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Abstract

Price comparison websites (PCWs) or ‘web aggregators’ are poised to benefit consumers by increasing competitive pressure between firms, in turn lowering prices faced by consumers. But this comes at a cost, as these sites charge firms for sales. This article investigates the impact of the introduction of PCWs to the market for a homogeneous good to analyze this trade-off. I find that the introduction of a PCW increases prices for all consumers. Under competing aggregators, the welfare of shoppers who use the sites does not improve when they check only one, and is only guaranteed to improve when they check all of them. Furthermore, when shoppers do not check all, increasing the number of aggregators can increase prices. All equilibria feature price dispersion. *JEL: L11, D43

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1. Introduction

Over the past two decades a new industry of price comparison websites (PCWs) or ‘web aggregators’ has emerged. The industry has enabled consumers to check prices of many firms selling a particular service or product simultaneously in one place. The sites are popular in many countries, and in many markets including utilities, financial services, hotels, flights and durable goods. These sites command billions of dollars of revenue annually. In the UK, PCWs for utilities and financial services have been particularly successful. There are roughly 48 such PCWs in the UK, where over 70% of internet users have used such a site. The biggest four generated approximately £800m ($1.2bn) in revenue during 2013, with average annual profit of the group increasing by 14% that year.

The internet has trivialized search costs, allowing consumers to compare prices across firms in a matter of clicks, intensifying the competitive pressure between firms. Where a consumer may not know all the firms in a market, a PCW has the ability to expose the full list of market offerings, maximizing inter-firm pressure. However, this increase in competition comes at a cost. The industry gleans substantial profits from fees paid by firms who sell their products through the websites. In the UK these are understood to be between £44-60 ($69-95) for a customer switching gas and electricity provider. These fees, in turn, act as a marginal cost faced by producers, affecting their pricing decisions. As such, it is not clear...

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1 Examples for utilities and services include Moneysupermarket.com, Google Compare and Gocompare.com; for flights Skyscanner.net and Flights.com; for hotels Hotels.com and Booking.com; and for durable goods Amazon Marketplace, Pricerunner.com and Pricegrabber.com.
2 Regarding travel services, Priceline Group (who own Booking.com and Priceline.com) and Expedia Inc. (who own Expedia.com and Hotels.com) made approximately $6bn in total agency revenues in 2014. Regarding durable goods, Amazon Marketplace sold 2 billion items from third-party sellers. See their 2014 Annual Reports for details.
3 Number of PCWs taken from Consumer Focus (2013) report into PCWs in the UK. The ‘big four’ refers to Money Supermarket, Compare the Market, Go Compare and Confused.com. Number of PCWs Usage data from the 2013 Mintel Report on Web Aggregators. Financial information taken from the companies’ own annual reports where available, otherwise inferred from parent group reports or newspaper articles, the details are in the appendix.
4 For example, see BBC (2015). Fees are significant in other sectors too e.g., in the hotel-reservation sector they are reported to be 15-25% of the purchase price (see Daily Mail 2015).
5 In practice, PCWs in some markets charge per-click, some per-sale. I abstract from this difference and model the latter, assuming that the two are correlated.
whether the central premise that PCWs lower prices is valid. This fundamental tension is encapsulated in a quote from the BBC (2014):

“There’s another cost in the bill. It’s hidden, it’s kept confidential, and yet it’s for a part of the industry that appears to be on the consumers’ side. This is the cut of the bill taken by price comparison websites, in return for referring customers. The recommendation to switch creates churn in the market, and it is seen by supplier companies as worth paying high fees to the websites. Whether or not customers choose to use the sites, the cost to the supplier is embedded within bills for all customers.”

This article examines this “churn” and addresses the fundamental question of when consumers are better off with a PCW in the marketplace. I provide a model of a market for a homogeneous good and investigate when all consumers are made worse off following the introduction of PCWs.

The model builds on the elegant framework of Baye and Morgan (2001), who were the first to investigate the strategic incentives of a PCW or ‘information gatekeeper’. Without the PCW in their model, each consumer is served by a single ‘local’ firm which sells at the monopoly price (it is too costly for consumers to travel to another store). This forces the result that consumers benefit from the introduction of a PCW where firms must compete for the business of consumers who enjoy free access to the site. In the modern online marketplace however, firms have their own websites in addition to their presence on PCWs. In the absence of an aggregator, consumers do not need to physically travel to purchase the good; they can visit a firm’s website just as easily as they could an aggregator’s.

There are two types of consumer in the present model: shoppers, who use PCWs in equilibrium, and inactive consumers, who buy directly from a particular firm. Although shoppers are always better off compared to inactive consumers in equilibrium, my primary result is that both types are worse off after the introduction of a PCW. This is the first article

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6Often, a PCW simply re-directs users to the selected firm’s own website to complete the purchase.
in this setting to show such a result, and is the opposite to that of the existing literature, which I show can be seen as a special case. The model supposes each shopper is informed of the prices from \( q > 1 \) of the \( n \) firms’ websites in the absence of a PCW, rather than \( q = 1 \) as in Baye and Morgan. Without a PCW, shoppers see \( q \) prices; after the introduction of the PCW, they will see all of them in equilibrium. I show that adding competition to the setting without an aggregator reverses their finding: expected equilibrium prices are raised by the introduction of a PCW, making consumers worse off.

Upon the introduction of a single PCW into a market, the equilibrium fee it charges is sufficiently high to negate any benefits from the increased firm competition. At first glance, it may seem that allowing aggregator competition will undo this result, akin to textbook Bertrand competition. I extend the model to allow for multiple PCWs, and for shoppers to check any number of them, unlike the preceding literature. My characterization shows that both the number of PCWs and the number of them that shoppers check (or are informed of), are crucial variables. Firstly, when shoppers only check one of many PCWs, they all effectively remain monopolists and so consumer welfare does improve with any number of PCWs. At the other extreme where shoppers check all PCWs, Bertrand-style reasoning at the aggregator level results as a special case: PCWs undercut each other’s fees to reach a unique zero-profit equilibrium and shoppers benefit from their existence. Where shoppers check an intermediate number of PCWs (more than one, but not all), increasing the number of PCWs can make both types of consumers worse off relative to a setting with fewer. Moreover, in this realistic case, there is a critical number of PCWs beyond which all consumers can be worse off than without any aggregators at all.

In other extensions, I consider the impact of different policies or market features. Generally PCW fees are either not disclosed or are detailed on a subpage of the websites. I extend the analysis to a setting where PCWs publicly announce fees so that consumers are made aware of them. This creates the possibility that firms and shoppers could coordinate on

\footnote{This is in the absence of any persuasion or direction of consumers to more expensive products by firms e.g., Armstrong and Zhou (2011), or biased intermediaries e.g., de Cornire and Taylor (2014).}
the cheapest PCW, which can result in equilibria that benefit shoppers relative to a world without PCWs.

The primary focus of this article is on settings where a firm sets the same price for a product on every website that it is sold. This is the directly applicable framework in many markets where PCWs operate including gas, electricity, mortgages and durable goods. However, I extend the analysis to consider markets where price discrimination, where a firm’s price can differ across sites, is more common e.g., the hotel-reservation sector. In this case, I show that both types of consumers can be worse off when shoppers only check one PCW in equilibrium. However, unlike markets without price discrimination, when shoppers check at least two PCWs, competitive pressure between aggregators is then shown to be active a la Bertrand.

2. Literature

This article contributes to the literature of ‘clearing-house’ models, see for example Salop and Stiglitz (1977), Varian (1980), Rosenthal (1980), Baye and Morgan (2001, 2009), Baye et al. (2004), Chioveanu (2008), Arnold et al. (2011), Arnold and Zhang (2014). These models rationalize price-dispersion in homogeneous goods markets contributing to our understanding of pricing on the internet.\(^8\) Price dispersion has persisted despite the advances of technology such the internet and comparison sites. Early studies documented marked dispersion in the online markets for goods e.g., Brynjolfsson and Smith (2000) and Baye et al. 2004. A recent study by Gorodnichenko et al. (2015) finds substantial cross-seller variation in prices and voices support for clearing-house models that categorize consumers into loyal and shopping consumers. The equilibria in my model feature price dispersion regardless of whether there is an aggregator (or ‘clearing-house’). Without a PCW, this is because there is some consumer

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\(^8\)The search cost literature also provides explanations of price dispersion e.g., Stigler (1961), Burdett and Judd (1983), Stahl (1989, 1996), Ellison and Ellison (2009), Ellison and Wolitzky (2012). Motivated by the rise of the internet, clearing-house models abstract from a direct modeling of consumer search costs. The frameworks are to some extent isomorphic, see Baye et al. (2006) for a discussion.
search. Producing price dispersion with a PCW that employs the pricing mechanism seen in practice, is a challenge. The shift in the aggregator industry away from charging one-off fixed fees, toward pay-per-sale fees to firms is rationalized as profit-maximizing PCW behavior by Baye et al. (2011). However, without the introduction of some other exogenous fixed cost to a firm of listing on the PCW (e.g., transaction costs), price-dispersion vanishes in equilibrium. I do not deny the existence of such additional costs, but emphasize that dispersion arises from my framework without the need for fixed listing costs.

The larger relevant literature is that of two-sided markets, pioneered by Rochet and Tirole (2003) (see also Caillaud and Jullien 2003, Ellison and Fudenberg 2003, Armstrong 2006, Reisinger 2014). These articles model platforms where buyers and sellers meet to trade, focusing on platform pricing and the effect of network externalities with differentiated products and platforms. These models do not explicitly model seller-side competition, which is central to my model. More recent contributions such as Hagiu (2009) and Belleflamme and Peitz (2010) do so, but they model the platform as the only available technology, which is not appropriate for the questions of this article. Edelman and Wright (2015) do allow sellers and buyers to conduct business off the platform, but they study environments of differentiated product markets where the intermediary directly offers some buyer-side benefits e.g., rebates. In contrast, I model a homogeneous good, isolating price as the determinant of consumer welfare, where the only benefit a ‘platform’ brings is informational: it lists available prices. The potential benefit of the PCW to consumers is that it can lower prices through the interaction of strategic, competing firms, and hence the size of the benefit is determined endogenously via the equilibrium actions of firms and consumers.
3. Model

A World Without a PCW

There are \( n \) firms and a unit-mass of consumers. Firms produce a homogeneous product at zero cost without capacity constraints. Consumers wish to buy one unit and have a common willingness to pay of \( v > 0 \). Each consumer is endowed with a ‘default’, ‘current’ or ‘preferred’ firm from which they are informed of the price. This assumption has many natural interpretations. In the market for services and utilities (e.g., gas and electricity tariffs, mortgages, credit cards, broadband, cellphone contracts, car, home and travel insurance etc.) consumers can be thought of as having a current provider for the service for which they know the price they pay and the renewal price should they remain with the same provider. In the market for flights, hotels or durable goods, consumers can be thought to have a carrier, hotel (or hotel chain) or producer which they prefer, perhaps bookmarked in their browser or for which they receive marketing emails that directly inform them of the price or prompt the user to find out via hyperlinks, in a matter of seconds.

