Risk Preference Discrepancy: A Prospect Relativity Account of the Discrepancy between Risk Preferences in Laboratory Gambles and Real World Investments

Ivo Vlaev\(^1\), Neil Stewart\(^2\), Nick Chater\(^1\)

\(^1\) Department of Psychology
University College London
26 Bedford Way
London, WC1H 0AP
United Kingdom
Tel: +44 207 679 7585
i.vlaev@ucl.ac.uk
n.chater@ucl.ac.uk

\(^2\) Department of Psychology
University of Warwick
neil.stewart@warwick.ac.uk

* corresponding author
Author Note

Ivo Vlaev, Nick Chater (Department of Psychology, University College London, UK), and Neil Stewart (Department of Psychology, University of Warwick, UK).

This work was supported by Economic and Social Research Council grant R000239351.

Correspondence concerning this article should be addressed to Ivo Vlaev, Department of Psychology, University College London, London, WC1H 0AP, United Kingdom. E-mail: i.vlaev@ucl.ac.uk.
Abstract

In this article, we presented evidence that people are more risk averse when investing in financial products in the real world than when they make risky choices between gambles in laboratory experiments. In order to provide an account for this discrepancy, we conducted experiments, which showed that the range of offered investment funds that vary in their risk-reward characteristics had a significant effect on the distribution of hypothetical funds to those products. We also showed that people are able to use the context provided by the choice set in order to make relative riskiness judgments for investment products. This context dependent relativistic nature of risk preferences is proposed as a plausible explanation of the risk preference discrepancy between laboratory experiments and real-world investments. We also discuss other possible theoretical interpretations of the discrepancy.

Keywords: risk aversion; decision making; investment risk
A Prospect Relativity Account of the Discrepancy Between Risk Preferences in Laboratory Gambles and Real World Investments

In psychology and experimental economics, experimental participants usually make choices between risky prospects, or assign values or certainty equivalents to risky prospects. Participants’ level of risk aversion can be deduced from these choices. A level of risk aversion can also be deduced by examining how people assign their real-world assets to different investments (e.g., a building society account, property, or the stock market). We show that there is a discrepancy between the level of risk aversion shown in laboratory experiments and the level or risk aversion implicit in people’s asset allocation. Typically, far more risk aversion is shown in the allocation of real-world assets.

This article also offers an account of this discrepancy: We suggest that the discrepancy is a result of people’s inability to make absolute judgments of the riskiness of a laboratory gamble or an investment. Recent evidence (Stewart, Chater, Stott, & Reimers, 2003) suggests that instead of making absolute judgments of the riskiness of prospects, people in fact judge the riskiness of prospects relative to the riskiness of other prospects in the choice set. Thus what matters in participants’ judgment of the riskiness of a prospect is not the overall level of riskiness, but rather how the riskiness of the prospect compares to that of other prospects on offer. In the remainder of this article we present evidence to support our claim that there is a discrepancy between the level of risk aversion demonstrated in the laboratory and that evident in real-world decision under risk. We will then outline some evidence that supports our assertion that absolute assessments of the value or utility of a prospect or gamble are not available and instead only relative judgments can be made before explaining how this allows the discrepancy between laboratory and real-world behavior. Finally, we present some experimental evidence that real-world risky outcomes are assessed relative to one another, and that although there is a discrepancy between these decisions and
laboratory decisions the two are related.

The Discrepancy between the Level of Risk Aversion Shown in the Laboratory and That Shown in the Real World

In this section we will examine the level of risk aversion shown in laboratory studies and that shown in the real world. First however, it is necessary to introduce a simple measure of risk aversion. Traditionally, risk aversion for gains is explained in terms of utility being a negatively accelerated power function of value, as in expected utility theory (von Neumann & Morgenstern, 1947; see Bell & Fishburn, 1999, Fishburn & Kochenberger, 1979, and Luce, 2000, for a discussion and comparison of alternate functions). Figure 1 shows a hypothetical utility function, where the utility is an increasing but decelerating power function of money with exponent $\gamma$ where, for a decelerating utility function $0<\gamma<1$. The smaller the value of $\gamma$ the more concave the function. Consider, for example, a choice between £50 and a 50% chance of winning £100, otherwise nothing. For $\gamma<1$, the utility of £100 is less than twice the utility of £50. Thus the expected utility of £50, $U(\£50)$ is greater than the expected utility of the gamble 50% chance of £100 otherwise nothing, $0.5U(\£0)+0.5U(\£100)$, and the sure £50 is preferred over the 50% chance of £100, even though both options have the same expected value. The curvature of the utility function or degree of risk aversion, as measured by $\gamma$, determines the difference in expected utility, and thus degree of risk aversion.

INSERT FIGURE 1 ABOUT HERE

Risk Aversion in the Laboratory

In laboratory experiments we often observe what appears to be risk-averse behavior over small amount of money (typical payment rates are less than $50/ hour, and play rarely lasts two hours). Much recent experimental research calculating coefficients of relative risk aversion finds reasonable coefficients between 0 and 1, although this research ignores initial wealth and income (i.e., utility is defined only over gains). For example, Holt and Laury
(2002) discovered that the average coefficient of relative risk aversion in the domain of gains is 0.4. We observed values of risk aversion in this range in an unpublished study from our laboratory, in which the measures were deduced from choices between simple prospects and sure amounts (the risk attitudes shown in our study indicated that the average risk aversion is around 0.6). Cardenas & Carpenter (2005) report estimates between 0.32 and 1.25 in a review of studies of risky choice in developing countries. Binswanger (1981) found similar coefficients of partial relative risk aversion (the coefficient of absolute risk aversion multiplied by the stake of the gamble) in experiments with rural farmers in rural India, which involved ‘high-stake’ gambles for a significant fraction of annual income. Schechter (in press) reports similar coefficients (0.81) derived from choices (involving modest-stakes) made by rural Paraguayans whose income levels are known but not taken into account. Two other studies report similar coefficients even when annual income is considered as a proxy of ‘total wealth’---one derived the coefficients from choices of deductibles in the car insurance market (Cohen & Einav, 2005) and another from play in a TV game show (Gertner, 1993).

These results also suggest that people isolate the risky decision in the game from considerations of their final wealth status, i.e., assumptions about asset integration do not hold (Schechter, 2006). This behavior is known as “choice bracketing” or “narrow framing.” Read, Loewenstein, and Rabin (1999) discuss the many ways choice bracketing may affect decision making in daily life. Kahneman and Lovallo (1993) hypothesize that people are overly uncomfortable to take risks in their choices because they evaluate risky prospects one at a time rather than pooling risks. In summary, theories show that people bracket instead of integrating the choice options into their wealth. This is as if their assumed wealth is contained to the gains presented by the current choice set (or context), for which we are going to present a more systematic account in this article.

*Risk Aversion in the Real World*
In the financial market there are a variety of products that vary in their position on a continuum where risk and return are traded off. At one end there are bank accounts offering a fixed return, and at the other end are stocks and shares that offer a larger return on average, but are more risky as the return is subject to some variation. Under standard assumptions, we deduce expressions for the utility of an investment in a product with a given level of risk and return after one year, before extending the result to investments for more than one year. We then show that investing in a given product is indicative of a specific level of risk aversion.

In the following analysis a financial product, $X$, is modeled by assuming the annual return is log normally distributed about a mean of $\mu_X$ with standard deviation of $\sigma_X$. Figure 2 illustrates the distribution of returns for several products differing in mean return, $\mu$, and standard deviation, $\sigma$. The expected utility for a product can be determined by calculating the utility of each possible return of the product, and then averaging these utilities together, weighted by their probability of occurring. Here utility is assumed to be a power function of money, with exponent $\gamma$.

