Cascade Decode-and-Forward: Spatial Diversity Reuse in Sensor Networks

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Abstract

In this paper, we consider a wireless sensor network that involves sensory data hoping through multiple wireless relays to reach a central collection hub. In particular we improve the decode-and-forward cooperative relaying scheme. In this paper, we propose the Cascade-Decode-and-Forward, where the number of successful relays increases with each additional cooperation stage. The achieved effect is a cascade of relays that contribute towards achieving full spatial diversity at the destination. A novel relationship between the achievable bit error rate and delay is derived for the proposed scheme. The results show that a small delay constraint relaxation, the proposed scheme can achieve full diversity. As the delay constraint relaxes further, the protocol can achieve full diversity at signals levels 10–100 orders magnitude lower than the decode-and-forward protocol. The proposed protocol can dynamically trade-off transmission reliability with delay and the analysis has shown that a certain node connectivity density is required to achieve a cascading cooperation chain with an arbitrarily low data extinction probability.

I. Introduction

A. Motivation

In the 21st century, the management of key systems and networks is increasingly dominated by data collection and interpretation. The communication of data between humans and between sensors is seen as a key economic enabler for developing and developed societies alike by the UN. Over the past 10 years, this has contributed to the unprecedented growth of wireless data traffic. Whilst the volume of wireless traffic is dominated by multimedia content, one of the fastest growing sectors is sensory data. A global trend is to make city management smarter through the mass deployment of sensors and the collection of data from homes, factories, transportation, and infrastructures. Due to the embedded nature of sensor networks, the channel can be lossy and stochastic, and the sensor nodes have a constrained energy supply.

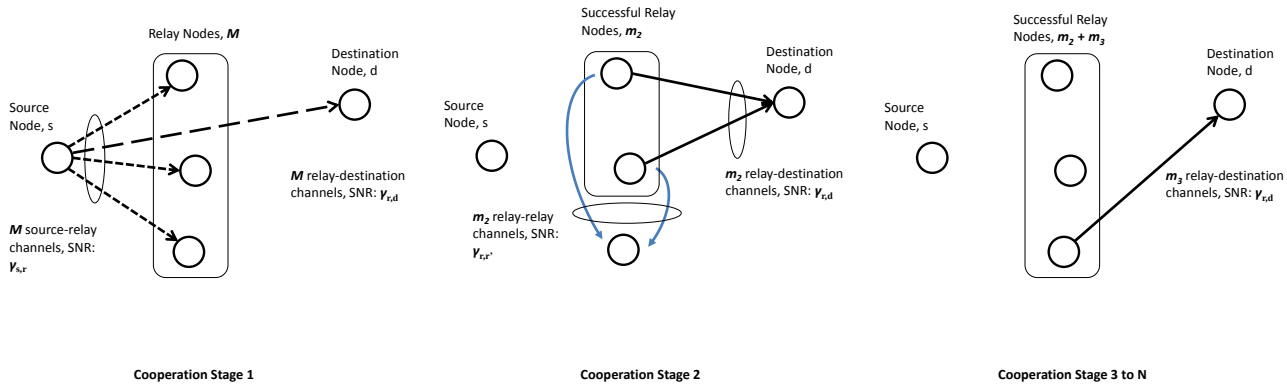

Fig. 1. Cascade Decode-and-Forward (C-DF) System setup, with M relay nodes. In stage 1, the source transmits the data to the relay nodes, of which only m_2 successfully decode its information. In stage 2, the successful relays transmit the data to the destination and the $M - m_2$ unsuccessful relays. In stage 3 to N, some of the unsuccessful relays attempt to become successful and improve the system diversity.

Cooperative transmission exploits any potential spatial diversity that exists in parallel channels. These parallel channels are typically between non-co-located transmitters and a common receiver [1]. The concept has been applied to a wide range of wireless communication research areas, including sensor networks [2]. In order to have high degree of spatial diversity, the fading on these parallel channels need to be sufficiently independent, which occurs when the paths are spatially apart. Whilst this can be achieved by utilising multiple antennas with sufficient inter-antenna separation, the solution is impractical for wireless sensor nodes that are typically small. Cooperative communications investigates how nodes can cooperate with each other to achieve a distributed multiple antenna solution.

B. Related Work

There are numerous cooperation schemes in literature and they can be generally categorised as those that do one of the following at the relay node [3], [4]: amplify the received signal, and those that decode and re-encode the signal. Whilst it has been shown that amplify-and-forward (AF) schemes can achieve full diversity, compared to other schemes, it requires significantly more bandwidth and transmission power for channel-state-information (CSI) estimation of the relay channels [5], [6]. Therefore, decode-and-forward (DF) schemes are attractive due to their simplicity and greater feasibility. The challenge is how to achieve the maximum diversity level with the DF scheme.

Indeed, opportunistic relaying has shown that maximum diversity order can be achieved [7], [8]. However, opportunistic scheme requires a strong understanding of the CSI in order to make the right relay selection choices. In terms of analysis, most existing literature employs Gaussian signal inputs [9], [10], which assumes that sensors are equipped with a full dynamic range modulation-coding-scheme (MCS) physical layer. In reality, simple sensors are likely to operate with a single modulation scheme such as QPSK, which has a low capacity saturation value. Assuming a Shannon channel capacity can potentially lead to an optimisation solution such as the Water-filling power allocation policy [11], which has been proven to be inaccurate at medium to high channel strengths [12]. The analysis of this paper employs a single generic PSK modulation and forward-error-correction (FEC) code in its analysis, which is a better reflection of reality in sensor networks.

C. Contribution

The paper considers a quasi-static (slow fading) channel, where the fading duration (coherence) is much greater than the packet lengths and delay tolerance. In order to improve spatial diversity, simply repeating transmissions on the same channel is insufficient. Reliable spatial diversity needs to be achieved from uncorrelated channels. In quasi-static fading channels, the reliable capacity is strictly zero [13], and error probability is often used as a metric for optimisation.

In the first part of the paper, the analysis obtains the error rate expressions for a Decode-and-Forward (DF) scheme that comprises of M relay nodes. This is then expanded to encompass the Cascade Decode-and-Forward (C-DF) scheme. The results demonstrate that the C-DF scheme achieves an increased diversity order with a delay penalty. The benefit of this novel trade-off is that the system designer can trade-off transmission reliability with delay tolerance, using the novel expression.

In the second part of the paper, the analysis examines the extinction probability of increased diversity with each additional cooperation stage. The analysis shows that there needs to be a sufficient sensor node density in order to achieve a decreasing extinction probability. The benefit of the analysis is that the system designer can relate the maximum cooperation duration with the node density.

II. System Model

In this paper, we consider a multi-hop network that comprises of sensors, relay nodes, and a common destination hub for data aggregation. In particular, we examine one sensor source and its multi-hop path to the destination via a set of potential relay nodes. We assume a static hidden routing table in relays, so that the relays only accept packets that it can potentially relay to the destination successfully, and our comparison between schemes consider all such relays. Whether all such relays should participate in relaying is a complex relay-selection optimisation matter which we do not consider.

A. Channel Model

The cooperative system we consider has the following salient features: repetition cooperation with N stages [13], with diversity achieved by Maximum-Ratio-Combining (MRC) at the common destination, and Decode-and-Forward (DF) relaying scheme is employed at each node.

The analysis observes the system from the perspective of a single source node (s) and its desired destination (d). In the vicinity, there are M sensor nodes acting as potential relay nodes (r), such that: the number of successful cooperating relays (decoded message correctly) at any time stage n is m_n. The source aims to transmit its data reliably to the destination, via the assistance of the relay nodes. A cooperation cycle consists of N stages, where each stage involves the transmission of a single data frame from one or more of the nodes.

The paper defines the instantaneous channel signal-to-noise ratio (SNR) as:

$$\gamma = |\overline{h}|^2 \frac{P_{\text{tx}}\Lambda}{N_0}, \tag{1}$$

where $|\overline{h}|$ is the magnitude of the complex fading coefficient h over many data frames, each of b bits long, P_{tx} is the transmit signal power, Λ is the pathloss constant, and N_0 is the additive white Gaussian noise power. For a scatter-rich environment with Rayleigh fading, the instantaneous channel SNR fluctuates about a mean SNR of $\overline{\gamma} = \frac{P_{\text{tx}}\Lambda}{N_0}$.

Given that the cooperating sensor nodes (SNs) in a small area are approximately equal distant to their common serving-BS, the paper assumes that the SN-BS channels have the same average SNR ($\overline{\gamma}$), but each channel's fading gain is completely de-correlated from other channels.

