NONLINEAR ENERGY HARVESTING FROM RANDOM NARROW-BAND EXCITATIONS

NATALIA KHOVANOVA
School of Engineering, University of Warwick, Library Road
Coventry, CV4 7AL, UK.
n.khovanova@warwick.ac.uk

IGOR KHOVANOV
School of Engineering, University of Warwick, Library Road
Coventry, CV4 7AL, UK.
i.khovanov@warwick.ac.uk

ZARINA DAVLETZHANOVA
School of Engineering, Nazarbayev University, 53 Kabanbay Batyr Ave
Astana, 010000, Republic of Kazakhstan.
zarina.davletzhanova@nu.edu.kz

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The efficiency of linear and nonlinear harvesters with different types of nonlinearity is compared. Narrow band ambient vibrations are modelled by harmonic Gaussian noise. We show that performance of nonlinear harvesters strongly depends on both the form of nonlinearity and the properties of the noise. Particular forms of nonlinearities which can produce a better than linear response are identified, and these depend on the spectral width of the harmonic noise.

Keywords: energy harvesting; hard and soft nonlinearities; bistability; narrow-band vibrations; harmonic noise.

1. Introduction

Energy harvesting from ambient vibrations attracts great attention from the scientific community due to many promising applications[1,2]. The idea of harvesting consists of using an auxiliary mechanical system as a source of energy and converting energy extracted from mechanical vibrations into electrical form. A piezoelectric transducer is typically used for the conversion because of its efficiency[1]. Properties of the mechanical system is the key factor defining energy efficiency of a harvester, and the mechanical system should be correctly tuned to the properties of ambient vibrations. The latter vary significantly and often possess random character and a complex form[3]. Typically, a resonant design of harvester is implemented[1], which is based on a linear system and allows the use of analytical expressions for optimizing of harvesting efficiency (for example in Ref. 4). However, linear behaviour of

mechanical systems is observed in a limited range of excitations, whereas degree of nonlinearity is growing with the reduction of the size of harvesters[5]. Moreover, the resonant design has a strong limitation since it targets a harmonic form of vibration. The stochasticity and/or variability of ambient vibration obliges researchers to explore a nonlinear design, where nonlinearity of the stiffness of a mechanical system is the main variable parameter (see Refs. 1, 6, 7 and references therein). The benefits of nonlinear design have been demonstrated in a number of publications where different forms of nonlinearity have been considered (e.g. in Refs. 6 and 7).

Ambient vibration in the form of a harmonic signal and broadband noise have been applied mainly as an external source of vibrations with respect to various nonlinear harvesters[1,2]. In the case of broadband vibrations, the cut-off frequency is higher than the natural frequency of mechanical system and, therefore, the broadband noise can be considered as white, i.e. having uniform spectral density over the entire spectral range. The noise distribution is usually modelled as Gaussian. White Gaussian stochastic excitation on linear and non-linear harvesters have been considered in a number of theoretical, numerical and experimental investigations[8,9,10,11,12,13] and the overall outcome have been formulated recently by Halvorsen[12] as "nonlinear harvesters are not fundamentally better than linear ones", but nonlinear harvesters can be tuned to out-perform linear ones if the intensity of white noise is known and fixed. On the contrary, harmonic form is characterised by a single delta-peak in the spectrum. A harmonic signal and white noise represent the two opposite limits of the idealized forms of an ambient vibration. More realistic vibration models are more complex, e.g. a harmonic form represents a particular case of periodic vibrations, which generally may contain harmonics and sub-harmonics of the fundamental frequency. The role of harmonics and sub-harmonics for nonlinear designs is practically unexplored. Together with the spectral content, random vibrations are characterised by probability distribution which also significantly changes the performance of nonlinear harvesters[14]. This effect of the shape of probability distribution on efficiency of energy harvesting has been initially explored[14,9], however, it requires further comprehensive investigation.

