COFFEE, THE MONEY MARKET, THE REAL EXCHANGE RATE, AND ECONOMIC FLUCTUATIONS IN COLOMBIA

By

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To my parents
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DECLARATION


Lastly, a shorter version of chapter 5 has been accepted for publication as Jesús Otero, "The real exchange rate in Colombia: An analysis using multivariate cointegration," *Applied Economics* (forthcoming).
This thesis analyses the effects of coffee booms on the money market, the real exchange rate, and the business cycle in Colombia. Chapter 2 presents an overview of the coffee sector in the country, including a brief description of its macroeconomic role, and unique institutional structure.

Chapter 3 investigates, from a simulation perspective, two empirical difficulties that arise in econometric modelling when using quarterly data, as is done in chapters 4 and 5. The first practical concern is whether to conduct the econometric analysis on data that have been subjected to seasonal adjustment or in terms of unadjusted data. The simulation results provide a justification for using seasonally unadjusted data, as the use of filters reduces the power of the Dickey-Fuller and Phillips-Perron cointegration tests. The second difficulty concerns an empirical regularity encountered when analysing the Colombian quarterly series of money supply and GDP, both of which exhibit a structural break (or change) in the seasonal pattern. We find that these structural breaks bias both unit root and seasonal root tests, so that new critical values must be tabulated allowing for a change in either the level and/or the seasonal pattern of the underlying series.

Chapter 4 examines the monetary consequences of coffee booms. The theoretical work on this subject shows that under a regime of fixed exchange rates, export booms affect both the demand and the supply for money. Within this theoretical framework, we assess whether the coffee booms of the second half of the seventies and mid eighties led to excess money supply in Colombia. We find a direct association between coffee export booms and excess money supply, implying that external disturbances jeopardise the ability of the economic authorities to carry out successful monetary policy.

Chapter 5 uses the Johansen procedure to estimate a real exchange rate determination model for Colombia. We find one cointegrating vector, which can be thought of as a long-run real exchange rate equation. The deviations of the real exchange rate from its long-run equilibrium relationship, after correcting for the short-run dynamics, are interpreted as a measure of real exchange rate misalignment. The simulation performance of the model, during the period of estimation and three years into the future, is particularly good, with the simulated real exchange rate reproducing the general long-run behaviour of the actual series.

Chapter 6 develops an intertemporal disequilibrium model in order to analyse the effects of temporary, anticipated, and permanent coffee price shocks on a small open economy under Keynesian unemployment. Our results indicate that a coffee price boom (whether temporary, anticipated or permanent) increases nontradable output in the short and long run (a similar result is obtained when we discuss other disequilibrium regimes). The basic model is then extended by including a government sector that administers a coffee price stabilisation fund, and by allowing capital market imperfections. Our results indicate that when the government is able to borrow on more favourable terms in international capital markets than households, the stabilisation fund neutralises part of the short-term effect of a temporary coffee price boom. On the other hand, when the government and the private sector borrow on the same terms, the stabilisation fund turns out to be redundant.
Both developed and developing countries are major producers of a broad range of primary products. However, while developed countries have a diverse economic structure that allows them to have a wide spectrum of exports, developing countries have a relatively important primary sector, so that they are still largely dependent on primary-commodity markets for their principal export earnings. In 1991, for instance, the share of fuels, minerals, metals, and other primary commodities in the exports of low- and middle-income countries amounted to over 50%, compared with a world share of approximately 25% (these figures are from the World Bank 1993).

As indicated by Maizels (1992), the basic problem faced by exporters of primary products is that these products, unlike manufactures, usually have low supply and demand price elasticities (in absolute value), so that a given shift in one of the curves causes a much larger change in short-run equilibrium prices, than if the elasticities are larger in absolute value. Moreover, these price fluctuations tend to have more important effects on developing countries, since the relative importance of the commodity sector in these countries is much greater than in most developed countries.

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1Adams and Behrman (1982), for example, find a strong association between price inelasticities and price instabilities for a number of primary commodities.
Oil and coffee constitute two of the most common examples of primary products that have experienced major fluctuations in their world prices. In the case of oil, from 1973 to 1974 and from 1978 to 1979 supply restrictions by OPEC member countries increased the world price of oil from US$2.70 to US$9.76 per barrel and from US$12.70 to US$17.26 per barrel, respectively. In the early 1980s, the world price of oil continued rising reaching a maximum level of US$33.47 per barrel in 1982, but four years later, in 1986, it had decreased to the levels observed in the years between the two shocks of the 1970s, owing to increases in the production and exportation by Saudi Arabia, the major producer of oil in the world, and other Middle Eastern countries.

Regarding coffee, during the second part of the seventies Brazil, the largest producer in the world, suffered the consequences of frosts which severely damaged half of the coffee trees (see Junguito and Pizano 1993). As a result, while from 1970 to 1974 the world price of coffee averaged US$0.57 per pound, from 1975 to 1979 it averaged US$1.58 per pound reaching an unprecedented level of US$3.20 per pound in April 1977. In 1986, once again adverse weather conditions in Brazil, this time a drought, reduced the world supply of coffee leading to another situation of high prices in world markets. During the late eighties and early nineties, on the contrary, the world market of coffee was characterised by a situation of low prices, because of the large stocks held by consumers as well as the greater levels of production observed in some countries.

\[\text{The duration of this boom was shorter than that of the seventies. This can be explained by the fact that when there is a drought coffee trees can produce one year later; on the other hand, when there is a frost the recovery of coffee trees lasts from two to three years (see Junguito and Pizano 1993).}\]
Colombia has like many other developing countries not escaped from a long history of dependency on primary commodity exports, let alone the sharp fluctuations in the world prices of these products. For over a century, coffee has played a central role in the Colombian economy by constituting a considerable source of foreign exchange, government revenue, employment, and value added in the agricultural and industrial sectors. The economic impact of the coffee sector is also evident in its contribution to the development of what is known in the country as the coffee zone, by means of large investments in physical infrastructure, public services, health and education, among other aspects.

The objective of this thesis is to analyse the effects of coffee booms on the monetary market, the real exchange rate, and the business cycle in Colombia. The first two issues are analysed from an empirical perspective, using the relatively recent developments in the econometric analysis of nonstationary data and cointegration. On the other hand, the third issue is analysed theoretically, since relatively few studies have attempted to formalise the relationship between the world price of coffee and the business cycle in Colombia.

This thesis proceeds as follows. Chapter 2 offers the reader an overview of the coffee sector in Colombia. We begin by presenting some basic indicators that give us an idea of the economic importance of the sector. Next, we present a brief review of the literature that analyses the role of coffee in the macroeconomic context, as well as a brief description of the unique institutional structure of the coffee sector in the country. The last section of the chapter is dedicated to the role of sectors other than coffee, that have constituted an additional source of external boom for the Colombian economy.
Chapter 3 investigates, from a simulation perspective, two empirical difficulties that arise in econometric modelling when using quarterly data, as is done in the empirical analyses of chapters 4 and 5. The first practical concern is whether to conduct the econometric analysis on data that have been subjected to seasonal adjustment or in terms of unadjusted data. We examine the impacts of seasonal adjustment filters on integrated-cointegrated data. The results of the Monte Carlo simulations provide a justification for using seasonally unadjusted data, as the use of filters reduces the power of the residual-based cointegration tests of Dickey and Fuller and Phillips and Perron, which implies that one may wrongly conclude that a static regression between nonstationary series is spurious.

The second difficulty concerns an empirical regularity encountered when analysing the Colombian quarterly time series of money supply and GDP. These two series, which are used later on in the thesis, exhibit a structural break (or change) in the seasonal pattern. Since Perron (1989), researchers have been particularly cautious about interpreting the results of unit root tests in the presence of apparent exogenous changes in either the level or growth rate of a series. It seems then useful to extend the seasonal unit root testing framework to allow for the presence of structural breaks in the level or seasonal pattern of the data. The Monte Carlo simulations indicate that seasonal unit root tests can be severely biased by these structural breaks, so that new critical values must be tabulated allowing for a change in either the level and/or the seasonal pattern of the underlying series.

Chapter 4 investigates the monetary consequences of coffee booms in Colombia. The theoretical work on this subject shows that under a regime of fixed exchange rates, export booms affect not only the demand for money, via real income, but also the supply for money, via foreign exchange accumulation. As a result, the
money market is likely to be in disequilibrium in the short run, although it is not possible to determine its nature, since it is not clear which of the two effects dominates. The main drawback of existing empirical work on the subject for Colombia, is that the effects on the demand for money have not been considered, despite the fact that theoretical models identify both money demand and money supply effects. In order to overcome this deficiency, we propose a two-step procedure in which a) we estimate a measure of excess money supply that corresponds to the residuals of a long-run money demand equation, assuming the money supply is given, and b) we examine if there is a direct association between this measure of excess money supply and the evolution of the coffee market.

Chapter 5 presents a real exchange rate determination model for Colombia, based on the theoretical framework pioneered by Edwards (1989a). The Edwards model belongs to a class of models that identifies variations in the terms of trade as an important determinant of the real exchange rate. This literature has been developed mainly to understand the process of determination of the real exchange rate in developing countries, which have been historically subjected to substantial changes in their terms of trade (in the case of Colombia, most of the changes in the country's terms of trade have been the result of variations in the world price of coffee). Our empirical analysis includes the formulation of a model to find the determinants of the real exchange rate in the short and long run, the estimation of a measure of exchange rate misalignment, and the evaluation of the performance of the model, in terms of its ability to predict the behaviour of the real exchange rate during the estimation period, and three years into the future.

Chapter 6 develops an intertemporal (two-period) disequilibrium macroeconomic model with microeconomic foundations, in order to analyse the
effects of variations in the world price of coffee on a small open economy. In a subsequent stage of the analysis, the basic model is extended in two ways. Firstly, we include a government sector that is in charge of the administration of a coffee price stabilisation fund, which is aimed at reducing the effects of variations in the world price of coffee on the economy. Secondly, we allow for capital market imperfections, motivated by the belief that this is a relevant assumption in the case of a developing country such as Colombia. The assumption of capital market imperfections is modelled in two alternative ways: a) we assume, as in van Wijnbergen (1987), that the government can borrow (on behalf of the stabilisation fund) on more favourable terms in international capital markets than individuals; and b) we assume that both private agents and the government face individual upward-sloping supply of capital curves, so that the rate at which an individual can borrow only depends on the amount of his own borrowing (we also examine the case where the private sector and the government face an aggregate supply of capital curve).

Finally, in chapter 7 we present a summary of the main conclusions of the thesis.
CHAPTER 2

THE IMPORTANCE OF COFFEE TO THE COLOMBIAN ECONOMY

2.1 SOME BASIC INDICATORS

Coffee was first introduced to Colombia during the XVIII century, and by the latter part of the XIX century the position of this commodity as an export crop had been consolidated. Palacios (1980) identifies the period between 1870 and 1897 as the "take-off" period of the product. Indeed, during this period coffee production increased fivefold. Moreover, in the 1870s, when tobacco exports were in decline, coffee exports accounted for 17% of the total value of exports; by the end of the last century, this contribution had increased to 40%.

At present, the Colombian coffee zone has an approximate area of 4.8 million hectares (approximately 4% of the total area of the country), 1 million of which is dedicated to coffee plantations. These plantations, typically in lands between 1,300 and 2,000 meters in altitude, are located in 16 out of the 33 departments in which the country is divided.1 Nearly 90% of the coffee farms are less than 5 hectares, and approximately 4 million people (12% of the Colombian population), are considered

1These departments are (in order of importance): Antioquia (18%), Caldas (13%), Tolima (12%), Valle (10%), Quindío (9%), Risaralda (8%), Cundinamarca (8%), Huila (5%), Santander (5%), Cauca (4%), Norte de Santander (3%), Nariño (1%), Boyaca (1%), Meta (1%), Cesar (1%) and Magdalena (1%).
to depend directly on production, processing, and primary marketing of coffee for their livelihood.²

The coffee sector has been a substantial source of value added. Figure 2.1 shows that from 1970 to 1994 the production of pergamino (or parchment) coffee averaged 12% of agricultural GDP, whereas processed (or threshed) coffee averaged 13% of industrial GDP.³ During the same period, both the agricultural and industrial coffee processes accounted for around 5.5% of GDP, reaching the highest average levels during the second half of the seventies (5.5%) and first half of the eighties (6%). Jaramillo (1994) indicates that this dynamism can be explained by the large investments carried out throughout the seventies, favourable internal prices, and the substitution of traditional coffee varieties ("arábigo" and "borbón") by a more productive variety ("caturra").

Figure 2.1: Coffee as percentage of GDP

![Graph showing coffee as percentage of GDP from 1970 to 1994.]

Source: Departamento Administrativo Nacional de Estadística (DANE), Departamento Nacional de Planeación (DNP), and calculations by the author.

²These figures are from the National Federation of Coffee Growers.
³Pergamino (or parchment) coffee corresponds to coffee in a stage of semi-processing, whereas processed (or threshed) coffee corresponds to coffee once the thin skin or parchment has been removed.
Coffee has also been an important source of employment, because its production is labour-intensive; indeed, it is estimated that 70% of the total cost of production corresponds to payments to labour. Additionally, the labour requirements of the cultivated area with the new coffee variety are of particular importance not only during the maintenance stage (the "caturra" variety requires, for instance, a more frequent use of fertilisers and involves more intense weeding activities), but also during the harvest season.\(^4\) Figure 2.2 shows that during the last decade, the coffee sector accounted for approximately 36.4% of the employment in the agricultural sector, and 5.7% of the total employment in the economy.

\[\text{Figure 2.2: Employment in the coffee sector}\]

\begin{figure}[h]
\centering
\includegraphics[width=0.7\textwidth]{coffee_employment_graph.png}
\caption{Employment in the coffee sector}
\end{figure}

Source: Clavijo et al. (1994) Table 4 (p.9), and calculations by the author.

In addition, the coffee sector has historically been a significant source of foreign exchange. From 1970 to 1992 coffee exports averaged 44.7% of export revenues, reaching a maximum level of 63.8% in 1978 (see Figure 2.3). In recent years, however, the relative importance of coffee in total exports has decreased whereas that of oil and minor exports, such as sugar, banana and flowers, has increased.

Figure 2.3: Exports by major commodities

Source: Banco de la República (1993) Table II 7, and calculations by the author.

Regarding the relative importance of Colombia in the world coffee market, the country has been the second largest producer in the world, supplying approximately 14% of the world exportable production during the period 1934-1992. Brazil ranks first with 37% of the world total; other Latin American countries supply 22%; all African producing countries supply about 20%; and countries in Asia and
Oceania supply 7%. The last sixty years have witnessed a change in the structure of world exportable production of coffee. Indeed, the Brazilian share of world exportable production has declined from 60% to 26%, while the relative importance of African countries has increased from 6% to 22%, and producing areas in Asia and Oceania have also expanded from 7% to 13%; the relative importance of Colombia and other Latin American countries has remained relatively constant. In addition, the international coffee market has been characterised by substantial instability of the world supply, which has been mainly the result of weather conditions in Brazil.

With reference to the main export markets of Colombian coffee, Junguito and Pizano (1993) indicate that there has been an important diversification. In the 1930s, the United States imported 80% of Colombian coffee and Western European countries the remaining 20%. In the 1980s and early 1990s, the United States decreased its share to 20%, Western European countries increased their share to 60%, and other countries such as Japan, former socialist economies and other Latin American countries were importing the remaining 20%. Junguito and Pizano argue that this diversification basically responded to the need to find more dynamic markets, as the American market was not expanding.

At this stage, it is worth dedicating some lines to the future role of the coffee sector given the recent discovery of the oil fields of Cusiana and Cupiagua, which have been regarded as a challenge in terms of economic policy because of their expected magnitude. At present, both public and private institutions are performing simulations to quantify the possible economic impact of these oil fields. For instance, Posada et al. (1994), from the National Department of Planning, expect the discoveries to increase the oil share in total exports to a level of approximately 40%. Also, the oil sector is expected to increase its share of GDP from 1.8% in the late
1980s, to a range between 3.5% and 4% of GDP at the end of the 1990s. On the fiscal side, the exploitation of the oil fields is expected to generate approximately 15% of central government revenues.

At the same time, Gómez (1993) argues that the coffee share will probably represent no more than 8% of total exports at the end of the nineties, and 5% of GDP. In this sense, even though the importance of the coffee sector as a source of foreign exchange is expected to decrease, the sector will remain a substantial source of employment and value added in the foreseeable future. Moreover, one may be tempted to argue that the expected "oil boom" can be regarded as a temporary phenomenon, lasting around 10 or 15 years, because of the uncertainties involved in the oil industry, where all depends on a successful exploratory phase.

2.2 Coffee and the Macroeconomic Context

The importance of the coffee sector in the macroeconomic context has been well documented for the Colombian case. Díaz-Alejandro (1976) and Cuddington (1986), for instance, relate the behaviour of the price of coffee to economic phases in Colombia during the post second world war period. In turn, Ocampo (1989) indicates that in economies with an important coffee sector, variations in the world price of coffee have both supply and demand effects. On the supply side, Ocampo argues that for an economy that has been subject to foreign exchange constraints, an increase in the world price of coffee relaxes this constraint allowing a greater amount of imported goods. Moreover, in the event that the composition of imports is biased in favour of intermediate and capital goods, this greater availability of imported inputs may, in turn, increase the production level of the economy. In the Colombian case,
both Diaz-Alejandro (1976) and Ocampo (1989) note that during the years after the second world war, when the world price of coffee exhibited an impressive upward trend, the volume of imports as well as the fixed capital formation were pulled up.

On the demand side, an increase in the world price of coffee raises the income accruing to coffee producers, which has an important positive effect on aggregate demand. In the Colombian context, the importance of the coffee sector as a source of aggregate demand is traditionally measured as the value of the coffee crop as a percentage of GDP. Figure 2.4 shows that from 1956 to 1993 this indicator averaged 4.7%, with peaks in 1977 and 1978 corresponding to the coffee price boom of the second half of the seventies. During the nineties, this percentage has decreased mainly because of reductions in the volume of the coffee crop as well as in the real price paid to producers.

**Figure 2.4: Value of the coffee crop as percentage of GDP**

![Graph showing the percentage of GDP from 1956 to 1992.]

Source: Federación Nacional de Cafeteros, Departamento Administrativo Nacional de Estadística (DANE), and calculations by the author.

The Colombian coffee sector has also been analysed in the context of the Dutch disease literature. Wunder (1991) classifies the studies in this area in three main groups. The first group comprises models that investigate the changes in the
economic structure brought about by a coffee price boom, and the real exchange rate appreciation associated with it. The second group comprises models that look into the monetary consequences of coffee price booms, more specifically, whether they lead to excess supply or demand in the money market. Lastly, the third group comprises those studies that incorporate the world price of coffee as one of the determinants of the real exchange rate. The findings of the first group of studies are not conclusive: while some present evidence that coffee booms affect the economy in the manner predicted by the Dutch disease theory (e.g. Kamas, 1986), others argue that production adjustments have been limited due to short-run rigidities (e.g. Wunder, 1991). The results of the studies in the second and third groups will be discussed in chapters 4 and 5, respectively.

Coffee has been a key determinant of the business cycle of Colombia. Ocampo (1989), for instance, finds that the behaviour of coffee in international markets has been the main driving force of the country’s business cycle. Cárdenas (1991) provides evidence indicating that a substantial part of the business cycle of Colombia, Costa Rica, Ivory Coast and Kenya, can be explained by temporary fluctuations in the world price of coffee. The author also detects that the business cycles of these coffee-producing countries are highly correlated, suggesting that these otherwise structurally different countries have been subject to the same kind of external shock. Despite this empirical evidence, relatively few studies have attempted to formalise the relationship between the world price of coffee and the business cycle. Therefore, in chapter 6 we develop an intertemporal (two-period) macroeconomic model with microeconomic foundations that intends to provide a formal theory of the possible transmission mechanism from coffee prices to output fluctuations.
Finally, it is also worth mentioning that the coffee sector has been one of the factors that contributed to the adoption of trade liberalisation policies in Colombia, as well as the subsequent policy reversals. García (1991), for example, indicates that in 1956-57 and 1966, declining world coffee prices caused a drastic reduction in the country's foreign exchange earnings, so that in order to avoid a balance-of-payments crisis, the government reversed the trade liberalisation policies adopted in the preceding years. In 1967 a gradual process of trade liberalisation began, which was characterised by the increasing use of tariffs, rather than quantitative restrictions, as a more appropriate tool of government intervention. This liberalisation process was accelerated in the late 1970s, mainly because of the need to sterilise the large inflow of foreign exchange resulting from the coffee boom. In the early 1980s, the poor performance of exports and the rapid increase in the import of consumer durables, make it difficult to sustain the move towards trade liberalisation.

2.3 COFFEE INSTITUTIONS

Since 1927, the National Federation of Coffee Growers, a private non-profit making organisation of coffee producers, has been charged by the government with directing coffee policy. This institutional arrangement constitutes a distinctive feature of the Colombian case since, as indicated by Cárdenas (1991), apart from Costa Rica, Ivory Coast and Kenya, Colombia is the only country where coffee producers have a direct influence on coffee policy.

Among other functions, the Federation administers the National Coffee Fund (NCF), a public account originally created, in 1940, with the purpose of purchasing

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5See Júnguito and Pizano (1997) for a discussion of coffee institutions and policy in Colombia.
excess production over the international export quota arrangement between the United States and the main Latin American producers. Since the 1950s, the Federation has been using the NCF as a price stabilisation device, keeping the variations in the domestic coffee price paid to farmers proportionally lower than the variations in the world price of coffee. Consequently, the NCF has played a countercyclical role in the economy, generating surpluses (deficits) when the international coffee price is higher (lower) than the internal price paid to producers. During the coffee price boom of 1986, for instance, the rise of the internal price paid to producers was proportionally lower than the rise in the world market price. As a result, the surplus accumulated by the Fund came to 3.2% as a proportion of GDP, a level without historical precedent (see Banco de la República 1986).

The Federation and the government jointly set the domestic coffee price, which constitutes one of the most important elements of coffee policy in Colombia. Indeed, the domestic coffee price is a variable that affects producers’ income, future coffee production, and the financial position (deficit or surplus) of the NCF. The link between coffee and macroeconomic policies can be observed through the financial position of the NCF, which is in turn mainly determined by the volume of the coffee crop, the evolution of world coffee prices, and, as already indicated, by the level of the domestic price of coffee.

Because the NCF is an account of public character, and in spite of the fact that its financial position is mainly determined by coffee variables, the Fund’s surplus or deficit has traditionally been added to that of the central government and other non-financial public enterprises, in order to produce the surplus or deficit of the public sector as a whole. Figure 2.5 plots the financial position of the NCF, the central government and other non-financial public enterprises from 1980 to 1993. As
can be observed from the figure, in some years the financial position of the Fund has been of considerable importance. For instance, the surplus generated in 1986 reached 3.2% of GDP, a magnitude that, even in the presence of deficits in the central government (1.6%) and other public enterprises (2%), led to an overall public sector deficit of just 0.3% of GDP. During the early 1990s, however, the Fund has been running deficits in order to support the domestic price paid to producers.⁶

**Figure 2.5:** Non-financial public sector surplus as percentage of GDP

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⁶These deficits can be explained by the break down of the International Coffee Agreement (on July 4, 1989), which caused the world price to fall to less than US$1 (the price remained low until early 1994).
2.4 The Role of Other Economic Sectors

Since the 1970s, coffee has not been the only source of external boom for the Colombian economy. In fact, the economy has had additional sources of boom, such as the hydrocarbons sector (which comprises oil and its derivatives, and coal), non-traditional exports, capital inflows, and the illegal sector (which corresponds to drug-related activities).

Regarding hydrocarbons, Cárdenas and Correa (1994) indicate that until the late sixties these were the second highest exported items, representing between 13.5% and 16.1% of total exports. However, owing to unsuccessful exploratory activity, production reduced considerably so that since the mid seventies, some years after the first oil shock, the country became a net importer. Since the mid eighties, both the oil and coal sectors have been gaining importance, basically as a result of the discovery of oil fields (e.g. Caño Limón), coal mines (e.g. Cerrejón) and the improvements in the sectors’ exports infrastructure (e.g. pipelines, railways and ports). Figure 2.6 presents the evolution of the trade balance of hydrocarbons as percentage of total exports from 1970 to 1992. As can be seen, there is an initial period where moderate surpluses were obtained (1970-1976), then follows a period of deficits (1977-1983), and finally, a new period of surpluses this time very substantial (1984-1992).
Non-traditional exports and capital inflows have constituted an important source of foreign exchange for some years over the period 1970-1992. During the early seventies and early nineties they exhibit great dynamism, mainly as a result of a real exchange rate depreciation that improved the competitiveness of these products in international markets. Concerning capital inflows, they originate from the increase in the peso-dollar interest rate differential observed since 1991 in the context of an economy more financially related to the rest of the world. Following Lora (1994), during 1991 the economic authorities adopted a series of measures in order to control the expansion of the monetary aggregates as it was thought to be fuelling inflation; as a result of the severe monetary contraction, interest rates increased considerably attracting foreign capital.

Lastly, with reference to illegal activities, their impact is more difficult to quantify because of their clandestine character. These activities probably originated

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7During the early seventies, the favourable behaviour of the world economy was also another important factor that explains the dynamism of non-traditional exports (see Carrasquilla and Suescún, 1987).
in the mid sixties when they were primarily based on the production and exportation of marijuana; in the eighties, they include the production and exportation of cocaine.\(^8\) The size and quantification of the Colombian illegal economy has been the subject of a number of different papers; nonetheless, because of its character some authors refer to "guesstimates" instead of estimates or, in terms of Caballero (1988), "science fiction exercises".\(^9\) In any case, and without pretending to assess the validity of such estimates, it is interesting to present some figures.


Regarding the amount of drug-related income that effectively enters the economy, Kalmanovitz (1990) estimated a yearly average of US$1,500 millions during the eighties, whereas Caballero (1988) estimated around US$1,000 millions in 1988. Gómez (1990) and Urrutia (1990) argue that the share of drug-related income that is repatriated is not likely to have a significant effect on the aggregate demand. Indeed, they observe that part of the proceeds of this illegal activity is either invested in financial institutions in the United States, Panama, and Switzerland, or used to buy

\(^8\)The Colombian Police Department estimates that there are approximately 65,000 hectares cultivated with marijuana, coca and poppy (Revista Semana No.662, January 10-17, 1995).

\(^9\)Wunder (1991) presents an interesting discussion about the drug economy in Colombia and suggests that it had a "...significant supplementary, but not a dominating impact on the economic development during (the period 1967-1988)..." (p. 360).
luxurious urban and rural properties in the country and abroad. Also, drug-related income is used to finance smuggling activities, which naturally has an adverse effect on industrial and commercial activities, and to buy livestock, lands and durable imported goods. Finally, it is also worth mentioning that the inflow of drug money has constituted a source of foreign exchange in the black market. As a result of this, from 1975 to 1981 the black market premium, that is the proportion by which the black market rate exceeds the official rate, was negative (see Figure 2.7). Among other effects, a negative black market premium increases the profitability of smuggling goods into the country.

**Figure 2.7: Black market premium**

![Graph showing black market premium over time.](image)

Source: Banco de la República (1993) Table II.46 for the official rate, Herrera (1990) for the black market rate, and calculations by the author.

In summary, during the last three decades the Colombian economy not only experienced coffee booms, but also a boom associated with the illegal economy, consisting of both marijuana and cocaine exports. Bearing in mind the constraints imposed by data availability, in section 4.3.6 we (attempt to) assess whether drug-related income has been one of the determinants of the disequilibrium in the

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Although such purchases may not have a considerable effect on the aggregate demand, they have certainly modified the relative prices of land, buildings and livestock.
monetary market. Hydrocarbons exports, non-traditional exports and capital inflows are not considered because they have only constituted a significant source of foreign exchange in some particular years (hence, the overall influence of the coffee and illegal sectors on the Colombian economy will probably be overestimated).
CHAPTER 3

SOME ISSUES IN ECONOMETRIC MODELLING USING
QUARTERLY DATA: SEASONAL ADJUSTMENT AND
STRUCTURAL BREAKS

3.1 INTRODUCTION

The empirical analyses of the monetary effects of coffee export booms (chapter 4), and the determinants of the real exchange rate (chapter 5), are performed using quarterly data, which naturally raises the question of whether to conduct the econometric analysis on data that have been subjected to seasonal adjustment, or in terms of unadjusted data. This chapter starts off by examining, from a simulation perspective, the impacts of seasonal adjustment filters on integrated-cointegrated data, paying particular attention to the size and power of the residual-based cointegration tests of Dickey and Fuller (1979, 1981), and Phillips and Perron (1988). The results of the Monte Carlo simulations provide a justification for using seasonally unadjusted data, since the power of the cointegration tests is adversely affected by the use of seasonal adjustment filters. Thus, one may wrongly conclude that a static regression between nonstationary series is spurious.

In the second part of this chapter we examine an empirical regularity that we encountered when analysing the Colombian quarterly time series of money supply and GDP. These two series, which are used later on in the thesis, exhibit a structural break (or change) in the seasonal pattern. Since Perron (1989) it is well-known that a
one-time shift in the mean or trend can generate a time series that seems to display a unit root. In the case of seasonal variables, there is the possibility that they may have unit roots not only in the long run (or zero frequency), but also at seasonal frequencies. The standard approach for seasonal unit root testing in quarterly time series is that of Hylleberg, Engle, Granger and Yoo (HEGY, 1990), who extend previous work by Dickey, Hasza and Fuller (1984). However, the HEGY test for both unit and seasonal unit roots is not applicable in the presence of an exogenous change in the level or seasonal pattern of the data. Consequently, it seems useful to extend the unit root testing framework to investigate the behaviour of the HEGY test in the presence of such exogenous changes. The evidence suggests that both unit root and seasonal unit root tests can be severely biased by these changes. New critical values for the HEGY test are then presented allowing a change in either the level and/or the seasonal pattern of the underlying series. Applying these findings to the Colombian series of money supply and GDP overturns initial results suggesting the presence of unit roots at the seasonal frequencies.

3.2 THE EFFECTS OF SEASONAL ADJUSTMENT LINEAR FILTERS ON COINTEGRATING EQUATIONS

The effects of seasonal adjustment filters on linear regression models have been analysed by Wallis (1974). He shows that as long as all the variables in a regression are adjusted with the same filter, the underlying relation among them is not altered, although the error term is no longer white noise but a high-order moving average process; nonetheless conducting inference may be problematic. If, on the other hand,

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1An alternative test for seasonal unit roots has been proposed by Osborn et. al. (1988).
the variables are adjusted using different filters, or if some of the explanatory variables are left unadjusted, then the estimated relationship among the variables will differ from the true relationship.

In the context of nonstationary series, Ghysels (1990) conducts a Monte Carlo investigation to assess the effects of the Henderson moving average filter and the linear approximation of the X-11 filter, in both their quarterly and monthly versions, on the power of the augmented Dickey and Fuller (ADF) and Phillips and Perron (PP) unit root tests. His main result is that these filters substantially reduce the power of the tests. Additionally, he also reports that while the null hypothesis for the presence of a unit root in the series of post-war seasonally adjusted quarterly U.S. GNP is strongly accepted, the evidence is “far less conclusive” when seasonally unadjusted data are used. Ghysels and Perron (1993) explore, in more detail, the effects of seasonal adjustment filters from both analytical and simulation perspectives. They find that both the ADF and PP unit root tests exhibit a considerable reduction in power compared to the benchmark cases where the data are seasonally unadjusted.

Within the cointegration framework, Ericsson, Hendry and Tran (1994) and Hendry (1995, chapter 15) show that for a given \( I(d) \) variable, \( y_t \), with \( d \leq 2 \), if the weights of a seasonal adjustment linear filter sum to unity, then the unadjusted and adjusted series are cointegrated with vector \([1, -1]\). The authors further show that if the filter satisfies the assumptions that it is symmetric and eliminates deterministic seasonals, then the number of cointegrating vectors and the cointegrating vectors themselves are invariant to the type of data; in the short run, however, the use of seasonally adjusted observations may not only distort the dynamics of the system but also whether or not a set of variables can be regarded as weakly exogenous.
Taking the above aspects into consideration, in what follows we investigate the effects of some seasonal adjustment linear filters on static cointegrating regressions via Monte Carlo simulations.

3.2.1 Design of the Monte Carlo Simulations

Let us consider the following data-generation process (DGP):

\[ x_i = x_{i-1} + u_i \]  \hspace{1cm} [3.1]

and

\[ y_i = x_i + v_i, \]  \hspace{1cm} [3.2]

where \( u_i \) and \( v_i \) are initially assumed to be white noise.

As can be seen, equations [3.1] and [3.2] indicate that \( x_i \) follows a random walk and \( y_i \) follows a random walk plus noise, and that both series are cointegrated with vector \([1, -1]\). The question is then, to what extent the null hypothesis of a unit root in the error term \( v_i \) is rejected when both \( x_i \) and \( y_i \) are subjected to a seasonal adjustment linear filter, or when only \( y_i \) alone is filtered? Ghysels and Perron (1993, p.63) argue that "...studying the effect of seasonal adjustment filtering procedures on series that have no seasonal components also has its advantages. Indeed, in this context, the issue concerning whether the seasonal part has been removed adequately does not occur. Hence, it permits a more specific investigation of the properties of the filters and their effects on the correlation structure of the data".

For the purpose of the Monte Carlo experiment, we generated 1,000 replications of the series \( \{y_i\} \) and \( \{x_i\} \) of length \( n = 64 \) as defined by equations [3.1] and [3.2], with the initial condition that \( x_1 = 0 \); the sample size was selected in order to match that of the empirical application that will be presented in section 3.2.3. The innovation term \( u_i \) is assumed to be \( \sim \text{i.i.d } N(0,1) \), whereas \( v_i \) is assumed to be \( \sim \text{i.i.d } \).
N(0,10). In a further set of experiments, $v_t$ is given by the following seasonal autoregressive seasonal moving average SARSMA (0,1,1,0) process:

$$v_t = 0.6v_{t-1} + \varepsilon_t + 0.25\varepsilon_{t-4},$$

[3.3]

where $\varepsilon_t \sim \text{i.i.d } N(0,6)$; this is the form of the error term in the empirical application.

Within this framework, we analyse four linear filters:

- A moving average of fourth order which can be written as $B(L) = (0.25 + 0.25L + 0.25L^2 + 0.25L^3)$, where $L$ denotes the lag operator.

- A filter that satisfies the assumptions in Ericsson, Hendry and Tran (1994) which we referred to as the “simple filter”, and is given by the following polynomial in the lag operator: $B(L) = 0.25 + 0.125\sum_{j=3}^{1}L^j$, for $j \neq 0$.

- The linear approximation of the quarterly version of the X-11 filter, as given by Laroque (1977, Table 1); see also Ghysels and Perron (1993, Table A.2).

- The quarterly version of the Henderson moving average filter, as given by Shiskin et al. (1967, Appendix B, Table 3), and also used by Ghysels and Perron (1993, Equation 2.7). This filter is actually a sub-filter of the X-11 filter, and provides an estimate of the trend component of a series.

Before describing the results, it is worth mentioning the following aspects. The first filter is asymmetric, the other three are symmetric. We refer to the case where the linear filter is applied on both sides of the cointegrating equation as 2-sided, whereas the 1-sided case corresponds to that where the filter is only applied to the dependent variable. Furthermore, in order to obtain the sample size of 64 for the filtered series, it was necessary to generate additional data points before and after the actual sample; this is also important for reducing the impact of the initial condition. Hence both the unadjusted and adjusted versions of the series begin with the 101st
observation. Finally, the filters used in the simulations are "time-invariant" filters, and we assume that there are enough data before and after each point in time at which the adjusted value is required.

### 3.2.2 Description of the Results

To begin with, we look at the effects of the seasonal adjustment filters on the cointegrating equation by examining the t-ratio on the cointegrating parameter being equal to 1. Given that the filters introduce serial correlation in the error term of the cointegrating equation, we also use the fully modified estimation method (FM-OLS) of Phillips and Hansen (1990) and Hansen and Phillips (1990).

Figure 3.1 plots the distributions of the t-ratio for OLS and FM-OLS on the filtered series, denoted OLSAdj and FM-OLS, respectively, as well as the distribution of the t-ratio on the unfiltered series, denoted OLS, which is close to that of a standard normal distribution. Table 3.1 reports summary statistics on the OLSAdj and FM-OLS t-ratios. In the 2-sided case the distributions of the OLS t-ratio are centred around zero, whereas in the 1-sided case the distributions are negatively biased for MA[4] and simple filters and, to a lesser extent, for the Henderson filter. The X-11 filter does not appear to distort the distribution seriously. The FM-OLS improves the distribution of the t-ratios for the MA[4] and simple filters, both partially correcting the bias and reducing the variance. Furthermore it does not appear to have a substantial effect in the case of the Henderson filter, although it

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2 All the simulations were performed using the econometrics software package RATS version 4.20.
3 The number of covariances used in FM-OLS was set equal to 7. As it is known, this procedure relies on the fact that although OLS produces "superconsistent" estimates of the cointegrating parameters, in finite samples they may be severely biased, either because the static regression omits the short-run dynamics, or because some of the series may be jointly determined. Thus, Phillips and Hansen's
actually produces a less normally distributed distribution for the X-11 filter, presumably because the number of lagged terms used in calculating the FM-OLS estimators, that is, 7, is excessive.

estimator aims to correct the bias originated from these two sources (see also Banerjee et. al. 1993, chapter 7; Davidson and MacKinnon 1993, chapter 20).
Figure 3.1: Distribution of t-ratios when $v_i$ is white noise

Simple filter (2-sided)

Simple filter (1-sided)
Figure 3.1 (Continued): Distribution of t-ratios when $v_i$ is white noise

MA[4] filter (2-sided)

MA[4] filter (1-sided)
Figure 3.1 (Continued): Distribution of t-ratios when \( v_t \) is white noise

**X-11 (2-sided)**

**X-11 (1-sided)**
**Figure 3.1 (Continued):** Distribution of t-ratios when $\nu_i$ is white noise

Henderson filter (2-sided)

Henderson filter (1-sided)
Table 3.1: Distribution of t-ratios

Summary statistics

<table>
<thead>
<tr>
<th>Filter</th>
<th>( \nu_t ) is white noise</th>
<th>( \nu_t ) is SARSMA (0,1,1,0)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2-sided</td>
<td>1-sided</td>
</tr>
<tr>
<td>MA[4]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.032 0.017 -1.296 -0.694</td>
<td>0.017 0.002 -1.001 -0.403</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.078 -0.030 -0.193 -0.198</td>
<td>-0.035 -0.039 -0.188 -0.098</td>
</tr>
<tr>
<td>Excess Kurtosis</td>
<td>0.102 0.273 0.113 0.221</td>
<td>0.297 0.396 0.491 0.412</td>
</tr>
<tr>
<td>Simple</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.042 0.013 -1.808 -0.109</td>
<td>-1.231 -0.072</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.062 -0.062 -0.292 -0.254</td>
<td>-0.070 -0.082 -0.342 -0.200</td>
</tr>
<tr>
<td>Excess Kurtosis</td>
<td>0.178 0.498 0.203 0.417</td>
<td>0.686 0.763 0.719 0.795</td>
</tr>
<tr>
<td>X-11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.017 0.021 -0.054 -0.012</td>
<td>0.019 0.000 -0.058 -0.019</td>
</tr>
<tr>
<td>Variance</td>
<td>1.240 2.260 1.220 2.244</td>
<td>5.876 3.491 5.762 3.460</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.022 -0.004 -0.032 -0.016</td>
<td>-0.029 -0.045 -0.040 -0.050</td>
</tr>
<tr>
<td>Excess Kurtosis</td>
<td>-0.092 0.142 -0.123 0.126</td>
<td>0.089 0.409 0.090 0.418</td>
</tr>
<tr>
<td>Henderson</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.018 0.007 -0.199 -0.052</td>
<td>0.022 0.000 -0.168 -0.034</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.054 -0.055 -0.080 -0.077</td>
<td>-0.031 -0.052 -0.068 -0.059</td>
</tr>
<tr>
<td>Excess Kurtosis</td>
<td>-0.028 0.140 -0.058 0.118</td>
<td>0.140 0.366 0.138 0.385</td>
</tr>
</tbody>
</table>
To examine the effects of the various filters on the power of cointegrating tests, we calculate the number of times the ADF and PP unit root tests correctly reject, at a 5% significance level, the null hypothesis of a unit root in the residuals of the cointegrating equation. Both the ADF and PP tests are calculated for up to $p=10$ lags. For the ADF test, the optimal number of lags included in the test regression is chosen in two alternative ways: a) estimating the test regression for a given maximum lag length, say $p$, and testing whether the last coefficient of the augmented part is statistically different from zero; if this coefficient is not significant, then the order of the autoregression is reduced by one until the last coefficient is significant (this procedure is suggested by Campbell and Perron 1991, and will be referred to as the Significance of the Last Coefficient SLC); and b) using the model selection procedure based on the minimisation of the Akaike Information Criterion (AIC).

Table 3.2 reports the power of the ADF and PP tests for cointegration for 1-sided and 2-sided filters, for all lags, $p = 1, \ldots, 10$, as well as the proportion of times each of the ADF tests is selected according to the SLC and AIC criteria described above. For the ADF test, in five out of eight cases both the SLC and AIC yield the same number of optimal lags to include in the test regression; when they do not coincide, the latter procedure is more parsimonious. When the MA[4] and simple filters are applied to both the dependent and explanatory variables there is a low probability of finding a cointegrating relationship, at 8.2% and 11.2% for the MA[4]

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4When the variables are not filtered, results not reported here indicate that the power of the DF and the PP tests is 100%.

5For $p<4$ an LM(4) test for serially correlated errors rejects the null hypothesis of no serial correlation, a fact that yields size distortions in the ADF test.
and simple filter, respectively, at their optimal lag length. When these filters are applied to just the dependent variable there is an increase in power. For the Henderson and X-11 filters there is a loss in power for $p>3$, although only for SLC for the X-11 filter (1-sided and 2-sided) is a value of $p>3$ actually selected. The power of the PP test is high and robust to changes in $p$ for both the X-11 and Henderson filters; this is expected as the PP test is able to model the MA error structure more precisely. For the MA[4] and simple filters the power of the test is highly dependent on $p$, although, in general, power does not fall by as much as the results ADF for $p>3$.

The use of a higher ordered MA process, such as an MA[8], lowers the power of the ADF and PP tests for cointegration below that of the MA[4] filter.

In practice the use of the linear approximation of the X-11 filter is very limited because its long tails lead to the loss of 22 observations at every end of the sample period. The loss of observations can be overcome by using the actual X-11 filter instead of its linear approximation. In the context of univariate unit root tests, Ghysels and Perron (1993, p.86) find that "...filtering with the actual X-11 filter reduces the power of the ADF test more than the linear X-11 filter does".

If the residuals of the cointegrating regression contain strong negative MA components, the critical values of the PP test are far below the Dickey-Fuller distributions reported in Fuller (1976), or MacKinnon (1991), and as a result the power of the test decreases.
Table 3.2: Power of tests for cointegration (\(v_t\) is white noise)

<table>
<thead>
<tr>
<th>Filter And Lag</th>
<th>2-sided AIC</th>
<th>2-sided SLC</th>
<th>2-sided Power</th>
<th>1-sided ADF</th>
<th>1-sided SLC</th>
<th>1-sided Power</th>
<th>1-sided PP</th>
<th>1-sided Power</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ADF PP</td>
<td></td>
<td></td>
<td>ADF PP</td>
<td></td>
<td></td>
<td>ADF PP</td>
<td></td>
</tr>
<tr>
<td>MA[4]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.7 1.1</td>
<td>34.5</td>
<td></td>
<td>19.8</td>
<td>24.6</td>
<td>48.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.0 1.1</td>
<td>56.3</td>
<td>50.4</td>
<td>12.3</td>
<td>10.3</td>
<td>65.3</td>
<td>64.3</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.2 0.4</td>
<td>71.5</td>
<td>58.9</td>
<td>9.4</td>
<td>6.6</td>
<td>68.6</td>
<td>70.0</td>
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</tr>
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Let us now turn to the more realistic scenario that the residuals of the cointegrating regression are not white noise but, for example, a SARSMA (0,1,1,0) process. The distributions of the t-ratio that the cointegrating parameter is equal to 1 are plotted in Figure 3.2; this time we include the density function of a standard normal random variable to facilitate comparison. Table 3.1 reports summary statistics on these distributions. Again there is only a bias in the mean of the distribution when the filter is applied to the dependent variable. All distributions are markedly fatter than the standard normal distribution. For the Henderson and X-11 filters all distributions are reasonably similar, although perhaps that of FM-OLS is closer to normality. For the MA[4] and simple filters, OLS on the adjusted series yields serious distortions, although the use of FM-OLS partially corrects these distortions.