**INSERT FIGURE 2 ABOUT HERE**

The derivation of the utility of a product as a function of the mean return, $\mu$, the standard deviation, $\sigma$, and an individual’s level of risk aversion, $\gamma$, is given in the Appendix. Note that it is possible to talk of the utility of a product because utility is assumed to be a power function of money. Strictly speaking talk should be of the utility of a given investment in a product. However, when comparing the relative expected utility of a given investment across two products, the actual amount of money invested does not matter - if the investment amount were multiplied by an arbitrary constant, the ratio of the utilities of the two products will remain the same. In the remainder of this paper the expression "utility of a product" will be used as shorthand for "the utility of a £1 investment in that product".

Figure 3 plots the trade-off between return, $\mu$, and risk, $\sigma$ observed for a set of
financial products in the market place. Return is a linear function of risk because modern portfolio theory assumes that the required return is a linear function of the risk free rate of return, the market risk premium, and an index of systematic risk. This model is known as the Capital Asset Pricing Model (CAPM) (Lintner, 1965; Sharpe, 1964). It is also the equation for the Security Market Line and the different lines in Figure 3 correspond to different hypothetical markets, where the trade-off between risk and return varies.

INSERT FIGURE 3 ABOUT HERE

For a given market, the utility of each product can be determined. Each panel in Figure 4 plots the utility of a product, \( U(X) \), against the product’s annual return, \( \mu_X \), for a range of risk aversions \( 0.2 \leq \gamma \leq 1.0 \). The panels differ in the trade-off between risk and return assumed - there is one panel for each market illustrated in Figure 3.

INSERT FIGURE 4 ABOUT HERE

The results in Figure 4A, for the market represented by the shallowest line in Figure 3 (where high return products are relatively low risk compared to the high return products of other markets), show utility is an increasing function of return for the range of products \( 1.05 \leq \mu \leq 1.20 \) when \( 0.2 \leq \gamma \leq 1.0 \). Thus, of the products with returns in the range explored, the preferred product is the one with the highest return independent of an individual’s degree of risk aversion. Figure 4D explores the case where the trade-off between risk and return is less favorable, that is, risk increases more quickly as return increases. Here expected utility does not increase as risk and return increase. Initially expected utility does increase with expected return, but then it begins to decrease. Here, for a product with high risk and return the concave downward utility function means that the reduced utility of possible low returns is no longer offset by the increased utility from possible high returns. The location of the product with maximum utility is different for different degrees of risk aversion. The more risk averse an individual, the lower the risk and return of the product with the greatest utility.
**Investments Over Longer Time Periods.** Investments for longer time periods require further assumptions. Over a longer time period the return from a product will compound. This is modeled by assuming the annual return for a product \( X \) is drawn from a log normal distribution mean \( \mu_X \) and standard deviation \( \sigma_X \), and that the return in each year is independent of previous years. Expressions for the distribution of returns and utility of a product in terms of the mean and standard deviation of the product, the duration of the investment in years, \( n \), and the level of risk aversion, \( \gamma \), are derived in the Appendix. Figure 5 illustrates how the distribution of returns for a product changes with time. The mean return increases exponentially over time, and the standard deviation also increases (in a more complicated way).

**INSERT FIGURE 5 ABOUT HERE**

In Figure 6, the utility of a product is plotted against the return of that product with each curve representing the expected utility for the products in the market after different amounts of time have elapsed. The top panel is for a market where risk increases more slowly with time than the bottom panel. The product with the maximum utility is constant across time. In other words, the product with the maximum utility after one year is the same product as the product with the maximum utility after \( n \) years. Proof that the duration of the investment does not alter the product with maximum utility is given in the Appendix.

**INSERT FIGURE 6 ABOUT HERE**

**The Level of Risk Aversion in the Real World.** There is surprisingly little systematic evidence about how risk averse people are. Still, there is enough convincing evidence that people’s risk aversion, or gamma, is much higher in the real-world financial decision making. Well known evidence that people’s risk aversion is higher in the real-world financial decision making is the equity premium puzzle, which shows that the risk premium investors put on risky assets (the extra return/profit they expect from a risky asset, i.e. a reward that is
Risk Preference Discrepancy

appropriate to the risk, relative to the risk-free return from safe assets like bonds or cash) is unrealistically high (Mehra & Prescott, 1985). The size and persistence of the excess return on stocks over fixed income securities and the observed disparity implies implausibly high degrees of risk aversion in standard models of asset pricing (although since the return on stocks is more variable, standard theory is consistent with some difference in the long-run rates of return). In other world, the puzzle exists because the excess of stock returns over returns on investments in bills or bonds is larger than can be explained by standard models of ‘rational’ asset prices. For example, a person with enough risk aversion, to explain the equity premium would be indifferent between a coin flip paying either $50,000 or $100,000 and a sure amount of $51,209.

Figure 7 shows risk premiums plotted against gamma. The risk premiums are taken from classic studies which relate gamma to risk premium in the market (Friend & Blume, 1975; Hildreth & Knowles, 1982; Kydland & Prescott, 1982; Tobin & Dolde, 1978). Figure 7 gives some idea of the range of $\gamma$ values in the real world because these studies have used a representative set of investment portfolios. Figure 7 also displays the slope and intercept of the risk against return line derived from (projected on the basis of) the real world studies and the laboratory experiments. Therefore, using the method described in this section, we can work out individual’s exposure to risk, which is substituted for a single point on the risk return line, and then work back to $\gamma$ (we know the distribution of assets in these portfolios and their return, and hence can infer the beta coefficient for each portfolio). Here we used a hypothetical power law utility function, in which $\gamma$ is the exponent which describes the curvature of this function. $\gamma = 1$ for a risk-neutral person. Smaller values of $\gamma$ denote greater risk aversion. People are risk prone if their $\gamma$ is over 1.

As you can see the gammas for the investment portfolios are negative. Such negative
exponent means super risk averse preferences, which are much more risk averse than the risk attitudes shown in the laboratory studies where the average $\gamma$ is around 0.6. These levels of risk aversion shown in the laboratory are derived from the studies described in the previous section here (entitled *Risk Aversion in the Laboratory*). In summary, Figure 7 demonstrates that there is a big discrepancy between the level of risk aversion shown in laboratory experiments and the level or risk aversion implicit in people’s asset allocation. Typically, far more risk aversion is shown in the allocation of real-world assets, which is reflected in the demanded higher risk premium.

This discrepancy could have very important consequences in terms of asset prices and stock returns. Figure 8 shows the implication of various risk aversion levels for asset values in terms of stock prices of a portfolio containing the Dow Jones Industrial Average. The curve represents the market valuation of a portfolio containing these assets. This projected curve shows that if people become less risk averse, i.e. like the level shown in lab tests, then the stocks will be much more attractive (say, as opposite to bonds). In other words, if $\gamma$ were higher, such portfolio would be perceived as having more attractive risk-return characteristics, or as worth more. As a result, investors would buy more shares and the stock prices will increase. And this potential price increase across all stocks represented in the index is shown by the line connecting the current levels of risk aversion (gammas) in the stock market (as we estimated it before) and the risk aversion levels shown in laboratory experiments. The curve was made in two steps. First we used a linear formula used in modern portfolio theory, which links gamma to investment rates of return: $\gamma = \frac{dE(r)}{d\sigma}$. In modern portfolio theory, risk aversion is measured as the marginal reward an investor wants to receive $E(r)$ if he takes for a new amount of risk $\sigma$. Since we know the values of both $\gamma$ and $\sigma$ (for real world and lab studies), then we can infer the rate of return for various levels of these variables. Once the expected return, $E(r)$, is calculated, the future cash flows of the assets can
be discounted to their present value using this rate to establish the correct price for the assets. This is a CAPM formula (Lintner, 1965; Sharpe, 1964), which links rates of return to market valuation. (These are all standard formulas that can be found in most finance textbooks and hence we do not discuss them in detail here.)

