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# Essays on Markets with Asymmetries of Information and Strategic Experimentation

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# Contents

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# Introduction

## § 0.1 Competitive Markets with Adverse Selection

Part 1 of the thesis is a theoretical investigation of (strategic) competitive markets with adverse selection. In this part, I propose possible optimal policies to correct inefficiencies that may be caused in competitive markets because of asymmetries of information among agents. Specifically Part 1 contains two distinct chapters.

In Chapter 1, the impact of redistributive taxation in a credit market with adverse selection is examined. The market consists of different types of entrepreneurs who need to borrow from banks to invest in stochastic technologies. Adverse selection leads to credit rationing, and the economy is characterised by low aggregate investment and constrained suboptimal allocations. It is shown that an anonymous, redistributive budget-balanced tax system can increase aggregate investment and lead to Pareto improvements. Unlike what is usually believed, it is shown that every entrepreneur benefits from the tax system, even if, in expectation, high-productivity entrepreneurs pay in taxes more than they receive in subsidies.

In Chapter 2, I model a competitive insurance market with adverse selection as an "*informed-principal game*". The informed buyer offers a set of contracts to all uninformed sellers, who accept or reject. If all sellers reject, then there is no trade. Otherwise, each one of the sellers who accepted has the right to add more contracts to the already existing offer if he wishes so. The buyer can choose one contract from one seller at the last stage of the game. I characterise the set perfect Bayesian equilibria of this game. I show that the well-known Rothschild-Stiglitz-Wilson (RSW) allocation places a lower bound in the equilibrium payoff of each type and is the unique equilibrium allocation (or a "*strong solution*") when it is not Pareto dominated. Every interim incentive efficient allocation, that weakly Pareto dominates the RSW allocation, is an equilibrium allocation. Bertrand-type competition among sellers drives expected profits to zero and demands every equilibrium allocation to be interim incentive efficient. The approach extends to any finite number of types and states, and to other similar markets like credit, labour or informed seller markets.

## § 0.2 Strategic Experimentation in R&D Races

Part 2 of the thesis is a joint work with Abhinay Muthoo, Alex Gershkov and Motty Perry with equal shares in all aspects of the paper.

In this part, we study a patent race between two firms as a two-armed bandit model. The first firm that successfully completes two phases (R&D) acquires a patent license. Each firm can learn from its rival's actions and outcomes. We show that two possible inefficiencies can be observed in equilibrium. On the one hand, spill-overs of information reduce the expected profitability of the patent and therefore firms may invest inadequately in R&D, in the fear of releasing good news to the market. On the other hand, an opposite, "tragedy-of-the-commons", effect may prevail, according to which R&D investment is socially excessive.

# Part

## Competitive Markets with Adverse Selection

# - Chapter 1 -

## Investment, Adverse Selection and Optimal Redistributive Taxation

### § 1.1 Introduction

It is commonly believed that corporate taxation distorts social efficiency and decreases the level of investment in the economy. The usual argument is that high taxation discourages the most productive entrepreneurs from investing in risky technologies because they are most likely to bear the cost of subsidising the tax system. Therefore, the economy is likely to fall into a trap of low investment and output.

This paper shows that this argument does not necessarily apply to credit markets with adverse selection. In particular, it is shown that an opposite (counterintuitive) effect may prevail: The government, by increasing taxes, can not only increase economy's investment, but also create Pareto improvements compared to the market outcome.

I analyse a simple credit market with entrepreneurial investment and banks. Entrepreneurs are endowed with stochastic investment technologies which can be either high- or low-productivity. Given that they do not possess any wealth, they need to borrow from wealthy banks in order to invest. The type of each entrepreneur is his private information leading to an adverse selection problem. The market is formulated as a signaling game in which entrepreneurs apply to a bank for a loan contract and the bank decides either to accept or reject. The only sorting devices are the amount of loan and the interest rate.[1] For instance, an entrepreneur can apply for a contract with a lower amount of loan at a lower interest rate in order to signal that he is a good type.

[1] Interest rates here refer to the per unit of borrowing/investment payment in case of success and failure. These two may be different depending on the contract signed.

Given that the amount of loan of the high-productive entrepreneur is distorted downwards, there is some kind of credit rationing in the market.[2] As it is shown, this form of credit rationing creates inefficiencies; equilibrium aggregate investment is relatively low and the equilibrium allocation is constrained Pareto suboptimal. I show that a carefully designed redistributive tax system can increase aggregate investment and create Pareto improvements. Evidently, the government does neither possess any superior information than banks, nor, does it provide any additional financing to entrepreneurs at the time of contracting. The unique intervention is in action only after the realisation of uncertainty and it takes the form of redistribution of wealth. This redistribution not only, as expected, does it benefit the low-productive entrepreneurs (who are cross-subsidised), but also allows the high-productive ones to apply for contract that are contingent on the subsidy they will receive from the government. This relaxes the incentive constraint and allows them to leverage/invest more and also increase their payoff.

The tax system is not unique, but there is a set of taxes that results to equilibria that are Pareto superior to the no tax equilibria. This is in contrast to what is usually believed that redistributive taxation is distortionary and harms the most productive types. Interestingly, it is shown that economy's aggregate investment is monotonically increasing in the level of taxes when these increase welfare. This provides a rationale of why governments increase taxes and provide "stimulus packs" in order to spark the economy in times of recession and when banks refuse to provide high loans to applicants.

**Related Literature.** The paper builds on the rich literature of credit markets with asymmetric information. Stiglitz and Weiss [57] show that when banks cannot observe the riskiness of the projects they are asked to finance and entrepreneurs have no signaling device, other than the interest rate, the latter may be declined financing. Bester [7, 8] highlights the role of collateral as a signaling device in this type of credit markets. De Meza and Webb [17] argue that, in contrast to Stiglitz and Weiss [57], a slight difference in the technologies of entrepreneurs can lead to excessive investment instead of credit rationing. Note that in my model investment is a continuous variable as opposed to all the papers mentioned above where it is indivisible.

The closest environment to this paper is Martin [41], who shows how entrepreneurial wealth affects economy's aggregate investment. Interestingly, in his model there may be a non-monotonic relationship between entrepreneurial wealth and investment. Intuitively, this happens because en-

[2] There are two forms of credit rationing discussed in the literature. One is similar to the one examined in this paper. The other one takes place when there is excess demand in the market and some applicants are declined credit. See Jaffee and Russel [33] and Stiglitz and Weiss [57].

trepreneurial wealth can be used as collateral by some types and therefore switch the equilibrium from pooling to separating causing a discontinuous change in the investment level.

Wilson [59] and Dahlby [13] show how government interventions can create Pareto improvements in competitive insurance markets with adverse selection. This paper differs from Wilson [59] and Dahlby [13] in at least two respects: First, the approach concerns primarily credit markets (as opposed to insurance markets), where risk-neutral entrepreneurs invest in stochastic investment technologies. Second, in this paper, it is shown how the composition of aggregate investment is affected by taxes and it is rather not straightforward how decisions taken after the realisation of production can affect the decisions for investment at the ex ante stage.

Bisin and Gottardi [9] show how, in the Rothschild and Stiglitz [55] insurance market, property rights can alleviate the incentive constraints and help the economy attain Pareto efficiency. Similar to Bisin and Gottardi [9] but in a credit market, Martin [42] shows that the government, by establishing a new market at the ex ante stage in which entrepreneurs can borrow without conditioning their loans, Pareto efficiency can be attained. Along the same lines, Innes [32] shows how the government can increase social welfare by offering subsidised debt contracts in credit markets with adverse selection. Martin[42] and Innes [32] are the closest environments to my paper and, in fact, they should be considered as complementary. The main difference is on the type of intervention. In these papers any intervention occurs in the ex ante stage (where investment decisions take place) in the form of creation of a new lending market regulated by the government, whereas in the present paper ex post (when entrepreneurs have realised any production) in the form of taxation.

In Section 1.2, the economy is described. In section 1.3, the equilibria without taxes under perfect and imperfect information are characterised. In Section 1.4, the equilibria of the economy with taxes are analysed and the Pareto improving taxes are proposed. In Section 1.5, I discuss some of the important assumptions as well as other relevant topics.

## § 1.2 The Economy

**Entrepreneurs.** There are only two periods $t = 0, 1$ and one consumption-investment good which is perishable.[3] There is a continuum of mass one of entrepreneurs, who are categorised into two types denoted H and L for "high productivity" and "low productivity" respectively. Therefore, $i = H, L$. A set of measure $\lambda$ is of type H and $1 - \lambda$ is of type L. They do not possess any

[3] The economy is similar to that of Martin [41] with the only difference that entrepreneurs do not possess any initial wealth at the time of contracting. This is only for simplicity and without loss of generality.

initial wealth but they own stochastic investment technologies which take the following form: By investing $X$ units in period $t = 0$ an entrepreneur can realise $\gamma_i f(X)$ units in period $t = 1$ with probability $\pi_i$ and zero otherwise. Uncertainty is purely idiosyncratic.

**Assumption 1.2.1.** *$f(\cdot)$ is twice continuously differentiable, strictly increasing, concave and satisfies Inada's conditions namely* $\lim_{X\to 0} f'(X) = \infty$ *and* $\lim_{X\to\infty} f'(X) = 0$. $f(0) = 0$

**Assumption 1.2.2.** $\pi^H > \pi^L$, $\gamma^H < \gamma^L$ *but* $\pi^H\gamma^H > \pi^L\gamma^L$

The investment technology of type H second-order stochastically dominates this of type L.[4] According to Assumption 1.2.1, $f(\cdot)$ is a normal decreasing-returns-to-scale production function. Inada's conditions are necessary to guarantee interior solutions. On the other hand, to validate Assumption 1.2.2, one can think that entrepreneurs of type L adopt riskier technologies, but conditional on success, they have higher marginal productivity.

The type of every entrepreneur is his private information. Denote as $\overline{\pi} = \lambda\pi^H + (1-\lambda)\pi^L$ the population's average probability of success. By an appropriate version of the law of large numbers, a set of measure $\overline{\pi}$ will be in the success state, whereas, a set of measure $1-\overline{\pi}$ will be in the failure state. Assume that only the individual state is observable and verifiable by a court of law. That way, entrepreneurs' types cannot be inferred after production has taken place, by observing the level of production. Lastly, all entrepreneurs are risk-neutral and indifferent between consuming in period $t = 0$ and $t = 1$.

**Banks.** There is a finite number of banks in this economy $j = \{1, ..., n\}$, where $n$ is also the number of banks. They own enough endowment of the consumption good in period $t = 0$, and they can lend funds to entrepreneurs.[5] All banks have the same endowment in period $t = 0$, are risk neutral, and indifferent between consuming in period $t = 0$ and $t = 1$. Therefore, the risk-free interest rate is zero.

Contracts take the following form: $(X_i, R_{s,i}, R_{f,i})$, where $X_i$ is the amount of loan, $R_{s,i}$ is the per unit of loan payment in case of success and $R_{f,i}$ is the per unit of loan payment in case of failure. The expected profit of each entrepreneur from such a contract can be written as:

$$V^i(X_i, R_{s,i}, R_{f,i}) = \pi_i(\gamma_i f(X_i) - R_{s,i}X_i) - (1-\pi_i)R_{f,i}X_i,$$

**Credit Market.** The market is formulated as a signaling game. All contracts are available from all banks, and each entrepreneur applies to anyone

[4] All these assumptions are very common in the credit rationing literature.

[5] To be precise, banks own enough endowment to finance all entrepreneurs under complete information.

he wants. Each bank then either accepts the application (in which case the entrepreneur can acquire funds at the predetermined interest rate) or rejects (in which case the entrepreneur who applied for this contract does not acquire any funds and therefore does not invest). Contracts are exclusive so each entrepreneur can acquire financing from only one bank. This is a pure signaling game like the one analysed by Spence [56]. Because signaling games admit too many equilibria, we will only examine equilibria that pass the "intuitive criterion" of Cho and Kreps [11].

## § 1.3 Equilibria without Taxes

**Perfect Information.** As a benchmark case, I characterise the equilibria of the economy under perfect information. The equilibrium contracts are straightforward to be calculated:[6]

$$f'(X_i^{FB}) = \frac{1}{\gamma_i \pi_i} \tag{1.1}$$

$$\pi_i R_{s,i}^{FB} X_i^{FB} + (1-\pi_i) R_{f,i}^{FB} X_i^{FB} = X_i^{FB} \tag{1.2}$$

Note that because of risk neutrality, there is a continuum of Nash equilibria that are all payoff equivalent. Any contract $(X_i^{FB}, R_{s,i}^{BF}, R_{f,i}^{FB})$, with $R_{s,i}^{BF}, R_{f,i}^{FB}$ satisfying (1.2) can be sustained as a Nash equilibrium since at all of them banks make zero profits and there is no other contract that makes positive profits and gives each type higher payoff. Because of the single-crossing property, any equilibrium is fully separating. As expected, entrepreneurs of type H invest more in the production technology and repay less, in expectation, per unit of investment. Given that no type has positive wealth in the state of failure, $R_{f,i}^{FB} \leq 0$, in the sense that entrepreneur of type $i$ receives a transfer from the bank.

**Imperfect Information.** Now consider the case where no outsider, other than each entrepreneur, can observe his true type. The above full-information contracts cannot constitute an equilibrium under incomplete information anymore. This is because entrepreneurs of type L have an incentive to misrepresent their types and also apply for type H's contract which gives them higher profits.[7] But that would violate the zero profit condition for the banks and therefore it cannot constitute an equilibrium.

The set of PBE of the signaling game includes any incentive compatible and positive profit allocation with the requirement that type L's payoff is at least the payoff from $(X_L^{FB}, R_{s,L}^{BF}, R_{f,L}^{FB})$. The only allocation that survives

[6] For a formal proof of existence and uniqueness of equilibrium under complete information see Appendix A.1

[7] See Appendix A for a formal proof.

the intuitive criterion is the "least-costly" allocation; the separating allocation that maximises the payoffs of both types within the set of incentive compatible allocations that make zero profits for all possible beliefs.

Proposition 1.3.1 combines all the above observations and characterises a separating equilibrium of our economy. Note that there exists a continuum of separating equilibria that are all payoff equivalent.[8]

**Proposition 1.3.1.** *The only PBE that passes the intuitive criterion is separating and characterised by a pair of contracts satisfying:*

$$(X^L, R_s^L, R_f^L) = (X_L^{FB}, R_{s,L}^{FB}, R_{f,H}^{FB}), \tag{1.3}$$

*and,*

$$(X^H, R_s^H, R_f^H) = (X_H^{NT}, \frac{1}{\pi_H}, 0), \tag{1.4}$$

*where $X_H^{NT}$ is the smallest root of $\gamma_L f(X_H) - \frac{X_H}{\pi_H} = \gamma_L f(X_L^{FB}) - \frac{X_L^{FB}}{\pi_L}$.*

Proof: See Appendix A.3.

When information is asymmetric, type H is restricted by the incentive constraint. He cannot invest as much as he could in the case of complete information. Actually, given the maintained assumptions, type H invests even less than type L. Because screening must take place through the level of investment, type H's investment must be distorted in such a way that his contract is not anymore desirable by type L.

Some comparative static analysis might be useful to better comprehend Proposition 1.3.1. Intuitively, *ceteris paribus*, an increase (decrease) in $\gamma_L$ would relax (tighten) the incentive constraint of type L, and therefore more (less) investment would flow into the technology of type H.

## § 1.4 Optimal Redistributive Taxes

In this section, I assume that there is a government who can perfectly commit to a redistributive tax system. This tax system is relatively simple and takes the following form: Every entrepreneur, regardless his type, is taxed $t_s$ in case he succeeds, and receives $t_f$ as a subsidy in case he fails, in period $t = 1$. Notably, the government does not know the true type of each entrepreneur but can fully observe the realisation of the individual

[8]As it is proved in Appendix A.3 in any equilibrium, the contract for type H is unique but there are many equilibrium contracts for type L that are all payoff equivalent. Intuitively, this happens because the incentive constraint of type H is not binding and therefore there are many contracts that make zero profits and provide type L with the same payoff.

state in period $t = 1$ as it is the case with banks.[9] It is important to be clear that the government does not impose or suggests any contracts or allocations. Its only tool is the tax system. We will only consider "budget-balanced" redistributive taxes, or: $\overline{\pi}\ t_s = (1-\overline{\pi})\ t_f$. In other words, the government just redistributes the wealth in period $t = 1$ and does not provide any additional resources to entrepreneurs in either period. Moreover, taxes are non-discriminatory (or anonymous) in the sense that all types of entrepreneurs pay the same tax and receive the same subsidy. In fact, the proof of Pareto improving taxes is divided into two steps. In the first step, the impact of taxation in the equilibrium contracts is examined. As it was argued above, even though taxation takes place *ex post* and contracts are signed *ex ante*, any subsidy expected to be paid by the government in period $t = 1$ can now be used in the equilibrium contracts. This is enough to influence the behaviour of all players in the game. Indeed, one can easily show that any subsidy small enough will be used in the equilibrium contract by type H in order to borrow more. This will relax type L's incentive constraint and allow for more investment to flow into type H's production technology. Intuitively, this happens because type H is more willing to give up any subsidy in the failure state in order to decrease his payment in the success state, making, that way, her contract less desirable by type L.

The proof of Pareto improving taxes is divided into two steps. In the first step, the impact of taxation in the equilibrium contracts is examined. As it was argued above, even though taxation takes place *ex post* and contracts are signed *ex ante*, any subsidy expected to be paid by the government in period $t = 1$ can now be used in the equilibrium contracts. This is enough to influence the behaviour of all players in the game. Indeed, one can easily show that any subsidy small enough will be used in the equilibrium contract by type H in order to borrow more. This will relax type L's incentive constraint and allow for more investment to flow into type H's production technology. Intuitively, this happens because type H is more willing to give up any subsidy in the failure state in order to decrease his payment in the success state, making, that way, her contract less desirable by type L.

