Modelling the hepatitis B vaccination programme in prisons

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SUMMARY

A vaccination programme offering hepatitis B (HBV) vaccine at reception into prison has been introduced into selected prisons in England and Wales. Over the coming years it is anticipated this vaccination programme will be extended. A model has been developed to assess the potential impact of the programme on the vaccination coverage of prisoners, ex-prisoners, and injecting drug users (IDUs). Under a range of coverage scenarios, the model predicts the change over time in the vaccination status of new entrants to prison, current prisoners and IDUs in the community. The model predicts that at baseline in 2012 57% of the IDU population will be vaccinated with up to 72% being vaccinated depending on the vaccination scenario implemented. These results are sensitive to the size of the IDU population in England and Wales and the average time served by an IDU during each prison visit. IDUs that do not receive HBV vaccine in the community are at increased risk from HBV infection. The HBV vaccination programme in prisons is an effective way of vaccinating this hard-to-reach population although vaccination coverage on prison reception must be increased to achieve this.

INTRODUCTION

In 2002, 827 acute cases of hepatitis B (HBV) infection were reported to the Health Protection Agency with the most frequent of the reported risk factors being intravenous drug use [1]. The Unlinked Anonymous Prevalence Monitoring Programme reported that in 2001 61% of surveyed injecting drug users (IDUs) were found to have been in prison [2]. In recent years the vaccination coverage of current IDUs in the community has steadily increased from 27.9% in 1998 to 39.6% in 2001. However, more needs to be done to vaccinate this high-risk group and vaccination in prison offers another opportunity to achieve this.

A vaccination programme offering HBV vaccine at reception into prison began in England and Wales in June 2001. During 2002 ~13% of new receptions at 42 prisons received HBV vaccine. This equates to ~5% of all male new receptions in prisons. With a steady increase in vaccine coverage throughout 2003 it is anticipated that the average figure for 2003 will be 10%. Over the coming years as the programme intensifies and expands this vaccination coverage of
initial receptions into prison is expected to continue to increase (R. Gilbert, unpublished observations).

The current prison policy [3] for adults is to use the super-accelerated programme with injections at 0, 7, and 21 days and a booster at the 12-month stage. This programme is not licensed for juveniles (<18 years), and so in their case an accelerated programme is administered at 0, 1, and 2 months with a booster again at the 12-month stage. A person who stays in prison for less than the time it takes to administer three doses will not complete the vaccination programme. Ensuring that prisoners complete the full course of injections before they leave prison is an ongoing problem.

Prisoners will be eligible for vaccination on each reception into a prison that is participating in the HBV vaccination programme. Over time persistent offenders will eventually begin returning to prison having already received vaccination in prison. As more persons return to prison that have been vaccinated, less doses of vaccine will be required to maintain vaccination coverage.

To judge the effectiveness of the prison vaccination programme, it is important to estimate how quickly the prison and IDU populations will be vaccinated and the maximum proportion of IDUs that can be captured by the programme over time under different vaccination scenarios. To address these questions a model is required which describes the turnover of IDUs and non-IDUs through prison. The model can then be used to estimate what proportion of the total current IDU population in England and Wales will receive HBV vaccination from alternative vaccination scenarios over time.

The work here describes the parameterization and sensitivity analysis of this model. The model is used to investigate the evolution of the vaccination status of the IDU and prison populations under a range of scenarios describing the vaccination coverage on prison reception.

METHODS
Population

The purpose of this paper is not to make detailed projections regarding the prison population over time and so in the base case we assume that both the male prison population and the total male population (females are not considered) of England and Wales are stable. In the sensitivity analysis we relax this assumption and explore how an increasing prison population affects the final results obtained from the model. Mortality is assumed to be zero until 74 years and infinite thereafter producing a fixed life expectancy of 75 years. The model considers only males aged 15–74 years, stratified into 60 age cohorts. Persons enter the model at the start of the year into the first age cohort (at 15 years of age). Thereafter individuals change age cohorts at the beginning of each new year. It is assumed that the total number of persons in each age group is the same and is constant over time.

