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Natural Expectations and Home Equity Extraction

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Abstract

In this paper we propose a novel explanation for the increase in households’ leverage during the recent boom in U.S. housing prices. We use the U.S. housing market’s boombust episode that led to the Great Recession as a case study, and we show that biased long-run expectations of both households and, especially, financial intermediaries about future housing prices had a large impact on households’ indebtedness. Specifically, first we show that it is likely that financial intermediaries used forecasting models that ignored the long-run mean reversion of housing prices after a short-run momentum, thus leading to an overestimation of future households’ housing wealth. We frame this finding in the theory of natural expectations, proposed by Fuster et al. (2010), to the housing market. Then, using a tractable model of collateralized credit market populated by households and banks, we find that: (1) mild variations in long-run forecasts of housing prices result in quantitatively considerable differences in the amount of home equity extracted during a housing price boom; (2) the equilibrium levels of debt and interest rate are particularly sensitive to financial intermediaries’ naturalness; (3) home equity extraction data are better matched by models in which agents are fairly natural.

JEL Classification: E21, E32, E44, D84.

Keywords: Natural expectations, Home equity extraction, Consumption/saving decision, Housing price.

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1 Introduction

From 1999 to the end of 2006, U.S. household debt relative to income grew sharply, from 64 percent to more than 100 percent.\(^1\) The increase in debt was accompanied by a sharp appreciation in housing prices: the real house price Standard & Poor’s Case-Shiller Home Price Index soared by 65 per cent in the same time span. Unlike previous episodes of heated housing markets, the recent housing price boom has been characterized by a surge in households’ extraction of home equity, through cash-out refinancing of mortgages, second lien home equity loans, or home equity lines of credit (henceforth, HELOCs). In 1990, the value of these home equity extraction instruments recorded in the balance sheet of U.S. commercial banks was about $58 billion; at the end of 1999, their value doubled to $103 billion; and in 2006, when housing prices were at their peak, it had more than quadrupled.\(^2\) Also, Greenspan and Kennedy (2005) document that households’ gross home equity extraction as a fraction of disposable income increased by 7 percentage points (from less than 3 percent to about 10 percent) between 1997 and 2005.

In this paper we propose a novel explanation for the increase in households’ leverage during a housing price boom. We show that long-run expectations about future house prices of both households and, especially, financial intermediaries have a large impact on households’ indebtedness. Our story relates to the work of Fuster et al. (2010) and Fuster et al. (2012) and the concept of natural expectations as follows. In their setting: (1) fundamentals of the economy are truly hump-shaped, exhibiting momentum in the short run and partial mean reversion in the long run, which, however, is hard to identify in small samples. And (2) agents do not know that fundamentals are hump-shaped and, instead, base their beliefs on parsimonious models that fit the available data.\(^3\) Following a similar approach, we assume that our economy’s homeowners, taking housing prices as given, have to forecast house price realizations to quantify their future housing wealth and to decide how much equity to extract. Similarly, financial intermediaries need to forecast future house prices to choose the supply of home equity loans. Which model do agents use to forecast housing prices? We consider a set of parsimonious models that replicate empirically observed patterns in housing prices. Hence, these models are similar in terms of in-sample fit and short-run forecasts. However, they differ in their ability to capture the long-run hump-shaped dynamics that characterize

\(^1\) Source: US. Bureau of Economic Analysis (GDP, BEA Account Code: A191RC1) and Federal Reserve System, Flow of Funds (Households and nonprofit organizations; total mortgages; liability, id: Z1/Z1/FL153165005.Q).

\(^2\) Source: Federal Reserve System, Flow of Funds.

\(^3\) These assumptions are able to generate empirically observed patterns in asset prices, such as asset price volatility, mean-reversion, and large equity premium.
housing prices. We are interested in assessing how the behavior of agents in the credit market is affected by natural expectations - that is, by using simplified models that fail to take into account the long-run mean reversion of house prices after a positive short-run momentum when making forecasts.\footnote{As in Fuster et al. (2010), for tractability we abstract from learning and give agents a fixed, simple model estimated using available data.} After all, as shown by Fuster et al. (2010), long-run mean reversion is a property of a process that is hard to detect in small samples. Then, using a tractable model of a collateralized credit market populated by households and banks and calibrated to the most recent house price boom, we find that: (1) mild variations in long-run housing price forecasts result in quantitatively considerable differences in the amount of home equity extracted during a housing price boom; (2) the equilibrium level of debt and its interest rate are particularly sensitive to financial intermediaries’ naturalness; (3) home equity extraction data are better matched by models in which agents are fairly natural. Our findings, hence, support the theory of Case et al. (2012), which highlights the role of future housing price expectations among other several explanations of market dynamics.\footnote{In this respect, our paper is also in line with Burnside et al. (2011), which show that boom and bust dynamics in the housing market are affected by “social dynamics” that lead agents to change beliefs about future housing prices.} The assumption that households behave in line with natural expectations when confronting house prices is largely supported by empirical work. For example, Goodman and Ittner (1992) surveys the early literature about the excessive optimism of homeowners in assessing the future values of their homes and documents that households overestimate home price by between 4 percent and 16 percent. Even more drastically, homeowners appear to significantly overes-

\footnote{Other theories proposed in the literature focus on: growing complacency of lenders in the face of declining loan quality (Mian and Sufi, 2011, Demyanyk and Van Hemert, 2011); money illusion on the part of homebuyers that led to flawed comparisons of home purchase prices with rents (Brunnermeier and Julliard, 2008); an agency problem affecting the credit rating agencies (Mathis et al., 2009); and government failure to regulate an emerging shadow banking system (Gorton, 2010).}
timate the current value of their houses: Agarwal (2007) considers panel data from 2002 to 2005 and concludes that homeowners overestimate their house value by on average 3.1 percent. Also, using questionnaire survey data in the period 2002-2012, Case et al. (2012) find that households’ forecasts were accurate in the short-run (one year) but “abnormally high” in the long run (10 years).\(^7\) Nevertheless, households are only one side of the housing-related debt market. In fact, financial institutions supply credit to households and, if they did not share the same optimistic forecasts, they would be reluctant to provide home equity at low interest rates. One novelty of this paper is its insight in documenting that financial experts also fell victim to natural expectations when they made their housing price forecasts - in the sense that they, too, ignored any form of long-run mean reversion in housing prices after the positive and strong short-run momentum. In addition, using our model, we highlight that banks’ natural expectations were particularly important for the recent home equity extraction boom. Specifically, the equilibrium level of debt in the economy owes more to the natural expectations of financial intermediaries than of households. The natural expectations of financial intermediaries seems a necessary component in any explanation of the reduction of the interest rate for home equity loans as observed during the housing price boom. Yet, surprisingly, investigations of the role of the supply side on the surge of housing-related debt is a relatively unexplored issue. An exception is Justiniano et al. (2014), who, however, focus on the loosening of lending constraints in the mortgage markets.

Thus, the first contribution of our paper is to document that financial experts also likely ignored hump-shaped dynamics of housing prices in their forecasts, and thus wound up being excessively optimistic about long-run housing price appreciation in the recent price boom. Specifically, we gather a unique dataset of out-of-sample housing price forecasts made by a professional forecasting company in the period 1995-2011 and show that these forecasts do not display any sort of adjustment after a period of short-run positive momentum: forecasts made prior to 2006 predict overall constant and large increases in long-run housing price until 2030. These findings are in line with other studies about the behavior of housing market experts during the boom phase.\(^8\) We argue, then, that financial experts can also be treated as natural agents and that their inability to account for hump-shaped housing price dynamics affected the supply of credit during the recent boom.

As a second contribution of the paper, we apply the theory of natural expectations to the housing market. Specifically, first we show that housing prices are characterized by hump-

\(^7\)As the authors state: “it may be a general expectation about the vague and distant future that helps explain why people behaved in the 2000s as if they thought that home prices could never fall: perhaps they thought so only about the long run, as our 10-year expectations data seem to confirm”.

\(^8\)See Foote et al. (2012) and Cheng et al. (2014).
shaped dynamics, which imply a large momentum in the short run and partial mean reversion in the long run. Then, we compare four models to estimate and forecast housing price dynamics. We consider two possible dimensions that lead to natural expectations: (1) an inner tendency of agents to incorporate a small set of explanatory variables when estimating a model, in line with the findings in Beshears et al. (2013); and (2) a limited ability of agents to consider a large set of data when estimating the model, in line with the assumption of extrapolative expectations applied to the housing market.\footnote{See Goetzmann et al. (2012), Abraham and Hendershott (1994), Muellbauer and Murphy (1997) and Piazzesi and Schneider (2009).} We also consider two rigorous and more sophisticated statistical approaches to modeling and forecasting housing prices, which differ in the information criterion used to select the most appropriate specification. We find that models that incorporate hump-shaped dynamics are not preferred, in terms of in-sample fit, to more parsimonious models that ignore long-run mean reversion. As a result, the use of simple models leading to natural beliefs is fully justifiable in terms of in-sample performance. Finally, we demonstrate that models that have diverse degrees of ability to capture hump-shaped dynamics in housing price market may differ in their long-run forecasts, while leading to similar short-run predictions. Hence, agents that make use of simple models fail to take into account the partial mean reversion of housing prices in the long run.\footnote{As discussed in Fuster et al. (2010): “there are several reasons that justify the use of simple models: they are easy to understand, easy to explain, and easy to employ; simplicity also reduces the risks of over-fitting”.}

The third contribution of the paper is to link long-run housing price forecasts to the optimal behavior of agents in the credit market. We therefore introduce a tractable model of a collateralized credit market populated by a representative household and bank. The household can obtain credit from the bank by pledging its house as collateral.\footnote{The model is related to Cocco (2005), Yao (2005), Li and Yao (2007), Campbell and Cocco (2011), and Brueckner et al. (2012).} In each period, the household decides how much to consume and how much to borrow and, given the realization of the stochastic exogenous housing price, whether to repay its debt or to default and lose the ownership of the house. The amount of debt demanded crucially depends on the expected realizations of the housing price. The bank borrows resources at a prime rate and lends them to the household charging a margin. The bank gains either from debt repayment, in the case of no default from the household, or from the sale of the housing stock, in the case of default. Hence, the bank faces a trade-off when offering a large amount of debt: on the one hand, it might increase its revenue in case of no-default; on the other, it provides incentives to the household to default. Obviously, the banks’ expected future house price is a key determinant of its supply of credit.

