself (subject to its conditionality requirements). In the literature on debt forgiveness there are two approaches to debt forgiveness:[17] one which seeks to link the amount of debt forgiveness to an endogenous variable such as output, and the other which makes the amount forgiven dependent on an exogenous magnitude such as the world price of oil. The former often leads to an incentive problem as the country may aim to affect a variable under its control in order to influence the amount of forgiveness.[18]

Whilst the latter proposal fares better in this respect, it would not seem to solves the ills afflicting a rent-seeking economy. When the debt falls due, the political parties are forced into a policy of high taxation to repay the debt and attempt to elicit contributions from the lobbies, displeasing the voters. It may be thought that reducing the repayment burden would allow the parties to set a lower tax rate but, in this rent-seeking society, the parties may keep the tax rate unchanged in order to fund more transfers to the lobbies and gain contributions. Debt forgiveness would therefore do very little for the ordinary citizen or the level of rent-seeking.

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[17] See for example, Krugman (1988) and Froot, Scharfstein and Stein (1988).

[18] By reducing output if forgiveness depends inversely on output, for example.

With the debt already in place,[19] one way of avoiding this problem would be to link the level of forgiveness to the amount of lobbying contributions in the economy:[20] the lower the level of contributions, that is the less the amount of rent-seeking, the more the forgiveness would be. In other words the proposal makes forgiveness conditional on reform of the political process. The debt forgiveness schedule would then be a function F(c) with $F' < 0$. In this situation, the larger the proposed redistributive transfer, the larger is the lobby contribution and the lower would be the amount of forgiveness. By reducing the initial transfer, a party may be able to 'buy' an increased election probability through a larger level of forgiveness making the taxation policies more palatable to the voters. The fact that the debt repayment burden has been reduced means that even though the tax rate is lower, the lobbies may not lose out too much since transfers are the difference between taxation revenue and repayment obligations. The overall winner from this scheme is the ordinary citizen who pays less tax.

Without the outside option of forgiveness then, any welfare improvement for the taxpayers comes at the expense of the lobbies and vice versa. With the forgiveness option, it may be possible to simultaneously increase the welfare of both groups, with the welfare increase being 'funded' by the IMF for instance. An additional beneficial effect occurs if the resources which are not spent on rent seeking are released into productive rather than DUP activities so that the economy will be

[19] If the agents were able to anticipate the amount of debt forgiveness then this would affect their behaviour and the amount of debt contracted in the first period. Here we join Krugman (1988) in examining the case in which the debt is already in place.

[20] This idea arose from conversations with Prof. W. A. Brock.

able to grow faster.

This type of debt forgiveness schedule introduces another mitigating effect into the policy selection process: previously, the only force keeping the redistributive policies 'low' was the displeasure of the voters, but now in addition, we have the possibility that announcing less redistributive policies can lower the repayment obligation. The parties may receive less contributions but will be able to reduce their announced tax rates to placate the hostile voters.

If the aim of the forgiveness schedule is to reduce lobbying contributions and hence provide less of a positive impulse to taxation policy, care must be taken in the design of the schedule. If the level of forgiveness falls quickly to zero at low lobbying levels, then the existence of forgiveness will do little to alter the incentives of the interest groups so that the original (high contribution) equilibrium will obtain. If, on the other hand, the level of forgiveness is positive even at high lobbying levels, we may get the undesired result that the introduction of a contribution linked forgiveness schedule will, by reducing the repayment obligation, increase the level of transfers and hence the amount of lobbying leading possibly to a higher tax rate and lower welfare for the ordinary citizens. In order to have the desired effect of reducing lobbying and hence the tax rate, the forgiveness schedule must balance these two concerns.

Before proceeding to an algebraic statement of these points, some diagrammatic analysis can be used to illustrate in the lobby sub-game. When the debt

becomes due, assuming the absence of any forgiveness schedule, the marginal cost to the lobbies of contributing is $MC^{NF} = 1$ and the marginal benefit (assuming a positive symmetric tax rate for simplicity) is

$$MB^{NF} = \frac{\tau\ (2N + n) - R(B)}{4N\ c^{NF}} \tag{7.38}$$

so that the lobby sub-game solution equates marginal cost and marginal benefit from contributing. This is depicted as c^{NF} in Figure 7.1. When a contributions-linked forgiveness schedule is in place, F(c) with $F'(.) < 0$, $F''(.) > 0$, $F'(0) = 0$ and $F(c \geq x) = 0$, the marginal cost of contributing is

