

Original citation:

Bernhardt, Dan and Rastad, Mahdi. (2016) Collusion under risk aversion and fixed costs. The Journal of Industrial Economics, 64 (4). pp. 808-834.

Permanent WRAP URL:

<http://wrap.warwick.ac.uk/76987>

Copyright and reuse:

The Warwick Research Archive Portal (WRAP) makes this work by researchers of the University of Warwick available open access under the following conditions. Copyright © and all moral rights to the version of the paper presented here belong to the individual author(s) and/or other copyright owners. To the extent reasonable and practicable the material made available in WRAP has been checked for eligibility before being made available.

Copies of full items can be used for personal research or study, educational, or not-for profit purposes without prior permission or charge. Provided that the authors, title and full bibliographic details are credited, a hyperlink and/or URL is given for the original metadata page and the content is not changed in any way.

Publisher's statement:

"This is the peer reviewed version of the following article: Bernhardt, Dan and Rastad, Mahdi. (2016) Collusion under risk aversion and fixed costs. The Journal of Industrial Economics, 64 (4). pp. 808-834. which has been published in final form at <http://doi.org/10.1111/joie.12111> . This article may be used for non-commercial purposes in accordance with [Wiley Terms and Conditions for Self-Archiving](#)."

A note on versions:

The version presented here may differ from the published version or, version of record, if you wish to cite this item you are advised to consult the publisher's version. Please see the 'permanent WRAP URL' above for details on accessing the published version and note that access may require a subscription.

For more information, please contact the WRAP Team at: wrap@warwick.ac.uk

COLLUSION UNDER RISK AVERSION AND FIXED COSTS *

Dan Bernhardt[†]

Mahdi Rastad[‡]

We analyze collusion under demand uncertainty by risk-averse cartels that care about the utility derived from profits. With sufficient risk aversion and non-trivial fixed operating costs, it becomes difficult for cartels to collusively restrict output both when demand is low and marginal dollars are highly-valued, and when demand is high and potential defection profits are high: output relative to monopoly levels becomes a U-shaped function of demand. Greater risk aversion or higher fixed operating costs make collusion more difficult to support in recessions, but easier to support in booms.

I. INTRODUCTION

IN THEIR PIONEERING ANALYSIS OF COLLUSION under demand uncertainty, Rotemberg and Saloner [1986] show that when demand is independently and identically distributed over time, and firms observe demand before taking actions, collusion is harder to support when demand is higher. The intuition is compelling: the period incentive to cheat on the cartel rises with demand, but expected continuation payoffs are unchanged. While this setting is stark, Kandori [1991] shows that the essence of the Rotemberg and Saloner result extends to serially-correlated demand—the key is that the period incentive to cheat on the cartel is more sensitive to current market conditions than are continuation payoffs. Haltiwanger and Harrington [1991] establish related results for deterministic cyclical demand—collusion is hardest to support at the peak of a cycle.¹ Bagwell and Staiger [1997] generalize these results melding a Markov demand growth process on top of the i.i.d. transitory shocks.

*We thank Odilon Camara, Volker Nocke (the editor), two anonymous referees and seminar participants at California Polytechnic State University for helpful comments.

[†]Department of Economics, University of Illinois, David Kinley Hall, Urbana, IL 61820, and University of Warwick. E-mail: danber@illinois.edu

[‡]Department of Finance, Orfalea College of Business, California Polytechnic State University, San Luis Obispo, CA 93407. E-mail: mrastad@calpoly.edu

In contrast to this robust theoretical prediction, we have Scherer [1980]’s summary of his empirical work: ‘Yet it is precisely when business conditions turn sour that price cutting runs most rampant among oligopolistic firms with high fixed costs,’ Staiger and Wolak [1992]’s assertion that ‘the conventional empirical wisdom [is] that tacit collusion tends to break down when business conditions turn sour,’ and Aiginger et al. [1998]’s survey of 113 experts, which found that most believed that price wars are more likely when demand is low. Empirical studies providing support for the premise that collusion is harder to support in downturns include Porter [1985], Scherer and Ross [1990], Suslow [2005], and Ellison [1994]. This research primarily focuses on prices and price-cost margins; but with output competition, procyclical price-cost margins remain consistent with collusion being more difficult to support when demand is high, making conclusions harder to draw. Still, even among commodity cartels, there is evidence that collusion is difficult to support in downturns. For example, Scherer [1980] provides examples of cartels in the rayon, steel and cement industries that experienced breakups in recessions. So, too, among country cartels—cartels where production decisions are made by leaders who care about the utility citizens derive from the profits earned—the two largest production wars in OPEC occurred in 1986 and 1997 when demand was extremely low. In contrast to this literature, Rotemberg and Saloner [1986], Domowitz et al. [1987], Chevalier and Scharfstein [1996], Borenstein and Shepard [1996], Rosenbaum and Sukharomana [2001], and Mariuzzo and Walsh [2013] provide empirical evidence consistent with cartels being likely to breakdown in booms.

One approach to trying to reconcile these empirical findings is to argue that, as is likely, different factors are relevant in different markets. Thus, for example, Rotemberg and Saloner’s mechanism may underlie why cartels sometimes breakdown in booms, but a completely different mechanism may underlie why cartels sometimes breakdown when demand is low. For example, in the imperfect monitoring literature (e.g., Green and Porter [1984]) firms might see equilibrium prices, but not demand realizations or outputs, and low prices trigger price wars because cartel members cannot distinguish whether they are due to low demand or to cheating by a cartel member. However, given the premised lack of observabil-

ity of demand, the literature delivers a limited link between output and demand. Moreover, among commodity cartels like OPEC, outputs are fairly well observed. Another approach is to introduce capacity constraints for firms, which limit defection gains when demand is high, reducing incentives to cheat, but also weaken the ability to punish in high demand states. Staiger and Wolak [1992] add a capacity-building stage to the Rotemberg and Saloner model and predict (seemingly contrary to the empirical evidence), that price wars are more likely in *intermediate* demand states than in either low demand states or high demand states (see Fabra [2006] for a related result in the Haltiwanger and Harrington [1991] cyclical demand model, and Nocke [1999] for how excess capacity in bad times can bring instability to a cartel).

We take a different approach. We return to the insights implicit in the empirical analysis of Scherer [1980], and explore collusion by *risk averse* (CRRA or DARA preferences) cartel members that face positive fixed *operating costs*. Because most commodity cartels choose output levels rather than prices, we model output competition when defections from cartel quotas are deterred by threats to revert to static Nash equilibrium output levels.² We otherwise focus on the classical i.i.d. demand, constant marginal cost setting studied by Rotemberg and Saloner. We measure the extent of collusion by the ratio of output relative to monopoly levels that can be supported in different demand states—a higher ratio indicates that collusion is more difficult to support. We show that our model can reconcile the collective set of empirical findings: collusively restricting output toward monopoly levels can be more difficult *both* when demand is unusually low *and* when it is unusually high.

Risk averse cartel members value a marginal dollar of profit by more when profits are lower. This might lead one to conjecture that with power utility, cartels could find collusion more difficult to support in low demand states. This conjecture is false: to overturn the Rotemberg-Saloner result that higher demand always makes it more difficult for firms to collusively restrain output, cartel members must not only be risk averse, but fixed operating costs of production must be positive. Absent fixed costs, the extent of risk aversion just scales the period incentive to cheat on the cartel, preserving monotonicity of the incentive in demand. Fixed operating costs (or large operational/non-oil budget deficits in country

cartels that care about the utility citizens derive from the profits) amplify the asymmetric valuation of a dollar in good times and bad, inducing cartel members with CRRA preferences to behave as if they have decreasing absolute risk averse (DARA) preferences. This sharply raises the incentive to cheat on the cartel when demand is especially low.

More provocatively, we establish that collusion is easiest to support when demand is intermediate, neither too low, nor too high. That is, risk aversion together with fixed operating costs give rise to the opposite predictions of those in Staiger and Wolak [1992]. When demand is especially low, the very high marginal valuation of an additional dollar of profit induced by the fixed operating costs makes the incentive to cheat on the cartel very high; and when demand is very high, decreasing risk aversion implies that the classical effect dominates—as demand increases, there are more and more dollars to be gained from cheating on the cartel, and with DARA preferences, the marginal utility of those dollars declines by less and less. We further establish that although the incentive to cheat on the cartel is a U-shaped function of the level of demand, the incentive rises more sharply in low demand states (as demand gets worse) than in higher demand states (as demand gets better).

We then show that greater fixed operating costs or risk aversion make it *harder* to support collusion when demand is low, but *easier* to support collusion when demand is high. Greater fixed costs or risk aversion raise the net continuation payoff from collusion by enhancing the threatened Nash reversion punishment for cheating on the cartel. However, greater fixed costs or risk aversion also raise the potential period utility gains from cheating on the cartel. The impact of higher operating costs on period incentives dominates when demand is especially low, making collusion more difficult to support; but the higher *net* continuation payoffs dominate when demand is especially high, making collusion easier to support.

Our central premise that cartel members are risk averse captures two types of scenarios. First, risk-averse preferences may better describe the objective function of a firm. For example, a firm may be run by a risk-averse manager, or a financially-constrained firm with high debt obligations may value a marginal dollar in bad times by more because it may stave off liquidation.³ Second, many commodity cartels consist of ‘country cartels’ that do

not care about profits per se, but rather about the utility that (possibly a subset of) their citizens derive from those profits. In addition to OPEC, country cartels have existed in many manufacturing and commodity cartels (natural resources such as minerals, chemicals, raw materials, metals, etc.; see Suslow [2005] for a list).

The literature has long argued that DARA utility functions better capture a firm’s behavior. Sandmo [1971], and Appelbaum and Katz [1986] are the first to study firm behavior under uncertainty with risk averse preferences. Greenwald and Stiglitz [1990] propose micro-foundations for DARA preferences. Spagnolo [1999] argues that non-linearities in the corporate tax schedule, convexity of external financing costs, and managers’ capped monetary incentives together with their fears of losing jobs or reputations give rise to strictly concave firm objective functions. Nocke and Thanassoulis [2014] show how bad demand realizations can cause a risk-neutral credit-constrained firm to under-invest in assets/collateral, endogenously becoming risk-averse. Asplund [2002] highlights the impact of costly financial distress, liquidity constraints, and non-diversified owners. Using the intuition that ‘firms give relatively greater weight to realizations with low profits,’ Asplund looks at how the degree of risk aversion affects competition intensity. He shows that with DARA preferences, ‘fixed costs influence best-response strategies by increasing firms risk aversion.’ Spagnolo [1999, 2005] shows that it is easier for firms to support collusive outcomes when they are risk averse.⁴

We next present the model and analysis. A conclusion follows. Proofs are in an appendix.

