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Abstract

N. Stewart, G. D. A. Brown, and N. Chater (2005) presented a relative judgment model (RJM) of absolute identification, in which the current stimulus is judged relative to the preceding stimulus. S. Brown, A. A. J. Marley, and Y. Lacouture (2007) found that the RJM does not predict their finding of increased accuracy after large stimulus jumps, except at the expense of other effects. In fact, the RJM does predict both the core effects and also increased accuracy after large jumps (though it underestimates this effect) when better constrained parameters are estimated from the trial-by-trial raw data rather than from summary plots. Further, a modified RJM, in which the stimulus from two trials ago is sometimes used as a referent, provides a better fit.
Absolute Identification is Relative

Stewart, Brown, and Chater (2005) presented a relative judgment model (RJM) of absolute identification. In absolute identification, participants are presented with a series of stimuli drawn from a set that varies along a single dimension (e.g., tones varying in their loudness). Stimuli are normally evenly spaced along the dimension. Participants are asked to identify each stimulus with its rank position in the set. Stewart et al.'s RJM differs from previous accounts in assuming that identifications are made without reference to long-term memories of absolute stimulus magnitudes (see also Laming, 1984). Almost all existing models utilize long-term absolute magnitude information. For example, Thurstonian models represent long-term absolute magnitude information in the positioning of criteria along a perceptual continuum (Durlach & Braida, 1969; Luce, Green, & Weber, 1976; Treisman, 1985). Exemplar models represent long-term absolute magnitude information in the stored stimulus magnitude-stimulus label pairs (Kent & Lamberts, 2005; Nosofsky, 1997; Petrov & Anderson, 2005). Connectionist models represent long-term absolute magnitude information in the mapping between stimulus and response nodes (Lacouture & Marley, 2004). Anchor models represent long-term absolute magnitude information as a memory for anchors at the edges of the stimulus range (Karpiuk, Lacouture, & Marley, 1997; Marley & Cook, 1984).

In the RJM, long-term memories of absolute magnitudes are assumed to be unavailable and judgment is instead relative to the immediately preceding stimulus. In conjunction with the feedback for the previous stimulus, the difference between the current stimulus and the previous stimulus is used to identify the current stimulus. For example, if the feedback on the previous trial is Stimulus 4 and the current stimulus is 3 response scale units higher in magnitude, then Stimulus 7 will be given as a response. By assuming that the difference between stimuli are confused and that there is a capacity limit in mapping between stimulus differences and the response scale, the RJM can account for the main phenomena observed in absolute identification (reviewed in detail by Stewart et al., 2005).
Brown, Marley, and Lacouture (2007) presented a more detailed examination of the sequential effects in Lacouture's (1997) absolute identification of line length experiment. Although the RJM accounts for the average effects well, Brown et al. claimed that it does not account for the more detailed pattern. Specifically, Brown et al. plotted accuracy on the current trial as a function of the difference between the current stimulus and the previous stimulus (Figure 1A). There is a small accuracy advantage when the previous stimulus is the same as the current stimulus and there is a larger accuracy advantage when the difference between stimuli is as large as possible. Brown et al. suggested that this larger advantage is problematic for the RJM. In the RJM, the number of responses represented in the mapping process depends upon the previous feedback and the sign of the difference between the current and previous stimuli (see Stewart et al., 2005, Equations 4 and 5). For example, when Stimulus 1 is followed by Stimulus 10, constant noise in the mapping process produces a large amount of noise in responding because more possible responses must be represented (Responses 2-10) within the limited capacity. In contrast, when Stimulus 5 is followed by Stimulus 10, fewer responses must be represented (Responses 6-10) and so the constant noise in the mapping process produces less noise in responding. The RJM is predicting more noise when there is a big difference between the previous and current stimuli, but Lacouture's data show increased accuracy - not reduced accuracy - in this situation. This observation led Brown et al. to question whether judgment is always relative.