$$MC^F = 1 - \frac{F'(c)}{2N} \geq 1 \qquad c \leq x$$

$$MC^F = 1 \qquad c \geq x \tag{7.39}$$

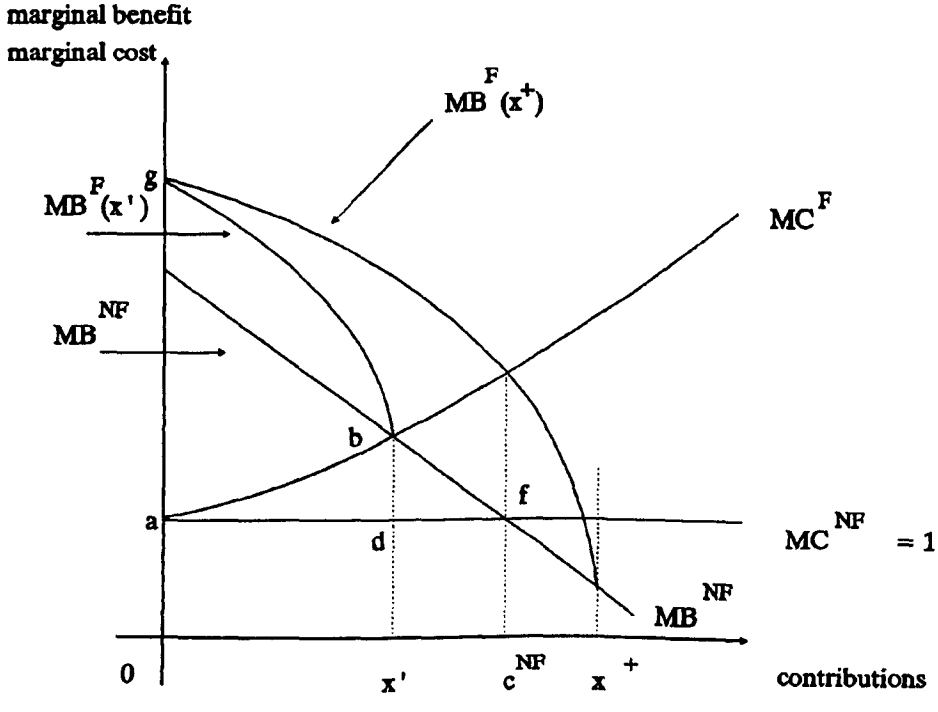
The extra marginal cost for $c \leq x$ reflects the fact that by contributing (up to x) the lobbies reduce the amount of forgiveness and hence, for a given tax rate, the amount of the transfer which is given by $\tau(2 + n/N) - R(B)/N + F(c)/N$ per lobby member. After x, the amount of forgiveness is zero so that the marginal cost of contributing reverts to one. Notice that there is a discontinuity in the marginal cost schedule at $c = x$ (which, as indicated below, leads to the possibility of two solutions to the lobby sub-game for a symmetric tax rate).

With a forgiveness schedule in place, the marginal benefit to the lobbies of contributing, again for a given symmetric tax rate is

$$MB^F = \frac{\tau\ (2N + n) - R(B) + F(c^F)}{4N\ c^F} \tag{7.40}$$

For a given tax rate, from (7.38) and (7.39) it is clear that $MB^F = MB^{NF}$ when $c = x$, so that for any contribution less than x the MB^F schedule lies above the MB^{NF} line but at and after x they are equal as forgiveness has fallen to zero.

Figure 7.1



We can use Figure 7.1 to illustrate the earlier point that the level of forgiveness must not be set 'too high' or 'too low'. As indicated, the critical magnitude appears to be x, the contribution level at which forgiveness falls to zero. Assume that x = x' then the marginal benefit schedule is $gbMB^{NF}$ and the marginal cost is discontinuous at b so that it follows ab and then dMC^{NF}. Two equilibria are now possible - one with c = x' and one with $c = c^{NF}$. The forgiveness schedule in this case is insufficient to rule out the high lobbying (and hence high taxation) equilibrium. For all x < x', the MC^F and MB^F do not cross until they have reverted to MC^{NF} and MB^{NF} so that c^{NF} is the only equilibrium. The forgiveness proposal is completely ineffective in this case.