With reference to the power of the tests for cointegration, results not reported here indicate that both the ADF and the PP tests exhibit reasonable power when neither of the variables have been filtered, for small $p$. In particular, for a DF test the probability of correctly rejecting the null hypothesis of non-cointegration is 86.2%, and the probability of rejecting a LM[4] test for residual serial correlation is slightly greater than the nominal size of 5%; in the case of the PP test, the power is always above 85% and the statistic does not appear to be sensitive to the number of autocovariances considered when constructing it.
Figure 3.2: Distribution of t-ratios when $v_i$ is SARSMA (0,1,1,0)

Simple filter (2-sided)

Simple filter (1-sided)
Figure 3.2 (Continued): Distribution of t-ratios when \( \nu_i \) is SARSMA \((0,1,1,0)\)

MA[4] filter (2-sided)

MA[4] filter (1-sided)
Figure 3.2 (Continued): Distribution of t-ratios when $v_i$ is SARSMA (0,1,1,0)
Figure 3.2 (Continued): Distribution of t-ratios when $v_t$ is SARSMA $(0,1,1,0)$

Henderson filter (2-sided)

Henderson filter (1-sided)
The results presented in Table 3.3 indicate that when the filters are applied to both sides of the cointegrating equation, or only on the left hand side, the power performance of the ADF and PP tests for the existence of a unit root is very low; for example, the MA[4] and simple filters at best correctly find cointegration on 18.5% and 14.3% of occasions when both the dependent variable and the explanatory variable are filtered. The power of the ADF test is markedly lower than that observed in Table 3.2 for both the X-11 and Henderson filters. The power of the PP test is extremely low when the MA[4] and simple filters are used and is also much worse than those observed in Table 3.2 for the X-11 and Henderson filters.

The basic conclusion of the Monte Carlo simulations is that the use of linear filters for seasonal adjustment in cointegrating equations has adverse consequences in terms of the power of the ADF and PP tests for cointegration (Ghysels 1990 and Ghysels and Perron 1993 report similar findings in the context of univariate unit root tests). Consequently, considerable care must be exercised when using linear filters for seasonal adjustment, as one may wrongly conclude that a static regression between nonstationary series is spurious.

\[9\text{ Again for } p<4 \text{ there exists substantial serial correlation problems and hence the size of the tests will be severely distorted.}\]
Table 3.3: Power of tests for cointegration ($v_t$ is SARSMA(0,1,1,0))

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<td>2.4</td>
<td>2.4</td>
<td>6.9</td>
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</tr>
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<td>9.6</td>
<td>2.8</td>
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<td>9.9</td>
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<td>2.6</td>
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<td>12.6</td>
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<tr>
<td>7</td>
<td>8.4</td>
<td>22.5</td>
<td>25.4</td>
<td>7.6</td>
<td>20.6</td>
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<tr>
<td>8</td>
<td>13.4</td>
<td>23.3</td>
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<td>61.9</td>
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<td>16.8</td>
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<td>9</td>
<td>11.0</td>
<td>9.8</td>
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<td>4.1</td>
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</tr>
<tr>
<td>10</td>
<td>6.6</td>
<td>4.0</td>
<td>4.0</td>
<td>5.2</td>
<td>4.1</td>
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<tr>
<td>Henderson</td>
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<td>2.1</td>
<td>5.3</td>
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<td>0.0</td>
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<td>31.9</td>
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<td>1.1</td>
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</tr>
<tr>
<td>4</td>
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<td>3.2</td>
<td>16.0</td>
<td>11.9</td>
<td>20.3</td>
<td>21.8</td>
<td></td>
</tr>
<tr>
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<td>4.8</td>
<td>3.8</td>
<td>7.7</td>
<td>12.2</td>
<td>5.4</td>
<td>4.1</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>12.2</td>
<td>21.2</td>
<td>23.2</td>
<td>9.7</td>
<td>8.6</td>
<td>8.9</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>12.5</td>
<td>14.2</td>
<td>2.8</td>
<td>8.1</td>
<td>7.3</td>
<td>5.9</td>
<td></td>
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<tr>
<td>8</td>
<td>21.0</td>
<td>25.6</td>
<td>10.7</td>
<td>6.6</td>
<td>5.3</td>
<td>4.3</td>
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<tr>
<td>9</td>
<td>12.0</td>
<td>6.7</td>
<td>4.1</td>
<td>6.5</td>
<td>3.9</td>
<td>5.4</td>
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<tr>
<td>10</td>
<td>11.8</td>
<td>6.4</td>
<td>2.9</td>
<td>6.0</td>
<td>3.1</td>
<td>3.4</td>
<td></td>
</tr>
</tbody>
</table>
3.2.3 **Empirical Application: A Money Demand Modelling Exercise**

In this section we look at the effects of linear filters for seasonal adjustment in cointegrating equations, by re-examining the results of the money demand modelling exercise for Colombia performed by Carrasquilla and Galindo (1994). It is important to highlight that it is our interest to assess if the results of their cointegration analysis change when one attempts to model the variables’ seasonal pattern using simple methods, instead of removing it by filtering the data.

Carrasquilla and Galindo estimate a long-run money demand equation of the form:

\[
\frac{m^d}{p} = f(y, R),
\]

where \(m^d\) is money in nominal terms, \(p\) is an appropriate price level, \(y\) is a measure of the volume of real transactions,\(^{10}\) and \(R\) is a vector of interest rates on the alternatives of money. The \(f\) function is expected to be increasing in \(y\) as well as decreasing in the elements of \(R\); however, if some of the components of the monetary aggregate bear interest, their interest rate should also be present in \(R\) and the \(f\) function should be increasing in these elements.

For their analysis, Carrasquilla and Galindo use quarterly information for the period 1978:1-1993:4. The monetary aggregate corresponds to M1, which is deflated by the consumer price index to produce real money balances. The scale variable corresponds to the GDP series, and the proxy for the opportunity cost of holding money is denoted by \(R\). All series are considered in logarithms, and denoted LRM1,

\(^{10}\)The variables commonly used as proxies are real GNP (see e.g. Goldfeld and Sichel, 1990; and Hendry and Ericsson, 1991), real total final expenditure (see e.g. Hendry and Ericsson, 1991; and Hendry 1995, chapter 16), and real consumer expenditure (see e.g. Mankiw and Summers, 1986).
LGDP and LR, respectively (see Figure 3.3). Instead of modelling the seasonal behaviour of LRM1 and LGDP, Carrasquilla and Galindo choose to adjust all series using a moving average of fourth order which, although unnecessary in the case of the interest rate series, can be justified on the grounds that as long as all the variables in a regression are adjusted with the same filter, the underlying relation among them is not altered, although the error term is no longer white noise but a high-order moving average process (see Wallis, 1974). Accordingly, we apply the same filter to the series under consideration, and the resulting adjusted series are denoted LRM1A, LGDPA and LRA (see Figure 3.3).

---

11 We use the same series in our money demand model of chapter 4, although the sample period is different. Precise definitions of the data series are provided in Appendix 4.1.
12 The filter used by Carrasquilla and Galindo transforms a white noise error term into a MA(3) process, whose theoretical autocorrelation function is given by \( \rho_1 = 0.75, \rho_2 = 0.5, \rho_3 = 0.25 \) and \( \rho_k = 0 \) for all \( k > 3 \).
13 The filter is applied since 1977:2 in order to avoid the loss of the first three observations.
Figure 3.3: Real M1, output and interest rates in Colombia – Not seasonally adjusted and adjusted data
3.2.3.1 Testing for Unit Roots

The order of integration of the series under consideration is investigated by means of the ADF tests for unit roots, which we apply as indicated by Perron (1988). The number of lags of the dependent variable to include in the test regressions is selected following Campbell and Perron (1991), starting with an upper bound of 5 lags, and then we perform the LM[4] test for serial correlation on the residuals of the test regressions. Lastly, when dealing with the unadjusted versions of LRMI and LGDP we also include centred seasonal dummies to capture some of the seasonal pattern; Dickey, Bell and Miller (1986) show that this procedure does not have any effect on the limiting distributions of the unit root tests statistics.

In the top half of Table 3.4 we summarise the results of the ADF unit root tests for LRMI, LGDP and LR, whereas in the bottom half we report those for the filtered series. Regardless of the type of data, the results suggest that LGDP seems to contain a unit root and a non-zero drift term, whereas LRMI and LR may also contain a unit root with a zero drift term; in the case of LR, however, this conclusion seems to contradict what the correlogram of the series (not reported here) shows.

---

14 Given that our interest is to discuss the modelling of cointegrated variables at the zero, or long-run, frequency, we do not perform tests for seasonal unit roots nor tests for cointegration at seasonal frequencies (see e.g. Dickey, et. al., 1984; Hylleberg, et. al., 1990; and Engle, et. al., 1993).

15 These centred seasonal dummy are defined as CSD<sub>t</sub> = 0.75 if t is the ith quarter of the year and -0.25 otherwise.

16 These results must be interpreted with caution due to the low power of the unit root tests (Schwert, 1989). In addition, it is important to bear in mind that the order of integratedness is not an inherent property of a time series, that is the order of integration of a time series may differ for different sample periods (see Hendry, 1995).
Table 3.4: Dickey and Fuller unit root tests  
(sample period 1978:1 – 1993:4)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model</th>
<th>Lags of Dep. Var.</th>
<th>LM[4]</th>
<th>$\tau_\tau$</th>
<th>$\Phi_3$</th>
<th>$\Phi_2$</th>
<th>$\tau_\mu$</th>
<th>$\Phi_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LRM1</td>
<td>A</td>
<td>1</td>
<td>F_{4,52} 0.632</td>
<td>-1.776</td>
<td>2.189</td>
<td>2.180</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LRM1</td>
<td>B</td>
<td>1</td>
<td>F_{4,53} 0.656</td>
<td>-0.013</td>
<td>1.020</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LGDP</td>
<td>A</td>
<td>3</td>
<td>F_{4,48} 1.108</td>
<td>-1.440</td>
<td>1.078</td>
<td>**11.988</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LR</td>
<td>A</td>
<td>1</td>
<td>F_{4,55} 0.781</td>
<td>-3.070</td>
<td>5.648</td>
<td>3.766</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LR</td>
<td>B</td>
<td>1</td>
<td>F_{4,56} 0.691</td>
<td>-2.750</td>
<td>3.782</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LRM1A</td>
<td>A</td>
<td>5</td>
<td>F_{4,46} 0.564</td>
<td>-2.765</td>
<td>5.260</td>
<td>3.961</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LRM1A</td>
<td>B</td>
<td>5</td>
<td>F_{4,47} 0.207</td>
<td>0.186</td>
<td>0.591</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LGDPA</td>
<td>A</td>
<td>5</td>
<td>F_{4,46} 0.972</td>
<td>-2.099</td>
<td>3.191</td>
<td>*5.311</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LRA</td>
<td>A</td>
<td>4</td>
<td>F_{4,48} 0.697</td>
<td>-2.106</td>
<td>4.666</td>
<td>3.153</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LRA</td>
<td>B</td>
<td>4</td>
<td>F_{4,49} 0.686</td>
<td>-1.556</td>
<td>1.269</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:
Model A is $\Delta Y_t = \gamma_0 + \gamma_1 Y_{t-1} + \gamma_2 Y_{t-1} + \text{lags dep. variable}$.  
Model B is $\Delta Y_t = \beta_0 + \beta_1 Y_{t-1} + \text{lags dep. variable}$.  
The LM[4] test is reported in its F version. The critical values for the $\tau$ statistics are reported in MacKinnon (1991). The critical values for the $\Phi_1$, $\Phi_2$, and $\Phi_3$ statistics are reported in Dickey and Fuller (1981). * denotes significance at the 5% level. ** denotes significance at the 1% level.
3.2.3.2 Cointegration Analysis for Pairs of Unadjusted and Adjusted Series

One important property of seasonal adjustment filters is that they should only remove the seasonal behaviour of a series, without affecting its long-run properties. Ericsson, Hendry and Tran (1994) and Hendry (1995, chapter 15) show that for a given $I(d)$ variable $y_t$ with $d \leq 2$, if the weights of the seasonal filter sum to unity, then the pair of unadjusted and adjusted series are cointegrated with vector $[1, -1]$. Accordingly, it is of particular interest to test whether this result holds for the adjusted and unadjusted versions of LRMI, LGDP and LR.

In order to do this, we test for cointegration between the pairs of series LRMI-LRMIA, LGDP-LGDPA and LR-LRA, using Johansen's maximum likelihood procedure (see Johansen 1988, and Johansen and Juselius 1990). In particular, we consider the following two-dimensional VAR models: a) LRMI-LRMIA (VAR 1); b) LGDP-LGDPA (VAR 2); and c) LR-LRA (VAR 3). All VAR models include a $\mu_t$ vector of constant terms, and the first two also include a $3 \times 1$ $D_t$ matrix containing centred seasonal dummy variables. Both the centred seasonal dummy variables and the constant terms are entered unrestricted. It is worth indicating that given the nature of the filter for seasonal adjustment, we use a lag length of two for the estimations, as longer lags yield perfect multicollinearity.

In Table 3.5 we report the main diagnostic tests for the three models as well as the cointegration analysis results. With regard to VAR 1, the regression for LRM1A fails the LM[4] test for serial correlation at the 5% significance level; the other tests for misspecification are easily passed. Under other circumstances, it would have been desirable to include additional lags to remove the autocorrelation;
however, as it is not possible to do this because of perfect multicollinearity, we proceed with the cointegration analysis. Both the trace and maximal-eigenvalue test statistics indicate the presence of one cointegrating vector. Moreover, the null hypothesis that LRM1 and LRM1A are cointegrated with a unit coefficient is easily accepted ($\chi^2_1 = 0.014$).

Concerning VAR 2, both the regressions for LGDP and LGDPA fail the LM[4] test for serial correlation at the 5% significance level; in addition, LGDP fails White’s test for heteroscedasticity at the 1% level. Similar to VAR 1, the outcome of the cointegration analysis not only indicates the presence of one cointegrating vector, but also that LGDP and LGDPA are cointegrated with a unit coefficient ($\chi^2_1 = 0.002$).

Lastly, the estimation of VAR 3 with two lags produces residuals that fail all misspecification tests. Since it is not possible to increase the order of the VAR in order to remove the serial correlation, because of perfect multicollinearity, we proceed with the cointegration analysis. The trace and maximal-eigenvalue test statistics suggest the presence of two cointegrating vectors, at the 5% significance level, which is consistent with LR and LRA being integrated of order zero. The rejection of the unit root hypothesis in the interest rate series contradicts the results of the Dickey-Fuller unit root tests, but is nonetheless consistent with the correlogram evidence (not reported here).
Table 3.5: Cointegration analysis for pairs of adjusted and unadjusted series

<table>
<thead>
<tr>
<th>Model diagnostic tests</th>
<th>VAR 1</th>
<th>VAR 2</th>
<th>VAR 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LRM1</td>
<td>LRM1A</td>
<td>LGDP</td>
</tr>
<tr>
<td>Normality</td>
<td>0.363</td>
<td>1.900</td>
<td>0.034</td>
</tr>
<tr>
<td>Heteroscedasticity</td>
<td>0.535</td>
<td>0.425</td>
<td>*3.374</td>
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</table>

<table>
<thead>
<tr>
<th>Cointegration analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximal eigenvalue test</td>
</tr>
<tr>
<td>Null hypothesis</td>
</tr>
<tr>
<td>Alternative hypothesis</td>
</tr>
<tr>
<td>Test value</td>
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</table>

<table>
<thead>
<tr>
<th>Trace test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Null hypothesis</td>
</tr>
<tr>
<td>Alternative hypothesis</td>
</tr>
<tr>
<td>Test value</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>β' eigenvectors (Standardized)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
</tr>
<tr>
<td>2.550</td>
</tr>
</tbody>
</table>

Notes:
The LM[4], ARCH[4] and Heteroscedasticity tests are reported in their F versions. The test for normality is distributed as χ². The number of cointegrating vectors is denoted by r. Critical values for the maximal eigenvalue and trace tests are reported in Osterwald-Lenum (1992). * denotes significance at the 5% level. ** denotes significance at the 1% level.
3.2.3.3 A Long-Run Money Demand using the Seasonally Adjusted Series

The results of estimating the long-run money demand equation using the seasonally adjusted series are presented below:

\[ LRM1A = 8.799 + 0.493 \text{LGDPA} - 0.228 \text{LRA} + \hat{u}_i. \quad [3.5] \]

It is important to recall that the seasonally adjusted and unadjusted versions of LRM1 and LGDP may contain a unit root, and that the interest rate may be integrated of order zero. Thus, it may seem peculiar that for estimating the long-run money demand equation we are combining variables of different order of integration. The reason is that even if asymptotically the inclusion of I(0) variables should not affect the superconsistency results (provided it is not the regressand), nor the asymptotic critical values of the test statistics, in finite samples, as in our case, they may affect the outcome.

Returning to the cointegrating equation, the signs of the estimated coefficients correspond to those of a money demand equation, with the coefficient associated to LGDPA suggesting scale economies in the holding of money.

The residuals of [3.5] were then tested for a unit root using the ADF and PP tests. When we use the first method, it is necessary to introduce 5 lags of the dependent variable in order to whiten the residuals, as indicated by the LM[4] test for serial correlation; the following results are obtained:

\[ \Delta \hat{u} = -0.118 \hat{u}_{t-1} + \sum_{i=1}^{5} \alpha_i \Delta \hat{u}_{t-i} \]

\[ \text{t - Stat}(-2.781) \]

\[ [3.6] \]
Thus, it is not possible to reject the null hypothesis of non-cointegration at traditional significance levels (critical value of -3.552 at the 10% significance level). On the basis of this outcome, Carrasquilla and Galindo conclude that the demand equation for real balances is spurious, and proceed to formulate it in first differences.\(^\text{17}\)

With regard to the Phillips and Perron test, we use different truncation lag parameters, that determine the number of autocovariances to be considered when constructing the statistic; however, regardless of this the null hypothesis of non-cointegration cannot be rejected.

Lastly, regression [3.5] is estimated using Phillips and Hansen’s fully modified estimator, for selected truncation lag parameters:

<table>
<thead>
<tr>
<th>Trunc. Lag</th>
<th>Const.</th>
<th>LGDPA</th>
<th>LRA</th>
<th>ADF(5)</th>
<th>PP(4)</th>
<th>PP(6)</th>
<th>PP(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>8.503</td>
<td>0.527</td>
<td>-0.258</td>
<td>-2.902</td>
<td>-2.296</td>
<td>-2.337</td>
<td>-2.387</td>
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<tr>
<td>4</td>
<td>8.536</td>
<td>0.528</td>
<td>-0.271</td>
<td>-2.845</td>
<td>-2.329</td>
<td>-2.368</td>
<td>-2.416</td>
</tr>
<tr>
<td>6</td>
<td>8.473</td>
<td>0.532</td>
<td>-0.266</td>
<td>-2.870</td>
<td>-2.289</td>
<td>-2.329</td>
<td>-2.378</td>
</tr>
<tr>
<td>8</td>
<td>8.415</td>
<td>0.534</td>
<td>-0.258</td>
<td>-2.914</td>
<td>-2.248</td>
<td>-2.290</td>
<td>-2.339</td>
</tr>
<tr>
<td>10</td>
<td>8.308</td>
<td>0.535</td>
<td>-0.232</td>
<td>-3.043</td>
<td>-2.161</td>
<td>-2.208</td>
<td>-2.260</td>
</tr>
<tr>
<td>12</td>
<td>8.156</td>
<td>0.539</td>
<td>-0.200</td>
<td>-3.185</td>
<td>-2.049</td>
<td>-2.102</td>
<td>-2.157</td>
</tr>
</tbody>
</table>

The results indicate that the estimated coefficient on LGDPA does not change markedly in comparison to the OLS estimate in equation [3.5], although the same cannot be said about the estimated coefficient on LRA.

The residuals of the fully modified regressions are then tested for a unit root using the ADF(5) test, and the PP test for selected truncation lag parameters; regardless of the test, the null hypothesis of non-cointegration is not rejected at

\(^\text{17}\)The null hypothesis of non-cointegration is also accepted when the series are adjusted using the “simple filter” defined in the Monte Carlo simulations; these results, however, will not be reported.
traditional significance levels. It shall be remembered from the Monte Carlo simulations reported previously, that both the ADF and PP tests for cointegration suffer from low power when the variables have been previously filtered.

### 3.2.3.4 A Long-Run Money Demand Using the Seasonally Unadjusted Series

Let us now consider the results when one attempts to model the seasonal pattern of LRM1 and LGDP by including a set of centred seasonal dummy variables in the cointegrating equation. Firstly, the estimated cointegrating equation is:

\[
\text{LRM1} = 9.080 + 0.478 \text{LGDP} - 0.256 \text{LR} + \text{CSD} + \hat{u}_t, \tag{3.7}
\]

where CSD indicates the set of centred seasonal dummies. As can be seen, the estimated coefficients change little with respect to those obtained using the adjusted variables.\(^{18}\)

The residuals of (3.7) were then tested for a unit root using the Dickey and Fuller and Phillips and Perron tests. With regard to the former, we begin by including 5 lags of the dependent variable, although none of them proved significant. Thus, we end up with the following regression:

\[
\Delta \hat{u} = -0.372 \hat{u}_{t-1}, \tag{3.8}
\]

\(t-\text{Stat}(-3.662)\)

which easily passes the LM[4] test for residual serial correlation \(F_{4,58}=1.093\). Unlike the previous case, the null hypothesis of non-cointegration can be rejected at a 10% significance level. In the case of the PP test, the statistics are not greatly affected by

---

\(^{18}\)After examining the autocorrelation and partial autocorrelation functions of the residuals of (3.7), we end up with the SARSMA \((0,1,1,0)\) specification utilised in the Monte Carlo simulations.
the selection of the truncation parameter, and the hypothesis of non-cointegration can be rejected at the 10% significance level.

3.3 **STRUCTURAL BREAKS AND SEASONAL INTEGRATION**

Since the article by HEGY (1990), there has been a growing interest in tests for unit roots at seasonal frequencies. In the case of quarterly data, empirical evidence often confirms the presence of seasonal unit roots at least at one of the seasonal frequencies. For example, HEGY find that UK quarterly consumption expenditure on nondurables contains a unit root at both the biannual and annual frequencies, whereas personal disposable income contains a unit root only at the biannual frequency. Engle et. al. (1993) report that (real) disposable income and (real) consumption for Japan contain unit roots at the seasonal frequencies, although in the case of the latter the existence of a unit root at the annual frequency depends upon the set of deterministic regressors included. Similar findings are also reported by Otto and Wirjanto (1990) and Lee and Siklos (1991) when analysing a large set of Canadian macroeconomic time series, and by Ghysels et al. (1993) and Beaulieu and Miron (1993) when using US data.

However, in a recent paper Ghysels et. al. (1994, p.436), note that “We may indeed tend to find unit roots at seasonal frequencies because of changes in seasonal pattern unaccounted for in tests like HEGY...”. In what follows, we use a similar approach to that of Perron (1989) to investigate the behaviour of the test of HEGY for both unit and seasonal unit roots, in the presence of an exogenous change in the level or seasonal pattern of the data. The evidence suggests that both unit root and

---

19An independent piece of research on this topic has been written by Franses and Vogelsang (1995).
seasonal root tests can be severely biased by these changes. The HEGY test is then applied to the Colombian series of M1 and GDP, which exhibit evidence of a break in the seasonal pattern, in order to determine whether the order of integration of these series can be changed by allowing for a change in the seasonal pattern over the period of analysis.

3.3.1 HEGY TESTS

The HEGY test, tests for the existence of both a unit root as well as a seasonal root in a series \( Y_t \), by estimating the equation [3.9]

\[
\bar{Y}_{4t} = \pi_1 \bar{Y}_{t-1} + \pi_2 \bar{Y}_{2t-1} + \pi_3 \bar{Y}_{3t-2} + \pi_4 \bar{Y}_{3t-1} + \nu_t, \quad t = 5, \ldots, T, \tag{3.9}
\]

where

\[
\bar{Y}_{lt} = \bar{Y}_1 + \bar{Y}_{t-1} + \bar{Y}_{t-2} + \bar{Y}_{t-3}
\]

\[
\bar{Y}_{2t} = -\bar{Y}_1 + \bar{Y}_{t-1} - \bar{Y}_{t-2} + \bar{Y}_{t-3}
\]

\[
\bar{Y}_{3t} = -\bar{Y}_1 + \bar{Y}_{t-2}
\]

\[
\bar{Y}_{4t} = \bar{Y}_t - \bar{Y}_{t-4}
\]

and \( \bar{Y}_t \) are the OLS residuals from the auxiliary regression

\[
Y_t = \hat{\mu} + \sum_{j=2}^{4} \hat{\delta}_j D_{jt} + \hat{\beta} t + \bar{Y}_t, \quad t = 1, 2, \ldots, T. \tag{3.10}
\]

The methodology suggested by HEGY is to test for the existence of a unit root, by testing \( H_0: \pi_1 = 0 \) against the one-sided alternative \( H_1: \pi_1 < 0 \). To test for the existence of a seasonal unit root HEGY (p.223) note that "there will be no seasonal unit roots if \( \pi_2 \) and either \( \pi_3 \) or \( \pi_4 \) are different from zero". Consequently, they

\(^{20}\)The analysis is limited to the a of quarterly data; see Beaulieu and Miron (1993) for a monthly analysis.
recommend testing $H_0: \pi_2 = 0$ against the one-sided alternative $H_1: \pi_2 < 0$ and simultaneously testing the joint hypothesis $H_0: \pi_3, \pi_4 = 0$ against the alternative $H_1: \pi_3 < 0, \pi_4 \neq 0$. A null hypothesis of a seasonal unit root is only rejected when, both the $t$-test for $\pi_2$ and the joint $F$-test for $\pi_3$ and $\pi_4$ are rejected.

Using this suggested methodology, we present power probabilities of the HEGY test to reject both a unit root and a seasonal root, when the underlying Data Generating Process (DGP) is an AutoRegressive (AR) model, of the form

$$ Y_t = \sum_{j=1}^{4} \mu_j D_{j,t} + \alpha_1 Y_{t-1} + \epsilon_t. $$

where, $\mu_j = 0 \forall j$. The auxiliary regression [3.10] used to construct $\bar{Y}_t$ in equation [3.9] is heavily over-parameterised by the inclusion of an intercept, seasonal dummies and trend.

The results are presented in Table 3.6 for the sample sizes considered by HEGY ($T = 48, 100, 136, 200$) and a range of values of $\alpha \leq 1$. For $\alpha = 1$, and $T = 200$, the empirical size probabilities approach their theoretical significance level for the test of a unit root. However, the power of this test to reject the unit root null hypothesis when $1 > \alpha > 0.9$ is considerably less than that associated with the simple Dickey-Fuller test (see e.g. Dickey and Fuller, 1981). The power of this model to reject the null hypothesis of a seasonal unit root is always high and approaches 100% for $T > 100$.

---

21 All simulations are based upon 10,000 replications.
Table 3.6: Power of HEGY test - DGP is an AR(1)

<table>
<thead>
<tr>
<th>T</th>
<th>α</th>
<th>Pr[Reject Unit Root] 1%</th>
<th>2.5%</th>
<th>5%</th>
<th>10%</th>
<th>Pr[Reject Seasonal Unit Root] 1%</th>
<th>2.5%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
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<td>1.00</td>
<td>0.30 0.85 1.86 4.43</td>
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<td>41.60 63.65 80.46 92.83</td>
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<tr>
<td></td>
<td>0.95</td>
<td>0.26 0.93 2.20 5.15</td>
<td></td>
<td></td>
<td></td>
<td>40.01 63.57 80.44 92.09</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.90</td>
<td>0.42 1.42 3.23 7.06</td>
<td></td>
<td></td>
<td></td>
<td>39.88 62.47 79.99 92.32</td>
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<tr>
<td></td>
<td>0.80</td>
<td>0.94 2.59 5.61 11.66</td>
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<td></td>
<td>38.80 61.49 78.73 91.59</td>
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</tr>
</tbody>
</table>

Table 3.7 reports power probabilities when the underlying DGP is a Seasonal AR (SAR) model of the form

\[ Y_t = \sum_{j=1}^{4} \mu_j + \alpha_4 Y_{t-4} + \varepsilon_t, \]  

[3.12]

where, \( \mu_j = 0 \ \forall \ j \). Using the auxiliary regression \[3.10\], for \( \alpha = 1 \) and \( T \to \infty \), the empirical size probabilities again approach the theoretical significance level for the unit root null hypothesis. However, for the test of a seasonal unit root the empirical size probabilities are always too small, compared with their theoretical p-values, even as \( T \to \infty \). This implies that the rule for rejecting the null hypothesis of a seasonal unit root, when both null hypotheses \( H_0: \pi_2 = 0 \) and \( H_0: \pi_3 = 0 \) are jointly rejected, is too stringent a hypothesis to impose. For \( \alpha < 1 \) power of the seasonal unit root is quite small, which is to be expected as the size probabilities are too small. The unit
root null hypothesis is rejected far less than when the DGP was a simple AR model
(see Table 3.6).

Table 3.7: Power of HEGY test - DGP is a SAR(1)

<table>
<thead>
<tr>
<th>T</th>
<th>( \alpha_4 )</th>
<th>Pr[Reject Unit Root]</th>
<th>Pr[Reject Seasonal Unit Root]</th>
</tr>
</thead>
<tbody>
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<td></td>
<td></td>
<td>1%</td>
<td>2.5%</td>
</tr>
<tr>
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<td>0.20</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td>0.95</td>
<td>0.27</td>
<td>0.71</td>
</tr>
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<td></td>
<td>0.90</td>
<td>0.21</td>
<td>0.71</td>
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<tr>
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<td>0.80</td>
<td>0.26</td>
<td>0.96</td>
</tr>
<tr>
<td>100</td>
<td>1.00</td>
<td>0.63</td>
<td>1.57</td>
</tr>
<tr>
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<td>0.95</td>
<td>0.60</td>
<td>1.49</td>
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<tr>
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<td>0.90</td>
<td>0.99</td>
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<td>0.80</td>
<td>1.44</td>
<td>3.04</td>
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<td>1.51</td>
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<td></td>
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<td>0.68</td>
<td>1.72</td>
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<td>0.90</td>
<td>0.72</td>
<td>2.21</td>
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<td>0.80</td>
<td>1.86</td>
<td>4.67</td>
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<tr>
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<td>0.88</td>
<td>2.09</td>
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<td>0.95</td>
<td>1.05</td>
<td>2.32</td>
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<td>1.45</td>
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<tr>
<td></td>
<td>0.80</td>
<td>4.36</td>
<td>10.02</td>
</tr>
</tbody>
</table>

Perron (1989) investigated the performance of the Augmented Dickey-Fuller (ADF) test, when there was an exogenous change in the level or growth of a trend stationary process, at some point \( T_B \). When the DGP is of the form,

\[
Y_t = \sum_{j=1}^{4} \delta_j D_j + \sum_{j=1}^{4} \gamma_j D_j DU_t + \beta t + \epsilon_t, \tag{3.13}
\]

\[
DU_t = \begin{cases} 
0 & \text{if } t \leq T/2 \\
1 & \text{if } t > T/2
\end{cases},
\]

with \( \delta_j = 0 \ \forall j \), \( \beta = 1 \) and \( \gamma_j = 0, 1, 2, 5, 10 \ \forall j \), this corresponds to case A in Perron (1989), that is, a change in the level of a series. The results in Table 3.8 confirm the finding of Perron (1989) (see Figure 4, p.1369), that as the "break" becomes increasingly large, so the ability of a unit root test to distinguish between stationarity
and nonstationarity declines. Unsurprisingly, the power of the seasonal unit root tests to reject the null hypothesis is approximately 100% for $T \geq 100$, as this does not affect the spectrum of the series at the seasonal frequency. Figure 3.4 plots the power probabilities at the 5% significance level for $T=100$, for the unit root test, when the break point ($T_B$) varies over the sample from $T_B = 1, ..., T - 1$, for a variety of values for $\mu_2$. The figure shows that the power probabilities follow a symmetric but flat W-shaped function with minima at $T_B = T/4, 3T/4$, and power increasing quickly at the extreme points, that is for $90 \leq T_B \leq 10$.

Table 3.8: Power of HEGY test - Change in the level

<table>
<thead>
<tr>
<th>$T$</th>
<th>$\mu_2$</th>
<th>Pr[Rejecting a Unit Root]</th>
<th>Pr[Rejecting a Seasonal Unit Root]</th>
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<tr>
<td></td>
<td></td>
<td>1%</td>
<td>2.5%</td>
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<tr>
<td>48</td>
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<td>11.03</td>
<td>24.42</td>
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<td>6.02</td>
<td>13.97</td>
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<tr>
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<td>2.0</td>
<td>0.92</td>
<td>2.74</td>
</tr>
<tr>
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<td>5.0</td>
<td>0.00</td>
<td>0.00</td>
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<td></td>
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<td>0.00</td>
<td>0.00</td>
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<tr>
<td>100</td>
<td>0.0</td>
<td>93.05</td>
<td>97.52</td>
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<td>74.34</td>
<td>86.80</td>
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<td>36.91</td>
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<td>0.00</td>
</tr>
</tbody>
</table>
Table 3.9 reports the case when there is a change in the seasonal pattern of the series, that is, in equation [3.13], $\delta_j = 0 \forall j$, $\beta = 1$, $\gamma_j = 0$, $j = 1, 2$, and $-\gamma_3 = \gamma_4 = 0, 1, 2, 5, 10$. The test of a unit root at the zero frequency is unaffected as there is no actual change in the level of the series. However, the change in the seasonal pattern affects the spectrum at the seasonal frequency, and this adversely affects power performance of the HEGY test for a seasonal unit root; for example, for $T = 100$ and $\gamma_4 = 5$, power is 0% at the 5% significance level, compared with approximately 100% when there is no change in the seasonal pattern. Again increasing $T$, for a given size of break increases the power of the test. Figure 3.5 reports the power probabilities at the 5% significance level for $T = 100$, for the seasonal unit root hypothesis, for all values of $T_b$ and a range of values for $\gamma_4$. The power function is a symmetric flat U-shaped curve with a minimum at $T_b = T/2$, and power increasing at the end points, that is, $80 \leq T_b \leq 20$. 

Figure 3.4: Power of HEGY unit root test as a function of break period
Table 3.9: Power of HEGY test - Change in the seasonal pattern (Level unchanged)

<table>
<thead>
<tr>
<th>T</th>
<th>(-\gamma_3 = \gamma_4)</th>
<th>1%</th>
<th>2.5%</th>
<th>5%</th>
<th>10%</th>
<th>Pr[Rejecting a Unit Root]</th>
<th>1%</th>
<th>2.5%</th>
<th>5%</th>
<th>10%</th>
<th>Pr[Rejecting a Seasonal Unit Root]</th>
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<tbody>
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<td>48</td>
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<td>11.03</td>
<td>24.42</td>
<td>39.72</td>
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<td>16.48</td>
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<td>2.44</td>
<td>7.69</td>
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</tr>
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</table>

Figure 3.5: Power of HEGY seasonal unit root test as a function of break period
Table 3.10 reports the case when both the level and seasonal pattern of a series change, that is, \( \delta_j = 0 \ \forall j, \beta = 1, \gamma_j = 0, j = 1, 2, 3, \) and \( \gamma_4 = 0, 1, 2, 5, 10 \) in equation [3.13]. For both the unit root and seasonal unit root tests power falls as the size of the break increases. The power of the seasonal unit root tests to correctly reject the null hypothesis is slightly higher than in Table 3.9. Direct comparison of Table 3.8 with Table 3.10 is not possible because in Table 3.10 the change in the level of the series is \( \gamma_4 / 4 \). Setting \( \gamma_4 = 4, 8 \), the power of unit root test to reject the null hypothesis at the 5% significance level for \( T = 100 \), is 75.95% and 12.95%, respectively (compared with 94.19% and 56.14% for \( \mu_2 = 1, 2 \) in Table 3.8).

### Table 3.10: Power of HEGY Test - Change in the Seasonal Pattern and Level

<table>
<thead>
<tr>
<th>( T )</th>
<th>( y_4 )</th>
<th>( 1% )</th>
<th>( 2.5% )</th>
<th>( 5% )</th>
<th>( 10% )</th>
<th>( 1% )</th>
<th>( 2.5% )</th>
<th>( 5% )</th>
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Finally, Table 3.11 reports the power probabilities when the exogenous growth of the process changes at $T/2$. This corresponds to the case when the GDP is

$$Y_t = \sum_{j=1}^{4} \delta_j D_{jt} + \beta_1 t + (\beta_2 - \beta_1) DT_t + \epsilon_t, \quad DT_t = \begin{cases} 0 & \text{if } t \leq T/2 \\ t - T/2 & \text{if } t > T/2 \end{cases}$$ [3.14]

The unit root test is adversely affected, with power falling to zero for $\beta_2 < 0.975$. As a change in the trend does not affect the spectral density at the seasonal frequencies, the seasonal unit roots tests have high power for $T \geq 100$, irrespective of the value for $\beta_2$.

**Table 3.11: Power of HEGY test - Change in the growth rate**

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<th>Pr[Rejecting a Seasonal Unit Root]</th>
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### 3.3.2 HEGY Test and Structural Break

In this section, we present new critical values for the HEGY test given a change in the level or change in the seasonal pattern of a series. As a point of comparison with the existing critical values, Table 3.12 reports the critical values for $\pi_1, \pi_2, \pi_3, \pi_4$, and the joint F-test at the 1%, 2.5%, 5%, 10%, 90%, 95%, 97.5% and 99% significance level when $T = 1000$.

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Table 3.12: Critical values for HEGY test (T=1000)

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22 This sample size is used because, following Perron (1989), we calculate the critical values of the HEGY tests allowing for a structural break using $T = 1000$. 

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Two different cases are considered. Table 3.13 presents the critical values of the HEGY tests when there is an assumed change in the level of the process, and there are no deterministic seasonal dummies included in the auxiliary regression [3.10]. This case is similar to Perron's (1989) Case A. The critical values are sensitive to the position of the break and are presented for a range of known alternative break points, which are assumed to be some proportion, \( \lambda \), of the sample size, \( T \). The critical values in Table 3.13 are comparable with those in the third block in Table 3.12 (Intercept, No Seasonal Dummies and Trend). Only those critical values corresponding to \( \pi_1 \) have changed to any substantive extent.

Finally, Table 3.14 reports the critical values, when there is an assumed change in the seasonal pattern and level of the data, and seasonal dummies are included in the auxiliary regression equation [3.10]. The critical values in Table 3.14 should again be compared to those in the last block of Table 3.12 (Intercept, Seasonal Dummies and Trend). The critical values for both the unit root test (\( \pi_1 \)) and the seasonal unit root tests (\( \pi_2 \) and \( \pi_3 \)) have become substantially more leftward skewed compared with those when no change is permitted (Table 3.12), and there has been a marked increase in the critical values of the F-statistic.
Table 3.13: Critical values for HEGY test - Change in level  
(Intercept, trend, no seasonal dummies)

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3.3.3 **Empirical Application**

In this section we illustrate our approach of testing for seasonal unit roots in presence of changes in the seasonal pattern of a series, by analysing the Colombian quarterly time series of M1 and GDP over the period 1970:1-1992:4.23

Figure 3.6 plots the log of Colombian M1, denoted LM1, and ΔLM1. It can be seen from the figure that LM1 exhibits an upward trend as well as a seasonal pattern, consisting of peaks during the fourth quarter. Moreover, it is possible to notice that after 1979:4 these peaks tend to be more pronounced, suggesting a change in the seasonal pattern of the series. According to Montenegro et. al. (1987), this change in the seasonal pattern of the series obeys the fact that since the early 1980s, the higher demand for currency, at the end of the year, is being satisfied by some of the components of the monetary base, instead of by a reduction in demand deposits.

**Figure 3.6**: Money supply and money supply growth in Colombia

---

23See Appendix 4.1 for precise definitions of the data series.
As a second illustration, Figure 3.7 plots the log of Colombian GDP, denoted LGDP, and ΔLGDP. As can be seen, LGDP exhibits an upward trend with a seasonal pattern consisting of peaks during the fourth quarter. Furthermore, a closer inspection of the series also allows us to notice that after 1985:4 a peak in the second quarter is also present. The change in the seasonal pattern of the GDP series after 1985 may be the result of a change in the measurement technique used to calculate the series.

Figure 3.7: Real GDP and GDP growth in Colombia

\[ \bar{Y}_{4t} = \pi_1 \bar{Y}_{1t-1} + \pi_2 \bar{Y}_{2t-1} + \pi_3 \bar{Y}_{3t-2} + \pi_4 \bar{Y}_{3t-1} + \sum_{i=1}^{\delta_p} \delta_i \bar{Y}_{4t-i} + \nu_t, \]  

[3.15]

is determined in a sequential fashion starting at \( p = 4 \), dropping terms until the null hypothesis \( H_0: \delta_p = 0 \) is rejected. This general to specific model selection rule has been suggested by Ghysels et. al. (1993). Using critical values corresponding to \( T = \)
100, both LM1 and LGDP reject the joint hypothesis \( H_0: \pi_3, \pi_4 = 0 \), at the 5% significance level but not at the 1%. Consequently, the two series are believed to be \( I(1,1) \), where the first number reflects the order of integration at frequency zero, and the second number the order of integration at the seasonal frequency.

**Table 3.15: HEGY tests for LM1 and LGDP**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Break Period</th>
<th>( \lambda )</th>
<th>( p )</th>
<th>( \pi_1 )</th>
<th>( \pi_2 )</th>
<th>( F )-test</th>
<th>( \pi_3 \cap \pi_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>LM1</td>
<td>No break</td>
<td>0.0</td>
<td>1</td>
<td>-1.746</td>
<td>-0.695</td>
<td>*7.076</td>
<td></td>
</tr>
<tr>
<td>LGDP</td>
<td>No break</td>
<td>0.0</td>
<td>1</td>
<td>-2.155</td>
<td>-2.262</td>
<td>*6.900</td>
<td></td>
</tr>
<tr>
<td>LM1</td>
<td>1979:4</td>
<td>0.4</td>
<td>0</td>
<td>-1.735</td>
<td>*-3.798</td>
<td>**46.640</td>
<td></td>
</tr>
<tr>
<td>LGDP</td>
<td>1985:4</td>
<td>0.7</td>
<td>0</td>
<td>-1.832</td>
<td>**-4.540</td>
<td>**27.226</td>
<td></td>
</tr>
</tbody>
</table>

Notes:
* denotes significance at the 5% level. ** denotes significance at the 1% level.

Allowing for a change in the seasonal pattern, define a change variable for LM1 and LGDP as:

\[
DUM1 = \begin{cases} 
0 \text{ if } t < 1979:4 \\
1 \text{ if } t \geq 1980:1 
\end{cases}
\]

and

\[
DUGDP = \begin{cases} 
0 \text{ if } t < 1985:4 \\
1 \text{ if } t \geq 1986:1 
\end{cases}
\]

Interacting these dummy variables with the seasonal dummy variables, \( \tilde{Y}_t \) is obtained as the residuals from the auxiliary regression [3.10], with the addition of the 4 interaction dummy variables. Applying equation [3.16] to \( \tilde{Y}_t \),
\[
\tilde{Y}_{4t} = \pi_1 \tilde{Y}_{1t-1} + \pi_2 \tilde{Y}_{2t-1} + \pi_3 \tilde{Y}_{3t-2} + \pi_4 \tilde{Y}_{3t-1} + \sum_{j=1}^{4} \theta_j D(TB)_{j,t} \\
+ \sum_{i=1}^{p} \delta_i \tilde{Y}_{4t-i} + \sum_{i=1}^{p} \eta_i D(TB)_{4,t-i} + \nu_t,
\]

where \(D(TB)_{j,t} = 1\) if \(t = TB + j\), yields the results presented in the bottom half of Table 3.15. Comparing the test statistics reported in this table with the critical values tabulated in Table 3.14, the results now indicate that LM1 and LGDP are both I(1,0).

