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Price Comparison Websites

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Abstract

The large and growing industry of price comparison websites (PCWs) or 'web aggregators' is poised to benefit consumers by increasing competitive pricing pressure on firms by acquainting shoppers with more prices. However, these sites also charge firms for sales, which feeds back to raise prices. I investigate the impact of introducing PCWs to a market for a homogeneous good. I find that introducing a single PCW *increases* prices for all consumers, both shoppers and non-shoppers. More generally, in the most profitable equilibrium for competing PCWs, prices tend to *rise* with the number of PCWs. (*JEL: L11, L86, D43*)

Keywords: online markets; price comparison sites; competition; price dispersion;

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1. Introduction

Over the past two decades a new industry of price comparison websites (PCWs) or 'web aggregators' has emerged. The industry has enabled consumers to check prices of many firms selling a particular service or product simultaneously in one place. The sites are popular in many countries, and in many markets including utilities, financial services, hotels, flights and durable goods.[2] These sites command billions of dollars of revenue annually.[3] In the UK, PCWs for utilities and financial services have been particularly successful. There are roughly 48 such PCWs in the UK, where over 70% of internet users have used such a site. The largest four aggregators generated approximately £800m ($1.2bn) in revenue during 2013, with average annual profit of the group increasing by 14% that year.[4]

The internet has altered search costs, allowing consumers to compare prices across firms in a matter of clicks, intensifying competitive pricing pressure between firms. While a consumer may not know all the firms in a market, a PCW can expose the full list of market offerings, maximizing inter-firm pressure. However, underlying this increased competition are the fees paid by firms who sell their products through the websites. In the UK these are understood to be between £44-60 ($69-95) for a customer switching gas and electricity provider.[5] These fees, in turn, act as a marginal cost faced by producers, affecting their pricing decisions.[6] The

[2]Examples for utilities and services include Moneysupermarket.com, Google Compare and Gocompare.com; for flights Skyscanner.net and Flights.com; for hotels Hotels.com and Booking.com; and for durable goods Amazon Marketplace, Pricerunner.com and Pricegrabber.com.

[3]Regarding travel services, Priceline Group (which owns Booking.com and Priceline.com) and Expedia Inc. (which owns Expedia.com and Hotels.com) made approximately $6bn in total agency revenues in 2014. Regarding durable goods, Amazon Marketplace sold 2 billion items from third-party sellers. See their 2014 Annual Reports for details.

[4]Number of PCWs taken from Consumer Focus (2013) report into PCWs in the UK. The 'big four' refers to Money Supermarket, Compare the Market, Go Compare and Confused.com. Number of PCWs Usage data from the 2013 Mintel Report on Web Aggregators. Financial information taken from the companies' own annual reports where available, otherwise inferred from parent group reports or newspaper articles. See appendix for details.

[5]For example, see BBC (2015). Fees are significant in other sectors too e.g., in the hotel-reservation sector they are reported to be 15-25% of the purchase price (see Daily Mail, 2015).

[6]In practice, PCWs in some markets charge per-click, but many per-sale. I abstract from this difference and

industry gleans substantial profits from these fees. As such, it is not clear whether the central premise that PCWs lower prices is valid. This fundamental tension is encapsulated in a quote from the BBC (2014):

> "There's another cost in the bill. It's hidden, it's kept confidential, and yet it's for a part of the industry that appears to be on the consumers' side. This is the cut of the bill taken by price comparison websites, in return for referring customers. The recommendation to switch creates churn in the market, and it is seen by supplier companies as worth paying high fees to the websites. Whether or not customers choose to use the sites, the cost to the supplier is embedded within bills for all customers."[7]

This article examines this "churn" and addresses the fundamental question of when consumers are better off with a PCW in the marketplace. In homogeneous good markets, I characterize when all consumers are made worse off following the introduction of PCWs.

My model builds on the elegant framework of Baye and Morgan (2001), who investigate the strategic incentives of a PCW or 'information gatekeeper'. Without the PCW in their model, each consumer is served by a *single* 'local' firm which sells at the monopoly price (it is too costly for consumers to travel to another store). This leads to the result that consumers benefit from the introduction of a PCW, because firms must compete for the business of consumers who enjoy free access to the site. In the modern online marketplace however, firms also have their own websites.[8] In the absence of an aggregator, consumers do not need to physically travel to purchase the good; they can visit another firm's website just as easily as they could an aggregator's.

My model features two types of consumers: shoppers, who use PCWs in equilibrium, and

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model the latter, assuming that the two are correlated.

[7]The concern was also expressed by US senator Amy Klobuchar regarding mergers of hotel-reservation sites: "The whole idea of cheaper hotels is very good, but if it all starts to come under one company, you can easily foresee the situation where they can charge higher commissions that are then passed on to consumers." (New York Times, 2015)'

[8]Often, a PCW simply re-directs users to the selected firm's own website to complete the purchase.

inactive consumers, who buy directly from a particular firm. Although shoppers are always better off than inactive consumers in equilibrium, my primary result is that *both* types are always made worse off by the introduction of a single PCW. I then provide conditions under which consumers are harmed by the introduction of multiple competing PCWs. This is the first article in this setting to show such results, reversing those in the existing literature, which I show can be seen as a special limiting case.[9] My model supposes each shopper is informed of the prices from $q > 1$ of the n firms' websites in the absence of a PCW, rather than $q = 1$ as in Baye and Morgan. Without a PCW, shoppers see q prices; after the introduction of the PCW, they will see all of them in equilibrium. I show that adding competition ($q > 1$) to the setting without an aggregator reverses their finding: expected equilibrium prices are raised by the introduction of a single PCW, making all consumers worse off.

What happens is that the equilibrium fee a single PCW charges for a sale through its site is so high that it more than negates any benefits from the increased firm competition. One may conjecture that allowing multiple competing aggregators will undo this result, akin to textbook Bertrand competition. I extend the model to allow for multiple PCWs, and for shoppers to check any number of them. My characterization shows that both the number of PCWs and the number of PCWs that shoppers check (or are informed of) matter. Concretely, when shoppers only check one of many PCWs, they all effectively remain monopolists and so consumer welfare does improve with any number of PCWs. At the other extreme, where shoppers check all PCWs, Bertrand-style reasoning at the aggregator level results as a special case: PCWs undercut each other's fees to reach a unique zero-profit equilibrium and shoppers benefit from their existence. When shoppers visit some, but not all PCWs, consumer welfare tends to rise with the number of PCWs checked, but it falls with the number of PCWs. In particular, in this realistic scenario, there is a critical number of PCWs beyond which all consumers can be worse off than without any aggregators at all.

I then investigate the impact of different policies and other market features. Generally, PCW fees are either not disclosed or are detailed on a subpage of the websites. When PCWs

[9] This is in the absence of any persuasion or direction of consumers to more expensive products by firms (e.g., Armstrong and Zhou, 2011), or biased intermediaries (e.g., de Cornire and Taylor, 2014).

publicly announce fees so that consumers are made aware of them, I show that this creates the possibility that firms and shoppers could coordinate on the cheapest PCW, which can result in equilibria that benefit shoppers relative to a world without PCWs.

My primary focus is on settings where a firm sets the same price for a product on every website that it is sold. This assumption describes many markets where PCWs operate including gas, electricity, mortgages and durable goods. However, I extend the analysis to consider markets such as the hotel reservation sector, where a firm's price can differ across sites. I show that both types of consumers can be worse off when shoppers only check one PCW in equilibrium. However, unlike markets without price discrimination, when shoppers check at least two PCWs, competitive pressure between aggregators can work à la Bertrand. I also consider markets where consumers may face some non-negligible search cost in order to retrieve prices e.g., home insurance quotes. Here, the number of shopping consumers is endogenously determined by the distribution of search costs and the expected savings consumers can make by using it. I show that despite this activity at the 'extensive search margin', an aggregator can still make all consumers worse off, including those who decide to start shopping.

Section 2 reviews the literature; Section 3 presents the model; Section 4 conducts comparative statics with a monopolist PCW; Section 5 models competing aggregators; Sections 6-8 consider settings with publicly observable fees, price discrimination and search costs; Section 9 concludes.

2. Literature

This article contributes to the literature on 'clearing-house' models, see for example (Salop and Stiglitz, 1977; Varian, 1980; Rosenthal, 1980; Baye and Morgan, 2001, 2009; Baye et al., 2004; Chioveanu, 2008; Arnold et al., 2011; Arnold and Zhang, 2014). These models rationalize price-dispersion in homogeneous goods markets.[10] Indeed, price dispersion has persisted

[10]The search cost literature also provides explanations of price dispersion (e.g., Burdett and Judd, 1983; Ellison and Ellison, 2009; Ellison and Wolitzky, 2012; Stahl, 1989; 1996; Stigler, 1961). Motivated by the rise of the internet, clearing-house models abstract from a direct modeling of consumer search costs. The frameworks are to some extent isomorphic (see Baye et al., 2006).

despite the advances of technology such the internet and comparison sites. Early studies documented marked dispersion in the online markets for goods (e.g., Brynjolfsson and Smith, 2000; Baye et al., 2004). A recent study by Gorodnichenko et al. (2015) finds substantial cross-seller variation in prices, and voices support for clearing-house models that categorize consumers into loyal and shopping consumers. The equilibria in my model feature price dispersion regardless of whether there is an aggregator (or 'clearing-house'). Without a PCW, this is because there is *some* consumer search. Producing price dispersion with a PCW that employs the pricing mechanism seen in practice, is a challenge. The shift in the aggregator industry away from charging one-off fixed fees, toward pay-per-sale fees to firms is rationalized as profit-maximizing PCW behavior by Baye et al. (2011). However, without the introduction of some other exogenous fixed cost to a firm of listing on the PCW (e.g., transaction costs), price-dispersion vanishes in equilibrium. I do not deny the existence of such additional costs, but emphasize that dispersion arises in my framework without appeal to fixed listing costs.

The larger relevant literature is that of two-sided markets, pioneered by Rochet and Tirole (2003) (see also Caillaud and Jullien, 2003; Ellison and Fudenberg, 2003; Armstrong, 2006; Reisinger, 2014). These articles model platforms where buyers and sellers meet to trade, focusing on platform pricing and the effect of network externalities with differentiated products and platforms. These models do not explicitly model seller-side competition, which is central to my setting. More recent contributions do (e.g., Belleflamme and Peitz, 2010; Hagiu, 2009) but they model the platform as the only available technology, which is not appropriate for the questions I address. Edelman and Wright (2015) allow sellers and buyers to conduct business off platform, but they study environments with differentiated product markets, where the intermediary directly offers buyer-side benefits such as rebates. In contrast, I model a homogeneous good, isolating price as the determinant of consumer welfare, where the only benefit a 'platform' brings is informational: it lists available prices. The potential benefit of a PCW to consumers is that it can lower prices via the interaction of strategic, competing firms, and hence the size of the benefit is determined endogenously via the equilibrium actions of firms and consumers.

3. Model

A World Without a PCW

There are n firms and a unit-mass of consumers. Firms produce a homogeneous product at zero cost without capacity constraints. Consumers wish to buy one unit and have a common willingness to pay of $v > 0$. Each consumer is endowed with a 'default', 'current' or 'preferred' firm from which they are informed of the price. This assumption has many natural interpretations. In a market for services or utilities (e.g., gas and electricity tariffs, mortgages, credit cards, broadband, cellphone contracts, car, home and travel insurance etc.) consumers can be thought of as having a current provider for the service for which they know the price they pay and the renewal price should they remain with the same provider.[11] In the market for flights, hotels or durable goods, consumers can be thought to have a carrier, hotel (or hotel chain) or producer which they prefer, perhaps bookmarked in their browser or for which they receive marketing emails that directly inform them of the price or prompt the user to find out via hyperlinks, in a matter of seconds.

A proportion $\alpha \in (0, 1)$ of consumers are 'shoppers'. Casual empiricism suggests that many people enjoy browsing or looking for a bargain. Shoppers check (are aware of) at least one rival firm's website and therefore know $q > 1$ of the n prices. A shopper sticks with his or her default firm if its price is not beaten by another price. If a rival firm's price is cheaper, the shopper switches. The remaining proportion of $1 - \alpha$ consumers are 'auto-renewing', 'loyal', 'offline' or 'inactive' consumers who do not shop around. In markets where firms are service providers, such consumers simply allow their contract with their existing provider to continue with their default firm. Much of the furore surrounding PCWs has been directed at those operating in the services and utilities sectors. In the presentation that follows I adopt terminology suited to that sector, of 'current' firms or providers with consumers referred to as 'shoppers' and 'auto-renewers'.

I assume that each firm has an equal share of current consumers of each type ($\frac{\alpha}{n}$ shoppers and $\frac{1-\alpha}{n}$ auto-renewers). Firms set prices and shoppers simultaneously decide which rival-

[11] Providers usually provide renewal price information directly to consumers.

firm websites to visit. I focus on equilibria in which firms adopt identical pricing strategies and shoppers employ symmetric shopping strategies. Going forward I refer to such symmetric equilibria simply as equilibria.

This setup generalizes Varian (1980), nesting his equilibrium.[12] He motivates the two types as being completely 'informed' about all n prices, or 'uninformed'. The informed buy the cheapest on the market, while the uninformed buy from their default firm. In my model, shoppers see q prices where $1 < q \leq n$ and I derive the unique equilibrium. When $q = n$, Varian's equilibrium corresponds to that derived in Proposition 1 below.