A proportion \( \alpha \in (0, 1) \) of consumers are ‘shoppers’. Casual empiricism suggests that many people enjoy browsing or looking for a bargain. Shoppers check (are aware of) at least one rival firm’s website and therefore know \( q > 1 \) of the \( n \) prices. A shopper sticks with his or her default firm if its price is not beaten by another price. If a rival firm’s price is cheaper, the shopper switches. The remaining proportion of \( 1 - \alpha \) consumers are ‘auto-renewing’, ‘loyal’, ‘offline’ or ‘inactive’ consumers who do not shop around. In markets where firms are service providers, such consumers simply allow their contract with their existing provider to continue with their default firm. Much of the recent furore surrounding PCWs has been directed at those operating in the services and utilities sectors. In the presentation that follows I adopt terminology suited to that sector, of ‘current’ firms or providers with

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\(^9\)Providers usually provide renewal price information directly to consumers.
consumers referred to as ‘shoppers’ and ‘auto-renewers’.

I assume that each firm has an equal share of current consumers of each type ($\frac{\alpha}{n}$ shoppers and $\frac{1-\alpha}{n}$ auto-renewers). Firms set prices and shoppers simultaneously decide which rival-firm websites to visit. I focus on equilibria in which firms adopt identical pricing strategies and shoppers employ symmetric shopping strategies. Going forward I refer to such symmetric equilibria simply as equilibria.

This setup generalizes Varian (1980), nesting his equilibrium.\(^{10}\) He motivates the two types as being completely ‘informed’ about all $n$ prices, or ‘uninformed’. The informed buy the cheapest on the market, while the uninformed buy from their default firm. In my model, shoppers see $q$ prices where $1 < q \leq n$ and I derive the unique equilibrium. When $q = n$, Varian’s equilibrium corresponds to that derived in Proposition 1 below.

Shoppers can be characterized by $(n \choose q)$ groups. Each group is a list of the firms checked by that consumer. For example, consider $q = 2$, $n = 4$ with firms indexed 1,2,3,4. Then there are 6 possible comparisons shoppers could make: $\{12, 13, 14, 23, 24, 34\}$, where the two digits refer to which firms’ prices are checked. Each firm is involved in $\binom{n-1}{q-1}$ = 3 price comparisons. Shoppers employ symmetric shopping strategies, and hence are evenly distributed across these groups i.e. $\frac{1}{6}$ compare the prices of firms 1 and 2, $\frac{1}{6}$ compare firms 1 and 3, and so on. Equivalently, one could interpret a consumer as randomizing uniformly over which rival-firm websites she or he checks.\(^{11}\)

Now consider the best-response of firms. Without loss of generality, let $F : p \rightarrow [0, 1]$ denote the cumulative distribution function of prices charged by firms in equilibrium. Proposition 1 describes the equilibrium without a PCW.

**Proposition 1.** In the unique equilibrium firms mix according to the distribution

$$F(p) = 1 - \left[ \frac{(v-p)(1-\alpha)}{q\alpha} \right]^{\frac{1}{1-q}} \text{ over the support } p \in \left[ \frac{v(1-\alpha)}{1+\alpha(q-1)}, v \right].$$

\(^{10}\)The only differences being that I assume zero costs and do not employ his zero-profit condition.

\(^{11}\)Note that although I focus on a symmetric distribution across these pairs, this shopper behavior is not an assumption per se; it is a best response in equilibrium to symmetric firm behavior because consumers are then indifferent to which firm they check.
Price dispersion is a central feature in clearing-house models and showcases how they rationalize price dispersion in homogeneous goods markets. The limited-search assumption made in this article does not alter the intuition for why. Bertrand-style reasoning of undercutting down to the marginal cost (here normalized to 0) does not play out. Because of the existence of auto-renewers, firms can guarantee themselves profit of at least $v(1-\alpha)/n > 0$. Therefore any point mass existing in equilibrium strategies must be for some $p > 0$. Any such mass would always be undercut by firms to gain a discrete number of shoppers for an arbitrary $\epsilon$-loss in price.

Models building on Varian’s assume that in a world without a clearing-house, consumers cannot check other prices, so the pure monopolistic-price equilibrium of $p = v$ results. Similarly to Stahl (1989), if there were no shoppers here in my model, I would obtain such a Diamond (1971) equilibrium with each firm charging $v$, and if there were only shoppers, then the Bertrand outcome of $p = 0$ would result. Unlike Stahl, shoppers do not necessarily know the prices of all firms, but they know at least two. Proposition 1 shows that with some (but non-zero) search, price dispersion still emerges in equilibrium.

It is also instructive to note that equilibrium pricing does not vary with the number of firms, $n$, given $q < n$. Each shopper compares $q$ prices, regardless of the total number of firms. When making pricing decisions, a firm is not concerned about the number of other firms per se, but rather about the other prices shoppers know. As all firms price symmetrically and independently in equilibrium, it is as if each firm only faces $q-1$ rivals. In other words, what matters is the number of comparisons shoppers make, not the number of firms in the market.

**A World With a PCW**

Suppose an entrepreneur creates a price comparison website. I add a preliminary stage to the game at which the PCW sets a ‘click-through fee’ $c \in \mathbb{R}_+$ that a firm must pay to the aggregator per sale made via the site. Consumers do not learn $c$, although they will have
correct expectations in equilibrium. Each firm sees the fee c and must choose a price, and whether or not to post it on the PCW.

Clearing-house models typically have many equilibria, see for instance Baye et al. (1992) for their characterization of the full set of equilibria of Varian (1980). Throughout the paper, I focus on symmetric equilibria in which shoppers only check PCWs. With a monopoly PCW, this results in a unique equilibrium that one can explicitly solve for. In the equilibrium, firms list on the PCW with probability one and prices are dispersed, so shoppers have a strict incentive to check the PCW.

To find the equilibrium, I use the following lemma which takes c as fixed and characterizes the ensuing mutual best-responses of firms. Define $G(p; c)$ to be the cumulative distribution function of prices charged by firms for a given click through fee c:

$$G(p; c) = 1 - \left[ \frac{(v - p) (1 - \alpha)}{\alpha(np - (n - 1)c)} \right]^{\frac{1}{n-1}}$$

**Lemma 1.** The mutual best-responses of firms as a function of c:

1. If $c \in [0, v(1 - \alpha))$, firm responses have no point masses, and are described by $G(p; c)$.
2. If $c = v(1 - \alpha)$, there are two classes of response, one with no point masses described by $G(p, c)$, and those that are degenerate.
3. If $c \in (v(1 - \alpha), v]$, firm responses are degenerate.

Where pricing is described by $G(p; c)$ each firm always lists its price on the PCW.

When c is in the lower interval described by Lemma 1, firms undercut each other until the point at which they would be better off charging the maximum price v and only selling to

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12 As the BBC quote in the Introduction notes, the exact fee is “kept confidential”, not publicly announced.
13 Recall this is in addition to knowing their current or preferred firm’s price, which upon sale, continues to count as a direct purchase e.g., the auto-renewal letter, the bookmarked airline or the hotel’s marketing email.
14 Note more generally that there exist other equilibria e.g., trivial equilibria in which no firms or shoppers attend the PCW. The set-up is then identical to that of the previous section and the equilibrium is given by Proposition 1 with the vacuous addition that the PCW can charge any c. There also exist asymmetric equilibria where not all firms list on the PCW (see Footnote 15).
their auto-renewers. However, if $c$ exceeds $v(1 - \alpha)$, undercutting does not reach this point, so no firm would jump to $v$. Rather, it reaches a point where all firms charge the same price, selling to all their consumers directly, where no firm undercuts further or jumps to $v$. The reason no further undercutting occurs is that doing so would mean charging a price $p < c$ that would win all rival firms' shoppers, but would do so at a loss. The reason no firm jumps to $v$ is that they are making more in this pure-pricing equilibrium where they sell to all their consumers directly. If $c = v(1 - \alpha)$ then both mixed and pure equilibria obtain as threshold cases.

To derive the equilibrium, consider now incentives of the PCW. The PCW will make zero profit if firms all charge the same price, as then no shoppers switch, but positive profit in any mixing equilibrium if $c > 0$. The PCW therefore has a strong incentive to induce price dispersion because shoppers switch when they can obtain a strictly lower price with a new firm. A feature of distributions with no point masses is that the probability of a tie in price is zero. As a result, shoppers at $n - 1$ of the $n$ firms will switch. Given that firms mix in this way, the PCW will raise $c$ as high as possible before reversion to a pure equilibrium, which here happens for $c > v(1 - \alpha)$. Proposition 2 characterizes the equilibrium.\footnote{Note that in the symmetric equilibrium, all firms list on the PCW. In practice however, some firms do not always list on PCWs. Note that here there exist asymmetric equilibria where $m \geq 2$ firms list on the PCW, mixing over prices $[v(1 - \alpha), v]$ in a similar way to the CDF of Proposition 2, with the other $n - m$ firms not listing, charging $v$ and selling only to their auto-renewers with all shoppers uniformly spread over the mixing firms. The price-rising result of this article also holds in any of these asymmetric equilibria.}

**Proposition 2.** In the unique equilibrium where shoppers check the PCW, the PCW sets a click-through fee of $c = v(1 - \alpha)$, firms list on the PCW and mix over prices according to $G(p; v(1 - \alpha))$ over the support $p \in [v(1 - \alpha), v]$.

That only pure equilibria exist for $c > v(1 - \alpha)$ is what limits the PCW fee and allows there to be price dispersion at equilibrium without fixed costs for firms. It is the outside option of current shoppers for firms that limits the fee that the PCW charges. Shoppers know their current provider’s price regardless of where they check prices and stay with their current firm if there are no lower prices to be found on the PCW. Hence, a firm can always
guarantee itself its own shoppers and avoid the aggregator’s fee by charging a price low enough to undercut the market and not list on the PCW. It is this threat that discourages the PCW from charging fees that are even higher in equilibrium. If firms did not have this outside option, the PCW could raise its fee to \( c = v \), firms would charge \( p = v \), price dispersion would be lost and the PCW would be able to extract all the surplus.

4. Comparative Statics

Comparing the equilibria of Propositions 1 and 2 reveals how the PCW affects consumer welfare.

**Proposition 3.** *Both types of consumer are worse off with the PCW than without.*

The proof of Proposition 3 shows that the expected shopper-price under \( F \) is less than the lower bound of the support of \( G \). It follows that shoppers expect to pay more under \( G \), as the expected lowest price is higher. That \( G \) first-order stochastic dominates \( F \) shows that auto-renewers expect to pay more under \( G \), as the expected price is higher. Figure 1 illustrates these features for a calibrated example.

With the introduction of a PCW, the two effects stated in Corollaries 1 and 2 give rise to Proposition 3:

**Corollary 1.** *Within the mixed-price equilibrium firm responses of Lemma 1, as \( c \in [0, v(1 - \alpha)] \) increases, the expected price paid by both types of consumer increases.*

**Corollary 2.** *As the number of firms increases, the expected price paid by shoppers decreases but the expected price paid by auto-renewers increases.*

Corollary 1 shows that when the PCW sets a higher fee, the expected price paid by both shoppers and auto-renewers rise. The fee is passed on by firms to consumers through a first-order stochastic shift in the prices set in equilibrium. Upon winning, a firm must pay
Figure 1: Equilibrium price distributions calibrated with $q = 2, v = 1, \alpha = .7$

--- $F(p)$  $G(p; v(1 - \alpha))$ for $n = 2, 3, 5, 10, 50$

the PCW for all the $\left(\frac{n-1}{n}\right)$ shoppers who purchase through the site. Because the amount paid rises with the fee, the price charged in equilibrium also rises with the fee.