In the second step, we will express the equilibrium payoffs of both types as a function of the tax. This very fact will allow us to compare the payoffs before and after taxation. Given that $\overline{\pi}\ t_s = (1-\overline{\pi})\ t_f$, it is clear that type L always benefits from the tax system, whereas type H seems to lose at first sight. Nonetheless, given that, as I argued above, type H is able to borrow and invest more using the subsidy in the equilibrium contract, it may be the case that the marginal increase in his production more than offsets what he pays up in taxes and that way he may also be better off. This is, in fact, the main idea driving the result.

[9] If this is not observable then there is no element the contracts can be contingent to and investment is zero.

**Proposition 1.4.1.** *For any tax small enough, the equilibrium contracts are given by:*

$$(X_L, R_{s,L}, R_{f,L}) = (X_L^{FB}, R_{s,L}^{FB}, R_{f,L}^{FB}),$$

$$(X_L, R_{s,L}, R_{f,L}) = (X_L^T, R_{s,L}^T, \frac{t_f}{X_H^T})$$

*with $X_H^T$ and $R_{s,H}^T$ satisfying:*

$$\gamma_L f(X_H^T) - \frac{X_H^T}{\pi_H} = \pi_L\left(\gamma_L f(X_L^{FB}) - \frac{X_L^{FB}}{\pi_L}\right) + \left(\frac{(1-\pi_L)}{\pi_L} - \frac{(1-\pi_H)}{\pi_H}\right)\frac{\overline{\pi}}{1-\overline{\pi}}t_s \tag{1.5}$$

$$\pi_H R_{s,H}^T X_H^T + (1-\pi_H)t_f = X_H^T$$

Proof: See Appendix A.4.

As I mentioned previously, type L applies for his perfect information contract whereas type H applies for a contract with $R_{f,H}X_H = t_f$. This decreases $R_{s,H}$ and increases $X_H$. The fact that $f(\cdot)$ is a concave function is enough to guarantee that an increase in $X_H$ increases the net payoff of type H in the success state. Therefore type H's payoff is higher if he uses the subsidy in his equilibrium contract.

Note now that type L is always better off after the tax-subsidy scheme because he is just cross-subsidised and benefits from the presence of type H. Therefore, we only need to examine whether there is indeed a set of parameters, such that the equilibrium payoff of type H, after taxation, is greater than his equilibrium payoff without taxation. To do so, denote as $X_H(t_s)$ the equilibrium investment level of type H as a function of the tax and note that $X_H(t_s) = X_H^T$. $X_H(t_s)$ is the solution of (1.5) in terms of $X_H$ as a function of the tax $t_s$. As it is shown in Lemma A.5.1 (Appendix A.5), $X_H(t_s)$ is a strictly increasing function in some interval. Define now the following function:

$$G(t_s) = \gamma_H f(X_H(t_s)) - \frac{X_H(t_s)}{\pi_H} + \left(\frac{(1-\pi_H)}{\pi_H}\frac{\overline{\pi}}{1-\overline{\pi}} - 1\right)t_s \tag{1.6}$$

which corresponds to the equilibrium payoff of type H in the success state as a function of the tax. We consider taxes that make the feasibility constraint bind and therefore the equilibrium payoff of type H in the failure state is zero.

Because $f(\cdot)$ is a concave function, it should be expected that $G(\cdot)$ is also concave in $t_s$. In fact, this is the first part proved in Proposition 1.4.2. In the second part, it is proven that there is a positive tax $\tilde{t}_s$ that increases the payoff of type H relative to $t_s = 0$. According to Proposition 1.4.2,there is some threshold in $\overline{\pi}$, denote this as $\overline{\pi}^{min}$, such that, for any $\overline{\pi} \geq \overline{\pi}^{min}$, $G(t_s)$ has always an interior maximiser.

Given the concavity of $G(\cdot)$, there must be a closed interval of taxes that make type H better off relative to the economy without taxes. However, the tax should not be very high to make type H's after taxation less than his payoff before taxation. Denote this upper limit as $\bar{t}_s$. Any positive tax less than $\bar{t}_s$ makes both types better off. Proposition 1.4.2 formalises the behaviour of $G(\cdot)$ and all the above observations.

**Proposition 1.4.2.** *$G(t_s)$ is concave in $t_s$, for any $\overline{\pi} \in [\overline{\pi}^{min}, \pi_H]$, for some $\overline{\pi}^{min} \in [\pi_L, \pi_H]$, and it attains an interior maximum. There is an interval of taxes that, when applied, correspond to a separating equilibrium that is Pareto superior to the no tax equilibrium.*

Proof: See Appendix A.5.

When $\overline{\pi}$ is high enough, type H may benefit from the tax as well, since the increase in his production, using the subsidy in the equilibrium contract, more than offsets what he pays up in the success state. In other words, the use of the tax relaxes type L's incentive constraint and this allows more investment to flow into type H's technology which is enough to make him better off as well. Denote as $A(t_s)$ the equilibrium aggregate investment level as a function of the tax. This is now given by the following formula: $A(t_s) = \lambda X_H(t_s) + (1-\lambda)X_L^{FB}$.

**Corollary 1.4.3.** *$A(t_s)$ is a strictly increasing function in any interval of taxes that increase welfare.*

Perhaps, as we did at the end of Section 1.3, some comparative statics will help us understand better the intuition of this result. Again, *ceteris paribus*, an increase (decrease) in $\gamma^L$ will relax (tighten) the incentive constraint of type L and allow for more (less) investment to flow into type H's technology. This fact will make the use of taxes for Pareto improvements less (more) frequent. $\overline{\pi}^{min}$ will the higher (lower) than before the increase in $\gamma^L$ occurs.[10] In other words, the more (less) diverted are the technologies of the two types, the less (more) frequent is the government intervention for Pareto improvements.

[10]In fact, when $\gamma^L$ surpasses a threshold, $\overline{\pi}^{min}$ will be even higher than $\overline{\pi}^{RS}$, which means that the government cannot improve upon the market allocation when the equilibrium is separating in the reference economy.

## § 1.5 Discussion

1. For simplicity, an elementary two period model with investment, banking and production was analysed. Furthermore, the analysis was restricted to only two types and two individual states. The use of two individual states comes without loss of generality. The argument could go through with more than two states. On the other hand, the restriction to two types is crucial. The analysis with more than two types seems much more complicated.

2. The assumption that entrepreneurs own no initial wealth is not important. The result would remain the same even if entrepreneurs had some initial wealth in either of the two periods. There is always room for Pareto improvements as long as the initial wealth is not enough to give entrepreneurs the luxury to use it and produce at the efficient (perfect information) level. Note however that an interesting further result is that optimal taxes are non-monotonic in the level of initial wealth of entrepreneurs.

3. As it was shown, there is a interval of taxes that increases the welfare of both types in the economy (high- and low -productivity). An interesting question is what is right level of taxation. In fact, this is a question that concerns the policy the government establishes. For instance, in a direct democracy in which entrepreneurs could vote by majority rule about their preferred level of taxation, this would depend upon the population of each type in the economy. From the Median Voter Theorem, high-taxation ($\bar{t}_s$) would be chosen in case low-productivity entrepreneurs formed the majority ($\lambda < 1/2$), whereas moderate taxation ($\tilde{t}_s$) in case high-productivity entrepreneurs formed the majority ($\lambda > 1/2$). See Acemoglu and Robinson (2005) [1].

4. A more realistic model would be one with multiple periods and aggregate uncertainty, where the productivity of entrepreneurs would depend on the state of the world. In that more realistic model the role of government in providing long-term tax plans would become more clear. This is left for future work.

5. A policy implication of this paper could be that in financial markets, where asymmetries of information are pervasive, the government (or the state) must have an active role. Even though, the presence of competitive banks is necessary to allocate resources in the best way, there are still inefficiencies and therefore the government should intervene and redistribute the wealth using the tax system. As it was shown, the use of the tax system could be interpreted as a “stimulus pack” that is necessary to correct many of the imperfections in the market and result in efficient allocation of

resources.

# - Chapter 2 -

## Strategic Foundations for Efficient Competitive Markets with Adverse Selection

### § 2.1 Introduction

One of the main open questions in information economics regards the "*right*" game-theoretic modelling of efficient competition in markets with asymmetric information. Early contributions in this literature[1] highlighted that in this type of markets, the "*usual modeling*" of price or/and quantity competition is not sufficient; equilibria may not exist or may be Pareto dominated. The present paper builds on the seminal contributions of Myerson [46] and Maskin and Tirole [43] to construct a novel noncooperative game that possesses all the nice properties of a competitive market: (i) Equilibria always exist, (ii) Competitors earn zero expected profits in equilibrium, and (iii) All equilibrium allocations are interim incentive efficient, in the sense of Holmstrom and Myerson [30].

To formalise this argument, I employ a generalisation of the standard insurance market with adverse selection analysed in Rothschild and Stiglitz [55] and Wilson [59]. A risk-averse, privately informed buyer with a random endowment seeks for insurance, which is supplied in the market by uninformed, risk-neutral sellers. The market takes the following simple extensive form: Initially, the buyer proposes a set of contracts to all sellers, who accept or reject. If all sellers reject, there is no trade. Otherwise, all those sellers who accepted have the right to add a new set of contracts in the already existing offer. A menu of contracts between the buyer and seller X is defined as the union of the set of contracts proposed by both parties. After the buyer observes all the menus of contracts that have been formed

[1] The most important contributions in this literature Spence [56] and Rothschild and Stiglitz [55].

in the market, he[2] can choose any of the available contracts contained in any of the menus. He is restricted to contract with only one seller- i.e. contracts are exclusive.

I characterise the set of perfect Bayesian equilibria (PBE) of this game. The well-known Rothschild-Stiglitz-Wilson (or RSW) allocation plays a crucial role in the analysis. It is defined as the allocation that maximises the payoff of each type of the buyer within the set of incentive compatible allocations that make positive profits, irrespective of the beliefs of the sellers. The main results of the paper are the following: First, the RSW allocation is the unique equilibrium allocation when it is contained in the set of interim incentive efficient allocations.[3] Second, any interim incentive efficient allocation that weakly Pareto dominates the RSW allocation is an equilibrium allocation. Last, only interim incentive efficient allocations can be sustained as equilibrium allocations in pure strategies.[4]

The three main elements in the game that drive the result are the following: (i) The stage in which the informed buyer proposes a set of contracts. (ii) The stage in which the competing uninformed sellers have the right to add new sets of contracts. (iii) The fact that every menu of contracts is the union of the set of contracts proposed by both parties, and specifically it must contain the set of contracts proposed by the buyer as an option.

To begin with, following Maskin and Tirole [43], by allowing the informed party to propose a set of contracts in the first stage, he can always guarantee his RSW allocation in any equilibrium. In fact, the RSW allocation is the unique equilibrium allocation, or a *"strong solution"* in the sense of Myerson [46], when it is interim incentive efficient. Moreover, this stage along with the fact that every menu of contracts must contain the set of contracts proposed by both the buyer and any seller is important for the existence of equilibria. I show that any interim incentive efficient allocation that Pareto dominates the RSW allocation is an equilibrium allocation and corresponds to a *"neutral optimum"* in the sense of Myerson [46].[5] The intuition behind this result is as follows: Assume that in equilibrium, every type proposes the same set of contracts that consists of an interim incentive efficient allocation, and contracts with seller X. In case some other seller Y proposes any other set of contracts, then according to the equilibrium strategies, all types contract with seller Y. Note that all types are as well off as they are by contracting with seller X, because every menu of contracts

[2] I will use masculine pronouns (he or him) for the buyer and feminine pronouns (she or her) for the sellers.

[3] Unless otherwise stated, interim incentive efficiency is defined with respect to the prior beliefs of the sellers about the type of the buyer.

[4] "Pure strategies" are strategies where the buyer and sellers do not randomise over their proposals.

[5] The RSW allocation is itself a neutral optimum if it is contained in the set of interim efficient allocations.

must include the set of contracts proposed by the buyer. In this case, given that the proposal made by the buyer was interim incentive efficient, any possible *"cream-skimming"* offer attracts all types and must therefore be loss-making. For an appropriate set of off-the-equilibrium path beliefs, no type has an incentive to deviate either and therefore any interim incentive efficient allocation that strictly Pareto dominates the RSW allocation can always be sustained in equilibrium.

Lastly, the stage in which the uninformed sellers can add contracts in the already existing set of contracts proposed by the buyer is used to exploit Bertrand-type competition among sellers and eliminate profits and allocations that are not incentive efficient. Indeed, as I show, competition induces any equilibrium allocation to be interim incentive efficient.

**Related Literature.** My work is related to several strands in the literature. To begin with, the seminal paper on competitive screening markets with adverse selection is Rothschild and Stiglitz [55]. They analyse an insurance market with adverse selection and show that for some parameter values a "competitive" equilibrium fails to exist.[6] Wilson [59] and Riley [53] place restrictions on the set of possible contracts insurance firms can offer and show that an equilibrium always exists. Miyazaki [45] extends the idea of Wilson [59] to a model where insurance firms can offer menus of contracts (instead of single contracts) and proves that an equilibrium always exists and the equilibrium allocation is always constrained efficient. From those two authors, this allocation is often called the Miyazaki-Wilson (or MW) allocation. Hellwig [29] provides a game-theoretic foundation for the idea of Wilson [59]. Along with the equilibrium allocation of Wilson [59], he shows that there is a continuum of other equilibrium allocations.[7] Engers and Fernadez [20] also propose a game with an infinite number of moves to provide foundations for Riley's equilibrium allocation. Similarly to Hellwig [29], a continuum of other allocations can be supported as equilbria. Another strand in the literature that analyses the existence of mixed strategy equilibria in the elementary Bertrand game of Rothschild and Stiglitz [55] is Rosenthal and Weiss [54], and Dasgupta and Maskin [14, 15].

Netzer and Scheuer [47] propose the following three-stage game: In the first stage, insurance firms offer menus of contracts. In the second stage, each firm decides either to stay in the market, or become inactive, in which case it must pay an exogenously-given withdrawal cost. In the last stage, insurees select from the set of contracts offered by all active firms. Depending on

[6] Rothschild and Stiglitz's [55] definition of competition was vague and it was heavily criticised. In fact, most of the early contributions in this literature were towards defining the right notion of competition in screening markets with adverse selection.

[7] The set of equilibria of Hellwig's game coincides with the set of equilibria of a signaling game in which the informed party moves first and can propose a unique contract that is accepted or rejected by some firm. See also Maskin and Tirole[43] pp. 30.

the value of the withdrawal cost, there may be an equilibrium, where the equilibrium allocation coincides with the MW, or not. Mimra and Wambach [44] examine a game in which in the first stage insurance firms offer menus of contracts and there is an infinite number of rounds in the second stage in which each firm can withdraw contracts out of those it has proposed. Insurees choose from the set of contracts that have not been withdrawn after the end of this process. Without further restrictions, the equilibrium set of this game contains every incentive compatible and positive profit allocation. If, however, there are firms ready to enter the market after all incumbent firms have made their moves, the equilibrium allocation coincides with MW. Diasakos and Koufopoulos [18] adopt a similar approach to Hellwig [29] but introduce endogenous commitment in the first stage. It is claimed that the MW allocation is the unique equilibrium of the game. However, neither the action/strategy space nor the contract space are well-defined in this paper.

Asheim and Nilssen [3], Faynzilberg [21] and Picard [48] also examine different games and show that the equilibrium allocation coincides with MW. For instance, in Asheim and Nilssen [3] it is possible for insurance firms to renegotiate the contracts they have signed with their customers, imposing the constraint that they can not discriminate among the different types in the renegotiation stage. Faynzilberg [21] examines a model in which insurance firms can become insolvent, which introduces an externality between agents in a contract. Picard [48] examines a similar externality model in which insurance firms can offer "participating contracts" such that any insuree who signs a contract needs to "participate" in the profits of the firm who offered it. It seems that these models significantly depart from the original formulation of Rothschild and Stiglitz [55] by imposing hardly justifiable theoretical assumptions. Moreover, apart from Picard [48], all the above papers analyse the case of two types. In this paper, I analyse a more general environment with any finite number of types and states of nature, and the assumptions imposed are easier to be justified. Moreover, the equilibrium set of this paper significantly differs from the equilibrium set of all the aforementioned papers.

The seminal papers on informed principal models are Spence [56], Myerson [46] and Maskin and Tirole [43]. Myerson [46] examines a general environment in which a principal with private information designs a mechanism to coordinate his subordinates. The focus of the paper is on the development of a theory of inscrutable mechanism selection for the principal, and what axioms desirable mechanisms must satisfy. The principal's neutral optima are defined as the smallest possible set of "*unblocked*" mechanisms. Spence [56] examines a labour market in which workers can acquire costly education before applying for jobs. He shows that a continuum of equilibria exist most of them Pareto dominated. This multiplicity of equilibria gave rise to an extensive literature examining possible equilibrium refinements. These refinements tried to put restrictions on the off-the-equilibrium path beliefs that are used to support undesirable equilibria. Among others, the most i well-known refinements are Kohlberg and Mertens [37], Cho and Kreps [11] and Banks and Sobel [4]. Maskin and Tirole [43] analyse an informed principal environment with an extended set of contracts (or mechanisms). They consider a three stage game (proposal- acceptance/rejection- execution) and, similarly to this paper, they show that in any equilibrium of the game, the informed principal can guarantee his RSW allocation in contrast to Spence [56]. Naturally, a wealth of other allocations that weakly Pareto dominate the RSW contract can be supported in equilibrium for some set of beliefs.

