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A BRIEF SUMMARY OF DIGITAL IMAGE ENHANCEMENT

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Digital image processing is a modern branch of computer application. With the help of computers the visual quality of original images which were blurred by external interferences or optical transfer function of transmission channel can be improved so that the human eye can obtain more authentic information from the processed image. The major research objectives of digital image processing are image enhancement, image restoration, image encoding, image segmentation and image description.

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A Brief Summary of Digital Image Enhancement

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ABSTRACT

Digital image processing is a modern branch of computer application.With the help of computers the visual quality of original images which were blurred by external interferences or optical transfer function of transmission channel can be improved so that the human eye can obtain more authentic information from the processed image.The major research objectives of digital image processing are image enhancement,image restoration, image encoding,image segmentation and image description.

Image Enhancement

The principal objective of image enhancement is to strengthen some information of the original image for a specific application.Because of the variety of the different sorts of images there is no general theory and approach for enhancing pictures.A image which has such low contrast that it is barely visible,for example,by using histogram modification techniques its contrast can be improved.The results of applying a lowpass filter is image smoothing. Image sharpening can be achieved in the frequency domain by a highpass filtering process.

Histogram Modification Techniques

A histogram is a statistical figure which shows the graylevel distribution of the pixels in a image.A narrow range of graylevel values occupied by the pixels of a original image indicates that the pixels in the original image have little dynamic range and the contrast of this picture is very low. Naturally,we want to spread the histogram throughout the full range of graylevels to enhance the contrast of the image.In other words,it is expected that the probability density function of the transformed graylevels is a constant in the interval of definition.

Consider the transformation function

$$S = T(r) = \int_0^r p_r(w)dw$$

May 8, 1988

$$0 \le r \le 1$$

where $p_r(r)$ is graylevel probability density function

$$p_r(r) = \frac{ds}{dr}.$$

It follows from elementary probability theory that the probability density function of the transformed gray levels is given by the relation

$$p_s(s) = \left[p_r(r) \frac{dr}{ds} \right]_{r=T^{-1}(s)} = \left[p_r(r) \frac{1}{p_r(r)} \right] = 1$$

$$0 \le s \le 1$$

In the discrete case,the graylevel transformation function is

$$S_k = T(r_k) = \sum_{j=0}^{k} \frac{n_j}{n} = \sum_{j=0}^{k} p_r(r_j)$$

$$0 \le r_k \le 1$$

$$k = 0,1,\cdots,L-1$$

Related probability density function $p_s(s_k)$ can be yielded by dividing the number of the pixels which were mapped to the same s_k by the total number of pixels in the image.That is,

$$p_s(s_k) = \frac{n_k}{n}$$

$$0 \le r_k \le 1$$

$$k = 0,1,\cdots,L-1$$

It is sometimes descriable to be able to specify interactively particular histograms capable of highlighting certain graylevel ranges in a image.The procedure can be summarized as follows:

(1) Equalize the levels of the original image using Eg.

$$s_k = T(r_k) = \sum_{j=0}^{k} \frac{n_j}{n} = \sum_{j=0}^{k} p_r(r_j)$$

$$0 \le r_k \le 1$$

$$k = 0,1,\cdots,L-1$$

(2) Specify the desired density function $p_z(z)$ obtain the transformation function $g(z)$ using Eg.

$$v_k = g(z_k) = \sum_{j=0}^{k} p_z(z_j)$$

(3) Apply the inverse transformation function $Z = G^{-1}(s)$ to the levels obtained in step (1).

Image Smoothing

Smoothing operations are used primarily for diminishing spurious effects that may be present by noise.