Shoppers can be characterized by $\binom{n}{q}$ groups. Each group is a list of the firms checked by that consumer. For example, consider $q = 2$, $n = 4$ with firms indexed 1,2,3,4. Then there are 6 possible comparisons shoppers could make: $\{12, 13, 14, 23, 24, 34\}$, where the two digits refer to which firms' prices are checked. Each firm is involved in $\binom{n-1}{q-1} = 3$ price comparisons. Shoppers employ symmetric shopping strategies, and hence are evenly distributed across these groups: $\frac{1}{6}$ compare the prices of firms 1 and 2, $\frac{1}{6}$ compare firms 1 and 3, and so on. Equivalently, one could interpret a consumer as randomizing uniformly over which rival-firm websites she or he checks.[13]

Now consider the best-response of firms. Without loss of generality, let $F : p \to [0, 1]$ denote the cumulative distribution function of prices charged by firms in equilibrium. Proposition 1 describes the equilibrium without a PCW.

Proposition 1. *In the unique equilibrium firms mix according to the distribution*

$$F(p) = 1 - \left[\frac{(v-p)(1-\alpha)}{qp\alpha}\right]^{\frac{1}{q-1}} \text{ over the support } p \in \left[\frac{v(1-\alpha)}{1+\alpha(q-1)}, v\right].$$

Price dispersion is a central feature in clearing-house models and showcases how they rationalize price dispersion in homogeneous goods markets. The limited-search assumption does not alter the intuition for why. Bertrand-style reasoning of undercutting down to the marginal

[12] The only differences being that I assume zero costs and do not employ his zero-profit condition.

[13] Although I focus on a symmetric distribution across these pairs, this shopper behavior is not an assumption per se; it is a best response in equilibrium to symmetric firm behavior because consumers are then indifferent to which firm they check.

cost (here normalized to 0) does not play out. A firm can guarantee itself a profit of at least $\frac{v(1-\alpha)}{n} > 0$ from its auto-renewers. Therefore, any point mass in equilibrium strategies must be for some $\dot{p} > 0$. Any such mass would always be undercut by firms to gain a discrete number of shoppers for an arbitrary ϵ-loss in price.

Models building on Varian's assume that in a world without a clearing-house, consumers cannot check other prices, so the pure monopolistic-price equilibrium of $p = v$ results. As in Stahl (1989), were there no shoppers, I would obtain such a Diamond (1971) equilibrium with each firm charging v; and were there only shoppers, then the Bertrand outcome of $p = 0$ would result. Unlike Stahl, shoppers do not necessarily know the prices of all firms, but they know at least two. Proposition 1 shows that with some (but non-zero) search, price dispersion still emerges in equilibrium.

It is also instructive to note that equilibrium pricing does not vary with the number of firms, n, as long as $q < n$. Each shopper compares q prices, regardless of the total number of firms. When making pricing decisions, a firm is not concerned about the number of other firms per se, but rather about the other prices shoppers know. As all firms price symmetrically and independently in equilibrium, it is as if each firm only faces $q - 1$ rivals. In other words, what matters is the number of comparisons shoppers make, not the number of firms in the market.

A World With a PCW

Suppose an entrepreneur creates a price comparison website. I add a preliminary stage to the game at which the PCW sets a 'click-through fee' $c \in \mathbb{R}_+$ that a firm must pay to the aggregator per sale made via the site. Consumers do not learn c, although they will have correct expectations in equilibrium.[14] Each firm sees the fee c and must choose a price, and whether or not to post it on the PCW.

Models with a clearing-house that does not charge a fixed fee typically have many equilibria, see for instance Baye et al. (1992) for their characterization of the full set of equilibria of Varian (1980). Throughout the paper, I focus on symmetric equilibria in which shoppers only

[14]As the BBC quote in the Introduction notes, the exact fee is "kept confidential", not publicly announced.

check PCWs.[15] With a monopoly PCW, this results in a unique equilibrium. In this equilibrium, firms list on the PCW with probability one and prices are dispersed, providing shoppers a strict incentive to check the PCW.[16]

To find the equilibrium, I use the following lemma which takes c as fixed and characterizes the ensuing mutual best-responses of firms. Define $G(p;c)$ to be the cumulative distribution function of prices charged by firms for a given click through fee c:

$$G(p;c) = 1 - \left[\frac{(v-p)(1-\alpha)}{\alpha(np-(n-1)c)}\right]^{\frac{1}{n-1}}$$

Lemma 1. *The mutual best-responses of firms as a function of c:*

1. *If $c \in [0, v(1-\alpha))$, firm best-responses are described by $G(p;c)$, and have no point masses.*

2. *If $c = v(1-\alpha)$, there are two classes of responses, one with no point masses described by $G(p,c)$, and those in which all firms charge the same price.*

3. *If $c \in (v(1-\alpha), v]$, all firms charge the same price.*

Where pricing is described by $G(p;c)$ each firm always lists its price on the PCW.

When c is in the lower interval described by Lemma 1, firms undercut each other until the point at which they would be better off charging the maximum price v and only selling to their auto-renewers. However, if c exceeds $v(1-\alpha)$, undercutting does not reach this point, so no firm would jump to v. Rather, it reaches a point where all firms charge the same price, selling to all their consumers directly; no firm undercuts further or jumps to v. The reason no further undercutting occurs is that doing so would mean charging a price $p < c$ that would win all rival firms' shoppers, but would do so at a loss. The reason no firm jumps to v is that they are

[15] Recall this is in addition to knowing their current or preferred firm's price, which upon sale, continues to count as a direct purchase e.g., the auto-renewal letter, the bookmarked airline or the hotel's marketing email.

[16] More generally there exist other equilibria e.g., trivial equilibria in which no firms or shoppers attend the PCW. The set-up is then identical to that of the previous section and the equilibrium is given by Proposition 1 with the vacuous addition that the PCW can charge any c. There also exist asymmetric equilibria where not all firms list on the PCW (see Footnote 17).

making more in this pure-pricing equilibrium where they sell to all their consumers directly. If $c = v(1-\alpha)$ then both mixed and pure equilibria obtain as threshold cases.

To derive the equilibrium fee set by the PCW, consider its incentives. The PCW will make zero profit if firms all charge the same price, as then no shoppers switch. In contrast, the PCW earns positive profit in any mixing equilibrium wherever $c > 0$. The PCW thus has a strong incentive to induce price dispersion because shoppers switch when they can obtain a strictly lower price with a new firm. A feature of distributions with no point masses is that the probability of a tie in price is zero. As a result, shoppers at $n-1$ of the n firms will switch. Given that firms mix in this way, the PCW will raise c as high as possible before reversion to a pure equilibrium, which here happens for $c > v(1-\alpha)$. Proposition 2 characterizes the equilibrium.[17]

Proposition 2. *In the unique equilibrium where shoppers check the PCW, the PCW sets a click-through fee of* $c = v(1-\alpha)$*, firms list on the PCW and mix over prices according to* $G(p; v(1-\alpha))$ *over the support* $p \in [v(1-\alpha), v]$.

That only pure equilibria exist for $c > v(1-\alpha)$ is what limits the PCW fee and allows there to be price dispersion at equilibrium without fixed costs for firms. It is the outside option of current shoppers for firms that limits the fee that the PCW charges. Shoppers know their current provider's price regardless of where they check prices and stay with their current firm if there are no lower prices to be found on the PCW. Hence, a firm can always guarantee itself its own shoppers and avoid the aggregator's fee by charging a price low enough to undercut the market and not list on the PCW. It is this threat that discourages the PCW from charging fees that are even higher in equilibrium. If firms did not have this outside option, the PCW could raise its fee to $c = v$, firms would charge $p = v$, price dispersion would be lost and the PCW would be able to extract all the surplus.

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[17] Note that in the symmetric equilibrium, all firms list on the PCW. In practice, some firms do not always list on PCWs. Note that here there exist asymmetric equilibria where $m \geq 2$ firms list on the PCW, mixing over prices $[v(1-\alpha), v]$ in a similar way to the CDF of Proposition 2, with the other $n-m$ do not list, charging v and selling only to their auto-renewers with all shoppers uniformly spread over the mixing firms. The price-rising result of this article also holds in any of these asymmetric equilibria.

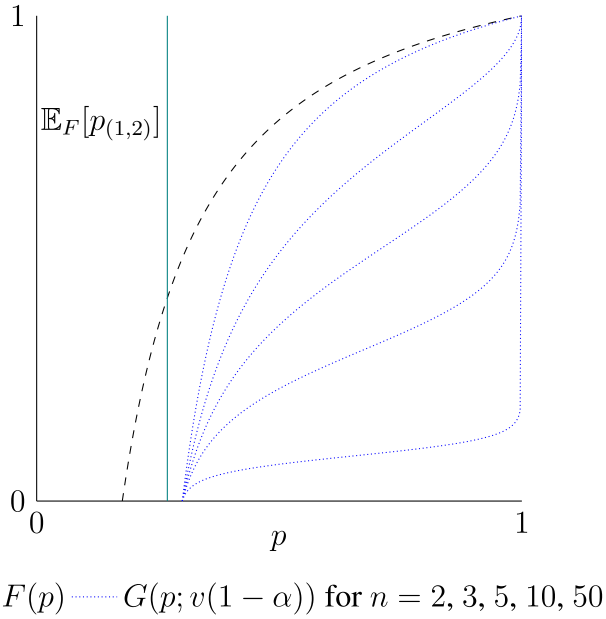
4. Comparative Statics

Comparing the equilibria of Propositions 1 and 2 reveals how the PCW affects consumer welfare.

Proposition 3. *Both types of consumer are worse off with the PCW than without.*

The key to the proof is to show that the expected shopper-price under F is *less* than the *lower* bound of the support of G (see Figure 1). It immediately follows that shoppers expect to pay more under G, as the expected lowest price is even higher. That G first-order stochastic dominates F shows that auto-renewers expect to pay more under G, as the expected price is higher.

Figure 1: Equilibrium price distributions calibrated with $q = 2$, $v = 1$, $\alpha = .7$



With the introduction of a PCW, the two effects stated in Corollaries 1 and 2 give rise to Proposition 3:

Corollary 1. *Within the mixed-price equilibrium firm responses of Lemma 1, as* $c \in [0, v(1-\alpha)]$ *increases, the expected price paid by both types of consumer increases.*

Corollary 2. *As the number of firms increases, the expected price paid by shoppers falls, but the expected price paid by auto-renewers rises.*

Corollary 1 shows that when the PCW sets a higher fee, the expected price paid by both shoppers and auto-renewers rise. The fee is passed on by firms to consumers through a first-order stochastic shift in the prices set in equilibrium. Upon winning, a firm must pay the PCW for all of the $(\frac{n-1}{n})$ shoppers who purchase through the site. Because the amount paid rises with the fee, the price charged in equilibrium also rises with the fee.

The second effect is that the PCW increases competitive pressure among firms to fight for all $n-1$ rival firms' shoppers. In contrast, without the PCW, firms effectively competed against only $q-1$ rivals. Different models in the clearing house literature offer different predictions about the effect of n on equilibrium prices. Some derive distributions for which an increase in n raises prices for both types of consumer (e.g., Rosenthal, 1980); while in other models it lowers prices for shoppers and raises prices for captive consumers (e.g., Varian, 1980; Morgan et al., 2006).[18] My model belongs to this second category. An increase in n has two effects on equilibrium prices. First, increased competition pushes probability mass to the high-price extreme of the distribution, as Figure 1 shows. This results in a first-order stochastic ordering in n: expected price thus increases in n and auto-renewers pay more. Second, shoppers now pay the lowest of $n+1$ prices rather than n, which reduces the expected lowest price. Corollary 2 reveals that this second effect more than offsets the first, implying that shoppers pay less in expectation when there are more firms.[19]

In order to relate these two effects of a PCW to Proposition 3, I make the following remark.

Remark 1. *If* $q = n$, $G(p; 0) = F(p)$

The entrance of a PCW increases both the fee firms pay (from 0 to $v(1-\alpha)$) and the number of prices that shoppers compare (from q to n). The first effect raises the expected price auto-

[18]Also see Baye et al. (2006) for a discussion on this point and a wider review of clearing-house models.

[19]Although the result of Corollary 2 is common to clearing-house models, it may seem a rather nuanced prediction of equilibrium pricing. Morgan et al. (2006) conduct an experiment with participants playing the role of firms against computerized buyers and found that when n was increased, prices paid by inelastic consumers indeed increased whereas those paid by shoppers decreased.

renewers pay, which is compounded further by $n > q$, so they are unambiguously worse off with a PCW. The effects pull shopper welfare in opposite directions, but Proposition 3 shows that no matter how large n is, it fails to undo the effect of the optimally chosen PCW fee c. Hence, shoppers are also worse off in expectation with a PCW.

Due to the constant-sum nature of the game, welfare necessarily sums to v in equilibrium. As firms make the same expected profit in both worlds, there is a one-to-one relation between the decrease in consumer welfare and the profits of the PCW. That the PCW does not reduce firm profits comes from the fact that (with or without the PCW) firms have $\left(\frac{1-\alpha}{n}\right)$ auto-renewers. Firms can therefore guarantee themselves $v\left(\frac{1-\alpha}{n}\right)$ in both worlds. Although this article focuses on consumer rather than producer welfare, it is important to point out that the incentives of the PCW and firms are not aligned. This is because there is exactly one cheapest price and hence $\alpha\left(\frac{n-1}{n}\right)$ shoppers who switch from non-cheapest prices, which increases with n. An increase in n however, would of course, squeeze per firm profit. Therefore, a PCW would always encourage market entry if it could.