### 3.4 Concluding Remarks

In this chapter we have assessed the effects of some seasonal adjustment linear filters on static cointegrating regressions, via Monte Carlo simulations. We found that the use of filters has adverse consequences in terms of the power of the ADF and PP tests for cointegration, so that they are not likely to reject the hypothesis of a unit root in the residuals of a cointegrating equation correctly. Consequently, considerable care must be exercised when using filters for seasonal adjustment, as one may wrongly conclude that a static regression between nonstationary series is spurious.

As an empirical application, we re-examined the results of the money demand modelling exercise for Colombia performed by Carrasquilla and Galindo (1994). We found that when one attempts to model the variables' seasonal pattern using simple methods, instead of removing it by filtering the data, the null hypothesis of non-cointegration is no longer accepted.

We have also shown that the HEGY tests for both unit roots and seasonal unit roots can be adversely affected by a change in either the level or seasonal pattern of a series. The unit root test is affected by a change in the level, although not by a change in the seasonal pattern of a series. In contrast, the seasonal root test is affected by a
change in the seasonal pattern and remains unaffected by a change in the level of the process. The position of the "break" can have a substantial effect on the power of the test statistic. As an empirical application, we investigated the time series properties of the Colombian series of money supply and GDP, both of which exhibit a change in the seasonal pattern. These series, which appear to be I(1,1) when no account is taken of the change in the seasonal pattern, are shown to be better described as I(1,0), if a change in the seasonal pattern is allowed.
CHAPTER 4

COFFEE BOOMS AND MONETARY DISEQUILIBRIUM

4.1 INTRODUCTION

Various studies have analysed the effects of coffee price booms on the Colombian economy in the context of the Dutch disease literature. In particular, the theoretical models that analyse the monetary consequences of export booms show that, under a regime of fixed exchange rates they affect not only the demand for money, via real income, but also the money supply, via foreign exchange accumulation. Thus, in the short run the monetary sector is likely to be in disequilibrium, although it is not possible to determine its nature (i.e. excess demand or excess supply) since, a priori, it is not clear which of the two effects dominates.

Existing empirical works on the monetary consequences of coffee booms in Colombia, confirm a positive relationship between fluctuations in the price of coffee and the money supply. However, the main drawback of the approach that has been taken, is that the effects on the demand for money have not been considered. That is, despite the fact that theoretical models identify both money supply and money demand effects, only money supply effects have been tested.

Taking the above aspects into consideration, in this chapter we use an alternative approach in order to determine whether the coffee booms of the 1970s and 1980s led to excess money supply in Colombia. This alternative approach consists of two steps. First, we estimate a measure of excess money supply which corresponds to the residuals of a long-run money demand equation, assuming that the money supply
is given. Second, we examine if there is a direct association between this measure of excess money supply and the evolution of the coffee market. We use information for the period 1970-1992, during which the country had a crawling peg exchange rate system.

The outline of the chapter is as follows. In section 2 we present a brief literature review of some of the models that have dealt with the macroeconomic consequences of export booms, sometimes referred to as Dutch disease models. In section 3 we summarise the studies that have analysed the monetary effects of coffee export booms on the Colombian economy. Then, we assess whether the coffee booms of the 1970s and 1980s led to excess money supply in the country. Finally, in section 4 we present some concluding remarks.

4.2 DUTCH DISEASE MODELS: THEORETICAL OVERVIEW

One of the most commonly used models for explaining the Dutch disease phenomenon is Corden and Neary (1982). The authors assume a small open economy that produces a booming tradable good, a non-booming tradable good and a nontradable good. Initially, all three goods are used for final consumption, although Corden and Neary also consider the case where the booming tradable good is used as an input into the production process of the other two goods. The prices of the two tradable goods are exogenously given, whereas the price of the nontradable good (PN) is domestically determined by supply and demand. Additionally, the model assumes that the relative price of the two traded goods does not change, so that they

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1Strictly speaking, since 1991 there have been a series of policy reforms aimed at allowing the exchange rate to be determined by the market. Since January 1994, after a transition period, Colombia adopted an exchange rate system in which the exchange rate fluctuates within a predetermined band.

2In the original formulation, these goods were energy, manufactures, and services, respectively.
can be aggregated into a single Hicksian composite good whose price is denoted PT. The real exchange rate is defined as the relative price of nontraded to traded goods (i.e. PN/PT) so that a rise (fall) corresponds to a real appreciation (depreciation). There are no distortions in either factors or goods markets; in particular, wages are perfectly flexible ensuring full employment. Lastly, on the production side each sector uses two factors of production, namely labour and capital, the latter assumed fixed and sector specific.

On this basis, Corden and Neary analyse the effects of a boom in one of the tradable sectors. Specifically, they consider a once and for all Hicks neutral improvement in technology, although the framework can also be applied to other kind of shocks such as a discovery of natural resources, or a rise in the price of one of the tradable goods. The distinctive feature of the model is that the favourable shock leads to a "resource movement effect" and a "spending effect". The first effect arises from the fact that the improvement in technology, or alternatively the natural resource discovery or the price boom, increases the profitability of the booming tradable sector so that it draws resources out of the other two non-booming sectors. The second effect arises from the higher real income resulting from the favourable shock, which increases the demand for both non-booming tradables and nontradables.

The pre-boom equilibrium in the goods market is depicted in Figure 4.1, where nontradables and tradables (both booming and non-booming) are measured on

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3This assumption is valid in the short run; in the long run capital becomes flexible.
4Corden and Neary (1982) point out that if the booming sector is an "enclave", so that it does not compete for production factors with other sectors of the economy, the resource movement effect is negligible, and the major impact of a resource discovery comes instead through the spending effect. On the other hand, if the boom is due to an increase in the price of one of the tradable goods, the resource movement effect works as indicated, but the spending effect will depend on whether the
the horizontal and vertical axes, respectively.\(^5\) PP denotes the production possibility frontier of the economy and IC represents the income-consumption path which is an upward-sloping curve assuming that both nontradables and tradables are normal in demand. Therefore, the initial equilibrium is attained at a, where the production possibility frontier is tangential to the highest social indifference curve I; at this point, the slope of the common tangent is equal to PN/PT, that is, the real exchange rate.

Figure 4.1: Real effects of an export boom

Following a resource discovery boom the production possibility frontier shifts out asymmetrically from PP to PP', indicating that the resource discovery boom allows the economy to increase its maximum output of tradables without affecting the maximum output of nontradables. Next, we will look at the resource movement effect and the spending effect separately.
First, let us consider the resource movement effect assuming that there is no spending effect (i.e. the income elasticity of demand for nontradables is zero). On the production side, because the favourable shock draws resources out of the non-booming sectors to the booming sector, there will be a new production point \( b \), to the left of \( j \), indicating that the production of nontradables is less than that observed originally. On the consumption side, as we abstract from the income effect, the income-consumption path is not IC but a vertical line that passes through points \( a \) and \( j \); accordingly, the new consumption point will be at \( j \). Consequently, at the initial real exchange rate there will be excess demand for nontradables, given by the horizontal distance between \( b \) and \( j \), that increases PN. As PN increases, the equilibrium in the nontradable goods market is gradually restored, and the final equilibrium will be at some point between \( b \) and \( j \), with a higher (appreciated) real exchange rate.

Second, let us consider the spending effect assuming that there is no resource movement effect. In this case, the new production point will be at \( j \) where neither the production of nontradables nor the production of non-booming tradables decreases with respect to the initial equilibrium. On the consumption side, the demand for nontradables will move along the IC income-consumption path until the intersection with the new production possibility frontier, at \( c \). Once again, there will be an excess demand for nontradables, given by the horizontal distance \( j-c \), so that PN has to increase in order to restore the equilibrium. The final equilibrium will be at some point between \( j \) and \( c \), e.g. \( g \), where the real exchange rate will be higher (PN/PT)\(^6\) compared to the initial equilibrium (PN/PT).\(^6\)

\(^6\) Since the model assumes a small open economy, any excess demand for tradable goods does not increase the price of these products. Therefore, the disequilibrium will be cleared through adjustments.
In conclusion, the Corden and Neary model predicts that both the resource movement effect and the spending effect lead to a real exchange rate appreciation. On the other hand, both effects increase the production of booming tradables and decrease the production of non-booming tradables. However, the impact on the production of nontradables is uncertain: while the resource movement effect tends to reduce the production of nontradables, the spending effect tends to increase it.

Bevan, Collier and Gunning (1987) have extended the Corden and Neary model in two important respects. Firstly, they consider the time dimension of the commodity boom, that is whether it is perceived to be permanent or temporary. Secondly, they argue that the nontradables sector must be disaggregated into capital (K) and consumer (C) goods. Within this extended framework, the authors find that a permanent commodity boom should leave the relative price $P_K/P_C$ unaffected, since the demands for these type of goods expands proportionally. In contrast, when the commodity boom is perceived to be temporary, a large proportion of it would be devoted to asset accumulation, and so the relative price $P_K/P_C$ should rise. This relative price change not only has distributional consequences, but also reduces the proportion of the commodity boom that is invested.

Neary and van Wijnbergen (1986) develop a model that incorporates monetary aspects in the analysis of the Dutch disease phenomenon. The Neary and van Wijnbergen model assumes that the real side of the economy is represented by the equilibrium in the nontraded goods market, which is given by

$$X_N\left(\frac{PN}{PT}, \tilde{B}\right) = C_N\left(\tilde{PN}, \tilde{PT}, \tilde{Y}, \tilde{M}/P\right),$$

in quantities which might lead to a trade deficit; that is, exports will be redirected to the domestic market whereas imports will increase.
where $X_N$ and $C_N$ denote supply and demand, respectively. The supply depends positively on the relative price of nontraded to traded goods (i.e. the real exchange rate), and negatively on $B$, a variable representing the resource movement effect. On the demand side, there is a negative own price effect and a positive cross-substitution effect; the spending effect is captured through the real income variable ($Y$) and there is also a real-balance effect ($M/P$). The domestic price level corresponds to $P$, a homogeneous-of-degree-one function in the prices of both nontradables and tradables (i.e. $P=P(P_N,P_T)$). In addition, the foreign price of tradable goods is set equal to unity so that their domestic price is equal to the nominal exchange rate ($e$).

The equilibrium in the nontraded goods market is depicted in Figure 4.2 by the NN locus, an upward-sloping curve. Intuitively, an increase in $P_N$ induces an excess supply of nontraded goods requiring an increase in $P_T$ to restore equilibrium. Points above (below) the NN locus correspond to situations of excess supply (demand) for nontradables.

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**Figure 4.2: Real and monetary effects of an export boom**

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7 Throughout the thesis, the signs of the responses of a function to changes in the variables on which it depends are indicated by the signs over those variables.
The model is then completed by incorporating the equilibrium condition in the monetary market, in an economy where money is the only asset. The money demand equation is given by \(\frac{M}{P} = \alpha Y\) which, as can be seen, incorporates the effect of the export boom in the money demand via real income. This is referred to as the "liquidity effect".

The equilibrium in the money market is depicted in Figure 4.2 by the MM locus, a downward-sloping curve. Intuitively, an increase in either PN or PT increases the domestic price level, which in turn reduces the real money supply leading to an excess money demand; consequently, a decrease in the other price is required to restore equilibrium. At this stage, Neary and van Wijnbergen incorporate into the analysis the implications of the exchange rate regime. In particular, they indicate that if the exchange rate is flexible and the money market clears at all times, equilibrium must lie on the MM locus; that is, for given levels of M and Y, an increase in PN must be offset by a decrease in PT (i.e. e). On the contrary, if the exchange rate is fixed, equilibrium in the short run might be, for instance, at points above (below) the MM locus so that there would exist a situation of excess real money demand (supply) that can be offset via foreign exchange accumulation (depletion).

On this basis, the initial equilibrium is given at point A where NN and MM intersect. What are the effects of a resource discovery boom? As far as NN is concerned, the boom leads, via the spending and resource movement effects, to an excess demand for nontradables at the initial prices; hence, the NN locus shifts upwards to \(\text{NN}'\). Regarding MM, the boom leads, via the liquidity effect, to an excess money demand at the initial prices; therefore, the MM locus shifts inwards to \(\text{MM}'\).
The final equilibrium, however, will depend on the exchange rate regime. If the nominal exchange rate is flexible, then the final equilibrium will be at point C where the NN' and MM' loci intersect. As can be seen, the favourable shock leads to a nominal exchange rate appreciation, from $e'$ to $e''$, as well as a real exchange rate appreciation given by the greater slope of the ray OCD compared to the ray OA. The effect on PN is ambiguous as it depends on the strength of the shift of the NN locus. Figure 4.2 depicts the case where PN reduces as a result of the boom.

If, on the other hand, the nominal exchange rate is fixed, for example at a level $e'$, the adjustment process is different. Given a constant nominal money supply, the two loci shift as mentioned. PN increases until J to restore equilibrium in the nontradable goods market, but at this point there is a situation of excess money demand so that the economy begins to accumulate foreign exchange which, if not sterilised, increases the money supply. As the money supply increases, both the MM' and NN' loci shift upwards so that the economy moves from J to a final equilibrium at D, where the real exchange rate has appreciated and PN has unambiguously risen.

In short, a resource discovery boom appreciates the real exchange rate in the long run. Under flexible exchange rates, the mechanism whereby this comes about is through a deflationary process, while under a fixed exchange rate is through an inflationary process.

Next, Neary and van Wijnbergen examine the effects of price rigidities in the economy, particularly the case where PN is downward rigid so that the nontradable goods market does not necessarily clear. Accordingly, given that under a flexible exchange rate a resource discovery boom might lead to a decrease in PN, the new adjustment process involves an initial movement from A to B where the money market is in equilibrium, but there is excess supply in the nontraded goods market; as
time passes, this excess supply leads to a reduction in PN so that the economy moves from B to the final equilibrium at C. The interesting part of the analysis is the fact that at B both the nominal and real exchange rates have overshot their long-run equilibrium levels. The nominal exchange rate overshooting is given by the horizontal distance between points B and C; the real exchange rate overshooting is reflected in the greater slope of the ray OB relative to the ray OC.

So far, we have considered a model where the export boom, via the liquidity effect, leads to an increase in the money demand which, provided the money supply does not change, results in an excess money demand. However, other authors argue that under a regime of fixed exchange rates, an export boom also leads to an increase in the money supply which arises from the monetisation of the additional export revenues. Consequently, it is not clear whether the export boom leads to excess money demand or excess money supply.

Edwards and Aoki (1983) is one example of such kind of models. Their aim is to analyse the effects of an increase in the price of oil in a small open economy with a fixed exchange rate. They assume that factors of production are perfectly mobile between non-oil sectors, in both the short and long run, whereas the oil sector, owned by the government, is assumed to use specific factors of production (i.e. the oil sector is an enclave). In addition, oil is not consumed domestically, and the government allocates its expenditure of the oil revenue between tradables and nontradables in the same way as private agents do. Lastly, money is the only asset in the economy, the capital account is assumed to be closed, and the money market clears slowly.

*A similar result is obtained by Neary (1985) using a money demand equation that includes the nominal interest rate as an additional argument. Other examples of real exchange rate overshooting
Unlike the Neary and van Wijnbergen model, where the export boom led to excess money demand via the liquidity effect, in the Edwards and Aoki model there exists the possibility of excess money supply if the government monetises the additional oil revenue. Nevertheless, in the long run an excess money supply (demand) is removed through trade deficits (surpluses). More formally, excess money supply is defined as

\[ M_E = M_S - M_D = M_E \left( M_S, \hat{P_N}, \hat{P_T}, \hat{Y} \right), \]

where the variables have the traditional meaning, and \( Y \) stands for real income, measured in terms of nontradables. This equation states that the excess supply of money (measured in nominal terms) is a positive function of the money supply (also measured in nominal terms), and a negative function of the price of nontradables, the price of tradables, and real income.

On the other hand, the excess demand for nontradables is assumed to depend not only on the relative price of tradables to nontradables (i.e. \( PT/PN \)) \(^9\) and real income, but also on the excess supply of money due to Walras Law; specifically

\[ N = N \left( \hat{PT}/\hat{PN}, \hat{M_E}, \hat{Y} \right). \]

The equilibrium in the nontraded goods market is depicted by NN, a downward-sloping curve in Figure 4.3. Points above (below) the NN locus correspond to excess demand (supply) for nontradables. The relative price of tradable goods in terms of oil is depicted by T, a ray from the origin. The initial equilibrium is attained at A, where the NN curve intersects T.

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\(^9\) According to this definition, an increase (decrease) of this relative price corresponds to a depreciation (appreciation) of the real exchange rate.
Figure 4.3: Effects of an export boom when there is excess money supply in the short run

Let us now look at the effects of an increase in the price of oil that reduces the relative price of tradable goods in terms of oil from $T$ to $T'$. Initially the economy moves from A to B given a constant nominal price of nontradables. But, at B there is excess demand for nontradables which requires an increase in their price to restore equilibrium; therefore, the economy moves along the $T'$ line reaching a final equilibrium at D, on the NN locus. As can be seen, at this final equilibrium the relative price of tradables in terms of nontradables is lower (that is, a real exchange rate appreciation), which moves resources out of tradables production into nontradables production.

In the short run, however, the money market is likely to be in disequilibrium as the export boom affects not only the money demand, through the liquidity effect, but also the money supply via foreign exchange accumulation. A priori, it is not clear which effect dominates. In any case, the disequilibrium in the money market translates into a disequilibrium in the nontraded goods market shifting the NN curve. Specifically, if there is excess money supply, the NN locus shifts inwards to $NN'$ so that at the original prices there is excess demand for nontradables; the short-run
equilibrium is then reached at C where the relative price of tradable goods in terms of nontradables has overshot its long-run equilibrium.10

As can be seen, the analysis shows that even in the absence of rigidities in the economy, an export boom might result in a short-run real exchange rate overshooting if the government monetises the extra foreign revenue. Regarding the adequate policy response, Edwards and Aoki suggest three alternatives. First, sterilisation of the balance of trade surplus resulting from the export boom. Second, a reduction of the proportion of oil revenue that the government spends in the nontradable goods market. Third, a nominal devaluation which increases the domestic price of tradable goods, and leads to an increase in the domestic price level that eliminates the excess money supply.

Harberger (1983) constructs a model to study the effects of variations in the world price of oil in a small open economy with a fixed exchange rate, that is quite similar to the Edwards and Aoki model, but he introduces plausible dynamics into the model. More specifically, in the Harberger model the demands for nontradables and tradables depend upon last period’s monetary disequilibrium and last period’s income, and the wage equation is based on last period’s price levels of nontradables and tradables. The dynamic simulations of the model indicate that in the long run the price of nontradables increases as a result of an increase in the world price of oil; in the short run, the price of nontradables overshoots its long-run equilibrium level.

10Conversely, if the export boom leads to excess money demand (i.e. excess supply for nontradables), the NN locus shifts outwards (not depicted) so that in the short run the relative price of tradables in terms of nontradables has undershot its long-run level.
4.3 COFFEE BOOMS AND MONETARY DISEQUILIBRIUM IN COLOMBIA

Perhaps one of the earliest studies that analyses the monetary consequences of coffee booms in Colombia is Urrutia (1981, p.217), who observes that “with the Brazilian coffee frost of 1975, ... international reserves ... started to grow at a very rapid rate, and this caused excessive rates of increase in the money supply. The increase in coffee prices started to produce increases in money supply which were, unfortunately not neutralised rapidly enough through changes in monetary and fiscal policy...”.

The most extensive empirical work in this field has been done by Edwards (1984, 1985, 1986a, 1986b). The starting point of Edwards is the assumption that the export boom is likely to generate an excess supply of money. Then, he proceeds to estimate, in a simultaneous equations context, a money creation equation for MO (Edwards, 1985) and M2 definitions of money (Edwards 1984, 1986a, 1986b) using three lags of the respective monetary aggregate, the price of coffee and the fiscal deficit as explanatory variables.\(^\text{11}\) Edwards finds a positive link between the price of coffee and the money supply (either MO or M2), which in turn leads to additional inflationary pressures and, other things being equal, results in a real exchange rate appreciation.

Carkovic (1986) builds an econometric model that aims to capture the effects of a coffee price boom on the monetary sector, as well as on the dynamic behaviour of the real exchange rate. The model consists of a real exchange rate equation, a real income equation, a balance of payments equation, and a money demand equation. Within this framework, Carkovic concludes that “in the monetary sector, a coffee

\(^{11}\)In the previous formulations of Hanson (1980) and Edwards (1983), however, the price of coffee is not considered as one of the determinants of the money supply process in Colombia. In fact, Hanson (1980) estimates an autoregression of order three, whereas Edwards (1983) includes the fiscal deficit in addition to the three lags of the dependent variable.
bonanza increases reserves and affects desired cash balances in an ambiguous way presumably producing an excess supply in the money market..." (p. 84). Nonetheless, it is worth mentioning that from an econometric perspective Carkovic’s results are subject to some criticisms: the overall fit of the balance of payments equation is poor with only one estimated coefficient statistically different from zero, and the author does not indicate the reason why the interest rate is not included in the money demand equation (presumably lack of information).12

Reinhart and Reinhart (1991) postulate a VAR model in order to analyse the dynamic responses among output, prices, the money supply, interest rates, the exchange rate, both general wages and the minimum wage, and the world price of coffee. They find that the money supply can be placed at the top of a causal ordering as no lagged variables were significant in explaining its dynamic behaviour. Moreover, they find no evidence of any contemporaneous or lagged relationship between the world price of coffee and inflation, which contradicts previous findings by Edwards.

Wunder’s (1991) analysis of the Dutch disease phenomenon is based on the consideration that it is “superimposed” on long-run trends, so that he measures some of the variables by deviation from their long-run (deterministic) trend. His results also indicate that the monetary base equation depends positively on the trend deviation of the real value of coffee exports,13 among other variables.

12Carkovic assumes that the money demand is a function of income, inflation (which measures the opportunity cost) and lagged money demand. However, studies on money demand for Colombia confirm the importance of the interest rate (see e.g. Steiner 1988, Carrasquilla and Renteria 1990, and Herrera and Julio 1993).
13He uses the real value of coffee exports instead of the coffee price on the grounds that the Colombian coffee boom of the second half of the seventies was initially a price boom which was then followed by a quantity boom.
In summary, with the exception of Reinhart and Reinhart (1991), existing empirical analyses on the monetary consequences of coffee booms in Colombia, find a positive relationship between fluctuations in the price of coffee and the money supply. However, the main limitation of the approach that has been taken, is that despite the fact that theoretical models identify both money supply and money demand effects, only money supply effects have been tested. Accordingly, in what follows we propose a two-step approach in order to determine whether the coffee booms of the 1970s and 1980s led to excess money supply in Colombia. More specifically, in the first step we estimate a measure of excess money supply which corresponds to the residuals from estimating a long-run money demand equation, assuming that money supply is exogenous. In the second step, we relate this measure of excess money supply to the evolution of the coffee market as well as other possible sources of disequilibrium.

4.3.1 MODELLING A MONEY DEMAND FUNCTION

Given that we are interested in a measure of money market disequilibrium rather than portfolio considerations, we use a standard money demand equation of the form:

\[ m^d = f(y, p, R), \]  

[4.1]

where, unlike the specification stated in equation [3.4], the restriction of price homogeneity has not been imposed, but will be formally tested.

The money demand modelling exercise is performed using the relatively recent developments of cointegration analysis (see Engle and Granger, 1987). The basic idea of cointegration is that, even though individual time series may not be stationary, one or more linear combinations of them may be. The relevance of this
concept is that if these linear combinations are interpreted as a long-run equilibrium, the existence of cointegration implies that deviations from this equilibrium are stationary; put another way, there will be economic forces that do not let these deviations become increasingly large.

The method of estimation we use is that of Johansen (1988) and Johansen and Juselius (1990), who develop a maximum likelihood estimation of cointegration vectors and likelihood ratio tests of hypotheses about cointegration vectors. One of the advantages of Johansen's approach is that in a multivariate context it allows us to estimate all possible cointegrating vectors.

It is worth mentioning that we did not rely on the residuals of existing money demand functions for three main reasons. Firstly, we intend to provide a more detail analysis of the time series properties of the variables involved. Secondly, previous money demand modelling exercises usually assume price homogeneity without providing a formal test of this hypothesis. Lastly, we explicitly test whether the set of variables (p, y, R) are weakly exogenous for the parameters of the money demand model.14

### 4.3.2 Data

We use seasonally unadjusted quarterly data for the period 1970-1992, since the use of seasonal adjustment filters reduces the power of cointegration tests (as shown in the previous chapter), and affects the results of weak exogeneity tests (see Ericsson, Hendry and Tran 1994). The monetary aggregate corresponds to M1, the price level

---

corresponds to the consumer price index, and the scale variable corresponds to the GDP series. The proxy for the opportunity cost of holding money is constructed by combining two different interest rate series: from 1970:1 to 1980:1 the yield of 120-day CAT certificates, and from 1980:2 to 1992:4 the yield of 90-day CDT certificates offered by banks and financial corporations. The first three series are considered in logarithms and denoted LM1, LCPI and LGDP, respectively; on the other hand, the interest rate series, denoted R, will not be considered in logarithms in order to allow the interest rate elasticity to vary with the level of the interest rate. In Appendix 4.1 we present a more detailed description of the series and their sources.

In Figure 4.4, we plot the levels and first differences of the variables, whereas in Figure 4.5, we present their autocorrelation functions. As can be noticed, LM1 exhibits an upward trend indicating that it may be nonstationary in levels; the correlogram of the series decays slowly as the lag length increases, which also supports this conclusion. LM1 has a seasonal pattern with peaks during the fourth quarter. In addition, it is possible to notice that after 1979:4 these peaks tend to be more pronounced, suggesting a change in the seasonal pattern; as shown in the previous chapter, accounting for this change in the seasonal pattern of the series overturns initial results suggesting the presence of unit roots at the seasonal frequencies. Plots of ΔLM1 show no evidence of a changing mean nor of a changing variance, except for the wider fluctuations observed since the eighties, which are in turn associated with the change in the seasonal pattern already indicated. The correlogram of ΔLM1 illustrates the presence of significant autocorrelation at seasonal frequencies even after some years.

\footnote{It was also tried to estimate a money demand function including real gross domestic income as scale variable. However, in this case we did not find evidence of cointegration.}
In the case of LCPI, it is possible to observe that the series exhibits an upward trend suggesting that it may be nonstationary in levels; this conclusion is also supported when visually inspecting the correlogram of the series. ΔLCPI, on the other hand, shows no evidence of a changing mean nor of a changing variance. The correlogram of ΔLCPI shows significant autocorrelation at seasonal frequencies, although they are not as large as those observed in the case of ΔLM1.

With reference to LGDP, the clear upward trend suggests that the series may be nonstationary in levels; the correlogram of the series, which decays slowly as the lag length increases, also supports this conclusion. LGDP exhibits a seasonal pattern consisting of peaks during the fourth quarter of each year. Furthermore, it is possible to notice that after 1985:4 there is a peak in the second quarter, suggesting a change in the seasonal pattern of the series; it is worth recalling that in the previous chapter we found that the unit roots at the seasonal frequencies disappear once we account for this change in the seasonal pattern. A plot of ΔLGDP shows no evidence of a changing mean nor of a changing variance. The correlogram of ΔLGDP illustrates the presence of significant autocorrelation at seasonal frequencies even after some years.

Lastly, R exhibits an upward trend although not as clear as that of the previous variables. In addition, the series presents a change in its growth rate in 1980:2, as a result of the combination of two different series. At first sight, we might say that R may be nonstationary in levels, which appears to be confirmed by the correlogram of the series. However, in the next section we examine the nonstationarity of R in more detail, since the power of the unit root test is adversely affected when the growth rate of a series changes (see e.g. Perron 1989, and our results in Table 3.11). A plot of ΔR shows no evidence of a changing mean nor of
changing variance; nevertheless, it is important to highlight the wider fluctuations observed throughout the seventies. The correlogram of $\Delta R$ drops off to zero quickly as the lag length increases.
Figure 4.4: Nominal M1, prices, output and interest rates in Colombia -
Levels and first differences of the series
Figure 4.5: Correlograms for M1, prices, output, interest rates and their first differences
4.3.3 TESTING FOR UNIT ROOTS

The order of integration of the series is investigated by means of the Augmented Dickey and Fuller (ADF) tests for unit roots, which we apply as indicated by Perron (1988). The number of lags of the dependent variable to include in the test regressions is selected following Campbell and Perron (1991), starting with an upper bound of 8 lags, and then we perform the LM[4] test for serial correlation on the residuals of the test regressions.16

In the first panel of Table 4.1 we report the results of the ADF tests. As can be observed, LMI, LCPI, and LGDP seem to contain a unit root with a non-zero drift term.17 With reference to R, the results indicate that the series may contain a unit root with a zero drift term; however, from Figure 4.4 it is possible to distinguish a clear break when the two different measures of the opportunity cost of holding money are linked. Thus, it is worth examining if the non-rejection of the unit root hypothesis is because of this combination of series. In order to do this, we apply two different procedures.

Firstly, we test for the existence of a unit root in the two series of opportunity cost of holding money; that is, the yield of 120-day CAT certificates (available from 1970:1 to 1980:1), and the yield of 90-day CDT certificates (available from 1980:2 to 1992:4). Even though the results suggest that the two series appear to contain a unit root with a zero drift term (see the second panel of Table 4.1), they should be interpreted with caution given the reduced number of observations.

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16When dealing with LMI, LCPI and LGDP we also include centred seasonal dummies to capture some of the seasonal pattern. Dickey et. al. (1986) show that this procedure does not affect the limiting distributions of the unit root tests statistics.

17In the case of LMI and LGDP the null hypothesis of a unit root (at the zero frequency) is also accepted based on the HEGY test (see Table 3.15).
### Table 4.1: Dickey and Fuller unit root tests

<table>
<thead>
<tr>
<th>Series</th>
<th>Sample Period</th>
<th>Model</th>
<th>Lags</th>
<th>LM[4]</th>
<th>$\tau_t$</th>
<th>$\Phi_3$</th>
<th>$\Phi_2$</th>
<th>$\tau_\mu$</th>
<th>$\Phi_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LM1</td>
<td>70.1 - 92.4</td>
<td>A</td>
<td>4</td>
<td>F 4.73 1.080</td>
<td>-1.686</td>
<td>2.608</td>
<td><strong>5.495</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LCPI</td>
<td>70.1 - 92.4</td>
<td>A</td>
<td>3</td>
<td>F 4.75 0.380</td>
<td>-2.781</td>
<td>4.788</td>
<td><strong>8.603</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LGDP</td>
<td>70.1 - 92.4</td>
<td>A</td>
<td>4</td>
<td>F 4.73 1.800</td>
<td>-2.392</td>
<td>4.493</td>
<td><strong>7.858</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R</td>
<td>70.1 - 92.4</td>
<td>A</td>
<td>2</td>
<td>F 4.80 0.186</td>
<td>-1.638</td>
<td>2.488</td>
<td>1.753</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R</td>
<td>70.1 - 92.4</td>
<td>B</td>
<td>2</td>
<td>F 4.81 0.180</td>
<td>-2.227</td>
<td>2.624</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>70.1 - 80.1</td>
<td>A</td>
<td>2</td>
<td>F 4.29 0.558</td>
<td>-2.112</td>
<td>2.248</td>
<td>1.990</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>70.1 - 80.1</td>
<td>B</td>
<td>2</td>
<td>F 4.30 0.399</td>
<td>-1.096</td>
<td>1.293</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>80.2 - 92.4</td>
<td>A</td>
<td>0</td>
<td>F 4.43 2.556</td>
<td>-2.575</td>
<td>3.555</td>
<td>2.451</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>80.2 - 92.4</td>
<td>B</td>
<td>0</td>
<td>F 4.44 2.222</td>
<td>-2.079</td>
<td>2.279</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LCP</td>
<td>70.1 - 92.4</td>
<td>A</td>
<td>1</td>
<td>F 4.82 0.627</td>
<td>-2.277</td>
<td>2.898</td>
<td>2.028</td>
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<td></td>
</tr>
<tr>
<td>LCP</td>
<td>70.1 - 92.4</td>
<td>B</td>
<td>1</td>
<td>F 4.83 0.913</td>
<td>-1.588</td>
<td>1.401</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LCX</td>
<td>70.1 - 92.4</td>
<td>A</td>
<td>5</td>
<td>F 4.74 0.354</td>
<td>-1.373</td>
<td>2.173</td>
<td>1.453</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LCX</td>
<td>70.1 - 92.4</td>
<td>B</td>
<td>5</td>
<td>F 4.75 0.418</td>
<td>-1.372</td>
<td>0.948</td>
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<td></td>
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<tr>
<td>FS</td>
<td>70.1 - 92.4</td>
<td>A</td>
<td>6</td>
<td>F 4.69 0.958</td>
<td>-2.580</td>
<td>3.403</td>
<td>2.331</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FS</td>
<td>70.1 - 92.4</td>
<td>B</td>
<td>6</td>
<td>F 4.70 0.849</td>
<td>-2.360</td>
<td>2.879</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:

Model A is $\Delta Y_t = \gamma_0 + \gamma_1 t + \gamma_2 Y_{t-1} + \text{lags dep. variable}$.
Model B is $\Delta Y_t = \beta_0 + \beta_1 Y_{t-1} + \text{lags dep. variable}$.

The regressions of LM1, LCPI, LGDP and FS include centred seasonal dummies. The LM[4] test is reported in its F version. The critical values for the $\tau$ statistics are reported in MacKinnon (1991). The critical values for the $\Phi_1$, $\Phi_2$, and $\Phi_3$ statistics are reported in Dickey and Fuller (1981). * denotes significance at the 5% level. ** denotes significance at the 1% level.

**Figure 4.6:** R, non-linear trend function and $R^{DT}$
Secondly, we follow the analysis developed by Perron (1989) concerning the implications of structural breaks for the unit root tests. Based on the visual inspection of R, we decided to consider the model that allows for a change in the rate of growth of the series, or what Perron refers to as the "changing growth model". Figure 4.6 plots R, its kinked trend function and, at the bottom, the resulting de-trended series which will be denoted $R_{DT}^{t}$. Next, $R_{DT}^{t}$ is used to run a Dickey and Fuller type regression without a constant, from which the existence of a unit root, using the critical values tabulated in Perron (1993), is easily rejected for a break occurring in the middle of the sample period.

In summary, the important point to notice is that when we take into consideration that the interest rate series consists of two different measures, it is no longer completely clear that it is I(1). This result certainly contrasts with those of other authors who have found that the same interest rate series, or its logarithm, behaves as an I(1) process for different sample periods (see e.g. Gaviria and Uribe, 1993 and Herrera and Julio, 1993). However, we regard that our result is justified on the grounds that standard Dickey-Fuller unit root tests have low power under the presence of structural breaks, and on the fact that the order of integratedness is not an inherent property of a time series, that is the order of integration of a time series may differ for different sample periods (see Hendry, 1995; chapter 16).

---

$R_{DT}^{t}$ corresponds to the residuals of an OLS regression of R on a constant, a linear time trend and $DT_{t}^{t} = t - TB$ if $t > TB$ and 0 otherwise, where TB refers to the time of the break.
4.3.4 Cointegration Analysis

In this section we use the Johansen cointegration approach (see Johansen, 1988 and Johansen and Juselius, 1990) to estimate a long-run money demand function.\(^\text{19}\) As it is known, this approach is based on the following VAR model:

\[ X_t = A_1 X_{t-1} + \ldots + A_k X_{t-k} + \varepsilon_t, \]  

where \(X_n, X_{t-1}, \ldots, X_{t-k}\) are vectors that contain current and lagged values of all \(n\) variables of the model, which are assumed to be \(I(1): A_1, \ldots, A_k\) are \(n \times n\) matrices of coefficients; and \(\varepsilon_t\) is a vector of random errors.\(^\text{20}\) In practice, the order of the VAR model should be such that the estimated error terms pass tests of serial correlation, normality and heteroscedasticity. The VAR model can be written in error correction form as:

\[ \Delta X_t = \Gamma_t \Delta X_{t-1} + \ldots + \Gamma_{k-1} \Delta X_{t-k+1} + \Pi X_{t-k} + \varepsilon_t, \]  

where

\[ \Gamma_j = -(I - A_j - \ldots - A_k), \quad j = 1, 2, \ldots, k-1, \]

\[ \Pi = -(I - A_1 - \ldots - A_k), \]

and \(I\) is the identity matrix. The importance of the second representation is that the rank of the matrix of coefficients \(\Pi\) contains information about the number of long-run relationships in the multivariate model.

Within this framework, we first consider a four-dimensional VAR model consisting of LM1, LCPI, LGDP and R\(^\text{DT}\); we use R\(^\text{DT}\) instead of R in order to take into account the clear structural break observed in the interest rate series.\(^\text{21}\) As we are using seasonally unadjusted series, we also include a 3x1 \(D_t\) matrix containing

\(^{19}\)We did not consider the possibility of seasonal cointegration, since results in the previous chapter ruled out the presence of unit roots at the seasonal frequencies.

\(^{20}\)For simplicity we have excluded deterministic terms such as constants, trends, seasonal dummies.

\(^{21}\)The unit root test is affected by a change in the growth rate of a series, but not by a change in the seasonal pattern of a series.
centred seasonal dummy variables and a μ, vector of constant terms, which are entered unrestricted as LM1, LCPI and LGDP contain a non-zero drift term. In order to select the lag length, we estimate the VAR for lag lengths of 5 and 4, using the same sample period, and then we test whether the fifth lag is redundant, which is easily accepted (F_{16,183}=1.021). Both the Schwarz and Hannan-Quinn information criteria decrease when the order of the autoregression is reduced from 5 to 4.

The VAR model is then estimated adjusting the sample period to a lag length of 4, and the main diagnostic tests are summarised in Table 4.2. In general terms, the results of the diagnostic tests are satisfactory, despite that the equation for LGDP passes the LM[4] test for residual serial correlation at the two per cent significance level, and that the residuals in the equation for LCPI do not appear to be normally distributed due to outliers. Lastly, there is some evidence of heteroscedastic errors in the equation for RDT at the two per cent significance level. Parameter constancy is analysed by means of the one-step residuals test, 1-step F-tests (1↑-step Chow-tests), break-point F tests (N↓-step Chow-tests) and forecast F-tests (N↑-step Chow-tests), calculated from a recursive estimation of the model (see Figures 4.7 to 4.10). These tests suggest that the equations of the model are constant during the period under review.

---

22The inclusion of an extra lag in the VAR model did not improve this statistic.
23The 1↑-step Chow-tests are calculated between the estimates based on the first T₀ and T₀+1 observations, for T₀=1979:3, ..., 1992:3. The N↓-step Chow-tests evaluate the hypothesis that elements in the sequence of estimates from T=1979:3, 1979:4, ..., 1992:3 are equal to the estimates based on the sample up to 1992:4. The N↑-step Chow-tests are for constancy between the initial estimates (here based on the first sub-sample up to 1979:3) and all subsequent sample sizes. The alternative forms of the Chow test are scaled by their one per cent significance values at each possible point, so that values above the straight line at unity are significant. See Doornik and Hendry (1994a, 1994b) for details about these tests.
Table 4.2: Cointegration analysis

<table>
<thead>
<tr>
<th>Model diagnostic tests</th>
<th>LM1</th>
<th>RDT</th>
<th>LCPI</th>
<th>LGDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>LM [4]</td>
<td>F4,64</td>
<td>1.732</td>
<td>0.342</td>
<td>1.326</td>
</tr>
<tr>
<td>ARCH [4]</td>
<td>F4,60</td>
<td>0.849</td>
<td>2.424</td>
<td>1.152</td>
</tr>
<tr>
<td>Normality</td>
<td>$\chi^2_2$</td>
<td>1.268</td>
<td>3.421</td>
<td>**24.267</td>
</tr>
<tr>
<td>Heteroscedasticity</td>
<td>F32,35</td>
<td>0.849</td>
<td>*2.123</td>
<td>1.149</td>
</tr>
</tbody>
</table>

Cointegration analysis

Maximal eigenvalue test

<table>
<thead>
<tr>
<th>Null hypothesis</th>
<th>r = 0</th>
<th>r &lt;= 1</th>
<th>r &lt;= 2</th>
<th>r &lt;= 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alternative hypothesis</td>
<td>r = 1</td>
<td>r = 2</td>
<td>r = 3</td>
<td>r = 4</td>
</tr>
<tr>
<td>Test value</td>
<td>*31.370</td>
<td>23.500</td>
<td>11.370</td>
<td>0.576</td>
</tr>
</tbody>
</table>

Trace test

<table>
<thead>
<tr>
<th>Null hypothesis</th>
<th>r = 0</th>
<th>r &lt;= 1</th>
<th>r &lt;= 2</th>
<th>r &lt;= 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alternative hypothesis</td>
<td>r &gt;= 1</td>
<td>r &gt;= 2</td>
<td>r &gt;= 3</td>
<td>r = 4</td>
</tr>
<tr>
<td>Test value</td>
<td>**66.810</td>
<td>*35.440</td>
<td>11.940</td>
<td>0.576</td>
</tr>
</tbody>
</table>

$\beta'$ eigenvectors

(Standardized)

<table>
<thead>
<tr>
<th></th>
<th>1.000</th>
<th>1.933</th>
<th>-0.937</th>
<th>-0.956</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-2.860</td>
<td>1.000</td>
<td>2.714</td>
<td>2.519</td>
</tr>
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</table>

Adjustment coefficients $\alpha$

(Standardized)

<table>
<thead>
<tr>
<th></th>
<th>LM1</th>
<th>RDT</th>
<th>LCPI</th>
<th>LGDP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.280</td>
<td>0.029</td>
<td>-0.010</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>-0.147</td>
<td>-0.089</td>
<td>-0.011</td>
<td>-0.559</td>
</tr>
</tbody>
</table>

Notes:
The LM[4], ARCH[4] and Heteroscedasticity tests are reported in their F versions. The test for normality is distributed as $\chi^2_2$. The number of cointegrating vectors is denoted by r. Critical values for the maximal eigenvalue and trace tests are reported in Osterwald-Lenum (1992). * denotes significance at the 5% level. ** denotes significance at the 1% level.
Figure 4.7: 1-step residuals ±2 equation standard error

Figure 4.8: 1-step F tests
Figure 4.9: Break-point F tests

Figure 4.10: Forecast F tests
Before presenting the results of the cointegration analysis, it is worth indicating that because the interest rate series has been previously de-trended, the critical values used when analysing the VAR model, i.e. Osterwald-Lenum (1992) Table 1, are not applicable; in fact, the required critical values do not exist. To overcome this difficulty, we use the critical values reported in Osterwald-Lenum’s Table 2 corresponding to the case where the VAR model includes a constant and a time trend, which are larger than the respective critical values in Table 1. Compared to these larger critical values, the maximal eigenvalue and trace statistics suggest the presence of two cointegrating vectors: the first one with the expected signs of a money demand function, and the second one without a clear economic interpretation (see Table 4.2).

Nonetheless, it shall be remembered that in the previous section we had obtained ambiguous results as to whether the interest rate series was an I(1) variable; in other words, it may be the case that the first cointegrating vector corresponds to a money demand function, whereas the second one indicates that $R^{DT}$ is I(0). Consequently, we proceed to test that $LM_1$, $LCPI$ and $LGDP$ do not enter the cointegrating space of the second cointegrating vector which is easily accepted, indicating that $R^{DT}$ is stationary or, put another way, that the interest rate series is stationary around a non-linear time trend.

Next, we test whether the set of variables are weakly exogenous for the estimation of the parameters of the money demand model. The concept of weak exogeneity, originally proposed by Engle, Hendry and Richard (1983), states that whether a variable is exogenous or not will depend on the parameters of interest. Johansen (1992) shows that in the context of a p-dimensional VAR under the hypothesis of cointegration, weak exogeneity of $Z_t$ (for $Z_t$ either a single or a set of
variables) with respect to the long-run parameters, is equivalent to test that the corresponding adjustment coefficients of the $Z_i$ equation (or equations) are zero.

For the purposes of the VAR model we are dealing with, the results indicate that $L.M1$ cannot be regarded as weakly exogenous as its weighting coefficient is significant, although the same cannot be said about $LCPI$, $LGDP$ and $R^{DT}$, as their weighting coefficients are not statistically different from zero. If we then impose weak exogeneity for $LCPI$, $LGDP$ and $R^{DT}$ as well as long-run price homogeneity, the resulting statistic is also clearly accepted. In summary, the results suggest a money demand equation that satisfies long-run price homogeneity (i.e. no money illusion), and scale economies in the holding of money (i.e. long-run income elasticity of 0.628); the interest rate effect is equal to -1.769.

