Contracts with Endogenous Information*

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Abstract

I study covert information acquisition and reporting in a principal agent problem allowing for general technologies of information acquisition. When posteriors satisfy two dimensional versions of the standard First Order Stochastic Dominance and Concavity/Convexity of the Distribution Function conditions, a first-order approach is justified. Under the same conditions, informativeness and riskiness of reports are equivalent. High powered contracts, that make the agent's informational rents more risky, are used to increase incentives for information acquisition, insensitive contracts are used to reduce incentives for information gathering. The value of information to the agent is always positive. The value of information to the principal is ambiguous.

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1 Introduction

A vast literature on contracting and mechanism design has investigated the consequences of asymmetric information on the efficiency and distributive properties of allocations. In most of this literature the model's primitive is an information structure. However, in some economic problems it is reasonable to assume that economic agents do only possess information because they expect to make use of it. Moreover, their effort to gather information is often unobservable to others. Thus, an information acquisition technology rather than the information structure itself should be taken as the model's primitive, and contracts serve the double role of motivating the acquisition of information and ensuring its truthful revelation. How does this second role affect the nature of optimal screening contracts?

Since Demski and Sappington (1987) have raised this question, many investigations have followed. Notably, a prominent literature has investigated how optimal supply arrangements in procurement should be changed to account for costs of acquiring information about cost-of-production conditions (see, e.g., Crémer and Khalil (1992), Crémer, Khalil, and Rochet (1998a,b), Lewis and Sappington (1997), Sobel (1993), and Laffont and Martimort (2002) for a survey of these models). More recently, I myself (Szalay (2005)) have analyzed how decision-making in an advisor-advisee relationship should be structured to guarantee high quality advice.

The findings of this literature are as follows. If the buyer in the procurement context wants to make sure the seller is well informed, then he should offer "high powered" incentive contracts. Compared to a supply arrangement with a seller who is already well informed about his costs, the seller will benefit from an unusually high order if his marginal costs are lower than expected, but he will also receive an exceptionally low order if his costs are higher than ex ante expected. As a result, the quantity supplied is discontinuous and drops sharply when the seller's cost is higher than ex ante expected. If the buyer does not want the seller to become informed, then the supply arrangement should be rigid and should make little use of the seller's information. Both cases can occur, depending on the cost of information acquisition and the timing of events.[1] The structure of decision-making in Szalay's (2005) model of advice displays an exaggeration property that is akin to a high powered incentive contract. If the advisor recommends an action that is higher than the

[1] This result depends on the absence of competition. Compte and Jehiel (2002) reinvestigate the case studied by Crémer et al. (1998b) allowing many agents to compete. While Crémer et al. (1998b) showed that information acquisition is socially wasteful, Compte and Jehiel (2002) show that it may become desirable again when agents compete.

ex ante expected action then the advisee takes an action that is even higher than the recommended one; if the advisor's proposed action is lower than the ex ante expected one, then the advisee takes an even lower action. Similar to the procurement case, the decision schedule is discontinuous and increases sharply at the prior mean.

Information acquisition in all these papers is of an all-or-nothing nature, where the person who acquires information is in equilibrium either completely informed or does not receive additional information at all. I raise a simple question: how do the insights of this literature depend on this simplification?

I find that super powered incentive contracts and exaggeration are general features of contracts with endogenous information, discontinuities are not. To demonstrate these findings, I develop a general but still tractable model of information acquisition. Since the techniques I use can be applied to a wide class of problems with endogenous information, the model is of interest well beyond the context of procurement and the specific question I raise.

I study the procurement problem that Crémer et al. (1998a) have analyzed. A buyer wishes to obtain parts from a seller. Neither the seller nor the buyer knows ex ante how costly it is to produce these parts, say because they both engage in this particular kind of activity for the first time. The buyer begins by offering a menu of contracts to the seller. Before the seller has to accept or reject offers he can acquire information about his costs. In contrast to Crémer et al. (1998a), the seller can exert a continuous choice of effort and receive a continuum of noisy signals. An increase in the seller's effort improves the quality of the signal he receives stochastically. Both the seller's choice of effort and the signal he receives are known only to him but not to the buyer. After the seller has observed a signal he either accepts one of the contracts or walks away without further sanction. The seller learns the true cost of production only when he produces.