II. THE MODEL

Our framework features an infinitely repeated output game played by two agents 1 and 2 (e.g., risk-averse firms, or country members of OPEC) that sell a homogeneous good in a market where demand evolves stochastically according to an i.i.d. process. Date t demand is given by

$$(1) \quad P_t = \theta_t - Q_t,$$

where θ_t is identically and independently distributed, with associated distribution function $F(\theta)$ on its positive support $[a, b]$, with $a > 0$, and $Q_t = q_t^1 + q_t^2$ is aggregate output. Without

loss of generality, we normalize the constant marginal costs of production to zero. The agents also incur fixed operating costs each period of $f \geq 0$, where $f \leq \frac{a^2}{9}$. In the appendix, we extend the analysis to $N > 2$ agents, and to more general demand and preference structures, providing conditions under which our qualitative findings extend.⁵

In the classical Rotemberg and Saloner [1986] framework, the agents are risk-neutral firms whose period payoffs equal their period profits, $U^i(\pi_t^i) = \pi_t^i$. Thus, the marginal value that a firm derives from a dollar of profit does not vary with the level of profit, and fixed operating costs are irrelevant for a firm's decision-making (assuming that exit is not a strategic consideration). We depart from this setting to investigate collusion by risk-averse agents that face positive fixed operating costs. Agent i derives period utility from profit π_t^i of $U^i(\pi_t^i) = (\pi_t^i)^\alpha$, where $0 < \alpha \leq 1$. For simplicity, we assume that each period, a cartel member consumes its period profits, i.e., there is no saving and borrowing. As a result, a cartel member values an extra dollar of profit by more when profits are lower. Moreover, fixed operating costs enter decision-making non-trivially, as they especially magnify the marginal value of an additional dollar in bad times. Firms use a common discount factor $\beta \in (0, 1)$ to discount future payoffs.

Our assumption of output competition captures our real world motivating example of a commodity cartel, in which cartel members choose outputs rather than prices. We focus on the maximal period collusion profits that the cartel can support by threats to revert to the non-cooperative static Nash equilibrium outputs forever if a cartel member ever deviates from their collusive agreement. This assumption simplifies analysis. More generally, we establish that our qualitative findings extend (in symmetric settings) as long as the punishment path is not so harsh that monopoly profits are always sustainable. This means that we must preclude punishments that would result in consumption levels that are 'too close' to the effective subsistence level of f . With output competition and linear demand, the assumption that $f < \frac{a^2}{9}$ ensures that the profits from Nash outputs always cover the period fixed operating costs.

After observing period demand, cartel members simultaneously choose outputs. Define $q^C(\theta)$ to be the collusive firm output supported along the equilibrium path when the demand shock is θ . Given that deviations from collusive outputs result in static Nash outputs in the

future, an agent that cheats on the cartel agreement will produce the $q^F(\theta)$ that maximizes period profit, and hence period utility, solving

$$\max_{q(\theta)} (\theta - q^C(\theta) - q(\theta))q(\theta) - f \Rightarrow q^F(\theta) = (\theta - q^C(\theta))/2.$$

Let $\pi^C(\theta) = (\theta - 2q^C(\theta))q^C(\theta) - f$ and $\pi^F(\theta) = (\theta - q^C(\theta))^2/4 - f$ denote the respective period profits from cooperating and cheating on the cartel, and let $q^P(\theta) = \theta/3$ be the Nash output and $\pi^P(\theta) = \theta^2/9 - f$ be the associated Nash period profit. Finally, let $U^C \equiv E[U(\pi^C(\theta))]$ be the expected period utility from cooperation along the equilibrium path, and let $U^P \equiv E[U(\pi^P(\theta))]$ be the expected period utility along the punishment path. Then, for each given demand shock θ , incentive compatibility requires

$$(2) \quad U(\pi^C(\theta_t)) + \left(\frac{\beta}{1-\beta}\right) U^C \geq U(\pi^F(\theta_t)) + \left(\frac{\beta}{1-\beta}\right) U^P.$$

Equation (2) can be re-arranged in terms of the ‘incentive to cheat’:

$$(3) \quad U(\pi^F(\theta_t)) - U(\pi^C(\theta_t)) \leq \left(\frac{\beta}{1-\beta}\right) (U^C - U^P) \equiv v.$$

That is, for a cartel production schedule to be incentive compatible, the net period utility payoff from cheating when demand is θ , $U(\pi^F(\theta_t)) - U(\pi^C(\theta_t))$, cannot exceed the net expected payoff from future cooperation rather than punishment, v .

II.(i) *Cartel’s Problem.*

The cartel’s objective is to find the incentive compatible production schedule that maximizes their joint utility on the equilibrium path, $\sum_{t=1}^{\infty} \beta^{t-1} E[U(\pi_i^C(\theta_t)) + U(\pi_j^C(\theta_t))]$. With power utility, we can write the cartel’s problem as

$$(4) \quad \max_{q(\theta)} 2 \int_a^b (q(\theta)(\theta - 2q(\theta)) - f)^\alpha dF(\theta)$$

$$s.t. \quad \left(\frac{(\theta - q(\theta))^2}{4} - f\right)^\alpha - ((\theta - 2q(\theta))q(\theta) - f)^\alpha \leq \left(\frac{\beta}{1-\beta}\right) (U^C - U^P) \equiv v, \quad \forall \theta \in [a, b].$$

We measure a cartel's ability to support collusion in demand state θ by the ratio $q^C(\theta)/q^m(\theta) \geq 1$, i.e., by the ratio of output relative to monopoly levels. One can also motivate this measure by the observation that the square of this ratio equals the ratio of consumer surplus under the cartel vs. under monopoly. A higher ratio indicates that collusion is more difficult to support. Most empirical researchers measure collusion using price-cost margins. If price-cost margins fall with θ , then $q^C(\theta)/q^m(\theta)$ also rises with θ . However, with output competition, $q^C(\theta)/q^m(\theta)$ can rise with θ , indicating a reduced ability of the cartel to support collusion in higher demand states, even though price-cost margins rise uniformly with θ . Phrased differently, with output competition, the procyclical price-cost margins found empirically do *not* imply that collusion is easier to support in high demand states.

For the special case of linear utility, $U(\pi_i(\theta_t)) = \pi_i(\theta_t)$, the cartel's objective reduces to the output-competition variant of Rotemberg and Saloner [1986]. In that setting, it immediately follows that the incentive to cheat increases in θ , as with i.i.d. demand, expected continuation payoffs do not vary with θ , but the current payoffs from cheating on the cartel rise when the stakes are higher. As a result, $q^C(\theta)/q^m(\theta)$ is constant when demand is low enough that monopoly profits can be supported, and it is strictly increasing once demand is high enough that threats to deviate to Nash outputs cannot support monopoly profits.

One might conjecture that risk-aversion alone, i.e., $\alpha < 1$, would be enough to reverse the result that increases in demand make it more difficult for the cartel to collusively restrict output toward monopoly levels, i.e., to reverse the result that $q^C(\theta)/q^m(\theta)$ is non-decreasing in θ . That is, one might conjecture that since the marginal utility derived from another dollar of profit is higher when profits are lower, collusion might be more difficult to support when demand is low and cartel members are sufficiently risk averse. This conjecture is false. The following proposition establishes necessary conditions for it to be harder to support collusion when demand is low than when it is high: not only must cartel members be risk averse, $\alpha < 1$, but they must also have positive fixed costs of operation, $f > 0$.

Proposition 1. Suppose that either $f = 0$ or $\alpha = 1$. Then over-production relative to

monopoly levels rises with the level of demand, i.e., $q^C(\theta)/q^m(\theta)$ is non-decreasing in θ .

Thus, *both* risk-aversion *and* positive fixed operating costs are necessary for overproduction not to rise with θ . Intuition for why more than risk aversion is required can be gleaned from looking at those demand states θ where the net value of future cooperation v is high enough that the IC constraint is slack. For such θ , the cartel's optimization problem simplifies to a pointwise maximization of its objective. The associated first-order condition is

$$(q(\theta)(\theta - 2q(\theta)) - f)^{\alpha-1} (\theta - 4q(\theta)) = 0,$$

with solution $q^C(\theta) = \theta/4$. The two agents jointly produce the monopoly output, $\theta/2$, and each earns half of the monopoly profits net of operating costs, $\theta^2/8 - f$; and the associated firm output is $3\theta/8$, which delivers profits of $\pi^F(\theta) = 9\theta^2/64 - f$. To see how incentives to cheat on the cartel hinge on the level of demand, the extent of risk aversion and the fixed operating costs, define the (period) incentive to cheat on monopoly output as

$$(5) \quad h(\theta; \alpha, f) = U(\pi^F(\theta)) - U(\pi^C(\theta)) = \left(\frac{9\theta^2}{64} - f\right)^\alpha - \left(\frac{\theta^2}{8} - f\right)^\alpha.$$

When there are no fixed costs, $h(\theta, \alpha, f = 0)$ simplifies to

$$h(\theta; \alpha, f = 0) = \left(\frac{9\theta^2}{64}\right)^\alpha - \left(\frac{\theta^2}{8}\right)^\alpha = \left(\frac{\theta^2}{64}\right)^\alpha (9^\alpha - 8^\alpha).$$

Thus, without fixed costs, the extent of risk aversion scales the incentive to cheat, preserving monotonicity in θ . A similar result holds when monopoly output cannot be supported.

We now show that for the incentive to cheat on the cartel not to rise monotonically with the level of demand, the impact of risk aversion must be higher in low demand states than high, i.e., the preferences induced by the fixed operating costs must exhibit decreasing relative risk aversion. With positive fixed operating costs, preferences effectively take a subsistence utility form, and the associated Arrow-Pratt measure of relative risk-aversion, $RRA(W) = -WU''(W)/U'(W) = (1-\alpha)\frac{W}{W-f}$, decreases in W if and only if $\alpha < 1$ and $f > 0$. Then, when demand is low, the higher marginal valuation of an additional dollar of profit induced by the fixed operating costs cause the incentives to cheat to rise further when demand drops lower,

and agents become more desperate for another marginal dollar of profit. In contrast, when demand is much higher, the decreasing risk aversion implies that risk aversion matters less, with the result that the standard effect dominates—as demand rises, there are more dollars to be gained from cheating on the cartel. Putting these two observations together suggests that the incentive to cheat on the cartel will be a U-shaped function of θ . We now formalize this intuition and begin to address the question of exactly where the separation between good and bad times occurs. The theorem shows that to deliver the U-shaped relationship, agents must have intermediate levels of risk aversion: for the incentive to cheat on the cartel not to rise monotonically with θ , for a given level of fixed costs, agents must be sufficiently risk averse; and for the incentive not to fall monotonically with θ , they must not be too risk averse.

Proposition 2. There exist critical levels of risk aversion, $\underline{\alpha}(f)$ and $\bar{\alpha}(f)$, indexed by the fixed costs f , such that if and only if cartel members have intermediate levels of risk aversion, $\underline{\alpha}(f) < \alpha < \bar{\alpha}(f) < 1$, then h is a U-shaped function of θ , achieving a minimum at $\hat{\theta}(\alpha, f) \in (a, b)$. That is, $h'(\theta) < 0$ for $\theta < \hat{\theta}(\alpha, f)$, and $h'(\theta) > 0$ for $\theta > \hat{\theta}(\alpha, f)$. Further, $\underline{\alpha}(f)$, $\bar{\alpha}(f)$ and $\hat{\theta}(\alpha, f)$ rise with the fixed cost f , and $\hat{\theta}(\alpha, f)$ increases in risk aversion $(-\alpha)$.