INSERT FIGURE 8 ABOUT HERE

Thus, Figure 8 is showing the effect of shifting the risk aversion levels (or what we could call the “psychological foundations” for stock prices). Such a shifting is possible because, as we demonstrate later in our experiments, people’s risk aversion (gamma) is wildly manipulable by contexts and by distorting sampling in the environment or memory. In summary, if we suppose that the financial assets provide their own sample, or context, in relation to which people judge the riskiness of the assets, then the risk premium and stock values can be quite detached from any psychological fundamentals. The experiments presented in the next section aim to test and support this claim.

A Prospect Relativity Account of the Risk Aversion Discrepancy

Traditionally, models of decision under risk assume that every risk prospect can be assigned a value or utility and that for a given prospect this assignment is independent of other prospects that may be available in the choice set. EU theory (axiomized by von Neumann & Morgenstern, 1947), prospect and cumulative prospect theory (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992), and rank dependent utility theory (Quiggin, 1982, 1993) all make this assumption. An alternative is that the assignment of a utility to a given prospect is not independent of the utility of other prospects. In regret theory (Loomes & Sugden, 1982) the utility of a prospect is modified to take into account possible feelings of regret (or rejoicing) about outcomes that would have obtained if different choices had been made. Thus, the utility of a prospect is not independent of the other prospects in the choice set. Similarly, in the componential context model (Tversky & Simonson, 1993) utilities or
values are modified by the relative trade-off between object attributes, although this theory was not extended to decision under risk. The stochastic difference model (González-Vallejo, 2002) makes the strong assumption that the degree of preference for one prospect over another depends only on a relative comparison of prospect attributes (i.e., prospects are judged relative to one another and thus, the choice between options is made in relative rather than absolute terms). This is also a core assumption in multialternative decision field theory (Roe, Busemeyer, & Townsend, 2001) although this theory is of decisions between certain outcomes, and has not been extended to decision under risk.

Stewart, Stott, Chater, and Reimers (2003) provide evidence that prospects are judged or valued relative to one another. In one experiment, participants were offered a set of prospects and asked to select a single prospect to play. Within the set, the probability of winning a prospect was reduced as the amount that could be won was increased. Thus each participant faced a choice between prospects offering small amounts with high probability through to larger amounts with a lower probability. Different groups of participants were offered different sets. One group of participants were offered the full set of 10 prospects. Two additional groups were offered a restricted choice: one group was offered only the 5 most risky prospects (lower probabilities of higher amounts) and another group was offered only the 5 least risk prospects (higher probabilities of lower amounts). If these participants’ preferences were not affected by the set of options provided, they should simply choose the prospect closest to the prospect they would select if they were offered all 10 prospects. However, the distribution of choices differed significantly from those expected under this prediction. Instead, the set of options available seemed to determine participants’ preferences: When the available prospects were ranked from most risky to least risky, the distribution of choices not differ significantly between the group offered only the least risky prospects and the group offered only the most risky prospects. Stewart et al. concluded that the level of risk
aversion shown by a participant was determined by the level of risk in the set (range) of prospects offered. There have been other experiments that have also investigated the effect of the set of available options in decision under risk. Birnbaum (1992) investigated a rank dependence effect (while Stewart et al. tested a range based effect) and demonstrated that the skew of the distribution of options offered as certainty equivalents for simple prospects, (while the maximum and minimum are held constant), influences the selection of a certainty equivalent. In particular, prospects were less valued in the positively skewed option set where most offered choice values were small, compared to when the options were negatively skewed and hence most offered values were large.

Stewart et al.’s (2003) findings show the importance of the effects of the choice set on people’s preferences in various decision domains. One way to explain these effects is by assuming that people’s representations of the relevant dimensions (e.g., level of risk) are not stable, but are influenced by context. Parducci’s (1965, 1995) range-frequency theory of psychophysical judgment has been first proposed by Birnbaum (1992) and Stewart et al. (2003) as a possible explanatory model of this type of effect on risky choice. Range-frequency theory predicts that the subjective value given to a magnitude is a function of its position within the overall range and rank of distribution of magnitudes that have been observed. Specifically, the impact of range is captured by expressing the current magnitude as a fraction of the interval from the lowest to the highest magnitude that has been encountered.

In summary, there is reasonable evidence and some precedent for assuming that prospects are judged relative to other prospects in the choice set, rather than in isolation. Such prospect relativity can also offer an account of the discrepancy in risk aversion between laboratory studies and real-world investments. The argument is as follows. Rather than assume that each individual has an absolute level of risk aversion (or alternatively a utility
function), we assume that instead each individual has a relative level of risk aversion. In other words, an individual might, for example, know that they are relatively risk averse. When presented with a set of options, they would therefore tend to select a relatively less risky option. Without an absolute grip on the riskiness of an alternative, the individual would be unable to judge whether the entire set of options were all very risky or all somewhat safer. Thus when offered a set of laboratory gambles that are, compared to real-world financial products, quite risky, the person will select one of the safer prospects. When the same individual is presented with a set of real-world financial options, that, when compared to the laboratory gambles, are all far less risky, they will still select a relatively safe option. Thus, because real world options are less risky than laboratory options, people would select less risky options in the real-world compared to laboratory experiments.

Experiment 1A

The purpose of Experiment 1A was to investigate whether the choice set across which hypothetical investments are made influences the allocation made. Participants were offered a range of investment products that varied in the trade off between risk and return. The range of products offered varied between participants. One third of participants were offered the full range of products, and asked how they would split a given investment across the products. One third of participants completed the same task except they were only offered the less risky products, and one third of participants were offered only the more risky products. The logic in this design follows that of Stewart, Chater, Stott, and Reimers (2003).

Method

Participants. There were three between participant conditions, with 20 undergraduates participating in each condition for payment of £1.

Design and procedure. Participants were approached on the university campus. Participants were given a sheet containing a set of financial products, and asked to make a
hypothesised investment of £20,000 across the products, splitting the funds in any way they liked. The products offered were varied across three conditions. In the free choice condition, participants were offered all eight products described in Table 1. In the low risk condition, participants were only offered the four lowest risk-return products. In the high risk condition, participants were only offered the four highest risk-return products. The meaning of average return and the risk, or variation about that mean, was explained to participants. \( \beta \) (beta) was also provided, which is a measure of an investment’s return volatility in relation to the rest of the market. In other words, this is a risk measure indicating how does the stock’s price move relative to the overall market. The whole market, which for this purpose is considered the S&P 500, is assigned a beta of 1. Stocks with a beta of 1 fluctuate in price at the same rate as the market. Stocks that have a beta greater than 1 have greater price volatility than the overall market and are more risky, while stocks with a beta of less than 1 have less price volatility than the market and are less risky. Since risk also implies return, stocks with a high beta should have a higher return than the market (i.e., accepting more risk, should bring more reward). It was explained that \( \beta \) might be useful to finance students.