In this manuscript we discuss the performance of nonlinear harvesters excited by narrow-band vibrations centred around a particular frequency. As a model of a narrow-band vibration, we consider harmonic noise[16,17] which is limited by harmonic signal and white noise. Following previous publications[6,14], several types of non-linearities in stiffness have been considered here, aiming to explore the roles of stiffness hardening and softening as well as the effect of bistability on the performance of the harvesters. In section 1, the model, types of nonlinearity and properties of the harmonic noise are described. Section 2 contains the results of numerical simulations and comparative analysis of the performances of the harvesters. The summary and conclusions are presented in the last section of the manuscript.

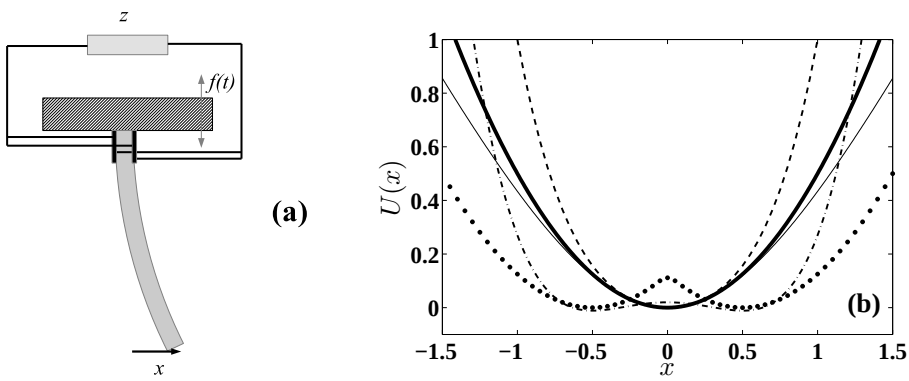

Fig. 1. (a) Schematics of a piezoelectric energy harvester under base excitation $f(t)$. (b) Shapes of potentials $U_L(x)$ (thick solid line), $U_M(x)$ (dashed line), $U_B(x)$ (dot-dashed line), $U_{BP}(x)$ (dotted line) and $U_S(x)$ (thin solid line) are shown.

2. Harvester and noise models

The harvester model consists of mechanical and electrical parts (Fig. 1(a)) forming the following three-dimensional system[18]

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -\alpha y - \frac{dU(x)}{dx} + \chi z + f(t) \\ \dot{z} &= -\lambda z - \kappa y \end{aligned} \tag{2.1}$$

The Mechanical part is defined by variables x and $y \equiv \dot{x}$, and describes the dynamics of a cantilever beam. Coordinate z corresponds to the electrical part and it is proportional to the voltage arisen from the interaction between piezoceramic layers and beam bending. This interaction is characterised by parameters κ and χ; λ describes reactivity and resistance of the electrical part. Properties of the mechanical part are regulated by damping α and stiffness $dU(x)/dx$; $U(x)$ is the potential energy of the beam. The power extracted can be defined as follows:

$$P = \rho \frac{1}{T} \int_0^T z^2(t) dt \tag{2.2}$$

where ρ corresponds to load conductivity[19] and it is set to 1 ($\rho = 1$) for simplicity. T is the time interval used to calculate the output power and the term $f(t)$ corresponds to ambient vibration. Values of other parameters are fixed as follows: $\alpha = 0.02$, $\chi = 0.05$, $\lambda = 0.05$, $\kappa = 0.5$[18]. The term $dU(x)/dx = k(x)$ describes nonlinearity of stiffness in (2.1) and can be adjusted by changing the beam geometry and/or by coupling the beam with external elements, e.g. magnets[20]. The potential energy $U(x)$ stored in the beam is related to the stiffness $U(x) = \int k(x) x dx$ and this potential energy (potential profile) is used here to characterize the system's

nonlinearity. In this way, the resonant (linear) harvester has parabolic potential

$$U_L(x) = k_0 \frac{x^2}{2} \tag{2.3}$$

and its resonant frequency depends on the value of stiffness k_0; we fix it to $k_0 = 1$. Monostable and bistable harvesters, which have been studied experimentally in Refs. 18-20, can be considered as having the following potential profiles, respectively

$$\begin{aligned} U_M(x) &= \frac{x^2}{2} + \frac{x^4}{2} \\ U_B(x) &= -\frac{x^2}{4} + \frac{x^4}{2} \end{aligned} \tag{2.4}$$