The proof shows that, as in Figure 1, there is a unique intermediate demand level $\hat{\theta}$ that minimizes the incentive to cheat. As demand falls below $\hat{\theta}$, the incentive to cheat rises due to the high marginal valuation of another dollar of profit; and as demand rises above $\hat{\theta}$, so too does the incentive to cheat due to the greater profit that can be gained. The comparative statics reveal that when agents are more risk averse or fixed costs are greater, demand does not have to be as bad for the incentive to cheat to begin to rise as demand drops lower.

[Place Figure 1 about here.]

Monopoly outputs are supportable when the period benefit from cheating, h , is less than the expected net value of future cooperation, v , which is independent of θ . When h is a U-shaped function of θ , it directly follows that monopoly outputs can only be sustained for intermediate values of demand whenever cartel members are neither so patient that they can

support monopoly outputs in every state, nor so impatient that they can support monopoly outputs in no state (see Figure 1). Corollary 1 formalizes the necessary conditions.

Corollary 1. There exist $\underline{\beta}, \bar{\beta}$ with $0 < \underline{\beta} < \bar{\beta} < 1$ such that if and only if $\beta \in [\underline{\beta}, \bar{\beta}]$ the cartel can support monopoly profits only if demand is neither too low nor too high: If and only if $\beta \in [\underline{\beta}, \bar{\beta}]$, there exist $\underline{\theta}(\beta), \bar{\theta}(\beta)$ with $a < \underline{\theta}(\beta) < \bar{\theta}(\beta) < b$ such that monopoly profits can be supported if and only if $\theta \in [\underline{\theta}(\beta), \bar{\theta}(\beta)]$.

We have identified two forces that can drive the cartel away from supporting monopoly output levels: temptations rooting from the larger potential profit gain when times are good, and desperateness for an extra dollar of profits when times are bad. But, which force is stronger? In Proposition 3, we show that the ability to support collusion drops off more quickly when demand falls below $\hat{\theta}$ than when it rises past $\hat{\theta}$.

Proposition 3. The incentive to cheat on monopoly output rises more quickly as low demand states become worse than as high demand states improve: $h(\hat{\theta} - \delta) > h(\hat{\theta} + \delta)$ for all $\delta > 0$.

The intuition for Proposition 3 devolves from the increasing desperation to obtain another dollar of profit when its marginal valuation is high that is implicit in Scherer [1980]’s summary that ‘Yet it is precisely when business conditions turn sour that price cutting runs most rampant among oligopolistic firms with high fixed costs.’ Proposition 3 goes beyond Proposition 2. Proposition 2 showed that the incentive to cheat on monopoly output rises not only when demand is larger, but also when market conditions turn sour. Proposition 3 documents an asymmetry in the incentive to cheat: monopoly can be supported in a narrower range of bad states than good ones. Put differently, $\hat{\theta}$ is closer to $\underline{\theta}$ than to $\bar{\theta}$.

We now characterize output levels following demand realizations—both high and low—that are sufficiently extreme that the cartel cannot support monopoly outputs. To prevent agents from cheating, cartel output must be increased to a level that makes agents indifferent between cheating and cooperation. More formally, at each $\theta \in [a, \underline{\theta}] \cup [\bar{\theta}, b]$ incentive

compatible quotas, $q(\theta)$, solve

$$(6) \quad \left(\frac{(\theta - q(\theta))^2}{4} - f\right)^\alpha - ((\theta - 2q(\theta))q(\theta) - f)^\alpha = \left(\frac{\beta}{1 - \beta}\right) (U^C - U^P) \equiv v.$$

Define the normalized production level $z \equiv q(\theta)/\theta$: z is an index for overproduction relative to monopoly output, as $4z = q(\theta)/(\theta/4) = q(\theta)/q^m(\theta)$. When monopoly output can be supported, there is no overproduction, so that $z = 1/4$; and when the cartel breaks down and agents revert to Nash outputs, then $z = 1/3$. That is, outside the monopoly support region, we have $z \in (1/4, 1/3)$, and profits decrease in z .

We rewrite the left-hand side of equation (6) in terms of z and define $H(z, \theta)$ to be this period incentive to cheat:

$$(7) \quad H(z, \theta) = \left(\theta^2 \frac{(1 - z)^2}{4} - f\right)^\alpha - (\theta^2(1 - 2z)z - f)^\alpha.$$

When $z = 1/4$, then $H(1/4, \theta; \alpha, f)$ reduces to the period incentive to cheat on monopoly output, $h(\theta; \alpha, f)$. As in Proposition 2, one can show that $H(z, \theta)$ is a U-shaped function of θ for every $z \in (1/4, 1/3)$. Proposition 4 shows that when demand realizations make it more attractive to cheat on the cartel, members must reduce this attraction by increasing output relative to the monopoly level, but that output increases become less and less effective at reducing this incentive. Further, collusion is harder to sustain both for more extreme low demand realizations and for more extreme high demand realizations, requiring greater overproduction:

Proposition 4. Outside the monopoly support region $[\underline{\theta}, \bar{\theta}]$, the period incentive to cheat is a continuously decreasing, convex function of output relative to monopoly levels: $\frac{\partial H(z, \theta)}{\partial z} < 0$, and $\frac{\partial^2 H(z, \theta)}{\partial z^2} > 0$. Overproduction relative to monopoly output rises when demand is further from the monopoly support region: $\partial \left(\frac{q(\theta)}{q^m(\theta)}\right) / \partial(\underline{\theta} - \theta) > 0$ for $\theta < \underline{\theta}$, and $\partial \left(\frac{q(\theta)}{q^m(\theta)}\right) / \partial(\theta - \bar{\theta}) > 0$ for $\theta > \bar{\theta}$.⁶

One might conjecture that when fixed operating costs, f , are higher, or cartel members are more risk averse (lower α), it becomes more difficult to support collusion in every demand state. The intuition underlying such a conjecture is that such changes raise the period utility gain from cheating on any given level of output. However, the intuition underlying this

conjecture is incomplete. The conjecture that greater fixed costs or increased risk aversion make collusion harder to support would follow directly *if* the net continuation payoffs from collusion versus punishment did not rise. However, as f is increased (or agents become more risk averse), the threat to punish cheating on the cartel by reverting to Nash equilibrium outputs becomes harsher relative to the gain from a given level of cooperation. If, as a result, v rises by enough with greater operating costs or risk aversion to offset the increased period incentive to cheat on the cartel, then greater collusion may be facilitated.

We next characterize how the extent of risk aversion or fixed costs affect the ability to support collusion in different demand states. We establish a single-crossing property characterizing which states collusion is easier to support. We show that provided that increases in operating costs f or in risk aversion (reductions in α) do not uniformly raise or lower the incentive to cheat on the cartel, then greater fixed costs and greater risk aversion make collusion harder to support when demand is low, but easier when demand is high.

[Place Figure 2 about here.]

Proposition 5. Consider $\alpha_2 < \alpha_1$. Suppose there exists a θ^* such that $z_1(\theta^*) = z_2(\theta^*) > \frac{1}{4}$. Then outside the monopoly support region, more risk averse agents find it harder to support collusion in bad times, but easier in good times: For $\alpha_2 < \alpha_1$, for all $\theta < \theta^*$, if $z_2(\theta) > 1/4$, then $z_2(\theta) > z_1(\theta)$; and for all $\theta > \theta^*$, if $z_1(\theta) > 1/4$ then $z_2(\theta) < z_1(\theta)$.

Proposition 6. Consider fixed operating costs, $f_2 > f_1$. Suppose there exists a θ^* such that $z_1(\theta^*) = z_2(\theta^*) > \frac{1}{4}$. Then outside the monopoly support region, greater fixed costs make it harder to support collusion in bad times, but easier in good times: For $f_2 > f_1$, for all $\theta < \theta^*$, if $z_2(\theta) > 1/4$, then $z_2(\theta) > z_1(\theta)$; and for all $\theta > \theta^*$, if $z_1(\theta) > 1/4$ then $z_2(\theta) < z_1(\theta)$.

[Place Figure 3 about here.]

The key to these proofs is to show that the impact of an increase in f or in risk aversion on the period gain from cheating, $H(z, \theta)$ falls with θ for a fixed $z = q(\theta)/\theta$, i.e., that $\frac{\partial^2 H(z, \theta; f)}{\partial f \partial \theta} < 0$ and $\frac{\partial^2 H(z, \theta; \alpha)}{\partial \alpha \partial \theta} > 0$. Hence, if there exists a θ^* at which $z_1(\theta^*) = z_2(\theta^*)$, then it is unique.

A grid search of the parameter space in the neighborhood of the values used in Figures 2 and 3 indicates that when (a) demand is uniformly distributed, and (b) agents are sufficiently risk averse with high enough operating costs that the monopoly support region is interior, then continuation payoffs always rise with f or with risk aversion by amounts that, consistent with Figures 2 and 3 and the two propositions, give rise to asymmetric effects on the cartel’s ability to support collusion. That is, with uniform uncertainty, the single-crossing property always holds. Numerically, we find that the ‘crossing point’ is always at a high demand state.⁷ That is, the effect of an increase in f or reduction in α on the increased incentive to cheat dominates the impact on net continuation payoffs for sufficiently low demand shocks where agents are especially desperate for another dollar of profit. However, net continuation payoffs rise with increased operating costs and increased risk aversion, and this effect dominates once demand is sufficiently high, making collusion easier to sustain. These results reflect the induced decreasing relative risk aversion in preferences—the effect of an increase in operating costs or risk aversion on the period utility gain from cheating on a given level of collusion falls as demand, and hence profits, rise.

[Place Figure 4 about here.]

II.(ii) *Asymmetric Cartels.*

Although we do not analyze it formally, Propositions 5 and 6 have suggestive implications for how heterogeneous agents with different levels of fixed operating costs or risk aversion should collude. For example, in practice, OPEC countries do not rely solely on oil revenues, and higher non-oil revenues effectively imply lower fixed operating costs. From this perspective, Saudi Arabia with \$1,789 per capita in non-oil export revenues may effectively have lower fixed operating costs than other OPEC members such as Venezuela which only

has \$127 per capita in non-oil export revenues.⁸ Then the propositions would suggest that Saudi Arabia should have a lower share of output in low demand states (where high operating cost cartel members find cheating more attractive), but a higher share of output when demand is high (and high operating cost cartel members mind ceding share by less, and are willing to do so in order to obtain greater shares in low demand states where they care more about their shares). That is, high operating cost countries gain relatively more utility from a marginal dollar when demand is low, and their incentive to cheat on the cartel in low demand states is higher. As a result, a cartel that maximizes a weighted sum of its members' utilities will allocate relatively greater shares to high operating cost/more risk-averse countries when demand is low, and relatively lower shares when demand is high. Consequently, the output of a low operating cost/low non-oil budget deficit cartel member should be more sensitive to the level of demand than the output of higher operating cost cartel members. Thus, Saudi Arabia should be the swing producer, with a smaller share of output in bad times, and a larger share in good times, so that its output would appear to be the primary driver determining cartel outcomes, as the popular press regularly emphasizes.

[Place Table I about here.]