Model structure

The model describes the flow of IDUs and non-IDUs through prison. It is an age-structured deterministic model based on a set of ordinary differential equations (Appendix 1) illustrated in Figure 1 with parameter definitions and baseline values described in Tables 1 and 2. Each mutually exclusive compartment represents a different status of imprisonment and injecting, and the arrows represent the flow between the states.

![Flow diagram of the age-specific turnover of prisoners in England and Wales (including IDUs) repeated for each of the 60 age groups. The mutually exclusive compartments represent the different imprisonment and IDU states. Arrows represent the flow between the states.](image-url)
prison (\(\theta\)) are assumed to be the same for non-IDUs and past-IDUs but may be different for current IDUs.

### Vaccination

Throughout the model the vaccination status of each compartment is stratified by the number of doses received, 0, 1, 2, or 3+. The prison vaccination programme is modelled by assuming 0, 1, 2, or 3 doses are administered to prisoners at the beginning of their sentence. Prison vaccine coverage (any dose) is given by parameter \(\alpha\) and is defined as the proportion of prisoners that receive at least one dose of HBV vaccine on prison reception. The parameters \(a\), \(b\), and \(c\) represent the proportion of those vaccinated who receive one dose, two, or three doses of vaccine respectively (\(c = 1 - a - b\)).

IDUs may be vaccinated outside prison in the community, implemented in the model as an annual event. It is assumed that a proportion \(\lambda\) of IDUs are given three doses and that this proportion is constant over time, with age, and with injecting career length. The model is run to steady state prior to the introduction of prison vaccination.
Model parameterization

Where possible parameters were directly calculated from specific data sources ($\gamma$, $\beta$, $\rho$), otherwise were indirectly estimated by maximum likelihood from data on IDUs and prisons ($\theta$, $\mu$). Age-specific rates at which IDUs start and stop using drugs ($\gamma(a)$ and $\beta(a)$) were taken from analysis of the current and reported starting age of injecting in IDUs in contact with services [4].

Prison discharge rates [$p(a)$ and $p_I(a)$]

Data were taken from a survey of prisoners during 1997–1998 [5] describing the sentence length, the age, and the IDU status of prisoners from a cross-section of prisons in England and Wales. There are two difficulties in calculating the average length of imprisonment per spell of imprisonment for IDUs and non-IDUs from these data; the actual time served for a sentence will be less than the sentence length imposed, and, a cross-sectional survey of prisoners’ current sentence lengths is not representative of all admissions because the prisoners serving longer sentences will be over-represented.

Prison statistics 2001 [6] provides data on the average time served by length of sentence, and so data were adjusted from length of sentence to average time served. To allow for the over-representation of longer sentences in the prison survey, the number of prisoners for each length of sentence was weighted by $1/(\text{average time served})$. The discharge rate for each age group was then calculated (Table 2).

Imprisonment rates [$\mu(a)$, $\mu_I(a)$ $\theta(a)$, and $\theta_I(a)$]

The rate of first-time imprisonment for IDUs and non-IDUs, and the rate of repeat imprisonment for IDUs and non-IDUs were estimated concurrently by fitting the model to data described in Figure 2 by maximum likelihood (Appendix 2) keeping previously estimated parameters (discharge rates, IDU start and stop rates) at their base-case level.

Figure 2(d) shows the proportion of the male prison population that are current IDUs [5]. The original data for the 15–20 years age group was 1.6% (11/708). From the UA Survey 2001 35% of IDUs aged 15–17 years had been to prison and for the 18–20 years group the figure was even higher at 44%. Coupled with the fact that the young offender data was only obtained from one institution, it may be that this figure is not a true reflection of the proportion of imprisoned young offenders that are IDUs.
Additional data of arrestees aged between 17 and 24 years that were interviewed and urine-tested for drugs whilst on arrest in police custody suites showed that rates of last-year use of heroin, crack and cocaine all stood at around 20% [7]. On the basis of these concerns an alternative arbitrary value of 14.1% was selected (100/708) as the base case. The effects of this selection are examined in the sensitivity analysis.