The second effect is that the PCW increases competitive pressure among firms to fight for all $n - 1$ rival firms’ shoppers. In contrast, without the PCW, firms effectively competed against only $q - 1$ rivals. Different models in the clearing house literature offer different predictions about the effect of $n$ on equilibrium prices. Some derive distributions for which an increase in $n$ raises prices for both types of consumer (e.g., Rosenthal (1980)); while in other models it lowers prices for shoppers and raises prices for captive consumers (e.g., Varian (1980), Morgan et al. (2006)).\(^{16}\) My model belongs to this second category. An increase in $n$ has two effects on equilibrium prices. Firstly, increased competition pushes probability mass to the high-price extreme of the distribution, as Figure 1 shows. This results in a first-order stochastic ordering in $n$, so expected price increases in $n$ and auto-renewers pay more. The second effect however, is that shoppers now pay the lowest of $n + 1$ prices rather than $n$, which reduces the expected lowest price. Corollary 2 shows that this second effect more than

\(^{16}\)Also see Baye et al. (2006) for a discussion on this point and a wider review of clearing-house models.
offsets the first, implying that shoppers pay less in expectation.\textsuperscript{17}

In order to relate these two effects of a PCW to Proposition 3, I make the following remark.

**Remark 1.** If $q = n$, $G(p; 0) = F(p)$

The entrance of a PCW increases both the fee firms pay (from 0 to $v(1 - \alpha)$) and number of prices that shoppers compare (from $q$ to $n$). The first effect raises the expected price auto-renewers pay, which is compounded further by $n > q$, so they are unambiguously worse off with a PCW. The effects pull shopper welfare in opposite directions, but Proposition 3 shows that there does not exist $n$ large enough to undo the effect of the optimally chosen PCW fee $c$. Hence, shoppers are also worse off in expectation with a PCW.

Due to the constant-sum nature of the game, welfare necessarily sums to $v$ in equilibrium. As firms make the same expected profit in both worlds, there is a one-to-one relation between the decrease in consumer welfare and the profits of the PCW. That the PCW does not reduce firm profits comes from the fact that firms have $(\frac{1 - \alpha}{n})$ auto-renewers with or without the PCW. Firms can therefore guarantee themselves $v \left( \frac{1 - \alpha}{n} \right)$ in both worlds. Although this article focuses on consumer rather than producer welfare, it is important to point out that the incentives of the PCW and firms are not aligned. This is because there is exactly one cheapest price and hence $\alpha \left( \frac{n - 1}{n} \right)$ shoppers who switch from non-cheapest prices, which increases with $n$. An increase in $n$ however, would of course, squeeze per firm profit. Therefore, a PCW would encourage market entry if it could.

One channel through which all consumers would gain is by more auto-renewers becoming shoppers:\textsuperscript{18}

\textsuperscript{17}Although the result of Corollary 2 is common to clearing-house models, it may seem a rather nuanced prediction of equilibrium pricing. Morgan et al. (2006) conduct an experiment with participants playing the role of firms against computerized buyers and found that when $n$ was increased, prices paid by inelastic consumers indeed increased whereas those paid by shoppers decreased.

\textsuperscript{18}This is a prediction common to clearing-house models, for which Morgan et al. (2006) find experimental support.
Corollary 3. When pricing is determined by \( G(p;c) \), as the proportion of shoppers \( \alpha \) increases, expected prices for both types of consumer decrease.

One may conjecture that a PCW would want to maximize \( \alpha \) (the number of shoppers) in order to obtain more referral fees. However, this logic is incomplete. Expanding the PCWs action set to include the determination of \( \alpha \) (one can think of the PCW determining \( \alpha \) through advertising) yields the following result:\(^{19}\)

Corollary 4. If the PCW can determine \( \alpha \) as well as \( c \) in the preliminary stage, then the PCW sets \( \alpha = \frac{1}{2} \).

PCW revenue is hump-shaped in profit. As \( \alpha \to 0 \) it receives less and less traffic, and hence vanishing revenue. As \( \alpha \to 1 \), firms have fewer auto-renewers to exploit, which pushes \( v(1-\alpha) \), the maximum fee for which firms are willing to mix, to zero. Indeed, when \( \alpha = 1 \) all consumers are shoppers and the PCW removes any incentive for firms to increase prices as there are no auto-renewers to exploit. This leads any equilibrium to have all firms charging the same price, and such pure-strategy equilibria leave the PCW with zero profit. So even if the PCW could bring all consumers online, it has a strict incentive to ensure an interior solution for \( \alpha \).

5. Competing Aggregators

Suppose that there are now \( K > 1 \) PCWs indexed \( k = 1, ..., K \) which move simultaneously in the first period each setting a fee \( c_k \). Here I show that allowing competition between PCWs is not meaningful, per se. A crucial variable and measure of competitive pressure is the number of aggregators shoppers check, \( r \), where \( 1 \leq r \leq K \). In the setting without PCWs, shoppers were informed of \( q \leq n \) firms. In the world with PCWs, one can interpret \( r \leq K \) as the number of aggregators shoppers are aware of. In the equilibria derived, firms

\(^{19}\)\( \alpha \) is determined costlessly for the PCW. If this advertising costs were a convex function of \( \alpha \), it would not change the results qualitatively.
list on all PCWs so shoppers will be indifferent to which set of PCWs they check. However, as PCW fees and firm-pricing strategy depend on $r$, there are different sets of equilibria for each $r$. Consequently, to investigate equilibria I parameterize them by $r$. I first look at the case of $r = 1$, where shoppers check just one of the PCWs.

**Proposition 4.** With $K > 1$ competing PCWs, if shoppers check $r = 1$ PCW then both types of consumer are worse off with the PCWs than without.

Proposition 4 obtains because the high-fee, single-PCW equilibrium of Proposition 2 with $c = v(1 - \alpha)$ remains the unique equilibrium fee level.\(^{20}\) The introduction of competing PCWs exerts no downward pressure on fees. There are no equilibria at lower fee levels because shoppers only check one PCW, which means there is no incentive for PCWs to undercut each other’s fees. If they did, they would not increase their volume of sales, but would receive lower fees. In contrast, for any candidate equilibrium with $c < v(1 - \alpha)$, there is an incentive to raise the fee because PCWs can maintain the volume of sales and achieve a higher fee level.

In the simplest textbook Bertrand result, there is an immediate fall in the equilibrium price between that of a monopoly firm and the marginal-cost pricing of two firms. At the aggregator level however, one can see that this logic is incomplete as Proposition 4 shows, with $K > 1$ such undercutting does not even necessarily begin. The persistence of the consumer-welfare-decreasing equilibrium in Proposition 4 is artefact of each shopper only checking one PCW which effectively makes each PCW a monopoly. Because all PCWs offer the same information, shoppers have no incentive to check other PCWs. In this respect, the equilibrium is reminiscent of Diamond (1971), but at the aggregation rather than firm level. The Bertrand remedy for shoppers would be to require them to check at least two PCWs, causing PCWs to undercut each other until an equilibrium with all PCWs charging $c = 0$ is reached. I now explain how this logic is incomplete. Suppose that the model is as before

\(^{20}\)As in the monopolist case, firms’ outside option gives rise to pure-pricing firm responses at higher fee levels, which bounds equilibrium fee levels from above at $c = v(1 - \alpha)$; see the Appendix for details.
with \( K > 1 \) aggregators, but shoppers now check \( r > 1 \) PCWs. First, I examine the special case of \( r = K > 1 \):

**Proposition 5.** With \( K > 1 \) competing PCWs, shoppers are guaranteed to be better off than before the introduction of PCWs if and only if they check all PCWs: The unique equilibrium PCW fee-level is \( c = 0 \) if and only if \( r = K > 1 \).

The spirit of this result resembles that of Bertrand. To see why there cannot be some other equilibrium with \( c > 0 \), suppose so and consider an undercutting deviation by PCW\(_1\) to some \( \hat{c}_1 = c - \epsilon \). Shoppers do not detect the deviation and so do not change their behavior. As for firms, notice that for any \( p \) they strictly prefer to list exclusively on PCW\(_1\): When a firm is the cheapest, it will sell to all shoppers so long as it lists on some PCW. This is precisely because \( r = K \). Hence by listing on PCW\(_1\) only, there is no reduction in the number of shoppers switching to them when they are cheapest, but there is a reduction in the fee the firm pays as \( \hat{c}_1 < c \). The PCW finds this deviation strictly profitable because it receives a discrete gain in the number of shoppers switching through it, for an arbitrary loss in price.

Where shoppers check more than one PCW, but not necessarily all PCWs:

**Lemma 2.** When \( K > r > 1 \), there exist equilibria in which PCWs charge \( c \in [0, \bar{c}] \), firms list on all PCWs, mixing over prices by \( G(p; c) \) where

\[
\bar{c} = \frac{v(1 - \alpha)K r (K - r)}{K(1 + r(K - 2)) + \alpha r (K - 1)(r - 1)(n - 1)}. 
\]

Substituting \( r = K \) in Lemma 2 yields \( \bar{c} = 0 \), and one obtains Proposition 5. One can see now that the Bertrand-style reasoning underlying Proposition 5 was a special case. To see why the principle does not apply more generally, consider a fee level \( c > 0 \) and an undercutting deviation by PCW\(_1\) to some \( c_1 = c - \epsilon > 0 \). Unlike when \( K = r > 1 \), when \( K > r > 1 \) it is not necessarily better for a firm to only list on the cheaper PCW\(_1\). By listing a price exclusively on PCW\(_1\), there are now \( \frac{K - r}{K} > 0 \) shoppers who do not see the
firm’s price, and so will not buy from it even if it is the cheapest. Firms now face a trade-off:
Exclusively listing on PCW₁ means that any sales incur only the lower fee \( \hat{c}_1 \) upon a sale, but
there will be a reduction in sales volume because not all shoppers check PCW₁. Which force
is stronger in this trade-off depends on the size of the undercut \( \epsilon \). If PCW₁ undercuts by
enough, firms will deviate to exclusively list on PCW₁, breaking the symmetric equilibrium.
Unlike the simpler logic of Proposition 5, it is no longer true that any \( \epsilon > 0 \) undercut will
attract firms to exclusively list on the cheapest PCW. Hence, PCWs do not always have an
incentive to undercut each other and higher-price equilibria are sustained.

I now discuss how the set of equilibria varies with how many PCWs shoppers check \( r \)
and the number of aggregators \( K \). Firstly, the highest sustainable equilibrium fee from
Lemma 2 \( \hat{c} \) is mitigated by a higher \( r \): \( \frac{dc}{dr} < 0 \). That is, as shoppers check more of the
PCWs in equilibrium, the incentive for PCWs to undercut each other’s fees increases, and
only lower fee-levels can be sustained in equilibrium.

However, as the number of aggregators increases, the incentive is reversed. The number
of shoppers checking a given PCW \( \frac{r}{K} \) falls. Accordingly, in equilibrium each firm receives
less of its expected revenue from any single PCW. It then requires a more severe undercut
from a PCW to get firms to exclusively list on it and forgo the business available from
the other aggregators. For undercuts that are too severe, it is unprofitable for a PCW to
deviate, even if it were to win exclusive arrangements with all firms as a result. Thus, as
\( K \) increases, higher equilibrium fees can be sustained in equilibrium: \( \frac{dc}{dK} > 0 \). This allows
for the result that a higher number of aggregators can lead to higher fees, and hence higher
prices. Furthermore:

**Proposition 6.** For any \( r \), there exists a \( \tilde{K} \) such that as long as there are more than \( \tilde{K} \)
aggregators both types of consumer can be worse off than before the introduction of any
PCWs.