Figure 4 offers evidence consistent with such a premise.⁹ Production shares are very asymmetric, with four countries—Iran, Kuwait, Libya and Venezuela—consistently having sharply higher shares when oil prices are very low; and three countries—Saudi Arabia, United Arab Emirates and Nigeria—consistently having sharply lower production shares when oil prices are very low, and higher shares when oil prices are at their highest.¹⁰ Table I summarizes this figure by measuring the slope of the price-share relationship using regression analysis. It shows that Saudi Arabia among the pro-cyclical producers and Venezuela among the counter-cyclical producers have the highest (absolute) price sensitivity. On average, for every dollar drop in real oil price, in the following year Saudi Arabia cuts back on its share

by 0.158%, whereas Venezuela increases its share by 0.085%. Obviously, conclusions about why these patterns obtain are speculative in nature, and a full analysis is beyond the scope of our paper. Nonetheless, the patterns are quite suggestive.

III. CONCLUSION

A robust prediction of the theoretical literature on collusion under demand uncertainty when cartel members observe demand and can monitor each other's actions is that collusion is more difficult when demand is higher. In contrast to this theoretical prediction, most empirical researchers have concluded that price wars are more common when demand is low; but a few find cartels also break down in booms.

We provide a simple theory of collusion by risk averse agents that face positive fixed operating costs that can reconcile these literatures by providing conditions under which (1) it is most difficult to collusively restrict output when demand is especially low, but (2) it also becomes difficult to support collusion when demand is high. The literature has long advocated that a firm's objective is better described by risk aversion. Moreover, many effective cartels are comprised of countries that may care about the utility derived by their risk-averse citizens from those profits. As a result, the marginal value of a dollar of profit is greater when demand, and hence profits, are lower; and this high marginal valuation is magnified by the large fixed operating costs that Scherer [1980] cites as playing a vital role in making collusion difficult.

We show that for aggregate cartel output relative to monopoly levels to be a U-shaped function of the level of demand, both ingredients are necessary—cartel members must be risk averse, and operating costs must be positive, inducing DARA preferences. We further establish that when cartel members are more risk averse or fixed operating costs are higher, then it becomes more difficult to support collusion in bad demand states, but easier in good ones.

APPENDIX

Proof of Proposition 1.

When $\alpha = 1$, it is immediate from Rotemberg and Saloner [1986] that $q(\theta)/q^m(\theta)$ is non-decreasing in θ .

If $f = 0$, and the IC constraint does not bind, then $q^C(\theta)/q^m(\theta) = 1$. Now suppose that the IC constraint binds, and let $\theta_1 < \theta_2$ be two arbitrary values of θ outside the monopoly support region. Since $q^m(\theta) = \theta/4$, to show that $q(\theta)/q^m(\theta)$ increases in θ we must show that $q(\theta_2)/\theta_2 > q(\theta_1)/\theta_1$, where $q(\theta_i)/\theta_i \equiv z_i \in (1/4, 1/3)$. To prove that $z_2 > z_1$, suppose instead that $z_1 \geq z_2$. Rewrite the IC constraint in terms of z_i when $f = 0$ as:

$$\left(\theta_i^2 \frac{(1-z_i)^2}{4}\right)^\alpha - (\theta_i^2 (1-2z_i)z_i)^\alpha = v.$$

Since v is independent of θ ,

$$\left(\theta_1^2 \frac{(1-z_1)^2}{4}\right)^\alpha - (\theta_1^2 (1-2z_1)z_1)^\alpha = \left(\theta_2^2 \frac{(1-z_2)^2}{4}\right)^\alpha - (\theta_2^2 (1-2z_2)z_2)^\alpha.$$

Since $\theta_1 < \theta_2$, it follows that

$$\left(\frac{(1-z_1)^2}{4}\right)^\alpha - ((1-2z_1)z_1)^\alpha > \left(\frac{(1-z_2)^2}{4}\right)^\alpha - ((1-2z_2)z_2)^\alpha.$$

Calling the four terms in this inequality from left to right as A, B, C and D , rewrite the inequality as: $A - B > C - D$. Under the assumption $z_1 > z_2$, and recalling that cooperation profits decrease in z , i.e., $(1-2z)z$ decreases in $z > 1/4$, we have $B/D < 1$. Therefore, $A - B > C - D$ implies that $\frac{A-B}{B} > \frac{C-D}{B} > \frac{C-D}{B} \frac{B}{D} = \frac{C-D}{D}$. Therefore, $\frac{A}{B} > \frac{C}{D}$, i.e.,

$$\left(\frac{\frac{(1-z_1)^2}{4}}{(1-2z_1)z_1}\right)^\alpha > \left(\frac{\frac{(1-z_2)^2}{4}}{(1-2z_2)z_2}\right)^\alpha,$$

or equivalently, $\frac{(1-z_1)^2}{(1-2z_1)z_1} > \frac{(1-z_2)^2}{(1-2z_2)z_2}$ for $z_2 \geq z_1$. But $g(z) = \frac{(1-z)^2}{(1-2z)z}$ is a decreasing function of z , i.e., $g'(z) < 0$, a contradiction. \square

Proof of Proposition 2.

The first-order condition is

$$h'(\theta) = 2\alpha\theta \left(\frac{9}{64} \left(\frac{9\theta^2}{64} - f \right)^{\alpha-1} - \frac{1}{8} \left(\frac{\theta^2}{8} - f \right)^{\alpha-1} \right) = 0.$$

Solving yields

$$\frac{\frac{\theta^2}{8} - f}{\frac{9\theta^2}{64} - f} = \left(\frac{8}{9} \right)^{\frac{1}{1-\alpha}} \equiv k \Rightarrow \hat{\theta} = 8\sqrt{\frac{(1-k)f}{8-9k}}.$$

Notice that $k = \left(\frac{8}{9} \right)^{\frac{1}{1-\alpha}} < \frac{8}{9} < 1$. Clearly, $\theta < \hat{\theta}$ implies that

$$\frac{\frac{\theta^2}{8} - f}{\frac{9\theta^2}{64} - f} < \left(\frac{8}{9} \right)^{\frac{1}{1-\alpha}} \Rightarrow 9 \left(\frac{9\theta^2}{64} - f \right)^{\alpha-1} < 8 \left(\frac{\theta^2}{8} - f \right)^{\alpha-1}.$$

Therefore, $h'(\theta) < 0$. A similar argument holds for $\theta > \hat{\theta}$.

The requirement that $a < \hat{\theta} < b$ imposes bounds on the range of α . We require

$$a < \hat{\theta} = 8\sqrt{\frac{(1-k(\alpha))f}{8-9k(\alpha)}} < b.$$

Solving yields the upper and lower bounds:

$$\bar{\alpha}(f) = 1 + \frac{\log(9/8)}{\log\left(\frac{8a^2-64f}{9a^2-64f}\right)} \quad \text{and} \quad \underline{\alpha}(f) = 1 + \frac{\log(9/8)}{\log\left(\frac{8b^2-64f}{9b^2-64f}\right)}.$$

Since $\alpha(x; f) = 1 + \log(9/8)/\log\left(\frac{8x^2-64f}{9x^2-64f}\right)$ is a decreasing function of x , with a limit of zero as x goes to infinity, $\hat{\theta} \in (a, b)$ exists as long as $\underline{\alpha}(f) < \alpha < \bar{\alpha}(f)$.

Finally, differentiating $\bar{\alpha}(f)$ (or similarly $\underline{\alpha}(f)$) and $\hat{\theta}(\alpha, f)$ with respect to f and α delivers the comparative statics results:

$$\frac{\partial \bar{\alpha}(f)}{\partial f} = \frac{8a^2 \log(9/8)}{(9a^2 - 64f)(a^2 - 8f) \left(\log\left(1 - \frac{a^2}{9a^2-64f}\right) \right)^2} \geq 0,$$

which is non-negative since $f < \frac{a^2}{9}$; and

$$\frac{\partial \hat{\theta}(\alpha, f)}{\partial \alpha} = - \frac{2^{\frac{2\alpha-1}{\alpha-1}} 9^{\frac{1}{1-\alpha}} \log\left[\frac{9}{8}\right] f}{(\alpha-1)^2 \left(2^{\frac{3\alpha}{\alpha-1}} 9^{\frac{1}{1-\alpha}} - 9 \right)^2 \sqrt{\frac{\left(\left(\frac{9}{8}\right)^{\frac{1}{1-\alpha}} - 1\right) f}{2^{\frac{3\alpha}{\alpha-1}} 9^{\frac{1}{1-\alpha}} - 9}}} < 0; \quad \frac{\partial \hat{\theta}(\alpha, f)}{\partial f} = \frac{4}{f} \sqrt{\frac{\left(\left(\frac{8}{9}\right)^{\frac{1}{1-\alpha}} - 1\right) f}{8^{\frac{1}{1-\alpha}} 9^{\frac{\alpha}{\alpha-1}} - 8}} > 0. \quad \square$$

Proof of Corollary 1.

Let $\underline{\theta} < \bar{\theta}$ be the two roots of $h(\theta; \alpha, f) = v$ when it has two roots for $a \leq \theta \leq b$. Note that v is independent of θ . Since $h(\theta; \alpha, f)$ is a U-shaped function of θ (Proposition 2), for intermediate values of θ where $h(\theta; \alpha, f) < v$ the IC constraint (3) is slack. Therefore, monopoly profits can be supported for $\theta \in [\underline{\theta}, \bar{\theta}]$.

For $h(\theta; \alpha, f) = v$ to have two roots, v can be neither too small nor too large. Since $v \equiv \left(\frac{\beta}{1-\beta}\right)(U^C - U^P)$, there is one-to-one mapping between v and β ¹¹. Thus, we must bound β appropriately: β must exceed the $\underline{\beta}$ that solves

$$h(\hat{\theta}; \alpha, f) = \left(\frac{\underline{\beta}}{1-\underline{\beta}}\right)(U^C - U^P),$$

and be less than the $\bar{\beta}$ that solves

$$\text{Min}\{h(a; \alpha, f), h(b; \alpha, f)\} = \left(\frac{\bar{\beta}}{1-\bar{\beta}}\right)(U^C - U^P).$$

Hence, $h(\theta; \alpha, f) = v$ has two roots for $\theta \in [a, b]$ if and only if $\beta \in [\underline{\beta}, \bar{\beta}]$. \square

Proof of Proposition 3.

We must show that $h(\hat{\theta} - \delta) > h(\hat{\theta} + \delta)$, i.e.,

$$\left(\frac{9}{64}(\hat{\theta} - \delta)^2 - f\right)^\alpha - \left(\frac{1}{8}(\hat{\theta} - \delta)^2 - f\right)^\alpha > \left(\frac{9}{64}(\hat{\theta} + \delta)^2 - f\right)^\alpha - \left(\frac{1}{8}(\hat{\theta} + \delta)^2 - f\right)^\alpha.$$

Let $m_1 = \frac{1}{8}(\hat{\theta}^2 + \delta^2) - f$, and $m_2 = \frac{9}{64}(\hat{\theta}^2 + \delta^2) - f$. Also let $n_1 = \frac{1}{8}(2\hat{\theta}\delta)$, and $n_2 = \frac{9}{64}(2\hat{\theta}\delta)$.