Lastly, this paper is also related to the literature in general equilibrium with adverse selection starting from Prescott and Townsend [50, 49]. Gale [23, 24, 25], and Dubey and Geanakoplos [19] explore different notions of competition to prove existence of equilibria, and propose refinements of beliefs to pin down the equilibrium set. Bisin and Gottardi [9] also analyse a Walrasian market with adverse selection and introduce markets for property rights that agents can trade. They show that property rights help the implementation of constrained efficient allocations. Citanna and Siconolfi [12] also provide a different notion of competition and prove, under mild restrictions, that for any finite number of types an equilibrium always exists and it is constrained efficient. In fact, their analysis is closely related to my paper, even though it is more general and it is applied to a Walrasian market.

In Section 2.2, I present the model. In Section 2.3, I provide some preliminary properties of incentive compatible and interim incentive efficient allocations that prove to be important for the analysis. I also provide an algorithm to characterise the RSW allocation which plays an important role in this paper. In Section 2.4, I define the game- i.e. market structure, contract space and strategies of the players. Moreover, I give a definition of a perfect Bayesian equilibrium. In Section 2.5, the main results of the paper are presented. Lastly, in Section 2.6 some extensions of the model are suggested.

## § 2.2 The Model

There is a risk-averse buyer with a finite number of possible types $t = 1, ..., T$. There is a finite number of possible states $\omega = 0, 1, ..., \Omega$. The endowment of the buyer is risky and is denoted by $e = (W - d_0, W - d_1, ..., W - d_\Omega)$, where $d_0 = 0$ and $d_\omega > 0$, for any $\omega \geq 1$. For simplicity, I assume that the endowment is type-invariant. Type $t$'s objective probability distribution over the states is denoted by $\theta^t = (\theta^t_0, ..., \theta^t_\Omega)$. The type of the buyer is his private information. Assume that $\sum_{\omega=0}^{\Omega} \theta^t_\omega d_\omega < \sum_{\omega=0}^{\Omega} \theta^{t'}_\omega d_\omega$ for any $t > t'$; the expected endowment is increasing in the index of types. The prior beliefs about the type of the buyer are $\lambda_0^{\cdot} = \{\lambda_0^t\}_{t=1}^T$, with $\sum_{t=1}^T \lambda_0^t = 1$. Furthermore, I assume that the state of nature is perfectly observable and verifiable by a court of law. This is the minimum requirement for contracts to be enforceable. The von Neuman-Morgenstern utility index of all types is state- and type- independent and is represented by $u : X \to \mathbb{R}$, where $u$ is continuous, strictly increasing and strictly concave.

Sellers are denoted by $i \in N$, where $N \geq 2$ is also the number of sellers in the market. They are all risk-neutral, expected utility maximisers and they have enough wealth in order to provide insurance to the buyer if he wishes so. The number of sellers must be at least two so there is competition in the market.[8] Denote as $V^i$ the expected utility of seller $i$.

An insurance contract is denoted by $\psi = (p, b_1, ..., b_\Omega) \in \mathbb{R}_+^{\Omega+1}$ with $p$ denoting the premium paid and $b_\omega$ the benefit received by the buyer in state $\omega$. The space of *feasible insurance contracts* is given by $\Psi = \{(p, b_1, ..., b_\Omega) : 0 \leq p \leq \min\{W, W - d_\omega + b_\omega\}, b_\omega \geq 0, p - b_\omega \leq \bar{A} \text{ for all } \omega = 0, ...\Omega\}$, where $\bar{A}$ is an arbitrarily large constant representing the wealth of sellers in every state. $\Psi$ is a compact set. The expected utility of type $t$ from insurance contract $\psi$ is given by: $U^t(\psi) = \sum_{\omega=0}^{\Omega} \theta_\omega^t u(W - d_\omega - p + b_\omega)$. Denote the null contract by $\psi_o = (0, ..., 0)$ and the status quo utility of type $t$ as: $\underline{U}^t = \sum_{\omega=0}^{\Omega} \theta_\omega^t u(W - d_\omega)$. The net expected profit (cost) of insurance contract $\psi$ when taken up by type $t$ is given by $\pi^t(\psi) = p - \sum_{\omega=1}^{\Omega} \theta_\omega^t b_\omega$.

Sorting Assumption: Whenever $\psi$, $\psi'$ are such that $U^t(\psi) \geq U^t(\psi')$ and $U^{t+1}(\psi') > U^{t+1}(\psi)$, then $U^{t+h}(\psi') > U^{t+h}(\psi)$ and $U^{t-h}(\psi) > U^{t-h}(\psi')$ for any $h \geq 1$.

In words, sorting says that for any two contracts $\psi$ and $\psi'$, if some type $t$ weakly prefers $\psi$ to $\psi'$ and the immediate successor type $t+1$ strictly prefers $\psi'$ to $\psi$, then all types lower in the rank from type $t$ strictly prefer $\psi$ to $\psi'$ and those types higher in the rank from type $t+1$ strictly prefer $\psi'$ and $\psi$. Note that when there are only two states of nature $\Omega = 1$, this condition is vacuously satisfied.

A special case of the model when $\Omega = 1$ is known in the literature of competitive insurance markets with adverse selection as *Wilson's model*, from WIlson [59]. In this model, there are only two states $\omega = 0, 1$ which can be interpreted as no accident and accident respectively. A graphical illustration of this model is provided in Figure 2.1. In this figure, the benefit

[8] The assumption of one buyer and many sellers is without loss of generality. Usually in this type of models, it is assumed that there is a continuum of informed parties (buyers). The reason I chose to analyse a single buyer model is that it greatly simplifies the notation and analysis and it is closer to the contract theory framework. The results would remain the same even if we assumed that there was a continuum of buyers and no aggregate uncertainty. In this model, insurance would be provided by sellers who could freely start operating insurance firms.

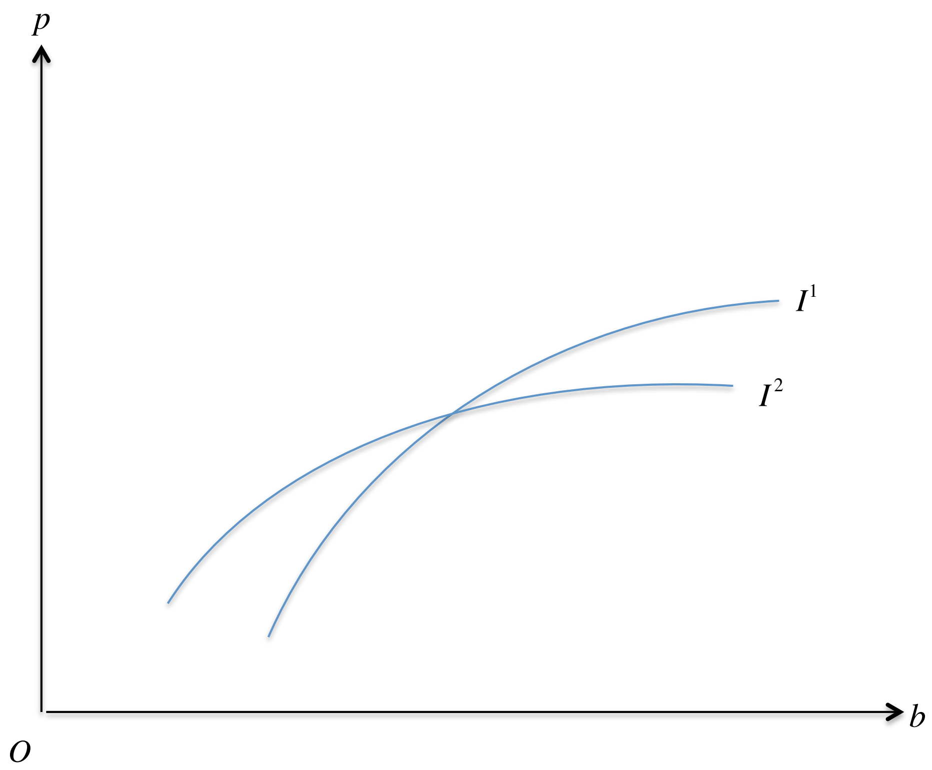


**Figure 2.1:** *A graphical illustration of Wilson's model for $T = 2$.*

is represented on the horizontal axis and the premium on the vertical axis. The indifference curve of type $t$ is labeled by $I^t$. In the figure, there are only two possible types. Although not depicted, the profit functions are upward straight lines. The zero-profit line for type $t$ is the line with slope $\theta_1^t$ passing through the origin. Because of the single-crossing property, the indifference curve of type $t$ is always steeper than the one of type $t+1$ for any $b$. Utility increases as we move south-east and profit increases as we move north-west for all types. We can easily establish that all indifference curves are tangential to the zero-profit lines at the same point $b_1^t = d_1$ (the point of full insurance), for all $t$ with a higher premium for the high-risk types.[9] This is because the utility indexes are type- and state- independent.

Definition: An allocation $\psi^{\cdot} = \{\psi^t\}_{t=1}^{T}$ is a set of contracts one for each type of the buyer. Denote by $\mathbf{\Psi} \subseteq \Psi_+^{T\times(\Omega+1)}$ the set of all feasible allocations.

Definition: $\Pi(\psi^{\cdot}) = \sum_{t=1}^{T} \lambda_0^t \pi^t(\psi^t)$ denotes the expected profit of al-

[9] For a detailed exposition of Wilson's model see Jehle and Reny [34].

location $\psi^{\cdot}$.

Definition: An allocation $\psi^{\cdot} = \{\psi^t\}_{t=1}^T$ is *incentive compatible* (IC) if and only if: $U^t(\psi^t) \geq U^t(\psi^{t'}), \forall \; t, t'$. Denote by $\mathbf{\Psi}^{IC} \subseteq \mathbf{\Psi}$ the set of all incentive compatible allocations.

Definition: An allocation $\psi^{\cdot} = \{\psi^t\}_{t=1}^T$ is *interim individually rational* (IR) if and only if: (i) $U^t(\psi^t) \geq \underline{U}^t, \quad \forall \quad t = 1, ..., T$, and, (ii) $\Pi(\psi^{\cdot}) \geq 0$. Denote the set of all incentive compatible and interim individually rational allocations as $\mathbf{\Psi}^{ICR} \subseteq \mathbf{\Psi}^{IC}$.

Definition: An alocation $\psi^{\cdot} = \{\psi^t\}_{t=1}^T \in \mathbf{\Psi}^{ICR}$ weakly Pareto dominates allocation $\hat{\psi}^{\cdot} = \{\hat{\psi}^t\}_{t=1}^T \in \mathbf{\Psi}^{ICR}$ if and only $U^t(\psi^t) \geq U^t(\hat{\psi}^t)$ for all $t = 1, ..., T$ with the inequality being strict for at least one $t$. Strict Pareto dominance is defined by taking all inequalities to be strict.

Definition: An allocation $\psi^{\cdot} = \{\psi^t\}_{t=1}^T$ is a weak (strong) interim incentive efficient (IE) allocation, in the sense of Holmstrom and Myerson [30], if and only if $\psi^{\cdot} \in \mathbf{\Psi}^{ICR}$, and there exists no other $\tilde{\psi}^{\cdot} = \{\tilde{\psi}^t\}_{t=1}^T \in \mathbf{\Psi}^{ICR}$ that strictly (weakly) Pareto dominates $\psi^{\cdot}$. Denote by $\mathbf{\Psi}^{WIE}$ the set of weak incentive efficient allocations and $\mathbf{\Psi}^{SIE}$ the set of strong incentive efficient allocations.

Note that interim efficiency is defined with respect to the buyer's payoff and the prior beliefs of the sellers. In fact, an incentive efficient allocation maximises the welfare of all types of the buyer guaranteeing at the same time that no seller is worse off than not participating in any transaction with the buyer. The efficiency concept demands only that allocations are individually rational on average and not necessarily ex post individually rational. Evidently, $\mathbf{\Psi}^{SIE} \subseteq \mathbf{\Psi}^{WIE} \subseteq \mathbf{\Psi}^{ICR}$.

## § 2.3 Some Preliminary Properties of Incentive Compatible Allocations

In this section, I provide some preliminary properties of the set of incentive compatible allocations that will prove useful in the analysis. Note that a special case of the model is the Rothschild and Stiglitz [55] model, where $T = 2$ and $\Omega = 1$. Type 1 is the high-risk type and type 2 is the low-risk type. To better illustrate the argument, I will sometimes employ this special case. Figure 2.2 provides a graphical illustration of the Rothschild and Stiglitz [55] model. In the same figure, we can also see the well-known Rothschild and Stiglitz [55] allocation, often called the Rothschild and Stiglitz

[55] equilibrium.

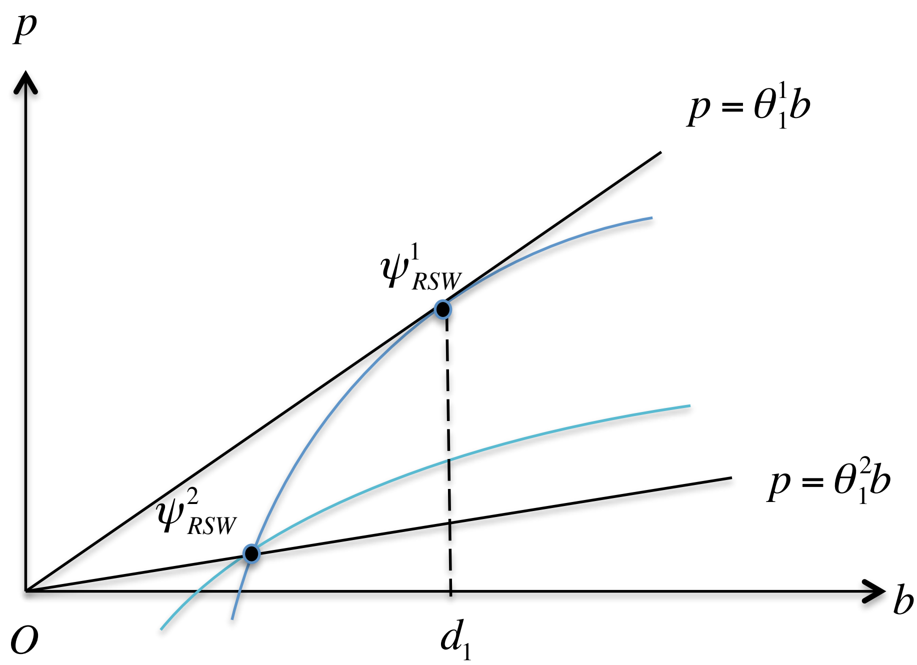


**Figure 2.2:** *The RSW allocation for the special case of $T = 2$ and $\Omega = 1$.*

Following Maskin and Tirole [43], the generalisation of the Rothschild and Stiglitz [55] allocation will be called from now on the RSW allocation; an acronym for Rothschild-Stiglitz-Wilson.[10]

Definition: The RSW allocation is denoted by $\psi_{RSW} = \{\psi^t_{RSW}\}_{t=1}^T$ and can be derived by solving the following recursive program:

$$\text{Program R(1):} \quad \max_{\psi^1} \quad U^1(\psi^1) \quad \text{subject to}$$

$$\pi^1(\psi^1) \geq 0$$

and for every $t = 2, ..., T$:

$$\text{Program R(t):} \quad \max_{\psi^t} \quad U^t(\psi^t) \quad \text{subject to}$$

$$U^{t-1}(\psi^{t-1}_{RSW}) \geq U^{t-1}(\psi^t)$$
$$\pi^t(\psi^t) \geq 0$$

For each $t = 1, ..., T$, the constraint set of Program R(t) is a closed subset of a compact set and therefore is compact. Therefore a solution

[10]See also Maskin and Tirole [43] for more on RSW allocations.

always exists. It is easy to show that, with strictly increasing utility indexes, for each $t = 1, ..., T$ all constraints are satisfied with equality. In fact, for $t = 1$, $\psi^1_{RSW} = (\sum_{\omega=1}^{\Omega} \theta^t_\omega d_\omega, d_1, ..., d_\Omega)$; the lowest in the rank type's RSW contract coincides with his "perfect-information" contract. For each type $t = 2, ..., T$, $\psi^t_{RSW}$ provides less than full insurance and moreover, $U^t(\psi^t_{RSW}) > U^t(\psi^{t-1}_{RSW})$. Note that because of the sorting assumption, we can neglect global incentive constraints and solve each program using the incentive constraint for the upward-adjacent type. Because of strict concavity of the utility index, $U^t(\psi^t_{RSW}) > \underline{U}^t$

The RSW allocation plays a significant role in this paper as well as in any competitive market with adverse selection. This is because it is the only incentive compatible allocation that maximises the utility of all types and it is also ex post individually rational. In the spirit of Myerson [46], the RSW allocation is a "*safe*" or incentive compatible "*type-by-type*" allocation (or mechanism). A safe mechanism is one which would be incentive compatible even if the sellers knew the type of the buyer.