In the limit, the proportion of firm income that comes from sales on any one aggregator
becomes vanishingly small. As it does so, sustainable equilibrium fee levels approach the
monopoly-PCW level, so both types of consumer can again be worse off than before the introduction of the sites.

6. Publicly-Observable Fees

One reason that competing aggregators do not ensure fees are undercut to zero is that shoppers do not detect changes in the fees of PCWs in equilibrium. This precludes a coordinated response between firms and shoppers that could punish a PCW that charges higher fees. If fees were publicly announced so that shoppers are aware of them, then credible subgame equilibria could follow the fee-setting decision in which the PCW charging the lowest fee is attended by all firms and shoppers. Any higher-fee equilibrium would then be undercut until \( c = 0 \), leading to lower shopper-prices. Indeed:

**Proposition 7.** When \( K > 1 \) and PCW fees are observed by shoppers, there exists an equilibrium where \( c = 0 \).

There are multiple equilibria because there are many subgame equilibria that can follow each value of the vector of PCW fees, including somewhat unintuitive ones where coordination occurs at more expensive PCWs.\(^{21}\) If one suggests the plausible refinement that requires firms and consumers to all attend only the lowest-fee PCWs, then one obtains the zero-fee equilibrium as the unique equilibrium.

However, even with sufficient refinement criteria to implement the zero-fee equilibrium, in some markets one may question a policy of fee-announcement on a more fundamental level. If fees can be publicly announced, then surely so can firm prices, which would extinguish the role of a PCW in the first place.

In reality PCW fees are not publicized directly. However, some PCWs do advertise summary statistics of the price information of firms that list on them. PCWs frequently advertise

\(^{21}\)Proposition 7 excludes the monopoly-PCW case \((K = 1)\), where the unique equilibrium is still that of Proposition 2 because of course no coordination between firms and consumers over which PCW to attend is possible when there is only one PCW.
the average savings a consumer using their site is expected to make, which could direct shoppers to the cheapest PCW. By Proposition 7, this could lead to a shopper-welfare-improving equilibrium. However, many PCWs do not advertise based on purchase-relevant information.\textsuperscript{22} PCWs spend large sums on such advertising, which has been shown to correlate with the number of unique visitors they experience, suggesting that many shoppers are directed to PCWs based on information other than price.\textsuperscript{23} If such ‘persuasive’ advertising caused all shoppers to loyally visit one site each i.e., \(r = 1\), Proposition 2 applies and both types of consumers would still have been better off without the PCW industry.

7. Price Discrimination

So far, I have considered the impact of web services that list or ‘aggregate’ the available information (prices) charged by firms offering a product or service. In practice, this is often the case in the markets for gas, electricity, financial products such as mortgages, and durable goods.\textsuperscript{24} However, in other markets price discrimination occurs, where a firm may set many prices, \(p_0\) which it charges for a direct purchase, and \(p_k\) for each PCW it lists on. Where a PCW operates by referring shoppers back to a firm’s website to complete the purchase, the fact that the click came from a PCW is recognized by the firm’s site, which then offers the price seen on the PCW that attracted the click. When \(K \geq r = 1:\)

**Proposition 8.** With price discrimination, if \(r = 1\), there exists an equilibrium in which PCWs set \(c = v(1 - \alpha)\), firms list on all PCWs, \(p_0 = v\) and \(p_1 = \cdots = p_K = v(1 - \alpha)\).

The ability of firms to price discriminate does not prevent PCWs setting fees at the same high level as Proposition 4 because \(r = 1\), so as before, there is effectively no competitive pressure between PCWs. However, price discrimination does lead to firms listing a common

\textsuperscript{22}See the campaign of Comparethemarket.com, based on a story about meerkats.

\textsuperscript{23}The big four PCWs in the UK spent approximately £110m in 2010 on advertising. The evidence here is from Nielsen Company (findings reported by the This is Money 2015 and Marketing Magazine 2011).

\textsuperscript{24}In the example of the UK gas and electricity markets, regulation has enforced that each energy company may only offer a maximum of four tariffs in total.
price on PCWs in equilibrium because PCWs no longer have an incentive to keep prices posted on it dispersed. This is because firms can set a high direct purchase price $p_0 = v$ and a lower price through the PCWs. Given this, shoppers always purchase through a PCW. Because all $\alpha$ shoppers now purchase through PCWs, rather than in the case without discrimination, where only $\alpha \left(\frac{n-1}{n}\right)$ did so, total PCW profit is now $\alpha v(1 - \alpha)$ rather than $\left(\frac{n-1}{n}\right) \alpha v(1 - \alpha)$. As firm profit is unaffected, it follows then that relative to the case without price discrimination, total consumer welfare falls. There are however opposite effects on each type. Auto-renewers are worse off, as now firms charge them the monopoly price $v$. Shopper welfare has improved as they now face $p_k = v(1 - \alpha)$ for sure, whereas before this was just the minimum of the support of equilibrium prices. More importantly though, shopper welfare does not improve sufficiently to overturn Proposition 3, which continues to hold for this equilibrium and both types are worse off than in a world with no PCWs. The ability of firms to discriminate allows them to fully extract surplus from their captive auto-renewers; but PCWs can now extract the revenue from sales to all shoppers through their sites.$^{25}$

In equilibria with $r > 1$, the incentive for aggregators to undercut is present. Corollary 5 describes a best response of firms following a unilateral downward deviation by a PCW from a symmetric fee level.

**Corollary 5.** When firms price discriminate, following $c_1 \in (0, c_2)$ and $c_2 = c_3 = \ldots = c_K \equiv c \in (0, v(1 - \alpha)]$ there exist mutual best-responses of firms such that they list on all PCWs, setting $p_0 = v$ and $p_k = \ldots = c_k$ for all $k$.

When $r = 1$, there is no incentive for a PCW to make such an undercutting deviation as suggested by the equilibrium described in Proposition 8. When $r > 1$ however, PCWs have this incentive to undercut because they can enjoy a discrete gain in fee revenue from the $\frac{r-1}{K}$ of shoppers who were checking their PCW but buying though another site in the symmetric equilibrium. When firms can price discriminate across websites, firms can compete in prices

$^{25}$As before, in the equilibrium of Proposition 8, PCWs cannot raise fees further, to say $c' > v(1 - \alpha)$ because of firms’ outside option. Following such a unilateral PCW deviation, firms would set $p_0 = v(1 - \alpha)$ so that their shoppers purchase directly from them, reducing PCW profit to zero.
on PCW without changing their prices on other PCWs. This shows how price discrimination can unleash undercutting at the PCW level whenever consumers check \( r > 1 \) PCWs, which can in turn lead to zero-fee equilibria.\(^{26}\) This contrasts with markets where PCWs aggregate price-information, where Proposition 5 showed that this undercutting was only fully unlocked when \( r = K \).

8. Conclusion

The analysis shows that the introduction of PCWs may not in fact benefit consumers by reducing expected prices. The introduction of a single aggregator facilitates comparison of the whole marketplace for shoppers, which exerts competitive pressure on firm pricing. However, the aggregator charges a fee which, in turn, places upward pressure on prices. The net effect is that prices increase for all consumers, who would be better off without the site.

Competition at the aggregator level need not lead to a reduction in fees. There are many equilibria, which I parameterize by the number of PCWs shoppers check, or are informed of. If shoppers only check one PCW each, consumers are no better off than in the monopoly setting. A greater number of aggregators is only guaranteed benefit shoppers if they check all of them. If shoppers check an intermediate number, increasing the number of aggregators can lead to higher prices; for a sufficiently high number, all consumers can again be better off without the industry. Hence, when there are many aggregators in the market, how many of them shoppers check is a crucial variable. As a result, regulatory bodies may wish to consider incentivizing consumers to check more, alongside stronger actions such as limiting the fees charged by aggregators and limiting the number of PCWs in the market.

If competing PCWs publicly announce fees, this can result in low-fee equilibria, but it relies on coordination between firms and shoppers. In markets with price discrimination, if shoppers only check one PCW, the monopoly fee level can still be sustained, making consumers worse off. However, with competing aggregators where shoppers check multiple

\(^{26}\)The existence of zero-fee equilibria when \( r > 1 \) are shown in the Appendix.
PCWs, there is then the incentive for the sites to undercut each other’s fees. As such, regulators may also wish to consider whether it is possible to introduce price discrimination into markets in which it is not currently present.

Appendix

Lemmas only appearing in the appendix are prefixed with ‘A’.


The figures quoted for turnover and profit in the introduction for the UK utilities and services industry are estimates for 2013 for the largest four such companies. The estimate for each site was taken from the following sources:

- Money Supermarket: Their own 2013 annual report.

For the first three, the figures were taken directly from the cited sources. For Compare the Market, estimated to be the largest of the four sites, the estimate is particularly rough as BGL Group offer no breakdown of their accounts. I assumed that the proportion of BGL’s total revenue and profit due to Compare the Market was the same where the estimate for annual profit due to Compare the Market is taken from the BBC article. Even if the estimated £800m of total turnover and 14% for average annual profit growth for these sites is off by a margin, the industry can still be considered large and growing.
A2. A World Without a PCW

The domain of prices is $\mathbb{R}$. The equilibrium pricing strategy can always be described by its cdf (denoted $F$ in this section, and with other letters later on). In what follows, either equilibrium pricing distributions will be pure (so that $F$ is flat, with one jump discontinuity at this price); or will have no point masses so that $F$ is continuous, which implies the density $f$ exists and $f = F'$, wherever $F'$ exists.

**Lemma A1.** In any equilibrium, there are no prices $p$ charged s.t. $p \leq 0$ or $p > v$.

**Proof:** This is a variant of Varian (1980) Proposition 1.

**Lemma A2.** In any equilibrium, there are no point masses.

**Proof:** This is a variant of Varian (1980) Proposition 3.

**Lemma A3.** In any equilibrium, the maximum of the support of $f$ must be $v$.

**Proof:** This is a variant of Varian (1980) Proposition 7.

**Proposition 1.** In the unique equilibrium firms mix according to the distribution

$$F(p) = 1 - \left[\frac{(v - p)(1 - \alpha)}{q\alpha}\right]^\frac{1}{q-1} \text{ over the support } p \in \left[\frac{v(1 - \alpha)}{1 + \alpha(q - 1)}, v\right].$$

**Proof:** By Lemma A2, there is more than one element of the equilibrium support, and by Lemma A3 $v$ is the maximal element. In equilibrium, a firm must be indifferent between all elements of the support $p$, hence profit must equal $v\left(\frac{1-\alpha}{n}\right)$ for all $p$, that is:

$$v\left(\frac{1-\alpha}{n}\right) = p\left[1 - \frac{\alpha}{n} + \frac{\alpha}{n}X(p)\right]$$

(1)

$$X(p) \equiv \binom{n-1}{q-1} \binom{n-1}{n-1} F(p)^0 (1 - F(p))^{n-1} + ... + \binom{q-1}{q-1} \binom{n-1}{q-1} F(p)^{n-q} (1 - F(p))^{q-1}$$