Now rewrite the inequality to be established as

$$(m_1 + n_1)^\alpha - (m_1 - n_1)^\alpha > (m_2 + n_2)^\alpha - (m_2 - n_2)^\alpha.$$

Given that $m_2 > m_1$ and $n_2 > n_1$, to prove the above inequality it suffices to show that the cross-derivative of $L = (m + n)^\alpha - (m - n)^\alpha$ with respect to m and n is negative, and we have:

$$\frac{\partial^2 L}{\partial m \partial n} = -\alpha(1 - \alpha) \left((m + n)^{\alpha-2} + (m - n)^{\alpha-2} \right) < 0. \quad \square$$

Proof of Proposition 4.

$$\frac{\partial H(z, \theta)}{\partial z} = \frac{1}{2} \alpha \theta^2 \left(2(4z - 1)(\theta^2(1 - 2z)z - f)^{\alpha-1} - (1 - z) \left(\frac{\theta^2}{4}(1 - z)^2 - f \right)^{\alpha-1} \right).$$

To show $\frac{\partial H(z, \theta)}{\partial z}$ is negative, equivalently we must prove:

$$\frac{1 - z}{2(4z - 1)} > \left(\frac{\theta^2 \frac{1}{4}(1 - z)^2 - f}{\theta^2 z(1 - 2z) - f} \right)^{1-\alpha}.$$

Since the cheat payoff, $(1 - z)^2/4$ always exceeds the cooperation payoff, $z(1 - 2z)$, the right-hand side exceeds one. Therefore, it suffices to show that

$$\frac{1 - z}{2(4z - 1)} > \frac{\theta^2 \frac{1}{4}(1 - z)^2 - f}{\theta^2 z(1 - 2z) - f}.$$

Define $f' \equiv f/\theta^2$ and rearrange the above inequality as

$$\frac{1 - z}{2(4z - 1)} - \frac{\frac{1}{4}(1 - z)^2 - f'}{z(1 - 2z) - f'} = \frac{\frac{1}{2}(3z - 1)(z - 1 + 6f')}{2(4z - 1)(z(1 - 2z) - f')} > \frac{\frac{1}{6}(3z - 1)^2}{2(4z - 1)(z(1 - 2z) - f')} > 0,$$

for $1/4 \leq z < 1/3$. The next to the last inequality follows since the above expression decreases in f' and thus is minimized when f' equals its upper bound of $Max(c/\theta^2) = (a^2/9)/a^2 = 1/9$, implying that $3(z - 1 + 6f') > (3z - 1)$.

To prove convexity of H , we bound the second derivative of H/α strictly away from zero (we divide by α because the derivative of H goes to zero as α goes to zero). We also write H/α in terms of $f' = f/\theta^2 \in [0, \frac{1}{9}]$ to make the domain compact:

$$\frac{1}{\alpha} H(z; \alpha, f') = \frac{1}{\alpha} \left[\left(\frac{(1 - z)^2}{4} - f' \right)^\alpha - ((1 - 2z)z - f')^\alpha \right],$$

with associated second derivative

$$(8) \quad \frac{1}{\alpha} \frac{d^2 H}{dz^2} = \frac{1}{4} (1 - \alpha) \left[4(4z - 1)^2 ((1 - 2z)z - f')^{\alpha-2} - (1 - z)^2 \left(\frac{1}{4}(1 - z)^2 - f' \right)^{\alpha-2} \right] \\ + \frac{1}{2} \left[\left(\frac{1}{4}(1 - z)^2 - f' \right)^{\alpha-1} + 16((1 - 2z)z - f')^{\alpha-1} \right].$$

The compact domain has $z \in [1/4, 1/3]$, $\alpha \in [0, 1]$ and $f' \in [0, 1/9]$. Further, $\frac{1}{\alpha} \frac{d^2 H}{dz^2}$ is continuous and twice differentiable on its domain, with derivatives bounded from below, so that in an ϵ ball around any point (z, α, f') , $\frac{1}{\alpha} \frac{d^2 H}{dz^2}$ cannot drop too far below its value at (z, α, f') . Therefore, to establish convexity, it suffices to bound $\frac{1}{\alpha} \frac{d^2 H}{dz^2}$ strictly away from zero on an appropriately fine grid. An exhaustive search on a grid with increments of 0.001 for z , α and f' reveals that it achieves a lower bound of 9/2 when $\alpha = 1$. See Figure 5.

[Place Figure 5 about here.]

We now establish that over-production relative to monopoly increases in $\theta - \bar{\theta}$ for $\theta > \bar{\theta}$; and in $\underline{\theta} - \theta$, for $\theta < \underline{\theta}$. First consider any $\theta_2 > \theta_1 \in (\bar{\theta}, b]$. To establish that $q(\theta)/q^m(\theta)$ increases in $(\theta - \bar{\theta})$, we show that $z_2 > z_1$. Suppose instead that $z_1 > z_2$. We have:

$$H(z_i, \theta_i) = \left(\theta_i^2 \frac{(1 - z_i)^2}{4} - f \right)^\alpha - \left(\theta_i^2 (1 - 2z_i)z_i - f \right)^\alpha = v \quad \text{for } i = 1, 2.$$

Consider the two functions $H(\cdot, \theta_1)$ and $H(\cdot, \theta_2)$. From Proposition 2 for $\theta > \bar{\theta}$, h increases in θ , so

$$h(\theta_2; \alpha, f) > h(\theta_1; \alpha, f) \Rightarrow H(1/4, \theta_2) > H(1/4, \theta_1).$$

Also from incentive compatibility,

$$H(z_2, \theta_2) = H(z_1, \theta_1) = v,$$

at the premised $z_1 > z_2$, and since $H(z, \theta)$ is decreasing in z for any θ , this implies that $H(z_2, \theta_1) > H(z_2, \theta_2)$. But if $H(1/4, \theta_2) > H(1/4, \theta_1)$ and $H(z_2, \theta_2) < H(z_2, \theta_1)$ then by the intermediate value theorem there exists a z' with $1/4 < z' < z_2$ such that $H(z', \theta_2) = H(z', \theta_1)$, a contradiction of $\theta_2 > \theta_1$ and $z' < 1/3$.

An identical proof by contradiction establishes that if $\theta_1 < \theta_2 < \underline{\theta}$, then $z_1 > z_2$. That is, $z_1 < z_2$ would imply $H(z_2, \theta_2) = H(z_1, \theta_1)$ at $z_1 < z_2$ (by incentive compatibility), and hence $H(z_1, \theta_2) > H(z_1, \theta_1)$, but here, $H(1/4, \theta_1) > H(1/4, \theta_2)$ yields a contradiction via the

intermediate value theorem. \square

Proof of Proposition 5.

Let $H_1(z, \theta) \equiv H(z, \theta; \alpha_1, f)$ and $H_2(z, \theta) \equiv H(z, \theta; \alpha_2, f)$ for $\alpha_1 > \alpha_2$. We prove that if there exists a θ such that $z_1(\theta) = z_2(\theta)$, then it is unique. Call these values θ^* and z^* . To establish this single-crossing result, we prove that for a fixed z , $H_1 - H_2$ increases in θ by showing that $\frac{\partial H}{\partial \theta}$ increases in α . Therefore, there exists a neighborhood of θ^* and z^* , such that for a fixed z , H_1 is a steeper function of θ than H_2 .

When both IC constraints bind (i.e., $H_i = v_i$ for $i = 1, 2$) then $H_1 - H_2 = v_1 - v_2$ does not vary with θ , i.e., $\frac{\partial H}{\partial \theta} = \frac{\partial v}{\partial \theta} = 0$.

(9)

$$H(z, \theta) = \left(\theta^2 \frac{(1-z)^2}{4} - f \right)^\alpha - (\theta^2(1-2z)z - f)^\alpha = v \equiv v(\alpha, f),$$

$$\frac{\partial H}{\partial \theta} = \frac{1}{2} \theta \alpha \left((1-z)^2 \left(\theta^2 \frac{(1-z)^2}{4} - f \right)^{\alpha-1} - 4z(1-2z) (\theta^2(1-2z)z - f)^{\alpha-1} \right) = 0.$$

Defining $\gamma_F \equiv (1-z)^2 \left(\theta^2 \frac{(1-z)^2}{4} - f \right)^{\alpha-1}$ and $\gamma_C \equiv 4z(1-2z) (\theta^2(1-2z)z - f)^{\alpha-1}$, we must have $\gamma_F = \gamma_C \equiv \gamma$. We now prove that $\frac{\partial H}{\partial \theta}$ increases in α , i.e., $\frac{\partial^2 H}{\partial \theta \partial \alpha} > 0$:

$$\begin{aligned} \frac{\partial^2 H}{\partial \theta \partial \alpha} &= \frac{1}{2} \theta \left((1-z)^2 \left(\theta^2 \frac{(1-z)^2}{4} - f \right)^{\alpha-1} - 4z(1-2z) (\theta^2(1-2z)z - f)^{\alpha-1} \right. \\ &\quad \left. + \alpha \left((1-z)^2 \left(\theta^2 \frac{(1-z)^2}{4} - f \right)^{\alpha-1} \log \left(\theta^2 \frac{(1-z)^2}{4} - f \right) - 4z(1-2z) (\theta^2(1-2z)z - f)^{\alpha-1} \log \left(\theta^2(1-2z)z - f \right) \right) \right). \end{aligned}$$

Substituting γ_F and γ_C , and using $\gamma_F = \gamma_C \equiv \gamma$, rewrite this as:

$$\begin{aligned} \frac{\partial^2 H}{\partial \theta \partial \alpha} &= \frac{1}{2} \theta \left(\gamma_F - \gamma_C + \alpha \left(\gamma_F \log \left(\theta^2 \frac{(1-z)^2}{4} - f \right) - \gamma_C \log \left(\theta^2(1-2z)z - f \right) \right) \right) \\ &= \frac{1}{2} \theta \alpha \gamma \left(\log \left(\theta^2 \frac{(1-z)^2}{4} - f \right) - \log \left(\theta^2(1-2z)z - f \right) \right) > 0. \end{aligned}$$

The inequality holds since $\frac{(1-z)^2}{4} > (1-2z)z$ for $z \in [1/4, 1/3]$. When monopoly output cannot be supported in both environments, then $\frac{\partial^2 H}{\partial \theta \partial \alpha} > 0$, implies that for $\theta > \theta^*$, we need $z_1(\theta) > z_2(\theta)$ to retrieve $H_1 = v_1$ and $H_2 = v_2$; and $\theta < \theta^*$ demands $z_1(\theta) < z_2(\theta)$.

\square

Proof of Proposition 6.

Let $H_1(z, \theta) \equiv H(z, \theta; \alpha_1, f)$ and $H_2(z, \theta) \equiv H(z, \theta; \alpha_2, f)$ for $f_1 < f_2$. We prove that if there exists a θ such that $z_1(\theta) = z_2(\theta)$, then it is unique. Call these values θ^* and z^* . To establish this single-crossing result, we prove that for a fixed z , $H_1 - H_2$ increases in θ by showing that $\frac{\partial H}{\partial \theta}$ decreases in f . Therefore, there exists a neighborhood of θ^* and z^* , such that for a fixed z , H_1 is a steeper function of θ than H_2 .

