Discussion of "Stability assessment of slopes with cracks using limit analysis"*

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1 Introduction

The discusser has recently published a paper in *Geotechnique* entitled "Investigation by limit analysis on the stability of slopes with cracks" (Utili, 2013) which included for the first time, to the discusser's knowledge, a systematic investigation on the influence of the presence of cracks in uniform slopes for rotational failure mechanisms via the limit analysis upper bound approach. Looking at the discusser's paper and the discussed paper, (Michalowski, 2013), a reader may note that the aim of the two papers is the same, namely to assess quantitatively the effect of the presence of cracks on the stability of slopes, and the methodology of using the upper bound approach of limit analysis. The discusser's paper was sent to *Geotechnique* when the discusser had no knowledge of either the author's conference paper (Michalowski, 2012), or of the discussed paper published in July 2013. On the other hand, the discusser's paper was published after the publication of the author's conference paper (Michalowski, 2012). Hence, it can be concluded that the discusser and the author had independently developed an original formulation for the calculation of upper bounds based on rotational failure mechanisms for cracked uniform slopes at similar times.

However, regarding the findings and the formulation of the problem, (Utili, 2013) and the discussed paper, (Michalowski, 2013), are rather different. In this discussion, the discusser wants to highlight the main complementary and different findings between the two papers and point to some aspects of the discussed paper that in the discusser's opinion need clarifications especially with regard to the following three topics each being a section of the present discussion: the calculation

24 of the external rate of work for rotational failure mechanism; the failure mechanisms

25 analyzed for pre-existing cracks; and the failure mechanisms with crack formation.

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Calculation of the external rate of work for rotational failure mechanisms

29 Concerning the calculation of the external rate of work in case of a rotational failure

30 mechanism, the author reports neither the derivation nor the final analytical

31 expressions of the functions employed to calculate the rate of the external work done

32 by the soil mass sliding away, wedge BOCDB in figure 6. The calculation of the

33 external work is as important as the calculation of the energy dissipated since both

34 appear in the energy balance equation from which the stability factor, $\gamma H/c$ (Taylor,

35 1948; Chen, 1975), is calculated. In this regard, the author seems to justify the lack

36 of detail provided making reference to the works of Chen (e.g. Chen & Giger, 1969;

37 Chen & Giger, 1971) and stating that "without a crack, this mechanism has been

38 described in the literature multiple times". However, this is not the case. In fact, in

39 the referenced Chen's publications, the calculation of the external work rate is

40 reported only for failure surfaces made entirely by a log-spiral (wedge BOCADB in

41 figure 6) with either the logspiral passing through the slope toe (see Fig. 6a) or

42 below (see Fig. 6b). Here instead, the failure surface is composite: partly log-spiral

43 and partly planar (wedge BOCDB in figure 6). The calculation of the external work

44 in this case of a composite partly log-spiral partly linear failure surface requires the

45 calculation of the work done by the fictitious wedge BOCADB minus the work of

46 the fictitious wedge DCAD (Utili, 2013). The analytical expressions for the

47   calculation of the external work done by soil masses sliding along composite log-

48   spiral failure surfaces, which requires the use of fictitious wedges bordered by a log-

49   spiral, was first presented in (Utili, 2005; and Utili & Nova, 2008) for the case of

50   slopes with horizontal upper part subject to a sequence of landslides, and in (Utili

51   and Crosta, 2011) for the more general case of slopes with an inclined upper part. In

52   (Utili and Nova 2007), the calculation of the work done by a wedge enclosed by two

53   log-spirals is presented. In these publications the calculation of the external work is

54   reported in detail together with the related analytical expressions.

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56   **Failure mechanisms for pre-existing cracks**

57   In the analysis of rotational failure mechanisms for slopes with pre-existing cracks,

58   rightly the author states that the minimization of the function providing the stability

59   number is a problem of constrained minimization because of the constraint on the

60   maximum depth of the crack. In the search for the failure mechanism of a slope of

61   given inclination and friction angle (β and ϕ respectively), the length and location of

62   the crack is free, *i.e.* the minimization of the function is sought over 4 independent

63   variables, the angles θ_0, θ_h, θ_C (χ, ν, ζ in (Utili, 2013) with $\chi=\theta_0, \nu=\theta_h, \zeta=\theta_D$) and β'

64   with the additional constraint that "the crack cannot be deeper than the maximum

65   depth of the crack discussed". Concerning the variable β', the discusser has shown

66   that for $\phi>5°$, all the failure mechanisms pass through the slope toe, i.e. $\beta'=\beta$,

67   whatever values of β and ϕ are considered (Utili, 2013), so that for the drained

68   analyses presented in the discussed paper with $\phi=10°$ or greater the number of

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69   variables to be considered in the unconstrained minimization can be reduced to three:

70   θ_0, θ_h, θ_C.

