State Censorship†

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We characterize a ruler’s decision of whether to censor media reports that convey information to citizens who decide whether to revolt. We find: (i) a ruler gains (his ex ante expected payoff increases) by committing to censoring slightly less than he does in equilibrium: his equilibrium calculations ignore that censoring less causes citizens to update more positively following no news; (ii) a ruler gains from higher censorship costs if and only if censorship costs exceed a critical threshold; (iii) a bad ruler prefers a very strong media to a very weak one, but a good ruler prefers the opposite. (JEL D72, D74, D83)

We analyze the strategic choices by a ruler (an authoritarian state) of when to censor the information available to citizens to avoid revolution. In particular, we investigate how a ruler’s expected payoffs are influenced by his ability to commit to a censorship law (censorship strategy); communication technologies that raise censorship costs; and the media’s ability to uncover news about the regime.

The ruler tries to manage information transmission by preventing dissemination of unfavorable news by the media to citizens to mitigate the likelihood of revolt. In practice, most censorship takes this simple form. For example, Tai’s (2014, 195) analysis of the Chinese government’s directives to the media from March 2007 to April 2013 shows that most of these directives were outright bans on reporting particular news events. Censoring a news event can benefit a ruler whenever the likelihood of revolt following that news exceeds that following no news. Citizens understand a ruler’s incentives to conceal bad news, so they update negatively about a regime when they see no news, inferring that there might have been bad news that was censored. In equilibrium, provided that the direct censorship cost is not too high, there exists a unique threshold level of news such that the ruler censors a news event if and only if it is worse than that threshold. At the threshold, the gains from the reduced probability of revolution just equal the direct cost of censorship.

Moreover, when the media is more likely to uncover politically-relevant news, the news must be worse for a ruler to censor, as citizens update to conclude that an absence of news was more likely due to censorship. Thus, in a country with a
strong media that often uncovers news, a ruler censors only very bad news; while in a country with a weak media, a ruler censors even modestly bad news. So, too, when there is more uncertainty about the possible news, a ruler ceases to censor marginally bad news in order to prevent citizens from drawing inferences that the news could have been far worse.

This initial analysis presumes that a ruler cannot commit to a censorship cutoff, and hence only weighs the direct consequences of censorship decisions. In particular, a ruler does not internalize that even though a freer media is risky because it raises the probability of revolt following bad news, the gains from improving citizens’ trust in media may offset those risks because citizens update less negatively following no news. This observation leads us to consider a ruler who can credibly set up institutions that allow him to commit to censoring at other than the “equilibrium” level of censorship. By delegating censorship decisions to bureaucrats and threatening punishment if they deviate from censorship laws, a ruler might be able to commit to censoring slightly more or less than what he otherwise would without those institutions. Still, large deviations are likely infeasible—for example, a bureaucrat has strong incentives to censor very bad news that would inevitably result in revolt and hence severe punishment of members of the regime, or a ruler would find other ways to get good news out. We address whether and when a ruler would be better off if he could commit to censoring slightly more or less.

Remarkably, we find that from an ex ante perspective, a ruler would always be strictly better off if he censored slightly less than he does in the equilibrium where he cannot commit—a ruler would always benefit from a slightly freer press. A ruler’s equilibrium tradeoff equates the marginal costs and benefits of censoring, ignoring the impact of his censorship cutoff on how citizens update when the media does not report politically-relevant news. Paradoxically, were a ruler to censor slightly less, citizens would draw more favorable inferences about the regime following no news, and hence would be less likely to revolt. We show that the likelihood citizens revolt following no news is a single-peaked function of a ruler’s censorship cutoff, and that the equilibrium censorship cutoff is always to the left of the peak whenever censorship is costly.

This analysis yields insights into how changes in censorship costs or in the media’s ability to uncover news affect a ruler. One may think that a ruler must be harmed by new technologies such as the Internet or cell phones that raise censorship costs and reduce the probability that censorship succeeds, or by the entry of a media organization such as Al Jazeera that uncovers more news. However, our initial analysis suggests that such changes could benefit a ruler: we showed that a ruler would be better off if he could commit to censoring less, and higher censorship costs or a stronger media cause a ruler to censor less in equilibrium. Thus, the ruler may observably delegate censorship to agents with higher censorship costs, so that citizens understand that censorship level is lower than what the ruler would choose—when he cannot commit.

In fact, the consequences of higher censorship costs or of a stronger media for a ruler’s expected payoffs are subtle. We prove that if censorship is almost costless, then slight increases in censorship costs always harm a ruler. Intuitively, when censorship is inexpensive, a ruler censors a lot, and hence is likely to incur the incremental
censorship cost; but the probability of revolt given no news is insensitive to a marginal reduction in censorship. However, when censorship costs are higher, a ruler is less likely to censor, and the probability of revolt is more sensitive to the censorship threshold. We show that, as a result, increases in censorship costs benefit a ruler (increase his ex ante expected payoff) if and only if censorship costs are already high enough.

Finally, we prove that when the revolution payoff is low, a ruler prefers a weak media that uncovers almost no news—in effect censoring both bad news and good—to a strong media that uncovers almost all news. In contrast, when the revolution payoff is high, the ruler prefers a strong media that serves to commit the ruler to censoring almost nothing to a weak media. Intuitively, when revolution payoffs are high, a ruler values a strong media that might uncover good news about the status quo that then forestalls a revolution; but when revolution payoffs are low, a ruler fears a strong media that might uncover bad news about the status quo that then precipitates a revolution.

**Literature Review.**—The closest paper is Besley and Prat (2006), which analyzes media capture in a democracy. The media can send a binary (good versus bad) signal to voters about the incumbent who can bribe the media to conceal the bad signal. They find that an incumbent bribes the media if and only if the costs of bribery (which reflect the number of outlets that must be bribed, and transactions costs) are not too large. Binary signals simplify characterizations, but are not rich enough to permit an analysis of the effects of commitment on citizen beliefs, or the effects of changes in censorship costs and media strength on a ruler’s/incumbent’s expected payoff, issues that are at the heart of our paper.

Based on Besley and Prat’s model, Gehlbach and Sonin (2012) explore media bias caused by a government that cares both about inducing citizens to take an action and generating advertising revenue. In their two state model, citizens can obtain a costly binary signal about the true state. They show that media bias is greater if the government owns the media and hence does not have to give bribes, or if it cares less about advertising revenue.

In Egorov, Guriev, and Sonin (2009), a ruler can manipulate a binary (good versus bad) signal sent by media about the regime’s economic performance that directly affects the likelihood citizens revolt, and indirectly affects this likelihood via the effect on incentives of bureaucrats to exert effort that increases taxable economic output. They assume that the ruler does not see the actual economic performance, but, rather, only observes the possibly distorted media reports. As a result, a ruler trades off between inducing bureaucrats to perform well and directly discouraging citizens from revolting.

Edmond (2013) studies information manipulation in dictatorships when a revolution succeeds if and only if the measure of citizens revolting exceeds the regime’s “strength.” Citizens decide whether to revolt after receiving private signals about the regime’s strength, and the regime can take a costly hidden action that increases these private signals. In equilibrium, the probability of successful revolution falls discontinuously from one to zero at some regime strength threshold. Edmond’s focus is on altering a signal, rather than concealing it, and he does not consider how commitment affects a ruler’s payoffs.
Our finding that a ruler censors sufficiently bad news (Proposition 2) has analogues in the literature on the disclosure of accounting information (Dye 1985; Verrecchia 1983). In these papers, like ours, complete unraveling as in Grossman and Hart (1980), Grossman (1981), Milgrom (1981), or Milgrom and Roberts (1986), does not occur because there is a chance that nothing happened or that no information was uncovered. That literature studies precommitment to disclose information in the context of information sharing in oligopoly markets, a very different setting from ours (see Beyer et al. 2010; Dye 2001; and Verrecchia 2001 for comprehensive reviews).

There are related analyses of information disclosure by an expert/advisor to a decision maker. For example, in Che and Kartik (2009), an advisor must decide whether to reveal his signal to a decision maker. The advisor and decision maker share the same preferences, but their priors over the state of the world have different means, so that the advisor’s preferred action always differs by a constant from the decision maker’s. This leads the advisor to censor a bounded interval of signals. See also de Bettignies and Zabojnik (2014). In particular, our result that, even when censorship is costless, the ruler does not censor all the bad news (Corollary 1) resonates with Dziuda (2011) who shows that a biased expert may provide information unfavorable to his preferences to convince the decision maker that the expert is not biased.

Our result that the ruler always gains if he can commit to censoring slightly less that the equilibrium level (Proposition 3) resonates with the result of Khalil (1997) in the context of optimal contracts with auditing/monitoring, who shows that the principal audits less often if he can precommit to an auditing strategy. However, the underlying reason in that paper is that “when threats are credible, they need not be used as often” (Khalil 1997, 630), while in our paper, precommitment allows the ruler to manipulate the citizen’s beliefs following no news.

A more distantly-related literature looks at the incentives of the media to selectively report news about candidates that affects how citizens update, and hence electoral outcomes. In Balan, DeGraba, and Wickelgren (2004); Duggan and Martinelli (2011); or Anderson and McLaren (2012), the incentives of media outlets to selectively report news devolves from their partisan views, which lead them to have a preferred candidate or policy outcome; in Bernhardt, Krasa, and Polborn (2008), media compete in their news mix for audiences that value hearing news that conforms with their views; in Chan and Suen (2008), media outlets commit to binary editorial recommendation cutoffs on the state of nature for recommending a left or right party that maximize their viewers’ welfare, and these cutoffs feed back to influence party policy choice; and in Gentzkow and Shapiro (2006), the media care about reputation, which can lead to censoring of stories that do not conform with reader expectations. This literature focuses on consumer choice of media outlet, the competition between media outlets for audiences, including the effects of mergers and merger policy (Anderson and McLaren 2012, or Balan, DeGraba, and Wickelgren 2004), and the role of endogenous costs of production. Then, they engage in Cournot or Bertrand competition. Precommitment amounts to determining whether to disclose their information prior to receiving it (Vives 2001, 237–62).

For a model where voters do not update about media bias (sequential rationality does not hold) see Mullainathan and Shleifer (2005).
media bias on electoral outcomes (Bernhardt, Krasa, and Polborn 2008, Duggan and Martinelli 2011) or policy outcomes (Anderson and McLaren 2012).

I. Model

A representative citizen must decide whether or not to revolt against a ruler. If the citizen does not revolt, the status quo prevails; but if he revolts, the revolution succeeds with probability $r \in (0, 1]$. We normalize the ruler’s payoff from preserving the status quo to 1 and his payoff after a successful revolution to 0. The citizen’s net payoff from successful revolution is $R - \gamma - \epsilon$, where $R$ is known, but $\gamma$ and $\epsilon$ are uncertain. We normalize the means of $\epsilon$ and $\gamma$ to zero. Thus, $R$ captures the ex ante expected net payoff from successful revolution. A larger $R$ can reflect a society in which the revolution payoff is higher; and/or a society in which, from an ex ante perspective, citizens have a lower assessment of the status quo. If the citizen revolts and the revolution fails, the citizen incurs an expected punishment $\mu > 0$, i.e., $\mu$ is the probability a failed revoler is caught times the punishment.