The full model was fitted using maximum likelihood (Appendix 2). For the functions describing the first-time and repeat reception rates, each was examined separately to find an appropriate functional form. An iterative process in which various functional forms were applied and tested was used. Resultant deviances for each test were compared, with the functional form exhibiting the lowest deviance being selected.

It was found as a consequence of applying the fitted model that the calculated values for the discharge rates for the IDU population resulted in lower repeat reception rates for IDUs compared to non-IDUs across all ages. IDUs are generally more likely to re-offend than non-IDUs [8], and it was found by assuming that the discharge rate for IDUs was the same for all age groups (at the 15–19 years age group rate) that a higher re-imprisonment rate for IDUs could be achieved. An argument for this can be made when considering data from different types of prison from the prison survey. Considering data from only local prisons the average time spent in prison for an IDU across all age groups is 3.8 months, this comparable to the 2.64 months for the 15–19 years age group IDUs. The effect of this is tested in the sensitivity analysis.

The final fitted function for the first-time reception rate was found to follow a gamma function, and an exponential function for the repeat reception rate (Table 2) as shown in Figure 3(a, b). It can be seen that the repeat reception rate was found to be constant for IDUs although the exponential function was used during sensitivity analysis.

**Prison vaccination**

Data were collected from the HBV vaccination programme in prisons from January to June 2003 (not shown) on vaccination doses administered to prisoners on reception during this period. To reduce the effects of truncation only data from March and April was considered. From this data, 38% of prisoners received one dose, 28% received two doses, and 34% received three doses. This is applied to the model representing vaccination parameters \(a\), \(b\), and \(c\) respectively.

**Community vaccination**

Data was considered from the Unlinked Anonymous (UA) survey 2001 [2] of male IDUs (who injected in the previous 4 weeks prior to the survey) reporting whether they have been vaccinated against HBV by career length. Assuming both that this data is representative of the IDU population in the community, and that all reported vaccinations were administered in the community, the model was fitted to the data (UA survey 2001) using maximum likelihood (Appendix 2) (fit not shown). The vaccination rate of IDUs in the community (\(l\)) was estimated to be 0.106/IDU per year.

Running the model to steady state results in a fixed value of 38.4% for the proportion of IDUs vaccinated in the community in 2001 prior to the introduction of the prison vaccination programme. This compares with the 39.6% of IDUs found in the UA survey 2001.
Sensitivity analysis

The sensitivity analysis presented here investigates how the following factors affect the final results from the model. In each case by substituting alternative values to the baseline (Tables 1 and 2) as follows:

1. The total assumed size of the male IDU population inside and outside prison was chosen to be 80,000 and 160,000.
2. The average injecting career of an IDU was assumed to be 5.3 years, 6.1 years (by changing the age-dependent injecting start and stop rates), and 20 years (assuming an age-dependent start rate and constant stop rate across all ages) respectively.
3. The percentage of imprisoned young offenders that are IDUs was assumed to be 1.6% (data, see parameterization), 4.9, 10 and 20%.
4. The average time an IDU spends in prison was assumed to be 2.0, 3.0, 4.0, 5.0, and 6.0 months.
5. The community vaccination rate ($l$) was assumed to be 0.00, 0.05, 0.075, 0.12 and 0.15 respectively.
6. To simulate the effects of a growing prison population, the first-time and repeat reception rates into prison for IDUs and non-IDUs across all ages are assumed to increase by 1, 2.5, and 5% per year.

The first four factors that were tested during sensitivity analysis were used during the concurrent calculation of first-time and repeat reception rates, these reception rates must be recalculated during each test of sensitivity.

As (in the base case) the prison population size is assumed to be constant over time, i.e. the total receptions is equal to the total discharges, it is important that any changes to the model maintains this equilibrium. Thus, if for instance the percentage of young offenders that are IDUs is altered, then the average sentence lengths (and therefore the discharge rates) of the non-IDUs must also be adjusted.

