Distance Estimation Schemes for Diffusion Based Molecular Communication Systems

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Abstract—Molecule concentration is often used as the information carrier to accomplish Diffusion-based Molecular Communications (DMC) among nano-machines. In order to achieve the optimal functionality, knowing the distance between the transmitter nano-machine (TN) and the receiver nano-machine (RN) is of high importance. In this paper, two distance estimation schemes are proposed based upon the RN-sensed concentration which changes with regards to the time and distance. The RN estimates the distance by means of measuring either the concentration-peak time or received concentration energy. Simulations are performed to compare the accuracy of each scheme and to discover how the diffusion channel and noise may influence the accuracy. Results show that both schemes will provide a beneficial enhancement to molecular communication systems.

Index Terms—Distance estimation, Diffusion-Based Molecular Communications, Molecular Concentration.

I. INTRODUCTION

Diffusion based molecular communications (DMC) is a promising paradigm, which is accomplished by means of the diffusion of molecules among nano-machines [1]. At the transmitter nano-machine (TN), molecules are encoded with information, released into the surrounding environment, and diffuse to the receiver nano-machine (RN) to enable the exchange of messages. The distance between the TN and RN is an essential parameter that has great influence on the performance of DMC systems. If nano-machines have the distance information beforehand, they can accordingly coordinate their functionality such as the transmission rate and the number of released molecules. For example, either an excessive or insufficient transmission rate will result in the loss of the channel capacity [2], [3]. With the distance pre-known, nano-machines can adjust the transmission rate to achieve an optimal trade-off between these two features. Another example is given in the implementation of the protocols presented in the work of [4]. Here, prior knowledge of the distance allows the TN to make a better decision on the selection of the relevant parameters to balance the time and energy costs of the transmission scheme. Furthermore, in application areas such as drug-delivery systems [5], it is highly important to know the current location of the nano-machine-bound drug and its destination, which could therefore be achieved by estimating the relative distances to certain pre-deployed beacons [6].

Four distance estimation schemes based on feedback protocols were introduced in [7], subsequently expanded in [8], [9]. The schemes were designed in 2D environments by means of measuring the round-trip time or the fading of the signal amplitude (or frequency). These schemes are heuristic and the estimation accuracy is quite low, and furthermore, suffer from a high time cost. Another two estimation schemes proposed in [10] were also considered only in a 1D situation. Here, the nano-machines measured either the peak concentration or the time interval between the first and second peak to estimate the distance. By using these two schemes, the time cost is significantly decreased compared to [7]–[9], but no intuitive benefit on accuracy is presented. In [11], an approach for bounding the estimation accuracy was described to provide a possible method to evaluate distance estimation schemes. However, this bound is only accessible within certain optimal criteria, which means this approach can be only served as the guide for finding optimal estimation protocols rather than as a specific estimation scheme. Motivated by the aforementioned papers, there is a clear and timely requirement for the continued development of distance estimation schemes that are shown to be both accurate, fast and proven in 3D scenarios.

In this letter, the following contributions are presented. Firstly, two algorithmic concentration-based schemes are proposed to estimate the distance between two nano-machines in a 3D scenario. Secondly, for the first time, the input concentration is considered to be a rectangular pulse, rather than being simplified as an impulse, which is therefore closer to practical situations. The difference in estimation accuracy between using rectangular pulses and impulses has thus also been analyzed. Thirdly, it is shown that compared with previous work, the estimation accuracy is significantly increased.

The remainder of this paper is organized as follows. In Section II the communication model is introduced as well as the system structure. The distance estimation schemes and their simplification methods are explained in Section III. Simulation results are provided in Section IV. Finally in Section V, the paper is concluded.

II. THE DIFFUSION-BASED MOLECULAR COMMUNICATIONS MODEL

As illustrated in Fig. 1, the DMC system considered herein consists of two transceiver nano-machines in a duplex diffusion channel. One of the nano-machines encodes the binary message bits using certain molecules, which can be modulated in various ways, Concentration Shift Keying (CSK), Molecule Shift Keying (MoSK), and Isomer-based Ratio Shift
Keying (IRSK), to name a few [12]. Among these modulation techniques, CSK is the simplest so that it is chosen for the first trial to find proper distance estimation schemes. The targeted nano-machine determines the transmitted message sequence based on the samples of the sensed concentration. However, before the establishment of the communications, the distance between each other needs to be estimated. During the estimation process, the targeted nano-machine is viewed as the TN and the other nano-machine is regarded as the RN. In the system, the size of TN is negligible compared with the relative distance between each other [13]. The concentration at the RN is calculated by averaging within the sensing range ($R$), which can be considered as the concentration at the center of sphere [14]. The coordinates of the TN and RN are respectively (0,0,0) and ($x$,$y$,$z$), and they are assumed to be static within the time for estimation, $T_o$.