[1] Neighbourhood averaging

Heighbourhood averaging is a straightforward spatial-technique for image smoothing.The smoothed image is obtained by using the relation

$$g(x,y) = \frac{1}{M}\sum f(n,m) + \frac{1}{M}\sum \eta(n,m)$$

The variance of the noise is

$$V_{ar}\left[\frac{1}{M}\sum \eta(n,m)\right] = \frac{1}{M}\sigma^2$$

It is obvious that after smoothing operation the variance of the noise is down to $1/M$ of the original variance σ^2. For a given radius, the smoothed image is obtained by using the relation

$$g(x,y) = \begin{cases} \frac{1}{M}\sum f(m,n) & \text{if } |f(x,y) - \frac{1}{M}\sum f(m,n)| > T \\ f(x,y) & otherwise \end{cases}$$

where T is a specified nonnegative threshold.

[2] Median filtering

Median filter technique is a straightforward smoothing approach.When the template

$$\begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$$

covers the nine elements the central element is substituted by median which is the median of the sequence of the nine elements ordered by magnitude.

[3] Lowpass filtering

Sharp transitions (such as noise and edges) in the gray levels of an image contribute heavily to the high-frequency content of its Fourier transform.The function of lowpass filtering approaches is to filter out a specified range of high-frequency components in the transform of a given image and to let the information in the low-frequency range pass without attenuation.

The transfer function of ideal lowpass filter is

$$H(u,v)=\begin{cases}1 & \text{if } D(u,v)\le D_0\\ 0 & \text{if } D(u,v)>D_0\end{cases}$$

where D_0 is the cut-off frequency locus,D(u,v) is the distance from point (u,v) to the origin of the frequency plane.

For avoiding "ringing" phenomenon and passing a fairly high amount of high-frequency information to preserve more of the edge content in the picture following several filters were improved.

Butterworth filter

$$H(u,v)=\frac{1}{1+(\sqrt{2}-1)\,[D(u,v)/D_0]^{2n}}$$

Exponential filter

$$H(u,v)=e^{-\left[\ln(1/\sqrt{2})\right]\left[D(u,v)/D_0\right]^n}$$

Trapezoidal filter

$$H(u,v)=\begin{cases}1 & \text{if } D(u,v)<D_0\\ \dfrac{[D(u,v)-D_1]}{[D_0-D_1]} & \text{if } D_0\le D(u,v)\le D_1\\ 0 & \text{if } D(u,v)>D_1\end{cases}$$

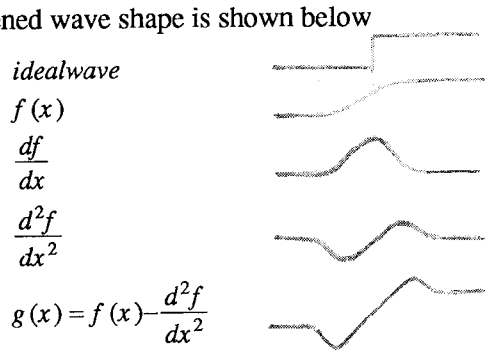
where D_0 and D_1 are specified,and it is assumed that $D_0<D_1$.

Image sharpening

The following sharpening techniques are useful as the enhancement tools for highlighting edges in an image which was blurred after transmission or transform.

[1] Sharpening by differentiation

In the one-dimensional case, the sharpened wave shape is shown below



Similar expression holds for two dimensions using Laplacian

$$g(x,y)=f(x,y)-a\nabla^2(x,y)$$

where a is specified.

The discrete form of above expression is given by the relation

$$g(x,y)=f(x,y)-[f(x+1,y)+f(x,y+1)+f(x,y-1)+f(x-1,y)-4f(x,y)]$$

This Eq. can be expressed by the template

$$\begin{bmatrix} 0 & -a & 0 \\ -a & 1+4a & -a \\ 0 & -a & 0 \end{bmatrix}$$

By properly selecting a threshold, it is possible to emphasize significant edges without destroying the characteristics of smooth backgrounds

[2] Highpass filtering

Since edge is a abrupt graylevel change which includs a large amount of high-frequency components,image sharpening can be achieved in the frequency domain by a highpass filtering process which attenuates the low-frequency components without disturbing high-frequency information.

Ideal highpass filter

Its transfer function can be defined by the relation

$$H(u,v)=\begin{cases} 0 & \text{if } D(u,v) \le D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$$

with cut-of frequency locus at a distance D_0 from the origin.