**Results**

The mean funding allocated to each product is shown in Figure 9A. In the free choice condition, where all products were offered, there is a slight bias towards low risk-return products. Product \( \beta = 0.4 \) was also unpopular. In the low and high conditions, there was also a bias towards low risk-return products, and in addition, the highest risk product in each condition was also favored, with the distribution of funds following a “U” shape.

The effect of context was measured in two ways. First, for each participant a weighted mean \( \beta \) value was calculated by summing the mathematical product of the \( \beta \) value for each financial product and the funds invested, and then dividing by the total funds invested. \( \beta \), then, provides a measure of the level of risk-reward for each participant’s investment. If
participants are unaffected by the context, the mean $\beta$ across participants in each condition should not differ between conditions. The mean $\beta$ values were: 0.26 (S.E. 0.04) for the low condition; 0.67 (S.E. 0.08) for the free choice condition; 1.24 (S.E. 0.06) for the high condition. Planned $t$ tests found a significant difference between the mean $\beta$ values low and free choice conditions, $t(38)=4.52$, $p<.0001$, and a between the free choice and high conditions, $t(38)=5.52$, $p<.0001$.

An alternate analysis is to assume that participants in the restricted range low and high conditions, who would otherwise choose products outside that range, should make up for this by placing all funding that had wanted to place outside the range into the nearest available product. So, for example, in the low condition, funding that participants would normally have assigned to the higher risk-return products ($0.8 \leq \beta \leq 2.0$) should be assigned into the highest risk-return product available ($\beta=0.6$). If participants in the low condition were following this strategy, then their allocation to the $\beta=0.6$ product should not differ significantly from free choice condition participants allocation to products with $\beta \geq 0.6$. In fact the funding allocated to product $\beta=0.6$ in the low condition is significantly less than the funding allocated to products $\beta \geq 0.6$ in the free choice condition, $t(38)=2.94$, $p=.0130$. Similarly, funding allocated to product $\beta=0.8$ in the high condition is significantly less than the funding allocated to products $\beta \leq 0.8$ in the free choice condition, $t(38)=4.08$, $p=.0001$.

**Discussion**

The range of risk-reward of financial products offered was found to have a significant effect on the distribution of hypothetical funds to those products. Participants behaving optimally should seek the same weighted mean $\beta$, a measure that represents the overall level of risk-reward for each participant’s investment, no matter what financial options are available to them. Instead participants offered only low risk-return products had a significantly lower $\beta$ than those given a free choice, and participant offered only high risk-
return products had a significantly higher $\beta$. Participants were also found not to be using a simpler but sub-optimal strategy to compensate for being constrained to choose from a restricted set of products. This strategy involved lumping all funding they would have liked to invest at a higher risk-reward level into the highest risk reward level available, and all funding they would have liked to invest in lower risk-reward products into the lowest risk reward available. In conclusion, the range or risk-reward spanned by the products offered strongly affects participants’ hypothetical investments.

Experiment 1B

An alternative account of the results of Experiment 1A is that participants simply did not pay attention to the risk and return of each product. For this reason, we attempted to replicate the results of Experiment 1A, while omitting the prose describing each product.

**Method**

**Participants.** There were three between participant conditions, with 20 new undergraduates participating in each condition for payment of £1.

**Design and procedure.** The design and procedure was the same as that of Experiment 1A, except that only the return (expressed as the percentage average annual return) and risk (expressed as a confidence interval about the average return, e.g., ±4%) were presented for each product.

**Results and Discussion**

The mean funding allocated to each product is shown in Figure 9B. In the free choice condition, where all products were offered, there is a stronger bias towards low risk-return products than seen in Experiment 1A. In the low and high conditions, the “U” shaped distribution of funding was not as prevalent as in Experiment 1A.

As in Experiment 1A the effect of context was measured in two ways. First, for each participant a weighted mean $\beta$ value was calculated by summing the mathematical product of
the $\beta$ value for each financial product and the funds invested, and then dividing by the total funds invested. $\beta$, then, provides a measure of the level of risk-reward for each participant’s investment. If participants are unaffected by the context, the mean $\beta$ across participants in each condition should not differ between conditions. The mean $\beta$ values were: 0.22 (S.E. 0.02) for the low condition; 0.61 (S.E. 0.11) for the free choice condition; 1.37 (S.E. 0.08) for the high condition. Planned $t$ tests found a significant difference between the mean $\beta$ values low and free choice conditions, $t(38)=3.61, p=.0009$, and a between the free choice and high conditions, $t(38)=5.71, p<.0001$.

As in Experiment 1A, an alternate analysis is to assume that participants in the restricted range low and high conditions, who would otherwise choose products outside that range, should make up for this by placing all funding that had wanted to place outside the range into the nearest available product. Participants allocated significantly less funding to the $\beta=0.6$ product in the low condition that to products with $\beta \geq 0.6$ in the free choice condition, $t(38)=3.38, p=.0017$. Similarly, funding allocated to product $\beta=0.8$ in the high condition is significantly less than the funding allocated to products $\beta \leq 0.8$ in the free choice condition, $t(38)=6.08, p<.0001$.

The effect of the choice set offered on asset allocation seen in Experiment 1A has been replicated when participants were presented with only the return and risk of a product.

Experiment 1C

The purpose of Experiment 1C was to attempt to replicated the results of Experiments 1A and 1B using a non-student population.

Method

The method is as for Experiment 1A, except participants were conference guests attending conferences at Warwick.

Results and Discussion
Figure 9C shows the mean funding allocated to each product for the free choice condition and the two restricted choice conditions. The distribution of funding for of Experiments 1A and 1B has been replicated, with a bias towards low risk-return products in the free choice condition, and the “U” shaped distribution in each of the restricted conditions. The weighted mean $\beta$ for the low condition (mean=0.24, S.E.=0.02) is significantly lower than the weighted mean $\beta$ for the free choice condition (mean=0.55, S.E.=0.05), $t$(38)=5.23, $p<.0001$. The weighted mean $\beta$ for the high condition (mean=1.23, S.E.=0.04) is significantly higher than the weighted mean $\beta$ for the free choice condition, $t$(38)=17.05, $p<.0001$.

Following the alternate analysis, the funding allocated to product $\beta=0.6$ in the low condition is significantly less than the funding allocated to products $\beta \geq 0.6$ in the free choice condition, $t$(38)=3.17, $p=.0030$. Similarly, funding allocated to product $\beta=0.8$ in the high condition is significantly less than the funding allocated to products $\beta \leq 0.8$ in the free choice condition, $t$(38)=6.67, $p<.0001$.

Experiment 1C replicated the context effect from Experiment 1A and 1B in a non-student population.

Experiment 2

Our claim in this article is that people have difficulty assessing the absolute riskiness of a financial product, and instead make relative riskiness judgments for each product using the context provided by the choice set. The purpose of this second experiment is to test whether the participants are really able to make such relative comparisons, and also we aimed to verify that they are not just responding randomly in Experiments 1A-1C due to a lack of sensitivity to, or understanding of, the choice options. For example, it is still possible that the relativistic effect observed in these experiments could be due to people not been sensitive to the relative riskiness of the investment options in high range vs. low range condition. Note that our main claim is that human risk preferences are unstable and relativistic, not that
people cannot perceive the relative riskiness of the options in the choice set.

It is typically assumed that, in a financial market, that risk and return are linearly related: As the risk increases, so does the return. A product is particularly good therefore, if it lies below the line of risk plotted against return for that market (see Figure 3), as, for the product’s level of return, the associated risk associated is lower than for other products in the market. Similarly, a product that lies above the risk against return is less efficient than other products in the market. It is not necessary to abstract absolute notions of the utility of each product in order to identify favorable or poor products in this way. Even if one has no idea of the market trade-off between risk and return, one may infer it from the choice set, and then identify outlying products. Indeed there is evidence that people are able to make such judgments in other contexts. Simonson and Tversky (1992) found effects of the trade-off between two attributes, either in previous choices or within the current choice set, for attribute pairs such as cash and gift coupons, price and amount of memory for personal computers, or tires varying in price and warranty duration.