These forms are selected to match the resonant (natural) frequency of the linear harvester in linearized versions of the profiles. The monostable potential represents a harvester with hard nonlinearity, whereas the bistable profile has two states (bistability) and demonstrates soft nonlinearity for vibration in the vicinity of one of the states. In order to separate these two properties of softening and bistability, two additional potentials have been considered: a piece wise parabolic bistable potential $U_{PB}(x)$ with minimized softening of nonlinearity and a monostable harvester with soft nonlinearity $U_S(x)$

$$U_{PB}(x) = \begin{cases} (x-0.5)^2/2 & \text{if } \ x < -x_s \\ -9x^2/2 + c_1 & \text{if } \ |x| \le x_s \\ (x+0.5)^2/2 & \text{if } \ x > x_s \end{cases} \quad x_s = 0.05 \tag{2.5}$$

$$U_S(x) = \begin{cases} \log[\cosh(x)] & \text{if } \ |x| < x_b \\ x^2/2 + c_2 & \text{if } \ |x| > x_b \end{cases} \quad x_b = 3. \tag{2.6}$$

Here, c_1 and c_2 are constants: $c_1 = 5x_s^2 - 0.5x_s + 0.125$, $c_2 = \log[\cosh(x_b)]x_b^2/2$. Note that softening nonlinearity in $U_S(x)$ is observed in a limited range of deviation x to avoid instability of the system. All potentials ((Fig. 1(b))) have same natural frequency of intra-well motion: $\omega_0 \approx 0.988$.

For modelling, a narrow-band vibration harmonic noise $f(t)$ as an ambient source has been applied which is an output of a second-order linear system driven by random noise $\xi(t)$

$$\ddot{f}(t) + \Gamma \dot{f}(t) + \omega_h^2 f(t) = \sqrt{D}\xi(t) \ . \tag{2.7}$$

Here, $\xi(t)$ is white Gaussian noise with zero mean, $\langle \xi(t) \rangle = 0$ and delta-correlated central moment, $\langle \xi(t)\xi(0) \rangle = \delta(0)$, D is intensity of noise, Γ and ω_h are parameters defining width and location of the peak in $f(t)$ spectrum. Intensity of the harmonic noise is D/Γ, and spectrum $S_{ff}(\omega)$ of the system (2.7) has the following form[16]

$$S_{ff}(\omega) = \frac{D}{2\pi \left[\omega^2\Gamma^2 + (\omega^2 - \omega_h^2)^2\right]}, \tag{2.8}$$

with a peak located at frequency $\omega_p = \sqrt{\omega_h^2 - \Gamma^2/2}$ and of width $\Delta\omega \approx \Gamma$. This expression is valid for an underdamped case $\omega_h^2 \ge \Gamma^2/2$ which is considered below.

This harmonic noise represents stochastic fluctuations of various civil structures which normally have a distinct natural frequency. Therefore, this is a more realistic model than white noise. Note that in the limit $\Gamma \to 0$ spectrum $S_{ff}(\omega)$ tends to a delta-peak, i.e. the spectrum of a harmonic signal, whereas in the limit $\Gamma \gg \omega_h$ function $S_{ff}(\omega)$ becomes flat and approaches the spectrum of white noise.