B. Cooperation Scheme

As shown in Figure 1, the paper considers repetition cooperation, whereby each cooperation cycle has the following stages:

1) **Stage 1:** source node (s) transmits its data to all M relay nodes (r) as well as the destination (d).
2) **Stage 2:** those m_2 relays that successfully decoded the source node transmission, retransmit the data to the destination (d).
3) **Stage 3:** exploit the fact that the transmission in Stage 2, is also received by the $M - m_2$ unsuccessful relays. The number of successful relays in stage 3 is $m_3 \leqslant M - m_2$. This signal can be combined with the source transmission from Stage 1, to improve the probability of successful decoding in the previously unsuccessful relays. This yields the opportunity that additional diversity can be achieved in the $n > 2$ cooperation stages, improving the diversity order of the system to $m_2 + m_3$ for stage 3.

The system can cascade this effect to Stage N if necessary to achieve full diversity, and this is the motivation for naming it Cascade Decode-and-Forward (C-DF). Potentially the C-DF system can achieve the full diversity order of M after $n > 2$ stages, which could not have been readily achieved in the conventional DF scheme.

In terms of channel frequency bands, we assume cooperation exists in the basic form, where each link operates on orthogonal channels to avoid co-channel interference. For each cooperation time stage (i.e., stage 1, stage 2, stage 3), the separate transmissions by any node exist on an orthogonal channel. In Stage 2, the recycled relays will have to listen in on a wide number of pre-defined channels and attempt to decode the information received on each of the channels.

III. Decode-and-Forward Error Probability

A. M Channel Bit Error Rate

Wireless communication systems such as fixed broadband access, rural cellular networks, and sensor networks often experience quasi-static (slow) fading, where the achievable capacity at an arbitrary level of reliability, is zero [13]. Error rate metrics are usually employed to measure the performance of quasi-static fading channels. The paper now presents the Bit Error Rate (BER) of a direct point-to-point channel, and also for M parallel quasi-static fading channels received at the Maximum-Ratio-Combiner (MRC) at the destination.

TABLE I
PARAMETERS FOR COOPERATIVE NETWORK

Parameter	Symbol
Cooperation Stage	n
Number of Relay Nodes	M
Number of Cooperating Relays (n stage)	m_n
Average SNR	$\overline{\gamma}$
Instantaneous SNR	γ
BER	p
Probability of Decoding Success	ρ
Probability of No. of Successful Relays	$\wp$
FEC Code SNR Threshold	T
BER of M Channels	$p_{T,M}$

Each channel is assumed to employ a PSK modulation scheme and a turbo FEC code, and this modulation and coding combination is identified by the variable T. Assuming that the BER in a Gaussian noise channel is $p_{\text{AWGN},r}$, the average quasi-static fading channel BER is [14]:

$$p_{\text{fading},\overline{\gamma},T,M} = \int_0^\infty p_{\text{AWGN},\gamma,T} f_{\text{fading},\overline{\gamma},\gamma,M} \, \mathrm{d}\gamma, \tag{2}$$

where $p_{\text{AWGN},r,\gamma}$ is the BER of the transmission scheme in an AWGN channel, and $f_{\text{fading},\overline{\gamma},\gamma,M}$ is the fading SNR distribution for M channels. A full list of symbols used in this paper and what they represent is given in Table I.

The BER of a PSK modulated turbo FEC coded symbol in an AWGN channel is approximately given by a *water-fall* threshold effect, modelled as a step function (U) [15], [16]:

$$p_{\text{AWGN},\gamma,T} = U(T-\gamma) = \begin{cases} 1 & \gamma \leqslant T \\ 0 & \gamma > T \end{cases}, \tag{3}$$

where T is the SNR threshold for a certain combination of modulation and FEC code rate [15]. The stronger the code performance, the lower the value of T in the heavy-side step function [16].

The fading SNR distribution for Rayleigh channels of a similar SNR (symmetrical) is as follows [17]:

$$f_{\text{fading}}(\overline{\gamma},\gamma,M) = \frac{\gamma^{M-1}}{(M-1)!\overline{\gamma}^M} e^{-\gamma/\overline{\gamma}}. \tag{4}$$

A similar expression for arbitrary channel strengths (asymmetrical) can also be obtained in the same way using expressions found in [14], [17], but the equal channel strength is employed to gain performance insights. **Verification** of the above theoretical expressions using simulation results can be found in [18].

Therefore, by combining Eq. (3) and (4), the BER [19] in a point-to-point Rayleigh fading channel is given by Eq. (5), which is:

$$\begin{aligned} p_{\text{fading},\overline{\gamma},T,1} &= \int_0^\infty \frac{U(T-\gamma)}{\overline{\gamma}} e^{-\frac{\gamma}{\overline{\gamma}}} \, \mathrm{d}\gamma \\ &= 1 - e^{-\frac{T}{\overline{\gamma}}}. \end{aligned} \tag{5}$$

Similarly, the BER for M quasi-static channels, that are combined by MRC is [17]:

$$\begin{aligned} p_{\text{fading},\overline{\gamma},T,M} &= \int_0^\infty \frac{U(T-\gamma)\gamma^{M-1}}{(M-1)!\overline{\gamma}^M} e^{-\frac{\gamma}{\overline{\gamma}}}\ \mathrm{d}\gamma \\ &= 1 - \sum_{m=0}^{M-1} \frac{\left(\frac{T}{\overline{\gamma}}\right)^m}{m!} e^{-\frac{T}{\overline{\gamma}}} \\ &\approx \frac{\left(\frac{T}{\overline{\gamma}}\right)^M}{M!} \quad \text{for: } M \gg 1. \end{aligned} \tag{6}$$

Note that when $M = 1$ in Eq. (6), the un-approximated expression is the same as the direct channel expression given in Eq. (5). The approximated MRC expression holds true when $T/\overline{\gamma}$ is small, such that the following approximation holds accurate: $1 - e^{-T/\overline{\gamma}} \approx T/\overline{\gamma}$.

The paper will now omit the fading notation from probability expressions and note that all the BER are for a fading channel.

B. System Error Rate

So far, the paper has considered the BER of M parallel quasi-static fading channels received at the MRC of the destination. In order to complete the multi-node cooperative system, one must consider the inter-node (relay) channels and probability of successful decoding at each node.

For a DF system of M relays in attempted cooperation over $n = 2$ transmission stages, the full BER is comprised of the direct channel and up to $M + 1$ combined channel performances, with each weighted by their probability of successful occurrence. Table I lists the probabilities of each channel in such a system, and the probability of the number of successful relays in stage n of cooperation is ($\wp$):

$$\wp_{m_n} = \binom{M}{m_n} \rho_{s,r}^{m_n} (1 - \rho_{s,r})^{M - m_n}, \tag{7}$$

where the parameter $\rho_{s,r}$ is the probability of successful decoding in any given source-relay channel and is a function of the single channel BER:

$$\rho_{s,r} = 1 - p_{\overline{\gamma}_{s,r},T,1} = e^{-T/\overline{\gamma}_{s,r}}. \tag{8}$$

The DF system's BER ($p_{\text{DF},n,T,M}$) for $n = 2$ stages, M relays and a transmission code T, is effectively a modified binomial distribution:

$$\begin{aligned} p_{\text{DF},2,T,M} &= \sum_{m_2=0}^{M} \wp_{m_2} p_{\overline{\gamma}_{r,d},T,1+m_2} \\ &\approx \sum_{m_2=0}^{M} \binom{M}{m_2} \rho_{s,r}^{m_2} (1 - \rho_{s,r})^{M - m_2} \frac{\left(\frac{T}{\overline{\gamma}_{r,d}}\right)^{1+m_2}}{(1+m_2)!}, \end{aligned} \tag{9}$$

where it is assumed that the source-destination (s, d) and the relay-destination (r, d) channels have the same average SNR. In the absence of a direct channel, the Eq. (9) can be modified to set $p_{\overline{\gamma}_{s,d},T,1} = 1$, and the MRC BER to be $p_{\overline{\gamma}_{r,d}T,m_2}$.