The representative citizen privately observes the realization of $\epsilon$, which we assume is distributed according to a strictly positive, continuously differentiable density $g$ on $\mathbb{R}$, with associated cdf $G$. The citizen does not directly observe $\gamma$, but has a prior that $\gamma \sim f$, where $f$ is a strictly positive, continuously differentiable density on $\mathbb{R}$. To ease exposition, we assume that $\epsilon$ and $\gamma$ are independently distributed.3

The citizen learns the realization of $\gamma$ if and only if the media observe $\gamma$ and report it. With probability $q(\gamma)$, the media and ruler observe the realization of $\gamma$, where $q(\cdot) : \mathbb{R} \rightarrow (0, 1)$ is a continuously differentiable function, mapping the level of news $\gamma$ to a likelihood that it is discovered by the media; but with probability $1 - q(\gamma)$ the realization is unobserved by the media. Thus, $q(\gamma)$ captures the probability that an event occurs that is informative about $\gamma$ and the media learn about it. We allow the probability $q(\gamma)$ that the media discover information about $\gamma$ to vary with the magnitude of the news, $\gamma$. For example, the media may be more likely to uncover very bad or very good news. The analysis extends if a ruler may sometimes learn $\gamma$ when the media do not, and when $\gamma$ is verifiable, the ruler can sometimes credibly convey the news $\gamma$ to the media at his discretion, in which case the media may be more likely to observe good news.4

To focus on a ruler’s censorship decisions, we abstract from strategic dissemination decisions by the media (see e.g., Duggan and Martinelli (2011) or the other papers in our literature review). Thus, we assume that when the media observes $\gamma$ and the ruler does not censor, the media publicly reveals $\gamma$.5 However, the ruler can censor the media at a cost $c \geq 0$, preventing the media from conveying $\gamma$ to the representative citizen, in which case the citizen does not receive any further information about $\gamma$. For example, if a ruler censors a prison massacre of dissidents, the

3 With added parametric structure (e.g., $\epsilon$ and $\gamma$ are jointly normally distributed), incorporating correlation is straightforward. Note that $\epsilon$ can also be interpreted as a valence parameter or an idiosyncratic political preference.

4 What matters is that the ruler observes $\gamma$ any time the media does (or else he has no censorship decision).

5 Our findings are qualitatively unaffected if the media and the ruler observe $\gamma$ with noise; or if a citizen observes a media report with noise.
citizen never learns about it. Given his information, which includes $\gamma$ only if the media learned about it and the ruler did not censor, the citizen decides whether to revolt.

The timing is as follows. First, $\gamma$ and $\epsilon$ are realized and the representative citizen observes $\epsilon$. Then with probability $q(\gamma)$ the media learn $\gamma$, which they publicly report unless the ruler (who also observes the media’s information) censors it. After the ruler makes his censorship decision, the citizen decides whether to revolt. Finally, payoffs are realized.

We next establish the existence of a unique Perfect Bayesian Equilibrium, and derive its properties. Following this analysis, we characterize outcomes when the ruler has limited commitment powers, and can commit to levels of censorship that differ from what he does in the Perfect Bayesian Equilibrium without commitment.

**Analysis.**—The representative citizen’s strategy is a function $\sigma_\epsilon$ mapping his private information $\epsilon$ and any public information into a decision about whether to revolt, where $\sigma_\epsilon = 1$ indicates that he revolts, and $\sigma_\epsilon = 0$ indicates that he does not. A strategy for the ruler is a function $\sigma_r$ mapping $\gamma$ into a decision about whether to censor, where $\sigma_r(\gamma) = 1$ indicates that the ruler censors $\gamma$, and $\sigma_r(\gamma) = 0$ indicates that he does not.

We begin with the representative citizen’s choice of whether to revolt. The citizen has private information $\epsilon$ and public information $\Omega \in \mathbb{R} \cup \{\emptyset\}$ about $\gamma$, where $\emptyset$ indicates there has not been a media report about $\gamma$. The citizen revolts if and only if the net expected payoffs from doing so are positive, i.e., if and only if $0 < rE[R - \gamma - \epsilon|\Omega] - (1 - r)\mu$. Rearranging, the citizen revolts if and only if

\[ \epsilon < R - E[\gamma|\Omega] - \mu_r, \]

where $\mu_r \equiv \frac{1 - r}{r} \mu$. Denoting the right-hand side of (1) by $\rho(R, \Omega) = R - E[\gamma|\Omega] - \mu_r$, it follows that the probability of successful revolution $P(\rho)$ is

\[ P(\rho) = rG(\rho). \]

Good news for the ruler is any news that decreases the citizen’s expectation of the net payoff from successful revolution, while bad news is the opposite. Thus, bad news raises $\rho$. Proposition 1, which directly follows from equation (2), characterizes the properties of the citizen’s equilibrium behavior.

**PROPOSITION 1:** Bad news raises the likelihood of successful revolt: $\frac{\partial P(\rho)}{\partial \rho} > 0$. The citizen almost always revolts when the news is very bad, $\lim_{\rho \to -\infty} P(\rho) = r$; but almost never revolts when it is very good, $\lim_{\rho \to \infty} P(\rho) = 0$.

Proposition 1 contains all of the key structure that we exploit in our analysis of censorship. Specifically, the equilibrium likelihood of revolution is a differentiable function of the media reports about the net payoff from preserving the status quo versus successful revolution with the properties that revolt almost always occurs following extremely bad news about the status quo, but almost never occurs after extremely good news.
Our substantive findings only rely on this structure together with modest distributional assumptions such as log concavity. Appendix A presents a multicitizen model, where citizens must coordinate for a revolution to succeed. The natural equilibrium to this multicitizen model delivers exactly the same key properties. Section V shows that our qualitative conclusions extend when the ruler does not always succeed in concealing news when he attempts to censor it. That is, we allow the likelihood that censorship succeeds to depend continuously on the resources that the ruler devotes to concealing the news. We also establish robustness to the possibility that the citizen cares differently about $\epsilon$ and $\gamma$.

Censorship Equilibrium.—Let $P_\gamma$ be the probability of revolution following a report $\gamma$, and $P_\emptyset$ be the probability of revolution following no report about $\gamma$: $P_\gamma = rG(R - \gamma - \mu_e)$ and $P_\emptyset = rG(R - E[\gamma|\emptyset] - \mu_e)$. The ruler censors a report $\gamma$ if and only if his expected payoff from censoring, $1 - P_\emptyset - c$, exceeds that of not censoring, $1 - P_\gamma$. That is,

$$\sigma_r(\gamma) = 1 \text{ if and only if } P_\gamma - P_\emptyset > c.$$ 

$P_\gamma$ decreases monotonically in $\gamma$ and $P_\emptyset$ does not depend on $\gamma$. Thus, the ruler adopts a cutoff strategy in equilibrium, censoring $\gamma$ if and only if it is below some critical cutoff, $\bar{\gamma}$.

**LEMMA 1:** The ruler’s equilibrium censorship strategy takes a cutoff form: there exists a $\bar{\gamma} \in \mathbb{R} \cup \{-\infty, \infty\}$ such that $\sigma_r(\gamma) = 1$ if and only if $\gamma < \bar{\gamma}$ for some $\bar{\gamma} \in \mathbb{R} \cup \{-\infty, \infty\}$, where $\bar{\gamma} = -\infty$ corresponds to never censoring, and $\bar{\gamma} = \infty$ corresponds to censoring everything.

The ruler censors reports of $\gamma$ whenever the likelihood of revolution following that report, $P_\gamma$, exceeds the probability of revolution following no report $P_\emptyset$ by a margin that exceeds the censorship cost $c$. In equilibrium, the citizen’s beliefs must be consistent with the ruler’s equilibrium censorship cutoff $\bar{\gamma}_e$: $P_\emptyset$ implicitly depends on the equilibrium censorship cutoff $\bar{\gamma}_e$. Thus, $\bar{\gamma}_e$ is an equilibrium strategy of the ruler if and only if

$$P_\gamma - P_\emptyset(\bar{\gamma}_e) > c \text{ for all } \gamma < \bar{\gamma}_e, \text{ and } P_\gamma - P_\emptyset(\bar{\gamma}_e) \leq c \text{ for all } \gamma \geq \bar{\gamma}_e.$$ 

The equilibrium threshold depends on how the representative citizen updates when there is no media report concerning $\gamma$. Given a censorship cutoff $\bar{\gamma}$, the representative citizen’s beliefs about $\gamma$ following no media report depend on $\bar{\gamma}$ via Bayes’ rule:

$$E[\gamma|\emptyset; \bar{\gamma}] = \frac{\int_{-\infty}^\gamma \gamma \, dF(\gamma) + \int_\gamma^\infty \gamma(1 - q(\gamma)) \, dF(\gamma)}{1 - \int_\gamma^\infty q(\gamma) \, dF(\gamma)}$$

$$= \frac{E[\gamma] - \int_\gamma^\infty \gamma q(\gamma) \, dF(\gamma)}{1 - \int_\gamma^\infty q(\gamma) \, dF(\gamma)} = -\frac{\int_\gamma^\infty \gamma q(\gamma) \, dF(\gamma)}{1 - \int_\gamma^\infty q(\gamma) \, dF(\gamma)},$$
where $f$ and $F$ are the pdf and cdf of the prior distribution over $\gamma$, and the last equality reflects the normalization of $E[\gamma]$ to zero. When a ruler always censors, i.e., when $\bar{\gamma} = \infty$, the absence of a media report conveys no information about $\gamma$, so that $E[\gamma|\emptyset; \bar{\gamma} = \infty] = E[\gamma] = 0^–$, i.e., $E[\gamma|\emptyset; \bar{\gamma} = \infty]$ approaches 0 from below. When, instead, a ruler never censors, i.e., when $\bar{\gamma} = -\infty$, the absence of a media report can contain information about $\gamma$ because the likelihood $q(\gamma)$ that the media discovers news can hinge on the nature of the news:

(6) \[
\bar{\gamma} \equiv \lim_{\gamma \to -\infty} E[\gamma|\emptyset; \bar{\gamma}] = -\frac{\int_{-\infty}^{\gamma} q(\gamma) \ dF(\gamma)}{1 - \bar{q}},
\]

where $\bar{q} \equiv \int_{-\infty}^{\gamma} q(\gamma) \ dF(\gamma)$ is the mean likelihood that the media uncovers some news. For example, if the media is more likely to discover good news and the ruler never censors, then the absence of news makes the citizen update negatively, i.e., when $q(\gamma)$ is increasing in $\gamma$, then $\bar{\gamma} < E[\gamma] = 0$. Alternatively, if the ruler never censors and $q(\gamma)$ is constant or $q(\gamma)$ and $f(\gamma)$ are both symmetric, then the absence of media report conveys no information about $\gamma$, so that $\bar{\gamma} = E[\gamma] = 0$. Differentiating equation (5) with respect to $\bar{\gamma}$ yields