For all x such that $c^{NF} \geq x \geq x'$, there are two possible outcomes of the lobby sub-game - one with c^{NF} and the other with a lower level of contributions. On the other hand, if we have $x^+ \geq x > c^{NF}$, then there is only one outcome, c^F, and furthermore this has $c^{NF} \geq c^F$ (with equality at $x = x^+$). The level of contribution x^+ is such that the MB^F and MC^F schedules cross at c^{NF}. For x in this range, the MB^F line reverts to MB^{NF} after the no-forgiveness equilibrium c^{NF} has been surpassed, ruling out this original outcome and hence leading to a lower level of contributions and tax rate. Although additionally there is at most one equilibrium for $x > x^+$, this equilibrium yields a crossing point for the marginal benefit and cost schedules to the right of c^{NF} implying a higher level of lobbying so that the forgiveness schedule does not lead to a smaller impulse to taxation policy. Setting x too high therefore leads to even more lobbying! This occurs in the lobby sub-game because the amount of forgiveness increases the transfer available (the tax rate is taken as constant in the lobby sub-game) and the larger the transfer, the greater the lobbying contributions.

The optimal forgiveness schedule will be indexed by a cut-off point x^* such that $x^+ \geq x^* > c^{NF}$ in order that the level of forgiveness is high enough to impose a cost on the lobbies, but low enough that the transfer is not increased at high levels of contributions. In fact, as long as $x^+ > x^* > c^{NF}$, then the outcome of the lobby sub-game will yield less contributions to the political system, that is, less of a positive impulse to taxation policy. A decrease in the tax rate makes the n ordinary citizens better off, but of course this improvement in their welfare has been funded by the creditor through its forgiveness. If one views the commercial banks as not being quite so benevolent, then the same result could be achieved by an organization such as the International Monetary Fund repaying (or writing off if the debt is owned by the IMF) part of the obligations for this country conditional on the level of rent-seeking. If one of the aims of the IMF is to improve the well-being of the ordinary citizen in a borrowing country, then such a contribution-based system appears to be effective for lobbying economies. The fact that less resources will be spent on lobbying, means additionally that more can be released into productive activities, with implications for the growth of an economy.[21]

Let us turn now to some more formal analysis of the debt forgiveness issue in this lobbying economy. For simplicity assume that the lender (or IMF) implements a forgiveness schedule which writes off an amount of debt F modified by a constant ($f > 0$) amount of the total lobbying in the economy in the repayment period. The forgiveness schedule is then $\Gamma = F - Nf(c_1+c_2) \geq 0$, so that Γ/N benefits each lobby member. If lobbying is too large so that $F < Nf(c_1+c_2)$ then $\Gamma = 0$ as forgiveness

[21] See Terrones (1990) for the effect of rent-seeking on growth.

cannot be negative. The problem for lobby 1 is now to choose its level of contribution to maximize its expected income per member given by

$$\pi\,[1 - \tau_1 + \tau_1(2 + n/N) - R/N + F/N - f(c_1 + c_2)] + (1-\pi)(1-\tau_2) - c_1$$

where R is the size of the obligation due. The first order conditions for an optimum are

$$\frac{\partial \pi}{\partial c_1}\,[\tau_2 + \tau_1\,(1+n/N) - R/N + F/N - f\,(c_1+c_2)] \le 1 + \pi f \qquad c_1 \ge 0 \tag{7.41}$$

with complementary slackness. Notice that the forgiveness schedule has increased the marginal cost of contributing to $1+\pi f$ for lobby 1 (and $(1 + (1-\pi)f)$ for lobby 2). It is straightforward to show that at an interior optimum we have

$$\frac{c_1}{c_2} = \frac{(1 + (1-\pi)f)\,Y_1}{(1 + \pi f)\,Y_2} \tag{7.42}$$

where $Y_1 = [\tau_2 + \tau_1(1+n/N) - R/N + \Gamma/N]$ and $Y_2 = [\tau_1 + \tau_2(1+n/N) - R/N + \Gamma/N]$. Using (7.41) and (7.42), the following optimal interior contributions can be derived

$$c_1 = \frac{(1-\tau_1)\,(1-\tau_2)\,(1+ (1-\pi)f)\,Y_1^2\,Y_2}{[(1+(1-\pi)f)\,Y_1\,(1-\tau_1) + (1+\pi f)\,Y_2\,(1-\tau_2)]^2}$$

