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DECLARATION AND INCLUSION OF MATERIAL
FROM PRIOR THESIS

This Thesis is submitted to the University of Warwick in support of my application for the degree of Doctor of Philosophy. It has been composed by myself and has not been submitted in any previous application for any degree.
ABSTRACT

In the first chapter of this Thesis, we examine an incumbent monopolist’s incentives to upgrade his durable, network product in the subsequent period while facing a potential rival who may also produce a version of identical quality. We show that the incumbent firm may commit not to upgrade because he can charge sufficiently patient, forward looking consumers more in the present market when entry is certain and compatibility between the competitors’ versions is mandatory. In fact, his commitment could be an additional factor of inefficiency while a potential or actual competitive threat could dissolve social optimality. When sequential, non-drastic innovation occurs with certainty, we show in the second chapter that the dominant market player may voluntarily support compatibility when he anticipates a moderately large quality improvement by the competitor for a fairly general set of assumptions regarding consumers’ (un)willingness to postpone their purchase and the rival’s (in)ability to price discriminate between the different customers’ classes. This happens as strategic pricing allows the dominant firm to extract more of the higher total expected surplus that emerges when interoperability is present. Furthermore, we find that mandatory compatibility does not de-facto maximise social welfare, decreases consumer welfare and we identify no market failure when network effects are not particularly strong. For sufficiently innovative products and although compatibility is not supported by the dominant firm, consumers’ welfare is maximised because of the lower prices that emerge.
due to the higher degree of competition that arises when interoperability is not present. In the third chapter, we consider discrete time, stochastic Research and Development [R&D] processes where both an initially dominant and a smaller rival are potential inventors. For sufficiently innovative future products, our first key result is that the dominant firm invests more when compatibility is present and voluntarily decides to supply interoperability information. This happens as the probability that he is the only inventor in the market increases when products are compatible, allowing him to enjoy a higher expected future profit that outweighs the lost current revenue. For economies whose existing market size is considerably large, the rival also demands compatibility while this is no longer true in industries with a relatively smaller number of existing consumers. For less innovative new versions, the dominant firm rejects compatibility and we also find that there is a cutoff in network externalities below which the dominant firm invests more when compatibility is not present. Regarding welfare, we find that a laissez faire Competition Law with respect to the Intellectual Property Rights holders is socially preferable.
CHAPTER 1

EFFICIENT UPGRADING IN DURABLE NETWORK GOODS; IS COMMITMENT ALWAYS GOOD?

1.1 Introduction

While competition is in general socially efficient, this chapter identifies a scenario where this may not necessarily be true. In particular, we find that potential or actual competition are sources of too frequently introduced new products in durable network goods markets. Moreover, we show that although the incumbent may currently commit not to upgrade, his commitment power may in fact add an additional source of inefficiency.

The model we use is similar to that of Ellison and Fudenberg (2000). More specifically, we explore a market leader’s incentives to provide an upgrade of his software in the presence of forward looking consumers and a competitor who could potentially offer a good of the same quality. We do this by considering both the cases when the incumbent may or may not be able to commit to whether he will sell the superior product in the future period. We also give the rival the power to offer compatibility with the leader’s versions. On the demand side, forward
looking consumers incur both the monetary cost and a cost of learning how to use any software product. It is the latter cost that determines whether introducing the new product into the market is socially optimal: society would be better off without it if the social benefit from the quality improvement from its introduction is lower than the social cost and nevertheless, consumers of the old version are "forced" to purchase it due to network externalities.

Our results recognize potential or actual competition as well as the incumbent’s commitment power as sources of inefficiency. More precisely, although the introduction of the new product may not be socially efficient, the market leader will always commit to upgrade if this choice can deter the competitor from investing. If the rival’s entry is certain, the incumbent may commit not to upgrade because such a choice enables him to charge sufficiently patient customers more in the present market. Furthermore, if the rival cannot price discriminate between the old and the new users, the incumbent’s commitment may lead the old consumers to stick to the old product, although the new version is an important improvement while social efficiency is obtained when he lacks commitment power. Thus, forbidding the incumbent to commit may in fact raise social welfare.

1.1.1 Related literature

This chapter links to the literature on durable goods by examining how durability affects the pricing and innovation behaviour of an incumbent firm and a potential
competitor. It also relates to the discussion regarding whether a durable goods monopolist implements the socially optimal level of technological progress when he faces potential or actual competition. Waldman (1993, 1996) as well as Fudenberg and Tirole (1998) and Choi (1994) examined whether the time inconsistency problem faced by a durable goods monopolist might be overcome if the firm introduces a new product. Although these papers recognized the linkage between the present and future market on the monopolist’s pricing and investment decisions, they do not allow for potential or actual competition. Hoppe and Lee (2003) show that the intertemporal linkage may introduce inefficiency in investment if there is a potential entrant that may also innovate. Unlike Bucovetsky and Chilton (1986) and Bulow (1986), Hoppe and Lee (2003) consider a competitor who can come up with a new generation of the good currently supplied by the incumbent monopolist. They identify limit pricing as a source of inefficiency, and they also shed light on Microsoft’s puzzling pricing strategy in Operating systems as a virtual monopolist would charge much more than the technology giant. Fudenberg and Tirole (2000) show that an incumbent monopolist may use limit pricing for his network good to deter entry of a potential entrant’s incompatible product. Our work differs because, unlike Fudenberg and Tirole (2000), we consider durability coupled with network externalities, allowing for compatibility between the competitors’ products. Meanwhile, contrary to Hoppe and Lee (2003) and Fudenberg and Tirole (2000), we no longer identify limit pricing but rather, potential
or actual competition as a source of inefficiency. Ellison and Fudenberg (2000) is the paper that is closest to this work. The authors consider a durable goods monopolist’s incentives to offer an upgrade of his product in the future period. If consumers are homogeneous, the lack of the firm’s commitment power is a source of potential inefficiency because the monopolist always sells the new version in the second period even if its improvement is negligible and thus, not profitable overall. So, his inability to commit may hurt his profits as well as social welfare. This chapter is different because by adding a rival who could introduce a superior version in the future just like the market leader, we find that unlike Ellison and Fudenberg (2000) where commitment is socially desirable, this is no longer true in a scenario when competition is present.

1.2 The model

Consider an industry where a software, durable product of quality $q_1$ is currently supplied by a market leader.\textsuperscript{1} He is considering whether to upgrade his product in the next period by selling a good of superior quality $q_2$. The choice of upgrading does not involve any cost of development as it is assumed that previous investment provides the incumbent with the technology to launch the new product. The incumbent knows that there is a serious threat of a rival firm\textsuperscript{2} that can also develop a good of the same quality $q_2$ after bearing a fixed cost of development $F$

\textsuperscript{1}We follow Ellison and Fudenberg who also consider quality as a positive, real number $q$.

\textsuperscript{2}She will be called rival, competitor or entrant interchangeably throughout the paper.
In a two period environment, the incumbent sets the price(s) for his product(s) while he is able to offer an upgrade price to old users. If he has commitment power, he also has an additional simultaneous choice to make: whether to commit to upgrade or not.

At the same time, the competitor needs to decide whether she will enter the market in the following period. If entry occurs, the two firms engage in price competition (a la Bertrand) and they incur a zero marginal cost of production for all product versions. I investigate both the cases where the rival can price discriminate between the different consumers’ classes and when she cannot while when firms are indifferent, they make a decision that lowers the opponent’s expected profits.

On the demand side, consumers are assumed identical and arrive in constant flows \( \lambda_t \) \((t = 1, 2)\). Customers’ utility takes the form \( U = \sum_{t=1}^{2} \delta^{t-1} V \) where \( \delta \) is the common discount factor and \( V \) is positively dependent on network effects captured by the parameter \( \alpha \). Thus, if the buyer joins a network of mass \( x \) (including himself), the network benefit is \( \alpha x \). In addition to the monetary cost,

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\( F \geq 0 \).³

³Imitation of product functionality is consistent with the software industry in the United States in late 80s. It is also consistent with cases that have recently appeared in Europe (see for example http://curia.europa.eu/jcms/upload/docs/application/pdf/2012-05/cp120053en.pdf for the SAS Institute Inc. versus World Programming Ltd) verifying that software product functionality can be imitated.

⁴This corresponds to the semi-anonymous case in Fudenberg-Tirole (1998).

⁵This assumption is consistent with the applications in the computer software market industry.

⁶We follow Ellison and Fudenberg who also assume the same type of utility \( U \). In their paper, \( V \) is linear in money.
consumers also incur a cost of learning the new technology. Each consumer incurs a cost $c$ the first time he starts to use the incumbent monopolist’s product followed by an additional lower cost ($c_u < c$) when learning to use a new product.

Customers who arrive in the market in the first period are assumed to be forward looking and, depending on their expectations, they may either buy the initial good immediately after observing its price or postpone their decision to the future. Their expectations reflect the information available to consumers at the time they are called to make their buying decision and are fully aligned in equilibrium; that is, they possess perfect foresight. In the second period and if there is a new product in the market, the old customers are not guaranteed to buy it because of the durability of the initial version. These customers’ purchasing decision given announced prices resembles a coordination game and can have multiple equilibria. Following the literature, old consumers coordinate to the Pareto optimal outcome.\footnote{See Ellison and Fudenberg (2000).} In the similar coordination problem related to the new customers’ purchasing decisions, the standard assumption is that buyers with the same preferences act as if they were a single player. Thus, after observing the prices, they coordinate to what is best for all of them. All consumers make their purchasing decisions simultaneously where we assume that they prefer a better than an inferior product even if their net utility is equivalent. Also note that the same discount factor $\delta$ applies to all the agents in the economy.
The model makes the strong assumption that the competitors’ superior quality products are compatible. Thus, a buyer of a high quality good can interact with all the superior product buyers, independently of whether they purchase it from the incumbent or the entrant firm. Backward compatibility makes the buyers of any new product able to open and save a document that was created with the lower quality product. Thus, the high quality good buyers are part of a network which also consists of the low quality good users. On the other hand, non-forward compatibility prevents the buyers of the initial product from working with documents that are created with the superior version.

1.3 Results

1.3.1 Social Welfare

I begin by considering the problem faced by a planner who maximises social surplus. He needs to decide whether it is socially beneficial if the new product is introduced or the old version is used for both periods. In the former case, it may be efficient if the new product is used either by both the new and the old consumers or only by the new potential comers.

If all old and new customers use the new product, social welfare is:

\[ W_U = \lambda_1 (q_1 + \delta q_2 + \alpha \lambda_1 + \delta \alpha - c - \delta c_u) + \lambda_2 \delta (q_2 + \alpha - c), \]

where without loss of generality, the market size in the second period is normalized
to unity and consumers’ utility is linear in money. If the new product is introduced but only the second period customers use it, social welfare is:

\[ W_I = \lambda_1[(1 + \delta)q_1 + (1 + \delta)\alpha\lambda_1 - c] + \lambda_2\delta(q_2 + \alpha - c). \]

If the lower quality product is used for both periods, social welfare is given by the expression:

\[ W_N = \lambda_1[(1 + \delta)q_1 + \alpha\lambda_1 + \delta\alpha - c] + \lambda_2\delta(q_1 + \alpha - c). \]

Comparing the above expressions yields the next proposition that summarizes the socially efficient outcome.

**Proposition 1** Let \( \Delta q = q_2 - q_1 \) denote the quality improvement from the introduction of the new product. The socially efficient outcome is:

(a) keep the lower quality good for two periods if \( a > c_u \) and \( \Delta q < \lambda_1 c_u \) or if \( \alpha < c_u \) and \( \Delta q < \alpha\lambda_1 \),

(b) use the incompatible regime, that is, introduce the new product but only the second period potential customers use it if \( \Delta q > \alpha\lambda_1 \) and \( \Delta q + \alpha\lambda_2 < c_u \) and (c) introduce the new product and everyone uses it in the second period if \( \alpha > c_u \) and \( \Delta q > \lambda_1 c_u \) or if \( \alpha < c_u \) and \( \Delta q > c_u - \alpha\lambda_2 \).

Think of the case that the network effects are large relative to the adoption cost (\( \alpha > c_u \)). It is then beneficial for society to maintain the lower quality good if the cost of learning how to use the new product for the old users exceeds the gain in every customer’s second period utility (\( \Delta q < \lambda_1 c_u \)) and it is socially efficient.
for everyone to purchase the new product if the sign of the inequality is reversed
($\Delta q > \lambda_1 c_u$). When network effects are weak ($\alpha < c_u$), it is socially optimal
to withhold the superior product when the loss from incompatibility is greater
than the benefit the new users enjoy from the new version ($\Delta q \lambda_2 < \alpha \lambda_1 \lambda_2$).
It is also socially efficient if everyone uses the new product when the quality
improvement and the gains from a larger network are greater than the adoption
cost ($\Delta q + \alpha \lambda_2 > c_u$), whereas it is socially optimal if only the new buyers use it
when the last inequality is reversed. Figure 1 provides a graphical representation
of the socially optimal outcome.

Figure 1.1: The socially optimal outcome: The red area indicates values in the
parameter space where using the old product for two periods is socially optimal.
The yellow area represents parameter values where it is socially efficient if only
the new customers use the new product. The green area captures the case where
it is socially optimal if everyone uses the superior good.
1.3.2 Market outcome/ Incumbent’s commitment

I consider a scenario of potential entry when the incumbent has already acquired the technology allowing him to commit to choose to upgrade in the following period. I analyze first the case when entry is certain and the entrant may or may not have the ability to price discriminate between the old and the new customers.

1.3.2.1 Certain entry/ The rival can price discriminate

The entrant is assumed to be able to costlessly imitate the superior good ($F = 0$) and this fact allows her to always enter the market.\(^8\) If the incumbent commits to upgrade, in the second period, Bertrand competition drives all the prices to zero.\(^9\) This is no longer true if he commits to keep the initial product. In this case, the entrant can exploit the quality improvement and charge a strictly positive price either to the whole market or only to the new comers.\(^10\) Since the first period potential customers’ outside opportunity is higher when the incumbent commits to sell the superior version, he could charge them more if he committed to maintain the initial product in the market. Thus, the incumbent could be better-off if he committed not to upgrade. The next proposition summarizes the incumbent’s choice and the equilibrium market outcome for the different parameter values.

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\(^8\)The same result of certain entry could be alternatively generated if the entrant needed to bear a fixed cost $F$ to develop the product and she could offer a superior version with sufficiently smaller adoption cost than the incumbent’s upgrade.

\(^9\)A complete characterization of the prices set and the market outcome is given in the Appendix.

\(^10\)Again, see the Appendix for a complete characterization of the market outcome and the prices set.
Proposition 2  The incumbent always commits not to upgrade. In the second period, either the whole market (if $\Delta q + \alpha \lambda_2 - c_u \geq 0$) or only the new comers (if $\Delta q + \alpha \lambda_2 - c_u < 0$) buy the superior rival’s version of quality $q_2$. In the former scenario, old customers buy the new product for free.

The proposition above suggests that in equilibrium, the higher quality good is always sold in the second period and is purchased either by the whole market or only by the new customers. This fact already highlights the potential inefficiency that may arise in the market as it could be socially beneficial if there is no new product in the economy. The next proposition summarizes the comparison between the market equilibrium and the socially optimal outcome:

Proposition 3  It is socially optimal if there is no new product in the second period and nevertheless: (a) both new and old customers buy the rival’s superior product if $\Delta q + \alpha \lambda_2 - c_u \geq 0$, $\Delta q < \lambda_1 c_u$ (these parameter values imply that $\alpha \geq c_u$). (b) only the new potential customers purchase the entrant’s product if $\Delta q + \alpha \lambda_2 - c_u < 0$, $\Delta q < \alpha \lambda_1$ (these parameter values imply that $\alpha < c_u$).

Society would be better-off if the initial version is used for both periods when the network benefit is relatively large ($\alpha \geq c_u$) and the adoption cost for the old users exceeds the gain in every customer’s second period utility ($\Delta q < \lambda_1 c_u$). Nevertheless, the superior product is always sold in the market and everyone buys it if the quality improvement and the gains from a larger network are greater than
the costs of learning how to use it ($\Delta q + \alpha \lambda_2 - c_u \geq 0$). For relatively weak network benefits compared to the adoption cost ($\alpha < c_u$), it is socially efficient to withhold the high quality product if the loss from incompatibility is greater than the utility benefit the new users enjoy from the new version ($\Delta q \lambda_2 < \alpha \lambda_1 \lambda_2$). However, the entrant sells the superior product and only the new potential customers purchase it when the cost of learning how to use it for the old users is higher than their benefit from upgrading ($\Delta q + \alpha \lambda_2 - c_u < 0$). Therefore, inefficiency may occur as a result of actual competition when the incumbent can commit to his future actions and figure 2 represents diagrammatically the potential inefficiency that may arise in the market.

![Diagram](image)

Figure 1.2: Market outcome and efficiency: The red and yellow shaped areas in the parameter space represent inefficient use of a new product by all consumers and only the new users, respectively.
1.3.2.2  Certain Entry/ The rival cannot price discriminate

If the entrant is unable to price discriminate, the analysis when the incumbent commits to upgrade leads to the same prices set by the competitors.\textsuperscript{11} If the incumbent monopolist commits not to upgrade, the entrant needs to decide whether to serve all the market in the second period or sell the superior product only to the new comers.\textsuperscript{12} The next proposition summarizes the incumbent’s choices as well as the market equilibrium outcome.

**Proposition 4**  (a) If \( \Delta q + \alpha \lambda_2 - c_u < 0 \) or \( \Delta q + \alpha \lambda_2 - c_u \geq 0, \Delta q \lambda_1 + \alpha \lambda_2 < c_u, \alpha \lambda_2 < c_u \), the incumbent commits not to upgrade and the entrant serves only the new comers. (b) If \( \Delta q + \alpha \lambda_2 - c_u \geq 0, \Delta q \lambda_1 + \alpha \lambda_2 - c_u \geq 0, \alpha \lambda_2 < c_u \), the incumbent commits not to upgrade and the entrant serves the whole market. (c) If \( \Delta q + \alpha \lambda_2 - c_u \geq 0, \alpha \lambda_2 > c_u \), the incumbent is indifferent between committing to upgrade or not.

The proposition above suggests that in equilibrium and similar to the case that the entrant can exercise price discrimination, the higher quality good is always sold and is purchased either by the whole market or only by the new customers. Note that under most parameter values, the incumbent commits not to sell the higher quality good because if he sold the upgrade, actual competition would lower

\textsuperscript{11}See the appendix for the complete characterization of the equilibrium prices and market outcome.
\textsuperscript{12}Again, the Appendix contains all the different cases.
his total profits. The next proposition highlights the potential inefficiency that may arise in the market.

**Proposition 5** A) It is socially optimal if the initial product is used for both periods and nevertheless, (a) the higher quality product is sold to the whole market if \( \Delta q + \alpha \lambda_2 - c_u \geq 0, \Delta q \lambda_1 + \alpha \lambda_2 - c_u \geq 0, \Delta q < \lambda_1 c_u \). (b) the entrant’s higher quality good is sold only to the new customers if \( \Delta q + \alpha \lambda_2 - c_u < 0, \Delta q < \alpha \lambda_1 \) or if \( \Delta q + \alpha \lambda_2 - c_u \geq 0, \Delta q \lambda_1 + \alpha \lambda_2 - c_u, \alpha \lambda_2 < c_u \) and \( \Delta q < \lambda_1 c_u \). B) It is socially optimal for everyone to use the new product but the entrant sells the new product only to the new potential customers if \( \Delta q + \alpha \lambda_2 - c_u \geq 0, \Delta q \lambda_1 + \alpha \lambda_2 < c_u, \alpha < c_u \).