(7) \[
\frac{dE[\gamma|\emptyset; \bar{\gamma}]}{d\bar{\gamma}} = \frac{[\gamma q(\gamma) f(\gamma)] [1 - \int_{-\infty}^{\gamma} q(\gamma) \ dF(\gamma)] - [q(\gamma) f(\gamma)] [\int_{-\infty}^{\gamma} q(\gamma) \ dF(\gamma)]}{[1 - \int_{-\infty}^{\gamma} q(\gamma) \ dF(\gamma)]^2} \]

\[
= \frac{q(\gamma) f(\gamma)}{1 - \int_{-\infty}^{\gamma} q(\gamma) \ dF(\gamma)} (\gamma - E[\gamma|\emptyset; \bar{\gamma}]).\]

Figure 1 illustrates the two key features describing how the citizen updates following no media report: (i) how $E[\gamma|\emptyset; \bar{\gamma}]$ varies with the censorship threshold $\bar{\gamma}$, and (ii) how the citizen would update were he to see the threshold news $\bar{\gamma}$. The citizen always updates more negatively following extremely bad news than following no media report, because there is always a chance that no news is due to the media not observing $\gamma$ rather than censorship: $E[\gamma|\emptyset; \bar{\gamma}] > \bar{\gamma}$ for sufficiently low $\bar{\gamma}$. That is, when a ruler only censors very bad news, the marginal news censored $\bar{\gamma}$ is worse than the average $E[\gamma|\emptyset; \bar{\gamma}]$. Thus, when a ruler censors more, then, as equation (7) shows, the citizen updates more negatively following no news up to the point where the marginal news equals the average, i.e., up to the $\gamma_m$ that solves $E[\gamma|\emptyset; \gamma_m] = \bar{\gamma}_m$. Thereafter, the marginal news $\bar{\gamma}$ exceeds the average $E[\gamma|\emptyset; \bar{\gamma}]$, so censoring more causes the citizen to update less negatively following no media report. Moreover, since $0 = E[\gamma] > E[\gamma|\emptyset; \bar{\gamma}]$, for all $\gamma > 0$, it must be that at the unique value of $\gamma$ solving $\bar{\gamma} = E[\gamma|\emptyset; \gamma]$ that $\gamma_m < 0$, implying that the marginal news censored is bad news, and no news is bad news.

**LEMMa 2:** The citizen’s estimate $E[\gamma|\emptyset; \bar{\gamma}]$ of $\gamma$ following no media report and given the ruler’s censorship cutoff $\bar{\gamma}$ has a unique extremum (a minimum) at $\gamma_m < 0$, where $E[\gamma|\emptyset; \gamma_m] = \gamma_m$. $E[\gamma|\emptyset; \bar{\gamma}] > \bar{\gamma}$ if $\gamma < \gamma_m$, and $E[\gamma|\emptyset; \bar{\gamma}] < \bar{\gamma}$ if $\gamma > \gamma_m$. Moreover, $E[\gamma|\emptyset; \bar{\gamma}]$ is continuous in $\bar{\gamma}$, with $\lim_{\gamma \to -\infty} E[\gamma|\emptyset; \bar{\gamma}] = E[\gamma] = 0$ and $\lim_{\gamma \to +\infty} E[\gamma|\emptyset; \bar{\gamma}] = \bar{\gamma}$.
These features of the representative citizen’s belief imply that censoring extremely bad news benefits a ruler because the citizen updates less negatively following no media report than following extremely bad news:

\[
\bar{c} \equiv \lim_{\gamma \to -\infty} [P_{\bar{\gamma}} - P_\emptyset(\bar{\gamma})] = r - P_{\bar{\gamma} = \bar{\gamma}} = r - rG(R - \hat{\gamma} - \mu_r).
\]

As a ruler censors more, this gain \( P_{\bar{\gamma}} - P_\emptyset(\bar{\gamma}) \) falls until it becomes zero at \( \bar{\gamma} = \bar{\gamma}_m \). Beyond this point, a ruler is better off allowing the threshold news \( \bar{\gamma} \) to reach the citizen rather than censoring it. It follows that there is a unique, finite, equilibrium censorship cutoff, \( \bar{\gamma}_e \), at which \( P_{\bar{\gamma}_e} - P_\emptyset(\bar{\gamma}_e) = c \) as long as the censorship cost is low enough relative to the payoff from preventing successful revolution, i.e., as long as \( c < \bar{c} \):

**PROPOSITION 2:** There is a level of censorship costs \( \bar{c} \in (0, r) \) above which the ruler does not censor. If \( c < \bar{c} \), then in the unique equilibrium, the ruler censors all media reports worse than a threshold news \( \bar{\gamma}_e \in \mathbb{R} \), where \( \bar{\gamma}_e \) is the unique solution to \( P_{\bar{\gamma}_e} - P_\emptyset(\bar{\gamma}_e) = c \).

The next corollary highlights that it is the structure of the citizen’s belief following no media report, and not the cost of censorship, that discourages a ruler from censoring all bad news: revealing modestly bad news that slightly lowers the citizen’s expectation below the prior (i.e., \( \bar{\gamma}_m < \gamma < E[\gamma] = 0 \)) is better for a ruler than no news, because news that is concealed could be far worse.

**COROLLARY 1:** Even when censorship is costless, a ruler never censors all bad news. That is, \( \bar{\gamma}_e(c = 0) = \bar{\gamma}_m < E[\gamma] = 0 \).
In the analysis that follows, we assume that $c < \bar{c}$. We next show how the primitives of the economy affect which media reports are censored.

**COROLLARY 2:** Increases in the cost $c$ of censorship reduce a ruler’s equilibrium censorship cutoff, $\bar{\gamma}_e$. Suppose that the likelihood that the media uncovers news does not depend on the nature of the news, i.e., $q(\gamma) = q$. Then, increases in $q$ reduce a ruler’s equilibrium censorship cutoff, $\bar{\gamma}_e$; in addition, suppose that for $\bar{\gamma} < E[\gamma] = 0$, both $F(\bar{\gamma})$ and $E[\gamma|\gamma < \bar{\gamma}]$ decrease with the variance $\sigma_0^2$ of $\gamma$. Then, increases in $\sigma_0^2$ (which captures the amount of potential news about $\gamma$), reduce a ruler’s equilibrium censorship cutoff, $\bar{\gamma}_e$.

That higher censorship costs reduce censorship is obvious. A ruler also censors less when the media is more likely to uncover news. This result can be generalized: raising $q(\gamma)$ for $\gamma > 0$ or $\gamma < \bar{\gamma}$ causes the citizen to update more negatively following no news. That is, fix any censorship cutoff, and raise the probability on “bad news,” i.e., on news worse than the cutoff. Then, following no media report, the citizen updates more pessimistically about concealed news, making it less attractive to conceal the news. Next, consider raising the probability of “good news,” $\gamma > 0$. Then, following no media report, the citizen believes that it is more likely due to censorship (as it is less likely that the good, uncensored, news was not revealed), so that he again updates more negatively. In sum, when the media is more likely to uncover news that is either good or sufficiently bad that it is censored, then ceteris paribus, the citizen updates more negatively following no reported news, reducing the attraction of censorship to the ruler.\(^6\) So, too, a ruler censors less when there is more potential news: a higher $\sigma_0^2$ (e.g., when news is normally distributed) causes the citizen to update more negatively when there is no report because there is more probability mass on bad tail news. Therefore, a ruler ceases to censor more marginally bad news to prevent the citizen from drawing inferences that the news might have been far worse.

The environment may place enough structure on how $q(\gamma)$ varies with $\gamma$ that we can characterize the consequences for censorship. For example, if the ruler sometimes directly observes $\gamma$ when the media do not, and can verifiably convey $\gamma$ to the media (possibly at a cost), then the ruler censors less. The ruler never has an incentive to convey bad news to the media, but he may have an incentive to convey good news. This has an effect analogous to raising $q(\gamma)$ for $\gamma$ sufficiently greater than $\bar{\gamma}$. It follows directly for any fixed censorship cutoff $\bar{\gamma}$ that $E[\gamma|\emptyset, \bar{\gamma}]$ is reduced—the citizen updates more negatively when no news is reported. In turn, this means that bad news $\gamma$ must be worse for the ruler to censor it.

We next describe how the extreme limits of a very strong and very weak media affect a ruler’s censorship decisions.

\(^6\)Only when $q(\gamma)$ is increased for $\gamma \in [\gamma, 0]$ can a more active media cause the citizen to update more positively following no media report, raising the attraction of censorship. When $q(\gamma) = q$, the effect of raising $q$ on this interval is swamped by raising it for other $\gamma$s. More generally, let $\frac{\delta E[\gamma|\emptyset, \bar{\gamma}, q(\gamma)]}{\delta q(\gamma)} \equiv \lim_{\epsilon \to 0} E[\gamma|\emptyset, \bar{\gamma}, q(\gamma) + \epsilon] - E[\gamma|\emptyset, \bar{\gamma}, q(\gamma)]$. One can show that as long as $\int_{\gamma}^{\infty} q(\gamma) \, dF(\gamma) \geq 0$, we have $\frac{\delta E[\gamma|\emptyset, \bar{\gamma}, q(\gamma)]}{\delta q(\gamma)} < 0$. Note that when $q(\gamma)$ is constant, $\int_{\gamma}^{\infty} q(\gamma) \, dF(\gamma) = q \int_{\gamma}^{\infty} dF(\gamma) = q(1 - F(\bar{\gamma}))E[\gamma|\gamma > \bar{\gamma}] > 0$. 


COROLLARY 3: Let \( q_s = \sup q(\gamma) \) and \( q_i = \inf q(\gamma) \). A ruler censors almost nothing if the media almost always uncover \( \gamma \). Conversely, a ruler censors almost all bad news if censorship is almost costless and media almost never uncover \( \gamma \). That is, \( \lim_{q_i \to 1} \gamma_e = -\infty \) and \( \lim_{c, q_s \to 0} \gamma_e = 0 \).

II. Commitment in Censorship

A ruler’s equilibrium censorship decisions weigh the gains from reducing the likelihood of revolution when a (bad) media report is censored against the costs of censorship, ignoring the consequences for the level of the citizen’s trust in the media. We now show that a ruler would always be better off were he able to commit to censoring slightly less than he does in equilibrium, because the citizen would then update more positively when there is no media report. We first calculate a ruler’s ex ante expected payoff from an arbitrary censorship cutoff \( \gamma \):

\[
W(\gamma, c, q(\gamma)) = \int_{-\infty}^{\gamma} (1 - P_\emptyset(\gamma) - c)q(\gamma) \, dF(\gamma) + \int_{\gamma}^{\infty} (1 - P_\gamma)q(\gamma) \, dF(\gamma) \\
+ \int_{-\infty}^{+\infty} (1 - P_\emptyset(\gamma))(1 - q(\gamma)) \, dF(\gamma) \\
= 1 - \left[ \int_{-\infty}^{\gamma} q(\gamma) \, dF(\gamma) + (1 - \bar{q}) \right] P_\emptyset(\gamma) - \int_{\gamma}^{\infty} P_\gamma q(\gamma) \, dF(\gamma) \\
- c \int_{-\infty}^{\gamma} q(\gamma) \, dF(\gamma).
\]

With probability \( 1 - q(\gamma) \) the media do not observe \( \gamma \), so the ruler’s payoff is \( 1 - P_\emptyset(\gamma) \). With probability \( q(\gamma) \) the media observe \( \gamma \). But the ruler censors any report of \( \gamma < \bar{\gamma} \), in which case his payoff is \( 1 - P_\emptyset(\gamma) - c \). If \( \gamma \) exceeds \( \bar{\gamma} \), the ruler does not censor, so the citizen learns \( \gamma \), and the ruler’s payoff is \( 1 - P_\gamma \).