Extension to Other Data Sets

I have examined three other data sets (Table 1) and repeat Brown et al.'s analysis in the other panels of Figure 1. Kent and Lamberts (2005) used a task in which the distance between two dots was identified (a task very similar to the identification of line lengths used by Lacouture, 1997). Stewart et al. (2005) and Brown, Neath, and Chater (2002) both used absolute identification of the frequency of pure tones. The same qualitative pattern can be seen in all of the data sets. There is an accuracy advantage for stimulus repetitions and for the
largest jump sizes compared to intermediate jump sizes. Finding this pattern in data from four different laboratories (Lacouture's, Kent's, Stewart's, and Neath's) with two different continua (length and pitch) shows it is reliable and I would tentatively suggest that this effect be included in the set of benchmark phenomena that any absolute identification model should explain. The relative ordering of performance on stimulus repetitions compared to the largest jump sizes differs between data sets. Performance is better for large jumps in the Lacouture data set (by .070) and the Kent and Lamberts data set (by .029), but performance is better for repetitions in the Stewart et al. data set (by .303) and the Brown et al. data set (by 0.127).

Relation to Other Effects

Increased accuracy on stimulus repetitions can be seen in other summary plots of the data. For example, when accuracy plotted as a function of stimulus is parameterized by the number of trials since a stimulus repetition (e.g., Petrov & Anderson, 2005; Rouder, Morey, Cowan, & Pfaltz, 2004; Siegel, 1972, Stewart et al., 2005) very high accuracy is observed when stimuli are repeated (i.e., at 0 trials since a repetition). This shows that the advantage for repetitions is seen for all stimuli. When $d'$ (a measure of the confusion between adjacent stimuli) plotted as a function of stimulus is parameterized by the difference between the current and previous stimulus higher $d'$ is observed when the current and previous stimuli are similar (e.g., Stewart et al., 2005) though the effect is sometimes small or null (e.g., Luce, Nosofsky, Green, & Smith, 1982; Nosofsky, 1983; Purks, Callahan, Braida, & Durlach, 1980).

Increased accuracy after large stimulus jumps can also be seen in other summary plots of the data. For example, when accuracy or $d'$ is plotted as a function of stimulus, a bow is observed with better performance for the smallest and largest stimuli (see Stewart et al., 2005, for a review of the bow effect). Because the largest jumps are necessarily between the smallest and largest stimuli, high accuracy for the smallest and largest stimuli is linked with high accuracy for the largest jumps. The bow effect in accuracy can be attributed, in part, to the
restricted opportunity to make mistakes at the edges of the stimulus range. Increased accuracy after large jumps can also be attributed to this restricted opportunity to make mistakes. When the error in responding is plotted as a function of the current stimulus and the previous stimulus, assimilation of the current response to the previous stimulus is seen for all stimulus combinations (see Stewart et al., 2005, for a review of assimilation and contrast effects). The amount of assimilation is roughly a constant proportion of the difference between the current stimulus and the previous stimulus. However, assimilation is reduced for the largest stimulus jumps - and this is directly linked to the increase in accuracy for the largest jumps.

Fits of the RJM

To examine whether the RJM predicts the effect in Figure 1, I have fitted the RJM to each data set. Because the trial-by-trial raw data were available, I have estimated the model’s parameters separately for each participant by maximizing the likelihood of the response on each trial using the Nelder-Mead (1965) simplex algorithm. This method is exactly equivalent to fitting the full conditional (upon all previous stimuli) confusion matrix. (This method should be preferred to fitting summary data, because the raw data will constrain the parameters better and prevent the model fitting summarized effects at the expense of other, unsummarized effects.) For each participant, the best fitting parameters were used to plot accuracy as a function of stimulus difference. Figure 1 shows these predictions averaged across participants (dashed line marked with circles).

The RJM captures the qualitative pattern of increased accuracy for stimulus repetitions and for large stimulus jumps. However, for the Lacouture (1997) and Kent and Lamberts (2005) data sets, the original model overestimates accuracy after repetitions and underestimates accuracy after large jumps. The overestimation in accuracy after stimulus repetitions occurs because the RJM predicts very little noise in responding when stimuli are repeated. As described above, the underestimation in accuracy after large stimulus jumps occurs because a large number of responses must be represented within the limited mapping
capacity when there is a large stimulus jump and this leads to more noise in responding. For the Stewart et al. (2005) and Brown et al. (2002) data sets, the RJM provides a much better quantitative fit. The model captures the high accuracy after stimulus repetitions well, though it still underestimates accuracy after large stimulus jumps.