The following lemma is a preliminary result which will be extensively used in all the lemmas that follow:

**Lemma 2.3.1.** *For every $\psi^\cdot, \tilde{\psi}^\cdot \in \Psi^{IC}$, such that $\tilde{\psi}$ strictly Pareto dominates $\psi^\cdot$, there exist $0 < \epsilon < 1$ and $\bar{\psi}^\cdot \in \Psi^{IC}$ that also strictly Pareto dominates $\psi^\cdot$ and $\Pi(\bar{\psi}^\cdot) > \epsilon\Pi(\psi^\cdot) + (1-\epsilon)\Pi(\tilde{\psi}^\cdot)$.*

Proof: Take $\psi^\cdot, \tilde{\psi}^\cdot \in \Psi^{IC}$, such that $\tilde{\psi}$ strictly Pareto dominates $\psi^\cdot$. Consider the following random allocation: Every type $t$ is offered a contract that after the realisation of the state of nature $\omega$, there is a lottery which with probability $\epsilon$ pays $-p^t + b^t_\omega$ and with probability $1-\epsilon$, $-\tilde{p}^t + \tilde{b}^t_\omega$. The expected utility of type $t$ from this random contract can be written as: $U^t = \sum_{\omega=1}^{\Omega} \theta^t_\omega[\epsilon u(W - d_\omega - p^t + b^t_\omega) + (1-\epsilon)u(W - d_\omega - \tilde{p}^t + \tilde{b}^t_\omega)]$. For every $0 < \epsilon < 1$ and every $\omega$ we can find $-\bar{p}^t + \bar{b}^t_\omega$ (the certainty equivalent) such that

$$\epsilon u(W - d_\omega - p^t + b^t_\omega) + (1-\epsilon)u(W - d_\omega - \tilde{p}^t + \tilde{b}^t_\omega) = u(W - d_\omega - \bar{p}^t + \bar{b}^t_\omega)$$

Because of the strict concavity of the utility function and by Jensen's inequality,

$$W - d_\omega - \bar{p}^t + \bar{b}^t_\omega < \epsilon(W - d_\omega - p^t + b^t_\omega) + (1-\epsilon)(W - d_\omega - \tilde{p}^t + \tilde{b}^t_\omega)$$

or

$$\bar{p}^t - \bar{b}^t_\omega > \epsilon(p^t - b^t_\omega) + (1-\epsilon)(\tilde{p}^t - \tilde{b}^t_\omega)$$

Therefore,

$$\pi^t(\bar{\psi}^t) \equiv \bar{p}^t - \sum_{\omega=1}^{\Omega} \theta^t_\omega \bar{b}^t_\omega > \epsilon(p^t - \sum_{\omega=1}^{\Omega} \theta^t_\omega b^t_\omega) + (1-\epsilon)(\tilde{p}^t - \sum_{\omega=1}^{\Omega} \theta^t_\omega \tilde{b}^t_\omega) \equiv \epsilon\pi^t(\psi^t) + (1-\epsilon)\pi^t(\tilde{\psi}^t)$$

Summing up over $t$:

$$\sum_{t=1}^{T} \lambda_0^t \pi^t(\bar{\psi}^t) > \epsilon \sum_{t=1}^{T} \lambda_0^t \pi^t(\psi^t) + (1-\epsilon) \sum_{t=1}^{T} \lambda_0^t \pi^t(\tilde{\psi}^t)$$

or

$$\Pi(\bar{\psi}^{\cdot}) > \epsilon \Pi(\psi^{\cdot}) + (1-\epsilon)\Pi(\tilde{\psi}^{\cdot})$$

Since $\psi^{\cdot}, \tilde{\psi}^{\cdot} \in \mathbf{\Psi}^{IC}$, for any $\epsilon$, the random allocation $(\epsilon \otimes \psi^{\cdot}, 1-\epsilon \otimes \tilde{\psi}^{\cdot})$ is also incentive compatible or $(\epsilon \otimes \psi^{\cdot}, 1-\epsilon \otimes \tilde{\psi}^{\cdot}) \in \mathbf{\Psi}^{IC}$. This necessarily means that $\bar{\psi}^{\cdot} \in \mathbf{\Psi}^{IC}$. Moreover, for any $0 < \epsilon < 1$, $U^t(\psi^t) < U^t(\bar{\psi}^t) < U^t(\tilde{\psi}^t)$, therefore $\bar{\psi}^{\cdot}$ strictly Pareto dominates $\psi^{\cdot}$. Q.E.D.

Many of the proofs will be based on the following important property of IC allocations:

**Lemma 2.3.2.** *For every $\psi^{\cdot} \in \mathbf{\Psi}^{ICR}$ and $\delta > 0$ with $\Pi(\psi^{\cdot}) > 0$, there exists $\bar{\psi}^{\cdot} \in \mathbf{\Psi}^{ICR}$ that strictly Pareto dominates $\psi^{\cdot}$ and $\Pi(\bar{\psi}^{\cdot}) > \Pi(\psi^{\cdot}) - \delta$.*

Proof: Take allocation $\psi^{\cdot} \in \mathbf{\Psi}^{ICR}$ with $\Pi(\psi^{\cdot}) > 0$. Consider the following complete-risk-pooling allocation $\tilde{\psi}^{\cdot}$, where $\tilde{\psi}^t = (\tilde{p}, d_1, ..., d_\Omega)$ for each $t = 1, ..., T$ and $\tilde{p} < \sum_{t=1}^{T} \lambda_0^t \sum_{\omega=1}^{\Omega} \theta_\omega^t d_\omega$. From Lemma 2.3.1, there exists $0 < \epsilon < 1$ and $\bar{\psi}^{\cdot}$ that strictly Pareto dominates $\psi^{\cdot}$ such that $\Pi(\bar{\psi}^{\cdot}) > \epsilon\Pi(\psi^{\cdot}) + (1-\epsilon)\Pi(\tilde{\psi}^{\cdot})$. For $\delta = (1-\epsilon)[\Pi(\psi^{\cdot}) - \Pi(\tilde{\psi}^{\cdot})]$, $\epsilon$ and $\tilde{p}$ appropriately chosen, we obtain the result. Q.E.D.

If we recall the definition of incentive efficiency, it is not hard to see that given risk-neutrality on behalf of the sellers, and Lemma 2.3.2, every (weak) incentive efficient allocation must be zero-profit.

**Corollary 2.3.3.** *Every $\psi^{\cdot} \in \mathbf{\Psi}^{WIE}$ is such that $\Pi(\psi^{\cdot}) = 0$.*

Another important property of incentive compatible allocations is the following:

**Lemma 2.3.4.** *For every $\psi^{\cdot} \in \mathbf{\Psi}^{ICR}$, with $\Pi(\psi^{\cdot}) = 0$ and $\psi^{\cdot} \notin \mathbf{\Psi}^{SIE}$, there exists $\bar{\psi}^{\cdot} \in \mathbf{\Psi}^{ICR}$ that strictly Pareto dominates $\psi^{\cdot}$ with $\Pi(\bar{\psi}^{\cdot}) > 0$.*

Proof: Case 1. Assume first that $\psi^{\cdot} \notin \mathbf{\Psi}^{WIE}$, with $\Pi(\psi^{\cdot}) = 0$. By definition, there exists $\tilde{\psi}^{\cdot} \in \mathbf{\Psi}^{WIE}$ that strictly Pareto dominates $\psi^{\cdot}$. From Corollary 2.3.3, $\Pi(\tilde{\psi}^{\cdot}) = 0$. From Lemma 2.3.1, there exists $\bar{\psi}^{\cdot} \in \mathbf{\Psi}^{IC}$ that strictly Pareto dominates $\psi^{\cdot}$ with $\Pi(\bar{\psi}^{\cdot}) > \epsilon\Pi(\psi^{\cdot}) + (1-\epsilon)\Pi(\tilde{\psi}^{\cdot}) = 0$.

Case 2. Assume that $\psi^{\cdot} \in \mathbf{\Psi}^{WIE}$ but $\psi^{\cdot} \notin \mathbf{\Psi}^{SIE}$. There exists $\tilde{\psi}^{\cdot} \in \mathbf{\Psi}^{SIE}$ that weakly Pareto dominates $\psi^{\cdot}$. Let the set of types whose utility remains the same in both allocations be $T_1$ and those whose utility is strictly higher under $\tilde{\psi}^{\cdot}$ be $T_2$. By following the same logic as in the proof of Lemma 2.3.1, we can find $\bar{\psi}^{\cdot} \in \mathbf{\Psi}^{IC}$ with $U^t(\bar{\psi}^t) > U^t(\psi^t)$ for all $t \in T_2$ and

$U^t(\bar{\psi}^t) = U^t(\psi^t)$, for all $t \in T_1$. Moreover, $\Pi(\bar{\psi}^{\cdot}) > 0$. From Lemma 2.3.2, for any $\delta > 0$, there exists $\hat{\psi}^{\cdot} \in \mathbf{\Psi}^{IC}$ that strictly Pareto dominates $\bar{\psi}^{\cdot}$ and $\Pi(\tilde{\psi}^{\cdot}) > \Pi(\bar{\psi}^{\cdot}) - \delta \geq 0$, for $\delta$ small enough. Q.E.D.

**Corollary 2.3.5.** *The sets of strict and weak incentive efficient allocations coincide, or:* $\mathbf{\Psi}^{SIE} = \mathbf{\Psi}^{WIE}$.

## § 2.4 The Game

In this section, I describe the interaction of economic agents in the market. I specify a noncooperative game- i.e. a *market mechanism*- in which all economic participants meet and exchange contracts. The most important element is that all agents are allowed to offer any set of contracts they wish.

In the market, Nature moves first and decides the type of the buyer. This is neither observable nor verifiable by any third party. Then, the buyer makes a first proposal to all sellers in the form of a set of a finite number of contracts.[11] Denote the set of contracts proposed by the buyer as $\mu_b^{\cdot}$. After the buyer's proposal, each one of the sellers decides whether to participate in the game or not (accept or reject the proposal). Any seller who decides to participate can propose a new set of a finite number of contracts. Denote the set of contracts proposed by seller $i$ as $\mu_i^{\cdot}$. A menu of contracts between the buyer and some seller $i$ is defined as the union of the proposals of the set of contracts proposed by the buyer and seller $i$. Denote such a menu as $m^i = (\mu_i^{\cdot}, \mu_b^{\cdot})$. The buyer has access to any menu traded in the market. Note that one of the most important features of the game is that every menu of contracts between the buyer and seller $i$ must always includes as options both the buyer's as well as the seller's proposals. If any of the sellers decides not to participate, then she cannot make any proposal and she is excluded from the game irreversibly.

An allocation can be formed by combining contracts from all (or some) menus of contracts in the market or out of a single menu. Evidently, every equilibrium allocation must be incentive compatible. The buyer can sign only one contract with only one seller. This is one of the most common and most important assumptions used in this literature. Relaxing this assumption leads to *common agency* problems.[12] Denote the "winning" seller- i.e. the seller who contracts with the buyer- by $i^*$.

To formally describe the timing of events, the market is formulated as an extensive form game, denoted as $\Gamma^e$. To simplify notation, denote as $a_k^j$, $j = \{N, b, i\}$, $k = 0, 1, 2, 3, 4$, an action from nature, the buyer, or seller $i$ in stage $k$:

[11] For technical reasons any set of contracts can only contain a finite number of contracts. This is without loss of generality.

[12] For more on common agency models, see Martimort [40] and Stole [58].

*Stage 0*: Nature decides the type of the buyer: $a_0^N \in \{1, ..., T\}$.

*Stage 1*: The buyer proposes a set of contracts $a_1^b = \mu_b^{\cdot}$.

*Stage 2*: Each seller $i \in N$ accepts or rejects the proposal of the buyer, $a_2^i = \{1, 0\}$, for all $i \in N$. $\dot{N}$= {set of sellers who accepted}. If $\dot{N} = \{\emptyset\}$, the game ends with payoffs $V^i = 0$, for each $i \in N$ and $\underline{U}^t$ for type $t = 1, ..., T$. If $\dot{N} \neq \{\emptyset\}$, the game moves to Stage 3. For each $i \notin \dot{N}$, $V^i = 0$.

*Stage 3*: Each seller $i \in \dot{N}$ proposes a new set of contracts $a_3^i = \mu_i^{\cdot}$. A menu of contracts between the buyer and some seller $i$ is formed by taking the union of the proposals of the set of contracts proposed by the buyer and seller $i$. Denote such a menu as $m^i = (\mu_i^{\cdot}, \mu_b^{\cdot})$.

*Stage 4*: The buyer signs one of the available contracts $a_4^b \in \cup_{i \in \dot{N}} m^i$. If $a_4^b = \psi^{t'}$, for some $t'$, the game ends with payoffs $U^t(\psi^{t'})$ for the buyer of type $t$, $V^{i^*} = \pi^t(\psi^s)$ for seller $i^*$.

A strategy for the buyer is denoted by $\sigma^b$ and consists of a proposal of a set of contracts $\mu_b^{\cdot}$ in Stage 1 and, for any possible menu of contracts, a choice of a contract proposed by the sellers or a contract out of his proposed set of contracts in Stage 4. A strategy for seller $i$ is denoted by $\sigma^i$ and, for any possible proposal of a set of contracts by the buyer, consists of a decision (accept/ reject) in Stage 2 and, a proposal of a new set of contracts in Stage 3 (conditional on acceptance). A mixed strategy is defined by taking probability measures over the set of pure strategies.

After Stage 1 each seller observes an action from the buyer. This action may disclose some information regarding the buyer's type. Therefore, all sellers revise their beliefs appropriately after the buyer's action. The *common posterior beliefs* of all sellers after Stage 1 are denoted as $\lambda_1^{\cdot} = \{\lambda_1^t\}_{t=1}^T$.

I will be interested in the *perfect Bayesian equilibria* (PBE) of the overall game. Given the dynamic nature of the game and the fact that sellers take actions after observing a move from the buyer, this is a plausible assumption. A perfect Bayesian equilibrium is a vector of strategies for the buyer and all the sellers and a vector of beliefs at each information set such that (i) the strategies of the players are optimal at every node of the game tree (sequential rationality), (ii) interim beliefs about the type of the buyer are the same in nodes where he does not take an action and are derived by Bayes' rule from the strategies of the players (Bayesian updating).

## § 2.5 Equilibria and their Properties

We can now turn our interest to the set PBE of game $\Gamma^e$. In Section 2.3, we characterised properties of allocations with no reference to any game or market mechanism. In fact, the main goal of this paper is to examine the relationship between incentive efficient allocations and the set of equilibrium allocations for game $\Gamma^e$. We will only be interested in the set of pure strategy

PBE, refraining from examining strategies in which agents randomise over pure strategies.

First, note that Game $\Gamma^e$ has a strong signaling feature.[13] This is because the informed party offers a set of contracts which may perhaps reveal some of his information to the uninformed parties. In fact, the game is an enriched "*Informed Principal Game*", in which the buyer is the informed principal and the sellers are the subordinates. The difference from the usual informed principal model of Myerson [46] and Maskin and Tirole [43] is that the subordinates have the right to influence the mechanism with their offers. Another significant observation is that Myerson's [46] *inscrutability principle* holds in game $\Gamma^e$. According to this principle, the informed party (buyer) never needs to disclose his type with his proposal, because he can always build such communication into the process of the mechanism itself. What it is meant by this is that for every possible equilibrium in which there is partial revelation of information- e.g. an equilibrium in which partitions of different types offer different sets of contracts- there is another equilibrium in which all types offer the same set of contracts and the equilibrium payoffs are equivalent.[14] Therefore, there is no loss of generality to concentrate on possible equilibria in which all types offer the same set of contracts and no belief updating takes place in Stage 2 of the game, or: $\lambda_1^{\cdot} = \lambda_0^{\cdot}$. Given this simplifications, the first result of the paper is stated as follows:

**Proposition 2.5.1.** *If $\hat{\psi}^{\cdot}$ is an equilibrium allocation of game $\Gamma^e$, then* $\Pi(\hat{\psi}^{\cdot}) = 0$.

[13] However, in pure signaling games, there exist *pooling* equilibria, in which no information is transmitted from the informed to the uninformed parties, as well as *separating* equilibria in which all relevant information is transmitted. In these games nonetheless, interaction among the uninformed and the informed parties ends after the transmission of this information. Moreover, the mechanisms that are allowed to be traded in these models are very limited- e.g. in Spence [56]; every worker can propose only one contract. Our game differs from these models, because the uninformed parties have the right to make a proposal of contracts to the informed party and the informed party can offer richer mechanisms. Therefore, the set of equilibria has a different structure.

[14] Note that full revelation is never a possible equilibrium scenario. The intuition behind this result is the following: Assume (for simplicity) that there exists an equilibrium in which every different type makes a distinct proposal and his type is fully revealed. It must necessarily be (because of sequential rationality) that the proposal of any type must be utility maximising for this type (or incentive compatible). In other words no type must have an incentive to propose something that some other type proposes in equilibrium. Given that the buyer's type becomes publicly known after his proposal, in the continuation of the game, and because of Bertrand competition, at least one seller must propose to this type a menu that contains as a contract, the contract that this type would get under complete information. Otherwise, the equilibrium fails to be sequentially rational. Note that that with state and type independent utility functions the first-best contract of type $t$ is $(\sum_{\omega=1}^{\Omega} \theta_\omega^t d_\omega, d_1, ..., d_\Omega)$. Every type is perfectly insured. Because by assumption $\sum_{\omega=1}^{\Omega} \theta_\omega^T d_\omega < \sum_{\omega=1}^{\Omega} \theta_\omega^t d_\omega$ for any $t = 1, ..., T-1x$, all types strictly prefer the first-best contract of type $T$ over their contract. Therefore, there is a contradiction with the assumption that the first initial offer was utility maximising for all types.