Allowing for a continuous quality of noisy information introduces considerable technical difficulties, and one of the contributions of this paper is to demonstrate an elegant way over these hurdles. A rich model of information acquisition leads naturally into a problem of multi-dimensional screening. Ex post, when the seller has acquired a noisy signal, his entire posterior, a multi-dimensional object, may be relevant for contracting. Thus, the buyer faces a problem of multi-dimensional screening, which is potentially quite nasty to solve[2]. However, when the seller's utility is linear in his information variable (e.g., his constant marginal costs), then the seller's preference over

[2] See McAfee and McMillan (1988) for a screening problem where types have more dimensions than the principal has screening instruments available. See also Armstrong and Rochet (1999) and Rochet and Stole (2003) for overviews of multidimensional screening problems.

contracts depends effectively only on the mean calculated from the posterior distribution. Since this is a one dimensional statistic, the problem at the reporting stage is reduced to the well known one-dimensional screening problem. To understand the seller's ex ante problem of how much effort to invest in information acquisition, one has to study the dependence of the ex ante distribution of the conditional expectation on effort. One can resort to standard differentiability methods to describe the optimal amount of effort spent on information acquisition only if the seller's effort influences the ex ante distribution of the conditional mean in a particular way. The seller's optimal choice of information acquisition is adequately described by a first-order condition for any contract that ensures truthful communication of information, if the seller's effort increases the riskiness of the ex ante distribution of the posterior expectation at a decreasing rate, where riskiness is understood in the sense of Rothschild and Stiglitz (1970).[3]

The second contribution of this paper is to provide statistical foundations for increasing risk at a decreasing rate in the distribution of conditional expectation in terms of the primitives of the experiment structures. I obtain an influence of the desired sort when I impose the following assumptions. First, the marginal distributions of signals and true costs are given and the sellers effort influences only the joint distribution of these two variables.[4] Second, for signals below the prior expected signal value, an increase in effort increases the posterior in the sense of First Order Stochastic Dominance (FOSD), and the posterior is concave in signal and effort. For signals above the prior expected signal value, an increase in effort decreases the posterior in the sense of FOSD and the posterior is convex in signal and effort.

It is interesting to contrast these conditions with those used to justify the traditional "first-order approach" in problems of pure moral hazard (Rogerson (1985) and Jewitt (1988)). My conditions are two dimensional versions of the standard FOSD and Convexity/Concavity of the Distribution Function condition (CDFC). Moreover, the qualitative impact of effort on the posterior is reversed as the signal is increased above its expected value. For this reason I term my conditions mean reversing FOSD and CDFC, respectively. The rationale for having mean reversing rather than

[3] Note that this notion of riskiness is somewhat different from Blackwell's, which states that one information structure is Blackwell-better than another if it gives rise to a more risky distribution of the posterior. Riskiness of the posterior expectation is a less restricting condition. Heuristically, while Blackwell requires the distribution of all moments to be more risky, the present concept requires only that the distribution of the first moment is more risky. The difference arises because I impose restrictions on the seller's utility function, while Blackwell's criterion orders information structures for all decision makers whose utility function belongs to a class. For more recent approaches that order information structures, see Karlin and Rubin (1956), Lehmann (1988), and Athey and Levin (2001).

[4] A statistical structure of essentially this type is called a copula (see, e.g. Nelsen (2006))).

standard FOSD and CDFC is that the latter imply changing means (Milgrom (1981)), which is a rather undesirable feature of a model of information acquisition; the law of iterated expectations requires that the means be independent of the amount of information acquisition. My conditions are less restrictive than the ones used to justify the traditional first-order approach. In problems of pure moral hazard one has to ensure the monotonicity of contracts by imposing in addition the Monotone Likelihood Ratio Property (MLRP), which makes the specification overall rather restrictive. In contrast, there is no need to ensure the monotonicity of contracts when there is adverse selection, because monotonicity of contracts is a necessary condition for implementability (Guesnerie and Laffont (1984)). Therefore, it is fair to say that the first-order approach goes through more easily than in a problem of pure moral hazard.