Thus, there may be a superior product in the market even though society would be better-off without it for the same parameter values as in the case the entrant can price discriminate. There is also an additional inefficiency (B): when old second period customers’ benefit from using the new product offsets their adoption cost and the first period market size is relatively small, the social optimum is achieved when everyone uses the new product and nevertheless, the entrant sells the superior good only to the new buyers. Figure 3 represents the potential inefficiency that may arise in the market.

1.3.2.3 Potential entry

Consider now the case that the potential entrant needs to pay a fixed cost \( F > 0 \) to develop the superior good. If the incumbent commits to upgrade, the potential
Figure 1.3: Market outcome and efficiency: The red and yellow shaped areas in the parameter space represent inefficient use of a new product. The green area represents the additional inefficiency when the new product is purchased only by the new comers while it is socially optimal for everyone to use a new version.

If the incumbent commits not to upgrade, the analysis is identical with the scenario when the entrant can costlessly imitate the high-quality good under the condition that her development cost is not prohibitively high and this guarantees her entry. The incumbent compares the profit gained by her commitment to either withhold the high quality good or sell it in the second period and the next result summarizes his choice as well as the market outcome. Note that these results are independent of whether the entrant has the ability to price discriminate:

**Proposition 6** The incumbent monopolist always commits to sell the superior
product in the second period and the potential entrant is deterred to enter. If a) 
\[ \Delta q + \alpha \lambda_2 - c_u \geq 0, \] all the market purchases the upgrade, b) \[ \Delta q + \alpha \lambda_2 - c_u < 0, \] only the new customers upgrade.

The incumbent firm’s choice to always commit to upgrade may be socially inefficient as it could be socially optimal if there was no upgrade in the market. This potential inefficiency is highlighted in the next result.

**Proposition 7** It is socially optimal for the low quality good to be sold in the market in both periods and nevertheless, (a) the incumbent commits to sell the upgrade and the whole market buys it when \[ \Delta q + \alpha \lambda_2 - c_u \geq 0 \text{ and } \Delta q < \lambda_1 c_u, \] (b) the incumbent commits to sell the superior good and only the new customers purchase it when \[ \Delta q + \alpha \lambda_2 - c_u < 0 \text{ and } \Delta q < \alpha \lambda_1. \]

Note that inefficiency may arise for the same parameter values as in the case when the entrant’s entry is certain and she can price discriminate between the old and the new users.

**1.3.3 Market outcome/ No commitment for the incumbent**

In this subsection, I will discuss the case when the incumbent firm faces the threat of entry and cannot commit to its future actions.

Consider first the case when the potential rival’s entry is certain and she also has the ability to price discriminate between the different consumers’ classes. Due
to Bertrand competition, the incumbent’s second period profits are zero independently of whether he decides to sell the low or the superior product. In this scenario, he will choose to sell the upgrade in the market, because otherwise, the entrant would enjoy positive profits. Therefore, there will be a high quality good in the second period sold by both competitors and this may be socially inefficient when is optimal for society if there is no new product. In fact, the inefficiency range is the same as in the commitment case analyzed in the previous subsection (proposition 3).

If the entrant lacks the power to price discriminate, the range of inefficiency is the same as in the scenario when the incumbent has commitment power except for one additional case: while it may be socially optimal if all customers purchase the superior product, unlike the scenario that the incumbent lacks the power to commit, the market leads the old customers to keep the initial version when the incumbent can commit ($\Delta q + \alpha \lambda_2 - c_u \geq 0$, $\Delta q \lambda_1 + \alpha \lambda_2 - c_u < 0$, $\alpha < c_u$). Therefore, contrary to the monopolistic environment where social optimality is achieved under the incumbent firm’s commitment power, lack of commitment may raise social welfare when the market is open to competition.

Consider now the situation when the development cost for the entrant is positive. The entrant firm would not invest in developing the higher quality good. To see this fact, consider the post entry game. The incumbent would be indifferent between selling the lower or the superior product because (due to Bertrand com-
petition) his profits would be zero in both cases. He would then prefer to upgrade, because this would guarantee that the entrant would incur losses. The potential entrant anticipates the incumbent’s post entry behaviour and she rationally does not pay the fixed development cost. This fact allows the incumbent to be the sole supplier of the upgrade in the second period. Thus, the range of inefficiency appears to be exactly the same as in the case when the incumbent can commit to his future actions. To summarize, the inefficiency range when the incumbent firm enjoys or lacks commitment power and the fixed development cost for the entrant is strictly positive or zero, respectively, are highlighted in the following table:
<table>
<thead>
<tr>
<th></th>
<th>Commitment for the incumbent</th>
<th>No commitment for the incumbent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monopoly</td>
<td>Social efficiency</td>
<td>Inefficiency: The monopolist always upgrades even though it could be socially optimal if there is no upgrade in the market</td>
</tr>
<tr>
<td>Potential Competition</td>
<td>Inefficiency: The same range as in the monopoly case under no commitment</td>
<td>Inefficiency: Same range as in the monopoly non-commitment case</td>
</tr>
<tr>
<td>(F &gt; 0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Actual competition/</td>
<td>Inefficiency: The same range as in the monopoly case under no commitment</td>
<td>Inefficiency: Same range in the monopoly non-commitment case</td>
</tr>
<tr>
<td>The entrant can price</td>
<td></td>
<td></td>
</tr>
<tr>
<td>discriminate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Actual competition/</td>
<td>Inefficiency: The range of inefficiency is larger than the monopoly non-commitment case.</td>
<td>Same range of inefficiency as in the monopoly non-commitment case</td>
</tr>
<tr>
<td>The entrant cannot</td>
<td></td>
<td></td>
</tr>
<tr>
<td>price discriminate</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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1.4 Applications/ Conclusion

This chapter serves as a small step towards understanding the role of a competitive threat in the frequency of new product introductions in durable network goods. The message of this work is that better versions of such products may arise too often and this inefficiency may be due to potential or actual competition. Going one step further, it is suggested that it may be beneficial for society if the incumbent is forbidden to commit to whether he will upgrade or not. This contrasts sharply with the monopolistic scenario where the first best is achieved under the firm’s commitment power.

The model applies to scenarios where an incumbent monopolist is threatened by a potential competitor and is considering whether to upgrade his product in the subsequent period. It predicts that the superior good is always introduced in the market and this may not be socially beneficial. Such a scenario may occur in technology markets where we observe frequent new versions sold either by the same firm or a competitor. A prime example that fits proposition 2 comes from the spreadsheet market for personal computers. In 1988, Lotus was the dominant player with 70% market share. In 1989 it sold its software program 1-2-3 version 3 in IBM high-end computers and also committed not to upgrade in the Windows platform. Microsoft sold Excel 3 in 1990 offering backward compatibility to the

17 See http://archive.computerhistory.org/resources/access/text/2012/04/102658156-05-01
1-2-3 version 3, free upgrade prices to Lotus customers and a runtime Windows
version shipped free of charge with Excel. Consumers switched to Excel and by
1993, Microsoft had outplaced Lotus as the market leader.

Although the model matches well with the real world example identified above,
there are other reasons that may affect an incumbent monopolist’s decision to
upgrade when he faces a competitive threat. For example, it may be the case
that he is unsure about the quality improvement introduced by the competitor.
It could also be that the success of the new platform (Windows) was ex-ante
questionable. Although these situations are acknowledged to be possible, they
are not considered in this paper.

1.5 Appendix

1.5.1 Market outcome/ Certain Entry/ The incumbent commits to
upgrade and the entrant can price discriminate

If the incumbent commits to upgrade, in period two, perfect compatibility between
the superior products and backward compatibility of the new versions ensure that
the new potential customers join a network of size 1 if they buy from either the

\[\text{See http://books.google.co.uk/books?id=8s3aAAAAMAAJ&pg=PA40&dq=january+1991+
infoworld+excel+3+vs+lotus+1-2-3+compatibility&hl=en&sa=X&ei=zubjUruJOvGf7gbqYDwDQ&ved=0CDsQ6AEwAv#v=onepage&q=january%201991%20infoworld%20excel%20%20compatibility&f=false and
http://www.joelonsoftware.com/articles/fog000000052.html}\]
incumbent or the entrant. Their net utility if they buy either of the competitors’
superior good is $q_2 + \alpha - c - p'_2$, $q_2 + \alpha - c - p_2$ where $p'_2$, $p_2$ are the entrant’s
and the incumbent’s price choices, respectively. Old consumers are assumed to
coordinate to a ‘reluctant rule’; that is, they buy a product independently of what
the other period one customers do. So, they will purchase the entrant’s superior
good even if all the other period one customers either stick to the incumbent’s
initial or upgrade version if:

$$q_2 + \alpha - c_u - p'_u \geq \max \{q_1 + \alpha \lambda_1, \ q_2 + \alpha - c_u - p_u \},$$

where $p_u$, $p'_u$ are the competitors’ price choices. Since Bertrand competition drives
all prices to zero, the new comers purchase the superior product for free from either
of the competitors. If $\Delta q + \alpha \lambda_2 - c_u < 0$, the old customers stick to the incumbent’s
initial version. If $\Delta q + \alpha \lambda_2 - c_u \geq 0$, the whole market purchases a new product
from either the incumbent or the rival. Working back in the first period, the
incumbent sets a price for the initial version to attract the incoming customers. If
the first period potential customers buy the initial version and expect to purchase
the new product in the second period (when $\Delta q + \alpha \lambda_2 - c_u \geq 0$), they will do so
by paying a price $p_1$ satisfying the equality:

$$q_1 + \delta q_2 + \alpha \lambda_1 + \delta \alpha - c - \delta c_u - p_1 = \delta(q_2 + \alpha - c)$$

or equivalently, $p_1 = q_1 + \alpha \lambda_1 - c(1 - \delta) - \delta c_u$. Similarly, if old customers expect
not to purcahse a new product (when $\Delta q + \alpha \lambda_2 - c_u < 0$), they are willing to pay
a price $p_1$ such that their total expected discounted benefit from buying the initial product and not upgrading is greater than or equal to their expected surplus if they postpone their decision for period two. Thus, the equilibrium period one price is set by the incumbent monopolist such that:

$$q_1 + \delta q_1 + \alpha \lambda_1 + \delta \alpha \lambda_1 - c - p_1 = \delta(q_2 + \alpha - c),$$

or

$$p_1 = q_1 + \delta q_1 + \alpha \lambda_1 - \delta \alpha \lambda_2 - c(1 - \delta) - \delta q_2.$$

1.5.2 Market outcome/ Certain Entry/ The incumbent commits not to upgrade and the entrant can price discriminate

Consider now the case that the incumbent commits not to upgrade. The new customers are assumed to act as if they are a single player. Thus, their net utility if they buy the entrant’s superior product is $q_2 + \alpha - c - p_2'$, where $p_2'$ is her price choice. If they all decide to purchase the incumbent’s initial version, their net utility is $q_1 + \alpha \lambda_2 + \alpha \lambda_1 x_1 - c - p_1'$, where $x_1$ is the old customers’ fraction that sticks to the old product and $p_1'$ is his price choice. Thus, the new comers will decide to purchase the entrant’s good if:

$$q_2 + \alpha - c - p_2' \geq q_1 + \alpha \lambda_2 + \alpha \lambda_1 x_1 - c - p_1'$$

Old customers prefer the entrant’s version even if all the other first period consumers stick to the old product if:

$$q_2 + a - c_u - p_u' \geq q_1 + \alpha \lambda_1 + \alpha \lambda_2 x_2,$$
where $x_2$ is the new consumers’ fraction that buys the old good and $p'_u$ is the entrant’s price choice. If $\Delta q + \alpha \lambda_2 - c_u < 0$, old customers don’t buy the new product independently of the entrant’s price choice. Bertrand competition leads to prices $p'_2 = \Delta q$, $p'_1 = 0$, $p'_u = 0$ and the new customers purchase the new product. If $\Delta q + \alpha \lambda_2 - c_u \geq 0$, Bertrand competition leads to equilibrium prices $p'_2 = \Delta q + \alpha \lambda_1$, $p'_1 = 0$, $p'_u = \Delta q + \alpha \lambda_2 - c_u$ and all the customers buy the new product. Going back to the initial period, the incumbent sets a price to attract the first period potential customers whose outside opportunity is to wait and make their purchase in the second period by paying a price $\Delta q$. If they expect that they will buy the superior product in the following period (when $\Delta q + \alpha \lambda_2 - c_u \geq 0$), they are willing to buy the initial version if their expected total net benefit is higher than their discounted payoff from postponing their decision for the following period. Thus, the total expected price they are willing to pay ($p_1 + p'_u$) is given by the equality:

\[
q_1 + \delta q_2 + \alpha \lambda_1 + \delta \alpha - c - \delta c_u - \delta(q_2 + \alpha - c - \Delta q) = p_1 + \delta p'_u,
\]

or equivalently $p_1 = q_1 + \delta q_2 + \alpha \lambda_1 + \delta \alpha - c - \delta c_u - \delta(q_2 + \alpha - c - \Delta q) = q_1 + \delta \Delta q + \alpha \lambda_1 - c(1 - \delta) - \delta c_u$ as the optimal incumbent’s choice is to force the competitor to set a zero price in the second period to the existing, incumbent’s old customers.

If old customers expect to stick to the old product (when $\Delta q + \alpha \lambda_2 - c_u < 0$), they are willing to buy the initial product by paying a price $p_1$ that satisfies the
equality:

\[ q_1 + \delta q_1 + \alpha \lambda_1 + \delta \alpha \lambda_1 - c - p_1 = \delta(q_2 + \alpha - c - \Delta q) \]

or \( p_1 = q_1 + \alpha \lambda_1 - \delta \alpha \lambda_2 - c(1 - \delta) \).

1.5.3 Market outcome/ Certain Entry/ Incumbent’s Commitment/

No price discrimination

If the incumbent commits to upgrade, perfect compatibility between the superior products and backward compatibility of the new version ensure that the new potential customers join a network of size 1 if they buy from either the incumbent or the entrant. Their net utility if they buy the entrant’s or the incumbent’s superior good is \( q_2 + \alpha - c - p_1 \), \( q_2 + \alpha - c - p_2 \) where \( p_1 \), \( p_2 \) are the entrant’s and the incumbent’s price choices, respectively. The old consumers will buy the entrant’s product even if all the other period one customers either stick to the incumbent’s initial or upgrade version if:

\[ q_2 + \alpha - c_u - p_2' \geq \max \{ q_1 + \alpha \lambda_1, \ q_2 + \alpha - c_u - p_u \} , \]

where \( p_u \) is the incumbent’s price choice for the old consumers who upgrade in period two. Bertrand competition drives all the prices to zero. If \( \Delta q + \alpha \lambda_2 - c_u < 0 \), the old customers stick to the incumbent’s initial version and the new comers purchase the superior good for free by either of the competitors. If \( \Delta q + \alpha \lambda_2 - c_u \geq 0 \), the whole market purchases for free either the incumbent’s or the entrant’s
high-quality product. In the first period, the incumbent sets a price for the initial version to attract the incoming customers. If the old consumers expect to purchase a new product in the second period \( (\Delta q + \alpha \lambda_2 - c_u \geq 0) \), the price in the first period satisfies the equality:

\[
q_1 + \delta q_2 + \alpha \lambda_1 + \delta \alpha - c - \delta c_u - p_1 - \delta p_u = \delta(q_2 + \alpha - c - p'_2)
\]

or equivalently, \( p_1 = q_1 + \alpha \lambda_1 - c(1 - \delta) - \delta c_u \), where \( p_u = p'_2 = 0 \). If these customers expect to keep the initial version (when \( \Delta q + \alpha \lambda_2 - c_u < 0 \)), the first period price satisfies the equality:

\[
q_1 + \delta q_1 + \alpha \lambda_1 + \delta \alpha \lambda_1 - c - p_1 = \delta(q_2 + \alpha - c),
\]

or \( p_1 = q_1 + \delta q_1 + \alpha \lambda_1 - \delta \alpha \lambda_2 - c(1 - \delta) - \delta q_2 \).

Consider now the case that the incumbent commits not to upgrade. The new customers choose the entrant’s superior good if:

\[
q_2 + \alpha - c - p'_2 \geq \max \left\{ q_1 + \alpha \lambda_2 + \alpha \lambda_1 x_1 - c - p'_1, 0 \right\}
\]

where \( p'_2, p'_1 \) are the entrant’s and the incumbent’s second period price choices for the high and the initial version, respectively. Old consumers prefer the entrant’s version and do not stick to the incumbent’s initial product if:

\[
q_2 + a - c_u - p'_2 \geq q_1 + \alpha \lambda_1 + \alpha \lambda_2 x_2
\]

or equivalently

\[
\Delta q + \alpha \lambda_2 - \alpha \lambda_2 x_2 - c_u - p'_2 \geq 0,
\]
where \( x_2 \) is the new consumers’ fraction that buys the old good. If \( \Delta q + \alpha \lambda_2 - c_u \geq 0 \), \( \Delta q > \Delta q + \alpha \lambda_2 - c_u \), \( \Delta q + \alpha \lambda_2 - c_u \geq \lambda_2 \Delta q \), Bertrand competition leads to \( p_2' = \Delta q + \alpha \lambda_2 - c_u \) and \( p_1' = 0 \) and the equilibrium market outcome is that everyone purchases the entrant’s new product. Otherwise, the prices are \( p_2' = \Delta q \) and \( p_1' = 0 \) with potentially different equilibrium market outcomes dependent on the parameter values. To be more precise, if \( \Delta q + \alpha \lambda_2 - c_u < 0 \) or if \( \Delta q + \alpha \lambda_2 - c_u \geq 0 \), \( \Delta q > \Delta q + \alpha \lambda_2 - c_u \), \( \Delta q + \alpha \lambda_2 - c_u < \lambda_2 \Delta q \), unlike the old consumers, the new comers purchase the entrant’s superior product, whereas if \( \Delta q + \alpha \lambda_2 - c_u \geq 0 \), \( \Delta q < \Delta q + \alpha \lambda_2 - c_u \), everyone buys the new product in the second period. In the initial stage, the incumbent sets a price \( p_1 \) for the lower quality good such that the potential customers buy it and do not postpone their purchase decision. First period customers’ outside opportunity is to purchase the superior entrant’s product by paying a price \( p_2'' = \Delta q \) in the future period. If they expect to buy the higher quality product, \( (\Delta q + \alpha \lambda_2 - c_u \geq 0, \Delta q > \Delta q + \alpha \lambda_2 - c_u, \Delta q + \alpha \lambda_2 - c_u \geq \lambda_2 \Delta q \)\), the first period price satisfies the equation:

\[
q_1 + \delta q_2 + \alpha \lambda_1 + \delta \alpha - c - \delta c_u - p_1 - \delta p_2' = \delta (q_2 + \alpha - c - p_2''), \quad \text{where} \quad p_2' = \Delta q + \alpha \lambda_2 - c_u, \quad p_2'' = \Delta q
\]

or equivalently \( p_1 = q_1 + \alpha \lambda_1 - c(1 - \delta) - \delta \alpha \lambda_2 \). They also expect to buy \( q_2 \) if \( \Delta q + \alpha \lambda_2 - c_u \geq 0, \Delta q < \Delta q + \alpha \lambda_2 - c_u \). In this case, the first period price is given by the equality:

\[
q_1 + \delta q_2 + \alpha \lambda_1 + \delta \alpha - c - \delta c_u - p_1 - \delta p_2' = \delta (q_2 + \alpha - c - \Delta q), \quad \text{where} \quad p_2' = \Delta q
\]

or \( p_1 = q_1 + \alpha \lambda_1 - c(1 - \delta) - \delta c_u \). If the old customers expect to stick to the old
version (if $\Delta q + \alpha \lambda_2 - c_u < 0$ or if $\Delta q + \alpha \lambda_2 - c_u \geq 0$, $\Delta q > \Delta q + \alpha \lambda_2 - c_u$, $\Delta q + \alpha \lambda_2 - c_u < \lambda_2 \Delta q$), the first period price satisfies the equation:

$$q_1 + \delta q_1 + \alpha \lambda_1 + \delta \alpha \lambda_1 - c - p_1 = \delta (q_2 + \alpha - c - \Delta q)$$

and thus, $p_1 = q_1 + \alpha \lambda_1 - c(1 - \delta) - \delta \alpha \lambda_2$.