$$c_2 = \frac{(1-\tau_1)\ (1-\tau_2)\ (1+\pi f)\ Y_2^2\ Y_1}{[(1+(1-\pi)f)\ Y_1\ (1-\tau_1) + (1+\pi f)\ Y_2\ (1-\tau_2)]^2}$$

which are valid for $Y_i > 0$, $1 = 1, 2$. As $F - Nf(c_1+c_2) \geq 0$ we have that $Y_i > 0$ as long as $\tau\ (2N+n) > R$. In other words there is only an interior symmetric solution to the lobby sub-game if a positive transfer will be granted. Using these contributions to evaluate the first order conditions of party 1 (7.9) at a symmetric interior equilibrium yields the following results for the lobby contributions and the tax rate under the proposed forgiveness schedule

$$c^F = \frac{n\ [\ 2N + n - R + F\ ]}{4N\ [(1+f/2)\ (2N+n) + n(1+f)]}$$

$$\tau^F = \frac{n\ (1+f) + (1+f/2)\ (R - F)}{(1+f/2)\ (2N+n) + n(1+f)}$$

Notice that if $f = F = 0$ then we return to the case of no forgiveness depicted in equations (7.19) and (7.20). To demonstrate the importance of the setting of the cut off point for the debt forgiveness schedule, we shall assume $f = 2$ and look for the critical setting of F. In this case we obtain

$$c^F = \frac{n\ [\ 2N + n - R + F\ ]}{4N\ (4N+5n)}$$

$$\tau^F = \frac{3n + 2R - 2F}{4N + 5n}$$

The fact that c^F is increasing in F demonstrates the point made graphically in Figure 7.1 that a forgiveness schedule which is too generous will lead to more lobbying in an interior symmetric equilibrium than if there were no forgiveness. As the tax rate is decreasing in F, this increase in lobbying will not increase the size of the tax rate further. In fact, the parties use the amount of forgiveness to reduce the tax rate and placate the voters.

For this scheme to have an effect, we require that the level of forgiveness be positive at the equilibrium. This requires that $F - 2Nfc^F > 0$. Furthermore, comparison of c^F and the contribution level without forgiveness from (7.19) (denote this contribution by c^{NF}) implies a further restriction upon the size of F. These two conditions imply

$$\frac{(2N + 3n)\,(2N + n - R)}{2(N + n)} > F > \frac{n\,(2N + n - R)}{4(N + n)} \tag{7.43}$$

Comparing τ^F with the tax rate without forgiveness from (7.20) indicates that the tax rate under forgiveness is indeed smaller as long as F is greater than the magnitude on the right hand side of (7.43). It is therefore possible to design a schedule of debt forgiveness for a lobbying economy which reduces the amount of lobbying and the size of the tax rate at an interior symmetric equilibrium.

As indicated earlier, for some values of F in the range given by (7.43), two interior equilibria exist: one with positive forgiveness and the other being the original

equilibrium of equations (7.19) and (7.20). From the discussion of Figure 7.1, in order to rule out the original (high lobbying) equilibrium requires $F - 2Nfc^{NF} > 0$, which yields a range for F in which there is only one equilibrium: in this equilibrium, both lobbying and the tax rate are less than in the case with no debt forgiveness. The range for F is given by

$$\frac{(2N + 3n)\,(2N + n - R)}{2(N + n)} > F > \frac{n\,(2N + n - R)}{2(N + n)}$$

In the lobbying economy considered here, debt forgiveness linked to an endogenous variable (lobby contributions) can improve the welfare of the ordinary citizen in an interior symmetric lobbying equilibrium by reducing the tax rate and additionally it allows resources to be channelled away from DUP towards productive activities. The effect of the introduction of a contributions-linked forgiveness schedule on the welfare of the lobbies is ambiguous in this model. There are two reasons why contributions fall in the presence of the proposed schedule of forgiveness: i) higher marginal cost of contributing; ii) lower transfer to be gained. So the lobbies make a saving (in terms of resources not spent rent seeking) but to balance this is the lower potential transfer should the favoured party be elected. Whether the saving outweighs the cost will depend upon the exact nature of the forgiveness schedule.

7.6 Summary

In this chapter, a lobbying model has been presented which incorporates direct voter hostility to the redistributive policies announced by the political parties. This was not a feature of the general model of chapter six. The model here was solved for the interior optimal lobbying contributions and the optimal policies (tax rates) arising from a politico-economic symmetric equilibrium under both limited and full information. Although the comparative statics of both interior symmetric solutions appeal to intuition, the limited information solution was shown to be unreasonable where the lobbies have symmetric incentives to contribute. Additionally, this solution predicted too much lobbying too often and policies which are too high in comparison with the 'true' model. This is due to the fact that the parties only consider the positive impulse from the own lobby under the limited information structure, ignoring the adverse reaction of the opposing lobby. This adverse reaction is explicitly accounted for in the full information solution.