Alongside the vaccination parameters previously described (parameterization), the baseline vaccination scenario (described below) will be used as an input into the model when testing model sensitivity.

Vaccination scenarios

To test the effectiveness of the HBV vaccination programme in prisons a number of vaccination scenarios are proposed. The total proportion of prisoners on reception that participated in the HBV vaccination programme across all prisons in England and Wales during 2002 was 5%, in 2003 this figure was expected to rise to 10% and so these values are applied to all vaccination scenarios (Table 3). For each vaccination scenario: the parameters are as the baseline unless otherwise stated, the vaccination coverage on reception increases linearly between 2003 and 2006 where appropriate, and remains constant from 2006 onwards. In some cases the proposed vaccination scenarios are not achievable in reality (i.e. maximum scenario), however these have been selected to assist in drawing conclusions from the model.

RESULTS

The results presented here consider the implications of various tests of sensitivity on the results obtained from the model when the baseline vaccination scenario is applied. This is followed by an examination of the potential impact of the HBV vaccination
programme in prisons depending on the vaccination scenario implemented. In both cases the key result that has been considered is the proportion of the IDU population both inside and outside prison that has received two or more doses of vaccine. For clarity of exposition in the sensitivity analysis (excluding that examining the community vaccination rate), it has been assumed the IDU population is initially completely unvaccinated.

Results – sensitivity analysis

It can be seen from the results presented in Figure 4(a–f) that the factors that are most important in evaluating the success of the HBV vaccination programme in prisons are the total IDU population size, the average time that an IDU spends in prison during each prison visit, the increasing reception rate into prison over time, and the rate that IDUs are vaccinated in the community. These are the factors that contribute towards the greatest variation in results obtained from the model.

Results – vaccination scenarios

The vaccination scenarios, Baseline (50% by 2006), Up to 33% by 2006, Up to 66% by 2006 and No Expansion, all represent the potential future of the HBV vaccination programme in prisons. It can be seen (Table 4) that greater vaccination coverage on
prison reception results in an increase in the total proportion of IDUs vaccinated (2+ doses), an increase in the vaccination status of the prison population (2+ doses), and an increase in the number of IDUs appearing on prison reception that receive at least one dose of vaccine for the first time. However an increase in vaccination coverage on reception also requires a greater number of vaccine doses to administer the vaccination scenario. Figure 5 shows the vaccination status of the IDU population with variations in age when the Baseline and ‘Up to 66% by 2006’ scenarios have been applied.

Assuming pessimistically that the coverage on reception does not increase from 2004 onwards, 43·1% of the total IDU population will be vaccinated against HBV by 2012. This is only an additional 5% of the current IDU population being vaccinated by the prison vaccination programme, with the majority being captured by vaccination within the community. Increasing the vaccination coverage up to 66% by 2006 results in 61·3% of the total IDU population being vaccinated, an additional 22·9% of the current IDU population being vaccinated in prison.

If from 2004 only young offenders are eligible for vaccine, and by 2006 100% of them are vaccinated on prison reception, then by 2012 48·3% of the IDU population will be vaccinated (2+ doses) and only 24714 IDUs will receive vaccine in prison for the first time during this period (Table 4). When compared to the baseline it can be seen that even if 100% of young offenders are receiving vaccination on prison reception (as is the case here), this is still less

Table 4. Summary of results obtained from model with variations in the vaccination scenario applied, where vaccinated is assumed to be those persons receiving 2+ doses of vaccine