The citizen’s belief about \( \gamma \) in the absence of a media report depends on the ruler’s censorship policy. However, the ruler ignores this in his decision making: his censorship decisions maximize his expected payoff \textit{given} the citizen’s (equilibrium) belief. If, instead, a ruler can ex ante commit to a censorship level, he can “internalize” how his censorship rule affects updating by the citizen. For example, a ruler may be able to do this by passing censorship laws and delegating enforcement to bureaucrats, threatening to punish them if they adopt a different cutoff. Alternatively, a ruler may delegate to someone with a smaller stake in preserving the regime. Still, a ruler’s ability to commit is limited. For example, a bureaucrat who fails to censor very bad news is likely to be punished (either by the ruler or by citizens following the likely resulting successful revolution); or a ruler with good news may feel compelled to reveal it. Thus, such commitment mechanisms only allow a ruler to commit to censorship levels that differ slightly from what he would do without those mechanisms. The question becomes: Under what circumstances would a ruler want to increase or decrease his censorship cutoff marginally from its equilibrium level?
Remarkably, if censorship has any costs, i.e., if \( c > 0 \), then a ruler’s equilibrium censorship choice is never optimal from an ex ante perspective:

**PROPOSITION 3:** If censorship is costly, a ruler’s ex ante expected payoff would be raised if he could commit to censoring slightly less than he does in equilibrium:

\[
\frac{dW(\gamma)}{d\gamma} \bigg|_{\gamma = \gamma_e} < 0 .
\]

Small reductions in the censorship cutoff below the equilibrium threshold have three effects: (i) When there is a report, the citizen now sees marginal realizations of \( \gamma \) that are slightly worse than the equilibrium cutoff \( \gamma_e \). For these marginal news events, the probability of revolution rises from \( P(\gamma - \epsilon) \) to \( P(\gamma - \gamma_e) \). However, (ii) the ruler does not incur censorship costs. At equilibrium, fixing the citizen’s belief, these two effects just offset each other, as a ruler’s equilibrium cutoff \( \gamma_e \) equates these marginal costs and benefits of censorship. Thus, the net impact on a ruler’s ex ante expected payoff equals the third effect: (iii) A slight reduction in the censorship cutoff reduces the probability of revolution \( P(\gamma - \epsilon) \) when there is no media report. To see this, recall that as a ruler censors more news, the citizen updates more negatively following no media report, up to the point where the ruler censors all news worse than \( \gamma_m \) (see Figure 1); and \( \gamma_e < \gamma_m \) when censorship is costly. Therefore, the probability of revolution \( P(\gamma - \epsilon) \) is rising in \( \gamma \) at the equilibrium cutoff \( \gamma_e \). Hence, the net effect of marginally reducing the censorship cutoff is to raise the ruler’s ex ante expected payoff.

The sole restriction in the proposition is that censorship be costly. The reason is that when censorship is costless, the citizen’s belief becomes locally insensitive to changes in the equilibrium censorship cutoff: When \( c = 0 \), we have \( \gamma_e = \gamma_m \), i.e., the ruler’s equilibrium level of censorship minimizes \( E[\gamma|\emptyset; \gamma] \). Hence, marginal changes in \( \gamma \) do not affect the citizen’s belief about \( E[\gamma|\emptyset; \gamma] \) (see Lemma 2 and Figure 1), i.e., \( \frac{dE[\gamma|\emptyset; \gamma]}{d\gamma} = 0 \). This directly implies that \( \frac{dW(\gamma_e)}{d\gamma} = 0 \), which might lead one to conjecture that this censorship cutoff minimizes or maximizes the ruler’s ex ante expected payoff. However, this is not so:

**Remark 1:** When censorship is costless, i.e., when \( c = 0 \), a ruler’s equilibrium censorship cutoff is an inflection point of his ex ante expected payoff. That is, not only is \( \frac{dW(\gamma_e)}{d\gamma} = 0 \), but so, too is \( \frac{d^2W(\gamma_e)}{d\gamma^2} = 0 \).

### III. Censorship Costs and Media Strength

New technologies such as the Internet and cell phones have significantly increased the costs of censorship. The extensive use of these technologies in the Arab Spring, the Green Movement in Iran, and the Orange Movement in Kuwait (Wheeler 2010) has led many to conclude that technologies that make censorship difficult, i.e., increase censorship costs, must work against dictators. We show that such assessments are only partially correct. To the contrary, higher costs of censorship can sometimes increase a ruler’s chance of survival. In fact, these gains can be high enough to offset the greater direct costs of censorship and make the ruler better off. Similarly, one might suspect that dictators always prefer a weak media that rarely uncovers \( \gamma \) to a strong media.
This conjecture is also wrong. In fact, when the ruler faces a serious risk of revolution, he prefers a strong media that almost always uncovers $\gamma$ to a weak media.

A. Higher Censorship Costs

Higher censorship costs have direct and indirect effects. The direct effect reduces a ruler’s ex ante expected payoff because a ruler must pay more each time he censors. The indirect effect is that higher censorship costs cause a ruler to censor less in equilibrium. We just established that decreases in the equilibrium level of censorship cause the citizen to update less negatively following no media report—this indirect effect of increases in censorship costs raises a ruler’s ex ante expected payoff. Proposition 4 shows that when the costs of censorship are small, the direct effect always dominates, so that marginal increases in censorship costs reduce the ruler’s ex ante expected payoff. However, when censorship costs are higher, and the expected payoffs from successful revolution exceed the expected punishment from failed revolt, then the indirect effect eventually dominates, so that further increases in censorship costs benefit the ruler:

PROPOSITION 4: Suppose $F(\gamma)$ and $q(\gamma)$ are log-concave, $G(\epsilon)$ is strictly unimodal with a median that does not exceed its mean, and $r R \geq (1 - r) \mu$. Then there exists a critical censorship cost $\tilde{c} > 0$ such that increases in a ruler’s cost of censorship raise his ex ante expected payoff if and only if $c > \tilde{c}$.

When censorship is costless, i.e., when $c = 0$, the equilibrium level of censorship is at the minimum of $E[\gamma|\emptyset; \gamma_e]$, i.e., $\tilde{\gamma}_c = \tilde{\gamma}_m$. Hence, marginal changes in the extent of censorship do not affect the citizen’s belief about $\gamma$ following no media report. Therefore, a marginal increase in censorship cost from zero only has the direct effect of imposing positive censorship costs on the ruler. This reasoning extends whenever censorship costs are sufficiently low, in which case the ruler is likely to censor (thereby incurring the higher censorship cost), but the improvement in the citizen’s belief is modest.

However, the proposition identifies sufficient conditions such that once censorship costs are high enough, the indirect effect dominates: for censorship costs exceeding $\hat{c}$, further increases in censorship costs raise the ruler’s expected payoff due to the less pessimistic updating by the citizen, and to the fact that the ruler is less likely to incur the censorship cost. Intuitively, the indirect effect can dominate once censorship costs are intermediate so that $P_{\emptyset}(\hat{\gamma} = \tilde{\gamma}_c(c))$ is sensitive to extent of censorship, $\tilde{\gamma}_c$, and the censorship cutoff is low enough that the ruler is not too likely to censor (and thus incur the higher direct censorship costs). See Figure 2. It follows that there exists a cost threshold $\bar{c} < \hat{c}$ such that once censorship costs reach $\bar{c}$, the ruler would be strictly better off with censorship costs that are so high that they discourage all censorship. Log-concavity of $f(\gamma)q(\gamma)$ alone is enough to guarantee that the ruler’s ex ante expected payoff rises once $c$ is sufficiently high (close enough to $\bar{c} = r - P_{\emptyset}(\hat{\gamma} = \tilde{\gamma})$).

The remaining structure implies that $\frac{g(R - \mu_r - \tilde{\gamma}_e)}{g(R - \mu_r - E[\gamma|\emptyset; \gamma_e])}$ decreases in $c$, which is a sufficient condition that ensures $\frac{dW(\tilde{\gamma}_c(c), c)}{dc}$ has a single-crossing property.
B. Media Strength

Increases in censorship costs work as a commitment device for the ruler, making him censor less in equilibrium. This logic suggests a similar role for the likelihood $q(\gamma)$ that the media uncover $\gamma$. Suppose $q(\gamma) = q$. Then, marginal increases in $q$ benefit the ruler by making him censor marginally less in equilibrium, but increases in $q$ raise the expected censorship costs $qF(\gamma)e$.

However, there is also a third effect: given any censorship cutoff $\gamma$, increases in $q$ cause the representative citizen to update more negatively following no media report. This third effect makes a comprehensive analysis intractable absent strong distributional assumptions. This leads us, instead, to analyze the ruler’s ex ante expected payoff in the two polar cases where the media almost always uncovers $\gamma$ and where the media almost never uncovers it.

A ruler facing a weak media that almost never uncovers $\gamma$ is in the same situation as a ruler who commits to censoring almost everything—when censorship is costless. In both cases, the citizen updates only marginally negatively following no media report. Conversely, a ruler facing a strong media that almost always uncovers $\gamma$ is in the same situation as a ruler who commits to censor almost nothing. Proposition 5 compares a ruler’s ex ante expected payoff if he censors almost everything with his expected payoff if he censors almost nothing.

**PROPOSITION 5:** Suppose $G(\epsilon)$ is strictly unimodal and $q(\gamma)f(\gamma)$ is symmetric. When $R$ or $c$ is sufficiently large, a ruler’s ex ante expected payoff is higher if he censors nothing than if he censors everything: $\lim_{\gamma \to \infty} W(\gamma; R) < \lim_{\gamma \to -\infty} W(\gamma; R)$. Conversely, when $R$ and $c$ are sufficiently small, the opposite result obtains: $\lim_{\gamma \to \infty} W(\gamma; R) > \lim_{\gamma \to -\infty} W(\gamma; R)$.

When $g(\epsilon)$ is symmetric, one can strengthen the proposition: when $c = 0$, a ruler is better off censoring nothing than censoring everything if and only if $rR > (1 - r)\mu$.

The intuition is that when $R$ is high enough that the citizen is likely to revolt based on his prior, a ruler is better off rolling the dice on media reports, hoping for good news that would forestall revolution: unimodality of $G$ and symmetry of $f(\gamma)q(\gamma)$ imply that when $R$ is high, bad news only marginally increases the likelihood of revolt, but good news can sharply reduce that probability. If, instead, $R$ is low, it...
takes unusually bad news to make the citizen revolt—the effect of good news on
the probability of revolt is marginal, while the adverse effect of bad news is sharp,
so the ruler prefers to censor everything rather than nothing. Corollary 4 states the
implications for a ruler’s ex ante preference for the strength of media.