The model (under full information) was then extended to two periods linked by external borrowing which the parties could promise to use if elected to purchase the loyalty of their lobbies. Different cases were analyzed depending on whether the parties had both a tax rate and borrowing available as policy options or just one of these, and whether the voters realized that borrowing increased the tax rate in the second period. It was shown that economies in which voters are myopic (or have utility positively correlated with national income) are most likely to incur large external debts then those in which the voters react in a hostile way to borrowing for

redistributive purposes. When voters are myopic, borrowing is undertaken simply to please the lobbies, with the optimal level trading off between current lobby gains and future lobby losses due to repayment. The availability of a tax rate makes no difference to this calculation.

When voters are rationally hostile, the parties must trade off the interests of the lobbies and those of the voters. Here, the number of policy instruments can also affect the level of borrowing, with less debt being contracted if a tax rate is available as a means for redistributing income. With hostile voters and a choice of policy instruments, the parties set each policy to trade off the interests of the voters and the lobbies: borrowing yields a large gain to the lobbies in period one but imposes a cost in period two, whereas redistribution by taxation benefits the lobby whose party wins power without imposing a cost. It seems as if the lobbies would rather have income redistributed via the taxation system, but due to the hostile voter effect, the parties will not increase the tax rate above a certain level so they begin to borrow. The larger the tax rate, the less the borrowing component of redistributed income.

The amount of lobbying is greatest in myopic voter economies where parties have two policy instruments as this is the case in which the transfer is greatest. When voters are hostile, the number of instruments weakly decreases the level of lobbying: voters react negatively to both policies causing a lower transfer and hence less lobbying.

If the voters take the debt obligation to be non-binding, then they do not act

in a hostile manner to borrowing even if they are 'rational'. Furthermore, if the lobbies too regard debt repudiation as an option, then the parties will borrow up to the maximum permitted. This is the behaviour of a type P economy in chapter five which was able to obtain loans due to the informational imperfections in the international credit market.

Most debt forgiveness schedules appear to be designed for countries which borrow in order to invest. The picture is different in the type of lobbying economy depicted here. In order to make the average citizen (a taxpayer) better off, the positive impulse to high taxation policy through contributions needs to be reduced. Indexing forgiveness to an exogenous variable or granting unconditional forgiveness to a lobbying economy will decrease the repayment obligation and could actually increase the redistributive transfer which raises the amount of lobbying and will not decrease the tax rate. The proposal in this chapter has been to link the amount of debt forgiveness to the level of lobbying. Some insights on an optimal contributions-linked forgiveness schedule were gleaned with the help of a diagram before proceeding to an algebraic solution. The apparently crucial importance of the level of contribution at which forgiveness falls to zero was demonstrated. Forgiveness must strike a balance between restraint and generosity. If the schedule is too tight, then it has no effect, if it is too generous then an increase in lobbying will result (although it was shown that no increase in the tax rate will occur). Implementing such a forgiveness schedule can make the average citizen better off, in terms of paying less tax, and can also free resources for productive purposes which can increase growth.

CHAPTER 8. SUMMARY

In its treatment of sovereign debt, this thesis has focussed on a debtor which borrows to finance investment and one which borrows as a result of the working of the political system. Whilst a great deal of work has appeared in the literature concerning the former country type, chapter two was still able to highlight important issues for research.

An important feature of the work of chapters three, four and five is that a borrower, if it has access to the international credit market at all, cannot expect this access to be permanent or continuous. Chapters three and four assumed that the bank was aware of the identity of the customer which it was facing but was unable to observe the realized investment returns in that country. This feature built in a natural role for the willingness of the borrower to repay. The main differences between the model structures of these two chapters is that the former allows the bank to optimize over an infinite horizon, requiring repayment to be all or nothing, whilst the later assumes that the creditor must expect to break even each period with the country choosing the amount to repay on its debt.

In both of these models, the borrowing country must pass some strict criteria in order to obtain access to external funds; failing to meet these conditions leads to a no lending equilibrium. When the bank can optimize over the longer term, a lending equilibrium requires that it be patient enough not to close credit lines after an isolated case of default (which may just be caused by a poor investment return).

