Original citation:

Permanent WRAP url:
http://wrap.warwick.ac.uk/67632

Copyright and reuse:
The Warwick Research Archive Portal (WRAP) makes this work by researchers of the University of Warwick available open access under the following conditions. Copyright © and all moral rights to the version of the paper presented here belong to the individual author(s) and/or other copyright owners. To the extent reasonable and practicable the material made available in WRAP has been checked for eligibility before being made available.

Copies of full items can be used for personal research or study, educational, or not-for-profit purposes without prior permission or charge. Provided that the authors, title and full bibliographic details are credited, a hyperlink and/or URL is given for the original metadata page and the content is not changed in any way.

A note on versions:
The version presented here is a working paper or pre-print that may be later published elsewhere. If a published version is known of, the above WRAP url will contain details on finding it.

For more information, please contact the WRAP Team at: publications@warwick.ac.uk
Who Should Cast the Casting Vote?

Steve Alpern and Bo Chen
Warwick Business School, University of Warwick
Coventry, CV4 7AL, United Kingdom

May 13, 2015

Abstract

The use of a casting vote to break ties is a common feature of majority voting schemes. We consider the use of such schemes in amalgamating information in a common-interest setting, such as a jury or panel of experts. They vote between two states of Nature, say Innocent or Guilty. In a situation where jurors are heterogeneous in their ability to discern the truth, how does the choice of the ability of the casting voter affect the reliability of the verdict, that is, the probability that it is correct? Conventional wisdom, as followed in practice, says that the most able voter should have the casting vote. On the contrary, we show that for both honest and strategic voting, reliability for a three-person jury is always maximized when the casting vote is given to the juror of median ability. For high ability juries, but not otherwise, honest voting is as reliable as strategic voting. For somewhat larger juries, it may even be best to give the casting vote to the juror of least ability.

To obtain our results, we require a model of private information in which jurors receive signals in an interval, rather than simply binary signals. This allows modeling strength of opinion.

Keywords: jury; casting vote; group decision

1 Introduction

The use of a casting vote to break ties is widespread in voting schemes. In this paper we are concerned with whom the casting vote should be given to, where jury voting is used to amalgamate information. By a jury, we refer to a panel of experts (of varying expertise) who vote between two possible states of Nature, A or B. This might be “innocent” or “guilty” for a judicial jury or judicial review panel of judges, or “rain” or “dry” for an expert panel of weather forecasters. Also included in our framework is the peer review process for a journal, where an editor would exercise her casting “vote” in the case where the two referees disagree. The paper analyzes voting systems where an even number of jurors vote simultaneously for one of the alternatives A or B and, in case of a tie, it is broken by a casting voter, who votes in the knowledge of all the previous votes.

*Corresponding author: +44 2776524755 (Tel); b.chen@warwick.ac.uk (Email)
In particular, we are concerned with jurors who are heterogeneous in their ability to determine the truth (alternative state A or B), and where these abilities are known to the casting voter. A juror’s ability may relate to his expertise or experience. For a linesman in tennis, for example, ability may relate to his location on the court or his eyesight. In the case of a tie, the casting voter can bias his vote towards the alternative that had the higher ability jurors voting for it. Conventional wisdom holds that the wisest or most senior juror should have the casting vote. For example, a statute of the International Court of Justice (art. 12.4) states that “in the event of an equality of votes among the judges, the eldest judge shall have a casting vote”. This bias towards wisdom goes back as far as Aeschylus’ Eumenides, where Orestes is acquitted on the casting vote of Athena. (As goddess of wisdom, she would certainly fit this bias.) We find that conventional wisdom is wrong.

We address the titled question of “who should cast the casting vote?” in terms of the reliability of the verdict, which we define as the probability that the verdict agrees with the state of Nature, in other words, the probability that the jury “gets it right”. For a given set of jurors of known abilities, we ask which ability juror should be given the casting vote to maximize reliability. We model the ability of a juror in terms of the stochastic nature of private information sent to them as a signal. The private information available to jurors in our model is somewhat deeper than that of, say, the Condorcet model (Condorcet 1785), where this information is given in the form of binary signals, either for A or B. A juror’s ability in that model is simply the probability that his signal is correct. We require richer signals that measure strength of opinion, and come not in discrete form but as real numbers in the interval $[-1, +1]$. To simplify matters, we assume that the alternatives A and B are equiprobable, each having a prior probability of $1/2$, although our results still hold for some perturbation of these probabilities.

We mainly assume that jurors vote honestly, that is, for the alternative that is more likely, given their signal and (in the case of the casting voter) the votes of the other jurors. In later parts of the paper we also consider strategic voters, whose common aim is to maximize the reliability, even if this means that an individual voter may have to vote for the alternative he views as less likely to be true. For a jury of common interest of maximizing the reliability, whether the jurors are honest or strategic, we answer the question: who should cast the casting vote?

2 Examples and Literature

In this section we present some examples of casting vote schemes with known abilities of the jurors and discuss the small literature on the subject.

An example of interest to academics is the refereeing process of conference or journal paper submissions. In the latter, it is common to have two referees and an editor who can break ties. Usually the editor will know the referees by reputation and can bias her casting vote towards the one with more expertise in the area. For conference paper submissions, the expertise is made more explicit. For example, the conference system EasyChair explicitly asks referees to evaluate their relevant expertise by giving it one of five possible expertise levels.

There are numerous examples of three-person casting vote juries. For example, the selection
committee for the Master of the Rolls (a senior post in the United Kingdom) is mandated as follows, with the President of the Supreme Court given the casting vote: “The selection panel comprises the President of the Supreme Court or his nominee as Chair, the Lord Chief Justice or his nominee, the Chairman of the JAC or their nominee and a lay member of the JAC. The Chairman of the panel has a casting vote in the event of a tie.”

In boxing, three man juries are common. A famous example, reported by the BBC on February 20, 2000, was the following.

“Marco Antonio Barrera, WBO super-bantamweight champion, and WBC champ Erik Morales squared up in Las Vegas for what will go down as one of the greatest fights in boxing history. Unfortunately a fantastic contest was spoilt when Morales was handed a controversial split decision by the judges. One judge each voted for Morales and Barrera but the casting vote, of Dalby Shirley, was 115-113 in favour of the Tijuana man.”

The literature on what we call jury voting goes back to the so-called Condorcet Jury Theorem, Condorcet (1785). We have not found any analytical work on the casting vote, but sequential (or roll-call) voting has received some attention. Principal papers in this area are Dekel and Piccione (2000) and Ottaviani & Sorenson (2001). See also the working paper of Alpern & Chen (2014), which contains numerical work on voting order in roll-call voting. For large juries, roll-call voting is clearly far from casting voting, as the first \( n - 1 \) jurors vote sequentially rather than simultaneously. On the other hand, perhaps for a jury of three the similarities are stronger, as it is only the second juror who has different information in the two schemes. The importance of voting order on selection committees with vetoes is demonstrated in Alpern, Gal & Solan (2010). A general investigation of what we call jury voting is given in Ali et al. (2008). Our work would fit into the information amalgamation portion of the survey of Dewan & Shepsle (2011). In discussing the work of Dekel & Piccione (2000), they observe that “because voters condition on the same event, namely that of being pivotal, it makes no difference whether they cast their votes sooner or later.” Our contrary results, where voting order matters, is due to the heterogenous abilities of our voters and the continuous nature of their private signals, and so it matters to the later, or casting, voter which early voters went for \( A \) and which went for \( B \). If those voting for \( A \) were overall of significantly higher abilities, then the casting voter might vote against his weak signal for \( B \) (assuming a tie vote). Dewan and Shepsle take account of this fact in a footnote where they say that “the individual with the casting vote conditions her vote on the set of observed actions.”

Dekel & Piccione (2000) also take account of voting order (in sequential voting) and conclude that (p. 48):

“…if voters are endowed \( ex \ ante \) with differential information (some voters can be better informed than other) knowing which voters voted in favor and which against can affect the choice of a later voter. It can be shown that, in a common-value and two signal environment (as in Sec. IIIC above), if the player’s signals are completely ordered (in the sense of Blackwell), then it is optimal to have the better informed vote earlier. This provides an interesting contrast to the findings
of Ottaviani and Sorensen (1998). They obtain the opposite optimal order in an environment in which information providers care not about the outcome but about appearing to be well informed. It is not difficult, however, to construct examples in which having the best-informed voter vote first is not optimal. Hence in seems unlikely that general insights into this question can be obtained.”

3 The Model

Our model is one of majority voting between two alternative states of Nature, $A$ or $B$. There are an odd number $n$ of jurors, or voters. First $n - 1$ of them vote simultaneously. Then the casting juror votes, with knowledge of the earlier voting. It does not matter if the casting voter only votes in the case of a tie or always votes. To specify the model, we have to define what we mean by the ability of each juror, and how his ability (and the state of Nature) determines the distribution of signals that he receives as private information. We then define threshold strategies that determine a juror’s vote, based on his signal and any prior voting he is aware of (if he is the casting voter). Finally, we define what we mean by honest voting and strategic voting.

3.1 Signals and Abilities

We assume two states of Nature $A$ and $\sim A = B$, with a priori probability of $A$ given by $\Pr(A) = \theta_0$. To simplify the analysis we will assume the equiprobable case $\theta_0 = 1/2$, although our results are robust for $\theta_0$ values around $1/2$. Individuals have private information about the state of Nature modeled as a signal $s$ in the signal interval $[-1, +1]$. Positive signals are indications of $A$; negative signals $B$. The signal $s = 0$ is neutral. Higher positive signals indicate $A$ more strongly; similarly for negative signals and $B$. Thus a better signal is one with a higher absolute value.

Individual jurors have an ability $a$ in the ability interval $A = [0, 1]$, where individuals of higher ability are generally (but not always) able to make better guesses about the state of Nature. When Nature is in state $A$ (resp. $B$), jurors receive independent signals $s \in [-1, 1]$ with probability density given by $f_a(s)$ (resp. $g_a(s)$) if they have ability $a$. We make the simplest nontrivial assumption on $f_a(s)$ and $g_a(s)$, namely that they are linear in $s$. The slope of the density functions $f_a(s)$ and $g_a(s)$ for a juror of ability $a$ is proportional to $a$. Given that $f_a$ and $g_a(s)$ are density functions on $[-1, +1]$, they take the following form:

$$f_a(s) = \frac{(1 + as)}{2}, \quad -1 \leq s \leq +1,$$
$$g_a(s) = \frac{(1 - as)}{2}, \quad -1 \leq s \leq +1,$$

when Nature is $A$;

when Nature is $B$.

It is easily checked that $f_a(\cdot)$ and $g_a(\cdot)$ defined above are indeed density functions for any $0 \leq a \leq 1$. The density functions for ability $a = 2/3$ are shown in Figure 1. The probability of a correct signal, that is positive when Nature is $A$, is the area under the $A$ line (and above the $s$ axis) to the right of $s = 0$. When $a = 1/2$, this area is $1/2$, showing that a juror with ability 0 is just guessing (by flipping a fair coin to determine the state of Nature).
The corresponding cumulative distributions of the signal $s$ when Nature is $A$ or $B$ are given by

\[
\begin{cases}
  F_a(s) = (s + 1)(as - a + 2)/4, & -1 \leq s \leq +1, \quad \text{when Nature is } A; \\
  G_a(s) = (s + 1)(a - as + 2)/4, & -1 \leq s \leq +1, \quad \text{when Nature is } B.
\end{cases}
\]  

(1)

Given only his signal $s$, a juror of ability $a$ has a posterior probability $\theta'$ that Nature is $A$, given by Bayes’ Law as

\[
\theta' = \Pr(A|s) = \frac{\theta_0 f_a(s)}{\theta_0 f_a(s) + (1 - \theta_0) g_a(s)}
\]

(2)

\[
= \frac{\theta_0 + as\theta_0}{2as\theta_0 - as + 1} \quad \text{(where } \theta_0 \text{ is prior probability of } A) \\
= (as + 1)/2 \quad \text{(in the equiprobable case } \theta_0 = 1/2) .
\]

Note that for a juror of ability 0, we have $\theta' = \theta$ for any received signal $s$, reinforcing our notion that ability 0 is no ability at all. A juror of ability 0 can do no more than guess. If we wish to view our juror of ability $a$ as a Condorcet juror, we would say that his probability of a correct signal (positive when Nature is $A$ or negative when Nature is $B$) is given by

\[
\int_0^1 f_a(s) \, ds = (2 + a)/4.
\]

(3)

In particular a juror of ability 0 has only a 50% probability of a correct sign signal, while a boffin of maximum ability 1 gets it right 75% of the time.
3.2 Threshold Strategies and Honest Strategies

A strategy for a juror is a threshold $\tau$, depending on previous voting, if any, such that the juror votes $A$ with signal $s \geq \tau$ and $B$ with signal $s < \tau$ (see Figure 2 for an illustration). A strategy profile is a list of strategies for each juror. So in our model of three jurors, with a casting vote, the two first voters have single thresholds $x$ and $y$. We can number the jurors by their voting order, although in our model the order of the first $n - 1$ is arbitrary. The casting voter has two thresholds $z_{AB}$ (if the prior voting was $AB$) and $z_{BA}$ (if the prior voting was $BA$). We can ignore the case of prior voting $AA$ or $BB$, because in that case the last vote does not matter. So a strategy profile is a four-tuple $(x, y, z_{AB}, z_{BA})$.

