Enhancing the Efficiency of Constrained Dual-hop Variable-gain AF Relaying under Nakagami-m Fading

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***Abstract*—This paper studies power allocation for performance constrained dual-hop variable-gain amplify-and-forward (AF) relay networks in Nakagami-m fading. In this context, the performance constraint is formulated as a constraint on the end-to-end signal-to-noise-ratio (SNR) and the overall power consumed is minimized while maintaining this constraint. This problem is considered under two different assumptions of the available channel state information (CSI) at the relays, namely full CSI at the relays and partial CSI at the relays. In addition to the power minimization problem, we also consider the end-to-end SNR maximization problem under a total power constraint for the partial CSI case. We provide closed-form solutions for all the problems which are easy to implement except in two cases, namely selective relaying with partial CSI for power minimization and SNR maximization, where we give the solution in form a one-dimension equation which can be solved efficiently. Numerical results are then provided to characterize the performance of the proposed power allocation algorithms considering the effects of channel parameters and CSI availability.**

***Index Terms*—Amplify-and-forward, cooperative relaying, energy-efficiency, power allocation, Nakagami-m fading, variable-gain relays.**

I. Introduction

Recent years have seen energy-efficiency becoming an important metric in evaluating the performance of wireless networks [1]. This is mainly due to two reasons [2]. The first is the environmental impact. Base stations burn considerable amount of fuel to generate the required electrical power [3], [4]. Hence, there is an urgent need to lessen the carbon footprint of wireless networks. The second reason is the operating costs [5]. An increase in power consumption means a direct increase in operating costs which is undesirable for mobile vendors and operators. Moreover, mobile terminals have a limited battery life which needs to be preserved. Due to these reasons, many new protocols and strategies have been proposed to enhance the energy-efficiency of wireless networks. Moreover, previously known concepts have also been revisited.

One such concept is cooperative relaying in which source and destination terminals communicate with each other with the help of other terminals [6]. It has been shown that in addition to other benefits, cooperative relaying reduces energy consumption [7], [8]. Therefore, cooperative relaying strategies and power allocation for cooperative relaying have been the subject of much work recently [9]–[17]. However, cooperative relaying comes with its own challenges. Two challenges that are associated with cooperative relaying are the availability of CSI at the relay nodes and the processing power at the relay nodes [18]. These are particularly important in the context of energy-efficiency as enhancing energy-efficiency requires allocating the power as efficiently as possible which in turn requires availability of CSI and sophisticated signal processing hardware at the relay nodes. However, having more CSI available at the relays increases feedback overhead. Additionally, as feedback requires transmission of data, it also consumes power and adds to the carbon footprint. Moreover, to perform power allocation, relays might need to utilize sophisticated hardware which furthers increases their complexity and increases their power consumption. However, in practical scenarios, it is difficult to achieve this complexity as in cooperative relaying, the relays are small mobile nodes, which are traditionally simple and are constrained in power. Therefore, it is desirable to have efficient power allocation algorithms which are simple to implement and require less overhead. This work attempts to address these issues jointly in the context of dual-hop AF relaying.

Previous relevant works include the following. Optimal power allocation to minimize the outage probability of a variable-gain dual-hop AF relay network with multiple relays was studied in [19]. In this article, both an all-participate (AP) scheme in which all the relays forwards the signal from the source to the destination and a relay selection scheme in which only the selected relay forwards the signal were presented. Additionally, [19] considered both full CSI at the relays and knowledge of only channel statistics at the relays. Power allocation algorithms were proposed for both schemes

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and the optimal selection criterion for relay selection was also obtained under Rayleigh fading. Using a similar model but with different assumptions and approximations, [20] obtained power allocation to minimize the outage probability of a multiple AF relay network under Rayleigh fading. Both [19] and [20] used bounds on the outage probability as it was difficult to optimize the exact outage probability.

Reference [21] studied power allocation for multiple variable-gain AF relays relays with partial CSI and AP relaying. The authors made two assumptions on the available CSI at the relays and at the destination. Under both assumptions on the CSI, [21] maximized the end-to-end SNR by optimally allocating power to the relays for Rayleigh fading channels. We studied energy-efficient power allocation for an AP fixed-gain AF relay network requiring only the knowledge of the instantaneous channel responses of the second hop and channel statistics of the first hop, and requiring only the knowledge of the channel statistics of all the links in [2] and [22], respectively. In both works, we considered the problems of maximizing the end-to-end SNR under individual and overall power constraints and minimizing the total power consumed while ensuring that the end-to-end SNR remains above a certain threshold value.

The contribution of this work is summarized as follows:

- This work differs from [2] and [22] by considering variable-gain AF relays instead of fixed-gain AF relays.
- However, unlike [19] and [20], it studies the problem of minimizing the total consumed power under peak power constraints on the individual nodes and a maximum threshold constraint on the end-to-end SNR. Specifically the work in [19] minimized outage probability under a total power constraint, while this work minimizes the total consumed power under an end-to-end SNR constraint. Similarly, [21] considers SNR maximization.
- This work consider Nakagami-m fading while all these previous works considered the special case of Rayleigh fading.
- We consider two relaying strategies, AP and selective, and two assumptions on the available CSI at the relay, full knowledge of all the links and full knowledge of all the source-relay links and knowledge of channel statistics of the source-destination and relay-destination links.
- We obtain power allocation algorithms to minimize the consumed power while maintaining the end-to-end SNR over a certain threshold for the aforementioned two relaying strategies and under the two aforementioned assumptions on the available CSI at the relays for Nakagami-m fading which is a generalized distribution and contains the Rayleigh distribution as a special case.
- This work also generalizes the work done in [21] by considering the end-to-end SNR maximization problem for the partial CSI case for Nakagami-m fading.
- In keeping with the theme of this work, we provide efficient and simple to implement power allocation algorithms for all the considered optimization problems.

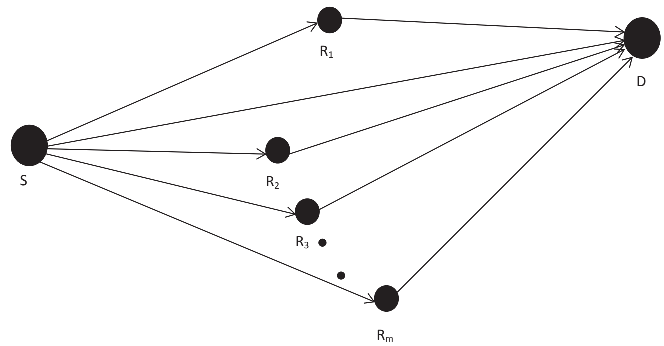

Fig. 1: System model

The rest of the paper is organized as follows. Section II describes the system model. Section III consider power allocation under the full CSI assumption for the power minimization problem. Consumed power minimization with partial channel state is considered in Section IV. Section V presents the power allocation schemes for SNR maximization with partial channel state information. Numerical results are discussed in Section VI to quantify the performance of the proposed algorithms. Finally, Section VII concludes the paper.

II. System Model

Consider the system shown in Fig. 1. The system comprises of a source node (S), a destination node (D) and M variable-gain AF relay nodes (R). The source node is connected to the destination node through a single-hop direct link and M dual-hop links through the relays. It is noted here that this work contains the case of no direct link as a special scenario which can be obtained by setting the S-D channel to 0. The relays are assumed to work in half-duplex mode and hence, cannot simultaneously transmit and receive over the same frequency at the same time. The relays are also assumed to be equipped with a single antenna. The system is assumed to operate in Time Division Multiple Access (TDMA) mode. Therefore, the source and the relays transmit on time orthogonal channel. However, the analysis and algorithm provided in this work are general and are applicable to both frequency and code orthogonal channels.

The signal is transmitted in two phases. In the first phase, the source broadcasts the signal to the relays and the destination. In the second phase, the relays forward the signal to the destination. For AP relaying, all the M relays forward the signal to the destination. Hence, to transmit one packet of information, $M + 1$ time slots are required. For selective relaying, only the selected relay[1] forwards the signal to the destination. Thus, selective relaying utilizes a total of 2 time slots. The destination combines the received signals using a maximal ratio combining (MRC) scheme.

A block-fading channel model is assumed here. It is assumed that the channel gain of each link remains constant for transmission of one packet and changes independently from one packet transmission phase to another. Furthermore, all the

[1]The criterion for relay selection will be considered in the next section when selective relaying is considered.

links are assumed to undergo independent fading. The fading gains of all the links are modeled as Nakagami-m random variables (RVs) [23]. It is assumed that the destination has full CSI of all the links so that it can perform MRC. At the relays, two CSI assumptions are considered. In Section III, it assumed that the relays have full CSI of all the links. This assumption is the same as in [19], [24]. In Section IV, it is assumed that the relays have full CSI of all the S-R links, but knowledge of only the channel statistics of the S-D link and all the R-D links. This is the same assumption as Assumption A in [21].

The primary objective of this work is to minimize the the total power consumed while achieving a targeted level of system performance. This targeted level of performance is characterized in the form of a constraint on the end-to-end SNR which has to be met. In addition to the SNR constraint, peak power constraints at the source and relay nodes are also considered. The objective can be written in the form of an optimization problem as

$$\begin{gathered}\min_{P_s, P_i} \ P_s + \sum_{i \in \mathbb{Z}} P_i, \ \text{ subject to} \\ \gamma \geq \gamma^{th}, \ 0 < P_s \leq P_s^{max}, \ 0 \leq P_i \leq P_i^{max},\end{gathered} \tag{1}$$

where P_s is the source power, P_i is the ith relay power, P_s^{max} specifies the peak power constraint at the source node, P_i^{max} specifies the peak power constraint at the ith relay node, γ is the end-to-end SNR, γ^{th} is the pre-specified threshold on γ and $\mathbb{Z}$ is the set of all relays which forward the signal from the source to the destination. For AP relaying, $\mathbb{Z}$ contains all the relays, while for selective relaying, $\mathbb{Z}$ contains only the selected relay. This problem is considered for the two assumptions on the CSI at the relays and the two relay participation schemes in the next two sections.

