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REASONING ABOUT HYBRID
REASONING: A DISCUSSION

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In this discussion we outline an approach to reasoning about hybrid reasoning systems. We believe that following this approach allows theoretical results to be obtained rapidly, allows a high level of understanding about the relationship between different hybrid reasoning systems, and allows advantage to be taken of results about classical systems. There are two aspects to the approach we propose. First we suggest that we should treat hybrid reasoning systems in as much generality as possible. Second, although we acknowledge that unification is of central importance in hybrid reasoning systems, we think it can be a mistake to centre theoretical treatments of hybrid reasoning systems on unification itself. Instead we suggest that one should consider proof systems without unification and then introduce unification as a purely syntactic measure to improve efficiency.

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Reasoning about hybrid reasoning: a discussion

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In this discussion we outline an approach to reasoning about hybrid reasoning systems. We believe that following this approach allows theoretical results to be obtained rapidly, allows a high level of understanding about the relationship between different hybrid reasoning systems, and allows advantage to be taken of results about classical systems.

There are two aspects to the approach we propose. First, we suggest that we should treat hybrid reasoning systems in as much generality as possible. Second, although we acknowledge that unification is of central importance in hybrid reasoning systems, we think it can be a mistake to centre theoretical treatments of hybrid reasoning systems on unification itself. Instead we suggest that one should consider proof systems without unification and then introduce unification as a purely syntactic measure to improve efficiency.

The ideas supporting our case arise out of our research. Both of us have been much more involved in theoretical work concerning hybrid reasoning systems than in actual hybrid reasoning. Our arguments therefore concern the processes of reasoning *about* hybrid reasoning systems rather than the process of reasoning *within* hybrid reasoning systems.

§1 Analytic tableau for hybrid reasoning

Although a lot of research has been done concerning hybrid reasoning systems using resolution (Stickel 85, Frisch 89, Bürckert 90), very little has considered analytic tableaux.

Presumably, the reason that resolution has dominated tableaux in the literature is that people consider that tableau proof systems must be more inefficient than resolution. In fact, this question is still open. Although it is known that there are cases where the shortest resolution proof is exponentially shorter than the shortest tableau proof, and that the reverse cannot hold (see for example D'Agostino 1990, Chapter 4) the same results are not known about *searching* for a proof, even in the classical propositional case. Reeves (1987) has argued that analytic tableaux remain a suitable framework for automated thoerem proving.

Without needing to claim that tableaux are more efficient than resolution, there remain two excellent reasons for investigating hybrid reasoning in analytic tableaux. The first is that many people consider analytic tableaux a very nice way for people to prove simple theorems. The second reason concerns the efficiency of automated theorem proving. Wallen (1989) has shown that by careful analysis of the inherent inefficiencies of analytic tableaux one can derive a more efficient proof system that is the same as Bibel's (1982) connection method. Gent's forthcoming thesis starts by considering a particular type of analytic tableau.[1] The syntax of the logic under consideration is normal except that all quantifiers are restricted by statements ρ from some special purpose theory. We read "$\forall x_{:\rho(x)}\, \phi(x)$" as "$\phi(x)$ is true for all x satisfying $\rho(x)$", and "$\exists x_{:\rho(x)}\, \phi(x)$" as "$\phi(x)$ is true for some x satis-

[1] In fact, a simple generalisation which includes not only tableaux but also Mondadori's (1989) KE.

fying $\rho(x)$". The restriction ρ must be an atomic formula, but the predicate may be of arbitrary arity. The quantifier rules in the tableau system are shown in Figure 1.

Figure 1: Quantifier rules in a theory tableau system

(†) and (‡) represent side conditions on applications of the rules. They restrict which terms may be introduced when the quantifier rules are applied.

(†)*a is some new name (i.e. a has not been used to date on the branch)*

(‡)$\rho(t)$ *is a* Σ*-consequence of the set* R *of* Σ*-literals on the branch.*

$T\forall$ (‡)	$T\forall x{:}\rho(x)\ \phi(x)$ $\mid$ $T\phi(t)$	$F\forall$ (†)	$F\forall x{:}\rho(x)\ \phi(x)$ $\mid$ $T\rho(a)$ $F\phi(a)$
$T\exists$ (†)	$T\exists x{:}\rho(x)\ \phi(x)$ $\mid$ $T\rho(a)$ $T\phi(a)$	$F\exists$ (‡)	$F\exists x{:}\rho(x)\ \phi(x)$ $\mid$ $F\phi(t)$

This tableau system is not complete for all first order theories Σ, but completeness holds if Σ satisfies a unique minimal model condition. A similar condition has arisen in the work of Frisch (1991) and Bürckert (1991). The condition is satisfied if Σ can be expressed in Horn clauses.