Butterworth filter

$$H(u,v) = \frac{1}{1 + [\sqrt{2}-1]\,[D_0/D(u,v)]^{2n}}$$

Exponential filter

$$H(u,v) = e^{-[\ln 1/\sqrt{2}]\,[D_0/D(u,v)]^n}$$

Trapezoidal filter

$$H(u,v)=\begin{cases} 0 & \text{if } D(u,v) < D_1 \\ \dfrac{[D(u,v)-D_1]}{[D_0-D_1]} & \text{if } D_1 \le D(u,v) \le D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$$

Homomorphic filtering approach

An image, $f(x,y)$,can be expressed in terms of its illumination,$i(x,y)$, and reflectance,$r(x,y)$,components by means of the relation

$$f(x,y) = i(x,y)r(x,y)$$

Let $$z(x,y) = \ln f(x,y) = \ln i(x,y) + \ln r(x,y)$$

then,

$$Z(u,v) = I(u,v) + R(u,v)$$

let

$$S(u,v) = H(u,v)I(u,v) + H(u,v)R(u,v)$$

then

$$s(x,y) = F^{-1}\left\{S(u,v)\right\} = i'(x,y) + r'(x,y)$$

where

$$i'(x,y) = F^{-1}\left\{H(u,v)I(u,v)\right\}$$

$$r'(x,y) = F^{-1}\left\{H(u,v)R(u,v)\right\}$$

The desired enhanced image is

$$g(x,y) = \exp s(x,y) = \exp i'(x,y) \exp r'(x,y) = i_0(x,y)\, r_0(x,y)$$

The illumination component which is characterized by slow spatial variations is associated with the low frequencies of the Fourier transform of the logarithm of an image and the reflective component which tends to vary abruptly with the high frequencies;therefore, the specified filter function , H(u,v),will tend to decrease the low-frequencice and amplify the high- frequencies.In fact,it is a highpass filter.

Pseudo-color and false-color processing

The human eye can discern thousands of color shades and intensities.This is in sharp contrast with the eye's poor performance with gray levels. The objective of pseudo-color technique is to assign a color to each pixel of a monochrome picture based on its gray level.

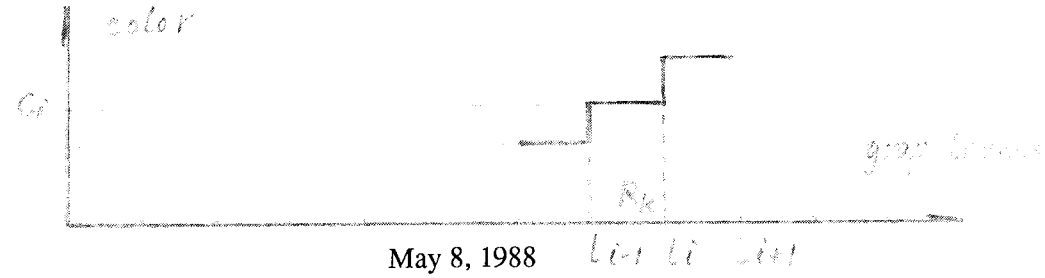
[1] Density slicing

Suppose that M planes partition the gray scale into M+1 regions and color assignments are made according to the relation

$$f(x,y) = C_k \quad \text{if } f(x,y) \in R_k$$

where C_k is the color associated with the kth region, R_k .

A alternative representation is shown below



[2] Gray-level-to-color transformations

This approach is to perform three independent transforms on the gray levels of any input pixel.The three elements of the results are shown below