**Definition 1.** We say a strategy profile is **honest (or naive)** if the thresholds are such that every juror votes for the alternative that he believes is more likely, given the a priori probability of $A$, his private signal, and any prior voting.

In our casting vote model, with neutral (equiprobable) alternatives $A$ and $B$, honest voting requires that the two first jurors have 0 thresholds ($x = y = 0$) since for $\theta_0 = 1/2$ we have by (2) that $\Pr(A/s) = 1/2 + as/2 > 1/2$ if and only if $s > 0$. This should be clear in any case from the symmetry of our model with respect to $A$ and $B$. In other words, with honest voting early jurors (all but casting voter) each vote $A$ with a positive signal and $B$ with a negative signal. The situation for the casting voter is a bit different. We may relabel the alternatives so that the higher-ability early juror voted $A$ and the lower-ability early juror voted $B$. If the casting voter gets a positive signal (for $A$) then obviously he votes $A$ and that is the majority verdict. If however he agrees with the early voter of lower ability (he gets a negative signal), then his honest vote depends on the strength of his signal versus the ability discrepancy of the early voters. Given their abilities, he will have a negative threshold $\tau$. If his own signal is even more negative, he follows his signal and vote $B$. Otherwise he will base his vote on the fact that while the early voters split their votes, those of higher ability voted for $A$. This is the crux of the matter — the only case in which the casting voter will vote differently than if he had been voting simultaneously with the others.

3.3 Honest Thresholds of the Casting Voter

In this section we analyze the problem faced by the casting voter, who knows the voting, abilities and thresholds of the first two voters. What is his honest threshold? We assume
the first two voters have ability \(a\) with threshold \(x\) and ability \(b\) with threshold \(y\). We now determine the optimal threshold of the third juror of ability \(c > 0\) (the case of \(c = 0\) will be considered shortly after), under the assumption that the a priori probability of \(A\) (before the casting voter receives his signal \(s\)), given the previous voting and thresholds, is \(\theta\). If the casting voter has signal \(s\), then his posteriori probability of \(A\) is given by \(\theta'\) where \(\theta' = \theta f_c(s)/(\theta f_c(s) + (1 - \theta) g_c(s))\), or

\[
\theta' = \frac{\theta + cs\theta}{2cs\theta - cs + 1}.
\]

The honest threshold \(z\) is the value of \(s\) for which \(\theta' = 1/2\), or

\[
\frac{1}{2} = \frac{\theta + cs\theta}{2cs\theta - cs + 1}.
\]

Solving for \(s\) and making this value the honest threshold \(z\) gives

\[
z = \frac{1 - 2\theta}{c}.
\] (4)

Of course, if \((1 - 2\theta)/c > 1\) this means always vote \(B\) (same as threshold \(z = 1\)), and if \((1 - 2\theta)/c > 1\) this means always vote \(A\) (same as threshold \(z = -1\)). We can use sided limit \(c \to 0^+\) to make the same arguments if \(c = 0\).

Now let us consider how to determine the above value of \(\theta\) given the prior voting sequence \(AB\) (i.e., the voter of ability \(a\) votes \(A\) and the voter of ability \(b\) votes \(B\)). This value is given by

\[
\theta(AB) = \frac{\theta_0(1 - F_a(x))F_b(y)}{\theta_0(1 - F_a(x))F_b(x) + (1 - \theta_0)(1 - G_a(x))G_b(y)}
\]

\[
= \frac{(2 + a + ax)(2 + b(-1 + y))}{2(4 + ab(1 + x)(-1 + y))},
\] (5)

where the second equality is due to (1) and \(\theta_0 = 1/2\). For the case of honest voting, the thresholds \(x\) and \(y\) of the early voters are 0, so (6) reduces to

\[
\theta(AB) = \frac{(2 + a)(2 - b)}{8 - 2ab}.
\] (7)

According to (4) we then have

\[
z_{AB} = \frac{1 - 2\theta(AB)}{c} = w(a, b, c) = \frac{2(b - a)}{c(4 - ab)},
\] (8)

and similarly

\[
z_{BA} = -z_{AB} = -w(a, b, c) = \frac{2(a - b)}{c(4 - ab)}.
\] (9)

To illustrate the importance of these calculations, consider the threshold of the casting voter when the early voters have similar abilities and the ability of the casting voter is large. For
example suppose the early voters have abilities 0.5 and 0.6 and the ability of the casting voter is 0.8. Then if the voter of ability 0.5 votes A and the voter of ability 0.6 votes B, the threshold for the casting voter of ability 0.8 is given by equation (8) as $2(0.1)/(0.8(4−0.3)) \simeq 0.068$. Thus the signal of the casting voter has to be just a bit above neutral 0 for him to vote A. However, if the early voters have widely different abilities, say 0.1 and 0.9, while the casting voter has ability 0.2, the threshold of the casting voter will be $2(0.8)/(0.2(4−(0.1)(0.9))) = 2.046$. Since this is greater than 1, it means the casting voter will always copy the vote of the stronger early voter, regardless of his own signal.

3.4 Reliability

We define the reliability of a voting scheme as the probability that the majority verdict is correct under this voting scheme. With equiprobable alternatives, a simple symmetry argument shows this is the same as the probability of majority verdict A when Nature is in state A. It is easy to calculate the reliability under honest voting scheme where the early voters have abilities $a_1, a_2, \ldots, a_{n−1}$ and the casting voter has ability $a_n$. We ask the simple question: Given a set of $n$ abilities, which one of these should have the casting vote if we wish to maximize the reliability of honest voting? For a jury of size three, we will show that honest-voting reliability is maximized when the juror of median ability has the casting vote. An alternative approach, also within the purview of our model, ascribes costs to each type of voting error (verdict A when Nature is B and vice versa) and minimizes the expected cost.

We now evaluate reliability $Q(a, b, c)$, the probability of a correct verdict when $\theta_0 = 1/2$, where the jurors have abilities $a, b$ and $c$ (in voting order) and honest thresholds $x = y = 0$. As the theoretical voting order of the early voters (who vote simultaneously) does not matter, we clearly have $Q(a, b, c) = Q(b, a, c)$. Let $q_A$ (resp. $q_B$) denote the probability of majority verdict $A$ (resp. $B$) when Nature is $A$ (resp. $B$). Then for an arbitrary a priori probability $\theta_0$ of $A$ we have that the reliability $Q(a, b, c)$ is given by

$$Q(a, b, c) = \theta_0 q_A(a, b, c) + (1 - \theta_0) q_B(a, b, c).$$

Hence with neutral alternatives $\theta_0 = 1/2$, we have

$$Q(a, b, c) = \frac{1}{2}(q_A(a, b, c) + q_B(a, b, c)),$$

and symmetry gives the simpler formula

$$q(a, b, c) = q_A(a, b, c) = q_B(a, b, c).$$

(10)

From now on we assume the case of neutral alternatives. As long as $|z_{AB}| < 1$, the formula for $q_A(a, b, c)$ is given by summing up the probabilities of voting patterns $AA, ABA$ and $BAA$, as

$$q_A(a, b, c) = (1 − F(a, 0))(1 − F(b, 0)) + (1 − F(a, 0))F(b, 0)(1 − F(c, z_{AB}))) + F(a, 0)(1 − F(b, 0))(1 − F(c, z_{BA}))),$$

(11)
with a similar formula for $q_B$. Then according to (8), (9), and (10) provided $|z_{AB}| < 1$, we have $Q(a, b, c) = q(a, b, c)$, where

$$q(a, b, c) = \frac{1}{32}(4(4 + a + b) + \frac{4(a-b)^2}{(4-ab)c} + (4-ab)c). \quad (12)$$

In the case $|z_{AB}| \geq 1$, the casting voter follows the vote of the early voter of maximum ability, that is, the one of ability $\max\{a, b\}$ in our notation. We have calculated the probability that is juror gets the correct sign signal in (3). So in the above calculation of $q(a, b, c)$, by replacing $z_{AB}$ with $-1$ and $+1$ respectively if $z_{AB} \leq -1$ and $z_{AB} \geq +1$, we get the more general reliability formula

$$Q(a, b, c) = \begin{cases} q(a, b, c), & \text{if } |w(a, b, c)| < 1; \\ (\max\{a, b\} + 2)/4, & \text{otherwise}. \end{cases} \quad (13)$$

Note that $q(a, b, c) = (\max\{a, b\} + 2)/4$ when $|z_{AB}| = 1$. This indicates the fact that in this case the early juror of higher ability is a dictator, whose private signal alone determines the verdict as his vote will be copied by the casting voter. To summarize, we have the single formula $q(a, b, c)$ for the reliability when the casting voter has a history-dependent threshold $|z_{AB}| < 1$, and a more complicated formula $Q(a, b, c)$ without this assumption. Thus $Q$ applies even when the casting voter has extreme thresholds whereby he can vote without looking at his private signal.

For example, suppose we partition the ability interval $[0, 1]$ into three subintervals of length $1/3$, and take a jury with one juror in the middle of each of these subintervals. That is, we have a uniformly distributed jury of abilities $1/6$, $1/2$ and $5/6$. If the high-ability juror has the casting vote ($c = 5/6$ in the above notation), then $z_{AB} \simeq 0.204$, which lies within the signal interval $[-1, 1]$ and so reliability is given by the formula $q(1/6, 1/2, 5/6) \simeq 0.690$. Similarly, if the juror of middle-ability $1/2$ has the casting vote, then $z_{AB} \simeq 0.691$. As this is also in the signal interval, the reliability is given by $q(1/6, 5/6, 1/2) \simeq 0.714$. Finally, if the weakest juror has the casting vote, then $|z_{AB}| = 48/43 > 1$. This means that the casting voter follows the vote of the juror of ability $5/6 = \max\{1/2, 5/6\}$, who is correct with probability $(2+5/6)/4 \simeq 0.708$ by formula (13). So in this case we have calculated that giving the casting vote to the median-ability juror is best, to the lowest-ability voter is second best, and to the highest-ability voter is worst. We plot these reliabilities in Figure 3, mainly for later comparison with our results for larger juries.

4 Mechanism Design

We now consider the main question of the paper, the problem faced by a designer who is given a fixed set of jurors with known abilities and must decide to whom to give the casting vote. (Perhaps he is organizing a sporting event and has three volunteers for refereeing, who come with eyesight certificates. Or maybe he is writing the constitution of the International Court of Justice and has to say which judge has the casting vote.) We suppose here that there are three jurors and their abilities $a, b, c$ are labeled so that $a \leq b \leq c$. (Note that we have changed our labelling conventions from the last section, where $c$ was always the ability of the casting
voter.) In the last section we gave an example with uniformly distributed abilities $1/6$, $1/2$ and $5/6$, where we showed that it was best for the juror of ability $1/2$ to have the casting vote, with the juror of ability $1/6$ second best. Here we will discuss the problem more generally.

To aid the intuition, we carry out a thought experiment where the casting vote scheme is conducted in another equivalent way. We let all the jurors vote simultaneously. If the verdict is close (2 to 1, or $(n + 1)/2$ to $(n - 1)/2$) we are allowed to pick one of the jurors and let him decide whether to change his vote after viewing the other votes (and with knowledge of everyone's ability). If he voted with the minority, changing his vote will not affect the verdict, so we assume he voted with the majority, say $A$. This means the others voted equally for $A$ and $B$. He will only change his vote if this increases the reliability of the verdict, that is, if he now thinks $B$ is more likely than $A$, despite his positive signal for $A$. This will occur if one of the following two things occurs:

1. He has a weak signal $s$ close 0.
2. The overall abilities of the $B$ voters are significantly higher than that of the $A$ voters.

In the case of a jury of three, condition 1 is most likely to be satisfied when the juror has the weakest ability $a$, or more generally when the juror has the smallest ability. This is because in our model small abilities are more likely to produce weak signals. Condition 2 is most likely, for a jury of three, when the abilities of the other two jurors are as far apart as possible. That is, when they have the two extreme abilities. This occurs when the selected juror has the middle ability. So this intuitive and qualitative analysis leads us to believe that in general the casting voter should have a low or middle signal. Our later analysis for larger juries indeed bears this out. Here we show that for a jury of three the forces (condition 2) favoring the middle or median ability juror outweigh the forces (condition 1) favoring the low ability juror. In particular we have our following main result.

![Figure 3: Reliability comparison](image-url)
Theorem 1. Suppose that $A$ and $B$ are equiprobable and we have three honest jurors of abilities $a, b, c$ with $0 \leq a \leq b \leq c \leq 1$. Then the reliability $Q$ is maximized when the juror of median ability $b$ has the casting vote.