The important point about the theory tableau system is that the only place in which the theory Σ needs to be considered is in the side condition (‡) on universal instantiation. This can be seen as a fundamental result for hybrid reasoning. It gives a hybrid reasoning system that can be used immediately for a wide range of theories Σ and explicitly separates special purpose reasoning from logical reasoning.

Modal logics are an example where the tableau system is directly applicable. The standard restrictions on the accessibility relation can easily be written in Horn clauses (for example symmetry, transitivity and reflexivity). On being shown an example of this, a colleague was surprised that all that was going on was explicit reasoning about the accessibility theory (personal communication from Nick Measor to Gent). This is exactly the point. We have a simple system for reasoning explicitly about underlying theories, and simple results about its correctness.

§2 Be General

We should reduce to a minimum what we have to study about each different hybrid reasoner. By stating the result in §1 in as much generality as possible, we achieve this. We need only consider how to fit in a special purpose reasoner with universal instantiation or unification. Furthermore, the isolation of the unification problem from the logical problem may make it easier to recognise that the unification problem has been solved, perhaps by comparison with the methods surveyed by Siekmann (1986).

Of course we get the usual advantages of generality: many systems are special cases. For example, many of Fitting's (1983) prefixed tableaux for modal logics are, with a little extra machinery, special cases of the system in §1. So is Schmitt and Wernecke's (1990) tableau calculus for order sorted logic.[2]

The generality of the system of §1 has further advantages. We can use it as a foundation to prove correct other proof systems for hybrid reasoning. For instance, following the arguments of Wallen (1986, 1989), it is possible to derive an analogue of the connection method which incorporates special purpose reasoning. Indeed, that is the reason why the first author

[2] Indeed Schmitt and Wernecke, by treating order sorted logic as a special case, have missed the wood for the trees: they propose as a research problem "What is an adequate general framework for establishing results ... for variants of many sorted predicate logic?". This is a problem that the system of §1 solves.

started studying the tableau system in the first place. The result generalises Wallen's connection based proof systems for modal logics. To prove correct the proof system for each modal logic, we need only show that a certain property holds of the accessibility theory of the modal logic. Proving this property is non trivial, but immensely much simpler to show than Wallen's direct proof for each logic. We also obtain connection based proof methods for, for example, many order sorted logics. This will be described in Gent's forthcoming thesis as well as in (Gent 1991).

The argument used to produce connection methods is too complex to give here. A much simpler argument shows how to prove correct a resolution system like Frisch's (1991) or Bürckert's (1990).

Consider tableau proofs of formulas known to be in clause form.[3] We will never need to use existential introduction. As existential introduction is the only way of changing the set of restrictions, the side condition (‡) is always equivalent to "$\rho(t)$ is a Σ-consequence of the empty set." So exactly when universal instantiation takes place does not affect the satisfaction of (‡). This means that we can move universal instantiation around in the proof arbitrarily. Since all universal quantifiers are at the outside of a clause, we can move all the universal instantiations to the top of a proof. This splits the tableau proof into two parts. In the first part, we choose suitable terms for each variable. In the second, we do some purely propositional reasoning to establish an inconsistency. However, we know that resolution is complete for propositional logic. Therefore, we can replace the second part of the proof by a propositional resolution proof. We now have a resolution system incorporating special purpose reasoning. All that remains is to observe that it is sensible to delay the choice of terms by introducing dummy variables. Maximal unifiers are defined syntactically. The result is a resolution system incorporating special purpose unification, yet with a very simple proof of correctness. The result can be obtained for any restriction of resolution which is complete for propositional logic.

Note that the systems we have discussed in §1 and §2 are *ground* systems: that is, unification plays no part. Unification is crucial, so we turn our attention to it now.

§3 The Role of Unification in Hybrid Reasoning

The system of §1 provides a beautiful explanation of the crucial role of unification in hybrid reasoning. Unification arises out of universal instantiation. But that is exactly where we attached special purpose reasoning in the new tableau system. Special purpose unification arises from the combination of special purpose reasoning and the desire to delay the choice of term to introduce. We introduce dummy variables instead of terms in universal instantiation. We then say that a Σ-unifier of two terms is a unifier of them that ensures that any instantiation would satisfy all relevant instances of the side condition (‡). A *maximally general* Σ-unifier is defined in a standard way (e.g. see Frisch 1991), but, as we point out below, is not necessarily unique, even up to renaming.

As in non hybrid reasoning, unification plays a crucial part in making hybrid reasoning efficient. This is for the same reason as in classical logic. That is, the choice of term for universal quantifier elimination is delayed. Indeed, most existing work in hybrid reasoning has concentrated, sometimes exclusively, on the role of unification in hybrid reasoning and how unification is affected by the background theory or theories.