One channel through which all consumers would gain is by more auto-renewers becoming shoppers:[20]

Corollary 3. *As the proportion of shoppers α increases, expected prices paid by shoppers and auto-renewers both decrease.*

One may conjecture that a PCW would want to maximize α (the number of shoppers) in order to obtain more referral fees. However, this logic is incomplete. Expanding the PCWs action set to include the determination of α (one can think of the PCW determining α through advertising) yields the following result:[21]

Corollary 4. *If the PCW can determine α as well as c in the preliminary stage, then the PCW sets $\alpha = \frac{1}{2}$.*

[20]This is a prediction common to clearing-house models, for which Morgan et al. (2006) find experimental support.

[21]α is determined costlessly for the PCW. If this advertising costs were a convex function of α, it would not change the results qualitatively.

PCW revenue is hump-shaped in profit. As $\alpha \to 0$ it receives less and less traffic, and hence vanishing revenue. As $\alpha \to 1$, firms have fewer auto-renewers to exploit, which pushes $v(1-\alpha)$, the maximum fee for which firms are willing to mix, to zero. Indeed, when $\alpha = 1$ all consumers are shoppers and the PCW removes any incentive for firms to increase prices as there are no auto-renewers to exploit. As a result, all firms charge the same price, leaving the PCW with zero profit. Thus, even if the PCW could bring all consumers online, it has a strict incentive to ensure that only some do so.

5. Competing Aggregators

Now suppose there are $K > 1$ PCWs indexed by $k = 1, ..., K$. Each PCW moves simultaneously in the first period with PCW$_k$ setting a fee c_k. A crucial measure of competitive pressure is the number, r, of aggregators shoppers check where $1 \leq r \leq K$. In the setting without PCWs, shoppers know the prices of $q \leq n$ firms. In the world with PCWs, one can interpret $r \leq K$ as the number of aggregators that shoppers are aware of. In the equilibria derived, firms list on all PCWs so shoppers are indifferent to which set of PCWs they check. However, PCW fees and firm-pricing strategies depend on r. Hence, there are different sets of equilibria for each r. This leads me to index equilibria by r. I first consider the case where shoppers check just one of the PCWs.

Proposition 4. *With $K > 1$ competing PCWs, if shoppers check $r = 1$ PCW then both types of consumer are worse off with the PCWs than without.*

Proposition 4 obtains because the high-fee, single-PCW equilibrium of Proposition 2 with $c = v(1-\alpha)$ remains the unique equilibrium fee.[22] Introducing competing PCWs exerts no downward pressure on fees. There are no equilibria at lower fee levels because shoppers only check one PCW, which means there is no incentive for PCWs to undercut each other's fees. If they did, they would not increase their volume of sales, but they would receive lower fees. In contrast, for any candidate equilibrium with $c < v(1-\alpha)$, there is an incentive to *raise* the fee because PCWs can maintain the same volume of sales and earn a higher fee.

[22] As in the monopolist case, firms' outside option gives rise to pure-pricing firm responses at higher fee levels, which bounds equilibrium fee levels from above at $c = v(1-\alpha)$; see the Appendix for details.

In the simplest textbook Bertrand result, there is an immediate fall in the equilibrium price between that of a monopoly firm and the marginal-cost pricing of two firms. At the aggregator level however, Proposition 4 shows that this logic is incomplete: with $K > 1$ such undercutting does not even necessarily begin. The persistence of the consumer-welfare-decreasing equilibrium in Proposition 4 is an artefact of each shopper only checking one PCW which effectively makes each PCW a monopoly, facing no competitive pressure. Because all PCWs offer the same information, shoppers have no incentive to check other PCWs. In this respect, the equilibrium is reminiscent of Diamond (1971), but at the aggregation level, not the firm level. One might think that the Bertrand remedy for shoppers would be to require them to check at least two PCWs, causing PCWs to undercut each other until an equilibrium with all PCWs charging $c = 0$ is reached. I now explain how this logic is incomplete. Suppose now that there are $K > 1$ aggregators, and shoppers check $r > 1$ PCWs. First, I examine the special case of $r = K > 1$:

Proposition 5. *With $K > 1$ competing PCWs, shoppers are guaranteed to be better off than before the introduction of PCWs if $r = K > 1$. Further, the unique equilibrium PCW fee-level is $c = 0$ if and only if $r = K > 1$.*

The spirit of this result resembles that of Bertrand. To see why there cannot be some other equilibrium with $c > 0$ when $r = K > 1$, suppose so and consider an undercutting deviation by PCW_1 to some $\hat{c}_1 = c - \epsilon$. Shoppers do not detect the deviation and so do not change their behavior. As for firms, notice that for any p they strictly prefer to list exclusively on PCW_1: When a firm is the cheapest, it will sell to *all* shoppers so long as it lists on *some* PCW. This is precisely because $r = K$. Hence by listing on PCW_1 only, there is no reduction in the number of shoppers switching to them when they are cheapest, but there is a reduction in the fee the firm pays as $\hat{c}_1 < c$. The PCW finds this deviation strictly profitable because it receives a discrete gain in the number of shoppers switching through it, for an arbitrarily small loss in price.

Where shoppers check more than one PCW, but not all PCWs, we have:

Lemma 2. *When $K > r > 1$, there exists an equilibrium in which PCWs charge $\bar{c} > 0$, and firms list on all PCWs, mixing over prices by $G(p; \bar{c})$ where*

$$\bar{c} = \frac{v(1-\alpha)Kr(K-r)}{K(1+r(K-2)) + \alpha r(K-1)(r-1)(n-1)}.$$

More generally, there exist equilibria in which all PCWs charge $c \in [0, \bar{c}]$. To understand how much consumer welfare can be reduced, I analyze the highest-fee equilibrium from this set. Because PCWs have a strong incentive to coordinate on this equilibrium, it may be especially relevant in practice.

Notice that substituting $r = K$ in Lemma 2 yields $\bar{c} = 0$, and one obtains Proposition 5. One can see now that the Bertrand-style reasoning underlying Proposition 5 was a special case. To understand why the principle does not apply more generally, consider a fee level $c > 0$ and an undercutting deviation by PCW_1 to some $c_1 = c - \epsilon > 0$. Unlike when $K = r > 1$, when $K > r > 1$ it is not necessarily better for a firm to only list on the cheaper PCW_1. By listing a price exclusively on PCW_1, there are now $\frac{K-r}{K} > 0$ shoppers who do not see the firm's price. These shoppers will not buy from it even if it is the cheapest. Firms now face a trade-off: Exclusively listing on PCW_1 means that any sales incur only the lower fee $\hat{c}_1$ upon a sale, but there will be a reduction in sales volume because not all shoppers check PCW_1. Which force is stronger in this trade-off depends on the size of the undercut ϵ. If PCW_1 undercuts by enough, firms will deviate to list exclusively on PCW_1, breaking the symmetric equilibrium. Unlike the simpler logic of Proposition 5, it is no longer true that any $\epsilon > 0$ undercut will attract firms to exclusively list on the cheapest PCW. Hence, PCWs do not always have an incentive to undercut each other and higher-price equilibria are sustained.

I now discuss how the set of equilibria varies with how many PCWs shoppers check (r) and the number of aggregators (K). Firstly, as shoppers check more PCWs in equilibrium, the incentive for a PCW to undercut the fees of other PCWs increases so that only lower fee-levels can be sustained in equilibrium. That is, $\bar{c}$ is limited by a higher r: $\frac{d\bar{c}}{dr} < 0$

However, as the number of aggregators increases, the incentive is reversed. The number of shoppers checking a given PCW ($\frac{r}{K}$) falls. Accordingly, in equilibrium each firm receives less of its expected revenue from any single PCW. It then requires a more severe undercut from a PCW to get firms to exclusively list on it and forgo the business available from the other aggregators. For undercuts that are too severe, it is unprofitable for a PCW to deviate, even if it were to win exclusive arrangements with all firms as a result. Thus, as K increases, higher equilibrium fees can be sustained in equilibrium: $\frac{d\bar{c}}{dK} > 0$. This allows for the result that a higher number of aggregators can lead to higher fees, and hence higher prices. Furthermore:

Proposition 6. *For any r, there exists a $\tilde{K}$ such that as long as there are more than $\tilde{K}$ aggregators both types of consumer are worse off than before the introduction of PCWs.*

In the limit, the proportion of firm income that comes from sales on any one aggregator becomes vanishingly small. As this happens, $\bar{c} \to v(1-\alpha)$ i.e., sustainable equilibrium fee levels approach the monopoly-PCW level, again making both types of consumer worse off than before the introduction of the sites.

6. Publicly-Observable Fees

One reason that competing aggregators do not drive fees to zero is that shoppers do not detect changes in the fees set by PCWs in equilibrium. This precludes a coordinated response between firms and shoppers that could punish a PCW that charges higher fees. If fees were publicly announced so that shoppers were aware of them, then credible subgame equilibria could follow the fee-setting decision in which the PCW charging the lowest fee is attended by all firms and shoppers. Any higher-fee equilibrium would then be undercut until $c = 0$, leading to lower shopper-prices. It follows immediately that:

Proposition 7. *When $K > 1$ and PCW fees are observed by shoppers, there exists an equilibrium with $c = 0$.*

There are multiple equilibria because there are many subgame equilibria that can follow any vector of PCW fees, including less intuitive ones where coordination occurs at more expensive PCWs.[23] If one adopts the plausible refinement that firms and consumers only patronize the lowest-fee PCWs, then one obtains the zero-fee equilibrium as the unique equilibrium.

However, even with sufficient refinement criteria to implement the zero-fee equilibrium, in some markets one may question a policy of fee-announcements on a more fundamental level.

[23]Proposition 7 excludes the monopoly-PCW case ($K = 1$), where the unique equilibrium is still that of Proposition 2 because of course no coordination between firms and consumers over which PCW to attend is possible when there is only one PCW.

If fees can be publicly announced, then surely so can firm prices, which would extinguish the role of a PCW in the first place.

In reality, PCW fees are not publicized directly. However, some PCWs do advertise summary statistics of the price information of firms that list on them. PCWs frequently advertise the average savings a consumer using their site is expected to make, which could direct shoppers to the cheapest PCW. By Proposition 7, this could lead to a shopper-welfare-improving equilibrium. However, many PCWs do not advertise based on purchase-relevant information.[24] PCWs spend large sums on such advertising, which has been shown to correlate with the number of unique visitors they experience, suggesting that many shoppers are directed to PCWs based on information other than price.[25] If such 'persuasive' advertising caused all shoppers to loyally visit one site each i.e., $r = 1$, Proposition 2 applies and all consumers would have been better off without the PCW industry.

7. Price Discrimination

So far, I have considered the impact of web services that list or 'aggregate' the available information (prices) charged by firms offering a product or service. In practice, this is often the case in the markets for gas, electricity, financial products such as mortgages, and durable goods.[26] However, in other markets, a firm may set a price p_0 for a direct purchase, and p_k different prices for each PCW$_k$ that it lists on. Where a PCW operates by referring shoppers back to a firm's website to complete the purchase, the fact that the click came from a PCW is recognized by the firm's site, which then offers the price seen on the PCW that attracted the click. When $K \geq r = 1$, we have:

Proposition 8. *With price discrimination, if $r = 1$, there exists an equilibrium in which PCWs set $c = v(1 - \alpha)$, firms list on all PCWs, $p_0 = v$ and $p_1 = \cdots = p_K = v(1 - \alpha)$.*

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[24]See the campaign of Comparethemarket.com, based on a story about meerkats.

[25]The big four PCWs in the UK spent approximately £110m in 2010 on advertising. The evidence here is from Nielsen Company (findings reported by the This is Money 2015 and Marketing Magazine 2011).

[26]For UK gas and electricity markets, regulation limits each energy company to offering a maximum of four tariffs in total.

The ability of firms to price discriminate does not prevent PCWs from setting fees at the same high level as in Proposition 4 because $r = 1$. As before, there is effectively no competitive pressure between PCWs. However, price discrimination does lead to firms listing a common price on PCWs in equilibrium because PCWs no longer have an incentive to keep prices posted on it dispersed. This is because firms can set a high direct purchase price $p_0 = v$ and a lower price through the PCWs. Given this, shoppers always purchase through a PCW. Because all α shoppers now purchase through PCWs, rather than in the case without discrimination, where only $\alpha\left(\frac{n-1}{n}\right)$ did so, total PCW profit is now $\alpha v(1-\alpha)$ rather than $\left(\frac{n-1}{n}\right)\alpha v(1-\alpha)$. As firm profit is unaffected, it follows then that relative to the case without price discrimination, total consumer welfare falls. There are, however, opposing effects on shoppers and auto-renewers. Auto-renewers are worse off, as firms charge them the monopoly price v. Shopper welfare though improves as they now face $p_k = v(1-\alpha)$ for sure, whereas before this was just the minimum of the support of equilibrium prices. More importantly, shopper welfare does not improve sufficiently to overturn Proposition 3, which continues to hold: all consumers are worse off than in a world with no PCWs. The ability of firms to discriminate allows them to fully extract surplus from their captive auto-renewers; but PCWs can now extract the revenue from sales to all shoppers through their sites.[27]

In equilibria with $r > 1$, the incentive for aggregators to undercut is present. Corollary 5 describes a best response of firms following a unilateral downward deviation by a PCW$_1$ from a symmetric fee level.