Once we have found evidence of a cointegrating relationship, the next step is to estimate the VAR model in error correction form, which allows us to model the short- and long-run behaviour simultaneously. Based on the previous results of the tests for weak exogeneity, the VAR model can be regarded as a one equation model conditioning on $LCPI$, $LGDP$ and $R^{DT}$; in other words, there is no need to model the system as a whole. Accordingly, the error correction model (ECM) is given by the equation:

$$\Delta L.M1_t = \text{constant} + \text{lags}(\Delta L.M1_t, \Delta LCPI_t, \Delta LGDP_t, R^{DT}_t) + \mu \text{ECT}_{t-1},$$

where

$$\text{ECT}_{t-1} = L.M1_{t-1} - \beta_1 LCPI_{t-1} - \beta_2 LGDP_{t-1} - \beta_3 R^{DT}_{t-1},$$

denotes the error correction term that is equal to the residuals of the cointegrating vector lagged once. Since there is evidence of cointegration among money, prices, output and interest rate, each component in the error correction equation is $I(0)$, and so the equation is balanced. The lag length for $\Delta L.M1$, $\Delta LCPI$ and $\Delta LGDP$ is equal to
three as we included four lags in the VAR model in levels, whereas that of \( R^\text{DT} \) is equal to four; furthermore, centred seasonal dummy variables are included.

Before estimating the ECM, as specified above, we examine the possibility that it is non-linear in the sense that the \( \mu \) parameter is different depending on whether \( \text{ECT}_{t-1} \) is positive or negative; in order to do this, we split this term up into \( \text{ECT}^{+}_{t-1} \) and \( \text{ECT}^{-}_{t-1} \), where:

\[
\text{ECT}_{t-1}^+ = \begin{cases} 
\text{ECT}_{t-1} & \text{if } \text{ECT}_{t-1} \geq 0 \\
0 & \text{otherwise}
\end{cases}
\]

and

\[
\text{ECT}_{t-1}^- = \begin{cases} 
\text{ECT}_{t-1} & \text{if } \text{ECT}_{t-1} < 0 \\
0 & \text{otherwise}
\end{cases}
\]

and then we test the hypothesis that the estimated coefficients on \( \text{ECT}_{t-1}^+ \) and \( \text{ECT}_{t-1}^- \) are equal. Results not reported here indicate that this hypothesis is easily accepted at the five per cent significance level, so that we proceed to estimate the ECM in its standard linear form.

The initial estimates of the ECM as well as some misspecification tests are presented in the top half of Table 4.3. As expected, the equation is overparameterised so that in order to obtain a more parsimonious representation we exclude some of the regressors based on Wald tests for zero restrictions. In particular, we first exclude \( \Delta \text{LGDP}_{t-1}, R^\text{DT}_{t-2}, R^\text{DT}_{t-3}, R^\text{DT}_{t-4}, \Delta \text{LCPI}_{t-1}, \Delta \text{LCPI}_{t-2} \) and \( \Delta \text{LCPI}_{t-3} \), and then \( \Delta \text{LM}_{1,2}, \Delta \text{LGDP}, \) and \( \Delta \text{LGDP}_{t-3} \). From the resulting equation we observe that the estimated coefficients of \( R^\text{DT} \) and \( R^\text{DT}_{t-1} \) have similar magnitudes and opposite signs, so that we impose this restriction which is easily accepted.

The resulting ECM is reported in the bottom half of Table 4.3. It passes the \( \text{LM}[4] \) test for residual serial correlation, Engle’s \( \text{LM}[4] \) test for ARCH, White’s test for heteroscedasticity as well as Ramsey’s \( \text{RESET} \) test; the test for normality,
however, is significant at the five per cent significant level. Figure 4.11 reports the recursive estimates of the individual coefficients, with their respective 95 per cent confidence intervals; the estimated coefficients appear to be relatively constant as later estimates lie inside previous confidence intervals. Figure 4.12 plots parameter constancy tests for the ECM. As can be seen, in very few occasions the one-step residuals lie outside of the error bars suggesting no evidence of significant changes in the equations standard error. The sequences of 1-step F-tests (1↑-step Chow-tests), break-point F tests (N↓-step Chow-tests), and forecast F-tests (N↑-step Chow-tests), provide further evidence in favour of a relatively stable model, with almost none of the tests rejecting at the one per cent significance level.
Table 4.3: Error correction model

<table>
<thead>
<tr>
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<th>2</th>
<th>3</th>
<th>4</th>
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<tbody>
<tr>
<td>ΔLM1</td>
<td>-1.000</td>
<td>-0.343</td>
<td>-0.192</td>
<td>-0.540</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.112)</td>
<td>(0.118)</td>
<td>(0.102)</td>
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<tr>
<td>ΔLCPI</td>
<td>0.471</td>
<td>0.117</td>
<td>0.219</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.196)</td>
<td>(0.209)</td>
<td>(0.193)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔLGDP</td>
<td>0.311</td>
<td>0.235</td>
<td>0.502</td>
<td>0.313</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.2110)</td>
<td>(0.2140)</td>
<td>(0.1970)</td>
<td>(0.1950)</td>
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<tr>
<td>R²Dt</td>
<td>-0.343</td>
<td>0.577</td>
<td>-0.122</td>
<td>-0.073</td>
<td>-0.032</td>
</tr>
<tr>
<td></td>
<td>(0.095)</td>
<td>(0.201)</td>
<td>(0.118)</td>
<td>(0.119)</td>
<td>(0.107)</td>
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<tr>
<td>ECT</td>
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<td>-0.350</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td>(0.096)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>R²</td>
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<td>0.930</td>
<td></td>
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<tr>
<td>LM[4]</td>
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<td>F₄₆,₂</td>
<td>0.942</td>
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<tr>
<td>ARCH[4]</td>
<td></td>
<td>F₄₅,₈</td>
<td>0.274</td>
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<td></td>
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<tr>
<td>Normality</td>
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<td>Z²₂</td>
<td>4.315</td>
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<tr>
<td>Heteroscedasticity</td>
<td></td>
<td>F₁₇,₂₄</td>
<td>0.456</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RESET</td>
<td></td>
<td>F₁₅,₅</td>
<td>1.727</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| ΔLM1| -1.000| -0.322| -0.416|       |
|     | (0.080)| (0.077)|       |       |
| ΔLCPI| 0.424 |       |       |       |
|     | (0.166)|       |       |       |
| ΔLGDP|       |       | 0.297|       |
|     |       |       | (0.131)|       |
| ΔR²Dt| -0.325|       |       |       |
|     | (0.084)|       |       |       |
| ECT |       | -0.246|       |       |
|     |       | (0.044)|       |       |
| R²  |       | 0.917 |       |       |
| ARCH[4] |       | F₄₆,₆₉ | 0.210 |       |
| Normality |       | Z₂² | '6.160 |       |
| Heteroscedasticity |       | F₁₅,₆₁ | 0.301 |       |
| RESET |       | F₁₃,₆ | 2.533 |       |

Notes:
Regressions include constant and centred seasonal dummies. Standard errors in parentheses. * denotes significance at the 5% level.
Figure 4.11: Recursive estimation of the parameters of the ECM

Figure 4.12: Parameter constancy tests for the ECM
4.3.5 **Main Determinants of the Excess Money Supply**

In the preceding section we found evidence of cointegration among money, prices, output and interest rates, which implies that these variables must be linked by a long-run equilibrium relationship. In the short run these variables may deviate from the long-run equilibrium relationship, although not by an ever-growing amount, since economic forces may be expected to act so as to restore equilibrium (i.e. the discrepancy in the relationship must be integrated of order zero). In what follows we aim to relate the short-run disequilibrium in the money market to the evolution of the coffee market. To do this, we assume that the residuals of the long-run money demand equation can be thought of as an approximate measure of short-run disequilibrium in the money market, with positive (negative) residuals denoting excess money supply (demand).

Figure 4.13 presents the estimated measure of excess money supply. Without pretending to provide exact dates, from the figure it is possible to identify six sub-periods. On the one hand, between 1970-1974, 1977-1980 and 1984-1988 the estimated measure of money market disequilibrium exhibits an increasing trend suggesting a movement from excess demand to excess supply; on the other hand, the declining trend between 1975-1976, 1981-1983 and 1989-1992 suggests a movement from excess supply to excess demand.
If we compare the estimated measure of excess money supply (EMS) with the logarithm of the price of coffee (LCP) and the logarithm of coffee exports (LCX) (see Figures 4.14 and 4.15, respectively), where the two coffee variables are expressed in constant dollars of 1975, one might say that there is a direct association between money disequilibria and the evolution of the coffee market for most of the period under review. For instance, the sub-periods 1977-1980 and 1984-1988, during which the economy is moving from excess demand to excess supply of money, coincide with the occurrence of the coffee booms of the second half of the seventies and mid eighties. During the subsequent phases of falling prices, i.e. between 1981-1983 and 1989-1992, EMS moves in the opposite direction.

---

24Both series were deflated by the CPI of the United States.
Another interesting aspect to be noticed is that between 1977-1980 the estimated monetary disequilibria were considerably greater than those observed during the coffee boom of the mid eighties. This result is consistent with the fact that the former boom was more intense and lasted longer than the latter. At the peak of the boom of the seventies (in April 1977) the real price of coffee reached US$2.91
per pound, which is not only greater than the maximum level reached during the boom of 1986, but also the highest level observed since the 1940's. Despite that during the boom of the seventies the phase of falling prices began around 1977, with a short interruption in 1979, the dynamism of coffee exports extended the phase of high revenues until 1980 (see e.g. Steiner 1983, Ocampo 1989, and Wunder 1991).

We now formally address whether there is a direct association between EMS and the evolution of the coffee market. In order to do this, we follow Edwards by postulating that the monetary disequilibria depend upon a coffee variable and a fiscal variable. The coffee variable is measured in two alternative ways. First, we consider the logarithm of the price of coffee (LCP), which is the traditional proxy of coffee boom. This variable is expressed in constant dollars of 1975 to account for the erosion of coffee purchasing power due to U.S inflation, as measured by this country's CPI. Second, we consider the logarithm of coffee exports (LCX), again expressed in constant dollars of 1975, where this variable is selected on the grounds that the coffee boom of the seventies had elements of both price and quantity booms (see e.g. Wunder, 1991). Concerning the fiscal variable, we aim to establish whether EMS responds to current or previous disequilibria in government finances; in this case, we use the fiscal surplus of the central government as a proportion of GDP, which we denote FS.

Given that the estimated measure of excess money supply is a stationary series, the econometric analysis that follows is performed in I(0) space. Accordingly, it is very important to establish the order of integration of the coffee and fiscal

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24 The source of the price of coffee is Banco de la República (1993).
25 The source of the series of coffee exports is the National Federation of Coffee Growers.
26 The data were obtained from various issues of the Revista del Banco de la República for the period 1970-1979, and from Ramos and Rodriguez (1995) for the period 1980-1992. The revenue side of the fiscal balance excludes revenues from the Cuenta Especial de Cambios.
variables, so that we deal with a balanced equation. In order to investigate the order of integration of the series, we first inspect their plots and correlogram, and then we perform the more formal tests for unit roots.

Regarding the graphical evidence, in Figures 4.16 and 4.17 we plot the levels and first differences of LCP, LCX and FS, as well as their autocorrelation functions, respectively. As can be noticed, LCP exhibits a slight downward trend, particularly visible since the mid seventies, suggesting that the series may be nonstationary in levels; the correlogram of the series decays slowly as the lag length increases, which also supports this conclusion. The periods of boom, which were associated with weather problems in Brazil, the major world coffee producer, can be easily seen by noting the sharp increases in the price of coffee during the years 1976-1978 and 1985-1986. Plots of ΔLCP show no evidence of a changing mean and there are wider fluctuations, particularly at the time of the two coffee booms.

In the case of LCX, the series shows a slight downward trend since the late seventies suggesting that LCX may be nonstationary in levels; this conclusion appears to be confirmed by inspecting the correlogram of the series. Similar to LCP, the periods of boom can be easily identified, and in this case the seventies coffee boom extends until 1980.28 A plot of ΔLCX shows no evidence of a changing mean with the series fluctuating around zero, and exhibiting considerable fluctuations during the periods of boom.

Lastly, FS does not exhibit a defined trend over the period under review, but the series fluctuates around a mean of -0.013. During the early 1970s government’s finances deteriorate, basically due to growing expenditures, and the fiscal deficit

reaches a maximum level of approximately 4% of GDP in 1972; after this year the magnitude of the deficit gradually reduces and the government eventually generates fiscal surpluses after 1976. In the early eighties there is once again a continuous deterioration of government’s finances, a situation that is corrected around 1984 with the introduction of a successful adjustment programme. During the late eighties and early nineties the government runs fiscal deficits, although their magnitude does not increased throughout the years. Hence, it is not clear that the series contains a unit root although it certainly does not revert to its mean very quickly. The correlogram of FS shows large autocorrelation coefficients at seasonal lags, suggesting that the series may follow a seasonal AR(1) process. ΔFS fluctuates around zero and does not present evidence of a changing mean; nonetheless, when one inspects the correlogram of ΔFS there seems to be some evidence that the series may have been overdifferenced, since the first autocorrelation coefficient is significant and close to -0.5 (i.e. -0.38).
Figure 4.16: The price of coffee, coffee exports and fiscal surplus in Colombia - Levels and first differences of the series
Figure 4.17: Correlograms for the price of coffee, coffee exports, fiscal surplus and their first differences
On the other hand, the results of the Dickey and Fuller tests, reported in the third panel of Table 4.1, indicate that LCP, LCX and FS may contain a unit root with a zero drift term. Thus, both the graphical evidence and the tests for unit roots suggest that LCP and LCX may contain a unit root during the period under review, although we obtain conflicting results concerning FS. In the case of the latter, however, the presence of a unit root implies that the series increases without bound, which is unlikely to occur in reality; therefore, we treat FS as an I(0) series based on the evidence provided by its sample autocorrelation function.

Once we have established the order of integration of the coffee and fiscal variables, we proceed to assess whether EMS depends upon them. In particular, we use two different autoregressive-distributed lag (ADL) models whose initial lag length was set equal to 4; more formally:

Model 1: \[ EMS = \alpha_0 + \sum_{i=1}^{4} \alpha_i EMS_{t-i} + \sum_{j=0}^{4} \beta_j \Delta LCP_{t-j} + \sum_{j=0}^{4} \gamma_j FS_{t-j} \]

and

Model 2: \[ EMS = \delta_0 + \sum_{i=1}^{4} \delta_i EMS_{t-i} + \sum_{j=0}^{4} \xi_j \Delta LCX_{t-j} + \sum_{j=0}^{4} \theta_j FS_{t-j} . \]

The results of estimating Model 1 are reported in the top half of Table 4.4.

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\*We do not consider a unique relationship incorporating both \( \Delta LCP \) and \( \Delta LCX \) in order to avoid multicollinearity problems.
Table 4.4: Determinants of the Excess Money Supply

<table>
<thead>
<tr>
<th>Lag</th>
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<th>3</th>
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<td></td>
<td>EMS</td>
<td>0.873</td>
<td>-0.255</td>
<td>0.182</td>
<td>-0.051</td>
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<td></td>
<td></td>
<td>(0.117)</td>
<td>(0.155)</td>
<td>(0.156)</td>
<td>(0.119)</td>
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<tr>
<td></td>
<td>ΔLCP</td>
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<td>0.014</td>
<td>-0.041</td>
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<tr>
<td></td>
<td></td>
<td>(0.025)</td>
<td>(0.026)</td>
<td>(0.025)</td>
<td>(0.025)</td>
</tr>
<tr>
<td></td>
<td>FS</td>
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<td>0.127</td>
<td>0.168</td>
<td>-0.576</td>
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<tr>
<td></td>
<td></td>
<td>(0.254)</td>
<td>(0.192)</td>
<td>(0.196)</td>
<td>(0.269)</td>
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<td>R²</td>
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<td>LM [4]</td>
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<td>ARCH [4]</td>
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<td>RESET</td>
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Model 2

<table>
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<td>EMS</td>
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<td>(0.118)</td>
<td>(0.156)</td>
<td>(0.157)</td>
<td>(0.120)</td>
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<tr>
<td>ΔLCP</td>
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<td>-0.010</td>
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<td>(0.016)</td>
<td>(0.016)</td>
<td>(0.016)</td>
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<tr>
<td>FS</td>
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<td>0.093</td>
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Notes:
Standard errors in parentheses.
As expected, the equation is overparameterised so that in order to obtain a more parsimonious representation we exclude some of the regressors based on Wald tests for zero restrictions. In particular, we first exclude $\text{EMS}_{t-2}$, $\text{EMS}_{t-3}$, $\text{EMS}_{t-4}$, $\Delta \text{LCX}_{t-3}$, $\Delta \text{LCX}_{t-1}$, $\text{FS}_{t-1}$, $\text{FS}_{t-2}$ and $\text{FS}_{t-3}$, and then $\Delta \text{LCX}_{t}$ and $\Delta \text{LCX}_{t+1}$. From the resulting equation we observe that the estimated coefficients of $\text{FS}_{t}$ and $\text{FS}_{t+4}$ have similar magnitudes and opposite signs, so that we impose this restriction which is easily accepted. The final form of Model 1 is:

$$\begin{align*}
\text{EMS} &= -0.0004 + 0.775 \text{EMS}_{t-1} + 0.028 \Delta \text{LCX}_{t-1} + 0.347 \Delta_{4} \text{FS}_{t} \\
&= \text{[4.4]} \\
R^2 &= 0.597 \\
\text{ARCH}[4] &= F_{4.75} 0.707 \\
\text{Hetero} &= F_{6.76} 1.189
\end{align*}$$

where $\Delta_{4}$ denotes the fourth difference operator (standard errors in parentheses).

On the other hand, in the bottom half of Table 4.4 we report the results of estimating Model 2. Similar to the previous case, the first estimation of the model produces various coefficients that are not significant, so that we proceed to exclude them. More specifically, in a first step we exclude $\text{EMS}_{t-3}$, $\text{EMS}_{t-4}$, $\Delta \text{LCX}_{t-1}$, $\Delta \text{LCX}_{t-2}$, $\Delta \text{LCX}_{t+3}$, $\text{FS}_{t-1}$, $\text{FS}_{t-2}$ and $\text{FS}_{t-3}$, and then $\text{EMS}_{t-2}$. A further simplification is possible by noting that the estimated coefficients of $\text{FS}_{t}$ and $\text{FS}_{t+4}$, and $\Delta \text{LCX}_{t}$ and $\Delta \text{LCX}_{t+4}$, have similar magnitudes and opposite signs (the $F$-test of this last simplification is easily accepted). The resulting final form of Model 2 is (standard errors in parentheses):

$$\begin{align*}
\text{EMS} &= -0.001 + 0.777 \text{EMS}_{t-1} + 0.025 \Delta_{4} \Delta \text{LCX}_{t} + 0.381 \Delta_{4} \text{FS}_{t} \\
&= \text{[4.5]} \\
R^2 &= 0.616 \\
\text{ARCH}[4] &= F_{4.75} 0.570 \\
\text{Hetero} &= F_{6.76} 0.923
\end{align*}$$
As can be seen from equations [4.4] and [4.5], the R^2's indicate that approximately 60% of the variance in EMS is accounted for by the regressions. Both regressions pass diagnostic tests for residual serial correlation of up to fourth order, ARCH of up to fourth order, heteroscedasticity, functional form misspecification (i.e. Ramsey's RESET test) as well as the test for normality.

The estimated coefficient on the coffee variable has the expected positive sign in the two equations, although in equation [4.4], when using the price of coffee, it is not statistically different from zero at traditional significance levels. In fact, the better econometric results are obtained using coffee exports, which suggests that it is relevant to consider both price and quantity elements when analysing the monetary consequences of export booms. The results thus provide evidence in favour of a direct association between the coffee variable and excess money supply, via foreign exchange accumulation. The main policy implication of this result, is that it shows that external disturbances have important short-run monetary effects, which jeopardises the ability of the economic authorities to carry out successful monetary policy.

In these terms, our findings suggest that in the short run the economic authorities were unable to fully sterilise the foreign exchange of the export booms. Recently Kamas (1995) investigated the effects of monetary policy under the crawling peg system in Colombia during the period 1975-1989. Kamas found, based on the estimation of vector autoregressions, that the five-year cumulative effect on domestic credit of a transitory change in foreign reserves varies from -0.77 to -0.88, depending on the criterion used to select the lag length of the VARs. A sterilisation coefficient of this magnitude, indicates that in the medium term the economic
authorities have been relatively successful in their aim of insulating the economy from the monetary effects of coffee export booms.\footnote{In a previous exercise, Kamas (1985) estimated a static regression of changes in domestic credit on changes in foreign reserves and other regressors, and found a sterilisation coefficient equal to -0.92. She then concluded that "...a sterilisation coefficient of this magnitude suggests that the authorities have been able to insulate the economy from the monetary effects of reserve flows..." (p.325). However, considerable care should be taken when interpreting this result, as the estimated regression is seriously misspecified; the reason is that there is evidence of serial correlation, and when Kamas attempts to correct it, she obtains an estimated autocorrelation coefficient equal to -0.99. Ocampo (1989) estimates for the period 1975-1987 a static regression of changes in domestic credit on changes in foreign reserves, and finds a sterilisation coefficient of -0.83; however, he does not report any diagnostic tests, so that it is not possible to comment about the specification of the regression.}

With regard to the fiscal variable, previous findings by Edwards (1983, 1984, 1985, 1986a, 1986b) imply the monetarist view that increasing fiscal deficits (i.e. $FS$) lead to excess money supply. However, the positive sign on the estimated coefficient of $FS$ suggests that, during the period under review, an increase in the government’s fiscal surplus, accomplished either through an increase in taxes or through a reduction in expenditures, decreases output, which in turn diminishes the demand for real balances, and leads to a less than proportional increase in the monetary disequilibrium. In this sense, our results indicate that during the period 1970-1992 the monetarist link between fiscal deficits and excess money supply does not hold for Colombia.

Concerning the estimated coefficient on $EMS_{t-1}$, it is approximately equal to 0.77 which yields a relatively small speed of adjustment parameter (i.e. 0.23). In Figure 4.18 we present actual and fitted values of $EMS$, when Model 2 is estimated; as can be observed from the figure, the actual and fitted plots are reasonably coincident.
As an additional exercise, we examine the possibility that Models 1 and 2 are non-linear, in the sense that the estimated coefficients differ depending upon the kind of monetary disequilibrium; if this is the case, it may occur, for instance, that the speed of adjustment is not the same for points above and below equilibrium. In order to allow for non-linearities, we define a dummy variable DX that takes the value of 1 when EMS$_{t-1}$ is positive and 0 otherwise, and then we create additional terms corresponding to the interaction of DX with the other regressors that are present in [4.4] and [4.5]. Next, we test whether these additional terms are statistically different from zero, in which case the linear specification should not be adopted. Results not reported here indicate that these additional terms are not significant at the five per cent significance level in either [4.4] or [4.5], so that it is valid to formulate the two models linearly; these results suggest that the estimated coefficients do not depend upon the type of monetary disequilibrium of the previous period.
4.3.6 The Role of the Illegal Economy

As indicated in chapter 2, during the last three decades the Colombian economy not only experienced coffee booms, but also a boom in the illegal economy. In this section we attempt to examine whether drug revenues have also been one of the determinants of the disequilibrium in the money market. Accordingly, we postulate that the monetary disequilibrium depends upon coffee exports, the fiscal surplus of the central government, and foreign exchange earnings from the exportation of illicit drugs.

Given data availability on the possible size of drug revenues, the analysis is performed using annual data for the sample period 1980-1992. We use Steiner's (1996) annual series on the net income received by Colombians from the exportation of cocaine, marijuana and heroin (see Figure 4.19).\textsuperscript{31} The estimates of Steiner suggest that during the first half of the 1980s, when drug trading was not intensely persecuted neither nationally nor internationally, drug revenues were of considerable importance accounting for approximately 6.5% of GDP. Since 1986, following a more intense persecution of drug dealers, the share of drug revenues fell to 4.9% as a percentage of GDP.\textsuperscript{32}

\textsuperscript{31}In the case of cocaine, transport costs and imported inputs have been deducted. Gómez (1990) and Kalmanovitz (1990) also present time series data of the drug revenues; however, we use Steiner's data because they cover a longer sample period.

\textsuperscript{32}Steiner estimates that in 1994-1995 this share was even lower (approximately 3.5%).
The outcome of estimating a static regression of the excess money supply $EMS'$, as measured by the yearly average of EMS, on drug revenues as a proportion of GDP (DREV), and coffee exports, by least squares is (standard errors in parentheses):

$$EMS' = -0.108 - 0.008 \text{DREV} + 0.027 \text{LCX}$$

(0.123) (0.003) (0.022)

<table>
<thead>
<tr>
<th>Obs</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>R2</td>
<td>0.454</td>
</tr>
<tr>
<td>DW</td>
<td>1.490</td>
</tr>
</tbody>
</table>

The fiscal surplus was not included in the regression as the coefficient on this variable was not significant.\(^3\) The coefficient on drug revenues is negative and statistically different from zero; this negative sign gives support to the view that the income generated in the illegal economy increases the demand for money which, assuming that the money supply is given, reduces the extent of the excess supply in

\(^3\)We also tried to estimate a dynamic version of the model including the first lag of $EMS'$, DREV and LCX, although the coefficients on these variables were not significant.
that market. The coefficient on coffee exports is once again positive and statistically different from zero at the 12 per cent significance level (based on a one-sided t-test).

In summary, these findings suggest that drug revenues have been one of the determinants of the disequilibrium in the money market. However, it is also worth mentioning that the results are subject to some criticisms. Firstly, the regression analysis was performed using a small number of observations. Secondly, the effect of drug revenues may be overstated, because we are using an estimate of the foreign exchange received from the exportation of illegal drugs, which implicitly assumes that the totality of this revenue is repatriated. Finally, it is very likely that the data used to construct DREV are subject to measurement errors, and some of the assumptions used in the calculation of this series may be subject to criticisms.

4.4 CONCLUDING REMARKS

In this chapter we have assessed whether the coffee booms of the 1970s and 1980s led to excess money supply in the Colombian economy. The empirical analysis was implemented in two steps. First, we obtained a measure of excess money supply as the residuals from a long-run money demand equation, assuming that money supply is given. Second, we related this measure of excess money supply to a coffee variable and a fiscal variable.

As far as the money demand modelling exercise is concerned, we found evidence of a long-run money demand equilibrium relationship that satisfies both long-run price homogeneity and scale economies in the holding of money. We also found that the price level, the scale variable, and the proxy for the opportunity cost of holding money can be regarded as weakly exogenous for the estimation of the
parameters of the money demand equation, so that it is valid to formulate a one equation model for M1 conditioning on the other variables.

Turning to the second part of the analysis, we found evidence of a direct association between real coffee exports and the estimated measure of excess money supply. The fact that we obtained better econometric results when using coffee exports, rather than e.g. other variables such as the price of coffee, suggests that when analysing the monetary consequences of export booms, it is important to consider both their price and quantity elements. The main policy implication of this result is that during the period 1970-1992, external disturbances had important short-run monetary effects, which jeopardised the ability of the economic authorities to carry out successful monetary policy. Our results also indicated that the monetarist link between fiscal deficits and excess money supply does not hold for Colombia, at least during the period under review.

Finally, we also tested an alternative specification for the excess money supply incorporating coffee exports and drug revenues for the sample period 1980-1992. These variables turned out to be significant and with the expected sign in our regression analysis.
APPENDIX 4.1

DATA SET USED IN THE MONEY DEMAND MODELLING EXERCISE

We use the traditional definition of M1 as monetary aggregate, that is currency plus demand deposits, and the consumer price index as an appropriate measure of the price level in the economy; the source of these two series is Banco de la República (1993). The proxy of the opportunity cost of holding money is constructed by combining two different interest rate series: from 1970:1 to 1980:1 the yield of 120-day CAT (certificados de abono tributario) certificates, and from 1980:2 to 1992:4 the yield of 90-day COT (certificado de deposito a término) certificates offered by banks and financial corporations; the sources of the two series are Toro (1987) and Banco de la República (1993), respectively.

With reference to the scale variable, we use the GDP series constructed by the National Department of Planning; the source of the series is Cubillos and Valderrama (1993) for the period 1980-1992, and worksheets of the National Department of Planning for the period 1975-1979. For the period 1970-1974 the series is estimated following Guerrero (1989), who provides a framework for obtaining optimal conditional ARIMA forecasts, and also considers the problem of testing whether they are compatible with the historical data.

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3CAT certificates were created in 1967 by Decree Law 444 in order to replace previous fiscal incentives granted to minor exporters. These certificates, originally set at 15% of the export value, could be used after one year of their date of issue for paying taxes and were freely negotiable in the stock market.

3Banco de la República (1993) also publishes two alternative measures of interest rates: the yield of 90-day CDT certificates offered by private financial corporations, and the yield of 90-day CDT certificates offered by banks, financial corporations, and other financial institutions. These series exhibit the same behaviour of the interest rate series we use, but are only available from 1984.

3We do not use the optimal forecasts resulting from fitting an ARIMA model, as we face the problem of constraining them so that they add up annual GDP.
Regarding the implementation of Guerrero's methodology, and without entering into its formal aspects, we initially fit an ARIMA model for the GDP series obtaining optimal forecasts for the next four quarters. Then we introduce the restriction that the four quarters must add up annual GDP obtaining optimal conditional forecasts. Lastly we test the hypothesis that both forecasts are compatible, which if not rejected permits us to use the conditional forecasts as input for repeating the process and obtaining those of the next period, and so on.\(^{37}\)

In practice, we use information for the period 1975-1985 in order to estimate the optimal conditional forecasts for the period 1970-1974.\(^{38}\) After examining the autocorrelation and partial autocorrelation functions of the GDP series, we end up with the following ARIMA specification: \((1-\phi L)(1-L)(1-L^4)\text{GDP}_t=(1-\theta L^4)\varepsilon_t\), where \(L\) denotes the lag operator (i.e. \(LY_t=Y_{t-1}\)). This specification results in uncorrelated random disturbances and minimises both Akaike's Information Criterion and Schwarz's Bayesian Criterion. The resulting conditional forecasts appear to be consistent with the historical data as the compatibility tests were not rejected at traditional significance levels.\(^{39}\)

\(^{37}\)If, on the contrary, the hypothesis is rejected Guerrero (1989) proposes to include a random term in the restriction. However, we will see below that this was not necessary in our case.

\(^{38}\)The observations for the period 1986-1992 are not included as there is a change in the seasonal pattern of the series.

\(^{39}\)Both the optimal conditional ARIMA forecast and the compatibility test were performed using the software package SAS based on a routine kindly provided by Martha Misas. See Melo and Oliveros (1991) for an application of the same methodology to the case of the inflation rate in Colombia.
CHAPTER 5

THE DETERMINANTS OF THE REAL EXCHANGE RATE IN
COLOMBIA

5.1 INTRODUCTION

Economists generally agree that the real exchange rate (RER) is a key relative price in the economic system. Through its changes, the RER affects the flows of foreign trade, the current account balance, the level and composition of production and consumption, the allocation of resources, and employment. Being a relative price, and unlike the nominal exchange rate, that in some countries constitutes a policy instrument, the RER is an endogenous variable that responds to exogenous shocks and policy induced disturbances. In this sense, it is particularly relevant to model the behaviour of the RER, in order to understand how it is determined in the short and long run.

There are three major models, in the monetary approach, to exchange rate determination. These are the flexible price model of Frenkel (1976), Mussa (1976) and Bilson (1978a, 1978b), the sticky price model of Dornbush (1976), and the real interest rate differential model of Frankel (1979). The distinctive feature of these models is that the demand and supply for money are the key determinants of the exchange rate: factors that increase the demand for money will appreciate the exchange rate, whereas an increase in the domestic money supply will depreciate it.

Meese and Rogoff (1983a, 1983b) have tested the out-of-sample forecasting accuracy of monetary models of exchange rate determination, and time series
exchange rate models.¹ They find that for major exchange rates against the dollar, the random walk model outperforms the monetary models at one-to-twelve-month horizons, despite the fact that the forecasts of the latter are based on actual realised values of the explanatory variables. In a subsequent paper, Meese and Rogoff (1988) investigate the empirical relationship between major currency real exchange rates and real interest rates over the recent floating-rate period, and fail to find any evidence of cointegration. Similar results are obtained by Edison and Pauls (1993), even after allowing for other variables, such as the cumulated current account balance, that may affect the real exchange rate in the long run.²

A second strand of literature identifies variations in the terms of trade, as another major determinant of the RER: Dornbusch (1980), Neary (1988), Ostry (1988), Edwards (1989a) and De Gregorio and Wolf (1994). This literature has been developed to understand the process of determination of the RER in developing countries, which have been historically subjected to substantial changes in the prices of the goods they export and import. Changes in the terms of trade can be expected to have sectoral resource reallocation effects on the supply side of the economy; also, because terms of trade changes affect a country’s real income, they can be expected to have demand-side effects. Among these models, the Edwards model is perhaps the most comprehensive. Indeed, according to this theoretical framework, real factors (referred to as “fundamentals”) and macroeconomic policies affect the RER in the short run, but in the long run only real factors affect the sustainable equilibrium level of the RER.

¹See Levich (1985) and Pilbean (1992, chapter 9) for a literature review of empirical studies.  
²Baxter (1994) argues that prior studies have not found a statistical link between real exchange rates and real interest rate differentials, because they have focused on the high-frequency component of the data. In her analysis, Baxter finds evidence of correlation between these two variables at trend and business cycle-frequencies, but not at high frequencies.
In this chapter, we build a RER determination model for Colombia based on the Edwards model. The use of the Edwards model for Colombia may be justified on the grounds that it captures the role of variables (such as terms of trade, government expenditure, import tariffs and capital controls) that previous studies have found help to explain the behaviour of the RER.

The implications of this theoretical framework have been tested by Edwards (1989a) and Elbadawi (1994): Edwards in the context of a partial adjustment model, using pooled data for a group of 12 developing countries including Colombia, and Elbadawi using cointegration analysis for the cases of Chile, Ghana and India. In contrast to Edwards, we follow Elbadawi’s use of cointegration analysis and interpret the deviations of the RER from its long-run equilibrium relationship, after correcting for the short-run dynamics, as a measure of RER misalignment. Elbadawi uses Engle and Granger’s two step procedure, and in doing so implicitly assumes that there is only one cointegrating vector and that all variables, except the RER, are weakly exogenous for the estimation of the parameters of the long-run RER equilibrium relationship. Under this approach, if the assumption of weak exogeneity does not hold for some of the variables, then a single-equation RER model is no longer appropriate, since there is information to be gained from estimating the other equations in the system. We therefore use Johansen’s maximum likelihood analysis of cointegrated systems which, in a multivariate context, allows us to determine and estimate all possible cointegrating vectors, and test for weak exogeneity.

1Ghura and Grennes (1993) also use pooled data for 33 countries in Sub-Saharan Africa.
In the second place, unlike Edwards and Elbadawi, we use neither moving averages nor the Beveridge-Nelson decomposition to correct for short-run dynamics, because the first method involves the loss of observations and the second one can not always be applied. Instead, we use the alternative method proposed by Johansen and Juselius (1992). Another interesting aspect of our modelling exercise is that the performance of the estimated model is evaluated with a simulation for the period of estimation, and a simulation three years into the future. Likewise, we perform a policy experiment in order to examine what might have happened to the RER as a result of alternative policies.

The outline of the chapter is as follows. In section 2 we state the Edwards model of RER determination, which constitutes the theoretical framework of our modelling exercise. In section 3 we examine alternative definitions of the RER, and discuss how empirical counterparts to the concept can be constructed. In section 4 we present our RER modelling exercise for Colombia which includes: a) the formulation of a model to find the determinants of the RER in the short and long run; b) the estimation of a measure of exchange rate misalignment; and c) the evaluation of the performance of the model, in terms of its ability to predict the behaviour of the RER during the period of estimation, and forward in time beyond the estimation period. Finally, section 5 offers some concluding remarks.

5.2 A MODEL OF THE REAL EXCHANGE RATE

This section introduces the Edwards model, which analyses the process of determination of the equilibrium RER, and how it reacts to a series of real
disturbances such as terms of trade shocks, imposition of import tariffs and changes in government expenditure. The concept of equilibrium RER used by Edwards, is closely related to that of “fundamental equilibrium exchange rate” (FEER) proposed by Williamson (1983). In particular, the equilibrium RER in a particular period is defined as the relative price of tradables to nontradables that, for given sustainable (equilibrium) values of other variables, such as taxes, international prices, and technology, equilibrates simultaneously the internal and external sectors.

The Edwards model is an intertemporal (two-period) general equilibrium model for a small open economy. Throughout the analysis we occasionally refer to period 1 as the present and period 2 as the future (a tilde – over a variable indicates a period 2 variable). The model starts off with an economy consisting of three agents: a representative consumer, a representative firm and the government. The first two are assumed to be optimising agents, whereas the government, a non-optimising agent, is subject to an intertemporal budget constraint. The model assumes that the economy produces nontradable (N), exportable (X) and importable (M) goods under conditions of perfect competition, and constant returns to scale; importables are subject to a tariff both in periods 1 and 2. In addition, there is perfect foresight and full employment, prices are flexible, and agents can borrow or lend in international markets; there is investment and it is also assumed that the labour force does not grow. In further developments of the model, some of these assumptions are relaxed; for instance, price rigidities are incorporated in the analysis. The model is real, so that monetary disturbances are not considered.

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*See Edwards (1989b, 1989c) for simplified versions of the model.*
The production side of the model is given by revenue functions in periods 1 and 2, denoted \( R \) and \( \tilde{R} \), which give us the maximum revenue that firms obtain from producing \( X \), \( M \) and \( N \), subject to the existing technology; more formally, the revenue functions for periods 1 and 2 are defined as:

\[
R = R(p, q, V)
\]

and

\[
\tilde{R} = \tilde{R}(\tilde{p}, \tilde{q}, \tilde{V}),
\]

respectively, where \( p \) (\( \tilde{p} \)) is the domestic price of importables relative to exportables in period 1 (2); \( q \) (\( \tilde{q} \)) is the price of nontradables relative to exportables in period 1 (2); and \( V \) (\( \tilde{V} \)) is a vector that summarises the factors of production in period 1 (2).

From the revenue functions it is possible to obtain the corresponding supply functions by taking the derivative with respect to their prices (i.e. Hotelling's lemma). Hence, the supply functions of \( M \) and \( N \) in periods 1 and 2 are:\(^5\)

\[
\frac{\partial R}{\partial p} = R_p = Q_M(p, q),
\]

\[
\frac{\partial R}{\partial q} = R_q = Q_N(p, q),
\]

\[
\frac{\partial \tilde{R}}{\partial \tilde{p}} = \tilde{R}_p = \tilde{Q}_M(\tilde{p}, \tilde{q}),
\]

and

\[
\frac{\partial \tilde{R}}{\partial \tilde{q}} = \tilde{R}_q = \tilde{Q}_N(\tilde{p}, \tilde{q}),
\]

---

\(^5\)To simplify notation we assume that revenues in periods 1 and 2 are function of prices alone.
where the signs over the variables reflect the assumptions that the supply functions are upward sloping and, assuming that the three goods compete for a given amount of factors of production, have negative cross price elasticities.

Turning to the consumers' choice problem, preferences are represented by a (time separable) utility function with current and future consumption of X, M and N as arguments; that is,

\[ W' U(C_X, C_M, C_N), \bar{U} (\bar{C}_X, \bar{C}_M, \bar{C}_N) \],

where \( W \) is the intertemporal utility function; \( U (\bar{U}) \) denotes the subutility function in period 1 (2); and \( C_X, C_M \) and \( C_N \) (\( \bar{C}_X, \bar{C}_M \) and \( \bar{C}_N \)) represent the consumption of the three goods in period 1 (2). The consumers' lifetime budget constraint is given by

\[ C_X + pC_M + qC_N + \delta' (\bar{C}_X + \bar{p}\bar{C}_M + \bar{q}\bar{C}_N) \leq \text{Wealth}, \]

where \( \delta' \) is the world discount factor which is equal to \((1 + r^* )^{-1}\), for \( r^* \) the world real interest rate, and wealth is the present value of consumers' income in periods 1 and 2; income in turn includes labour and capital income as well as transfers from the government. On this basis, the consumers' problem can be represented by the expenditure function

\[ E = E(p, q, \delta p, \delta q; W), \]

which gives us the minimum cost of achieving a fixed level of utility. From the expenditure function Hicksian (or compensated) demand functions can be obtained by taking the derivative of \( E \) with respect to prices. More formally, the Hicksian demand functions of M and N in periods 1 and 2 are given by:

\[
\frac{\partial E}{\partial p} = E_p = D_M \left( p, q, \bar{p}, \bar{q}; W \right),
\]

\[
\frac{\partial E}{\partial q} = E_q = D_N \left( p, q, \bar{p}, \bar{q}; W \right),
\]
\[ \frac{\partial E}{\partial p} = E_p = \tilde{D}_m(p,q,\tilde{p},\tilde{q};W), \]

and

\[ \frac{\partial E}{\partial q} = E_q = \tilde{D}_m(p,q,\tilde{p},\tilde{q};W). \]

The resulting compensated demand functions are downward sloping and, given the assumption of time separability, all intertemporal price effects are positive. Based on the previous equations, a simplified version of the Edwards model, in which there is neither government consumption nor investment decisions, is given by the following set of equations:

\[ R(p,q;V) + \delta' \tilde{R}(p,\tilde{q};\tilde{V}) + \tau(E_p - R_p) + \delta'\tilde{\tau}(E_p - R_p) = E(p,q,\delta'\tilde{p},\delta'\tilde{q};W) \]  

[5.1a]

\[ R_q = E_q \]  

[5.1b]

\[ \tilde{R}_q = E_q \]  

[5.1c]

\[ p = p + \tau \]  

[5.1d]

and

\[ \tilde{p} = \tilde{p} + \tilde{\tau}. \]  

[5.1e]

Equation [5.1a] states that the present value of revenues from production plus tariffs collection, must be equal to the present value of expenditure; given that there is no investment in the model, this equation represents the equilibrium condition in the external sector, with the current account defined as the difference between income and expenditure. Equations [5.1b] and [5.1c], on the other hand, represent the equilibrium condition in the nontradable goods market in periods 1 and 2, respectively. Lastly, equations [5.1d] and [5.1e] relate the domestic price of importables with the world price of imports and tariffs in periods 1 and 2, respectively. In the model, there are two real exchange rates in each period: the
relative price of importables relative to nontradables \((p/q\) and \(\tilde{p}/\tilde{q}\)), and the inverse of the relative price of nontradables relative to exportables \((1/q\) and \(1/\tilde{q}\)). To simplify the analysis, we focus on the second measure, with an increase (a decrease) in either \(q\) or \(\tilde{q}\) representing a RER appreciation (depreciation).

The equilibrium RER in periods 1 and 2, for given world prices, tariffs, and existing technology, corresponds to that relative price that equilibrates the internal and external sectors. In Figure 5.1 we depict the equilibrium conditions in the nontradable goods market in the two periods.⁶

**Figure 5.1:** Determination of time path of equilibrium RER and the effect of the imposition of a temporary import tariff

The locus \(H\) indicates the combination of \(q\) and \(\tilde{q}\) for which the nontraded goods market is in equilibrium in period 1; its slope is equal to:

\[
\frac{dq}{d\tilde{q}} = \frac{E_{q\tilde{q}}}{R_{qq} - E_{qq}} > 0,
\]

provided the intertemporal cross price effect (i.e. \(E_{q\tilde{q}}\)) is positive. Intuitively, an increase in \(\tilde{q}\) causes substitution effects in favour of current consumption of

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⁶Neary and van Wijnbergen (1986) use this type of diagram to analyse the effects of a resource discovery on current and future RERs.
nontradables, that in turn creates an incipient excess demand for such goods in period 1; thus, q will have to increase in order to restore equilibrium in that market. On the other hand, the locus \( \bar{H} \) depicts the combination of \( q \) and \( \bar{q} \) for which the nontradable goods market is in equilibrium in period 2; its slope is equal to

\[
\frac{dq}{d\bar{q}} = \frac{\bar{R}_{q\bar{q}} - E_{q\bar{q}}}{E_{q\bar{q}}} > 0,
\]

and the intuition for the positive slope of \( \bar{H} \) is also related to the assumption of intertemporal substitution in consumption, i.e. \( E_{q\bar{q}} > 0 \). Furthermore, it is possible to show that stability of the system requires the locus \( \bar{H} \) be steeper than the locus \( H \).