<table>
<thead>
<tr>
<th>Vaccination scenario</th>
<th>Total proportion of male prison population vaccinated</th>
<th>Total proportion of male IDU population vaccinated</th>
<th>Total doses administered</th>
<th>Total IDUs vaccinated for the first time*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>By 2007</td>
<td>By 2012</td>
<td>By 2007</td>
<td>By 2012</td>
</tr>
<tr>
<td>Baseline</td>
<td>53·5%</td>
<td>66·1%</td>
<td>52·4%</td>
<td>57·2%</td>
</tr>
<tr>
<td>No Expansion</td>
<td>26·0%</td>
<td>30·0%</td>
<td>42·2%</td>
<td>43·1%</td>
</tr>
<tr>
<td>Up to 33% coverage by 2006</td>
<td>42·8%</td>
<td>53·9%</td>
<td>48·4%</td>
<td>52·1%</td>
</tr>
<tr>
<td>Up to 66% coverage by 2006</td>
<td>63·0%</td>
<td>75·6%</td>
<td>56·0%</td>
<td>61·3%</td>
</tr>
<tr>
<td>Targeting young offenders</td>
<td>33·8%</td>
<td>44·6%</td>
<td>44·6%</td>
<td>48·3%</td>
</tr>
<tr>
<td>Baseline + campaign</td>
<td>58·8%</td>
<td>67·7%</td>
<td>53·8%</td>
<td>57·4%</td>
</tr>
<tr>
<td>Regular campaign</td>
<td>100%</td>
<td>38·6%</td>
<td>45·6%</td>
<td>43·3%</td>
</tr>
<tr>
<td>Up to 100% + campaign</td>
<td>99·0%</td>
<td>100·0%</td>
<td>69·4%</td>
<td>72·6%</td>
</tr>
</tbody>
</table>

* The number of IDUs who are completely unvaccinated on prison reception and subsequently receive at least one dose of vaccine in prison.

Fig. 5. The percentage of the IDU population that have received 2+ doses of vaccine by age (BL = baseline, 66% = up to 66% by 2006).
effective than expanding the vaccination coverage including all offenders on prison reception up to 50% by 2006.

While administering the Baseline vaccination scenario it can be seen (Table 4) that the introduction of an additional one-off vaccination campaign in 2004 has little long-term effect on the vaccinated IDU population. By 2012 only an additional 0.2% of IDUs would be vaccinated when compared to the Baseline scenario, this being due to the high-turnover rate of IDUs. The implementation of regular vaccination scenarios (in this case every 3 years) results in a fluctuation in the proportion of IDUs and prison populations vaccinated with high values after the vaccination campaign followed by a steady decrease as vaccinated persons leave the populations. This explains the 43.3% of the IDU population and 38.6% of the prison population being captured by vaccination by 2012, a decrease from 45.6% and 100% respectively in 2007.

Increasing the vaccination coverage of prisoners on reception into prison up to 100% by 2006 and at the same time administering a prison vaccination campaign at the end of 2004 results in 72.6% of the IDU population being vaccinated by 2012. An additional effect of this vaccination scenario is the 100% vaccinated status of the prison population. Having 100% vaccination coverage on prison reception will result in the prison population becoming completely vaccinated over time, the inclusion of the vaccination campaign results in this happening much sooner.

DISCUSSION
This paper considers the structure, design, parameterization, and results obtained from a model of the HBV vaccination programme in prisons. The flow of the IDU population through prison has been given particular attention, as this is a major at-risk group for HBV [9], it frequently encounters prison [5, 9], and is a difficult population to vaccinate in the community. The parameterization described here helps to inform as to the offending behaviour of IDUs when compared to non-IDUs and their flow through prison. Over a range of vaccination scenarios the model informs as to the effectiveness of various levels of vaccination coverage on prison reception. The model describes what proportion of both the IDU population and prison population will be vaccinated over time. As the vaccination programme is expanded to include more prisons a greater proportion of IDUs will be vaccinated.

A greater community vaccination rate results in more IDUs presenting themselves on prison reception having already being vaccinated. The proportion of IDUs that are being surveyed in the community having already been vaccinated against HBV has risen from 1998 to 2001 (see Introduction). The base case always assumed that the community vaccination rate (0.106/IDU per year) is constant with time although it is acknowledged that this assumption could be modified if the rate that IDUs are vaccinated does continue to rise.