A second statistical model that delivers the same reduced form is a stochastic experiment structure that is similar in nature to the spanning condition studied in Grossman and Hart (1983). In that specification, an experiment is the realization of two independent random variables; a signal which follows a given marginal distribution and an informativeness parameter whose distribution depends on the agent's effort. The posterior satisfies a mean reversing version of MLRP; for signals above the mean, a posterior arising from a relatively more informative experiment places relatively more weight on the high realization of costs, for signals below the mean, it places relatively more weight on the low realizations of costs. Finally, an increase in effort makes it more likely to observe a more informative experiment in the sense of FOSD, and the distribution of informativeness satisfies a CDFC condition.

The main insight arising from this analysis is that informativeness and risk are equivalent in any tractable model. It is in fact this equivalence result that explains the findings of the literature on the value of information and the structure of optimal contracts. The value of information depends on the seller's and the buyer's attitudes towards risk, that is, the shape of their indirect utility functions. It is well known that only convex indirect utility profiles of the seller are implementable (see Rochet (1985)). Thus, incentive compatibility makes the seller a quasi-risk lover so that he always likes to have more information. In contrast, the shape of the buyer's indirect utility function is a more complex issue. It depends both on his direct utility function and the distribution of types. More information can either be a blessing or a curse to the buyer[5], and I provide sufficient conditions for both cases. Similarly, the structure of the optimal supply arrangement is more risky than its exogenous information counterpart when the buyer provides the seller with extra incentives for

[5] This confirms results of Green and Stokey (1981), who do, however, not relate their results to risk.

information acquisition, and is less risky when the buyer reduces the seller's incentives to acquire information. In the former case, when the seller's expected cost is surprisingly low he is rewarded by an extra increase in production that increases his informational rent at the margin, and punished if his expected cost is surprisingly high. These results confirm and generalize those of Crémer et al. (1998a) and eliminate the undesirable discontinuity in their supply arrangement due to the all-or-nothing nature of information acquisition. But the analysis is of use beyond that context and can be applied to any model that relies on a linear environment.

Ordering better information by riskiness in the distribution of conditional expectations is an extremely useful concept. In contemporaneous work, Dai and Lewis (2005) have studied a model of sequential screening with two possible levels of precision of information that obey this ordering. They show that experts with differentially precise information can be screened by the extent of decision authority embodied in contracts. As in the present paper, the value of information to the principal is ambiguous. However, they show that this ambiguity can be overcome by varying the timing structure of the interaction between the principal and the expert. Dai and Lewis (2005) and the present paper complement each other. While their aim is to develop a model that is easily tractable, the current paper provides general statistical foundations for the reduced form they employ and thereby confirms the generality of their findings. Moreover, the justification of the first-order approach in terms of the primitives of the experiment structure is a novelty of my model. More recently, Shi (2006) has studied information acquisition in optimal auctions showing how the optimal reserve price is affected by the fact that information is endogenous. Shi studies information structures that are "rotation ordered", a concept that Johnson and Myatt (2006) have used to study general transformations of demand. The information structures used in this paper satisfy the rotation order. In contrast to Shi (2006), this paper derives more general statistical foundations in terms of experiment structures that induce the desired ordering in the ex ante distribution of conditional expectations.

Closest related to the present paper in terms of its aim to uncover the general principles of information acquisition are Gromb and Martimort (2004) and Malcomson (2004). Gromb and Martimort (2004) establish the Principle of Incentives for Expertise, according to which an agent should be rewarded when his advice is confirmed either by the facts or by the advice of other agents. There are two main differences to the present paper. First, their setup is simpler on the informational side but richer on the organizational side, in that they allow for multiple agents. Second, they allow for contracting contingent on advice and ex post realizations whereas I focus on the case where the agent's information is not verifiable ex post. Malcomson (2004) analyzes the standard principal agent problem, where the agent not only exerts some effort but also makes a decision. The main difference to the present paper is the role of communication. I allow for communication while Malcomson considers the case where the principal commits to a single contract in advance. Moreover, Malcomson's main interest is in characterizing conditions under which the addition of the agent acquiring a signal makes the problem and its solution any different from the standard principal agent problem, and its solution, respectively. In contrast, the present approach allows for a complete characterization of the optimal mechanism.