The interaction of the \( H \) and \( \bar{H} \) loci determine a set of equilibrium RERs (depicted as point A in Figure 5.1), for given world prices, tariffs, and existing technology. The next important aspect is to analyse how these determinants affect the equilibrium RERs. In his analysis, Edwards considers the effects of an anticipated future increase in import tariffs, a permanent terms of trade shock and, after incorporating the public sector, a change in the composition of government consumption, among others. In what follows, we will illustrate the functioning of the model by looking at how the anticipation of the imposition of import tariffs in the future affects the equilibrium RERs; then, we will briefly describe the effect of changes in other determinants. To simplify the analysis, we assume that at the initial equilibrium there are no tariffs in either period; this assumption allows us to concentrate on substitution effects ruling out first-order income effects. In terms of Figure 5.1, it is possible to show that the policy measure in question will shift both the \( H \) and \( \bar{H} \) loci by a magnitude equal to:

Formally, the stability condition is \( (R_{q\bar{q}} - E_{q\bar{q}})(\bar{R}_{q\bar{q}} - E_{q\bar{q}}) - E_{q\bar{q}}E_{q\bar{q}} > 0 \).
\[
dq = \frac{E_{q\hat{p}}}{R_{q\hat{q}} - E_{q\hat{q}}} d\hat{\tau} \]

\[
\bar{dq} = \frac{E_{q\hat{p}} - \tilde{R}_{q\hat{q}}}{R_{q\hat{q}} - E_{q\hat{q}}} d\hat{\tau}.
\]

respectively. As can be seen, provided there is intertemporal substitutability in the consumption of the goods, the $H$ schedule will shift upwards to $H'$, and the $\hat{H}$ schedule will shift to the right to $\hat{H}'$, although there is no certainty as to which of the two schedules shifts by more.\(^8\)

At the new equilibrium, let us say point B, the price of nontradables relative to exportables in periods 1 and 2 will be higher than before the imposition of the anticipated import tariff (i.e. the policy measure in question will cause a RER appreciation). From an economic perspective the imposition of the tariff makes future consumption of importables more expensive; this, in turn, makes consumers substitute away from these goods into nontradables, in the present and future, leading to an incipient excess demand for nontradables. Thus, there will be an increase in the relative price of nontradables in both periods in order to clear that market.

Regarding the imposition of either a temporary or a permanent import tariff, it is also possible to show that they appreciate the equilibrium RER, provided the assumption of intertemporal substitution in consumption holds; in fact, an interesting result that emerges from the analysis is that the imposition of a permanent tariff will appreciate the equilibrium RER in period 1 by more than the imposition of a temporary tariff. On the other hand, if the changes in tariffs are accomplished when

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\(^8\)In his original formulation, Edwards assumes that a 45° line passes through the initial equilibrium point A. Given that it is not possible to determine which of the two schedules shifts by more, it may
the tariffs are initially greater than zero, then the associated first-order income effects may compensate the substitution effects. Thus, a temporary increase in import tariffs may result in an equilibrium RER depreciation.

The effect of a terms of trade deterioration (whether temporary, anticipated or permanent) on the equilibrium RERs, can be decomposed into substitution and income effects. For instance, a temporary terms of trade worsening has a positive substitution effect on the equilibrium RERs, which is identical to that of imposing a temporary import tariff. The reason is that the terms of trade worsening can be viewed as an increase in the domestic price of imports (due to an increase in their world price), which in turn makes consumers alter their consumption. On the other hand, the temporary terms of trade worsening has a negative income effect on the equilibrium RERs, which in turn generates downward pressure on the price of all goods, including nontradables. A priori it is not possible to determine which of the two effects dominates; nonetheless, assuming that the income effect is stronger than the substitution effect, a terms of trade worsening decreases the price of nontradables relative to exportables in periods 1 and 2, that is a RER depreciation.

Fiscal policy is also an important determinant of the equilibrium path of the RER. In order to introduce the role of the public sector in the model stated in equations [5.1a] to [5.1e], it is convenient to introduce some simplifying assumptions. In particular, we assume that there are no tariffs ($\tau = \bar{\tau} = 0$), and that the terms of trade do not change, so that the two traded goods can be aggregated into a Hicksian composite good referred to as tradable. Under these assumptions, the model becomes a two-good model, with the RER measured as the inverse of the price.

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happen that the final equilibrium lies above or below the 45° line. If the final equilibrium lies above the 45° line, then there is an “equilibrium overshooting”.

*The same correspondence applies when the shock is either anticipated or permanent.
of nontradables relative to tradables. Next, the budget constraint stated in equation [5.1a] is modified by subtracting two terms from the left hand side, say $T$ and $\delta' \tilde{T}$, that denote nondistortionary taxes in periods 1 and 2, respectively. Further, a government’s intertemporal budget constraint is appended to the model; this states that the present value of government spending on both tradables and nontradables must equal the present value of taxes. Lastly, it is necessary to modify the equilibrium conditions in the nontradable goods market in periods 1 and 2; in particular, for each of these periods the total supply of nontradables has to equal the sum of the quantities demanded by the private and public sectors.

Within this modified framework, it is possible to study the effects of an increase in the government’s consumption of nontradables in period 1. In this case, the increased demand for these goods generates an upward pressure on the price of nontradables, which in turn appreciates the RER in period 1. However, given that the government has to increase taxes in period 2 in order to satisfy its intertemporal budget constraint, there will be a negative income effect that reduces the demand for nontradables in both periods. The final result on the equilibrium RER in period 1 will depend upon which of the two effects dominates; assuming that the substitution effect dominates, then a RER appreciation will occur in period 1. On the other hand, if the fiscal policy measure consists of an increase in the government’s consumption of tradables in period 1, then the RER will unambiguously depreciate in the current and future periods because of the negative income effect already described.

The model can also be used to investigate the effects of capital controls on the path of equilibrium RERs; capital controls are modelled as a tax on foreign borrowing, so that the domestic real interest rate exceeds the world real interest rate. In this context, a liberalisation of the capital account that reduces the extent to which
foreign borrowing is taxed, causes a RER appreciation in period 1. The rationale for this result is that the reduction in the tax on foreign borrowing increases the domestic discount factor, which in turn makes consumers care less about future consumption relative to current consumption. As consumers increase their first-period consumption in all goods, including nontradables, there will be an incipient excess demand for nontradables and a subsequent increase in their relative price (i.e. a RER appreciation).¹⁰

Transfers and exogenous capital flows constitute another kind of disturbances that affect the equilibrium path of the RER. More specifically, a positive transfer from abroad, such as foreign aid, allows the recipient economy to increase its expenditure above income. Consequently, there will be a RER appreciation in periods 1 and 2, provided the income elasticity of demand for nontradables in both periods are different from zero. In this sense, recipients of financial assistance may find that the foreign aid worsens the country's degree of international competitiveness. Lastly, technological progress appreciates the equilibrium RERs, since any productivity shock has a positive income effect, that in turn generates demand pressure on the nontradables market in periods 1 and 2.¹¹

So far, we have been concerned with the process of determination of the equilibrium RER, and how it responds to real disturbances. The existence of an equilibrium RER, however, does not imply that the actual exchange rate is at its equilibrium level at all times, as the actual rate may depart from the implied equilibrium rate in the short run. In this sense, sustained discrepancies between the actual and equilibrium RERs are referred to as situations of RER misalignment.

¹⁰Edwards also points out that the removal of the only distortion in the economy, generates a positive welfare effect that reinforces the RER appreciation.
¹¹See Balassa (1964) for an alternative explanation of the effects of technological progress.
In what follows, we state the dual nominal exchange rate model of Edwards (1989a), which analyses the interactions between RER misalignment and macroeconomic policies. The model is for a small open economy that produces and consumes tradables and nontradables, and where agents demand domestic money (M) and foreign money (F). There is a fixed nominal exchange rate for commercial transactions (E), and a freely floating nominal exchange rate for financial transactions (δ). Also, the foreign price of the traded goods (P′ T) is the numeraire, which implies that the domestic price of tradables (P T) is equal to the fixed nominal exchange rate E; the real exchange rate (e) is defined as the nominal fixed exchange rate (E) relative to the price of nontradables (P N). Perfect foresight is assumed throughout the analysis.

The government consumes both tradables (G T) and nontradables (G N), and finances its expenditure via nondistortionary taxes (t) and domestic credit creation (D); it is also assumed that there is no domestic public debt so that interest rates can be ignored. More formally, the budget constraint of the government is given by:

\[ P_N G_N + EG_T = t + D. \]  

Total assets in domestic currency are equal to domestic money plus foreign money times the freely floating nominal exchange rate for financial transactions, i.e. \( A = M + \delta F \). This equation can in turn be represented in terms of tradable goods as:

\[ a = m + \rho F, \]

where \( a = A/E \), \( m = M/E \) and \( \rho \) indicates the spread between the free and the commercial nominal exchange rates (i.e. \( \rho = \delta/E \)).

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12Edwards (1988b) presents a version of the model with three goods.
The desired ratio of domestic money to foreign money is expressed as a negative function of the expected rate of depreciation of the freely floating nominal exchange rate. Given that there is perfect foresight, the expected rate of depreciation is replaced by the actual rate; more formally

\[ \frac{M}{\delta F} = \sigma \left( \frac{\delta}{\delta} \right), \]

where \( \sigma < 0 \) is the derivative of the desired ratio of domestic money to foreign money with respect to the rate of depreciation of the free rate \( \delta \). Rearranging terms and dividing by \( E \) on both sides we get

\[ m = \sigma \left( \frac{\delta}{\delta} \right) \rho F. \]

Assuming that \( E \) is fixed, \( \delta / \delta \) can be replaced by the rate of change of the spread between the free and the commercial nominal exchange rates \( \hat{\rho} / \rho \). Hence, we have:

\[ m = \sigma \left( \frac{\hat{\rho}}{\rho} \right) \rho F. \]

Inverting this equation and solving for \( \hat{\rho} \) we obtain

\[ \hat{\rho} = \rho L \left( \frac{m}{\rho F} \right), \] [5.2c]

where \( L = \sigma^{-1} \), and \( L' < 0 \) is the derivative of \( \hat{\rho} \) with respect to the ratio of real domestic money to real foreign money.

On the other hand, the equilibrium condition in the nontradables market is

\[ C_N(e, a) + G_N = Q_N(e), \]
which states that the total demand for nontradables by both the private and public sectors has to equal output of nontradables. The private demand for nontradables depends positively on the RER, as well as on total assets; the inclusion of total assets as an additional argument on $C_N$ constitutes an important difference in comparison with the model described previously. On the other hand, the supply of nontradables depends negatively on the RER. From this equation, we find an expression for the equilibrium RER, which depends negatively on both total assets and government spending on nontradables; that is

$$e = v\left(\tilde{a}, G_N\right).$$  \[5.2d\]

The inverse relationship is explained by the fact that an increase in either total assets or $G_N$ generates a demand pressure in the market for nontradables, which in turn increases the price of these goods, appreciating the RER.

Turning to the external sector, it can be described by the following equations:

$$CA = Q_T(e^\ast) - C_T(e^\ast, \tilde{a}) - G_T = \tilde{R}$$  \[5.2e\]

and

$$M = \dot{D} + E \cdot \dot{R}.$$  \[5.2f\]

Equation [5.2e] defines the current account balance (CA), in foreign currency, as the difference between the output of tradables and private and public consumption of these goods; at the same time, this equation also states that the current account balance is equal to the change in international reserves ($\tilde{R}$). Equation [5.2f] indicates that changes in the domestic money supply ($\dot{M}$) are made up of changes in domestic credit ($\dot{D}$), and changes in international reserves ($\dot{R}$) measured in domestic currency (E). Replacing [5.2a] and [5.2e] in [5.2f], dividing by E, and observing that
\( e = \frac{E}{P_N} \), we obtain the following dynamic relationship for the equilibrium in the external sector

\[
\dot{m} = Q_T(e) - C_T(e, a) + \frac{G_N}{e} - \frac{t}{E}. \tag{5.2g}
\]

Equations [5.2c] and [5.2g] are a set of differential equations that jointly determine the time paths of \( \rho \) and \( m \). This system of equations can be solved graphically by means of a phase diagram (see Figure 5.2). Accordingly, we proceed to draw the demarcation curves \( \dot{\rho} = 0 \) and \( \dot{m} = 0 \), that serve to delineate the set of points in the \((m, \rho)\) space where the variables in question are stationary. In particular, it is possible to show that the \( \dot{\rho} = 0 \) curve has a positive slope. Intuitively, an increase in the spread \( \rho \) will lower the desired ratio of real domestic money to real foreign money; in order to maintain equilibrium, an increase in \( m \) is required. On the other hand, the \( \dot{m} = 0 \) curve has a negative slope reflecting the fact that an increase in \( m \) leads to higher real assets, and a subsequent deterioration of the current account balance; in order to reverse the worsening of the current account, it is necessary to reduce real assets through a decline in \( \rho \).

**Figure 5.2:** Determination of the equilibrium RER in a dual nominal exchange rate model
In Figure 5.2 we depict the system under consideration. The intersection of the $\rho = 0$ and $m = 0$ curves is attained at $S$, where the economy reaches its "long-run sustainable equilibrium" with $m = m_0$ and $\rho = \rho_0$. In addition, it is possible to show that the system exhibits saddle path stability, with the stable manifold given by the SS line. Once the economy is at the long-run sustainable equilibrium, from equation [5.2d] it is possible to obtain a "long-run equilibrium RER", for given levels of government spending on nontradable goods ($G_\gamma = G_{N_\gamma}$), and foreign currency ($F = F_\gamma$), that is

$$e_{LR} = v(m_0 + \rho_0 F_\gamma, G_{N_\gamma}).$$

As can be observed, given that the real side of the economy is highly simplified, the only real variable affecting the long-run equilibrium RER is government expenditure on nontraded goods. More specifically, an increase in government spending in nontradables will shift the $m = 0$ curve to the right, leaving the $\rho = 0$ curve unaffected (not depicted here). As a result, the economy will reach a new final equilibrium characterised by higher levels of $m$ and $\rho$, which in turn appreciates the equilibrium RER; this is the same result we obtained when we discussed the intertemporal RER determination model.

The model under consideration can also be used to investigate the effects of monetary disturbances on the long-run equilibrium RER, such as a once-and-for-all unanticipated increase in real balances caused by an increase in the stock of domestic credit (i.e. $\dot{D}$). This policy measure is illustrated in Figure 5.3 as an increase of real balances from $m_0$ to $m_1$, so that the economy then moves upwards along the stable manifold from $S$ to a point like $Q$. At this point, the RER has appreciated in comparison to its long-run equilibrium level due to the higher levels of $m$ and $\rho$. Nonetheless, this is only a temporary result as once the economy is at $Q$, the
dynamics of the system move the economy, along the stable manifold, back towards its initial long-run equilibrium. The important point to be noticed is that monetary disturbances have only temporary effects on the equilibrium long-run RER, whereas policy measures affecting RER fundamentals, such as the fiscal policy discussed above, have permanent effects on the equilibrium RER.

Turning to the duration of the adjustment process, it may be accelerated by implementing, at a point like Q', an "unanticipated discrete nominal devaluation" (this policy measure reduces the stock of real balances $m = M / E$). On the other hand, if the nominal devaluation is adopted when the economy is at its long-run equilibrium S, then it will cause a temporary RER depreciation (this result can be seen by noting that a nominal devaluation can be thought of as a policy that reduces the stock of domestic credit).

Figure 5.3: Effect of a once-and-for-all unanticipated increase in real balances in the dual nominal exchange rate model

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13 As the economy moves from Q to S, foreign reserves decline because agents reduce their excess real balances. Once the economy is back at S, the stock of real balances is the same as before the monetary disturbance, although its composition has changed in favour of domestic credit (this adjustment process assumes sufficient foreign reserves, otherwise agents will anticipate a balance of payments crisis).

14 Calvo and Rodriguez (1977) develop a model in which the RER is shown to depend upon monetary variables in the short run, while it is fully determined by real variables in the long run.

15 Calvo et. al. (1995) show that the steady-state RER is independent of changes in the rate of devaluation, so that policymakers can only target the RER for a limited period of time.
5.3 The Real Exchange Rate: Alternative Definitions and Measurement

Although economists agree that the RER is a key relative price in the economic system, there is no consensus as to which relative price should be called the RER. Following Edwards (1988a), there are least two definitions of the RER. Firstly, theoretical models in the tradition of the dependent economy model define the RER as the relative price between traded and nontraded goods, i.e.:

\[ \text{RER} = e = \frac{P_T}{P_N}. \] \[5.3a\]

This type of measure provides valuable information that affects resource allocation in the economy. For instance, when this ratio increases, that is when the RER depreciates, production of tradables is relatively more profitable, and factors of production tend to move out of the nontradables sector and into the tradable sector. At the same time, an increase in this relative price affects spending patterns, with consumers reallocating their budget and buying more of those goods that have become relatively cheaper. In addition, this definition provides an indicator of the degree of international competitiveness of the country’s tradable sector. In particular, a RER depreciation reflects a decrease in the domestic cost of producing tradable goods which, provided there are no changes in relative prices abroad, constitutes an improvement in the country’s international competitiveness. Assuming that the law of one price (LOOP) holds for tradable goods, equation [5.3a] can be expressed as:

\[ \text{RER} = e = \frac{E \cdot P_T^*}{P_N}, \] \[5.3b\]
where $E$ is the country’s nominal exchange rate, defined as the number of units of domestic currency needed to buy a unit of foreign currency, and $P_T^*$ is the world price of tradables.

Secondly, the RER can be defined in the tradition of the purchasing power parity (PPP) theory, as the country’s nominal exchange rate ($E$), multiplied by the ratio between the foreign price level ($P^*$) and the domestic price level ($P$), that is

$$RER_{ppp} = e_{ppp} = \frac{E \cdot P^*}{P}.$$ \[5.4\]

This definition aims to capture some indication of the evolution of foreign versus domestic prices. Thus, if over a period of time domestic prices have been rising more than foreign prices, a devaluation is called for in order to restore the country’s international competitiveness.

It is interesting to note that for short-run changes, the dependent economy definition of the RER can be expressed in terms of the PPP version. Indeed, as indicated by Edwards (1988a), assuming that a) the domestic country is small; b) the LOOP holds for tradable goods; c) there are no taxes on trade; d) the nominal exchange rate is fixed and equal to 1; and e) the domestic and foreign price levels are weighted averages of tradable and nontradable prices (i.e. $P = P_N^\alpha P_T^{1-\alpha}$ and $P^* = P_N^\beta P_T^{1-\beta}$, respectively), it is possible to obtain the following expression for \(\hat{e}\):

$$\hat{e} = \left(\frac{1}{\alpha}\right)\hat{e}_{ppp} + \left(\frac{\beta}{\alpha}\right)\left(\hat{p}_T^* - \hat{p}_N^*\right).$$ \[5.5\]

where the “hat” refers to percentage changes. As can be seen from [5.5], in the short run both $e$ and $e_{ppp}$ may change in opposite directions depending upon the behaviour of relative prices abroad. Later on, we will examine empirically whether there is a long-run relationship between these two definitions.
Turning to measurement issues, let us begin by noting that although the dependent economy definition of the RER is useful from the theoretical point of view, in practice it is highly sensitive to the definition of the tradable and nontradable sectors. Goldstein and Officer (1979) observe that conventional proxies for $P_T$ are the export price index (EPI), the import price index (IPI) and the wholesale price index (WPI), while typical proxies for $P_N$ are the consumer price index (CPI), the price deflator for the GDP (PGDP) and the price deflator for the GNP (PGNP). These proxies, however, are found to be deficient in several important respects. For example, one limitation of the EPI and the IPI is that they are used as alternative proxies for the price of all tradables, which comprise both exportables and importables. In turn, the WPI is often regarded as a suitable proxy for $P_T$ on the grounds that it is heavily weighted with traded goods, and does not include services which are typically considered within the nontradables category; another important aspect is that almost every country periodically publishes data on WPI. Nonetheless, Goldstein and Officer note that one of the deficiencies of this index is the possibility of double-counting, as it measures the price of commodities at different stages of production.

Regarding the proxies for $P_N$, one of the deficiencies of the CPI is that not all nontradable output is covered by the index, as it measures price movements of a basket of domestic consumption; moreover, the CPI not only incorporates price movements of nontradable goods, but also of tradables. An additional argument against the use of the CPI is that it may be subject to seasonal patterns or purely transitory shocks, which may be interpreted as a deterioration of the country’s degree of international competitiveness. On the positive side, the CPI, like the WPI, is readily available in almost every country at least periodically. As to the PGDP and
the PGNP, their basic limitation is that they include both tradable and nontradable output, as they measure price movements of all domestic or national output.

Another alternative methodology that has been suggested for constructing proxies of \( P_T \) and \( P_N \) is that of using the components of price indexes that are readily available, such as the CPI or the WPI. In this case, however, the main problems are selecting which components are to be included as part of the index, and determining the level of disaggregation; as a result, in some circumstances it may not be possible to be entirely objective.

Taking the above aspects into consideration, Goldstein and Officer (1979) propose new measures of prices for the tradable and nontradable sectors. In particular, they suggest that the first three categories of the standard industrial classification (SIC) of the United Nations (namely agriculture, mining and manufacturing) could be regarded as the tradable sector, while goods in the remaining categories could be regarded as the nontradable sector.\(^{16}\) For most developing countries, however, this methodology has important limitations because the basic source of data to calculate the proxies for \( P_T \) and \( P_N \) is the national accounts; consequently, the RER measure would be available on a yearly basis only and in some cases with a substantial delay.

On the other hand, CPIs, WPIs and wage rate indexes are typically used in the construction of RER measures in the tradition of the PPP theory. For the first two types of indexes, some of the deficiencies noted above are also applicable here. That is, when CPIs are used, seasonal patterns or purely transitory shocks may be interpreted as reflecting changes in the country’s degree of international

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\(^{16}\)The remaining categories are electricity, gas and water; construction; wholesale and retail trade; transport, storage and communication; finance, insurance and real estate; consumer services; business services and government.
competitiveness. Concerning the use of WPIs, Edwards (1988a) notes that the resulting RER may exhibit limited variation and not reflect actual changes in the country's competitiveness, because they include highly homogeneous tradable goods, whose prices tend to equate across countries under conditions of efficient arbitrage. Lastly, the main disadvantages of wage rate indexes are that they take into account only one factor of production, and that they may be of limited availability for developing countries.

So far, we have been dealing with bilateral RERs, that is measures that are concerned with the degree of international competitiveness of a country relative to its main trading partner. In reality, however, since most countries do not trade with a single foreign country, analysts are not so much concerned with what is happening to their exchange rate against a single foreign currency, but rather what is happening to the exchange rate in comparison to a basket of foreign countries with whom the country trades. The effective real, or multilateral trade-weighted, exchange rate is a measure of the country's international competitiveness relative to its trading partners.

Bearing in mind the limitations mentioned above, we construct two empirical counterparts to the concept of (effective) RER. The first one, which we refer to as RER1, is based on the tradition of the PPP theory, and uses WPIs as proxies for both foreign and domestic price levels. The second one, which we refer to as RER2, corresponds to the dependent economy version of the RER and assumes, following Edwards (1989a) and Helmers (1991), that the foreign country's WPI is a suitable proxy for \( P^*_f \), and that the domestic country's CPI is a suitable proxy for \( P^*_n \). Both WPIs and CPIs were selected on the grounds that the former are heavily weighted with traded goods, while the latter contain a large proportion of nontradable goods. An additional reason is that our modelling exercise is performed using quarterly data,
and these two indexes are available at that frequency in almost every country. For the calculations, we use a basket of 14 countries that accounted for approximately 80% of Colombia's foreign trade during the period 1970-1992. In Appendix 5.1 we present the list of countries as well as the time varying weights we actually use.

Figure 5.4 shows the logarithm of the two RER indexes, denoted LRER1 and LRER2.¹⁷ In general terms, these series exhibit the same long-run behaviour, and the plots are reasonably coincident. From the mid seventies to the early eighties, the declining trend exhibited by the indexes reflects a RER appreciation; during this period LRER1 is greater than LRER2. From 1983 to 1991 there is a period of continuous depreciation of the RER, and since 1987 LRER2 is consistently above LRER1. Lastly, in 1991 begins a new period of RER appreciation that seems to be more accentuated when using LRER2.

¹⁷The Banco de la República (Central Bank) calculates a RER index based on the PPP theory, with WPIs as deflators; this index, however, is available since 1975. For a presentation of the methodology used in the construction of this index see e.g. Banco de la República (1984).
Cointegration analysis provides a more powerful tool to explore the long-run behaviour of LRER1 and LRER2. Before turning to this analysis, we first investigate the order of integration of the series, for which we use ADF unit root tests. Table 5.1 reports the results of these tests, which suggest that LRER1 and LRER2 contain a unit root without a drift; the presence of a unit root is also confirmed when looking at the correlogram of the series (see Figure 5.5). Table 5.2 presents the results of Johansen’s maximum likelihood test statistics, when a fifth-order bivariate VAR is estimated for LRER1 and LRER2. As can be seen, the model passes all misspecification tests, and the maximal eigenvalue and trace test statistics support the choice of one cointegrating vector.

Consequently, the comparison between the empirical counterparts to the dependent economy and PPP definitions of the RER (i.e. LRER1 and LRER2, respectively) suggests that although the two measures can move in opposite directions in the short run (recall the result stated in equation [5.5]), they are subject to the same long-run trend.

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Table 5.1: Dickey and Fuller unit root tests

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model</th>
<th>Lags of Dep. Var.</th>
<th>LM[4]</th>
<th>$\tau$</th>
<th>$\Phi_3$</th>
<th>$\Phi_2$</th>
<th>$\tau_{\mu}$</th>
<th>$\Phi_1$</th>
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</thead>
<tbody>
<tr>
<td>LRER1</td>
<td>A</td>
<td>7</td>
<td>F 4.70 0.158</td>
<td>-1.995</td>
<td>2.147</td>
<td>1.446</td>
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<tr>
<td>LRER1</td>
<td>B</td>
<td>7</td>
<td>F 4.71 0.170</td>
<td>-1.643</td>
<td>1.372</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LRER2</td>
<td>A</td>
<td>7</td>
<td>F 4.70 0.036</td>
<td>-2.311</td>
<td>2.727</td>
<td>1.845</td>
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<tr>
<td>LRER2</td>
<td>B</td>
<td>7</td>
<td>F 4.71 0.036</td>
<td>-2.026</td>
<td>2.093</td>
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<td></td>
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<tr>
<td>LCP</td>
<td>A</td>
<td>1</td>
<td>F 4.82 0.627</td>
<td>-2.277</td>
<td>2.898</td>
<td>2.028</td>
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<tr>
<td>LCP</td>
<td>B</td>
<td>1</td>
<td>F 4.83 0.913</td>
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<tr>
<td>TAR</td>
<td>A</td>
<td>5</td>
<td>F 4.74 0.088</td>
<td>-2.478</td>
<td>3.855</td>
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<td>B</td>
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<td>F 4.75 0.068</td>
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<tr>
<td>LPFD</td>
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<td>GCOMP</td>
<td>A</td>
<td>4</td>
<td>F 4.76 1.170</td>
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<td>FS</td>
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<td>F 4.70 0.849</td>
<td>-2.360</td>
<td>2.879</td>
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</table>

Notes:
Model A is $\Delta Y_t = \gamma_0 + \gamma_1 t + \gamma_2 Y_{t-1} + \text{lags dep. variable}.$
Model B is $\Delta Y_t = \beta_0 + \beta_1 Y_{t-1} + \text{lags dep. variable}.$
The regressions of FS include centred seasonal dummies. The LM[4] test is reported in its F version. The critical values for the $\tau$ statistics are reported in MacKinnon (1991). The critical values for the $\Phi_1$, $\Phi_2$, and $\Phi_3$ statistics are reported in Dickey and Fuller (1981).
Figure 5.5: Time-series graphs of the variables used in the real exchange rate model

Autocorrelation function of the variables in levels and first differences
Table 5.2: Cointegration analysis between LRER1 and LRER2

<table>
<thead>
<tr>
<th>Model diagnostic tests</th>
<th>LRER1</th>
<th>LRER2</th>
</tr>
</thead>
<tbody>
<tr>
<td>LM [4]</td>
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<td>0.348</td>
</tr>
<tr>
<td>ARCH [4]</td>
<td>1.538</td>
<td>0.351</td>
</tr>
<tr>
<td>Normality</td>
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<td>2.532</td>
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<tr>
<td>Heteroscedasticity</td>
<td>1.288</td>
<td>1.057</td>
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<table>
<thead>
<tr>
<th>Cointegration analysis</th>
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</thead>
<tbody>
<tr>
<td>Maximal Eigenvalue Test</td>
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<tr>
<td>Null Hypothesis</td>
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<tr>
<td>Alternative Hypothesis</td>
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<tr>
<td>Test Value</td>
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</table>

<table>
<thead>
<tr>
<th>Trace Test</th>
</tr>
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<tbody>
<tr>
<td>Null Hypothesis</td>
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<tr>
<td>Alternative Hypothesis</td>
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<tr>
<td>Test Value</td>
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</table>

<table>
<thead>
<tr>
<th>$\beta'$ Eigenvectors (Standardized)</th>
<th>LRER1</th>
<th>LRER2</th>
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</thead>
<tbody>
<tr>
<td>1.000</td>
<td>-1.147</td>
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</table>

Notes:
The LM[4], ARCH[4] and Heteroscedasticity tests are reported in their F versions. The test for normality is distributed as $\chi^2_2$. $^*$ denotes significance at the 10% level. $^{**}$ denotes significance at the 5% level. The number of cointegrating vectors is denoted by $r$. Critical values for the maximal eigenvalue and trace tests are reported in Osterwald-Lenum (1992).
5.4 **Real Exchange Rate Determination Model for Colombia**

Since the mid 1980s there has been a growing interest in identifying the main determinants of the RER in Colombia: Carkovic (1986), Kamas (1986), García and Montes (1988), Herrera (1989, 1997), Ocampo (1989), Rodríguez (1989), Wunder (1991), Echavarría and Gaviria (1992), Langebaek (1993), Calderón (1995) and Cárdenas (1997). In these empirical analyses, the RER is assumed to depend upon a set of relevant variables, which are then analysed in terms of their significance and expected sign. The authors commonly find that the coffee price, which maintains a close relationship with the country's terms of trade, and fiscal variables such as the size of government expenditures and the fiscal deficit, are important determinants of the RER.

The performance of the models is typically assessed during the estimation period in terms of $R^2$s, and by looking at misspecification tests such as the DW statistic. Echavarría and Gaviria (1992), for example, is the only work that departs from this traditional approach by presenting the CUSUM test for testing model instability. In addition, few empirical applications examine the time series properties of the series under review. Neglecting this aspect could have led some authors to deal with unbalanced equations (e.g. Echavarría and Gaviria 1992; Langebaek 1993; and Herrera 1989, 1997). Lastly, it is rather unfortunate that the predictive ability of the models, either during the estimation period or beyond it, has never been assessed.

In what follows, we build a RER determination model for Colombia based on the Edwards model. The modelling exercise includes a) the formulation of a model to find the determinants of the RER in the short and long run; b) the estimation of a measure of exchange rate misalignment; and c) the evaluation of the performance of
the model, in terms of its ability to predict the behaviour of the RER during the period of estimation, and three years into the future. We use quarterly data for the period 1970-1992.19

5.4.1 DATA

The Edwards model states that in the long run only real factors affect the sustainable equilibrium level of the RER, whereas in the short run real factors and macroeconomic policies affect the RER. The set of real factors comprises variables such as the terms of trade, import tariffs, government expenditure, capital controls and technological progress. With regard to macroeconomic policies, we investigate the effects of monetary, fiscal and exchange rate policies. Next, we describe the variables to be used as well as their sources.

We use the price of coffee to capture the terms of trade, as this commodity has historically constituted Colombia's major export. In this sense, one would expect that changes in the price of coffee lead to changes in the country's terms of trade, which can be formally tested by means of Granger's causality tests. In particular, using quarterly data for the period 1970-1995, we find that changes in the price of coffee Granger-cause changes in the terms of trade (F_{2,96}=4.36), and that changes in the latter variable do not Granger-cause changes in the former (F_{2,96}=1.47). The (logarithm of the) price of coffee is denoted LCP, and is expressed in 1986 dollars to account for the erosion of coffee purchasing power due to U.S. inflation, as measured

---

19With the exception of Cárdenas (1997), existing RER models for Colombia use annual data. Cárdenas, however, uses a shorter sample period (1980:1-1996:3) and a different theoretical model.
20The order of the underlying VAR (i.e. 2 lags) was selected using a general to specific procedure.
by this country's CPI; the source of LCP is Banco de la República (1993) and for the CPI of the U.S. we use data from the International Monetary Fund.

Regarding import tariffs, we first calculate an implicit import tariff (defined as the ratio of tariff revenues to total imports) using annual data from national accounts. Then, we assume that tariff revenues are equally distributed throughout the year, so that the import tariff of any quarter can be set equal to that of the year as a whole. For our purposes, we set the import tariff of the second quarter equal to that estimated for the year as a whole, and then we interpolate in order to obtain a quarterly series of import tariffs, which is denoted TAR. It is worth bearing in mind, as argued by Edwards (1989), that TAR does not constitute a perfect measure of existing trade controls in the economy, since it ignores the role of nontariff barriers such as import quotas.

With reference to government expenditure, it is worth recalling that the theoretical model distinguished between government consumption on nontraded and traded goods. Given that in practice such data are not available, we assume that central government's current and capital expenditures provide suitable proxies for government consumption on nontraded and traded goods, respectively. On this basis, we calculate the variable GCOMP as the ratio of central government’s current expenditure to total expenditure, so that an increase in GCOMP reflects an increase in the share of government expenditure on nontradables, and consequently a decrease in the share of government expenditure on tradables. The data were obtained from various issues of the Revista del Banco de la República for the period 1970-1979.

---

21From 1980 to 1992, 24% of total imports took place during the first quarter, whereas 25%, 26% and 25% took place during the second, third and fourth quarters, respectively (calculations based on Cubillos and Valderrama, 1993).

22The results are practically the same had we set any of the other quarters equal to the import tariff of the whole year.
and from Ramos and Rodríguez (1995) for the period 1980-1992. Naturally, we are aware that GCOMP is not a comprehensive measure of the expenditure of the public sector, as it only includes the central government.

Regarding a proxy for the capital controls existing in the economy, we follow Herrera (1989, 1997) who uses the stock of foreign debt of the private sector. In this sense, a relaxation of the impediments to free borrowing and lending will be associated with an increase in the private sector's stock of foreign debt, which in turn allows private agents to increase their expenditure on traded and nontraded goods. The (logarithm of the) stock of foreign debt of the private sector is denoted LPFD, and is expressed in 1986 dollars using the U.S. CPI; the data on LPFD were obtained from various issues of the Revista del Banco de la República. Lastly, technological progress is measured as a time trend, although this variable is not significant.

Turning to the macroeconomic policies that might affect the behaviour of the RER in the short run, we consider the role of monetary, fiscal and exchange rate policies. In the case of monetary policy, we use a measure of money market disequilibrium (denoted EMS) that corresponds to the residuals of the long-run money demand function estimated in the previous chapter; it should be recalled that positive (negative) residuals denote excess money supply (demand). Concerning fiscal policy, we use the fiscal surplus of the central government as a proportion of GDP, which we denote FS. Exchange rate policy is denoted ΔLNER and corresponds to the rate of nominal devaluation; the source is Banco de la República (1993).

It is also worth mentioning that the foreign exchange from illegal drug exports that enters the economy could affect the behaviour of the real exchange rate.

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23This series exhibits outliers in 1979:3, 1981:2, 1982:4 and 1983:1, which correspond to dates of substantial variation in the stock of foreign debt.
through its effect on the aggregate demand. However, available evidence does not seem to support this claim. For example, Gómez (1990) and Urrutia (1990) argue that the small proportion of drug revenues that is repatriated is not likely to have a sizeable effect on the aggregate demand, because they have commonly been used to buy durable imported goods, livestock, lands, and (luxurious) urban and rural properties. In a more recent study, Cárdenas (1997) estimates a real exchange rate model for Colombia using a series of terms of trade that includes the price of the narcotics exported by the country, as taken from Steiner (1996). The results of Cárdenas are not robust to alternative definitions of the RER, as the coefficient on the “adjusted” terms of trade turns out to be statistically different from zero in only one of the five estimated models. Lastly, we also estimated an alternative version of our RER model, incorporating the “adjusted” terms of trade series used by Cárdenas, but the results were inferior to the model that uses the price of coffee.

5.4.2 TESTING FOR NONSTATIONARITY

The order of integration of the series under review, except EMS which we already know is I(0), is investigated with the use of graphical and correlogram evidence (see Figure 5.5), and with ADF tests for a unit root (see Table 5.1). Inspection of the plots of LRER1, LRER2, LCP, TAR and LPFD suggest that they may be nonstationary in levels, a conclusion that is also supported when looking at the correlogram of the series. On the other hand, the first differences of the series fluctuate around their sample means, and the correlograms decay rapidly to zero suggesting stationarity.

---

24 Given that Steiner’s data are annual and the model is quarterly, Cárdenas assumes that the prices and quantities of the narcotics exported by the country remain constant throughout the year.
25 The cointegration analysis suggests that LCP, LPFD and the “adjusted” terms of trade are stationary, contradicting the results of unit root tests on the individual variables, and the graphical evidence.
GCOMP and FS fluctuate around their sample means during the period under review; hence, it is not completely clear that the two fiscal variables contain a unit root, although it may take some time while they revert to their means. This tentative conclusion is also supported when one inspects the correlogram of these series. On the other hand, the correlograms of ΔGCOMP and ΔFS suggest that the series may have been overdifferenced (the first autocorrelation coefficient is significant and close to -0.5).

Inspection of the plots of LNER suggest a linearly trended series, consistent with the presence of a unit root (possibly with a non-zero drift term). The autocorrelation function dies down very slowly, further supporting the tentative conclusion that the series may be nonstationary. When one applies the first difference operator the upward trend disappears, although the autocorrelation function of ΔLNER still decays slowly, this time with the correlation at the first lag approximately equal to 0.8.

In Table 5.1 we report the ADF tests for a unit root in LRER1, LRER2, LCP, TAR, GCOMP, LPFD and FS. These tests indicate that all series contain a unit root without a drift term, which in the case of GCOMP and FS contradicts graphical and correlogram evidence. Given that the existence of a unit root implies that the variance of GCOMP and FS is unbounded, which is unlikely to occur in reality, we treat them as I(0) series based on graphical and correlogram evidence. In the case of LNER, although graphical and correlogram evidence suggests that only one difference is necessary to render the series to stationarity, we formally test for the existence of two unit roots using the testing procedure recommended by Dickey and Pantula (1987); the results, not reported here, indicate that LNER is I(1) while ΔLNER is I(0).
5.4.3 Cointegration Analysis

We consider two three-dimensional VAR models for the variable sets \{LRER_1, LCP, LPFD\} and \{LRER_2, LCP, LPFD\}, which are referred to as VAR-RER1 and VAR-RER2, respectively. In each VAR there is also a set of non-modelled variables, which comprises TAR, GCOMP, ΔLNER, EMS and FS. Based on the earlier theoretical model, TAR and GCOMP are assumed to enter in levels in the cointegrating space, which implies that they are regarded as exogenous to the system; this assumption allows us to reduce the dimension of the VAR enabling estimation. Concerning ΔLNER, EMS and FS, they are included in the short-run dynamics, along with a dummy variable that aims at removing the effects of the dates of substantial accumulation of foreign debt by the private sector; this dummy variable takes the value of one in 1979:3, 1981:2, 1982:4 and 1983:1, and zero otherwise.

The reduced form vector error correction (VEC) representation of the VAR models is:

\[
\Delta y_t = \Gamma_1 \Delta y_{t-1} + \ldots + \Gamma_k \Delta y_{t-k+1} + \Pi \bar{y}_{t-1} + \Psi X_t + \varepsilon_t \tag{5.7}
\]

where

\[
y_t = [LRER_1, \text{ or } LRER_2, LCP_t, LPFD_t],
\]

\[
\bar{y}_t = [y_t, TAR_t, GCOMP_t],
\]

and

\[
X_t = [\Delta LNER_{t-1}, EMS_t, EMS_{t-1}, FS_t, FS_{t-1}, \text{Dummy}].
\]

The importance of this representation is that the rank of the matrix of coefficients \(\Pi\) contains information about the number of long-run relations in the system. Following Johansen and Juselius (1990), in the particular case that
0 < \text{Rank}(\Pi) = r < p$, $p$ being the dimension of the VAR, there exist two $p \times r$ matrices $\alpha$ and $\beta$ such that it is possible to represent $\Pi = \alpha \beta'$, where $\beta$ is the cointegrating matrix that has the property to transform $\beta' \tilde{y}_{t-1}$ into a stationary process, even though the components of $\tilde{y}_{t-1}$ are not stationary. On the other hand, the elements of the matrix $\alpha$ measure the speed of the adjustment when there are disturbances in the equilibrium relationship.

The VAR models are estimated using three lags, and single-equation misspecification test statistics are reported in the first panel of Tables 5.3 and 5.4.
Table 5.3: Cointegration analysis VAR-RER1

<table>
<thead>
<tr>
<th>Model diagnostic tests</th>
<th>LRER1</th>
<th>LPFD</th>
<th>LCP</th>
</tr>
</thead>
<tbody>
<tr>
<td>LM[4]</td>
<td>0.109</td>
<td>1.159</td>
<td>0.868</td>
</tr>
<tr>
<td>Normality</td>
<td>0.080</td>
<td>1.166</td>
<td>4.275</td>
</tr>
<tr>
<td>ARCH[4]</td>
<td>1.306</td>
<td>0.889</td>
<td>0.961</td>
</tr>
<tr>
<td>Heteroscedasticity</td>
<td>0.610</td>
<td>0.951</td>
<td>0.683</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cointegration analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximal Eigenvalue Test</td>
</tr>
<tr>
<td>Null Hypothesis</td>
</tr>
<tr>
<td>Alternative Hypothesis</td>
</tr>
<tr>
<td>Test Value</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Trace Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Null Hypothesis</td>
</tr>
<tr>
<td>Alternative Hypothesis</td>
</tr>
<tr>
<td>Test Value</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\beta'$ Eigenvectors (Standardized)</th>
<th>LRER1</th>
<th>LPFD</th>
<th>LCP</th>
<th>GCOMP</th>
<th>TAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>LRER1</td>
<td>1.000</td>
<td>0.383</td>
<td>0.437</td>
<td>0.541</td>
<td>-3.871</td>
</tr>
<tr>
<td>LPFD</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LCP</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Standardized $\alpha$ Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>LRER1</td>
</tr>
<tr>
<td>LPFD</td>
</tr>
<tr>
<td>LCP</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Testing long-run exclusion (Test distributed as $\chi^2_1$)</th>
<th>LRER1</th>
<th>LPFD</th>
<th>LCP</th>
<th>GCOMP</th>
<th>TAR</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>10.779</strong></td>
<td><strong>12.494</strong></td>
<td><strong>19.794</strong></td>
<td>3.454</td>
<td><strong>18.317</strong></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Testing weak exogeneity (Test distributed as $\chi^2_1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.208</td>
</tr>
</tbody>
</table>

Notes:
* denotes significance at the 5% level. ** denotes significance at the 1% level. The number of cointegrating vectors is denoted by $r$. Critical values for the maximal eigenvalue and trace tests are reported in Osterwald-Lenum (1992).
Table 5.4: Cointegration analysis VAR-RER2

<table>
<thead>
<tr>
<th>Model diagnostic tests</th>
<th>LRER2</th>
<th>LPFD</th>
<th>LCP</th>
</tr>
</thead>
<tbody>
<tr>
<td>LM[4]</td>
<td>0.163</td>
<td>1.073</td>
<td>0.787</td>
</tr>
<tr>
<td>Normality</td>
<td>2.751</td>
<td>1.491</td>
<td>6.626</td>
</tr>
<tr>
<td>ARCH[4]</td>
<td>0.714</td>
<td>0.993</td>
<td>0.575</td>
</tr>
<tr>
<td>Heteroscedasticity</td>
<td>0.556</td>
<td>1.158</td>
<td>0.618</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cointegration analysis</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximal Eigenvalue Test</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Null Hypothesis</td>
<td>r = 0</td>
<td>r &lt;= 1</td>
<td>r = 2</td>
</tr>
<tr>
<td>Alternative Hypothesis</td>
<td>r = 1</td>
<td>r = 2</td>
<td>r = 3</td>
</tr>
<tr>
<td>Test Value</td>
<td>**37.410</td>
<td>20.900</td>
<td>3.100</td>
</tr>
</tbody>
</table>

| Trace Test                    |       |      |     |
| Null Hypothesis               | r = 0 | r <= 1| r = 2|
| Alternative Hypothesis        | r >= 1| r = 2| r >= 3|
| Test Value                    | **61.410 | 24.000 | 3.100|

<table>
<thead>
<tr>
<th>β' Eigenvectors (Standardized)</th>
<th>LRER2</th>
<th>LPFD</th>
<th>LCP</th>
<th>GCOMP</th>
<th>TAR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.000</td>
<td>0.284</td>
<td>0.389</td>
<td>0.099</td>
<td>-4.109</td>
</tr>
<tr>
<td></td>
<td>-0.663</td>
<td>1.000</td>
<td>0.390</td>
<td>0.264</td>
<td>-3.328</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Standardized α Coefficients</th>
<th>LRER2</th>
<th>LPFD</th>
<th>LCP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.117</td>
<td>0.049</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.193</td>
<td>-0.041</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.176</td>
<td>-0.156</td>
<td></td>
</tr>
</tbody>
</table>

| Testing long-run exclusion    | LRER2 | LPFD | LCP | GCOMP | TAR |
| (Test distributed as χ²₁)     | **15.367| **5.489| **13.609| 0.125 | **14.392|

| Testing weak exogeneity       |       |      |     |
| (Test distributed as χ²₁)     | *5.549| **10.808| 0.934|

| Restricted β' Eigenvector     | 1.000 | 0.222| 0.358| - - | -3.813|
| Restricted α Coefficients     |       |      |     |
| LRER2                         | -0.139|      |     |
| LPFD                          | -0.178|      |     |
| LCP                           | - -   |      |     |

Notes:
* denotes significance at the 5% level. ** denotes significance at the 1% level. The number of cointegrating vectors is denoted by r. Critical values for the maximal eigenvalue and trace tests are reported in Osterwald-Lenum (1992).
All equations pass the LM[4] test for residual serial correlation, Engle’s LM[4] test for ARCH, White’s test for heteroscedasticity, and the test for normality.\textsuperscript{26} The next important aspect is parameter constancy, for which we use a recursive estimation. In Figures 5.6 and 5.7 we plot one-step residuals (with corresponding equation standard errors) and sequences of 1-step F-tests (1\textsuperscript{st}-step Chow-tests), break-point F tests (N\textsuperscript{th}-step Chow-tests) and forecast F-tests (N\textsuperscript{th}-step Chow-tests), for the VAR models. As can be seen, the equations for LRERI, LRER2, LCP and LPFD are relatively constant during the period of estimation, as in very few occasions the F statistics are rejected at the 1\% significance level.