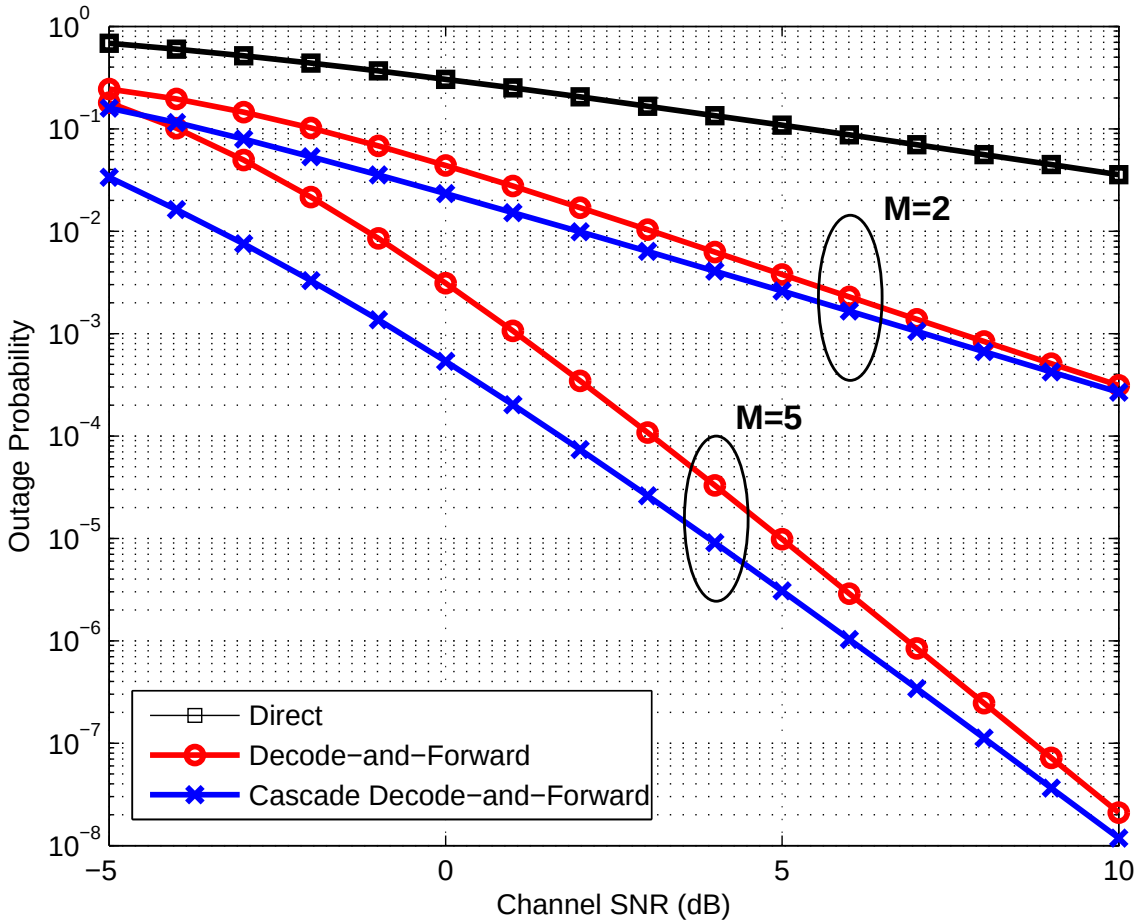

Fig. 2. System BER achieved by: Direct, Decode-and-Forward (DF), and Cascade Decode-and-Forward (C-DF) transmission. System configuration: $\overline{\gamma}_{s,d} = \overline{\gamma}_{s,r} = \overline{\gamma}_{r,r'} = \overline{\gamma}_{r,d} = -5$ to 15 dB, C-DF ($N = 3$), and $M = 2, 5$.

The mean diversity order of the DF system is defined as:

$$D_{\text{DF},2,T,M} = \sum_{m_2=0}^{M} \binom{M}{m_2} \rho_{s,r}^{m_2} (1 - \rho_{s,r})^{M-m_2} m_2, \tag{10}$$

which can be interpreted as the mean number of independent channels received at the destination. For a fixed mean channel SNR, the diversity order D determines the BER of the system.

The DF system BER for $M = 2, 5$ is shown in Fig. 2, along with the direct transmission BER. The proposed C-DF scheme will now be formally introduced in the paper.

IV. Cascade Decode-and-Forward (C-DF) BER

A. Introduction

In Cascade Decode-and-Forward (C-DF), the scheme prolongs the cooperation cycle to $N > 2$ stages in order to improve the diversity order achieved (Fig. 1). As mentioned previously in Section I(c), due to the quasi-static nature of the channel,

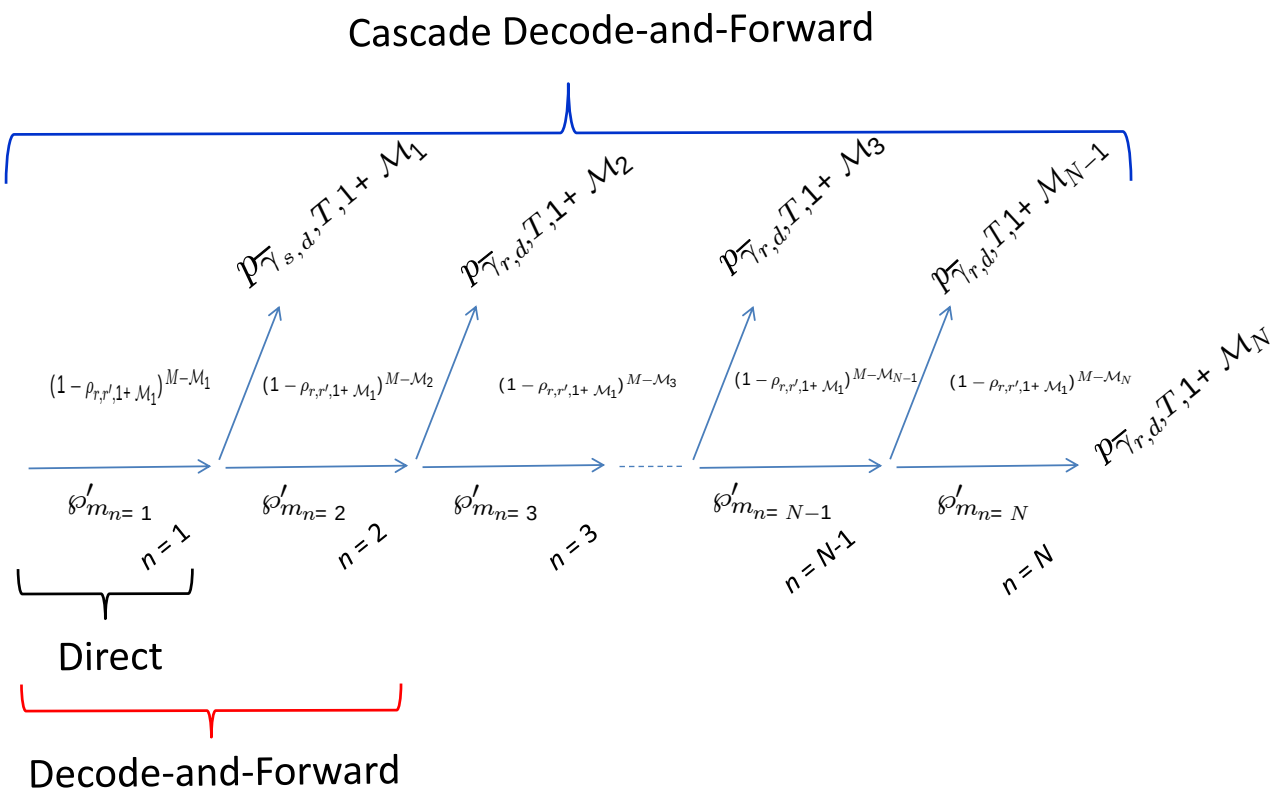

Fig. 3. Cascade Decode-and-Forward (C-DF) probability branching illustration.

repeated transmission from the source and already successfully relays in cooperation stages $n > 2$ can be futile. In order to improve diversity in the $n > 2$ stages, additional uncorrelated channels must be utilised. This section of the paper presents the BER of the system for the N stage general form. The expression reveals that there is a trade-off between delay tolerance and transmission reliability.

B. System BER

The C-DF scheme employs n cooperation stages, where the upper-limit of n is N, such that the number of successful relays in all previous stages is less than the maximum: $\sum_{n=1}^{N-1} m_n < M$. If the maximum number of successful relays (M) has been achieved in stage n, there will be no requirement for incurring stage $n+1$. The paper defines the total number of successful relays *up to* stage n as:

$$\mathcal{M}_n = \sum_{k=1}^{n-1} m_k \quad \text{for: } n \geqslant 2, \tag{11}$$

and $\mathcal{M}_{n<2}$ = 0, as there are no successful relays in the first stage of a new cooperation cycle. We assume that all values of n are strictly positive.

For DF, the probability of the number of successful relays in stage $n = 2$ of cooperation is ($\wp$), which was given in Eq. (7).