PROOF: It is trivial to see that there cannot exist a strictly negative profit equilibrium allocation. Assume therefore that there exists an equilibrium allocation $\hat{\psi}^{\cdot}$ such that $\Pi(\hat{\psi}^{\cdot}) > 0$. Assume, for simplicity, that all sellers have accepted the offer of the buyer: $\dot{N} \equiv N$. One can easily prove that there exists at least one seller $i' \in N$ such that $\hat{V}^{i'} < \Pi(\hat{\psi}^{\cdot})$, where $\hat{V}^{i'}$ is the equilibrium payoff of Seller $i'$. From Lemma 2.3.1, for any $\delta > 0$, there exists $\bar{\psi}^{\cdot}$ that strictly Pareto dominates $\hat{\psi}^{\cdot}$ and $\Pi(\bar{\psi}^{\cdot}) > \Pi(\hat{\psi}^{\cdot}) - \delta > 0$. Therefore, there exists $\delta'$ (arbitrarily small) such that $\Pi(\bar{\psi}^{\cdot}) > \Pi(\hat{\psi}^{\cdot}) - \delta' = \hat{V}^{i'}$ which simply means that seller $i'$ can increase his payoff by proposing $\bar{\mu}^{\cdot}_{i'} = \bar{\psi}^{\cdot}$ such that a new menu $\bar{m}^{i'} = (\hat{\mu}^{\cdot}_b, \bar{\mu}^{\cdot}_{i'})$ is formed. In this case, all types must contract with seller $i'$, given that $\bar{\psi}^{\cdot}$ strictly Pareto dominates $\hat{\psi}^{\cdot}$, otherwise the equilibrium fails to be sequentially rational- an immediate contradiction. $i'$ makes strictly higher profits than when offering allocation $\hat{\psi}^{\cdot}$ which contradicts the thesis that $\hat{\psi}^{\cdot}$ is an equilibrium allocation. Q.E.D.

This result highlights the competitive identity of the game. Because of the presence of many competing sellers equilibria are compatible with the zero-profit condition (due to constant-returns-to-scale). The following result is the first of the two main results of the paper:

**Theorem 2.5.2.** *If $\hat{\psi}^{\cdot}$ is an equilibrium allocation of game $\Gamma^e$, then $\hat{\psi}^{\cdot} \in \mathbf{\Psi}^{SIE}$.*

PROOF: Assume $\hat{\psi}^{\cdot} \notin \mathbf{\Psi}^{SIE}$ is an equilibrium allocation. From Proposition 2.5.1, we know that $\Pi(\hat{\psi}^{\cdot}) = 0$. Assume, again for simplicity, that all sellers have accepted the offer of the buyer: $\dot{N} \equiv N$, and all of them make zero profits in this equilibrium. Because $\hat{\psi}^{\cdot} \notin \mathbf{\Psi}^{SIE}$, from Lemma 2.3.4, there exists $\bar{\psi}^{\cdot} \in \mathbf{\Psi}^{ICR}$ that strictly Pareto dominates $\hat{\psi}^{\cdot}$ and $\Pi(\bar{\psi}^{\cdot}) > 0$. Therefore, at least one seller $i'$ can offer allocation $\bar{\psi}^{\cdot}$ and all types must contract with seller $i'$ because of sequential rationality. This contradicts the thesis that all types sign a contract from allocation $\hat{\psi}^{\cdot}$. Therefore, if there exists an equilibrium then the only possible equilibrium allocation must be such that $\hat{\psi}^{\cdot} \in \psi^{SIE}$. Q.E.D.

Not only equilibrium allocations must be zero-profit, but also, from Theorem 2.5.2, there cannot exist even weakly Pareto dominated equilibrium allocations. The stage where sellers can propose contracts is critical for both Proposition 2.5.1 and Theorem 2.5.2; it allows Bertrand-type competition among sellers in order to eliminate any strictly positive profits and to force allocations to be SIE. This seems to be an important departure from all the relevant papers in the literature since in their main parts, they are unable to exclude allocations that are not interim incentive efficient.

The following proposition is the third result concerning the set of equilibrium allocations of game $\Gamma^e$.[15] It places a minimum bound in the equilibrium

[15]Note that the equilibrium allocation may be a combination of contracts from $\mu^{\cdot}_b$ and $\mu^{\cdot}_i$

payoff of each type.

**Proposition 2.5.3.** *If $\hat{\psi}^{\cdot}$ is an equilibrium allocation of game $\Gamma^e$, then $U^t(\hat{\psi}^t) \geq U^t(\psi^t_{RSW})$, for all $t = 1, ..., T$.*

Proof: Assume that there exists an equilibrium in which the equilibrium allocation $\hat{\psi}^{\cdot}$ is such that $U^t(\hat{\psi}^t) < U^t(\psi^t_{RSW})$ for some $t$. From Proposition 2.5.1, we know that $\Pi(\hat{\psi}^{\cdot}) = 0$. Assume that the set of sellers participating is $\dot{N}$. Take $\delta > 0$ small enough. Let the buyer of type $t$ propose the following set of contracts in Stage 1: $\mu^{\cdot}_b = \tilde{\psi}^{\cdot}_b$ such that $\tilde{\psi}^{\cdot}_b$ is the solution of the perturbed recursive program:

$$\text{Program} \quad \tilde{R}(1): \quad \max_{\psi^1} \quad U^1(\psi^1) \quad \text{subject to}$$

$$\pi^1(\psi^1) \geq \delta$$

and for every $t = 2, ..., T$:

$$\text{Program} \quad \tilde{R}(t): \quad \max_{\psi^t} \quad U^t(\psi^t) \quad \text{subject to}$$

$$U^{t-1}(\tilde{\psi}^{t-1}) \geq U^{t-1}(\psi^t) + \delta$$
$$\pi^t(\psi^t) \geq \delta$$

Allocation $\tilde{\psi}^{\cdot}_b$ is strictly incentive compatible- every type strictly prefers his contract over the contract of any other type- and therefore the buyer will choose the right contract out of this allocation in Stage 4. Given that, allocation $\tilde{\psi}^{\cdot}_b$ makes strictly positive profit regardless of the posterior beliefs $\lambda^{\cdot}_0$ of the sellers. Thus, at least one seller $i \in \dot{N}$ has to accept the proposal of the buyer, otherwise the equilibrium fails to be sequentially rational. Therefore, at any continuation of the game, type $t$'s payoff is at least $U^t(\tilde{\psi}^t)$. This contradicts the initial hypothesis that $\hat{\psi}^{\cdot}$ was an equilibrium allocation. Because this is true for any $\delta > 0$, a lower bound in the utility of any type $t$ is $U^t(\psi^t_{RSW})$. Q.E.D.

**Corollary 2.5.4.** *If $\psi^{\cdot}_{RSW} \in \mathbf{\Psi}^{SIE}$, then $\psi^{\cdot}_{RSW}$ is the unique equilibrium allocation.*

Proposition 2.5.3 (and its proof) is similar to Proposition 5 of Maskin and Tirole [43]. There, it is proven that when the informed party is the one who makes the contract offer, he can always guarantee his RSW allocation in any equilibrium provided that the contract space is rich enough.[16] Maskin

for any $i \in \dot{N}$. This is the case for example when the RSW allocation is interim incentive efficient. The equilibrium allocation must always be incentive compatible. There exist equilibria in which some seller serves all types.

[16] By rich enough, Maskin and Tirole [43] consider the space which contains a contract for every type of the seller.

and Tirole [43] examine a general informed principal model as a three-stage noncooperative game (contract proposal- acceptance/rejection- execution). The model in this paper differs in at least two respects from Maskin and Tirole [43]. First, in this paper, there are multiple uninformed parties who compete for the same informed party in exclusive contracts, unlike Maskin and Tirole [43] where there is only one informed and one uninformed party. Moreover, after the buyer's (the informed party's) proposal, the sellers (uninformed parties) have the right to also propose a set of contracts. This dramatically changes the set of equilibria as it is proven in Proposition 2.5.1 and Theorem 2.5.2. In fact, in $\Gamma^e$ only weak incentive efficient allocations can be supported in equilibrium, unlike Maskin and Tirole [43], where any allocation that weakly Pareto dominates the RSW allocation can be supported as an equilibrium allocation for some set of beliefs.

In the spirit of Myerson [46], the RSW allocation is the only "*strong solution*" when it is interim incentive efficient. A strong solution is an allocation that is safe and undominated. When a strong solution exists, then it must always be the equilibrium allocation.

Besides their importance, Proposition 2.5.1 and Theorem 2.5.2 cannot have a bite unless we prove that equilibria exist for all possible parameter values. In Proposition 2.5.3, it is proven that the RSW allocation is an equilibrium allocation if and only if it is incentive efficient relative to the prior beliefs. Note that the same results hold even in the elementary model examined by Rothschild and Stiglitz [55]. However, one of the main difficulties in these environments is that incentive efficiency sometimes requires cross-subsidisation. This simply means that to increase the payoff of some type(s), some contracts must become loss-making- i.e. to violate the ex post individual rationality constraints of the sellers. In fact, that was the initial problem pointed out by Rothschild and Stiglitz [55], who showed that in this case, there are robust regions in which a pure strategy equilibrium fails to exist. According to their definition of competition, some new seller (insurance firm in their jargon) could enter and "*skim the cream*" in the market, by attracting only the low-risk types, creating losses to other active sellers. Therefore the main interest is to examine whether pure strategy equilibria exist even when the RSW is not incentive efficient. Note that, in this range of parameters, Maskin and Tirole [43] find a continuum of equilibrium allocations; some of them strictly Pareto dominated. However, as we showed in Theorem 2.5.2, this is never the case in game $\Gamma^e$; Bertrand-type competition eliminates any strictly positive profits and forces equilibrium allocations to be incentive efficient. It only remains to show that equilibria always exist when the RSW allocation does not belong to the set of incentive efficient allocations.

The key to construct equilibria is to notice that every seller who accepts in Stage 2 of the game, implicitly accepts to offer every contract contained in the set of contracts proposed by the buyer. Noticing so allows us to construct equilibria in which the buyer always offers a stong incentive efficient allocation that strictly Pareto dominates the RSW allocation and some seller accepts. Cream-skimming is not possible in $\Gamma^e$ because there are strategies for the buyer according to which all types contract with any entrant trying to skim the cream, and because of the nature of the set of contracts that have been proposed by the buyer, any entrant makes negative profits. The following theorem is the last main result of this paper.

**Theorem 2.5.5.** *If $\hat{\psi}^{\cdot} \in \mathbf{\Psi}^{SIE}$ and it strictly Pareto dominates $\psi^{\cdot}_{RSW}$ then it is an equilibrium allocation of game $\Gamma^e$.*

PROOF: Consider the following candidate equilibrium strategies: "The buyer, regardless his type, proposes a set contracts $\hat{\mu}^{\cdot}_b$ such that $\hat{\mu}^{\cdot}_b = \hat{\psi}^{\cdot}$, allocation $\hat{\psi}^{\cdot} \in \mathbf{\Psi}^{SIE}$ and strictly Pareto dominates $\psi^{\cdot}_{RSW}$. Agent $i = 1$ accepts the proposal and proposes $\hat{\mu}^{\cdot}_1 = \psi^{\cdot}_o$- i.e. all the contracts being the null contracts. No other seller accepts the proposal. The menu formed between the buyer and seller 1 is denoted by $\hat{m}^1 = (\hat{\mu}^{\cdot}_b, \hat{\mu}^{\cdot}_1)$. If the buyer proposes any different set of contracts, the posterior beliefs are updated to $\tilde{\lambda}^{\cdot}_1$. If any other seller $i' \in N/\{1\}$ decides to enter the market and makes a proposal, then all types contract with seller $i'$."

First, recall that any seller who enters the market has to include in any menu the set of contracts proposed by the buyer. Let us examine first all the possible subgames resulting after the buyer's "equilibrium proposal". If no seller, other than seller 1, enters the market, then given that allocation $\hat{\psi}^{\cdot}$ is incentive compatible, each type $t = 1, ..., T$ gains maximal payoff by sticking to contract $\hat{\psi}^t$ in Stage 4. Because $\hat{\psi}^{\cdot} \in \mathbf{\Psi}^{SIE}$, by Corollary 2.3.3, $\Pi(\hat{\psi}^{\cdot}) = 0$ so seller 1 is indifferent between participating in the game or not. Given the equilibrium strategies, no other seller has an incentive to enter the market and offer a different set of contracts (form a different menu). For assume not. Let seller 2 form a new menu $\tilde{m}^2 = (\hat{\mu}^{\cdot}_b, \tilde{\mu}^{\cdot}_2)$ where $\tilde{\mu}^{\cdot}_2$ contains contracts that are strictly preferred by some types and makes strictly positive profits. Given the equilibrium strategies, all types must contract with seller 2. This is sequentially rational for all types since, by construction of the game, they can always guarantee by any entrant the menu of contracts they have proposed in Stage 1. Because of Lemma 2.3.2, and given that $\hat{\psi}^{\cdot} \in \mathbf{\Psi}^{SIE}$, any allocation that Pareto dominates $\hat{\psi}^{\cdot}$ must make strictly negative expected profits which contradicts the thesis that seller 2 can make strictly positive profits by forming a new menu.

All that is left is to construct appropriate beliefs $\tilde{\lambda}^{\cdot}_1$, and continuation payoffs such that $\tilde{U}^t \leq U^t(\psi^t_{RSW})$ for all $t = 1, ..., T$. Consider the following candidate off-the-equilibrium path beliefs $\tilde{\lambda}^{\cdot}_1 = (1, 0, ...., 0)$. For these posterior beliefs, no set of contracts can be accepted by any of the sellers if it contains an element (contract) $\tilde{\psi}$ such that $U^1(\tilde{\psi}) > U^1(\psi^1_{RSW})$. This is because it must necessarily make negative profits under $\tilde{\lambda}^{\cdot}_1$. From the definition of $\psi^1_{RSW}$ from Program RSW, under the sorting assumption,

$U^t(\psi^t_{RSW}) > U^t(\psi^1_{RSW})$ for any $t = 2, ..., T$. This means that after a deviation by the buyer of any type, at any continuation of the game given beliefs $\tilde{\lambda}^{\cdot}_1$, the maximal payoff any type can approximate is the payoff he could have from $\psi^1_{RSW}$, which by definition is worse than the equilibrium payoff under $\hat{\psi}^{\cdot}$ given that the latter by assumption strictly Pareto dominates $\psi^{\cdot}_{RSW}$. Q.E.D.

An equilibrium always exists because of the nature of the menus of contracts allowed in the market. The key fact is that each menu of contracts must always include the set of contracts the buyer proposed in the first stage of the game. This is enough to create a credible threat for potential entrants who try to skim the cream in the market. Any equilibrium allocation of game $\Gamma^e$ is an *Undominated Mechanism* in the sense of Myerson [46]. No type can ever block such an allocation.

In Figure 2.3, a candidate equilibrium allocation is illustrated for Wilson's model for $\Omega = 1$. This allocation provides full insurance to both types, strictly Pareto dominates the RSW allocation and makes zero-expected profits on average.

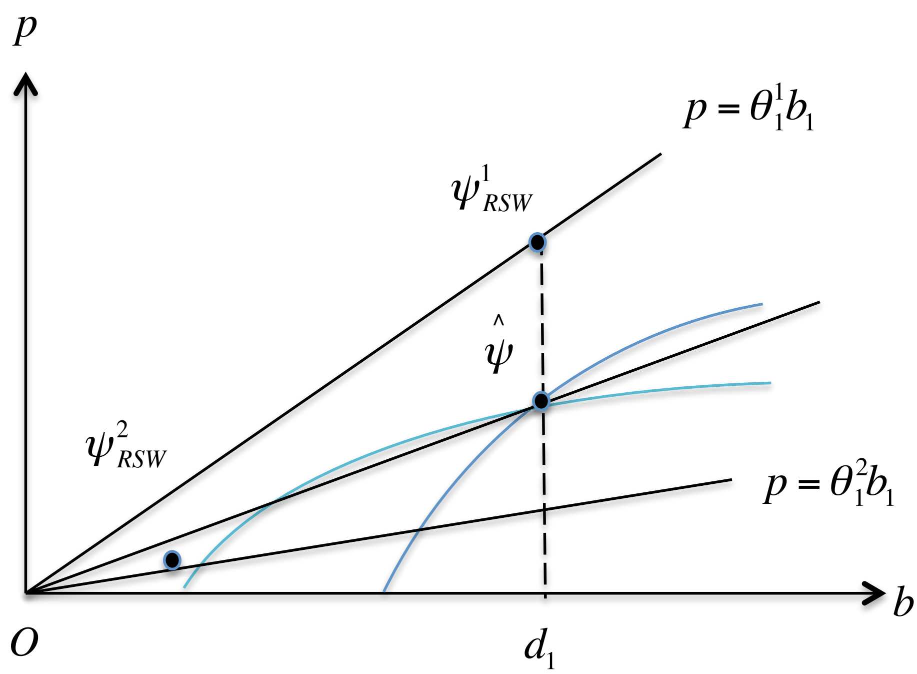


**Figure 2.3:** *A candidate equilibrium for the special case of $T = 2$ and $\Omega = 1$.*

## § 2.6 Extensions

In this section, I show how the results extend to other similar environments with adverse selection.

### 2.6.1 Managerial Compensation

A manager with a finite number of possible types $t = 1, ..., T$ bargains with potential identical employers. The productivity of the manager depends on his type $\theta^t$ with $\theta^t > \theta^{t'}$, for any $t > t'$. A manager of type $t$ has utility function: $u^t(m, q) = y - \frac{1}{\theta^t}c(q)$, where $y$ is money and $q$ is observable output. Assume that $c' > 0$, $c'' > 0$, $\lim_{z\to\infty} c'(z) = 0$, $\lim_{z\to 0} c'(z) = \infty$. All employers have the same linear utility function by employing a manager of type $t$: $\theta^t q - y$. A contract is denoted by $\psi = (y, q) \in X \subset \mathbb{R}^2_+$. Everything else is defined similarly to the insurance model.

### 2.6.2 Credit Markets

There are two periods and one consumption-investment good which is perishable.[17] An entrepreneur has a finite number of possible types: $t = 1, ..., T$. His initial wealth is zero but he owns a stochastic investment technology which take the following form: By investing $X$ units of the consumption good in period zero an entrepreneur of type $t$ can realize $\gamma^t f(X)$ units in period one with probabiliy $\theta^t$ and zero otherwise. Assume that$f(\cdot)$ is twice differentiable, strictly increasing, concave and satisfies Inada conditions namely $\lim_{z\to 0} f'(z) = \infty$ and $\lim_{z\to\infty} f'(z) = 0$. $f(0) = 0$.