Corollary 5. *When firms price discriminate, following $c_1 \in (0, c_2)$ and $c_2 = c_3 =, \ldots, = c_K \equiv c \in (0, v(1-\alpha)]$ there exist mutual best-responses of firms such that they list on all PCWs, setting $p_0 = v$ and $p_k = \cdots = c_k$ for all k.*

When $r = 1$, there is no incentive for a PCW to make such an undercutting deviation as suggested by the equilibrium described in Proposition 8. When $r > 1$ however, PCWs have this incentive to undercut because they can enjoy a discrete gain in fee revenue from the $\frac{r-1}{K}$

[27] As before, in the equilibrium of Proposition 8, PCWs cannot raise fees further, to say $c' > v(1-\alpha)$ because of firms' outside option. Following such a unilateral PCW deviation, firms would set $p_0 = v(1-\alpha)$ so that their shoppers purchase directly from them, reducing PCW profit to zero.

of shoppers who were checking their PCW but buying though another site in the symmetric equilibrium. When firms can price discriminate across websites, firms can compete in prices on PCW_1 without changing their prices on other PCWs. This shows how price discrimination can unleash undercutting at the PCW level whenever consumers check $r > 1$ PCWs, which can in turn lead to zero-fee equilibria.[28] This contrasts with markets where PCWs aggregate price-information, where Proposition 5 showed that this undercutting was only fully unlocked when $r = K$.

Aggregation and Discrimination in Large Markets

In the setting with a PCW and price discrimination, the equilibrium of Proposition 8 shows the price paid by the two types are as maximally separated: Shopper price is competed down to firms' marginal cost c, and auto-renewers pay v. In the setting with an aggregator and no price discrimination, the equilibrium is given by Proposition 2 where Corollary 2 explains that as the number of firms increases, the expected prices paid by shoppers and that paid by auto-renewers, diverge. This occurs because as the number of firms increases, so does the competitive pressure on pricing to win shoppers. As a result, more probability mass is placed on lower prices. Firms compensate for this by also increasing the mass placed on higher prices, increasing their expected profit from auto-renewers. In fact, for arbitrarily large markets, I show that these two settings are equivalent.

Proposition 9. *As $n \to \infty$, following the introduction of a PCW, the expected prices faced by both types of consumer in a setting without price discrimination (Proposition 2) approach those in a setting with price discrimination (Proposition 8).*

The result highlights the connection between the two market structures. Indeed, I find that all consumers can be worse off with a PCW with or without the possibility of price discrimination. I emphasize therefore that the key difference between the settings lies in their predictions under competition at the aggregator level.

[28]The existence of zero-fee equilibria when $r > 1$ are shown in the Appendix.

8. The Extensive Search Margin

This paper utilizes a clearing-house framework, where auto-renewers are inactive and can also be interpreted as being offline, loyal, uninformed or as having high search costs. Here, I focus on a search-cost rationalization, better applied to markets where obtaining a quote requires more information from the consumer e.g., home insurance. In an environment without a PCW where auto-renewers find it too costly to enter these details into a firm's website to retrieve *one* extra price, the introduction of a PCW offers to expose *all* prices to them, for the same single search cost. Depending on their search cost, the introduction of a PCW may then cause an auto-renewer to engage in comparison via the PCW. Some empirical studies have offered a similar argument to explain observed increases in market competitiveness (e.g., Brown and Goolsbee, 2002; Byrne et al., 2014). Their arguments are distinct from mine because they contrast a world with web-based aggregators relative to a world without the Internet, rather than a world with the Internet and firm websites. This engagement of new customers is commonly referred to as the 'extensive search margin' (for a recent discussion see Moraga-González et al., 2015).

The benefit of an additional search for a consumer in the world without a PCW is the difference between the expected price and the expected lowest of two prices drawn (from F). With a PCW, the benefit of a search on the PCW is the difference between the expected price and the expected lowest of n draws (from G).[29] I denote these search benefits with and without an aggregator respectively as,

$$\mathcal{B}_1 = \mathbb{E}_G[p] - \mathbb{E}_G[p_{(1,n)}], \qquad \mathcal{B}_0 = \mathbb{E}_F[p] - \mathbb{E}_F[p_{(1,2)}].$$

The model is as before save that each auto-renewer faces a search cost s. I assume these costs are heterogeneous, distributed by S over $s \in [\underline{s}, \infty)$.[30] I assume $\underline{s} > \mathcal{B}_0$ which means that without a PCW, no auto-renewers shop, preserving the equilibrium of Proposition 1. After the introduction of a PCW, the benefit of a search ($\mathcal{B}_1$) may outweigh the cost (s) for some auto-renewers, who then choose to use the site. I use the term 'converts' and 'non-converts' to

[29] These terms are analogous to the 'value of information' in Varian (1980).

[30] That there is no upper bound ensures that there are always some auto-renewers, and hence price dispersion, in equilibrium.

distinguish between auto-renewers who decide to shop or not in equilibrium with a PCW.

The total number of consumers using the PCW (shoppers and converts) is endogenously determined in equilibrium and is denoted $\tilde{\alpha} = \alpha + (1 - \alpha)S(\mathcal{B}_1)$. Given $\tilde{\alpha}$, the PCW sets its profit-maximizing fee $c = v(1 - \tilde{\alpha})$. As c and $\tilde{\alpha}$ are exogenous to firms, equilibrium pricing is as in Proposition 2 with $\tilde{\alpha}$ replacing α. In turn, pricing determines $\mathcal{B}_1$. There is an equilibrium when this value of $\tilde{\alpha}$ satisfies $S(\mathcal{B}_1) = \frac{\tilde{\alpha}-\alpha}{1-\alpha}$. When there exists such an S, $\tilde{\alpha}$ is said to be 'rationalized'.

Corollary 3 showed that a higher α increases the welfare of all consumers. Corollary 1 showed that a lower c has the same effect. As the equilibrium fee level is $v(1 - \tilde{\alpha})$, both forces work to benefit all types of consumer. However, whether consumers actually gain depends on how many auto-renewers are converted. In fact, relative to the world without a PCW, the presence of converts is not sufficient to guarantee lower prices for any consumer, not even converts themselves:

Proposition 10. *Shoppers, converts and non-converts can all be worse off with a PCW than without.*

The proof gives an example of such an equilibrium, along with a distribution S that rationalizes it. More generally, there can be many equilibria, each with a different $\tilde{\alpha}$. If the benefit ($\mathcal{B}_1$) is small or there are no types with low search costs so that $S(\mathcal{B}_1) = 0$, there are no converts ($\tilde{\alpha} = \alpha$) and the equilibrium of Proposition 2 applies. Proposition 10 shows that when *some* auto-renewers convert, all consumers can be worse off with a PCW. However, there may also exist equilibria where $\tilde{\alpha}$ is high enough such that some consumers benefit. Whether these equilibria exist depends on the distribution of search costs S. For a given PCW search benefit B_1, when more auto-renewers have low search costs (higher $S(\mathcal{B}_1)$), $\tilde{\alpha}$ is higher, c is lower and total consumer welfare is higher.

The number of firms also determines the size of the benefit the PCW offers ($\mathcal{B}_1$), and hence the number of converts. I now investigate which consumers benefit from an aggregator when the potential benefit it offers to consumers ($\mathcal{B}_1 - \mathcal{B}_0$) is as large as possible. For a given $\tilde{\alpha}$, a higher number of firms increases the equilibrium search benefit $\mathcal{B}_1$ (see Corollary 2). Specifically, as $n \to \infty$, $\mathcal{B}_1 \to v - c$ (see Proposition 9), which is as large as possible. To maximize $B_1 - B_0$, I let $q = 2$ in the world without a PCW, which makes $\mathcal{B}_0$ as low as possible, as $\mathcal{B}_0$ is increasing in q.[31] Accordingly, I define,

Definition. *A market has 'maximum potential' when $q = 2$ and $n \to \infty$.*

Proposition 11. *If the market has maximum potential: converts are better off; shoppers can be worse off; and non-converts are worse off with a PCW than without.*

When the market has maximum potential, converts are guaranteed to be better off but shoppers still may not be.[32] Shoppers may not be better off because even with maximum potential, there may still not be sufficiently many auto-renewers converting to PCW use.

9. Conclusion

The analysis shows that the introduction of PCWs may not in fact benefit consumers by reducing expected prices. The introduction of a single aggregator facilitates comparison of the whole marketplace for shoppers, exerting competitive pressure on firm pricing. However, the aggregator charges a fee which, in turn, places upward pressure on prices. The net effect is that prices increase for all consumers, who would be better off without the site.

Competition at the aggregator level need not lead to a reduction in fees. There are many equilibria, which I parameterize by the number of PCWs that shoppers check. If shoppers only check one PCW each, consumers are no better off than in the monopoly setting. More aggregators only guarantee to benefit shoppers if they check all of them. If shoppers check an intermediate number, increasing the number of aggregators can lead to higher prices; for a sufficiently high number, all consumers can again be better off without the industry. Hence, when there are many aggregators in the market, how many of them shoppers check is a crucial variable. As a result, regulatory bodies may wish to consider incentivizing consumers to check more, alongside stronger actions such as limiting the fees charged by aggregators and limiting the number of PCWs in the market.

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[31] See the end of the proof of Proposition 5.

[32] By Proposition 9, one can also use Proposition 11 to consider the effect of introducing a PCW into a market with search costs and price discrimination under the equilibrium of Proposition 8.

If competing PCWs publicly announce fees, this can result in low-fee equilibria, but it relies on coordination between firms and shoppers. In markets with price discrimination, if shoppers only check one PCW, the monopoly fee level can still be sustained, making consumers worse off. However, with competing aggregators where shoppers check multiple PCWs, there is then the incentive for the sites to undercut each other's fees. As such, regulators may also wish to consider whether it is possible to introduce price discrimination into markets in which it is not currently present. In online markets with non-negligible search costs, even those consumers who rationally start engaging in price comparison may be worse off following the introduction of a PCW. Helpful policies would help erode these costs where possible and encourage more inactive consumers to engage in comparison.

Appendix

A1. Summary Statistics of UK PCW Industry

The figures quoted for turnover and profit in the introduction for the UK utilities and services industry are estimates for 2013 for the largest four such companies. The estimate for each site was taken from the following sources:

- Money Supermarket: Their own 2013 annual report.
- Confused: The Admiral Group's Annual Reports 2012 and 2013.
- Go Compare: Newspaper 'The Guardian' 2014 article.
- Compare the Market: Parent BGL Group's Annual Report 2013 and BBC article 2015.

For the first three, the figures were taken directly from the cited sources. For Compare the Market, estimated to be the largest of the four sites, the estimate is particularly rough as BGL Group offer no breakdown of their accounts. I assumed that the proportion of BGL's total revenue and profit due to Compare the Market was the same where the estimate for annual profit due to Compare the Market is taken from the BBC article. Even if the estimated £800m of total turnover and 14% for average annual profit growth for these sites is off by a margin, the industry can still be considered large and growing.

A2. A World Without a PCW

The domain of prices is $\mathbb{R}$. The equilibrium pricing strategy can always be described by its CDF (denoted F in this section, and with other letters later on). In what follows, either equilibrium pricing distributions will be pure (so that F is flat, with one jump discontinuity at this price); or will have no point masses so that F is continuous, which implies the density f exists and $f = F'$, wherever F' exists. Lemmas A1-A3 are variants of Varian (1980)'s Propositions 1,3 and 7 respectively.

Lemma A1. *In any equilibrium, there are no prices p charged s.t.* $p \leq 0$ *or* $p > v$.