**Comparison to Brown et al.'s Fits**

Brown et al. (2007) also presented fits of the RJM to the Lacouture (1997) data set. Brown et al. used the best fitting parameters for Lacouture's data presented in Stewart et al. (2005). Using these parameters, the RJM fails to predict increased accuracy after large stimulus jumps. These parameters were estimated by fitting the model to a summary plot of the error in responding on the current trial as a function of the lag and magnitude of preceding stimuli (see Lacouture, 1997, Figure 5) to demonstrate that the model could predict the summary pattern. However, as Stewart et al. acknowledged, fitting only the summary data does not constrain some of the model parameters well, because information in the raw data is discarded in the summary. The fits to raw data presented above should be preferred because they constrain the parameters better.

Brown et al. (2007) also presented fits using a modified parameter set, which they obtained by fitting the RJM to several summary plots, including the plot in Figure 1. In the RJM, a random variable representing the response is partitioned into response categories by a set of criteria (see Stewart et al., 2005, Equations 6 and 7). The spacing of all of the criteria is controlled by a single $c$ parameter. The value of this parameter was smaller in Brown et al.'s fit, which shrunk the criteria towards the center of the response scale. This has the effect of increasing accuracy for the smallest and largest stimuli, because more of the response scale falls within these categories. In turn, this allows the model to predict increased accuracy after large stimulus jumps because these jumps necessarily finish on the smallest or largest stimuli. However Brown et al. found this modified parameter set caused the model to fail to predict the smooth bow seen when $d'$ between adjacent stimuli is plotted as a function of stimulus
rank. The fits to raw data presented in this article do all predict a smooth bow in $d'$, though they do underestimate the bow (as Stewart et al. consider in detail, p. 896).

As a follow up to Brown et al.'s (2007) investigation of criteria placement, I ran an additional set of model fits in which each criterion's location was a separate free parameter, instead of constraining all of the criteria with the single $c$ parameter as in the original RJM. Allowing the criteria to vary independently produced only a slightly better fit to the data (the improvement in fit was not significant). In fact, the free criteria took almost exactly the same values as when they were constrained by the $c$ parameter. Stewart et al. (2005) presented a related finding. They found that allowing the criteria to vary freely to maximize accuracy (rather than the fit to the data) did not increase accuracy. Together, these two findings support Stewart et al.'s assumption that the criteria in the RJM are optimally located to maximize response accuracy (or information transmission).

A Simple Modification of the RJM

Thus far, I have shown that the sequential effects found by Brown et al.'s (2007) more detailed analysis of Lacouture's (1997) data are robust and occur in three other data sets. I have also shown that the original RJM does predict increased accuracy after large stimulus jumps when parameters are estimated by fitting data at the trial-by-trial level rather than from a single summary plot. However, the RJM does still underestimate the accuracy after large stimulus jumps, particularly for the Lacouture (1997) and Kent and Lamberts (2005) data sets. Also, for these two data sets, the RJM does overestimate accuracy when stimuli are repeated. In the remainder of this article I show that two simple modifications of the RJM provide a much better quantitative fit. First, a parameter that was fixed in the original model is allowed to vary. Second, I assume that judgment is sometimes relative to the stimulus two trials ago instead of one trial ago.

Stimulus Repetitions

The discrepancy between Stewart et al.'s (2005) and Brown et al.'s data sets (in which
there is a large advantage for stimulus repetitions) and Lacouture's (1997) and Kent and Lamberts's (2005) data sets (in which the advantage for stimulus repetitions is smaller) is seen in other data sets. For example, Petrov and Anderson (2005), Rouder et al. (2004), and Siegel (1972) found a large accuracy advantage for stimulus repetitions. Luce et al. (1982) and Nosofsky (1983) found that $d'$ was only slightly higher when the current and previous stimuli were similar (though the difference was significant) and Purks et al. (1980) found no difference. These results are not necessarily inconsistent: an increase in accuracy on stimulus repetitions could be caused by movement of criteria within a Thurstonian framework away from the previous stimulus, resulting in increased accuracy but unchanged $d'$ (see Purks et al., 1980).

In the original RJM, a stimulus is identified as repeated whenever the magnitude of perception of the difference between the current stimulus and the previous stimulus was less than a criterion value $X$ (see Stewart et al., 2005, Equation 5). $X$ was assumed to be fixed at half the stimulus spacing in Stewart et al.'s model fitting. Stewart et al. suggest the discrepancy between data sets described above could be captured by allowing the $X$ parameter to vary (p. 905). A smaller value of $X$ represents a greater difficulty in (or reluctance to) identify stimulus repetitions. For this reason I allow $X$ to vary in the second set of fits.