The effectiveness of the vaccination scenario targeting only young offenders did not compare well with the vaccination scenarios advocating increased vaccination coverage of all prisoners on reception. This can be explained when considering the assumptions made during the model construction. It has been assumed due to the lack of data that persons start and stop using drugs at the same rates irrespective of prison status. Therefore, persons that are currently imprisoned or have previously been imprisoned are assumed to start injecting at the same age-specific rate as those that are not. The implication of this is that in reality targeting young offenders for vaccination on reception into prison may lead to a greater proportion of IDUs being vaccinated in the long term than has been presented here. This is because offenders, particularly young adults, are more likely to turn to drug use if they are currently in prison [10] or have previously been to prison.

The maximum vaccination scenario represents the highest vaccination coverage of the IDU population that can be achieved through the HBV vaccination programme in prisons (73%, assuming constant prison and IDU population sizes). The injectors unreachable by this programme are those who have not (yet) gone to prison.

The results here suggest that the effect of administering a vaccination campaign either as a ‘one-off’ or regularly in which all prisoners currently in prison are vaccinated over a short period of time has few long-term benefits, and the focus for vaccination should be towards capturing prisoners on reception into prison. This is because the majority of IDUs pass through prison so quickly (short sentence lengths) that at the time of the vaccination campaign, relatively few IDUs will be vaccinated.

Monitoring the HBV vaccination coverage of IDUs in the community will inform as to the impact
of the prison vaccination programme. This can be compared to the results from the model to see whether the predicted vaccinated status of the IDU population is comparable to actual data.

This work can be taken forward to consider the impact of the HBV vaccination programme in prisons on the prevalence and transmission of HBV within the IDU and non-IDU populations in England and Wales. IDUs are not the only high-risk group from HBV that may pass through prison, sex workers and those that engage in risky sexual practices may also benefit from a prison HBV vaccination programme. The vaccination coverage of these high-risk groups has not been considered although it is acknowledged that their vaccination may help to reduce HBV incidence in England and Wales. Finally the model could also be adapted to assess the impact of prison-based programmes aimed at controlling other infectious diseases, for instance vaccination against hepatitis A virus, or screening against tuberculosis.

**CONCLUSION**

The prison vaccination programme is an effective way of improving the HBV vaccination coverage of the IDU population. Increased vaccination coverage on prison reception is preferable to a vaccination campaign in which all prisoners are vaccinated as a ‘one-off’, as more IDUs with shorter sentence lengths will then be captured by vaccination.

**APPENDIX 1**

**Mathematical structure**

The differential equations for the deterministic model are as follows:

\[
\frac{dX_{ij}}{dt} = -(\mu_i + \beta_i)X_{ij},
\]

\[
\frac{dY_{ij}}{dt} = (1-\alpha)(X_{ij}\mu_i + Z_{ij}\theta_i) + \sum_{k=0}^{3} V_{jk}(X_{ik}\mu_i + Z_{ik}\theta_i) - (\rho_i + \beta_i)Y_{ij},
\]

\[
\frac{dZ_{ij}}{dt} = Y_{ij}\rho_i - (\theta_i + \beta_i)Z_{ij},
\]

\[
\frac{dXI_{ij}}{dt} = \beta_i(XP_{ij} + X_{ij}) - (\mu_i^* + \gamma_i)XI_{ij},
\]

\[
\frac{dYI_{ij}}{dt} = (1-\alpha)(XI_{ij}\mu_i^* + ZI_{ij}\theta_i^*) + \sum_{k=0}^{3} V_{jk}(XI_{ik}\mu_i^* + ZI_{ik}\theta_i^*) - (\rho_i^* + \gamma_i)YI_{ij} + \beta_i(Y_{ij} + YP_{ij}),
\]

\[
\frac{dZI_{ij}}{dt} = \beta_i(Z_{ij} + ZP_{ij}) + YI_{ij}\rho_i^* - ZI_{ij}\theta_i^* + \gamma_i),
\]

\[
\frac{dXP_{ij}}{dt} = XI_{ij}\gamma_i - (\beta_i + \mu_i)XP_{ij},
\]

\[
\frac{dYP_{ij}}{dt} = (1-\alpha)(XP_{ij}\mu_i + ZP_{ij}\theta_i) + \sum_{k=0}^{3} V_{jk}(XP_{ik}\mu_i + ZP_{ik}\theta_i) + YI_{ij}\gamma_i - (\beta_i + \rho_i)YP_{ij},
\]

\[
\frac{dZP_{ij}}{dt} = YP_{ij}\rho_i + ZI_{ij}\gamma_i - (\theta_i + \beta_i)ZP_{ij}.
\]