In our quantitative assessment, we are mainly interested in examining the extent to which
the equilibrium level of debt and its price vary with the ability of agents to take into account possible long run mean-reverting dynamics of housing prices. Hence, we select a housing price path in our model that matches the observed dynamics of the aggregate U.S. housing price in the period 2001-2010, and we vary the specification of the process the agents use to predict future house prices. We consider a large set of specifications (fifty) that are identical in terms of the short-run (one-year ahead) forecast, and in terms of magnitude of the unconditional variance of the housing price process, but that differ in terms of the long-run expectations. Hence, we can rank the different specifications according to their degree of naturalness: more natural processes ignore the long-run mean reversion of housing prices and predict a higher long-run price; less natural processes incorporate a certain degree of housing price adjustment after the short-run momentum and predict a lower long-run price. We find four results. First, the model predicts a positive relationship between the average equilibrium level of debt in the economy in the boom phase and the degree of naturalness of agents. Intuitively, after observing an increase in the house price, a more natural agent expects a longer-lasting housing price appreciation, which gives stronger incentive to demand/supply debt. Second, long-run expectations play a large role from a quantitative point of view: when the economy is populated by more natural agents, the debt to income ratio during a boom phase is about 55 percent; when the economy is populated by less natural agents it falls to 35 percent. Recall that the difference in these quantities is solely due to the contrasting long-run expectations of housing prices, since by construction agents have the same short-run expectations in each of the fifty specifications. Third, we show the importance of supply-side naturalness for the increasing household debt leverage during the housing price boom and for the interest rate reduction of home equity loans, as documented by Justiniano et al. (2014). In fact, by conducting a simple experiment where only the bank or the household (or both) are natural, we highlight that banks’ naturalness has a larger effect than households’ naturalness in increasing the equilibrium level of debt in the economy, and that, consistent with economic theory, it is the outward shift of the debt-supply schedule driven by banks’ naturalness that is able to generate lower interest rates in home-equity-related debt. Hence, whereas Justiniano et al. (2014) explain increased levels of household debt (at lower prices) with the relaxation of lending constraints, our paper proposes an alternative story for the outward shift in credit supply observed during the phase of rising housing price. As a last result, using data on Gross Home Equity Extraction as computed in Greenspan and Kennedy (2005), we show that the simulated process that better fits the observed debt dynamics during the 2000-2009 housing price boom is characterized by a rather high degree of naturalness.

The rest of the paper is organized as follows. In section 2 we document the tight rela-
tionship between housing prices and households’ economic behavior in the United States, and we provide evidence that financial experts’ forecasted future housing prices were not able to incorporate their long-run downward adjustment after a positive momentum. In section 3 we discuss the properties of natural expectations and their implications for long-run housing price forecasts. In section 4 we describe the theoretical model, and in section 5 we describe its calibration. In section 6 we discuss the quantitative results of the model. Section 7 concludes and summarizes the main findings.

2 Debt, Housing Prices, and Professional Forecasts

The goal of this paper is to analyze the interaction between housing price forecasts and private agents’ economic behavior in the credit market. This link is not obvious if one considers housing an illiquid asset. However, recent innovations in financial markets and, in particular, the growing popularity of home equity loans have contributed to making housing a liquid asset, thus strengthening the relationship between housing prices (and housing wealth) and agents’ consumption/saving decisions.\textsuperscript{12} To understand the rising popularity of such financial instruments in the last decades, in Figure 1 we plot the flow of home equity extracted by households (in billions of dollars, solid line), together with the Standard & Poor’s Case-Shiller Home Price Index (dashed line). The positive trend starting from the beginning of the ’90s, as well as the comovement between Home Equity Extraction (HEE) and housing prices, are evident: in 1992 the value of HEE was about $41 billion (in 2006 dollars); at the end of 1999, HEE value more than doubled to about $95 billion; and from 2000 to 2006, when housing price growth was at its peak, HEE almost tripled.

This evidence has been already examined in the literature. For example, Mian and Sufi (2011) estimate the aggregate impact of the home equity-based borrowing channel, finding that $1.25 trillion (i.e. about 2 percent of GDP per year) of the rise in household debt from 2002 to 2006 is attributable to existing homeowners borrowing against the increased value of their homes. Disney and Gathergood (2011) present evidence for the relationship between housing price growth and household indebtedness among homeowners in the United States from 1999 to 2007, and find that rising housing prices explain roughly 20 percent of the growth in indebtedness among U.S. households. Brown et al. (2013) find that all homeowner types increased their housing and non-housing debt in response to the housing price boom.

One possible explanation for the large exposure of households to home equity loans is their

\textsuperscript{12}Home equity loans allow households to borrow up to a maximum amount within a given term, pledging their home equity as a collateral.
inability to correctly forecast future housing prices, as largely suggested and documented in the literature.\textsuperscript{13} Importantly, Case et al. (2012) present evidence, based on annual household surveys between 2003 and 2012, showing that while households have been rather accurate in predicting short-term housing price appreciation, their long-term forecasts have been largely upward biased until 2005, when a strong revision of long-term forecasts occurred. Though households’ expectations about future housing prices are obviously important, they cannot be the sole ingredient for the high level of collateralized debt. In fact, if demand for debt increases (thanks to households’ upwardly biased long-term housing price forecasts) but supply of debt does not shift, basic economic theory suggests that the economy should experience an increased level of debt at higher costs -namely, higher interest rates. Nevertheless, as Justiniano et al. (2014) document, this is counterfactual since the recent home equity loan boom has been associated with low interest rates. To reconcile this evidence, in this paper we highlight that the supply side’s house price expectations were particularly important for the recent home equity extraction boom. Specifically, we provide evidence that financial experts behave as natural agents, in the sense that, when making forecasts, they did not take into account any sort of long-run mean reversion in housing prices after a large short-run momentum.

2.1 Financial Experts Forecasts

In this section we provide evidence that models used by financial experts to forecast future housing prices were not able to incorporate their long-run downward adjustment after a positive momentum, which led to too optimistic future housing price expectations. For this reason, it is not unreasonable to consider financial experts as natural agents, in the sense that, as Fuster et al. (2012) define, they have ignored the hump-shaped dynamics of the housing price process that indeed characterize the housing price data, as we document later in the paper.

Specifically, we analyze a unique data set that contains out-of-sample forecasts of quarterly housing prices up to a horizon of 30 years, produced by a professional forecasting company.\textsuperscript{14}

\textsuperscript{13}There is a large literature documenting that homeowners tend to overestimate both the current value of their house and their future values. For example, Goodman and Ittner (1992) state that households’ housing price estimates are between 4 percent and 16 percent larger than the actual realization. Shiller (2007) states that a significant factor in the housing boom was the perception that housing is a profitable investment and that housing price appreciation generated expectations of future price appreciation. Using panel data from 2002 to 2005 Agarwal (2007) finds that homeowners significantly overestimate the value of their home by an average 3.1 percent. Benitez-Silva et al. (2008) estimate a sale-price equation as a function of a self-reported housing wealth, concluding that homeowners in average overestimate the value of their home by 5 percent to 10 percent. In addition, data show that the overestimation was more likely after 1980.

\textsuperscript{14}This globally recognized professional forecasting company provided us with their nominal housing price out-of-sample forecasts generated by their models. Unfortunately, the company was willing to privately disclose to us point estimates only.
The model used for generating the forecasts is described as a rich demand-supply model that takes into account long-term influences on housing prices, such as income trends and demographics, and cyclical factors such as unemployment and changes in mortgage rates. These forecasts began in 1995 and were updated every quarter until the end of 2011. We take these forecasts as a proxy for the forecasts made by financial experts. We believe that since these forecasts were made by a professional forecasting company they are not subject to a “bad incentive” bias. Specifically, Barberis (2013) suggests that financial institutions might have had incentives to sell real estate financial instruments even when predicting a coming house price collapse. The fact that our dataset is not provided by a financial institution rules out the possible problem that these forecasts were simply strategic statements to sell specific product to clients. Our underlying assumption is, then, than these forecasts collect what the financial experts really knew about the future evolution of house prices.

Figure 2 shows the professional forecasts of a nominal housing price index for the period 1998-2020. In this figure we consider four forecasts made in the period 1998-2006, before the bust of the housing bubble. The red dotted line represents the forecast made in 2000Q1, the green circled line represents the forecast made in 2002Q1, the purple dashed line represent the forecast made in 2004Q1, and the blue dash-dotted line is the forecast made in 2006Q1. As the figure displays, the forecast made in 2000, 2002, and 2004 looking one to two quarters ahead were relatively accurate since they are very close to the actual realization of the housing prices (solid tick line). Nevertheless, the forecasts computed in those three years were not able to capture the steep price appreciation that characterized the period 2000-2007. Furthermore, and most importantly, all the forecasts were completely unable to predict the large housing price bust experience in 2006. Notice that the forecasters expected overall constant and large increases in long-run housing prices for the period 2000-2030.

We argue that these forecasts are consistent with the assumption that professional forecasters also failed to take into account any sort of long-run mean reversion in housing prices. To support this point, in Table 1 we report the average annualized growth rate from the year of the forecast (each row) to the horizon year (each column). Three main features are worth noticing. First, notice that all the forecasted average annualized housing price growth rates from the six dates in which forecasts were made (1995, 1998, 2000, 2002, 2004, 2006) to 2010 were much larger than their actually realized counterparts (in parenthesis). For example, the predicted average appreciation of housing price for the 12-years period between 1998 and 2010 was 3.97 percent per year, whereas the realized average was only 2.2 percent per year. Second, notice that the forecaster does not predict an adjustment in long-run housing prices, following a period of appreciation. In fact, all the annualized housing price growth rates in
Table 1 are large and range between than 3.1 percent and 4.1 percent. It is evident that periods of stagnation in housing prices are not expected. Finally, notice that the predicted average growth rate in the long run (2030) is very similar to the forecasted growth rate in the short run: this difference ranges from -0.66 percent (for the forecast made in 2006) to 0.07 percent (for the forecast made in 2000). This is further evidence that the model used to generate these forecasts does not take into account a high degree of mean reversion.

We can observe the stable dynamics of the forecasts by computing the $x$-quarters ahead forecasts for each year in which the forecast was made, as reported in Table 2. We consider both short-run forecasts ($x=1,4,8$) and long-run forecasts ($x=20,40,80$). We normalize the housing price in the quarter in which the forecast was made to be equal to 100, and we analyze the dynamics of the forecast in relation to that value. Three main properties of the forecasts emerge from Table 2. First, forecasts made throughout the period 1995-2006 expected housing prices to largely appreciate. Second, the dynamics of the forecasts as a function of the horizon are roughly independent from the period in which the forecast was made. In fact, all of the forecasts imply increasingly large appreciations of housing prices over time: the one-year-ahead forecasts imply increases of 2 percent to almost 4 percent; the five-year-ahead forecasts imply increases of 15 percent to 22 percent; the 10-year-ahead forecasts imply increases of 34 percent to 47 percent; and the 20-year ahead forecasts imply increases of 79 percent to 113 percent. Although the magnitude of the forecasted appreciation varies, we argue that throughout the period 1995-2006 there is no evidence of an adjustment in terms of housing price forecasts.