Additionally, the country must have an adequate investment technology which can produce sufficient returns sufficiently often to repay the debt; furthermore, the borrower must want to repay the debt with a high probability. In chapter four, where the bank is short-sighted, the investment technology of the debtor must produce returns more frequently than when the creditor is patient in order for a lending equilibrium to exist.

The model of chapter three is explicitly Markovian so that current actions depend upon the past only in as much as the past affects the current state of the system which evolves according to a first-order Markov process. Assuming that a Markov Perfect Lending Equilibrium exists, it is possible for a debtor to choose to default but such action will incur a punishment in the form of a reduced loan probability in the next period. The extent of the punishment depends upon the efficiency of the equilibrium played as multiple equilibria were shown to exist. A period of exclusion can thus result which is finite or infinite: the former class of equilibria are automatically missed by models which assume that default leads to an immediate and permanent credit embargo. By allowing the punishment periods to be endogenously determined, the lulls in activity in the international credit market described by Eichengreen (1989) and Cole, Dow and English (1989) can thus be modelled. Whether the market for external funds is in a phase of activity or inactivity depends upon the repayment action of the borrowing country. A default will most often lead to a period of exclusion - numerical analysis demonstrated that this may be long - but it is perfectly possible for voluntary lending to resume again after the punishment period has elapsed.

In chapter three, the lending action of the bank is state dependent, so that the probability of it granting a loan depends upon the current state, which in turn depends upon actions in the previous period. The extent to which default is punished hinges on the exact equilibrium played. In all cases, the bank must strike a balance between giving a high enough reward for repayment and making the punishment strong enough following default. There is a distance which the bank must keep between the 'reward' loan probability, the 'punishment' loan probability (following default) and the probability that lending resumes after a period in which no loan is made. If the bank reduces its loan probability following a repayment, then it must also reduce the loan probability following default and the loan resumption probability.

The most efficient equilibrium was demonstrated to be the one in which a repayment was rewarded with a loan for certain next period with the other two loan probabilities also being relatively high in striking the balance between reward and punishment. Consequently, the most efficient equilibrium was shown to be the one in which the country has the greatest degree of access to external funds, whereas in the less efficient equilibria, the country cannot guarantee itself a loan in the following period even by making a repayment. The least efficient equilibrium displays a high probability that the credit relationship will be terminated. This equilibrium appears to correspond most closely to those which have previously been examined in the sovereign debt literature as it prescribes a complete and permanent credit embargo following default. There exist a class of equilibria which are more efficient, however, as neither reputation nor access to the international credit market are ever permanently lost.

The extent to which default is excusable in the sense of Grossman and van Huyck (1988) was measured in chapter three by the difference between the loan probabilities following default and repayment. Whilst it is always the case that the latter is greater than the former, a numerical example demonstrated that the distance between the two is always large so that default can never be regarded as excused. Although it is still possible for a loan to be made in the period immediately after a default has occurred, this is a result of the bank's randomization process and does not imply that default has been excused.

Chapter four examined the relationship between excusable default and access to the international credit market in more detail. Whilst initially no restriction was placed upon the strategies of the players, it was demonstrated that an equilibrium existed in Markov strategies so that the results would be comparable to chapter three. The repayment options of the borrower were extended so that it could decide how much of its obligation to repay. In equilibrium, the country was found to repay a constant optimal amount when it received a good investment return, whilst it is unable to repay anything when investment fails. Such an optimal repayment was shown to lead to a loan for certain next period and default (or a non-optimal repayment) was punished through a lower loan probability.

Multiple lending equilibria were again shown to arise, distinguished by the punishment to default and the probability of lending resuming after a period of no lending. As optimal repayment is always rewarded with a loan for certain, giving the country the correct incentives to pay this optimal amount involves adjustment of the

default punishment and the loan resumption probability. In chapter three, the bank has three loan probabilities which it chooses according to its optimal strategy, setting the optimal distance between the reward loan probability and the other two probabilities. In chapter four, the bank cannot vary its reward loan probability (as it is equal to one in all equilibria) but can adjust the probabilities of a loan following default and no lending so that, if the immediate punishment to default is low (i.e. the loan probability following default is high) then the loan resumption probability is low. When default is punished least immediately, that is when default is most excusable, lending will never resume once it has ceased. The 'most excusable default' lending equilibrium therefore exhibits the greatest chance that the borrower will lose its access to the international credit market and consequently has the lowest payoff for the country. Excusing default actually harms the borrower! On the other hand, by punishing default through a low loan probability in the period following default, that is by excusing default the least, the equilibrium value of the loan resumption probability was found to be positive so that the country would not permanently lose access to the international credit market. Punishment intervals of exclusion could still be long, however, rendering commercial bank lending unsuitable for development purposes.