1.5.4 Post Entry game/ Potential entry/ The incumbent commits to upgrade

Think of the hypothetical post-entry scenario when the entrant needs to bear a fixed positive development cost when the incumbent commits to upgrade. Note that I consider the case where the entrant is able to price discriminate between the old and the new users. Under the assumption of compatibility between the rival firms’ products, the new customers’ net utility if they buy the high-quality product by either the incumbent or the entrant is $q_2 + \alpha - c - p_2$, $q_2 + \alpha - c - p_2'$, respectively. Old customers buy the new product even if every other old customer either chooses the entrant’s high-quality or the incumbent’s initial version when:

$$q_2 + \alpha - c_u - p_u \geq \max \left\{ q_2 + \alpha - c_u - p_u', q_1 + \alpha \lambda_1 \right\}.$$  

where $p_u$, $p_u'$ are the the competitors’ price choices and because they expect the new second period customers to purcahse a new version. If $\Delta q + \alpha \lambda_2 - c_u < 0$, the old consumers will not buy the upgraded version independently of the rival firms’
price choices. Bertrand competition leads to prices, \( p_2 = \frac{F}{\lambda_2} - \epsilon \), \( p_2' = \frac{F}{\lambda_2} \). New customers would purchase the superior good from the incumbent and thus, the potential entrant would incur losses after entry. Thus, she will optimally choose not to invest. Similarly, think of the post-entry game if \( \Delta q + \alpha \lambda_2 - c_u \geq 0 \). Bertrand competition would lead to prices \( p_2 = \frac{F}{\lambda_2} - \epsilon, \ p_2' = \frac{F}{\lambda_2}, \ p_u = \frac{F}{\lambda_1} - \epsilon, \ p_u' = \frac{F}{\lambda_1} \) and the whole market would upgrade. Thus, the potential entrant would be better-off if she stayed out of the market. Going back to the first period, the incumbent needs to attract the potential customers into buying the initial version of the product. If the first period customers expect to upgrade (when \( \Delta q + \alpha \lambda_2 - c_u \geq 0 \)), the first period price is given by the expression:

\[
p_1 = q_1 + \alpha \lambda_1 - c + \delta q_2 + \delta \alpha - \delta c_u - \delta p_u,
\]

where \( p_u = \Delta q + \alpha \lambda_2 - c_u \). If they expect to stick to the old version (when \( \Delta q + \alpha \lambda_2 - c_u < 0 \)), the first period price \( p_1 \) is such that:

\[
p_1 = q_1 + \delta q_1 + \alpha \lambda_1 + \delta \alpha \lambda_1 - c.
\]

1.5.5 Post entry game/ Potential entry/ The incumbent commits not to upgrade

I analyze the scenario where the entrant can price discriminate between the different consumers’ classes.

\(^{19}\)For \( \epsilon \) being any small positive number

\(^{20}\)When, without loss of generality, I assume that the development cost is not prohibitively high: \( F < (q_2 + \alpha \lambda_2 - c_u) \min \{\lambda_1, \lambda_2\} \).
**Case 1** \( \Delta q + \alpha \lambda_2 - c_u < 0, \lambda_2 \Delta q - F \geq 0. \)

In the second period, Bertrand competition leads to the entrant’s and the incumbent’s prices being \( p_2' = \Delta q, p_1' = 0 \), respectively and only the new potential customers purchase the superior product. The incumbent in period one sets a price \( p_1 \), such that:

\[
q_1 + \alpha \lambda_1 + \delta q_1 + \delta \alpha \lambda_1 - c - p_1 \geq \delta (q_2 + \alpha - c - p_2''),
\]

where the left hand side of the inequality is the customers’ net utility from purchasing the lower quality good in period one and retaining it in period two. Note that if all consumers postpone their purchase, the price they would face is \( p_2'' = \Delta q \). Thus, the first period price satisfies the above inequality as equality and is given by the expression:

\[
p_1 = q_1 - (1 - \delta)c + \alpha \lambda_1 - \delta \alpha \lambda_2.
\]

The incumbent’s and the entrant’s equilibrium profits are:

\[
\Pi_I = \lambda_1[q_1 - (1 - \delta)c + \alpha \lambda_1 - \delta \alpha \lambda_2],
\]

\[
\Pi_E = \lambda_2 \Delta q - F, \lambda_2 \Delta q - F \geq 0,
\]

respectively.

**Case 2** \( \Delta q + \alpha \lambda_2 - c_u \geq 0, \lambda_1(\Delta q + \alpha \lambda_2 - c_u) + \lambda_2(\Delta q + \alpha \lambda_1) - F \geq 0. \)

In the second period, Bertrand competition leads to the prices \( p_2' = \Delta q + \alpha \lambda_1, p_u' = \Delta q + \alpha \lambda_2 - c_u \), set by the entrant and \( p_1' = 0 \) set by the incumbent and everyone purchases the entrant’s superior good. Initially, the incumbent sets a
price \( p_1 \), such that:

\[
q_1 + \delta q_2 + \alpha \lambda_1 + \delta \alpha - c - \delta c_u - p_1 - \delta p'_u \geq \delta (q_2 + \alpha - c - p_2''),
\]

where \( p_2'' = \Delta q \) is the entrant’s price if the old customers wait and purchase the superior product in the second period. Thus, the equilibrium prices as well as the competitors’ profits are given by the expressions:

\[
p_1 = q_1 + \alpha \lambda_1 - c(1 - \delta) - \delta c_u, \quad p'_u = \Delta q + \alpha \lambda_2 - c_u, \quad p'_1 = 0, \quad p'_2 = \Delta q + \alpha \lambda_1.
\]

\[
\Pi_I = \lambda_1 [q_1 + \alpha \lambda_1 - c(1 - \delta) - \delta c_u],
\]

\[
\Pi_E = \lambda_1 (\Delta q + \alpha \lambda_2 - c_u) + \lambda_2 (\Delta q + \alpha \lambda_1) - F.
\]
CHAPTER 2

COMPATIBILITY, INTELLECTUAL PROPERTY, INNOVATION AND WELFARE IN DURABLE GOODS MARKETS WITH NETWORK EFFECTS

2.1 Introduction

This chapter relaxes the assumption of mandatory compatibility set previously and poses the question: Should dominant firms with durable network goods like markets for the applications in the software market industry have the obligation to provide technical compatibility information to direct competitors? This fundamental question lies at the intersection of Competition and Intellectual Property Law and different countries give different answers.

In the European Union, market leaders must provide interoperability\textsuperscript{1} information to rivals. Failure to do so is a potential violation of Article 102 (ex article 82) of the European Competition Law, and often leads to regulation forcing a dominant firm to allow compatibility.\textsuperscript{2} In a nutshell, the refusal to license Intel-

\textsuperscript{1}We will use the terms interoperability and compatibility interchangeably throughout the paper.
\textsuperscript{2}See http://ec.europa.eu/competition/antitrust/legislation/handbook_vol_1_en.pdf
lectual Property may, in itself, constitute a breach of Article 102 if all the following conditions are met: a) access is indispensable for carrying on a particular business, b) it results in the elimination of competition on a secondary market and c) it may lead to consumer harm. A famous example comes from the 2008 European Commission case against Microsoft, which was related to the firm’s refusal to provide competitors technical information about its Office suite so that they could craft interoperable software. The case followed a complaint from firms-members of the ECIS [European Committee of Interoperable Systems] and was put on hold in December 2009 after Microsoft’s commitment to comply.

In contrast, in the United States, a more laissez faire Competition Law with respect to the Intellectual Property Rights owners is favoured. For example, Thomas Barnett of the United States Department of Justice argues that: "U.S. courts recognize the potential benefits to consumers when a company, including a dominant company, makes unilateral business decisions, for example to add features to its popular products or license its intellectual property to rivals, or to refuse to do so". Indeed, the U.S antitrust authorities conclude that "antitrust liability for mere unilateral, unconditional refusal to license patents will not play a meaningful part in the interface between patent rights and antitrust protections".

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3 See http://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:52009XC0224(01)&from=EN
5 See http://www.techhive.com/article/124813/article.html
8 See http://www.justice.gov/atr/public/reports/236681_chapter7.htm
In this chapter I investigate dominant firms’ approaches towards interoperability, how these decisions affect their competitors’ incentives to invest in improving product quality and the welfare effects of refusals to supply interoperability that may occur in a laissez faire economy. More precisely, my paper provides answers to the following questions: When do market leaders block interoperability with a rival future innovator and under what conditions would they support compatibility? If there is incompatibility, does this de-facto mean that it is socially undesirable? Could a market where compatibility is voluntary converge to interoperability when it is socially efficient? Are consumers better-off in an economy that mandates compatibility?

To answer these questions, a sequential game is built in which a small, innovative rival initially decides whether to invest in product quality. The rival’s choice crucially depends on the importance of her idea and the dominant firm’s future anticipated support or refusal to supply interoperability information due to the leader’s large installed base of consumers. This model fits a common pattern in durable, technology goods markets, where a smaller rival may have valuable ideas that emerge as follow-on, non-drastic substitutable innovations, after the dominant firm’s invention hits the market in a Schumpeterian scenario of creative destruction. My analysis shows that given modest structure on consumer preferences, the market leader chooses to supply interoperability information if he anticipates a moderately large quality improvement by the rival. This reflects...
that strategic pricing allows him to extract more of the higher future surplus in
the present market when the competitor has the power to price discriminate in-
dependently of new first period customers’ (un)willingness to postpone purchase
decisions. An analogous result arises even when the rival cannot price discrimi-
nate as long as potential first period consumers can delay purchase decisions.

When compatibility is not supported and incompatible networks arise, consumer
surplus increases relative to the scenario that compatibility is mandatory due to
more fierce competitive forces that reduce equilibrium prices. I also demonstrated
that mandatory compatibility does not de-facto maximise social welfare, while
there is no market failure when network effects are weak. When a rival’s product
is more innovative, the dominant firm always refuses to offer compatibility. In
turn, when the rival lacks the ability to price discriminate, although the economy
that mandates compatibility or operates under a laissez faire Competition Law
may lead to inefficiency, the presence of incompatibility benefits new second pe-
riod consumers, as their surplus is higher when compatibility is not supported due
to the competitive pressures that reduce equilibrium prices. Our conclusions cast
doubts as to whether mandatory interoperability, while trying to support com-
petition, allow technological advancement and protect consumers from abusive
behaviour, may actually distort the market and lead to both socially undesirable
results and customers’ harm.

This chapter is organized as follows: I next discuss related literature. Section
2 presents the model. Section 3 considers the case where the anticipated second period quality improvements are moderately large. I characterize equilibrium outcomes when compatibility is mandatory and when the economy operates under a laissez faire Competition Law toward Intellectual Property Rights holders. I then analyze the problem of a social planner who wishes to maximise social surplus and I contrast equilibrium outcomes with the social optimum. Section 4 looks at the case where the superior second period product is expected to be more innovative. I first analyze the market equilibrium outcome and determine the economy that maximises consumers’ welfare because the first welfare measure fails to answer which economy is socially preferable. Section 5 discusses applications and concludes.

2.1.1 Related Literature

This work contributes to the literature regarding firms’ incentives toward compatibility with their competitors when network effects are present. In a seminal paper, Katz and Shapiro (1985) show that firms with a larger installed base prefer to be incompatible with their rivals. In the same vein, Cremer, Rey and Tirole (2000) analyze competition between Internet backbone providers and predict that a dominant firm may want to reduce the degree of compatibility with smaller market players. Malueg and Schwartz (2006) find that a firm with the largest installed base will not support connectivity with firms that are themselves compat-
ble when its market share exceeds fifty percent, or the potential to add consumers falls. Similar results appear in Chen, Doraszelski and Harrington (2009), who consider a dynamic setting with product compatibility and market dominance. They find that if a firm gets a larger market share, it may make its product incompatible. If, instead, firms have similar installed bases, they make their products interoperable to expand the market. Viecens (2009) distinguishes between direct and indirect network effects by studying platform competition between two firms where users buy a platform and its compatible applications. By allowing for applications to be substitutes, complements or independent, she considers compatibility in two dimensions: 1) compatibility of the complementary good, which she calls compatibility in applications, 2) inter-network compatibility with direct network externalities. She finds that the dominant firm never promotes compatibility in applications but both firms find inter-network compatibility profitable.

In contrast to this literature, I focus on durable goods with direct network effects. Both durability and network externalities are important features of most software products that are at the heart of this compatibility question. Contrary to the literature, I consider improvements in product quality and find that the dominant firm may support compatibility with its rival when the quality improvement expected to be introduced by the smaller firm is moderately large. However, the dominant firm does not offer compatibility to a rival that is expected to sell a sufficiently innovative product.
Economides (2006) argues that it is socially efficient to move toward compatibility. Similarly, in Katz and Shapiro (1985), interoperability would raise consumer surplus. In a static environment, Viecens (2009) concludes that compatibility in the applications may be harmful for users and social welfare, particularly when asymmetries are strong. Moreover, inter-network compatibility should not be supported by consumers. I find that interoperability could lead to losses in consumer welfare and dynamic inefficiency because unlike a market that operates under a laissez faire Competition Law toward Intellectual Property Rights holders, a regime of compulsory compatibility may result in both higher prices and in the inefficient introduction of a higher quality product, when the expected quality improvement is small relative to the network externalities, in which case society would be better-off without it. For sufficiently innovative expected product improvements, consumers gain when compatibility is not supported due to the lower prices that emerge compared to a regime of mandatory compatibility.

A second related strand of literature explores firms’ incentives to upgrade durable, network goods and how these decisions affect social welfare. In a monopolistic environment, Ellison and Fudenberg (2000) show that upgrades may occur too frequently due to a firm’s inability to commit to whether it will upgrade in the future or not. The present paper indicates that in a market that operates under a laissez faire Competition Law, the social and private incentives for producing better products are aligned when network effects are not too strong.
In the literature on sequential innovation, Scotchmer (1991, 1996), Scotchmer and Green (1996) and others study the case of single follow-on innovations. They focus on the breadth and length of patents needed to secure the initial innovator’s incentives to innovate when a second innovator threatens to innovate as well. They hold the view that patents for the first innovator should last longer when a sequence of innovative activity is undertaken by different firms than when innovation is concentrated in one firm. I am mainly interested in the interplay between Intellectual Property Rights protection and firms’ behaviour towards compatibility. In contrast to these papers, I find that the first innovator will voluntarily offer compatibility to rivals because strategic pricing enables him to absorb more of the expected future profit when he anticipates a moderately large improvement from the second innovator.

2.2 The Model

My objective is to provide insights into how dominant firms’ short-run compatibility and pricing decisions regarding their durable network goods relate to homogeneous, forward-looking consumers, and how these anticipated compatibility choices affect an innovative rival’s investment in R&D. The industry I have in mind is the market for computer software applications highlighted in the introduction.
2.2.1 Supply

In the three-period model, the sequence of events in the supply side is as follows: initially, the dominant firm has an installed base $\lambda_0$ of consumers. The market leader is marketing his product, which improves the level of quality from the previously sold version of quality $q_0$ to a higher level $q_1$.\(^9\) The upgraded version of quality $q_1$ is backward compatible, allowing its purchasers to interact with the users of the old version of quality $q_0$. In contrast, forward incompatibility prevents users of the initial version from saving and editing documents that are created with the upgrade.\(^{10}\)

At the beginning of period $t=1$, a rival firm can choose to incur a fixed cost $F$ to create a substitute product of higher unknown quality $q_2 > q_1$ and only undertakes R&D when the project has a positive net present value. The fact that the rival is the only firm that can build on the initial incumbent’s improved product of quality $q_1$ may raise the question: why is it not the dominant firm that is the further innovator?\(^11\) After all, he knows his products and he also knows that his improved good of quality $q_1$ could be improved further. The assumption is designed to capture the widely observed scenario in the high-tech and software industry that small rivals often have better ideas than the initially dominant firm. Thus, the model does not necessarily assume that the dominant firm has no further

\(^9\)We follow Ellison and Fudenberg (2000) who also assume quality as a positive, real number.
\(^{10}\)See Ellison and Fudenberg (2000) for a paper with backward compatibility but forward incompatibility.
\(^{11}\)In a related paper, I allow both firms to be future potential innovators.
ideas; rather, it captures the fact that a smaller competitor may have a better, future idea. All innovations occur with certainty and the magnitude of the quality improvements is treated as exogenous.\textsuperscript{12}

At the end of period $t=1$, the market leader sets the price(s) for his product(s) and decides whether to support compatibility by eliciting interoperability information about his version(s). Note that the quality of the new product $q_2$ is publically known when the dominant firm is called to make its decisions.\textsuperscript{13} This compatibility choice is a binary decision. Following Malheg and Schwartz (2006), I assume that compatibility requires both parties’ consent, and cannot be achieved unilaterally using converters or adapters. In addition, in order to avoid potentially collusive behaviour, licensing of Intellectual Property is free.\textsuperscript{14}

If the dominant firm chooses to be compatible, purchasers of a product of quality $q_2$ belong to a network of maximum size due to backward compatibility. In contrast, users of the product of quality $q_1$ cannot interact with the purchasers of the new version unless they buy the superior product.

If compatibility is not expected to be supported, the rival firm may still innovate. More precisely, if the market leader is expected to refuse to offer compatibility, the rival has an alternative route to innovate that does not use the dominant firms’ network of existing customers. This assumption accords with the

\textsuperscript{12}In a related paper, I consider the case where innovation does not occur with certainty and current work endogenizes the quality improvements.

\textsuperscript{13}Current work investigates the role of private information in this setting.

\textsuperscript{14}See Malheg and Schwartz (2006).
Microsoft Office case highlighted in the introduction: Microsoft’s refusal to offer compatibility did not, per se, prevent rivals from innovating as they could use the Open Document Format, which could allow product innovation.

At date $t=2$, both risk neutral, profit maximising firms simultaneously set prices. Moreover, products are functional for two periods. Consistent with software applications, I assume that the marginal cost of production is zero for all versions. Note that although I assume that the initial market leader has the power to price discriminate in period 1, I analyze both cases where in period 2, the rival has the ability or cannot price discriminate between old and new consumers (see figure 1 for the timing of the firms’ moves).

![Figure 2.1: Timing of the firms’ moves.](image)
2.2.2 Demand

Consumers are identical and have a per period unitary demand. At date t=0, there is a mass $\lambda_0$ of customers in the economy who have previously purchased the product of quality $q_0$, and future generations arrive in constant flows $\lambda_1, \lambda_2$ at dates $t=1,2$, respectively.