In addition to the power minimization problem, we also consider the end-to-end SNR maximization for the case of partial CSI. This problem had been previously considered for AP relaying in [21]. However, only for Rayleigh fading. In this work, we generalize it to Nakagami fading. Moreover, we also consider SNR maximization for selective relaying which is new enhancement to previous results. In the SNR maximization problem, we maximize the end-to-end SNR under a total power constraint and individual power constraint on all the nodes. Thus, the SNR maximization optimization problem can be formulated as

$$\begin{gathered}\max_{P_s, P_i} \ \bar{\gamma}, \ \text{ subject to} \\ P_s + \sum_{i \in \mathbb{Z}} P_i \leq P_{tot}, \ 0 \leq P_s \leq P_s^{max}, \ 0 \leq P_i \leq P_i^{max},\end{gathered} \tag{2}$$

where $\bar{\gamma}$ is the average end-to-end SNR for the partial CSI case which will be derived in Section IV and P_{tot} is the constraint on the overall power of the system.

III. Case of Full CSI with End-End SNR Constraint

In this section, we consider the problem of minimizing the overall power under individual power constraints and the constraint on the end-to-end SNR for the full CSI assumption. We first discuss AP relaying and them selective relaying. For AP relaying, as we will see, it is difficult to find the optimal solution. Hence, we propose a suboptimal, but simple and efficient power allocation algorithm. For selective relaying, we obtain the optimal solution. However, first we obtain the end-to-end SNR.

Under the assumptions described in Section II, the end-to-end SNR for variable-gain AF relaying with MRC at the destination considering the direct S-D link is given by [19]

$$\begin{aligned}\gamma &= P_s\alpha_0 + \sum_{i \in \mathbb{Z}} \frac{P_s P_i \alpha_i \beta_i}{P_s\alpha_i + P_i\beta_i + 1} \\ &= P_s\left(\alpha_0 + \sum_{i \in \mathbb{Z}} \alpha_i\right) - \sum_{i \in \mathbb{Z}} \frac{P_s^2\alpha_i^2 + P_s\alpha_i}{P_s\alpha_i + P_i\beta_i + 1},\end{aligned} \tag{3}$$

where P_s is the source power, P_i is the power of the ith relay, $\alpha_0 = \frac{|h_{sd}|^2}{\sigma_{sd}^2}$, $\alpha_i = \frac{|h_{si}|^2}{\sigma_{si}^2}$, $\beta_z = \frac{|h_{id}|^2}{\sigma_{id}^2}$, h_{sd} is the channel gain of the direct S-D link, h_{si} is the channel gain of the S-R link of the ith relay, h_{id} is the channel gain of the R-D link of the ith relay, σ_{sd}^2 is the variance of the additive white Gaussian noise (AWGN) of the S-D link, σ_{si}^2 is the variance of the AWGN of the S-R link of the ith relay and σ_{id}^2 is the variance of the AWGN of the R-D link of the ith relay. As all the channels are modeled as Nakagami-m RVs α_0, α_i and β_i are all Gamma RVs [25, Sec. 2.2.1.4].

A. AP Relaying

In the case of AP relaying, $\mathbb{Z}$ contains all the relays. Hence, the summation in (3) is from $i = 1$ to M. It is very difficult to find the optimal solution of the problem in (1) for the AP case. Hence, here a simple and suboptimal solution is provided.

First, the direct link is checked, if it can fulfill the constraint on the end-to-end SNR while also meeting its respective peak power constraint, then only the source transmits at power

$$P_s = \frac{\gamma^{th}}{\alpha_0}, \qquad \text{with } \ P_s \leq P_s^{max}, \tag{4}$$

and the relays don't transmit. This operation requires only one time slot but the D has to inform the relays about meeting the required γ^{th}. However, if the direct link cannot fulfil the constraint on the end-to-end SNR, then the source power is set at its peak power constraint, $P_s = P_s^{max}$, and the relay powers are obtained from

$$\begin{gathered}\min_{P_i} \ \sum_{i=1}^{M} P_i, \ \text{ subject to } \ 0 \leq P_i \leq P_i^{max} \text{ (and)} \\ P_s\sum_{i=0}^{M}\alpha_i - \sum_{i=1}^{M} \frac{P_s^2\alpha_i^2 + P_s\alpha_i}{P_s\alpha_i + P_i\beta_i + 1} \geq \gamma^{th}.\end{gathered} \tag{5}$$

It is easy to see that (5) is a convex optimization problem (P_i appears in the denominator of (3), which means that minimizing the term after -ve sign will improve γ). Moreover, the end-to-end SNR is a monotonically increasing functions of the relay powers. Hence, the optimal solution to (5) is achieved when the constraint on the end-to-end SNR is met with equality. As the problem is convex and one constraint is an equality and the other constraint is linear, the primal and dual problem will yield the same solution [26]. Therefore, the duality gap is 0 and solving (5) using the Lagrange dual method gives the optimal solution [26].

Ignoring the individual constraints, which will be incorporated later on, and forming the Lagrangian

$$\mathbb{L} = \sum_{i=1}^{M} P_i + \rho \left(\gamma^{th} - P_s \sum_{i=0}^{M} \alpha_i + \sum_{i=1}^{M} \frac{P_s^2 \alpha_i^2 + P_s \alpha_i}{P_s \alpha_i + P_i \beta_i + 1} \right), \tag{6}$$

where ρ is the Lagrange multiplier. From the Karush-Kuhn-Tucker (KKT) conditions for optimality [26], one obtains

$$\frac{\rho (P_s^2 \alpha_i^2 + P_s \alpha_i) \beta_i}{(P_s \alpha_i + P_i \beta_i + 1)^2} = 1 \quad i = 1, 2, \dots M, \tag{7a}$$

$$\rho \geq 0, \tag{7b}$$

$$P_s \sum_{i=0}^{M} \alpha_i - \sum_{i=1}^{M} \frac{P_s^2 \alpha_i^2 + P_s \alpha_i}{P_s \alpha_i + P_i \beta_i + 1} = \gamma^{th}. \tag{7c}$$

The optimal ith relay power can be obtained from (7a) as

$$P_i = \sqrt{\frac{\rho (P_s^2 \alpha_i^2 + P_s \alpha_i)}{\beta_i}} - \frac{P_s \alpha_i + 1}{\beta_i}, \tag{8}$$

where the Lagrange multiplier ρ can be obtained from (7c) as

$$\rho = \frac{\left(\sum_{i=1}^{M} \sqrt{\frac{P_s^2 \alpha_i^2 + P_s \alpha_i}{\beta_i}} \right)^2}{(P_s \sum_{i=0}^{M} \alpha_i - \gamma^{th})^2}. \tag{9}$$

Now, as both the objective function in (5) and the end-to-end SNR are monotonically increasing and convex functions of P_i, the optimal solution after incorporating the individual constraints lies at the boundary. Thus, the optimal ith relay power after including the individual constraints is given by

$$P_i = \left(\sqrt{\frac{\rho (P_s^2 \alpha_i^2 + P_s \alpha_i)}{\beta_i}} - \frac{P_s \alpha_i + 1}{\beta_i} \right)_0^{P_i^{max}}, \tag{10}$$

where $i = 1, 2, \dots M$ and $(x)_b^a = \max(b, \min(x, a))$. Thus, the power allocation policy follows a water-filling solution where the power is allocated in an iterative manner. On each iteration, the power is allocated according to (8), then the powers are checked to see if any violates their respective peak power constraints. If there are powers which violate their peak power constraints, then the maximum of these powers is set at its peak constraint and the power allocation algorithm is run again for the rest of the relays. If no power violates its respective peak constraint, then all the power are checked to see if any power violates the lower constraint of 0. If there are powers which are less than 0, then the minimum of these powers is set at 0 and the algorithm is run again for the rest of the relays. This iterative procedure is repeated until all the powers satisfy their respective individual constraint.

An important point to note is that as each node is restricted in its power, there can be scenarios where the constraint on the end-to-end SNR cannot be met due to bad channel conditions. In such a case, the source and the relays transmit at full power. Therefore, a check can be performed at the beginning of the power allocation algorithm to see if the constraint on γ can be fulfilled. If it can be, then the iterative power allocation procedure is run, otherwise the source and the relays all transmit at their peak powers. The complete power allocation algorithm is shown in Algorithm 1[2]. For the first iteration, the set $\mathbb{J}$ which contains the powers which violate their constraint and are set at their constraints, is empty

Algorithm 1 Power Allocation(M,$\boldsymbol{\alpha}$,$\boldsymbol{\beta}$,γ^{th},$\boldsymbol{P^{max}}$)