While chapters three and four suggest that lending breakdowns and resumptions can occur in equilibrium, the results of chapter five indicate that there is a limit to the tolerance of the creditor to default in a less certain environment. This chapter considered the possibility that there are certain types of borrowers who are inherently not creditworthy and inflict an externality on borrowers of the kind discussed in chapters three and four. The bank was assumed to be faced with a customer of

initially unknown identity but which could be learned through time. Naturally, the bank would only be willing to learn its type of customer if credit would be extended with complete information about the country's type. Therefore the credit market was assumed to be populated by countries which would receive loans according to the model of chapter three (type I) and those which would always default (type P). The problem was to characterize the actions of a type I country in optimally releasing its private information and the bank in attempting to learn the identity of its debtor.

The actions of both players in this framework depend on the degree of certainty with which the creditor believes that its customer is of type I; this belief is in effect the borrower's reputation for creditworthiness. A critical level of reputation was shown to exist in this model, below which the bank will permanently exclude the borrower from funds. A numerical example (carried over from chapter three) suggested that this critical value would be extremely high. The further the type I debtor's reputation is above this critical level, the greater is the temptation to default through choice so the lower the repayment probability is set. As the bank revises its belief in the country downwards following default, the type I country becomes more concerned with revealing itself, through repayment, in order to avoid the permanent embargo which ensues should reputation fall too low. The fact that the type I country can only reveal itself when it realizes a successful investment means that it sets a high repayment probability at all levels of belief, with this probability rising as the belief falls. It is still possible for a type I country to be permanently barred from receiving funds in this model if it fails to make the correct signal in time. This contrasts to the work of Cole, Dow and English (1989) where the stable borrower always has the

funds available to signal its type.

The number of loans granted until the lender believes the borrower's type to be revealed depends on the critical belief, the initial belief, the repayment actions and the size of the revisions of the prior. In this way, we have presented a dynamic model of sovereign lending and borrowing in the presence of a negative externality imposed by the existence of bad debtors. The size of the revisions in reputation following default is increasing in the interest rate charged, suggesting a reaction to adverse selection on the part of the bank. The higher the interest rate, the less willing is a good debtor to contract sovereign debt obligations so that the customer is most likely to be a bad debtor. To reflect this fact, the bank revises swiftly downwards its belief in the creditworthiness of the country.

Having considered the possibility that debtors are fundamentally dissimilar, chapters six and seven move towards a depiction of a borrowing country which does not necessarily contract external debt for investment purposes. Rather, the political parties in a country are persuaded to promise to take on external obligations in order to reward their support groups after an election. Prior to the election, these support groups spend resources in trying to get their favoured party elected in anticipation of a transfer from borrowed funds if this preferred party forms the government.

The underlying framework for the analysis of chapters six and seven is the probabilistic voting model of Brock and Magee (1978). In only taking the positive reaction of the support lobby into account when considering its policy, the parties in

the Brock and Magee model set high redistributive policies. Underlying their 'limited information' assumption is a desire to avoid computational complexity. Chapter six presented a general formulation of the political process in the presence of interest groups and non-hostile voters which allows the parties to account for the positive reaction of the own lobby and the negative reaction of the opposing lobby in deciding its policy (the 'full information' solution). Where voters are not directly hostile to redistributive policy (but may be indirectly hostile when their utility is positively correlated with the sum of the lobbies utility, if the latter is decreasing in the policy) it was shown that the outcome depends critically on the multiplicity of the maximizers of the sum of the lobbies' utilities. Where this maximizer is unique, both parties were shown to choose the single 'efficient' (in terms of maximizing the lobbies' utility sum) policy whilst no lobbying would take place in equilibrium. If this joint utility possesses multiple maximizers, the parties can choose different polices (but both efficient as they offer the same maximal utility sum). In this case, it was indicated that lobbying may occur in equilibrium.