For C-DF, a similar probability on the number of successful relays in stages $n > 2$ of cooperation is ($\wp'$):

$$\begin{aligned} \wp'_{m_n} &= \binom{M-\mathcal{M}_n}{m_n} \rho^{m_n}_{r,r',1+\mathcal{M}_n} (1-\rho_{r,r',1+\mathcal{M}_n})^{\nu} \\ \text{for: } \nu &= M - \mathcal{M}_n - m_n, \end{aligned} \tag{12}$$

where $\rho_{r,r'}$ is the probability of successful decoding at the relay in any given relay-relay channel (r, r'), which combines the source-relay transmission from stage 1 with $\mathcal{M}_n$ transmissions from successful relays in all previous stages:

$$\rho_{r,r',1+\mathcal{M}_n} = \frac{(\frac{T}{\overline{\gamma}_{r,r'}})^{1+\mathcal{M}_n}}{(1+\mathcal{M}_n)!}. \tag{13}$$

It is worth noting that when all the system channels have the same average SNR (symmetrical), Eq. (12) and (13) are general forms of Eq. (7) and (8) respectively.

For the C-DF scheme with N stages of cooperation, the full system BER has a structure which is illustrated in Fig. 3. The number of additional successful relays in each new stage of cooperation (m_n) is conditioned on the number of successful relays in the previous cooperation stage (m_{n-1}). The C-DF system BER for N stages of cooperation as the the general form of:

$$\begin{aligned} p_{\text{C-DF},N,T,M} = &\prod_{n=2}^{N} \left(\sum_{m_n=0}^{M-\mathcal{M}_n} \wp'_{m_n} \right) p_{\overline{\gamma}_{r,d},T,1+\mathcal{M}_{n+1}} \\ &+ \sum_{n=1}^{N} \left[\prod_{k=0}^{n-1} \left(\sum_{m_k=0}^{M-\mathcal{M}_{k-1}} \wp'_{m_{k-1}} \right) \right. \\ &\left. (1-\rho_{r,r',1+\mathcal{M}_n})^{M-\mathcal{M}_n} p_{\overline{\gamma}_{r,d},T,1+\mathcal{M}_k} \right], \end{aligned} \tag{14}$$

where the expressions for $p_{\overline{\gamma},T,m}$ and $\wp'_m$ are given in Eq. (6) and (12) respectively for different values of m. It can be observed that the C-DF BER given in Eq. (14), is identical to the DF case in the Eq. (9), for $N = 2$ and $\wp' = \wp$. The system BER for $M = 2, 5$ is shown in Fig. 2, along with the direct transmission and DF scheme's BER. The results show that at low-medium SNRs (-5 to 5 dB), the C-DF scheme offers a significant improvement in BER. As the system's channel conditions improve, the C-DF performance converges with the DF scheme, as the need for cascaded cooperation vanishes (all relays are successful in second stage of cooperation).

The C-DF system's mean diversity order is defined as:

$$\begin{aligned} D_{\text{C-DF},N,T,M} = &\prod_{n=2}^{N} \left(\sum_{m_n=0}^{M-\mathcal{M}_n} \wp'_{m_n} m_n \right) \\ &+ \sum_{n=1}^{N} \left[\prod_{k=1}^{n-1} \left(\sum_{m_k=0}^{M-\mathcal{M}_{k-1}} \wp'_{m_{k-1}} \right. \right. \\ &\left. \left. (1-\rho_{r,r',1+\mathcal{M}_n})^{M-\mathcal{M}_n} m_k \right) \right], \end{aligned} \tag{15}$$

which can be interpreted as the mean number of independent channels received at the destination. For a fixed mean channel

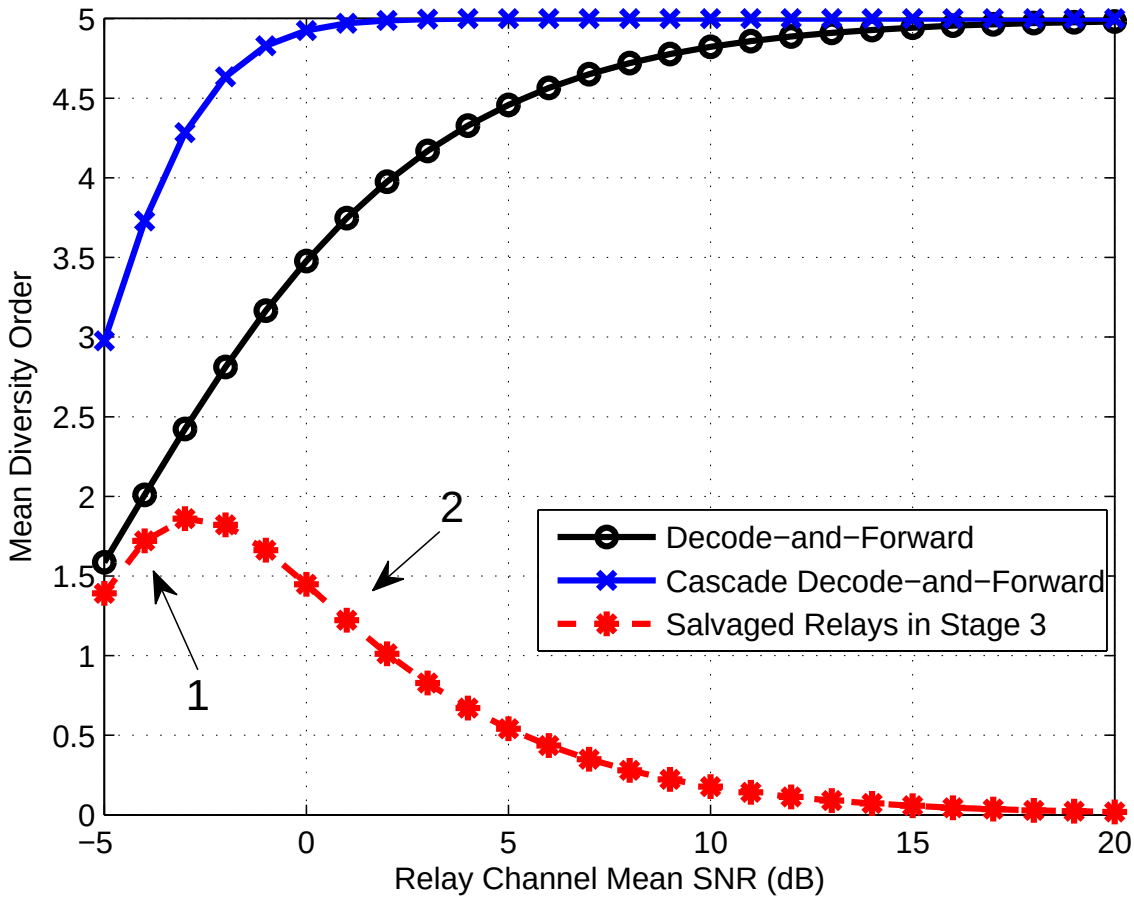

Fig. 4. Mean Diversity Order achieved by: Decode-and-Forward (DF), Cascade Decode-and-Forward (C-DF), and the Stage 3 of C-DF. System configuration: $\overline{\gamma}_{r,d} = 5$ dB, $\overline{\gamma}_{s,r} = \overline{\gamma}_{r,r'} = -5$ to 20 dB, C-DF ($N = 3$), $M = 5$, and no direct channel is considered.

SNR, the diversity order D determines the system BER.

C. C-DF Performance Results

The paper now presents the BER performance results for the no-cooperation, DF and C-DF cooperation schemes. The diversity order for DF and C-DF schemes is defined in Eq. (10) and (15) respectively. The results in Fig. 4 show that the proposed C-DF scheme can achieve nearly full diversity order at a lower average channel SNR (0 dB) than the DF scheme (15 dB). From the results it can be seen that the number of successfully recycled relays in stage 3 of the C-DF scheme varies in a non-monotonic relationship with the relay channel SNR:

1) Region 1: the number of successful relays in stage 2 of cooperation is low (30–40%) and the number of successful relays in stage 3 increases with relay channel SNR.
2) Region 2: the number of successful relays in stage 2 of cooperation is high (70–100%) and the number of successful relays in stage 3 decreases with relay channel SNR, because the decreasing upper-bound of the stage 3 recycled relays dominates.