At the end of period 1, new customers ($\lambda_1$) observe the dominant firm’s price of the product of quality $q_1$ and $q_0$. Their utility is partially dependent on direct network effects captured by a parameter $\alpha$. Thus, if they purchase the product of quality $q_1$, their utility (gross of the price that will be determined in the next section) is $q_1 + \alpha(\lambda_0 + \lambda_1) - c$ independently of what other customers do, where $\lambda_0 + \lambda_1$ is the market size at $t=1$ and $c$ is these customers’ cost of learning how to use the product.\footnote{Note that the utility function may not be necessarily linear in income (any monotonic transformation would suffice).} If they buy the product of quality $q_0$, their utility is $q_0 + \alpha \lambda_0 x_0 + \alpha \lambda_1 x_1 - c$, where $x_0, x_1$ are the old and new customers’ fractions that own $q_0$, respectively. I analyze both the case that new customers at $t=1$ ($\lambda_1$) cannot postpone their purchase, and where they are willing to wait. Old customers ($\lambda_0$) own the product of quality $q_0$ and observe the market leader’s upgrade price for its product $q_1$. If they buy it, their benefit is $q_1 + \alpha(\lambda_0 + \lambda_1) - c_u$, where $c_u$ is the additional adoption cost they need to incur ($c_u < c$). If they stick to the initial version, their utility is $q_0 + \alpha \lambda_0 x_0 + \alpha \lambda_1 x_1$. A date $t=1$ customer’s overall benefit
depends on his forecast of the prices at t=2, and the way that customers make purchase decisions. These forecasts reflect consumers’ information when they are called to make purchase decisions and are aligned, in equilibrium.

At date t=2, new consumers (\( \lambda_2 \)) observe the prices and decide which product to buy. If they purchase the dominant firm’s product of quality \( q_1 \), their utility is \( q_1 + \alpha \lambda_2 x_2 + \alpha(\lambda_0 + \lambda_1)x_1 - c \). If they buy the product of quality \( q_2 \) and compatibility is present, their utility is \( q_2 + \alpha - c \) while when compatibility is not supported, their benefit from getting the product of quality \( q_2 \) is \( q_2 + \alpha \lambda_2(1 - x_2) + \alpha(\lambda_0 + \lambda_1)(1 - x_1) - c \). These customers’ purchasing decision also resembles a coordination game and following the literature, we assume they behave as a single player.\(^{16}\) Thus, when compatibility is supported, they buy \( q_2 \) if \( \Delta q + \alpha(\lambda_0 + \lambda_1)(1 - x_1) > 0 \), where \( \Delta q = q_2 - q_1 \) is the quality improvement from introducing \( q_2 \). This expression is always positive and implies that new customers (gross of prices) are always better-off if they buy the product of quality \( q_2 \). In contrast, when there is incompatibility in the market, these customers buy the product of quality \( q_2 \) if \( \Delta q + \alpha(\lambda_0 + \lambda_1)(1 - 2x_1) > 0 \). This last expression may take a negative sign if old customers stick to the product \( q_1 \) in the second period.

In the similar old consumers’ coordination problem, while we assume that second period old customers coordinate on the Pareto optimum, old first period customers coordinate on the global Pareto optimum.\(^{17}\) All consumers make their purchasing

\(^{16}\)See Ellison and Fudenberg (2000).
\(^{17}\)See Ellison and Fudenberg (2000).
decisions simultaneously and prefer a new product rather than an older version when they gain the same net expected utility by either of these two choices.

2.3 Moderate expected quality improvements

2.3.1 Market outcome

We start by solving for equilibrium outcomes when compatibility is mandatory followed by an economy that operates under a laissez faire Competition Law. Although we solve for equilibria after the product of quality $q_1$ hits the market, we will not neglect to check the dominant firm’s initial investment decision at date $t=0$. Our benchmark case will consider the scenario that new first period customers cannot postpone their purchase and the rival firm has the power to price discriminate.

2.3.1.1 Mandatory compatibility

Date $t=2$ New customers in the second period ($\lambda_2$) can choose to buy either the rival firm’s product of quality $q_2$ or the market leader’s previous version $q_1$.

Given the price charged by the rival, their utility if they buy the version of quality $q_2$ is $q_2 + \alpha - c - p_{22}$, where the first and second subscripts in the price ($p_{22}$) are related to the quality level of the product purchased and the type of customers.

\footnote{Note that it will become apparent in the first period analysis that the product of quality $q_0$ is not sold in the second period and thus, there is not a third choice of purchasing $q_0$ for the new second period customers.}
buying the good, respectively. If they choose to buy $q_1$, their utility given the price set by the dominant firm ($p_{12}$) is $q_1 + \alpha \lambda_2 x_2 + \alpha (\lambda_0 + \lambda_1) x_1 - c - p_{12}$ where $x_1, x_2$ are the old and new second period customers’ fractions that use $q_1$.\footnote{It will become apparent in the first period analysis that the old second period customers ($\lambda_0 + \lambda_1$) purchased $q_1$ in the previous period.} New customers coordinate, given prices, to what is best for all of them and thus, they will choose to buy $q_2$ if:

\[
\begin{align*}
p_{22} - p_{12} &\leq \Delta q + \alpha (\lambda_0 + \lambda_1)(1 - x_1), \\
\end{align*}
\]

where $\Delta q = q_2 - q_1$ denotes the quality improvement from purchasing the product of quality $q_2$ instead of $q_1$.

Let’s now turn our attention to the old second period customers ($\lambda_0 + \lambda_1$). If they purchase the product of quality $q_2$, their utility given the rival’s price $p_{21}$ is $q_2 + \alpha - c_u - p_{21}$ independently of other customers’ choices. If they stick to $q_1$, their utility will be $q_1 + \alpha \lambda_2 x_2 + \alpha (\lambda_0 + \lambda_1) x_1$, where $x_1, x_2$ are the $\lambda_0 + \lambda_1, \lambda_2$ customers’ fractions that either stick or buy $q_1$ in the second period, respectively. If old consumers make their purchasing decisions independently of what other old customers do, they will purchase $q_2$ even if all other $\lambda_0 + \lambda_1$ stick to $q_1$ when:

\[
\begin{align*}
p_{21} &\leq \Delta q + \alpha \lambda_2 (1 - x_2) - c_u. \\
\end{align*}
\]

The next assumption holds:

**Assumption 1 (A1)** $\Delta q \geq \frac{c_u - \alpha \lambda_2}{\lambda_0 + \lambda_1}$.

This assumption says that the rival firm is better-off by serving the whole
market when she lacks the ability to price discriminate and it allows the linkage between the two periods. It also implies that at date \( t=2 \), old consumers (gross of prices) benefit from purchasing the product of quality \( q_2 \), allowing us to focus on the interplay between the extent of network externalities and the second period quality improvement.

Thus, Bertrand competition leads the second period equilibrium prices to

\[
p_{22} = \Delta q + \alpha(\lambda_0 + \lambda_1), \quad p_{21} = \Delta q + \alpha \lambda_2 - c_u, \quad p_{12} = 0.
\]

Date \( t=1 \) The next assumption holds regarding the development cost:

**Assumption 2 (A2)** \( F < \lambda_2 \Pr \{ \Delta q^e \geq \alpha(\lambda_0 + \lambda_1) \} \mid \Delta q^e = \alpha(\lambda_0 + \lambda_1) \}, \)

where \( \Delta q^e = q_2^e - q_1 \) is the expected quality improvement from the introduction of the product of quality \( q_2^e \) in the second period.

This assumption says that the cost of development does not, per se, deter the rival firm from investing into the new product of expected quality \( q_2^e \), as the probability of successfully producing a product of quality at least above a threshold and the market size are large enough to enjoy positive profits in the second period.

Let’s first think of the maximum price the dominant firm can charge to the new first period customers \( (\lambda_1) \) by selling the product of quality \( q_1 \). If these customers buy the superior version \( q_1 \) and they conjecture that they will coordinate on the Pareto optimum tomorrow, their total net discounted expected utility given the price set by the dominant firm \( (p_{11}) \) if they expect they will purchase
$q^c_2$ in the second period is $q_1 + \delta q^c_2 + \alpha(\lambda_0 + \lambda_1) + \delta \alpha - c - \delta c_u - p_{11} - \delta p^c_{21}$ independently of what other customers do where $p^c_{21}$ is the price they expect to pay in order to buy $q^c_2$ in the second period and $\delta$ is the common discount factor in the economy. Thus, the maximum total expected price they are willing to pay is $p_{11} + \delta p^c_{21} = q_1 + \delta q^c_2 + \alpha(\lambda_0 + \lambda_1) + \delta \alpha - c - \delta c_u$ if they cannot postpone their purchase.\textsuperscript{20} Note that for a sufficiently innovative product of quality $q_1$, the dominant firm’s maximum price to these consumers by selling the product of quality $q_0$ is strictly smaller.

Let’s now turn our attention to the old consumers ($\lambda_0$). By upgrading to $q_1$ and if they forecast that they will also coordinate to the Pareto optimum tomorrow, their total expected discounted utility given the price $p_{10}$ for upgrading is $q_1 + \delta q^c_2 + \alpha(\lambda_0 + \lambda_1) + \delta \alpha - c_u - \delta c_u - p_{10} - \delta p^c_{20}$ independently of what others do, where $p^c_{20}$ is the price they expect to pay in the second period to buy the superior product $q^c_2$ over the alternative of sticking to $q_1$. If they initially choose to stick to $q_0$, they may expect to keep it or either upgrade to $q_1$ or purchase $q^c_2$ in the second period. Their total discounted expected utility if they expect to purchase $q^c_2$ in the second period is $q_0 + \delta q^c_2 + a\lambda_0 x''_0 + \alpha \lambda_1 x''_1 + \delta \alpha - \delta c_u - \delta p^c_{20}$, where $p^c_{20} = \Delta q^c + \alpha \lambda_1(1 - x_1) + \alpha \lambda_2(1 - x_2)$ is the price they expect to pay in order to get $q^c_2$ in the second period if they conjecture that they will coordinate on the Pareto optimum tomorrow and $x''_0, x''_1$ are the $\lambda_0, \lambda_1$ customers’ fractions who

\textsuperscript{20}See the Appendix for the case these customers are willing to postpone their purchase.
stick or buy $q_0$ in the first period, respectively. So, they prefer to buy $q_1$ when they make their purchasing decisions independently of what other old customers do if:

$$p_{10} + \delta p_{20}^e \leq \Delta q + \delta \Delta q^e + \alpha \lambda_1 (1 - x_1^\prime) + \delta \alpha \lambda_2 (1 - x_2) + \delta \alpha \lambda_1 (1 - x_1) - c_u$$

where from the first period perspective, $\Delta q = q_1 - q_0$, $\Delta q^e = q_2^e - q_1$ are the first and second period quality improvements, respectively. The price $p_{10}$ is a decreasing function of the number of the new customers who buy the product of quality $q_0$ in the first period ($x_1^\prime$). Thus, the optimal dominant firm’s choice is to stop selling the product of quality $q_0$ in the first period and the pricing decisions satisfy the expressions:

$$p_{11} + \delta p_{21}^e = q_1 + \delta q_2^e + \alpha (\lambda_0 + \lambda_1) + \delta \alpha - c - \delta c_u,$$  \hspace{1cm} (3)

$$p_{10} + \delta p_{20}^e = \Delta q + \delta \Delta q^e + \alpha \lambda_1 + \delta \alpha \lambda_2 + \delta \alpha \lambda_1 - c_u.$$  \hspace{1cm} (4)

We observe that the total maximum expected payment that new and old customers are willing to pay is fixed. The dominant firm’s optimal choice is to set:

$$p_{11} = q_1 + \delta q_2^e + \alpha (\lambda_0 + \lambda_1) + \delta \alpha - c - \delta c_u - \delta p_{21}^e$$  \hspace{1cm} (3’)

$$p_{10} = \Delta q + \delta \Delta q^e + \alpha \lambda_1 + \delta \alpha \lambda_2 + \delta \alpha \lambda_1 - c_u - \delta p_{20}^e;$$  \hspace{1cm} (4’)

where $p_{21}^e = p_{20}^e = \Delta q^e + \alpha \lambda_2 - c_u$.

\footnote{Check the Appendix for the price these customers expect to pay to buy $q_2^e$.}
The next proposition summarizes the market equilibrium outcome in an economy where compatibility is mandatory and it holds independently of the rival’s ability to price discriminate and new first period customers’ willingness to wait:

**Proposition 8** Under assumptions A1, A2, the dominant firm stops selling the old version of quality \( q_0 \) in the first period. Instead, he sells the product \( q_1 \) to the new and the old first period customers. In the second period, the product of quality \( q_2 \) is sold by the rival to the market.

### 2.3.1.2 Laissez faire Competition Law

We will solve for equilibrium outcomes when the economy operates under a laissez faire Competition Law after discussing the firms’ and customers’ optimal choices when there is incompatibility in the market.

**Date t=2** If there is a product of quality \( q_2 \) in the market and all new second period customers (\( \lambda_2 \)) buy it, their utility given the price charged by the rival firm \( p_{22} \) is \( q_2 + \alpha \lambda_2 + \alpha(\lambda_0 + \lambda_1)(1 - x_1) - c - p_{22} \) and if they all buy \( q_1 \), their utility given the dominant firm’s price \( p_{12} \) is \( q_1 + \alpha \lambda_2 + \alpha(\lambda_1 + \lambda_0)x_1 - p_{12} \), where \( x_1 \) is the old customers’ fraction that sticks to \( q_1 \) in the second period.\(^{22}\) Thus, \( \lambda_2 \) customers buy \( q_2 \) if:

\(^{22}\) Note that the facts that the product of quality \( q_0 \) is not sold in the second period and thus, there is not a third choice of purchasing \( q_0 \) for the new second period customers as well as that old second period customers (\( \lambda_0 + \lambda_1 \)) have purchased \( q_1 \) in the first period will become apparent in the first period analysis.
\[ p_{22} - p_{12} \leq \Delta q + \alpha (\lambda_0 + \lambda_1)(1 - 2x_1). \]  \hspace{1cm} (5)

Let’s turn our attention to the old second period customers \((\lambda_0 + \lambda_1)\). Their utility if they purchase \(q_2\) is \(q_2 + \alpha (\lambda_0 + \lambda_1)(1 - x_1) + \alpha \lambda_2 (1 - x_2) - c_u - p_{21}\) given the rival’s price \(p_{21}\) while if they stick to \(q_1\), their utility is \(q_1 + \alpha \lambda_2 x_2 + \alpha (\lambda_1 + \lambda_0)x_1\), where \(x_1, x_2\) are the old and new customers’ fractions that stick or buy \(q_1\) in the second period. If old customers make their purchase decision independently of what other old customers do, they will choose to buy \(q_2\) even if all other old customers stick or buy \(q_1\) \((x_1 = 1)\) when:

\[ p_{21} \leq \Delta q + \alpha \lambda_2 (1 - 2x_2) - \alpha (\lambda_0 + \lambda_1) - c_u. \]  \hspace{1cm} (6)

We make the following assumption with respect to the quality improvement from the introduction of the product of quality \(q_2\):

**Assumption 3 (A3)** (a) \(\Delta q + \alpha \lambda_2 (1 - 2x_2) - \alpha (\lambda_0 + \lambda_1) - c_u < 0, \ 0 \leq x_2 \leq 1,\)

(b) \(q_2 - q_1 \leq q_1 - q_0.\)

The first part of the assumption (a) says that when compatibility is not supported, first period customers expect not to buy the rival’s product of anticipated quality \(q_2^*\) and thus, we restrict attention to moderately high values of quality improvements relative to network effects. The second part of the assumption (b) says that a given investment leads to a smaller quality improvement in the second rather than in the first period when the product of quality \(q_1\) is an important
innovation. In the next section, we will relax assumption 3 when we consider a sufficiently innovative future product compared to the extent of network effects.

**Case 1:** $\Delta q < \alpha(\lambda_0 + \lambda_1)$.

In this scenario, the product quality $q_2$ does not allow the rival to generate any revenue in the second period. Instead, new second period customers purchase the product of quality $q_1$, as price competition leads to

$$p_{12} = q_1 - q_2 + \alpha(\lambda_0 + \lambda_1) - \varepsilon,$$

(7)

for a small positive number $\varepsilon$.

Date $t=1$  
If new customers ($\lambda_1$) buy the good of quality $q_1$, their total expected discounted utility given the price charged by the dominant firm $p_{11}$ and their forecast of coordinating on the Pareto optimum tomorrow is $q_1 + \delta q_1 + \alpha(\lambda_0 + \lambda_1) + \delta \alpha - c - p_{11}$.\(^{23}\)

Let’s now turn our attention to the old consumers in the first period ($\lambda_0$). If they buy the product of quality $q_1$, their total expected discounted utility given the upgrade price charged by the dominant firm ($p_{10}$) is $q_1 + \delta q_1 + \alpha(\lambda_0 + \lambda_1) + \delta \alpha - c_u - p_{10}$, independently of what other customers do. If they stick to $q_0$, their expected utility by upgrading in the second period to $q_1$ is $q_0 + \delta q_1 + \alpha \lambda_0 x_0 + \alpha \lambda_1 x_1 + \delta \alpha - \delta c_u - \delta p_{10}$, where $p_{10}^{e} = \Delta q + \alpha \lambda_1 (1 - x_1^{'}) - \alpha \lambda_0 + \alpha \lambda_2 (1 - x_2^{'}) - c_u - \epsilon$.

\(^{23}\)Similarly with the case when compatibility is present, the dominant firm’s optimal price to these customers by selling $q_0$ is strictly smaller for a sufficiently innovative product of quality $q_1$. 

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is the price they expect to pay tomorrow in order to upgrade if they forecast they will coordinate on the Pareto optimum tomorrow. If old customers make their purchasing decisions independently of what other old customers do, they upgrade to $q_1$ in the first period even if all other old $\lambda_0$ consumers stick to $q_0$ if:

$$p_{10} \leq \Delta q + \alpha \lambda_1 (1 - x_1) - c_u + \delta c_u + \delta p_{10}^e.$$ 

Notice that since the dominant firm’s profits are a decreasing function of the number of $\lambda_1$ customers that buy $q_0$ in the first period ($x_1$), its optimal choice is to stop selling the initial version in the first period (and $x_1 = 0$ in the inequality above). Thus, the first period prices are given by the expressions:

$$p_{11} = q_1 + \delta q_1 + \alpha (\lambda_0 + \lambda_1) + \delta \alpha - c,$$  \hspace{1cm} (8)

$$p_{10} = \Delta q + \alpha \lambda_1 - c_u + \delta c_u + \delta p_{10}^e,$$ \hspace{1cm} (9)

where

$$p_{10}^e = \Delta q + \alpha \lambda_1 + \alpha \lambda_2 - \alpha \lambda_0 - c_u - \varepsilon.$$ \hspace{1cm} (10)

**Case 2:** $\Delta q > \alpha (\lambda_0 + \lambda_1)$.

In this scenario, the prices at date $t=2$ are given by the expressions:

$$p_{22} = \Delta q - \alpha (\lambda_0 + \lambda_1), \ p_{12} = 0.$$ \hspace{1cm} (11)

Date $t=1$  Let’s first think of the new customers in the first period ($\lambda_1$). If they buy the product of quality $q_1$, their total expected discounted utility given the

\footnote{See the Appendix for the determination of this price.}
dominant firm’s price $p_{11}$ is $q_1 + \delta q_1 + \alpha (\lambda_0 + \lambda_1) + \delta \alpha \lambda_1 + \delta \alpha \lambda_0 - c - p_{11}$ if they conjecture that they will coordinate on the Pareto optimum tomorrow.\(^{25}\)

Let’s now turn our attention to the old customers in the first period ($\lambda_0$). If they upgrade to $q_1$, given the dominant firm’s upgrade price $p_{10}$, their total expected discounted utility is $q_1 + \delta q_1 + \alpha (\lambda_0 + \lambda_1) + \delta \alpha \lambda_0 + \delta \alpha \lambda_1 - c_u - p_{10}$ if they forecast that they will coordinate on the Pareto optimum tomorrow. If they keep $q_0$, their total discounted expected utility if they expect to purchase $q_2$ is $q_0 + \delta q_0 + \alpha \lambda_0 x_0^\prime + \alpha \lambda_1 x_1^\prime + \delta \alpha \lambda_1 x_2 + \delta \alpha \lambda_2 x_1 + \delta \alpha \lambda_0 x_0 - \delta c_u - \delta p_{20}^\prime$ where $p_{20}^\prime = \Delta q^e - \alpha \lambda_0 + \alpha \lambda_1 (2x_1 - 1) + \alpha \lambda_2 (2x_2 - 1)$ is the price they expect to pay in order to purchase $q_2^e$ tomorrow if they conjecture they will also coordinate to the Pareto optimum in the following period and $x_0^\prime$, $x_1^\prime$ are the old and the new first period customers’ fractions that stick or buy $q_0$ today, respectively while $x_0$, $x_1$, $x_2$ are the old and new second period customers that are expected to purchase $q_2$ tomorrow.\(^{26}\)

Old first period consumers buy $q_1$ in the first period even if all other customers of the same class choose $q_0$ ($x_0^\prime = 1$) if:

$$p_{10} \leq \Delta q + \alpha \lambda_1 (1 - x_1^\prime) - c_u (1 - \delta) - \delta \alpha \lambda_0 + \delta \alpha \lambda_1 x_1 - \delta \alpha \lambda_2 (1 - x_2).$$

Since $p_{10}$ is a decreasing function of the number of $\lambda_1$ customers who buy $q_0$ in the first period ($x_1^\prime$), the dominant firm’s optimal choice is to stop selling $q_0$ in the first period (thus, $x_1^\prime = 0$ in the inequality above) and the first period price

\(^{25}\)For a sufficiently innovative product of quality $q_1$, the maximum price the dominant firm could charge these customers for $q_0$ is strictly smaller.