Proof. The idea of our proof is simple: starting with the case where the juror of ability $b$ has the casting vote, we show that reliability cannot increase when he is replaced in that role by the juror of either higher ability $c$ or lower ability $a$. Let $S = \{(a, b, c) : 0 \leq a \leq b \leq c \leq 1\}$. Denote

$$
\Delta_1(a, b, c) = Q(a, c, b) - Q(a, b, c);
$$

$$
\Delta_2(a, b, c) = Q(a, c, b) - Q(b, c, a);
$$

Then our aforementioned simple idea is implemented by showing that both $\Delta_1(a, b, c)$ and $\Delta_2(a, b, c)$ are non-negative for any $(a, b, c) \in S$. First of all, the following are straightforward according to definition (8) for any given $(a, b, c) \in S$:

$$
w(b, c, a) \geq 1 \Leftrightarrow P_1(a, b, c) \equiv abc - 4a - 2b + 2c \geq 0; \quad (14)$$

$$
w(a, c, b) \geq 1 \Leftrightarrow P_2(a, b, c) \equiv abc - 2a - 4b + 2c \geq 0; \quad (15)$$

$$
w(a, b, c) = 2(b-a)/(c(4-ab)) \leq (2/3)(b/c) < 1. \quad (16)$$

Depending on the magnitude of $w(a, c, b) \geq 0$ for any given $(a, b, c) \in S$, we consider two possible cases separately.

Case 1: $w(a, c, b) \geq 1$.

According to (13), we have

$$
Q(a, c, b) = (c+2)/4. \quad (17)
$$

Since $P_1(a, b, c) - P_2(a, b, c) \geq 2(b-a) \geq 0$, it follows from (14) and (15) that

$$
w(a, c, b) \geq 1 \Rightarrow w(b, c, a) \geq 1,
$$

which together with (13) implies that $Q(b, c, a) = (c+2)/4$, and hence $\Delta_2(a, b, c) = Q(a, c, b) - Q(b, c, a) = 0$.

Next we prove the more difficult result $\Delta_1(a, b, c) \geq 0$. According to (16) and (13) we have $Q(a, b, c) = q(a, b, c)$. It follows from (17) that

$$
\Delta_1(a, b, c) = \frac{c+2}{4} - q(a, b, c) = \frac{d_3(a, b, c)}{32c(4-ab)};
$$

where

$$
d_3(a, b, c) \equiv -4a^2 + 8ab - 4b^2 - 16ac - 16bc
$$

$$
+4a^2bc + 4ab^2c + 16c^2 - a^2b^2c^2.
$$

So it remains only to establish that $d_3(a, b, c) \geq 0$ given the additional condition $2b \leq c$, which is implied by (15). In Section A.2 of the Appendix we show that $d_3(a, b, c)$ has a minimum of 0 over the set $S \cap \{(a, b, c) : 2b \leq c\}$, which is attained uniquely at the point $a = b = c = 0$. 

11
Case 2: $w(a, c, b) < 1$.

According to (13), we have $Q(a, c, b) = q(a, c, b)$, which together with (16) implies that

$$
\Delta_1(a, b, c) = q(a, c, b) - q(a, b, c) = \frac{(c - b)d_1(a, b, c)}{8bc(4 - ab)(4 - ac)},
$$

where

$$
d_1(a, b, c) \equiv 4a^2 - 8ab + 4b^2 - 8ac + 4bc + 2a^2bc - ab^2c + 4c^2 - ab^2c + 8bc.
$$

By taking partial derivatives we can easily see that $d_1(a, b, c)$ is monotonically decreasing in $b$ and $c$, which implies that $d_1(a, b, c) \geq d_1(a, a, a) = 0$ and hence $\Delta_1(a, b, c) \geq 0$. Let us now show $\Delta_2(a, b, c) \geq 0$. If $w(b, c, a) \geq 1$, then $Q(b, c, a) = (c + 2)/4$ and we have

$$
\Delta_2(a, b, c) = q(a, c, b) - q(b, c, a) = \frac{P_2(a, b, c)^2}{32b(4 - ac)} \geq 0.
$$

If $w(b, c, a) < 1$ (note that we always have $w(b, c, a) \geq 0$), with (13) we have $Q(b, c, a) = q(b, c, a)$ and hence

$$
\Delta_2(a, b, c) = q(a, c, b) - q(b, c, a) = \frac{(b - a)d_2(a, b, c)}{8ab(4 - ac)(4 - bc)},
$$

where

$$
d_2(a, b, c) \equiv -4a^2 - 4ab - 4b^2 + 8ac + 8bc + a^2bc + ab^2c - 4c^2 - 2abc^2.
$$

In Section A.1 of the Appendix we show that the minimum of $d_2(a, b, c)$ over the intersection of $S$ and the set $\{(a, b, c) : w(b, c, a) \leq 1\}$ is 0. Hence $\Delta_2(a, b, c) \geq 0$. $\square$

Minimizing Expected Cost

Let us consider an alternative voting goal. Instead of maximizing the reliability, suppose we wish to minimize the expected cost of making both types of error: (I) verdict $B$ when Nature is $A$ (e.g., acquittal of a guilty defendant); and (II) verdict $A$ when Nature is $B$ (e.g., conviction of an innocent defendant). Similar to (11), we can calculate the probability of making either type of error, $Pr[B/A]$ or $Pr[A/B]$. Recall that in Section 3.4 we defined $q_A$ and $q_B$ as the probability of correct verdict when Nature is in state $A$ and $B$, respectively. Therefore,

$$
Pr[B/A] = 1 - q_A, \quad Pr[A/B] = 1 - q_B.
$$

Let $k_1, k_2 \geq 0$ denote the cost of type-I and type-II error, respectively. Then the total expected cost of making both types of error under honest voting with voting order $(a, b, c)$ (casting vote to $c$) is given by

$$
K(a, b, c) = k_1 Pr[B/A] + k_2 Pr[A/B]
= k_1(1 - q_A(a, b, c)) + k_2(1 - q_B(a, b, c)) = (k_1 + k_2)(1 - q(a, b, c)),
$$

where the last equality is due to (10), with which we have shown the following.
Proposition 2. For a three-person jury under honest voting, the voting order with maximum reliability also minimizes the expected cost of incorrect verdict, making both types of error.

5 Larger Juries with Uniformly Distributed Abilities

Our main result of Theorem 1 has been established algebraically only for juries of size three. Such analysis seems out of reach for larger juries with arbitrary sets of abilities. However, if we take a jury of size \( n \) with uniformly distributed abilities, we can determine numerically which juror should be given the casting vote to maximize reliability under honest voting. We divide the ability interval \([0, 1]\) into \( n \) subintervals of length \( 1/n \) and give one juror \( i \) the ability of the midpoint of the \( i \)th interval, so that \( a_i = (2i - 1)/(2n) \) for the \( i \)th juror in the jury of size \( n \). As an example, when \( n = 5 \), the abilities of the five jurors are 0.1, 0.3, 0.5, 0.7 and 0.9. For each jury of size \( n \), let \( Q_n \) denote the reliability of simultaneous voting and \( Q_n[i] \) denote the reliability of casting voting with the casting vote given to the \( i \)th juror, the one of ability \( a_i \). We then define the non-negative quantities

\[
\delta(n, a_i) = Q_n[i] - \bar{Q}_n
\]

as the incremental reliability of casting voting. It turns out that calculating \( \delta(n, a_i) \) is easier than calculating \( Q_n[i] \) directly. For fixed \( n \), the reliability of giving the casting vote to juror \( i \) is maximized when \( \delta(n, a_i) \) is maximized over \( a_i \). Figure 4 plots for \( n = 3, 5, 7 \) the incremental reliability \( \delta(n, a_i) \) when the casting vote on the jury of size \( n \) is given to the juror of ability \( a_i \), \( i = 1, \ldots, n \). For each \( n \), the plotted points are connected by straight lines to make the plot easier to read.

The curve for \( n = 3 \) has three plot points at abilities 1/6, 1/2 and 5/6. As known from Theorem 1, the highest value will be for the median ability 1/2, as shown clearly in the curve for \( n = 3 \). We have already seen this curve, on its own, in Figure 3, without the subtraction of \( Q_3 \). For the jury of size \( n = 5 \), the abilities of the jurors are 0.1, 0.3, 0.5, 0.7, 0.9, and the incremental reliability (and hence the absolute reliability) is maximized when the second lowest ability juror (ability 0.3) has the casting vote. For \( n = 7 \), reliability is maximized when the juror of lowest ability is given the casting vote. The pattern for \( n = 7 \) is continued for larger juries, as shown in Figure 5 for juries of size \( n = 9, 11, 13, 15 \), where incremental (or absolute) reliability is decreasing in the ability of the casting voter. To distinguish between the curves for different values of \( n \), note that at their left points, the curves are \( n = 9, 11, 15, 13 \), counting from the top. Also observe that these figures are not useful for comparing reliability of different size juries, as they have different base points \( \bar{Q}_n \). The idea that larger juries have higher reliability goes back to Condorcet, but that is not our point of discussion here.

The mathematical analysis required to calculate the incremental radiabilities is presented in Section A.3 of the Appendix.

6 Strategic Voting

Up to this point in the paper we have been assuming that jurors vote honestly, for the alternative they believe is more likely at the time of their vote. In particular, this assumption sets
Figure 4: Incremental reliability as a function of casting voter ability

Figure 5: Plots of $\delta(n, \cdot)$, $n = 9, 11, 13, 15$
\( x = y = 0 \) for the early voter thresholds. However, sometimes removing this restriction and allowing what we call strategic voting can increase the reliability of the majority verdict. In this section we consider how to optimize the early voter thresholds \( x \) and \( y \) in the three-person jury in order to maximize the resulting reliability, i.e., the reliability under strategic voting scheme, which we denote by \( \hat{Q} \).

Let us consider the following example of a jury consisting of two yokels (of near 0 abilities) who are early voters, and a boffin (of ability near 1) who has the casting vote. With honest voting and the boffin as casting voter, the two yokels might make the same wrong vote (the wrong guess). This will occur with probability roughly \( \frac{1}{4} \). Even if the yokels split their votes (which occurs roughly half the time), the boffin will still get it wrong \( \frac{1}{4} \) of the time, so the reliability of such an honest vote is only about \( 1 - \frac{1}{4} - \frac{1}{8} = \frac{5}{8} \). This can also be seen as \( q(0, 0, 1) = \frac{5}{8} \) from equation (12). If the thresholds of the early voters (in this case the yokels) are chosen strategically, the reliability of the majority verdict can be improved. Clearly reliability is maximized if the two yokels vote against each other and thus let the boffin, who votes honestly, decide the verdict. This means one yokel has threshold +1 (always votes B) and the other yokel has threshold \(-1\) (always votes A). When the abilities of the two early voters are sufficiently low with respect to that of the casting voter, so that optimally they vote against each other, we call such a situation Two Yokels and a Boffin (2Y1B). Such strategic voting has reliability near \( \frac{3}{4} \). If we view this as an optimization problem (which we do) where the foreman simply chooses the strategy profile and tells everyone what thresholds to adopt, there is no problem. If we view the problem as a common interest game where every player has as his utility the probability of a correct verdict, then the above profile is a Nash equilibrium, and there is a corresponding coordination problem to determine which equilibrium (which yokel votes A) to adopt. But we can let the jurors talk to each other before they receive their signals.

In any case, it is useful to observe that the casting voter should always vote honestly, as for him there is no distinction between having his vote correct and having the majority vote correct, as they are identical.

We restrict our analysis to a jury of \( n = 3 \) voters. For a given voting order of abilities \((a, b, c)\) (voters with abilities \(a\) and \(b\) vote first and the casting voter has ability \(c\)) the reliability of the majority verdict depends on the thresholds of the jurors. Recall that the early voters have single thresholds \( x \) and \( y \), which are now no longer required to be 0. In general, they should not vote honestly. The casting voter has two thresholds \( z_{AB} \) (if the first votes were \( AB \)) and \( z_{BA} \) (if the first votes were \( BA \)). Note that, in honest voting the casting voter wants to maximize the probability that his own vote is correct, while in strategic voting he wants to maximize the probability that the majority verdict is correct. Since for the casting voter, after a tie, his vote is the majority verdict, we conclude that a strategic casting voter is also honest. Therefore, the optimal thresholds for the casting voter are his honest thresholds, which we have already calculated in (4) as \((1 - 2\theta)/c\), where \(c\) is his ability and \(\theta\), as calculated in (5), is his a priori probability for alternative \(A\), before receiving his signal. The reliability of the majority verdict under strategic voting can be calculated as follows:

\[
\hat{Q}(a, b, c) = \frac{1}{2} \max_{-1 \leq x, y \leq +1} \left( \hat{Q}_A(a, b, c, x, y) + \hat{Q}_B(a, b, c, x, y) \right), \tag{18}
\]
where \( \hat{Q}_A(a, b, c, x, y) \) (resp. \( \hat{Q}_B(a, b, c, x, y) \)) is the probability that the verdict is \( A \) (resp. \( B \)) when nature is in state \( A \) (resp. \( B \)), that is, the sum of the probabilities of voting sequences \( AA \) (resp. \( BB \)), \( ABA \) (resp. \( BAB \)) and \( BAA \) (resp. \( BAB \)) when Nature is \( A \) (resp. \( B \)), or

\[
\hat{Q}_A(a, b, c, x, y) = (1 - F_a(x))(1 - F_b(y)) + (1 - F_a(x))F_b(y)(1 - F_c(z_{AB})) \\
\quad + F_a(x)(1 - F_b(y))(1 - F_c(z_{BA}));
\]

\[
\hat{Q}_B(a, b, c, x, y) = G_a(x)G_b(y) + G_a(x)(1 - G_b(y))G_c(z_{BA}) \\
\quad + (1 - G_a(x))G_b(y)G_c(z_{AB});
\]

where \( z_{AB} \) and \( z_{BA} \) are known functions (8) and (9) of \( x \) and \( y \) according to (6).