The concentration distribution is obtained by solving Fick’s laws of diffusion, which can be viewed as the impulse response for the diffusion channel in a 3D medium [15]:

$$h(r, t) = \frac{m}{(4\pi D t)^{3/2}} \exp\left(-\frac{r^2}{4D t}\right),$$

(1)

where $D$ is the diffusion coefficient, $m$ is the number of molecules emitted, and $r^2 = x^2 + y^2 + z^2$.

The molecular concentration emitted by the TN can be considered as a rectangular pulse given as:

$$s(t) = A \text{rect}\left(\frac{t - T_a/2}{T_e}\right), 0 \leq t \leq T_p,$$

(2)

where $A$ is the concentration amplitude, $T_e$ is the emission duration, and $T_p$ is the molecules emission period. Therefore, the noiseless concentration at the RN, $y_k(t)$, can be obtained by $y_k(t) = s(t) \ast h(t)$, that is:

$$y_k(t) = \begin{cases} \frac{Am}{4\pi D} \text{erfc}\left(\frac{r}{\sqrt{4D t}}\right), & t \leq T_e \\ \frac{Am}{4\pi D} \left[ \text{erfc}\left(\frac{r}{\sqrt{4D t}}\right) - \text{erfc}\left(\frac{r}{4(t-T_e)D}\right)\right], & t > T_e \end{cases}$$

(3)

However, molecules will not vanish at the RN within the period $T_p$. The remaining molecules will have an influence on the concentration distribution of the newly emitted molecules, which causes Inter-Symbol Interference (ISI). Considering the ISI, the noiseless concentration at the RN can be regarded as the sum of the current signal concentration and previous ones:

$$y(t) = \sum_{i=0}^{l} y(t - i \times T_p) a_{k-i} = \sum_{i=0}^{l} y_i a_{k-i},$$

(4)

where $l$ is the ISI length, which means the newly emitted molecules will be eliminated by the time $(l+1) \times T_p$. During the diffusion, an additive signal-dependent noise, $n(t)$, will also affect the concentration at the RN. The expression of the noise has zero mean and standard deviation given by [15]:

$$\sigma = \sqrt{3y(t)/(4\pi R^2)}$$

(5)

Thus, referring to Fig. 1, the concentration at the RN can be derived as:

$$r(t) = y(t) + n(t) = \sum_{i=0}^{l} y_i a_{k-i} + n(t).$$

(6)

Considering (4), (5), and (6), the Signal-to-Noise Ratio (SNR) at the RN for this DMC system can be calculated by:

$$\text{SNR} = \frac{P_y}{P_n} = \frac{1}{\text{var}[\sum_{i=0}^{l} y_i a_{k-i}^2]} = \frac{1}{3\sqrt{\pi R^2}} \sum_{i=0}^{l} y_i a_{k-i}^2$$

(7)

III. THE DISTANCE ESTIMATION SCHEME

A. Using the peak time to estimate the distance

Taking (3) into consideration, for $t \leq T_e$, $y_k(t)$ is a strictly increasing function with respect to $t$, which means the concentration increases within this period of time. However, after the TN finishes the emission process, the concentration will peak at a certain time ($t_{max}$) before decreasing. Therefore, when $t = t_{max}$, the derivative of $y_k(t)$ with respect to $t$ should be zero, and then the distance $r$ can be derived as:

$$r = \sqrt{\frac{6D}{T_e} \cdot (t_{max} - T_e) \cdot t_{max} \cdot \ln\left(\frac{t_{max}}{t_{max} - T_e}\right)},$$

(8)

If $T_e$ is small, (8) can be reduced to:

$$r_e = \lim_{T_e \to 0} r = \sqrt{6t_{max}D}$$

(9)

When $T_e$ is small, the input of the system can be viewed as the impulse signal. Thus, by solving the derivative of (1), the relationship between the distance $r$ and the peak time $t_{max}$ can be obtained, which agrees with (9).

This method enjoys great benefit for its simplicity. The RN senses the concentration and picks out the peak time $t_{max}$. Using (8) or (9), the distance can be estimated. However, due to the channel noise, it is quite hard to precisely find the peak time, which will influence the estimation accuracy.

B. Using the energy to estimate the distance

The energy of the signal in DMC system is defined as the sum of the molecular concentration [14]. Given (3), the received noiseless energy within $T_p$ can be obtained by:

$$E_0 = \int_0^{T_p} y(t) dt = \frac{4\pi D}{A} \int_0^{T_p} \left[ \text{erfc}\left(\frac{r}{\sqrt{4Dt}}\right) dt - \int_0^{T_p} \text{erfc}\left(\frac{r}{\sqrt{4(t-T_e)D}}\right) dt \right]$$

(10)
If $T_e$ is quite small, the input can be viewed as an impulse signal. Thus, the energy can be computed as:

$$E_n = \int_0^{T_o} c(r, t)dt = \frac{m}{4\pi D o} \text{erfc} \left( \frac{r}{2\sqrt{D T_o}} \right)$$