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check ⇐ P_s^max Σ_{i=0}^M α_i − Σ_{i=1}^M ((P_s^max)^2 α_i^2 + P_s^max α_i) / (P_s^max α_i + P_i^max β_i + 1)
if check < γ^th then
  P_s ⇐ P_s^max
  P_i ⇐ P_i^max   ∀i
else
  P_s ⇐ γ^th / α_0
  if P_s ≤ P_s^max then
    P_i ⇐ 0   ∀i
  else
    P_s ⇐ P_s^max
    loopind=0
    while loopind=0 do
      P_i ⇐ sqrt(ρ(P_s^2 α_i^2 + P_s α_i) / β_i) − (P_s α_i + 1) / β_i   ∀i ≠ 𝕁
      (∼, l) ⇐ max(P_i)   ∀i ≠ 𝕁
      if P_l > P_l^max then
        Add l to 𝕁
        P_l ⇐ P_l^max
      else
        ∼, l ⇐ min(P_i)   ∀i ≠ 𝕁
        if P_l < 0 then
          Add l to 𝕁
          P_l ⇐ 0
        else
          loopind⇐1
        end if
      end if
    end while
  end if
end if
return P_s, P_i ∀i
```

B. Selective Relaying

For selective relaying, $\mathbb{Z}$ only contains the relay selected to forward the signal from the source to the destination. Hence, the optimization problem is now given by

$$\begin{gathered} \min_{P_s, P_z} \; P_s + P_z, \quad \text{subject to} \\ \gamma \geq \gamma^{th}, \; 0 \leq P_s \leq P_s^{max}, \; 0 \leq P_z \leq P_z^{max}, \end{gathered} \tag{11}$$

[2]Boldface letters represent vectors. For example, $\boldsymbol{\alpha} = [\alpha_0 \; \alpha_1 \dots \alpha_m]$.

P_z^{max} is the peak power constraint at the selected relay node and (special case of (3) when selected relay is only considered)

$$\gamma = P_s(\alpha_0 + \alpha_z) - \frac{P_s^2\alpha_z^2 + P_s\alpha_z}{P_s\alpha_z + P_z\beta_z + 1}, \tag{12}$$

It can be observed from (12) that the end-to-end SNR, γ, is a monotonically increasing function of both P_s and P_z. Thus, the optimal solution of (11) is achieved when $\gamma = \gamma^{th}$. Using this fact, P_z can be written as a function of P_s

$$P_z = \frac{P_s^2\alpha_z^2 + P_s\alpha_z}{\beta_z(P_s(\alpha_0 + \alpha_z) - \gamma^{th})} - \frac{P_s\alpha_z}{\beta_z} - \frac{1}{\beta_z}, \tag{13}$$

where $P_s(\alpha_0 + \alpha_z) \neq \gamma^{th}$ and the individual power constraints have been ignored. The individual power constraints are incorporated later on after addressing the convexity of the optimization problem. Also, $P_s(\alpha_0 + \alpha_z) = \gamma^{th}$ can only be the case when $\alpha_z = 0$ as evident from (12). The first condition, $\alpha_z = 0$, means that there is no path between the source and the relay. Hence, there is only the direct link and there is no need to allocate power to the relay. Therefore, for all scenarios involving non-zero power allocation to the relay, $P_s(\alpha_0 + \alpha_z) \neq \gamma^{th}$.

Using (13) and ignoring the individual constraints, the problem in (11) can be re-formulated as

$$\min_{P_s} \quad \frac{P_s^2\alpha_z^2 + P_s\alpha_z}{\beta_z(P_s(\alpha_0 + \alpha_z) - \gamma^{th})} - \frac{P_s\alpha_z}{\beta_z} - \frac{1}{\beta_z} + P_s. \tag{14}$$

Let $\zeta_z = \frac{P_s^2\alpha_z^2 + P_s\alpha_z}{\beta_z(P_s(\alpha_0+\alpha_z)-\gamma^{th})} - \frac{P_s\alpha_z}{\beta_z} - \frac{1}{\beta_z} + P_s$. Taking the double derivative of ζ_z to check the the convexity of the problem,

$$\frac{\partial^2\zeta_z}{\partial P_s^2} = \frac{2\alpha_z\gamma^{th}(P_s(\alpha_0 + \alpha_z) - \gamma^{th})\left(\alpha_z\gamma^{th} + \alpha_0 + \alpha_z\right)}{\beta_z(P_s(\alpha_0 + \alpha_z) - \gamma^{th})^4}. \tag{15}$$

It is evident from (15) that in the range $P_s < \frac{\gamma^{th}}{\alpha_0+\alpha_z}$, ζ_z is concave and in the range $P_s > \frac{\gamma^{th}}{\alpha_0+\alpha_z}$, ζ_z is convex. Hence, solving the problem in (14) by taking the derivative of ζ_z and equating it to 0 will yield two solutions. However, the lesser of the two solutions will always give $P_z < 0$ as can observed from (13). Thus, the greater one is the optimal solution which can be obtained as

$$P_s = \frac{\gamma^{th}\phi + \sqrt{\gamma^{th}\phi(\alpha_0\alpha_z + \alpha_z^2 + \alpha_z^2\gamma^{th})}}{(\alpha_0 + \alpha_z)\phi}, \tag{16}$$

where $\phi_z = \alpha_0\beta_z + \alpha_z\beta_z - \alpha_0\alpha_z$. If $\phi_z < 0$, then (16) yields an imaginary solution. This means that the derivative of ζ_z is never equal to 0. As ζ_z is a convex function in the range of interest, $P_s > \frac{\gamma^{th}}{\alpha_0+\alpha_z}$, the optimal solution in this case lies on the boundary. Thus, the optimal value of P_s which solves (14) is

$$P_s = \begin{cases} \frac{\gamma^{th}\phi_z + \sqrt{\gamma^{th}\phi_z(\alpha_0\alpha_z + \alpha_z^2 + \alpha_z^2\gamma^{th})}}{(\alpha_0+\alpha_z)\phi_z} & \phi_z > 0 \\ \min\left(P_s^{max}, \frac{\gamma^{th}}{\alpha_0}\right) & \phi_z < 0. \end{cases} \tag{17}$$

Now if $P_s = \frac{\gamma^{th}}{\alpha_0}$, then $P_z = 0$ and the optimal solution is obtained. Otherwise P_z is obtained by substituting P_s into (13). If both powers satisfy their respective individual constraints, then the power allocation algorithm exits. However, if one of the powers exceeds its peak constraint then it is set at its respective peak constraint and the other power is calculated again. If P_s exceeds P_s^{max}, then P_s is set at P_s^{max} and P_z is obtained from (13). If P_z exceeds P_z^{max}, then P_z is set at P_z^{max} and P_s can then be obtained from the constraint, $\gamma = \gamma^{th}$, as the solution to the following quadratic equation

$$aP_s^2 + bP_s + c = 0, \tag{18}$$

where a, b, and c are defined as

$$a = \alpha_0\alpha_z$$
$$b = P_z^{max}\alpha_0\beta_z + \alpha_0 + P_z^{max}\alpha_z\beta_z - \alpha_z\gamma^{th}$$
$$c = -P_z^{max}\beta_z\gamma^{th} - \gamma^{th}.$$

It can be easily seen that $b^2 - 4ac > b^2$. Hence, one of the roots of (18) is less than 0. So, P_s can be obtained as

$$P_s = \frac{-b + \sqrt{b^2 - 4ac}}{2a}. \tag{19}$$

If no power exceeds their respective peak constraint, then P_z is checked to see if it is below 0. If it is then P_z is set at 0 and $P_s = \frac{\gamma^{th}}{\alpha_0}$.

The above power allocation algorithm is performed for each relay and the relay which minimizes the total power consumed is selected to forward the signal to the destination, i.e. P_s and P_z are calculated for each relay through the algorithm described above and the relay which has the minimum $P_s + P_z$ is selected. Similar to the AP case, there can be scenarios where the constraint on γ cannot be met. In such cases, the source and the selected relay transmit at full power, otherwise the above algorithm is run. Moreover, if all the relays cannot meet the constraint on γ, then the relay which maximizes γ is selected, where as stated previously $P_s = P_s^{max}$ and $P_z = P_z^{max}$. The complete relay selection and power allocation algorithm is shown in Algorithm 2.

IV. Case of Partial CSI with End-to-End SNR Constraint

In the previous section, it was assumed that each relay node has complete CSI of all the links. Even though the full CSI assumption is interesting to study as it provides insight into the problem and provides a benchmark to which all other suboptimal schemes can be compared, it is still difficult to implement in practice. Hence, in this section we consider a case when there is only partial CSI at the relays. Each relay has knowledge of the instantaneous channel gain of all the first hop links and only the knowledge of the channel statistics of all the second hop links and the direct link. Thus, as the relays don't have the knowledge of the instantaneous CSI of the second hops, the end-to-end SNR needs to be averaged over them.