Proof: Any $p \leq 0$ generates firm profits $\pi(p) \leq 0$ which is dominated by $p = v$ which gives profit of at least $v\frac{(1-\alpha)}{n} > 0$ because firms always sell to their auto-renewers. For $p > v$, $\pi(p) = 0$ because no one will buy at such a high price, hence again $p = v$ dominates. ■

Lemma A2. *In any equilibrium, there are no point masses.*

Proof: Suppose not. Then the there is a point mass in equilibrium, $\dot{p}$ s.t. $pr(p = \dot{p}) > 0$. Note that $\dot{p} \in (0, v]$ from Lemma A1. Because the number of point masses must be countable, there exists an $\varepsilon > 0$ small such that $\dot{p} - \varepsilon > 0$ and is charged with probability zero. Consider a deviation of a firm from the equilibrium F to a distribution over prices where the only difference is that the new distribution charges $\dot{p} - \varepsilon$ with probability $pr(p = \dot{p})$ and $\dot{p}$ with probability zero. Note that a firm appears in $\binom{n-1}{q-1}$ of the groups of shoppers. Index these groups $z = 1, ..., \binom{n-1}{q-1}$ (this is without loss as F is symmetric). Let $pr(\dot{p}; t, z)$ be the probability under F that a firm is cheapest in group z along with $t = 0, ..., q-1$ others. Call the difference in profit due to the deviation d and note that:

$$\lim_{\varepsilon \to 0} d = \sum_{z=1}^{\binom{n-1}{q-1}} \sum_{t=0}^{q-1} pr(\dot{p}; t, z) \frac{\alpha}{\binom{n}{q}} \left(1 - \left(\frac{1}{q} + \frac{q-(t+1)}{q(t+1)}\right)\right) \dot{p}$$

The term in parentheses is the difference in the amount of shoppers won under the deviation and under F, in given group, when the firm along with t others charge $\dot{p}$, and $\dot{p}$ is the lowest price in that group. Due to symmetry of the groups, $pr(\dot{p}; t, z)$ is the same for all groups, so let this more simply be termed $pr(\dot{p}; t)$. Also given $pr(\dot{p}; t) > 0$, this simplifies to:

$$\frac{\binom{n-1}{q-1}}{\binom{n}{q}} \sum_{t=1}^{q-1} pr(\dot{p}; t) \alpha \left(\frac{t}{t+1}\right) \dot{p} > 0$$

hence for some $\varepsilon > 0$ there exists a profitable deviation, so F could not have been an equilibrium. ■

Lemma A3. *In any equilibrium, the maximum of the support of f must be v.*

Proof: Suppose not. Define $\bar{p}$ as the maximum element of the support and note that by Lemma A2 the probability of a tie at any price is zero. By Lemma A1, $\bar{p} \in (0, v)$. Consider a firm called upon to play $\bar{p} < v$ in equilibrium. They only sell to their auto-renewers, making $\bar{p}\frac{(1-\alpha)}{n}$ but would strictly prefer to deviate to v and make $v\frac{(1-\alpha)}{n}$, a contradiction. ■

Proposition 1. *In the unique equilibrium firms mix according to the distribution*

$$F(p) = 1 - \left[\frac{(v-p)(1-\alpha)}{qp\alpha}\right]^{\frac{1}{q-1}} \text{ over the support } p \in \left[\frac{v(1-\alpha)}{1+\alpha(q-1)}, v\right].$$

Proof: By Lemma A2, there is more than one element of the equilibrium support, and by Lemma A3 v is the maximal element. In equilibrium, a firm must be indifferent between all elements of the support p, hence profit must equal $v\left(\frac{1-\alpha}{n}\right)$ for all p, that is:

$$v\left(\frac{1-\alpha}{n}\right) = p\left[\frac{1-\alpha}{n} + \frac{\alpha}{\binom{n}{q}}X(p)\right] \tag{1}$$

$$X(p) \equiv \binom{n-1}{q-1}\binom{n-1}{n-1}F(p)^0(1-F(p))^{n-1} + ... + \binom{q-1}{q-1}\binom{n-1}{q-1}F(p)^{n-q}(1-F(p))^{q-1}$$

The first term on the RHS of (1) is the profit from ARs, who always purchase at $p \leq v$. The second term is the expected proportion of shoppers that a firm will win, charging price p. Shoppers can be characterized by $\binom{n}{q}$ groups, where the set of groups is given by $\{1, ..., n\}^q$. $X(p)$ describes the expected number of groups a firm expects to win given it charges p. By Lemma A2 there are no ties in price, so label prices by $p_1 < ... < p_n$. p_1 will be the cheapest in every group in which it appears, and it appears in $\binom{n-1}{q-1}$ of the groups. The probability of being the lowest price is given by $\binom{n-1}{n-1}F(p)^0(1-F(p))^{n-1}$, which accounts for the first term in $X(p)$. The observation that p_i is the cheapest in $\binom{n-i}{q-1}$ groups if $i \leq n-(q-1)$, zero groups otherwise, accounts for the remaining terms of $X(p)$. The following manipulations to simplify the second term on RHS of (1) this make use of the binomial theorem:

$$= p\frac{\alpha}{\binom{n}{q}} \sum_{j=q-1}^{n-1} \binom{j}{q-1}\binom{n-1}{j} F(p)^{n-j-1}(1-F(p))^j$$

which after some manipulations can be shown to be:

$$= p\alpha\frac{q}{n}(1 - F(p))^{q-1}$$

rearranging for $F(p)$ gives:

$$F(p) = 1 - \left[\frac{(v-p)(1-\alpha)}{qp\alpha}\right]^{\frac{1}{q-1}}$$

Notice that this is a well-defined c.d.f. over:

$$p \in \left[\frac{v(1-\alpha)}{1+\alpha(q-1)}, v\right]$$

Notice that v is strictly preferred to any $p \in \left[0, \frac{v(1-\alpha)}{1+\alpha(q-1)}\right)$. ■

A3. A World With a PCW

I look for symmetric equilibria where PCWs charge some fee level $c \geq 0$ and shoppers check PCWs in equilibrium. I do not look at equilibria where firms never list on PCWs, where the setting without a PCW applies. To derive equilibria, one needs to know the mutual best-responses of firms to unilateral deviations of PCWs. To do so, take $\mathbf{c}$ and equilibrium shopper strategy as given, and consider the firm responses.

Let $K \geq 1$ denote the number of PCWs and $r : K \geq r \geq 1$ the number of PCWs checked by shoppers. In symmetric equilibrium, a proportion $\frac{r}{K}$ of each firm's shoppers check any given PCW. Define a vector of PCW fees as $\mathbf{c} = (c_1, ..., c_K) \in \mathbb{R}_+^K$ labeled such that $c_1 \leq ... \leq c_K$. Let $\beta_k \in [0,1]$ be the probability with which a firm enters PCW$_k$ and define the event E: "all PCWs are empty". Denote $(a_1, a_2) = (p, \mathcal{K})$ as a firm's action, where p is the price charged and $2^{\{1,...,K\}}$ is set of all combinations of PCWs they could choose to list in where $\mathcal{K}$ a typical element, and $\emptyset$ denotes not listing on any PCW. Define the following CDF, which is used throughout:

$$G(p;\mathbf{c}) = 1 - \left[\frac{(v-p)(1-\alpha)}{\alpha\left(np - (n-1)\frac{1}{K}(c_1 + \cdots + c_K)\right)}\right]^{\frac{1}{n-1}} \tag{2}$$

which is well-defined over the support $[\underline{p}(\mathbf{c}), v]$ where $\underline{p}(\mathbf{c}) = \frac{v(1-\alpha)+\frac{1}{K}(c_1+\cdots+c_K)\alpha(n-1)}{1+\alpha(n-1)}$. When $c_1 = \cdots = c_K \equiv c$, let $G(p;\mathbf{c})$ and $\underline{p}(\mathbf{c})$ also be written $G(p;c)$ and $\underline{p}(c)$.

Lemma A4. *In any equilibrium, there are no prices p charged s.t. $p \leq 0$ or $p > v$.*

Proof: No $p \leq 0$ or $p > v$ because they yield negative and zero profit respectively, whereas $(v, \emptyset)$ yields $\frac{v(1-\alpha)}{n} > 0$. ■

Lemma A5. *If $c_1 \in [0, v(1-\alpha))$, $pr(E) = 0$.*

Proof: Suppose $pr(E) > 0$. Denote the infimum of prices charged when no PCW is listed on and that of prices ever listed on a PCW as p_0 and $\underline{p}$ respectively. [Note the infima exist because prices are a bounded from below by Lemma A4]. Note that $p_0 \geq \underline{p}$ because $(\underline{p}, \emptyset)$ is strictly preferred to any lower price as a firm faces no competition for prices below $\underline{p}$ off the PCWs. Consider when the firm is called upon to play $(p_0, \emptyset)$ (or a price arbitrarily closely above p_0):

If $p_0 > c_1$, a deviation to $(p_0 - \epsilon, 1)$ is strictly profitable. This is because with probability $pr(E)^{n-1} > 0$ PCWs are empty with other firms charging at least p_0. By listing the firm then has a positive probability of winning $\alpha \frac{r}{K} \frac{n-1}{n}$ new shoppers. For a sufficiently small $\epsilon > 0$, this will offset the arbitrary loss in revenue from its own consumers.

If $p_0 \leq c_1$, firm profit must be at least $\frac{\underline{p}}{n}$ which can be guaranteed by $(\underline{p}, 1)$, because $p_0 \geq \underline{p}$. In turn, this must be at least as much as $\frac{v(1-\alpha)}{n}$ which the firm can guarantee by playing $(v, \emptyset)$. Putting these together, $p_0 \geq \underline{p} \geq v(1-\alpha) > c_1$ which contradicts $p_0 \leq c_1$. ■

Lemma A6. *If $c_1 \in [0, v(1-\alpha))$, firm strategies have no point masses.*

Proof: Define $\underline{p}_1(c_1)$ as the price at which a firm is indifferent between selling to all shoppers exclusively through the cheapest PCW(s) and charging v, only sell to auto-renewers:

$$\underline{p}_1(c_1) = \frac{v(1-\alpha) + \alpha c_1(n-1)}{1 + \alpha(n-1)}$$

By Lemma A5, some $(p, \mathcal{K})$ is played. Note that firms would by construction not play $(p, \mathcal{K})$ where $p < \underline{p}_1(c_1)$, strictly preferring $(v, \emptyset)$. To see that there are no point masses:

If there were a point mass at $(\underline{p}_1(c_1), \mathcal{K})$ then there is a positive probability of being tied for the lowest price at $(\underline{p}_1(c_1), \mathcal{K})$. By definition of $\underline{p}_1(c_1)$ firms would strictly prefer to deviate to $(v, \emptyset)$.

If there were a point mass at $(\dot{p}, \mathcal{K})$ s.t. $\dot{p} > \underline{p}_1(c_1)$ then there is a positive probability of being tied for the lowest price at $(\dot{p}, \mathcal{K})$. A firm would strictly prefer to shift that probability mass to $(\dot{p} - \epsilon, \mathcal{K}')$ where $\mathcal{K}' = \mathcal{K} \setminus \{k : c_k \geq \dot{p}\}$. Here, the firm would sell to $\alpha \frac{r}{K} \frac{n-1}{n}$ other firms' shoppers at an arbitrary loss in revenue from its own consumers. There is always an $\epsilon > 0$ small enough to ensure this is profitable because $\underline{p}_1(c_1) > c_1 \iff c_1 < v(1-\alpha)$. ■

Lemma A7. *If there are no point masses, the maximum of the support f must be v.*

Proof: This is a variant of Varian (1980) Proposition 7. ■

Lemma A8. *If $c_1 \in [0, v(1-\alpha))$ and $c_1 < c_2$, $\beta_1 = 1$.*

Proof: By Lemma A5 it is never the case that all PCWs are empty. Suppose $\beta_1 < 1$. Lemma A6 implies there is more than one price, $\hat{p}$, that is listed on some other PCWs. By Lemma A7 there is one such that $\hat{p} < v$. Consider a firm being called upon to play this $(\hat{p}, \mathcal{K}_{\text{-}1})$ where $1 \notin \mathcal{K}_{\text{-}1}$ and $\bar{m} = \max\{\mathcal{K}_{\text{-}1}\}$. As this price has a positive probability of being the lowest of all firms, it will generate sales through the PCWs in $\mathcal{K}_{\text{-}1}$. But as PCW$_1$ is the unique cheapest PCW, there is a strictly profitable deviation to $(\hat{p}, \mathcal{K}_{\text{-}1} \cup 1 \setminus \bar{m})$. ■

Lemma A9. *If $c_1 = \ldots, c_K \equiv c \in (v(1-\alpha), v]$, firm pricing strategies are pure where $p \in [v(1-\alpha), c]$. Any $(\beta_1, \ldots, \beta_K) \in (0,1]^K$ can be supported.*

Proof: Either there is a point mass or there is not.

1. Suppose there is a point mass at price $\dot{p}$. If $\dot{p} > c$, firms have a strict incentive to shift this mass to $(\dot{p} - \epsilon, \{1, \ldots, K\})$. If $\dot{p} < v(1-\alpha)$, firms prefer to shift this mass to $(v, \emptyset)$. These leaves $\dot{p} \in [v(1-\alpha), c]$ as the only points that can be point masses. There can be at most one point mass: If not, then there a second point mass $\ddot{p} < c$, which if played with $\mathcal{K} \neq \emptyset$ would generate negative profit, so $(\ddot{p}, \emptyset)$ is preferred; if $\mathcal{K} = \emptyset$, then $(\ddot{p} + \epsilon, \emptyset)$ for some sufficiently small $\epsilon > 0$ is preferred. To see that this pure pricing at $p \in [v(1-\alpha), c]$ can be part of firm strategies, note that firm profit is $\pi = \frac{p}{n} \geq \frac{v(1-\alpha)}{n}$, so there is no strict incentive to sell only to auto-renewers instead. Because shoppers buy directly when prices are all the same, there are no sales through PCWs and so firms are indifferent between any $(\beta_1, \ldots, \beta_K) \in (0,1]^K$.

2. Suppose there is no point mass. By Lemma A7 the maximum of the support is v, where v is not the only element of the support, else it would be a point mass. There is therefore a positive probability of a firm being the cheapest at some p. There can be no $(p, \mathcal{K})$ s.t. $p < c$ charged: If $\mathcal{K} \neq \emptyset$ played profit from these sales is negative, so $(p, \emptyset)$ is preferred; if $\mathcal{K} = \emptyset$, then $(p + \epsilon, \emptyset)$ for some sufficiently small $\epsilon > 0$ is preferred. Given $p \in [c, v]$, $pr(E) = 0$.