All the evidence provided in this section should convey that it is not unreasonable to assume that financial experts might also have been exposed to some source of bias that led them to ignore the mean-reversion component of housing prices growth. These findings are in line with other studies on the behavior of housing market experts during the boom phase. Gerardi et al. (2008) show that analysts and experts attached a very low probability to a significant reduction in house prices, while Cheng et al. (2014) find that securitization agents were on average not aware of the overvaluation of the housing market. The optimism about house prices is reflected in the risk (and the subsequent losses) borne by financial intermediaries, which kept the vast majority of second liens on their balance sheets, while securitizing first-lien mortgages.16

15Interestingly, their study finds that “certain groups of agents - those living in bubblier areas, working on the sell side, or at firms with greater exposure to subprime mortgages - may have been particularly subject to potential sources of belief distortions, such as job environments that foster group think, cognitive dissonance, or other sources of over-optimism.”

16See for instance Figure 4 from "Residential Credit Losses - Going into Extra Innings?" Lehman Brothers U.S. Securitized Products, April 11, 2008 (reprinted in Acharya et al. (2009)), where it is shown that a relevant
The main conclusion we draw from this section is that professional forecasters were most likely making use of models that were not able to capture any sort of mean reversion in long-run housing price dynamics. In this regard, we can state that financial experts displayed natural expectations, as we will formally define in the next section. Even though financial experts- unlike households - commonly make use of large and convoluted models to generate forecasts, it seems evident that the internal propagation mechanisms of these models are inadequate to the task of capturing the long-run mean reversion pattern that characterizes housing prices. In this sense, our evidence supports the hypothesis proposed by Barberis (2013) that financial experts used “bad models” for predicting future housing prices and that these models let them to be too optimistic about future values of collateral. This has likely affected the supply of credit, as we show in the next sections.

3 Natural House Price Expectations

The main goal of this paper is to link the inability of agents to take into account the long-run hump-shaped dynamics of housing prices when making forecasts, and the amount of housing-related debt demanded or supplied. In this section we show three results that establish this linkage. First, it is, indeed, likely that housing prices are characterized by hump-shaped dynamics, which imply a large momentum in the short run and partial mean reversion in the long run. Second, we document that models that incorporate hump-shaped dynamics are not preferred, in terms of in-sample fit, to more parsimonious models that ignore long-run mean reversion. As a result, the use of simple models leading to natural beliefs is perfectly justifiable in terms of in-sample performance. Third, we demonstrate that, nevertheless, models with diverse degrees of ability in capturing the hump-shaped dynamics of housing prices differ in their long-run forecasts, although they have similar short-run predictions. Hence, if agents use simple models (for a wide range of good reasons\(^{17}\)), they fail to forecast the partial mean reversion in housing prices over the long run (this is in line with the pattern shown by the financial experts’ forecasts documented in the previous section). Following Fuster et al. (2010), we call the resulting beliefs of these agents natural expectations.

\(^{17}\)As Fuster et al. (2010) put: “simple models are easier to understand, easier to explain, and easier to employ; simplicity also reduces the risks of overfitting. Whatever the mix of reasons -pragmatic, behavioral, and statistical- economic agents usually do use simple models to understand economic dynamics”.

\[\text{fraction of HELOCs and second-liens were kept in the balance sheets of US banks.}\]
3.1 Modeling Natural Expectations for Housing Prices

In this section we examine data for the aggregate real U.S. housing price index, and we analyze how different modeling approaches vary in their ability to capture hump-shaped long-run dynamics. The series of interest is the quarterly Standard & Poor’s Case-Shiller Home Price Index for U.S. real housing prices in the sample 1951:1-2010:4. The logarithm of the raw series is plotted in the upper panel of Figure 3. The series displays at least four episodes of boom and bust: the first one in the early ’70s, the second one later in the decade, the third one in the ’80s, and, finally, the most recent and significant from 1997 to 2005.

The series is statistically characterized by the presence of a unit root\(^{18}\). We therefore consider as a variable of interest its year-over-year growth rate, displayed in the bottom panel of Figure 3. Notice also that the growth rate of housing prices is characterized by relatively long periods of increase followed by abrupt declines, which indicate the presence of a rich autocorrelation structure.

We then assume that the process for housing price growth rate, \(g_t\) is autoregressive,\(^{19}\) i.e.:

\[
(1 - \Phi_p(L)) g_t = \mu + \varepsilon_t,
\]

where \(\Phi_p(L)\) is a lag polynomial of order \(p\), \(\mu\) is a constant, and \(\varepsilon_t\) are iid innovations.

We assume that an agent could estimate the model in equation (1) using four different criteria that gather a spectrum of different approaches to estimation and forecasting. Initially, we propose two simple models that capture natural expectations on housing prices. Recall that, as in Fuster et al. (2010), we define natural expectations as the beliefs of agents that fail to incorporate hump-shaped long-run dynamics of the fundamentals. We explore two possible dimensions that lead to natural expectations: (1) a limited ability of agents to incorporate a large set of explanatory variables when estimating a model; and (2) a limited ability of agents to consider a large set of data when estimating the model. Regarding the first model, we assume that an agent naively considers a first order polynomial, that is \(p = 1\) and \(\Phi_p(L) = 1 - \phi_1 L\) when estimating equation (1). This assumption captures behavioral biases, such as a natural attitude to use over-simplified models, as reported in Beshears et al. (2013).

\(^{18}\)To formally test the null hypothesis of presence of a unit root in the house price level, we run the Phillips and Perron (1988) unit root test. We allowed the regression to incorporate from one to 15 lags. For any of these specifications the test could not reject the null hypothesis of a presence of a unit root. To check whether the presence of a unit root is driven by the 1997–2007 price boom, we run the test for the shorter sample 1953:1–1996:4. Also in this case, the Phillips-Perron test could not reject the null hypothesis at a 5 percent significance level for any model specifications. In addition, there is no statistical evidence that the house price of growth rate contains unit roots.

\(^{19}\)Our modeling choice is justified by, Crawford and Fratantoni (2003) who show that linear (ARMA) models are preferred to non-linear housing price models for out-of-sample forecasts.
and analyzed in Hommes and Zhu (2014). We refer to this model as intuitive expectations, consistently with Fuster et al. (2010). Regarding the second model, we assume that an agent has finite memory and accordingly forecasts the model in equation (1) by considering only the most recent observations. In particular, we assume that agents consider only the last $T_{lim} = 100$ observations when estimating the model. The underlying assumption is that agents using this model do not take into account the earlier historical housing price dynamics, either because they do not have access to those data, or because they ignore them, or simply because they assign much lower weight to older observations. We refer to this model as finite memory. Notice that the finite memory model captures a source of bias that does not emerge because of a possible model mis-specification (as for the intuitive expectations model), but the bias depends upon the limited amount of information that is relevant for the agent when estimating the model.

We then compare the implications of these natural expectations models with the ones produced by more rigorous and sophisticated statistical approaches. In fact, an agent could, to the contrary, make use of more sophisticated econometric techniques to estimate the more appropriate lag polynomial in equation (1). When choosing how many parameters to include, a modeler faces a trade-off between improving the fit of the model in-sample and the risk of overfitting the available data, which may result in poor out-of-sample forecasts. Two of the most popular criteria are the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). It is not clear which criterion should be preferred by practitioners in small samples. Generally, the BIC imposes a larger penalty for increasing the number of parameters, and thus will tend to select models with fewer parameters than the AIC. As a result, as shown by Fuster et al. (2012), when the true model is characterized by hump-shape patterns, the BIC selects models that are not able to capture the true dynamics. Hence, these two approaches might lead to different specifications of the model in equation (1). Therefore, we consider as third and fourth models the specification of equation (1) obtained when an econometrician uses respectively the AIC criterion and the BIC criterion.

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20 We obtain similar results when varying $T_{lim}$ in the range 80-120.

21 There are other interpretation for this approach. For example, agents might have adopted a “new-era thinking”, which refers to agents deliberately excluding older observation because they believe they are not relevant anymore. Alternatively this approach can also capture the assumption of extrapolative expectations in the housing market employed by Goetzmann et al. (2012), Abraham and Hendershott (1994), Muellbauer and Murphy (1997), Piazzesi and Schneider (2009), and it relates to the findings of Agarwal (2007) and Duca and Kumar (2014), which state that younger individuals have statistically significant more propensity to overestimate house prices and to withdraw housing equity, respectively.

22 We assume that the agent with finite memory estimates the model by maximizing information criteria. Since the BIC and AIC select the same length for the lag-polynomial, the two approaches deliver the same results.

23 See McQuarrie and Tsai (1998) and Neath and Cavanaugh (1997) for opposite arguments.
In Table 3 (left panel for the whole sample 1953:1-2010:4) we report point estimates (standard errors in brackets) for four models: $p = 1$, estimated by an *intuitive* model; $p = 6$, estimated by a *finite memory* model; $p = 5$, estimated by the BIC model; $p = 16$, estimated with the AIC model. Notice that there is a remarkable difference in the number of lags selected by the last two models: since the BIC criterion largely penalizes overfitting, it select much fewer lags than the AIC criterion. Furthermore, the large number of significant parameters for lags greater than one, in particular for the AIC model, confirms that the process of housing price growth has a relatively rich autoregressive structure. Consequently, an agent who makes use of a simpler autoregressive model is likely to ignore important dynamics of house price growth. The different long-run implications of the models are summarized by their resulting long-run persistence, as discussed in detail below. Notice that these findings are robust to considering only a more limited sample (1953:1-1996:4) that does not include a recent housing price boom, as reported on the right panel of Table 3.

### 3.2 In-sample Fit and Long-Run Predictions

In the previous section we have reported the estimates of four different specifications of a linear model for housing price growth. In this section we provide evidence that, although drastically contrasting in their underlying assumptions, these specifications have similar in-sample properties, and they are hardly distinguishable from a statistical point of view. Table 4 reports statistics about the goodness of fit of the four models. The Root Mean Squared Error (RMSE), the unadjusted coefficient of determination ($R^2$), and the adjusted coefficient of determination ($\bar{R}^2$) are very similar across the models. Since the *intuitive* model, the BIC model, and the AIC model are all nested models, we can formally test whether the data can formally reject the null hypothesis that the three models are observationally similar by comparing the log-likelihood evaluated at the unrestricted model parameter estimates and the restricted model parameter estimates. As Table 4 displays, the resulting Likelihood Ratio (LR) test statistics when assuming that the restricted model corresponds to $p = 1$ and the unrestricted model corresponds to $p = 5$ and $p = 16$, respectively, confirm that the models cannot be distinguished on the basis of goodness-of-fit alone. Since the *finite memory* model considers a different sample, it cannot be nested in the other three models. Hence, the LR test cannot be applied. Nevertheless, notice that its likelihood is very similar to the one of the other three models. Notice, too, that the one-quarter-ahead forecasts produced by these

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24 Although we do not report them here, the historical in-sample fitted values of the four model are basically indistinguishable. Therefore, they the different empirical models have a very similar ability to capture the in-sample boom-and-bust episodes.
models are also similar.