We proceed with a cointegration analysis which, in terms of model [5.7], involves testing the hypothesis of reduce rank in the matrix of coefficients $\Pi$. The determination of the number of cointegrating vectors is based on the results of the maximal eigenvalue and trace tests (see Tables 5.3 and 5.4), the graphs of the cointegrating relations (see Figures 5.8 and 5.9) and the interpretability of the obtained coefficients. In the case of VAR-RER1, the maximal eigenvalue test would lead us to accept, at the 5\% level, the hypothesis of one cointegrating vector, although the trace test leaves open the possibility of a second one. With reference to VAR-RER2, the test statistics illustrate the presence of two cointegrating vectors. If we are willing to accept that there are two cointegrating vectors, then we would expect the first two plots in Figures 5.8 and 5.9 to look like I(0) processes; however, the graphical evidence presented there suggests the presence of one cointegrating vector, as only the cointegrating relation labelled “vector 1” appears stationary.

\textsuperscript{26}In VAR-RER2, the equation for LCP fails normality at the 5\% significance level, but not at the 1\%.
Figure 5.6: System recursive evaluation statistics VAR-RER1
1-step residuals ±2 equation standard error

1-step (↑), break-point (↓) and forecast (↑) F tests
Figure 5.7: System recursive evaluation statistics VAR-RER2
1-step residuals ±2 equation standard error

1-step (↑), break-point (↓) and forecast (↑) F tests
Figure 5.8: Cointegrating relations VAR-RER1

Figure 5.9: Cointegrating relations VAR-RER2
Turning to the interpretability of the results, the first cointegrating vector can be thought of as a long-run RER equation (see Tables 5.3 and 5.4). Accordingly, the RER appreciates as a result of increases in the price of coffee, in the stock of foreign debt of the private sector, and in the ratio of central government’s current expenditure to total expenditure. On the other hand, the RER depreciates as a result of increases in import tariffs; it shall be remembered that the theoretical model predicted that if the changes in tariffs are accomplished when the tariffs are initially greater than zero, then the associated first-order income effects may compensate the substitution effects, conducing to a RER depreciation. It is also worth noting that the estimation of the two models produce coefficients on LPFD, LCP and TAR that are very close, although this is not the case for the estimated coefficient on GCOMP. As to the second cointegrating vector, the obtained coefficients do not have a clear economic interpretation.

Taking the above aspects into consideration, the subsequent analysis is based on the assumption of one cointegrating vector, i.e. that the rank of the matrix of coefficients Π is equal to one, since the test statistics, the graphical analysis and the economic interpretation of the results support this choice.

Having determined the number of cointegrating relations, we continue with the formulation and testing of hypotheses about the cointegration vectors and the adjustment coefficients. Regarding the first type of hypotheses, we test whether each individual variable can be excluded from the long-run relation. The null hypothesis of long-run exclusion is then rejected for all variables, except GCOMP (see Tables 5.3 and 5.4). In other words, the ratio of central government’s current expenditure to

[27] Echavarria and Gaviria (1992) also obtain that import tariffs affect positively the RER in Colombia.
total expenditure is not needed in the cointegration space; later on it will be shown that GCOMP is nonetheless significant for modelling the short-run dynamics.\(^{28}\)

With reference to the tests on the adjustment coefficients, we test whether the set of variables are weakly exogenous for the estimation of the parameters of the long-run RER equation (see Johansen, 1992). Accordingly, in the first VAR, LRER1 can be regarded as weakly exogenous, but this is not true for LPFD and LCP. In the second VAR, weak exogeneity is accepted for LCP, but not for LRER2 and LPFD. These conflicting results concerning the status of LRER1 and LRER2 may be due to short-run dynamics, as we previously found that they maintain a long-run equilibrium relationship. On the other hand, given that historical evidence suggests that the behaviour of the coffee price in the world market has been determined by variables other than those included in the VARs (e.g. weather conditions in Brazil), we regard that it is valid to assume that LCP is weakly exogenous for the estimation of the parameters of the long-run RER equation, as found in VAR-RER2. Thus, based on the results of the tests for weak exogeneity, we discard VAR-RER1 and continue the modelling exercise with VAR-RER2.

Lastly, we test in VAR-RER2 the joint hypothesis that GCOMP can be excluded from the cointegration space, and that LCP is weakly exogenous for the estimation of the parameters of the long-run RER equation. The test statistic of this joint hypothesis is \(\chi^2 = 1.048\), which is easily accepted. The restricted cointegrating vector and adjustment coefficients are reported in the last panel of Table 5.4. As can be seen, the coefficients in the cointegrating vector indicate that the RER appreciates as a result of increases in LCP and LPFD, while it depreciates as a result of increases

\(^{28}\)We also estimated variants of the models including terms of trade and a trend term (the latter as proxy for technological progress), but these variables were not significant. In the specification with
in TAR; GCOMP is not needed in the cointegration space. In Figure 5.10 we plot the restricted cointegrating relation, which looks very much like the unrestricted one (reported as "vector 1" in Figure 5.9).

5.4.4 REAL EXCHANGE RATE MISALIGNMENT

We derive a measure of RER misalignment that corresponds to the deviations of the RER from its long-run equilibrium relationship. At first glance, it appears that the cointegrating relation depicted in Figure 5.10, that is the linear combination $\beta' \tilde{y}_t$, can be interpreted as a measure of RER misalignment. However, the limitation of $\beta' \tilde{y}_t$ is that it is not corrected for the short-run dynamics of the model. In contrast to Edwards (1989a) and Elbadawi (1994), we use neither moving averages nor the Beveridge-Nelson decomposition to correct for the short-run dynamics, because the first method involves the loss of observations and the second one can not always be applied. Instead, we follow Johansen and Juselius (1992), who recommend calculating the cointegrating relations as $\beta'r_{it}$, where $r_{it}$ are the residuals from regressing $\tilde{y}_{t-1}$ on the short-run dynamics ($\Delta y_{t-1}, \ldots, \Delta y_{t-k+1}$) and the set of non-modelled variables entering in the short run ($X_t$). In other words, we use the linear combination $\beta'r_{it}$ as a more precise measure of RER misalignment, with positive (negative) residuals denoting undervaluation (overvaluation) of the RER.

Figure 5.11 plots the cointegrating relation corrected for short-run dynamics (i.e. $\beta'r_{it}$). It can be seen that the linear combination $\beta'r_{it}$ appears more stationary than $\beta'\tilde{y}_t$, which illustrates the importance of specifying the short-run dynamics of the terms of trade, we found a long-run relationship between this variable and LCP.
the model. Further inspection of the graph reveals that the magnitude of the misalignment fluctuates within a range of approximately ±20%.

**Figure 5.10:** Restricted cointegrating relation VAR-RER2

**Figure 5.11:** Restricted cointegrating relation VAR-RER2 (correcting for the short-run dynamics)
The behaviour of the indicator illustrates that from 1970 to around 1977 the RER passes from being overvalued to being undervalued. After this year and up to around 1985 negative residuals are predominant, suggesting overvaluation of the RER; during this period, the country experienced a coffee price boom, accumulation of foreign debt, and a significant expansion of the public sector. In 1986, after a period where fiscal imbalances and the deterioration of the foreign sector led the government to adopt an adjustment programme that included, among other reforms, fiscal austerity and acceleration of the rate of crawl of the nominal exchange rate, our measure of misalignment suggests that the RER was undervalued; in the following year, the RER appears to be overvalued once again. From 1988 to 1992 our estimates indicate a period of RER undervaluation. as most of the residuals are positive; it is worth noticing that the magnitude of the misalignment in 1992 is less than that observed in the previous two years.29

5.4.5 SHORT-RUN DYNAMICS

Once we have found evidence of a cointegrating relationship, we estimate the VAR-RER2 in error correction form. But, before doing this, it is worth recalling from the outcome of the tests of hypotheses about the adjustment coefficients, that only LCP can be regarded as weakly exogenous for the estimation of the parameters of interest. Given that ΔLPFD, is not weakly exogenous, a one-equation model for ΔLRER2, conditioning upon it and the remaining variables is not valid; instead, we need to model ΔLRER2, and ΔLPFD, jointly.

29Similar results are obtained by Cárdenas (1997) using a different methodology.
We thus proceed by conditioning upon the price of coffee, so that $\Delta LCP_t$ is included as a regressor in the reduced form error correction models (ECMs). The lag length of the ECMs is equal to two because we included three lags in the VAR model of the variables in levels. Least-squares estimates for the ECMs are reported in Table 5.5 along with the associated standard errors. As expected, the two equations are initially overparameterised, so that a more parsimonious representation could be obtained by removing the insignificant regressors. The estimated coefficient on the error correction term is negative and statistically different from zero in the two equations. It also appears that the ECMs are well specified as none of the diagnostic tests are failed.

Based on the reduced form ECMs, we formulate a system of simultaneous equations to model $\Delta LRER_{2,t}$ and $\Delta LPFD_t$. To define a structural equation for $\Delta LRER_{2,t}$, we include $\Delta LPFD_t$ as explanatory variable, and exclude $\Delta LRER_{2,t-1}$, $\Delta LPFD_{t-1}$, $\Delta LPFD_{t-2}$, $\Delta LCP_{t-1}$, $\Delta LCP_{t-2}$, $\Delta TAR_{t-1}$, $GCOMP_t$, $EMS_t$, $EMS_{t-1}$, $FS_t$, and the dummy variable. These restrictions are based on the earlier theoretical analysis, which states that in the short run the RER is influenced by real factors and macroeconomic policies, and the significance of the coefficients in the reduced form ECMs. To restrict the equation determining $\Delta LPFD_t$, we exclude, based on the data evidence in Table 5.5, $\Delta LRER_{2,t-1}$, $\Delta LRER_{2,t-2}$, $\Delta LPFD_{t-2}$, $\Delta LCP_{t-1}$, $\Delta LCP_{t-2}$, $\Delta TAR_{t-1}$, $\Delta TAR_{t-2}$, $GCOMP_t$, $GCOMP_{t-1}$, $EMS_t$, $EMS_{t-1}$, $FS_t$, $FS_{t-1}$ and $\Delta LRER_{t-1}$. Two stage least squares estimation of the resulting equations, using as instruments the explanatory variables of the reduced form ECMs, yields the results presented in Table 5.5.
Table 5.5: Estimates of the ECMs conditioning upon \( \Delta LCP \)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Ordinary Least Squares</th>
<th>Two Stages Least Squares</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \Delta LRER )</td>
<td>( \Delta LPFD )</td>
</tr>
<tr>
<td></td>
<td>Coeff. Std. Error</td>
<td>Coeff. Std. Error</td>
</tr>
<tr>
<td>Constant</td>
<td>0.685 0.240</td>
<td>0.788 0.284</td>
</tr>
<tr>
<td>( \Delta LRER ) t-1</td>
<td>0.151 0.130</td>
<td>-0.069 0.154</td>
</tr>
<tr>
<td>( \Delta LRER ) t-2</td>
<td>-0.193 0.109</td>
<td>-0.119 0.129</td>
</tr>
<tr>
<td>( \Delta LPFD ) t</td>
<td>-0.129 0.063</td>
<td>0.129 0.074</td>
</tr>
<tr>
<td>( \Delta LPFD ) t-1</td>
<td>-0.002 0.064</td>
<td>-0.068 0.076</td>
</tr>
<tr>
<td>( \Delta LPFD ) t-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta LCP ) t</td>
<td>-0.026 0.026</td>
<td>0.055 0.030</td>
</tr>
<tr>
<td>( \Delta LCP ) t-1</td>
<td>-0.003 0.031</td>
<td>0.060 0.036</td>
</tr>
<tr>
<td>( \Delta LCP ) t-2</td>
<td>0.036 0.029</td>
<td>-0.004 0.034</td>
</tr>
<tr>
<td>( \Delta TAR ) t</td>
<td>0.646 1.220</td>
<td>-0.130 1.446</td>
</tr>
<tr>
<td>( \Delta TAR ) t-1</td>
<td>1.024 1.263</td>
<td>1.841 1.497</td>
</tr>
<tr>
<td>( \Delta GCOMP ) t</td>
<td>0.037 0.073</td>
<td>-0.020 0.087</td>
</tr>
<tr>
<td>( \Delta GCOMP ) t-1</td>
<td>-0.152 0.075</td>
<td>0.029 0.088</td>
</tr>
<tr>
<td>( \Delta EMS ) t</td>
<td>-0.036 0.134</td>
<td>-0.176 0.159</td>
</tr>
<tr>
<td>( \Delta EMS ) t-1</td>
<td>0.036 0.128</td>
<td>0.164 0.152</td>
</tr>
<tr>
<td>( \Delta FS ) t</td>
<td>0.172 0.226</td>
<td>0.236 0.268</td>
</tr>
<tr>
<td>( \Delta FS ) t-1</td>
<td>0.662 0.242</td>
<td>0.068 0.287</td>
</tr>
<tr>
<td>( \Delta LNER ) t-1</td>
<td>0.610 0.188</td>
<td>-0.165 0.223</td>
</tr>
<tr>
<td>CVector t-1</td>
<td>-0.112 0.043</td>
<td>-0.143 0.051</td>
</tr>
<tr>
<td>Dummy</td>
<td>-0.010 0.020</td>
<td>0.239 0.024</td>
</tr>
</tbody>
</table>

**Diagnostic tests**

<table>
<thead>
<tr>
<th>Test</th>
<th>Coeff. Std. Error</th>
<th>Coeff. Std. Error</th>
<th>Coeff. Std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>LM [4]</td>
<td>0.039 1.627</td>
<td>2.096</td>
<td>**5.434</td>
</tr>
<tr>
<td>ARCH [4]</td>
<td>0.155 1.652</td>
<td>0.868</td>
<td>0.611</td>
</tr>
<tr>
<td>Normality</td>
<td>4.231 1.522</td>
<td>2.875</td>
<td>4.514</td>
</tr>
<tr>
<td>Heteroscedasticity</td>
<td>0.433 0.550</td>
<td>0.478</td>
<td>0.753</td>
</tr>
</tbody>
</table>

Notes:
The LM[4], ARCH[4] and Heteroscedasticity tests are reported in their F versions. The test for normality is distributed as \( \chi^2_2 \). ** denotes significance at the 1% level.
As far as the first equation is concerned, the estimated coefficients on ΔLPFD, and ΔLCP, have the expected negative sign, although the second one is not significant, and the coefficient on ΔTAR, is positive. The public sector affects the RER through changes in the composition of government expenditure and changes in the fiscal surplus; in particular, the RER appreciates when GCOMP increases and FS decreases. Hence, even when the government financial balance is equal to zero, the RER may appreciate or depreciate as a result of changes in the composition of government expenditure. The coefficient on ΔLNER,, is positive and relatively large, so that in the short run a policy of nominal devaluation causes a depreciation of the RER. In contrast, variations in the RER do not depend upon monetary disequilibria, as the coefficients on current and lagged values of EMS are not statistically different from zero. Lastly, the equation determining ΔLRER2, seems well specified, as none of the diagnostic tests are failed.

Turning to the second equation, the main economic determinants of ΔLPFD, are ΔLCP, and CVector.30 Although we are not particularly concerned with this equation, it is of some interest to notice that changes in LPFD respond negatively to deviations of the RER from its implied long-run relationship (lagged once). Put another way, the stock of foreign debt held by the private sector would increase when the actual value of the RER is below its equilibrium level, and vice versa.31 The equation passes the tests for ARCH[4], normality and heteroscedasticity, but not the LM[4] test for residual serial correlation. Despite this, we proceed the analysis with the estimated system of equations as the vector error autocorrelation test of up to

30 Changing the simultaneous model around so that ΔLRER2, enters as a regressor in the equation determining ΔLPFD, is unsuccessful, because the coefficient on ΔLRER2, is not statistically different from zero in the ΔLPFD, equation.
31 Dornbusch (1985) stresses that for some Latin American countries overvalued exchange rates were often important causes of excessive foreign borrowing.
fourth order, that is the multivariate equivalent of the single equation LM[4] test, is easily accepted ($F_{16,140} = 1.216$).

5.4.6 Solution of the Model and Policy Analysis

In this section we simulate (or solve) the system of simultaneous equations, in order to obtain the predictions of the system for the values of its endogenous variables, that is $\Delta \text{LRER}_t$, $\Delta \text{LPFD}_t$, $\text{LRER}_t$, and $\text{LPFD}_t$. The simulation of the model begins in 1972 and runs forward dynamically until 1995. Actual values in the year 1971 are supplied as initial conditions for the endogenous variables, and historical series beginning in 1971 and ending in 1995 are used for the non-modelled variables. For the period up to 1992, that is the period of estimation, the simulation corresponds to an "ex-post" or "historical" simulation, while for the period 1993-1995 the simulation corresponds to an "ex-post" forecast.

In Figure 5.12 we compare actual values of LRER with the predictions obtained from the ex-post simulation over the period of estimation. Inspection of the graph shows that the simulation performance of the model is particularly good, with the simulated series of LRER reproducing the long-run behaviour of the actual series. The simulated series predicts the main turning points in the historical data, and tracks the historical data closely since the mid 1980s.
The ability of the model to predict beyond the estimation period is evaluated with an ex-post forecast, in which the model is simulated forward starting in year 1993, and continuing as long as historical data of the non-modelled variables are available, that is 1995. The results of this simulation are presented in Figure 5.13. The first important aspect to be noticed is that the plots of the simulated and actual series are reasonably coincident, with the former reproducing the downward trend that the latter has been exhibiting since the early 1990s. Between 1993 and 1994 the decline predicted by the model is less accentuated than what actually occurred. In 1995, the ex-post forecast of the index of the RER is on average 83.91, which is very close to the actual value (i.e. 85.06).
Lastly, it is of some interest to change the time path of some non-modelled policy variables, in order to examine what might have taken place as a result of alternative policies. In particular, since the public sector affects the behaviour of the RER through changes in GCOMP and FS, we examine the economic consequences that would have resulted had these fiscal variables followed a different time path. We thus perform a simulation experiment for the period 1993-1995, in which we assume that GCOMP and FS remain at the average levels observed over the period 1970-1992. The results indicate that if the two fiscal variables had followed the alternative time paths, the RER would have appreciated less than what actually occurred; more specifically, the model predicts an average RER of 98.18 and 91.94 in 1994 and 1995, respectively, compared to ex-post forecasts for the same years of 95.16 and 83.91 (see Figure 5.14).

Historical data show that GCOMP increases from an average of 66% during 1970-1992, to 74% during 1993-1995. On the other hand, from 1970 to 1992 the fiscal balance of the central government was on average close to zero; in 1993 and 1995 the deficit amounted to 0.8% and 2.7% of GDP, respectively, and in 1994 the fiscal balance showed a surplus of 0.7% of GDP. The favourable fiscal performance of the government in 1994 was linked to the increased revenues accruing from privatisations and concessions.
5.5 CONCLUDING REMARKS

In this chapter we have used Johansen's analysis of cointegrated systems to build a model of the Colombian RER. The modelling exercise was built upon the Edwards model of RER determination, which states that real factors and macroeconomic policies affect the RER in the short run, but in the long run only real factors affect the sustainable equilibrium level of the RER. In general terms our results coincided with the Edwards model. In particular, we found one cointegrating vector, which can be thought of as a long-run RER equation. The RER appreciates as a result of increases in the price of coffee and in the stock of foreign debt held by the private sector, while it depreciates as a result of increases in import tariffs. The ratio of central government's current expenditure to total expenditure is not needed in the cointegration space, but it is nonetheless significant for modelling the short-run dynamics. Technological progress, proxied by a time trend, is not significant.

The modelling of the RER short-run dynamics was based on a system of two equations, given that the stock of foreign debt held by the private sector is not weakly exogenous. Estimation of the system indicated that a) the set of real factors affect the
short-run behaviour of the RER; b) the RER appreciates when the ratio of central
government’s current expenditure to total expenditure increases, and when the fiscal
surplus as a percentage of GDP decreases; c) in the short run a policy of nominal
devaluation causes a depreciation of the RER; and d) variations in the RER do not
depend upon monetary disequilibria.

We interpreted the deviations of the RER from its long-run equilibrium
relationship, after correcting for the short-run dynamics, as a measure of RER
misalignment. The actual method to estimate such measure is already contained
within Johansen’s estimation procedure, and does not require the use of either
moving averages or the Beveridge-Nelson decomposition to correct for the short-run
dynamics. The derived measure of exchange rate misalignment fluctuates within a

Finally, we simulated the system of simultaneous equations, in order to obtain
predictions for the RER. The simulation performance of the model during the period
of estimation is particularly good, with the simulated series of the RER reproducing
the long-run behaviour of the actual series. More importantly, the simulation of the
model beyond the estimation period is also successful, with the simulated series
reproducing the downward trend that the actual series has been exhibiting since the
early 1990s. A policy analysis experiment indicated that during the period 1993-
1995, the RER would have appreciated less than what actually occurred, if the fiscal
deficit and the ratio of central government’s current expenditure to total expenditure
had remained at their average levels for the period 1970-1992.
APPENDIX 5.1

CONSTRUCTION OF THE REAL EXCHANGE RATE INDEXES

Two indexes of the RER were used in the empirical calculations:

\[
\text{RER}_1 = \left( \sum_{j=1}^{k} \alpha_{ij} \cdot E_{ij} \cdot WPI_{j1}^* \right) / \text{WPI}_t
\]

and

\[
\text{RER}_2 = \left( \sum_{j=1}^{k} \alpha_{ij} \cdot E_{ij} \cdot WPI_{j1}^* \right) / \text{CPI}_t,
\]

where \( \alpha_{ij} \) is the share of country \( j \) in Colombia’s trade; although the trade weights have the subscript \( t \), in practice they are kept constant during the periods 1970-1974, 1975-1979, 1980-1984, 1985-1989 and 1990-1992. \( E_{ij} \) is an index of the nominal exchange rate between country \( j \) and Colombia in period \( t \); \( WPI_{j1}^* \) is the wholesale price index of country \( j \) in period \( t \); and \( \text{CPI}_t \) is the consumer price index of Colombia in period \( t \). The base year is 1986 so that the RER is equal to 100 in that year.

The trade weights are calculated with data from the Directions of Trade Statistics of the IMF. The figures in parentheses are percentages of average trade weights, before normalisation, for the periods 1970-1974, 1975-1979, 1980-1984, 1985-1989 and 1990-1992: U.S. (39.6, 34.6, 32.1, 36.3 and 39.2); Germany (11.4, 13.2, 10.9, 10.5 and 7.9); Venezuela (1.3, 5.6, 7.1, 3.8 and 5.9); Japan (5.4, 5.9, 7.8, 7.0 and 5.6); Netherlands (3.1, 3.7, 2.7, 3.5 and 2.7); Spain (4.4, 3.0, 2.7, 2.4 and 2.0); France (2.4, 3.1, 2.8, 3.0 and 2.7); UK (3.2, 2.6, 2.1, 2.4 and 2.6); Ecuador (1.9, 2.1, 1.9, 1.2 and 1.4); Italy (2.0, 2.0, 2.9, 1.8 and 1.7); Canada (2.2, 2.0, 2.7, 2.4 and 1.9); Sweden (2.2, 2.6, 1.9, 1.6 and 0.9); Switzerland (1.9, 1.3, 1.2, 1.3 and 1.5); and Mexico (1.2, 1.0, 1.2, 1.7 and 1.6). The trade weights of these 14 countries add up 82.2%, 82.7%, 79.8%, 79.0% and 77.8%.
The nominal exchange rates and the WPIs of the 14 countries listed above were obtained from the International Financial Statistics of the IMF; it is worth mentioning that for France and Ecuador we use their respective CPIs, as the WPIs were not available. The source for Colombia’s CPI is Banco de la República (1993).
CHAPTER 6

COFFEE, ECONOMIC FLUCTUATIONS AND STABILISATION:
AN INTERTEMPORAL DISEQUILIBRIUM MODEL WITH
CAPITAL MARKET IMPERFECTIONS

6.1 INTRODUCTION

The analysis of the effects of variations in the world price of coffee on output fluctuations in Colombia, has been a subject of extensive research. Díaz-Alejandro (1976) and Cuddington (1986), for instance, relate the behaviour of the price of coffee to economic phases in Colombia during the post second world war period. Carkovic (1986) and Gaviria and Uribe (1993) find a positive relationship between the world price of coffee and GDP fluctuations. Ocampo (1989) finds that the behaviour of coffee in international markets, has been the primary driving force of the country's business cycle. Cárdenas (1991) finds that a substantial part of the business cycle of Colombia, Costa Rica, Ivory Coast and Kenya, can be explained by temporary fluctuations in the world price of coffee. In addition, Cárdenas detects that the business cycles of these coffee-producing countries are highly correlated, suggesting that these otherwise structurally different countries have been subject to the same kind of external shock. Lastly, within the group of coffee-producing countries, business cycle fluctuations seem to be less pronounced in Colombia, which can be explained by the degree to which the domestic price of coffee is
stabilised (in Colombia coffee producers have direct influence on coffee policy), as well as by the countercyclical response of government expenditure.¹

Despite the extensive empirical evidence, relatively few studies have attempted to formalise the relationship between the world price of coffee and the business cycle. Cárdenas (1991) presents two models of export-led business cycles. The first model focuses on the real aspects, and is based on the Keynesian view of macroeconomic fluctuations. The key distinctive aspect of this output determination model, is that government expenditure is allowed to respond endogenously to external shocks; the response of the government is thus crucial in the model, since a procyclical (countercyclical) government expenditure reinforces (offsets) the effect of a coffee export boom on output. The second model, a modified version of the models used by Eastwood and Venables (1982), Neary and van Wijnbergen (1984, 1986), and Edwards (1986b), focuses on the effects of foreign exchange accumulation on the money supply, and emphasises the role of the real exchange rate in determining the production structure of the economy.

Another theoretical model is that of Montenegro (1991), who addresses the effects of external shocks and stabilisation policy measures, on the allocation of resources (more precisely labour) in a small open economy, which is assumed to have some of the characteristics of the Colombian economy. Montenegro’s starting point is the (static) tradables-nontradables model with microeconomic foundations, in both its flexible and fixed price versions; the model is augmented with the inclusion

¹During the coffee boom of 1976-79, the Kenyan government chose to pass the price increase on to domestic producers. In addition, government expenditure did not play a countercyclical role in the economy, as the temporary windfall induced a larger increase in public expenditure than in government revenue. See Bevan et. al. (1987, 1990) for an analysis of the effects of this coffee boom in Kenya.
of a “booming” tradable sector, as in standard Dutch disease models (see e.g. Corden and Neary 1982, and Neary and van Wijnbergen 1986).

The theoretical models of Cárdenas and Montenegro are clearly important, since they provide a formal theory of the possible transmission mechanism from export prices to output fluctuations. Nonetheless, both Cárdenas and Montenegro note that their models have important drawbacks. First, in the models of Cárdenas the functional forms are ad-hoc in the sense that they are not derived from microeconomic principles. Second, Montenegro models in a relatively simple way the utility maximising problem, since consumers do not take into account their whole life-time utility functions and budget constraints. Third, the models of both Cárdenas and Montenegro are static, so that it is not possible to distinguish between temporary, anticipated and permanent coffee price shocks; that is, intertemporal issues are not treated.

Taking the above aspects into consideration, in this chapter we develop an intertemporal (two-period) disequilibrium macroeconomic model with microeconomic foundations, in order to analyse the effects of a coffee price boom on a small open economy. The basic model, built on previous work by Cuddington and Viñals (1986), Fender and Nandakumar (1987) and Rankin (1994), assumes that some prices are fixed above their market clearing levels, so that agents in the long side of the market will be rationed. We opt for the disequilibrium (or quantity-constrained) approach, in preference to the more classical view of Walrasian equilibrium in all markets, because, as indicated by Rankin (1994), it adds the realism of unemployment. The intertemporal dimension of the model implies that there will be at least twelve possible market configurations, depending on the type of disequilibrium in the labour and goods markets. Instead of analysing all possible
rationing regimes, which might become tedious, we consider three configurations that may well capture some of the characteristics not only of the Colombian economy, but also of other developing countries that rely on exports of primary commodities. In particular, following the terminology used in the analysis of disequilibrium models (see Cuddington et al., 1984), these configurations are: a) Keynesian unemployment in the short and long run; b) Keynesian unemployment in the short run and Walrasian equilibrium in the long run; and c) orthodox Keynesian unemployment in the short and long run.

The first variant of the basic model is then extended by including a government sector that is in charge of the administration of a coffee price stabilisation fund (or commodity marketing board), which is aimed at reducing the effects of commodity price instability on the economy. In addition, we allow for the presence of capital market imperfections, motivated by the belief that this is a relevant assumption for the Colombian economy. With reference to the earlier static models of Cárdenas (1991) and Montenegro (1991), one might want to argue that their assumption that agents cannot borrow and lend at all is too strong, in view of the major liberalising reforms that the Colombian government has been implementing since 1989. But, on the other hand, to assume perfect borrowing and lending opportunities is also too strong.

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2See Urrutia (1994) for a description of these reforms.
3Recent empirical evidence for Colombia by López (1994) indicates that the fraction of income accruing to consumers who are liquidity constrained, often referred to as the excess sensitivity parameter, varies between 60 to 75%. Vaidyanathan's (1993) estimate of this parameter for Colombia is 62%. These results fail to provide much support for the permanent income hypothesis and the Ricardian equivalence proposition. Haque and Montiel (1989) identify the existence of liquidity constraints as the factor that explains the rejection of Ricardian equivalence in fifteen out of a group of sixteen developing countries (not including Colombia).
Capital market imperfections are initially modelled as in van Wijnbergen (1987), that is, we assume that the government can borrow on more favourable terms in international capital markets than individuals. In a subsequent stage of the analysis, we use an alternative modelling strategy in which we assume that individuals face an upward-sloping supply of capital funds. As will be seen, when individuals cannot borrow at the same interest rate as the government, the private sector’s intertemporal budget constraint is nevertheless still linear, so that there is no impediment to “consumption smoothing” by private agents. On the contrary, when individuals face an upward-sloping supply of capital funds, the private sector’s intertemporal budget constraint turns out to be non-linear.

The outline of the chapter is as follows: Section 2 describes the structure of the basic model, and analyses the effects of temporary, anticipated and permanent coffee price shocks on a small open economy under the three rationing regimes mentioned above. Section 3 extends the first variant of the basic model, by including a coffee price stabilisation fund, and capital market imperfections as modelled by van Wijnbergen (1987). Section 4 presents a simplified version of the basic model, and introduces the assumption that the economy faces an upward-sloping supply of capital funds. Section 5 examines the basic model in the presence of a coffee price stabilisation fund, and an upward-sloping supply of capital funds. Section 6 offers some concluding remarks.

6.2 THE BASIC MODEL

The basic model is an intertemporal (two-period) macroeconomic model with microeconomic foundations of a small open economy. Throughout the analysis we
occasionally refer to period 1 as the present and period 2 as the future, although they can be alternatively thought of as the short and long run, respectively. The economy produces nontradable goods, tradable goods, and a booming tradable good which we refer to as coffee. For simplicity, all three goods are assumed to be nonstorable and used for final consumption; coffee, however, is not consumed domestically. Given that we are not interested in the effects of monetary policy, we assume that there is no money in the economy, which implies that all prices will be relative. The price of tradables is chosen as the numeraire, and the real exchange rate is defined as the relative price of nontraded to traded goods, so that a rise (fall) corresponds to a real exchange rate appreciation (depreciation). We initially assume that there is no public sector, although this assumption is relaxed later on by incorporating a government that is in charge of the administration of a coffee price stabilisation fund (or commodity marketing board). All economic agents are assumed to have perfect foresight.

The small open economy assumption implies not only that the country's aggregate supply of tradables (and coffee) can always be sold at the prevailing world prices, but also that the economy never faces quantity constraints when buying or selling tradable goods in the world market. In this sense, an excess demand for tradables is therefore cleared through adjustments in quantities: exports are redirected to the domestic market and imports are increased, that is, a trade deficit is opened up with the rest of the world. In contrast, in the nontradable goods market prices are assumed to be arbitrarily fixed not necessarily at the clearing market level, which brings the possibility of excess supply and excess demand in the model. As the

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4This captures the common assumption in intertemporal models that current prices are fixed, and future prices are flexible ensuring Walrasian equilibrium. In the preceding chapters, we have used the
relative price of nontradables may not equate supply and demand, it becomes possible to have rationing in the nontradable goods market; as a result, the short side rule, i.e. trade = min (demand, supply), determines the actual quantity traded in that market, and agents in the long side of the market face rationing.

In the model labour is the only variable factor of production; it is freely mobile within the economy, but internationally immobile. The wage rate (in terms of tradables) is assumed to be arbitrarily fixed at a level that does not necessarily equate supply and demand, so that it is also possible to have rationing in the labour market. For example, when the wage rate is fixed below the market clearing level, the shortage of the factor of production affects the three sources of demand for labour, that is nontradable producers, tradable producers and coffee producers.

In the context of our intertemporal model, there are various possible configurations (or regimes) depending upon the type of disequilibrium in the nontradables and labour markets. For our purposes, however, we consider the following three variants of the basic model. In the first variant, we assume that in periods 1 and 2 the wage rate and the relative price of nontradables are fixed above their market clearing levels, which implies excess supply in the labour and nontradables markets. Following the terminology used in the analysis of disequilibrium (or quantity-constrained) models, this specification corresponds to the case where there is Keynesian unemployment in the short and long run.¹

¹Rankin (1994) uses this specification in a model with two goods that are assumed to be imperfect substitutes.
In a second variant of the model we aim at capturing the notion of short-run price rigidity and long-run market clearing. In order to do this, we assume that in period 1 the wage rate and the relative price of nontradables are fixed at a level such that there is excess supply in the labour and nontradables markets. In period 2, the wage rate and the price of nontradables fully adjust ensuring market clearing. This specification corresponds to the case where there is Keynesian unemployment in the short run and Walrasian equilibrium in the long run.³

Lastly, in the third variant of the model we assume that in periods 1 and 2 the wage rate is fixed above the market clearing level, which results in excess supply in the labour market. The relative price of nontradables, on the other hand, fully adjusts ensuring market clearing. Cuddington et. al. (1984) refer to this configuration as orthodox Keynesian unemployment.

6.2.1 THE HOUSEHOLD SECTOR

The model assumes that consumers derive utility from consuming nontradable and tradable goods in periods 1 and 2; as we indicated above, coffee is not consumed domestically but exported to the rest of the world. In order to ease the derivation of the results we assume, as is often the case in intertemporal disequilibrium models, that consumers' preferences can be adequately summarised with an additive time-separable, log linear utility function (U); more specifically:

\[ U = \alpha \log C_{N1} + (1 - \alpha) \log C_{T1} + \beta \{ \alpha \log C_{N2} + (1 - \alpha) \log C_{T2} \}, \]  

[6.1]

³This specification is often adopted in intertemporal disequilibrium models. See e.g. Cuddington and Vihals (1986), Fender and Nandakumar (1987), Fender and Yip (1989), and van Wijnbergen (1985, 1987), among others.
where $C_{NI}$ ($C_{N2}$) represents consumption of the nontradable good in period 1 (2); $C_{T1}$ ($C_{T2}$) denotes consumption of the tradable good in period 1 (2); $\beta = (1 + \delta)^{-1}$, where $\delta$ is the time preference rate which is assumed to be constant; and $0 < \alpha < 1$. As it is known, the use of this particular functional form implies zero cross-price effects in the resulting commodity demand functions.

In period 1 consumers derive income from the sale of nontradables output ($Y_{NI}$), tradables output ($Y_{T1}$) and coffee ($Y_{C1}$), and this income is in turn used to buy nontradables ($C_{NI}$) and tradables ($C_{T1}$). We assume that consumers can lend or borrow at the interest rate (in terms of tradables) $\bar{r}$, which is equal to the world interest rate. Thus, if consumers' income in period 1 is greater than the value of goods they want to consume in that period, then they acquire domestic and/or foreign bonds by an amount equal to $B_1$; more formally,

$$ p_{NI} C_{NI} + C_{T1} + B_1 = p_{NI} Y_{NI} + Y_{T1} + p^*_{C1} Y_{C1}, \quad [6.2] $$

where $p_{NI}$ denotes the relative price of nontradables in period 1, and $p^*_{C1}$ denotes the world relative price of coffee in period 1. By saving $B_1$ consumers will receive in period 2 an amount equal to $B_1 (1 + \bar{r})$, so that their consumption in period 2 can exceed their income by that amount. Defining the gross interest rate $\bar{R} = 1 + \bar{r}$, then:

$$ p_{N2} C_{N2} + C_{T2} = p_{N2} Y_{N2} + Y_{T2} + p^*_{C2} Y_{C2} + B_1 \bar{R}. \quad [6.3] $$

As can be observed, we are implicitly assuming that consumers end period 2 holding no bonds.7 Solving [6.3] for $B_1$, and substituting the resulting expression in [6.2], we derive the household's intertemporal budget constraint:

7If consumers borrowed in period 1, so that $B_1 < 0$, they have to repay $B_1 (1 + \bar{r})$. 

which simply restricts the present value of consumption spending on nontradables and tradables to equal the present value of output, that is wealth $W$. The household’s problem reduces then to maximising (6.1) subject to the intertemporal budget constraint stated in (6.4). The solution of this constrained maximisation problem yields the following commodity demand functions for nontradables and tradables in periods 1 and 2:

$$C_{N1} = \frac{\alpha}{(1 + \beta)} \frac{W}{p_{N1}}$$  \hspace{1cm} (6.5a)

$$C_{T1} = \frac{(1 - \alpha)}{(1 + \beta)} W$$  \hspace{1cm} (6.5b)

$$C_{N2} = \frac{\alpha \beta R}{(1 + \beta)} \frac{W}{p_{N2}}$$  \hspace{1cm} (6.5c)

and

$$C_{T2} = \frac{(1 - \alpha) \beta R}{(1 + \beta)} W.$$  \hspace{1cm} (6.5d)

### 6.2.2 The Production Sector

Turning to the production side of the model, in each of the two periods nontradables, tradables and coffee are produced using labour, which constitutes the only variable factor of production. In the absence of market distortions, labour mobility within the economy guarantees that the wage rate (in terms of the numeraire) must be the same across sectors; that is, labour will move from low-wage sectors to high-wage sectors
until wages are equalised. In each period, the wage rate is determined by the requirement that total employment equals total labour supply; hence, the labour market equilibrium condition in period \( t \) is given by:

\[
L_t = L_{N,t} + L_{T,t} + L_{C,t},
\]

where \( L \) is the exogenous total labour supply, and \( L_N, L_T \) and \( L_C \) denote the demand for labour in nontradables, tradables and coffee sectors, respectively. The exogeneity of the total labour supply is a result of assuming no utility of leisure, so that households supply all of their time endowment to the labour market.

The problem faced by producers is that of choosing labour input to maximise profits, subject to the sector's production function. Taking first the case of nontradable producers in period \( t = 1, 2 \), the firm's problem is given by:

\[
\text{Max } \Pi_{N,t} = \rho_{N,t} Y_{N,t} - w_t L_{N,t} \quad \text{subject to } \quad Y_{N,t} = f_{N,t}(L_{N,t}),
\]

where \( \Pi_{N,t} \) denotes profits in the nontradables sector, \( w_t \) is the wage rate, and \( f_{N,t} \) represents the twice continuously differentiable and strictly concave production function in the nontradables sector. The first order condition for this maximisation problem is given by:

\[
f_{N,t}'(L_{N,t}) = \frac{w_t}{\rho_{N,t}},
\]

which can be inverted to produce the usual labour demand function.

\[
L_{N,t} = f_{N,t}^{-1} \left( \frac{w_t}{\rho_{N,t}} \right) = L_{N,t} \left( \frac{w_t}{\rho_{N,t}} \right),
\]

that is a negative function of the real product wage. The firm's supply of nontradables can be then obtained by substituting the previous labour demand into the production function, i.e.:
so that the supply of nontradables depends negatively on the wage rate, and positively on the product's price.

Turning to the maximisation problem of tradable producers, it can be stated as follows (in $t = 1, 2$):

$$\max_{L_{1,t}} \Pi_{T,t} = Y_{T,t} - w_t L_{T,t} \quad \text{subject to} \quad Y_{T,t} = f_{T,t}(L_{T,t}),$$

where $\Pi_{T,t}$ denotes profits in the tradables sector, and $f_{T,t}$ represents the sector's production function, assumed to be twice continuously differentiable and strictly concave. From this maximisation problem we obtain the sector's labour demand function

$$L_{T,t} = f'_{T,t}^{-1}(w_t) = L_{T,t}(w_t),$$

that is a decreasing function of the wage rate. The firm's supply of tradables is thus given by:

$$Y_{T,t} = Y_{T,t}(w_t),$$

so that the supply of the commodity is a decreasing function of the wage rate (it should also be noticed that $Y_{T,t}$ is exogenous as long as $w_t$ is exogenous).