(11)

The additive noise is a main factor that affects the accuracy of the distance estimation. To minimize the influence, adding all values of the sensed concentrations is a simple solution, because the noise has a zero mean [15]. Thus, given (6), the energy at the RN can be derived as:

$$E_n = \int_0^{T_o} r(t)dt = \int_0^{T_o} y(t)dt + \int_0^{T_o} u(t)dt \approx \int_0^{T_o} y(t)dt$$

(12)

where $T_o$ is the time for estimation. If $T_o > (I + 1)T_p$, the upper limit of the integral in (12) will be $(I + 1)T_p$. When $T_o = T_p$, then $E_n \approx E_0$. With sufficient samples of the sensed concentration, the impact of the noise can be offset. Based on (10) or (11), the distance can be computed.

Theoretically, using energy to estimate the distance provides better performance on the accuracy than using the peak time, but it has a higher requirement of the complexity in the DMC system. Both schemes can be simplified by decreasing the emission time, $T_e$, so that the system can be viewed as the impulse response. The performance comparisons between the two estimation schemes, respectively using the peak time and energy, and their simplification methods will be shown in the next section.

IV. Simulation Results

The main aim is to find the accuracy of these two distance estimation schemes by means of obtaining the absolute deviations between the real distance and the estimated distance. During simulations, the global parameters are set as: $T_p = 100$ s, $T_o = 1000$ s, $R = 10$ µm, and $D = 100$ µm² s⁻¹. All the results are obtained by averaging after $10^3$ trials.

A. Using the peak time to estimate the distance

In Fig. 2, it is clearly shown that this estimation scheme works well. It should be noticed that the absolute deviations get larger when the TN releases less molecules or becomes further from the RN. The main reason of this scenario is that either larger distances or less molecules will result in the decrease of the SNR given by (7). In this case, the concentration distribution at the RN has less resistance against the noise, which makes it hard to find the accurate peak time. Thus, the accuracy becomes worse for longer distance or less molecules. What is not so intuitive is that with same amount of molecules ($mT_e$) used, if the molecules are emitted rapidly, the performance will be better. This is also due to the difference in the SNR, that is, faster releasing molecules leads to a slightly higher SNR, which will provide a better performance.

In Fig. 3, the results of the simplified estimation scheme are compared with the original scheme. Obviously, when $T_e$ is so small to be negligible (in this case $T_e = 0.1$), the performance using the simplified scheme is almost as good as the one using the original scheme; but when $T_e$ is not able to be neglected, the average absolute deviation increases greatly, which means the simplified method is not suitable for this case.

Results obtained from Fig. 2 and Fig. 3 show that both the original and simplified estimation schemes taking advantage of the peak time work well in the DMC system. To get the best performance, the TN is required to emit molecules as fast as possible, and the simplified estimation scheme according to (9) can perform as well as the original scheme using (8) if $T_e$ is quite small.

B. Using the energy to estimate the distance

As is shown in the Fig. 4, the performance of this estimation scheme is desirable. Similarly, the accuracy for this scheme goes down when the distance between the TN and RN increases. However, the decrease of the accuracy is not significantly great. This is mainly due to the superiority that the RN offsets the influence of the noise by adding all the sensed concentrations. Another feature is that the absolute deviation increases slightly with the ISI length getting larger.
According to (7), the SNR becomes less with the increase of the ISI length, which make the estimation less accurate.

Performance of the simplified energy-used estimation scheme is shown in Fig. 5. What is quite interesting is that for long $T_e$, the absolute deviation becomes very small with a large ISI length. The difference between the simplified and original scheme (in Fig. 4) is almost negligible. For example, when $T_e = 10\text{s}$, the performance of the simplified scheme is as good as the original scheme with $I = 4$ and $I = 8$. However, the absolute deviation becomes quite large if $I = 0$. When the ISI length increases, the SNR will become lower, which should have lead to a poorer performance. But the increase of the ISI length means the molecules can exist for a longer time. Taking equations (1), (3) and (11) into consideration, it is easy to prove that the energy difference between the impulse emission and the rectangular shape emission with $T_e$ gets smaller when molecules can exist for a longer time. The benefit of such scenario may outweigh the decrease of the accuracy estimation caused by a lower SNR, which leads to a better performance.

**V. CONCLUSIONS**

In this paper, two algorithmic distance estimation schemes and their own simplification methods have been introduced. The performance is analyzed mainly with regards to the accuracy in terms of the absolute deviation. Simulation results show that these proposed schemes can provide reasonable accuracy with well-designed parameters such as $T_e$ and $m$. Furthermore, using energy as the measure, enjoys a much more accurate estimation, but suffers from a much higher requirement of the system complexity to compute the distance, which may limit the implementation of this estimation scheme. When designing a communication system, the selection on the distance estimation schemes should be made according to the specific requirement of the accuracy, the amount of the molecules and the complexity of the system.

**REFERENCES**