$$I_R(x,y) = T_R f(x,y)$$

$$I_G(x,y) = T_G f(x,y)$$

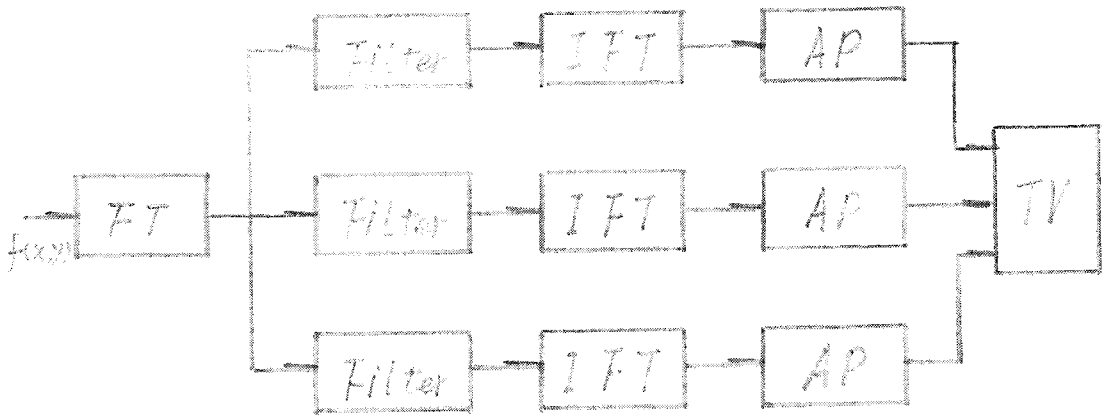
$$I_B(x,y) = T_B f(x,y)$$

where T_R , T_G and T_B are three spesified transformation functions,respectively.

Then the three color elements overlap in TV monitor and produce a composite image whose color content is modulated by the nature of the transformation function.

[3] Filtering approach

The objective of this color processing technique which is shown in following figure is to color-code regions of an image based on frequency content.



However,the different composite images are obtained by using the different filters.

[4] False color processing

This method is based on a special case that the original image is a color picture.The procedure of the false color processing can be described by means of a simple example.Consider the relation

$$\begin{bmatrix} R_g \\ G_g \\ B_g \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R_f \\ G_f \\ B_f \end{bmatrix}$$

where R_f , G_f and B_f are the three color elements of the original image,R_g , G_g and B_g are the color elements of the false color image .

In fact, this is a sort of color coordinate transforms from the original three primary colors to a new set of three primary colors.

Image Restoration

The expression describing a linear degradation model of an input image, $f(x,y)$,is

$$g(x,y) = \iint_{-\infty}^{\infty} f(\alpha,\beta)\, h(x,\alpha,y,\beta)\, d\alpha d\beta + \eta(x,y)$$

If H is position invariant, this expression becomes

$$g(x,y) = \iint_{-\infty}^{\infty} f(\alpha,\beta)\, h(x-\alpha,y-\beta)\, d\alpha d\beta + \eta(x,y)$$

The discrete degradation model is

$$g_e(x,y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f_e(m,n)\, h_e(x-m,y-n) + \eta_e(x,y)$$

It can be expressed in the following vector-matrix form

$$g = H f + n$$

where f ,g and n represent MN-dimension column vectors formed by stacking the rows of the $M \times N$ functions $f_e(x,y)$,$g_e(x,y)$ and $\eta_e(x,y)$, the matrix H is a block-circulant matrix.

Unconstrained restoration

We wish to find a $\hat{f}$ such that $||n||^2 = ||g - H\hat{f}||^2$ is minimum.

Let

$$\frac{\partial[||g - H\hat{f}||^2]}{\partial \hat{f}} = -2H'(g - H\hat{f}) = 0$$

We have $\hat{f} = (H'H)^{-1}H'g$

If H is a square matrix,then

$$\hat{f} = H^{-1}(H')^{-1}H'g = H^{-1}g$$

Constrained restoration

This is a least-squares restoration problem.We seek an $\hat{f}$ which minimizes the criterion function

$$J(\hat{f}) = ||Q\hat{f}||^2 + \alpha(||g - H\hat{f}||^2 - ||n||^2)$$

where α is a constant called the Lagrange multiplier,Q is a linear operator on f, $\alpha(||g - Hf\ hat||^2 + ||n||^2)$ is the constraint.