We denote the optimizing values for the thresholds \( x \) and \( y \) of the early voters as \( \bar{x} = \bar{x}(a, b, c) \) and \( \bar{y} = \bar{y}(a, b, c) \). For example, when \( a = 0.22 \), \( b = 0.6 \) and \( c = 0.7 \) the strategic thresholds are approximately \( \bar{x} = 0.68 \) and \( \bar{y} = -0.30 \). Notice that the early juror with smaller ability of 0.22 is thrown off the neutral threshold of 0 more than the one with larger ability. A useful way of measuring the discrepancy of strategic thresholds from their honest counterparts is via what we call the dishonesty function \( d \) given by

\[
d = d(a, b, c) = |\bar{x}(a, b, c)| + |\bar{y}(a, b, c)|.
\]

In this notation a strategy profile is honest if it has \( d = 0 \). In such cases we have \( Q(a, b, c) = \hat{Q}(a, b, c) \), although in general the right-hand side can be higher. The 2Y1B situation described earlier, where \( a \) and \( b \) are small and \( c \) is close to 1, has thresholds which result in the two yokels (the early voters with small abilities) voting against each other. That is, one always votes \( A \) (threshold \(-1\)) while the other always votes \( B \) (threshold \(+1\)), so that \( d = 2 \), the maximum dishonesty. Figure 6 plots \( d \) on the vertical axis for \( c = 1 \) and \( a \) and \( b \) on the interval \([0, 1]\).

Note that there are two flat surfaces. One has height \( d = 0 \) for high values of both \( a \) and \( b \), which corresponds to honest voting (\( Q = \hat{Q}, \bar{x} = \bar{y} = 0 \)). The other has height \( d = 2 \), for small values of \( a \) and \( b \) (yokels), which corresponds to the region of 2Y1B. When the ability of the casting voter is lowered to 0.6, the plot of dishonesty \( d \) changes as shown in its contour plot in Figure 7, where the bounding curves of the regions \( d = 0 \) and \( d = 2 \) are drawn. We see that the region of honest strategic voting is enlarged and the region of 2Y1B gets smaller.

The figures seem to suggest that when all the abilities are high, honest voting is strategic. That is, \( Q = \hat{Q} \) with \( \bar{x} = \bar{y} = 0 \). Indeed we establish this for a discrete set of abilities in Theorem 4. It should be noted that dishonesty in our setting is not a bad thing. Rather \( d \) is the optimal amount of total individual dishonesty required for the jury as a whole to maximize the reliability, a good thing.

We present our further findings in the following two theorems with details of the supporting computational results presented in Section A.4 of the Appendix. Let \( \mathcal{A}_0 = \{0, 1/10, \ldots, 1\} \) be a set of discrete abilities.

**Theorem 3.** Given a three-person jury of abilities \( a, b, c \in \mathcal{A}_0 \) with \( a \leq b \leq c \), reliability is maximized under strategic voting when the juror of median-ability \( b \) has the casting vote.

**Theorem 4.** Given a three-person jury of abilities \( a, b, c \in \mathcal{A}_0 \) with \( a, b, c \geq 0.6 \), honest voting is strategic for any fixed voting order. In other words, the maximum in (18) is achieved for \( x = y = 0 \).
Figure 6: Dishonesty function $d(a, b)$ ($0 \leq a, b \leq 1$) with $c = 1$

Figure 7: Contour of dishonesty function $d(a, b)$ ($0 \leq a, b \leq 1$) with $c = 0.6$
Note that Theorem 3 gives the same conclusion as Theorem 1 but with different setting: strategic rather than honest voting. This result, logically distinct from Theorem 1, is established for discrete abilities. For abilities all high enough so that Theorem 4 applies, Theorem 1 and Theorem 4 do have as a consequence of Theorem 3.

7 Conclusions

Majority voting schemes with a casting vote in the case of a tie are widespread. The rationale for such schemes is rarely if ever laid out. Of course, such a scheme avoids an inconclusive verdict. We have observed in this paper that one effect of such a scheme with respect to fully simultaneous voting is that the reliability of the verdict, the probability that it is correct, is increased. Conventional wisdom has it that the casting vote should be given to the ablest, or senior, voter (juror), and such a stipulation can be found in many situations. We do not know the reasoning behind this stipulation, but if it is to optimize the reliability of the verdict then it is clearly wrong. In some cases (large, honest juries) the ablest juror is in fact the worst person to whom to give the casting vote. On a positive note, we have established algebraically that for an honest three-person jury with a fixed set of abilities, the casting vote should be given to the juror of median ability. We have established the same fact numerically for the case where the jury votes strategically to maximize the reliability of their verdict. We have also shown that when all jurors have a high ability, they can simply vote in an honest fashion for the alternative they deem most likely, and this still results in the optimal reliability of the verdict. Of course, as shown with the example of two yokels and a boffin, other juries may have to vote very strategically to optimize the reliability of their verdict. We remark that, the aforementioned results are robust for perturbations of the a priori probability \( \theta_0 = \frac{1}{2} \) of alternative \( A \). We believe that the techniques introduced in this paper can be successfully adapted to other sequential voting schemes, such as roll-call voting.

References


Appendix

A.1 Proof of $d_2(a, b, c) \geq 0$ in Theorem 1

For minimization of $d_2(a, b, c)$ subject to $(a, b, c) \in S$ and $P_2(a, b, c) \leq 0$, the Kuhn-Tucker conditions for potential minimizers are as follows:

\[
2abc - 8a + b^2c - 2bc^2 + \lambda_1(bc - 4) - 4b + 8c - \lambda_2 + \lambda_3 = 0, \\
a^2c + 2abc - 2ac^2 + \lambda_1(ac - 2) - 4a - 8b + 8c - \lambda_3 + \lambda_4 = 0, \\
a^2b + ab^2 - 4abc + \lambda_1(ab + 2) + 8a + 8b - 8c - \lambda_4 + \lambda_5 = 0, \\
\lambda_1(abc - 4a - 2b + 2c) = 0, \\
a\lambda_2 = 0, \lambda_3(b - a) = 0, \lambda_4(c - b) = 0, (1 - c)\lambda_5 = 0. \\
\lambda_1, \ldots, \lambda_5 \geq 0; (a, b, c) \in S \text{ and } P_2(a, b, c) \leq 0.
\]

All solutions $(a, b, c, \lambda_1, \ldots, \lambda_5)$ of the above system projected to the $(a, b, c)$ space form the following set $S_{\text{min}}$:

\[
S_{\text{min}} = \{(0, b, b) : 0 \leq b \leq 1\} \cup \{(a_\lambda, a_\lambda, a_\lambda) : 0 < \lambda \leq 3\} \\
\cup\{(a, a, a/(a^2 + 2)) : 0 < a \leq 3 - \sqrt{7}\}
\]

where $a_\lambda$ is the middle root of equation $a(4 - a^2) = \lambda$. Since $d_2(a, b, c) = 0$ for any $(a, b, c) \in S_{\text{min}}$, all elements of $S_{\text{min}}$ are minimizers of $d_2(a, b, c)$. 


A.2 Proof of \(d_3(a, b, c) \geq 0\) in Theorem 1

For minimization of \(d_3(a, b, c)\) subject to \((a, b, c) \in S\) and \(c-2b \geq 0\), the Kuhn-Tucker conditions for potential minimizers are as follows:

\[
-2ab^2c^2 + 8abc - 8a + 4b^2c + 8b - 16c - \lambda_2 + \lambda_3 = 0, \\
-2a^2bc + 4a^2c + 8abc + 8a - 8b - 16c + 2\lambda_1 - \lambda_3 = 0, \\
-2a^2b^2c + 4a^2b + 4ab^2 - 16a - 16b + 32c - \lambda_1 + \lambda_4 = 0, \\
\lambda_1(c - 2b) = 0, \ a\lambda_2 = 0, \ c(1 - c)\lambda_4 = 0. \\
\lambda_1, \ldots, \lambda_4 \geq 0; \ (a, b, c) \in S \text{ and } c - 2b \geq 0.
\]

The above system has a unique solution of \((0, 0, 0, 0, 0, 0, 0)\) and \(d_3(0, 0, 0) = 0\). Hence we have \(d_3(a, b, c) \geq 0\).

A.3 Analysis of Large Juries for Section 5

Here we give the analysis of the casting vote scheme for an arbitrary odd number of jurors, which is used to derive Figures 4 and 5 in Section 5. Suppose we have \(n = 2m + 1\) jurors with abilities, in nondecreasing order, given as the \(n\)-vector \(\vec{a} = (a_1, a_2, \ldots, a_n)\). Let \(N = \{1, \ldots, n\}\) and \(N^i = N \setminus \{i\}\). If juror \(i\) has the casting vote, then the jurors \(N^i\) who vote first are ordered as the \((n-1)\)-vector \(\vec{a}^i = (a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n)\). Let \(S^i\) denote the set of all \(m\)-subsets of \(N^i\). A set \(S \in S^i\) can be interpreted as the set of jurors who vote for alternative \(A\) in the first round when there is a tie vote and juror \(i\) has the casting vote. The conditional probability of \(A\) in this case is denoted \(\theta_S\). If \(\theta_S = 1/2\) then \(Q[i] = \bar{Q}\), where \(Q\) is the reliability of simultaneous voting (with abilities \(\vec{a}\)) and \(Q[i]\) is the reliability of casting voting where juror \(i\) has the casting vote. Of course, we have \(Q[i] \geq \bar{Q}\) for all \(i \in N\). If \(\theta_S > 1/2\) then for negative signals close to 0, the casting voter \(i\) will still vote for \(A\). The condition \(\theta_S > 1/2\) says roughly that those \(m\) jurors who voted for \(A\) have collectively stronger abilities than those \(m\) jurors who voted for \(B\). For any \(j \in S\), let \(r_j = 1 - F(a_j, 0)\) be the individual reliability of juror \(j\), the probability that he gets a positive signal and hence votes \(A\) when \(A\) is the state of Nature (or the probability that juror \(j\) gets a negative signal given \(B\)). The probability that he gets a negative signal given \(A\) (or a positive signal given \(B\)) is \(F(a_j, 0) = 1 - r_j\). Therefore, we have

\[
\theta_S = \frac{\prod_{j \in S} r_j \prod_{k \in N^i \setminus S} (1 - r_k)}{\prod_{j \in S} r_j \prod_{k \in N^i \setminus S} (1 - r_k) + \prod_{j \in S} (1 - r_j) \prod_{k \in N^i \setminus S} r_k}. \quad (A-1)
\]

Consequently, the honest threshold for casting voter \(i\), given \(S\), is

\[
\tau_i(S) = \left. \frac{1 - 2\theta_S}{a_i} \right|_{[-1,1]}, \quad (A-2)
\]

where \(z|_{[-1,1]}\) denotes the projection of \(z\) onto \([-1,1]\). We can also calculate the probability that those voting \(A\) in the first round constitute a particular set \(S \in S^i\), given Nature is \(A\):

\[
\Pr(S/A) = \prod_{j \in S} r_j \prod_{k \in N^i \setminus S} (1 - r_k), \text{ for } S \in S^i. \quad (A-3)
\]
To evaluate $Q[i] - Q$, we see that the verdict with casting voter $i$ will be different from that of simultaneous voting only if both of the following two conditions hold: (i) a tie vote (i.e., those voting $\{j : s_j > 0\}$, form a set $S \in S^i$, and (ii) small signal for casting voter (i.e., juror $i$ gets a signal $s_i$ between his threshold $\tau_i(S)$ and 0). Taking equiprobable alternatives $\theta_0 = 1/2$ with juror $i$ as the casting voter, let $S = |\{j \in N^i : s_j > 0\}$, those who vote $A$ in the first round. Then the verdict is $A$ if either $|S| > m$ or $|S| = m$ and $s_i \geq \tau_i(S)$. Therefore, we have the following formula for the reliability of voting with casting voter $i$:

$$Q[i] = \sum_{S \subseteq N^i, |S| \geq m+1} \Pr(S/A) + \sum_{S \subseteq N^i, |S| = m} \Pr(S/A)(1 - F(a_i, \tau_i(S))).$$

Similarly, for simultaneous voting we can separate out the voting of juror $i$ to obtain the asymmetric formula for the reliability $Q$:

$$Q = \sum_{S \subseteq N^i, |S| \geq m+1} \Pr(S/A) + \sum_{S \subseteq N^i, |S| = m} \Pr(S/A)(1 - F(a_i, 0)).$$

The difference between the above two formulae is that the latter involves only individual reliabilities, whereas the former takes into account linear density functions on the full signal distribution of the casting voter. In particular we have (noting that the conditions $S \subseteq N^i$ and $|S| = m$ are the same as $S \subseteq S^i$):

$$Q[i] - \tilde{Q} = \sum_{S \subseteq S^i} \Pr(S/A)((1 - F(a_i, \tau_i)) - (1 - F(a_i, 0)))$$

$$= \sum_{S \subseteq S^i} \Pr(S/A)(F(a_i, 0) - F(a_i, \tau_i(S))).$$

In the case of $n = 3$ with $i = 3$ having the casting vote and $a_1 = a$, $a_2 = b$, $a_3 = c$, there are two sets in $S^3$, namely $\{1\}$ (which is voting pattern $AB$) and $\{2\}$ (which is $BA$). There is one set $S \subseteq \{1, 2\}$ with $|S| = m + 1 = 2$, namely $\{1, 2\}$. So evaluating the general formula for $Q[i] = Q[3]$ with the terms $S$ in order $\{1, 2\}, \{1\}, \{2\}$ gives (with $\tau_3(\{1\}) = z_{AB}$)

$$Q(a, b, c) = (1 - F(a, 0))(1 - F(b, 0)) + [(1 - F(a, 0))(F(b, 0))(1 - F(c, z_{AB}))$$

$$+ (F(a, 0))(1 - F(b, 0)))(1 - F(c, z_{BA}))$$

Note that the advantage of formula (A-4) is that we have fewer terms to evaluate to see which is the best juror $i$ to have the casting vote.