When both IC constraints bind then $\frac{\partial H}{\partial \theta} = \frac{\partial v}{\partial \theta} = 0$. We have

$$\frac{\partial H}{\partial \theta} = \frac{1}{2}\theta\alpha \left((1-z)^2 \left(\theta^2 \frac{(1-z)^2}{4} - f \right)^{\alpha-1} - 4z(1-2z) (\theta^2(1-2z)z - f)^{\alpha-1} \right) = 0.$$

We now prove that $\frac{\partial H}{\partial \theta}$ decreases in f , i.e., $\frac{\partial^2 H}{\partial \theta \partial f} < 0$:

$$\frac{\partial^2 H}{\partial \theta \partial f} = \frac{1}{2}\alpha(1-\alpha)\theta \left((1-z)^2 \left(\theta^2 \frac{(1-z)^2}{4} - f \right)^{\alpha-2} - 4z(1-2z) (\theta^2(1-2z)z - f)^{\alpha-2} \right).$$

Substituting γ_F and γ_C , and using $\gamma_F = \gamma_C \equiv \gamma$, rewrite this as:

$$\begin{aligned} \frac{\partial^2 H}{\partial \theta \partial f} &= \frac{1}{2}\alpha(1-\alpha)\theta \left(\frac{\gamma_F}{\theta^2 \frac{(1-z)^2}{4} - f} - \frac{\gamma_C}{\theta^2(1-2z)z - f} \right) \\ &= \frac{1}{2}\alpha(1-\alpha)\theta\gamma \left(\frac{1}{\theta^2 \frac{(1-z)^2}{4} - f} - \frac{1}{\theta^2(1-2z)z - f} \right) < 0. \end{aligned}$$

The inequality holds since $\frac{(1-z)^2}{4} > (1-2z)z$ for $z \in [1/4, 1/3)$, and hence its reciprocal is smaller. When monopoly output cannot be supported in both environments, then $\frac{\partial^2 H}{\partial \theta \partial f} > 0$, implies that for $\theta > \theta^*$ we need $z_1(\theta) > z_2(\theta)$ to retrieve $H_1 = v_1$ and $H_2 = v_2$, and $\theta < \theta^*$ demands $z_1(\theta) < z_2(\theta)$. \square

EXTENSIONS

$N > 2$ Agents. With $N > 2$ agents, in a symmetric cartel ($q_t^i = q_t$) date t demand simplifies to $P_t = \theta_t - Nq_t$. Cooperation, cheating and Nash profits become:

$$\begin{aligned}\pi_C(\theta) &= (\theta - Nq_C(\theta))q_C(\theta) - c \\ \pi_F(\theta) &= (\theta - (N-1)q_C(\theta))^2/4 - c \\ \pi^P(\theta) &= \theta^2/(N+1)^2 - c.\end{aligned}$$

When each firm produces one N^{th} of monopoly output, i.e., $q_C(\theta) = \theta/(2N)$, each member earns one N^{th} of monopoly profits net of operating costs, i.e., $\pi_C(\theta) = \theta^2/(4N) - c$, and the associated firm output is $(\frac{N+1}{4N})\theta$, resulting in firm profit $(\frac{N+1}{4N})^2\theta^2 - c$. Now we extend Proposition 2 to $N > 2$.

The incentive to cheat on monopoly output becomes:

$$(10) \quad h(\theta; \alpha, c) = U(\pi_F(\theta)) - U(\pi_C(\theta)) = \left(\left(\frac{N+1}{4N} \right)^2 \theta^2 - c \right)^\alpha - \left(\frac{\theta^2}{4N} - c \right)^\alpha,$$

which we establish is a U-shaped function of θ . To see this, differentiate h with respect to θ and solve the resulting first-order condition for the interior minimizer:

$$\frac{\frac{\theta^2}{4N} - c}{\left(\frac{N+1}{4N} \right)^2 \theta^2 - c} = \left(\frac{4N}{(1+N)^2} \right)^{\frac{1}{1-\alpha}} \equiv k_N \Rightarrow \hat{\theta}_N = 4N \sqrt{\frac{(1-k_N)c}{4N - k_N(1+N)^2}}.$$

Note that $k_N = \left(\frac{4N}{(1+N)^2} \right)^{\frac{1}{1-\alpha}} < \frac{4N}{(1+N)^2} < 1$. As when $N = 2$, the fact that $\frac{\frac{\theta^2}{4N} - c}{\left(\frac{N+1}{4N} \right)^2 \theta^2 - c}$ increases in θ , implies that for $\theta < \hat{\theta}_N$

$$\frac{\frac{\theta^2}{4N} - c}{\left(\frac{N+1}{4N} \right)^2 \theta^2 - c} < \left(\frac{4N}{(1+N)^2} \right)^{\frac{1}{1-\alpha}} \Rightarrow \left(\frac{N+1}{4N} \right)^2 \left(\left(\frac{N+1}{4N} \right)^2 \theta^2 - c \right)^{\alpha-1} < 4N \left(\frac{\theta^2}{4N} - c \right)^{\alpha-1},$$

or equivalently $h'(\theta) < 0$. A similar argument holds for $\theta > \hat{\theta}_N$.

General Demand and Utility Functions. We now generalize the setting under which the analogue of Proposition 2 holds, providing more general conditions under which the incentive to cheat on the cartel is a U-shaped function of θ . Because firm and monopoly profits,

$\pi_F(\theta) > \pi_C(\theta)$, are increasing in θ , we drop the explicit dependence on θ and write $\pi_F(\pi_C)$.

We assume that:

1. Net (gross minus fixed cost) firm profit is (a) a positive, increasing convex function of net monopoly profit with $\pi_F(\pi_C) > \pi_C$, $\pi'_F(\pi_C) > 1$ and (b) strictly positive when net monopoly profit goes to zero, i.e., $\pi_F(0) > 0$.
2. The utility function U is such that (a) $-U''(x)/U'(x)$ is non-increasing in x (CARA/DARA), where x is net profit, and (b) $U'(0) = \infty$.
3. $\lim_{\pi_C \rightarrow \infty} \frac{dU(\pi_F)}{d\pi_F} \frac{d\pi_F(\pi_C)}{d\pi_C} > \frac{dU(\pi_C)}{d\pi_C}$.

When $\lim_{\pi_C \rightarrow \infty} U'(\pi_F)/U'(\pi_C) = \lim_{\pi_C \rightarrow \infty} U'(\pi_F/\pi_C)$ (e.g., power utility over net profit), assumption 3 can be replaced by $-(U')^2/U'' > 1$.

We now show that the key result of proposition 2 extends with this structure. The analogue of equation (5) becomes:

$$(11) \quad h(\pi_C) = U(\pi_F(\pi_C)) - U(\pi_C).$$

Differentiating yields

$$\begin{aligned} h'(\pi_C) &= U'(\pi_F)\pi'_F(\pi_C) - U'(\pi_C) \\ &= U'(\pi_C) \left(\frac{U'(\pi_F)}{U'(\pi_C)} \pi'_F(\pi_C) - 1 \right) \equiv U'(\pi_C)e(\pi_C), \end{aligned}$$

where we write π_F instead of $\pi_F(\pi_C)$. We now show that $h(\pi_C)$ is a U-shaped function of π_C (and hence θ), whenever $\underline{\pi}_C \equiv \pi_C(\underline{\theta})$ is sufficiently small, and $\bar{\pi}_C \equiv \pi_C(\bar{\theta})$ is sufficiently large.

We use the intermediate value theorem to prove that there is a $\hat{\pi}_C$ such that $e(\hat{\pi}_C) = 0$. We first show that $e(\pi_C)$ is a continuous, strictly increasing function of π_C . The sign of the derivative of $e(\pi_C)$ is given by the sign of

$$\begin{aligned} &(U''(\pi_F)(\pi'_F)^2 + U'(\pi_F)\pi''_F)U'(\pi_C) - U'(\pi_F)\pi'_F U''(\pi_C) \\ &> \pi'_F (U''(\pi_F)\pi'_F U'(\pi_C) - U'(\pi_F)U''(\pi_C)) \\ &> 0, \end{aligned}$$

where the first inequality holds since π_F is convex, and the second inequality holds since $\pi'_F > 1$ and $U''(\pi_F)/U'(\pi_F) > U''(\pi_C)/U'(\pi_C)$, since $-U''(x)/U'(x)$ is non-increasing in x by assumption 2.

As $\pi_c \rightarrow 0$, $\pi_F(0) > 0$ implies $U'(\pi_F)/U'(\pi_C) \rightarrow 0$. Therefore, there exists $\underline{\pi}_C > 0$ such that $e(\pi_C) < 0$ for all $\pi_C < \underline{\pi}_C$, and by assumption 3, $\pi'_F > \frac{U'(\pi_C)}{U'(\pi_F)}$ for π_C sufficiently large. Thus, by the intermediate value theorem there exists a $\hat{\pi}_C$ at which $e(\hat{\pi}_C) = 0$, and hence $h'(\pi_C) = 0$. Because $e(\pi_C)$ is increasing in π_C we have $h'(\pi_C) < 0$ for $\pi_C < \hat{\pi}_C$ and $h'(\pi_C) > 0$ for $\pi_C > \hat{\pi}_C$. Thus, $h(\pi_C)$ is a U-shaped function of π_C , and hence θ .

REFERENCES

- Aiginger, K., Mueller, D. C., Weiss, C., November 1998. 'Objectives, topics and methods in industrial organization during the nineties: Results from a survey'. *International Journal of Industrial Organization* 16 (6), pp. 799–830.
- Appelbaum, E., Katz, E., 1986. 'Measures of Risk Aversion and Comparative Statics of Industry Equilibrium'. *The American Economic Review* 76 (3), pp. 524–529.
- Asplund, M., 2002. 'Risk-averse firms in oligopoly'. *International Journal of Industrial Organization* 20 (7), pp. 995 – 1012.
- Bagwell, K., Staiger, R., 1997. 'Collusion over the business cycle'. *RAND Journal of Economics* 28 (1), pp. 82–106.
- Borenstein, S., Shepard, A., 1996. 'Dynamic pricing in retail gasoline markets'. *RAND Journal of Economics* 27 (3), pp. 429–451.
- Buschena, D. E., Perloff, J. M., 1991. 'The Creation of Dominant Firm Market Power in the Coconut Oil Export Market'. *American Journal of Agricultural Economics* 73 (4), pp. 1000–1008.