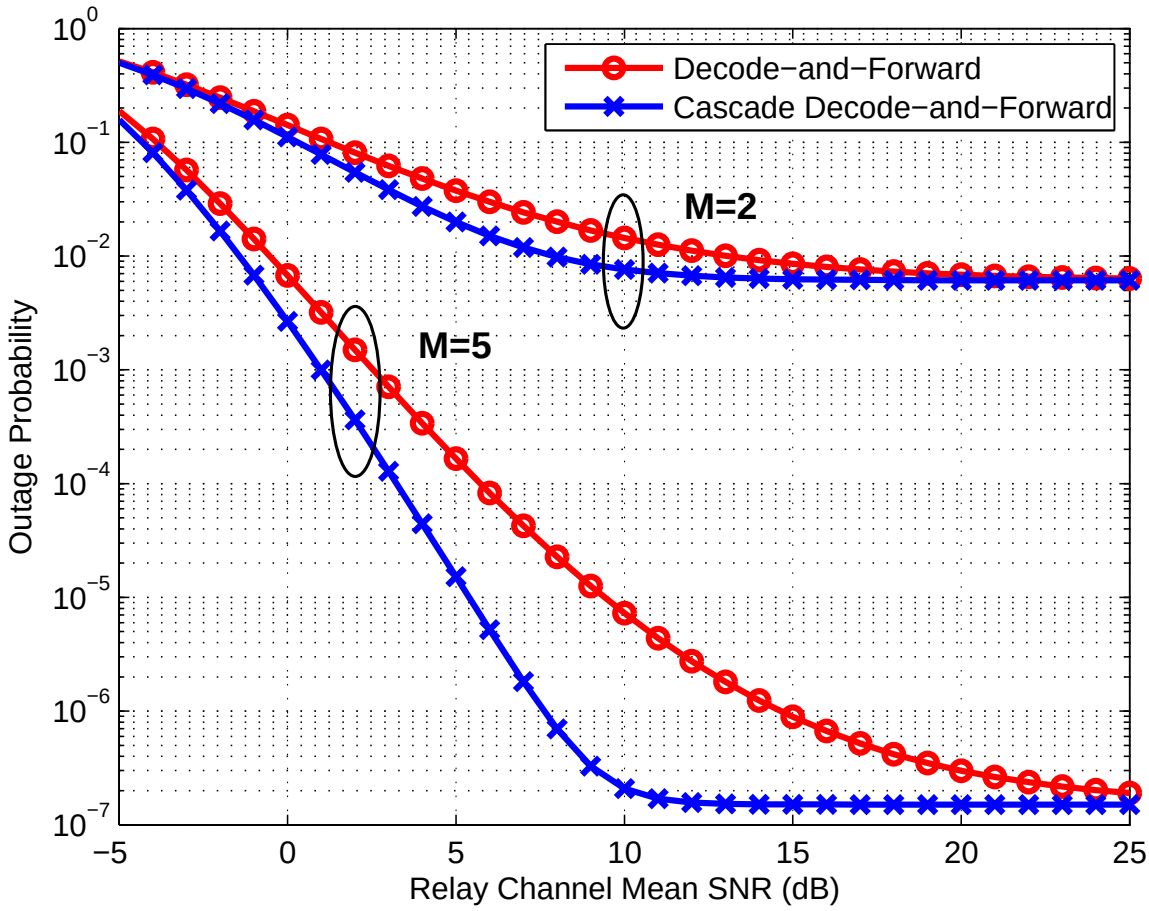

Fig. 5. System BER achieved by: Decode-and-Forward (DF), and Cascade Decode-and-Forward (C-DF). System configuration: $\overline{\gamma}_{r,d} = 5$ dB, $\overline{\gamma}_{s,r} = \overline{\gamma}_{r,r'} = -5$ to 25 dB, C-DF ($N = 3$), $M = 5$, and no direct channel is considered.

In terms of BER of the system, Fig. 5 shows that the BER saturates in the C-DF system at approximately 10 dB, whereas the DF system requires 25 dB for a similar performance. The relay-destination channels are fixed at 5 dB, which explains why the BER saturates. Therefore, in terms of both diversity order and error performance, the C-DF with $N - 2 = 1$ additional stage of cooperation has achieved a 15 dB improvement over the DF scheme.

D. BER and Delay Trade-off

So far, the paper has considered the improved BER performance of the proposed C-DF scheme. It was found that by increasing the cooperation duration (delay), the C-DF scheme can achieve the full diversity and the minimum BER at a lower channel SNR than the traditional DF scheme. However, the diversity improvement incurs both a cost in delay and transmission resource efficiency.

The delay of a data packet varies depending on the route it took to the receiver. For example a direct transmission, if successful will incur the minimal delay. Therefore, we should say the delay is upper-bounded by N (the number of cooperative transmission stages shown in Fig. 2), and this is because all packets need to arrive before the receiver performs maximum-ratio-

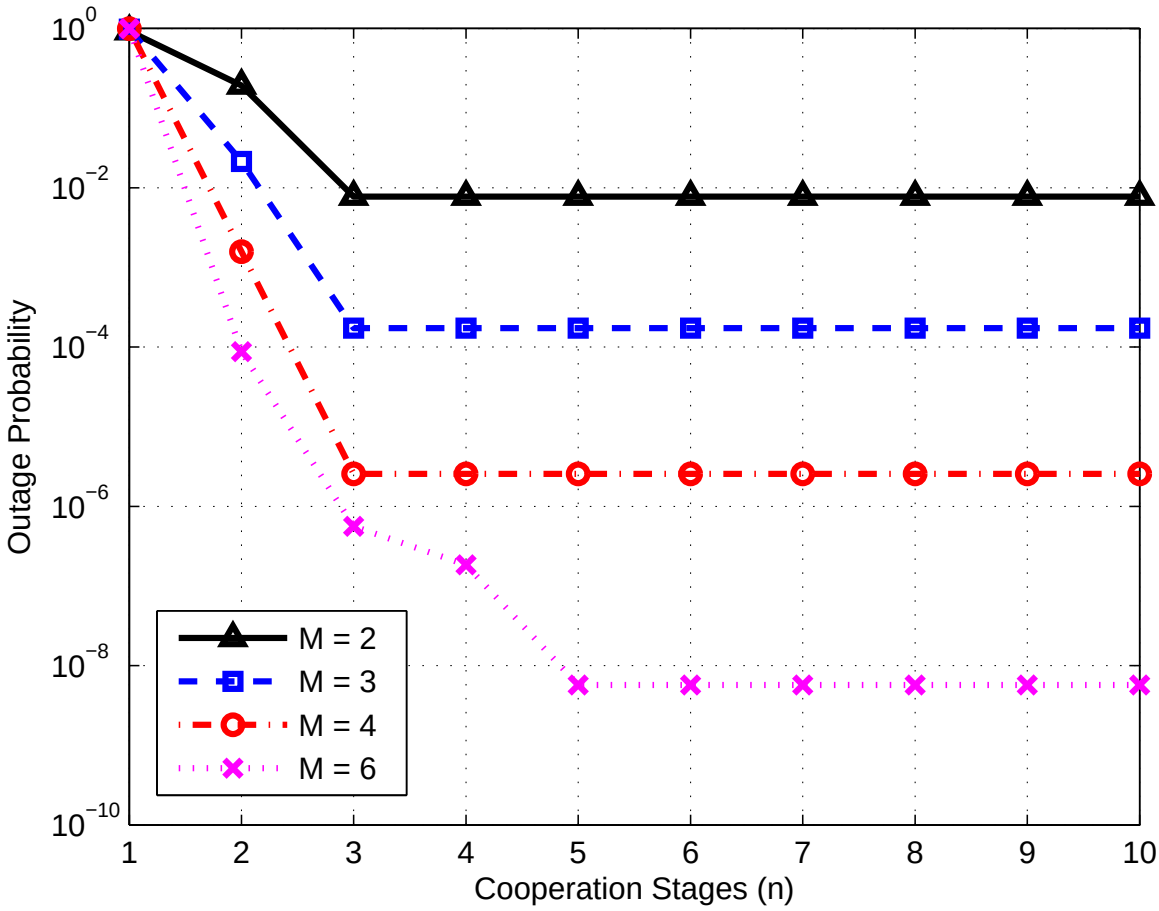

Fig. 6. System BER as a function of the Cooperation duration / delay (n stages) for different number of relays. System configuration: $\overline{\gamma}_{s,d} = \overline{\gamma}_{r,d} = \overline{\gamma}_{s,r} = \overline{\gamma}_{r,r'}$ =5 dB, and $M = 2, 3, 4, 6$.

combining (MRC). The BER Eq. (14) has presented a novel trade-off expression between the achievable BER $p_{\mathrm{C-DF},N,T,M}$ and the additional packet delay $(N-1)$. Fig. 6 shows this relationship between BER and the number of total cooperation stages N. The results show that generally speaking, *a single additional step* of cooperation is required on top of the DF scheme to achieve minimum BER. From the results presented, it can be observed that the additional diversity gained from each new cooperation stage decreases rapidly. That is to say, after $n \leqslant N$ stages, there is no need for additional cooperation.