71   With regard to the maximum crack depth allowable, unfortunately, the author

72   does not state when the constraint turns out to be active, *i.e.* for what values of the

73   parameters β and ϕ. In case of dry slopes, employing the formula given in Eq. (5),

74   the discusser has verified that this limit on the maximum crack depth is never

75   exceeded by the crack depth resulting from the unconstrained optimization of the

76   function expressing the stability factor for all the considered values of β and ϕ (see

77   Figure 1). The interested reader can find the analytical expression of the function

78   reported in Eq. (25) in (Utili, 2013). This implies that the inequality of Eq. (5) is not

79   active so that the minimization presented in the discussed paper is actually an

80   unconstrained minimization rather than a constrained one providing the solution to

81   the problem of determining the critical failure mechanism for slopes with cracks of

82   unspecified location and depth (problem *c* in Utili, 2013). This solution is a

83   particular case of the solutions found for the other two dual problems tackled in

84   (Utili, 2013): determination of the critical failure mechanism for slopes with a crack

85   of known length but unspecified location (see Figure 2a) and determination of the

86   critical failure mechanism for slopes with a crack of known location but unknown

87   depth (see Figure 2b), problems *a* and *b* respectively in (Utili 2013), which are not

88   tackled in the discussed paper. The solution to these problems is provided by a

89   genuine constrained optimization where the minimum for the function expressing the

90   stability factor, is sought with the additional constraint of satisfying a non-linear

91   equality prescribing, in case of problem *a*, the crack depth, and in case of problem *b*,

92   the crack location, so that the number of independent variables in both problems is

93   reduced to two. The stability factors found for these two problems, are a function of

94   the crack depth and of the crack distance respectively (the imposed constraints), and

95   their minimum with respect to crack depth and crack distance corresponds to the

96   solution presented in the discussed paper for the case of cracks of any depth and

97   location (see Figure 3).

98   With regard to the geometry of the failure mechanisms, it is important to note

99   that in the discussed paper, failure mechanisms are assumed to pass either through

100 the toe or below without consideration for mechanisms daylighting on the slope face

101 above the toe. However, unlike the case of intact slopes, the presence of cracks

102 implies that mechanisms passing above the slope toe are no longer self-similar (see

103 figure 4) and therefore need to be considered in the calculation of the upper bounds.

104 From the calculations in (Utili, 2013), it turns out that in case of dry slopes with

105 either dry or water filled cracks, the failure mechanisms pass through the slope toe.

106 However, for different hydraulic conditions as the ones considered in the discussed

107 paper and in case of failure mechanisms accounting for crack formation, mechanisms

108 daylighting on the slope face could still turn out to be more critical than the

109 mechanisms passing through the toe assumed in the discussed paper. Hence, it could

110 be interesting to know if the mechanisms considered by the author are still the most

111 critical once potential failure mechanisms daylighting on the slope face are

112 accounted for in the calculations.

113   Finally, concerning how good the achieved upper bounds are, the following

114 remark in the paper "Of all admissible two-dimensional slope collapse mechanisms

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115   for soils considered in the literature, it is the rotational one that has been found most

116   critical for uniform slopes (Chen, 1975)" overlooks the fact that (Bekaert 1995)

117   found an upper bound of 1.0% lower for a vertical uniform slope with ϕ=0, by

118   considering a multiple rotation mechanisms made of several log-spiral blocks.

119   However, although it has to be pointed out that Chen's upper bounds obtained

120   assuming a rigid rotation may no longer be the best upper bounds in the light of

121   more recent works in the literature, they are very close to the true collapse load: for

122   instance (Krabbenhoft *et al.,* 2005) achieved lower bounds by finite element limit

123   analyses which are on average 1.5% and in the most unfavourable case 2.5% less

124   than the upper bounds obtained for β ranging from 50° to 90° and ϕ from 10° to 40°.

125   Conversely, it is crucial, in the discusser's view, to point out that when cracked

126   slopes are considered, no lower bound solutions are available in the literature to

127   bracket the true collapse values; therefore in case of cracked slopes it cannot be

128   taken for granted that the upper bounds obtained for rigid rotational mechanisms are

129   still close to the true collapse load. In this regard, it is reasonable to expect that at

130   low crack depths, the upper bounds remain close to the true values whereas for high

131   values of crack depths, they may diverge substantially. This limitation of the

132   presented solutions should be acknowledged. Also in the conclusions, the author

133   remarks that "for slopes with an inclination of 30° or less, the calculated critical

134   height is little or not affected by the presence of a crack. The influence of the crack

135   presence becomes significant however with an increase of the slope inclination". On

136   this point it is interesting to note that if the newly found upper bounds for rotational

137   failure mechanisms are compared to the upper bounds relative to planar mechanisms,

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138   the reduction on the stability factor determined, *i.e.* the improvement of the upper

139   bounds of the new solution in comparison with the available bounds for planar

140   mechanisms (Hoek and Bray, 1977), the trend is opposite with the upper bound

141   reduction being higher for shallow slopes (Utili, 2013).