Lastly, the profit maximisation problem of coffee producers takes the form (in $t = 1, 2$):

$$\max_{L_{C,t}} \Pi_{C,t} = p_{C,t}^* Y_{C,t} - w_t L_{C,t} \quad \text{subject to} \quad Y_{C,t} = f_{C,t}(L_{C,t}),$$

where $\Pi_{C,t}$ denotes profits in the coffee sector, and $f_{C,t}$ is the production function in the coffee sector, assumed to be well behaved. From this maximisation problem we derive the labour demand function:
Let \( f(c_t) \) be a decreasing function of the real wage rate. Substituting the previous labour demand into the production function we obtain the corresponding firm’s supply of coffee:

\[
Y_{C,t} = Y_{C,t} \left( \frac{W_t}{P_{C,t}} \right),
\]

At this stage of the analysis, we assume that in the coffee sector the demand for labour is completely inelastic to changes in the real wage rate. This assumption aims at capturing the idea that once coffee trees have been planted, coffee producers will employ in each period the labour force required to collect the coffee fruits, no matter the real wage rate. In terms of Corden and Neary’s (1982) analysis of the Dutch disease phenomenon, this assumption allows us to focus on the so-called “spending effect”, and rule out the “resource movement effect”; i.e., the latter can be regarded as negligible. Hence, in the model the coffee sector may be alternatively thought of as an “enclave” type sector, that is, a sector that uses very specific factors of production, so that it does not compete for production factors (in this case labour) with the other sectors of the economy. Within this framework, the labour market equilibrium condition given by (6.6) must be modified to

\[
L_t - L_{C,t} = \hat{L}_t = L_{N,t} + L_{T,t},
\]

\[6.6a\]

It should be remembered that the resource movement effect arises from the fact that a price boom (or alternatively an improvement in technology, or a natural resource discovery) increases the profitability of the booming tradable sector, so that it will draw resources out of the other two non-booming sectors. The spending effect arises from the higher real income resulting from the favourable shock, which increases the demand for both non-booming tradables and nontradables.
where $\bar{L}_t$ is the total labour supply available to the nontradables and tradables sectors. Also, the labour demand functions in the coffee sector become independent of the real wage rate, that is $L_{ct} = \bar{L}_{ct}$ ($t = 1, 2$). Substituting these labour demand functions into the production functions we derive the firm's supply of coffee as $Y_{ct} = Y_{ct}\left(\bar{L}_{ct}\right)$, for $t = 1, 2$.

### 6.2.3 Market Equilibrium: The KK Configuration

Having described the microeconomic behaviour of the households and firms, we proceed to analyse the effects of different types of coffee price shocks in the presence of Keynesian unemployment in the short and long run, which we refer to as the KK configuration (or regime). As noted previously, the KK configuration arises when we assume that $w_t$ and $p_{ni}(t = 1, 2)$ are fixed above their market clearing levels. The short side rule thus tells us that under these circumstances the output level of nontradables will be demand determined; more formally, the $[IS1]$ equation

$$Y_{ni} = C_{ni},$$

denotes the market equilibrium condition in period 1, and the $[IS2]$ equation

$$Y_{n2} = C_{n2},$$

is the corresponding expression in period 2. Substituting the commodity demand functions for nontradables given in [6.5a] and [6.5c] into the $[IS1]$ and $[IS2]$ equations we obtain:

$$Y_{ni} = \frac{\alpha W}{(1 + \beta) p_{ni}}$$  [IS1]

and
This pair of equations, jointly with the expression for wealth given in [6.4], constitute a system of equations that determine three endogenous variables: $Y_{N1}$, $Y_{N2}$, and $W$. The dimension of the system can be easily reduced by substituting [6.4] into the IS1 and IS2 equations, which yields:

\[ p_{N1} Y_{N1} = \frac{\alpha \bar{R}}{(1 + \beta)} \left\{ p_{N1} Y_{N1} + p_{C1} Y_{C1} + \frac{p_{N2} Y_{N2} + p_{C2} Y_{C2}}{\bar{R}} + \bar{Y}_t \right\} \]  \[ \text{[6.9a]} \]

and

\[ p_{N2} Y_{N2} = \frac{\alpha \bar{R}}{(1 + \beta)} \left\{ p_{N1} Y_{N1} + p_{C1} Y_{C1} + \frac{p_{N2} Y_{N2} + p_{C2} Y_{C2}}{\bar{R}} + \bar{Y}_t \right\}, \]  \[ \text{[6.9b]} \]

where tradables output is treated as constant since $w_1$ and $w_2$ are assumed to be exogenous, i.e. $\bar{Y}_t = Y_{T1} + \bar{R}^{-1} Y_{T2}$.

After regrouping terms the IS1 equation may be alternatively written as:

\[ \left[ 1 - \frac{\alpha}{(1 + \beta)} \right] p_{N1} Y_{N1} - \frac{\alpha}{(1 + \beta)\bar{R}} p_{N2} Y_{N2} = \frac{\alpha}{(1 + \beta)} \left\{ p_{C1} Y_{C1} + \frac{p_{C2} Y_{C2}}{\bar{R}} + \bar{Y}_t \right\} \]

and the IS2 equation as:

\[ -\frac{\alpha \bar{R}}{(1 + \beta)} p_{N1} Y_{N1} + \left[ 1 - \frac{\alpha \beta}{(1 + \beta)} \right] p_{N2} Y_{N2} = \frac{\alpha \bar{R}}{(1 + \beta)} \left\{ p_{C1} Y_{C1} + \frac{p_{C2} Y_{C2}}{\bar{R}} + \bar{Y}_t \right\}.

Differentiating this pair of equations, setting $dY_{C1} = dY_{C2} = d\bar{Y}_T = 0$, and imposing the mild condition that $Y_{C1} = Y_{C2} = Y_c$ in order to simplify the algebra, we obtain the following expressions for the IS1 and IS2 schedules:

\[ \left[ 1 - \frac{\alpha}{(1 + \beta)} \right] p_{N1} dY_{N1} - \frac{\alpha}{(1 + \beta)\bar{R}} p_{N2} dY_{N2} = \frac{\alpha Y_C}{(1 + \beta)} \left[ dp_{C1} + \frac{dcp_{C2}}{\bar{R}} \right] \]  \[ \text{[6.10a]} \]

and
respectively. In Figure 6.1 we present a diagrammatic representation of the model in 
\( (Y_{N1}, Y_{N2}) \) space, similar to Rankin's (1994) variant of the van Wijnbergen (1987) model. As can be observed, the IS1 and IS2 equations give a pair of upward-sloping curves, so that there are two possibilities depending on whether the IS1 locus is steeper than the IS2 locus. Throughout the analysis we consider the case where the former is steeper than the latter, as required for the system to have a stable equilibrium (see Appendix 6.1 for an analysis of the stability of the system).

\[
- \frac{\alpha \beta R}{(1 + \beta)} p_{N1} dY_{N1} + \left[ 1 - \frac{\alpha \beta}{(1 + \beta)} \right] p_{N2} dY_{N2} = \frac{\alpha \beta R Y_C}{(1 + \beta)} \left[ \frac{dp_{C1}^*}{R} + \frac{dp_{C2}^*}{R} \right], \tag{6.10b}
\]

**Figure 6.1:** Equilibrium under the KK configuration

The initial equilibrium of the economy is depicted as point E in Figure 6.1. Consider now the effects of a temporary increase in the world price of coffee (i.e. \( dp_{C1}^* > 0, dp_{C2}^* = 0 \)). As can be seen from [6.10a] and [6.10b], the temporary price shock shifts the IS1 schedule to the right in a magnitude equal to:

\[
dY_{N1} \bigg|_{dY_{N2}=0}^{IS1} = \frac{\alpha Y_C}{(1 - \alpha + \beta) p_{N1}} dp_{C1}^*.
\]
and also induces an upward shift in the IS2 schedule in a magnitude equal to:

\[ dY_{N2}^{IS2}_{dY_{x1,0}} = \frac{\alpha \beta \bar{Y}_C}{[1 + \beta(1 - \alpha)]p_{N2}} dp_{c1}^*. \]

The new equilibrium of the economy is reached at point T, where the IS1* and the IS2* schedules cross each other, and this point is characterised by a greater level of nontradables output in both the current period and the future. The transition of the economy from E to T can be explained following similar lines to that of Neary and Stiglitz (1983) and Rankin (1994). In particular, a temporary increase in the world price of coffee raises people’s income, which in turn raises demand for nontradables consumption in period 1, and we thus have a “first round” multiplier effect because we have excess supply of labour and home goods; the process already described corresponds to the movement from E to F depicted in Figure 6.1. The problem at point F is that people’s expectations about domestic output in period 2 are too pessimistic because domestic output in period 1 has gone up, and it is part of households’ wealth; when households get more wealth, they would like to spend some of it in the future. Hence, when people adjust their expectations the economy moves from F to G. The increase in domestic output in period 2, by increasing households’ wealth, induces higher spending in period 1, and the economy then moves from G to H (“second round” multiplier effect), and so on until it reaches T. A temporary increase in the world price of coffee thus has a spill-over effect in raising domestic output in the future.

Let us now consider the effects of the anticipation of a future increase in the world price of coffee (i.e., \( dp_{c1}^* = 0 \) and \( dp_{c2}^* > 0 \)). From [6.10a] and [6.10b] it is apparent that this type of external shock induces changes in the same direction in the IS1 and IS2 schedules, so that domestic output in the current period and the future
increase as a result (the adjustment process could be described in the same terms as for the temporary shock). Accordingly, it is of some interest to note that this simple model suggests that domestic output in period 1 responds not only to temporary increases of the price of coffee, but also to the anticipation of a future increase.

Lastly, we look at the effects of a permanent increase in the world price of coffee, which can be thought of as the sum of the temporary and the anticipated shocks (that is \( \Delta p_c^1 = \Delta p_c^2 = \Delta p_c > 0 \)). As expected, domestic output increases in the current period and the future as a result of the shock. A diagrammatic representation of the effects of the three types of external shocks is depicted in Figure 6.2, where the initial equilibrium of the economy is at \( E \). An anticipated coffee price shock moves the economy from \( E \) to \( A \), the temporary shock from \( E \) to \( T \), and the permanent shock from \( E \) to \( P \). It is worth noting that Figure 6.2 is drawn under the (mild) condition that \( Y_{c1} = Y_{c2} = Y_c \), so that the effect of an anticipated coffee price shock on both \( Y_{NI} \) and \( Y_{N2} \) is less than that of a temporary shock, and this in turn is less than the effect of a permanent shock. In Table 6.1 we present the results of the comparative statics, which indicate that when the economy is under Keynesian unemployment in the short and long run, an increase in the world price of coffee (that can be either anticipated, temporary, or permanent) has a positive effect on domestic output in the current period and the future.
**Figure 6.2:** Effects of a temporary, anticipated and permanent increase in the price of coffee under the KK configuration

**Table 6.1:** Comparative statics under the KK configuration

<table>
<thead>
<tr>
<th>TYPE OF SHOCK</th>
<th>$dY_{N1}$</th>
<th>$dY_{N2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temporary</td>
<td>$\frac{\alpha Y_C}{(1 - \alpha)(1 + \beta)P_{N1}} &gt; 0$</td>
<td>$\frac{\alpha \beta Y_C}{(1 - \alpha)(1 + \beta)P_{N2}} &gt; 0$</td>
</tr>
<tr>
<td>$dp_{C1} &gt; 0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Anticipated</td>
<td>$\frac{\alpha Y_C}{(1 - \alpha)(1 + \beta)\bar{P}_{N1}} &gt; 0$</td>
<td>$\frac{\alpha \beta Y_C}{(1 - \alpha)(1 + \beta)P_{N2}} &gt; 0$</td>
</tr>
<tr>
<td>$dp_{C2} &gt; 0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Permanent</td>
<td>$\frac{\alpha (1 + \bar{R}) Y_C}{(1 - \alpha)(1 + \beta)\bar{P}_{N1}} &gt; 0$</td>
<td>$\frac{\alpha \beta (1 + \bar{R}) Y_C}{(1 - \alpha)(1 + \beta)P_{N2}} &gt; 0$</td>
</tr>
<tr>
<td>$dp_{C1} = dp_{C2} = dp_{C} &gt; 0$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
6.2.4 Market Equilibrium: The KW Configuration

In the second variant of the model we deal with the configuration that has been usually favoured in disequilibrium models: \( w_1 \) and \( p_{ni} \) are assumed fixed at a level such that there is Keynesian unemployment in the short run, and \( w_2 \) and \( p_{n2} \) are fully flexible ensuring Walrasian equilibrium in the long run. In the first period the level of output in the nontradable sector is thus demand determined, whereas in the second period it will be at its full employment level, so that external shocks will vary domestic relative prices. The market equilibrium condition in period 1 is, once again, given by the [IS1] equation:

\[
Y_{N1} = C_{N1},
\]

and the equilibrium condition in period 2 is given by the [GM2] equation:

\[
Y_{N2}\left(\frac{w_2}{p_{n2}}\right) = C_{N2}.
\]

Substituting the commodity demand functions for nontradables given in [6.5a] and [6.5c] into [IS1] and [GM2] schedules we obtain:

\[
Y_{N1} = \frac{\alpha}{1 + \beta} \frac{W}{p_{ni}} \tag{IS1}
\]

and

\[
Y_{N2}\left(\frac{w_2}{p_{n2}}\right) = \frac{\alpha \beta R}{1 + \beta} \frac{W}{p_{n2}} \tag{GM2}
\]

Equations [IS1], [GM2], [6.4] and the labour market equilibrium condition [6.6a] determine four endogenous variables: \( Y_{N1}, p_{n2}, w_2 \) and \( W \). We know that labour market equilibrium implies a unique relationship between \( p_{n2} \) and \( w_2 \), and also that an increase in \( p_{n2} \) induces a less than proportional increase in \( w_2 \), so that \( w_2 \) relative to \( p_{n2} \) decreases, and this in turn increases \( Y_{N2} \) (see Appendix 6.2).
Consequently, using this positive relationship between $p_{N2}$ and $Y_{N2}$, and substituting the expression for wealth into the [IS1] and [GM2] equations, it is possible to transform the four-variable system into a pair of equations that determine two endogenous variables: $Y_{N1}$ and $p_{N2}$; more formally, the [IS1] and [GM2] schedules are given by:

$$Y_{N1} = \frac{\alpha}{(1+\beta)p_{N1}} \left( p_{N1}Y_{N1} + Y_{T1} + p_{C1}Y_{C1} + \frac{p_{N2}Y_{N2}^{+}(p_{N2}) + Y_{T2}^{+}(p_{N2}) + p_{C2}Y_{C2}}{R} \right)$$

and

$$Y_{N2}^{+}(p_{N2}) = \frac{\alpha\beta\bar{R}}{(1+\beta)p_{N2}} \left( p_{N1}Y_{N1} + Y_{T1} + p_{C1}Y_{C1} + \frac{p_{N2}Y_{N2}^{+}(p_{N2}) + Y_{T2}^{+}(p_{N2}) + p_{C2}Y_{C2}}{R} \right)$$

respectively. As can be seen, $Y_{T2}$ is endogenous since it depends on $p_{N2}$; hence, it is now ambiguous whether an increase in $p_{N2}$ increases wealth. Rearranging terms the system can be rewritten as:

$$\left[ 1 - \frac{\alpha}{(1+\beta)} \right] p_{N1}Y_{N1} - \frac{\alpha}{(1+\beta)\bar{R}} \left[ p_{N2}Y_{N2}^{+}(p_{N2}) + Y_{T2}^{+}(p_{N2}) \right] = \frac{\alpha}{(1+\beta)} \left[ p_{C1}Y_{C1} + \frac{p_{C2}Y_{C2}}{R} + Y_{T1} \right]$$

and

$$-\frac{\alpha\beta\bar{R}}{(1+\beta)p_{N1}}Y_{N1} + \left[ 1 - \frac{\alpha\beta}{(1+\beta)} \right] p_{N2}Y_{N2}^{+}(p_{N2}) - \frac{\alpha\beta}{(1+\beta)} Y_{T2}^{+}(p_{N2}) = \frac{\alpha\beta\bar{R}}{(1+\beta)} \left[ p_{C1}Y_{C1} + \frac{p_{C2}Y_{C2}}{R} + Y_{T1} \right]$$

Differentiating the system, setting $dY_{C1} = dY_{C2} = dY_{T1} = 0$, and imposing the condition that $Y_{C1} = Y_{C2} = Y_{C}$ in order to simplify the algebra, we obtain:

$$\left[ 1 - \frac{\alpha}{(1+\beta)} \right] p_{N1}dY_{N1} - \frac{\alpha}{(1+\beta)\bar{R}} \left[ \bar{R} + Y_{T2}^{+}(p_{N2}) \right] dp_{N2} = \frac{\alpha Y_{C}}{(1+\beta)} \left[ dp_{C1} + \frac{dp_{C2}}{R} \right] \quad [6.11a]$$

and
These equations define a set of curves in $(Y_{N1}, p_{N2})$ space. As can be seen, although the sign of the slope of the IS1 schedule cannot be determined, since $\tilde{\Gamma} + Y_{T2}'(p_{N2})$ can be either positive or negative, the GM2 schedule has a positive slope. If we assume for a moment that $Y_{T2}'(p_{N2}) = 0$, then the IS1 will be an upward-sloping curve, and the system will be stable provided the IS1 schedule is steeper than the GM2 schedule. As the term $Y_{T2}'(p_{N2})$ gets larger in absolute value, the IS1 curve rotates anti-clockwise, so that its slope may become negative, and the GM2 curve rotates clockwise, so that its slope tends to zero. More formally, it can be shown that the stability condition of the model is $(1 - \alpha)\tilde{\Gamma} - \frac{\alpha \beta Y_{T2}'}{1 + \beta} > 0$, which is satisfied.  

Figure 6.3 depicts the equilibrium in the KW configuration when $\tilde{\Gamma} + Y_{T2}'(p_{N2}) > 0$. In this case, both the IS1 and GM2 are upward-sloping curves, and the former is steeper than the latter, as required for the system to have a stable equilibrium. The initial equilibrium of the economy is depicted as point E. Consider now the effects of a temporary increase in the world price of coffee. As can be seen from [6.11a] and [6.11b], the temporary price shock shifts the IS1 schedule to the right, and also induces an upward shift in the GM2 schedule, so that the new equilibrium schedules are IS1' and GM2'.

---

9This stability condition can be derived by postulating an adjustment mechanism of the form:

$\dot{Y}_{N1} = \xi_1 (C_{N1} - Y_{N1})$, $\dot{p}_{N2} = \xi_2 (C_{N2} - Y_{N2})$, where $\xi_1$ and $\xi_2$ are adjustment coefficients.
The transition of the economy from E to T is somewhat similar to that described for the economy under the KK configuration, although it is worth recalling that this time the endogenous variables are \((Y_{N1}, p_{N2})\) rather than \((Y_{N1}, Y_{N2})\). In particular, the temporary increase in the world price of coffee raises people's income which raises demand for nontradables in period 1, and we thus have a "first round" multiplier effect because we have excess supply of labour and nontraded goods in the short run. This process corresponds to the movement from E to F depicted in Figure 6.3, for a constant expected relative price of nontradables in period 2. Given that output in period 1 has gone up, households get more wealth, and this in turn allows them to increase their consumption of nontradables in period 2. The increased demand for nontradables raises the relative price of nontradables in period 2, which corresponds to the movement from F to G. Since production of nontradables is more profitable in period 2, domestic output increases in period 2, and this will induce higher spending in period 1 because households wealth has increased; this process corresponds to the movement from G to H ("second round" multiplier effect). The adjustment process continues until the economy reaches a final equilibrium at T. In
this sense, it is interesting to notice that under the KW configuration, a temporary increase in the world price of coffee results in an increase in the level of domestic output in the short run, and in a real exchange rate appreciation in the long run.

The formal expressions for the equilibrium changes in $Y_{N1}$ and $p_{N2}$ as a result of a temporary increase in the world price of coffee are presented in Table 6.2, along with the multipliers when the price shock is anticipated, and when it is permanent.

Table 6.2: Comparative statics under the KW configuration

<table>
<thead>
<tr>
<th>TYPE OF SHOCK</th>
<th>$dY_{N1}$</th>
<th>$dY_{N2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temporary</td>
<td>$\alpha Y_C \frac{\tilde{\Gamma}}{(1 + \beta) p_{N1} \Theta}$</td>
<td>$\frac{\alpha \beta Y_C}{(1 + \beta) \Theta}$</td>
</tr>
<tr>
<td>Anticipated</td>
<td>$\alpha Y_C \frac{\tilde{\Gamma}}{(1 + \beta) R p_{N1} \Theta}$</td>
<td>$\frac{\alpha \beta Y_C}{(1 + \beta) \Theta}$</td>
</tr>
<tr>
<td>Permanent</td>
<td>$\alpha (1 + \tilde{R}) Y_C \frac{\tilde{\Gamma}}{(1 + \beta) R p_{N1} \Theta}$</td>
<td>$\frac{\alpha \beta (1 + \tilde{R}) Y_C}{(1 + \beta) \Theta}$</td>
</tr>
</tbody>
</table>

where $\Theta = (1 - \alpha) \tilde{\Gamma} - \frac{\alpha \beta Y_{T2}}{\tilde{\Theta}} > 0$.

Indeed, it is possible to show that under the mild condition that $Y_{C1} = Y_{C2} = Y_C$, it follows that

\[
\frac{dY_{N1}}{dp_{C2}} < \frac{dY_{N1}}{dp_{C1}} < \frac{dY_{N1}}{dp_C} < \frac{dY_{N2}}{dp_{C1}} < \frac{dY_{N2}}{dp_{C2}}< \frac{dY_{N2}}{dp_C}
\]
It is also worth noting, that the results obtained in period 2 are consistent with the predictions of the models in the tradition of the Dutch disease (see e.g. Corden and Neary 1982, and Neary and van Wijnbergen 1986). That is, in the absence of the "resource movement effect", the real exchange rate appreciation induced by the favourable price shock, makes nontradable production more profitable relative to non-booming tradable production, so that nontradable output rises while non-booming tradable output falls.

6.2.5 Market Equilibrium: A Modified KK Configuration

In this part, we consider a modified KK configuration in which \( w_1 \) and \( w_2 \) are assumed to be fixed above their market clearing level, and \( p_{n1} \) and \( p_{n2} \) are fully flexible. In order to understand how this configuration might arise, let us begin by noting that in the absence of money, the idea that wages are fixed but prices flexible must refer to "real" wages. This, in turn, raises the question of what is the appropriate numeraire in terms of which the real wage is fixed. The natural answer is that it should be the "real consumption wage" which is fixed, which then suggests that \( w_i \) should not be treated as exogenous, but as increasing in \( p_{N,t} \). However, for the sake of simplicity we shall continue to treat \( w_i \) as exogenous: this is a reasonable assumption if the share of nontradable goods in the consumption-based price index (i.e. the parameter \( \alpha \)) is sufficiently small.

Within this framework, the market equilibrium conditions in the short and long run will be given by the following "modified" [IS1] and [IS2] schedules:

\[
Y_{N1}^{(*)} = \frac{\alpha}{(1+\beta)p_{N1}} \left[ p_{N1} Y_{N1}^{(*)} + p_{C1} Y_{C1} + \frac{p_{N2} Y_{N2}^{(*)}}{R} + p_{C2} Y_{C2} + \bar{Y}_T \right]
\]
and

\[ Y_{N2}^{(*)}(p_{N2}) = \frac{\alpha \beta R}{(1 + \beta)p_{N2}} \left\{ p_{N1} Y_{N1}(p_{N1})^{(*)} + p_{C1} Y_{C1} + \frac{p_{N2} Y_{N2}(p_{N2})^{(*)} + p_{C2} Y_{C2}}{R} + \bar{Y}_T \right\}, \]

respectively, into which we have already substituted the commodity demand functions for nontradables given in [6.5a] and [6.5c], and the expression for wealth given in [6.4]. As can be seen, provided the wage rate is fixed, an increase in the relative price of nontradables will increase nontradables output, but will leave tradables output unaffected (this explains why tradables outputs can be conveniently grouped as the constant term \( \bar{Y}_T \)). Rearranging terms, the pair of equations can be alternatively written as:

\[
\left[ 1 - \frac{\alpha}{(1 + \beta)} \right] p_{N1} Y_{N1}(p_{N1}) - \frac{\alpha}{(1 + \beta)R} p_{N2} Y_{N2}(p_{N2}) = \frac{\alpha}{(1 + \beta)} \left[ p_{C1} Y_{C1} + \frac{p_{C2} Y_{C2}}{R} + \bar{Y}_T \right]
\]

and

\[
-\frac{\alpha \beta R}{(1 + \beta)} p_{N1} Y_{N1}(p_{N1}) + \left[ 1 - \frac{\alpha \beta}{(1 + \beta)} \right] p_{N2} Y_{N2}(p_{N2}) = \frac{\alpha \beta R}{(1 + \beta)} \left[ p_{C1} Y_{C1} + \frac{p_{C2} Y_{C2}}{R} + \bar{Y}_T \right].
\]

Differentiating the system, and after imposing the usual simplifications, we obtain:

\[
\left[ 1 - \frac{\alpha}{(1 + \beta)} \right] \Gamma dp_{N1} - \frac{\alpha}{(1 + \beta)R} \tilde{\Gamma} dp_{N2} = \frac{\alpha Y_{C1}}{(1 + \beta)} \left[ dp_{C1} + \frac{dp_{C2}}{R} \right] \quad [6.12a]
\]

and

\[
-\frac{\alpha \beta R}{(1 + \beta)} \Gamma dp_{N1} + \left[ 1 - \frac{\alpha \beta}{(1 + \beta)} \right] \tilde{\Gamma} dp_{N2} = \frac{\alpha \beta R Y_{C1}}{(1 + \beta)} \left[ dp_{C1} + \frac{dp_{C2}}{R} \right], \quad [6.12b]
\]

where \( \Gamma = Y_{N1}(p_{N1}) + p_{N1} Y'_{N1}(p_{N1}) > 0 \).

The pair of equations [6.12a] and [6.12b] determines two endogenous variables: \( p_{N1} \) and \( p_{N2} \). These two loci are depicted in Figure 6.4, under the
assumption that the former is steeper than the latter, as required for the system to have a stable equilibrium. The initial equilibrium is attained at point E, and the comparative statics is qualitatively the same as in the first two versions of the model; that is, an increase in the world price of coffee (whether temporary, anticipated or permanent) shifts the modified IS1 schedule to the right, and also induces an upward shift in the modified IS2 schedule.

**Figure 6.4**: Effects of a temporary, anticipated and permanent increase in the price of coffee under the “modified” KK configuration

The formal expressions for the multiplier effect are presented in Table 6.3. In particular, a temporary increase in the world price of coffee increases the relative price of nontradables (i.e. appreciates the real exchange rate) in periods 1 and 2, and this in turn induces an increase in the production of nontradables in both periods. An anticipated price shock also has a positive effect on the relative price of nontradables in the short and long run, but it is less than the effect of the temporary shock. The
effect of the permanent price shock is positive, as it is the sum of the previous two shocks.

Table 6.3: Comparative statics under the modified KK configuration

<table>
<thead>
<tr>
<th>TYPE OF SHOCK</th>
<th>$dY_{N1}$</th>
<th>$dY_{N2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temporary</td>
<td>$\frac{\alpha Y_c}{(1-\alpha)(1+\beta)\Gamma} &gt; 0$</td>
<td>$\frac{\alpha \beta R Y_c}{(1-\alpha)(1+\beta)\Gamma} &gt; 0$</td>
</tr>
<tr>
<td>$dp_{c1}^* &gt; 0$</td>
<td>[\text{<strong>Temporary</strong>} \quad dp_{c1}^* &gt; 0]</td>
<td>[\text{<strong>Temporary</strong>} \quad dp_{c1}^* &gt; 0]</td>
</tr>
<tr>
<td>Anticipated</td>
<td>$\frac{\alpha Y_c}{(1-\alpha)(1+\beta)\Gamma} &gt; 0$</td>
<td>$\frac{\alpha \beta Y_c}{(1-\alpha)(1+\beta)\Gamma} &gt; 0$</td>
</tr>
<tr>
<td>$dp_{c2}^* &gt; 0$</td>
<td>[\text{<strong>Anticipated</strong>} \quad dp_{c2}^* &gt; 0]</td>
<td>[\text{<strong>Anticipated</strong>} \quad dp_{c2}^* &gt; 0]</td>
</tr>
<tr>
<td>Permanent</td>
<td>$\frac{\alpha (1 + R) Y_c}{(1-\alpha)(1+\beta)\Gamma} &gt; 0$</td>
<td>$\frac{\alpha \beta (1 + R) Y_c}{(1-\alpha)(1+\beta)\Gamma} &gt; 0$</td>
</tr>
<tr>
<td>$dp_{c1}^* = dp_{c2}^* = dp_c^* &gt; 0$</td>
<td>[\text{<strong>Permanent</strong>} \quad dp_{c1}^* = dp_{c2}^* = dp_c^* &gt; 0]</td>
<td>[\text{<strong>Permanent</strong>} \quad dp_{c1}^* = dp_{c2}^* = dp_c^* &gt; 0]</td>
</tr>
</tbody>
</table>

In summary, perhaps one of the most striking points about the comparative static effects presented in this section, is the robustness of the results to the three different disequilibrium regimes. They are also robust to the timing of the coffee price boom: the multiplier effects reported above indicate that booms, whether temporary, anticipated or permanent, increase the production of nontradables in the short and long run.

6.3 The Role of Capital Market Imperfections and a Coffee Price Stabilisation Fund

Perhaps the central notion of the intertemporal, maximising model developed in the previous section is that current consumption (of both tradables and nontradables) depends on lifetime resources ($W$), and not on current resources ($Y_t$); the latter only
affects consumption insofar as it affects the former. This result relies on the assumption that individuals can borrow (or lend) as much as they desire at a fixed rate of interest, so long as the intertemporal budget constraint [6.4] is satisfied. Nonetheless, as indicated by Deaton (1990), to assume that individuals are able to borrow freely to smooth their consumption does not seem to be appropriate in the context of developing countries, since consumers in these countries are likely to be subject to borrowing constraints.

Engel and Meller (1993) indicate that the existence of borrowing constraints, and the fact that developing countries derive a substantial part of their income from the sale of primary commodities, whose prices are extremely volatile in the world markets, motivate considering mechanisms like commodity price stabilisation funds, because the incentives to save in "good" times are even greater, for there will be limited access to international borrowing in "bad" times.

Taking these aspects into consideration, in this section we augment the basic model under the KK configuration, by incorporating a government sector that is in charge of the administration of a coffee price stabilisation fund (or commodity marketing board), which is aimed at reducing the effects of commodity price instability on the national economy. More specifically, the fund is said to act as a stabilising mechanism, when it is able to reduce the magnitude of the response of \( Y_{N1} \) to variations in the world price of coffee.\(^{10}\) The structure of the government sector is highly simplified as we abstract from taxes and government expenditure; hence, the government's only role is to manage the stabilisation fund.\(^{11}\) The fund buys coffee at

\(^{10}\)As should be recalled, in the KK regime the variables of interest are \( Y_{N1} \) and \( Y_{N2} \). We do not present the results for either the KW or the modified KK regimes, as the analysis may become repetitive.

\(^{11}\)This is equivalent to assuming that the intertemporal budget constraint for the government, which limits the present value of the government's consumption to the present value of its revenues, is satisfied.
price $p_c$, and sells it in the world market of coffee at $p_c^*$, which is assumed given. A further simplifying assumption is that coffee producers sell the totality of their product to the stabilisation fund, although the model can be easily modified to deal with the case where coffee producers sell part of their crop in the world market.

In addition, we allow for the presence of capital market imperfections, which are initially modelled as in van Wijnbergen (1987). That is, we assume that the government can borrow, on behalf of the fund, on more favourable terms in international capital markets than households; more formally, $r_g < \bar{r}$ and both interest rates are in terms of tradables.\footnote{In van Wijnbergen's (1987) paper, the assumption that $r_g < \bar{r}$ breaks down Ricardian equivalence. See also Rankin's (1994) variant of the van Wijnbergen model.} Within this framework, the fund's intertemporal budget constraint, derived in a similar fashion as that for consumers, is given by:

$$
(p_{c1} - p_{c1}^*)y_{c1} + \frac{(p_{c2} - p_{c2}^*)y_{c2}}{R^G} = 0.
$$

where $R^G = 1 + r_g$ denotes the gross interest rate at which the government can borrow and lend. Setting $Y_{c1} = Y_{c2} = Y_c$, in order to simplify the algebra, yields

$$
(p_{c1} - p_{c1}^*) + \frac{(p_{c2} - p_{c2}^*)}{R^G} = 0.
$$

Equation [6.13] (or [6.13a]) simply restricts the present value of the purchases of coffee made by the fund to equal the present value of its sales in the world market. In this equation, $p_{c1}^*$ and $p_{c2}^*$ are treated as exogenous, since the country is small in the world market of coffee, and $p_{c1}$ and $p_{c2}$ are the set of policy instruments. It is worth noting that the possibility of systematic taxation of coffee producers does not arise in the model, since the fund is assumed to break even at the end of the second
period; put another way, if the government lowers the domestic price of coffee by \( dp_{c1} \) on date 1, it must raise it by \((1 + r_e)dp_{c1}\) on date 2.

For the purposes of the analysis that follows, we consider the case where the government sets \( p_{c1} \), so that \( p_{c2} \) acts as the residual that guarantees that the fund’s intertemporal budget constraint holds. Solving for \( p_{c2} \), equation [6.13a] can be rewritten as:

\[
p_{c2} = \frac{\mu dp_{c1}}{1 + \mu R_c}.
\]

In addition, we introduce the policy rule \( dp_{c1} = \mu dp_{c1} \), \( 0 \leq \mu \leq 1 \), so that in the initial equilibrium, the government is not intervening at all, and instead government intervention only occurs in response to the shock in the world price of coffee. This policy rule for the domestic price of coffee gives rise to three possibilities: a) when \( \mu = 0 \), the domestic price is independent of the world price, which implies an extreme form of stabilisation; b) when \( \mu = 1 \), variations in the world price are fully transferred to domestic producers, which implies that stabilisation is not pursued; and c) when \( 0 < \mu < 1 \), variations in the world price of coffee are partially transferred to domestic producers. Cárdenas (1991, 1994) estimated the value of the elasticity of the domestic price of coffee with respect to the world price for four coffee producer countries. He obtained a value close to one in Kenya and Costa Rica, a small value in Ivory Coast, and an intermediate value (around 0.5) in Colombia.13

---

13Cárdenas used World Bank and FAO annual data. He also included the RER in the specification of the equation of the domestic price of coffee, although the coefficient on this variable was not significant in Ivory Coast (regardless of the data source) nor in Kenya (for the FAO data). In a previous exercise for Colombia, Ocampo (1989) estimated a value for the elasticity of domestic coffee prices with respect to world prices equal to 0.68. He also included the surplus of coffee (defined as production minus consumption minus exports) as one of the determinants of the domestic price of coffee.
The presence of the coffee price stabilisation fund modifies the expression for wealth given in [6.4], since individuals now sell coffee to the fund at \( p_c \), and not in the world market at \( p_c^* \). Consequently, the equilibrium conditions of the economy under the KK configuration are given by:

\[
\begin{align*}
\frac{p_{N1} Y_{N1}}{p_{N2} Y_{N2}} &= \frac{\alpha}{(1 + \beta)} \left\{ p_{N1} Y_{N1} + p_{C1} Y_C + \frac{p_{N2} Y_{N2} + p_{C2} Y_C}{R} + \bar{\gamma}_t \right\} \quad \text{[6.14a]} \\
\frac{p_{N2} Y_{N2}}{p_{N1} Y_{N1}} &= \frac{\alpha \beta R}{(1 + \beta)} \left\{ p_{N1} Y_{N1} + p_{C1} Y_C + \frac{p_{N2} Y_{N2} + p_{C2} Y_C}{R} + \bar{\gamma}_t \right\}. \quad \text{[6.14b]}
\end{align*}
\]

Substituting equation [6.13b] into the pair of equations [6.14a] and [6.14b], we obtain the following set of “augmented” [IS1] and [IS2] schedules:

\[
\begin{align*}
\frac{p_{N1} Y_{N1}}{p_{N2} Y_{N2}} &= \frac{\alpha}{(1 + \beta)} \left\{ p_{N1} Y_{N1} + p_{C1} Y_C + \frac{p_{N2} Y_{N2} + p_{C2} Y_C}{R} + \left[ p_{C2} + R^G (p_{C1} - p_{C1}) \right] Y_C + \bar{\gamma}_t \right\} \\
\frac{p_{N2} Y_{N2}}{p_{N1} Y_{N1}} &= \frac{\alpha \beta R}{(1 + \beta)} \left\{ p_{N1} Y_{N1} + p_{C1} Y_C + \frac{p_{N2} Y_{N2} + p_{C2} Y_C}{R} + \left[ p_{C2} + R^G (p_{C1} - p_{C1}) \right] Y_C + \bar{\gamma}_t \right\}.
\end{align*}
\]

Rearranging terms, differentiating, substituting the policy rule \( dp_{C1} = \mu dp_{C1} \), and setting \( dY_C = d\bar{Y}_t = 0 \), we obtain:

\[
\begin{align*}
1 - \frac{\alpha}{(1 + \beta)} p_{N1} dY_{N1} - \frac{\alpha}{R (1 + \beta)} p_{N2} dY_{N2} &= \frac{\alpha Y_C}{(1 + \beta)} \left\{ \left[ \mu (1 - R^D) + R^D \right] dp_{C1} + \frac{dp_{C2}^*}{R} \right\} \\
- \frac{\alpha \beta R}{(1 + \beta)} p_{N1} dY_{N1} + \left[ 1 - \frac{\alpha \beta}{(1 + \beta)} \right] p_{N2} dY_{N2} &= \frac{\alpha \beta R Y_C}{(1 + \beta)} \left\{ \left[ \mu (1 - R^D) + R^D \right] dp_{C1} + \frac{dp_{C2}^*}{R} \right\},
\end{align*}
\]

where \( 0 < R^D = \frac{R^G}{R} = \frac{1 + r_g}{1 + \bar{r}} < 1 \), provided \( r_g < \bar{r} \).
This set of equations can be used to investigate the response of $Y_{N1}$ and $Y_{N2}$ to variations in the world price of coffee, whether temporary, anticipated or permanent. The formal expressions are presented in Table 6.4, and should be compared with the multiplier effects when stabilisation is not pursued, which are presented in Table 6.1. The following results are particularly interesting. To begin with, the coffee stabilisation fund reduces the multiplier effect of temporary and permanent coffee price shocks not only on $Y_{N1}$ (as one might expect), but also on $Y_{N2}$. The stabilisation fund does not have any effect in the presence of anticipated coffee price shocks; the ineffectiveness of the fund in this case can be explained by equation [6.13b], which shows that variations in the world price of coffee on date 2 are fully translated to the domestic price in that period.

Table 6.4: Comparative statics under the KK configuration

<table>
<thead>
<tr>
<th>TYPE OF SHOCK</th>
<th>$dY_{N1}$</th>
<th>$dY_{N2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temporary</td>
<td>$\alpha Y_C \left[ \frac{\mu(1 - R^D) + R^D}{(1 - \alpha)(1 + \beta)p_{N1}} \right] &gt; 0$</td>
<td>$\alpha \beta R Y_C \left[ \frac{\mu(1 - R^D) + R^D}{(1 - \alpha)(1 + \beta)p_{N2}} \right] &gt; 0$</td>
</tr>
<tr>
<td>$d\p^*_C1 &gt; 0$</td>
<td>$\alpha Y_C \left[ \frac{\mu(1 - R^D) + R^D}{(1 - \alpha)(1 + \beta)p_{N1}} \right] &gt; 0$</td>
<td>$\alpha \beta Y_C \left[ \frac{\mu(1 - R^D) + R^D}{(1 - \alpha)(1 + \beta)p_{N2}} \right] &gt; 0$</td>
</tr>
<tr>
<td>Anticipated</td>
<td>$\alpha Y_C \left[ \frac{\mu(1 - R^D) + R^D}{(1 - \alpha)(1 + \beta)p_{N1}} \right] &gt; 0$</td>
<td>$\alpha \beta Y_C \left[ \frac{\mu(1 - R^D) + R^D}{(1 - \alpha)(1 + \beta)p_{N2}} \right] &gt; 0$</td>
</tr>
<tr>
<td>$d\p^*_C2 &gt; 0$</td>
<td>$\alpha Y_C \left[ \frac{\mu(1 - R^D) + R^D}{(1 - \alpha)(1 + \beta)p_{N1}} \right] &gt; 0$</td>
<td>$\alpha \beta Y_C \left[ \frac{\mu(1 - R^D) + R^D}{(1 - \alpha)(1 + \beta)p_{N2}} \right] &gt; 0$</td>
</tr>
<tr>
<td>Permanent</td>
<td>$\alpha Y_C \left[ \frac{\mu(1 - R^D) + R^D}{(1 - \alpha)(1 + \beta)p_{N1}} \right] &gt; 0$</td>
<td>$\alpha \beta Y_C \left[ \frac{\mu(1 - R^D) + R^D}{(1 - \alpha)(1 + \beta)p_{N2}} \right] &gt; 0$</td>
</tr>
<tr>
<td>$d\p^<em>_C1 = d\p^</em>_C2 = d\p^*_C &gt; 0$</td>
<td>$\alpha Y_C \left[ \frac{\mu(1 - R^D) + R^D}{(1 - \alpha)(1 + \beta)p_{N1}} \right] &gt; 0$</td>
<td>$\alpha \beta Y_C \left[ \frac{\mu(1 - R^D) + R^D}{(1 - \alpha)(1 + \beta)p_{N2}} \right] &gt; 0$</td>
</tr>
</tbody>
</table>

In the second place, the coffee stabilisation fund does not play any role when the assumption of capital market imperfections is lifted; indeed, when $r_g = \bar{r}$, so that
R^D = 1, the multiplier effects presented in Table 6.4 are identical to the ones reported in Table 6.1. The model thus predicts that when both the private sector and the government (on behalf of the fund) can borrow on the same terms in international capital markets, the presence of the stabilisation fund is redundant.

Lastly, when there are capital market imperfections, the effectiveness of the stabilisation fund will depend upon the coffee policy rule. In particular, the highest degree of stabilisation occurs when variations in the world price of coffee are not transferred to domestic producers (i.e. \( \mu = 0 \)). As \( \mu \) increases the fund's stabilisation power gets weaker, and when \( \mu = 1 \) the multiplier effects become identical to the ones obtained when stabilisation is not pursued (see Table 6.1).

As indicated above, one surprising result is that the stabilisation fund dampens the multiplier effect of \( p_{c1}^* \) (or \( p_c^* \)) on \( Y_{N2} \). By contrast, one might have expected that since the government has to raise \( p_{c2} \) in order to satisfy its intertemporal budget constraint, the effect of the fund would be to shift some of the effect of the increase in \( p_{c1}^* \) (or \( p_c^* \)) from period 1 to period 2. What the model predicts is that in the presence of the stabilisation fund, output increases by less in both periods; as a result, with the fund the country is worse off, since the present value of its income is lower with it than without it. On further reflection, this paradoxical result can be explained by the van Wijnbergen method of modelling capital market imperfections. Indeed, the presence of the fund means that when there is an increase in \( p_{c1}^* \) (or \( p_c^* \)), the government lends some of the additional revenue in period 1; nonetheless, given that \( r_g < \bar{r} \), it gets a worse return on it than the private sector would. This reduces the lifetime wealth of the country, and hence reduces demand and output in both periods. If we look instead at the case of a decrease in
the country would gain from the stabilisation fund, since the government would have to borrow in period 1, and it could do so at a lower interest rate than the private sector.

6.4 An Alternative Strategy for Modelling Capital Market Imperfections

In the previous section, we argued that the reason why the stabilisation fund was unable to shift some of the effects of the coffee price shocks between periods, was related to the form in which we modelled capital market imperfections. In what follows, we attempt to remedy this by modelling capital market imperfections in an alternative way. In particular, we assume that the private sector faces an upward-sloping supply of capital funds or, put another way, that the interest rate at which private agents can borrow, is an increasing function of the amount borrowed. Within this framework, it turns out that private agents face a non-linear intertemporal budget constraint; it should be remembered that when we assumed that $r_g < \bar{r}$, the private sector's intertemporal budget constraint remained linear, so that there was no impediment to "consumption smoothing" by private agents.

Perhaps not surprisingly, under the alternative strategy for modelling capital market imperfections, the resulting multiplier effects turn out to be relatively more complicated than those obtained previously. For this reason, we will initially look at the effects of capital market imperfections in a simplified, classical, version of the basic model we have been dealing with. Consider then a small open economy that produces and consumes only one good, and whose output levels are exogenous. Consumers are assumed to derive utility from consuming this good in periods 1 and
2, and their preferences can be adequately summarised by the following additive
time-separable, log linear utility function (U):

\[ U = \log C_1 + \beta \log C_2, \]

where \( C_1 \) (\( C_2 \)) denotes consumption in period 1 (2), and \( \beta = (1 + \delta)^{-1} \). The
consumer's intertemporal budget constraint is

\[ RC_1 + C_2 = RY_1 + Y_2, \]

or alternatively

\[ C_2 - Y_2 = -R(C_1 - Y_1), \]

where \( Y_1 \) (\( Y_2 \)) denotes output in period 1 (2), and \( R = (1 + r) \) is the domestic gross
interest rate.