In previous experiments, the products offered to participants always lay on the same risk-return line. In this experiment, for some participants a single product in the choice set lay above the line, and for other participants a single product lay below the line. If participants are able to make relative comparisons between products they should be sensitive to this manipulation and assign less of their funding to the above the line product and more of their funding to a below the line product.

The experiment described here investigates whether people are sensitive to the other options in the choice set by using an investment situation, in which people are asked to divide a lump sum defined as a cash windfall between three different investment products differing in their increasing risk-return characteristics. The three financial products offered different expected return from a unit of investment, and higher average return was associated with
Risk Preference Discrepancy

higher variability of this return (which is also a measure of the investment risk). In this scenario, the attractiveness of the middle option was manipulated by increasing and decreasing its expected return in comparison to the risk-return line and keeping its risk properties fixed.

Method

Participants. There were 42 participants in each of the two conditions in this study, 84 in total. The participants were recruited from the University of Oxford student population via the experimental economics research group mailing list of people who have asked to be contacted participated in this experiment.

Design. The task was formulated as an investment decision problem. The participants had to decide how to allocate £25,000 between three risky investment products that differ in the increasing expected degree of risk and return embedded in each of them. The risk here is understood as a possible variation of the capital return around (i.e., plus/minus) some expected average return. The least risky product was a Unit Trust Fund with mixed portfolio of stocks and bonds, which offers 8% annual average return and the annual return varies by ±12% around the average. The second product was Index Tracking Fund. The context manipulation was created by manipulating the expected return of the Index Tracking Fund (the middle option). In the bad investment condition the expected average return was 9.5%, and in the good investment condition the expected return was 12.5%. The third and riskiest product was a Hedge Fund offering expected annual average return of 15%, which however may vary ±40% around this average (characteristic of this fund is that it uses investor funds to borrow more money to put in the markets).

Figure 10 presents graphically the products that were displaced from the risk-return line. Thus, there were two between-participants conditions. In the bad investment condition the second product is above the line, while in the good investment condition the same product
is below the line. The expected difference between the mean return of the good and the bad product was 3%, which however small, still changed the underlying economic rationality of the decision problem because if people are rational optimizers, they should put more of their funds in the product above the line, and less in a product below the line (and in this case they should distribute more of their capital between the other two products). Our manipulation was expected to prompt such behavior because of the empirical evidence by Stewart et al. (2003) that people are sensitive mainly to the set of available options.

**Procedure.** The design of the materials was the same as that of Experiment 1A, except that only three products were used in each condition. The participants were given a sheet of paper with written instruction explaining that the purpose of the experiment is to allocate the funds by indicating how much they would invest in each of the products described in the table below the instruction (splitting funds between products as they see fit). Participants were informed also that some investments offer a smaller return, but are relatively low risk, while other investments offer a larger average return, but carry more risk, but all investment products are efficient in a capital markets sense in that they represent the best rates available for the risk being undertaken. There was a table below the instruction, which contained the name of the investment products, and their expected return and risk (variability). There was also an explanation involving an example how to interpret the return and risk figures in the table.

**Results and Discussion**

The allocations to each financial product in each of the two conditions are presented in Figure 11 (the two conditions are presented as separate lines). There is a clear tendency that people tend to invest less as the product becomes more risky, which shows that people are risk averse in general. It appears also that the participants were most sensitive to the manipulation in the bad investment condition for the Index Fund, which received on average
less investment than the good investment condition. The average allocation to the Index tracking fund in the bad investment condition was £5,708, while in the good investment condition the average allocation in the Index Fund was £7,518. The difference between these average investments allocated to the Index Fund in the bad condition and the good condition was statistically significant, t(82) = 2.07, p = .042. This result confirms our initial predictions about the existence of contrast effect due to the relative comparison of the middle option with the other two available options in each condition.

The results demonstrated that the manipulation of the attractiveness of the middle investment option in comparison with the two neighbouring options affected participants’ investment decisions. People invested significantly more in the middle option if it was above the control condition compared to the case when it was below it, although in none of the conditions the middle option stochastically dominated anyone of the two other options or was dominated by them. This result confirms that people are able to make relative riskiness judgments for each product using the context provided by the choice set. This experiment also demonstrates that people do make such relative comparisons. Thus, this conclusion rejects the hypothesis that the relativistic effects observed in Experiments 1A-1C could be caused by random responding due to a lack sensitivity to, or understanding of, the choice options.

One mechanism by which people may be identifying the favorable and poor products is to fit a regression line of risk against return using the products, and then note the degree to which each product differs from this line. This is consistent with the componential context account of trade-off contrast offered by Tversky and Simonson (1993). In this account the advantage for one product over another is the sum of all the attribute differences in its favor, and the disadvantage is the sum of all of the attribute differences against it. Together the advantage and disadvantage scores are used to construct a relative preference for one product.
over the other. Extending choices beyond binary sets, the relative advantage for one product over the other is the sum of all of the pair-wise relative advantages. Thus, for example, products below the risk against return line are assigned higher relative preference because the relative improvement in return is not offset by the increase in risk.

**General Discussion**

First we showed that laboratory behavior with gambles is more risky than people’s allocation of funds to financial products in more realistic setting. This is a result with important practical consequences. Then we demonstrated that the choice set across which hypothetical investments are made influences the allocation made. In particular, we found that the range of risk-reward of financial products offered has a significant effect on the distribution of hypothetical funds to those products. We also showed that people are able to make relative riskiness judgments for investment products using the context provided by the choice set. Thus on the whole, we demonstrated that prospect relativity can explain the risk aversion discrepancy between the real world and laboratory setting because choices in both domains are contingent on the context. Thus as a result the really risk averse choice in real world investments might be due to the low risk set of investment options with which people are presented. Note that actual financial decisions are also assumed to be influenced by accessibility (availability) of more risky investment options (e.g., how easy it is to buy shares). Our experiments indicate that had the investors been presented with more risky options (e.g., hedge funds, in which ordinary small investors do not have access to invest because of legal constraints), then they would have selected riskier investments. In other words, if people select the safer options from the safer range of investment options, then their overall risk aversion compared the whole set (i.e., all options available on the market, or relative to the total market portfolio) would appear to be extremely risk averse.

These findings illustrate that investors have ill-formed preferences about their
investments, which is consistent with Stewart et al.’s (2003) claims. We are not the first study to show the relativity of investment preferences. Benartzi and Thaler (2002) asked individuals to choose among investment programs that offer different ranges of retirement income (for instance, a certain amount of $900 per month versus a 50/50 chance to earn either $1,100 per month or $800 per month). When they presented individuals with three choices ranging from low risk to high risk, they found a significant tendency to pick the middle choice. For instance, people viewing choices A, B, and C, will often find B more attractive than C. However, those viewing choices B, C, and D, will often argue that C is more attractive than B. Benartzi and Thaler (2002) give a hypothetical scenario that illustrates what could possibly happen if people persistently avoid extremes and choices between alternatives depend on the other options available. For example, if there are three possible portfolios, which offer different proportions of stocks---0% (conservative), 40% (moderate), and 80% (aggressive), respectively, then choosing the middle portfolio implies a moderate risk taking. However, when the three options are 40%, 70%, and 100%, respectively, then choosing the middle option implies aggressive portfolio. Such effects of the choice set are analogous to Benartzi and Thaler’s (2002) demonstration that investors prefer the middle fund in a range of funds ranked according to risk level, only due to its relative position. Similarly, Benartzi and Thaler (2001) showed that participants invest more money in equity funds when the offered plan contains mostly equity funds. These results also imply that financial advisors may ‘guide’ investment decisions simply by selecting the array of funds offered to investors.