\(^{26}\)See the Appendix for the calculation of this expected price.
is:
\[ p_{10} = \Delta q + \alpha \lambda_1 - c_u + \delta c_u - \delta \alpha \lambda_0. \] (12)

Note that this price is strictly smaller than the optimal price that the dominant firm would set when there is compatibility in the market. The optimal dominant firm’s price to \( \lambda_1 \) customers if they cannot postpone their purchase is:
\[ p_{11} = q_1 + \delta q_1 + \alpha(\lambda_0 + \lambda_1) + \delta \alpha(\lambda_0 + \lambda_1) - c. \] (13)

Depending on the quality improvement expected to be introduced by the competitor (\( \Delta q^c \)) and for different values of the investment (\( F \)), we identify the following (ex-post) scenarios:

**S1** \( \Delta q^c - (\lambda_0 + \lambda_1)c_u < F < (\lambda_0 + \lambda_1)(\Delta q^c + \alpha \lambda_2 - c_u) \). This scenario implies that the expected quality improvement in the second period is relatively small relative to network externalities (\( \Delta q^c < \alpha(\lambda_0 + \lambda_1) \)).

**S2** \( F \leq \Delta q^c - (\lambda_0 + \lambda_1)c_u, \Delta q^c \geq \alpha(\lambda_0 + \lambda_1) \). This scenario occurs when the quality differential anticipated to be introduced by the competitor is relatively large compared to the extent of network effects.

**S3** \( F \leq \Delta q^c - (\lambda_0 + \lambda_1)c_u, \Delta q^c < \alpha(\lambda_0 + \lambda_1) \). This scenario necessarily implies that the network parameter is greater than the upgrading cost (\( \alpha \geq c_u \)).

In a laissez faire economy, the dominant firm compares its expected profit under the two regimes and decides whether to support compatibility or not. The

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\(^{27}\)See the Appendix for the price these customers are willing to pay if they can postpone their purchasing decision.
next proposition summarizes the equilibrium outcome in the economy that operates under a laissez faire Competition Law. When old customers coordinate on the Pareto optimum, the proposition holds when the rival can price discriminate independently of whether new first period customers cannot postpone their purchasing decision or are willing to wait and also holds when the rival lacks the ability to price discriminate and first period customers can postpone their purchase. 

**Proposition 9** (a) If A1-A3 and S1 or S3 hold, the dominant firm does not support compatibility and all customers purchase the product of quality $q_1$ in both periods. (b) If A1-A3 and S2 hold, the dominant firm supports compatibility and consumers in the second period buy the rival’s product of quality $q_2$.

The dominant firm’s optimal choice is to refuse to offer compatibility for a relatively small quality improvement (a). On the other hand, if the quality differential by the competitor is large relative to the network externalities (b), the dominant firm’s optimal strategy is to offer interoperability to its competitor because it can absorb in the first period more of the expected discounted future total surplus which is higher when compatibility is present.

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28If these customers coordinate on what all the other members of their class prefer, the dominant firm would be indifferent between supporting or impeding compatibility.

29See the Appendix for the dominant firm’s compatibility and price choices when consumers can postpone their purchase and the rival cannot price discriminate.
2.3.2 Social Optimum

It is important to analyze the social efficiency of the results obtained previously and this subsection considers the problem faced by a planner that maximizes social surplus after the important version of quality $q_1$ is produced. He chooses whether using the product of quality $q_1$ for two periods or introducing the new product $q_2$ is socially optimal. Although consumers’ welfare is also of first order of importance, social surplus is a fuel for economic growth and will be our initial welfare measure.

If the product of quality $q_1$ is used in both periods, social welfare is:

$$W_N = \lambda_0[q_1+\delta q_1+\alpha(\lambda_0+\lambda_1)+\delta \alpha-c_u]+\lambda_1[q_1+\delta q_1+\alpha(\lambda_0+\lambda_1)+\delta \alpha-c]+\delta \lambda_2(q_1+\alpha-c).$$

If the superior product of quality $q_2^c$ is sold to everyone\footnote{Because of assumption 1, the third potential case of having incompatible products is not optimal.}, social welfare becomes:

$$W_U = \lambda_0[q_1+\delta q_2^c+\alpha(\lambda_0+\lambda_1)+\delta \alpha-c_u-\delta c_u]+\lambda_1[q_1+\delta q_2^c+\alpha(\lambda_0+\lambda_1)+\delta \alpha-c-\delta c_u]+$$

$$+\delta \lambda_2(q_2^c+\alpha-c) - F,$$

where in the second period, all customers join a network of maximum size. Comparing the expressions above yields the socially optimal outcome:

**Proposition 10** It is socially efficient if (a) the product of quality $q_1$ is sold for two periods when $A1$ and $S1$ hold, (b) the product of quality $q_2^c$ is introduced and purchased by the whole market if $A1$ and $S2$ hold.
It is socially efficient if the good of quality \( q_1 \) is sold for both periods when the benefit from everyone purchasing it is smaller than the total investment and the cost of learning how to use the new product \( (\Delta q^e < F + c_u(\lambda_0 + \lambda_1)). \) When the last inequality is reversed, social optimality is achieved when the superior product is introduced in the second period and is purchased by both the new and the old consumers.

Depending on the industry characteristics and the expected quality improvement, the market outcome may lead to socially undesirable results. More precisely, the next proposition highlights the potential inefficiency that may arise in markets that operate under a laissez faire Competition Law towards Intellectual Property Rights or under mandatory compatibility:

**Proposition 11** (a) If \( S_1 \) holds, an economy that mandates interoperability leads to the inefficient introduction of the product of quality \( q_2 \).
(b) There is no inefficiency in the laissez faire market if the network parameter is smaller than the cost of upgrading \((\alpha < c_u)\).
(c) If network effects are strong \((\alpha \geq c_u)\), the market that operates under a laissez faire Competition Law may lead to an inefficient technological slowdown when \( S_3 \) holds.

Independently of the extent of network externalities relative to the adoption cost, an economy that mandates compatibility may lead to the inefficient introduction of the product of quality \( q_2 \) (\( S_1 \)). A laissez faire market leads to social efficiency when network effects are relatively weak \((\alpha < c_u)\) and consumer wel-
fare is always maximised when compatibility is not supported due to lower prices set to new second period customers. These results are important contributing in the discussion among academics and policy makers regarding the desirability of a more interventionist Competition Law, as they indicate that mandated compatibility by Competition Authorities decreases consumer welfare and may also lead to inefficiency. On the other hand, a laissez faire market may lead to inefficiency for relatively strong network effects \( \alpha > c_u \) and for small values of the cost of development \( c \). In particular, it may be socially efficient to introduce the product of quality \( q_2^e \) in the second period and nevertheless, a laissez faire market leads to technological slowdown withholding the product of quality \( q_2^e \) from the economy.

2.4 Sufficiently innovative quality improvements

In this section, we relax the assumption that old second period customers only buy the rival’s product when compatibility is present and after we solve for equilibrium outcomes in an economy that operates under a laissez faire Competition Law, we will investigate whether an economy that mandates compatibility or the one that allows firms to choose is preferable. Our benchmark case considers that the rival cannot price discriminate between the different customers’ classes and the new first period customers are not willing to postpone their purchase.
2.4.1 Market outcome

When compatibility is mandatory, the rival firm’s possible choices are either to serve the whole market or only the new second period consumers. For quality improvement satisfying (A1), her optimal choice is to serve the whole market by setting an equilibrium price \( p_{21} = \Delta q + \alpha \lambda_2 - c_u \). In the first period, the market leader extracts customers’ expected total surplus by setting a price to the new and the old first period comers (from (3) and (4)):

\[
p_{11} + \delta p_{21} = q_1 + \delta q_2 + \alpha (\lambda_0 + \lambda_1) + \delta \alpha - c - \delta c_u,
\]

\[
p_{10} + \delta p_{20} = \Delta q + \delta \Delta q^e + \alpha \lambda_1 + \delta \alpha \lambda_2 + \delta \alpha \lambda_1 - c_u,
\]

respectively, where \( p_{21}^e = p_{20}^e = \Delta q^e + \alpha \lambda_2 - c_u \) is the rival’s expected second period price.

When compatibility is not supported, in the second period, there are again two candidate prices for the rival: she can either charge \( p_{21} = \Delta q + \alpha \lambda_2 - \alpha (\lambda_0 + \lambda_1) - c_u \) and serve the whole market or \( p_{22} = \Delta q - \alpha (\lambda_0 + \lambda_1) \) and only serve the new comers. When network effects are strong \( (\alpha \lambda_2 > c_u) \), her optimal choice is to choose \( p_{21} \) while for relatively weaker network externalities, she will decide to offer the new product to everyone when the quality improvement from its introduction is sufficiently large, satisfying the following inequality:

\[
\Delta q - \alpha (\lambda_0 + \lambda_1) + \frac{1}{\lambda_0 + \lambda_1} (\alpha \lambda_2 - c_u) > 0.
\]
In this case, the first period prices to the new and the old first period consumers are calculated using the equations:

\[
p_{11} + \delta p_{21}^e = q_1 + \delta q_2^e + \alpha(\lambda_0 + \lambda_1) + \delta \alpha - c - \delta c_u,
\]

\[
p_{10} + \delta p_{21}^e = \Delta q + \delta \Delta q^e + \alpha \lambda_1 + \delta \alpha \lambda_2 + \delta \alpha \lambda_1 - c_u - \delta a \lambda_0,
\]

where \( p_{21}^e = \Delta q^e + \alpha \lambda_2 - \alpha(\lambda_0 + \lambda_1) - c_u \).

In a laissez faire Competition Law, the dominant firm compares its expected profit by supplying or not allowing interoperability to the rival. The next proposition summarizes the equilibrium outcome independently of the rival’s ability to price discriminate and new customers’ (un)willingness to delay their purchasing decision:

**Proposition 12** *In equilibrium, the dominant firm never supports compatibility with the smaller firm in the second period. Consumers purchase her product of quality \( q_2 \) in the second period.*

### 2.4.2 Consumers’ Welfare maximization

We will focus on consumers’ welfare because the economies that operate under mandatory compatibility or a laissez faire Competition Law are equivalent with respect to the first welfare measure and may also lead to inefficiency. Thus, in this case our welfare measure will be consumers’ surplus and the next proposition summarizes the comparison between the economy that mandates compatibility
and the one that operates under a laissez faire Competition Law when the rival cannot price discriminate.\footnote{If she can price discriminate, consumers’ surplus is equivalent under compatibility and a laissez faire economy.}

**Proposition 13** Consumers’ Welfare is maximised under a laissez faire Competition Law. In particular, if $S1$ holds and $A3$ is relaxed, both a laissez faire market and an economy that mandates compatibility lead to the inefficient introduction of the product of quality $q_2$ while consumers are better-off in a laissez faire economy.

For sufficiently innovative second period products ($A3$ does not hold), the dominant firm does not support compatibility with his rival. This fact leads to a higher degree of competition in the second period and to lower prices, benefiting the new second period consumers.

### 2.5 Applications/ Discussion/ Future Research

This chapter analyses firms’ behaviour towards compatibility and the relation of these decisions with their incentives to invest in improving their durable network goods. By using a sequential game, we give a smaller rival the ability to build on innovations previously introduced by the market leader. Recognizing the intertemporal linkage in forward-looking customers’ purchasing choices, we find that in anticipation of a moderately large quality improvement by the rival, strategic pricing leads the dominant firm to support compatibility even if it could exclude
its rival from using its network. On the other hand, the market leader does not support compatibility when the new products are sufficiently innovative.

Regarding welfare, an economy that mandates compatibility may lead to the inefficient introduction of a relatively less innovative product. We also find that when network effects are weak, a laissez faire market converges to social efficiency while when network effects are strong, the refusal to supply interoperability information may lead to the inefficient slowdown of technological progress. For sufficiently large expected quality improvements, a laissez faire Competition Law leads to incompatibility and fierce competitive forces benefit consumers through lower prices.

An important application captured by the model comes from the 2008 European Commission case against Microsoft regarding its office suite highlighted earlier in the introduction. Although Microsoft’s compliance to compatibility was enforced by the European Commission, this mandate in favour of interoperability may have been harmful for society. In particular, Microsoft Office 2007 was followed by Corel’s WordPerfect Office suite in 2008 that introduced negligible quality improvements. In anticipation of this, Microsoft decided not to support compatibility in the first place. This denial to support compatibility would have a twofold effect: First, it would lead to an increase in consumer welfare due to a higher degree of competition between incompatible networks. Second, as proposition 4 shows, a market operating under a laissez faire competition policy towards
intellectual property rights would lead to social efficiency, assuming that network effects are weak relative to the cost of learning the new product. Moreover, mandating compatibility would lead to inefficiency, as society would have been better-off without the new product (4a).

The policy implication of these findings is that competition Authorities should investigate whether mandating compatibility may sometimes be socially unwelcome without necessarily benefiting consumers or even harming them. Instead, markets that allow unilateral refusals to supply interoperability information may possibly lead to efficient outcomes and even improve consumers’ welfare. In an economy where network effects are present, this exercise is not trivial but if network effects are not too strong and quality improvements are moderate, an economy operating under a laissez faire Competition Law generates social efficiency and maximises consumer welfare. For sufficiently innovative new products, a laissez faire economy leads to incompatibility and maximises consumer surplus through lower prices when the rival lacks the ability to price discriminate between the different consumers’ classes.

Nevertheless, there are a number of issues that are important and are not addressed in this paper. Firstly, a model that will test empirically our results could validate our predictions. It would also be interesting to study the competitors’ interoperability/investment decisions in the presence of stochastic demand.
2.6 Appendix

2.6.1 New first period customers ($\lambda_1$) can postpone their purchase/

The rival can price discriminate

2.6.1.1 Compatibility

Given the first period price, $p_{11}$, new first period customers’ total expected utility if they buy $q_1$ is $q_1 + \delta q_2^e + \alpha(\lambda_0 + \lambda_1) + \delta \alpha - c - \delta c_u - p_{11} - \delta p_{21}^e$. If they all postpone their purchase, they would belong to a network of size $\lambda_1 + \lambda_2$ (new second period customers) and the rival’s expected second period price can be computed by the equality:

$$q_2^e + \alpha - c - p_{22}^e = q_1 + \alpha(\lambda_1 + \lambda_2) - c - p_{12}^e,$$

or equivalently $p_{22}^e = \Delta q^e + \alpha \lambda_0$. Thus, their outside opportunity if they wait in the first period is $\delta(q_2^e + a - c - p_{22}^e) = \delta[q_1 + \alpha(\lambda_1 + \lambda_2) - c]$ and the maximum expected total price they are willing to pay to buy the product of quality $q_1$ or equivalently the dominant firm’s maximum first period price is:

$$p_{11} + \delta p_{21}^e = q_1 + \delta \Delta q^e + \alpha(\lambda_0 + \lambda_1) + \delta \alpha \lambda_0 - c(1 - \delta) - \delta c_u.$$

2.6.1.2 Incompatibility

We focus on the case that $\Delta q^e \geq \alpha(\lambda_0 + \lambda_1)$.

New first period customers’ total expected utility if they purchase $q_1$ is $q_1 + \ldots$
\( \delta q_1 + \alpha(\lambda_0 + \lambda_1) + \delta \alpha(\lambda_0 + \lambda_1) - c - p_{11}. \) If they postpone their purchase, they will subsequently belong to a network of size \( \lambda_1 + \lambda_2 \) and their expected discounted utility is \( \delta(q_2^e + \alpha(\lambda_1 + \lambda_2) - c - p_{22}^{''}) \), where \( p_{22}^{''} \) is the expected second period price that can be computed in the following equation:

\[
q_2^e + \alpha(\lambda_2 + \lambda_1) - c - p_{22}^{''} = q_1 + \alpha - c - p_{12},
\]

or equivalently \( p_{22}^{''} = \Delta q_e - \alpha \lambda_0 \). Their net expected utility if they wait to make their purchasing decision tomorrow is \( \delta(q_1 + a - c) \). Thus, the maximum price these customers are willing to pay today to buy \( q_1 \) is \( p_{11} = q_1 + \alpha(\lambda_0 + \lambda_1) - \delta \alpha \lambda_2 - c(1-\delta) \).

If \( \Delta q_e < \alpha(\lambda_0 + \lambda_1) \), the dominant firm’s optimal choice is to impede compatibility. In particular, the dominant firm compares his expected profit and he does not support compatibility if the following inequality is satisfied:

\[
\lambda_1 \alpha \lambda_2 + \lambda_2(q_1 + \alpha - c) + \lambda_0(\Delta q + \alpha \lambda_2 - c_u) > 0,
\]

which is always true since \( \Delta q > \Delta q_e \).