Types are ranked in the following way: $\theta^T > ... > \theta^1$, $\gamma^T < .... < \gamma^1$ but $\theta^T\gamma^T > ... > \theta^1\gamma^1$.

Assume that only the state of nature is observable by any outsider and verifiable by a court of law. That way, entrepreneur's type is not observable, before or after production has taken place, by either the banks or the government. Lastly, all entrepreneurs are risk neutral and indifferent between consuming in period any period.

Bankers are risk-neutral and they have enough wealth to finance the entrepreneur. A loan contract takes the following form: $(X, R_s, R_f)$, where $X$ is the amount of loan, $R_s$ is the per unit of loan payment in case of success and $R_f$ is the per unit of loan payment in case of failure. The rest is similar to the basic insurance model.

[17]The economy is similar to that of Martin (2009) [41] with the difference that entrepreneurs do not possess any initial wealth at the time of contracting. This is only for simplicity and without loss of generality.

### 2.6.3 The Informed Seller Model

There is one informed seller endowed with one object. There is a large number of potential buyers who are interested in acquiring the object at some price. Assume for simplicity that there are no financial constraints. The seller has a finite number of types $t = 1, ..., T$. In addition, he can invest in some technology in order to improve the quality of the object if he wishes so. Investment is perfectly observable and verifiable and therefore it can be used as a contractible variable along with the selling price. The preferences of the seller of type $t$ are represented by: $U^t(p, y)$, where $p$ is money (the selling price) and $y$ investment. $U^t$ is strictly increasing in $p$ and strictly decreasing in $y$. Moreover, assume that the sorting assumption holds. Buyers have linear preferences represented by $V^t(p, y) = \theta^t y - p$. $\theta^t$ is the marginal valuation of a buyer if he trades with a seller of type $t$. Assume, furthermore, that $\theta^t$ is increasing in $t$, so marginally, the buyer always prefers to trade with a seller of higher type. A contract in this model is denoted by $\psi = (p, y)$ and specifies a price for the object and a level of investment.[18] The rest is similar to the basic insurance model.

## § 2.7 Conclusion

In this paper, I studied a market with multiple competing uninformed risk-neutral sellers, one informed risk-averse buyer with a risky endowment and common values. The buyer could be any of a finite number of types and this was his private information. I proposed a game form (market mechanism or platform) to model the interaction among the buyer and sellers. In particular, I allowed the buyer to propose a set of contracts in the first stage and the sellers to also propose sets of contracts if they accepted the offer. After the proposals, a menu of contracts was formed between the buyer and each one of the sellers who decided to participate consisting of all the contracts proposed in both sets. The buyer had the discretion to choose any contract out of any mechanism in the market.

The main findings of the paper could be summed up as follows: First, the RSW allocation is an equilibrium allocation if and only if it is interim incentive efficient. Indeed, it is the unique strong solution. Second, only interim incentive efficient allocations can be possible equilibrium allocations of the game studied. Last, any interim incentive efficient allocation that strictly Pareto dominates the RSW allocation can be an equilibrium allocation and it corresponds to a neutral optimum. Surprisingly, no refinement was needed in out-of-equilibrium beliefs to eliminate unwanted equilibria.

[18] Note that in case there is no technology to improve the quality of the object, this model is identical to Akerlof's [2] lemons market. Even in this case, the market mechanism is able to produce only efficient allocations.

As I claimed, the extensive form game is not restricted only to the basic insurance model but could potentially extend in other competitive models such as managerial compensation models, credit models, franchising models, informed seller models etc.

# Part

## Strategic Experimentation in R&D

# - Chapter 3 -

## Strategic Experimentation in R&D Races

### § 3.1 Introduction

One of the main questions in "*patent races*" concerns the level of R&D activity. Ever since Kamien and Schwartz [35] and Loury [39], many studies have stressed the inefficiencies caused by competition in this type of races. A central result they establish is that aggregate investment in R&D is usually excessive, which relies on the well-known "tragedy-of-the-commons" effect. Nonetheless, one can imagine situations in which an opposite inefficiency prevails. For, if the competing firms can learn from each other's activities, information spill-overs could cause a "free-riding" effect. Our main contribution is to model a patent race in which both effects are possible equilibrium phenomena.

As an example, consider the development of a new medicine. If the pharmaceutical industry is highly concentrated, each company has almost perfect information with regards to the R&D activities undertaken by the other companies. Therefore, the release of good news by some company could spark excessive competition and therefore decrease the expected profitability of the patent for the medicine. Each company, rationally expecting that, would refrain from undertaking the risky venture.

In this article, we attempt to model a simple multistage patent race in order to stress this type of information externalities. By building on the literature of "*strategic experimentation*", started by Bolton and Harris [10] , we analyse a dynamic race between two firms using two-armed bandit technologies. Our model is meant to capture possible inefficiencies that are caused by information externalities (spillovers) as well as by strategic externalities as those discussed in the already existing literature.

The race is between two firms. The winner of the race is the firm that completes two phases: Research & Development. The winner acquires a patent and therefore fixed monopoly profits for a long period of time. For instance in the development of a new medicine, the research phase may refer to the identification of the mechanisms that cause a certain disease, as well as the substances that may control its symptoms or help the immune system in the fight against it. On the other hand, the development phase may refer to the trials of the medicine in animal or human subjects before the new product is released to the market. Both firms start the race equipped with similar bandit technologies which are all armed in the same state (either good or bad). However, successes occur independently. In fact, one success is enough for each stage to be completed.

In the research phase, the technologies can be either armed in a good state, and generate a success with positive probability, or in a bad state, and never generate a success. Therefore, with some positive probability, the research phase can never be completed successfully, regardless of the efforts of the two firms. In the medicine example discussed above, this idea captures the uncertainty that pharmaceutical companies usually face when they start investigating the possible causes of a disease. On the contrary, and without loss of generality, we assume that in the development phase, the technologies are always armed in a good state (and therefore known to be able to generate successes) but successes occur only with some positive probability.

The central results we establish are the following: In the research phase, the two firms may *under-invest* relative to the socially efficient level of investment because of information spillovers. After the release of a success from one firm in the research phase, information becomes a public good. At that point, the lagging firm realises that advancement to the development phase is feasible and therefore has an incentive to get back into the race in order to catch up and pass the finish line first. This reduces the profitability of the leading firm and each firm, rationally expecting that, decides not to experiment at all in the research phase.

On the other hand, the *over-investment* (or duplication of efforts) effect that has been widely stressed in the R&D literature can also be uncovered in equilibrium. In our model, this is translated to duplication of efforts (investments) compared to social optimality. In fact, this effect can be realised in any phase but it is more detrimental towards the end of the race, or in other words, during the development phase. As we show, there are cases in which both firms invest, even though social optimality requires only one of them to invest. Thus, while the current R&D literature identifies the inefficiency involved in over-investment as a result of externalities ignored by the rival firms, and the experimentation literature identifies the fundamental inefficiency of information acquisition because of free-riding, in our model both of these forces are at play.

□ **Related Literature.** Loury [39], Lee and Wilde [38] and Dasgupta and Stiglitz [16] analyse static strategic patent races in which investment in

R&D is irreversible and takes place only once. Each firm's investment determines its speed of innovation. Loury [39] and Lee and Wilde [38] do not explicitly model a market after the innovation occurs. They are mostly interested in the individual and aggregate levels of R&D investment as well as the degree of market concentration. On the contrary, Dasgupta and Stiglitz [16] explicitly model the market after the innovation. They proceed to comparative static analysis in order to stress the effect market demand has in the innovation process. One of the central results of all these articles is that aggregate investment is socially excessive.

Reinganum [51, 52], Fudenberg et al. [22] and Harris and Vickers [28, 27] extend the above static models to dynamic models in which firms have the right to change their level of investment during the race. Even though they make severe restrictions of how firms can alternate their investments, the analysis seems turns out to be complicated. Reinganum [51, 52] analyse a race in which each firm accumulates knowledge through time by investing in R&D. Innovation is stochastic and the time of successful completion increases with the stock of relevant knowledge. Part of Reinganum's contribution is technical since this game is a representative differential game with many technical difficulties. When knowledge is a private good, i.e. it can only be used by the firm who accumulates it, then Reinganum shows that in equilibrium there is over-investment and early innovation. On the other hand, when knowledge accumulated by one firm can be used by other firms, then there is a free-riding problem and therefore under-investment. Fudenberg et al. [22] analyse a multistage patent race. They provide conditions under which a race will be characterised by vigorous competition or will degenerate into monopoly. The key idea is whether the lagging firm in the race has time to "leapfrog" and catch up. In Harris and Vickers [27], it is shown that the leader in the race exerts higher effort than the follower, and effort increases as the gap between competitors decreases.

Grossman and Shapiro[26] is perhaps closer to our approach. They extend the model of Lee and Wilde [38] by adding one more phase in the development of the project. Therefore, a firm, in order to win the race, must, as in our model, complete two phases R&D. Firms have complete information about each other's actions. In their model, there is over-investment in equilibrium relative to the socially efficient level of investment. The main difference from our model is that we introduce uncertainty in the bandit technology of the research phase and therefore informational externalities. This allows us to stress the fundamental under-investment in the research phase along with the over-investment in the development phase.

Lastly, there is by now a vast literature on strategic experimentation started from Bolton and Harris [10] . Bolton and Harris [10] analyse two-armed bandit models with several agents. Each player can learn from the actions and outcomes of the other players and therefore, similarly to our approach, information becomes a public good. They show that, in equilibrium, a free-riding effect arises. Similarly to our approach, experimentation in their article is below the socially efficient level. Keller, Rady and Krips [36] analyse a similar model but with exponential bandits. The use of exponential bandits in contrast to Bolton and Harris (1999) allows for analytical tractability and therefore Keller, Rady and Krips [36] uncover also asymmetric equilibria. Our model is way simpler than Bolton and Harris [10] and Keller, Rady and Krips [36] first because investments are indivisible and second because after a success by one player the game ends. Therefore our game is more like a winner-takes-all race in contrast to papers on strategic experimentation in which after a success by one player, the other players continue investing in order to succeed as well. Lastly, the results between our article and these of Bolton and Harris [10] and Keller, Rady and Krips [36] are partly different because we ucncover both free-riding as well as over-investment. Bergemann and Hege [5, 6] introduce the two-armed bandit technologies into a dynamic moral hazard model.[1] In their model an entrepreneur seeks financing in order to run experiments needed to complete a project. The technologies are similar to our available technologies in the research phase. They show that because of moral hazard, and funds diversion, investment is relatively low compared to the socially efficient counterpart and financing stops prematurely. In contrast to our model, those models has only one firm experimenting, relying however to external financing to run the experiments. In our model there are at least two firms experimenting and they self-finance their experiments.

In Section 3.2 we present the model and notation. In Section 3.3 we analyse the equilibria of the game In Section 3.4 we extend the model to accommodate more than one stage in the development phase.

## § 3.2 The Model

**Timing and Payoffs.** Time is discrete and infinite: $t \in \mathbb{N}$. There are two firms, denoted by $i = \{1, 2\}$, that are engaged in an R&D race. Both firms are risk-neutral, Bayesian expected utility maximisers. There is no discounting. There are two phases to be completed, which can be considered as Research and Development (R&D). Completion of every phase requires investment (or effort), and the fixed investment cost to be paid every period is $c$. The first firm that successfully completes both phases receives a monetary payoff equal to $V$.[2,3] As soon as some firm succeeds in the research

[1] See also Horner and Samuelson [31].

[2] The cost $c$ can be thought either as a monetary cost or a disutility cost of effort. This distinction is irrelevant in our model.

[3] The expected payoff $V$ can be considered as the value of the product to be developed when sold to some external agent. For instance, if the product is a drug, then we can interpret the two firms as medical research firms and the external agent as a pharmaceutical

phase, it can proceed to the last (and final) phase of development. Even if one firm passes the research phase, the other firm has still time to invest and potentially qualify to the development phase as well. If the development phase is completed as well, then the first firm receives $V$ and the second firm 0. In case both firms finish at the same time, then each firm receives $V$ with equal probability.[4] Note that for a firm to qualify to the development phase, it is a prerequisite to first pass the research phase.[5] Each firm observes the actions and outcomes of the rival firm, therefore the game is considered as one of complete information.[6]

**Notation and Technologies.** In every phase there is a technology that can be used to provide the breakthrough needed, and this is available to both firms. The technology in the research phase is uncertain and risky. When both firms are in the research phase, each one of them decides either to use the technology or not, but, as we mentioned above, this is costly and the fixed monetary amount to be paid is constant and denoted by $c$. On the other hand, there is always the possibility of no experimentation at zero cost. In this case, the probability of success is zero. There is uncertainty regarding the suitability of the technology to provide a success in the research phase when in use. Specifically, the technology can be either in a good state with probability $\alpha$ or in a bad state with probability $1-\alpha$. If in a good state, the technology, after experimentation in the research phase, can generate a success with probability $\theta$ or a failure with probability $1-\theta$. On the other hand, if in a bad state, the technology can only generate a failure with probability 1. If no success takes place after one round of experimentation, then the posterior is updated downwards. If a success occurs then the posterior is updated to one since success is a fully informative signal.

On the contrary, the technology in the development phase is risky but not uncertain. We assume that by investing $c$, each firm it can realise a success with probability $\eta$ and a failure with probability $1-\eta$. Therefore, there is a fundamental difference between the available technologies in the two phases. In the development phase the probability of success in each stage is fixed every period. In the research phase the probability of success is not fixed but can vary depending on the outcome of experimentation.

company.

[4] $V$ can be considered as the monetary transfer from the seller of the product to the firm that made the breakthrough. We assume that this is not observable or verifiable by any outsider apart from the two firms. This simply means that no explicit contract can be written between the two firms contingent on the breakthrough since its value is not verifiable by a court of law.

[5] This assumption is easily justifiable if we consider the project as a product where both phases must be completed in order to be sold to the market. If one phase has not been completed then the product is considered as problematic and it has no market value.

[6] Given that the two firms are specialising on the same product, they have inside information of each other's actions.

This captures the fundamental uncertainty firms face in the initial stages of R&D activity.

$W_X^n(Y,Z)$ denotes the individual value of the project for a firm in phase $X \in \{R, D\}$ if there are $n$ firms active in some period, one firm is in phase $Y = \{R, D\}$ and the other one in phase $Z = \{R, D\}$.

## § 3.3 Equilibrium in Research and Development

It is easy to see that, due to no discounting, it can never be socially optimal to operate both technologies in any of the periods, in any of the phases. This is because the social cost of operating one technology, given that the other one is active is c, and the social benefit is zero (the payoff is zero if one succeeds given that the other succeeds). In fact, if both technologies, are operated by a benevolent social planner possessing the same information as the two firms, it is strictly dominant to operate one technology after the other instead of both together.[7] The assumption of no discounting is without loss of generality. All our results hold even if the discount factor is strictly less than one.

### 3.3.1 Both Firms in the Development Phase

Assume that both firms have qualified to the development phase. We will first examine the socially efficient investment level and we will compare this with the individual investment levels.

First assume that both technologies, being in the development phase, are operated by a benevolent social planner possessing the same information as the two firms. As we claimed earlier, it is never socially optimal to operate both technologies in the same period. Therefore we will examine when it is optimal to operate one technology at a time. Note that because of the stationarity of the model, if it is ever optimal to operate one technology in some period, then it will be optimal in any period thereafter. Therefore, it is socially efficient to invest in one technology in the development phase if and only if:

$$\eta V \geq c$$

We can define a cutoff point in the profitability of the project, such that it is socially efficient to operate one technology, if and only if the "payoff-to-cost" ratio is weakly above this cutoff point, or:

$$\frac{V}{c} \geq \frac{1}{\eta}$$

[7] We could equally assume that both firms merge and operate together.

This is a general necessary condition for any of the technologies to be active. In any other case, i.e. $\frac{V}{c} < \frac{1}{\eta}$, it is never socially optimal to operate any of the technologies in the development phase.

We can easily establish that the social value of the project in any period $t$ if at least one technology is in the development phase is given by:[8]

$$V - \frac{c}{\eta}$$

Turning our interest to the individual behaviour of the two firms being both in the same phase, note that they will both be active if and only if:

$$[\eta(1-\eta) + 0.5\eta^2)]V - c \geq 0$$

Perhaps, as we did above, we could define a cutoff point in the profitability of the product, such that both firms will be certainly active in the development phase, as:

$$\frac{V}{c} \geq \frac{1}{\eta(1-0.5\eta)}$$

In case the profitability of the product is such that:

$$\frac{1}{\eta(1-0.5\eta)} > \frac{V}{c} \geq \frac{1}{\eta}$$

then only one firm will be active in equilibrium. Even though there are equilibria in which the identity of the firm investing is determined randomly, we will concentrate, without loss of generality, in equilibria in which the identity of the firm who invests remains the same.[9]

The value of any firm, if both firms are active in the development phase is denoted by $W_D^2(D,D)$ and is given by:

$$\begin{aligned} W_D^2(D,D) &= \eta(1-0.5\eta)V - c + (1-(1-\eta)^2)(\eta(1-0.5\eta)V - c) + ... \\ &= \frac{\eta(1-0.5\eta)V - c}{1-(1-\eta)^2} \end{aligned}$$

On the other hand, the value of the only active firm in the development phase is denoted as $W_D^1(D,D)$ and is given by:

$$W_D^1(D,D) = V - \frac{c}{\eta}$$

Given that it is never socially optimal both technologies to be active, but as we saw there are cases where both firms acting independently are indeed

[8] The expected value is $\eta V - c + (1-\eta)(\eta V - c) + (1-\eta)^2(\eta V - c) + .... = \frac{\eta V - c}{1-(1-\eta)} = V - \frac{c}{\eta}$

[9] There are two symmetric SPNE: If each firm believes that the other one will invest, then it is best response for it not to invest. The subgame is similar to the "Battle-of-Sexes" game. All along we will not consider mixed-strategy equilibria. We only examine equilibria in which both firms play degenerate strategies.

active, we can easily deduce that there may be an inefficient allocation of resources in equilibrium. Note that both firms would be better off using only one technology, bear one cost and share the payoff with equal probability.[10]

**Proposition 3.3.1.** *If both firms have advanced to the development phase and* $\frac{V}{c} \geq \frac{1}{\eta(1-0.5\eta)}$*, then there is over-investment in equilibrium.*

The above result is relatively intuitive. Over-investment occurs in the last phase because in probability $\eta^2$ both firms make a breakthrough and one of the costs $c$ is, in this event, a social waste which the firms do not internalise. The externalities caused by both firms to each other lead to socially inefficient levels of investment.