For Nakagami-m fading, α_0 and β_i are Gamma random

Algorithm 2 Power Allocation(M,$\boldsymbol{\alpha}$,$\boldsymbol{\beta}$,γ^{th},$\boldsymbol{P^{max}}$)

for $r = 1$ to M **do**
  check$\Leftarrow P_s^{max}(\alpha_0+\alpha_r) - \frac{(P_s^{max})^2\alpha_r^2 + P_s^{max}\alpha_r}{P_s^{max}\alpha_r + P_r^{max}\beta_r + 1}$
  if check $< \gamma^{th}$ **then**
    $P_s \Leftarrow P_s^{max}$
    $P_r \Leftarrow P_r^{max}$
  else
    $\phi_r \Leftarrow \alpha_0\beta_r + \alpha_r\beta_r - \alpha_0\alpha_r$
    if $\phi_r < 0$ **then**
      $P_s \Leftarrow \min\left(P_s^{max}, \frac{\gamma^{th}}{\alpha_0}\right)$
      if $P_s = \frac{\gamma^{th}}{\alpha_0}$ **then**
        $P_r \Leftarrow 0$
      else
        $P_r \Leftarrow \frac{P_s^2\alpha_r^2 + P_s\alpha_r}{\beta_r(P_s(\alpha_0+\alpha_r)-\gamma^{th})} - \frac{P_s\alpha_r}{\beta_r} - \frac{1}{\beta_r}$
      end if
    else
      $P_s \Leftarrow \frac{\gamma^{th}\phi_z + \sqrt{\gamma^{th}\phi_z(\alpha_0\alpha_r + \alpha_r^2 + \alpha_r^2\gamma^{th})}}{(\alpha_0+\alpha_r)\phi_z}$
      if $P_s > P_s^{max}$ **then**
        $P_s \Leftarrow P_s^{max}$
      end if
      $P_r \Leftarrow \frac{P_s^2\alpha_r^2 + P_s\alpha_r}{\beta_r(P_s(\alpha_0+\alpha_r)-\gamma^{th})} - \frac{P_s\alpha_r}{\beta_r} - \frac{1}{\beta_r}$
      if $P_r > P_r^{max}$ **then**
        $a = \alpha_0\alpha_r$
        $b = P_r^{max}\alpha_0\beta_r + \alpha_0 + P_r^{max}\alpha_r\beta_r - \alpha_r\gamma^{th}$
        $c = -P_r^{max}\beta_r\gamma^{th} - \gamma^{th}$.
        $P_s \Leftarrow \frac{-b+\sqrt{b^2-4ac}}{2a}$
      else
        if $P_r < 0$ **then**
          $P_r \Leftarrow 0$
          $P_s \Leftarrow \frac{\gamma^{th}}{\alpha_0}$
        end if
      end if
    end if
  end if
  $\boldsymbol{P_s}(r) = P_s$
  $\boldsymbol{\gamma}(r) = P_s(\alpha_0+\alpha_r) - \frac{P_s^2\alpha_r^2 + P_s\alpha_r}{P_r\alpha_r + P_r\beta_r + 1}$
  $\boldsymbol{P_{tot}}(r) = P_s + P_r$
end for
indices=find($\boldsymbol{\gamma} > \gamma^{th}$)
if length(ind)=0 **then**
  [value,index]$\Leftarrow \max(\boldsymbol{\gamma})$
  $z \Leftarrow$index
else
  [value,index]$\Leftarrow \min(\boldsymbol{P_{tot}})$
  $z \Leftarrow$index
end if
return z, $\boldsymbol{P_s}(z)$, P_z

variables with probability density functions

$$
\begin{aligned}
f_{\alpha_0}(x) &= \frac{1}{\Gamma(m_{\alpha_0})\bar{\gamma}_{\alpha_0}^{m_{\alpha_0}}} x^{m_{\alpha_0}-1} e^{-\frac{x}{\bar{\gamma}_{\alpha_0}}} \\
f_{\beta_i}(x) &= \frac{1}{\Gamma(m_{\beta_i})\bar{\gamma}_{\beta_i}^{m_{\beta_i}}} x^{m_{\beta_i}-1} e^{-\frac{x}{\bar{\gamma}_{\beta_i}}} \qquad x \geq 0,
\end{aligned}
\tag{20}
$$

where m_{α_0} and m_{β_i} are the shape parameters of the direct link and second hop of the ith link, respectively, $\bar{\gamma}_{\alpha_i}$ and $\bar{\gamma}_{\beta_i}$ are the average SNRs of the first and second hop of the ith link respectively, and $\Gamma(.)$ is the gamma function [27, Eq. (8.310.1)]. Hence, the average end-to-end SNR can be obtained from (21) shown on top of the next page. where (21) comes from the independence of all the links. Solving (21) gives the end-to-end SNR averaged over the direct link and the second hop of the direct links in (22). Following on from the previous section, we will first consider AP relaying and then move on to selective relaying.

A. AP Relaying

With partial CSI, the power minimization problem is given by

$$
\begin{aligned}
&\min_{P_s, P_i} \; P_s + \sum_{i=1}^{M} P_i, \quad \text{subject to} \\
&\bar{\gamma} \geq \gamma^{th}, \; 0 \leq P_s \leq P_s^{max}, \; 0 \leq P_i \leq P_i^{max}.
\end{aligned}
\tag{23}
$$

Note that (23) follows identical form to (1) but with $\bar{\gamma}$ replacing γ. It can readily seen from (22) that the $\bar{\gamma}$ is a convex function of the relay powers. Thus, (23) is a convex optimization problem for the relay power. However, the joint source and relay power optimization problem seems intractable. Hence, we adopt the same approach as in the full CSI case where we fix the source power and optimize the relay powers.

Even though the relay power optimization problem is convex, it is still difficult to find efficient algorithms to find the solution. This is due to the multiplication of the exponential and upper incomplete Gamma function. Due to the multiplication of these functions, there are points at which the value of the function becomes too high as to be calculated by most softwares. These points are known as critical points [21]. Hence, we now use a bound to approximate the averaged end-to-end SNR. In this regard, we distinguish two cases: 1) Integer Nakagami shape parameter and 2) Generalized Nakagami shape parameter that can take any value $\geq \frac{1}{2}$. We discuss these two cases in turn below.

1) Integer Nakagami Parameter: For an integer n [28, Eq. (6.5.9)]

$$
\Gamma(1-n, x) = \frac{1}{x^{n-1}} E_n(x) \qquad x > 0, n = 1, 2, 3, \ldots, \tag{24}
$$

$$\bar{\gamma} = \int_0^\infty \int_0^\infty \dots \int_0^\infty \left(P_s \left(x_0 + \sum_{i=1}^{M} \alpha_i \right) - \sum_{i=1}^{M} \frac{P_s^2 \alpha_i^2 + P_s \alpha_i}{P_s \alpha_i + P_i y_i + 1} \right) \frac{1}{\Gamma(m_{\alpha_0}) \bar{\gamma}_{\alpha_0}^{m_{\alpha_0}}} x^{m_{\alpha_0}-1} e^{-\frac{x_0}{\bar{\gamma}_{\alpha_0}}} \times \left(\prod_{i=1}^{M} \frac{1}{\Gamma(m_{\beta_i}) \bar{\gamma}_{\beta_i}^{m_{\beta_i}}} y_i^{m_{\beta_i}-1} e^{-\frac{y_i}{\bar{\gamma}_{\beta_i}}} \right) dx_0 dy_1 \dots dy_M, \tag{21}$$

$$\bar{\gamma} = P_s \left(m_{\alpha_0} \bar{\gamma}_{\alpha_0} + \sum_{i=1}^{M} \alpha_i \right) - \sum_{i=1}^{M} \frac{P_s \alpha_i (P_s \alpha_i + 1)^{m_{\beta_i}}}{\bar{\gamma}_{\beta_i}^{m_{\beta_i}} P_i^{m_{\beta_i}}} e^{\left(\frac{P_s \alpha_i + 1}{\bar{\gamma}_{\beta_i} P_i} \right)} \Gamma \left(1 - m_{\beta_i}, \frac{P_s \alpha_i + 1}{\bar{\gamma}_{\beta_i} P_i} \right) \tag{22}$$

---

where $E_n(x)$ is the generalized exponential integral [28, Eq. (5.1.4)]. Applying (24) to (22), $\bar{\gamma}$ can be re-written as

$$\bar{\gamma} = P_s \left(m_{\alpha_0} \bar{\gamma}_{\alpha_0} + \sum_{i=1}^{M} \alpha_i \right) - \sum_{i=1}^{M} \frac{P_s \alpha_i (P_s \alpha_i + 1)}{\bar{\gamma}_{\beta_i} P_i} e^{\left(\frac{P_s \alpha_i + 1}{\bar{\gamma}_{\beta_i} P_i} \right)} E_{m_{\beta_i}} \left(\frac{P_s \alpha_i + 1}{\bar{\gamma}_{\beta_i} P_i} \right). \tag{25}$$

From [28, Eq. (5.1.19)], the product of an exponential and an exponential integral can be upper bounded as

$$e^x E_n(x) \leq \frac{1}{x + n - 1} \quad x > 0, n = 1, 2, 3, \dots. \tag{26}$$

Using (26), $\bar{\gamma}$ can be lower bounded as

$$\bar{\gamma} \geq \left(P_s \left(m_{\alpha_0} \bar{\gamma}_{\alpha_0} + \sum_{i=1}^{M} \alpha_i \right) - \sum_{i=1}^{M} \frac{P_s \alpha_i (P_s \alpha_i + 1)}{P_s \alpha_i + 1 + P_i \bar{\gamma}_{\beta_i} (m_{\beta_i} - 1)} \right) = \bar{\gamma}_{lb}, \tag{27}$$