### A.4 Computational Results for Theorems 3 and 4

Computational results for reliability values of different casting voters are listed in Table A-1, in which both $\hat{Q}$ of strategic voting and $Q$ of honest voting are put in the same row for easy comparison for different ability sets. To ensure accurate comparison, exact (fractional) numbers are also provided (in Table A-2) for honest voting in addition to decimal numbers. For strategic voting, not every reliability is a rational number. However, in all cases where six decimal places are not sufficient for relevant comparison stated in Theorems 3 and 4, the corresponding
reliability values are all rational numbers and they are the same as those corresponding values in Table A-2. For simplicity we use \( i \) to represent ability \( i/10 \) \((i = 0, 1, \ldots, 10)\) in the two tables. In both Tables A-1 and A-2, symbol "−" denotes the same reliability value as that of median-ability casting voter on the same row.

Table A-1: Reliability comparison for different casting voters

<table>
<thead>
<tr>
<th>{a, b, c} ( a \leq b \leq c )</th>
<th>Honest Voting</th>
<th>Strategic Voting</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( a ) last</td>
<td>( b ) last</td>
</tr>
<tr>
<td>{0,0,0}</td>
<td>-</td>
<td>0.5</td>
</tr>
<tr>
<td>{0,0,1}</td>
<td>-</td>
<td>0.525</td>
</tr>
<tr>
<td>{0,0,2}</td>
<td>-</td>
<td>0.55</td>
</tr>
<tr>
<td>{0,0,3}</td>
<td>-</td>
<td>0.575</td>
</tr>
<tr>
<td>{0,0,4}</td>
<td>-</td>
<td>0.6</td>
</tr>
<tr>
<td>{0,0,5}</td>
<td>-</td>
<td>0.625</td>
</tr>
<tr>
<td>{0,0,6}</td>
<td>-</td>
<td>0.65</td>
</tr>
<tr>
<td>{0,0,7}</td>
<td>-</td>
<td>0.675</td>
</tr>
<tr>
<td>{0,0,8}</td>
<td>-</td>
<td>0.7</td>
</tr>
<tr>
<td>{0,0,9}</td>
<td>-</td>
<td>0.725</td>
</tr>
<tr>
<td>{0,0,10}</td>
<td>-</td>
<td>0.75</td>
</tr>
<tr>
<td>{0,1,1}</td>
<td>0.525</td>
<td>0.528125</td>
</tr>
<tr>
<td>{0,1,2}</td>
<td>-</td>
<td>0.55</td>
</tr>
<tr>
<td>{0,1,3}</td>
<td>-</td>
<td>0.575</td>
</tr>
<tr>
<td>{0,1,4}</td>
<td>-</td>
<td>0.6</td>
</tr>
<tr>
<td>{0,1,5}</td>
<td>-</td>
<td>0.625</td>
</tr>
<tr>
<td>{0,1,6}</td>
<td>-</td>
<td>0.65</td>
</tr>
<tr>
<td>{0,1,7}</td>
<td>-</td>
<td>0.675</td>
</tr>
<tr>
<td>{0,1,8}</td>
<td>-</td>
<td>0.7</td>
</tr>
<tr>
<td>{0,1,9}</td>
<td>-</td>
<td>0.725</td>
</tr>
<tr>
<td>{0,1,10}</td>
<td>-</td>
<td>0.75</td>
</tr>
<tr>
<td>{0,2,2}</td>
<td>0.55</td>
<td>0.55625</td>
</tr>
<tr>
<td>{0,2,3}</td>
<td>0.575</td>
<td>0.576562</td>
</tr>
<tr>
<td>{0,2,4}</td>
<td>-</td>
<td>0.6</td>
</tr>
<tr>
<td>{0,2,5}</td>
<td>-</td>
<td>0.625</td>
</tr>
<tr>
<td>{0,2,6}</td>
<td>-</td>
<td>0.65</td>
</tr>
<tr>
<td>{0,2,7}</td>
<td>-</td>
<td>0.675</td>
</tr>
<tr>
<td>{0,2,8}</td>
<td>-</td>
<td>0.7</td>
</tr>
<tr>
<td>{0,2,9}</td>
<td>-</td>
<td>0.725</td>
</tr>
<tr>
<td>{0,2,10}</td>
<td>-</td>
<td>0.75</td>
</tr>
<tr>
<td>{0,3,3}</td>
<td>0.575</td>
<td>0.584375</td>
</tr>
<tr>
<td>{0,3,4}</td>
<td>0.6</td>
<td>0.604167</td>
</tr>
<tr>
<td>{0,3,5}</td>
<td>0.625</td>
<td>0.626042</td>
</tr>
</tbody>
</table>

Continued on next page
<table>
<thead>
<tr>
<th>{a, b, c}</th>
<th>Honest Voting</th>
<th>Strategic Voting</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a \leq b \leq c)</td>
<td>(a) last</td>
<td>(b) last</td>
</tr>
<tr>
<td>{0,3,6}</td>
<td>-</td>
<td>0.65</td>
</tr>
<tr>
<td>{0,3,7}</td>
<td>-</td>
<td>0.675</td>
</tr>
<tr>
<td>{0,3,8}</td>
<td>-</td>
<td>0.7</td>
</tr>
<tr>
<td>{0,3,9}</td>
<td>-</td>
<td>0.725</td>
</tr>
<tr>
<td>{0,3,10}</td>
<td>-</td>
<td>0.75</td>
</tr>
<tr>
<td>{0,4,4}</td>
<td>0.6</td>
<td>0.6125</td>
</tr>
<tr>
<td>{0,4,5}</td>
<td>0.625</td>
<td>0.632031</td>
</tr>
<tr>
<td>{0,4,6}</td>
<td>0.65</td>
<td>0.653125</td>
</tr>
<tr>
<td>{0,4,7}</td>
<td>0.675</td>
<td>0.675781</td>
</tr>
<tr>
<td>{0,4,8}</td>
<td>-</td>
<td>0.7</td>
</tr>
<tr>
<td>{0,4,9}</td>
<td>-</td>
<td>0.725</td>
</tr>
<tr>
<td>{0,4,10}</td>
<td>-</td>
<td>0.75</td>
</tr>
<tr>
<td>{0,5,5}</td>
<td>0.625</td>
<td>0.640625</td>
</tr>
<tr>
<td>{0,5,6}</td>
<td>0.65</td>
<td>0.66</td>
</tr>
<tr>
<td>{0,5,7}</td>
<td>0.675</td>
<td>0.680625</td>
</tr>
<tr>
<td>{0,5,8}</td>
<td>0.7</td>
<td>0.7025</td>
</tr>
<tr>
<td>{0,5,9}</td>
<td>0.725</td>
<td>0.725625</td>
</tr>
<tr>
<td>{0,5,10}</td>
<td>-</td>
<td>0.75</td>
</tr>
<tr>
<td>{0,6,6}</td>
<td>-</td>
<td>0.65</td>
</tr>
<tr>
<td>{0,6,7}</td>
<td>0.675</td>
<td>0.688021</td>
</tr>
<tr>
<td>{0,6,8}</td>
<td>0.7</td>
<td>0.708333</td>
</tr>
<tr>
<td>{0,6,9}</td>
<td>0.725</td>
<td>0.729688</td>
</tr>
<tr>
<td>{0,7,7}</td>
<td>0.75</td>
<td>0.752083</td>
</tr>
<tr>
<td>{0,7,8}</td>
<td>0.75</td>
<td>0.760625</td>
</tr>
<tr>
<td>{0,7,9}</td>
<td>0.725</td>
<td>0.765829</td>
</tr>
<tr>
<td>{0,7,10}</td>
<td>0.75</td>
<td>0.772222</td>
</tr>
<tr>
<td>{0,8,8}</td>
<td>0.75</td>
<td>0.775143</td>
</tr>
<tr>
<td>{0,8,9}</td>
<td>0.75</td>
<td>0.78125</td>
</tr>
<tr>
<td>{0,8,10}</td>
<td>0.75</td>
<td>0.78125</td>
</tr>
<tr>
<td>{0,9,9}</td>
<td>0.725</td>
<td>0.753125</td>
</tr>
<tr>
<td>{0,9,10}</td>
<td>0.75</td>
<td>0.772222</td>
</tr>
<tr>
<td>{0,10,10}</td>
<td>0.75</td>
<td>0.78125</td>
</tr>
<tr>
<td>{1,1,1}</td>
<td>-</td>
<td>0.537469</td>
</tr>
<tr>
<td>{1,1,2}</td>
<td>-</td>
<td>0.553078</td>
</tr>
<tr>
<td>{1,1,3}</td>
<td>-</td>
<td>0.575</td>
</tr>
<tr>
<td>{1,1,4}</td>
<td>-</td>
<td>0.6</td>
</tr>
<tr>
<td>{1,1,5}</td>
<td>-</td>
<td>0.625</td>
</tr>
<tr>
<td>{1,1,6}</td>
<td>-</td>
<td>0.65</td>
</tr>
</tbody>
</table>
Table A-1: continued from previous page

<table>
<thead>
<tr>
<th>{a, b, c}</th>
<th>Honest Voting</th>
<th>Strategic Voting</th>
</tr>
</thead>
<tbody>
<tr>
<td>a ≤ b ≤ c</td>
<td>a, b, c last</td>
<td>a, b, c last</td>
</tr>
<tr>
<td>{1,1,7}</td>
<td>- 0.675 0.612281</td>
<td>- 0.675 -</td>
</tr>
<tr>
<td>{1,1,8}</td>
<td>- 0.7 0.62475</td>
<td>- 0.7 -</td>
</tr>
<tr>
<td>{1,1,9}</td>
<td>- 0.725 0.637219</td>
<td>- 0.725 -</td>
</tr>
<tr>
<td>{1,1,10}</td>
<td>- 0.75 0.649687</td>
<td>- 0.75 -</td>
</tr>
<tr>
<td>{1,2,2}</td>
<td>0.562375 0.563945</td>
<td>- 0.562375 0.563945 -</td>
</tr>
<tr>
<td>{1,2,3}</td>
<td>0.577985 0.58111 0.575859</td>
<td>0.578765 0.58111 0.57716</td>
</tr>
<tr>
<td>{1,2,4}</td>
<td>0.6 0.601455 0.588035</td>
<td>0.600986 0.601455 0.6</td>
</tr>
<tr>
<td>{1,2,5}</td>
<td>- 0.625 0.600316</td>
<td>- 0.625096 0.625</td>
</tr>
<tr>
<td>{1,2,6}</td>
<td>- 0.65 0.612648</td>
<td>- 0.65 -</td>
</tr>
<tr>
<td>{1,2,7}</td>
<td>- 0.675 0.625011</td>
<td>- 0.675 -</td>
</tr>
<tr>
<td>{1,2,8}</td>
<td>- 0.7 0.637393</td>
<td>- 0.7 -</td>
</tr>
<tr>
<td>{1,2,9}</td>
<td>- 0.725 0.649786</td>
<td>- 0.725 -</td>
</tr>
<tr>
<td>{1,2,10}</td>
<td>- 0.75 0.662189</td>
<td>- 0.75 -</td>
</tr>
<tr>
<td>{1,3,3}</td>
<td>0.587219 0.591417</td>
<td>0.588889 0.591417 -</td>
</tr>
<tr>
<td>{1,3,4}</td>
<td>0.602847 0.609095 0.602774</td>
<td>0.60629 0.609095 0.605864</td>
</tr>
<tr>
<td>{1,3,5}</td>
<td>0.625 0.628909 0.61455</td>
<td>0.627356 0.628909 0.62642</td>
</tr>
<tr>
<td>{1,3,6}</td>
<td>0.65 0.650876 0.626537</td>
<td>0.650587 0.650876 0.65</td>
</tr>
<tr>
<td>{1,3,7}</td>
<td>- 0.675 0.638643</td>
<td>- 0.675046 0.675</td>
</tr>
<tr>
<td>{1,3,8}</td>
<td>- 0.7 0.650824</td>
<td>- 0.7 -</td>
</tr>
<tr>
<td>{1,3,9}</td>
<td>- 0.725 0.663056</td>
<td>- 0.725 -</td>
</tr>
<tr>
<td>{1,3,10}</td>
<td>- 0.75 0.675322</td>
<td>- 0.75 -</td>
</tr>
<tr>
<td>{1,4,4}</td>
<td>0.612 0.619102</td>
<td>0.618519 0.619102 -</td>
</tr>
<tr>
<td>{1,4,5}</td>
<td>0.627664 0.637033 0.630057</td>
<td>0.635 0.637033 0.635</td>
</tr>
<tr>
<td>{1,4,6}</td>
<td>0.65 0.656579 0.641485</td>
<td>0.654544 0.656579 0.654321</td>
</tr>
<tr>
<td>{1,4,7}</td>
<td>0.675 0.677751 0.653183</td>
<td>0.67665 0.677751 0.676058</td>
</tr>
<tr>
<td>{1,4,8}</td>
<td>0.7 0.700563 0.665051</td>
<td>0.700376 0.700562 0.7</td>
</tr>
<tr>
<td>{1,4,9}</td>
<td>- 0.725 0.677032</td>
<td>- 0.72502 0.725</td>
</tr>
<tr>
<td>{1,4,10}</td>
<td>- 0.75 0.689091</td>
<td>- 0.75 -</td>
</tr>
<tr>
<td>{1,5,5}</td>
<td>0.636719 0.646845</td>
<td>- 0.648148 -</td>
</tr>
<tr>
<td>{1,5,6}</td>
<td>0.652441 0.664925 0.657501</td>
<td>0.664321 0.664925 0.664321</td>
</tr>
<tr>
<td>{1,5,7}</td>
<td>0.675 0.684307 0.66864</td>
<td>0.682857 0.684307 0.682857</td>
</tr>
<tr>
<td>{1,5,8}</td>
<td>0.7 0.705 0.680079</td>
<td>0.703522 0.705 0.703426</td>
</tr>
<tr>
<td>{1,5,9}</td>
<td>0.725 0.727014 0.69172</td>
<td>0.726213 0.727014 0.725844</td>
</tr>
<tr>
<td>{1,5,10}</td>
<td>0.75 0.750361 0.703501</td>
<td>0.750241 0.750361 0.75</td>
</tr>
<tr>
<td>{1,6,6}</td>
<td>0.661375 0.674594</td>
<td>- 0.677778 -</td>
</tr>
<tr>
<td>{1,6,7}</td>
<td>0.677179 0.692771 0.685018</td>
<td>- 0.693739 -</td>
</tr>
<tr>
<td>{1,6,8}</td>
<td>0.7 0.712042 0.695914</td>
<td>0.711728 0.712042 0.711728</td>
</tr>
<tr>
<td>{1,6,9}</td>
<td>0.725 0.732413 0.707125</td>
<td>0.731481 0.732413 0.731481</td>
</tr>
<tr>
<td>{1,6,10}</td>
<td>0.75 0.753894 0.718556</td>
<td>0.75286 0.753894 0.75284</td>
</tr>
</tbody>
</table>