2.6.2 Old first period customers’ expected second period prices af-
ter sticking to $q_0$ in the first period.

2.6.2.1 Compatibility

Old customers expect to buy $q_2^e$ when all customers of their class either buy $q_1$ or stick to $q_0$ if:

$$q_2^e + a - c_u - p_{20}^e \geq \max\{q_0 + \alpha \lambda_0 + \alpha \lambda_1 x_1'' + \alpha \lambda_2 x_2'', q_1 + \alpha \lambda_0 + \alpha \lambda_1 x_1 + \alpha \lambda_2 x_2 - c_u - p_{10}^e\},$$

or equivalently: $p_{20}^e = \Delta q^e + \alpha \lambda_1 (1 - x_1) + \alpha \lambda_2 (1 - x_2), p_{10}^e = 0.$

2.6.2.2 Incompatibility/ $\Delta q^e < \alpha(\lambda_0 + \lambda_1)$

If $\lambda_0$ customers stick to $q_0$ in the first period, there are some possibilities in the following period: if they keep $q_0$, their second period utility is $q_0 + \alpha \lambda_0 x'_0 + \alpha \lambda_1 x'_1 + \alpha \lambda_2 x'_2$ while if they buy $q_1$, their second period utility will be $q_1 + \alpha - c_u - p_{10}.$

Thus, they will buy the higher quality product in the second period when they make their purchasing decisions independently of what other $\lambda_0$ customers do if:

$$p_{10} \leq \Delta q + \alpha \lambda_1 (1 - x'_1) + \alpha \lambda_2 (1 - x'_2) - c_u.$$

Thus, the price expected to be set by the dominant firm in the second period is $p_{10}^e = \Delta q + \alpha \lambda_1 (1 - x'_1) + \alpha \lambda_2 (1 - x'_2) - c_u.$
2.6.2.3 Incompatibility/ \( \Delta q^e \geq \alpha(\lambda_0 + \lambda_1) \)

\( \lambda_0 \) customers expect to pay a price \( p_{20}^{e'} \) to purchase \( q_2 \) in the future period even if all other customers of their class either buy \( q_1 \) or stick to \( q_0 \) for both periods if we solve the following equation:

\[
q_2 + \alpha \lambda_2 x_2 + \alpha \lambda_1 x_1 - c_u - p_{20}^{e'} = \\
\max\{q_1 + \alpha \lambda_0 + \alpha \lambda_2 (1 - x_2) + \alpha \lambda_1 (1 - x_1) - c_u - p_{10}^{e'},
q_0 + \alpha \lambda_0 + \alpha \lambda_1 x_1' + \alpha \lambda_2 x_2'\},
\]

or equivalently:

\[
p_{20}^{e'} = \Delta q^e + \alpha \lambda_2 (2x_2 - 1) + \alpha \lambda_1 (2x_1 - 1) - \alpha \lambda_0
\]

while their utility if they stick to \( q_0 \) for both periods is strictly dominated by purchasing \( q_1 \) tomorrow (\( q_1 \) is a sufficiently innovative product).

2.6.3 Market outcome/ The rival cannot price discriminate/ Consumers can postpone their purchase

2.6.3.1 Compatibility

Think of new consumers in the first period (\( \lambda_1 \)): their total expected utility if they purchase the product of quality \( q_1 \) is \( q_1 + \delta q^e_2 + \alpha(\lambda_0 + \lambda_1) + \delta \alpha - c - \delta c_u - p_{11} - \delta p_{21}^{e} \)

where \( p_{21}^{e} = \Delta q^e + \alpha \lambda_2 - c_u \) (for moderately large expected quality improvements, she is better-off by choosing to serve all the customers instead of only the new

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second period potential consumers). If they all postpone their purchasing decision, their total discounted expected utility is $\delta(q_2^e + \alpha - c - p_{21})$, where $p_{21}$ is the expected rival’s second period price. This price can be computed using the equation:

$$q_2^e + \alpha - c - p_{21} = q_1 + \alpha(\lambda_1 + \lambda_2) - c - p_{11}^e,$$

or equivalently $p_{21} = \Delta q^e + \alpha \lambda_0$. Thus, the maximum first period price $p_{11}$ that these customers are willing to pay to purchase the product of quality $q_1$ is such that:

$$q_1 + \delta q_2^e + \alpha(\lambda_0 + \lambda_1) + \delta \alpha - c - \delta c_u - p_{11} - \delta p_{21} = \delta(q_2^e + \alpha - c - p_{21}^e)$$

or equivalently $p_{11} = q_1 + \alpha(\lambda_0 + \lambda_1) + \delta \alpha \lambda_0 - \delta \alpha \lambda_2 - c(1 - \delta)$. Regarding the old first period customers ($\lambda_0$), they will buy $q_1$ if:

$$q_1 + \delta q_2^e + \alpha(\lambda_0 + \lambda_1) + \delta \alpha - c - \delta c_u - p_{10} - \delta p_{21}^e \geq q_0 + \alpha \lambda_0 + \delta \alpha + \delta q_2^e - \delta c_u - \delta p_{21},$$

where $p_{21}^e = \Delta q^e + \alpha \lambda_2 - c_u$ and $p_{21}^e$ is computed using the equation:

$$q_2^e + \alpha - c_u - p_{21}^e = q_1 + \alpha \lambda_0 - c_u - p_{11}^e$$

or equivalently $p_{21}^e = \Delta q^e + \alpha(\lambda_1 + \lambda_2)$. Thus, the maximum first period price that induces old customers to buy $q_1$ is $p_{10} = \Delta q + \alpha \lambda_1 + \delta \alpha \lambda_1 - c_u + \delta c_u$.

2.6.3.2 Incompatibility/ A3 holds

We start with the scenario that $\Delta q^e \geq \alpha(\lambda_0 + \lambda_1)$. 

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New customers’ total expected utility if they purchase the product of quality $q_1$ is $q_1 + \delta q_1 + \alpha(\lambda_0 + \lambda_1) + \delta \alpha(\lambda_0 + \lambda_1) - c - p_{11}$ while if they all wait, their outside opportunity is $\delta[q_2^e + \alpha(\lambda_1 + \lambda_2) - c - p_{21}']$, where $p_{21}'$ is the competitor’s expected second period price choice and is computed if we use the equation:

$$q_2^e + \alpha(\lambda_1 + \lambda_2) - c - p_{21}' = q_1 + \alpha - c - p_{11}$$

or equivalently: $p_{21}' = \Delta q^e - \alpha \lambda_0$. Thus, the dominant firm’s optimal first period choice is $p_{11} = q_1 + \alpha(\lambda_0 + \lambda_1) - \delta \alpha \lambda_2 - c(1 - \delta)$.

Old customers will purchase $q_1$ and will not stick to $q_0$ if:

$$q_1 + \delta q_1 + \alpha(\lambda_0 + \lambda_1) + \delta \alpha(\lambda_0 + \lambda_1) - c_u - p_{10} \geq q_0 + \delta q_2^e + \alpha \lambda_0 + \delta \alpha \lambda_2 - \delta c_u - \delta p_{20}'$$

where the expected price $p_{21}'$ is computed if we use the equation

$$q_2^e + \alpha \lambda_2 - c_u - p_{21}' = q_1 + \alpha(\lambda_0 + \lambda_1) - c_u - p_{10},$$

or equivalently $p_{20}' = \Delta q^e + \alpha \lambda_2 - \alpha(\lambda_0 + \lambda_1)$. Thus, the dominant firm’s first period equilibrium price choice, $p_{10} = \Delta q + \alpha \lambda_1 - c_u(1 - \delta)$.

If $\Delta q^e < \alpha(\lambda_0 + \lambda_1)$, the dominant firm impedes compatibility as the following inequality holds:

$$\delta \lambda_1[q_1 + \alpha(\lambda_1 + \lambda_2) - c] + \lambda_0[c - c_u + \delta(\Delta q + \alpha \lambda_2 - c_u)] + \delta \lambda_2(q_1 + \alpha - c) > 0$$

which always holds when $\Delta q \geq \Delta q^e$.

Proposition 9 summarizes the dominant firm’s optimal strategy and the market equilibrium outcome.
Note that if A3 does not hold, the dominant firm is indifferent between supporting and impeding compatibility.
CHAPTER 3

INCENTIVES TO INNOVATE, COMPATIBILITY AND EFFICIENCY IN DURABLE GOODS MARKETS WITH NETWORK EFFECTS

3.1 Introduction

The discussion in the previous chapter is now expanded to include situations where both an initially dominant and a smaller rival may innovate and innovations no longer occur with certainty but they follow a discrete time stochastic process. More precisely, we aim to provide some answers to the following questions: Why do dominant firms decide to supply interoperability information of their durable network products? Even if they refuse to support compatibility, does this necessarily imply that their R&D incentives are curbed?\(^1\) Could smaller rivals be better-off if they did not support compatibility with the current market leader? Which economy offers the socially preferable balance of aggregate R&D incentives: one that operates under mandatory compatibility or under a laissez faire

\(^1\) We will use the terms interoperability and compatibility interchangeably throughout the paper.
competition law? These questions are certainly not new but this is the first paper to examine them in an environment where technological progress is modelled in a scenario with sequential innovations of durable network products.

Although standard economic theory predicts that dominant firms may refuse to reveal interoperability information to smaller rivals\textsuperscript{2}, there are many cases in technology markets where firms with leading market shares welcome compatibility even from direct competitors.\textsuperscript{3} In the absence of network effects, a potential explanation of dominant firms’ support to competition and imitation is sequential, important innovation: the initial inventor allows imitation instead of getting a patent as the (exogenous) probability of future inventions increases, allowing him to enjoy a higher expected payoff which outweighs the loss from a lower current profit.\textsuperscript{4}

In this chapter, we provide an explanation of dominant firms’ support to compatibility: we endogenize firms’ probability of successful innovation by studying the competitors’ R&D incentives as well as their compatibility choices in the presence of durable, network goods and we show that sequential important innovation may lead the dominant firm to voluntarily support compatibility even if it may compete directly with its rival in the future. In this case, dominant firms invest less if the intellectual property rights system is very strong. In particular, we

\textsuperscript{2}See Chen, Doraszelski and Harrington (2009).
\textsuperscript{3}See Viecens (2009) and Bessen and Maskin (2009) for examples concerning dominant firms’ welcoming compatibility and competition.
\textsuperscript{4}See Bessen and Maskin (2009).
consider a model where substitutable, sequential innovations result from a discrete time R&D stochastic process and technological progress is modelled with exogenous quality improvements.\textsuperscript{5} We give both an initially dominant and a rival firm the ability to come up with valuable ideas following a commonly observed scenario of creative destruction in the Hi-tech and software industry where smaller innovative rivals often displace initial market leaders.\textsuperscript{6} We find that for important innovative products, the leader invests more when compatibility is present and in fact, he voluntarily chooses to offer interoperability information to the rival, who in turn accepts it only when the current market size is relatively large. For less innovative products, we find that when network effects are larger than a cutoff, a laissez faire competition law with respect to intellectual property holders leads the dominant firm to reject compatibility and also invest less than in an economy where compatibility is mandatory. Our welfare analysis, based on a more "economically sound" comparison of the market outcome with the socially optimal level of investment, shows that a laissez faire competition law is socially preferable compared to an economy operating under mandatory compatibility, especially when network effects are relatively weak. These results cast doubts as to whether mandated compatibility by Competition Authorities may lead to socially undesirable results.

This chapter is organized as follows: the next subsection discusses the related

\textsuperscript{5}Further work will endogenize the quality improvements.

\textsuperscript{6}For example, Microsoft Excel replaced Lotus 1-2-3 and Microsoft Word replaced WordPerfect.
Section 2 presents the Model. In Section 3, we solve for equilibrium outcomes when compatibility is either mandatory and under a laissez faire competition law. Section 4 provides the socially efficient investment level that a social planner would induce and a comparison with the market equilibrium investment under the economies that operate under mandatory compatibility or under a laissez faire competition law. Section 5 concludes.

3.1.1 Related Literature

This chapter relates first to the literature regarding firms’ attitude towards compatibility. In an economy where network effects exist and product quality is constant, although compatibility increases the number of potential buyers because of a larger network, the market leader prefers not to support it because otherwise, he would lose the advantage of the larger installed base.\(^7\) When sequential innovation occurs with certainty and products are substitutable, Athanasopoulos (2014) showed that a dominant market player offers interoperability information of his durable products to a smaller innovative rival when he expects a moderately large, future quality improvement from his competitor. Thus, strategic pricing allows the market leader to extract more of the higher expected total surplus when he supports compatibility. An important assumption in this model is that the rival is the only firm that can innovate in the future. Our work differs because

\(^7\)See the previous chapter for papers in this literature.
innovation no longer happens with certainty and both competitors are potential future inventors. For sufficiently innovative future products, we also find that the dominant firm voluntarily supports compatibility.

When network effects are not present and innovations are sequential and complementary, Bessen and Maskin (2009) showed that the initial innovator may welcome imitation because it allows both competitors to invest, increasing the exogenous probability of successful innovation and thus his second period profit, outweighing the loss from the foregone first period revenues. We depart from their work in a number of ways: first, we assume that direct network effects as well as product durability are present. Second, we assume that there is an alternative process that allows for product innovation even if there is incompatibility in the market. Third, unlike their paper where the probability of successful innovation is a parameter, we adopt a game theoretical approach where firms’ R&D cost is a function of the probability of success. We also consider forward looking customers and their role in determining the market equilibrium outcome and the social optimum. We agree with the message of their paper: dominant firms welcome compatibility when future products are sufficiently innovative while interestingly, we also find that the smaller rival may reject compatibility if the initial market size is relatively small. We also show that the initial market leader rejects interoperability for less important expected new products.

In addition, this work relates to a threatened incumbent’s and a smaller rival’s
R&D incentives when the economy operates under mandatory compatibility or a laissez faire Competition Law with respect to Intellectual Property Rights holders when network effects are present and substitutable products are durable. The Literature has focused mostly on the initial market structure and assesses whether a monopolist with perfectly exclusive Property Rights has higher or lower R&D incentives than his counterpart under perfect or imperfect competition. In this work, we assume an initial monopolist who is threatened to be displaced by a smaller innovative rival. We find that when network effects are relatively weak and for less innovative products, the dominant firm invests more when he does not supply interoperability, not allowing the rival to use his network. When the new versions are relatively important, the market leader initially invests more when compatibility is supported.

Regarding welfare, Bessen and Maskin (2009) showed that for important complementary innovations, imitation raises welfare and patents may impede innovation. When network effects are present, Economides (2006) also found that compatibility raises social and consumers’ welfare. We find that a laissez faire Competition Law either leads to compatibility or offers a socially preferable balance of both competitors’ R&D incentives compared to the economy that operates under mandatory compatibility.

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8 See Gilbert (2006) for an excellent survey on issues related to the initial market structure and the firms’ incentives.
9 Current work looks at the initial market structure and the competitors’ incentives when network effects are present and products are durable.
3.2 The Model

Consider the market for computer software applications where the current market leader must choose how much to invest into improving its durable, network product. The firm also needs to decide whether to support compatibility of its current and future version with a smaller rival that can also potentially innovate and has the same set of possible strategies regarding the compatibility of her future product and investment decisions.

On the supply side, the sequence of events is as follows: at date $t=0$, competitors simultaneously decide their investment levels as well as their attitude towards compatibility. Compatibility is a binary decision, is achieved bilaterally and comes free of charge. The two research lines are independent and no firm has a cost advantage in its R&D process over its opponent. More precisely, we assume that R&D spending is quadratic in the probability of successfully improving product quality.

At date $t = 1$, the dominant firm chooses the price for its initial version of quality $q_1$ while in the second ($t = 2$), the two firms compete a la Bertrand. If both research lines are successful, firms sell an improved product of expected quality $q^e_2$ ($q^e_2 > q_1$), which is considered as exogenous in the model. Forward

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\[10\] See Malueg and Schwartz (2006).

\[11\] We follow Ellison and Fudenberg (2000) who also considered quality as a positive, real number $q$.

\[12\] Current work endogenizes the quality improvement.
incompatibility of the product of quality $q_1$ prevents its users from working with a file that is created with a product of higher quality $q_2$. If compatibility is supported and because of backward compatibility, buyers of a product of quality $q_2$ join a network of maximum size.\textsuperscript{13} In contrast, when there is incompatibility, purchasers of a product of quality $q_2$ join only their seller’s network. Note that the goal of both firms is to maximise their expected profits where the marginal cost of production for all product versions is normalized to zero.\textsuperscript{14}

On the demand side, consumers are identical and arrive in constant flows $\lambda_t (t = 1, 2)$. At date $t=1$, $\lambda_1$ may observe the price for the product of quality $q_1$ and must decide whether to buy it. Their utility is partially dependent on network effects, captured by the parameter $\alpha$. Thus, if they buy a product of quality $q_1$, their utility (gross of price) is $q_1 + \alpha \lambda_1 x_1 - c$, where $x_1$ is the $\lambda_1$ customers’ fraction that also buys $q_1$ and $c$ is these customers’ adoption cost.\textsuperscript{15} Of course, these customers’ overall benefit depends on their forecasts regarding the second period play.

At date $t=2$ and if there is a new product in the market, the new ($\lambda_2$) and the old ($\lambda_1$) customers make their purchasing decisions after they observe the rival firms’ prices. Old customers’ purchasing decision, given announced prices, resembles a coordination game and can have multiple equilibria. Following the

\textsuperscript{13}See Ellison and Fudenberg (2000) for a paper where backward compatibility and forward incompatibility are present.

\textsuperscript{14}Zero marginal cost is consistent with the applications in the computer software market industry.

\textsuperscript{15}Note that the utility function may not be necessarily linear in income (any monotonic transformation would suffice) but linear utility simplifies the analysis.
literature, old consumers may be able to coordinate either to the Pareto optimal outcome or to what all the members of their class prefer.\textsuperscript{16} In the similar coordination problem related to the new customers’ purchasing decisions, the standard assumption is that buyers with the same preferences act as if they were a single player. Thus, after observing the prices, they coordinate to what is best for all of them. Since price discrimination is possible, both competitors can offer lower prices to old customers. We restrict attention to pure firms’ strategies and all consumers make their purchasing decisions simultaneously while we assume that they decide to purchase a superior product rather than an old version and join a network of superior rather than a smaller size even when their net utility may be equivalent. We also assume the same discount factor $\delta$ for all the agents in the economy.

Figure 1 summarizes the timing of the agents’ moves.

3.3 Market outcome

In this section, we will solve for equilibrium outcomes; that is, firms’ investment decisions, their prices in both the first and the second period as well as customers’ choices. We will start our analysis by considering the case where compatibility is mandatory.

\textsuperscript{16}See Ellison and Fudenberg (2000).
3.3.1 Mandatory compatibility

We will solve the model using backwards induction, starting from the second period firms’ pricing decisions (\( t = 2 \)), going back to calculating the dominant firm’s price for his initial version of quality \( q_1 \) (\( t = 1 \)) as well as the competitors’ optimal investment and compatibility decisions (\( t = 0 \)).
3.3.1.1 Second period ($t = 2$)

Imagine that both firms innovate and think first of the new customers ($\lambda_2$) who join a network of maximum size independently of where they purchase their new product of quality $q_2$. Thus, given the competitors’ prices and if we restrict attention to linear utility in income, their utility by purchasing any of the two new products is $q_2 + \alpha - c - p_{22i}$, after normalizing the second period market size to unity where the three subscripts in the price charged denote the quality of the product ($q_2$), the type of consumers ($\lambda_2$) and the product maker ($i = 1$ for the leader and $i = 2$ for the smaller rival), respectively. If all these customers purchase the dominant firm’s initial version, their utility given his price $p_{12}$ is $q_1 + \alpha \lambda_1 x_1 + \alpha \lambda_2 - c - p_{12}$, where $x_1$ is the $\lambda_1$ customers’ fraction that sticks to $q_1$.

Old customers ($\lambda_1$) observe the prices set by the competitors and their utility is $q_2 + \alpha - c_u - p_{21i}$; if they buy $q_2$ from competitor $i$ and $q_1 + \alpha \lambda_1 x_1 + \alpha \lambda_2 x_2$ if they stick to the product of quality $q_1^{17}$, where $x_1$, $x_2$ are the customers’ fractions that either stick or buy $q_1$ in the second period. If old customers make their purchasing decisions independently of what other old customers do, they will buy either the dominant or the smaller firm’s product when:

$$p_{21i} \leq \Delta q + \alpha \lambda_2 (1 - x_2) + c_u, \ \forall i = 1, 2. \ ^{18}$$

$^{17}$These customers are induced to buy the initial product of quality $q_1$ at $t = 1$ (see the Appendix for the first period analysis).