Let

$$\frac{\partial J(\hat{f})}{\partial \hat{f}} = 0 = 2Q'Q\hat{f} - 2\alpha H'(g - H\hat{f})$$

then

$$\hat{f} = (H'H + \gamma Q'Q)^{-1}H'g$$

where $\gamma = \frac{1}{\alpha}$.

Inverse filtering

Based on the discussion that the discrete degradation model is

$$g_e(x,y) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f_e(m,n)\, h_e(x-m, y-n) + \eta_e(x,y)$$

we have that the following frequency-domain relation holds

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

where G,F and N are the Fourier transforms of the degraded image, original image , and noise, H is a "filter" function.

Let

$$F(u,v) = \frac{G(u,v)}{H(u,v)}$$

Considering noise term this equation can be expressed in the following form

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

The restored image is obtained by using the relation

$$f(x,y) = F^{-1}\left[\hat{F}(u,v)\right]$$

It is important to note that if H(u,v) is zero or becomes very small the term could dominate the restoration result.In order to avoid small values of H(u,v), in practice,we carry out the restoration in a limited neighborhood about the origin.

Removal of blur caused by uniform linear motion

Suppose that the imge in question undergoes uniform linear motion in the positive x-direction only and y is time invariant.The total exposure at any point can be expressed in this form

$$g(x) = \int_0^T f[x - x_0(t)]dt = \int_{x-a}^{x} f(\tau)d\tau$$

$$\tau = x - at/T \quad , \quad 0 \le X \le L$$

then,by differentiation, $g'(x) = f(x) - f(x-a)$.

It will be convenient to assume that $L = Ka$,then,$x = z + ma$, where $0 < z < a$, $m = 0,1,2,....,K-1$, m is the integral part of $\frac{x}{a}$.

Then, we have $\qquad f(z+ma) = g'(z+ma) + f[z+(m-1)a]$.

Let

$$\phi(z) = f(z-a)$$

It is evident that $f(z+ma)$ can be expressed in the form

$$f(z+ma) = \sum_{k=0}^{m} g'(z+ka) + \phi(z)$$

or

$$\phi(x-ma) = f(x) - \sum_{k=0}^{m} g'(x-ka)$$

then,

$$K\phi(x) = \sum_{k=0}^{K-1} f(x+ka) - \sum_{k=0}^{K-1}\sum_{j=0}^{m} g'(x+ka-ja)$$

Let

$$\sum_{k=0}^{K-1} f(x+ka) = KA$$

then,

$$\phi(x-ma) = A - \frac{1}{K}\sum_{k=0}^{K-1}\sum_{j=0}^{m} g'(x+(k-m)a-ja)$$

The final result becomes

$$f(x) = A - \frac{1}{K}\sum_{k=0}^{K-1}\sum_{j=0}^{m} g'(x-ma+(k-j)a) + \sum_{j=0}^{m} g'(x-ja)$$

Least-squares(Wiener) filter

Consider the least-squares restoration problem, we seek an $\hat{f}$ which minimizes the criterion function and the solution is

$$\hat{f} = (H'H + \gamma Q'Q)^{-1}H'g.$$

By defining $Q'Q = R_f^{-1}R_n$

where

$$R_F = E\left\{ff'\right\} = WAW^{-1}$$

$$R_n = E\left\{nn'\right\} = WBW^{-1}$$

We yield

$$\hat{f} = (WD^*DW^{-1} + \gamma WA^{-1}BW^{-1})^{-1}WD^*W^{-1}g$$

or

$$W^{-1}\hat{f} = (D^*D + \gamma A^{-1}B)^{-1}D^*W^{-1}g$$

The elements of this equation can be written in the form

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2 + \gamma[S_\eta(u,v)/S_f(u,v)]}\right]G(u,v)$$

Constrained least-squares restoration

The constrained least-squares restoration procedure can be summarized as follows.

Step 1

Choose an initial value of γ and obtain an estimate of $||n||^2$ using

$$||n||^2 = (M-1)(N-1)[\sigma_\eta^2 + \bar{\eta}_e^2]$$

Step 2

Compute $\hat{F}(u,v)$ using

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma |P(u,v)|^2} \right] G(u,v)$$

Obtain $\hat{f}$ by taking the inverse Fourier transform of $F(u,v)$.