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143   Failure mechanisms including crack formation

144   Concerning translational mechanisms, the discusser points out to a typographical

145   error in the equation provided for the dilation angle, $\delta = \frac{\pi}{2} - \theta - \phi$, which instead needs

146   to be $\delta = \frac{\pi}{2} - \theta + \phi$ for the mechanisms to be kinematically admissible.

147   Concerning rotational mechanisms with crack formation, the paper does not

148   specify what physical phenomena cause the envisaged formation of the cracks. This

149   is an essential point if the analysis is to be realistic. In the presented analysis, a non-

150   zero shear stress state underneath the crack tip has been assumed for respect of the

151   normality rule, given the direction of the velocity vectors underneath the crack tip as

152   the author's points out: "The stress associated with this kinematics is described by

153   the circular portion of the yield condition. This is not necessarily the true stress state

154   but it is consistent with the selected kinematics". However, if one considers the

155   starting point where the crack begins to form, *i.e.* at the ground level on the

156   horizontal upper part of the slope, the presence of shear stresses violates equilibrium

157   since no loads are applied on the slope. Moreover, the author does not specify how

158   the envisaged shear stress relates to any of the several different possible physical

159   phenomena leading to crack formation: e.g. desiccation, wetting, and drying cycles,

160   weathering.

161   Finally, the first statement in the conclusions "It was demonstrated that crack

162   formation is an important factor affecting the outcome of stability analyses of

163   slopes." appears unjustified for the fact that when crack formation is considered, the

164   failure mechanisms turn out to be less critical than the case of pre-existing cracks, so

165   in the stability analysis of uniform slopes, consideration of crack formation is not

166   critical according to the analysis performed. More importantly, the usefulness of the

167   whole stability analysis with crack formation is rather debatable since the crack

168   formation mechanisms considered are driven by an unrealistic state of stress in the

169   ground for the aforementioned violation of the equilibrium at the boundary of the

170   slope (where the crack begins to form) and it has not been related to any physical

171   phenomenon causing the formation of cracks.

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173   **Summary**

174   The discussed paper (Michalowski, 2013) presents an interesting analysis of the

175   stability of slopes subject to vertical tension cracks. The authors considered both pre-

176   existing cracks and forming cracks, focusing considerable attention on the maximum

177   possible crack depth and seepage effects. These findings, when considered in

178   conjunction with the independently obtained findings of the discusser's paper (Utili,

179   2013), are likely to provide a comprehensive analysis of the effect of the presence of

180   cracks in various scenarios. The fact that two independent authors developed these

181   original formulations at simultaneous times demonstrates how strong the interest of

182   the geotechnical community is in this area. I hope that this discussion will contribute

183   to the advancement of this area of research.