The first term on the RHS of (1) is the profit from ARs, who always purchase at $p \leq v$. The second term is the expected proportion of shoppers that a firm will win, charging price $p$. Shoppers can be characterized by $\binom{n}{q}$ groups, where the set of groups is given by $\{1, ..., n\}^q$. $X(p)$ describes the expected number of groups a firm expects to win given it charges $p$. By
Lemma A2 there are no ties in price, so label prices by \( p_1 < \ldots < p_n \). \( p_1 \) will be the cheapest in every group in which it appears, and it appears in \( \binom{n-1}{q-1} \) of the groups. The probability of being the lowest price is given by \( \binom{n-1}{q-1} F(p)^0 (1 - F(p))^{n-1} \), which accounts for the first term in \( X(p) \). The observation that \( p_i \) is the cheapest in \( \binom{n-1}{q-1} \) groups if \( i \leq n - (q - 1) \), zero groups otherwise, accounts for the remaining terms of \( X(p) \). The following manipulations to simplify the second term on RHS of (1) this make use of the binomial theorem:

\[
= p\alpha \frac{q}{n} \sum_{j=1}^{n-1} \binom{j}{q-1} \binom{n-1}{j} F(p)^{n-j-1} (1 - F(p))^j
\]

which after some manipulations can be shown to be:

\[
= p\alpha \frac{q}{n} (1 - F(p))^{q-1}
\]

rearranging for \( F(p) \) gives:

\[
F(p) = 1 - \left[ \frac{(v - p)(1 - \alpha)}{q\alpha} \right]^{q-1}
\]

Notice that this is a well-defined c.d.f. over:

\[
p \in \left[ \frac{v(1 - \alpha)}{1 + \alpha(q - 1)}, v \right]
\]

Notice that \( v \) is strictly preferred to any \( p \in \left[ 0, \frac{v(1 - \alpha)}{1 + \alpha(q - 1)} \right] \). ■

A3. A World With a PCW

I look for symmetric equilibria where PCWs charge some fee level \( c \geq 0 \) and shoppers check PCWs in equilibrium. I do not look at equilibria where firms never list on PCWs, where the setting without a PCW applies. To derive equilibria, one needs to know the mutual best-responses of firms to unilateral deviations of PCWs. To do so, take \( c \) and equilibrium shopper strategy as given, and consider the firm responses.

Let \( K \geq 1 \) denote the number of PCWs and \( r : K \geq r \geq 1 \) the number of PCWs checked by shoppers. In symmetric equilibrium, a proportion \( \frac{r}{K} \) of each firm’s shoppers
check any given PCW. Define a vector of PCW fees as \( c = (c_1, ..., c_K) \in \mathbb{R}_+^K \) labeled such that \( c_1 \leq ... \leq c_K \). Let \( \beta_k \in [0, 1] \) be the probability with which a firm enters PCW \( k \) and define the event \( E \): “all PCWs are empty”. Denote \((a_1, a_2) = (p, \mathcal{K})\) as a firm’s action, where \( p \) is the price charged and \( 2^{\{1, ..., K\}} \) is set of all combinations of PCWs they could choose to list in where \( \mathcal{K} \) a typical element, and \( \emptyset \) denotes not listing on any PCW. Define the following CDF, which is used throughout:

\[
G(p; c) = 1 - \left[ \frac{(v - p)(1 - \alpha)}{\alpha (np - (n - 1) \frac{1}{K} (c_1 + \cdots + c_K))} \right]^{\frac{1}{n-1}}
\]

which is well-defined over the support \([p(c), v]\) where \( p(c) = \frac{v(1-\alpha) + \frac{1}{K} (c_1 + \cdots + c_K) \alpha (n-1)}{1 + \alpha (n-1)} \). When \( c_1 = \cdots = c_K \equiv c \), let \( G(p; c) \) and \( p(c) \) also be written \( G(p; c) \) and \( p(c) \).

**Lemma A4.** In any equilibrium, there are no prices \( p \) charged s.t. \( p \leq 0 \) or \( p > v \).

**Proof:** No \( p \leq 0 \) or \( p > v \) because they yield negative and zero profit respectively, whereas \((v, \emptyset)\) yields \( \frac{v(1-\alpha)}{n} > 0 \). \( \blacksquare \)

**Lemma A5.** If \( c_1 \in [0, v(1-\alpha)) \), \( pr(E) = 0 \).

**Proof:** Suppose \( pr(E) > 0 \). Denote the infimum of prices charged when no PCW is listed on and that of prices ever listed on a PCW as \( p_0 \) and \( p \) respectively. [Note the infima exist because prices are a bounded from below by Lemma A4]. Note that \( p_0 \geq p \) because \((p, \emptyset)\) is strictly preferred to any lower price as a firm faces no competition for prices below \( p \) off the PCWs. Consider when the firm is called upon to play \((p_0, \emptyset)\) (or a price arbitrarily closely above \( p_0 \)):

If \( p_0 > c_1 \), a deviation to \((p_0 - \epsilon, 1)\) is strictly profitable. This is because with probability \( pr(E)^{n-1} > 0 \) PCWs are empty with other firms charging at least \( p_0 \). By listing the firm then has a positive probability of winning \( \alpha \frac{r}{K} \frac{n-1}{n} \) new shoppers. For a sufficiently small \( \epsilon > 0 \), this will offset the arbitrary loss in revenue from its own consumers.

If \( p_0 \leq c_1 \), firm profit must be at least \( \frac{p}{n} \) which can be guaranteed by \((p, 1)\), because \( p_0 \geq p \). In turn, this must be at least as much as \( \frac{v(1-\alpha)}{n} \) which the firm can guarantee by playing \((v, \emptyset)\). Putting these together, \( p_0 \geq p \geq v(1-\alpha) > c_1 \) which contradicts \( p_0 \leq c_1 \). \( \blacksquare \)
Lemma A6. If \( c_1 \in [0, v(1 - \alpha)) \), firm strategies have no point masses.

Proof: Define \( p_1(c_1) \) as the price at which a firm is indifferent between selling to all shoppers exclusively through the cheapest PCW(s) and charging \( v \), only sell to auto-renewers:

\[
p_1(c_1) = \frac{v(1 - \alpha) + \alpha c_1(n-1)}{1 + \alpha(n-1)}
\]

By Lemma A5, some \((p, \mathcal{K})\) is played. Note that firms would by construction not play \((p, \mathcal{K})\) where \( p < p_1(c_1) \), strictly preferring \((v, \emptyset)\). To see that there are no point masses:

If there were a point mass at \((p_1(c_1), \mathcal{K})\) then there is a positive probability of being tied for the lowest price at \((p_1(c_1), \mathcal{K})\). By definition of \( p_1(c_1) \) firms would strictly prefer to deviate to \((v, \emptyset)\).

If there were a point mass at \((\hat{p}, \mathcal{K'})\) s.t. \( \hat{p} > p_1(c_1) \) then there is a positive probability of being tied for the lowest price at \((\hat{p}, \mathcal{K'})\). A firm would strictly prefer to shift that probability mass to \((\hat{p} - \epsilon, \mathcal{K}' \setminus \{k : c_k \geq \hat{p}\})\). Here, the firm would sell to \( \alpha \frac{r}{n} \frac{n-1}{n} \) other firms’ shoppers at an arbitrary loss in revenue from its own consumers. There is always an \( \epsilon > 0 \) small enough to ensure this is profitable because \( p_1(c_1) > c_1 \iff c_1 < v(1 - \alpha) \).

Lemma A7. If there are no point masses, the maximum of the support \( f \) must be \( v \).

Proof: This is a variant of Varian (1980) Proposition 7.

Lemma A8. If \( c_1 \in [0, v(1 - \alpha)) \) and \( c_1 < c_2, \beta_1 = 1 \).

Proof: By Lemma A5 it is never the case that all PCWs are empty. Suppose \( \beta_1 < 1 \). Lemma A6 implies there is more than one price, \( \hat{p} \), that is listed on some other PCWs. By Lemma A7 there is one such that \( \hat{p} < v \). Consider a firm being called upon to play this \((\hat{p}, \mathcal{K}-1)\) where \( 1 \notin \mathcal{K}-1 \) and \( \bar{m} = \max\{\mathcal{K}-1\} \). As this price has a positive probability of being the lowest of all firms, it will generate sales through the PCWs in \( \mathcal{K}-1 \). But as PCW is the unique cheapest PCW, there is a strictly profitable deviation to \((\hat{p}, \mathcal{K}-1 \cup 1 \setminus \bar{m})\).

Lemma A9. If \( c_1 = \ldots, c_K \equiv c \in (v(1 - \alpha), v] \), firm pricing strategies are pure where \( p \in [v(1 - \alpha), c] \). Any \((\beta_1, \ldots, \beta_K) \in (0, 1]^K \) can be supported.
Proof: Either there is a point mass or there is not.

1. Suppose there is a point mass at price $\hat{p}$. If $\hat{p} > c$, firms have a strict incentive to shift this mass to $(\hat{p} - \epsilon, \{1, \ldots, K\})$. If $\hat{p} < v(1 - \alpha)$, firms prefer to shift this mass to $(v, \emptyset)$. These leaves $\hat{p} \in [v(1 - \alpha), c]$ as the only points that can be point masses. There can be at most one point mass: If not, then there a second point mass $\hat{p} < c$, which if played with $\mathcal{K} \neq \emptyset$ would generate negative profit, so $(\hat{p}, \emptyset)$ is preferred; if $\mathcal{K} = \emptyset$, then $(\hat{p} + \epsilon, \emptyset)$ for some sufficiently small $\epsilon > 0$ is preferred. To see that this pure pricing at $p \in [v(1 - \alpha), c]$ can be part of firm strategies, note that firm profit is $\pi = \frac{p}{n} \geq \frac{v(1 - \alpha)}{n}$, so there is no strict incentive to sell only to auto-renewers instead. Because shoppers buy directly when prices are all the same, there are no sales through PCWs and so firms are indifferent between any $(\beta_1, \ldots, \beta_K) \in (0, 1]^K$.

2. Suppose there is no point mass. By Lemma A7 the maximum of the support is $v$, where $v$ is not the only element of the support, else it would be a point mass. There is therefore a positive probability of a firm being the cheapest at some $p$. There can be no $(p, \mathcal{K})$ s.t. $p < c$ charged: If $\mathcal{K} \neq \emptyset$ played profit from these sales is negative, so $(p, \emptyset)$ is preferred; if $\mathcal{K} = \emptyset$, then $(p + \epsilon, \emptyset)$ for some sufficiently small $\epsilon > 0$ is preferred. Given $p \in [c, v]$, $pr(E) = 0$. This follows because firms strictly prefer $(p, \{1, \ldots, K\})$ to $(p, \emptyset)$ for all $p \in (c, v)$. For any $1 \leq r \leq K$ firms are content to list prices on at least as many PCWs as is necessary to make sure every shopper sees their price e.g., for $r = K = 1$ all of them; for $r = K$, just one of them. There can therefore, be different configurations of $\beta_K$'s depending on $r, K$ so long as $pr(E) = 0$. To determine firm pricing strategy it must be that firms are indifferent between every $p$ they are called upon to play:

$$v \left(1 - \frac{\alpha}{n}\right) = p \left(1 - \frac{\alpha}{n}\right) + (1 - G(p; c))^{n-1} \left[\frac{\alpha}{n}p + \frac{\alpha(n-1)}{n} (p - c)\right]$$

which can be re-arranged to give $G(p; c)$ from (2). However, $p(c) < c$ because $c > v(1 - \alpha)$, so firms would make strictly negative profits at prices $p \in (p(c), c)$, preferring not to list. This provides a contradiction, so there do not exist strategic firm responses with no point
A3.1. Results for $K = 1$ or $r = 1$

Lemma A10. If $r = 1$, $c_1 = \cdots = c_{K-1} \in [0, v(1-\alpha))$ and $c_K \in [c_1, p(c))$: $\beta_k = 1$ for all $k$ where firms mix by CDF $G(p; c)$.