The number of individuals of age i who have received j doses of vaccine who have never been imprisoned and are non-IDUs (X), imprisoned non-IDUs (Y), previously imprisoned non-IDUs (Z), never been imprisoned IDUs (XI), imprisoned IDUs (YI), previously imprisoned IDUs (ZI), never been imprisoned previous IDUs (XP), imprisoned previous IDUs (YP), previously imprisoned previous IDUs (ZP). The different parameters determining the rates of flow between imprisonment states and IDU status are described in Table 1.

Vaccination is implemented in the model on reception into prison. A proportion \( a(t) \) of prisoners entering prison are included in the vaccination programme. The proportions of those who receive 1, 2, or 3 doses are denoted by \( a, b, \) and \( c \) respectively. The vaccination status transition matrix \( V_{jk} \) gives the proportion of persons that had received \( k \) doses before reception into prison and \( j \) doses following reception. Where:

\[
V_{jk} = \begin{pmatrix}
1-\alpha & 0 & 0 \\
aa & 1-\alpha & 0 \\
ab & aa & 1-\alpha \\
ac & ab & c & 1
\end{pmatrix}.
\]

**Computer details**

Numerical results were generated by a ModelMaker version 4.0 program (ModelKinetix, Wallingford,
Oxfordshire, UK). The system was solved using Runge–Kutta integration of ordinary differential equations with adaptable time steps. Simulations were performed on a PC.

APPENDIX 2

The model was fitted to the data using maximum likelihood. As the data are binomial the saturated likelihood and the model likelihood are:

\[
L^* = \sum_i (a_i \ln(P_i) + b_i \ln(1 - P_i)),
\]

\[
L = \sum_i (a_i \ln(M_i) + b_i \ln(1 - M_i)),
\]

where \(a_i\) is the observed number of positives (data) in age group \(i\); \(b_i\) is the observed number of negative (data) in age group \(i\).

\[
P_i = \frac{a_i}{a_i + b_i}.
\]

\(M_i\) is the modelled proportion positive in age group \(i\).

Deviance for each data set is:

\[
D = 2(L^* - L).
\]

The above is repeated for each independent dataset, with the deviance for each dataset being summed to provide an overall deviance.

Over-dispersion

Due to the size of the numerator in the prison population size, this dominates the binomial likelihood. The practical effect is that the model fit is also dominated by this part of the data. Despite this, no model gave a good fit to these data when using the binomial distribution. Therefore, we introduced an over-dispersion parameter to this part of the data using the negative binomial distribution [11]. The over-dispersion parameter \(\phi\) was chosen to fix the deviance of that part of the data equal to its degrees of freedom.

Negative binomial maximum likelihood

\[
Y_i = \text{observed number of positives (data) for age group } i;
\]

\[
\mu_i = \text{expected number of positives (model) for age group } i.
\]

\[
L = \sum_i \left[ \log(\Gamma(y_i + \phi \mu_i)) - \log(\Gamma(\phi \mu_i)) + \phi \mu_i \log\left(\frac{\phi}{1 + \phi}\right) \right].
\]

Saturated likelihood \(\mu_i = y_i\).

\[
L^* = \sum_i \left[ \log(\Gamma(y_i + \phi \mu_i)) - \log(\Gamma(\phi y_i)) + \phi y_i \log\left(\frac{\phi}{1 + \phi}\right) \right].
\]

The deviance for each model is

\[
D = 2(L^* - L).
\]

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DECLARATION OF INTEREST

None.

REFERENCES