Therefore, for a target BER and a given number of relays M, the C-DF analysis can provide the required number of cooperation stages to meet the BER target.

E. Diversity and Resource Efficiency Trade-off

Previously, we have not considered a resource budget. In terms of resource efficiency, previous work by the authors in [20] has shown that under a fixed energy and bandwidth budget, repetition cooperation achieves a non-monotonic trade-off between resource utilisation efficiency and transmission reliability.

Consider a node transmitting over M transmission frames, whereby the first frame is used to transmit its own source data

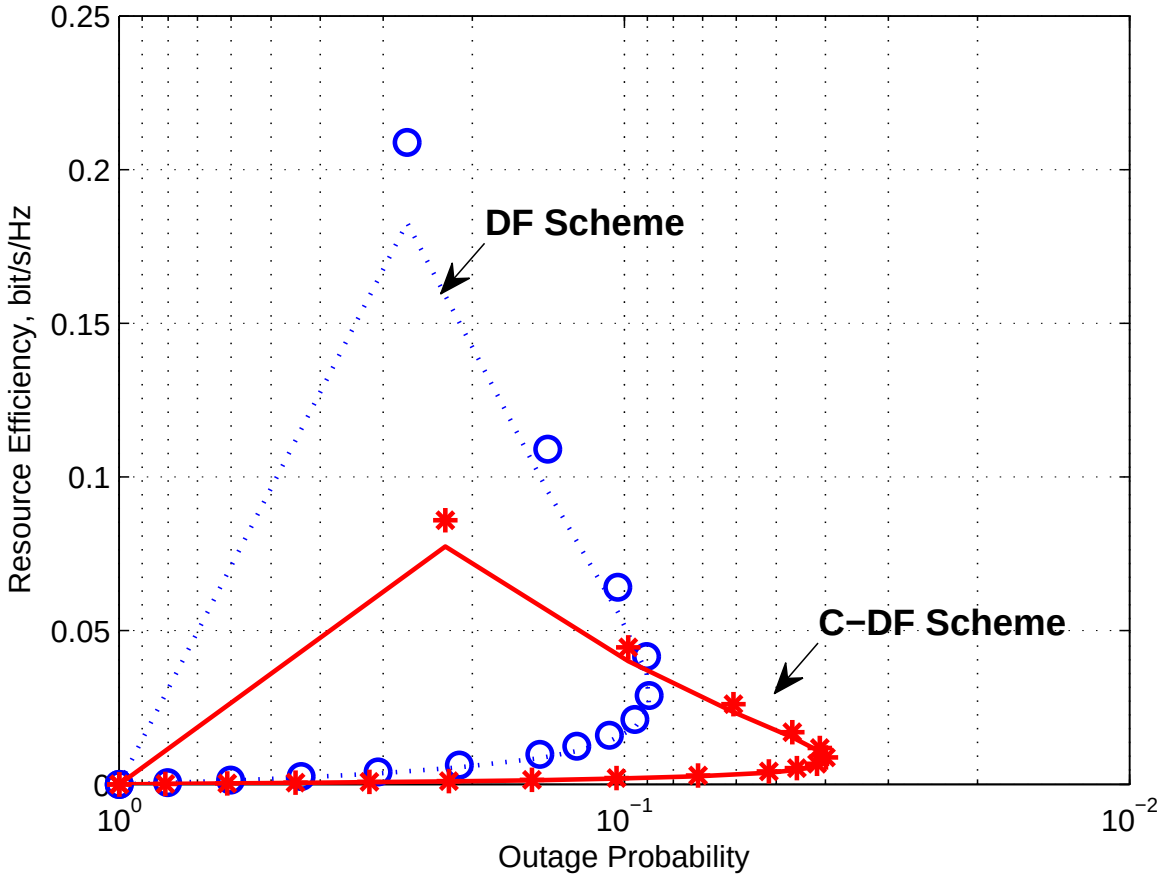

Fig. 7. System Resource Efficiency as a function of the BER for DF and C-DF schemes. System configuration: $\overline{\gamma}_{s,d} = \overline{\gamma}_{r,d} = \overline{\gamma}_{s,r} = \overline{\gamma}_{r,r'} = 5$ dB and $M = 3$. Symbols show simulation results and lines show theoretical results.

and the remaining $M-1$ frames is used to relay other nodes' data. Under a constant power budget of $P_{\sum}$, the constraint is:

$$\sum_n M P_{\text{tx},n} = P_{\sum}. \tag{16}$$

That is to say, the greater the number of cooperation frames transmitted, the lower the power per frame and hence the lower the channel SNR γ.

Similarly, repetition cooperation also causes a resource or spectral efficiency that is inversely proportional to the number of successful cooperation partners [13]. Therefore, for any DF system, the mean *resource efficiency* is:

$$\frac{1}{\mathbb{E}[\mathcal{M}_n]^2} = \frac{1}{\mathbb{E}[\sum_{k=1}^{n-1} m_k]^2} \quad \text{for: } n \geqslant 2, \tag{17}$$

where n is the fixed number of cooperation stages in the C-DF scheme. By inserting the energy budget into Eq. (14), and assuming a equal power allocation scheme, the trade-off between BER and resource efficiency is presented in Fig. 7. The trade-off is inherent in the fact that the greater number of cooperation stages, and the lower the resource utilization efficiency, but the greater the transmission reliability. More details on the formulation of this trade-off for DF networks can be found in [20]. Symbols show simulation results and lines show theoretical results. The results show that whilst C-DF leads to decreased BER,

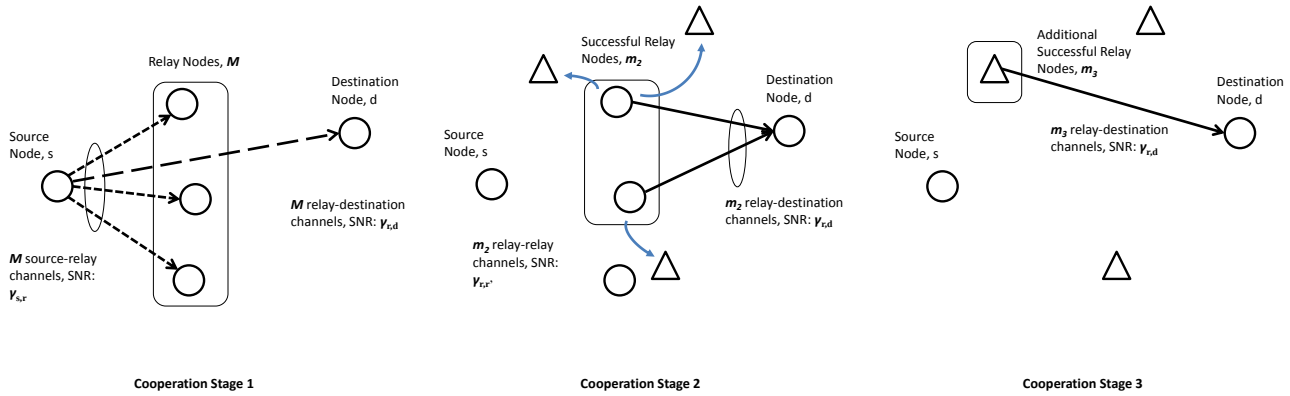

Fig. 8. Branching Cascade Decode-and-Forward (B-C-DF) System setup, where each node has potential connectivity to M nodes, which is a random-value. In stage 1, the source transmits the data to M relay nodes, of which only m_2 relays successfully decode its information. In stage 2, each successful relays transmit the data to the destination and a new set of M relays. This is repeated for up to N stages. Circles relay nodes denote those that received data from the first hop, and triangular relay nodes denote those that received data from the second hop.

it has led to a fall in resource or spectral efficiency.

F. Practicality

One area of concern with the proposed C-DF scheme is for each data packet the source sensor sends: how would the relays know how long (number of stages) it should participate in the relaying extension or cascade process? One suggestion is to adopt a fixed strategy that is a function of the average channel conditions, so that an average network performance is achieved in the long run. An alternative is to adopt a feedback scheme whereby the destination requests further relaying stages until a certain quality-of-service is achieved. We leave this for future researchers to investigate.

The paper now examines an alternative form of the C-DF scheme system setup, in order to gain insight into what the requirements are for allowing increasing diversity gain with increased cooperation.

V. Branching Network Analysis

A. Introduction

So far, this paper has considered the problem of how to trade-off BER (transmission reliability) and cooperation duration (packet delay). This analysis was conducted for a fixed pool of relays (M) over a 2-hop system. The results showed that increased cooperation leads to a saturation of the system diversity level (limited by M). The paper now considers a larger pool of relays over a multiple-hop network, and where each node has connectivity to a sub-set of the total number of relays. The challenge addressed is, how much connectivity to relays is needed per node, in order ensure that each new stage of cooperation can be increasingly beneficial in terms of improving system diversity.