Proof: By Lemma A5 there is always some price posted on some PCW(s). By Lemma A6 these prices are not point masses and by Lemma A7 the maximum of the support is $v$, where only auto-renewers are sold to. There is therefore a positive probability of a sale through some PCW(s) at some $(p, \mathcal{K})$. If $k \in \mathcal{K}$ where $k < K$ and sales there are profitable (when $p > c_1$), then $\{1, \ldots, K-1\} \in \mathcal{K}$; if $K \in \mathcal{K}$ and sales there are profitable $p > c_K$ (whether or not $p > c_K$ is the only consideration because $r = 1$), it follows that $\{1, \ldots, K\} \in \mathcal{K}$. To be part of firm strategy, it must also be that firms prefer to play $(p, \mathcal{K})$ than to charge $v$ and sell only to their auto-renewers. Note that $p(c)$ from (2) is the price at which firms are indifferent between selling through $\{1, \ldots, K\}$ with certainty and selling only to their auto-renewers. Similarly, denote $p_K(c)$ as the indifference point between selling for sure on $\{1, \ldots, K-1\}$ and charging $v$, and note $p_K(c) < p(c)$. There will then be no $(p, \{1, \ldots, K-1\})$ played s.t. $p < p_K(c)$ and no $(p, \{1, \ldots, K\})$ s.t. $p < p(c)$. For prices close to $v$, $\mathcal{K} = \{1, \ldots, K\}$. To determine firm pricing strategy, it must be that firms are indifferent between every $(p, \{1, \ldots, K\})$ they are called upon to play:

$$v \left(1 - \frac{\alpha}{n}\right) = p \left(1 - \frac{\alpha}{n}\right) + (1 - G(p; c))^{n-1} \left[\frac{\alpha}{n} p + \frac{\alpha(n-1)}{n} \left(p - \left(c_1 \frac{K-1}{K} + c_K \frac{1}{K}\right)\right)\right]$$

which can be re-arranged for $G(p; c)$ to give the CDF from (2). Because the minimum of the support is $p(c) > c_K \geq c_1$ (the first relation follows because $c_1 < v(1 - \alpha)$), all prices in the support generate profitable sales through all the PCWs, there is no price charged s.t. $p \in [p_K(c), p(c))$. It follows that $\beta_k = 1$ for all $k$. For $K = 1$, let $c_1 \in [0, v(1 - \alpha))$ and set $K = 1$ in the expressions of the Lemma. ■

Lemma A11. If $r = 1$ and $c = v(1 - \alpha)$ there are the following firm responses:
1. Pure-price strategies where \( p = v(1 - \alpha) \) is the only price ever charged. Here, any \((\beta_1, \ldots, \beta_K) \in (0,1]^K\) can be supported.

2. Mixed-price strategies where \( \beta_k = 1 \) for all \( k \) and prices are distributed according to the CDF \( G(p; v(1 - \alpha)) \) where \( p(v(1 - \alpha)) = v(1 - \alpha) \).

**Proof:** Either there is a point mass or there is not.

1. Suppose there is a point mass at price \( \hat{p} \). If \( \hat{p} > c \), firms have a strict incentive to shift this mass to \((\hat{p} - \epsilon, \{1, \ldots, K\})\). If \( \hat{p} < c \), firms have a strict incentive to shift this mass to \((v, \emptyset)\). These leaves \( \hat{p} = c \) as the only point that can be a point mass. To see that this pure pricing can be part of firm strategies, note that firm profit is \( \pi = \frac{\alpha}{n} = \frac{v(1-\alpha)}{n} \), so there is no strict incentive to sell only to auto-renewers instead. Because shoppers buy directly when prices are all the same, there are no sales through PCWs and so firms are indifferent between any \((\beta_1, \ldots, \beta_K) \in (0,1]^K\).

2. Suppose there is no point mass. By Lemma A6, \( v \) is the maximum of the support of prices. No prices \( p < c \) are charged because \((v, \emptyset)\) is strictly preferred. For \( p > c \), sales through all PCWs are profitable so \( \beta_k = 1 \) for all \( k \). When \( v \) is played, firm profit is \( \pi(v) = \frac{v(1-\alpha)}{n} \). To determine firm pricing strategy, it must be that firms are indifferent between every \( p \) they are called upon to play:

\[
\pi(v) = p \left( \frac{1-\alpha}{n} \right) + \frac{\alpha}{n} (1 - G(p; v(1 - \alpha)))^{n-1} [np - (n - 1)v(1 - \alpha)].
\]

which can be re-arranged to give the CDF from (2).


\[\text{Lemma A12.}\] If \( r = 1 \), \( c_1 = \cdots = c_{K-1} = v(1 - \alpha) \) and \( c_K \in (c_1, v] \) there exist firm responses in pure-price strategies where \( v(1 - \alpha) \) is the only price ever charged. Here, any \((\beta_1, \ldots, \beta_K) \in (0,1]^K\) can be supported.

**Proof:** In such an equilibrium, firm profit is \( \pi = \frac{v(1-\alpha)}{n} \). Consider a deviation to \( p \). If \( p \in (v(1 - \alpha), v] \), deviation profit is \( \hat{\pi} = \frac{p(1-\alpha)}{n} \leq \pi \). If \( p < v(1 - \alpha) \), firms have a
strict incentive to shift this mass to $(v, \emptyset)$. Because shoppers buy directly when prices are all the same, there are no sales through PCWs and so firms are indifferent between any $(\beta_1, \ldots, \beta_K) \in (0, 1]^K$. ■

**Lemma 1.** The mutual best-responses of firms as a function of $c$:

1. If $c \in [0, v(1 - \alpha))$, firm responses have no point masses, and are described by $G(p; c)$.

2. If $c = v(1 - \alpha)$, there are two classes of response, one with no point masses described by $G(p, c)$, and those that are degenerate.

3. If $c \in (v(1 - \alpha), v]$, firm responses are degenerate.

Where pricing is described by $G(p; c)$ each firm always lists its price on the PCW.

**Proof:** See Lemmas A10, A11 and A9 respectively. ■

**Proposition 2.** In the unique equilibrium where shoppers check the PCW, the PCW sets a click-through fee of $c = v(1 - \alpha)$, firms list on the PCW and mix over prices according to $G(p; v(1 - \alpha))$ over the support $p \in [v(1 - \alpha), v]$.

**Proof:**

If $c \in [0, v(1 - \alpha))$, Lemma A10 shows that there is a profitable upward deviation to $c \in (c_1, p(c))$. It is profitable because there is an increase in fee-level but no reduction in the quantity of sales.

If $c \in (v(1 - \alpha), v]$, Lemma A9 shows that firms will play pure-price strategies and so $u = 0$. If $c = v(1 - \alpha)$ and firms play pure-pricing strategies, then $u = 0$ again. In these cases, Lemma A10 shows that a deviation to $c \in (0, v(1 - \alpha))$ will generate $u_1 > 0$, so there are no equilibria where $c \in (v(1 - \alpha), v]$ or for $c = v(1 - \alpha)$ when firms respond with pure-pricing strategies.

If $c = v(1 - \alpha)$ and firms mix over prices by $G(p; v(1 - \alpha))$ as in Lemma A11, equilibrium PCW profit is $u^* = v(1 - \alpha) \frac{n-1}{K^2} > 0$. Lemma A9 shows that any upward deviation would yield $u = 0$. There can be no profitable deviation downwards because, as Lemma A10 shows, the fee would be reduced for no gain in the quantity of sales. ■
Proposition 3. Both types of consumer are worse off with the PCW than without.

Proof of Proposition 3: First I show that auto-renewers are worse off under $G(p; v(1-\alpha))$ than $F(p)$ (referred to as $G$ and $F$ here). Auto-renewers pay the price quoted by their current firm. To show they are worse off with the PCW, I show that $E_F[p] < E_G[p]$ by showing that $G$ first-order stochastic dominates (FOSDs) $F$. The distributions share the same upper bound on their supports, with $F$ having a lower lower bound. Hence, $G$ FOSDs $F$ if $G(p; v(1-\alpha)) \leq F(p)$ for $p \in [v(1-\alpha), v]$, which can be re-arranged as

$$\left[\frac{(v-p)(1-\alpha)}{\alpha q p}\right]^{\frac{1}{q-1}} \leq \left[\frac{(v-p)(1-\alpha)}{\alpha n p - \alpha v(1-\alpha)(n-1)}\right]^{\frac{1}{n-1}}.$$ 

This holds because the terms in parentheses are in $[0,1]$, the power on of the LHS is larger and the denominator on the LHS is larger iff $p(n-q) \leq v(1-\alpha)(n-1)$ which is satisfied because $p \leq v(1-\alpha)$ and $n-q < n-1$.

To show shoppers are worse off under $G$ than $F$, first show that $E_F[p_{(1)}]$ is lower than the lower bound of the support of $g$ in the case of $q = 2$. Then, I use Proposition 3 of Morgan et al. (2006) which corresponds to my setup (the only difference is that they have $v = 1$), which states that $E_F[p_{(1)}]$ is decreasing in $q$. Hence I show the first step here to prove that $E_F[p_{(1)}]$ is below $v(1-\alpha)$ for all $q$. For $q = 2$,

$$E_F[p_{(1)}] = \int_{v[\frac{1-\alpha}{1+\alpha}]}^v f(p_{(1)}) p dp$$

where $f(p_{(1)}) = 2(1-F(p)) f(p)$ is the density function of the lower of the two draws shoppers receive from $F$. Computing yields

$$E_F[p_{(1)}] = \left(\frac{1-\alpha}{\alpha}\right)^2 \frac{v}{2} \left[\log \left(\frac{1-\alpha}{1+\alpha}\right) + \frac{2\alpha}{1-\alpha}\right].$$

Then $E_F[p_{(1)}] < v(1-\alpha)$ can be rearranged to obtain

$$\log \left(\frac{1-\alpha}{1+\alpha}\right) + 2\alpha < 0.$$ 

The left hand side of this is a decreasing function in $\alpha$ and when $\alpha = 0$, it is 0. Hence this
expression holds for $\alpha \in (0,1)$. ■

**Corollary 1.** Within the mixed-price equilibrium firm responses of Lemma 1, as $c \in [0,v(1-\alpha)]$ increases, the expected price paid by both types of consumer increases.

**Proof:** From Lemma 1 the pricing strategy for $c \in [0,v(1-\alpha)]$ is given by, $G(p;c)$ over $[p(c),v]$. Differentiating,

$$\frac{dG(p;c)}{dc} = \frac{1}{c(n-1) - np} \left( \frac{(v-p)(1-\alpha)}{\alpha(n-1)c} \right)^{\frac{1}{n-1}}.$$ 

The second term is $\geq 0$ else $G(p;c) > 1$. The first term is $\leq 0 \iff c^{\frac{n-1}{n}} < p$ which is ensured because $p \geq c > c^{\frac{n-1}{n}}$ when $c \leq v(1-\alpha)$ (this follows because $p(c) \geq c \iff v(1-\alpha) \geq c$). Then for any $c,c' \in [0,v(1-\alpha)]$, if $c > c'$ then the equilibrium pricing distribution under $c$ first order stochastic dominates that under $c'$. Hence the expected price (paid by auto-renewers) and the expected lowest price from $n$ draws (paid by shoppers) are higher under $c$. ■

**Corollary 2.** As the number of firms increases, the expected price paid by shoppers decreases but the expected price paid by auto-renewers increases.