This follows because firms strictly prefer $(p, \{1, \ldots, K\})$ to $(p, \emptyset)$ for all $p \in (c, v)$. For any $1 \leq r \leq K$ firms are content to list prices on at least as many PCWs as is necessary to make sure every shopper sees their price e.g., for $r = K = 1$ all of them; for $r = K$, just one of them. There can therefore, be different configurations of β_K's depending on r, K so long as $pr(E) = 0$. To determine firm pricing strategy it must be that firms are indifferent between every p they are called upon to play:

$$v\left(\frac{1-\alpha}{n}\right) = p\left(\frac{1-\alpha}{n}\right) + (1 - G(p;c))^{n-1}\left[\frac{\alpha}{n}p + \frac{\alpha(n-1)}{n}(p-c)\right]$$

which can be re-arranged to give $G(p; c)$ from (2). However, $\underline{p}(c) < c$ because $c > v(1-\alpha)$, so firms would make strictly negative profits at prices $p \in (\underline{p}(c), c)$, preferring not to list. This provides a contradiction, so there do not exist strategic firm responses with no point masses.

■

A4. Results for $\mathbf{K = 1}$ or $\mathbf{r = 1}$

Lemma A10. *If $r = 1$, $c_1 = \cdots = c_{K-1} \in [0, v(1-\alpha))$ and $c_K \in [c_1, \underline{p}(\mathbf{c}))$: $\beta_k = 1$ for all k where firms mix by CDF $G(p; \mathbf{c})$.*

Proof: By Lemma A5 there is always some price posted on some PCW(s). By Lemma A6 these prices are not point masses and by Lemma A7 the maximum of the support is v, where only auto-renewers are sold to. There is therefore a positive probability of a sale through some PCW(s) at some $(p, \mathcal{K})$. If $k \in \mathcal{K}$ where $k < K$ and sales there are profitable (when $p > c_1$), then $\{1, \ldots, K-1\} \in \mathcal{K}$; if $K \in \mathcal{K}$ and sales there are profitable $p > c_K$ (whether or not $p > c_K$ is the only consideration because $r = 1$), it follows that $\{1, \ldots, K\} \in \mathcal{K}$. To be part of firm strategy, it must also be that firms prefer to play $(p, \mathcal{K})$ than to charge v and sell only to their auto-renewers. Note that $\underline{p}(\mathbf{c})$ from (2) is the price at which firms are indifferent between selling through $\{1, \ldots, K\}$ with certainty and selling only to their auto-renewers. Similarly, denote $\underline{p}_{\text{-}K}(\mathbf{c})$ as the indifference point between selling for sure on $\{1, \ldots, K-1\}$ and charging v, and note $\underline{p}_{\text{-}K}(\mathbf{c}) < \underline{p}(\mathbf{c})$. There will then be no $(p, \{1, \ldots, K-1\})$ played s.t. $p < \underline{p}_{\text{-}K}(\mathbf{c})$ and no $(p, \{1, \ldots, K\})$ s.t. $p < \underline{p}(\mathbf{c})$. For prices close to v, $\mathcal{K} = \{1, \ldots, K\}$. To determine firm pricing strategy, it must be that firms are indifferent between every $(p, \{1, \ldots, K\})$ they are called upon to play:

$$v\left(\frac{1-\alpha}{n}\right)=p\left(\frac{1-\alpha}{n}\right)+(1-G(p;\mathbf{c}))^{n-1}\left[\frac{\alpha}{n}p+\frac{\alpha(n-1)}{n}\left(p-\left(c_1\frac{K-1}{K}+c_K\frac{1}{K}\right)\right)\right]$$

which can be re-arranged for $G(p;\mathbf{c})$ to give the CDF from (2). Because the minimum of the support is $\underline{p}(\mathbf{c})>c_K\geq c_1$ (the first relation follows because $c_1<v(1-\alpha)$), all prices in the support generate profitable sales through all the PCWs, there is no price charged s.t. $p\in[\underline{p}_{\text{-}K}(\mathbf{c}),\underline{p}(\mathbf{c}))$. It follows that $\beta_k=1$ for all k. For $K=1$, let $c_1\in[0,v(1-\alpha))$ and set $K=1$ in the expressions of the Lemma. ■

Lemma A11. *If $r=1$ and $c=v(1-\alpha)$ there are the following firm responses:*

1. *Pure-price strategies where $p=v(1-\alpha)$ is the only price ever charged. Here, any $(\beta_1,\ldots,\beta_K)\in(0,1]^K$ can be supported.*
2. *Mixed-price strategies where $\beta_k=1$ for all k and prices are distributed according to the CDF $G(p;v(1-\alpha))$ where $\underline{p}(v(1-\alpha))=v(1-\alpha)$.*

Proof: Either there is a point mass or there is not.

1. Suppose there is a point mass at price $\dot{p}$. If $\dot{p}>c$, firms have a strict incentive to shift this mass to $(\dot{p}-\epsilon,\{1,\ldots,K\})$. If $\dot{p}<c$, firms have a strict incentive to shift this mass to $(v,\emptyset)$. These leaves $\dot{p}=c$ as the only point that can be a point mass. To see that this pure pricing can be part of firm strategies, note that firm profit is $\pi=\frac{c_1}{n}=\frac{v(1-\alpha)}{n}$, so there is no strict incentive to sell only to auto-renewers instead. Because shoppers buy directly when prices are all the same, there are no sales through PCWs and so firms are indifferent between any $(\beta_1,\ldots,\beta_K)\in(0,1]^K$.

2. Suppose there is no point mass. By Lemma A6, v is the maximum of the support of prices. No prices $p<c$ are charged because $(v,\emptyset)$ is strictly preferred. For $p>c$, sales through all PCWs are profitable so $\beta_k=1$ for all k. When v is played, firm profit is $\pi(v)=\frac{v(1-\alpha)}{n}$. To determine firm pricing strategy, it must be that firms are indifferent between every p they are called upon to play:

   (3) $$\pi(v)=p\left(\tfrac{1-\alpha}{n}\right)+\tfrac{\alpha}{n}(1-G(p;v(1-\alpha)))^{n-1}\left[np-(n-1)v(1-\alpha)\right].$$

   which can be re-arranged to give the CDF from (2).

■

Lemma A12. *If $r = 1$, $c_1 = \cdots = c_{K-1} = v(1-\alpha)$ and $c_K \in (c_1, v]$ there exist firm responses in pure-price strategies where $v(1-\alpha)$ is the only price ever charged. Here, any $(\beta_1, \ldots, \beta_K) \in (0,1]^K$ can be supported.*

Proof: In such an equilibrium, firm profit is $\pi = \frac{v(1-\alpha)}{n}$. Consider a deviation to p. If $p \in (v(1-\alpha), v]$, deviation profit is $\hat{\pi} = \frac{p(1-\alpha)}{n} \leq \pi$. If $p < v(1-\alpha)$, firms have a strict incentive to shift this mass to $(v, \emptyset)$. Because shoppers buy directly when prices are all the same, there are no sales through PCWs and so firms are indifferent between any $(\beta_1, \ldots, \beta_K) \in (0,1]^K$. ■

Lemma 1. *The mutual best-responses of firms as a function of c:*

1. *If $c \in [0, v(1-\alpha))$, firm best-responses are described by $G(p;c)$, and have no point masses.*
2. *If $c = v(1-\alpha)$, there are two classes of responses, one with no point masses described by $G(p,c)$, and those in which all firms charge the same price.*
3. *If $c \in (v(1-\alpha), v]$, all firms charge the same price.*

Where pricing is described by $G(p;c)$ each firm always lists its price on the PCW.

Proof: See Lemmas A10, A11 and A9 respectively. ■

Proposition 2. *In the unique equilibrium where shoppers check the PCW, the PCW sets a click-through fee of $c = v(1-\alpha)$, firms list on the PCW and mix over prices according to $G(p; v(1-\alpha))$ over the support $p \in [v(1-\alpha), v]$.*

Proof:

If $c \in [0, v(1-\alpha))$, Lemma A10 shows that there is a profitable upward deviation to $c \in (c_1, \underline{p}(c))$. It is profitable because there is an increase in fee-level but no reduction in the quantity of sales.

If $c \in (v(1-\alpha), v]$, Lemma A9 shows that firms will play pure-price strategies and so $u = 0$. If $c = v(1-\alpha)$ and firms play pure-pricing strategies, then $u = 0$ again. In these cases, Lemma A10 shows that a deviation to $c \in (0, v(1-\alpha))$ will generate $u_1 > 0$, so there are no equilibria where $c \in (v(1-\alpha), v]$ or for $c = v(1-\alpha)$ when firms respond with pure-pricing strategies.

If $c = v(1-\alpha)$ and firms mix over prices by $G(p; v(1-\alpha))$ as in Lemma A11, equilibrium PCW profit is $u^* = v(1-\alpha)\frac{\alpha}{K}\frac{n-1}{n} > 0$. Lemma A9 shows that any upward deviation would yield $u = 0$. There can be no profitable deviation downwards because, as Lemma A10 shows, the fee would be reduced for no gain in the quantity of sales. ■

Proposition 3. *Both types of consumer are worse off with the PCW than without.*

Proof of Proposition 3: First I show that auto-renewers are worse off under $G(p; v(1-\alpha))$ than $F(p)$ (referred to as G and F here). Auto-renewers pay the price quoted by their current firm. To show they are worse off with the PCW, I show that $\mathbb{E}_F[p] < \mathbb{E}_G[p]$ by showing that G first-order stochastic dominates (FOSDs) F. The distributions share the same upper bound on their supports, with F having a lower lower bound. Hence, G FOSDs F if $G(p; v(1-\alpha)) \leq F(p)$ for $p \in [v(1-\alpha), v]$, which can be re-arranged as

$$\left[\frac{(v-p)(1-\alpha)}{\alpha q p}\right]^{\frac{1}{q-1}} \leq \left[\frac{(v-p)(1-\alpha)}{\alpha n p - \alpha v(1-\alpha)(n-1)}\right]^{\frac{1}{n-1}}.$$

This holds because the terms in parentheses are in [0,1], the power on of the LHS is larger and the denominator on the LHS is larger iff $p(n-q) \leq v(1-\alpha)(n-1)$ which is satisfied because $p \leq v(1-\alpha)$ and $n - q < n - 1$.

To show shoppers are worse off under G than F, first show that $\mathbb{E}_F[p_{(1,2)}]$ is lower than the lower bound of the support of g in the case of $q = 2$. Then, I use Proposition 3 of Morgan et al. (2006) which corresponds to my setup (the only difference is that they have $v = 1$), which states that $\mathbb{E}_F[p_{(1,2)}]$ is decreasing in q. Hence I show the first step here to prove that $\mathbb{E}_F[p_{(1,2)}]$ is below $v(1-\alpha)$ for all q. For $q = 2$,

$$\mathbb{E}_F\left[p_{(1,2)}\right] = \int_{v\left[\frac{1-\alpha}{1+\alpha}\right]}^{v} f\left(p_{(1,2)}\right) p dp$$

where $f\left(p_{(1,2)}\right) = 2\left(1 - F(p)\right) f(p)$ is the density function of the lower of the two draws shoppers receive from F. Computing yields

$$\mathbb{E}_F\left[p_{(1,2)}\right] = \left(\frac{1-\alpha}{\alpha}\right)^2 \frac{v}{2}\left[\log\left(\frac{1-\alpha}{1+\alpha}\right) + \frac{2\alpha}{1-\alpha}\right].$$

Then $\mathbb{E}_F\left[p_{(1,2)}\right] < v\left(1-\alpha\right)$ can be rearranged to obtain

$$\log\left(\frac{1+\alpha}{1-\alpha}\right) > 2\alpha$$

which holds for $\alpha \in (0,1)$. ■

Corollary 1. *Within the mixed-price equilibrium firm responses of Lemma 1, as $c \in [0, v(1-\alpha)]$ increases, the expected price paid by both types of consumer increases.*

Proof: From Lemma 1 the pricing strategy for $c \in [0, v(1-\alpha)]$ is given by, $G(p;c)$ over $[\underline{p}(c), v]$. Differentiating,

$$\frac{dG\left(p;c\right)}{dc} = \frac{1}{c(n-1)-np}\left(\frac{\left(v-p\right)\left(1-\alpha\right)}{\alpha np - \alpha\left(n-1\right)c}\right)^{\frac{1}{n-1}}.$$

The second term is ≥ 0 else $G(p;c) > 1$. The first term is $\leq 0 \iff c\frac{n-1}{n} < p$ which is ensured because $p \geq c > c\frac{n-1}{n}$ when $c \leq v(1-\alpha)$ (this follows because $\underline{p}(c) \geq c \iff v(1-\alpha) \geq c$). Then for any $c, c' \in [0, v(1-\alpha)]$, if $c > c'$ then the equilibrium pricing distribution under c first order stochastic dominates that under c'. Hence the expected price (paid by auto-renewers) and the expected lowest price from n draws (paid by shoppers) are higher under c. ■

Corollary 2. *As the number of firms increases, the expected price paid by shoppers falls, but the expected price paid by auto-renewers rises.*

Proof: Using an observation from Morgan et al. (2006), industry profit of firms is given by $\alpha\mathbb{E}_G[p_{(1,n)}] + (1-\alpha)\mathbb{E}_G[p] = v(1-\alpha)$, where $\mathbb{E}_G[p_{(1,n)}]$ denotes the lowest price of n draws from $G(p; v(1-\alpha))$ and $\mathbb{E}_G[p]$ denotes the expected price from $G(p; v(1-\alpha))$. The RHS is the industry profit of firms if it charged v and only sold to auto-renewers. Differentiating and rearranging,

$$\frac{d\mathbb{E}_G[p_{(1,n)}]}{dn} = -\left(\frac{1-\alpha}{\alpha}\right)\frac{d\mathbb{E}_G[p]}{dn}$$

so the derivatives have opposite signs. Now show that $\frac{\mathbb{E}_G[p]}{dn} \geq 0$. To do this, show that $G(p, v(1-\alpha))$ is stochastically ordered in n:

$$\frac{dG(p; v(1-\alpha))}{dn} \leq 0 \iff \log\left(\frac{(v-p)(1-\alpha)}{\alpha(np - v(1-\alpha)(n-1)}\right) + \frac{(n-1)(p-v(1-\alpha))}{np - v(1-\alpha)(n-1)} \equiv X(p) \leq 0.$$

Note that $X(v(1-\alpha)) = 0$, and $\frac{dX}{dp} < 0$:

$$\frac{dX(p)}{dp} \leq 0 \iff p \geq v(1-\alpha)\left[\frac{(2n-2+\alpha(n-1)^2)}{(2n-1+\alpha(n-1)^2)}\right].$$

Notice that the term on RHS in square brackets is below 1. Recall that $p \geq v(1-\alpha)$ as this is the lower bound of the support, hence this is satisfied. ■

Proposition 4. *With $K > 1$ competing PCWs, if shoppers check $r = 1$ PCW then both types of consumer are worse off with the PCWs than without.*

Proof: There are no equilibria where $c \in [0, v(1-\alpha))$: Lemma A10 shows that there is a profitable upward deviation to $c_K \in (c_1, \underline{p}(c))$. It is profitable because there is an increase in fee-level but no reduction in the quantity of sales. Note that there can be no profitable deviation downwards given $r = 1$ because the fee would be reduced for no gain in the quantity of sales.