- Busse, M., 2002. 'Firm financial condition and airline price wars'. *Rand Journal of Economics* 33, pp. 298–318.
- Chevalier, J., Scharfstein, D., 1996. 'Capital-Market Imperfections and Countercyclical Markups: Theory and Evidence'. *American Economic Review* 86 (4), pp. 703–725.
- Domowitz, I., Hubbard, R. G., Petersen, B. C., 1987. 'Oligopoly Supergames: Some Empirical Evidence on Prices and Margins'. *Journal of Industrial Economics* 35 (4), pp. 379–98.
- Ellison, G., 1994. 'Theories of cartel stability and the joint executive committee'. *RAND Journal of Economics* 25 (1), pp. 37–57.
- Fabra, N., 2006. 'Collusion with capacity constraints over the business cycle'. *International Journal of Industrial Organization* 24 (1), pp. 69 – 81.
- Green, E., Porter, R., 1984. 'Noncooperative collusion under imperfect price information'. *Econometrica* 52 (1), pp. 87–100.
- Greenwald, B. C., Stiglitz, J. E., 1990. 'Asymmetric Information and the New Theory of the Firm: Financial Constraints and Risk Behavior'. *The American Economic Review* 80 (2), pp. 160–165.
- Haltiwanger, J., Harrington, J., 1991. 'The impact of cyclical demand movements on collusive behavior'. *RAND Journal of Economics* 22 (1), pp. 89–106.
- Kandori, M., 1991. 'Correlated demand shocks and price wars during booms'. *Review of Economic Studies* 58, pp. 171–180.
- Karp, L. S., Perloff, J. M., 1989. 'Dynamic Oligopoly in the Rice Export Market'. *The Review of Economics and Statistics* 71 (3), pp. 462–470.
- Karp, L. S., Perloff, J. M., 1993. 'A Dynamic Model of Oligopoly in the Coffee Export Market'. *American Journal of Agricultural Economics* 75 (2), pp. 448–457.

- Knittel, C. R., Lepore, J. J., 2010. 'Tacit collusion in the presence of cyclical demand and endogenous capacity levels'. *International Journal of Industrial Organization* 28 (2), pp. 131–144.
- Mariuzzo, F., Walsh, P. P., 2013. 'Commodity Market Dynamics and the Joint Executive Committee, 1880-1886'. *Review of Economics and Statistics* 95 (5), pp. 1722–1739.
- Nocke, V., 1999. Cartel stability under capacity constraints: The traditional view restored. Working Paper, Nuffield College, Oxford.
- Nocke, V., Thanassoulis, J., 2014. 'Vertical Relations Under Credit Constraints'. *Journal of the European Economic Association* 12 (2), pp. 337–367.
- Porter, R., 1985. 'On the incidence and duration of price wars'. *Journal of Industrial Economics* 33 (Jun 85), pp. 415–26.
- Rosenbaum, D. I., Sukharomana, S., 2001. 'Oligopolistic pricing over the deterministic market demand cycle: some evidence from the US Portland cement industry'. *International Journal of Industrial Organization* 19 (6), pp. 863 – 884.
- Rotemberg, J., Saloner, G., 1986. 'A supergame-theoretic model of price wars during booms'. *American Economic Review* 76 (3), pp. 390–407.
- Sandmo, A., 1971. 'On the Theory of the Competitive Firm Under Price Uncertainty'. *The American Economic Review* 61 (1), pp. 65–73.
- Scherer, F., Ross, D., 1990. *Industrial Market Structure and Economic Performance*. Boston, Houghton Mifflin.
- Scherer, F. M., 1980. *Industrial Market Structure and Economic Performance*, 2nd Edition. Boston, Houghton Mifflin.
- Spagnolo, G., 1999. 'On Interdependent Supergames: Multimarket Contact, Concavity, and Collusion'. *Journal of Economic Theory* 89 (1), pp. 127 – 139.

Spagnolo, G., 2005. 'Managerial incentives and collusive behavior'. *European Economic Review* 49 (6), pp. 1501 – 1523.

Staiger, R. W., Wolak, F. A., 1992. 'Collusive Pricing with Capacity Constraints in the Presence of Demand Uncertainty'. *RAND Journal of Economics* 23 (2), pp. 203–220.

Suslow, V. Y., 2005. 'Cartel contract duration: empirical evidence from inter-war international cartels.'. *Industrial and Corporate Change* 14 (5), pp. 705 – 744.

Notes

¹This prediction need not hold if firms face capacity constraints (see Fabra [2006] which features exogenous constraints or Knittel and Lepore [2010], which features endogenous constraints.)

²Papers that model international commodity markets as output competition include Karp and Perloff [1989], Buschena and Perloff [1991], and Karp and Perloff [1993] which employ quantitative competition structure with a homogenous good to estimate the degree of competitiveness in the markets for rice, coffee, and coconut oil.

³Consistent with this, many large commodity providers in the middle 20th century were family-owned big companies/oligarchists (e.g., Brazilian coffee farmers); and Busse [2002] provides evidence that price wars in airline industry were unilaterally initiated by financially troubled firms.

⁴Spagnolo's conclusion reflects the intertemporal income smoothing by firms that are averse to profit variance. More risk aversion helps collusion because the collusion path is less volatile than the paths for defection and punishment. In contrast, in our set up, risk aversion alters the marginal utility of period gains from defection vs. collusion: higher risk aversion makes collusion harder to support in recessions (when the difference in marginal utilities is highest) but easier in booms (when the net continuation payoff is highest).

⁵In particular, we need the net profits from cheating on the cartel to rise convexly with demand, and to be strictly positive when net per firm monopoly profit goes to zero, together with CARA/DARA preferences that have appropriate Inada assumptions. In sum, we need that the high marginal utility of a dollar to dominate the small profit available from cheating in low demand states, and the convex, growing dollar gains from cheating on the cartel to dominate the decreasing marginal utility of those dollars in high demand states.

⁶The proof establishes convexity of H using an exhaustive grid search algorithm on the compact set of parameters, establishing a lower bound on $\frac{1}{\alpha} \frac{d^2 H}{dz^2}$ of $\frac{9}{2} > 0$.

⁷Not surprisingly, one can construct very asymmetric distributions such that on some parameter range, changes in parameters have uniform effects in all demand states on the incentives to cheat on the cartel.

⁸Data source: 2013 OPEC Statistical Bulletin and 2013 IMF World Economic Outlook

⁹The pattern is robust to including year-fixed effects to control for events in this period that can impact all OPEC members such as the Iraq or Gulf war, the Iranian revolution, etc.

¹⁰We exclude Iraq's share due to the impact of the war and the fact that Iraq did not fully participate in OPEC's production agreements in this period. We omit Indonesia because their oil output was so low that they eventually became importers, and exited OPEC.

¹¹We ignore the endogeneity of q with respect to β and its effect on U^C . As β rises there is an indirect effect on v via changes in U^C . That is, U^C is a function of $q(\theta)$ at each state θ , but its effect is reinforcing: (i) from $v \equiv \left(\frac{\beta}{1-\beta}\right)(U^C - U^P)$, we see that fixing q state-by-state, increasing β raises v , i.e., $dv/d\beta > 0$ for fixed q 's, and (ii) from the constrained optimization in (4), increasing v relaxes the IC constraint, weakly increasing period payoffs, i.e., $dU^C/dv \geq 0$. It follows from (i) and (ii) that $dU^C/d\beta \geq 0$. Since $U^P = E[U(\theta^2/9 - f)]$ is independent of q and therefore of β , the indirect effect of β on v must be reinforcing, i.e., $d(U^C - U^P)/d\beta > 0$. Therefore, there is a one-to-one relationship between β and v .

	Venezuela	Iran	Kuwait	Libya	Qatar	Algeria	Nigeria	UAE	Saudi
Oil Price	-0.085*** (0.00)	-0.07*** (0.00)	-0.066*** (0.00)	-0.043** (0.02)	0.01** (0.01)	0.012* (0.06)	0.025** (0.03)	0.03** (0.02)	0.158*** (0.00)
Constant	15.119*** (0.00)	17.899*** (0.00)	11.299*** (0.00)	8.888*** (0.00)	1.905*** (0.00)	5.079*** (0.00)	6.291*** (0.00)	6.561*** (0.00)	25.02*** (0.00)
R^2	0.29	0.24	0.23	0.13	0.14	0.08	0.11	0.11	0.36
N	44	44	44	44	44	44	44	44	44

Table I: Price sensitivity of OPEC members shares. This table presents the regression result for each of 7 OPEC members. In each regression the dependent variable is the share of OPEC production for each country and the independent variable is the lag of 2009 real oil price. Countries are sorted from left to right based on their price sensitivity (slope coefficient). The numbers in parenthesis are p-values. ***, **, * denote significance at the 1%, 5%, and 10% levels, respectively.

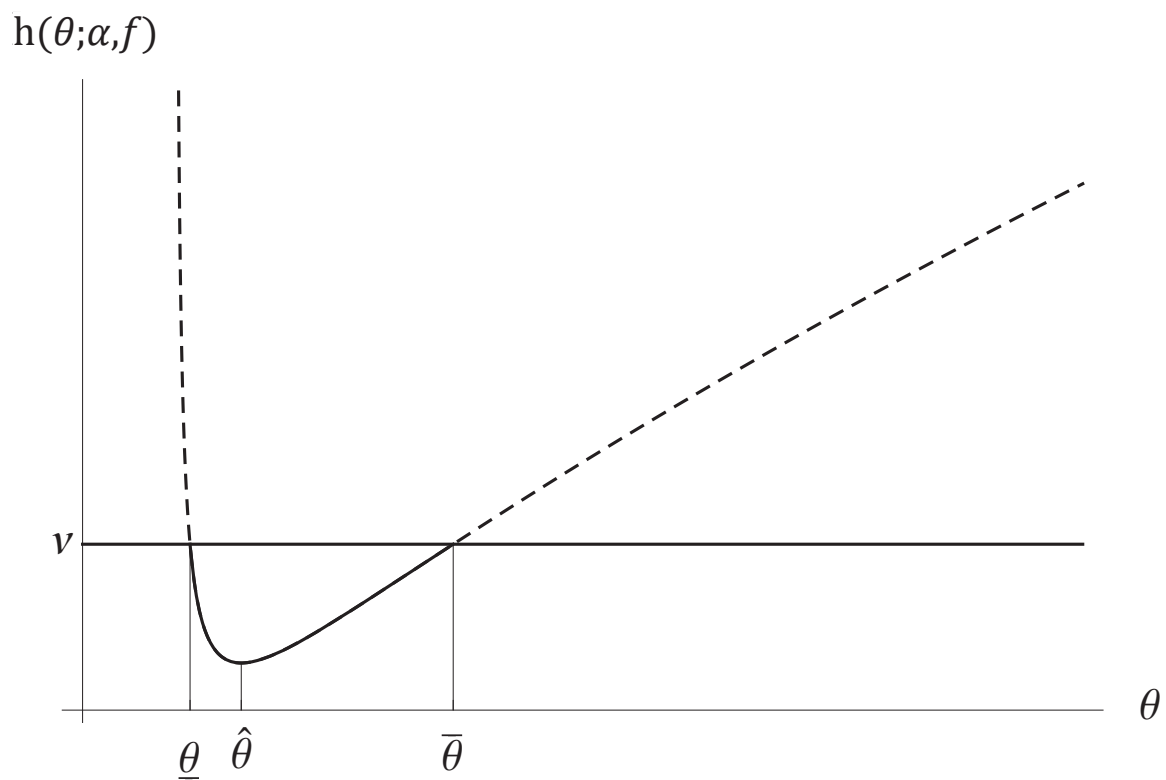


Figure 1: Incentive to cheat $h(\theta; \alpha, f)$ as a function of θ . Parameters are: $\alpha = 1/3$ and $f = 1/9$.