Although the models imply a similar fit to the data and similar short-run predictions, their long-run out-of-sample forecast implications are different. We can observe these features of the models by plotting the impulse response functions for a 1 percent positive shock in the housing price growth rate, as displayed in the top panel of Figure 4. The intuitive model (solid blue line) estimates a very persistent process, as indicated by the value of the parameter of the AR(1) process, equal to 0.96 as reported in Table 3. Consequently, it predicts a long-lasting positive effect of a shock on housing price growth. In contrast, the BIC model (dashed red line) and the AIC model (dotted green line) predict larger short-run responses of housing prices, but they estimate faster reversions after 10-15 quarters. Notice, also, that the practitioner who uses the AIC criterion estimates a negative medium-run response of price-growth after the large boom, but even this model does not particularly succeed of incorporating a large mean reversion component of house price. This fact shows that it is hard to obtain mean-reversion dynamics even with more sophisticated models when estimated in small samples. Finally, the finite memory model (dotted purple line) has a very large short-run response and implies a persistence of the positive shock for about 30 quarters, without any sort of mean reversion.

We can obtain insights about the different long-run predictions of the models by plotting the impulse responses of the level of the housing prices, as displayed in the lower panel of Figure 4. These responses are given by the cumulative sum of the impulse responses of the growth rate. An agent using the finite memory model (dotted purple line) predicts that, after a positive shock, the housing prices will largely increase for about 25-30 quarters and then stabilize at a high level. An agent using the intuitive model (solid blue line) expects a longer persistence of the housing price appreciation, which leads to a similar long-run forecasts as with the finite memory model. The two more sophisticated models (BIC model, dashed red line, and AIC model, dotted green line) predict a much lower degree of persistence, which leads to lower expected long-run prices. In fact, they prove better in capturing the mean-reversion feature of housing prices than both the intuitive model and the finite memory model. Notice also, that an econometrician using the AIC criterion expects a depreciation following the initial boom. Furthermore, since the four models are hardly distinguishable in the sample, as pointed out above, it is legitimate to conjecture that these impulse responses are associated with a large degree of uncertainty. Not surprisingly, this is indeed the case, as described in Appendix A.

The long-run dynamics of housing prices are particularly important for the purpose of this paper. In fact, we conjecture that households’ consumption-saving decisions are affected by
the perceived long-run housing wealth. This presumption is motivated by the long durability of housing as an asset, and by the nature of home equity loans, which have repayment periods of up to 25 years. It is therefore reasonable to assume that long-run forecasts of housing prices matter for households’ present decisions. A measure of the long-run price estimated after a shock is the long-run persistence of the price level, defined as the long run steady state level after a 1 percent shock. Given that the price level is assumed to follow an ARIMA($p$,1,0) model, the long-run persistence (LRP) can be computed as:

$$LRP = \frac{1}{1 - \sum_{j=1}^{p} \phi_j}$$  \hspace{1cm} (2)

where $\phi_j$, $j = 1, ..., p$ are the coefficients of the lag polynomial of order $p$, $\Phi_p(L)$. Table 4 reports the LRP of the processes estimated by the four models as well as their confidence band.

As Table 4 reports, the LRP estimated with an intuitive model is larger than the one estimated by agents using a more rigorous statistical approach. In particular, the AR(1) model delivers a long-run persistence that is 30 percent higher than the AR(5) model selected by the BIC, and 80 percent higher than the AR(16) model selected by the AIC.\textsuperscript{25} Also, the LRP estimated by the finite memory model is similar to the one estimated by the intuitive model. This an important result since it shows that agents who use oversimplified models (because of behavioral biases or sample selection) tend to have more optimistic expectations about long-run housing price resulting after a positive shock than agents using more sophisticated models. In Table 9 in Appendix B we report similar results obtained when considering annual data, confirming that our findings are not an artifact of data frequencies.

This section shows that there is a spectrum of approaches to a linear model for house price growth that are fairly equivalent in terms of their capability of fitting the data. These approaches range from capturing behavioral biases to including sophisticated and more rigorous statistical perspective. Although these models are hardly distinguishable by their in-sample properties, their long-run forecasts implications are different. In fact, the more sophisticated approaches are more capable (although at different degrees) of incorporating mean reversion dynamics in their forecasts, whereas natural models (as intuitive model and the finite memory

\textsuperscript{25}As a robustness check, we have alternatively assumed that the housing price growth rate $g_t$ is an autoregressive-moving average process, as in $(1 - \Phi_p(L))g_t = \mu + (1 + \Theta_q(L))\varepsilon_t$, where $\Theta_q(L)$ is a lag polynomial of order $q$. The BIC estimates an ARMA(1,4), whereas the AIC estimates an ARMA(18,5). Since the Long Run Persistence (18.6 for ARMA(1,4) and 12.9 for the ARMA (18,5)) and the Impulse Response functions estimated with the ARMA processes are very similar to the one estimated with the AR processes we decided to present only the latter.
4 A Model for Home Equity Loans and Natural Expectations

In this section we propose a model in which a representative household and a representative bank interact in a credit market and in which the household may obtain credit by pledging its house as collateral. This feature captures the role of home equity loans in the economy. Importantly, we allow agents to have a range of expectations upon the evolution of the exogenous housing price. This range of expectations varies with the degree to which agents are able to incorporate long run mean reversion of house prices. Hence, the expectations vary from more natural (lower ability to incorporate long-run mean reversion) to less natural (greater ability to incorporate long-run mean reversion). Our theoretical model can be used as a laboratory to investigate the extent to which naturalness of households and banks has affected the level of debt in the economy during the housing price boom.

4.1 Household

The economy lasts $T < \infty$ periods. The economy is populated by two representative agents: a household and a bank. There are a non-storable consumption good and two assets: housing and debt claims. The household starts at $t = 0$ with an endowment of housing stock $h$ worth $p_0 h$, where $p_t$ denotes the real housing price at time $t$, and the household is allowed to sell the house only in the final period, at a price $p_T$, unless it decides to default in any time $t = 1, \ldots, T - 1$. In case of default, the household loses the ownership of the house and becomes a renter. When the household decides to default, it is excluded from the debt market as it does not have any collateral to pledge. Because the household starts with an owned housing stock and with no previous debt, and because it does not engage in buying or selling of its housing stock, we can interpret the debt claims in the economy as home equity extraction.\footnote{Though this interpretation is made simply to relate our model to the evidence reported in section 2, our results clearly extend more generally to any type of collateralized borrowing.}

We assume that the household is endowed in each period with a constant income $y_t = y > 0$. The housing price is an exogenous variable for the agents in our economy.\footnote{This simplifying assumption is justified by this paper’s goal of understanding how different expectations about the evolution of housing prices affect agents’ economic behavior. Moreover, this same assumption is used in several studies on the effects of housing on macroeconomic or financial decisions, as in Campbell and Cocco (2011) or Cocco (2005).}
The household is allowed to borrow resources from the bank with the house serving as collateral. Subject to the repayment of debt accumulated in the past, in period \( t \) the household is allowed to borrow new debt \( d_t \) which it will eventually repay in the next period at an interest rate of \( r_t \). The household has the option of defaulting from \( t = 1 \) onwards. Hence, the budget constraint of a household that repays its debt at time \( t \) is:

\[
c_t + (1 + r_{t-1})d_{t-1} = y + d_t;
\]

whereas, the budget constraint of a household that decides to default at time \( t \) is:

\[
c_t + \gamma p_th = y,
\]

where \( \gamma p_th \) represents the renting cost, which is assumed, for simplicity, to be a fraction \( \gamma \) of the house’s value.

The household, then, maximizes its intertemporal utility:

\[
E_0 \sum_{t=0}^{T} \beta^t u(c_t, h),
\]

subject to the period-by-period budget constraint, which is conditional on the default decision. Later, we will discuss in depth how agents’ expectations are formed. In each period the household’s choice defines a debt demand schedule \( d_t (r_t) \) and a related default decision.

We can rewrite the problem recursively. Since the economy lasts for a finite number of periods, the model can be solved by backward induction. Let us then start from period \( t = T \): if the household has never defaulted in the past, in the last period it is entitled to sell its housing stock; hence the only decision variable is whether to default or not to default. Since the household sells the housing stock in the last period, there is no possibility of getting new debt, and, thus, consumption is simply determined by the exogenous income and housing value.

In case of a good credit history (i.e. no past default), the problem in period \( T \) can be then written as:

\[
V^*_T (r_{T-1}, d_{T-1}, p_T) = \max \{ u(y - \gamma p_th); u(y - (1 + r_{T-1})d_{T-1} + p_th) \}.
\]  

(3)

Provided that the household did not default in the past, it has the option of defaulting in periods \( t = 1, ..., T - 1 \). Hence, for \( t = 1, ..., T - 1 \) the household has to compare two value
functions: if it decides to default (or did so in the past), the value function writes:

\[ V_t^D(p_t) = u(y - \gamma ph) + \beta \mathbb{E}_t V_{t+1}^D(p_{t+1}), \]  

(4)

with \( d_\tau = 0 \) for \( \tau \geq t \). In the event that the household did not default in the past and is not defaulting in the current period \( t \), the value function writes instead:

\[ V_t^C(r_{t-1}, d_{t-1}, p_t) = \max_{d_t} \left[ u(y - (1 + r_{t-1})d_{t-1} + d_t) + \beta \mathbb{E}_t \{ V_{t+1}^* (r_t, d_t, p_{t+1}) \} \right]. \]  

(5)

Hence, in each period \( t = 1, \ldots, T - 1 \), the household compares the two value functions to pin down its default choice:

\[ V_t^* (r_{t-1}, d_{t-1}, p_t) = \max \{ V_t^D(p_t); V_t^C(r_{t-1}, d_{t-1}, p_t) \}. \]  

(6)

Finally, in period \( t = 0 \) there is no default choice, since the household is assumed to start with no debt; hence in \( t = 0 \) its value function reads:

\[ V_0^* (p_0) = \max_{d_0} \left[ u(y + d_0) + \beta \mathbb{E}_t \{ V_1^* (r_0, d_0, p_1) \} \right], \]  

(7)

with the initial stock of debt \( d_{-1} = 0 \) given.

4.2 Bank

The bank seeks to maximize its intertemporal stream of profits, taking into account the probability of the household’s default. In other words, in each period the bank obtains loans from outside the model at a risk-free rate, \( i_t \). The bank then supplies credit to the household, at a market interest rate \( r_t \). In case of default, the bank obtains revenue from liquidating the household’s housing stock. The bank’s problem can also be expressed in recursive form. Let’s start from the last period, \( t = T \). The profits for the bank write:
\[ \pi_T (r_{T-1}, d_{T-1}, p_T) = \begin{cases} 
(1 + r_{T-1})d_{T-1} - (1 + i_{T-1})d_{T-1} & \text{if the household does not default} \\
\kappa p_T h - (1 + i_{T-1})d_{T-1} & \text{if the household defaults} \\
0 & \text{if the household defaulted in the past.} 
\end{cases} \] 

Here \( \kappa \) represents the fraction of the collateral that the bank can recover after the household’s default.