**COROLLARY 4:** Suppose \( G(\epsilon) \) is strictly unimodal and \( f(\gamma) \) and \( q(\gamma) \) are symmetric. Then there exists a critical threshold \( R^* \) such that if \( R > R^* \), a ruler prefers a strong media that almost always uncover \( \gamma \) to a weak media that almost never does: if \( R > R^* \), \( \lim_{q_s \to 0} W(\bar{\gamma}_e(q); R) < \lim_{q_i \to 1} W(\bar{\gamma}_e(q); R) \). If, instead, \( R < R^* \), a ruler prefers a weak media to a strong one: if \( R < R^* \), then \( \lim_{q_s \to 0} W(\bar{\gamma}_e(q); R) > \lim_{q_i \to 1} W(\bar{\gamma}_e(q); R) \).

When revolution is likely, i.e., when the revolution payoff appears high relative
to the ex ante status quo payoff, a ruler’s best hope for survival is that the citizen
receives good news and updates positively. The chances that media uncovers and
reports such good news is highest when the media is very strong \( (q_i \equiv \inf q(\gamma) \to 1) \).
Of course, a strong media may uncover bad news that further raises the likelihood
of revolt, but because this likelihood is already very high, the possible gains from
sharply positive news outweigh any loss from more bad news that can only raise the
likelihood of revolt marginally. This result offers a rationale for why dictators some-
times relax censorship amidst unfolding revolutions. For example, Milani (2011,
388) notes that the Shah relaxed censorship in 1978 during the unfolding of the 1979
Iranian Revolution (see also Milani 1994, 117; Arjomand 1984, 115).

**IV. Extensions**

We study two extensions of the model. First, we make the state’s choice of censor-
ship continuous, so that an attempt to censor news is more likely to succeed when the
state devotes more resources to censoring. We then generalize the citizen’s payoff (from
successful revolution) from \( R - \gamma - \epsilon \) to \( R - \lambda \gamma - \epsilon \), to capture how the degree of
relative importance of news in the citizen’s payoff affects censorship decisions.

**Continuous Choice of Censorship.**—In practice, a ruler’s attempts to censor a media
report may fail. Advances in communication technologies have not only increased
the cost of censorship, but have also raised the possibility that censorship may fail.
For example, a ruler may not be able to conceal beatings of protesters if videos can
be taken with cell phones and uploaded to YouTube. One can model this by assuming
that a ruler who tries to censor succeeds in concealing the news with probability \( \zeta \).
We focus on \( \zeta = 1 \), but the effects of \( \zeta < 1 \) are straightforward to derive. In par-
ticular, when \( \zeta \) is smaller, the ruler reduces his cutoff, as censorship is less effect-
tive. Consequently, the citizen updates less negatively following no media report

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7 Kamenica and Gentzkow (2011) analyze a sender’s design of a signal technology that provides a decision maker (receiver) a signal about the state of the world. Our result contrasts with their finding that it can be optimal for the sender to design a signal technology that reveals everything to the receiver if interests are closely aligned, and to reveal nothing if interests are less aligned (Kamenica and Gentzkow 2011, 2604–06).
because it is less likely due to censorship. Qualitatively, the impact of a higher $\zeta$ is similar to that of a stronger media that uncovers $\gamma$ with a higher probability $q$.

To illustrate more nuanced implications, we now suppose that the likelihood the ruler succeeds in censoring a media report depends on the level of resources he devotes to censoring that news. In particular, if the ruler expends $c$, he succeeds in censoring with probability $\zeta(c)$, where $\zeta(0) = 0$, $\zeta'(\cdot) > 0$, $\zeta''(\cdot) < 0$, and $\zeta(1) < 1$ means that it is never optimal to spend enough to guarantee that censorship succeeds. Thus, with probability $1 - \zeta(c) > 0$, censorship fails and the citizen sees the media report of $\gamma$. The ruler’s expected payoff if he devotes $c$ to censoring $\gamma$ is $\zeta(c)(1 - P_\emptyset) + (1 - \zeta(c))(1 - P_\gamma) - c$. Hence, the ruler’s optimization problem simplifies to

$$\max_{c \geq 0} \zeta(c)(P_\gamma - P_\emptyset) - c.$$ 

Let $C(\gamma)$ be the function indicating the resources devoted to censorship when the media would reveal $\gamma$, and let $P_\emptyset(C)$ be the equilibrium probability of revolution following no report. Focusing on the case where $\zeta'(0)$ is large enough that some efforts at censorship are optimal when the news is extremely bad, there exists a news threshold $\bar{\gamma}$ at which $\zeta'(0)(P_\gamma - P_\emptyset) = 1$. Then, the resources $C(\gamma)$ that the ruler devotes to censoring $\gamma$ solve

$$\zeta(C(\gamma))(P_\gamma - P_\emptyset) = 1 \text{ for } \gamma < \bar{\gamma}, \text{ and } C(\gamma) = 0 \text{ for } \gamma \geq \bar{\gamma}. \quad (9)$$

Because $P_\gamma$ decreases in $\gamma$, it follows that $C(\gamma)$ decreases in $\gamma$: the ruler devotes more resources to censoring worse news. To ease presentation, suppose that $q(\gamma) = q$, for all $\gamma$ (the analysis extends routinely). Then, given the ruler’s strategy $C(\gamma)$, the citizen’s belief about the expected value of $\gamma$ following no news becomes:

$$E[\gamma|\emptyset; C(\gamma)] = \frac{q \int_{-\infty}^{\bar{\gamma}} \gamma \zeta(C(\gamma)) \, dF(\gamma)}{1 - q + q \int_{-\infty}^{\bar{\gamma}} \zeta(C(\gamma)) \, dF(\gamma)}, \quad (10)$$

which determines the equilibrium $P_\emptyset(C)$. The ruler’s ex ante expected payoff is

$$W(C) = (1 - q)(1 - P_\emptyset(C)) + q \int_{-\infty}^{\bar{\gamma}} \left[ \zeta(C(\gamma))(1 - P_\emptyset(C)) + (1 - \zeta(C(\gamma)))(1 - P_\gamma) \right] \, dF(\gamma) - q \int_{-\infty}^{\bar{\gamma}} C(\gamma) \, dF(\gamma) + q \int_{\bar{\gamma}}^{\infty} (1 - P_\gamma) \, dF(\gamma).$$

Algebra reveals that $1 - W(C)$ simplifies to

$$(1 - q)P_\emptyset(C) - q \int_{-\infty}^{\bar{\gamma}} \left\{ \zeta(C(\gamma))[P_\gamma - P_\emptyset(C)] - C(\gamma) \right\} \, dF(\gamma) + q \int_{-\infty}^{\infty} P_\gamma \, dF(\gamma).$$
Taking the pointwise derivative with respect to $C$ yields: \[ \frac{\delta W}{\delta C} = q \int_{-\infty}^{\gamma} \left\{ \zeta'(C(\gamma)) [P_\gamma - P_\emptyset(C)] - 1 \right\} dF(\gamma) \]

\[ - \left( q \int_{-\infty}^{\gamma} \zeta'(C(\gamma)) \frac{dP_\emptyset(C)}{dC} dF(\gamma) + (1 - q) \frac{dP_\emptyset(C)}{dC} \right) \]

\[ = - \left( (1 - q) \frac{dP_\emptyset(C)}{dC} + q \int_{-\infty}^{\gamma} \zeta'(C(\gamma)) \frac{dP_\emptyset(C)}{dC} dF(\gamma) \right) \]

at equilibrium, where the second equality follows from the equilibrium condition (9). Moreover, from equation (10), \( \frac{\delta E[\gamma|\emptyset;C]}{\delta C} < 0 \) for $\gamma < \bar{\gamma}$. Thus, \( \frac{\delta P_\emptyset(C)}{\delta C} > 0 \), and hence \( \frac{\delta W}{\delta C} < 0 \) at equilibrium. For each $\gamma < \bar{\gamma}$, the ruler equates the marginal benefit from greater censorship to its marginal cost, ignoring the adverse consequences for how the citizen updates following no report. It follows that, facing a bad report that would reveal $\gamma < \bar{\gamma}$, the ruler expends excessive resources trying to reduce the probability that the citizen receives that report: the ruler would be better off if he devoted slightly less resources to censoring bad news, i.e., if he slightly reduced $C(\gamma)$ pointwise for each $\gamma < \bar{\gamma}$.

**Relative Importance of News.**—We assumed that the citizen values $\gamma$ and $\epsilon$ equally. Now, we introduce a parameter $\lambda > 0$ that captures the relative importance of $\gamma$ and $\epsilon$ for citizen payoffs: the citizen’s net payoff from successful revolution becomes $R - \lambda \gamma - \epsilon$. Then, the citizen’s decision of whether to revolt hinges on whether $\epsilon < R - \lambda E[\gamma|\Omega] - \mu_r$. Because the decision of whether to censor depends on the difference in the citizen’s updating following news $\gamma$ versus following no report (see Figure 1), we only need to derive the impact of $\lambda$ on $\lambda (E[\gamma|\emptyset] - \gamma)$. But $\lambda$ enters as a scaling factor: increases in $\lambda$ widen the gap between $\gamma$ and $E[\gamma|\emptyset]$, which, in turn, widen the gap between $P_\gamma$ and $P_\emptyset$. It follows directly that raising the payoff weight on $\gamma$ results in greater censorship: $\frac{d^2 \epsilon}{d\lambda} > 0$.

Our paper investigates the dilemma of an unpopular ruler facing citizens who are deciding whether or not to revolt based on the public and private information that they gather. A ruler can manipulate their information-gathering process through censorship, preventing the media from disseminating information about the regime that would raise the likelihood of revolt. Unfortunately for the ruler, citizens account for his incentives to conceal bad news. Not knowing whether an absence of news was due to censorship or because the media failed to uncover politically-relevant news, citizens update negatively when no news is reported—“no news” becomes “bad news.” And yet, because citizens are not sure whether an absence of news is due to censorship, when bad news arrives, the ruler can increase his chances of survival by censoring it.

\[ ^{8} \text{By } \frac{\delta W}{\delta C} \text{ we mean } \lim_{\epsilon \to 0} \frac{W[C(\gamma) + \epsilon] - W[C(\gamma)]}{\epsilon}. \]
Remarkably, we show that a ruler’s ex ante expected payoffs are always increased if he can commit to censoring slightly less than what he does in equilibrium in order to induce citizens to update less negatively in the absence of news, i.e., so that “no news” becomes “not quite so bad news.” This does not mean that less censorship always maximizes the ruler’s ex ante expected payoffs, only that slightly less censorship is always better than the equilibrium level of censorship. Ironically, when ex ante expected revolution payoffs are high so that revolution is likely, a ruler is better off with a strong media that uncovers almost all news, causing the ruler to censor almost nothing, than with a weak one that uncovers almost nothing. However, when ex ante expected revolution payoffs are low, so that revolution is unlikely, a ruler prefers a weak media, in effect costlessly censoring almost all news, to a strong media. So, too, higher censorship costs (induced by new communication technologies) can make a ruler better off, increasing his ex ante chances of survival. Indeed, a ruler’s ex ante expected payoffs first fall and then rise with the costs of censorship.