A simple one period taxation game was used to illustrate this general model. In the context of this example, the assumption regarding the symmetry of the lobbies was relaxed so that one of the interests groups had a lower marginal cost of making political campaign contributions. It was shown that in equilibrium there will be no lobbying and both parties will set policies favouring the strong lobby. This contrasts strongly with Brock and Magee (1978) and Young and Magee (1986) where each party will still favour its own lobby in this asymmetric case. The crucial difference between these two models is that those authors present limited information solutions

whilst chapter six makes the more reasonable assumption of full information.

Continuing this theme in chapter seven, the one period taxation game was solved for the case where the voters are directly opposed to the political parties making redistributive transfers to their support lobbies on taking governmental office. In equilibrium, the parties were shown to trade off the interests of the voters and those of the lobbies in attempting to get elected. In the context of this model, the relationship between Brock and Magee's limited information solution and the full information outcome was demonstrated. With a limited information structure, lobbying was predicted at too high a level, leading to redistributive policies being too large in relation to the full information solution.

This model, with a full information structure, was then extended to two periods connected by a debt obligation. In the first period, the party which formed the government was able to borrow externally in order to transfer funds to its support lobby; in anticipation of these funds, the lobbies would contribute to the political campaigns of the parties in order to help them get elected into governmental office. The debt had to be repaid in the second period out of taxation revenue. The amount of borrowing taken on by a lobbying economy of this kind was shown to depend upon whether the voters were directly hostile to borrowing or not, and whether the government in the first period was able to fund its loyalty transfer by means of an endowment tax.

Reasonably, economies in which voters are hostile to borrowing tend to

contract less debt than economies with non-hostile voters (or where voter utility is positively correlated with national income). Additionally, it is only where voters are hostile that the availability of a redistributive tax makes any difference to the amount of borrowing. An external debt imposes a second period obligation on all agents whether they are ordinary citizens (voters) or lobby members, whereas taxation in the first period does not impose an obligation on the lobby which receives the government transfer (it has its tax payments reimbursed). The lobbies then prefer to have income redistributed via a tax but the hostile voter effect places an upper limit on how high the parties are prepared to set the tax rate. Once the maximum politically viable tax rate is reached, the government funds its transfers through external borrowing. It was shown that the larger the promised transfer, the greater is the amount expended by the lobbies on capturing this transfer. The amount of lobbying is greatest in non-hostile voter economies where the parties have access to borrowing and a tax rate with which to redistribute income. When voters are hostile, the number of instruments weakly decreases the level of lobbying: voters react negatively to both of the redistributive policies which leads to a lower level of transfer and so less lobbying.

In the type of lobbying economy depicted in chapter seven, it is the ordinary citizen who must share in the burden of the debt repayment, without realizing any direct benefits from the borrowing. The higher a party sets its tax rate, the more the support lobby will contribute to this party, increasing its chances of election. If one wished to make the ordinary citizens better off, it would be necessary to reduce the positive impulse to tax policy provided by the lobbies. In other words, the level of rent-seeking activity by the lobbies needs to be reduced. Linking the amount of the

required debt repayment to the level of political contributions was suggested as a means for doing this. The idea is that the creditor, or an organization such as the IMF, would agree to give the country some debt forgiveness depending inversely upon the rent-seeking impulse to taxation policy. If the amount of the repayment required were reduced unconditionally or linked to an exogenous variable, this would only increase the size of the potential transfer, increasing the amount of lobbying and possibly the tax rate. By making debt relief inversely dependent upon the level of political campaign contributions, it is possible to impose an additional cost on the lobbies: by lobbying an extra unit they not only spend that extra unit but increase the amount of the debt obligation due, reducing the potential transfer.

Critical in designing a debt forgiveness schedule for this type of economy appears to be the level of contributions at which the level of forgiveness sinks to zero. It was shown that if this value were too high, then the original high lobbying/high taxation equilibrium could still obtain. By setting the cut-off point too high, the potential transfer is increased so that the lobbies actually contribute more, so that the impulse to taxation policy is not reduced. A diagram was used to demonstrate a range for the cut-off point which would lead to the desired effect of reducing the amount of lobbying and hence the tax rate. It was shown algebraically that a contributions-linked debt forgiveness schedule can, in principle at least, improve the well being of a the ordinary citizens in an economy which borrows as a result of lobby pressure. There are two benefits of such a scheme: firstly, less resources are spent on DUP lobbying activities so that more is available to channel towards production; secondly, lower lobbying contributions lead to a reduced tax rate which benefits the ordinary citizens

in the economy.

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