For a given interest rate \( r_t \), in periods \( t = 1, ..., T - 1 \) the bank sets \( d_t \) in such a way as to maximize its profits:

\[ \max_{d_t} \pi_t (r_{t-1}, d_{t-1}, p_t) = \begin{cases} 
(r_{t-1} - i_{t-1})d_{t-1} + \delta E_t \pi_{t+1} (r_t, d_t, p_{t+1}) & \text{if the household does not default} \\
\kappa p_t h - (1 + i_{t-1})d_{t-1} & \text{if the household defaults} \\
0 & \text{if the household defaulted in the past.} 
\end{cases} \] 

By assumption, the bank cannot default on its obligations.\(^{28}\) Finally, the profit function in \( t = 0 \) writes:

\[ \pi_0 (p_0) = \delta E_0 \pi_1 (r_0, d_0, p_1). \] 

### 4.3 Recursive equilibrium

A recursive equilibrium in our economy can be defined, for \( t = 0, ..., T - 1 \), as an interest rate function \( r_t(p_t, d_{t-1}, r_{t-1}) \), a debt function \( d_t(p_t, d_{t-1}, r_{t-1}) \) and value functions \( V_t^D (p_t) \), \( V_t^C (r_{t-1}, d_{t-1}, p_t) \) and \( \pi_t (r_{t-1}, d_{t-1}, p_t) \) such that in each period \( t = 0, ..., T - 1 \) and for each realization of the housing price \( p_t \) and realizations of \( r_{t-1} \) and \( d_{t-1} \):

- given \( r_t, d_t(p_t, d_{t-1}, r_{t-1}) \) and value functions \( V_t^D (p_t), V_t^C (r_{t-1}, d_{t-1}, p_t) \) solve the house-

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\(^{28}\)To ensure limited liability, one can assume that the bank has access to a fixed amount of extra resources (equity) that allows it to repay the debt when revenues fall short of liabilities.
hold recursive maximization problem.

- given \( r_t \) and providing that no default has occurred up to period \( t \), \( d_t(p_t, d_{t-1}, r_{t-1}) \) and the profit function \( \pi_t(r_{t-1}, d_{t-1}, p_t) \) solve the bank maximization profit.

- markets for the consumption good and debt clear.

In period \( t = T \) the household maximizes its utility under the budget constraint, choosing whether or not to default.

### 4.4 Expectation Formation

In our model we treat housing prices as exogenous and assume that the growth rate of the housing price follows a stochastic process. Accordingly, given a price of housing in the initial period, \( p_0 \), the evolution of the house price is given by:

\[
p_{t+1} = p_t \left( 1 + r_{t+1}^h \right),
\]

with:

\[
(1 - \Theta^p(L)) r_{t+1}^h = \sigma \varepsilon_{t+1},
\]

Here, \( r_{t+1}^h \) denotes the growth rate of housing price, \( \Theta^p(L) \) is a lag polynomial of order \( p > 1 \), and \( \varepsilon_{t+1} \) is a mean-zero stochastic variable. This specification links the expectation of future house price growth rate to the autoregressive structure of the process, i.e.:

\[
E_t r_{t+1}^h = \Theta^p(L)r_t^h.
\]

As it will be clear next section, we examine the predictions of the model when varying the form of perceived expectation on future house prices by varying the properties of the lag polynomial \( \Theta^p(L) \).

### 5 Calibration

By using the model described in the previous section, we now assess the quantitative effects of natural expectations in the consumption/saving decision. We are mainly interested in examining the extent to which the equilibrium level of housing-related debt and its price vary with the ability of agents to take into account possible long-run mean-reverting dynamics of house prices.
We consider an economy that last $T=10$ periods (years). The length of the simulation is a computationally restricted parameter, since in a non-stationary model the number of state-variables quickly explodes when increasing the number of periods in the model. However, a 10-period time span is appealing for two reasons. First, it is long enough to fully capture a boom-bust episode such as the one observed in the U.S. housing market in the 2000s. Second, a large portion of HELOCs started during the boom years had a duration of around 10 years.

We conduct the following experiment. We feed the model with a given path of housing prices for 10 periods, which aims to replicate the boom-bust episode as experienced in the U.S. in the period 2001-2010. Then, we vary the agents’ beliefs about the process generating the observed evolution of housing prices. Therefore, after observing the same initial housing price appreciation, different beliefs about the housing price data generating process affect the agents’ optimal economic behavior.

The imposed evolution of housing price (solid line) is displayed in Figure 5. In the boom phase, from $t=1$ up to $t=6$, the housing price grows by 83 percent, whereas in the bust phase, from $t=7$ up to $t=10$, the housing price drops by 39 percent. This evolution of the housing price reflects the dynamics of the Shiller real house price index in the U.S. (dashed line in Figure 5) in the decade 2000-2009. Ultimately, we assume that agents in our model always observe the same evolution of housing prices and they rely on an autoregressive specification for the housing price growth rate in equation (12) of the form:

$$r_{t+1}^h = \Theta^p(L)r_t^h + \sigma \epsilon_{t+1},$$

where $\Theta^p(L)$ is a lag polynomial of order $p > 1$. To investigate the impact of different forms of expectations, we consider a large set of specifications of $\Theta^p(L)$ that generate forecasts that are similar in the short run but different in the long run. It is important to note that we are completely silent about the true process that generated the observed housing price series. This is outside the scope of our analysis. In fact, in the empirical sections above, we showed that a large set of theoretical processes are consistent with the observed historical housing price time series. In our theoretical experiment, we investigate how macroeconomic variables are affected by agents taking actions based on a diverse spectrum of plausible data generating

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29Campbell and Cocco (2011), one of the closest models to ours, is simulated over a 20-years span. However, in order to keep the state space confined, Campbell and Cocco (2011) consider an iid housing price growth process, approximated by a bimodal Markov process. By reducing the length of the simulation to 10 periods, we are able to consider richer housing price dynamics, allowing for an autoregressive process approximated by a tri-modal Markov process, whereas Campbell and Cocco (2011) consider only a bi-modal process.

30From the Semiannual Risk Perspective From the National Risk Committee, U.S. Department of Treasury, 2012, it can be inferred that this portion was equal to at least 58 percent of loans outstanding in 2012.
5.1 Calibrating Expectations

We consider 50 specifications for the model in equation (12) to generate agents’ expectations of future housing prices. This large number of specifications allows us to investigate how macroeconomic variables respond to rather small differences in expectation formation. For computational feasibility, we limit our investigation to processes of order two, i.e.:

\[ r_{t+1}^h = \mu (1 - \theta_1 - \theta_2) + \theta_1 r_t^h + \theta_2 r_{t-1}^h + \sigma \varepsilon_{t+1}. \]  

Even if parsimonious, this specification is flexible enough to capture features of the U.S. housing price index observed during the last boom-bust episode, and, above all, it allows us to incorporate different degrees of ability to embody hump-shaped dynamics. As a result, each specification is a function of four parameters: \( \mu, \theta_1, \theta_2, \sigma \). We assume that the average growth rate of housing prices, \( \mu \), is known, and it is constant across each specification. In particular, we fix \( \mu = 0 \), which is consistent with the historical average growth rate of the real Shiller index between 1953 and 2000, which is equal to 0.00016. We make use of three criteria to pin down the remaining three parameters (\( \theta_1, \theta_2, \sigma \)) for each specification. First, each specification should produce the same short-run (one-year-ahead) forecasts. This assumption is motivated by the evidence in Case et al. (2012), which find that, in the short run, homebuyers were generally well informed, that their short-run expectations were not largely different from the actual realized home prices, and that most of the root causes of the housing bubble can be reconnected to their long-term home price expectations. Also this assumption is motivated by the fact that natural expectations are able to capture short-run momentum, but fail to predict more subtle long-run mean reversion. Second, each specification should imply the same unconditional variance. As a consequence, the different behavior implied by each specification does not depend upon the magnitude of the housing-price variance, but only upon its propagation. Third, and most important, each specification should be characterized by different long-run forecasts. As a result, each specification differs only for the degree by which it is able to capture some sort of long-run mean reversion, when keeping fixed the short-run predictions and the overall variance of the process. Specifically, we set the first order autoregressive parameter, \( \theta_1 \), to be equal to 0.6, which is the persistence of an AR(1) process estimated using the Case-Shiller index annual growth rate. Since the one-step-ahead forecasts of an AR(2) process is only a function of \( \theta_1 \), each specification implies the same one-year forecast. The long-run predictions of a model can be summarized by its long-run
persistence (LRP). When considering annual data (see Table 9 in Appendix B), the LRP estimate range from the 1.5 (as estimated by the AIC model) to 2.8 (as estimated with the intuitive model). As Table 9 displays, there is a substantial degree of uncertainty around the estimated LRP. To capture this uncertainty, we consider specifications for process in (14) such that their LRP ranges between 1.4 and 4.5. Since the long-run persistence is given by \( LRP = \frac{1}{1-\theta_1-\theta_2} \), the values of LRP in this range pin down the different values of \( \theta_2 \). Finally, the parameter \( \sigma \) is set to such that all specifications imply a constant standard deviation equal to the estimated value from Case-Shiller index annual growth rate, which is equal to 0.049. This approach allows us to isolate the effects of a change in the perceived persistence of the house price growth rate process from changes in its perceived unconditional variance.  

Table 5 reports the resulting calibration for six specifications of the model in equation (12) among the 50 that we consider in our simulation, together with the implied long-run persistence. Notice that the degree of naturalness of an agent is driven by the second order autoregressive parameter, \( \theta_2 \): when this parameter is negative, agents are not natural since they expect a long-run mean reversion of housing prices after a positive short-run momentum; when \( \theta_2 \) is positive, agents are natural since they expect the short-run momentum to persist in the long-run.

Figure 6 displays the impulse response functions and their cumulative values for three of the above-described processes. More precisely, we plot the IRFs and CIRFs of the AR(1) process (Specification 5, cross-line), as a reference, along with the two “extreme” processes: process 1 (solid line) representing the process with the lowest degree of naturalness and which accordingly displays the strongest long-run mean reversion; process 50 (triangle-line) representing the process with highest degree of naturalness. Notice that the forecasted long-run price by process 50 is almost double the one implied by an AR(1) process.