where $\bar{\gamma}_{lb}$ refers to the lower-bound value of $\bar{\gamma}$ obtained after using (26). Hence, now instead of using $\bar{\gamma}$ in (23), we utilize the lower bound found in (27). This leads to a suboptimal solution, but as we show now, a simple closed-form solution which can be efficiently implemented in practice. Also, as we lower bound $\bar{\gamma}$, the constraint on $\bar{\gamma}$ is satisfied.

It is easily seen that (27) is similar to (3). Hence, the solution to (23) for the integer Nakagami parameter case can be obtained as shown in (28) where

$$\rho = \frac{\left(\sum_{i=1}^{M} \sqrt{\frac{P_s^2 \alpha_i^2 + P_s \alpha_i}{\bar{\gamma}_{\beta_i} (m_{\beta_i} - 1)}} \right)^2}{(P_s \left(m_{\alpha_0} \bar{\gamma}_{\alpha_0} + \sum_{i=1}^{M} \alpha_i \right) - \gamma_{ub}^{th})^2}. \tag{29}$$

The power allocation algorithm for this case is the same as Algorithm 1 with the expressions of the relay power and the Lagrange multiplier replaced in for the partial CSI case.

The case of $m_{\beta_i} = 1, \ \forall \ i$, is a special case as can be seen from the lower bound on $\bar{\gamma}$ in (27). For Rayleigh fading, all the relay terms cancel out and only the direct link term remains. Therefore, for Rayleigh fading, a different approximation is required. For $m_{\beta_i} = 1 \ \forall \ i$, the upper incomplete Gamma function simplifies to the exponential integral of the first order. Then using the inequality From [28, Eq. (5.1.20)]

$$e^x E_1(x) \leq \ln \left(1 + \frac{1}{x} \right) \quad x > 0, \tag{30}$$

a lower bound on $\bar{\gamma}$ can be obtained as

$$\bar{\gamma}|_{\text{Rayleigh}} \geq \left(P_s \left(\bar{\gamma}_{\alpha_0} + \sum_{i=1}^{m} \alpha_i \right) - \sum_{i=1}^{m} \frac{P_s \alpha_i (P_s \alpha_i + 1)}{\bar{\gamma}_{\beta_i} P_i} \ln \left(1 + \frac{\bar{\gamma}_{\beta_i} P_i}{P_s \alpha_i + 1} \right) \right) = \bar{\gamma}_{lb} \tag{31}$$

It can be shown that convexity of the problem is still retained. Solving (23) with the lower bound in (31) using the Lagrange dual method will yield $M + 1$ non-linear convex equations equations with $M + 1$ variables which are $P_i \forall i$. These can be solved using the iterative bisection procedure described in [21]. Alternatively, the method described below for the generalized Nakagami parameter can be used for Rayleigh fading.

2) Generalized Nakagami Parameter: Now, we consider the case of the generalized Nakagami shape parameter. It is difficult to obtain a lower bound on the upper incomplete Gamma function for the generalized Nakagami parameter. This is due to the fact that, here, we are interested in the upper incomplete Gamma function for negative values of the first parameter for which, to the best of the author's knowledge, good upper bounds are neither available in the literature nor easy to obtain. Hence, it is difficult to solve the problem fo the generalized Nakagami fading parameter.

To address this, we obtain a lower bound on the upper incomplete Gamma function which, as we show below, can be utilized to obtain simple closed-form expressions for the relay powers, $P_i \forall i$, and the Lagrange multiplier. However, as we now lower bound the upper incomplete Gamma function, we obtain an upper bound on $\bar{\gamma}$. Hence, now as we use an upper bound on $\bar{\gamma}$, it is not guaranteed that $\bar{\gamma} \geq \gamma^{th}$. As a means to ensure that the constraint is satisfied, the constraint on the upper bound can be changed to $\gamma_{ub}^{th} = \gamma^{th} + e$, where e is the additional added term and γ_{ub}^{th} is the modified upper bound on γ^{th}.

Returning to the solution of the problem. The upper incom-

$$P_i = \left(\sqrt{\frac{\rho(P_s^2\alpha_i^2 + P_s\alpha_i)}{\bar{\gamma}_{\beta_i}(m_{\beta_i}-1)}} - \frac{P_s\alpha_i}{\bar{\gamma}_{\beta_i}(m_{\beta_i}-1)} - \frac{1}{\bar{\gamma}_{\beta_i}(m_{\beta_i}-1)} \right)_0^{P_i^{max}}, \quad i = 1, 2, \dots M \tag{28}$$

plete Gamma function can be re-written as

$$\begin{aligned}\Gamma(a,x) = \int_x^\infty t^{a-1}e^{-t}dt = \int_0^\infty (t+x)^{a-1}e^{-(t+x)}dt \\ = e^{-x}\mathbb{E}[(T+x)^{a-1}],\end{aligned} \tag{32}$$

where T is an exponential random variable with mean 1 and $\mathbb{E}[.]$ is the expectation operator. The above is a convex function of T for $a < 1$ which is the case for the problem here. Hence, using Jensen's inequality, one can write

$$\Gamma(a,x) \geq e^{-x}(1+x)^{a-1}. \tag{33}$$

Thus an upper bound on $\bar{\gamma}$ can be achieved by substituting (33) in (22) as

$$\begin{aligned}\bar{\gamma} \leq P_s\left(m_{\alpha_0}\bar{\gamma}_{\alpha_0} + \sum_{i=1}^M \alpha_i\right) - \sum_{i=1}^M \frac{P_s\alpha_i(P_s\alpha_i+1)^{m_{\beta_i}}}{(P_s\alpha_i + P_i\bar{\gamma}_{\beta_i}+1)^{m_{\beta_i}}} \\ = \bar{\gamma}_{ub}\end{aligned} \tag{34}$$

Hence, now the the optimization problem for $\sum_{i=1}^M P_i$ can be formulated as

$$\begin{aligned}&\min_{P_i} \sum_{i=1}^M P_i, \quad \text{subject to} \\ &\bar{\gamma}_{ub} \geq \gamma_{ub}^{th}, \ 0 \leq P_i \leq P_i^{max}.\end{aligned} \tag{35}$$

Note that, we have already assumed a fixed source power, $P_s = P_s^{max}$, like in the previous cases of AP relaying due to the complexity and difficulty of joint optimization of source and relay powers. It can be easily verified that the above problem is a convex optimization problem. Hence, using the Lagrange dual formulation and ignoring the individual constraints, the Lagrangian is formulated as

$$\begin{aligned}\mathbb{L} = \sum_{i=1}^m P_i + \rho\left(\gamma_{ub}^{th} - P_s\left(m_{\alpha_0}\bar{\gamma}_{\alpha_0} + \sum_{i=1}^M \alpha_i\right) + \right. \\ \left. \sum_{i=1}^M \frac{P_s\alpha_i(P_s\alpha_i+1)^{m_{\beta_i}}}{(P_s\alpha_i + P_i\bar{\gamma}_{\beta_i}+1)^{m_{\beta_i}}}\right).\end{aligned} \tag{36}$$

From the KKT conditions and incorporating the individual constraints, the optimal water-filling solution can be obtained as in (37) where ρ is the Lagrange multiplier can be obtained using standard algorithms such as subgradient algorithms and ellipsoid method. However, if the Nakagami parameters of all the relay-destination links are equal, i.e. $m_{\beta_i} = m_\beta \forall i$, then the Lagrange multiplier can be obtained in closed-form as

$$\rho = \left(\frac{\left(\sum_{i=1}^M \frac{(P_s\alpha_i)^{\frac{1}{m_\beta+1}}(P_s\alpha_i+1)^{\frac{m_\beta}{m_\beta+1}}}{(m_\beta\bar{\gamma}_{\beta_i})^{\frac{m_\beta}{m_\beta+1}}} \right)}{P_s\left(m_{\alpha_0}\bar{\gamma}_{\alpha_0} + \sum_{i=1}^M \alpha_i\right) - \gamma_{ub}^{th}} \right)^{\frac{m_\beta+1}{m_\beta}}. \tag{38}$$

The power allocation algorithm is the same as in Algorithm 1. However, we need to replace the relay power and Lagrangian multiplier expressions by those in (37) and (38) under special case of $m_{\beta_i} = m_\beta \forall i$.

B. Selective Relaying

Now we move onto selective relaying with partial CSI at the relays. We again first use the lower bound on $\bar{\gamma}$ to solve the problem for integer Nakagami parameter and then give the generalized solution.

1) Integer Nakagami Parameter: In the case of selective relaying, with integer Nakagami parameter, $\bar{\gamma}$ can be lower bounded as special case of (22) to give

$$\bar{\gamma} \geq P_s\left(m_{\alpha_0}\bar{\gamma}_{\alpha_0} + \alpha_z\right) - \frac{P_s\alpha_z(P_s\alpha_z+1)}{P_s\alpha_z + 1 + P_z\bar{\gamma}_{\beta_z}(m_{\beta_z}-1)}. \tag{39}$$