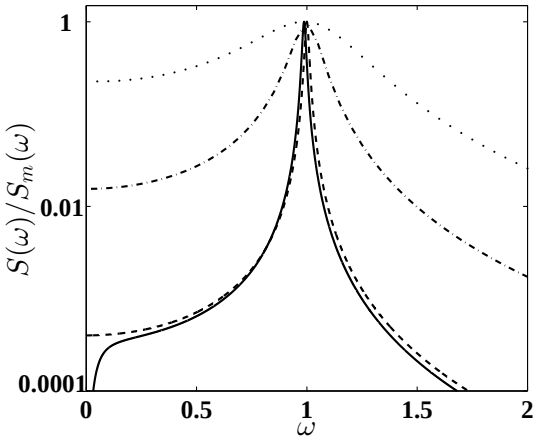

Fig. 2. Frequency response of the linear harvester, $S(\omega)$ (solid line) and spectra of harmonic noise, $S(\omega) \equiv S_{ff}(\omega)$ for different widths are shown: $\Delta\omega \approx 0.02$ (dashed line), $\Delta\omega \approx 0.125$ (dash-dotted line), $\Delta\omega \approx 0.49$ (dotted line). All curves are normalized by maximal value $S_{max}(\omega)$ to have maximum equal to 1. Ordinate is logarithmic.

Influence of the width of the spectral band of the harmonic noise on harvester efficiency is considered here. The location of the spectral peak of (2.7) is fixed and equal to $\omega_p = 1.0$ and the following three values for the width are discussed: $\Delta\omega \approx 0.02, 0.125, 0.49$. Corresponding spectra for different widths are shown in Fig. 2 alongside with frequency response of a linear harvester. The frequency response was calculated analytically (see Ref. 4 for an example of derivation of frequency response). The performance of linear and nonlinear harvesters under excitation in the form of harmonic noise was analyzed via numerical simulations; The Heun numerical scheme (see Refs. 21 and 22 for details) was used for simulations of coupled systems (2.1) and (2.7).

3. Results

3.1. *Harmonic signal*

Before considering system's response to random excitations, let us discuss responses of system (2.1) with different nonlinearities to harmonic signal $f(t) \equiv f_s(t) = A\sin(\omega t)$. Gain $\gamma = P/A^2$ as a function of the signal frequency ω (frequency response) was calculated, and a comparison of the responses of the linear harvesters with responses of monostable and bistable harvesters was performed. The frequency

response $\gamma(\omega)$ of the linear harvester with $U_L(\omega)$ (solid line in Fig. 2) does not depend on amplitude A and coincides with the responses of nonlinear harvesters for small values of amplitude, i.e. when a linearized model of the harvesters is valid. Nonlinearities change the frequency responses significantly with an increase of A (Fig. 3). A region of hysteresis with co-existing multiple solutions for the same external force is observed for both potentials $U_M(x)$ and $U_B(x)$. However, nonlinearity hardening in monostable harvester leads to shifting of the resonant frequency (a maximum of γ) to the higher frequency region (Fig. 3(a)), whereas a softening of nonlinearity in bistable systems shifts the resonant frequency to the low frequency region (Fig. 3(b)). The narrow peak of linear case (solid line in Fig. 2) is replaced by a wider peak when nonlinearity starts to play its crucial role (Fig. 3). This widening of the frequency response was suggested by a number of authors[6,7] to be considered as beneficial for using in nonlinear harvesters. A rational behind this is that, if the frequency is varied in some range, then an integral response of nonlinear system exceeds the narrow-band response of the corresponding linear harvester. However, this conclusion only holds true if the system stays on the upper branch of the response, otherwise the integral response of the lower branch is significantly less than the response for linear case. Thus, stability of the upper branch is important and it has to be taken into account for an efficient harvester design. The stability can be assessed via the consideration of an additional stochastic term in (2.1) and by nonlocal stability analysis (see for example Ref. 15). Majority of the research articles devoted to energy harvesting has so far failed to consider the stability issue. It is particularly important as the presence of additional perturbations can easily lead to a switch between the upper and lower branches of the resonant curve and, therefore, a procedure for recovering the high amplitude solution needs to be implemented.