There are no equilibria where $c \in (v(1-\alpha), v]$: Lemma A9 shows $u_k = 0$ for all k. Lemmas A6 and A8 show that a deviation to $c_1 \in (0, v(1-\alpha))$ will generate $u_1 > 0$.

The only remaining option is $c_1 = \cdots = c_K \equiv c = v(1-\alpha)$, where firm responses are described by Lemma A11. If firms play a pure-pricing strategy, then as in the previous case, PCWs make zero profit and there is a profitable deviation to $c_1 \in (0, v(1-\alpha))$. If however, firms respond with the mixed-price strategy, this fee-level is an equilibrium: There can be no profitable deviation downwards for PCWs because the fee would be reduced for no gain in the quantity of sales as $r = 1$. Following an upward deviation from PCW$_K$ to $c_K \in (v(1-\alpha), v]$, when firms respond with a pure-pricing strategy as detailed in Lemma A11, PCW$_K$'s profit falls to zero. There is then an equilibrium at this fee level, and it is the unique such level where there exists an equilibrium.

At this fee-level, firms play just as they did when $K = 1$ with the same CDF over prices and all PCWs list all n firm prices. Both types of consumer are therefore left with the same level of surplus they had under $K = 1$. ■

Corollary 3. *As the proportion of shoppers α increases, expected prices paid by shoppers and auto-renewers both decrease.*

Proof: Differentiating $G(p;c)$ by α,

$$\frac{dG\left(p;c\right)}{d\alpha}=\frac{1}{\alpha(n-1)(1-\alpha)}\left(\frac{\left(v-p\right)\left(1-\alpha\right)}{\alpha np-\alpha\left(n-1\right)c}\right)^{\frac{1}{n-1}}$$

The first term is >0. The second term is always ≥ 0 or $G(p;c)>1$. Then for any $\alpha,\alpha'\in(0,1)$, if $\alpha>\alpha'$ then the equilibrium pricing distribution under α' first order stochastic dominates that under α. Hence the expected price (paid by auto-renewers) and the expected lowest price from n draws (paid by shoppers) are lower under α. ■

Corollary 4. *If the PCW can determine α as well as c in the preliminary stage, then the PCW sets $\alpha=\frac{1}{2}$.*

Proof: I expand the PCW's action set to include $\alpha\in(0,1)$. Notice that for any choice of $\alpha\in(0,1)$, by the reasoning as in the proof of Proposition 2, the PCW will avoid the pure equilibria of Lemma 1 so $c\in[0,v(1-\alpha)]$, firms mix and the PCW is given by $c\alpha\left(\frac{n-1}{n}\right)$. The PCWs optimization problem can hence be solved by,

$$\max_{c,\alpha}\ c\alpha\left(\tfrac{n-1}{n}\right)\ \text{ s.t. } c\in[0,v(1-\alpha)] \text{ and } \alpha\in(0,1)$$

where the solution is $c=v(1-\alpha)$, $\alpha=\frac{1}{2}$. ■

A5. Results for K $>$ 1 and r $>$ 1

Lemma A13. *If $r>1$, $c_1=\cdots=c_{K-1}\in[0,v)$, $c_K\in(c_1,v]$, $u_K=0$.*

Proof: This Lemma says that there is no profitable upwards deviation for a PCW from any equilibrium. Consider such a unilateral deviation by PCW$_K$. Firm response can either be pure or mixed pricing. If pure, then $u_k=0$ for all k. If mixed, then at any price p that has positive probability of sales through PCWs where $K\in\mathcal{K}$, firms have a strict preference to play $(p,\{1,\ldots,K-1\})$ instead. This is because $r>1$: Every shopper who sees the prices on PCW$_K$ also sees the prices on another PCW. Firms can therefore avoid PCW$_K$'s higher fee by not listing there while facing no reduction in the quantity of sales. ■

Lemma A14. *If $r=K$, $c_2=\cdots=c_K\in(0,v(1-\alpha)]$ and $c_1\in[0,c_2)$: $\beta_1=1$, $\beta_k=0$ for $k=2,\ldots K$ and prices are distributed according to the CDF $G(p,c_1)$.*

Proof: There are no point masses by Lemma A6. By Lemma A8 $\beta_1 = 1$. $\beta_k = 0$ for $k = 2, \ldots, K$ follows because all shoppers check every PCW ($r = K$). Therefore, at any price p that has positive probability of generating sales through PCWs, firms have a strict preference only to list on PCW_1 without any reduction in the quantity of sales. By Lemma A7 the maximum of the support is v. Firms must be indifferent to all $(p, 1)$ they are called upon to play, hence:

$$v\left(\tfrac{1-\alpha}{n}\right) = p\left(\tfrac{1-\alpha}{n}\right) + \tfrac{\alpha}{n}(1 - G(p; c_1))^{n-1}\left[np - (n-1)c_1\right].$$

which can be re-arranged to give $G(p, c_1)$ from (2). ■

Proposition 5. *With $K > 1$ competing PCWs, shoppers are guaranteed to be better off than before the introduction of PCWs if $r = K > 1$. Further, the unique equilibrium PCW fee-level is $c = 0$ if and only if $r = K > 1$.*

Proof: Sufficiency: Suppose not. Then there exists an equilibrium with $c > 0$. By Lemma A13, there is no profitable upward deviation. Now consider a downward deviation. If $c \in [v(1-a), v]$ and firms respond with a pure-pricing equilibrium, as detailed in Lemma A9, then $u_k = 0$. A deviation by PCW_1 to $c_1 \in (0, v(1-\alpha))$ would lead to the responses detailed in Lemma A14 and deviation profit of $u_1 = c\alpha\frac{n-1}{n} > 0$. If $c \in (0, v(1-a)]$ and firms respond with by mixing by $G(p, c)$, as in Lemma A14, then $u_k = c\frac{\alpha}{K}\frac{n-1}{n} > 0$ for all k. But a deviation by PCW_1 to $c_1 < c$ exists s.t. $u_1 = c_1\alpha\frac{n-1}{n} > c\frac{\alpha}{K}\frac{n-1}{n} = u_k$. This deviation is strictly profitable, so c could not have been an equilibrium. To see that $c = 0$ is an equilibrium, recall that by Lemma A13 there is no profitable upward deviation.

Necessity: Lemma A11 shows that for $K \geq r = 1$ the unique equilibrium fee level is $c = v(1-\alpha)$. Lemma 2 shows that for $1 < r < K$ there are multiple equilibria. Hence $K = r > 1$ is the only case where the unique equilibrium of $c = 0$ obtains.

Consumer welfare: As noted in the text, as firms make the same expected profit in both worlds, there is a one-to-one relation between consumer welfare and PCW profit. To see the difference in the changes to shopper and auto-renewer welfare from a move to a world with a PCW but $c = 0$, Proposition 3 of Morgan et al. (2006) (the only difference is that they have $v = 1$) shows that the increase from q to n results in a reduction in the expected price paid by shoppers, and an increase for auto-renewers. ■

Lemma 2. *When $K > r > 1$, there exists an equilibrium in which PCWs charge $\bar{c} > 0$, and firms list on all PCWs, mixing over prices by $G(p;\bar{c})$ where*

$$\bar{c} = \frac{v(1-\alpha)Kr(K-r)}{K(1+r(K-2)) + \alpha r(K-1)(r-1)(n-1)}.$$

Proof: I show that there exist equilibria such that $c \in [0,\bar{c}]$. Note that $\bar{c} \in [0, v(1-\alpha))$ hence any $c \in [0, v(1-\alpha))$. Take such a c as a candidate equilibrium fee level. There are no point masses by Lemma A6. By Lemma A7 the maximum of the support is v. By Lemma A5 $pr(E) = 0$ and as all PCWs charge the same fee, firms are content to list in all of them i.e., $\beta_k = 1$ for all k. Firms must be indifferent to all $(p, \{1, \dots, K\})$ they are called upon to play, hence

$$v\left(\tfrac{1-\alpha}{n}\right) = p\left(\tfrac{1-\alpha}{n}\right) + \tfrac{\alpha}{n}(1-G(p;c))^{n-1}\left[np-(n-1)c\right]$$

which can be re-arranged to give $G(p;c)$ in (2).

To confirm c is an equilibrium fee level, ensure there is no profitable PCW deviation. By Lemma A13, there is no profitable upward deviation. However, unlike Lemma A14, when $1 < r < K$ it is no longer true that any undercut by PCW$_1$ to $c_1 < c$ will result in all firms listing on it exclusively. This is because consumers see $r > 1$ PCWs: when a firm sells having played $(p, \{1, \dots, K\})$ it pays $c_1\frac{1}{K} + c\frac{K-1}{K}$, but were it to have played $(p, 1)$ it would have paid $c_1\frac{r}{K}$ i.e., the firm faces a trade-off between lower fees and higher sales volume. By Lemma A8 $\beta_1 = 1$. PCW$_1$'s deviation profit is therefore determined by β_k, $k > 1$. For small-enough undercuts of c, firms will still be content to list on all PCWs. Suppose however, that PCW$_1$ undercuts by just enough such that such that firms are no longer content to list all their prices on the other PCWs. In the best case for PCW$_1$, $\beta_k = 0$ for all k so that all shoppers checking PCW$_1$ buy only through PCW$_1$. If in this case, PCW$_1$ still does not make more than its equilibrium profit $u^* = c\frac{\alpha}{K}\frac{n-1}{n}$, then the undercut is never profitable and c constitutes an equilibrium fee level. I now carry out this logic.

Denote $c_1 < c$ as the undercut of PCW$_1$ and $\tilde{c}_1$ as the threshold level required for c_1 to entice firms to de-list some of their prices from PCWk s.t. $k > 1$. Given $\mathbf{c} = (c_1, c, \dots, c)$, similarly to the derivation above, one can show firms will respond by $G(p;\mathbf{c})$ over $[\underline{p}(\mathbf{c}), v]$ as in (2). Firm deviation profit from this response to $(p,1)$ is given by

$$\pi(p) = p\left(\frac{1-\alpha}{n}\right) + \frac{(v-p)(1-\alpha)\left[Kp - r(p-c_1)(n-1)\right]}{n\left(Knp-(n-1)(c_1+c(K-1))\right)}.$$

Note that this is valid for $p \in [\underline{p}(\mathbf{c}), v]$, but $p < \underline{p}_1$ give strictly less profit than $p = \underline{p}(\mathbf{c})$. One can show that $\pi(p)$ is convex in p. Together with the observation that $(v,1)$ gives the equilibrium profit $\frac{v(1-\alpha)}{n}$, this says that the optimal deviation is to $(\underline{p}(\mathbf{c}), 1)$ and is profitable if and only if $\pi(\underline{p}(\mathbf{c})) > \frac{v(1-\alpha)}{n}$, which can be rearranged to give

$$c_1 < \frac{c(K-1)(K+r\alpha(n-1)) - Kv(K-r)(1-\alpha)}{r\alpha(K-1)(n-1)+K(r-1)} \equiv \tilde{c}_1.$$

Suppose that the firm response following the undercut was such that $\beta_k = 0$ for all $k > 1$ when PCW_1 sets the highest such undercut just below $\tilde{c}_1$. PCW_1 prefers not to make the deviation whenever

$$u^* \geq \tilde{c}_1 \frac{\alpha r}{K}\frac{n-1}{n} \iff \frac{c}{r} \geq \tilde{c}_1$$

which can be rearranged to give

$$c \leq \frac{v(1-\alpha)Kr(K-r)}{K(1+r(K-2)) + \alpha r(K-1)(r-1)(n-1)} \equiv \bar{c}.$$

Therefore at fee levels $c \in [0, \bar{c}]$ there exist equilibria where $\beta_k = 1$ for all k and firms mix by the CDF given in the Lemma. ■

Proposition 6. *For any r, there exists a $\tilde{K}$ such that as long as there are more than $\tilde{K}$ aggregators both types of consumer are worse off than before the introduction of PCWs.*

Proof: To show for shoppers, let the equilibrium be given as in Lemma 2 with $c = \bar{c}$ and see that

(4) $$\lim_{K\to\infty} \bar{c} = v(1-\alpha).$$

For shopper welfare notice that for $K = r$, $\bar{c} = 0$ and

(5) $$\mathbb{E}_F[p_{(1,q)}] \geq \mathbb{E}_{G(0)}[p_{(1,n)}]$$

where $G(c)$ and F denote $G(p;c)$ and $F(p)$. This is because the PCWs effectively increase the number of firms competing for each shoppers from q to n, and from the last point of the proof of Proposition 5, this lowers the price paid by shoppers. Proposition 3 shows that both types are worse off with PCWs when $c = v(1-\alpha)$ for $K \geq r = 1$, but notice that if $c = v(1-\alpha)$ in equilibrium with $K > r \geq 1$, then this will be true a fortiori, because it can be shown that $G(v(1-\alpha))$ first-order stochastically dominates F for all q. By (4) then as $K \to \infty$, $\mathbb{E}_F[p_{(1,q)}] < \mathbb{E}_{G(v(1-\alpha))}[p_{(1,n)}]$. Because $\bar{c}$ is continuous in K, $\frac{d\bar{c}}{dK} > 0$ and $\frac{d\mathbb{E}_{G(\bar{c})}[p_{(1,n)}]}{dK} > 0$ by Corollary 1, there exists $\tilde{K}$ s.t. for $K > \tilde{K}$ there is always an equilibrium fee level that makes shoppers worse off relative to a world without PCWs.