It can be easily seen that the lower bound in (39) is similar to the end-to-end SNR for the full CSI case in (12). We can thus obtain the optimal relay power allocation by replacing β_z in (12) by $\bar{\gamma}_{\beta_z}(m_{\beta_z}-1)$. Thus, the relay selection and power allocation for selective relaying with partial CSI and integer Nakagami parameter is obtained by substituting $\bar{\gamma}_{\beta_z}(m_{\beta_z}-1)$ in place of β_z in Algorithm 2.

2) Generalized Nakagami Parameter: For the generalized case, and following a similar procedure to the one that leads to (34) we have the upper bound on $\bar{\gamma}$ as

$$\bar{\gamma}_{ub} = P_s\left(m_{\alpha_0}\bar{\gamma}_{\alpha_0} + \alpha_z\right) - \frac{P_s\alpha_z(P_s\alpha_z+1)^{m_{\beta_z}}}{(P_s\alpha_z + P_z\bar{\gamma}_{\beta_z}+1)^{m_{\beta_z}}}. \tag{40}$$

Expressing P_z as a function of P_s

$$P_z = \frac{P_s^{\frac{1}{m_{\beta_z}}}\alpha_z^{\frac{1}{m_{\beta_z}}}(P_s\alpha_z+1)}{\bar{\gamma}_{\beta_z}\left(P_s\left(m_{\alpha_0}\bar{\gamma}_{\alpha_0}+\alpha_z\right) - \bar{\gamma}_{ub}^{th}\right)^{\frac{1}{m_{\beta_z}}}} - \frac{P_s\alpha_z}{\bar{\gamma}_{\beta_z}} - \frac{1}{\bar{\gamma}_{\beta_z}}, \tag{41}$$

where $P_s\left(m_{\alpha_0}\bar{\gamma}_{\alpha_0}+\alpha_z\right) - \bar{\gamma}_{ub}^{th} \neq 0$. Using a similar reasoning as in Section III-B, it can easily be seen that $P_s\left(m_{\alpha_0}\bar{\gamma}_{\alpha_0}+\alpha_z\right) - \bar{\gamma}_{ub}^{th} = 0$ can only be zero when there is no connection between the source and the relay. Moreover, it can also be seen that for $P_s\left(m_{\alpha_0}\bar{\gamma}_{\alpha_0}+\alpha_z\right) - \bar{\gamma}_{ub}^{th} < 0$, the relay power lies outside its constraints. Hence, the power

$$P_i = \left(\frac{\left(\rho m_{\beta_i} P_s \alpha_i (P_s\alpha_i + 1)^{m_{\beta_i}} \bar{\gamma}_{\beta_i}\right)^{\frac{1}{m_{\beta_i}+1}}}{\bar{\gamma}_{\beta_i}} - \frac{P_s\alpha_i}{\bar{\gamma}_{\beta_i}} - \frac{1}{\bar{\gamma}_{\beta_i}} \right)_0^{P_i^{max}} \qquad i = 1, 2 \ldots M \tag{37}$$

minimization problem can be written as

$$\min_{P_s} \; \frac{P_s^{\frac{1}{m_{\beta_z}}} \alpha_z^{\frac{1}{k_{\beta_z}}} (P_s\alpha_z + 1)}{\bar{\gamma}_{\beta_z} \left(P_s \left(m_{\alpha_0}\bar{\gamma}_{\alpha_0} + \alpha_z\right) - \bar{\gamma}_{ub}^{th}\right)^{\frac{1}{k_{\beta_z}}}} - \frac{P_s\alpha_z}{\bar{\gamma}_{\beta_z}} - \frac{1}{\bar{\gamma}_{\beta_z}} + P_s \tag{42}$$

Denoting the objective function in (42) by ψ_z and taking its double derivative gives (43). It can be seen from (43) that for the domain of interest, i.e $P_s\left(m_{\alpha_0}\bar{\gamma}_{\alpha_0} + \alpha_z\right) - \bar{\gamma}_{ub}^{th} > 0$, is convex. Hence, the optimization problem in (42) yields a unique solution in the domain of interest. Taking the derivative of ψ_z and equating to 0 gives (44). Equation (44) can be solved numerically by initializing it in the domain of interest to give the optimal value of the source power. After obtaining the source power, the relay power can be obtained from (41). Now, both powers are checked and if they satisfy their constraints, the power allocation is complete. However, if one of the powers violates its constraints then it is set at its constraint and the other power is obtained from the constraint on the end-to-end SNR. If the source power exceeds its constraint, then it is set P_s^{max} and P_z can be obtained from (41). If the relay power exceeds its constraint, then it is set at P_z^{max} and then P_s is obtained from the solving the non-linear equation which is obtained by replacing P_z^{max} on the constraint on $\bar{\gamma}_{ub}$. Thus, the optimal power allocation algorithm is similar to the Algorithm 2. However, the source power allocation is now obtained by solving an equation numerically rather than from a closed-form expression.

V. End-to-End SNR Maximization

In the previous two sections, we focused on the energy-efficiency problem, i.e. minimizing the consumed power while maintaining the SNR over a threshold. In this section, we consider the SNR maximization problem where the end-to-end SNR is maximized under total and individual power constraints. This section is an extension of the work in [21]. Reference [21] considered this problem for the partial CSI assumption for Rayleigh fading. We now extend the solution to the more generalized Nakagami fading. The SNR maximization problem is given by

$$\begin{gathered} \max_{P_i} \quad \bar{\gamma} \quad \text{subject to} \\ 0 \le P_i \le P_i^{max}, \; \sum_{i=1}^{M} P_i \le P_{tot} - P_s. \end{gathered} \tag{45}$$

Note that we do not optimize the source power as joint source and relay power optimization seems quite complicated. Hence, we assume a fixed power and optimize the relay powers only.

A. AP Relaying

We first consider the problem for AP relaying. We again identify the two cases of integer Nakagami parameter and generalized Nakagami parameter and consider each in turn.

1) Integer Nakagami Parameter: For integer Nakagami parameter, we maximize the lower bound on $\bar{\gamma}$ given in (31). It is evident that (45) with the lower bound on $\bar{\gamma}$ is a convex optimization problem. Hence, forming the Lagrangian without the individual constraints

$$\mathbb{L} = -P_s\left(m_{\alpha_0}\bar{\gamma}_{\alpha_0} + \sum_{i=1}^{M}\alpha_i\right) + \sum_{i=1}^{M}\frac{P_s\alpha_i(P_s\alpha_i+1)}{P_s\alpha_i + 1 + P_i\bar{\gamma}_{\beta_i}(m_{\beta_i}-1)} + \nu\left(\sum_{i=1}^{M}P_i - P_{tot} + P_s\right) \tag{46}$$

Now using the KKT conditions, we can obtain the optimal ith relay power in (47) where

$$\nu = \left(\frac{\sum_{i=1}^{M}\sqrt{\frac{P_s\alpha_i(P_s\alpha_i+1)}{(m_{\beta_i}-1)\bar{\gamma}_{\beta_i}}}}{P_{tot} - P_s + \sum_{i=1}^{M}\frac{P_s\alpha_i+1}{(m_{\beta_i}-1)\bar{\gamma}_{\beta_i}}}\right)^2. \tag{48}$$

The power allocation algorithm is similar to Algorithm 1. However, with the expressions for the relay powers substituted in and no check performed at the beginning of the algorithm.

2) Generalized Nakagami Parameter: For the generalized Nakagami shape parameter, we maximize the upper bound in (34). Ignoring the individual constraints and forming the Lagrangian

$$\mathbb{L} = -P_s\left(m_{\alpha_0}\bar{\gamma}_{\alpha_0} + \sum_{i=1}^{M}\alpha_i\right) + \sum_{i=1}^{M}\frac{P_s\alpha_i(P_s\alpha_i+1)^{m_{\beta_i}}}{(P_s\alpha_i + P_i\bar{\gamma}_{\beta_i}+1)^{m_{\beta_i}}} + \nu\left(P_s + \sum_{i=1}^{M}P_i - P_{tot}\right). \tag{49}$$

Taking derivative with respect to P_j and equating to 0 yields the optimal value of P_i in (50) where we have incorporated the individual constraints and ν is the Lagrange multiplier which can be calculated using the ellipsoid method. In the special case when all the second hops have the same Nakagami parameter, $m_{\beta_i} = m_\beta$, the Lagrange multiplier is given by

$$\nu = \left(\frac{m_\beta^{m_\beta}\sum_{i=1}^{M}\frac{1}{\bar{\gamma}_{\beta_i}}\left(P_s\alpha_i(P_s\alpha_i+1)^{m_\beta}\bar{\gamma}_{\beta_i}\right)^{\frac{1}{m_\beta+1}}}{P_{total} - P_s^{max} + \sum_{i=1}^{M}\frac{1}{\bar{\gamma}_{\beta_i}}(P_s\alpha_i+1)}\right)^{m_\beta+1}. \tag{51}$$