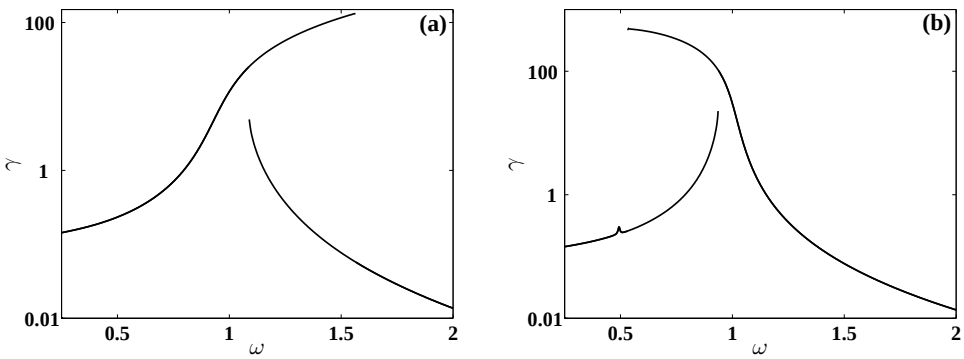

Fig. 3. Examples of frequency responses (output voltage gain versus excitation frequency) of nonlinear harvesters with $U_M(x)$ (figure (a)) and $U_B(x)$ (figure (b)) are shown. Ordinate is logarithmic.

Note, that the maximal possible response is larger for soft nonlinearity and

smaller for hard nonlinearity, so this fact may lead to a conclusion that the soft nonlinearity is more efficient (see for instance Ref. 6). This difference, however, results from the frequency response being scaled by $1/\omega^2$; such scaling is observed in linear case too. Therefore, for an unambiguous comparison of different system's performances, the same frequency range should be considered.

3.2. *Harmonic noise*

Harmonic noise possesses stochastic nature, i.e. includes perturbations. This fact, in contract to harmonic signal, allows for the implicit characterization of the stability of the system and its solution. Stochastic noise requires longer simulations and the use of statistical quantities, which are based on averaging, for characterization of the system's response. As in the previous section, the gain $\gamma = P\Gamma/D$ describing the efficiency of harvesting from vibrations is considered here. The value P has been calculated from a single trajectory , $z(t)$, within an extended time interval $T = 10^7$.

In linear case the efficiency of energy harvesting depends on both Γ and ω_h, but does not depend on intensity D. For the nonlinear cases, all three parameters are important. Note, that the output frequency response is defined by the product of the frequency responses of harmonic noise and the harvester and, therefore, in contrast to the harmonic signal, we cannot conclude that a linear system is more efficient for any finite width of noise spectrum.

Let us start the analysis with a discussion of the performance of the harvester within frequency range $\Delta\omega = 0.02$ which is close to the width of the peak in the frequency response of corresponding linear harvester (Fig. 2). The gain for all nonlinear harvesters, except for the monostable one, is equal or less than the gain of the linear system (Fig. 4(a)). The gain of both nonlinear and linear harvesters coincides within the range of noise intensities where harvesters can be linearised, and it decreases when nonlinearity contributes to the responses. However, the gain of the monostable harvester demonstrates improvement, as compared to linear case, for a region of noise intensity (between $D = 10^{-6}$ and 10^{-8}). In this region, the spectral peak (Fig. 5(a)) becomes wider and its height is very close to the height of the peak of the linear system. A similar widening of spectral peak is observed for all the other nonlinear harvesters; the heights of the peaks, however, reduce simultaneously with widening, and the resulting output becomes poorer than for the linear harvester (Fig. 5(b)). Both bistable harvesters show gain decrease as transitions between the states begin. Later, with the transitions becoming more frequent and the amplitude of noise excitations comparable to the barrier between the states (i.e. when fluctuations do not feel the presence of the barrier), the gain increases. In the bistable harvester $U_B(x)$ further increase of the noise intensity leads to gain decrease due to the form of nonlinearity of the potential profile, whereas the parabolic bistable harvester $U_{PB}(x)$ tends to show a linear response for larger noise intensities because its profile becomes parabolic for larger deviations. Note, that similar responses were observed for all harvesters with the width of

spectral band reduced by a factor 2, i.e. $\Delta\omega = 0.01$.