We focus on the censorship of media. However, our model can be reposed to study elections in dictatorships, where a dictator has incentives to conceal public information. Researchers have pointed out that one role of elections in dictatorships is to send a public signal to potential protesters (Egorov and Sonin 2012), opposition (Rozenas 2010), or government officials (Gehlbach and Simpser forthcoming) about a regime’s support. Electoral fraud aside, a decision of whether to hold an election is related to the choice of which news to censor: A ruler has private information about the extent of his support, and holding an election gives potential protesters, etc., a signal about this support.

APPENDIX

A. Multicitizen Game

Our previous analysis abstracts from coordination among citizens. To analyze strategic interactions among citizens, we modify the game analyzed in Shadmehr and Bernhardt (2011) by adding an uncertain element to citizen payoffs about which they may receive a public signal. We show this framework also delivers the minimal structure (Proposition 1) that our results rely on, namely that better net news about the status quo reduces the likelihood of revolution, and there is almost always a revolution following extremely bad news, but almost never a revolution following extremely good news.

Two representative citizens, A and B, can challenge a ruler by mounting a revolution. A revolution succeeds if and only if both citizens revolt; otherwise, the status quo prevails. The expected value to citizens of the status quo is $\gamma + \epsilon$, and their expected payoff from successful revolution is $R$. If a citizen revolts and revolution fails, she receives the status quo payoff minus an expected punishment cost $\mu > 0$.

$R$ is known, but $\gamma$ and $\epsilon$ are uncertain: $\gamma \sim f$ and $\epsilon \sim g$, where $f$ and $g$ are strictly positive, continuously differentiable densities on $\mathbb{R}$. Without loss of generality, we normalize the means of $\epsilon$ and $\gamma$ to zero because only considerations of the net expected payoff from revolt (versus the status quo) determine whether citizens
revolt. Each citizen $i \in \{A, B\}$ receives a noisy private signal $s^i$ about $\epsilon$ that is independent of $\gamma$. Given their private signals about $\epsilon$ and the public information about $\gamma$, citizens simultaneously decide whether to revolt. A strategy for citizen $i$ is a function $\sigma_i$ mapping $i$'s private signal $s^i$ and any public information into a decision about whether to revolt, where $\sigma_i = 1$ indicates that citizen $i$ revolts, and $\sigma_i = 0$ indicates that $i$ does not.

Let $\Omega \in \{\gamma, \emptyset\}$ be the public information about $\gamma$, where $\emptyset$ indicates that citizens do not observe any news. Then $\rho(R, \Omega) \equiv E[R - \gamma|\Omega]$ is the citizens' public knowledge about the expected value of $R - \gamma$. In this setting, a citizen’s natural strategy takes a cutoff form: citizen $i$ revolts if and only if his private signal is less than some threshold $k_i(\rho)$. We focus on this class of strategies.

We characterize the equilibrium behavior of citizens (Lemma 5) using the following minimal structure on citizen signals:

**ASSUMPTION 1:** $s^i, s^{-i},$ and $\epsilon$ are strictly affiliated with a strictly positively, continuously differentiable density on $\mathbb{R}^3$. Let $\sigma^2_\nu$ be the variance of $s^i|\epsilon$ and $s^{-i}|\epsilon$. For every $i$ and $k$,

(a) $\lim_{s^i \to \infty} E[\epsilon|s^{-i} < k, s^i] = \infty$, \quad $\lim_{s^i \to -\infty} E[\epsilon|s^{-i} < k, s^i] = -\infty$

(b) $\lim_{s^i \to \infty} \Pr(s^{-i} < k|s^i) = 0$, \quad $\lim_{s^i \to -\infty} \Pr(s^{-i} < k|s^i) = 1$

(c) $\lim_{\sigma_\nu \to 0} \Pr(s^{-i} < k|s^i = k) = \frac{1}{2}$, \quad $\lim_{\sigma_\nu \to 0} E[\epsilon|s^i = k, s^{-i} < k] = k$.

Assumption 1 is obviously satisfied by the additive, normal noise signal structure with $s^i = \epsilon + \nu^i$, where the error $\nu^i$ is normally distributed, $\nu^i \sim N(0, \sigma_\nu)$, and is independent from both $\epsilon$ and the error $\nu^{-i}$ in the other citizen’s signal.

**LEMMA 3:** The best response to a cutoff strategy is a cutoff strategy.

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9 Outcomes would be identical if citizens received signals about $R$ because optimal actions hinge on the expected difference, $E[\gamma + \epsilon - R|\Omega]$, not the source of the difference.
PROOF OF LEMMA 3:
Citizen $i$ revolts if and only if

$$\text{Pr}(\sigma_{-i} = 1 | s^i, \Omega)R + \text{Pr}(\sigma_{-i} = 0 | s^i, \Omega)\left( E[\gamma + \epsilon | s^i, \Omega, \sigma_{-i} = 0] - \mu \right)$$

$$- E[\gamma + \epsilon | s^i, \Omega] > 0.$$  

$\gamma$ and $\epsilon$ are independently distributed, so the expected net payoff from revolt simplifies to

$$\text{Pr}(\sigma_{-i} = 1 | s^i, \Omega)R + \text{Pr}(\sigma_{-i} = 0 | s^i, \Omega)\left( E[\gamma | \Omega] + E[\epsilon | s^i, \sigma_{-i} = 0] - \mu \right)$$

$$- E[\gamma | \Omega] - E[\epsilon | s^i].$$

Collecting terms that include $E[\gamma | \Omega]$ with $R$, write this expected net payoff as

$$\text{Pr}(\sigma_{-i} = 1 | s^i, \Omega)(R - E[\gamma | \Omega]) + \text{Pr}(\sigma_{-i} = 0 | s^i, \Omega)\left( E[\epsilon | s^i, \sigma_{-i} = 0] - \mu \right) - E[\epsilon | s^i].$$

Substituting $\rho(R, \Omega) = E[R - \gamma | \Omega]$ and using the cutoff strategy structure yield

$$\text{Pr}(s^{-i} < k^{-i} | s^i)\rho(R, E[\gamma | \Omega]) + \text{Pr}(s^{-i} \geq k^{-i} | s^i)\left( E[\epsilon | s^i, s^{-i} \geq k^{-i}] - \mu \right) - E[\epsilon | s^i],$$

which can be rewritten as

$$\Delta(s^i; k^{-i}) \equiv \text{Pr}(s^{-i} < k^{-i} | s^i)\left( \rho(R, E[\gamma | \Omega]) - E[\epsilon | s^i, s^{-i} < k^{-i}] + \mu \right) - \mu.$$  

We have (a) $0 < \text{Pr}(s^{-i} < k^{-i} | s^i)$, (b) $\rho(R, E[\gamma | \Omega])$ is finite, and (c) $E[\epsilon | s^i, s^{-i} < k^{-i}]$ is strictly increasing in $s^i$. Thus, if $\Delta(x; k^{-i}) \geq 0$, then $\Delta(s^i; k^{-i}) > 0$ for all $s^i < x$. Thus, $\Delta(s^i; k^{-i}) = 0$ has the single-crossing property. Moreover,

$$\lim_{s^i \to +\infty} E[\epsilon | s^i, s^{-i} < k^{-i}] = +\infty, \quad \lim_{s^i \to -\infty} E[\epsilon | s^i, s^{-i} < k^{-i}] = -\infty,$$

so $\lim_{s^i \to +\infty} \Delta(s^i; k^{-i}) < 0$, and $\lim_{s^i \to -\infty} \Delta(s^i; k^{-i}) > 0$. These properties together with the continuity of $\Delta(s^i; k^{-i})$ imply there is a unique $k'(k^{-i})$ such that $\Delta(s^i; k^{-i}) > 0$ for all $s^i < k'$, $\Delta(s^i; k^{-i}) = 0$, and $\Delta(s^i; k^{-i}) < 0$ for all $s^i > k'$. $\blacksquare$

LEMMA 4: When the noise in private signals is vanishingly small, all cutoff equilibria are symmetric.

PROOF OF LEMMA 4:
Suppose to the contrary that an equilibrium exists in which citizens $i$ and $-i$ adopt cutoff strategies with cutoffs $k^i$ and $k^{-i}$, where without loss of generality $k^i > k^{-i}$. Suppose $s^i \in [(k^i + k^{-i})/2, k^i]$. Then, when the noise in private signals is
vanishingly small, $i$ is almost sure that $s^{-i} > k^{-i}$ and hence almost sure that $-i$ will not revolt. But then $i$ is almost surely punished if he revolts at $s^i$, so it is not a best response, a contradiction. ■

Lemma 5 shows that the citizens’ (sub)game has a unique equilibrium in finite-cutoff strategies when the noise in private signals is vanishingly small.\(^{10}\)

**LEMMA 5:** Suppose the noise in citizens’ private signals is vanishingly small: $\sigma^i_\nu \to 0$. Then there is a unique, symmetric equilibrium in finite-cutoff strategies, characterized by the equilibrium cutoff $k = \rho(R, \Omega) - \mu$, where $\rho(R, \Omega) = E[R - \gamma|\Omega]$ is the citizens’ forecast of $R - \gamma$ given public information.

**PROOF OF LEMMA 5:**

Since all cutoff strategy equilibria are symmetric, equilibria in which citizens revolt with positive probability are characterized by the zeros (roots) of the symmetric expected net payoff function $\Delta_1(k)$, where citizens adopt the common cutoff $k^{-i} = k^i = k$:

$$\Delta_1(k) \equiv \Pr(s^{-i} < k|k)\rho(R, \Omega) + \Pr(s^{-i} \geq k|k)(E[\epsilon|k, s^{-i} \geq k] - \mu) - E[\epsilon|k]$$

$$= \Pr(s^{-i} < k|s^i = k)(\rho(R, \Omega) - E[\epsilon|s^i = k, s^{-i} < k] + \mu) - \mu.$$

From Assumption 1, $\lim_{\sigma^i_\nu \to 0} \Pr(s^{-i} < k|s^i = k) = \frac{1}{2}$ and $\lim_{\sigma^i_\nu \to 0} E[\epsilon|s^i = k, s^{-i} < k] = k$, so

$$\lim_{\sigma^i_\nu \to 0} \Delta_1(k) = \Pr(s^{-i} < k|s^i = k)(\rho(R, \Omega) - k + \mu) - \mu = \frac{1}{2}(\rho(R, \Omega) - \mu - k),$$

and hence $\Delta_1(k)$ has a unique root at $k = \rho(R, \Omega) - \mu$. ■

In addition to this finite-cutoff equilibrium, there is a cutoff strategy equilibrium in which citizens never revolt, i.e., $k = -\infty$. The finite-cutoff equilibrium yields higher citizen welfare than the no-revolution equilibrium. Moreover, as recent events highlight, the never-revolution equilibrium does not describe the real world. This leads us to assume that citizens coordinate on the finite-cutoff equilibrium. Otherwise, citizens could coordinate on different equilibria after different public news reports so that, for example, if $\gamma = -1$, citizens coordinate on the equilibrium that features revolution (and high welfare), but if $\gamma = -10$, citizens coordinate on the no-revolution, low-welfare equilibrium. This would create perverse censorship incentives, possibly leading a ruler to censor $\gamma = -1$, but not $\gamma = -10$.