### 3.3.2 One Team in the Research and the Other in the Development Phase

In this section, we assume that one firm has already qualified to the development phase and the other one is still lagging in the race. Given the success of the the leading firm however, the firm that is now lagging knows that advancing to the development phase is possible or $\alpha = 1$.

First, given that one technology has qualified to the development phase, it is never socially optimal to invest in the technology that is still in the research phase. However, from an individual point of view, there are SPNE depending on the parameter values, in which in some cases, the firm that is still in the research phase invests.

Even though there may be equilibria in which the leading firm stops investing when the lagging also advances to the development phase, we find as more reasonable equilibria those at which the leading firm never does. Given this, it is easy to verify that if $\frac{1}{\eta(1-0.5\eta)} > \frac{V}{c} \geq \frac{1}{\eta}$, then it is not optimal for the lagging firm to invest. This is because in this case, only one firm will be active in the development phase. Therefore, a necessary condition for the lagging firm to be active is $\frac{V}{c} > \frac{1}{\eta(1-0.5\eta)}$. The expected

[10] Of course, it is assumed that no explicit contracts can be written that allow cooperation, exactly as in the Cournot-Nash equilibrium or any game in which the Nash equilibrium is inefficient. We can further justify this assumption on the following grounds: Assume that the payoff of successfully developing the product and selling it in the market is $X$, distributed in the interval $[0, A]$ with a cdf function $F(X)$. Moreover, $\mathbf{E}(X) = V$ where $\mathbf{E}$ is the mean expectational operator of the distribution. Therefore, both firms believe that the ex-ante expected payoff of selling the product in the market is $V$, but the true payoff can be anything from 0 to $A$. The true payoff can be only realised when the product is developed. Assuming that no outsider can observe or verify the true payoff, the firm that develops the product and promises to deliver half the payoff to the other firm will never do so by claiming that the realised payoff is zero. Given that this is not verifiable, no firm will agree to pay half the cost to the other firm and then expect to receive half the payoff after realisation of uncertainty in the fear of being "held-up". This ex-post opportunistic behaviour makes cooperation impossible and therefore leads to inefficient Nash equilibria.

value of the firm that is in the research phase, while the other firm is in the development phase and they are both active is denoted by $W_R^2(R,D)$. This value can be calculated as:

$$W_R^2(R,D) = (1-\theta)(1-\eta)W_R^2(R,D) + \theta(1-\eta)W_D^2(D,D) - c$$

or

$$W_R^2(R,D) = \frac{-c + \theta(1-\eta)\frac{\eta(1-0.5\eta)V-c}{1-(1-\eta)^2}}{1-(1-\theta)(1-\eta)}$$

We can see that as long as this value is positive, or $\theta(1-\eta)\frac{\eta(1-0.5\eta)V-c}{1-(1-\eta)^2} \geq c$, then in the unique SPNE the lagging firm always invests because it is always profitable to do so.

As previously, we can find a cutoff in the profitability of the project such that there is an inefficiency because the firm that has lagged in the race does not internalise the externality that imposes to the other firm. This cutoff is given by:

$$\frac{V}{c} \geq \frac{1-(1-\eta)^2 + \theta(1-\eta)}{\theta(1-\eta)} \cdot \frac{1}{\eta(1-0.5\eta)}$$

We conclude this section with the following proposition that highlights the inefficiencies caused in the Development phase.

**Proposition 3.3.2.** *If one firm has advanced to the development phase, the other is still in the research phase and* $\frac{V}{c} \geq \frac{1-(1-\eta)^2+\theta(1-\eta)}{\theta(1-\eta)} \cdot \frac{1}{\eta(1-0.5\eta)}$ *there is over-investment in equilibrium.*

For later results, it is useful to also characterise the value of the firm that is leading the race- i.e. the firm that is in the development phase given that the other firm is in the research phase- when both firms are active. This is denoted by $W_D^2(R,D)$ and is given by:

$$W_D^2(R,D) = \eta V + (1-\theta)(1-\eta)W_D^2(R,D) + \theta(1-\eta)W_D^2(D,D) - c$$

or

$$W_D^2(R,D) = \frac{-c + \eta V + (1-\eta)\,\theta\left(\frac{1}{2}V - c - \frac{c}{2\eta-\eta^2}\right)}{1-(1-\theta)\,(1-\eta)}$$

Vacuously, the value of the leading firm, when the lagging firm is not active any more is denoted by $W_D^1(R,D)$ and equals the social value or $W_D^1(R,D) = V - \frac{c}{\eta}$.

### 3.3.3 Research Phase

We are now ready to analyse the socially optimal behaviour as well as the equilibrium behaviour when both firms (or technologies) are in the research phase. We know that if one of the two firms advances to the development phase, then the equilibrium is given in Subsection B. On the other hand, in case both firms advance to the development phase, then the equilibrium is given in Subsection A. In the two previous cases, we saw that over-investment is possible, since there are parameter values according to which only one technology should be active, but both are, in equilibrium. In this section we would like to establish that if both firms are in the research phase, another fundamental inefficiency commonly observed in R&D races is possible. This is the well known under-investment, due to free-riding. Free-riding is possible in this model since success in the research phase by one firm releases good news to the rival firm. Information therefore becomes a public good and a free-riding effect arises in equilibrium. On the other hand, it is possible to have over-investment in this phase as well, for some parameter values. Thus, in the research phase, both over-investment as well under-investment are possible equilibrium phenomena.

Given the uncertainty about the quality of the technology in the research phase, one needs to specify the evolution of the belief after experimentation. There are three possible cases to consider: If no firm experimented, then it is apparent that no updating takes place and the posterior probability remains the same (as the prior). If, however, one firm experimented and succeeded in some period then $\alpha = 1$, since success is a fully informative signal. On the other hand, if some firm experimented and failed then this brings bad news regarding the suitability of the technology to complete the state and the posterior probability is downgraded, by Bayes' rule, to:

$$\alpha' = \frac{\alpha(1-\theta)}{(1-\alpha)+\alpha(1-\theta)} < \alpha$$

Note that the behaviour of the two firms in the research phase crucially depends on the continuation value of the project, as this has been determined in the previous two sections. It is common sense to note that the more lucrative the project is ($\alpha$ is higher ceteris paribus), the higher the experimentation will be in the research phase. Note that even in this phase, it is never socially optimal to experiment in both technologies for the same reason it was never optimal in the development phase. It seems rather complicated to analyse the equilibrium given that there are too many cases to be considered in the continuation of the game as these have been examined in the two previous sections. To stress the result of the paper as clear as possible we will analyse only the two most interesting cases.

The first case is when $\frac{V}{c} < \frac{1-(1-\eta)(\theta-(1-\eta))}{\theta(1-\eta)} \cdot \frac{1}{\eta(1-0.5\eta)}$. From Subsection B, we know that as long as one firm advances to the development phase, the firm that remained behind will not experiment anymore. For some parameter values, both firms would be active in the development phase, if both advanced to it. Thus, it is rather straightforward to see that the two firms compete in the research phase exactly as they do in the development phase in this range of parameters. In other words, the first firm that advances to the development phase is considered the "winner" of the phase. We should, therefore, expect that over-investment is possible in this phase for the exact same reason that it was possible in the development phase. The two firms do no internalise the externalities impose through their actions. Indeed, as we prove in the following lemma, over-investment is the only source of externality in this range of parameters.

**Lemma 3.3.3.** *If both firms are in the research phase, $\frac{V}{c} < \frac{1}{\eta(1-0.5\eta)}$ and $\alpha\theta(1-0.5\theta)W_D^1(D,D) \geq 0$ then there is over-investment in equilibrium.*

*Proof.* First, note that when $\frac{V}{c} < \frac{1}{\eta(1-0.5\eta)}$, from the analysis above, the lagging firm will not invest. Therefore, in this range of parameters the race in the research phase is similar to the race in the development phase. The winner is the first one that succeeds in the research phase. If both succeed simultaneously then we assume that each firm has equal probability to keep on investing in the development phase. The other firm becomes inactive.

Substituting for $W_D^1(D,D) = V - \frac{c}{\eta}$, the question boils down to whether

$$\frac{c}{\theta(1-0.5\theta)(V-\frac{c}{\eta})} \leq 1$$

Rearranging, we can find that over-investment occurs if and only if

$$\frac{V}{c} \geq \frac{\theta(1-0.5\theta)+\frac{1}{\eta}}{\theta(1-0.5\theta)}$$

To complete the proof, we have to check that for some parameter values

$$\frac{\theta(1-\eta)+1-(1-\eta)^2}{\theta(1-\eta)} \cdot \frac{1}{\eta(1-0.5\eta)} > \frac{\theta(1-0.5\theta)+\frac{1}{\eta}}{\theta(1-0.5\theta)} > \frac{1}{\eta(1-0.5\eta)}$$

For the first inequality, by rearranging we get that

$$(1-(1-\eta)^2+\theta(1-\eta))(1-0.5\theta) > (1-\eta)(1-0.5\eta)$$

which for each value of $\eta$ is a quadratic equation with respect to $\theta$. One can easily see that there are parameter values that this inequality is satisfied and moreover $\frac{\theta(1-0.5\theta)+\frac{1}{\eta}}{\theta(1-0.5\theta)} > \frac{1}{\eta(1-0.5\eta)}$ (for instance $\eta = 0.5$, $\theta = 0.9$). □

The intuition behind the above result is that when $\frac{V}{c} < \frac{1-(1-\eta)(\theta-(1-\eta))}{\theta(1-\eta)} \cdot \frac{1}{\eta(1-0.5\eta)}$, there is no room for under-investment because there is no informational externalities and therefore no free-riding effect. This is because as long as one firm remains behind in the race, the game is over because it is not profitable anymore to experiment in order to catch up.

The most interesting case, however, and one of the distinguishing features of this paper, is when $\frac{V}{c} \geq \frac{1-(1-\eta)(\theta-(1-\eta))}{\theta(1-\eta)} \cdot \frac{1}{\eta(1-0.5\eta)}$. Note that in this range of parameter values, it is never socially optimal to operate the technology that has lagged in the race, but it is individually optimal. However, we will show that it is possible, when both firms are in the beginning of the race to be socially optimal to operate one of the two technologies, but none to be active in equilibrium. This is indeed true as it is proved in the proposition below.

**Proposition 3.3.4.** *If both firms are in the research phase*

1. $\frac{V}{c} \geq \frac{1-(1-\eta)(\theta-(1-\eta))}{\theta(1-\eta)} \cdot \frac{1}{\eta(1-0.5\eta)}$
2. $\alpha\ \theta\ (\eta V - c + (1-\eta)(V - \frac{c}{\eta})) - c \geq 0$
3. $\alpha\ \theta\ [(\eta V - c) + (1-\eta) W_D^2(R,D)] - c < 0$

*then there is under-investment in equilibrium.*

*Proof.* Assume that (1) holds. We will show that there are parameter values such that (2) and (3) hold. If both equations hold for some range of parameters, then it is necessarily the case that under-investment happens in equilibrium. The question boils down to whether there are parameter values such that $V - \frac{c}{\eta} > W_D^2(R,D)$. By manipulating $W_D^2(R,D)$, we can find that

$$\begin{aligned} W_D^2(R,D) &= \frac{-c + \eta V + (1-\eta)\,\theta\left(\frac{1}{2}V - c - \frac{c}{2\eta-\eta^2}\right)}{1-(1-\theta)(1-\eta)} \\ &= \frac{V\left(\eta + \frac{\theta(1-\eta)}{2}\right) - c\left(1 + (1-\eta)\,\theta + \frac{(1-\eta)\theta}{2\eta-\eta^2}\right)}{\eta + \theta - \theta\eta} \\ &= \frac{\eta + \frac{\theta-\eta\theta}{2}}{\eta+\theta-\theta\eta}\left(V - c\frac{1 + (1-\eta)\,\theta + \frac{(1-\eta)\theta}{2\eta-\eta^2}}{\eta + \frac{\theta(1-\eta)}{2}}\right) \end{aligned}$$

Let

$$\gamma = \frac{\eta + \frac{\theta-\eta\theta}{2}}{\eta+\theta-\theta\eta}, \quad \beta = \frac{1 + (1-\eta)\,\theta + \frac{(1-\eta)\theta}{2\eta-\eta^2}}{\eta + \frac{\theta-\eta\theta}{2}}$$

Thus, our goal is to show that

$$V - \frac{c}{\eta} > \gamma[V - \beta c]$$

Given that $\frac{\eta+\frac{\theta-\eta\theta}{2}}{\eta+\theta-\theta\eta} < 1$, for any $\eta \in (0,1)$, it is sufficient to show that

$$\frac{1 + (1-\eta)\,\theta + \frac{(1-\eta)\theta}{2\eta-\eta^2}}{\eta + \frac{\theta-\eta\theta}{2}} > \frac{1}{\eta}$$

which holds for all $\theta \in (0,1)$ and $\eta \in (0,1)$. $\square$

The intuition behind the above result is the following: Assume that the prior probability $\alpha$ is small enough such that it is socially optimal for only one technology to be active in the research phase and none to be active after a failure. It may be that because the firm that decides to invest in some period in the research phase knows that the other firm will get into the race after a success refrains from doing so. Therefore, for $\alpha$ small enough, it may be socially optimal for one firm to be active in some period, but none is in equilibrium. This is a well-known free-riding effect commonly observed in practice, especially in patent races. When some research firm is planning to invest in the development of some product (drug, high-technology product, etc.) and cannot control the information to be released to the competitors, it may decide not to do so. Research firms refrain from developing new risky products, in the fear of vast competition after the announcement of good news that would decrease profitability of the product considerably.

## § 3.4 Conclusion

In this paper we studied a winner-takes-all R&D race as a two-armed bandit model. Two firms competed on the development of a new product but they first had to pass two phases (Research & Development) before they could sell the product to the market. Firms possessed identical bandit technologies in every phase and could observe each other's actions and outcomes. The technology in the research phase was uncertain, in the sense that it was armed either in a good state, in which case after experimentation it could produce the required breakthrough with some probability, or in a bad state, in which case no breakthrough could ever occur. On the other hand, in the development phase the technology was always in a good state and it could produce the required breakthrough with some positive probability. Experimentation was costly and the monetary cost to be paid for every experiment was fixed. The winner of the race was the one completing both phases successfully.

We analysed the SPNE of this dynamic game and compared the levels of investment of the two firms with the socially optimal counterparts. We showed that, in equilibrium, two possible inefficiencies are likely to occur. On the one hand, in the initial phase (research), there were parameter values such that under-investment could occur in equilibrium. Even though it was socially optimal for at least one firm to be active, none was in equilibrium. On the other hand, another fundamental inefficiency, this of over-investment, that has been extensively examined in the R&D literature was uncovered in equilibrium. This is a phenomenon according to which both teams experiment, even though one of the costs of experimentation is a social waste.

Over-investment could occur in any of the phases, but it was more frequent towards the end of the race.

The model we analysed was very elementary with many simplifying assumptions. In a previous draft of this paper, we had analysed a more complicated model with discounting and both phases to be uncertain.[11] Even though this model was more general the analytical tractability was prohibitive. However, all our results go through even in this more general model and, hence, we traded-off generality for simplicity and tractability. In case of discounting, the planner would trade-off the time cost and wasting one cost every period (in case he experimented in both technologies). On the other hand, the individual teams would not take into account this trade-off and would only experiment as long as the expected payoff exceeded the cost (if the rival firm was active). Henceforth, it is clear that even there we could find parameter values such that over-investment could occur in equilibrium, at least in the development phase. On the other hand, in the research phase, under-investment would be possible in this model for the exact same reason under-investment can happen in the more basic model. Again we could find parameter values, such that the planner would not invest in the lagging technology after a success in the research phase but, acting individually, the lagging team would do. Therefore, for $\alpha$ low enough, no team would be active (even if it would be socially optimal one to be), because of the free-riding effect.

[11] By this we mean that in the development phase, the technology was either in a good or in a bad state.