$$\frac{\partial^2 \psi_z}{\partial P_s^2} = \frac{1}{m_{\beta_i}} P_s^{\frac{1}{m_{\beta_i}}-2} \frac{\frac{1}{m_{\beta_i}}\left(P_s\alpha_i+1\right)\bar{\gamma}_{ub}^2 + \left(P_s\left(m_{\alpha_0}\bar{\gamma}_{\alpha_0}+\alpha_i\right)-\bar{\gamma}_{ub}\right)\bar{\gamma}_{ub} + P_s\alpha_i\bar{\gamma}_{ub}^2 + P_s\left(k_{\alpha_0}\bar{\gamma}_{\alpha_0}+\alpha_i\right)\bar{\gamma}_{ub}}{\left(P_s\left(m_{\alpha_0}\bar{\gamma}_{\alpha_0}+\alpha_i\right)-\bar{\gamma}_{ub}\right)^{\frac{1}{k_{\beta_i}}+2}} \tag{43}$$

$$\frac{\partial \psi_z}{\partial P_s} = \frac{-\frac{1}{m_{\beta_i}} P_s^{\frac{1}{m_{\beta_i}}-1} \alpha_i^{\frac{1}{m_{\beta_i}}} (P_s\alpha_i+1)\bar{\gamma}_{ub}^{th} + P_s^{\frac{1}{m_{\beta_i}}} \alpha_i^{\frac{1}{m_{\beta_i}}+1}\left(P_s\left(m_{\alpha_0}\bar{\gamma}_{\alpha_0}+\alpha_i\right)-\bar{\gamma}_{ub}^{th}\right)}{\bar{\gamma}_{\beta_i}\left(P_s\left(m_{\alpha_0}\bar{\gamma}_{\alpha_0}+\alpha_i\right)-\bar{\gamma}_{ub}^{th}\right)^{\frac{1}{m_{\beta_i}}+1}} - \frac{\alpha_i}{\bar{\gamma}_{\beta_i}} + 1 = 0 \tag{44}$$

$$P_i = \left(\sqrt{\frac{P_s\alpha_i(P_s\alpha_i+1)}{\nu(m_{\beta_i}-1)\bar{\gamma}_{\beta_i}}} - \frac{P_s\alpha_i}{(m_{\beta_i}-1)\bar{\gamma}_{\beta_i}} - \frac{1}{(m_{\beta_i}-1)\bar{\gamma}_{\beta_i}}\right)_0^{P_i^{max}} \qquad i = 1, 2\ldots M \tag{47}$$

$$P_i = \left(\frac{1}{\bar{\gamma}_{\beta_i}}\left(\left(\frac{m_{\beta_i}P_s\alpha_i(P_s\alpha_i+1)^{m_{\beta_i}}\bar{\gamma}_{\beta_i}}{\nu}\right)^{\frac{1}{m_{\beta_i}+1}} - P_s\alpha_i - 1\right)\right)_0^{P_i^{max}} \qquad i = 1, 2, \ldots M \tag{50}$$

B. Selective Relaying

1) Integer Nakagami Parameter: In this case, the power allocation problem between the selected relay and the source can be formulated as in (52), where we have replaced the source and relay powers by $P_s = \eta_z P_{tot}$ and $P_z = (1 - \eta_z)P_{tot}$. It is noted here that the above formulation implicitly assumes that $P_s^{max} + P_z^{max} > P_{tot}$. Otherwise, the power allocation is trivial and both the source and relay transmit at their peak constraints.

The problem formulation is (52) has the same structure as [19, (31)]. Thus, using [19][3], the optimal value of η can be obtained as in (53), where $\Phi_z = \alpha_z\bar{\gamma}_{\beta_z}(m_{\beta_z} - 1) + \alpha_0\bar{\gamma}_{\beta_z}(m_{\beta_z}-1) - \alpha_0\alpha_z$. The power allocation is performed for each relay and then the relay which maximizes $\bar{\gamma}$ is selected for transmission.

2) Generalized Nakagami Parameter: In the case of the generalized Nakagami parameter, we utilize the upper bound on $\bar{\gamma}$. Thus the optimization problem is

$$\begin{gathered}\max_{P_s,P_z} \quad P_s\left(m_{\alpha_0}\bar{\gamma}_{\alpha_0}+\alpha_z\right) - \frac{P_s\alpha_z(P_s\alpha_z+1)^{m_{\beta_z}}}{\left(P_s\alpha_z+P_z\bar{\gamma}_{\beta_z}+1\right)^{m_{\beta_z}}} \\ \text{subject to} \\ 0 \le P_s \le P_s^{max},\ 0 \le P_z \le P_z^{max},\ P_s + P_z \le P_{tot}.\end{gathered} \tag{54}$$

Substituting in the value of P_z and ignoring the individual constraints, the problem can be re-formulated as

$$\max_{P_s}\ P_s\left(m_{\alpha_0}\bar{\gamma}_{\alpha_0}+\alpha_z\right) - \frac{P_s\alpha_z(P_s\alpha_z+1)^{m_{\beta_z}}}{\left(P_s(\alpha_z-\bar{\gamma}_{\beta_z})+P_{tot}\bar{\gamma}_{\beta_z}+1\right)^{m_{\beta_z}}} \tag{55}$$

Denoting the objective function in (55) as Π_z and taking the double derivative of with respect to P_s gives (56). It can be seen from (56) that for $P_{tot} > P_s$, the optimization problem in (55) in concave. Thus, taking the derivative of Π_z and equating to zero gives (57). Equation (57) can be solved numerically to yield the optimal value of P_s from which the optimal value of P_z can be obtained.

[3]There is a small error in [19, (32)], most likely a typo. Equation (31) in [19] should read $\rho^{opt} = \min\left(1, \frac{B_i+1}{B_i-A_i} - \frac{1}{B_i-A_i}\sqrt{\frac{C_i}{D_i}}\right)$

VI. Numerical Results

In this section, we present numerical results to characterize the performance of the studied power allocation algorithms. For the numerical results, we set all the noise variance the same, $\sigma_{sd}^2 = \sigma_{si}^2 = \sigma_{id}^2 = \sigma^2\ \forall\ i = 1, 2\ldots M$. The channel power gains are set as $\bar{\gamma}_{\alpha_0} = \frac{A_0}{\sigma^2}$, $\bar{\gamma}_{\alpha_i} = \frac{A_i}{\sigma^2}$ and $\bar{\gamma}_{\beta_i} = \frac{B_i}{\sigma^2}$. The number of relays is taken to be 3, i.e. $M = 3$mW. The vector $[A_0\ A_1\ A_2\ A_3]$ is set as [-20 -10 -15 -5], where all the values are in dB. Similarly, the vector $[B_1\ B_2\ B_3]$ are set as [-5 0 -5]. All the results are plotted against $\gamma_s = \frac{1}{\sigma^2}$ which is a measure of the SNR. The peak power constraints of the source and all the relay are set at 3. All the Nakagami parameters are taken to be the same, i.e. $m_{\alpha_0} = m_{\alpha_i} = m_{\beta_i} = m$. For the integer case, $m = 2$ and for the generalized case, $m = 2.5$. We first discuss the results for the power minimization problem and then the SNR maximization problem.

A. Power Minimization

For the power minimization problem, it si set that $\gamma^{th} = 5$ dB and $e = 1$ dB and thus, $\bar{\gamma}_{ub}^{th} = \bar{\gamma}^{th} + e = 6$ dB. The proposed schemes are compared with a benchmark scheme in which all the relays and the source transmit at their peak powers. It is noted that the benchmark scheme will always satisfy the constraint on the end-to-end SNR as long as any of the proposed schemes do.

Fig. 2 shows the power savings for the integer Nakagami parameter case. Power savings is defined as the difference of the power consumed by the benchmark scheme and power consumed by the respective proposed scheme. Hence, Power savings is equal to $P_s^{max} + \sum_{i=1}^{M} P_i^{max} - P_s - \sum_{i\in\mathbb{Z}P_i}$ where the first quantity refers to the power consumed by the benchmark scheme, while the second quantity refers to the power consumed by the respective proposed scheme. Fig. 2

$$\max_{0<\eta_z\leq\frac{P_s^{max}}{P_{tot}}} \eta_z P_{tot} m_{\alpha_0}\bar{\gamma}_{\alpha_0} + \frac{(m_{\beta_z}-1)\eta_z P_{tot}(1-\eta_z)P_{tot}\alpha_z\bar{\gamma}_{\beta_z}}{\eta_z P_{tot}\alpha_z + 1 + (1-\eta_z)P_{tot}\bar{\gamma}_{\beta_z}(m_{\beta_z}-1)}, \tag{52}$$

$$\eta_z = \begin{cases} \frac{P_s^{max}}{P_{tot}} & \Phi_z < 0 \\ \min\left(\frac{P_s^{max}}{P_{tot}}, \frac{P_{tot}\bar{\gamma}_{\beta_z}(m_{\beta_z}-1)+1}{P_{tot}(\bar{\gamma}_{\beta_z}(m_{\beta_z}-1)-\alpha_z)} - \sqrt{\frac{\alpha_z\bar{\gamma}_{\beta_z}(m_{\beta_z}-1)(P_{tot}\alpha_z+1)(P_{tot}\bar{\gamma}_{\beta_z}(m_{\beta_z}-1)+1)}{P_{tot}^2(\bar{\gamma}_{\beta_z}(m_{\beta_z}-1)-\alpha_z)^2\Phi_z}}\right) & \Phi_z > 0 \end{cases} \tag{53}$$

$$\frac{\partial^2\Pi_z}{\partial P_s^2} = -\frac{m_{\beta_z}\alpha_z\bar{\gamma}_{\beta_z}(P_s\alpha_z+1)^{m_{\beta_z}-2}(P_{tot}\alpha_z+1)}{(P_s(\alpha_z-\bar{\gamma}_{\beta_z})+P_{tot}\bar{\gamma}_{\beta_z}+1)^{m_{\beta_z}+2}}((P_s\alpha_z+1)(P_{tot}\bar{\gamma}_{\beta_z}+1)+P_s(\alpha_z-\bar{\gamma}_{\beta_z})+P_{tot}\bar{\gamma}_{\beta_z}+1+ \\ m_{\beta_z}P_s\bar{\gamma}_{\beta_z}(P_{tot}\alpha_z+1)) \tag{56}$$