Continued on next page
Table A-1: continued from previous page

<table>
<thead>
<tr>
<th>{a, b, c}</th>
<th>Honest Voting</th>
<th>Strategic Voting</th>
</tr>
</thead>
<tbody>
<tr>
<td>a ≤ b ≤ c</td>
<td>a last</td>
<td>b last</td>
</tr>
<tr>
<td>{1,7,7}</td>
<td>0.685969</td>
<td>0.702326</td>
</tr>
<tr>
<td>{1,7,8}</td>
<td>0.701884</td>
<td>0.720571</td>
</tr>
<tr>
<td>{1,7,9}</td>
<td>0.75</td>
<td>0.7599</td>
</tr>
<tr>
<td>{1,7,10}</td>
<td>0.7705</td>
<td>0.730031</td>
</tr>
<tr>
<td>{1,8,9}</td>
<td>0.726561</td>
<td>0.748325</td>
</tr>
<tr>
<td>{1,8,10}</td>
<td>0.75</td>
<td>0.767452</td>
</tr>
<tr>
<td>{1,9,9}</td>
<td>0.75122</td>
<td>0.776034</td>
</tr>
<tr>
<td>{1,10,10}</td>
<td>0.759375</td>
<td>0.785337</td>
</tr>
<tr>
<td>{2,2,2}</td>
<td>-</td>
<td>0.57475</td>
</tr>
<tr>
<td>{2,2,3}</td>
<td>-</td>
<td>0.588711</td>
</tr>
<tr>
<td>{2,2,4}</td>
<td>-</td>
<td>0.605878</td>
</tr>
<tr>
<td>{2,2,5}</td>
<td>-</td>
<td>0.626298</td>
</tr>
<tr>
<td>{2,2,6}</td>
<td>-</td>
<td>0.65</td>
</tr>
<tr>
<td>{2,2,7}</td>
<td>-</td>
<td>0.675</td>
</tr>
<tr>
<td>{2,2,8}</td>
<td>-</td>
<td>0.7</td>
</tr>
<tr>
<td>{2,2,9}</td>
<td>-</td>
<td>0.725</td>
</tr>
<tr>
<td>{2,2,10}</td>
<td>-</td>
<td>0.75</td>
</tr>
<tr>
<td>{2,3,3}</td>
<td>0.599438</td>
<td>0.600495</td>
</tr>
<tr>
<td>{2,3,4}</td>
<td>0.613361</td>
<td>0.616002</td>
</tr>
<tr>
<td>{2,3,5}</td>
<td>0.630556</td>
<td>0.633678</td>
</tr>
<tr>
<td>{2,3,6}</td>
<td>0.6511</td>
<td>0.653557</td>
</tr>
<tr>
<td>{2,3,7}</td>
<td>0.675</td>
<td>0.675674</td>
</tr>
<tr>
<td>{2,3,8}</td>
<td>-</td>
<td>0.7</td>
</tr>
<tr>
<td>{2,3,9}</td>
<td>-</td>
<td>0.7</td>
</tr>
<tr>
<td>{2,3,10}</td>
<td>-</td>
<td>0.75</td>
</tr>
<tr>
<td>{2,4,4}</td>
<td>0.624</td>
<td>0.627189</td>
</tr>
<tr>
<td>{2,4,5}</td>
<td>0.637895</td>
<td>0.643462</td>
</tr>
<tr>
<td>{2,4,6}</td>
<td>0.655149</td>
<td>0.661387</td>
</tr>
<tr>
<td>{2,4,7}</td>
<td>0.675871</td>
<td>0.680999</td>
</tr>
<tr>
<td>{2,4,8}</td>
<td>0.7</td>
<td>0.702297</td>
</tr>
<tr>
<td>{2,4,9}</td>
<td>0.725</td>
<td>0.725335</td>
</tr>
<tr>
<td>{2,4,10}</td>
<td>-</td>
<td>0.75</td>
</tr>
<tr>
<td>{2,5,5}</td>
<td>0.648438</td>
<td>0.654207</td>
</tr>
<tr>
<td>{2,5,6}</td>
<td>0.662314</td>
<td>0.670934</td>
</tr>
<tr>
<td>{2,5,7}</td>
<td>0.679662</td>
<td>0.689004</td>
</tr>
<tr>
<td>{2,5,8}</td>
<td>0.700625</td>
<td>0.708438</td>
</tr>
<tr>
<td>{2,5,9}</td>
<td>0.725</td>
<td>0.729256</td>
</tr>
</tbody>
</table>

Continued on next page
<table>
<thead>
<tr>
<th>{a, b, c}</th>
<th>Honest Voting</th>
<th>Strategic Voting</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a) last</td>
<td>(b) last</td>
</tr>
<tr>
<td>{2,5,10}</td>
<td>0.75</td>
<td>0.75148</td>
</tr>
<tr>
<td>{2,6,6}</td>
<td>0.67275</td>
<td>0.681341</td>
</tr>
<tr>
<td>{2,6,7}</td>
<td>0.686621</td>
<td>0.698368</td>
</tr>
<tr>
<td>{2,6,8}</td>
<td>0.704102</td>
<td>0.716531</td>
</tr>
<tr>
<td>{2,6,9}</td>
<td>0.725382</td>
<td>0.735848</td>
</tr>
<tr>
<td>{2,7,7}</td>
<td>0.67275</td>
<td>0.681341</td>
</tr>
<tr>
<td>{2,7,8}</td>
<td>0.686621</td>
<td>0.698368</td>
</tr>
<tr>
<td>{2,7,9}</td>
<td>0.704102</td>
<td>0.716531</td>
</tr>
<tr>
<td>{2,7,10}</td>
<td>0.725382</td>
<td>0.735848</td>
</tr>
<tr>
<td>{2,8,8}</td>
<td>0.704102</td>
<td>0.716531</td>
</tr>
<tr>
<td>{2,8,9}</td>
<td>0.725382</td>
<td>0.735848</td>
</tr>
<tr>
<td>{2,8,10}</td>
<td>0.75</td>
<td>0.756338</td>
</tr>
<tr>
<td>{2,9,9}</td>
<td>0.67275</td>
<td>0.681341</td>
</tr>
<tr>
<td>{2,9,10}</td>
<td>0.686621</td>
<td>0.698368</td>
</tr>
<tr>
<td>{3,3,3}</td>
<td>-</td>
<td>0.611656</td>
</tr>
<tr>
<td>{3,3,4}</td>
<td>-</td>
<td>0.624949</td>
</tr>
<tr>
<td>{3,4,4}</td>
<td>-</td>
<td>0.636</td>
</tr>
<tr>
<td>{3,5,5}</td>
<td>0.649221</td>
<td>0.651372</td>
</tr>
<tr>
<td>{3,5,6}</td>
<td>0.664683</td>
<td>0.667613</td>
</tr>
<tr>
<td>{3,5,7}</td>
<td>0.682456</td>
<td>0.685568</td>
</tr>
<tr>
<td>{3,5,8}</td>
<td>0.702616</td>
<td>0.705278</td>
</tr>
<tr>
<td>{3,5,9}</td>
<td>0.725242</td>
<td>0.726786</td>
</tr>
<tr>
<td>{3,5,10}</td>
<td>0.75</td>
<td>0.750135</td>
</tr>
<tr>
<td>{3,6,6}</td>
<td>0.660156</td>
<td>0.662754</td>
</tr>
<tr>
<td>{3,6,7}</td>
<td>0.673314</td>
<td>0.678078</td>
</tr>
<tr>
<td>{3,6,8}</td>
<td>0.688785</td>
<td>0.694773</td>
</tr>
<tr>
<td>{3,6,9}</td>
<td>0.706667</td>
<td>0.712872</td>
</tr>
<tr>
<td>{3,6,10}</td>
<td>0.727661</td>
<td>0.73241</td>
</tr>
<tr>
<td>{3,7,7}</td>
<td>0.750074</td>
<td>0.753421</td>
</tr>
<tr>
<td>{3,7,8}</td>
<td>0.684125</td>
<td>0.689033</td>
</tr>
<tr>
<td>{3,7,9}</td>
<td>0.697226</td>
<td>0.704858</td>
</tr>
</tbody>
</table>