The proof for auto-renewers is similar but simpler to the above steps for shoppers and has p replacing $p_{(1,q)}$ and $p_{(1,n)}$: From Proposition 3 of Morgan et al. (2006), the effect of increasing the number of firms competing for each shopper increases prices for auto-renewers. Then by Corollary 1, their prices only rise further under $G(c)$. Hence auto-renewers are worse off for any c and hence any K. ■

A6. Results under Publicly Observable Fees

Definition (Coordinated Subgame). *Given* **c***, a 'coordinated subgame' is when shoppers attend and firms list in only the cheapest PCWs i.e., shoppers in PCW*$_k$ *if* $c_k = c_1$*, and for firms* $\beta_k = 1$ *if* $c_k = c_1$ *else* $\beta_k = 0$*. Firms mix by* $G(p, c_1)$*.*

Lemma A15. *When fees are observed by shoppers, and all subgames are 'coordinated subgames' there exists a subgame perfect equilibrium where* $c = 0$*,* $\beta_k = 1$ *for all* k *and prices are distributed according to the CDF* $G(p, 0)$*.*

Proof: There are no profitable deviations for firms: $G(p, 0)$ ensures they are indifferent between listing all prices in the support $[\underline{p}(0), v]$, listing any price less than $\underline{p}(0)$ is less profitable than listing v, and any $(p, \emptyset)$ generates at most $\frac{v(1-\alpha)}{n}$ when $p = v$. To see there is no profitable upward deviation for PCWs, note that due to the 'coordinated subgame' shoppers and firms would not attend this PCW following such a deviation, so the PCW receives zero profit. To see that these coordination subgames are Nash equilibria: for firms one can conduct the same round of checks as at the beginning of this proof; for consumers notice that as the non-cheapest PCWs are all empty, there is no incentive to check them. ■

Proposition 7. *When $K > 1$ and PCW fees are observed by shoppers, there exists an equilibrium with $c = 0$.*

Proof: That there exists an equilibrium with $c = 0$ follows directly from Lemma A15. For shopper welfare at $c = 0$, see the last point of the proof of Proposition 5. ■

A7. Results under Price Discrimination

Now assume that different prices can be charged directly and through the PCW. Denote such prices p_0 and p_k respectively, where $k = 1, \dots, K$ indexes the PCW as before.

Lemma A16. *If $r = 1$, $c_1 = \cdots = c_{K-1} = v(1-\alpha)$ and $c_K \in [c_1, v]$ then there exist firm responses s.t. $p_0 = p_1 = \cdots = p_K = v(1-\alpha)$ and $\beta_k = 1$ for all k.*

Proof: Under these strategies, firm profit is $\pi = \frac{v(1-\alpha)}{n}$. There is no profitable deviation involving p_k for any k: lower would prompt sales at $p_k < c_k$, higher would never attract any shoppers. There is no profitable deviation involving p_0: lower would still sell to auto-renewers and own-shoppers but at a lower price, higher would only sell to auto-renewers for which the optimal such deviation is to v generating profit $\frac{v(1-\alpha)}{n} = \pi$. ■

Proposition 8. *With price discrimination, if $r = 1$, there exists an equilibrium in which PCWs set $c = v(1-\alpha)$, firms list on all PCWs, $p_0 = v$ and $p_1 = \cdots = p_K = v(1-\alpha)$.*

Proof: Given $c_1 = \cdots = c_K \equiv c = v(1-\alpha)$, let us confirm there are no profitable deviations for firms. Equilibrium firm profit is $\pi = \frac{v(1-\alpha)}{n}$. There is no profitable deviation involving p_k for any k: lower would prompt sales at $p_k < c_k$, higher would never attract any shoppers. There is no profitable deviation involving p_0: lower would either sell only to auto-renewers at a lower price, or to both auto-renewers and own-shoppers but only for prices at least as low as $v(1-\alpha)$, generating profit no greater than $\frac{v(1-\alpha)}{n}$; higher would be above v and hence make zero profits. Now consider a PCW deviation. Equilibrium PCW profit is $u_k = \frac{\alpha v(1-\alpha)}{K} > 0$. There is no profitable deviation to a lower fee: as $r = 1$ doing so would at best sell to the same proportion of shoppers ($\frac{\alpha}{K}$) but at a lower fee. As for a deviation to a higher fee: assume that firms' mutual best responses are given by those in Lemma A16 where PCW profit is zero. ■

Corollary 5. *When firms price discriminate, following* $c_1 \in (0, c_2)$ *and* $c_2 = c_3 =, \ldots, = c_K \equiv c \in (0, v(1-\alpha)]$ *there exist mutual best-responses of firms such that they list on all PCWs, setting* $p_0 = v$ *and* $p_k = \cdots = c_k$ *for all* k.

Proof: Given $\mathbf{c}$ as in the Lemma, let us confirm there are no profitable deviations for firms. Firm profit is $\pi = \frac{v(1-\alpha)}{n}$. There is no profitable deviation involving p_k for any k: lower could prompt sales, but only at $p_k < c_k$, higher would never attract any shoppers. There is no profitable deviation involving p_0: lower would either sell only to auto-renewers at a lower price, or to both auto-renewers and own-shoppers but only for prices at least as low as c_2, generating profit no greater than $\frac{c_2}{n} < \pi$; higher would be above v and hence make zero profits. ■

Lemma A17. *If* $r > 1$, $c_1 = \cdots = c_{K-1} = 0$ *and* $c_K \in [0, v]$ *then there exist firm responses s.t.* $p_0 = v$, $p_1 = \cdots = p_{K-1} = 0$, $\beta_k = 1$ *for* $k < K$ *and* $\beta_K = 0$.

Proof: Under these strategies, firm profit is $\pi = \frac{v(1-\alpha)}{n}$. There is no profitable deviation involving p_K: $p_K < 0$ would give sell at a loss whereas $p_K > 0$ would never attract any shoppers as $r > 1$. There is no profitable deviation involving p_k for any $k < K$: Lower would only create sales at a loss, higher would never attract any shoppers. There is no profitable deviation involving p_0: lower would still sell to auto-renewers and (for $p_0 \leq 0$) own-shoppers but at a lower price, higher would sell to no-one. ■

Lemma A18. *If* $r > 1$, *there exists an equilibrium where* $c = 0$, *firms list on all PCWs,* $p_0 = v$ *and* $p_1 = \cdots = p_K = 0$..

Proof: One can follow the proof of Proposition 8 to confirm there are no profitable deviations for firms. For PCWs, one only need consider an upward deviation in fee level in which case, assume firms respond as in Lemma A17, which yields zero profit for the deviating PCW. ■

Proposition 9. *As* $n \to \infty$, *following the introduction of a PCW, the expected prices faced by both types of consumer in a setting without price discrimination (Proposition 2) approach those in a setting with price discrimination (Proposition 8).*

Proof: Taking limits,

$$\lim_{n \to \infty} G(p; c) = 0$$

which shows $\lim_{n\to\infty} \mathbb{E}_G[p] = v$. The CDF for the lowest of n draws is given by

$$H(p;c) = 1 - (1 - G(p;c))^n = 1 - \left[\frac{(v-p)(1-\alpha)}{\alpha\,(np-(n-1)c)}\right]^{\frac{n}{n-1}}$$

for $p \in [\underline{p}(c), v]$. Taking limits,

$$\lim_{n\to\infty} H(p;c) = 1 \text{ and } \lim_{n\to\infty} \underline{p}(c) = c$$

which shows $\lim_{n\to\infty} \mathbb{E}_G[p_{(1,n)}] = c$. These prices are the same as those in the equilibrium of Proposition 8. ■

A8. Results with Search Costs

In the world without a PCW under $q = 2$, one can compute

$$\begin{aligned} \mathbb{E}_F[p] &= \frac{v}{2}\left[\frac{1-\alpha}{\alpha}\right]\left[\log\left(\frac{1+\alpha}{1-\alpha}\right)\right] \\ \mathbb{E}_F[p_{(1,2)}] &= \frac{v}{2}\left[\frac{1-\alpha}{\alpha}\right]^2\left[\log\left(\frac{1-\alpha}{1+\alpha}\right) + \frac{2\alpha}{1-\alpha}\right] \\ \mathcal{B}_0 &= \frac{v}{2}\left[\frac{1-\alpha}{\alpha^2}\right]\left[\log\left(\frac{1+\alpha}{1-\alpha}\right) - 2\alpha\right]. \end{aligned}$$

In the world with a PCW under $n \to \infty$, Proposition 9 shows that,

$$\mathbb{E}_G[p] = v, \qquad \mathbb{E}_G[p_{(1,\infty)}] = c, \qquad \mathcal{B}_1 = v - c$$

where $c = v(1-\tilde{\alpha}) \le v(1-\alpha)$ in equilibrium.

Proposition 10. *Shoppers, converts and non-converts can all be worse off with a PCW than without.*

Proof: This is a proof by example. Let $v = 1$, $\alpha = 0.7$, $n = 3$ and $q = 2$. Then, in the world without a PCW, it can be computed that,

$$\mathbb{E}_F[p] = 0.3717, \qquad \mathbb{E}_F[p_{(1,2)}] = 0.2693, \qquad \mathcal{B}_0 = 0.1024.$$

In the world with a PCW, assume $\tilde{\alpha} = 0.71$ (i.e., the PCW attracts 0.01 more consumers i.e.,

$\frac{1}{30}$ auto-renewers, to compare prices), so $c = v(1-\alpha) = 0.29$ and

$$\mathbb{E}_G[p] = 0.5557, \qquad \mathbb{E}_G[p_{(1,3)}] = 0.3748, \qquad \mathcal{B}_1 = 0.1808.$$

Comparing, one can see that if there is an S such that this can be rationalized as an equilibrium, then all types of consumer will be worse off with a PCW than without. To rationalize, I construct an S such that:

$$S(\mathcal{B}_1) = \begin{cases} 0 & \text{if } \mathcal{B}_1 < \underline{s} \\ \dfrac{\mathcal{B}_1 - \underline{s}}{\mathcal{B}_1} & \text{else} \end{cases}$$

where $\underline{s}$ is determined by

$$S(0.1808) = \frac{1}{30} \iff \underline{s} = 0.1748.$$

Finally, notice that $\underline{s} > \mathcal{B}_0$. ■

Lemma A19. *Under maximum potential, any $\tilde{\alpha}$ can be rationalized by an S.*

Proof: Note that any $\tilde{\alpha}$ can be rationalized with an S (as in the proof of Proposition 10) because $\mathcal{B}_0 < \mathcal{B}_1$. To see this holds, one can show that $\mathcal{B}_0 < v\alpha$ holds for any $\alpha \in (0,1)$ and hence so does $\mathcal{B}_0 < v\tilde{\alpha}$. ■

Proposition 11. *If the market has maximum potential: converts are better off; shoppers can be worse off; and non-converts are worse off with a PCW than without.*

Proof: As $v(1-\alpha) > v(1-\tilde{\alpha})$ when there are converts, to show converts are better off with a PCW under maximum potential one can show,

$$\mathbb{E}_F[p] > v(1-\alpha) \iff \log\left(\frac{1+\alpha}{1-\alpha}\right) > 2\alpha$$

which is satisfied for $\alpha \in (0,1)$. Note that $\tilde{\alpha}$ is rationalizable by some S (Lemma A19).

Shoppers are worse off under maximum potential wherever $v(1-\tilde{\alpha}) > \mathbb{E}_F[p_{(1,2)}]$. To show this can occur, note that $v(1-\alpha) > \mathbb{E}_F[p_{(1,2)}]$ holds for all $\alpha \in (0,1)$ (see Proposition 3). Hence for any α there exists some $\tilde{\alpha} > \alpha$ small enough such that shoppers are worse off. Note that such a $\tilde{\alpha}$ is rationalizable by some S (Lemma A19).

Non-converts are worse off under maximum potential because $\mathbb{E}_F[p] < \mathbb{E}_G[p] = v$. ■

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