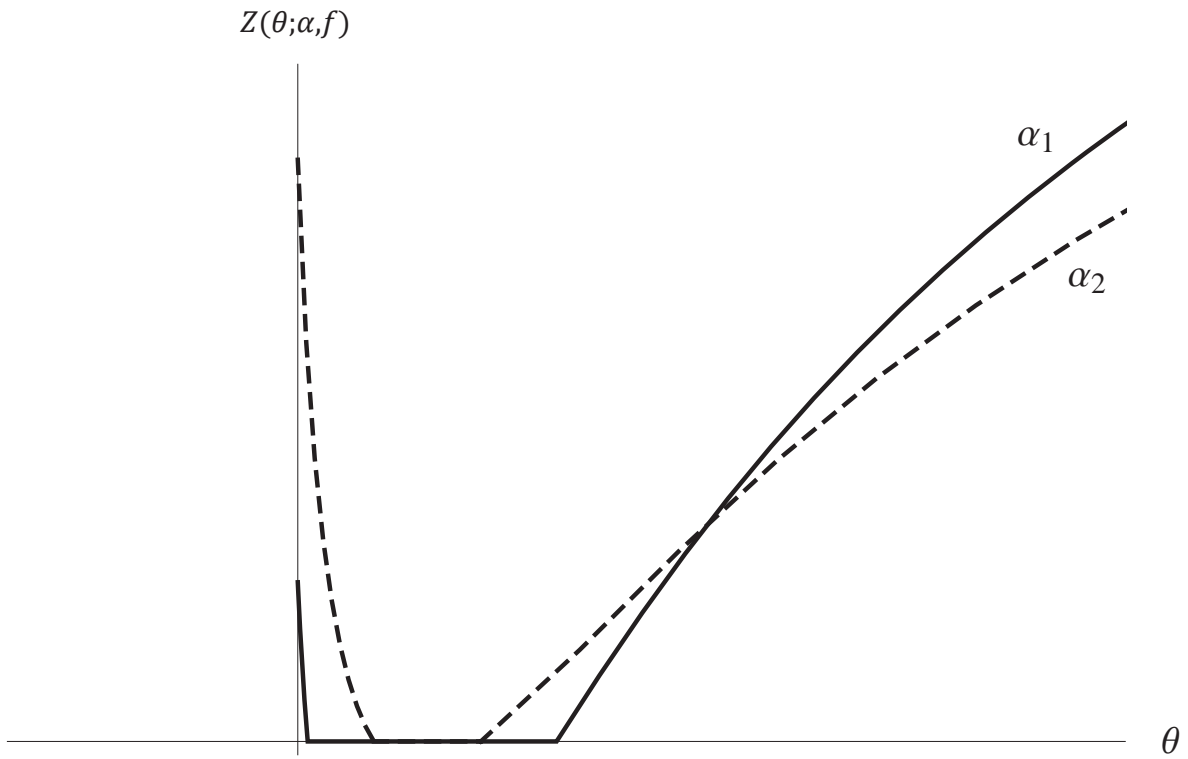


Figure 2: Dashed (solid) line presents the ratio of cartel-to-monopoly output for more (less) risk averse agents ($\alpha_2 = 2/5 < \alpha_1 = 1/2$). Other parameters: $f = \frac{1}{9}$, $\beta = 0.43$, $\theta \in [1, 5]$.

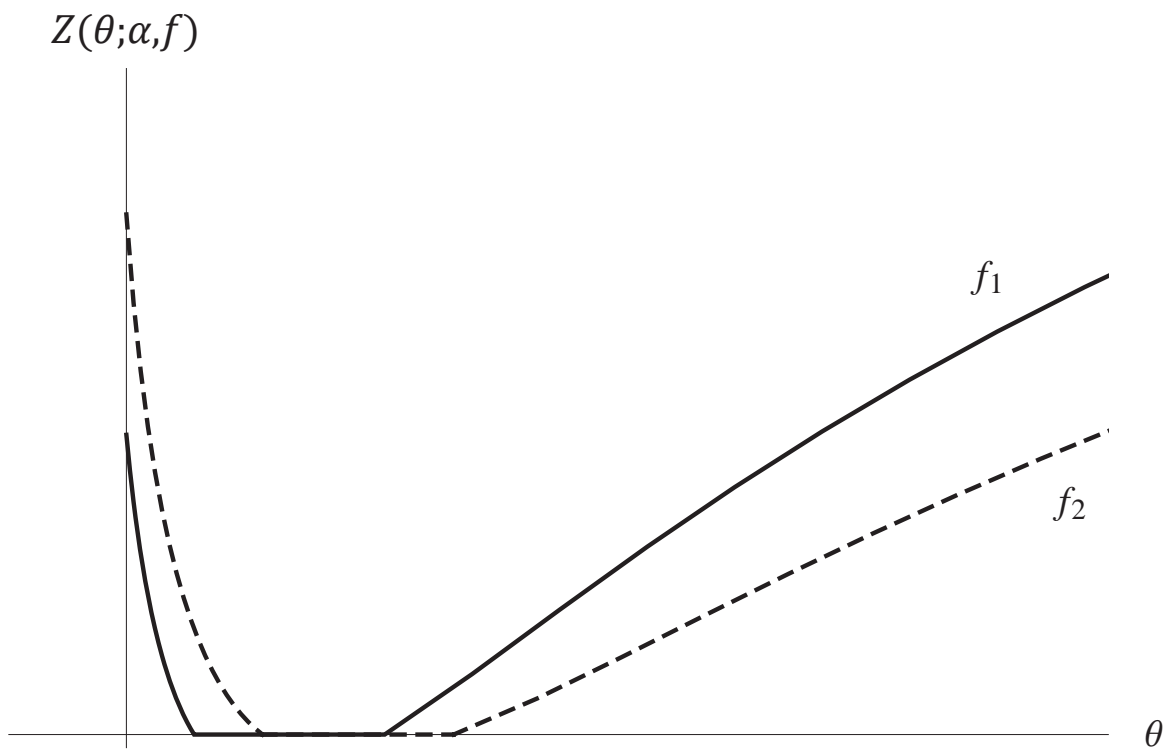


Figure 3: Dashed (solid) line presents the ratio of cartel-to-monopoly output for lower (higher) fixed costs ($f_2 = 1/9 > f_1 = 1/13$). Other parameters: $\alpha = .5$, $\beta = .4$, $\theta \in [1, 5]$.

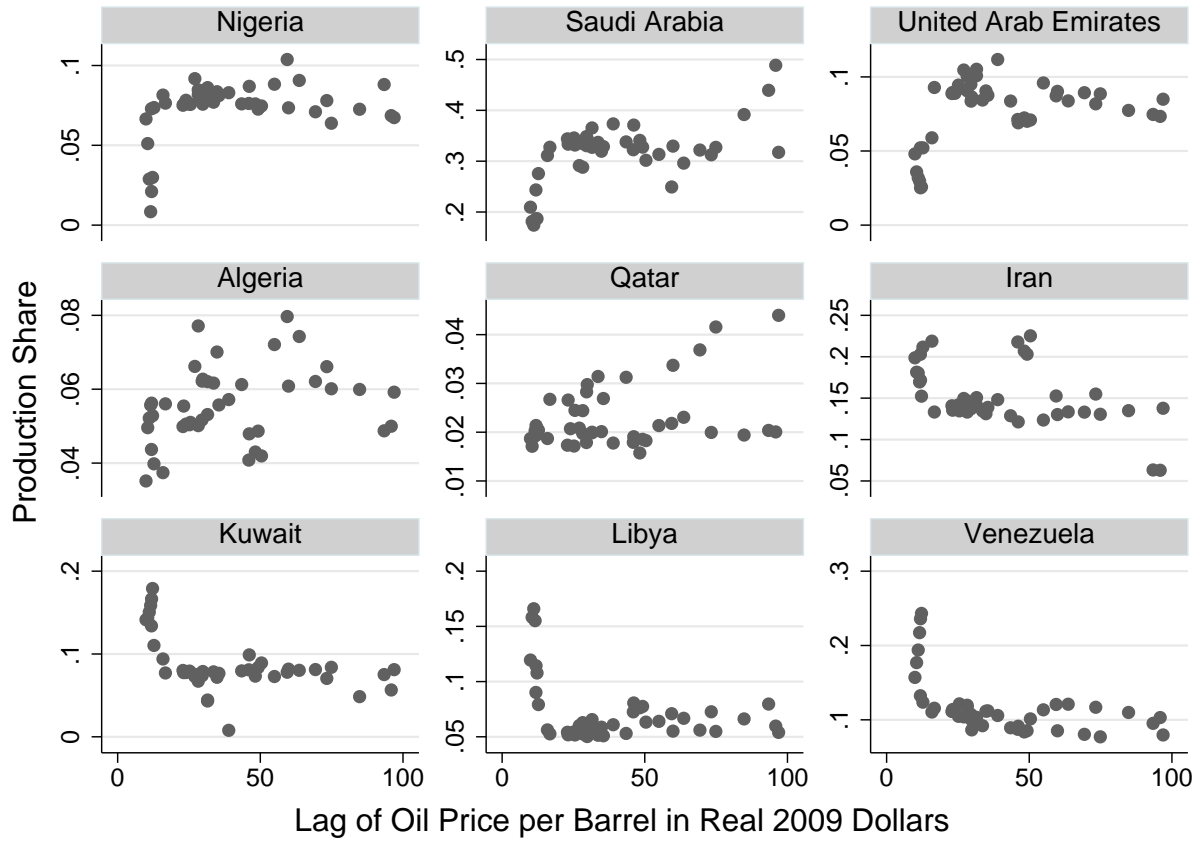


Figure 4: Production shares of selected OPEC countries as a function of price, 1965-2009, excluding Iraq's production.

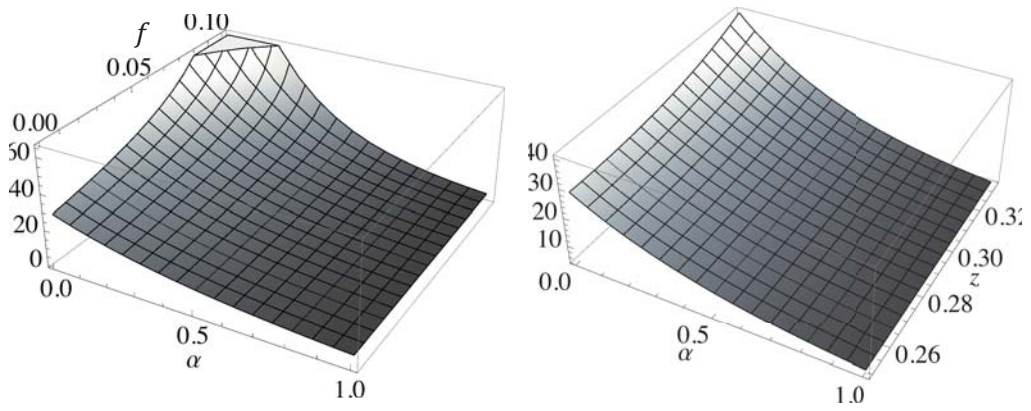


Figure 5: 3-dimensional plot of the second derivative of H/α ; (a) the left panel shows H/α as a function of α and f at $z = 1/4$, and (b) the right panel shows H/α as a function of α and z at $f = 0$.