Our demonstration, that risk aversion in the real world is a product of the set of risky investment options with which people are presented, was also demonstrated by Benartzi and Thaler (2001) who found evidence of the same phenomenon by studying how people allocate their retirement funds across various investment vehicles. In particular, they find some
evidence for what they call the $1/n$ heuristic. This is a bias to divide the money evenly among the funds offered (when an employee is offered $n$ funds to choose from in her retirement plan), which implies that the asset allocation an investor chooses will depend strongly on the array of funds offered in the retirement plan. For example, in a plan that offered one stock fund and one bond fund, the average allocation would be 50% stocks, but if another stock fund were added, the allocation to stocks would jump to two thirds. In a sample of US 401 pension plans, Benartzi and Thaler regressed the percentage of the plan assets invested in stocks on the percentage of the funds that are stock funds and found a very strong relationship.

These results are particularly relevant to our claims in this article, because Benartzi and Thaler’s study used real-world data on the distribution of assets across pension funds with different levels of risk due to the different numbers of stocks and bonds offered by each particular employer. Thus, this study creates a natural experiment that allows us to compare the effects of offering many stocks and less number of bonds versus small number of stock and many bonds. The results showing a strong effect of the choice set on actual behavior demonstrates again that risk aversion is the world is probably due to the partial exposure to only a limited set of investment opportunities. Thus, risk aversion is shown to depend on the particular mix of fixed income and equity funds on offer. If the plan offers many fixed-income funds the participants might invest too conservatively and will appear very risk averse. Similarly, if the plan offers many equity funds the employees might invest more aggressively and appear much less risk averse. This evidence is in line with our demonstration that the risk aversion discrepancy is due to a biased sample of choice options.

*Alternative Explanations of the Risk Aversion Discrepancy*

There are some alternative explanations of the observation that laboratory behavior is somewhat more risky than people’s allocation of funds to real-world financial products.
Below we consider some. Note that some of these possibilities do not go against our theoretical point because they just provide a rational why people are exposed to a limited subset (biased sample) of investment options.

*There is no discrepancy.* It may be that people’s portfolio of investments is already quite risky because of property ownership (e.g., house mortgage), and so their remaining assets are rightly placed in less risky alternatives. We do not have data on all investments of the participants in the cited publications (Friend & Blume, 1975; Hildreth & Knowles, 1982; Kydland & Prescott, 1982; Tobin & Dolde, 1978), from which we derived our gamma estimates, in order to calculate the total risk exposure of each individual. Therefore we cannot reject this hypothesis right away.

*Accessibility of more risky financial products.* It may be that the only reason that people are not invested in more risky financial products is that people are either not aware that such products exist, or not aware of how to go about investing in them. Alternatively it may be that the products are simply not available to them. For example, investing in hedge funds requires an investment of minimum of $50,000, which is aimed at preventing small scale investors being exposed to higher risk because they might not be able to recover financially from the potential losses.

*Lack of knowledge.* The calculations involved in predicting possible returns from financial products are non-trivial (see the Appendix). It may be that if information about the distribution of potential returns from financial products could be made available to people, then people could be persuaded to make more risky investments. However, analysis of existing research literature on the role of learning and education, suggests that people cannot learn rational preferences, particularly with regard to risky financial decisions in experimental setting (Humphrey, 2001; Kagel & Levin, 1986; Loewenstein, 1999; Slovic & Tversky, 1974). In order to converge to a rational equilibrium, learning requires endless trials and
practical experience of success and failure. Relying on such learning is impractical for many aspects of consumer financial decision-making, because of the relative infrequency of having to make such decisions in real life. Therefore more knowledge will not change preferences for risk. Choices would still depend on the context.

Conversational pragmatics. It is possible that some people do not know anything about investments and the available opportunities in this respect. As a result, they take the experimenter’s or the salesman’s options. This seems to be the case when trading off unknown options like price of a computer and amount of memory (e.g., Simonson & Tversky, 1992). It could also matter whether participants think the financial information was compiled carefully by an expert. It is also important to understand whether the impact of the set of alternatives on people’s choices involves reasoning about the experimenter’s intentions (e.g., whether the options they are given as provided by a co-operative experimenter and hence infer that their response should naturally fall within the given range). This possibility could build connections between our research and pragmatic theory in linguistic communication (e.g., Grice, 1975; Levinson, 1983). Such pragmatic effects can be very subtle and can have implications for regulation of the financial services and the education of the public.

Note that this hypothesis does not change our basic claims that people do not have a stable notion of the utility of a risky option. In this case, context would still be expected to play a substantial role in determining participants’ choices. In this case, they may take the range of available options as a clue from the experimenter about what answers may be appropriate. Then, if a participant judges that she is, for example, more risk averse than the others, she may decide to choose an option lower than the average option available.

Separate utility scales. It is also possible that the utility scale is not common between laboratory gambles and real-world financial products. In other words, the utility of a
laboratory prospect may not be represented on the same scale as the utility of a financial product. There is some evidence that different goods are discounted at different rates (e.g., Loewenstein & Thaler, 1997). If this were the case then there is every reason to expect a discrepancy in risk aversion. However, our demonstration of the malleability of risky choice to context effects (see also Stewart et al., 2003) goes against any notion of the existence a stable utility scale. In other words, the demonstrated relativity of risk preferences rejects the hypothesis that people could develop more or less stable separate scales for gambles and financial products.

Subjective probability. It is generally accepted that the subjective representation of probability is distorted: small probabilities are overestimated and large probabilities are underestimated (e.g., Prelec, 1998; Tversky & Kahneman, 1992; Wu & Gonzalez, 1996; 1999). This would lead to relatively small probabilities for financial losses in risky decisions in the real world to be overestimated while relatively high probabilities for good returns to be underestimated. As a result, people are likely to be more averse to take risky investment decisions.

Temporal discounting. Similarly, there is good evidence that people discount future rewards hyperbolically (e.g., Kirby, 1997; Myserson & Green, 1995; Rachlin, Raineri, & Cross, 1991). However, discounting should be exponential because rates compound over time. Therefore investment risks, which are defined in the future (returns), will be subjected to such non-standard discounting. As a result, long term returns would be devalued too much and could be perceived as not high enough return for the amounts invested at present. This would require higher future returns, i.e. premiums, and would lead to seemingly more risk-averse behavior.

Risk and uncertainty. This explanation relates to the possibility that people might perceive stock investment as uncertainty instead of risk. For example, the risk of investments
are presented by financial advisors by simply stating stock prices can go as well up as down and the past performance is not guarantee for future performance. Investments funds are often classified on a point scale from, say, Secure to Adventurous (the later being higher risk find with potential for higher returns) with some historical performance data for the particular fund. Such statements hardly indicate nay quantifiable risk measures comparable to the ones used in laboratory experiments with gambles. Even more, in many countries, like UK for example, financial advisors are not allowed to state numeric projections of future performance. Therefore people would behave in the context of real investment choice as if facing uncertainty (which has been defined as unknown risk, Epstein, 1999). In this cases people display what is known as ambiguity aversion (also known as uncertainty aversion), which is preference for known risks over unknown risks. It is demonstrated in the Ellsberg paradox (Ellsberg, 1961), in which people prefer to bet on urns containing known proportions of balls with different colours compared to urn with unknown mix, even when this choices would lead to irrational decisions. The Ellsberg Paradox essentially states that we treat ambiguous choices as risky. This has been cited as one of the reasons for the high returns in the stock market (Epstein & Wang, 1994; Cagetti, Hansen, Sargent, & Williams, 2002; Skiadatas, 2005). The argument is that since stock price movements are ambiguous, people treat the stock market as risky and demand high returns.