5.2 Calibration of Structural Parameters

The calibrated structural parameters of the model and their values are reported in Table 6. We set the discount rate for both the household and the bank at 0.98, which is consistent with an annual risk-free rate of 2 percent. The housing stock, \( h \), can be interpreted as the housing value in the initial period, since we set the initial housing price \( p \) equal to one. Hence,

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31 We have also run our model under the alternative assumption that the standard deviation of the shocks, \( \sigma \), is constant across the 50 specification, thus implying that the unconditional variance of the house growth rate process increases with its perceived persistence. Since the results are very similar to the one described in this paper (and therefore not reported here for sake of brevity), we infer that the main driving force of our results is the different long-run responses of the house price to shock across specifications, more than its unconditional volatility.
$h$ relates to the housing value to income in 2000. This value is equal to 2.1 in the Survey of Consumer Finance data, whereas it is equal to 1.3 when considering national aggregate data. Hence, we set $h$ to be equal to the intermediate value of 1.5. We assume a constant relative risk aversion (CRRA) utility function, i.e. $u(c) = \frac{c^{1-\eta}}{1-\eta}$, with coefficient of risk aversion $\eta$ equal to 2, a value broadly in line with the literature. Annual income, $y$, is standardized at the level of 1. We assume that the rental rate, $\gamma$, is 5 percent of the current value of the housing stock, thus implying a price-to-rent ratio equal to 0.05, which is consistent with the setting in Garner and Verbrugge (2009) and in Hu (2005). Finally, we assume that when the household defaults, the bank is able to recover only 20 percent of the value of the house. Such a value is in line with our interpretation of the asset in the economy as an HELOC.32

6 Quantitative Effects of Natural Expectations

Given the calibration of the structural parameters, the 50 specifications of the housing price growth process used by agents to forecast future housing prices, and the realized evolution of housing price for the 10 periods, as shown in Figure 5, we can compute the equilibrium dynamics of the variables of the model. Specifically, we are interested in the debt-to-income ratio, $\frac{d}{y}$, the consumption-to-income ratio, $\frac{c}{y}$, the loan-to-value ratio, $\frac{d}{p_kh}$, and the interest rate associated with home equity loans, $r_t$. We now investigate how these variables vary with agents’ naturalness in the housing price boom and bust, separately.

6.1 Equilibrium in a boom

Figure 7 reports the average values of debt (upper left panel), LTV ratio (upper right panel), consumption (lower left panel) and interest rate (lower right panel) for each of the 50 specifications of expected housing price growth (x-axis) across the boom phase (from period 1 to period 6 in our model, which corresponds to the period 2000-2005 in the data, blue solid line) and across the bust (from period 7 to period 9 in our model, which corresponds to the period 2007-2009 in the data, green dashed line). As a reference point, we denote with a red circle the values associated with assuming the agents form expectations using an AR(1) process, which relates to the intuitive statistical model as presented in Section 3. First, we consider the average values of our variables of interest during the boom phase. Four results are worth highlighting. First, the model predicts a positive relationship between the average equilibrium level of debt in the economy in the boom phase and the degree of

32Since HELOCs are junior-liens, and the maximum loan-to-value ratio for a first-lien is 80 percent, we are then implicitly assuming that the bank is able to fully recover the value of the equity in the house sale.
naturalness of agents. Recall that the 50 specifications for the expectations range from higher ability of the model to incorporate long-run mean reversion (specification 1, low naturalness) to lower ability of the model to incorporate long-run mean reversion (specification 50, high naturalness). Intuitively, after observing an increase in the housing prices, a more natural agent expects a longer-lasting appreciation of housing prices, which gives higher incentive to demand/supply debt. In contrast, a less natural agent expects a short-run momentum in housing prices followed by a mean reversion adjustment after some periods, as it can be visualized by the impulse response function for specification 1 in Figure 6. As a result, the household is less willing to demand debt and the bank is less willing to supply it. A second important result relates to the magnitude of the role of long-run expectations. Notice when agents in the economy are characterized by the lowest degree of naturalness, the equilibrium level of debt is roughly 35 percent of income. In contrast, when the agents ignore hump-shaped dynamics of housing prices, the equilibrium level of debt in the economy escalates to 55 percent of income. We obtain a similar pattern when considering the loan-to-value ratio, which increases from 18 percent for the least natural agents to 28 percent for the most natural agents. The pronounced differences in these quantities is solely due to the contrasting long-run expectations of housing prices, since by construction agents have the same short-run expectations in each of the 50 specifications. These results strongly support the argument in Case et al. (2012): the role of homebuyers’ long-run housing price expectations is a crucial determinant of agents’ behavior in terms of the consumption/saving choice. As a third result, notice that the accumulation of debt fuels consumption in the short-run, since there is positive correlation among average consumption in a boom phase and the degree of naturalness of agents in the economy. Intuitively, when expecting higher future appreciation of house’s price, the resulting wealth effect provides incentives to consume in the current period. As a forth result, notice that debt is associated with a lower interest rate in economies where agents are more natural. Intuitively, since banks in the model share the same form of expectations of households, when banks expect both short-run and long-run momentum in housing prices, they are willing to lend at a lower equilibrium price.

The above findings can be summarized as follows: when housing prices start to increase, a natural agent (a household or a bank) overestimates the persistence of positive shocks and ignores the possible long-run mean reversion that follows a short-run momentum. As a consequence, the household or bank also overestimates the overall long-run appreciation of the housing stock. Given the availability of financial instruments to smooth future housing wealth, a natural household has, then, more incentive to extract a large portion of home equity to increase its consumption immediately. A natural bank will then be willing to provide loans
to the household at lower price. As a result, natural expectations leads to large leverage during a housing price boom.

6.2 Equilibrium in a bust

The second set of results concerns the adjustment that the economy makes during the house price bust (periods from 6 to 9). These results reflect the predictions of our model for the behavior of agents in the period 2007-2009 and they show that the relationships between debt, consumption and degree of naturalness described above for the boom period are reversed. More natural households deleverage their debt position and they drastically reduce their consumption. Specifically, in the economies with most natural agents (processes 47-50), the amount of debt the household is able to extract is null. Although quite drastic, this result is in line with evidence regarding the practice of HELOC freezes observed since 2008, when financial institutions realized the depth of the bust (WSJ, 2008). Notice that the adjustment if households were less natural households would be less sharp: they reduce their consumption to a lower degree and they are still allowed to borrow to smooth consumption, since they have previously accumulated relatively low levels of debt during the boom phase.

6.3 Welfare Cost of Naturalness

What is the overall welfare cost of being natural? The answer is not obvious, because, as shown above, more natural agents that expect a long-lasting house price appreciation over-borrow (and over-consume) during a housing price boom, but they need to reduce their consumption more sharply during a housing price bust. Hence, we consider the whole boom-bust episode (periods from 1 to 9) and compute the ex-post difference in utility (in consumption equivalent terms) between a household that uses any process in the range 2-50 for computing its housing price forecasts and the least natural household, which uses process 1 for forecasting. In other words, such a measure corresponds to the percentage of consumption that an agent would require in each period to equate the utility of the least natural agent. As Table 7 reports, the welfare cost (in percent) is monotonically increasing and large, since it reaches a value of about 40 percent when agents make use of the least natural process. Intuitively, although a more natural household enjoys higher consumption levels during the house price boom, fuelled by higher debt, its deleveraging process during the bust phase is very costly in

33Such sharp dynamics in the deleveraging process may be due to the absence of frictions (e.g., adjustment costs) in lending: in case of an abrupt decline in collateral values, banks in our model suddenly cut-off lending. However, note that in the above calibration in equilibrium the household never reaches the default region.

34We define as ex-post the realized utility of the agents in the 10 years of our simulation.
utility terms. One might wonder what would be instead such welfare cost if measured ex-ante (i.e. in the initial period $t=0$) with agents entertaining different types of expectations. Not surprisingly, results are reversed compared to the ex-post analysis: the higher the degree of naturalness, the higher is the expected future housing wealth after a positive realisation of the house price shocks, and the higher is then the ex-ante lifetime expected utility; as a result, the resulting ex-ante consumption equivalent is positive for higher degrees of naturalness. Intuitively, since a natural agent perceives the initial positive house price appreciation to be very persistence and long-lasting, she overestimates her future housing wealth. This effect increases the lifetime expected utility. However, the expectations about the house price path soon reveal to be over-optimistic, and when the house price collapse, the natural agent has to drastically adjust its consumption and debt paths. This effect leads to a large drop in ex-post utility. Finally, notice that whereas even an extremely natural agent feels better off ex-ante because the ex-post cost after the realisation of the boom-and-bust house price episode is much larger.

6.4 The role of bank’s expectations

In section 2.1 we documented that financial experts are likely to have held natural expectations during the housing price boom of the early 2000s, since their forecast do not show any long-run mean reversion after the short-run momentum. Since our theoretical model accounts for both the demand and supply of credit, we can now assess the impact of debt-supply naturalness on macroeconomic variables of interest. Specifically, we now perform some experiments to identify the contribution of banks’ and households’ expectations to the equilibrium outcome of debt and interest rate under the following four competing hypotheses: (a) both bank and household hold strong natural expectations; (b) bank and household do not hold natural expectations; (c) only the household is strongly natural, while the bank is not; (d) only the bank is strongly natural, while the household is not. In these experiments, for simplicity, we give the natural label to an agent that forecasts future housing prices using the most natural process (process 50), and we give the not-natural label to an agent that forecasts future housing prices using the least natural process (process 1). These extreme values are vehicles for understanding the role of expectations in regards to supply and demand. Table 8 displays the results. The most striking result of our experiment reflects the crucial importance of banks’ expectations for the equilibrium level of debt. Let’s analyze first the boom phase. When both agents are not natural, as in scenario (b), the equilibrium level of debt in the economy is relatively low (around 35 percent of income). If we assume that only
the household is natural, as in scenario (c), the equilibrium level of debt increases by only 5 percent, whereas if only the bank is natural, as in scenario (d), the equilibrium level of debt increases up to 48 percent. In other words, without assuming a bank expectation channel, a model in which only households are natural can only replicate a small portion of the leverage level in the economy during the house price boom.

The importance of banks’ expectations can also be observed in the effect on the price of debt, expressed as the interest rate. Consistent with standard economic theory, the scenario in which only households are natural leads to an increase in the interest rate, and the scenario in which only banks are natural leads to a decline in the interest rate. In fact, we can interpret households’ increase in naturalness as an outward shift of the debt demand, since agents with more natural expectations overestimate their future housing wealth, and are more willing to obtain debt to smooth their consumption as a result. On the other hand, the scenario in which only banks are natural is consistent with an outward shift of the supply. This is reinforced by our result. When neither agent is natural, the equilibrium interest rate is 2.4 percent. When only households are natural, the equilibrium interest rate rises to 3 percent (an indication in a shift in demand for debt). When only banks are natural, the interest rate falls to 2 percent, (an indication in a shift of supply of debt). When both agents - banks and households - exhibit natural expectations, the equilibrium interest rate still falls, but to 2.1 percent.

Although data on banks’ charges for HELOC instruments are unavailable (because that interest rate is usually privately agreed at subscription), Justiniano et al. (2014) document the decline of mortgage rates during the housing price boom as evidence for the decline of interest rate associated with home equity debt instruments. Whereas Justiniano et al. (2014) explain this phenomenon with the relaxation of the lending constraint, the evidence provided in section 2.1 and the results of our model propose an alternative story for the outward shift in credit supply observed during the phase of rising housing prices.