B. Galton-Watson Branching Stochastic Process

The paper now considers an alternative form of the proposed C-DF scheme: Branching C-DF (B-C-DF). The paper considers each successful relay is able to find additional new relays to cascade the information to, in order to improve the system diversity

TABLE II
ANALOGY WITH GALTON-WATSON (GW) PROCESS

C-DF Parameter	GW Parameter	Symbol
Cooperation Stage	Descendent Tier	n
Available Relays	No. of Children	M
Decoding Success	Male Probability	ρ
Successful Relays	No. of Male Children	m
Added Diversity	Descendants	X

order. In order to gain system-level insight, the relays themselves in B-C-DF no-longer benefit from diversity: $\rho_{r,r',1+\mathcal{M}_n} \rightarrow \rho_{r,r',1}$ in Eq. (13). The system model is shown in Fig. 8.

This process is a branching stochastic process, whereby with each increased cooperation stage, there is the possibility of a decreasing chance in achieving additional diversity. This is in fact akin to the Galton-Watson extinction process [21], which models the probability of family names dying out. The Galton-Watson stochastic process describes the probability of a family name in existence at time stage n (n-th generation). By assuming that only males are valid descendants of the family name, the number of descendants at $n+1$ is X_{n+1}:

$$X_{n+1} = \begin{cases} 1 & n = 0 \\ \sum_{j=1}^{x_n} m_{n,j} & n > 0 \end{cases}, \tag{18}$$

where $\{m_{n,j} : n, j \in M\}$ is the number of male children (in a set of M children) of the jth descendant, and this is modelled as a set of independent and identically distributed (i.i.d.) natural number-valued random variables. By providing the number of children of the jth descendant with a Poisson distribution ($M \sim \text{Pois}(\lambda)$) of mean λ children, moment generating functions can be employed to derive an extinction probability. The paper now considers the B-C-DF scheme as a Galton-Watson (GW) family name extinction process, whereby each successful relay can find its own set of relays to cascade the information to. The analogy between the B-C-DF relay extinction and the family name extinction process is given in Table II.

Let the probability of data extinction η_n at the nth hop [21] (i.e., no relay nodes decode successfully):

$$\eta_n = \text{P}(Z_n = 0). \tag{19}$$

Clearly $\{Z_n = 0\} \subset \{Z_{n+1} = 0\}$. Let the probability generating function of the number of successful data transmissions be denoted by:

$$G_n(s) = \mathbb{E}[s^{Z_n}]. \tag{20}$$

By conditioning the number of successful data transmissions in the first generation, we can obtain:

$$G_n(s) = \sum_{i=0}^{+\infty} p_i G_{n-1}(s)^i. \tag{21}$$

Therefore, the *recurrence relationship* is established for a Poisson moment generating function (MGF). This is the probability

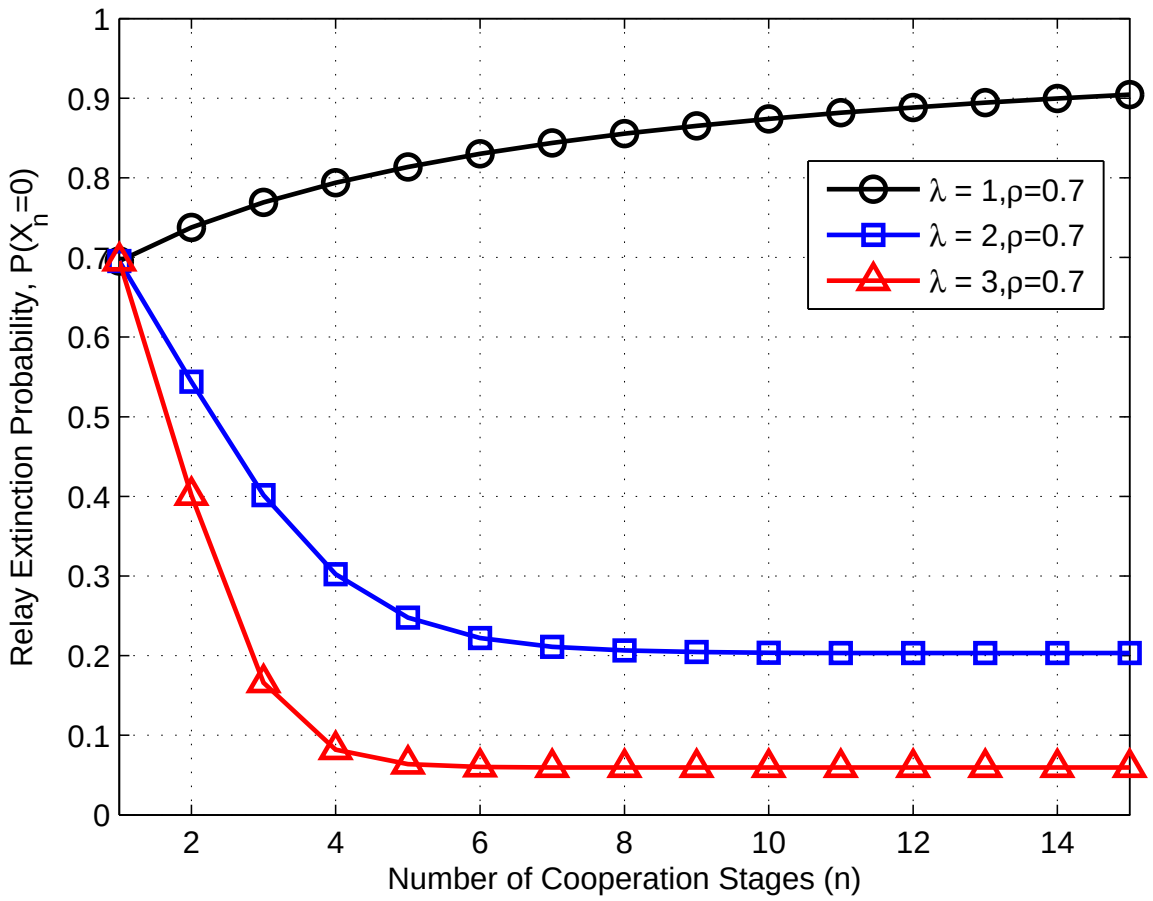

Fig. 9. Probability of Relay Extinction (no additional diversity order gained, $X_n = 0$), as a function of increasing cooperation duration (n) and relay node number per hop (λ).

of data extinction at the $(n+1)^{\text{th}}$ generation is:

$$\mathbb{P}(X_{n+1} = 0) = \rho e^{\lambda(\mathbb{P}(X_n=0)-1)}. \tag{22}$$

As a result, the added spatial diversity at the receiver from each additional cooperation stage is given by Eq. (22).

Fig. 9 shows the probability of achieving no additional diversity order ($X_n = 0$), as a function of increasing cooperation duration (n). The results employ Eq. (22) and show that for a mean connectivity relay number of $\lambda = 1$ and decode success probability of $\overline{\rho} = 0.7$, the relay extinction probability is a monotonically increasing function. For $\lambda \geqslant 2$, the extinction probability is monotonically decreasing and achieves rapid asymptotic convergence for $n < 6$ cooperation stages. As the channel quality improves, the results remain similar, but the extinction probability's rate of convergence decreases. Fig. 10 expands on the result to show the effect of channel SNR on the extinction probability. The results show that a poorer channel can cause a significant increase in extinction probability for the same set of relay density numbers.