Step 3

Form the residual vector r according to $r = g - H\hat{f}$ and compute $\phi(\gamma) = ||\gamma||^2$.

Step 4

If $\phi(\gamma) < ||n||^2 - a$, increment γ.

If $\phi(\gamma) > ||n||^2 + a$, decrement γ.

Step 5

Return to step 2 and continue unless $\phi(\gamma) = ||n||^2 \pm a$.

Interactive restoration

The Fourier transform of a two-dimensional coherent sinusoidal noise is given by the relation

$$N(u,v) = \frac{-jA}{2} \left[\delta(u-u_0/2\pi\ ,\ v-v_0/2\pi) - \delta(u+u_0/2\pi\ ,\ v+v_0/2\pi) \right]$$

By using a bandreject filter the pair of impulses in the frequency plane can be cleaned up.The transfer function of the bandreject filter is

$$H(u,v)=\begin{cases} 0 & \text{if } D_1(u,v) \le D_0 \;\; or \;\; D_2(u,v) \le D_0 \\ 1 & otherwise \end{cases}$$

where

$$D_1(u,v) = [(u-u_0)^2 + (v-v_0)^2]^{1/2}$$

$$D_2(u,v) = [(u+u_0)^2 + (v+v_0)^2]^{1/2}$$

It is a common problem that the interference pattern is composed of more than just one sinusoidal component.In this case,the restored image can be expressed in this form

$$\hat{f}(x,y) = g(x,y) - w(x,y)P(x,y)$$

where $g(x,y)$ is the corrupted image, $p(x,y)$ is the interference pattern, $w(x,y)$ is a weighting function to be determined.Then,

$$\sigma^2(x,y) = \frac{1}{(2x+1)(2y+1)} \sum_{m=-X}^{X} \sum_{n=-Y}^{Y} [\hat{f}(x+m,y+n) - \hat{f}(x,y)]^2$$

Let

$$\frac{\partial \sigma^2(x,y)}{\partial w(x,y)} = 0$$

The result is

$$w(x,y) = \frac{\overline{g(x,y)p(x,y)} - \bar{g}(x,y)\bar{p}(x,y)}{\overline{p^2(x,y)} - \bar{p}^2(x,y)}.$$

Conclusions

The above represents a brief summary of the large range of techniques for improving the quality of images , either to enhance their visual quality or to satisfy some objective criterion , such as m.s.e. Further details may be found in the references below .

References

[1] R.C. Gonzalez & P. Wintz , 'Digital Image Processing,' Reading , Addison-Wesley , 1977.

[2] H.E. Knutsson , R. Wilson , and G.H. Granlund , "Anisotropic Nonstationary Image Estimation and Its Applications: Part I--Restoration of Noisy Images," IEEE Transactions on Communications , March 1983.

[3] H.C.Andrews , "Monochrome Digital Image Enhancement," Applied Optics , February 1976.

[4] N.C. Gallagher , Jr. and G.L. Wise , "A Theoretical Analysis of the Properties of Median Filter," IEEE Transactions Acoustics , Speech , and Signal Processing , December 1981.

[5] M.P. Ekstrom , "Realizable Wiener Filtering in Two Dimensions," IEEE Transactions on Acoustics , Speech , and Signal Processing , February 1982.

[6] A.A. Sawchuk , "Space-Variant Image Motion Degradation and Restoration," Proceedings of the IEEE , July 1972.

[7] D.T. Kuan , A.A. Sawchuk , T.C. Strand , and P. Chavel , "Adaptive Noise Smoothing Filter for Images with Signal-Dependent Noise," IEEE Transactions on Pattern Analysis and Machine Intelligence , March 1985.

[8] R. Chellappa and R.L. Kashyap , "Digital Image Restoration Using Spatial Interaction Models," IEEE Transactions on Acoustics , Speech , and Signal Processing , June 1982.