Now let us discuss a larger peak width $\Delta\omega = 0.125$ (Fig. 4(b)). The harvesters with bistable profile $U_B(x)$ and soft nonlinearity $U_S(x)$ demonstrate qualitatively similar behaviour to the case considered above (Fig. 4(a)). However, the quantitative difference between the gain values for $U_B(x)$ and $U_S(x)$ profiles and for the linear case becomes smaller. The monostable harvester with $U_M(x)$ demonstrates a monotonic decline of the gain and the effect of the gain improvement disappears, whereas the parabolic bistable harvester shows a larger gain compared to linear case for a certain region of noise intensity. In this region, the parabolic bistable harvester performs relatively rare transitions between its states. As a result, a new spectral peak is observed around zero frequency, and this peak is effectively widening the peak around ω_h. On the other hand, the height of the peak at ω_h is comparable to one observed for the linear response (Fig. 6(a)). With a noise intensity increase, the peak at ω_h becomes wider, however its height reduces causing gain decrease (Fig. 6(b)).

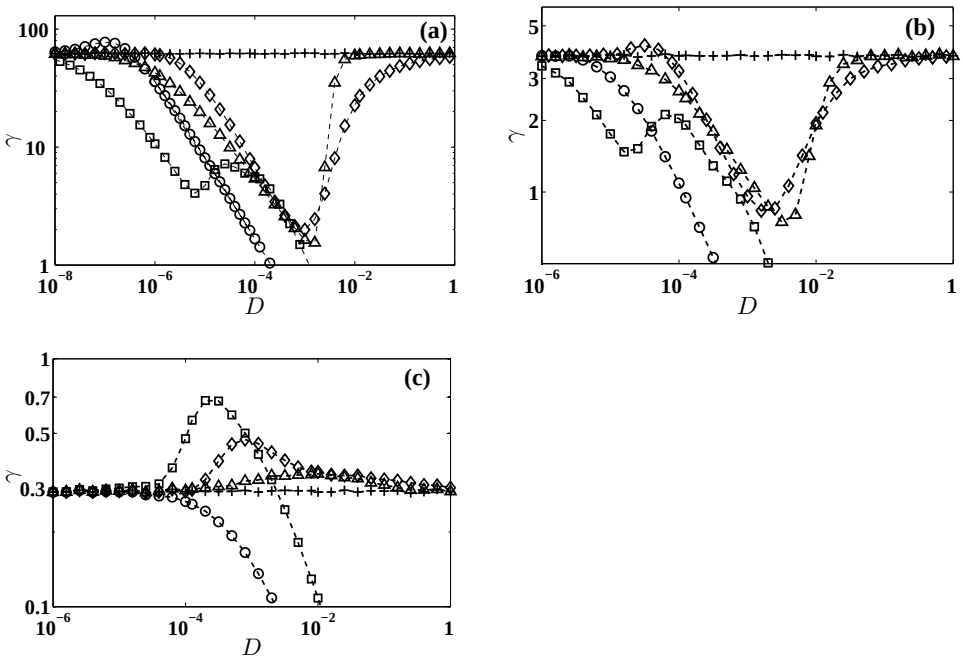

Fig. 4. Voltage gain of harvesters with $U_L(x)$ (marker +), $U_M(x)$ (marker ◯), $U_B(x)$ (marker □), $U_{BP}(x)$ (marker ◇) and $U_S(x)$ (marker △) versus noise intensity are shown for different values of width $\Delta\omega$: $\Delta\omega = 0.02$ (a), $\Delta\omega = 0.125$ (b), $\Delta\omega = 0.49$ (c). Axes are logarithmic.