Lack of incentive. Another possibility for the risk aversion discrepancy is that in the real world investments people, i.e. when real and substantial money are at stake, people more incentive to be cautious in relation to risk. Conversely, there would be lack of incentive to be careful with risk in laboratory setting where the stakes are usually rather small or hypothetical. Although some of the laboratory studies we reported here were conducted in developing countries for high payoff gambles (involving significant fraction of annual income), even Binswanger (1980) reported that most farmers exhibit a significant amount of
risk aversion that tends to increase as payoffs are increased. Holt and Laury (2002) present the results of a simple lottery-choice experiment that measured the degree of risk aversion over a wide range of payoffs, ranging from several dollars to several hundred dollars. In addition, they compared behavior under hypothetical and real incentives. With hypothetical payoffs, risky behavior is largely unaffected when hypothetical payoffs are scaled up, while with real payoffs, risk aversion increases sharply when payoffs are scaled up by factors of 20, 50, and 90. This result is qualitatively similar to that reported in different choice environments by Kachelmeier and Shehata (1992) who elicited buying and/or selling prices for simple lotteries, and Smith and Walker (1993) who reported increase in overbidding in private-value auctions as payoffs are scaled up (in auctions, overbidding relative to Nash predictions is usually attributed to risk aversion). Holt and Laury claim that these results imply that people facing hypothetical choices cannot imagine how they would actually behave under high-incentive conditions, which is contrary to Kahneman and Tversky’s (1979) belief to the contrary. In summary, there is convincing evidence that incentive levels do affect risk preferences and at this stage we cannot rule out the possibility that higher risk aversion in the real world might be caused by the higher gains and losses at stake.

**Losses loom larger than gains.** In the short term, for a risky investment, losses and gains are equally likely. But as losses loom larger than gains (cf., Kahneman & Tversky, 1979), people do not want to bare the potential downturns in the stock market and as a result might exhibit higher risk aversion. Such loss aversion combined with short-term horizon was proposed as an explanation of the well-known equity premium puzzle, which shows that the risk premium investors put on risky assets is unrealistically high (Mehra & Prescott, 1985).

**Discrepancy between expected return and actual return.** Fama & French (2002) conclude that the expected return for stocks using accepted valuation models is much lower than actual stock returns of the last half-century and lower than the average investor’s
expectation. Therefore, there is a discrepancy between the expected return from the stock market, and the actual return which has in fact turned out to be higher. Thus, if people were aware that the actual expected return was higher, then they would be more inclined to take more risk.

Final summary and comments

In this article, we presented a body of existing evidence that people are more risk averse when investing in financial products in the real world than when they make risky choices between gambles in laboratory experiments. In order to provide an account for this discrepancy, we investigated aspects of hypothetical decision behavior in one particular situation - long term investment, with the particular objective of seeing whether people can be motivated, by manipulating the decision context in which the options are presented to take more or less investment risk. We found that the range of offered financial products varying in their risk-reward characteristics had a significant effect on the distribution of hypothetical funds to those products. We also showed that people are able to use the context provided by the choice set in order the make relative riskiness judgments for investment products. This context dependent relativistic nature of risk preferences was then used as a very plausible explanation of the risk preference discrepancy between laboratory experiments and real-world investments. We also discussed other possible interpretations of the discrepancy and only future empirical research could select the most plausible explanation of risk preferences in various real-world situations and circumstances.
References


Humphrey, S.J. (2001). Do individuals learn not to make irrational choices? Working paper. Centre for Decision Research and Experimental Economics, School of Economics, University of Nottingham, UK.


Risk Preference Discrepancy


Schechter, L. (in press). Traditional trust measurement and the risk confound: An experiment


   
   *Management Science, 42,* 1676-1690.

   
   *Management Science, 45,* 74-85.
Appendix

Derivation of Log Normal Equation and Corresponding Means and Variances

If the return of product $X$ is normally distributed in log space

$$y(x) = \frac{1}{\beta \sqrt{2\pi}} e^{-\frac{(x-\alpha)^2}{2\beta^2}} \tag{A1}$$

with mean $\alpha$ and standard deviation $\beta$, then the distribution in linear space $z(w)$ of $w$ can be derived by observing that

$$x = \ln(w) \tag{A2}$$

and that the area under the curve between two fixed points must be conserved under the transformation. Thus

$$\Delta x \cdot y(x) = \Delta w \cdot z(w) \tag{A3}$$

Now from Equation A2

$$\frac{dw}{dx} = \frac{1}{w} \tag{A4}$$

Substituting Equations A2 and A4 into Equation A3 gives the distribution of $w$

$$z(w) = \frac{1}{w} y(\ln w) = \frac{1}{\beta \sqrt{2\pi w}} e^{-\frac{(\ln w - \alpha)^2}{2\beta^2}} \tag{A5}$$

The mean, $\mu$, of $w$ is given by

$$\mu = \int_{w=0}^{\infty} w \cdot z(w) \cdot dw = e^{\alpha + \frac{\sigma^2}{2}} \tag{A6}$$

and the variance, $\sigma^2$, by

$$\sigma^2 = \int_{w=0}^{\infty} (w - \mu)^2 \cdot z(w) \cdot dw = e^{\beta^2 + 2\alpha} \left(e^{\beta^2} - 1\right) \tag{A7}$$

Equations A6 and A7 may be rearranged to express the mean and standard deviation of the normal distribution in terms of the mean and standard deviation of the log normal distribution.
Risk Preference Discrepancy

\[ \alpha = \ln(\mu) - \frac{1}{2} \ln \left( \frac{\sigma^2}{\mu^2} + 1 \right) \quad (A8) \]

\[ \beta^2 = \ln \left( \frac{\sigma^2}{\mu} + 1 \right) \quad (A9) \]

The Utility of a Product

In the following analysis a financial product, \( X \), is modeled by assuming the annual return is log normally distributed about a mean of \( \mu_X \) with standard deviation of \( \sigma_X \). The choice of a log normal distribution is motivated by the observation that the return at the end of a given time period is a product of the returns for many subdivisions of that time period. Transforming return into log space, this product becomes a sum of random variables, which, by the central limit theorem, is normally distributed. If returns are normally distributed in log space, then they are log normally distributed in linear space.

If utility is assumed to be a power function of return with power \( \gamma \), then the utility of a product may be derived from the distribution in Equation A5 using the following integral.

\[ U[X] = \int_{0}^{\infty} w^\gamma z[w] d w = e^{\alpha y + \frac{\beta^2 y^2}{2}} = e^{\gamma \ln(\mu) + \frac{\gamma \sigma^2}{2} + \ln \left( \frac{\sigma^2}{\mu} + 1 \right)} \quad (A10) \]

Equation A10 first expresses the utility in terms of the mean and variance of the distribution in log space, and then in terms of the mean and variance of the distribution in real space.