6.5 Estimating Naturalness from the Data

Finally, we perform a comparison of our simulations with the debt-dynamics observed in the data to pin down which degree of naturalness better fits the debt data. The first step is to obtain a series that is comparable to the debt-to-income ratio as simulated in our model. We first consider the annualized series of Gross Home Equity Extraction in the U.S., as in Greenspan and Kennedy (2005). The series is available only until to 2008Q4. We divide the

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35The series is the sum of (a) cash-outs resulting from refinancings, (b) originations to finance purchases of existing homes minus sellers’ debt cancellation, and (c) changes in home equity debt outstanding less unscheduled repayments on regular mortgage debt outstanding.
series by nominal disposable personal income to compute the debt-to-income ratio. Because
the series is not directly comparable to the outcome of our simulated model, we need to correct
the former for the fraction of households effectively extracting home equity. Therefore, we
make use of the Survey of Consumer Finance data to compute the fraction of households with
an outstanding HELOC and interpolate via cubic splines for the years in which the survey
is not available. Such a percentage smoothly varies from 2.7 per cent in 2001 to 4.6 per
cent in 2008. We then compare the resulting debt-to-income series with the debt dynamics
of the model (where both household and bank can be natural) across the 50 specifications
and we select the process whose debt dynamics minimize the Euclidean distance with the
data. Figure 8 plots the selected process (black dotted line) and the debt-to-income ratio
in the data (blue solid line). The selected specification is the process 31, a fairly persistent
and natural one, since its second order autoregressive parameter is positive, \( \theta_2 = 0.08 \), and
its LRP is fairly large, equal to 3.15. Such an LRP is rather close to the one estimated on
yearly data with the intuitive model (see Table 9 in Appendix B). An interesting finding is
reported in Figure 9, where we plot, along with the debt dynamics observed in the data and
in the selected model, those that would arise under alternative assumptions on bank’s and
household’s expectations. More precisely, we plot the equilibrium debt that would arise if
banks had the least natural expectations (process 1) while households have expectations as
in process 31 (green circle-line), and if households had the least natural expectations (process
1) while banks have expectations as in process 31 (purple cross-line). These two scenarios
display the equilibrium debt-path that would realize if only one of the two agents displays
a certain degree of naturalness. Finally we plot the outcome of the simulation with both
bank and household being least natural. It can be observed that in order to closely match
the data we need a significant degree of naturalness both on the household and on the bank
side. In fact, the model that features both natural banks and households can replicate quite
well the observed dynamics of HELOCs. In contrast, if only one of the two agents is natural
the model predicts a much lower degree of leverage in the economy than the one observed.\footnote{We also tried feeding alternatively the bank and the household with the most natural expectations, while
keeping the expectations of the other agent least natural. In both cases we are not able to match the data as
closely as we are able to do it assuming that the two agents are both endowed with expectations of process
31. The results of this experiment, not reported here for space optimization, are available upon request.}

For comparison, in Figure 10 we plot the levels of debt and consumption arising under the
chosen model and we compare it with debt and consumption paths that one would get under
two extreme cases, assuming that agents are “most natural” and “least natural” (ie. taking
processes 50 and 1). It can be noted that the paths for debt and consumption up to period
4 for the selected model is very close to the most natural one. Then a deleveraging phase
occurs, less sharp than in the "most natural" case: consumption falls, but not as much as in the latter case. The reduction in consumption is however more significant than the one under "least natural" expectations.

7 Conclusions

The recent financial crisis has served as a reminder of the potential danger caused by undisciplined collateralized debt markets. In this paper, we use home equity extraction as a case study to explore the distortions arising from natural expectations about future values of collateral. We show that natural expectations arose during the period of the recent housing price boom because of the failure of households and financial experts to take into account the complex structure of house prices. We show that agents may end up overestimating long-run prices if they make use of models that fail to capture the rich autocorrelation structure of housing prices and its mean-reverting component. While the notion that households are likely to misestimate house prices has been documented in the literature, in this paper we provide evidence that financial experts also were too optimistic about long-run prices before and during the recent house price boom. Specifically, out-of-sample forecasts gathered from a professional forecaster largely overestimated long-run prices and did not capture any long-run mean reversion after the positive short-run momentum. We show the quantitative implications of natural expectations in a model where households and banks interact through a collateralized financial instrument. We feed the model with a set of expectations that differ in their ability to capture hump-shaped housing price dynamics. We document that after a positive shock on housing prices, less natural agents expect a lower persistence of the shock. In contrast, natural agents overestimate the persistence of the process, thus leading to overly optimistic long-run forecasts. We then simulate the model by considering housing price dynamics as observed during the 2000s. Our models predict a positive relationship between the amount of home equity extracted in a boom phase and the degree of naturalness of the agents in the credit market, while at the same time stressing the prominence of banks’ expectations in the equilibrium outcome. A version of the model in which agents hold natural expectations seems to captures the dynamics of U.S. home equity extraction during the recent boom and bust relatively well. Finally, we highlight that financial experts naturalness is a crucial component for observing a large accumulation of debt at low interest rates.
References


# Tables

<table>
<thead>
<tr>
<th>Date of Forecast</th>
<th>2010</th>
<th>2015</th>
<th>2020</th>
<th>2030</th>
</tr>
</thead>
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<td>1995</td>
<td>3.79 (3.3)</td>
<td>3.84</td>
<td>N.A.</td>
<td>N.A.</td>
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<tr>
<td>1998</td>
<td>3.97 (2.2)</td>
<td>4.06</td>
<td>4.12</td>
<td>N.A.</td>
</tr>
<tr>
<td>2000</td>
<td>3.89 (1.6)</td>
<td>3.93</td>
<td>3.94</td>
<td>3.96</td>
</tr>
<tr>
<td>2002</td>
<td>4.16 (0.4)</td>
<td>4.10</td>
<td>4.07</td>
<td>4.04</td>
</tr>
<tr>
<td>2004</td>
<td>3.28 (-1.7)</td>
<td>3.21</td>
<td>3.16</td>
<td>3.17</td>
</tr>
<tr>
<td>2006</td>
<td>3.77 (-6.3)</td>
<td>3.40</td>
<td>3.22</td>
<td>3.11</td>
</tr>
</tbody>
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Table 1 – Nominal Growth Forecasted House Price

Note: This table reports forecasted average housing price annualized growth rate by the professional forecaster company. The first column reports the year in which the forecasts were made. Numbers in parenthesis are actual realized values of the housing price growth rate.

| $t = \ \
| q = 1 | 4 | 8 | 20 | 40 | 80 |
|-------|---|---|---|----|----|----|
| 1995  | 103.2 | 103.8 | 106.9 | 119.9 | 145.8 | 215.9 |
| 1998  | 100.9 | 102.8 | 106.7 | 119.5 | 146.5 | 223.1 |
| 2000  | 100.9 | 103.3 | 106.9 | 120.7 | 147.4 | 218.1 |
| 2002  | 100.4 | 102.8 | 107.8 | 121.7 | 136.9 | 219.2 |
| 2004  | 100.8 | 101.9 | 103.5 | 114.8 | 134.0 | 181.8 |
| 2006  | 100.8 | 103.5 | 106.6 | 116.2 | 133.9 | 179.5 |

Table 2 – Nominal Growth Forecasted House Price

Note: This table reports $q$-quarters ahead normalized forecasts by the professional forecast company made in the first quarter of the year reported in the first column.
regarding the properties of the models about the short-run forecasts and long-run forecasts.

The bottom panel reports statistics for the model by using only the most recent 100 observations and select the order of the lag polynomial by considering the Bayesian Information Criterion. The BIC and AIC models are estimated by maximizing the two different information criteria when using observation from the whole sample (1953:1-2010:4) (left panel) and in the subsample (1953:1-1996:4) (right panel). The real housing price is the annual growth rate of the Shiller index. Standard errors are in brackets. Significance at 1 percent is indicated by ***, at 5 percent by **, at 10 percent by *.

### Table 3 – Estimation of House Price Growth

Note: In this table we report the estimates of the autoregressive process in equation (1) when considering four models. The intuitive expectations model assumes a first order autoregressive process. The finite memory model assumes an order of 5. The BIC and AIC models are estimated by maximizing the two different information criteria when using observation from the whole sample (1953:1-2010:4) (left panel) and in the subsample (1953:1-1996:4) (right panel). The real housing price is the annual growth rate of the Shiller index. Standard errors are in brackets. Significance at 1 percent is indicated by ***, at 5 percent by **, at 10 percent by *.

### Table 4 – In-Sample Fit and Forecasts

Note: The top panel of this table reports the in-sample fit statistics for the four models for model for housing prices (Intuitive expectations, finite memory model, and for the model selected by the BIC and by AIC). The bottom panel reports statistics regarding the properties of the models about the short-run forecasts and long-run forecasts.
Table 5 – Calibration of some processes

<table>
<thead>
<tr>
<th>Process</th>
<th>LRP</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\sigma$</th>
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<tbody>
<tr>
<td>1</td>
<td>1.4</td>
<td>0.6</td>
<td>-0.31</td>
<td>0.041</td>
</tr>
<tr>
<td>10</td>
<td>1.93</td>
<td>0.6</td>
<td>-0.12</td>
<td>0.041</td>
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<tr>
<td>20</td>
<td>2.51</td>
<td>0.6</td>
<td>0.002</td>
<td>0.039</td>
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<tr>
<td>30</td>
<td>3.10</td>
<td>0.6</td>
<td>0.08</td>
<td>0.037</td>
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<tr>
<td>40</td>
<td>3.73</td>
<td>0.6</td>
<td>0.13</td>
<td>0.035</td>
</tr>
<tr>
<td>50</td>
<td>4.48</td>
<td>0.6</td>
<td>0.18</td>
<td>0.033</td>
</tr>
</tbody>
</table>

Table 5 – Calibration of some processes

Note: This table reports the long-run persistence (LRP), the two autoregressive parameters ($\theta_1$ and $\theta_2$) and the standard deviation ($\sigma$) for six out of the 50 specifications of model as in (12).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\beta=\delta$</td>
<td>0.98</td>
<td>Discount rate for household and banks</td>
</tr>
<tr>
<td>$h$</td>
<td>1.5</td>
<td>Housing stock</td>
</tr>
<tr>
<td>$\eta$</td>
<td>2.0</td>
<td>CRRA coefficient</td>
</tr>
<tr>
<td>$y$</td>
<td>1.0</td>
<td>Income per year</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.05</td>
<td>Rental rate as a fraction of house value</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.2</td>
<td>Collateral value for the bank as a fraction of house value</td>
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</tbody>
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Table 6 – Calibration of structural parameters

<table>
<thead>
<tr>
<th>Process</th>
<th>1</th>
<th>10</th>
<th>20</th>
<th>30</th>
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<tr>
<td>ex-post Consumption equivalent (%)</td>
<td>0</td>
<td>-3</td>
<td>-7.1</td>
<td>-14.6</td>
<td>-25.9</td>
<td>-39.4</td>
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<tr>
<td>ex-ante Consumption equivalent (%)</td>
<td>0</td>
<td>0.5</td>
<td>1.0</td>
<td>1.2</td>
<td>1.4</td>
<td>1.5</td>
</tr>
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</table>

Table 7 – Consumption Equivalent throughout the cycle and ex-ante welfare

Note: This table reports the ex-ante and ex-post welfare cost in terms of consumption equivalent (in percent) of being natural. Welfare is computed as the percentage of consumption in every period that an agent that uses any process (2-50) to forecast future housing prices in the model requires to instead be endowed with beliefs described by the least natural process (process 1). The ex-post utility is computed as realised utility after the realisations of the 10-periods house price path. The ex-ante utility is computed as expected utility at time $t=1$.