More dramatic changes in frequency responses are observed for a peak with $\Delta\omega = 0.49$ (Fig. 4(c)). This peak is significantly wider than the peak in the frequency response of a linear harvester (see Fig. 2). The gain for the monostable har-

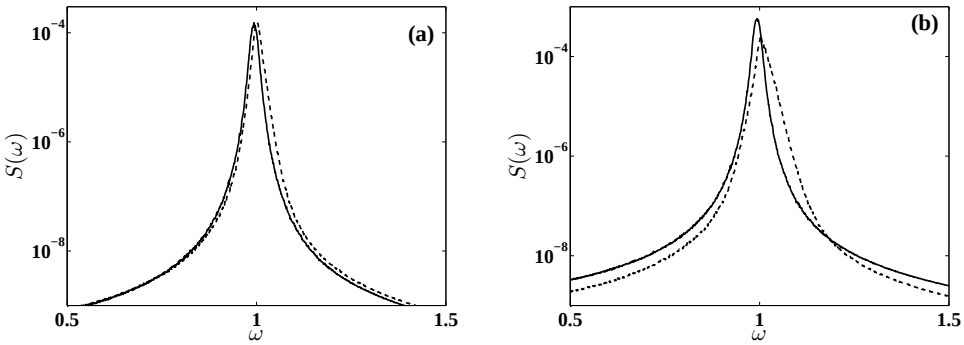

Fig. 5. Voltage output spectra of the linear (solid line) and monostable (dashed line) harvesters are shown for noise intensities $D = 10^{-7}$ (a) and $D = 10^{-6}$ (b). The spectral width of noise is $\Delta\omega = 0.02$. Ordinate is logarithmic.

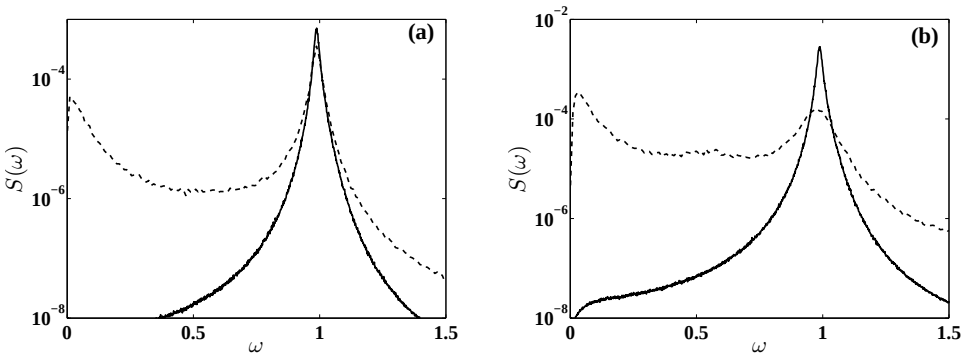

Fig. 6. Voltage output spectra of linear (solid line) and parabolic bistable (dashed line) harvesters are shown for noise intensities $D = 2.5 \cdot 10^{-5}$ (a) and $D = 10^{-4}$ (b). The spectral width of noise is $\Delta\omega = 0.125$. Ordinate is logarithmic.

vester monotonically decays. The bistable harvester $U_B(x)$ has significantly better (than linear) efficiency within a region of noise intensities, but the efficiency drops significantly for larger noise. Remarkably, efficiencies for harvesters with profiles $U_{PB}(x)$ and $U_S(x)$ are equal or better than in the linear case in the whole region of noise intensities considered above. An improvement with respect to linear response is observed in the regions where nonlinearities contribute to the response by widening the peaks and/or inducing new peaks in the spectrum. Note, that similar responses are observed when the peak width $\Delta\omega$ of harmonic noise is greater than 0.49 and up to the white noise limit.

4. Conclusions

Our results show a complex interplay between particular forms of nonlinearities and properties of noise excitations. For various widths of spectral peak $\Delta\omega$ of harmonic noise considered there was possible to select a nonlinear harvester with better effi-

ciency than for corresponding linear system. In other words, an efficiency of energy harvesting from vibrations in the form of harmonic noise can be improved by careful selection of the form of harvester nonlinearities.

Hard and soft nonlinearities define differences in the harvester's response for both the small and large widths $\Delta\omega$ of spectral band. The use of a bistable configuration is beneficial for a wide frequency peak and it is most pronounced in the white noise limit. Two considered profiles $U_{PB}(x)$ and $U_S(x)$, having a piece-wise shape and combining parabolic and non-parabolic parts, provide same or better efficiency than a linear harvester for wide band noise.

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