# Part

## Annexes

# - Appendix A -

## Chapter 1

### § A.1 Proof of existence of an interior maximum under complete information

First, note that in any equilibrium under complete information, banks must earn zero profits for every loan contract. This is because entrepreneurs possess all the bargaining power, being able to apply to any contract they like when their type is known.[1] Therefore, $\pi_i R_{s,i} X_i + (1-\pi_i) R_{f,i} X_i = X_i$. The equilibrium investment level can be found by solving the following unconstrained optimisation program for each $i$:

$$\max_{X_i} \ h_i(X_i) = \pi_i \gamma_i f(X_i) - X_i$$

**Lemma A.1.1.** *For each $i = H, L$, $h_i(X_i)$ has am unterior global maximum $X_i^{FB}$.*

Proof: Let $h_i(X_i) = \pi_i \gamma_i f(X_i) - X_i$. Since $f$ is twice continuously differentiable, $h_i$ is also twice continuously differentiable with $h_i'(X_i) = \pi_i \gamma_i f'(X_i) - 1$ and $h_i''(X_i) = \pi_i \gamma_i f''(X_i)$. Since $f^{''} < 0$, $h'' < 0$ and therefore $h_i$ is concave. From the Inada conditions $\lim_{X_i \to \infty} h_i'(X_i) = -1 < 0$ and $\lim_{X_i \to 0} h_i'(X_i) = \infty$. Since $h_i'$ is strictly decreasing, continuous and the limits are defined as above, from the intermediate value theorem, there exists exactly one $X_i^{FB}$ such that $h_i'(X_i^{FB}) = 0 \Rightarrow f'(X_i^{FB}) = \frac{1}{\gamma_i \pi_i}$. Given that $h_i(0) = 0$ and $h_i$ is concave $X_i^{FB}$ is a global maximum and moreover $h_i(X_i^{FB}) > 0$. Q.E.D.

[1] When information is perfect, the set of equilibria of this game is identical to the set of equilibria of a game in which banks compete a la Bertrand in loan contracts.

## § A.2 Proof that both types strictly prefer the first-best contract of type H over this of type L

Consider the payoffs of type L from the two contracts:

$$V^L(X_L^{FB}, R_{s,L}^{FB}, R_{f,L}^{FB}) = \pi_L\big(\gamma_L f(X_L^{FB}) - \frac{X_L^{FB}}{\pi_L}\big)$$

$$V^L(X_H^{FB}, R_{s,H}^{FB}, R_{f,H}^{FB}) = \pi_L\big(\gamma_L f(X_H^{FB}) - \frac{X_H^{FB}}{\pi_H}\big) - \frac{\pi_H - \pi_L}{\pi_H} R_{f,H}^{FB} X_H^{FB},$$

Recollect that $R_{f,H}^{FB} \leq 0$ (entrepreneurs do not possess any wealth in the failure state), the payoff of type L increases as $R_{f,H}$ decreases. Therefore, it suffices to show that type L's payoff is higher even for $R_{f,H} = 0$.

Define the following two functions:

$$\zeta(X) = \pi_L \gamma_L f(X) - X$$

and

$$\xi(X) = \pi_L \gamma_L f(X) - \frac{\pi_L}{\pi_H} X$$

Examining these two functions we can see that they are both concave and, given Assumption 1.2.1, they achieve a unique interior maximum. Moreover, the maximiser of $\xi(X)$ (call it $X'$) is greater than that of $\zeta(\cdot)$ (which is $X_L^{FB}$), and $X_L^{FB} < X_H^{FB} < X'$. $\xi'(X) > \zeta'(X)$ for any $X$. Since $\xi(0) = \zeta(0) = 0$, $\xi(X) > \zeta(X)$ for any $X$. Therefore, $\xi(X') > \xi(X_H^{FB}) > \zeta(X_L^{FB})$. Q.E.D.

## § A.3 Proof of Proposition 1.3.1.

Since type L is the one who is eager to misrepresent his type, and because banks must earn zero profits for each equilibrium contract (because of the intuitive criterion), type L receives the same contract as under perfect information. We can find the equilibrium contract of type H by solving the following maximisation program:

$$\max_{X_H, R_{s,H}, R_{f,H}} \pi_H(\gamma_H f(X_H) - R_{s,H} X_H) - (1-\pi_H) R_{f,H} X_H$$

s.t.

$$\pi_H R_{s,H} X_H + (1-\pi_H) R_{f,H} X_H = X_H$$

$$\pi_L \gamma_L f(X_L^{FB}) - X_L^{FB} = \pi_L(\gamma_L f(X_H) - R_{s,H} X_H) - (1-\pi_L) R_{f,H} X_H$$

$$X_H \geq 0$$

Substituting into the incentive constraint the zero profit condition:

$$\gamma_L f(X_H) - \frac{X_H}{\pi_H} - \frac{\pi_H - \pi_L}{\pi_H \pi_L} R_{f,H} X_H = \gamma_L f(X_L^{FB}) - \frac{X_L^{FB}}{\pi_L}$$

Note that as long as $R_f^H$ is negative, given that entrepreneurs have no wealth in the failure state, the incentive constraint is tighter. Therefore, the least costly contract that makes zero profits requires $R_f^H = 0$. At any separating equilibrium the following condition must be satisfied:

$$\gamma^L f(z^H) - \frac{z^H}{\pi^H} = \gamma^L f(\hat{z}^L) - \frac{\hat{z}^L}{\pi^L} \tag{A.1}$$

Note that the lower is $R_{f,H}$, the tighter is the incentive constraint. Therefore, the least costly contract that makes zero profits requires $R_{f,H} = 0$. At any separating equilibrium the following condition must be satisfied:

$$\gamma_L f(X_H) - \frac{X_H}{\pi_H} = \gamma_L f(X_L^{FB}) - \frac{X_L^{FB}}{\pi_L} \tag{A.2}$$

**Lemma A.3.1.** *Equation (A.2) has two solutions $(X_H^{NT}, X_H^*)$ with $X_H^{NT} < X_H^*$, and given the maintained assumptions the one that maximises the pay-off of type H is $X_H^{NT}$.*

PROOF: Let $g(X_H) = \gamma_L f(X_H) - \frac{X_H}{\pi_H} - [\gamma_L f(X_L^{FB}) - \frac{X_L^{FB}}{\pi_L}]$. We know that

$$g(0) = -[\gamma_L f(X_L^{FB}) - \frac{X_L^{FB}}{\pi_L}] < 0$$

and

$$g(X_L^{FB}) = X_L^{FB}(\frac{1}{\pi_H} - \frac{1}{\pi_L}) > 0$$

Therefore, since $g$ is continuous and strictly monotonic in the interval $(0, X_L^{FB})$, from the intermediate value theorem, there exists exactly one root of $g$ in the interval $(0, X_L^{FB})$. Let us call this root $X_H^{NT}$.

Consider now

$$\lim_{X_H\to\infty} g(X_H) = \lim_{X_H\to\infty} (\gamma_L f(X_H) - \frac{X_H}{\pi_H}) - [\gamma_L f(X_L^{FB}) - \frac{X_L^{FB}}{\pi_L}]$$

where.

$$\lim_{X_H\to\infty} (\gamma_L f(X_H) - \frac{X_H}{\pi_H}) = \lim_{X_H\to\infty} \frac{X_H}{\pi_H}(\frac{\gamma_L f(X_H)}{\frac{X_H}{\pi_H}} - 1) = \lim_{X_H\to\infty} \frac{X_H}{\pi_H} \times \lim_{X_H\to\infty} (\frac{\gamma^l f(X_H)}{\frac{X_H}{\pi_H}} - 1)$$

$\lim_{X_H\to\infty} \frac{\gamma_L f(X_H)}{\frac{X_H}{\pi_H}}$ is not defined, therefore, we can apply l'Hopital's rule:

$$\lim_{X_H\to\infty} \frac{\gamma_L f(X_H)}{\frac{X_H}{\pi_H}} = \lim_{X_H\to\infty} \frac{\gamma_L f'(X_H)}{\frac{1}{\pi_H}} = 0$$

because of Inada's conditions.

Hence, $\lim_{X_H\to\infty}\frac{X_H}{\pi_H}\times\lim_{X_H\to\infty}(\frac{\gamma_L f(X_H)}{\frac{X_H}{\pi_H}}-1)=-\infty$, and again from the intermediate value theorem, there is a another root of $g$ in the interval $(X_L^{FB},\infty)$. Call this root $X_H^*$.

To show that the payoff of type H is higher under the first root consider the following:

$$\gamma_L f(X_H^{NT})-\frac{X_H^{NT}}{\pi_H}-[\gamma_L f(X_L^{FB})-\frac{X_L^{FB}}{\pi_L}]=\gamma_L f(X_H^*)-\frac{X_H^*}{\pi_H}-[\gamma_L f(X_L^{FB})-\frac{X_L^{FB}}{\pi_L}]=0$$

Therefore:

$$f(X_H^{NT})-f(X_H^*)=\frac{X_H^{NT}-X_H^*}{\pi_H}\cdot\frac{1}{\gamma_L}$$

$$\gamma_H[f(X_H^{NT})-f(X_H^*)]=\frac{X_H^{NT}-X_H^*}{\pi_H}\cdot\frac{\gamma_H}{\gamma_L}$$

$$\gamma_H[f(X_H^{NT})-f(X_H^*)]-\frac{X_H^{NT}-X_H^*}{\pi_H}=\frac{X_H^{NT}-X_H^*}{\pi_H}\cdot(\frac{\gamma_H}{\gamma_L}-1)$$

But since $\frac{X_H^{NT}-X_H^*}{\pi_H}\cdot(\frac{\gamma_H}{\gamma_L}-1)>0$, $\gamma_H[f(X_H^{NT})-f(X_H^*)]-\frac{X_H^{NT}-X_H^*}{\pi_H}>0$ which proves the lemma. Q.E.D.

## § A.4 Proof of Proposition 1.4.1

Let the tax system be $(-t_s,t_f)$. We will show that type H is always better off using the subsidy in the failure state in order to borrow more.

In any separating equilibrium without taxes type L produces efficiently. His payoff after taxation is:

$$\begin{aligned}V^L(X_L^{FB},R_s^L,R_f^L)&=\pi_L(\gamma_L f(X_L^{FB})-\frac{X_L^{FB}}{\pi_L}-t_s)+(1-\pi_L)t_f\\&=\pi_L(\gamma_L f(X_L^{FB})-\frac{X_L^{FB}}{\pi_L})+\frac{\overline{\pi}-\pi_L}{1-\overline{\pi}}t_s\end{aligned}$$

which is increasing in $t_s$ given that $\bar{\pi}>\pi_L$. Intuitively this happens because type L is cross-subsidised by type H. On the other hand, the equilibrium contract of type H by solving the following optimisation program:

$$\max_{X_H,R_{s,H},R_{f,H}}\pi_H(\gamma_H f(X_H)-t_s-R_{s,H}X_H)+(1-\pi_H)(t_f-R_{f,H}X_H)$$

s.t.

$$\pi_H R_{s,H}X_H+(1-\pi_H)R_{f,H}X_H=X_H$$

$$\pi_L(\gamma_L f(X_L^{FB}) - t_s) + (1-\pi_L)(t_f -) - X_L^{FB} =$$
$$\pi_L(\gamma_L f(X_H) - R_{s,H}X_H - t_s) + (1-\pi_L)(t_f - R_{f,H}X_H)$$

$$R_{f,H}X_H \leq t_f$$
$$X_H \geq 0$$

Combining the zero profit condition with the incentive constraint, we obtain the following:

$$\gamma_L f(X_H) - \frac{X_H}{\pi_H} = (\gamma_L f(X_L^{FB}) - \frac{X_L^{FB}}{\pi_L}) + (\frac{1-\pi_L}{\pi_L} - \frac{1-\pi_H}{\pi_H})R_{f,H}X_H \quad \text{(A.3)}$$

Given that $\left(\frac{1-\pi_L}{\pi_L} - \frac{1-\pi_H}{\pi_H}\right)R_{f,H} > 0$ for $R_{f,H} > 0$, the smallest root of (A.3) $X_H^T$ is greater than $X_H^{NT}$. The payoff of type H from this contract is:

$$\pi_H\gamma_H f(X_H^T) - X_H^T + \frac{\overline{\pi} - \pi_H}{1-\overline{\pi}}t_s = h_H(X_H) + \frac{\overline{\pi} - \pi_H}{1-\overline{\pi}}t_s$$

From Lemma A.1.1, we know that $h_H(X_H)$ is a strictly increasing function in $[0, X_H^{FB}]$. Therefore, the payoff of type H increases when he uses all the subsidy (given that this small enough as defined below) in the equilibrium contract, or: $R_{f,H}X_H = t_f$. Q.E.D.

## § A.5 Proof of Proposition 1.4.2

Let $\gamma_L f(X_H) - \frac{X_H}{\pi_H} - (\gamma_L f(X_L^{FB}) - \frac{X_L^{FB}}{\pi_L}) - (\frac{(1-\pi_L)}{\pi_L} - \frac{(1-\pi_H)}{\pi_H})\frac{\overline{\pi}}{1-\overline{\pi}}t_s = 0$, and denote as $X_H(t_s)$ the smallest root of this equation. $X_H(t_s)$ defines a function $X_H(\cdot) : [0, t_s^*] \to [0, \infty)$, where $t_s^*$ is such that: $f'(X_H(t_s^*)) = \frac{1}{\gamma_H\pi_H}$.

**Lemma A.5.1.** *$X_H(t_s)$ is strictly increasing, twice continuously differentiable and convex in $[0, t_s^*]$.*

Proof: Denote as $q = \left(\frac{(1-\pi_L)}{\pi_L} - \frac{(1-\pi_H)}{\pi_H}\right)\frac{\overline{\pi}}{1-\overline{\pi}} > 0$ and define the following function:

$$\psi(X_H) = \frac{1}{q}\left(\gamma_L f(X_H) - \frac{X_H}{\pi_H} - \left(\pi_L f(X_L^{FB}) - \frac{X_L^{FB}}{\pi_H}\right)\right)$$

Note that $\psi(X_H) = \frac{g(X_H)}{q}$ with $q > 0$. We want $\psi(X_H) = t_s$. Since $\psi(\cdot)$ is continous, strictly increasing and twice differentiable we know that it is one-to-one and therefore invertible in the interval $[0, X_H^{FB})$. Denote the inverse function as $\psi^{-1}(\cdot)$. Therefore, $X_H = \psi^{-1}(t_s)$. The first and second derivatives of $\psi$ are well defined. Because $\psi$ is strictly increasing, its inverse function will also be strictly increasing. Moreover, since $\psi$ is differentiable, $\psi^{-1}$ will also be differentiable with $(\psi^{-1})'(t_s) = \frac{1}{\psi'(X_H)} > 0$ (because $\psi'(X_H) > 0$), and $(\psi^{-1})''(t_s) = -\frac{\psi''(X_H)}{(\psi'(X_H))^2} > 0$ (because $\psi''(X_H) < 0$). Therefore $\psi^{-1}(t_s)$ is convex. Q.E.D.

Let the function $G(t_s)$ be as in (1.6). Since $f(X_H)$ is continuous and the first and second derivatives exist and they are continuous for any $X_H > 0$, and $X_H(t_s)$ is also continuous and differentiable in $[0, t_s^*]$, $G(t_s)$ is also continuous and differentiable in $[0, t_s^*]$. Denote as $G'(t_s)$ the first derivative of $G(t_s)$ with respect to $t_s$. Then:

$$\begin{aligned} G'(t_s) &= (\gamma_H f'(X_H(t_s)) - \frac{1}{\pi_H}) \cdot (X_H)'(t_s) + (\frac{1-\pi_H}{\pi_H}\frac{\overline{\pi}}{1-\overline{\pi}} - 1) \\ &= \frac{\gamma_H f'(X_H(t_s)) - \frac{1}{\pi_H}}{\gamma_L f'(X_H(t_s)) - \frac{1}{\pi_H}} \cdot q + (\frac{1-\pi_H}{\pi_H}\frac{\overline{\pi}}{1-\overline{\pi}} - 1) \end{aligned} \quad \text{(A.4)}$$

We will show $G(t_s)$ is strictly concave, and there exists $\overline{\pi}^{min}$ such that $G'(\tilde{t}_s) = 0$, for some $\tilde{t}_s > 0$, $\forall\ \overline{\pi} \in [\overline{\pi}^{min}, \pi_H]$.

First, it is rather easy to show that $G'(t_s)$ is strictly decreasing for any $t_s \in [0, t_s^*]$. Evaluating $G'(t_s)$ at $t_s = t_s^*$ and $t_s = 0$ we have that:

$$G'(t_s^*) = (\frac{1-\pi_H}{\pi_H}\frac{\overline{\pi}}{1-\overline{\pi}} - 1) < 0$$

for any $\overline{\pi} \in [\pi_L, \pi_H)$ and

$$G'(0) = (\gamma_H f'(X_H(0)) - \frac{1}{\pi_H})(X_H)'(0) + (\frac{1-\pi_H}{\pi_H}\frac{\overline{\pi}}{1-\overline{\pi}} - 1)$$

Since $\gamma_H f'(X_H(0)) - \frac{1}{\pi_H}$ and $(X_H)'(0)$ are both positive, and $(\frac{1-\pi_H}{\pi_H}\frac{\overline{\pi}}{1-\overline{\pi}} - 1) < 0$ for any $\overline{\pi}$, there exists some $\overline{\pi}^{min}$ such that $G'(0) \geq 0$ for any $\overline{\pi} \in [\overline{\pi}^{min}, \pi_H)$. By the intermediate value theorem, because $G'(t_s)$ is strictly decreasing, when $\overline{\pi} \in [\overline{\pi}^{min}, \pi_H]$, there exists exactly one root ($\tilde{t}_s > 0$), such that $G'(\tilde{t}_s) = 0$. Given that $G'(\cdot)$ is strictly positive for any $t_s \in [0, \tilde{t}_s)$ and strictly negative for any $t_s \in [\tilde{t}_s, t_s^*]$, $G(t_s)$ is strictly concave and $\tilde{t}_s$ corresponds to a global maximum in $[0, t_s^*)$. Therefore, when $\overline{\pi} \in [\overline{\pi}^{min}, \pi_H]$ there exists some $\tilde{t}_s > 0$ that maximises the payoff of type H. Q.E.D.

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