<table>
<thead>
<tr>
<th></th>
<th>Boom</th>
<th>Bust</th>
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<tr>
<td></td>
<td>Debt</td>
<td>Rate</td>
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<tr>
<td>a) Bank and Household natural</td>
<td>54.5</td>
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</tr>
<tr>
<td>b) None natural</td>
<td>35.0</td>
<td>2.5</td>
</tr>
<tr>
<td>c) Only Household natural</td>
<td>36.2</td>
<td>2.8</td>
</tr>
<tr>
<td>d) Only Bank natural</td>
<td>42.2</td>
<td>2.1</td>
</tr>
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Table 8 – Debt dynamics under different assumptions

Note: This table reports the simulated average level of debt and interest rate across the boom phase (left panel, from period 1 to period 6 in our model, which correspond to the period 2000-2006) and bust phase (right panel, from period 1 to period 6 in our model, which correspond to the period 2000-2006 in the data) under the hypothesis that both the bank and household are natural (a), both bank and household are not natural (b), only the household is natural (c), and only the bank is natural (d). In this exercise, for simplicity, we assume that a natural agent uses process 50 to make forecasts, whereas a non natural agent uses process 1.
9 Figures

Figure 1 – Home equity extraction and house prices in the U.S.

Note: This figure displays the flows of home equity extraction (solid blue line, left scale) in the U.S. in billion of dollars along with the Shiller’ Real Home Price Index (dashed green line, right scale). Home equity extraction is computed as a four quarters moving average of Gross Equity Extraction divided by the Consumer Price Index. The series, computed according to the methodology in Greenspan and Kennedy (2005), is available at [http://www.calculatedriskblog.com/2009/03/q4-mortgage-equity-extraction-strongly.html](http://www.calculatedriskblog.com/2009/03/q4-mortgage-equity-extraction-strongly.html) (retrieved 7 August 2014). The Real Home Price Index is available at the Robert Shiller’s website ([http://www.econ.yale.edu/~shiller/data.htm](http://www.econ.yale.edu/~shiller/data.htm), retrieved 7 August 2014).

Figure 2 – Financial Expert’s Forecasts

Note: This figure displays the realized evolution of the house price index (solid black line) along with the financial expert forecasts made in different points in time. The four forecasts in the figure were made in 2000Q1 (red dotted line), in 2002Q1 (green circled line), in 2004Q1 (purple dashed line) and in 2006Q1 (blue dash-dotted line).
Figure 3 – Real U.S. Shiller House Price index
Note: This figure plots the Standard & Poor’s Case-Shiller Home Price Index U.S. real housing price index in its level (upper panel) and growth rate (lower panel).

Figure 4 – Comparison of Impulse Response Functions
Note: This figure reports the impulse response function (IRF) of housing price growth rate (upper panel) and housing price level (lower panel) to a positive unitary shock. The solid blue line represents the IRF implied by agents that estimate an AR(1) process for the housing price growth rate (intuitive model). The solid-dotted purple line represents the IRF implied by an agents that estimate a process for the housing price growth rate when using only the last 100 observations (finite memory model). The dotted red line represents the IRF for an agent that maximizes the Bayesian Information Criterion and, hence, estimates an AR(5) process for the housing price growth rate. The green dashed line represent the IRF for an agent that maximizes the Akaike Information Criterion and, hence, estimates an AR(16) process for the housing price growth rate.
Figure 5 – Simulated house price dynamics
Note: This figure plots the housing price series fed into the model (black solid line) along with the actual realization of the annualized Shiller index from 2001 to 2010 (dotted line). The Shiller index has been rescaled and set equal to 1 in 2004.

Figure 6 – IRFs and CIRFs for selected processes
Note: This figure plots the impulse response functions for the housing price growth rate (top-panel) and level (bottom panel) for three different processes used to solve the model: the one characterizing the most natural agents (green-triangle line), the AR1 model (blue-star line), and the one characterizing the least natural agents (black-solid line).
Figure 7 – Boom and bust dynamics for selected processes

Note: This figure displays the average values of debt-to-income (upper left panel), LTV ratio (upper right panel), consumption-to-income (lower left panel) and interest rate (lower right panel) for each of the fifty specifications of expected house price growth. The values displayed in the figure have been interpolated by a 3rd degree polynomial. The x-axis reports the number of each process, from the least (process 1) to the most (process 50) natural. Average values are computed both across the boom phase (from period 1 to period 6 in our model, which correspond to the period 2000-2006 in the data, blue solid line) and across the bust (from period 7 to period 9 in our model, which corresponds to the period 2007-2009 in the data, green dashed line).

Figure 8 – Actual v. simulated data

Note: The black solid line in this figure displays the ratio of gross Home Equity Extraction over Personal Disposable Income, weighted by the fraction of households with an active HELOC (source: Survey of Consumer Finance). The series is normalized at 0 in 2000. The blue crossed line is the simulated debt path arising from process 31, which is the process that minimize the Euclidian distance between the data and the dynamics of debt predicted by our model when varying the degree of naturalness of the agents (process 1 to 50). Sources: Greenspan and Kennedy (2005), FRED, Federal Reserve Economic Data, Federal Reserve Bank of St. Louis and SCF.
Debt dynamics under various assumptions on expectations

Figure 9 – Debt dynamics under various assumptions on expectations

Note: The black solid line in this figure displays the ratio of gross Home Equity Extraction over Personal Disposable Income, weighted by the fraction of households with an active HELOC (source: Survey of Consumer Finance). The series is normalized at 0 in 2000. The red-dashed line is the simulated debt path arising from both agents having expectations as in process 31, which is the process that minimize the Euclidian distance between the data and the dynamics of debt predicted by our model when varying the degree of naturalness of the agents (process 1 to 50). The blue-circle line represents the debt dynamics under the assumption that only the bank is natural (process 31) but the household is not natural (process 1). The green-circled line represents the debt dynamics under the assumption that only the household is natural (process 31) but the bank is not natural (process 1). Finally the purple-cross line represents the debt dynamics under the assumption that both bank and household are not natural. Sources for the data: Greenspan and Kennedy (2005), FRED, Federal Reserve Economic Data, Federal Reserve Bank of St. Louis and SCF.
Figure 10 – Actual v. simulated data

Note: The blue solid line in this figure represents the levels of debt and consumption arising from process 31, which is the process that minimize the Euclidian distance between the data and the dynamics of debt predicted by our model when varying the degree of naturalness of the agents (process 1 to 50). The green and red dotted lines are respectively the levels of debt and consumption arising under process 50 (most natural) and 1 (least natural).
A Appendix: Confidence Band Impulse Response House Price

The top panel of Figure 11 plots together the level impulse response of the intuitive model (blue solid line) and the AIC model (green dotted line) and their 95 percent confidence band (shaded area); the central panel plots together the level impulse response of the intuitive model (blue solid line) and the BIC model (red dashed line) and their 95 percent confidence band; and the bottom panel plots together the level impulse response of the intuitive model (blue solid line) and the finite memory model (purple circled line) and their 95 percent confidence band. As expected, the uncertainty around the impulse responses is large and the confidence bands largely overlap.

Figure 11 – Impulse Response Functions with confidence bands

Note: This figure reports the cumulative impulse response function (CIRF) of house price growth rate to a positive unitary shock. Shaded areas represent the 95 per cent confidence intervals. Top panel: intuitive model (blue solid line) and AIC model (green dotted line). Central panel: intuitive model (blue solid line) and BIC model (red dashed line). Bottom panel: intuitive model (blue solid line) and finite memory model (purple circled line).
Appendix: Long-Run Price for Annual Data

<table>
<thead>
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<th>p</th>
<th>Natural</th>
<th>BIC</th>
<th>AIC</th>
<th>Short Memory</th>
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</thead>
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<tr>
<td></td>
<td>2.76</td>
<td>1.72</td>
<td>1.52</td>
<td>2.29</td>
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<tr>
<td>(LRP)</td>
<td>[2.17; 4.49]</td>
<td>[0.85; 3.05]</td>
<td>[0.67; 2.91]</td>
<td>[0.25; 5.17]</td>
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<tr>
<td>Confidence Bands (95%)</td>
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<td></td>
<td></td>
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</table>

Table 9 – LRP and Confidence Band

Appendix: Numerical algorithm

The numerical algorithm for solving the model works as follows:

1. create a grid for debt and interest rate and assign values to parameters \((\beta, h, \eta, y, \gamma, \kappa, \{i_t\}_{t=0}^{T-1})\).

2. Define the true house price process \(\{p_t\}_{t=0}^{T}\) and the ones perceived by the agents: \(\{p_t^H\}_{t=0}^{T}\) for household and \(\{p_t^B\}_{t=0}^{T}\) for bank. Use Tauchen (1986)’s method for discretizing the shock and create a grid for debt and interest rate.

3. Start from period \(T\) and compute terminal value for both bank and household. For each value of the housing shock compute value function for household \(V_T^* (r_{T-1}, d_{T-1}, p_T)\) and bank \(\pi_T (r_{T-1}, d_{T-1}, p_T)\). More precisely, in order to recover \(V_T^* (r_{T-1}, d_{T-1}, p_T)\), we need to compute the default decision from the point of view of the household, while in order to recover \(\pi_T (r_{T-1}, d_{T-1}, p_T)\), we need to compute the default decision from the point of view of the bank. This is obtained by inserting the expectations of the bank into the household problem and solving it. More concisely, for each \(r_{T-1}, d_{T-1}\) and \(p_T\), the default decision is as follows:

\[
\text{default in } T = \begin{cases} 
1 & \text{if } p_T^B \leq \frac{(1+r_{T-1})d_{T-1}}{\gamma h} \\
0 & \text{otherwise}
\end{cases}
\]

4. Go back to period \(T - 1\). From now on, the algorithm is valid for periods from \(t = T - 1, \ldots, 1\). Fix an interest rate \(\tilde{r}_t\) and for each value of the housing shock compute the value function \(V_t^* (r_{t-1}, d_{t-1}, p_t)\) and the debt demand schedule \(d_H^t(\tilde{r}_t)\).

5. Feed the bank’s problem with the value of \(d_H^t\) and find the value of \(r_t^B\) that satisfies the value function of the bank.
6. If $r_t^B = r_t$ then stop and move to period $t - 1$, otherwise, replace $r_t$ with $r_t^B$ and repeat
the loop from 2 until convergence.

7. Given $r_1, d_1$ and expectations over the realization of $p_1$, compute the equilibrium values
of $d_0$ and $r_0$. 