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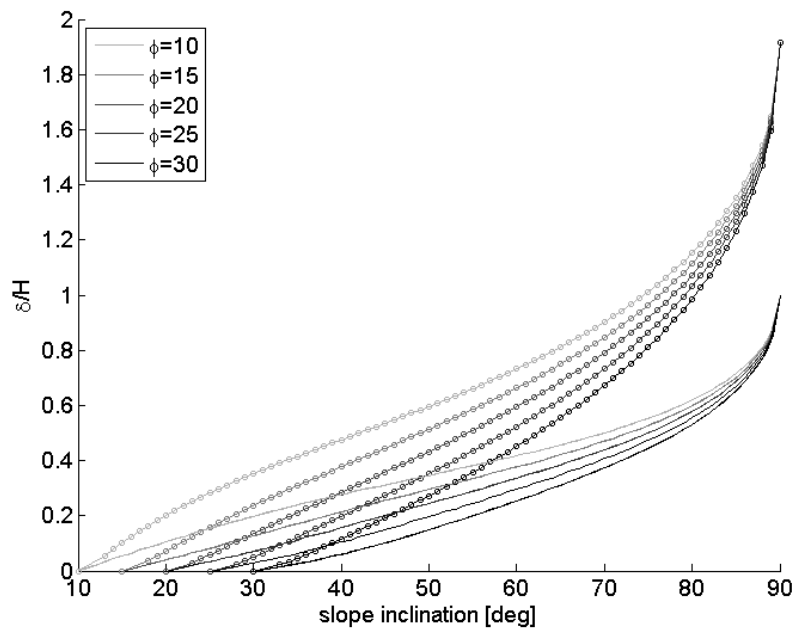
Figure 1. Failure mechanisms for cracks of any possible depth (δ) and location. The crack depth corresponding to the failure mechanism is plotted versus slope inclination for various friction angles: the lines without markers indicate the crack depths whilst the lines with markers indicate the maximum crack depth according to Eq (5) of the paper.
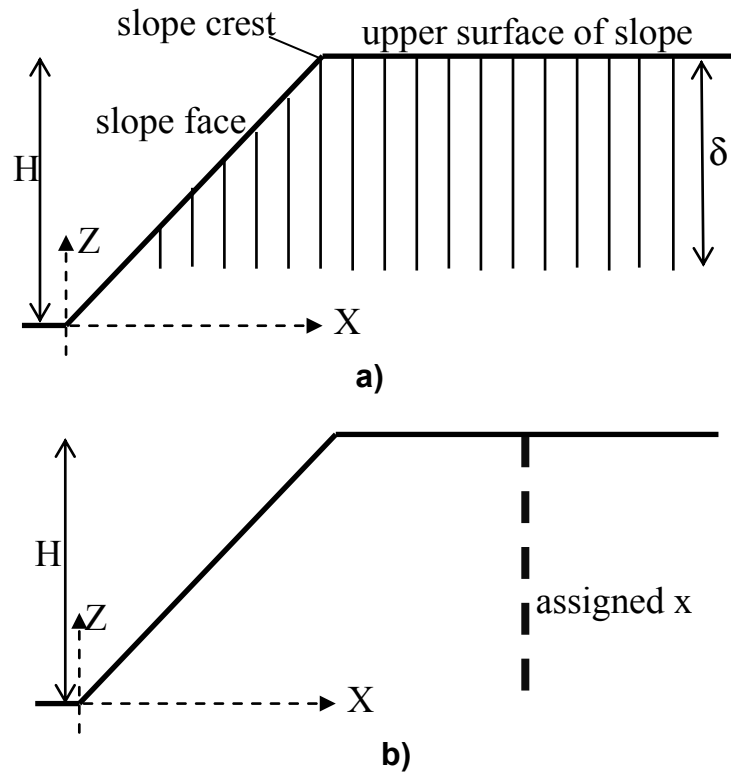
Figure 2 after (Utili, 2013). Problem a), the upper bound is sought for a fixed crack depth, δ, with the crack lying at any possible horizontal distance from the slope toe, x (the black lines representing the vertical cracks can be anywhere within the gray region). Problem b), the upper bound is sought for a crack of unknown depth (any δ is possible) located at a fixed horizontal distance from the slope toe, x.
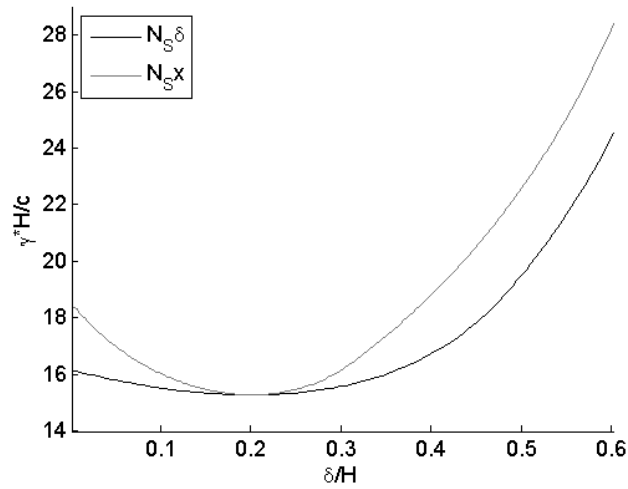
Figure 3 after (Utili, 2013). Stability factor obtained by constrained minimization vs. crack depth for a slope with ϕ=20° and β=45°: the gray line represents the stability factor, $N_S{}^x$, obtained for cracks of fixed location, x, whilst the black line represents the stability factor, $N_S{}^\delta$, obtained for cracks of fixed depth, δ. The minimum of the curves corresponds to the stability factor associated to the failure mechanism analyzed in the discussed paper for a crack of any depth and location, problem c in (Utili, 2013).

Figure 4. The gray log-spiral G-F represents a potential failure mechanism passing above the slope toe whilst the black one B-D the failure mechanism passing through the toe. The lack of self-similarity between the two mechanisms is due to the fact that the self-weight of the triangular region MOLM gives rise to a linearly distributed load on M-L whereas the rectangular region LOCDL to a uniformly distributed load on L-D (after Utili, 2013).