**Proof:** Using an observation from Morgan et al. (2006), industry profit of firms is given by $\alpha \mathbb{E}[p(1)] + (1-\alpha)\mathbb{E}[p] = v(1-\alpha)$, where $\mathbb{E}[p(1)]$ denotes the lowest price of $n$ draws from $G(p;v(1-\alpha))$ and $\mathbb{E}[p]$ denotes the expected price from $G(p;v(1-\alpha))$. The RHS is the industry profit of firms if it charged $v$ and only sold to auto-renewers. Differentiating and rearranging,

$$\frac{d\mathbb{E}[p(1)]}{dn} = -\left( \frac{1-\alpha}{\alpha} \right) \frac{d\mathbb{E}[p]}{dn}$$

so the derivatives have opposite signs. Now show that $\frac{\mathbb{E}[p]}{dn} \geq 0$. To do this, show that $G(p,v(1-\alpha))$ is stochastically ordered in $n$:

$$\frac{dG(p;v(1-\alpha))}{dn} \leq 0 \iff \log \left( \frac{(v-p)(1-\alpha)}{\alpha(np-v(1-\alpha)(n-1))} \right) + \frac{(n-1)(p-v(1-\alpha))}{np-v(1-\alpha)(n-1)} \equiv X(p) \leq 0.$$
Note that \( X(v(1 - \alpha)) = 0, \) and \( \frac{dX}{dp} < 0: \)

\[
\frac{dX(p)}{dp} \leq 0 \iff p \geq v(1 - \alpha) \left[ \frac{(2n - 2 + \alpha(n - 1)^2)}{(2n - 1 + \alpha(n - 1)^2)} \right].
\]

Notice that the term on RHS in square brackets is below 1. Recall that \( p \geq v(1 - \alpha) \) as this is the lower bound of the support, hence this is satisfied. ■

**Proposition 4.** With \( K > 1 \) competing PCWs, if shoppers check \( r = 1 \) PCW then both types of consumer are worse off with the PCWs than without.

**Proof:** There are no equilibria where \( c \in [0, v(1 - \alpha)) \): Lemma A10 shows that there is a profitable upward deviation to \( c_K \in (c_1, p(c)) \). It is profitable because there is an increase in fee-level but no reduction in the quantity of sales. Note that there can be no profitable deviation downwards given \( r = 1 \) because the fee would be reduced for no gain in the quantity of sales.

There are no equilibria where \( c \in (v(1 - \alpha), v] \): Lemma A9 shows \( u_k = 0 \) for all \( k \). Lemmas A6 and A8 show that a deviation to \( c_1 \in (0, v(1 - \alpha)) \) will generate \( u_1 > 0 \).

The only remaining option is \( c_1 = \cdots = c_K \equiv c = v(1 - \alpha), \) where firm responses are described by Lemma A11. If firms play a pure-pricing strategy, then as in the previous case, PCWs make zero profit and there is a profitable deviation to \( c_1 \in (0, v(1 - \alpha)) \). If however, firms respond with the mixed-price strategy, this fee-level is an equilibrium: There can be no profitable deviation downwards for PCWs because the fee would be reduced for no gain in the quantity of sales as \( r = 1 \). Following an upward deviation from PCW \( K \) to \( c_K \in (v(1 - \alpha), v] \), when firms respond with a pure-pricing strategy as detailed in Lemma A11, PCW \( K \)'s profit falls to zero. There is then an equilibrium at this fee level, and it is the unique such level where there exists an equilibrium.

At this fee-level, firms play just as they did when \( K = 1 \) with the same CDF over prices and all PCWs list all \( n \) firm prices. Both types of consumer are therefore left with the same level of surplus they had under \( K = 1 \). □

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Corollary 3. When pricing is determined by \( G(p; c) \), as the proportion of shoppers \( \alpha \) increases, expected prices for both types of consumer decrease.

Proof: Differentiating \( G(p; c) \) by \( \alpha \),

\[
\frac{dG(p; c)}{d\alpha} = \frac{1}{\alpha(n-1)(1-\alpha)} \left( \frac{(v-p)(1-\alpha)}{\alpha np - \alpha (n-1)c} \right)^{\frac{1}{n-r}}
\]

The first term is \( > 0 \). The second term is always \( \geq 0 \) or \( G(p; c) > 1 \). Then for any \( \alpha, \alpha' \in (0,1) \), if \( \alpha > \alpha' \) then the equilibrium pricing distribution under \( \alpha' \) first order stochastic dominates that under \( \alpha \). Hence the expected price (paid by auto-renewers) and the expected lowest price from \( n \) draws (paid by shoppers) are lower under \( \alpha \). ■

Corollary 4. If the PCW can determine \( \alpha \) as well as \( c \) in the preliminary stage, then the PCW sets \( \alpha = \frac{1}{2} \).

Proof: I expand the PCW’s action set to include \( \alpha \in (0,1) \). Notice that for any choice of \( \alpha \in (0,1) \), by the reasoning as in the proof of Proposition 2, the PCW will avoid the pure equilibria of Lemma 1 so \( c \in [0, v(1-\alpha)] \), firms mix and the PCW is given by \( c\alpha \left( \frac{n-1}{n} \right) \). The PCWs optimization problem can hence be solved by,

\[
\max_{c, \alpha} c\alpha \left( \frac{n-1}{n} \right) \text{ s.t. } c \in [0, v(1-\alpha)] \text{ and } \alpha \in (0,1)
\]

where the solution is \( c = v(1-\alpha), \alpha = \frac{1}{2} \). ■

A3.2. Results for \( K > 1 \) and \( r > 1 \)

Lemma A13. If \( r > 1 \), \( c_1 = \cdots = c_{K-1} \in [0, v), c_K \in (c_1, v], u_K = 0 \).

Proof: This Lemma says that there is no profitable upwards deviation for a PCW from any equilibrium. Consider such a unilateral deviation by PCW\(_K\). Firm response can either be pure or mixed pricing. If pure, then \( u_k = 0 \) for all \( k \). If mixed, then at any price \( p \) that has positive probability of sales through PCWs where \( K \in \mathcal{K} \), firms have a strict preference to play \((p, \{1, \ldots, K-1\})\) instead. This is because \( r > 1 \): Every shopper who sees the prices
on PCW\(_K\) also sees the prices on another PCW. Firms can therefore avoid PCW\(_K\)’s higher fee by not listing there while facing no reduction in the quantity of sales.

**Lemma A14.** If \( r = K, \ c_2 = \cdots = c_K \in (0, v(1 - \alpha)) \) and \( c_1 \in [0, c_2) \): \( \beta_1 = 1, \ \beta_k = 0 \) for \( k = 2, \ldots K \) and prices are distributed according to the CDF \( G(p, c_1) \).

**Proof:** There are no point masses by Lemma A6. By Lemma A8 \( \beta_1 = 1 \). \( \beta_k = 0 \) for \( k = 2, \ldots, K \) follows because all shoppers check every PCW \( (r = K) \). Therefore, at any price \( p \) that has positive probability of generating sales through PCWs, firms have a strict preference only to list on PCW\(_1\) without any reduction in the quantity of sales. By Lemma A7 the maximum of the support is \( v \). Firms must be indifferent to all \((p, 1)\) they are called upon to play, hence:

\[
v \left( \frac{1 - \alpha}{n} \right) = p \left( \frac{1 - \alpha}{n} \right) + \frac{\alpha}{n} (1 - G(p; c_1))^{n-1} [np - (n - 1)c_1].
\]

which can be re-arranged to give \( G(p, c_1) \) from (2).

**Proposition 5.** With \( K > 1 \) competing PCWs, shoppers are guaranteed to be better off than before the introduction of PCWs if and only if they check all PCWs: The unique equilibrium PCW fee-level is \( c = 0 \) if and only if \( r = K > 1 \).

**Proof:** Sufficiency: Suppose not. Then there exists an equilibrium with \( c > 0 \). By Lemma A13, there is no profitable upward deviation. Now consider a downward deviation. If \( c \in [v(1 - \alpha), v] \) and firms respond with a pure-pricing equilibrium, as detailed in Lemma A9, then \( u_k = 0 \). A deviation by PCW\(_1\) to \( c_1 \in (0, v(1 - \alpha)) \) would lead to the responses detailed in Lemma A14 and deviation profit of \( u_1 = c\alpha \frac{n-1}{n} > 0 \). If \( c \in (0, v(1 - \alpha)] \) and firms respond with by mixing by \( G(p, c) \), as in Lemma A14, then \( u_k = c\alpha \frac{n-1}{n} > 0 \) for all \( k \). But a deviation by PCW\(_1\) to \( c_1 < c \) exists s.t. \( u_1 = c_1\alpha \frac{n-1}{n} > c\alpha \frac{n-1}{n} = u_k \). This deviation is strictly profitable, so \( c \) could not have been an equilibrium. To see that \( c = 0 \) is an equilibrium, recall that by Lemma A13 there is no profitable upward deviation.

Necessity: Lemma A11 shows that for \( K \geq r = 1 \) the unique equilibrium fee level is \( c = v(1 - \alpha) \). Lemma 2 shows that for \( 1 < r < K \) there are multiple equilibria. Hence
$K = r > 1$ is the only case where the unique equilibrium of $c = 0$ obtains.

Consumer welfare: As noted in the text, as firms make the same expected profit in both worlds, there is a one-to-one relation between consumer welfare and PCW profit. To see the difference in the changes to shopper and auto-renewer welfare from a move to a world with a PCW but $c = 0$, Proposition 3 of Morgan et al. (2006) (the only difference is that they have $v = 1$) shows that the increase from $q$ to $n$ results in a reduction in the price paid by shoppers, and an increase for auto-renewers. ■

Lemma 2. When $K > r > 1$, there exist equilibria in which PCWs charge $c \in [0, \bar{c}]$, firms list on all PCWs, mixing over prices by $G(p; c)$ where

$$
\bar{c} = \frac{v(1 - \alpha)Kr(K - r)}{K(1 + r(K - 2)) + ar(K - 1)(r - 1)(n - 1)}.
$$

Proof: Note that $\bar{c} \in [0, v(1 - \alpha))$ hence any $c \in [0, v(1 - \alpha))$. Take such a $c$ as a candidate equilibrium fee level. There are no point masses by Lemma A6. By Lemma A7 the maximum of the support is $v$. By Lemma A5 $pr(E) = 0$ and as all PCWs charge the same fee, firms are content to list in all of them i.e., $\beta_k = 1$ for all $k$. Firms must be indifferent to all $(p, \{1, \ldots, K\})$ they are called upon to play, hence

$$
v \left( \frac{1 - \alpha}{n} \right) = p \left( \frac{1 - \alpha}{n} \right) + \frac{\alpha}{n} (1 - G(p; c))^{n-1} [np - (n - 1)c]
$$

which can be re-arranged to give $G(p; c)$ in (2).

To confirm $c$ is an equilibrium fee level, ensure there is no profitable PCW deviation. By Lemma A13, there is no profitable upward deviation. However, unlike Lemma A14, when $1 < r < K$ it is no longer true that any undercut by PCW$_1$ to $c_1 < c$ will result in all firms listing on it exclusively. This is because consumers see $r > 1$ PCWs: when a firm sells having played $(p, \{1, \ldots, K\})$ it pays $c_1 \frac{1}{K} + c \frac{K-1}{K}$, but were it to have played $(p, 1)$ it would have paid $c_1 \frac{r}{K}$ i.e., the firm faces a trade-off between lower fees and higher sales volume. By Lemma A8 $\beta_1 = 1$. PCW$_1$’s deviation profit is therefore determined by $\beta_k$, $k > 1$. For small-enough undercuts of $c$, firms will still be content to list on all PCWs. Suppose however,
that PCW1 undercuts by just enough such that such that firms are no longer content to list all their prices on the other PCWs. In the best case for PCW1, $\beta_k = 0$ for all $k$ so that all shoppers checking PCW1 buy only through PCW1. If in this case, PCW1 still does not make more than its equilibrium profit $u^* = cK^n / n$, then the undercut is never profitable and $c$ constitutes an equilibrium fee level. I now carry out this logic.