Continued on next page
<p>| {a, b, c} | Honest Voting | | Strategic Voting | |
|---|---|---|---|---|---|
| (a \leq b \leq c) | | | | | |
| {3,6,8} | 0.712735 | 0.721852 | 0.711681 | 0.714591 | 0.721852 | 0.712043 |
| {3,6,9} | 0.730776 | 0.740045 | 0.72321 | 0.734597 | 0.740045 | 0.731481 |
| {3,6,10} | 0.751483 | 0.759465 | 0.73482 | 0.755876 | 0.759465 | 0.75284 |
| {3,7,7} | 0.707906 | 0.715445 | - | 0.708711 | 0.715445 | - |
| {3,7,8} | 0.720961 | 0.731623 | 0.72634 | 0.723999 | 0.731623 | 0.72634 |
| {3,7,9} | 0.736539 | 0.748829 | 0.737457 | 0.742031 | 0.748829 | 0.7408 |
| {3,7,10} | 0.754801 | 0.767086 | 0.748829 | 0.755876 | 0.767086 | 0.759947 |
| {3,8,8} | 0.7452 | 0.75833 | 0.752485 | 0.752754 | 0.75833 | 0.752896 |
| {3,8,9} | 0.760208 | 0.775693 | 0.763311 | 0.770194 | 0.775693 | 0.77 |
| {3,9,9} | 0.754906 | 0.768311 | - | 0.766667 | 0.768311 | - |
| {3,9,10} | 0.778125 | 0.794679 | - | 0.796296 | - | - |
| {4,4,4} | - | 0.648 | - | 0.648 | - | - |
| {4,4,5} | - | 0.660822 | 0.66 | - | 0.660822 | 0.66 |
| {4,4,6} | - | 0.675324 | 0.672 | - | 0.675324 | 0.672 |
| {4,4,7} | - | 0.69156 | 0.684 | - | 0.69156 | 0.684 |
| {4,4,8} | - | 0.709587 | 0.696 | - | 0.709587 | 0.7 |
| {4,4,9} | - | 0.729463 | 0.708 | - | 0.729463 | 0.725 |
| {4,4,10} | - | 0.75125 | 0.72 | - | 0.752107 | 0.75 |
| {4,5,5} | 0.671875 | 0.672533 | - | 0.671875 | 0.672533 | - |
| {4,5,6} | 0.684595 | 0.68641 | 0.684298 | 0.684595 | 0.68641 | 0.684298 |
| {4,5,7} | 0.69905 | 0.701673 | 0.696095 | 0.69905 | 0.701673 | 0.696095 |
| {4,5,8} | 0.715313 | 0.71837 | 0.707911 | 0.715313 | 0.71837 | 0.707911 |
| {4,5,9} | 0.73346 | 0.736545 | 0.71974 | 0.73346 | 0.736545 | 0.725844 |
| {4,5,10} | 0.753571 | 0.75625 | 0.731579 | 0.75464 | 0.75625 | 0.75 |
| {4,6,6} | 0.6955 | 0.697716 | - | 0.6955 | 0.697716 | - |
| {4,6,7} | 0.708123 | 0.71229 | 0.70915 | 0.708123 | 0.71229 | 0.70915 |
| {4,6,8} | 0.722551 | 0.728058 | 0.720662 | 0.722551 | 0.728058 | 0.720662 |
| {4,6,9} | 0.738879 | 0.745059 | 0.732228 | 0.73888 | 0.745059 | 0.732689 |
| {4,6,10} | 0.757206 | 0.763333 | 0.74383 | 0.758559 | 0.763333 | 0.75284 |
| {4,7,7} | 0.718875 | 0.723195 | - | 0.718875 | 0.723195 | - |
| {4,7,8} | 0.731408 | 0.738264 | 0.73428 | 0.731408 | 0.738264 | 0.73428 |
| {4,7,9} | 0.745834 | 0.75439 | 0.745485 | 0.745985 | 0.75439 | 0.745485 |
| {4,7,10} | 0.762273 | 0.771607 | 0.756774 | 0.764227 | 0.771607 | 0.759947 |
| {4,8,8} | 0.742 | 0.748793 | - | 0.742014 | 0.748793 | - |
| {4,8,9} | 0.754453 | 0.764231 | 0.759539 | 0.755271 | 0.764231 | 0.759539 |
| {4,8,10} | 0.768906 | 0.780625 | 0.770435 | 0.772085 | 0.780625 | 0.77088 |
| {4,9,9} | 0.764875 | 0.774414 | - | 0.767326 | 0.774414 | - |</p>
<table>
<thead>
<tr>
<th>${a, b, c}$</th>
<th>Honest Voting</th>
<th>Strategic Voting</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a$ last</td>
<td>$b$ last</td>
</tr>
<tr>
<td>${4, 9, 10}$</td>
<td>0.777258</td>
<td>0.790139</td>
</tr>
<tr>
<td>${4, 10, 10}$</td>
<td>0.7875</td>
<td>0.8</td>
</tr>
<tr>
<td>${5, 5, 5}$</td>
<td>-</td>
<td>0.683594</td>
</tr>
<tr>
<td>${5, 5, 6}$</td>
<td>-</td>
<td>0.695088</td>
</tr>
<tr>
<td>${5, 5, 7}$</td>
<td>-</td>
<td>0.709771</td>
</tr>
<tr>
<td>${5, 5, 8}$</td>
<td>-</td>
<td>0.725</td>
</tr>
<tr>
<td>${5, 5, 9}$</td>
<td>-</td>
<td>0.741736</td>
</tr>
<tr>
<td>${5, 5, 10}$</td>
<td>-</td>
<td>0.760045</td>
</tr>
<tr>
<td>${5, 6, 6}$</td>
<td>0.706875</td>
<td>0.707438</td>
</tr>
<tr>
<td>${5, 6, 7}$</td>
<td>0.719136</td>
<td>0.720721</td>
</tr>
<tr>
<td>${5, 6, 8}$</td>
<td>0.732841</td>
<td>0.735208</td>
</tr>
<tr>
<td>${5, 6, 9}$</td>
<td>0.748065</td>
<td>0.750952</td>
</tr>
<tr>
<td>${5, 6, 10}$</td>
<td>0.76489</td>
<td>0.768006</td>
</tr>
<tr>
<td>${5, 7, 7}$</td>
<td>0.729844</td>
<td>0.731801</td>
</tr>
<tr>
<td>${5, 7, 8}$</td>
<td>0.741977</td>
<td>0.745714</td>
</tr>
<tr>
<td>${5, 7, 9}$</td>
<td>0.755624</td>
<td>0.760705</td>
</tr>
<tr>
<td>${5, 7, 10}$</td>
<td>0.778125</td>
<td>0.786161</td>
</tr>
<tr>
<td>${5, 8, 8}$</td>
<td>0.7525</td>
<td>0.756406</td>
</tr>
<tr>
<td>${5, 8, 9}$</td>
<td>0.770881</td>
<td>0.774048</td>
</tr>
<tr>
<td>${5, 8, 10}$</td>
<td>0.796744</td>
<td>0.802161</td>
</tr>
<tr>
<td>${5, 9, 9}$</td>
<td>0.774844</td>
<td>0.781104</td>
</tr>
<tr>
<td>${5, 9, 10}$</td>
<td>0.786744</td>
<td>0.795858</td>
</tr>
<tr>
<td>${5, 10, 10}$</td>
<td>0.796875</td>
<td>0.805804</td>
</tr>
<tr>
<td>${6, 6, 6}$</td>
<td>-</td>
<td>0.71825</td>
</tr>
<tr>
<td>${6, 6, 7}$</td>
<td>-</td>
<td>0.730207</td>
</tr>
<tr>
<td>${6, 6, 8}$</td>
<td>-</td>
<td>0.743367</td>
</tr>
<tr>
<td>${6, 6, 9}$</td>
<td>-</td>
<td>0.757794</td>
</tr>
<tr>
<td>${6, 6, 10}$</td>
<td>-</td>
<td>0.773554</td>
</tr>
<tr>
<td>${6, 7, 7}$</td>
<td>0.740813</td>
<td>0.741311</td>
</tr>
<tr>
<td>${6, 7, 8}$</td>
<td>0.752606</td>
<td>0.754029</td>
</tr>
<tr>
<td>${6, 7, 9}$</td>
<td>0.76566</td>
<td>0.767832</td>
</tr>
<tr>
<td>${6, 7, 10}$</td>
<td>0.780057</td>
<td>0.782777</td>
</tr>
<tr>
<td>${6, 8, 8}$</td>
<td>0.760045</td>
<td>0.742188</td>
</tr>
<tr>
<td>${6, 8, 9}$</td>
<td>0.774635</td>
<td>0.778064</td>
</tr>
<tr>
<td>${6, 8, 10}$</td>
<td>0.787604</td>
<td>0.792353</td>
</tr>
<tr>
<td>${6, 9, 9}$</td>
<td>0.78413</td>
<td>0.788425</td>
</tr>
<tr>
<td>${6, 9, 10}$</td>
<td>0.796297</td>
<td>0.802161</td>
</tr>
<tr>
<td>${6, 10, 10}$</td>
<td>0.80625</td>
<td>0.812132</td>
</tr>
<tr>
<td>${7, 7, 7}$</td>
<td>-</td>
<td>0.751781</td>
</tr>
</tbody>
</table>

Continued on next page
Table A-1: continued from previous page

<table>
<thead>
<tr>
<th>{a, b, c}</th>
<th>Honest Voting</th>
<th>{a, b, c}</th>
<th>Honest Voting</th>
</tr>
</thead>
<tbody>
<tr>
<td>a ≤ b ≤ c</td>
<td>a last</td>
<td>b last</td>
<td>c last</td>
</tr>
<tr>
<td>{7,7,8}</td>
<td>-</td>
<td>0.763269</td>
<td>0.76275</td>
</tr>
<tr>
<td>{7,7,9}</td>
<td>-</td>
<td>0.775838</td>
<td>0.773719</td>
</tr>
<tr>
<td>{7,7,10}</td>
<td>-</td>
<td>0.789558</td>
<td>0.784688</td>
</tr>
<tr>
<td>{7,8,8}</td>
<td>0.7735</td>
<td>0.773954</td>
<td>-</td>
</tr>
<tr>
<td>{7,8,9}</td>
<td>0.784794</td>
<td>0.786105</td>
<td>0.784654</td>
</tr>
<tr>
<td>{7,8,10}</td>
<td>0.79232</td>
<td>0.799261</td>
<td>0.795363</td>
</tr>
<tr>
<td>{7,9,9}</td>
<td>0.794781</td>
<td>0.79643</td>
<td>-</td>
</tr>
<tr>
<td>{7,9,10}</td>
<td>0.805889</td>
<td>0.8091</td>
<td>0.806796</td>
</tr>
<tr>
<td>{7,10,10}</td>
<td>0.815625</td>
<td>0.819034</td>
<td>-</td>
</tr>
<tr>
<td>{8,8,8}</td>
<td>-</td>
<td>0.784</td>
<td>-</td>
</tr>
<tr>
<td>{8,8,9}</td>
<td>-</td>
<td>0.794976</td>
<td>0.7945</td>
</tr>
<tr>
<td>{8,8,10}</td>
<td>-</td>
<td>0.806953</td>
<td>0.805</td>
</tr>
<tr>
<td>{8,9,9}</td>
<td>0.80475</td>
<td>0.805173</td>
<td>-</td>
</tr>
<tr>
<td>{8,9,10}</td>
<td>0.815504</td>
<td>0.816736</td>
<td>0.815381</td>
</tr>
<tr>
<td>{8,10,10}</td>
<td>0.825</td>
<td>0.826562</td>
<td>-</td>
</tr>
<tr>
<td>{9,9,9}</td>
<td>-</td>
<td>0.814719</td>
<td>-</td>
</tr>
<tr>
<td>{9,9,10}</td>
<td>-</td>
<td>0.825136</td>
<td>0.824687</td>
</tr>
<tr>
<td>{9,10,10}</td>
<td>0.834375</td>
<td>0.834778</td>
<td>-</td>
</tr>
<tr>
<td>{10,10,10}</td>
<td>-</td>
<td>0.84375</td>
<td>-</td>
</tr>
</tbody>
</table>

Table A-2: Exact reliability values for different casting voters under honest voting

<table>
<thead>
<tr>
<th>{a, b, c}</th>
<th>Honest Voting</th>
<th>{a, b, c}</th>
<th>Honest Voting</th>
</tr>
</thead>
<tbody>
<tr>
<td>a ≤ b ≤ c</td>
<td>a last</td>
<td>b last</td>
<td>c last</td>
</tr>
<tr>
<td>{0,0,0}</td>
<td>-</td>
<td>$\frac{1}{4}$</td>
<td>-</td>
</tr>
<tr>
<td>{0,0,2}</td>
<td>-</td>
<td>$\frac{1}{4}$</td>
<td>21</td>
</tr>
<tr>
<td>{0,0,4}</td>
<td>-</td>
<td>$\frac{1}{4}$</td>
<td>11</td>
</tr>
<tr>
<td>{0,0,6}</td>
<td>-</td>
<td>$\frac{1}{4}$</td>
<td>7</td>
</tr>
<tr>
<td>{0,0,8}</td>
<td>-</td>
<td>$\frac{1}{4}$</td>
<td>3</td>
</tr>
<tr>
<td>{0,1,0}</td>
<td>-</td>
<td>$\frac{1}{4}$</td>
<td>6</td>
</tr>
<tr>
<td>{0,1,2}</td>
<td>-</td>
<td>$\frac{1}{4}$</td>
<td>13</td>
</tr>
<tr>
<td>{0,1,4}</td>
<td>-</td>
<td>$\frac{1}{4}$</td>
<td>129</td>
</tr>
<tr>
<td>{0,1,6}</td>
<td>-</td>
<td>$\frac{1}{4}$</td>
<td>129</td>
</tr>
<tr>
<td>{0,1,8}</td>
<td>-</td>
<td>$\frac{1}{4}$</td>
<td>129</td>
</tr>
<tr>
<td>{0,1,10}</td>
<td>-</td>
<td>$\frac{1}{4}$</td>
<td>129</td>
</tr>
</tbody>
</table>