Large Stimulus Jumps

Siegel's (1972) data and Stewart et al.'s (2005) data showed that there is an advantage not only when the current stimulus ($S_n$) is the same as the previous stimulus ($S_{n-1}$), but also a smaller advantage when the current stimulus is the same as the stimulus two trials back ($S_{n-2}$). One possible explanation that Stewart et al. put forward (p. 904) is that people sometimes judge $S_n$ against $S_{n-2}$ instead of $S_{n-1}$. Stewart and Brown (2004) presented some evidence that is particularly suggestive of this possibility. They showed that people were very accurate in a binary categorization task if a recent stimulus was nearer the category boundary than the current stimulus. In this situation, relative judgment of the current stimulus against the recent
stimulus allows participants to determine the correct category. For example, if the previous
stimulus is in the Low Category and the current stimulus is perceived as lower in magnitude,
then the current stimulus must also be in the Low Category. Stewart and Brown found that
when $S_{n-2}$ was nearer the boundary than $S_n$, accuracy was high and unaffected by $S_{n-1}$. This
strongly suggests that participants can categorized $S_n$ by comparing it with $S_{n-2}$ instead of $S_{n-1}$.

In the second set of fits I assume that, on trials when $S_{n-2}$ is much nearer $S_n$ than $S_{n-1}$
is, $S_n$ is sometimes judged relative to $S_{n-2}$ instead of $S_{n-1}$. For example, if the sequence of
stimuli is $S_{n-2} = 9, S_{n-1} = 1, S_n = 10$ I assume that $S_n$ is compared to $S_{n-2}$ and not $S_{n-1}$. In these
fits, I define "much nearer" at half of the stimulus range. Using $S_{n-2}$ only when it is "much
nearer" represents the assumption that it is easier and preferable to compare $S_n$ to the relatively
strong trace of $S_{n-1}$ than to the relatively weaker trace of $S_{n-2}$. Revised equations are given in
the Appendix.

*Modified RJM Fit*

The procedure for fitting this modified version of the RJM was the same as described
above. Figure 1 shows these predictions averaged across participants (dashed line marked with
squares). In comparison to the original RJM fits, the fits of the modified RJM predict a greater
accuracy advantage after large stimulus jumps and a smaller accuracy advantage after stimulus
repetitions, producing a better overall fit to the data. For the Stewart et al. (2005) and Brown
et al. (2002) data sets, the model's fit to the data is good, though the model does slightly
underestimate accuracy for repetitions and overestimates accuracy for large jumps in the
Brown et al. (2002) data.

For the Lacouture (1997) and Kent and Lambert's (2005) data, the modified model
comes much closer to capturing the pattern in the data, though it still overestimates accuracy
on stimulus repetitions and underestimates accuracy after large stimulus jumps. Each of these
deficits can be addressed by further modification of the model. First, to address the
overestimation in accuracy after stimulus repetitions, a second, independent source of noise can be added to the model (e.g., perceptual noise, Stewart et al., 2005, p. 897, or noise in representing the difference between stimuli, $D_{n-1}^C$), allowing a closer fit to be obtained. Second, to address the underestimation in accuracy after the largest jumps, the RJM can be extended so that stimuli on trials further back in the sequence can be used to judge the current stimulus. Ultimately, one could extend the RJM so that any previous stimulus could be used. This, effectively, would turn the model into an exemplar model of absolute identification, and introduce long-term representation of magnitudes into the model. The RJM and the absolute magnitude models could be viewed as opposite ends of a continuum, with only very short-lived magnitude representations at the relative judgment end and very long-lived magnitude representations at the absolute judgment end. In practice, extending the model to use $S_{n+1}$ most of the time, $S_{n+2}$ sometimes and $S_{n+3}$ rarely allows the model to predict higher accuracy after large stimulus jumps.

It may be that the RJM is not capturing some aspects of the judgment process in Lacouture's (1997) and Kent and Lamberts's (2005) data sets. Some additional length cues may have been available in these experiments: In Kent and Lamberts's experiment, the edges of the screen were just visible after a little time to dark adapt. In Lacouture's experiment, distance between each response key and the home key was 101 mm, 2 mm longer than Stimulus 9 and 13 mm shorter than Stimulus 10. The (constant) stimulus width (20 mm) was similar in length to the shorter shorter stimulus lengths (33 mm, 38 mm, ...). Relative judgment of stimuli against these additional length cues (rather than the previous stimulus) would reduce accuracy after stimulus repetitions and improve accuracy for the longest and shortest stimuli, which in turn would also improve accuracy for the largest jumps (which are between the longest and shortest stimuli). However, Luce et al. (1982) and Purks et al. (1980) found large edge effects and small repetition effects using tones differing in loudness. Additional contextual cues were
probably not available in these experiments because they were conducted in soundproof chambers. So the availability of additional contextual cues is probably not the sole cause of the difference between the Brown et al. (2002) and Stewart et al. (2005) data sets and the Kent and Lamberts and Lacouture data sets.