---

\(^{10}\)With noisier signals, there can be two equilibria in finite-cutoff strategies. The equilibrium featuring more revolution (the one consistent with our equilibrium) is the sole stable one (Shadmehr and Bernhardt 2011). Strategic behavior is qualitatively unaffected as long as the ex ante revolution payoff is not so low that the probability of revolution following no report can drop discontinuously to zero as a function of the censorship cutoff. Lemma 5 looks similar to uniqueness results in Carlsson and van Damme (1993) and Morris and Shin (2003). However, our game does not feature two-sided limit dominance, which is central to their arguments.
A revolution succeeds if and only if both citizens revolt, i.e., if and only if both $s^i < k(\rho)$ and $s^{-i} < k(\rho)$. Moreover, as the noise in signals goes to zero, i.e., as $\sigma_\nu^2 \to 0$, the probability of a successful revolution, $P(\rho)$, goes to the probability that $\epsilon < k(\rho)$. That is,

\[(A1) \quad P(\rho) = G(k(\rho)), \quad \text{where } G(\epsilon) \text{ is the cdf of } \epsilon.\]

Good news is any news that raises citizens’ expectations of the status quo payoff, while bad news is the opposite. Recall that $\rho(R, \Omega) = E[R - \gamma|\Omega]$ is the citizens’ public knowledge about the expected payoff difference between revolution and the status quo. Thus, bad news increases $\rho$. Proposition 6, which directly follows from Lemma 5 and equation (A1), states the properties of the equilibrium to the citizens’ game that we exploit.

**PROPOSITION 6:** Bad news about the status quo raises the likelihood of revolution, $\frac{\partial}{\partial \rho} P(\rho) > 0$. Citizens almost always revolt following extremely bad news, $\lim_{\rho \to -\infty} P(\rho) = 1$; and almost never revolt following extremely good news about the status quo, $\lim_{\rho \to \infty} P(\rho) = 0$.

One can show that analogous results obtain in settings where citizens know payoffs, but are uncertain about the probability a revolution would succeed if a fraction $r$ of citizens revolt, or equivalently, about a regime’s ability to suppress revolt (Boix and Svolik 2013; Edmond 2013); or in settings with standard global games structure that feature super-modularity (Persson and Tabellini 2009) or a private value structure (Bueno de Mesquita 2010).

The multicitizen revolution framework endogenizes the probability that an attempted revolt succeeds because citizens must coordinate for successful revolution. As a result, this probability varies with the parameters of the model. Nonetheless, our results carry over from the representative citizen framework, where the probability $r$ that an attempted revolt succeeded is exogenous.

**B. Proofs**

**PROOF OF COROLLARY 2:**

Note that $c$ does not affect $P_\gamma$ or $P_\emptyset$, and $q$ and $\sigma_0^2$ only affect $P_\emptyset$. When $q(\gamma) = q$, equation (5) can be written as

\[(B1) \quad E[\gamma|\emptyset; \bar{\gamma}] = \frac{q \int_{-\infty}^{\bar{\gamma}} dF(\gamma) - \int_{-\infty}^{+\infty} dF(\gamma)}{1 - q(1 - F(\bar{\gamma}))} = \frac{q F(\bar{\gamma})}{1 - q(1 - F(\bar{\gamma}))} E[\gamma|\gamma < \bar{\gamma}] \leq 0,\]
where the second equality follows from our normalization $E[\gamma] = 0$. From (B1), $E[\gamma|\emptyset; \tilde{\gamma}]$ is decreasing in $q$, and hence increases in $q$ raise $P_\emptyset(\tilde{\gamma})$—for a fixed $\tilde{\gamma}$. To see the claim for $\sigma_0^2$, from equation (B1), write $E[\gamma|\emptyset; \tilde{\gamma}]$ as

$$E[\gamma|\emptyset; \tilde{\gamma}] = A(\tilde{\gamma})E[\gamma|\gamma < \tilde{\gamma}],$$

where $A(\tilde{\gamma}) \equiv \frac{qF(\tilde{\gamma})}{1 - q + q F(\tilde{\gamma})}$. Differentiating, we have $A'E[\gamma|\gamma < \tilde{\gamma}] + AE'[\gamma|\gamma < \tilde{\gamma}] < 0$, since both terms are negative ($A$ increases in $F(\gamma)$ and $E[\gamma|\gamma < \tilde{\gamma}] < 0$; and $A$ is positive and $E[\gamma|\gamma < \tilde{\gamma}]$ decreases in $\sigma_0^2$ for $\tilde{\gamma} < 0$ since greater $\sigma_0^2$ places more probability weight on lower values of $\gamma$). Thus, to retrieve equality of $c = P_{\gamma_e} - P_\emptyset(\tilde{\gamma_e})$, we must reduce $\tilde{\gamma_e}$. ■

**PROOF OF COROLLARY 3:**

From (5), $\lim_{q \to 1} E[\gamma|\emptyset; \tilde{\gamma}] = \lim_{q \to 1} \frac{\int_{-\infty}^{\gamma} df(\gamma)}{1 - \int_{\gamma}^{\infty} df(\gamma)} = E[\gamma|\gamma < \tilde{\gamma}] < \tilde{\gamma}$, and hence $\lim_{q \to 1} \bar{\gamma}_m = -\infty$. Because $\tilde{\gamma}_e \leq \bar{\gamma}_m$, this implies $\lim_{q \to 1} \tilde{\gamma}_e = -\infty$. From (5), $\lim_{q \to 0} E[\gamma|\emptyset; \tilde{\gamma}] = 0$, and hence $\lim_{q \to 0} \bar{\gamma}_m = 0$. Moreover, by continuity and Corollary 1, $\lim_{c \to 0} \bar{\gamma}_e = \bar{\gamma}_m$, and hence $\lim_{c, q \to 0} \tilde{\gamma}_e = 0$. ■

**PROOF OF PROPOSITION 3:**

Differentiating equation (8) with respect to $\tilde{\gamma}$ yields

$$\frac{dW}{d\tilde{\gamma}} = -q(\tilde{\gamma})f(\tilde{\gamma})P_\emptyset(\tilde{\gamma}) - \left[ \int_{-\infty}^{\tilde{\gamma}} q(\gamma) dF(\gamma) + (1 - \bar{q}) \right] \frac{dP_\emptyset(\tilde{\gamma})}{d\tilde{\gamma}} $$

$$+ P_{\gamma}(\tilde{\gamma})f(\tilde{\gamma}) - cq(\tilde{\gamma})f(\tilde{\gamma})$$

$$= q(\tilde{\gamma})f(\tilde{\gamma})[P_{\gamma} - P_\emptyset(\tilde{\gamma}) - c] - \left[ \int_{-\infty}^{\tilde{\gamma}} q(\gamma) dF(\gamma) + (1 - \bar{q}) \right] \frac{dP_\emptyset(\tilde{\gamma})}{d\tilde{\gamma}}.$$

From Proposition 2, $P_{\gamma_e} - P_\emptyset(\tilde{\gamma_e}) = c$. Since $c > 0$, we have $\tilde{\gamma}_e < \bar{\gamma}_m$, so $\frac{dP_\emptyset(\tilde{\gamma})}{d\tilde{\gamma}} > 0$. Thus,

$$\frac{dW}{d\tilde{\gamma}} \bigg|_{\tilde{\gamma}=\tilde{\gamma}_e} = -\left[ \int_{-\infty}^{\tilde{\gamma}_e} q(\gamma) dF(\gamma) + (1 - \bar{q}) \right] \frac{dP_\emptyset(\tilde{\gamma}_e)}{d\tilde{\gamma}} < 0. ■$$

**PROOF OF REMARK 1:**

From equation (B2),

$$\frac{d^2W}{d\tilde{\gamma}^2} = \frac{d[q(\tilde{\gamma})f(\tilde{\gamma})]}{d\tilde{\gamma}} \left[ P_{\gamma} - P_\emptyset(\tilde{\gamma}) - c \right] + q(\tilde{\gamma})f(\tilde{\gamma}) \left[ \frac{dP_{\gamma}}{d\tilde{\gamma}} - \frac{dP_\emptyset(\tilde{\gamma})}{d\tilde{\gamma}} \right]$$

$$- q(\tilde{\gamma})f(\tilde{\gamma}) \frac{dP_\emptyset(\tilde{\gamma})}{d\tilde{\gamma}} - \left[ \int_{-\infty}^{\tilde{\gamma}} q(\gamma) dF(\gamma) + 1 - \bar{q} \right] \frac{d^2P_\emptyset(\tilde{\gamma})}{d\tilde{\gamma}^2}.$$
At equilibrium, \( P_{\gamma} - P_{\psi}(\gamma_e) = c \). Further, when \( c = 0, \gamma_e = \gamma_m \), so \( \frac{dP_{\psi}(\gamma_e)}{d\gamma} = 0 \). Thus, equation (B4) simplifies to

\[
(B5) \quad \frac{d^2W}{d\gamma^2} = q(\gamma)f(\gamma) \frac{dP_{\gamma}}{d\gamma} - \left[ \int_{-\infty}^{\gamma} q(\gamma) \, dF(\gamma) + 1 - \bar{q} \right] \frac{d^2P_{\psi}(\gamma)}{d\gamma^2}.
\]

Moreover,

\[
(B6) \quad \frac{dP_{\gamma}}{d\gamma} = -rg(R - \mu_r - \gamma); \quad \frac{dP_{\psi}(\gamma)}{d\gamma} = -\frac{dE[\gamma|\theta; \gamma]}{d\gamma} \, rg(R - \mu_r - E[\gamma|\theta; \gamma]).
\]

Thus,

\[
(B7) \quad \frac{d^2P_{\psi}(\gamma)}{d\gamma^2} = \frac{d^2E[\gamma|\theta; \gamma]}{d\gamma^2} \, rg(R - \mu_r - E[\gamma|\theta; \gamma]),
\]

where we used the fact that when \( c = 0, \gamma_e = \gamma_m \), and hence \( \frac{dE[\gamma|\theta; \gamma]}{d\gamma} = 0 \). Moreover, when \( c = 0, \gamma_e = E[\gamma|\theta; \gamma] \). Thus, from equations (B6) and (B7),

\[
(B8) \quad \frac{d^2P_{\psi}(\gamma)}{d\gamma^2} = \frac{d^2E[\gamma|\theta; \gamma]}{d\gamma^2} \, \frac{dP_{\gamma}}{d\gamma}.
\]

Substituting equation (B8) into equation (B5) yields

\[
(B9) \quad \frac{d^2W}{d\gamma^2} = \frac{dP_{\gamma}}{d\gamma} \left( q(\gamma)f(\gamma) - \left[ \int_{-\infty}^{\gamma} q(\gamma) \, dF(\gamma) + 1 - \bar{q} \right] \frac{d^2E[\gamma|\theta; \gamma]}{d\gamma^2} \right) \text{(at } \gamma = \gamma_e = \gamma_m \text{)}.
\]