Denote $c_1 < c$ as the undercut of PCW1 and $\tilde{c}_1$ as the threshold level required for $c_1$ to entice firms to de-list some of their prices from PCWk s.t. $k > 1$. Given $c = (c_1, c, \ldots, c)$, similarly to the derivation above, one can show firms will respond by $G(p; c)$ over $[\bar{p}(c), v]$ as in (2). Firm deviation profit from this response to $(p, 1)$ is given by

$$\pi(p) = p \left(1 - \frac{\alpha}{n}\right) + \frac{(v-p)(1-\alpha)[Kp - r(p - c_1)(n-1)]}{n(Knp - (n-1)(c_1 + c(K-1)))}.$$ 

Note that this is valid for $p \in [\bar{p}(c), v]$, but $p < p_1$ give strictly less profit than $p = p(c)$. One can show that $\pi(p)$ is convex in $p$. Together with the observation that $(v, 1)$ gives the equilibrium profit $\frac{v(1-\alpha)}{n}$, this says that the optimal deviation is to $(p(c), 1)$ and is profitable if and only if $\pi(p(c)) > \frac{v(1-\alpha)}{n}$, which can be rearranged to give

$$c_1 < \frac{c(K-1)(K + r\alpha(n-1)) - K v(K - r)(1-\alpha)}{r\alpha(K-1)(n-1) + K(r-1)} \equiv \tilde{c}_1.$$

Suppose that the firm response following the undercut was such that $\beta_k = 0$ for all $k > 1$ when PCW1 sets the highest such undercut just below $\tilde{c}_1$. PCW1 prefers not to make the deviation whenever

$$u^* \geq \tilde{c}_1 \frac{\alpha r n - 1}{K} \iff c \geq \tilde{c}_1$$

which can be rearranged to give

$$c \leq \frac{v(1-\alpha)Kr(K-r)}{K(1 + r(K-2)) + \alpha r(K-1)(r-1)(n-1)} \equiv \tilde{c}.$$ 

Therefore at fee levels $c \in [0, \tilde{c}]$ there exist equilibria where $\beta_k = 1$ for all $k$ and firms mix by the CDF given in the Lemma. 

**Proposition 6.** For any $r$, there exists a $\tilde{K}$ such that as long as there are more than $\tilde{K}$
aggregators both types of consumer can be worse off than before the introduction of any PCWs.

**Proof:** To show for shoppers, let the equilibrium be given as in Lemma 2 with \( c = \bar{c} \) and see that

\[
\lim_{K \to \infty} \bar{c} = v(1 - \alpha). \tag{4}
\]

For shopper welfare notice that for \( K = r, \bar{c} = 0 \) and

\[
\mathbb{E}_F[p(1)] \geq \mathbb{E}_G(v(1 - \alpha)) \tag{5}
\]

where \( G(c) \) and \( F \) denote \( G(p; c) \) and \( F(p) \). This is because the PCWs effectively increase the number of firms competing for each shoppers from \( q \) to \( n \), and from the last point of the proof of Proposition 5, this lowers the price paid by shoppers. Proposition 3 shows that both types are worse off with PCWs when \( c = v(1 - \alpha) \) for \( K \geq r = 1 \), but notice that if \( c = v(1 - \alpha) \) in equilibrium with \( K > r \geq 1 \), then this will be true a fortiori, because it can be shown that \( G(v(1 - \alpha)) \) first-order stochastically dominates \( F \) for all \( q \). By (4) then as \( K \to \infty, \mathbb{E}_F[p(1)] < \mathbb{E}_G(v(1 - \alpha)) \). Because \( \bar{c} \) is continuous in \( K \), \( \frac{dc}{dK} > 0 \) and \( \frac{d\mathbb{E}_G(v(1 - \alpha))}{dK} > 0 \) by Corollary 1, there exists \( \bar{K} \) s.t. for \( K > \bar{K} \) there is always an equilibrium fee level that makes shoppers worse off relative to a world without PCWs.

The proof for auto-renewers is similar but simpler to the above steps for shoppers and has \( p \) replacing \( p(1) \): From Proposition 3 of Morgan et al. (2006), the effect of increasing the number of firms competing for each shopper increases prices for auto-renewers. Then by Corollary 1, their prices only rise further under \( G(c) \). Hence auto-renewers are worse off for any \( c \) and hence any \( K \). ■

**A4. Results under Publicly Observable Fees**

**Definition 1** (Coordinated Subgame). Given \( c \), a ‘coordinated subgame’ is when shoppers attend and firms list in only the cheapest PCWs i.e., shoppers in PCW \( k \) if \( c_k = c_1 \), and for firms \( \beta_k = 1 \) if \( c_k = c_1 \) else \( \beta_k = 0 \). Firms mix by \( G(p; c_1) \).
Lemma A15. When fees are observed by shoppers, and all subgames are ‘coordinated subgames’ there exists a subgame perfect equilibrium where $c = 0$, $\beta_k = 1$ for all $k$ and prices are distributed according to the CDF $G(p, 0)$.

**Proof:** There are no profitable deviations for firms: $G(p, 0)$ ensures they are indifferent between listing all prices in the support $[p(0), v]$, listing any price less than $p(0)$ is less profitable than listing $v$, and any $(p, \emptyset)$ generates at most $\frac{v(1-\alpha)}{n}$ when $p = v$. To see there is no profitable upward deviation for PCWs, note that due to the ‘coordinated subgame’ shoppers and firms would not attend this PCW following such a deviation, so the PCW receives zero profit. To see that these coordination subgames are Nash equilibria: for firms one can conduct the same round of checks as at the beginning of this proof; for consumers notice that as the non-cheapest PCWs are all empty, there is no incentive to check them. □

**Proposition 7.** When $K > 1$ and PCW fees are observed by shoppers, there exists an equilibrium where $c = 0$.

**Proof:** That there exists an equilibrium with $c = 0$ follows directly from Lemma A15. For shopper welfare at $c = 0$, see the last point of the proof of Proposition 5. □

### A5. Results under Price Discrimination

Now assume that different prices can be charged directly and through the PCW. Denote such prices $p_0$ and $p_k$ respectively, where $k = 1, \ldots, K$ indexes the PCW as before.

**Lemma A16.** If $r = 1$, $c_1 = \cdots = c_{K-1} = v(1-\alpha)$ and $c_K \in [c_1, v]$ then there exist firm responses s.t. $p_0 = p_1 = \cdots = p_K = v(1-\alpha)$ and $\beta_k = 1$ for all $k$.

**Proof:** Under these strategies, firm profit is $\pi = \frac{v(1-\alpha)}{n}$. There is no profitable deviation involving $p_k$ for any $k$: lower would prompt sales at $p_k < c_k$, higher would never attract any shoppers. There is no profitable deviation involving $p_0$: lower would still sell to auto-renewers and own-shoppers but at a lower price, higher would only sell to auto-renewers for which the optimal such deviation is to $v$ generating profit $\frac{v(1-\alpha)}{n} = \pi$. □
Proposition 8. With price discrimination, if $r = 1$, there exists an equilibrium in which PCWs set $c = v(1 - \alpha)$, firms list on all PCWs, $p_0 = v$ and $p_1 = \cdots = p_K = v(1 - \alpha)$.

Proof: Given $c_1 = \cdots = c_K \equiv c = v(1 - \alpha)$, let us confirm there are no profitable deviations for firms. Equilibrium firm profit is $\pi = \frac{v(1-\alpha)}{n}$. There is no profitable deviation involving $p_k$ for any $k$: lower would prompt sales at $p_k < c_k$, higher would never attract any shoppers. There is no profitable deviation involving $p_0$: lower would either sell only to auto-renewers at a lower price, or to both auto-renewers and own-shoppers but only for prices at least as low as $v(1 - \alpha)$, generating profit no greater than $\frac{v(1-\alpha)}{n}$; higher would be above $v$ and hence make zero profits. Now consider a PCW deviation. Equilibrium PCW profit is $u_k = \frac{\alpha v (1-\alpha)}{K} > 0$. There is no profitable deviation to a lower fee: as $r = 1$ doing so would at best sell to the same proportion of shoppers ($\frac{\alpha}{K}$) but at a lower fee. As for a deviation to a higher fee: assume that firms’ mutual best responses are given by those in Lemma A16 where PCW profit is zero. ■

Corollary 5. When firms price discriminate, following $c_1 \in (0, c_2)$ and $c_2 = c_3 = \cdots = c_K \equiv c \in (0, v(1 - \alpha)]$ there exist mutual best-responses of firms such that they list on all PCWs, setting $p_0 = v$ and $p_1 = \cdots = c_k$ for all $k$.

Proof: Given $c$ as in the Lemma, let us confirm there are no profitable deviations for firms. Firm profit is $\pi = \frac{v(1-\alpha)}{n}$. There is no profitable deviation involving $p_k$ for any $k$: lower could prompt sales, but only at $p_k < c_k$, higher would never attract any shoppers. There is no profitable deviation involving $p_0$: lower would either sell only to auto-renewers at a lower price, or to both auto-renewers and own-shoppers but only for prices at least as low as $c_2$, generating profit no greater than $\frac{\alpha v}{n} < \pi$; higher would be above $v$ and hence make zero profits. ■

Lemma A17. If $r > 1$, $c_1 = \cdots = c_{K-1} = 0$ and $c_K \in [0, v]$ then there exist firm responses s.t. $p_0 = v$, $p_1 = \cdots = p_{K-1} = 0$, $\beta_k = 1$ for $k < K$ and $\beta_K = 0$.

Proof: Under these strategies, firm profit is $\pi = \frac{v(1-\alpha)}{n}$. There is no profitable deviation involving $p_K$: $p_K < 0$ would give sell at a loss whereas $p_K > 0$ would never attract any
shoppers as $r > 1$. There is no profitable deviation involving $p_k$ for any $k < K$: Lower would only create sales at a loss, higher would never attract any shoppers. There is no profitable deviation involving $p_0$: lower would still sell to auto-renewers and (for $p_0 \leq 0$) own-shoppers but at a lower price, higher would sell to no-one. ■

**Lemma A18.** If $r > 1$, there exists an equilibrium where $c = 0$, firms list on all PCWs, $p_0 = v$ and $p_1 = \cdots = p_K = 0$.

**Proof:** One can follow the proof of Proposition 8 to confirm there are no profitable deviations for firms. For PCWs, one only need consider an upward deviation in fee level in which case, assume firms respond as in Lemma A17, which yields zero profit for the deviating PCW. ■

**References**


BGL Group Ltd. (2013), “Group Results and Review.”


Rickard-Straus, Rachel (2015), “We can’t act surprised that comparison sites make a mint from us switching - Snoop Dogg doesn’t work for free.” *This is Money*, URL http://www.thisismoney.co.uk/money/bills/article-2933401/Energy-price-comparison-sites-spend-110m-annoying-adverts.html.