$^{18}$See the Appendix for the prices these customers are willing to pay if they coordinate to what all the other members of their class prefer.
We make the following assumption:

**Assumption 1 (A1):** \( \Delta q + \alpha \lambda_2 x_2 - c_u \geq 0, \ 0 \leq x_2 \leq 1. \)

This assumption says that the old second period customers’ expected benefit from buying any new product is at least greater than the cost of learning how to use it and allows us to isolate the role of network externalities and the expected quality improvements in firms’ strategies and welfare.

Thus, in such a case, all customers buy or purchase any new version for free due to Bertrand competition.

If only the dominant firm innovates, he remains the sole supplier in both periods. Thus, given his prices, new customers’ utility if they purchase the new product \( (q_2) \) is \( q_2 + \alpha - c - p_{211}, \) while if they all buy his initial version \( (q_1) \), their utility is \( q_1 + \alpha \lambda_1 x_1 + \alpha \lambda_2 - c - p_{121}, \) where \( x_1 \) is the old customers’ fraction that sticks to the initial version. Old customers’ utility if they upgrade to the dominant firm’s \( q_2 \) is \( q_2 + a - c_u - p_{211} \) while their utility if they stick to the old version is \( q_1 + \alpha \lambda_1 x_1 + \alpha \lambda_2 x_2. \) If these customers coordinate on the Pareto optimal outcome, they will buy the new product even if everyone else sticks to \( q_1 \) \( (x_1 = 1) \) if:

\[
p_{211} \leq \Delta q + \alpha \lambda_2 (1 - x_2) - c_u.
\]

Thus, since the dominant firm’s second period profit is a decreasing function of the number of new customers who stick to the initial version, the market leader’s optimal choice is to stop selling his initial version and the prices he charges to customers are \( p_{221} = q_2 + \alpha - c, \ p_{211} = \Delta q + \alpha \lambda_2 - c_u. \)
When the rival firm is the sole inventor, the dominant firm can no longer stop selling his initial version in the second period as such a choice would imply a potentially collusive behaviour. Thus, in this scenario, the competitors’ optimal prices are $p_{222} = \Delta q + \alpha \lambda_1$, $p_{121} = 0$, $p_{212} = \Delta q + \alpha \lambda_2 - c_u$ and all customers buy the rival firm’s innovative product.

The last case occurs when none of the competitors innovates where the new customers face a price $p_{121} = q_1 + a - c$ from the dominant firm which extracts all their surplus.\(^{19}\)

3.3.1.2 First and initial period ($t = 1$ and $t = 0$)

In the first period ($t = 1$), the dominant firm decides on the optimal price of his initial version of quality $q_1$ wishing to extract consumers’ total expected surplus and potential buyers ($\lambda_1$) make their purchasing decisions, depending on their expectations regarding the market participants’ second period behaviour.

Moving to the initial period ($t = 0$), both firms decide their optimal investment taking into consideration that the rival is also maximising his/her expected total profits.\(^{20}\) We will consider the following possible scenarios:

**Scenario 2 (A2):** $\Delta q^e < \alpha \lambda_1$, $\Delta q^e \geq \lambda_1 c_u$. This scenario occurs when the expected quality improvement is smaller relative to the network effects.

**Scenario 3 (A3):** $\Delta q^e > \alpha \lambda_1$, $\Delta q^e \geq \lambda_1 c_u$. In this case, the expected quality

\(^{19}\)See the Appendix for the table containing the second period prices in the different scenarios.

\(^{20}\)See the Appendix for the rival’s maximization problems and their optimal investment levels.
improvement is larger than the extent of network externalities.

The next lemma summarizes the market equilibrium outcome when compatibility is mandatory:

**Lemma 14** Both competitors’ optimal choice is to invest into developing the product of quality $q_2$. If A2 holds, the dominant firm’s investment decision is an increasing function of the rival’s optimal choice. When A3 holds, the dominant firm’s investment is a decreasing function of the rival’s optimal choice. Customers in the first period purchase the product of quality $q_1$ and in the second, the whole market purchases the superior product of quality $q_2$.

When network effects are larger than the expected quality improvement (A2), the dominant firm’s reaction function is an increasing function of the rival’s investment decision.²¹ On the other hand, when network effects are relatively weak (A3), the dominant firm seems to free ride on the rival’s investment choice as his expected second period benefit by increasing his probability of success would be outweighed by the additional first period cost.

### 3.3.2 Laissez faire Competition Law

Under a laissez faire competition law, both firms initially choose their investment levels as well as whether they will support compatibility in the future period.

²¹See the Appendix for the graphical representation of the different cases.
3.3.2.1 Second period \((t = 2)\)

Think first of the scenario where only the rival innovates and consider the new second period customers \((\lambda_2)\). After they observe the prices, if they all purchase the rival’s product of quality \(q_2\), their utility is 
\[
q_2 + \alpha \lambda_2 + \alpha \lambda_1(1 - x_1) - c - p_{222},
\]
where \(1 - x_1\) is the old customers’ fraction that purchases \(q_2\). If they all buy \(q_1\), their utility is 
\[
q_1 + \alpha \lambda_2 + \alpha \lambda_1 x_1 - c - p_{121}.
\]
Thus, these customers prefer the rival’s superior product of quality \(q_2\) if:
\[
p_{222} - p_{121} \leq \Delta q + \alpha \lambda_1(1 - 2x_1).
\]

Old customers also observe the prices and decide whether to buy the superior product or stick to the initial version.\(^{22}\) If they purchase \(q_2\), their utility is 
\[
q_2 + \alpha \lambda_1(1 - x_1) + \alpha \lambda_2(1 - x_2) - c_u - p_{212}
\]
while if they stick to \(q_1\), their utility is 
\[
q_1 + \alpha \lambda_1 x_1 + \alpha \lambda_2 x_2,
\]
where \(x_1, x_2\) are the old and new customers’ fractions that stick or buy \(q_1\), respectively. If these customers coordinate on the Pareto optimal outcome, they will buy \(q_2\) even when all the other old customers stick to \(q_1\) if:
\[
p_{212} \leq \Delta q + \alpha \lambda_2(1 - 2x_2) - \alpha \lambda_1 - c_u.\(^{23}\)
\]

We will consider the following two scenarios:

**Scenario 4 (A4)**: \(\Delta q + \alpha \lambda_2 - \alpha \lambda_1 - c_u < 0\). In this scenario and in equilibrium, old customers do not buy the rival’s product of quality \(q_2\) as they are better-off

\(^{22}\) We consider here that these customers were already induced to buy \(q_1\) in the previous period (see the Appendix).

\(^{23}\) See the Appendix for the price these customers are willing to pay to purchase the product of quality \(q_1\) if they coordinate to what all the other members of their class prefer.
by retaining the dominant firm’s initial version.

**Scenario 5 (A5):** $\Delta q + \alpha \lambda_2 - \alpha \lambda_1 - c_u > 0$. In this case and in equilibrium, the first period customers are better off by purchasing the rival firm’s new product.

If the quality improvement from the rival’s new product is relatively small (A4), new customers prefer the product of quality $q_2$ if:

\[ p_{222} - p_{121} \leq \Delta q - \alpha \lambda_1, \]

and thus, the optimal firms’ prices are: $p_{222} = \Delta q - \alpha \lambda_1$, $p_{121} = 0$.

If old customers buy the rival’s version (A5 holds), new customers prefer the new product rather than the old if:

\[ p_{222} - p_{121} \leq \Delta q + \alpha \lambda_1, \]

and the competitors’ optimal choices are: $p_{222} = \Delta q + \alpha \lambda_1$, $p_{121} = 0$, $p_{212} = \Delta q + \alpha \lambda_2 - \alpha \lambda_1 - c_u$.

In the scenario that both competitors’ R&D processes are successful, consider first the new customers. After they observe the competitors’ prices, their utility if they all purchase the dominant firm’s or the rival’s $q_2$ is $q_2 + \alpha \lambda_2 + \alpha \lambda_1 x_1 - c - p_{221}$, $q_2 + \alpha \lambda_2 + \alpha \lambda_1 x'_1 - c - p_{222}$, respectively, where $x_1, x'_1$ are the old customers’ fractions that belong to either of the rivals’ network. If they all buy the dominant firm’s initial version, their utility is $q_1 + \alpha \lambda_2 + \alpha \lambda_1 (1 - x_1 - x'_1) - c - p_{121}$. Thus, these
customers will choose to purchase the dominant firm’s superior product if:

\[ q_2 + \alpha \lambda_2 + \alpha \lambda_1 x_1 - c - p_{221} \geq \]

\[ \geq \max\{q_2 + \alpha \lambda_2 + \alpha \lambda_1 x_1' - c - p_{222}, \quad q_1 + \alpha \lambda_2 + \alpha \lambda_1 (1 - x_1 - x_1') - c - p_{121}\} \]

Moving our attention to the old customers, their utility if they purchase \( q_2 \) from either the dominant or the rival firm is \( q_2 + \alpha \lambda_2 x_2 + \alpha \lambda_1 x_1 - c_u - p_{211}, \) \( q_2 + \alpha \lambda_2 x_2' + \alpha \lambda_1 x_1' - c_u - p_{212}, \) respectively while if they stick to the initial version \( q_1, \) their utility is \( q_1 + \alpha \lambda_2 (1 - x_2 - x_2') + \alpha \lambda_1 (1 - x_1 - x_1'). \) Thus, they will choose to buy the dominant firm’s product of quality \( q_2 \) even if all the other old customers either stick to the initial version or buy the rival’s new version if:

\[ q_2 + \alpha \lambda_2 x_2 - c_u - p_{211} \geq q_2 + \alpha \lambda_1 x_2 - c_u - p_{212} \]

and

\[ q_2 + \alpha \lambda_1 + \alpha \lambda_2 x_2 - c_u - p_{211} \geq q_1 + \alpha \lambda_1 + \alpha \lambda_2 (1 - x_2 - x_2'). \]

Note that when A4 holds and old customers coordinate on the Pareto optimum, their utility if they purchase the rival’s product of quality \( q_2 \) is not an option for them as it is strictly dominated by their alternative of sticking to \( q_1. \) The dominant firm’s optimal choice is to stop selling the initial version to the new second period customers, the equilibrium prices are \( p_{221} = \alpha \lambda_1, \) \( p_{222} = 0, \) \( p_{211} = \Delta q + \alpha \lambda_2 - c_u \)

and all customers buy the dominant firm’s superior product. When A5 holds, the whole market buys either the rival’s or the dominant firm’s new version and Bertrand competition drives all prices to zero.
The scenarios where the dominant firm is the only innovator as well as the case where no firm’s R&D process is successful lead to the same market outcome as in the economy that operates under mandatory compatibility.\textsuperscript{24}

3.3.2.2 First and initial period ($t = 1$ and $t = 0$)

In the first period ($t = 1$), the dominant firm decides the optimal price of his initial version of quality $q_1$ wishing to extract customers’ expected total surplus and potential buyers ($\lambda_1$) make their purchasing decisions, depending on their expectations regarding the market participants’ second period behaviour.

Moving to the initial period ($t = 0$), both competitors choose their investment levels aiming to maximise their expected profits.\textsuperscript{25}

The next proposition summarizes the market equilibrium outcome in an economy that operates under a laissez faire Competition Law:\textsuperscript{26}

\textbf{Proposition 15} (a) For relatively less innovative future products (A4), the dominant firm’s optimal choice is not to support compatibility. (b) For sufficiently innovative products (A5): (1) both firms welcome compatibility for a relatively large initial market size ($\lambda_1$), (2) if the first period market size is relatively small, the rival rejects to offer interoperability information to the initial market leader.

\textsuperscript{24}See the Appendix for the table containing the equilibrium second period prices under the different scenarios.
\textsuperscript{25}See the Appendix for the competitors’ maximization problems and their optimal investment choices as functions of the rival’s optimal choice.
\textsuperscript{26}Note that the dominant firm would be indifferent between supporting and impeding compatibility if old consumers coordinate to what all the other old consumers prefer.
Proof. See the Appendix.

For less innovative products (A4), incompatibility prevails in the market as the dominant firm prefers not to share his network with the smaller innovative rival. More precisely, for relatively weak network effects (A3) and unlike the rival, the dominant firm invests more under incompatibility while for stronger network externalities (A2), the dominant firm would have invested more if compatibility was compulsory.\(^{27}\) On the other hand, for sufficiently innovative products relative to network externalities (A5), the dominant firm both welcomes compatibility and invests more even if the rival is a direct future competitor. This happens as the gains from sharing its network outweigh the potential costs: more precisely, by supporting compatibility, the probability that he is the only inventor increases allowing him to enjoy a larger second period expected profit exceeding the loss from a lower first period profit. The rival faces a trade-off: if she supports compatibility, the probability of being the sole second period supplier decreases while it allows her to set a higher price to existing customers (\(\lambda_1\)). Thus, for a relatively large first period market size, her optimal choice is to offer compatibility to the market leader (b1). In a such a case, she also invests more than in a economy that incompatibility is mandatory. When the number of old customers is smaller (b2), unlike the dominant firm, the rival is better-off by not supplying interoperability information to the initial market leader.

\(^{27}\)See figures 2 and 3 in the Appendix.
3.4 Social Welfare Maximization

We consider the problem faced by a social planner who wishes to maximise the sum of consumers’ and producers’ total discounted expected surplus. He has access to the firms’ cost functions and can invest into the two research lines as well as choose his attitude towards compatibility.\(^{28}\)

If the planner supports compatibility, all customers are expected to buy the improved version of quality \(q_2\) in the second period (A1), joining a network of maximum size. For less innovative products (A4) and unlike the case where innovations are relatively important (A5), if compatibility is not supported, old customers only buy the Research line 1 new version.

The next proposition summarizes the socially optimal investment and compatibility choice and provides a comparison with the market equilibrium obtained in an economy operating under mandatory compatibility or a laissez faire competition law:

**Proposition 16**  a) If A4 and A2 hold, the social planner decides to support compatibility. Although the economy that operates under mandatory compatibility leads to overinvestment while a laissez faire competition law may lead to underinvestment, the laissez faire Competition Law is socially preferable. b) If A4 and A3 hold, the planner may choose to support compatibility. The market equilibrium

\(^{28}\)We call the initial line whose past R&D success produces \(q_1\) as Research line 1.
outcome in a laissez faire economy leads to incompatibility and is always socially preferable compared to the market equilibrium under mandatory compatibility. c) If A5 holds, the planner is indifferent between supporting and impeding compatibility.

Proof. See the Appendix

For less innovative products (A4), although a laissez faire economy leads to the dominant firm rejecting compatibility, the magnitude of the potential inefficiency is smaller compared to the economy that operates under mandatory compatibility. In particular, when network effects are relatively weak (A3), a laissez faire competition law leads to more balanced R&D incentives for both rivals and is certainly socially preferable compared to the economy that mandates compatibility where the dominant firm underinvests and the rival overinvests heavily.29 Similarly, when network externalities are relatively stronger (A2), a market where interoperability is compulsory leads to overinvestment and a laissez faire competition law is socially preferable although the rival is deterred to invest.30

In conclusion, we could say that a Laissez faire Competition Law is socially preferable compared to an economy that either mandates compatibility or imposes very strong intellectual property rights.

29 See figure 3 in the Appendix.
30 See figure 2 in the Appendix.
3.5 Conclusion

The first contribution of this work is that we give an alternative explanation as to why dominant firms may welcome compatibility. More precisely, we show that sequential innovation and sufficiently innovative products in an economy with durable network goods allow the market leader to voluntarily supply interoperability information even to direct future competitors. In fact, when compatibility is present, the dominant firm invests more increasing its probability of success as well as the probability that it is the only inventor in the market. On the other hand, the rival’s optimal choice depends on the market size: if the number of initial customers is sufficiently large, she will also support compatibility while this is no longer true for a smaller initial market size.

Our second contribution relates to the dominant firm’s R&D incentives, as we show that they are not curbed under incompatibility for less innovative products. In particular, we find a critical cutoff in network externalities below which the market leader invests more when refusing to support compatibility with his future potential rival.

Third, we hope to contribute to the discussion concerning the social desirability of a more interventionist competition law: we find that when network effects are weak, a laissez faire competition law is socially preferable compared to an economy that operates under mandatory compatibility, as a laissez faire market either
converges to compatibility or when this does not occur, incompatibility is socially beneficial. When network externalities are strong, unlike a laissez faire market, an economy that operates under mandatory compatibility leads to overinvestment.

We acknowledge the limitations of this piece of research. First, current work considers the interaction of different business models when the quality improvement is endogenous and is not modelled as a parameter when network effects and durability are present. Further research may also analyze the competitors’ R&D incentives and compatibility decisions in the face of stochastic demand.