From equation (7),

\[
\frac{q(\gamma)f(\gamma)}{1 - \int_{-\infty}^{\gamma} q(\gamma) \, dF(\gamma)} \left( \gamma - E[\gamma|\theta; \gamma] \right).
\]

\[
(B10) \quad \frac{d^2E[\gamma|\theta; \gamma]}{d\gamma^2} = \frac{d}{d\gamma} \left( \frac{q(\gamma)f(\gamma)}{1 - \int_{-\infty}^{\gamma} q(\gamma) \, dF(\gamma)} \right) \left( \gamma - E[\gamma|\theta; \gamma] \right)
\]

\[
+ \frac{q(\gamma)f(\gamma)}{1 - \int_{-\infty}^{\gamma} q(\gamma) \, dF(\gamma)} \left( 1 - \frac{dE[\gamma|\theta; \gamma]}{d\gamma} \right)
\]

\[
= \frac{q(\gamma)f(\gamma)}{1 - \int_{-\infty}^{\gamma} q(\gamma) \, dF(\gamma)} \text{(at } \gamma = \gamma_e = \gamma_m, \text{ when } c = 0 \text{)}
\]

\[
= \frac{q(\gamma)f(\gamma)}{1 - \int_{-\infty}^{\gamma} q(\gamma) \, dF(\gamma)} + \int_{-\infty}^{\gamma} q(\gamma) \, dF(\gamma).
\]
Substituting equation (B10) for \( \frac{d^2E[\gamma|\theta; \gamma]}{d\gamma^2} \) into equation (B9) yields

\[
\frac{d^2W}{d\gamma^2}\bigg|_{\gamma=\gamma_e} = \frac{dP_{\gamma_e}}{d\gamma} \left( qf(\gamma_e) - qf(\bar{\gamma}_e) \right) = 0. \]

**PROOF OF PROPOSITION 4:**

(B11) \[ \frac{dW(\bar{\gamma}(c), c)}{dc} \bigg|_{\gamma=\gamma_e} = \frac{\partial W(\bar{\gamma}(c), c)}{\partial \gamma} \bigg|_{\gamma=\gamma_e} \frac{d\gamma_e(c)}{dc} + \frac{\partial W(\bar{\gamma}_e(c), c)}{\partial c}. \]

From Proposition 2,

(B12) \[ \frac{d\gamma_e(c)}{dc} = \left( \frac{dP_{\gamma=\gamma_e}}{d\gamma} - \frac{dP_{\theta=\bar{\gamma}_e}}{d\gamma} \right)^{-1} = -\left( \frac{dP_{\theta=\bar{\gamma}_e}}{d\gamma} + \left| \frac{dP_{\gamma=\gamma_e}}{d\gamma} \right| \right)^{-1}. \]

From equation (8),

(B13) \[ \frac{\partial W(\bar{\gamma}_e(c), c)}{\partial c} = -\int_{-\infty}^{\gamma_e} q(\gamma) \ dF(\gamma). \]

Substituting from equations (B3), (B12), and (B13) into equation (B11) yields

\[
\frac{dW(\bar{\gamma}(c), c)}{dc} \bigg|_{\gamma=\gamma_e} = \left( (1 - \bar{q}) + \int_{-\infty}^{\gamma_e} q(\gamma) \ dF(\gamma) \right) \frac{dP_{\theta=\bar{\gamma}_e}}{d\gamma} \bigg|_{\gamma=\gamma_e} + \frac{dP_{\gamma=\gamma_e}}{d\gamma} - \int_{-\infty}^{\gamma_e} q(\gamma) \ dF(\gamma) \]

\[
= -\left| \frac{dP_{\gamma=\gamma_e}}{d\gamma} \right| \int_{-\infty}^{\gamma_e} q(\gamma) \ dF(\gamma) + (1 - \bar{q}) \frac{dP_{\theta=\bar{\gamma}_e}}{d\gamma} \bigg|_{\gamma=\gamma_e} + \frac{dP_{\gamma=\gamma_e}}{d\gamma} \bigg|_{\gamma=\gamma_e}, \]

which implies \( \frac{dW(\bar{\gamma}(c), c)}{dc} \bigg|_{\gamma=\gamma_e} > 0 \) if and only if \( \frac{\int_{-\infty}^{\gamma_e} q(\gamma) \ dF(\gamma)}{1 - \bar{q}} < \frac{dP_{\theta=\bar{\gamma}_e}}{d\gamma} \bigg|_{\gamma=\gamma_e} \).

**AQ 7** Substituting from equations (B6) and (7),

(B14) \[ \frac{\int_{-\infty}^{\gamma_e} q(\gamma) \ dF(\gamma)}{1 - \bar{q}} < \frac{q(\gamma_e)f(\gamma_e)}{1 - \int_{-\infty}^{\gamma_e} q(\gamma) \ dF(\gamma)} \left( E[\gamma|\theta; \gamma_e] - \bar{\gamma}_e \right) g(R - \mu_r - E[\gamma|\theta; \gamma_e]) \frac{g(R - \mu_r - \bar{\gamma}_e)}{g(R - \mu_r - \bar{\gamma}_e)}. \]
when \( c = 0 \), \( \tilde{\gamma}_e = \tilde{\gamma}_m \), so \( \tilde{\gamma}_e - E[\gamma|\theta; \tilde{\gamma}_e] = 0 \), and hence the right-hand side of equation (B14) is zero, so \( \frac{dW(\tilde{\gamma}_e; c)}{dc} \bigg|_{\tilde{\gamma} = \tilde{\gamma}_e} < 0 \). If \( c > 0 \), rearranging yields

\[
\frac{1}{E[\gamma|\theta; \tilde{\gamma}_e] - \tilde{\gamma}_e} \frac{g(R - \mu_r - \tilde{\gamma}_e)}{g(R - \mu_r - E[\gamma|\theta; \tilde{\gamma}_e])} < \frac{1 - \tilde{q}}{1 - \tilde{q} + \int_{-\infty}^{\gamma} q(\gamma) \, dF(\gamma)} \frac{q(\gamma) f(\gamma)}{\int_{-\infty}^{\gamma} q(\gamma) \, dF(\gamma)}.
\]

As \( c \to \tilde{c} = r - rG(R - \mu_r - \tilde{\gamma}) \), we have \( \tilde{\gamma}_e \to -\infty \), so the left-hand side of (B15) goes to zero, but the right-hand side is positive and bounded away from zero (by logconcavity of \( q(\gamma)f(\gamma) \)). Thus, \( \frac{dW(\tilde{\gamma}_e; c)}{dc} > 0 \) for \( c \in [\tilde{c} - \delta, \tilde{c}] \), \( \forall \delta > 0 \) sufficiently small.

To complete the characterization, we show that \( \frac{dW(\tilde{\gamma}_e; c)}{dc} \) has a single-crossing property. \( \tilde{\gamma}_e \) decreases in \( c \). Thus, logconcavity of \( q(\gamma)f(\gamma) \) implies that the right-hand side of (B15) increases in \( c \). Moreover, \( E[\gamma|\theta; \tilde{\gamma}_e] - \tilde{\gamma}_e \) also increases in \( c \) (see Figure 1). If \( R - \mu_r \geq 0 \), then \( R - \mu_r - E[\gamma|\theta; \tilde{\gamma}_e] \) is positive and decreasing in \( c \), while \( R - \mu_r - \tilde{\gamma}_e \) is positive and increasing in \( c \). Then strict unimodality of \( G(\epsilon) \) together with the median of \( \epsilon \leq E[\epsilon] = 0 \) imply that \( \frac{g(R - \mu_r - \tilde{\gamma}_e)}{g(R - \mu_r - E[\gamma|\theta; \tilde{\gamma}_e])} \) strictly decreases in \( c \), and hence so does the left-hand side of (B15).

**PROOF OF PROPOSITION 5:**

From equation (8),

\[
\lim_{\tilde{\gamma} \to +\infty} W(\tilde{\gamma}; R) = 1 - P\emptyset(+\infty) - c\tilde{q} = 1 - P_{\gamma=0} - c\tilde{q}.
\]

\[
\lim_{\tilde{\gamma} \to -\infty} W(\tilde{\gamma}; R) = 1 - \int_{-\infty}^{+\infty} P\emptyset(-\infty)\left[1 - q(\gamma)\right] \, dF(\gamma) - \int_{-\infty}^{+\infty} P_{\gamma} q(\gamma) \, dF(\gamma)
\]

\[
= 1 - P_{\gamma=\hat{\gamma}} - \int_{-\infty}^{+\infty} (P_{\gamma} - P_{\gamma=\hat{\gamma}}) q(\gamma) \, dF(\gamma).
\]

Recall that \( \mu_r \equiv \frac{1-r}{r} \mu \). Thus, \( \lim_{\tilde{\gamma} \to +\infty} W(\tilde{\gamma}; R) - \lim_{\tilde{\gamma} \to -\infty} W(\tilde{\gamma}; R) \) becomes:

\[
P_{\gamma=\hat{\gamma}} - P_{\gamma=0} + \int_{-\infty}^{+\infty} [P_{\gamma} - P_{\gamma=\hat{\gamma}}] q(\gamma) \, dF(\gamma) - c\tilde{q}
\]

\[
= P_{\gamma=\hat{\gamma}} - P_{\gamma=0} + \int_{-\infty}^{+\infty} [G(R - \mu_r - \gamma) - G(R - \mu_r - \hat{\gamma})] q(\gamma) \, dF(\gamma) - c\tilde{q}.
\]

When \( q(\gamma) \) is symmetric, \( \hat{\gamma} = 0 \), so the above equation simplifies to

\[
\int_{-\infty}^{+\infty} [G(R - \mu_r - \gamma) - G(R - \mu_r)] q(\gamma) \, dF(\gamma) - c\tilde{q}.
\]
Strict unimodality of $G(\epsilon)$ and symmetry of $q(\gamma)f(\gamma)$ imply that when $R$ or $c$ are sufficiently large, $\lim_{\gamma \to +\infty} W(\gamma; R) - \lim_{\gamma \to -\infty} W(\gamma; R) < 0$; and when $c = 0$ and $R$ is sufficiently low, $\lim_{\gamma \to +\infty} W(\gamma; R) - \lim_{\gamma \to -\infty} W(\gamma; R) > 0$. By continuity, this holds for sufficiently small $c$. Moreover, when $g$ is symmetric, $\int_{-\infty}^{+\infty} [G(R - \mu_r - \gamma) - G(R - \mu_r)] \, dF(\gamma)$ is negative if and only if $R - \mu_r > 0$.

**PROOF OF COROLLARY 4:**

From equation (8),

$$\lim_{q_r \to 1} W(\gamma_e(q(\gamma)), q(\gamma)) = 1 - \int_{-\infty}^{\infty} P_1 \, dF(\gamma).$$

$$\lim_{q_r \to 0} W(\gamma_e(q(\gamma)), q(\gamma)) = 1 - \lim_{q_r \to 0} P_1(\gamma_e(q(\gamma))) = 1 - P_{\gamma = 0}. $$

Thus, $\lim_{q_r \to 0} W(\gamma_e(q(\gamma)), q(\gamma)) - \lim_{q_r \to 1} W(\gamma_e(q(\gamma)), q(\gamma))$ becomes

$$\int_{-\infty}^{\infty} (P_1 - P_{\gamma = 0}) \, dF(\gamma) = \int_{-\infty}^{+\infty} [G(R - \mu_r - \gamma) - G(R - \mu_r)] \, dF(\gamma).$$

This is the same as equation (B16), and hence the results follow from the proof of Proposition 5.

**REFERENCES**


AUTHOR QUERIES

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