Continued on next page
### Table A-2: continued from previous page

<table>
<thead>
<tr>
<th>(a, b, c)</th>
<th>Honest Voting</th>
<th>(a, b, c)</th>
<th>Honest Voting</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a \leq b \leq c)</td>
<td>(a) last</td>
<td>(b) last</td>
<td>(c) last</td>
</tr>
<tr>
<td>(0.2, 3)</td>
<td>23</td>
<td>369</td>
<td>17</td>
</tr>
<tr>
<td>(0.2, 5)</td>
<td>-</td>
<td>5</td>
<td>19</td>
</tr>
<tr>
<td>(0.2, 7)</td>
<td>-</td>
<td>37</td>
<td>20</td>
</tr>
<tr>
<td>(0.2, 9)</td>
<td>-</td>
<td>40</td>
<td>23</td>
</tr>
<tr>
<td>(0.3, 3)</td>
<td>23</td>
<td>370</td>
<td>5</td>
</tr>
<tr>
<td>(0.3, 5)</td>
<td>27</td>
<td>690</td>
<td>100</td>
</tr>
<tr>
<td>(0.3, 7)</td>
<td>-</td>
<td>19</td>
<td>249</td>
</tr>
<tr>
<td>(0.3, 9)</td>
<td>-</td>
<td>30</td>
<td>234</td>
</tr>
<tr>
<td>(0.4, 4)</td>
<td>3</td>
<td>40</td>
<td>29</td>
</tr>
<tr>
<td>(0.4, 6)</td>
<td>20</td>
<td>350</td>
<td>19</td>
</tr>
<tr>
<td>(0.4, 8)</td>
<td>-</td>
<td>21</td>
<td>23</td>
</tr>
<tr>
<td>(0.4, 10)</td>
<td>-</td>
<td>15</td>
<td>24</td>
</tr>
<tr>
<td>(0.5, 6)</td>
<td>19</td>
<td>320</td>
<td>1,720</td>
</tr>
<tr>
<td>(0.5, 8)</td>
<td>19</td>
<td>300</td>
<td>1,720</td>
</tr>
<tr>
<td>(0.5, 10)</td>
<td>-</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>(0.6, 7)</td>
<td>27</td>
<td>1,220</td>
<td>19</td>
</tr>
<tr>
<td>(0.6, 9)</td>
<td>27</td>
<td>1,270</td>
<td>24</td>
</tr>
<tr>
<td>(0.7, 7)</td>
<td>19</td>
<td>320</td>
<td>1,720</td>
</tr>
<tr>
<td>(0.7, 9)</td>
<td>19</td>
<td>320</td>
<td>1,720</td>
</tr>
<tr>
<td>(0.8, 8)</td>
<td>19</td>
<td>320</td>
<td>1,720</td>
</tr>
<tr>
<td>(0.8, 10)</td>
<td>19</td>
<td>320</td>
<td>1,720</td>
</tr>
<tr>
<td>(0.9, 10)</td>
<td>19</td>
<td>320</td>
<td>1,720</td>
</tr>
<tr>
<td>(1.1, 1)</td>
<td>-</td>
<td>172</td>
<td>17</td>
</tr>
<tr>
<td>(1.1, 3)</td>
<td>-</td>
<td>23</td>
<td>179</td>
</tr>
<tr>
<td>(1.1, 5)</td>
<td>-</td>
<td>17</td>
<td>179</td>
</tr>
<tr>
<td>(1.1, 7)</td>
<td>-</td>
<td>23</td>
<td>179</td>
</tr>
<tr>
<td>(1.1, 9)</td>
<td>-</td>
<td>23</td>
<td>179</td>
</tr>
<tr>
<td>(1.2, 2)</td>
<td>499</td>
<td>897,900</td>
<td>1,220,000</td>
</tr>
<tr>
<td>(1.2, 4)</td>
<td>3</td>
<td>499</td>
<td>897,900</td>
</tr>
<tr>
<td>(1.2, 6)</td>
<td>-</td>
<td>23</td>
<td>179</td>
</tr>
<tr>
<td>(1.2, 8)</td>
<td>-</td>
<td>23</td>
<td>179</td>
</tr>
<tr>
<td>(1.2, 10)</td>
<td>-</td>
<td>23</td>
<td>179</td>
</tr>
<tr>
<td>(1.3, 4)</td>
<td>467,809</td>
<td>346,999</td>
<td>346,999</td>
</tr>
<tr>
<td>(1.3, 6)</td>
<td>19,189</td>
<td>24,709</td>
<td>24,709</td>
</tr>
<tr>
<td>(1.3, 8)</td>
<td>-</td>
<td>17</td>
<td>179</td>
</tr>
<tr>
<td>(1.3, 10)</td>
<td>-</td>
<td>17</td>
<td>179</td>
</tr>
<tr>
<td>(1.4, 5)</td>
<td>36,000</td>
<td>126,900</td>
<td>126,900</td>
</tr>
<tr>
<td>(1.4, 7)</td>
<td>-</td>
<td>17</td>
<td>179</td>
</tr>
<tr>
<td>(1.4, 9)</td>
<td>-</td>
<td>17</td>
<td>179</td>
</tr>
</tbody>
</table>

Continued on next page
Table A-2: continued from previous page

<table>
<thead>
<tr>
<th></th>
<th>Honest Voting</th>
<th></th>
<th>Honest Voting</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a ≤ b ≤ c</td>
<td></td>
<td>a</td>
</tr>
<tr>
<td></td>
<td>a last</td>
<td>b last</td>
<td>c last</td>
</tr>
<tr>
<td>(1,5.5)</td>
<td>163</td>
<td>65400</td>
<td>450</td>
</tr>
<tr>
<td>(1,5.7)</td>
<td>265</td>
<td>101130</td>
<td>54000</td>
</tr>
<tr>
<td>(1,5.9)</td>
<td>37</td>
<td>181280</td>
<td>374200</td>
</tr>
<tr>
<td>(1,6.6)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(1,6.8)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(1,6.10)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(1,7.8)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(1,7.10)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(1,8.9)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(1,9.9)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(1,10.10)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(2,2.3)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(2,2.5)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(2,2.7)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(2,2.9)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(2,3.3)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(2,3.5)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(2,3.7)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(2,3.9)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(2,4.4)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(2,4.6)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(2,4.8)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(2,4.10)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(2,5.6)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(2,5.8)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(2,5.10)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(2,6.7)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(2,6.9)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(2,7.7)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(2,7.9)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(2,8.8)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(2,8.10)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(2,9.10)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(3,3.3)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(3,3.5)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(3,3.7)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(3,3.9)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(3,4.4)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
<tr>
<td>(3,4.6)</td>
<td>30</td>
<td>181280</td>
<td>326400</td>
</tr>
</tbody>
</table>

Continued on next page
Table A-2: continued from previous page

<table>
<thead>
<tr>
<th>a, b, c</th>
<th>Honest Voting</th>
<th>a, b, c</th>
<th>Honest Voting</th>
</tr>
</thead>
<tbody>
<tr>
<td>a ≤ b ≤ c</td>
<td>a last</td>
<td>b last</td>
<td>c last</td>
</tr>
<tr>
<td>(3.4,8)</td>
<td>96961</td>
<td>530369</td>
<td>1062969</td>
</tr>
<tr>
<td>(3.4,10)</td>
<td>138000</td>
<td>752900</td>
<td>1552800</td>
</tr>
<tr>
<td>(3.5,6)</td>
<td>238161</td>
<td>118441</td>
<td>498841</td>
</tr>
<tr>
<td>(3.5,8)</td>
<td>33</td>
<td>6701</td>
<td>282809</td>
</tr>
<tr>
<td>(3.5,10)</td>
<td>10081</td>
<td>17841</td>
<td>35561</td>
</tr>
<tr>
<td>(3.6,7)</td>
<td>592801</td>
<td>1283569</td>
<td>4092169</td>
</tr>
<tr>
<td>(3.6,9)</td>
<td>2027872</td>
<td>4316587</td>
<td>22131229</td>
</tr>
<tr>
<td>(3.7,7)</td>
<td>228535</td>
<td>6073849</td>
<td>3056000</td>
</tr>
<tr>
<td>(3.7,9)</td>
<td>3238252</td>
<td>62566121</td>
<td>80494921</td>
</tr>
<tr>
<td>(3.8,8)</td>
<td>1468</td>
<td>111501</td>
<td>(3.8,9)</td>
</tr>
<tr>
<td>(3.8,10)</td>
<td>3649</td>
<td>95201</td>
<td>57401</td>
</tr>
<tr>
<td>(3.9,10)</td>
<td>59801</td>
<td>356200</td>
<td>665600</td>
</tr>
<tr>
<td>(4.4,4)</td>
<td>—</td>
<td>119</td>
<td>—</td>
</tr>
<tr>
<td>(4.4,6)</td>
<td>—</td>
<td>120581</td>
<td>84</td>
</tr>
<tr>
<td>(4.4,8)</td>
<td>—</td>
<td>32641</td>
<td>87</td>
</tr>
<tr>
<td>(4.4,10)</td>
<td>—</td>
<td>601</td>
<td>15</td>
</tr>
<tr>
<td>(4.5,6)</td>
<td>2533</td>
<td>25358</td>
<td>2801</td>
</tr>
<tr>
<td>(4.5,8)</td>
<td>3209</td>
<td>37020</td>
<td>11400</td>
</tr>
<tr>
<td>(4.5,10)</td>
<td>214</td>
<td>121</td>
<td>139</td>
</tr>
<tr>
<td>(4.6,7)</td>
<td>63377</td>
<td>22805</td>
<td>933241</td>
</tr>
<tr>
<td>(4.6,9)</td>
<td>68050</td>
<td>72000</td>
<td>1692000</td>
</tr>
<tr>
<td>(4.7,7)</td>
<td>7991</td>
<td>129998</td>
<td>354641</td>
</tr>
<tr>
<td>(4.7,9)</td>
<td>20980</td>
<td>159996</td>
<td>554641</td>
</tr>
<tr>
<td>(4.8,8)</td>
<td>4161</td>
<td>9261</td>
<td>89241</td>
</tr>
<tr>
<td>(4.8,10)</td>
<td>4307</td>
<td>10799</td>
<td>3575</td>
</tr>
<tr>
<td>(4.9,10)</td>
<td>4819</td>
<td>5089</td>
<td>3371</td>
</tr>
<tr>
<td>(5.5,5)</td>
<td>—</td>
<td>179</td>
<td>—</td>
</tr>
<tr>
<td>(5.5,7)</td>
<td>—</td>
<td>66231</td>
<td>181</td>
</tr>
<tr>
<td>(5.5,9)</td>
<td>—</td>
<td>38741</td>
<td>189</td>
</tr>
<tr>
<td>(5.6,6)</td>
<td>131</td>
<td>122641</td>
<td>(5.6,7)</td>
</tr>
<tr>
<td>(5.6,8)</td>
<td>4449</td>
<td>3529</td>
<td>43241</td>
</tr>
<tr>
<td>(5.6,10)</td>
<td>8900</td>
<td>3500</td>
<td>39200</td>
</tr>
<tr>
<td>(5.7,8)</td>
<td>4440</td>
<td>620</td>
<td>11800</td>
</tr>
<tr>
<td>(5.7,10)</td>
<td>8900</td>
<td>350</td>
<td>39200</td>
</tr>
<tr>
<td>(5.8,8)</td>
<td>2600</td>
<td>14781</td>
<td>1581</td>
</tr>
<tr>
<td>(5.8,9)</td>
<td>2600</td>
<td>14781</td>
<td>1581</td>
</tr>
<tr>
<td>(5.9,9)</td>
<td>4500</td>
<td>4089000</td>
<td>(5.9,10)</td>
</tr>
<tr>
<td>(5.10,10)</td>
<td>64</td>
<td>155</td>
<td>—</td>
</tr>
<tr>
<td>(6.6,7)</td>
<td>—</td>
<td>3185069</td>
<td>8537</td>
</tr>
<tr>
<td>(6.6,9)</td>
<td>—</td>
<td>1381000</td>
<td>8000</td>
</tr>
</tbody>
</table>

Continued on next page
Table A-2: continued from previous page

<table>
<thead>
<tr>
<th>{a, b, c}</th>
<th>Honest Voting</th>
<th>{a, b, c}</th>
<th>Honest Voting</th>
</tr>
</thead>
<tbody>
<tr>
<td>a ≤ b ≤ c</td>
<td>a last</td>
<td>b last</td>
<td>c last</td>
</tr>
<tr>
<td>{6,7,7}</td>
<td>11853</td>
<td>14861809</td>
<td>—</td>
</tr>
<tr>
<td>{6,7,9}</td>
<td>12585321</td>
<td>14871921</td>
<td>19681921</td>
</tr>
<tr>
<td>{6,8,8}</td>
<td>763</td>
<td>260201</td>
<td>—</td>
</tr>
<tr>
<td>{6,8,10}</td>
<td>7561</td>
<td>1441</td>
<td>13841</td>
</tr>
<tr>
<td>{6,9,10}</td>
<td>7684800</td>
<td>1469669</td>
<td>221129</td>
</tr>
<tr>
<td>{7,7,7}</td>
<td>—</td>
<td>24907</td>
<td>—</td>
</tr>
<tr>
<td>{7,7,9}</td>
<td>—</td>
<td>56956581</td>
<td>24759</td>
</tr>
<tr>
<td>{7,8,8}</td>
<td>1547</td>
<td>1063000</td>
<td>326000</td>
</tr>
<tr>
<td>{7,8,10}</td>
<td>1429000</td>
<td>1467000</td>
<td>—</td>
</tr>
<tr>
<td>{7,9,10}</td>
<td>1442909</td>
<td>1471991</td>
<td>890409</td>
</tr>
<tr>
<td>{8,8,8}</td>
<td>—</td>
<td>98</td>
<td>—</td>
</tr>
<tr>
<td>{8,8,10}</td>
<td>—</td>
<td>103929</td>
<td>161</td>
</tr>
<tr>
<td>{8,9,10}</td>
<td>1044900</td>
<td>1176000</td>
<td>200</td>
</tr>
<tr>
<td>{9,9,9}</td>
<td>—</td>
<td>26071</td>
<td>—</td>
</tr>
<tr>
<td>{9,10,10}</td>
<td>267</td>
<td>9281</td>
<td>—</td>
</tr>
</tbody>
</table>

33