3.6 Appendix

3.6.1 Tables regarding the second period prices

The next table summarizes the different potential cases as well as the rivals’ optimal second period prices charged to the new and the old customers under compatibility:

<table>
<thead>
<tr>
<th></th>
<th>Prices to $\lambda_2$</th>
<th>Prices to $\lambda_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both firms innovate</td>
<td>$p_{22i} = 0, \forall i = 1, 2$</td>
<td>$p_{21i} = 0, \forall i = 1, 2$</td>
</tr>
<tr>
<td>Only the Dominant innovates</td>
<td>$p_{221} = q_2 + \alpha - c$</td>
<td>$p_{211} = \Delta q + \alpha \lambda_2 - c_u$</td>
</tr>
<tr>
<td>Only the Rival innovates</td>
<td>$p_{222} = \Delta q + \alpha \lambda_1$</td>
<td>$p_{212} = \Delta q + \alpha \lambda_2 - c_u$</td>
</tr>
<tr>
<td>None innovates</td>
<td>$p_{121} = q_1 + \alpha - c$</td>
<td>already bought at $t = 1$</td>
</tr>
</tbody>
</table>
Under mandatory incompatibility, the following table summarizes all the potential second period cases as well as the rivals’ prices to the different customers’ classes under A4 when both firms invest into producing an improved version of quality $q_2$:

<table>
<thead>
<tr>
<th></th>
<th>Prices to $\lambda_2$</th>
<th>Prices to $\lambda_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both firms innovate</td>
<td>$p_{221} = \alpha \lambda_1$</td>
<td>$p_{211} = \Delta q + \alpha \lambda_2 - c_u$</td>
</tr>
<tr>
<td>Only the Dominant innovates</td>
<td>$p_{221} = q_2 + \alpha - c$</td>
<td>$p_{211} = \Delta q + \alpha \lambda_2 - c_u$</td>
</tr>
<tr>
<td>Only the Rival innovates</td>
<td>$p_{222} = \Delta q - \alpha \lambda_1$</td>
<td>$p_{212} = 0$</td>
</tr>
<tr>
<td>Noone innovates</td>
<td>$p_{121} = q_1 + \alpha - c$</td>
<td>already bought at $t = 1$</td>
</tr>
</tbody>
</table>

while under A5, the table becomes:

<table>
<thead>
<tr>
<th></th>
<th>Prices to $\lambda_2$</th>
<th>Prices to $\lambda_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both firms innovate</td>
<td>$p_{222} = 0$, $p_{221} = 0$</td>
<td>$p_{21i} = 0$, $\forall i = 1, 2$</td>
</tr>
<tr>
<td>Only the Dominant innovates</td>
<td>$p_{221} = q_2 + \alpha - c$</td>
<td>$p_{211} = \Delta q + \alpha \lambda_2 - c_u$</td>
</tr>
<tr>
<td>Only the Rival innovates</td>
<td>$p_{222} = \Delta q + \alpha \lambda_1$</td>
<td>$p_{212} = \Delta q + \alpha \lambda_2 - \alpha \lambda_1 - c_u$</td>
</tr>
<tr>
<td>Noone innovates</td>
<td>$p_{121} = q_1 + \alpha - c$</td>
<td>already bought at $t = 1$</td>
</tr>
</tbody>
</table>
3.6.2 Calculating firms’ investment decisions as a function of the rival’s optimal choices

3.6.2.1 Mandatory compatibility

Given the market leader’s price \( p_{11} \), first period customers’ total discounted expected utility if they purchase the product \( q_1 \) is:

\[
q_1 + \alpha \lambda_1 - c + \delta s_1(1 - s_2)(q_2^e + \alpha - c_u - p_{211}^e) + \\
+ \delta (1 - s_1)s_2(q_2^e + \alpha - c_u - p_{212}^e) + \delta s_1 s_2 (q_2^e + a - c_u) + \\
+ \delta (1 - s_1)(1 - s_2)(q_1 + a) - p_{11},
\]

where \( s_1, s_2 \) are the dominant firm’s and the rival’s probabilities of successfully innovating, respectively and the subscript \( e \) denotes the expectation for the quality improvement and the second period prices. Note that the market leader wishes to extract \( \lambda_1 \) customers’ expected total surplus by setting the highest price \( p_{11} \) that would induce them to buy \( q_1 \) and thus, his optimal first period choice is:

\[
p_{11} = q_1 + \alpha \lambda_1 - c + \delta s_1(1 - s_2)(q_2^e + \alpha - c_u - p_{211}^e) + \delta (1 - s_1)s_2(q_2^e + \alpha - c_u - p_{212}^e) + \\
+ \delta s_1 s_2 (q_2^e + a - c_u) + \delta (1 - s_1)(1 - s_2)(q_1 + a). \tag{1}
\]

Moving back to the initial period \( (t = 0) \), the two firms simultaneously choose their investment levels. Thus, the smaller firm’s maximization problem is:

\[
\max_{s_2 \geq 0} \begin{cases} 
\delta s_2(1 - s_1^*)(\lambda_2 p_{222} + \lambda_1 p_{212}) - s_2^2/2 & \text{if } s_2 > 0 \\
0, & \text{otherwise}
\end{cases}.
\] \tag{2}
The similar maximization problem for the dominant firm is:

\[
\max_{s_1 \geq 0} \left\{ \begin{array}{l}
\lambda_1 p_{11} + \delta \lambda_1 s_1 (1 - s_2^*) p_{211} + \delta \lambda_2 s_1 (1 - s_2^*) p_{221} + \\
+ \delta \lambda_2 (1 - s_1)(1 - s_2^*) p_{121} - s_1^2 / 2, \text{ if } s_1 > 0 \\
\lambda_1 p_{11} + \delta \lambda_2 (1 - s_2^*) p_{121}, \text{ otherwise}
\end{array} \right.
\]

(3)

where \( p_{11} \) is given in (1) and \( s_2^* \) is the rival’s optimal investment choice.

The rival and the dominant firm’s investment decisions as a function of the competitor’s optimal choice are:

\[ s_2 = \delta (1 - s_1^*)(\Delta q^e + 2\alpha \lambda_1 \lambda_2 - \lambda_1 c_u), \]  

(4)

\[ s_1 = -s_2^*(\delta \lambda_2 \Delta q^e - \delta \alpha \lambda_1 \lambda_2) + \delta (\Delta q^e - \lambda_1 c_u), \]  

(5)

respectively.

3.6.2.2 Mandatory incompatibility

If A4 holds, the rival’s optimization problem is:

\[
\max_{s_4 \geq 0} \left\{ \begin{array}{l}
\delta (1 - s_3^*) s_4 \lambda_2 p_{222}^e - s_4^2 / 2, \text{ if } s_4 > 0 \\
0, \text{ otherwise}
\end{array} \right.
\]

(6)

where \( s_4 \) is her investment choice and \( s_3^* \) is the dominant firm’s optimal investment decision. The similar maximization problem faced by the market leader is:

\[
\max_{s_3 \geq 0} \left\{ \begin{array}{l}
\lambda_1 p_{11} + \delta \lambda_1 s_3 (1 - s_4^*) p_{211}^e + \delta \lambda_2 s_3 (1 - s_4^*) p_{221}^e + \\
+ \delta \lambda_2 (1 - s_3)(1 - s_4^*) p_{121}^e + \delta \lambda_2 s_3 s_4 \alpha \lambda_1 - s_3^2 / 2, \text{ if } s_3 > 0 \\
\lambda_1 p_{11} + \delta \lambda_2 (1 - s_4^*) p_{121}^e, \text{ otherwise}
\end{array} \right.
\]

(7)
where the price $p_{11}$ extracts the first period customers expected surplus and is
given by the expression:

$$p_{11} = q_1 + \alpha \lambda_1 - c + \delta s_3 (1 - s_4) (q_2^e + \alpha - c_u - p_{211}^e) + \delta s_3 s_4 (q_2^e + \alpha - c_u - p_{211}^e) (8)$$

$$+ \delta (1 - s_3) (1 - s_4) (q_1 + \alpha) + \delta (1 - s_3) s_4 (q_1 + \alpha \lambda_1).$$

When old customers expect to purchase the product of quality $q_2$ in the second period independently of which firm innovates (A5 holds), the competitors’ problems become:

$$\max_{s_4 \geq 0} s_4 (1 - s_3^*) (\lambda_2 p_{222}^e + \lambda_1 p_{212}^e) - s_4^2 / 2, \quad (6')$$

$$\max_{s_3 \geq 0} \lambda_1 p_{11} + \delta \lambda_1 s_3 (1 - s_4^*) p_{211}^e + \delta \lambda_2 s_3 (1 - s_4^*) p_{221}^e + \delta \lambda_2 (1 - s_3) (1 - s_4^*) p_{121}^e - s_3^2 / 2, \quad (7')$$

for the rival and the dominant firm, respectively and the first period price is:

$$p_{11} = q_1 + \alpha \lambda_1 - c + \delta s_3 (1 - s_4) (q_1 + \alpha \lambda_1) + \delta s_3 s_4 (q_2^e - c_u) + \delta (1 - s_3) (1 - s_4) (q_1 + \alpha) + \delta (1 - s_3) s_4 (q_1 + 2\alpha \lambda_1). \quad (9)$$

Note that when both firms’ R&D is successful, all customers are expected to buy the product of quality $q_2^e$ from either of the incompatible competitors. Thus, they will be part of a network of size $x$, with $x$ being any non-negative number. Thus, in the first period, the dominant firm may risk losing these customers if he charges a price greater than $p_{11}$ defined above.
3.6.3 Proof of Proposition 15

a) If A3 and A4 hold, the dominant firm always refuses to support compatibility. To see this, let $E(\Pi_{\text{no compatibility}}) = f(s)$ and $E(\Pi_{\text{compatibility}}) = g(s)$ denote the dominant firm’s expected profit under incompatibility and compatibility, respectively, where $f(0) > g(0)$.

We take the derivative of the two functions with respect to the dominant firm’s choice ($s$): $f_s = -\delta[s_4(\Delta q^e - \alpha \lambda_1 \lambda_2 - \lambda_1 c_u) - (\Delta q^e - \lambda_1 c_u)] - s$ while $g_s = \delta[\Delta q^e - \lambda_1 c_u - s_2 \lambda_2 (\Delta q^e - \alpha \lambda_1)] - s$, where $s_4, s_2$ are the rival’s optimal choices under incompatibility and compatibility, respectively (see figure 3). The dominant firm is better-off by not supporting compatibility when:

$$s_2 \lambda_2 (\Delta q^e - \alpha \lambda_1) - s_4 (\Delta q^e - \alpha \lambda_1 \lambda_2 - \lambda_1 c_u) > 0,$$

(*)

where the rival’s choices lie on the lines:

$$s_2 = \delta(1 - s)(\Delta q^e + 2\alpha \lambda_1 \lambda_2 - \lambda_1 c_u), \text{ and } s_4 = \delta(1 - s)\lambda_2 (\Delta q^e - \alpha \lambda_1).$$

Without loss of generality, we assume that the discount factor is large ($\delta = 1$). After substituting $s_2$ and $s_4$ in * we get:

$$(1 - s)(\Delta q^e + 2\alpha \lambda_1 \lambda_2 - \lambda_1 c_u) \lambda_2 (\Delta q^e - \alpha \lambda_1) - (1 - s) \lambda_2 (\Delta q^e - \alpha \lambda_1) (\Delta q^e - \alpha \lambda_1 \lambda_2 - \lambda_1 c_u) > 0$$

which always holds and thus $f_s > g_s \forall s$. 

99
After solving for $s_1^*$, $s_3^*$, one gets:

$$s_1^* = \frac{\Delta q^e - \lambda_1 c_u - (\Delta q^e + 2\alpha \lambda_1 \lambda_2 - \lambda_1 c_u)(\lambda_2 \Delta q^e - \lambda_2 \alpha \lambda_1)}{1 - (\Delta q^e + 2\alpha \lambda_1 \lambda_2 - \lambda_1 c_u)(\lambda_2 \Delta q^e - \lambda_2 \alpha \lambda_1)}$$

and

$$s_3^* = \frac{\Delta q^e - \lambda_1 c_u - (\Delta q^e - \alpha \lambda_1 \lambda_2 - \lambda_1 c_u)(\lambda_2 \Delta q^e - \lambda_2 \alpha \lambda_1)}{1 - (\Delta q^e - \alpha \lambda_1 \lambda_2 - \lambda_1 c_u)(\lambda_2 \Delta q^e - \lambda_2 \alpha \lambda_1)}$$

and after substituting to the expressions for $s_2^*$, $s_4^*$, we get:

$$s_2^* = [1 - \frac{\Delta q^e - \lambda_1 c_u - (\Delta q^e + 2\alpha \lambda_1 \lambda_2 - \lambda_1 c_u)(\lambda_2 \Delta q^e - \lambda_2 \alpha \lambda_1)}{1 - (\Delta q^e + 2\alpha \lambda_1 \lambda_2 - \lambda_1 c_u)(\lambda_2 \Delta q^e - \lambda_2 \alpha \lambda_1)}] \lambda_2 (\Delta q^e - \alpha \lambda_1).$$

and

$$s_4^* = [1 - \frac{\Delta q^e - \lambda_1 c_u - (\Delta q^e - \alpha \lambda_1 \lambda_2 - \lambda_1 c_u)(\lambda_2 \Delta q^e - \lambda_2 \alpha \lambda_1)}{1 - (\Delta q^e - \alpha \lambda_1 \lambda_2 - \lambda_1 c_u)(\lambda_2 \Delta q^e - \lambda_2 \alpha \lambda_1)}] \lambda_2 (\Delta q^e - \alpha \lambda_1).$$

Thus, * becomes after some algebraic manipulation:

$$(\Delta q^e + 2\alpha \lambda_1 \lambda_2 - \lambda_1 c_u)[1 - (\Delta q^e - \alpha \lambda_1 \lambda_2 - \lambda_1 c_u)\lambda_2 (\Delta q^e - \alpha \lambda_1)] >$$

$$(\Delta q^e - \alpha \lambda_1 \lambda_2 - \lambda_1 c_u)[1 - (\Delta q^e + 2\alpha \lambda_1 \lambda_2 - \lambda_1 c_u)\lambda_2 (\Delta q^e - \alpha \lambda_1)]$$

which simply verifies that $s_3^* > s_1^*$.

Thus, the dominant firm impedes compatibility.

Note that if A4 and A2 hold, the rival’s optimal choice is not to invest ($s_4 = 0$) and the dominant firm chooses not to support compatibility. Think for example the following parameter values that satisfy A4 and A2: $\Delta q^e = 0.3$, $\alpha = 1$, $\lambda_1 = 0.7$, $\lambda_2 = 0.3$, $c_u = 0.1$, $c = 0.2$, $q_1 = 0.1$, $q_2 = 0.4$, $\delta = 1$. Direct comparison of the dominant firm’s values of maximised expected profit show that he impedes compatibility.
b1) Think for the example the case where: $\Delta q^e = 0.9$, $\alpha = 1$, $\lambda_1 = 0.8$, $\lambda_2 = 0.2$, $c_u = 0.2$, $c = 0.3$, $q_1 = 0.1$, $q_2 = 1$, $\delta = 1$.

Direct comparison of the two firms’ expected profits lead to the conclusion that they both support compatibility.

b2) Think of the parameter values: $\Delta q^e = 0.4$, $\alpha = 1$, $\lambda_1 = 0.3$, $\lambda_2 = 0.7$, $c_u = 0.2$, $c = 0.3$, $q_1 = 0.1$, $q_2 = 0.5$, $\delta = 1$.

Direct comparison of the firms’ expected profits yields that the dominant firm supports compatibility while the rival firm rejects it.
3.6.4 Proof of Proposition 16

Depending on whether the planner invests or not and whether he supports compatibility or not, the social welfare function if A4 holds is:

\[
\max_{\rho, \rho' \geq 0} \begin{cases} 
\lambda_1(q_1 + \alpha \lambda_1 - c) + \delta \lambda_1(\rho + \rho' - \rho \rho')(q_2^c + \alpha - c_u) + \\
\delta \lambda_1(1 - \rho)(1 - \rho')(q_1 + \alpha) + \delta \lambda_2(\rho + \rho' - \rho \rho')(q_2^c + \alpha - c) + \\
+\delta \lambda_2(1 - \rho)(1 - \rho')(q_1 + \alpha - c) - \rho^2/2 - \rho'^2/2, \ \rho, \rho' > 0 \text{ if he supports compatibility,} \\
\lambda_1(q_1 + \alpha \lambda_1 - c) + \delta \lambda_1 \rho(q_2^c + \alpha - c_u) + \delta \lambda_1 \rho(1 - \rho)(q_1 + \alpha \lambda_1) + \\
+\delta \lambda_1(1 - \rho)(1 - \rho')(q_1 + \alpha) + \delta \lambda_2(\rho + \rho' - \rho \rho')(q_2^c + \alpha - c) + \\
+\delta \lambda_2(1 - \rho)(1 - \rho')(q_1 + \alpha - c) - \rho^2/2 - \rho'^2/2, \ \rho, \rho' > 0 \text{ if he does not support compatibility,} \\
\lambda_1(q_1 + \alpha \lambda_1 - c + \delta q_1 + \delta \alpha) + \delta \lambda_2(q_1 + \alpha - c), \ \rho = \rho' = 0,
\end{cases}
\]

where \(\rho, \rho'\) are the planner’s investment choices in Research lines 1 and 2, respectively.

a) If A4 and A2 hold, the social planner will make the two products in the second period compatible if the maximum value of the social welfare function is higher compared to the scenario he makes incompatible products. If \(\max \text{SW}_{\text{com}}, \max \text{SW}_{\text{incom}}\) are the highest values in the social welfare if he supports compatibility or not, it is immediate to see that in the latter case, he only invests in improving the dominant firm’s product.
The planner supports compatibility when:

\[ maxSW_{com} > maxSW_{incom} \]

or equivalently when the expression:

\[
\begin{align*}
\lambda_1&(\rho + \rho' - \rho'' - \rho''')(q_2^2 + \alpha - c_u) + \lambda_1[(1 - \rho)(1 - \rho') - (1 - \rho'')(q_1 + \alpha)] + \\
\lambda_2&(\rho + \rho' - \rho'' - \rho''')(q_2^2 + \alpha - c) + \lambda_2[(1 - \rho)(1 - \rho') - (1 - \rho'')(q_1 + \alpha - c)]
\end{align*}
\]

is positive, where \( \rho, \rho' \) are his optimal investment choices when he chooses compatibility and \( \rho'' \) is his optimal investment if he chooses to have incompatible products satisfying the equations:

\[
\begin{align*}
\rho &= -\delta \rho'(\Delta q^e - \lambda_1 c_u) + \delta(\Delta q^e - \lambda_1 c_u), \\
\rho' &= -\delta \rho(\Delta q^e - \lambda_1 c_u) + \delta(\Delta q^e - \lambda_1 c_u), \\
\rho'' &= \delta(\Delta q^e - \lambda_1 c_u).
\end{align*}
\]

Note that \( \rho = \rho' = \frac{\kappa}{\kappa+1} \), where \( \kappa = \delta(\Delta q^e - \lambda_1 c_u), \ 0 < \kappa < 1 \).

Thus, we need to show that:

\[
\begin{align*}
\lambda_1\{(\frac{2\kappa}{\kappa+1} - \frac{\kappa^2}{(\kappa+1)^2} - \kappa)(q_2^2 + \alpha - c_u) + [(1 - \frac{\kappa}{\kappa+1})^2 - (1 - \kappa)](q_1 + \alpha)]\} &+ \\
\lambda_2\{(\frac{2\kappa}{\kappa+1} - \frac{\kappa^2}{(\kappa+1)^2} - \kappa)(q_2^e + \alpha - c) + [(1 - \frac{\kappa}{\kappa+1})^2 - (1 - \kappa)](q_1 + \alpha - c)\} &- 2\frac{\kappa^2}{(\kappa+1)^2} + \kappa^2 > 0
\end{align*}
\]

or equivalently:

\[
\begin{align*}
\lambda_1\frac{1 - \kappa^2}{(\kappa+1)^2}(\Delta q^e - c_u) + \lambda_2\frac{1 - \kappa^2}{(\kappa+1)^2}\Delta q^e + \kappa^2 - \frac{2\kappa^2}{(\kappa+1)^2} > 0.
\end{align*}
\]
For parameter values satisfying A1, A2, A4, the above expression takes a positive sign. Think for example the parameter values satisfying A1, A2 and A4 ($\alpha = 1, \lambda_1 = 0.7, \Delta q^e = 0.3, c_u = 0.01$). Direct calculation leads to the conclusion that the above expression is positive and thus the planner chooses compatibility.

b) If A4 and A3 hold, the social planner decides to support compatibility if:

$$max_{SW_{com}} > max_{SW_{incom}}$$

or equivalently the expression:

$$\delta \lambda_1 (\rho + \rho' - \rho \rho' - \rho'')(q_2^e + \alpha - c_u) + \delta \lambda_1 [(1 - \rho)(1 - \rho') - (1 - \rho')(1 - \rho'')(1 - \rho'')](q_1 + \alpha) -$$

$$- \delta \lambda_1 \rho''(1 - \rho'')(q_1 + \alpha \lambda_1) + \delta \lambda_2 (\rho + \rho' - \rho \rho' - \rho'' - \rho'' + \rho''\rho''')(q_2^e + \alpha - c) +$$

$$+ \delta \lambda_2 [(1 - \rho)(1 - \rho') - (1 - \rho'')(1 - \rho'')](q_1 + \alpha - c) - \rho^2/2 - \rho'^2/2 - \rho''^2/2 - \rho'''^2/2$$

takes a positive sign. It is straightforward to see that for parameter values satisfying A4 and A3 (for example, take $\Delta q^e = 0.4$, $\lambda_1 = 0.7$, $\alpha = 0.5$, $\delta = 1$, $c = 0.4$, $c_u = 0.3$, $q_1 = 0.1$), the planner supports compatibility as the social welfare function is maximised.
3.6.5 Figures regarding the competitors’ and the planner’s optimal investment decisions

The next figures summarize the market equilibrium outcome under a laissez faire Competition Law and under mandatory compatibility as well the social optimum level of investment:

![Diagram](image)

Figure 3.2: A4 and A2

3.7 Abbreviations

ECIS: European Committee of Interoperable Systems
Figure 3.3: A4 and A3

R&D: Research and Development

IPRs: Intellectual Property Rights
BIBLIOGRAPHY


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