$$\frac{\partial\Pi_z}{\partial P_s} = \frac{(P_s\alpha_z+1)^{m_{\beta_z}-1}}{(P_s(\alpha_z-\bar{\gamma}_{\beta_z})+P_{tot}\bar{\gamma}_{\beta_z}+1)^{m_{\beta_z}}}\times \\ \left(\frac{(P_s\alpha_z+1)(P_s(\alpha_z-\bar{\gamma}_{\beta_z})+P_{tot}\bar{\gamma}_{\beta_z}+1)+P_s m_{\beta_z}(\alpha_z(P_{tot}\bar{\gamma}_{\beta_z}+1)+\alpha_z-\bar{\gamma}_{\beta_z})}{(P_s(\alpha_z-\bar{\gamma}_{\beta_z})+P_{tot}\bar{\gamma}_{\beta_z}+1)}\right) - \frac{m_{\alpha_0}\bar{\gamma}_{\alpha_0}+\alpha_z}{\alpha_z} = 0 \tag{57}$$

shows that selective relaying provides better power savings than AP relaying for all values of γ_s. A similar observation was made in [22]. The reason selective reason seems to provide better power savings is that when the constraint on the end-to-end SNR is not satisfied, all the relays transmit at peak power. Thus, in these cases, AP relaying consumes significantly more power, depending upon the number of relays, and hence the power savings of selective relaying surpass that of AP relaying. Moreover, for selective relaying, we do joint source and relay power allocation, while for AP, we only do relay power allocation. However, it will be wrong to conclude that AP is worse than selective in general. This can be seen from Fig. 3 which shows the outage probability, $\mathrm{Prob}(\gamma \leq \gamma^{th})$, which is a measure of the number of instances the constraint on the end-to-end SNR is satisfied, of all the power minimization schemes. It can be observed from Fig. 3 that AP-full CSI provides significantly better outage probability than its selection counterpart even though, in selection, we do joint source and relay power optimization. However, when the relays have partial CSI, then selective relay can give better outage probability than the AP case in the high SNR regime. Thus, the benefit of joint source and relay power allocation can be seen in the high SNR regime for partial CSI. The reason for this seems to be that, when the channel conditions are bad, the source needs to be allocated more power. Hence, as we fix the source power to the peak value, it retains the inherent gain of AP over selective relaying. However, when the SNR is high, the optimal source power is not necessarily close to its peak value. Moreover, for the AP case, we check the direct link first and if it satisfies the constraint, we do not use relays. Thus, at high SNR, the direct link may satisfy the constraint on $\bar{\gamma}^{th}$, however the resulting γ may be lower than γ^{th}. This is not the case for selective relaying where we always jointly optimize source and relay power.

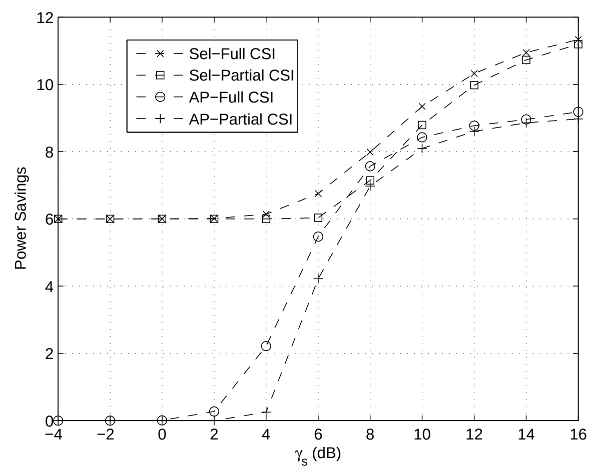

Fig. 2: Power Savings for AP and selective relaying with full and partial CSI with Nakagami parameter $m = 2$.

Another observation that can be made from Fig. 3 is that there seems to be an error floor for the partial CSI cases. This comes due to not having full CSI. As the constraint for the partial CSI cases involve $\bar{\gamma}$ instead of γ due to having less CSI, satisfying the constraint on $\bar{\gamma}$ does not mean that the γ is also greater than γ^{th}. Hence, in the partial CSI case, an error floor is seen.

A similar behaviour is seen for the generalized Nakagami parameter case in Figs. 4 and 5. However, here the difference between the power savings between the full CSI cases and the partial cases is not significant. In fact, it is almost the same, just the actual values are different due to less fading. However, the outage performance behaves as in the case of the integer case. For higher values of m, the behaviour will remain the same, just the actual value will differ due to decrease in fading.

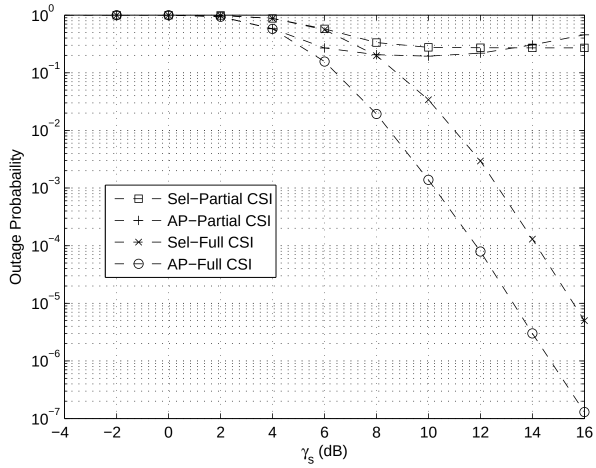

Fig. 3: Outage Probability for AP and selective relaying with full and partial CSI with Nakagami parameter $m = 2$.

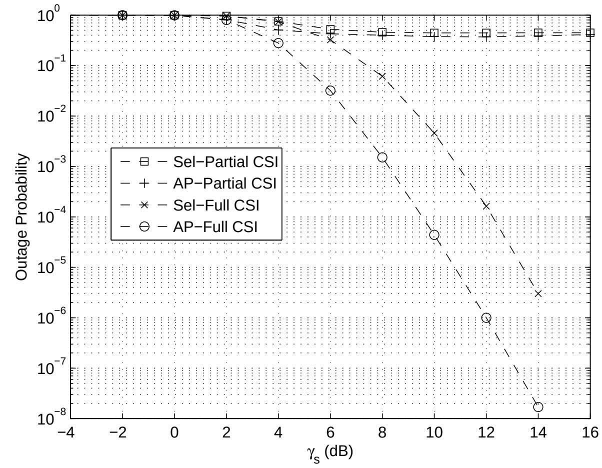

Fig. 5: Outage Probability for AP and selective relaying with full and partial CSI with Nakagami parameter $m = 2.5$.

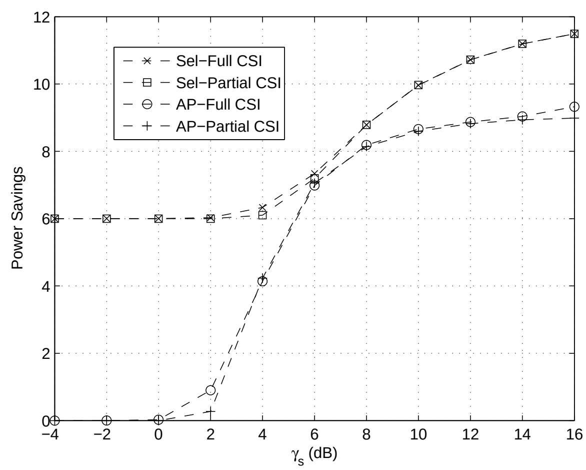

Fig. 4: Power Savings for AP and selective relaying with full and partial CSI with Nakagami parameter $m = 2.5$.

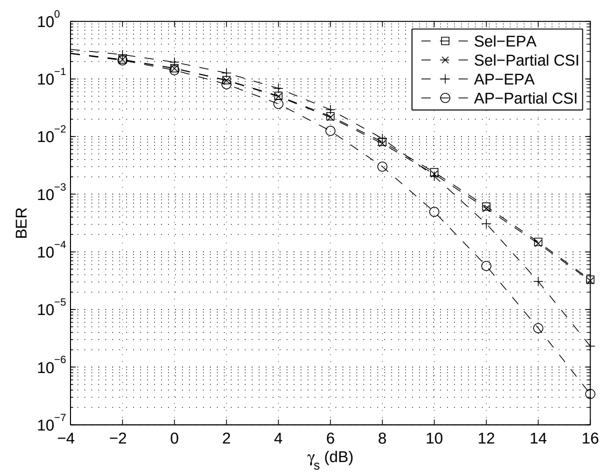

Fig. 6: BER for BPSK for AP and selective relaying with full and partial CSI with Nakagami parameter $m = 2$.

So, from these results it can be seen that, if the quality-of-service (QoS) requirements are strict, then full CSI is a must because even having partial CSI leads to significant degradation in outage performance.

B. SNR Maximization

For the SNR maximization problem, we compare the proposed power allocation schemes to one which assigns equal power to the source bit error rate (BER) of binary phase shift keying (BPSK) for comparison amongst the power allocation schemes.

Fig. 6 shows the BER for integer Nakagami parameter. The performance gap between EPA and the proposed power allocation algorithm for selective relaying is not significant. However, in the AP case, the proposed scheme significantly outperforms EPA. Moreover, in the low SNR regime, both selection schemes outperform AP-EPA scheme as also noted in [22]. However, as the noise decreases, the gain of AP is observed. As similar behaviour is seen for the $m = 2.5$ in Fig. 7, but a slight difference between Sel-EPA and Sel-Partial CSI is observed. The reason for this is that the Nakagami parameter is higher which leads to less fading and the gain of having partial CSI instead of no CSI is seen. Thus, for selective relaying, CSI feedback is only viable in good channel conditions.

One last remark for AP relaying is that, in the simulations it was assumed that the source power is fixed at its peak constraint. However, to improve performance and achieve the optimal solution, a search can be run over P_s to find the optimal source power. But, this will consume a significant amount of time. This time can be reduced by only searching over only a limited set of values of P_s such as $P_s = [P_s^{max}/4 \;\; P_s^{max}/2 \;\; 3P_s^{max}/4 \;\; P_s^{max}]$.

VII. CONCLUSIONS

This paper has studied power allocation strategies to enhance the efficiency of constrained dual-hop variable-gain AF relaying under Nakagami-m fading. Two optimization

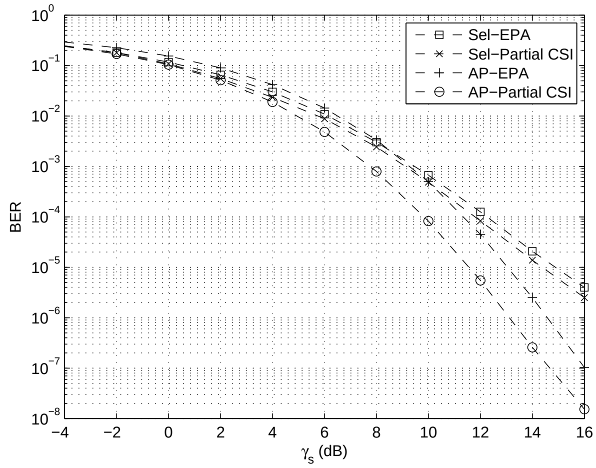

Fig. 7: BER for BPSK for AP and selective relaying with full and partial CSI with Nakagami parameter $m = 2.5$.

problems, power minimization under SNR constraint and SNR maximization under power constraint, have been formulated under different restrictions on the CSI available at the relays and for two different relaying protocols. Simple and efficient power allocation algorithms have been proposed for the two problems for all the scenarios considered. Numerical results have shown that significant power savings can be achieved through the proposed algorithms. Numerical results also show that for strict QoS requirements, partial CSI is not a good option for power minimization as having partial CSI leads to significant degradation in performance due to not meeting the constraint on the end-to-end SNR. However, for the SNR maximization problem, having partial CSI can enhance performance, particularly for AP relaying.

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