We briefly consider how unification is affected by the background theory of our hybrid reasoning system and how unification should be

[3] Our argument here omits the problem of translating formulas into clause form. For details see (Frisch 1991, Appendix B).

integrated into our calculus. The main question to consider is how the condition (‡) is to be integrated into unification. At the time of universal quantifier elimination, the constraint ρ is attached to the dummy variable. When unification subsequently substitutes for the variable the condition must be checked; however the instantiated variable may not be fully ground. Although, one could try to wait until this was the case, this is not computationally efficient since we may be able to detect early that the condition will not be satisfied. As is well known (e.g. Walther 1987, Schmidt-Schauss 1989, Frisch 1991), in general, it may be the case that the instantiated term may have to be further instantiated ('weakened') in order to ensure that the condition is satisfied. These works have also shown that some theories may give rise to more than one maximally general unifier and that there may even be infinitely many such unifiers. It also turns out that the minimal set of such unifiers cannot be computed without post filtering. It is worth pointing out that there are a number of techniques by which multiple unifiers may sometimes be compactly represented as a single, complex, constraint (Cohn 1987, Chaminade 1988).

If arbitrary theories are allowed (even arbitrary Horn clause theories, which obey the semantic restriction referred to earlier), unification can become very complex, as indicated above. The interest is in isolating those cases which have nice computational properties with regard to unification. Several such cases have been isolated for many sorted logic (Walther 1988, Schmidt-Schauss 1989, Meseguer et al 1989). Cases of theories whose predicates' arity is higher than one have been less investigated.

We are not making any claims for originality of work in hybrid unification here; the purpose of this section is merely to point out that we can capitalise on all the important existing work in this area.

§4 Think About Unification Last

We have argued that unification is crucial to hybrid reasoning. Yet in this paper we have thought about the proof systems first, and only then worried about unification. This is because we feel that the role of unification is made more clear by considering it last.

What happens if we take unification to be a basic operation? Of course one can obtain similar results to those we have outlined. However, how is one to define unification? In our earlier discussions, the definition arose naturally in each case as a way of abbreviating search in a known proof system. Taking unification as primary, how can one ensure that it is defined so that it will fit in exactly with the proof system? For instance, although Frisch (1991) proposes a unification based approach to hybrid reasoning, we find it difficult to see how one would apply his approach to tableau systems.

The objection in the last paragraph can be illustrated from the literature. Jackson and Reichgelt (1987, 1988) presented, without proofs, a proof method for predicate modal logics. This proof system is a resolution system, but considerably complicated by the fact that the authors included a particular unification algorithm as part of the proof system. Although Jackson and Reichgelt have presented a soundness proof (1989), they state that "We do not have a full completeness proof at the present time" (1989, page 190). No completeness proof had been found by January 1991 (personal communication from Reichgelt to Gent). We suggest that the completeness proof is elusive just because unification was introduced too early. We feel that Jackson and Reichgelt's system could be proved correct much more easily using our approach, that is to prove first a ground version of their system and then to check that their unification algorithm is correct. However, we have not actually carried out such a proof.

There is another objection to taking unification as primary. One then has to consider unification as a semantic operation. Yet we feel that unification is essentially a syntactic operation. The unifier of a number of terms just tells us a new term that captures the syntactic structure of all of them. To define unification semantically begs the question "what is the intuitive meaning of unification"? We find it difficult to answer this question. We accept happily that others may find it easier. For example, Frisch (personal communication) says that unification simply means solving a system of equations. Even then difficulties arise. Exactly which system of equations must be solved? Might this not depend on the details of the underlying proof system? With our approach one is not forced into defining the semantics of unification (though of course one may do so if desired).

§5 Conclusions

As already indicated, the approach to hybrid reasoning outlined here is not fully general. Theories which do not obey the unique minimal model condition are one example of systems not covered by our calculus. Also not covered are systems where theory literals also occur explicitly in normal formulae. The second author has argued (Cohn 1987,1989) for the utility of such expressiveness and has designed a special purpose, resolution based, system to perform hybrid reasoning in a many sorted logic of this kind: a rule akin to Stickel's *Theory Resolution* is needed in this case (Stickel 1985). Similar arguments have been made by Beierle et al (1989). An interesting research problem is to attempt to extend our approach to cover such systems in a way which is *enlightening*, i.e. which explains the computational advantages to be gained by a hybrid system.

We have made two claims. We suggest that when reasoning about hybrid reasoning, we should seek for as much generality as possible. We also suggest that thinking about the role of unification in hybrid reasoning systems should take second place to deciding on the correct proof system to use. We have made these claims in the context of our own research, which we have briefly discussed.

Although defending our claims in this paper, we appreciate that others might disagree with them. If so, we hope to provoke discussion.

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