Finally, I note that, in addition to capturing the effects considered here, the modified RJM predict the effects considered by Stewart et al. (2005): limits in information transmitted; bows in accuracy and $d'$; set size effects; sequence manipulation effects; and assimilation to the previous stimulus and contrast to those further back.

Conclusion

Brown et al. (2007) have drawn attention to a pattern of conditional accuracy that I have found to be robust in three other data sets: Accuracy is higher after stimulus repetitions or very large jumps compared to intermediate jumps. Here I have shown that the original RJM can capture this qualitative pattern, though it overestimates accuracy after repetitions and underestimates accuracy after large jumps. Exploring modifications suggested by Stewart et al. (2005), I present a modified RJM - that maintains the hypothesis that judgment is relative - that can capture these effects more fully. The direct experimental test of the relative judgment hypothesis presented by Stewart et al. strongly supports the idea that absolute identification is relative, and the data reviewed here are compatible with the relative judgment hypothesis.
References


Treisman, M. (1985). The magical number seven and some other features of category scaling:

Appendix

When $S_{n-2}$ is much nearer $S_n$ than $S_{n-1}$ is, $S_{n-2}$ is used as the base for relative judgment. Equations 1 and 2 implement this, and replace Equations 4 and 5 from Stewart et al. (2005).

$$R_n = \begin{cases} F_{n-1} + \frac{D_{n,n-1}^C \rho_{n-1} L}{\lambda} & \text{if } |S_n - S_{n-1}| \leq |S_n - S_{n-2}| + \frac{N}{2} \\ F_{n-2} + \frac{D_{n,n-2}^C \rho_{n-2} L}{\lambda} & \text{if } |S_n - S_{n-1}| > |S_n - S_{n-2}| + \frac{N}{2} \end{cases}$$

(1)

$$\rho_{n-i} = \begin{cases} N - F_{n-i} & \text{if } D_{n,n-i}^C > +X \\ 1 & \text{if } -X \leq D_{n,n-i}^C \leq +X \\ F_{n-i} - 1 & \text{if } D_{n,n-i}^C < -X \end{cases}$$

(2)

When $S_{n-2}$ is used as the base for judgment, the difference between $S_n$ and $S_{n-2}$ is assumed to be estimated, and is confused with recently encountered differences $D_{n,n-1}, D_{n-1,n-2}, ...$

$$D_{n,n-2}^C = \alpha_0 D_{n,n-2} + \sum_{i=1}^{n-2} \alpha_i D_{n-i,n-i-1}$$

(3)

In the modified model, generalization on the response scale is assumed to be exponential, not Gaussian, as in the original model. Thus, $L$ in Equation 1 is the double exponential or Laplace distribution, with mean 0 and scale parameter $\sigma$. There are good precedents for assuming exponential generalization (Shepard, 1957) and the exponential function provides a better fit for 80% of participants across the four data sets modeled here.

Note that similar fits to the Figure 1 data are obtained if Gaussian generalization is retained.
Author Note

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Source code and best-fitting parameters for the modified model are available from http://stewart.psych.warwick.ac.uk/.

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Table 1

*Experimental Details for the Four Data Sets*

<table>
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<tr>
<th>Authors</th>
<th>Stimulus</th>
<th>No. participants</th>
<th>No. trials</th>
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</table>
Figure Captions

*Figure 1.* Accuracy plotted as a function of the difference in ranks of the current and previous stimulus. Data (solid line) are from (a) Lacouture (1997), (b) Stewart et al. (2005), (c) Kent and Lamberts (2005), and (d) Brown et al. (2002). Error bars are standard error of the mean. Predictions of the RJM are shown as dashed lines.
Figure 1

A: Lacouture (1997)

B: Stewart, Brown, and Chater (2005)

C: Kent and Lamberts (2005)

D: Neath and Brown (2006)