Compounding of a Product’s Return

Equation A10 derived in the above section expresses the distribution of returns for a financial product after one year. In calculating the distribution of returns after \( n \) years, it is assumed that the return in any given year is distributed log normally as in Equation A5, and is independent from the returns in other years. Thus the return after \( n \) years is the product of \( n \) independent samples from the distribution in Equation A5. In log space, this product becomes the sum of \( n \) independent samples from a normal distribution mean \( \alpha \) and variance \( \beta^2 \) which
itself is normally distributed with mean $n\alpha$ and variance $n\beta^2$. Thus the distribution of returns for a product in real space is given by

$$z_n(w) = \frac{1}{\beta_n \sqrt{2\pi} w} e^{-\frac{(\ln(w) - \alpha)^2}{2\beta^2}}$$

(A11)

where

$$\alpha_n = \alpha^n$$

(A12)

$$\beta_n = \sqrt{n} \beta$$

(A13)

The mean and variance of this distribution are given by

$$\mu_n = e^{\frac{\alpha_n + \beta_n^2}{2}} = e^{\frac{n(\alpha + \beta)}{2}} = \mu^n$$

(A14)

$$\sigma_n^2 = e^{2\alpha + \beta^2} (e^{\mu^2} - 1) = e^{n(2 \alpha + \beta^2)} (e^{n \beta^2} - 1) = \mu^{2n} \left( \frac{\sigma^2}{\mu^2} + 1 \right)^n - 1$$

(A15)

Note that the mean expected value for a product after $n$ years is the annual average return raised to the power $n$. In other words, the mean expected value of a product will grow exponentially with time, as intuition suggests.

Utility of a Compounded Product

The derivation in Equation A10 may be generalized to the case of more than one year.

$$U_n[X] = \int_0^\infty w^\gamma z_n(w) dw = e^{\gamma \ln(\mu) + \frac{\gamma^2 \beta^2}{2}} = e^{\gamma \ln(\mu) + \frac{\gamma^2 \beta^2}{2}} = e^{\gamma \beta^2 (\mu + 1) - 1}$$

(A16)

If it is assumed that the standard deviation of a product’s annual return is a linear function of the product’s mean annual return, i.e.,

$$\sigma = m \mu + c$$

(A17)

then $\sigma$ can be substituted for in Equation A16. This substitution allows an expression for the product with maximum utility to be derived by calculating when the derivative of the utility with respect to $\mu$ is zero.

When
\[
\frac{d U_n | X}{d \mu} = 0 \tag{A18}
\]

\[
\mu = \frac{-c(3 - \gamma) m \pm c \sqrt{\gamma^2 m^2 - 2 \gamma m^2 + m^2 + 4 \gamma - 8}}{2(1 + m^2)} \tag{A19}
\]

Note the absence of the \( n \) parameter from Equation 19. The implication is that the product with maximum utility is independent of the duration of the investment. When

\[
\gamma^2 m^2 - 2 \gamma m^2 + m^2 + 4 \gamma - 8 < 0 \tag{A20}
\]

no real solutions exist for \( \mu \) and utility is always an increasing function of return.
### Table 1

**Financial products offered in Experiments 1A and 1B.**

<table>
<thead>
<tr>
<th>Category</th>
<th>Average Return Annual percent</th>
<th>Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Government bonds</td>
<td>5%</td>
<td>No risk, the income is guaranteed (beta = 0.0)</td>
</tr>
<tr>
<td>Building society account</td>
<td>6%</td>
<td>Variation of returns in the order of ±4% due to changes in interest rate environment (beta = 0.2)</td>
</tr>
<tr>
<td>Endowment fund savings account</td>
<td>7%</td>
<td>Funds are invested in a mixture of bonds and fixed income securities with some equity investments. Fluctuations in annual returns of ±8% are anticipated (beta=0.4)</td>
</tr>
<tr>
<td>Managed unit trust</td>
<td>8%</td>
<td>About two thirds of the fund is invested in equities and the remainder in government bonds. Annual return will vary by ±12% around this average (beta = 0.6)</td>
</tr>
<tr>
<td>FTSE 100 large corporate shares</td>
<td>9%</td>
<td>Invested in larger, lower volatility equities, annual returns are expected to vary by ±16% (beta=0.8)</td>
</tr>
<tr>
<td>All Share Index tracking fund</td>
<td>10%</td>
<td>Exposure to the whole equity market yields an annual return variation of about ±20% around the average (beta = 1.0)</td>
</tr>
<tr>
<td>Start-up and Venture Capital fund</td>
<td>13%</td>
<td>A specialist higher risk-return fund focussed on start-ups and buy-outs. Annual performance may vary by ±32% (beta=1.6)</td>
</tr>
<tr>
<td>Hedge fund</td>
<td>15%</td>
<td>This fund uses investor funds to borrow more money to put into the equity markets. Annual returns may vary ±40% around this average (beta = 2.0)</td>
</tr>
</tbody>
</table>
Figure Captions

Figure 1. The utility of money as a power function of money. The dashed lines illustrate that when the curve is downward concave ($\gamma<1$), the utility of a given amount is more than half the utility of twice that amount.

Figure 2. The distribution of returns for a set of financial products.

Figure 3. The relationship between risk, $\sigma$, and return, $\mu$, for four hypothetical markets.

Figure 4. The utility of a product $X$ as a function of the product’s average annual return, $\mu_X$, for various degrees of risk aversion. Each panel corresponds to a different market where the trade-off between risk and return differs as illustrated in Figure 3. Panel A corresponds to the line in Figure 3 with slope 2, panel B slope 3, panel C, slope 4, and panel D slope 5.

Figure 5. The distribution of returns for a given product (annual return $\mu_X=1.2$, standard deviation $\sigma_X=0.3$) after 1, 2, 4 and 8 years.

Figure 6. The utility of a product $X$ as a function of the product’s return, $\mu_X$, after 1, 2, 4 and 8 years for $\gamma=0.2$. In the left panel the market is as illustrated by the line of slope 2 in Figure 3, and in the right panel the slope is 4.

Figure 7. Risk premiums plotted against people’s risk aversion (gamma) according to their position on the risk against return line. The risk premiums are taken from classic studies that relate gamma to risk premium in the market by using a representative set of investment portfolios.

Figure 8. Implication of different levels of risk aversion for asset values in terms of stock prices of a portfolio containing the Dow Jones Industrial Average.

Figure 9. Average allocation of funds across financial products in Experiments 1A-C. (Error bars are standard error of the mean.)

Figure 10. Risk-return characteristics of the investment products.

Figure 11. Allocations to each financial product in each of the three conditions.
Figure 1
Figure 2
Figure 3

![Graph showing the relationship between Risk and Return with different slopes.]
Figure 4
Figure 5
Figure 6

(A) 8 Years

(B) 8 Years

Utility

Return
Risk Preference Discrepancy

Figure 7

Risk Premium

Gamma of Investment Community

Results derived from markets

Friend & Blume '75
Kydland & Prescott '82
Tobin & Dolde '78
Hildreth & Knowles '82

Typical lab result
Figure 8

Variability around typical lab result

Dow Jones Industrial Average

Current Level

Gamma of Investment Community

Stock prices

-1.0 -0.5 0.0 0.5 1.0
Figure 9

A Undergraduates

B Undergraduates (Numbers Only)

C Conference Guests
Figure 10

The diagram illustrates the relationship between return (%) and variability (Risk) for different product categories. It shows

- A Risk-Return line
- A Good product curve
- A Bad product curve
Figure 11

A line graph illustrating the fund allocation for different investment products, categorized into 'Bad product' and 'Good product'. The graph shows the variation in fund allocation across unit funds, index funds, and hedge funds.