As mentioned above, we assume that the interest rate at which agents can
borrow, is an increasing function of the amount they borrow, i.e.

\[ R = \bar{R} + \rho(C_1 - Y_1), \]

where \( \bar{R} = (1 + \bar{r}) \) is the intercept parameter in the capital supply function, and
\( \rho > 0 \). The reason why the capital supply function slopes upwards should be thought
of as risk of default, where the slope parameter \( \rho \) reflects the incentives of
individuals to default, although we do not model default explicitly. In other words, as
an individual borrows more, his incentive to, and likelihood of, defaulting increases,
and the interest rate he has to pay compensates the lender for this risk. The slope
parameter \( \rho \) can hence be thought of as a measure of capital market imperfection. In
particular, there are two extreme cases worth considering. On the one hand, \( \rho \to \infty \)
can be regarded as the case where borrowing becomes impossible. On the other hand,
\( \rho = 0 \) can be regarded as the case where there are no capital market imperfections, so
that private agents can borrow at the world gross interest rate \( \bar{R} \), and the model then
collapses to the standard two-period representative consumer model (see e.g. Obstfeld and Rogoff 1996, chapter 1).\footnote{It is worth mentioning that equation [6.17] also implies that when private agents are lenders in the first period (i.e. \( Y_1 > C_1 \)), then the domestic gross interest rate would fall the more they lend, which is not particularly plausible. In order to overcome this difficulty, one might postulate a quadratic version of equation [6.17]. Nonetheless, this would unnecessarily complicate the algebra, since the intertemporal budget constraint faced by the private agents would no longer be quadratic but of third order.}

Substituting [6.17] into [6.16] yields the following non-linear intertemporal budget constraint

\[
C_2 - Y_2 = -R(C_1 - Y_1) - \rho(C_1 - Y_1)^2, \tag{6.18}
\]

which is drawn in Figure 6.5.

**Figure 6.5:** The two-period representative consumer model with capital market imperfections

As can be seen, under the assumption of capital market imperfections, the intertemporal budget constraint of the private agents is an inverted parabola with a maximum at the point \( Y_1 - \frac{R}{2\rho} \). Moreover, the slope of the intertemporal budget constraint at \( C_1 = Y_1 \), that is where private agents neither borrow nor lend in the first period, is equal to \(-R\). Mathematically, the household’s problem is to maximise
[6.15] subject to the non-linear budget constraint stated in [6.18]. From the first order conditions of this maximisation problem we obtain

$$C_2 = \beta C_1 [\bar{R} + 2 \rho (C_1 - Y_1)]$$ \hspace{1cm} [6.19]

which can be then substituted in [6.18] to yield the following equation for $C_1$:

$$(1 + 2 \beta) \rho (C_1 - Y_1)^2 + [(1 + \beta) \bar{R} + 2 \rho \beta Y_1] (C_1 - Y_1) + \beta \bar{R} Y_1 - Y_2 = 0,$$ \hspace{1cm} [6.20]

that is a quadratic function with $Y_1, Y_2$ and the set of parameters $\beta, \rho$ and $\bar{R}$ as arguments. If we suppose, for analytical convenience, that $\beta \bar{R} Y_1 - Y_2 = 0$, then one solution of [6.20] is given by $C_1 = Y_1$. Assuming then that the initial equilibrium of the economy occurs at this point, we can derive, using the rule of the implicit function, the following comparative static results:

$$\frac{\partial C_1}{\partial Y_1} \bigg|_{C_1 = Y_1} = \frac{\bar{R} + 2 \rho \beta Y_1}{\bar{R}(1 + \beta) + 2 \rho \beta Y_1}$$ \hspace{1cm} [6.21a]

and

$$\frac{\partial C_1}{\partial Y_2} \bigg|_{C_1 = Y_1} = \frac{1}{\bar{R}(1 + \beta) + 2 \rho \beta Y_1}.$$ \hspace{1cm} [6.21b]

The economic interpretation of this pair of equations is particularly interesting. To begin with, [6.21a] shows that in the presence of capital market imperfections, the marginal propensity to consume out of current income is less than one. Moreover, dividing both numerator and denominator by $\rho$, we can see that as the degree of capital market imperfections increases (i.e. $\rho \to \infty$), the marginal order. Consequently, our analysis will be focused on the case where private agents are borrowers in the first period.

"The other solution is $C_1 = \frac{Y_1}{(1 + 2 \beta) \rho} - \frac{(1 + \beta) \bar{R}}{(1 + 2 \beta) \rho}$"
propensity to consume out of current income tends to one. Turning to [6.21b], current consumption is expected to increase in response to a change in future income, and this response tends to zero as \( \rho \to \infty \); put another way, as the degree of capital market imperfections increases, private agents’ consumption behaviour turns out to be linked only to current income rather than to past or future income.

Equations [6.21a] and [6.21b] also allow us to verify that:

\[
\frac{\partial C_t}{\partial Y_t} \bigg|_{C_t = Y_t} > R \frac{\partial C_t}{\partial Y_t} \bigg|_{C_t = Y_t}.
\]

that is, in the presence of capital market imperfections (i.e. \( \rho > 0 \)), the response of \( C_t \) to a change in \( Y_t \) (evaluated at \( C_t = Y_t \)) is greater than the response of \( C_t \) to a change of equal present value in \( Y_2 \) (evaluated at the same point). On the other hand, when the capital market is perfect (i.e. \( \rho = 0 \)),

\[
\frac{\partial C_t}{\partial Y_t} \bigg|_{C_t = Y_t} = R \frac{\partial C_t}{\partial Y_t} \bigg|_{C_t = Y_t}.
\]

Put another way, in the standard intertemporal choice problem with a linear budget constraint, the date at which income is received does not matter for its effect on consumption, provided we convert all income changes into present value terms. But in the presence of capital market imperfections, consumption at date \( t \) depends more on income at date \( t \), than on income at any other date.

Lastly, from [6.19] it is possible to state the additional result that:

\[
\frac{\partial C_t}{\partial Y_t} \bigg|_{C_t = Y_t} = \frac{\beta R^2}{R(1 + \beta) + 2\rho \beta Y_t},
\]

from which it is apparent that as \( \rho \to \infty \), the effect of a change in \( Y_t \) on \( C_t \) tends to zero. Similarly,

\[16\text{The same conclusion was reached by Flemming (1973), who modelled capital market imperfections.}\]
from which it follows that as \( \rho \to \infty \), the effect of a change in \( Y_2 \) on \( C_2 \) tends to one.

6.5 THE BASIC MODEL WITH A COFFEE PRICE STABILISATION FUND AND AN UPWARD SLOPING CAPITAL SUPPLY CURVE

In this section we go back to the basic model under the KK configuration, which is augmented with the presence of the coffee price stabilisation fund, and capital market imperfections as modelled in section 6.4. We start off by stating the problem of the consumers, who are assumed to maximise

\[
U = a \log C_{N_1} + (1 - a) \log C_{T_1} + \beta \{ a \log C_{N_2} + (1 - a) \log C_{T_2} \}, \tag{6.1}
\]

where \( \beta = (1 + \delta)^{-1} \), subject to the intertemporal budget constraint

\[
C_2 - Y_2 = -R(C_1 - Y_1), \tag{6.16}
\]

where \( C_t = p_{N_t} C_{N_t} + C_{T_t} \), for \( t = 1, 2 \).

The domestic gross interest rate is once again assumed to be an increasing function of the amount borrowed by private agents due to risk of default, so that

\[
R = \bar{R} + \rho (C_1 - Y_1). \tag{6.17}
\]

Within this extended framework, we formulate the Lagrangean function, from which we obtain the following solutions, written in compact form, for \( C_{N_1} \) and \( C_{N_2} \) (see Appendix 6.3):

\[
C_{N_1} = C_{N_1}(Y_1, Y_2, \nu) \tag{6.22a}
\]

and

as a divergence between lending and borrowing rates of interest.
where \( u \) is a vector that comprises the set of relative prices \( p_{n1} \) and \( p_{n2} \), as well as the parameters \( \beta \), \( \rho \) and \( \overline{R} \). The relative prices and the parameters are omitted from the analysis since they are assumed constant. Two relevant comments to make about the pair of equations [6.22a] and [6.22b] are that a) they are messy, non-linear functions, which is why we do not try to write them out explicitly here; and b) most importantly, unlike in the earlier model, \( Y_1 \) and \( Y_2 \) cannot be aggregated into a single “lifetime wealth” variable, \( W \).

Turning to the functioning of the coffee price stabilisation fund, we assume that it faces its own, individual, supply of capital curve. In addition, we assume that the stabilisation fund is perceived as a good risk, so that the supply of capital curve can be approximated as horizontal; that is, the stabilisation fund can borrow and lend at the world gross interest rate \( \overline{R} \), and the fund’s intertemporal budget constraint is:

\[
\left( p_{c1} - p_{\dot{c1}} \right) Y_{c1} + \left( p_{c2} - p_{\dot{c2}} \right) \frac{Y_{c2}}{\overline{R}} = 0,
\]

which, after setting \( Y_{c1} = Y_{c2} = Y_c \) in order to simplify the algebra, gives

\[
p_{c2} = p_{\dot{c2}} + \overline{R} \left( p_{\dot{c1}} - p_{c1} \right),
\]

[6.23]

where we are using the assumption that \( p_{c2} \) acts as the residual that guarantees that the fund’s intertemporal budget constraint holds. Since we assume that the economy is in the KK configuration, the market equilibrium conditions can be stated as:

\[
Y_{n1} = C_{n1} \left( p_{n1} Y_{n1} + \overline{Y}_{n1} + p_{c1} Y_c, p_{n2} Y_{n2} + \overline{Y}_{n2} + p_{c2} Y_c \right)
\]

and

\[
Y_{n2} = C_{n2} \left( p_{n1} Y_{n1} + \overline{Y}_{n1} + p_{c1} Y_c, p_{n2} Y_{n2} + \overline{Y}_{n2} + p_{c2} Y_c \right),
\]

where this pair of equations is determining \( Y_{n1} \) and \( Y_{n2} \).
Substituting equation [6.23] into the market equilibrium conditions, totally differentiating, using the policy rule \( dp_{c1} = \mu dp_{c1}^* \), and setting \( dY_c = dY_e = 0 \), yields:

\[
\begin{align*}
\left( 1 - \frac{\partial C_{N1}}{\partial Y_1} p_{N1} \right) dY_{N1} - \frac{\partial C_{N1}}{\partial Y_2} p_{N2} dY_{N2} \\
= Y_c \left[ \mu \frac{\partial C_{N1}}{\partial Y_1} + \bar{R}(1 - \mu) \frac{\partial C_{N1}}{\partial Y_2} \right] dp_{c1}^* + \frac{\partial C_{N1}}{\partial Y_2} dp_{c2}^* \quad [6.24a]
\end{align*}
\]

and

\[
\begin{align*}
- \frac{\partial C_{N2}}{\partial Y_1} p_{N1} dY_{N1} + \left( 1 - \frac{\partial C_{N2}}{\partial Y_2} p_{N2} \right) dY_{N2} \\
= Y_c \left[ \mu \frac{\partial C_{N2}}{\partial Y_1} + \bar{R}(1 - \mu) \frac{\partial C_{N2}}{\partial Y_2} \right] dp_{c1}^* + \frac{\partial C_{N2}}{\partial Y_2} dp_{c2}^* \quad [6.24b]
\end{align*}
\]

We can then use this system of equations to look at the effect of a change in the price of coffee on \( Y_{N1} \) (or \( Y_{N2} \)) in the neighbourhood of the initial equilibrium. For analytical convenience, we assume that the initial equilibrium is when \( C_t = Y_t \), so that the multiplier effects are evaluated at this point. For instance, it can be shown that the effects of a temporary increase in the price of coffee on \( Y_{N1} \) and \( Y_{N2} \) are:

\[
\begin{align*}
\left. \frac{dY_{N1}}{dp_{c1}^*} \right|_{C_t, Y_t} &= Y_c \left[ \mu \frac{\partial C_{N1}}{\partial Y_1} + \bar{R}(1 - \mu) \frac{\partial C_{N1}}{\partial Y_2} \right] \left( 1 - \frac{\partial C_{N2}}{\partial Y_1} p_{N1} \right) \left( 1 - \frac{\partial C_{N2}}{\partial Y_2} p_{N2} \right) - \frac{\partial C_{N2}}{\partial Y_1} p_{N1} \frac{\partial C_{N1}}{\partial Y_2} p_{N2} \\
\left. \frac{dY_{N2}}{dp_{c1}^*} \right|_{C_t, Y_t} &= Y_c \left[ \mu \frac{\partial C_{N2}}{\partial Y_1} + \bar{R}(1 - \mu) \frac{\partial C_{N2}}{\partial Y_2} \right] \left( 1 - \frac{\partial C_{N1}}{\partial Y_1} p_{N1} \right) \left( 1 - \frac{\partial C_{N1}}{\partial Y_2} p_{N2} \right) - \frac{\partial C_{N1}}{\partial Y_1} p_{N1} \frac{\partial C_{N2}}{\partial Y_2} p_{N2}
\end{align*}
\]
respectively. The partial derivatives of \(C_{N1}\) and \(C_{N2}\) with respect to \(Y_1\) and \(Y_2\), also evaluated at the point \(C_i = Y_i\), are presented in Appendix 6.3. After substituting in these partial derivatives, the multiplier effects can be conveniently simplified to:

\[
\frac{dY_{N1}}{dp_{Cl}^{*}}\bigg|_{C_{i} = Y_{i}} = \frac{Y_{C}}{p_{N1}} \left( \frac{\alpha \left( R + 2 \rho \beta Y_1 \mu (1 - \alpha) \right)}{(1 - \alpha) \left[ R (1 + \beta) + 2 \rho \beta Y_1 (1 - \alpha) \right]} \right) \quad [6.25a]
\]

and

\[
\frac{dY_{N2}}{dp_{Cl}^{*}}\bigg|_{C_{i} = Y_{i}} = \frac{Y_{C}}{p_{N2}} \left( \frac{\alpha \beta R^{2} + 2 \rho \beta Y_1 \alpha (1 - \mu) (1 - \alpha) R}{(1 - \alpha) \left[ R (1 + \beta) + 2 \rho \beta Y_1 (1 - \alpha) \right]} \right) \quad [6.25b]
\]

Before discussing the effects of the coffee price stabilisation fund, it is worth noting that equations [6.25a] and [6.25b] reveal something about the basic properties of the model prior to government intervention. In particular, when fluctuations in the world price of coffee are fully transferred to domestic producers, which implies that price stabilisation is not pursued since \(\mu = 1\), [6.25a] is clearly increasing in \(\rho\), while [6.25b] is clearly decreasing in \(\rho\); that is, as \(\rho \to \infty\), the response of \(Y_{N1}\) is greater than that in the absence of capital market imperfections, while the response of \(Y_{N2}\) tends to zero. Consequently, capital market imperfections shift some of the effects of temporary coffee price variations from period 2 to period 1. In other words, it is the capital market which shifts some of the effects of a temporary coffee price shock from period 1 to period 2; introducing the imperfection just blocks this.

Equations [6.25a] and [6.25b] also allow us to state that in the presence of capital market imperfections (i.e. \(\rho > 0\)), the stabilisation fund shifts some of the effects of a temporary coffee price shock from period 1 to period 2. That is, the fund reduces the multiplier of \(p_{Cl}^{*}\) on \(Y_{N1}\), but increases the multiplier of \(p_{Cl}^{*}\) on \(Y_{N2}\); more formally:
\[
\frac{dY_{N1}^{\mu=1}}{dp_{C1}^{*} \mid y_i} = \frac{Y_C}{P_{N1}} \left\{ \frac{\alpha [\bar{R} + 2\rho \beta Y_1 (1-\alpha)]}{(1-\alpha) [\bar{R}(1+\beta) + 2\rho \beta Y_1 (1-\alpha)]} \right\} 
\]

[6.26a]

\[
\frac{dY_{N1}^{0 < \mu < 1}}{dp_{C1}^{*} \mid y_i} = \frac{Y_C}{P_{N1}} \left\{ \frac{\alpha [\bar{R} + 2\rho \beta Y_1 \mu (1-\alpha)]}{(1-\alpha) [\bar{R}(1+\beta) + 2\rho \beta Y_1 (1-\alpha)]} \right\}
\]

and

\[
\frac{dY_{N2}^{\mu=1}}{dp_{C1}^{*} \mid y_i} = \frac{Y_C}{P_{N2}} \left\{ \frac{\alpha \beta \bar{R}^2}{(1-\alpha) [\bar{R}(1+\beta) + 2\rho \beta Y_1 (1-\alpha)]} \right\}
\]

[6.26b]

\[
\frac{dY_{N3}^{0 < \mu < 1}}{dp_{C1}^{*} \mid y_i} = \frac{Y_C}{P_{N2}} \left\{ \frac{\alpha \beta \bar{R}^2 + 2\rho \beta Y_1 \alpha (1-\mu)(1-\alpha) \bar{R}}{(1-\alpha) [\bar{R}(1+\beta) + 2\rho \beta Y_1 (1-\alpha)]} \right\}
\]

where \( \frac{dY_{N2}^{\mu=1}}{dp_{C1}^{*} \mid y_i} \) denotes the multiplier of \( p_{cl}^{*} \) on \( Y_{N1} \) (t = 1, 2) when stabilisation is not pursued (i.e. \( \mu = 1 \)), and \( \frac{dY_{N1}^{0 < \mu < 1}}{dp_{C1}^{*} \mid y_i} \) is the corresponding multiplier in the presence of the stabilisation fund (i.e. \( 0 \leq \mu < 1 \)).

With regard to anticipated variations, however, the fund is not able to stabilise nontradable output, since an increase in \( p_{c2}^{\mu} \) results in a proportional increase in the domestic price in that period (recall equation [6.23]).

Lastly, another result that can be derived from [6.25a] and [6.25b], is that when both the private sector and the government can borrow on the same terms in international markets, that is when the capital market is perfect \( (\rho = 0) \), the multiplier effects do not depend upon the parameter \( \mu \). In other words, in a context where private agents can borrow and lend at the world interest rate, so that there is no impediment to consumption smoothing by private agents, the stabilisation fund will be redundant.
Thus far, we have assumed that both private agents and the government face individual supply of capital curves, so that the rate at which an individual can borrow depends only on the amount of his own borrowing, not also on that of others. If, on the other hand, we assume that both the private sector and the government face an aggregate supply of capital curve, implying that extra borrowing by one individual raises the rate of interest which another individual must pay, the model predicts that the stabilisation fund will be redundant. One possible reason why the country as a whole may face an aggregate supply of capital curve, is that borrowers’ decisions may not be made independently. For example, the government may implicitly act to coordinate defaulting behaviour, by nationalising or expropriating foreign assets.

To see this result, let us begin by assuming that the supply of capital curve applies to the aggregate, private-plus-public-sector, borrowing of the country, that is:

\[ R = \bar{R} + \rho (C_1 - Y_1^*) \]  \[6.27\]

where \( Y_1^* \) is aggregate income in period 1, defined as:

\[ Y_1^* = Y_1 + Y_C (p_{C1}^* - p_{C1}) = p_{N1} Y_{N1} + \bar{Y}_{T1} + p_{C1}^* Y_C. \]

The corresponding expression for aggregate income in period 2 is:

\[ Y_2^* = Y_2 + Y_C (p_{C2}^* - p_{C2}) = p_{N2} Y_{N2} + \bar{Y}_{T2} + p_{C2}^* Y_C. \]

As can be seen, the capital supply curve slopes upwards because of risk of default, and the slope parameter \( \rho \) depends on the incentives of the country to default, which we do not model explicitly. We know, from equation \([6.16]\), that the intertemporal budget constraint of the private sector is:

\[ C_2 - Y_2 = -R(C_1 - Y_1). \]  \[6.16\]

In turn, the intertemporal budget constraint of the coffee price stabilisation fund is:
which differs from that stated in equation [6.23], because the stabilisation fund now borrows at the gross rate of interest R.

Next, substituting the definition of aggregate income (in periods 1 and 2) into [6.16] yields:

\[ C_2 - Y_2^* + R(C_1 - Y_1^*) + Y_C \left\{ (p_{C_2} - p_{C_1}) + R(p_{C_1} - p_{C_1}) \right\} = 0, \]

which can be further simplified to

\[ C_2 - Y_2^* = -R(C_1 - Y_1^*), \]  

[6.29]

since we know, from the intertemporal budget constraint of the stabilisation fund, that the term accompanying \( Y_C \) is equal to zero.

The household’s problem reduces then to maximising [6.1] subject to the intertemporal budget constraint [6.29], treating R as given, and the resulting demand functions will be as in equation [6.5]. It is worth emphasising that since the upward-sloping supply of capital curve applies to aggregate, i.e. all households’ and public sector, borrowing and lending, the individual household, which is small relative to the aggregate, takes R as given. R only becomes an endogenous variable when we combine the demand functions with the other equations of the model, to determine the general equilibrium. Hence, the market equilibrium conditions, assuming that the economy is in the KK configuration, can be stated as:

\[ p_{N_1} Y_{N_1} = \frac{\alpha}{(1 + \beta)} \left\{ p_{N_1} Y_{N_1} + Y_{T_1} + p_{C_1} Y_C + \frac{p_{N_2} Y_{N_2} + Y_{T_2} + p_{C_2} Y_C}{R} \right\}, \]  

[6.30a]

and

\[ p_{N_2} Y_{N_2} = \frac{\alpha \beta R}{(1 + \beta)} \left\{ p_{N_1} Y_{N_1} + Y_{T_1} + p_{C_1} Y_C + \frac{p_{N_2} Y_{N_2} + Y_{T_2} + p_{C_2} Y_C}{R} \right\}, \]  

[6.30b]
where \( R = \bar{R} + \rho \left( C_{1} - Y_{1}^{*} \right) \). From the first order conditions we know that
\[ C_{1} = \frac{p_{\text{NI}} C_{\text{NI}}}{\alpha}, \]
so that the upward-sloping supply of capital curve can be alternatively written as:

\[
R = \bar{R} + \rho \left\{ \left( \frac{1 - \alpha}{\alpha} \right) p_{\text{NI}} Y_{\text{NI}} - Y_{T1} - p_{C1}^{*} Y_{C} \right\}. \tag{6.30c}
\]

Using the system of equations [6.30], that define the equilibrium values of \( Y_{\text{NI}}, Y_{\text{N2}} \) and \( R \), it is possible to see directly that \( \mu \) does not appear in any of these three equations, which implies that the coffee price stabilisation fund is redundant. The model thus predicts that when both the private sector and the government (on behalf of the fund) can borrow on the same terms in international capital markets, whether at the rate of interest \( \bar{R} \) or at \( R \), the coffee price stabilisation fund is not able to dampen the multiplier effect of \( p_{C1}^{*} \) on \( Y_{\text{NI}} \). The stabilisation fund only neutralises part of the short-term effect of coffee price fluctuations, when it is able to borrow on more favourable terms in international capital markets than households.

6.6 **CONCLUDING REMARKS**

In this chapter we have developed an intertemporal (two-period) disequilibrium model with microeconomic foundations, in order to analyse the effects of temporary, anticipated, and permanent coffee price shocks on a small open economy. We considered three rationing regimes: Keynesian unemployment in the short and long run (KK configuration); Keynesian unemployment in the short run and Walrasian equilibrium in the long run (KW configuration); and orthodox Keynesian unemployment in the short and long run (modified KK configuration). Perhaps one of the most striking points about the comparative effects presented in the chapter, is the robustness of the results to the three different rationing regimes. They are also
robust to the timing of the coffee price boom: the multiplier effects indicate that a coffee boom, whether temporary, anticipated or permanent, increases nontradable output in periods 1 and 2.

We also extended the model under the KK configuration, by including a government sector that is in charge of the administration of a coffee price stabilisation fund, and by allowing capital market imperfections, in the form of a government that can borrow (on behalf of the stabilisation fund) on more favourable terms in international capital markets than individuals. Surprisingly, when there is an increase (decrease) in the coffee price, the model predicts that with the stabilisation fund the country is worse off (better off).

We argued that this rather paradoxical result is due to the van Wijnbergen method of modelling the imperfections in the capital market. To remedy this, we assumed that the private sector faces an upward-sloping supply of capital curve, while the government borrows and lends at the world interest rate. Under this alternative modelling strategy, the stabilisation fund shifts some of the effects of temporary coffee price shocks from period 1 to period 2.

Finally, when the private sector and the government can borrow on the same terms in international markets, either because the capital market is perfect or because the private sector and the government face an aggregate supply of capital curve, the stabilisation fund turns out to be redundant. In other words, the stabilisation fund neutralises part of the short-term effect of coffee price fluctuations, when it is able to borrow on more favourable terms in international capital markets than households.
Consider first the case where the IS1 locus is steeper than the IS2 locus, as depicted in the figure below, and say the economy is initially at point A, which lies on the IS1 locus and to the right of the point where the two loci cross each other. At this point, people's expectations about output in period 2 are too optimistic, so that when people adjust their expectations the economy moves from A to B. At this new point, there is excess supply of nontradables in period 1, so that the economy moves from B to C. At C people's expectations about output in period 2 are further adjusted, corresponding to the movement from C to D, and so on until the economy reaches Z.

Consider now the case when the IS2 curve is steeper than the IS1:
As is apparent from the figure, beginning from the same point $A$, that lies on the IS1 curve, the adjustment process this time moves the economy away from the point where the two loci cross each other.

From the expressions for the IS1 and IS2 loci, which we reproduce below to facilitate the analysis,

\[
\left[1 - \frac{\alpha}{(1 + \beta)}\right] p_{N1}dY_{N1} - \frac{\alpha}{(1 + \beta)R} p_{N2}dY_{N2} = \frac{\alpha \bar{Y}_C}{(1 + \beta)} \left[ dp_{C1} + \frac{dcp_{C2}}{R} \right] \tag{6.10a}
\]

and

\[
-\frac{\alpha \beta \bar{R}}{(1 + \beta)} p_{N1}dY_{N1} + \left[1 - \frac{\alpha \beta}{(1 + \beta)}\right] p_{N2}dY_{N2} = \frac{\alpha \beta \bar{Y}_C}{(1 + \beta)} \left[ dp_{C1} + \frac{dcp_{C2}}{R} \right] \tag{6.10b}
\]

it is possible to show that provided $1 - \alpha > 0$, the IS1 curve is steeper than the IS2 curve. This inequality constitutes the stability condition of the model, and is always satisfied.

An alternative procedure to look at the stability of the system is to assume that, in each period, nontradables output adjusts in response to excess demand. that is:

\[
\dot{Y}_{N1} = \theta_1 (C_{N1} - Y_{N1})
\]

and

\[
\dot{Y}_{N2} = \theta_2 (C_{N2} - Y_{N2})
\]

where $0 < \theta_1 < 1$ and $0 < \theta_2 < 1$ constitute adjustment coefficients. It is worth noting that as we are dealing with a two-period model, this adjustment mechanism should be viewed as one that occurs in “virtual” or fictitious time, rather than real time (the same is also true of the preceding mechanism). Hence, substituting the commodity demand functions for nontradables given in [6.5a] and [6.5c], and the expression for wealth given in [6.4], we have:
\[
\dot{Y}_{N_1} = \theta_1 \left\{ \frac{\alpha}{(1 + \beta)p_{N_1}} \left[ p_{N_1} Y_{N_1} + \frac{p_{N_2} Y_{N_2}}{R} + \Omega \right] - Y_{N_1} \right\}
\]

and

\[
\dot{Y}_{N_2} = \theta_2 \left\{ \frac{\alpha \beta R}{(1 + \beta)p_{N_2}} \left[ p_{N_1} Y_{N_1} + \frac{p_{N_2} Y_{N_2}}{R} + \Omega \right] - Y_{N_2} \right\}
\]

where \( \Omega = \frac{\dot{p}_{c_1}}{R} Y_{c_1} + \frac{\dot{p}_{c_2}}{R} Y_{c_2} + \frac{\dot{Y}_{t_1}}{R} + \frac{Y_{t_2}}{R} \)

This pair of equations can be rewritten in matrix form as:

\[
\begin{bmatrix}
\dot{Y}_{N_1} \\
\dot{Y}_{N_2}
\end{bmatrix} = \begin{bmatrix}
\theta_1 \left[ \frac{\alpha}{(1 + \beta)} - 1 \right] & \frac{\theta_1 \alpha p_{N_2}}{p_{N_1} (1 + \beta) R} \\
\frac{\theta_2 \alpha \beta R p_{N_1}}{(1 + \beta) p_{N_2}} & \theta_2 \left[ \frac{\alpha \beta}{(1 + \beta)} - 1 \right]
\end{bmatrix} \times \begin{bmatrix}
Y_{N_1} \\
Y_{N_2}
\end{bmatrix} + \begin{bmatrix}
\frac{\theta_1 \alpha \Omega}{(1 + \beta) p_{N_1}} \\
\frac{\theta_2 \alpha \beta R \Omega}{(1 + \beta) p_{N_2}}
\end{bmatrix}
\]

Denoting the right hand side matrix as \( A \), stability of the system requires \( \text{tr} A < 0 \), and \( \det A > 0 \). The first condition is easily met as the terms in parentheses along the diagonal are both negative. On the other hand, the second condition is also satisfied since \( \det A = 1 - \alpha > 0 \), which is identical to the condition we found when the IS1 is steeper than the IS2.
APPENDIX 6.2

THE LABOUR MARKET UNDER THE KW CONFIGURATION

In the long run the labour market equilibrium condition, i.e.

\[ \bar{L}_2 = L_{N2} \left( \frac{w_2}{p_{N2}} \right) + L_{T2} (w_2) \]

implies a unique direct relationship between \( w_2 \) and \( p_{N2} \). In particular, given a total labour supply available to the nontradables and tradables sectors (\( \bar{L}_2 \)), an increase in the relative price of nontradables reduces the real product wage, and this in turn increases the demand for labour in the nontradables sector. In order to restore equilibrium, the wage rate has to be increased, thus reducing demand for labour in both the nontradables and tradables sectors. It is worth noting that the wage rate must increase by less than the increase in the relative price of nontradables, since an equiproportionate increase in both variables leaves the demand for labour in the nontradables sector unchanged, but depresses the demand for labour in the tradable sector, giving then rise to unemployment.

The equilibrium in the labour market is illustrated in the figure below, where the horizontal axis measures the economy's endowment of labour. The marginal product of labour in the nontradables sector \( MPL_N \) slopes downward because of diminishing returns so that, for a given relative price of nontradables \( p_N \), the value of the marginal product of labour \( MPL_N \times p_N \) will also have a negative slope. We can therefore think of the equation \( MPL_N \times p_N = w \), as defining the demand curve for labour in the nontradables sector. Similarly, the demand for labour in the tradable sector is depicted by the negatively sloped line \( MPL_T \), whereas the demand for
labour in the coffee sector, which is assumed to be inelastic to changes in the wage rate, equals $L_C$. On this basis, we can see how the wage rate and employment in each sector are determined given the relative price of nontradables; in particular, the equilibrium wage rate and the allocation of labour between the non-tradable and tradable sectors is represented by point A in the figure below.

Consider now the effect of an increase in the relative price of nontradables from $p_N$ to $p_N^*$. The effect of the increase in $p_N$ is to shift up the nontradables labour demand curve in the same proportion as the price increase. This shifts the equilibrium in the labour market from A to B. The important point to notice is that although the wage rate rises, it does by less than the increase in the relative price of nontradables. As a result, labour shifts from the tradables sector to the nontradables sector, and the output of nontradables rises while that of tradables falls.
APPENDIX 6.3

THE PROBLEM OF THE CONSUMER FACING AN UPWARD-SLOPING SUPPLY OF CAPITAL FUNDS

The consumer maximises

\[ U = \alpha \log C_{N1} + (1 - \alpha) \log C_{T1} + \beta \{ \alpha \log C_{N2} + (1 - \alpha) \log C_{T2} \} \]

subject to the intertemporal budget constraint

\[ C_2 - Y_2 = -R(C_1 - Y_1) \]

which can be alternatively written as,

\[ RC_1 + C_2 = RY_1 + Y_2 \]

where \( R = \bar{R} + \rho (C_1 - Y_1) \), and \( C_t = p_{N,t} C_{N,t} + C_{T,t} \), for \( t = 1, 2 \).

To solve the formal optimisation problem we construct the Lagrangean expression:

\[ \max_{c_{N1}, c_{T1}, c_{N2}, c_{T2}, \lambda} \ell = \alpha \log C_{N1} + (1 - \alpha) \log C_{T1} + \beta \{ \alpha \log C_{N2} + (1 - \alpha) \log C_{T2} \} \]

\[ + \lambda \left( \bar{R}(p_{N1} C_{N1} + C_{T1} - Y_1) + \rho (p_{N1} C_{N1} + C_{T1} - Y_1)^2 + p_{N2} C_{N2} + C_{T2} - Y_2 \right) \]

From the first order conditions of this constrained maximisation problem, we establish the following relationships:

\[ C_{T1} = \frac{(1 - \alpha)}{\alpha} C_{N1} p_{N1} \] \hspace{1cm} [A.6.3.1]

\[ C_{N2} p_{N2} = \beta K C_{N1} p_{N1} \] \hspace{1cm} [A.6.3.2]

and

\[ C_{T2} = \frac{(1 - \alpha)}{\alpha} \beta K C_{N1} p_{N1} \] \hspace{1cm} [A.6.3.3]
where \( K = \bar{R} + 2\rho (p_{N1}c_{N1} + c_{T1} - Y_1) \). Substituting [A.6.3.1], [A.6.3.2] and [A.6.3.3] in the intertemporal budget constraint, and after some algebraic manipulations, we obtain the following (implicit) solution for \( C_{N1} \):

\[
(1 + 2\beta)\rho \left( p_{N1} \frac{c_{N1}}{\alpha} - Y_1 \right)^2 + \left[ (1 + \beta)\bar{R} + 2\rho \beta Y_1 \right] \left( p_{N1} \frac{c_{N1}}{\alpha} - Y_1 \right) + \beta \bar{R} Y_1 - Y_2 = 0
\]

Similar to the simplified model of section 4, it is analytically convenient to assume that \( \beta \bar{R} Y_1 - Y_2 = 0 \), so that one of the solutions of the previous equation is at the point where the economy neither lends nor borrows in the first period, i.e.

\[
C_1 = p_{N1} \frac{c_{N1}}{\alpha} = Y_1
\]

Using the rule of the implicit function, we then look at the effect of a change in \( Y_1 \) and \( Y_2 \) on \( C_{N1} \) in the neighbourhood of the initial equilibrium, obtaining the following results:

\[
\frac{\partial C_{N1}}{\partial Y_1} \bigg|_{C_1 = Y_1} = \frac{\alpha (\bar{R} + 2\rho \beta Y_1)}{p_{N1} \left( \bar{R}(1 + \beta) + 2\rho \beta Y_1 \right)} \tag{A.6.3.4} \]

and

\[
\frac{\partial C_{N1}}{\partial Y_2} \bigg|_{C_1 = Y_1} = \frac{\alpha}{p_{N1} \left( \bar{R}(1 + \beta) + 2\rho \beta Y_1 \right)} \tag{A.6.3.5}
\]

On the other hand, from equation [A.6.3.2] it is possible to show that:

\[
\frac{\partial C_{N2}}{\partial Y_1} \bigg|_{C_1 = Y_1} = \frac{\alpha \beta \bar{R}^2}{p_{N2} \left( \bar{R}(1 + \beta) + 2\rho \beta Y_1 \right)} \tag{A.6.3.6}
\]

and

\[
\frac{\partial C_{N2}}{\partial Y_2} \bigg|_{C_1 = Y_1} = \frac{\alpha (\bar{R} \beta + 2\rho \beta Y_1)}{p_{N2} \left( \bar{R}(1 + \beta) + 2\rho \beta Y_1 \right)} \tag{A.6.3.7}
\]
CHAPTER 7

CONCLUSIONS

The main objective of this thesis has been to analyse the effects of coffee booms on the money market, the real exchange rate, and the business cycle in Colombia. The first two issues have been tackled from an empirical perspective, using the relatively recent developments in the econometric analysis of nonstationary data and cointegration, while the third issue has been addressed theoretically.

The empirical analyses have been performed using quarterly data, which raises the practical concern of whether to conduct the econometric analysis on seasonally adjusted data, or in terms of unadjusted data. In chapter 3 we investigated the impacts of seasonal adjustment filters on integrated-cointegrated data using Monte Carlo simulations. We found that the use of filters has adverse consequences on the power of the ADF and PP cointegration tests, so that one may wrongly conclude that a static regression between nonstationary series is spurious. As an empirical application, we re-examined the money demand modelling exercise for Colombia carried out by Carrasquilla and Galindo (1994). We found that their result of noncointegration among real balances, output and interest rates may be due to the seasonal adjustment filter applied to the data, since a finding of cointegration among these variables is accepted for seasonally unadjusted data.

We have also investigated an empirical regularity encountered in the Colombian series of M1 and GDP, both of which exhibit a structural break (or change) in the seasonal pattern. Since Perron (1989) it is well-known that a one-time structural change in the trend function of a series, can generate a time series that seems to display a unit root. In the case of seasonal variables, there is the possibility
that they may have unit roots not only at the zero frequency, but also at seasonal frequencies. To decide on the number of seasonal unit roots in univariate time series, researchers usually apply the procedure described in HEGY (1990). However, given that the HEGY test is not applicable in the presence of an exogenous change in the level or seasonal pattern of the data, we extended the unit root testing framework to examine the behaviour of the HEGY test in the presence of such changes.

We found, on the basis of Monte Carlo experiments, that the HEGY test can be adversely affected by a change in either the level or seasonal pattern of a series. In particular, the unit root test is affected by a change in the level, although not by a change in the seasonal pattern of a series. In contrast, the seasonal root test is affected by a change in the seasonal pattern and remains unaffected by a change in the level of the process. The position of the “break” can have a substantial effect on the power of the test statistic. Applying these findings to the Colombian series of M1 and GDP overturns initial results suggesting the presence of unit roots at the seasonal frequencies.

Chapter 4 introduces the theoretical work on the monetary consequences of export booms, which demonstrates that under a regime of fixed exchange rates, they affect the demand and the supply for money, so that the money market is likely to be in disequilibrium in the short run. We argued that the main drawback of existing empirical work on the subject for Colombia, is that the effects on the demand for money have not been considered, despite the fact that theoretical models identify both money demand and money supply effects. In order to overcome this deficiency, we obtained a measure of excess money supply as the residuals from a long-run money demand equation, assuming that money supply is given, and related this measure of excess money supply to a coffee variable and a fiscal variable.
We estimated a money demand model using unadjusted data, as our results in chapter 3 indicated that seasonal adjustment filters reduce the power of cointegration tests. We did not consider the possibility of seasonal cointegration, since our previous results ruled out the presence of unit roots at seasonal frequencies, once we account for the change in the seasonal pattern of M1 and GDP. We found a long-run money demand equilibrium relationship that satisfies both long-run price homogeneity and scale economies in the holding of money. Also, we found evidence of a direct association between coffee exports and the estimated measure of excess money supply, implying that external disturbances jeopardise the ability of economic authorities to carry out successful monetary policy. Finally, we found that the monetarist link between fiscal deficits and excess money supply does not hold for Colombia, at least during the period 1970-1992.

In chapter 5 we have estimated a RER determination model based on the Edwards model, which states that real factors and macroeconomic policies affect the RER in the short run, but in the long run only real factors affect the sustainable equilibrium level of the RER. In general terms our results provided support for the Edwards model. In particular, we found a long-run relationship between the RER and real factors. The RER appreciates as a result of increases in the price of coffee, and in the stock of foreign debt held by the private sector, while it depreciates as a result of increases in import tariffs. The composition of government expenditure can be omitted from the cointegration space, but is nonetheless significant for modelling the short-run dynamics. Technological progress is not found to be significant.

The error correction estimation gives the short-run dynamic specification of the RER. It was found that a) the set of real factors affect the behaviour of the RER in the short run; b) the public sector affects the RER through changes in the composition of government expenditure and changes in the fiscal surplus, so that
even when the financial balance of the government is equal to zero, the RER may appreciate (or depreciate) as a result of changes in the composition of government expenditure; c) in the short run a nominal devaluation depreciates the RER; and d) the behaviour of the RER is not affected by disequilibria in the money market.

We have interpreted the deviations of the RER from its long-run equilibrium relationship, after correcting for the short-run dynamics, as a measure of RER misalignment. Our measure of this misalignment fluctuates within a range of approximately ±20% during the period 1970-1992. The simulation performance of the model during the estimation period, and three years into the future, was particularly good, with the simulated series of the RER reproducing the long-run behaviour of the actual series, and predicting the main turning points in the historical data.

In chapter 6 we have developed an intertemporal disequilibrium model in order to analyse the effects of temporary, anticipated, and permanent coffee price shocks on a small open economy. We considered three disequilibrium regimes: a) Keynesian unemployment in the short and long run; b) Keynesian unemployment in the short run and Walrasian equilibrium in the long run; and c) orthodox Keynesian unemployment in the short and long run. Perhaps one of the most striking points about the comparative static effects presented in the chapter, was the robustness of the results to the three different disequilibrium regimes. They were also robust to the timing of the coffee price boom: the multiplier effects indicated that booms, whether temporary, anticipated or permanent, increase the production of nontradables in the short and long run.

The first variant of the model was then extended by including a government sector that is in charge of the administration of a coffee price stabilisation fund, and allowing for capital market imperfections. The results have shown that when the
government is able to borrow on more favourable terms in international capital markets, the stabilisation fund neutralises part of the short-term effect of coffee price fluctuations. When both the government and individuals borrow on the same terms, the stabilisation fund turns out to be redundant.

Finally, let us look at the future development of the Colombian coffee sector. We have illustrated in the thesis that the coffee sector has been very valuable in Colombia’s development. Coffee production has represented a significant share of Colombian GDP, and has been a substantial source of foreign exchange, government revenue, and employment. In recent years, however, there has been a weakening of the coffee sector, particularly in terms of its relevance as a key foreign exchange earnings activity. Such weakening can be explained by two main factors. On the one hand, changes in preferences and production technologies have been such that the international price of coffee has been at historical low levels, despite transitory recoveries in 1994-1995 and 1997. On the other hand, the discovery of the oil fields of Cusiana and Cupiagua has increased the relative importance of hydrocarbons in total exports.

Despite the loss of relative importance of coffee export earnings in the Colombian economy, the future role of the coffee sector cannot be underestimated, as it will remain a source of employment and value added in the foreseeable future. Moreover, as mentioned in chapter 2, the expected oil boom might be regarded as a temporary phenomenon, depending on the evolution of the oil price, and the discovery of new oil fields. In fact, during the first eight months of 1997, the expected increase in the production of oil did not materialise due to delays in the exploitation of the oil fields of Cusiana and Cupiagua. Coffee exports compensated the decline of oil exports, owing to the transitory recovery in the international price of coffee.
Until now, the depreciation of the real exchange rate that follows a fall in the world price of coffee has defended in part the profitability of coffee producers in crises periods. During the early nineties, however, coffee producers faced a situation of low external prices, which coincided with an appreciated real exchange rate. We have shown in the thesis that the deterioration in the financial position of the government has been one of the factors behind this overvaluation of the real exchange rate. Accordingly, the economic authorities face the challenge of correcting the deterioration of public finances, since the macroeconomic stability of the country and the performance of coffee and other tradable sectors may be undermined. A careful management of the expected oil boom is equally important, in order to avoid the excessive rise of public spending that characterised many oil-exporting countries in the 1970s and 1980s (e.g. Nigeria and Mexico). Otherwise, the oil windfall might adversely affect the performance of the agricultural and industrial sectors.

Finally, the National Federation of Coffee Growers will likely continue using the National Coffee Fund as a price stabilisation device, since this mechanism has played a countercyclical role in the economy, and has been successful in isolating producers from the sharp fluctuations in the world price of coffee (at present the domestic price of coffee is adjusted more frequently following the signals of the international market). However, some authors argue that as a result of the new Colombian Constitution of 1991, which established the independence of the Central Bank, the ability of the National Coffee Fund to support the domestic price of coffee in a crisis period has been curtailed, because the Fund no longer has access to credit from the Central Bank. Similarly, the independence of the Central Bank also limits the availability of credit lines to the government, so that it will also be unable to support the coffee sector, which might increase coffee producers' exposure to fluctuations in the international price of coffee.
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