That is to say, for an average connectivity of $\lambda \geqslant 2$ relays per node, then the system will have a 20% asymptotic probability of extinction ($n \to$ large). For a system designer that wants the number of cooperation stages to scale with system diversity order,

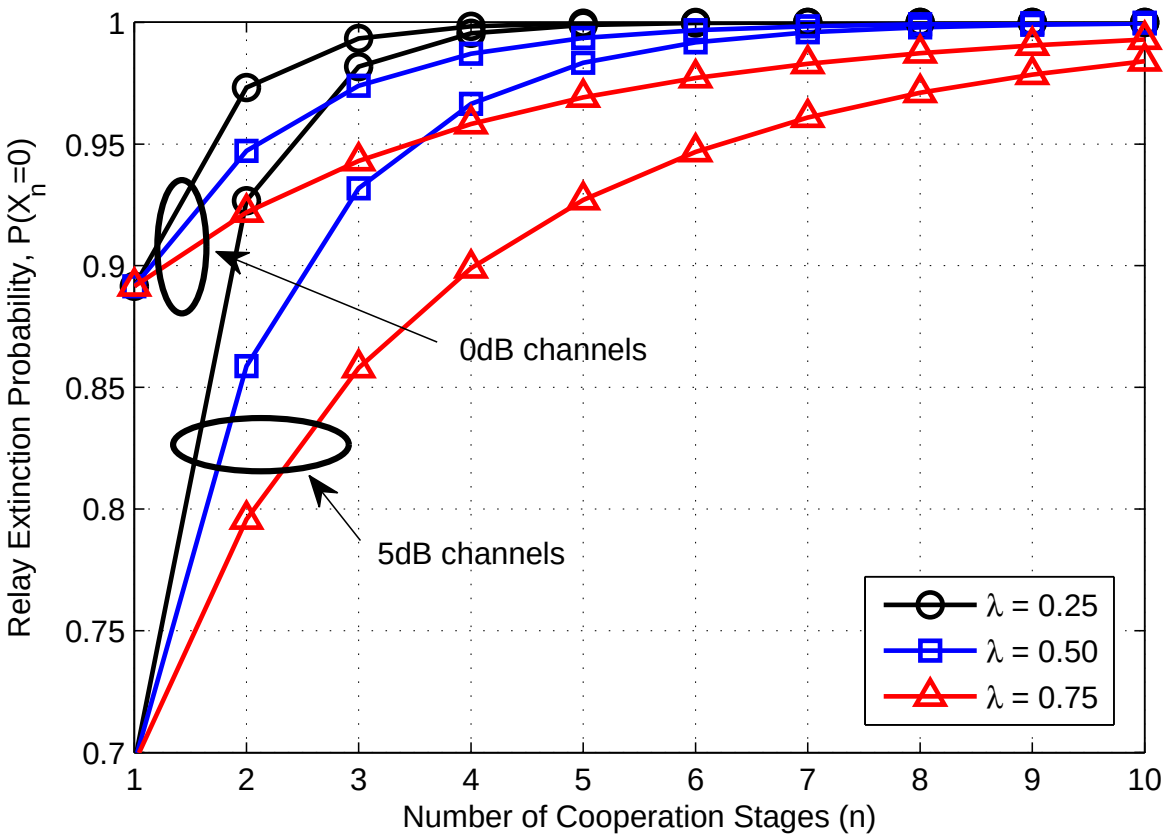

Fig. 10. Probability of Relay Extinction (no additional diversity order gained, $X_n = 0$), as a function of increasing cooperation duration (n) and relay node number per hop (λ) and mean channel SNR ($\overline{\gamma}$).

designing the node density (inter-connectivity) is a key parameter. Otherwise for $\lambda \leqslant 1$, the maximum beneficial cooperation duration (N) is limited.

C. System Error Rate Performance

The system BER can be written as a series, whereby the BER of the $n+1$ cooperation stage is a function of the n stage:

$$p_{\text{B-C-DF},n+1,T,\lambda} = 1 - \sum_{m_{n+1}=0}^{m_n+e^{\lambda(m_n-1)}-2} \frac{(\frac{T}{\overline{\gamma}})^{m_{n+1}}}{m_{n+1}!} e^{-T/\overline{\gamma}}, \tag{23}$$

which employs the extinction probability expression given in Eq. (22). From the previous results presented in Fig. 9, it can be observed that the overall system BER can improve if the threat of extinction reduces with increased cooperation duration.

Fig. 11 shows the BER for a B-C-DF network, where each channel has the same average SNR (6–20 dB). Two connectivity densities (λ relays per node) are considered with varying numbers of C-DF cooperation stage length. It shows that for a low connectivity ($\lambda = 0.3$), increased cooperation via C-DF doesn't yield significant improvements. However, for a medium connectivity ($\lambda = 1.3$), the BER improvement is significant, especially at low SNR regimes. This research relates to similar conclusions drawn in clustering relay studies for cellular networks [22]. For sensor networks, this result is especially beneficial

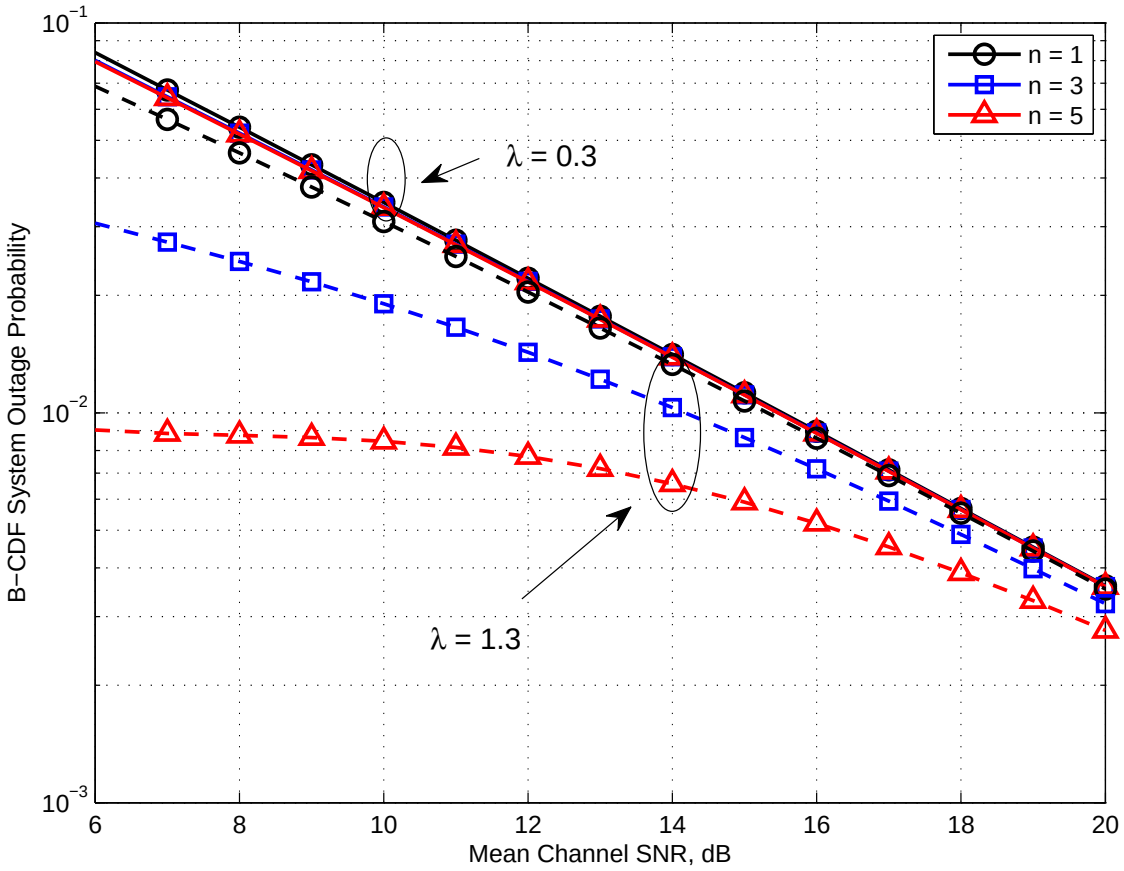

Fig. 11. BER of Branching-C-DF (B-C-DF) Scheme, as a function of increasing mean channel SNR. Solid lines represent a relay node number of 0.3 per hop, whereas dashed lines represent a relay node number of 1.3 per hop.

for systems that want to improve their lowest performing nodes in dark spots (deep shadow fading).

VI. Conclusions

This paper has examined how the basic DF scheme can be modified so that maximum diversity can be achieved at the cost of increased delay. The novelty is that a C-DF scheme is proposed and the trade-off between BER and delay is characterised.

In Section IV of the paper, it was found that if the delay constraint was relaxed for 1 additional data frame, the C-DF scheme can achieve full spatial diversity. Alternatively, for a longer delay constraint relaxations, the C-DF scheme can achieve full diversity in 10–20 dB lower channels than the DF scheme.

In Section V of the paper, the analysis examines the probability of diversity extinction with each additional cooperation stage. The results show that each node must be connected to a sufficiently high number of relays, otherwise there is an increasing chance that each additional stage of cooperation can lead to decreasing diversity gains. Typically, it was found that each node must be connected to an average of 2 or more relays to achieve a decreasing diversity extinction rate. Otherwise, the maximum beneficial cooperation duration (N) is limited.

In summary, the proposed novel C-DF scheme can dynamically trade-off transmission reliability with delay and the analysis has shown that a certain node connectivity density is required to achieve a cascading cooperation chain with an arbitrarily low extinction probability.

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