Bergemann and Välimäki (2002) analyze incentives for information acquisition in ex post efficient mechanisms. They show that incentives for information acquisition in a private value environment are related to supermodularity in the agents' payoff functions.[6] In contrast to the present paper contracts are only proposed after information has been acquired. As a result, information acquisition may be either excessive or insufficient although the seller's payoff function in the present model is submodular in the state and the contracting variable.

The information structures used in the present paper connect the contracting literature to a literature on the value of information in decision problems, a line of research that has been initiated by Blackwell (1951), and Karlin and Rubin (1956), and further pursued by Lehmann (1988), and most recently by Athey and Levin (2001). The combination of these two literatures delivers a powerful approach, that should prove useful to study further applications, because the predictions of the model are robust within a large class of information gathering technologies. One such application, already pursued by Shi (2006), is the study of optimal auctions with endogenous information (see Myerson (1981) for the case of exogenous information). His approach nicely complements the literature on auctions with endogenous information that has restricted attention to a class of mechanisms, e.g., first versus second price auctions (see Tan (1992), Hausch and Li (1993), Stegemann (1996), and more recently Persico (2000) on this).[7]

The paper is organized as follows. In section 2 I spell out the main model. Section 3 contains the main result on the validity of the first-order approach. Section 4 derives the statistical foundation of the second order stochastic dominance relation in the distribution of the conditional expectation. Sections 5 and 6 contain the main implications of the theory. Section 5 derives some results on the value of information, section 6 discusses the form of optimal contracts. Section 7 derives two

[6] They note that efficient mechanisms in the linear environment can be based on conditional expectations.

[7] Persico's result that the auction format with the higher risk sensitivity induces more information acquisition corresponds to the result that the marginal value of information for the agent is positive.

alternative formulations of experiments. In the first variation, I allow for moving supports, and show that the first-order approach is typically not valid in this framework but would deliver - if valid- essentially the same structural predictions except for distortions at the top. The second variation provides a particularly useful simplification of the main model which I term stochastic experiment structure. Section 8 concludes. All proofs are in the appendix.

2 The Model

The model is a variant of the Baron and Myerson (1982) model where I allow for general, endogenous information structures. A buyer (henceforth the principal) contracts with a seller (henceforth the agent) for the production of a good. The good is divisible, so output can be produced in any quantity, q. q is observable and contractible. The agent receives a monetary transfer t from the principal and has costs of producing the quantity q equal to βq. Both parties are risk neutral with respect to transfers. The principal derives gross surplus $V(q)$ from consumption, where $V(q)$ is defined on $[0,\infty)$ and satisfies the conditions[8] $V_q(q) > 0$, $V_{qq}(q) < 0$, $\lim_{q\to 0} V_q(q) = \infty$, $\lim_{q\to\infty} V_q(q) = 0$. Thus the principal's net utility is

$$V(q) - t.$$

The agent's payoff from receiving the transfer t and producing the amount q is given as

$$t - \beta q.$$

Ex ante the principal and the agent do not know the precise value of β, but share a common prior about it, which is supported on $[\underline{\beta}, \overline{\beta}]$ with cdf $P(\beta)$, where $\underline{\beta} > 0$. Once the principal has committed himself to the terms of the contract but before production takes place, the agent may acquire additional information about β. Information acquisition is modeled as a costly choice of effort e, that influences the informativeness of certain experiments.

An *experiment* is a joint distribution of β and and a random variable Σ. This distribution depends on the agent's effort. The marginal distributions of β and Σ are both independent of e, so effort influences only the joint distribution of the two variables (so roughly speaking the correlation between the two variables) but not their marginal distributions[9]. The random variable

[8] Throughout the paper subscripts will denote derivatives of functions with respect to their argument.

[9] The assumption that the marginal of Σ is independent of e will be important for the results in sections 4 through 6, but is not needed for the results in section 3. Since the changes to incorporate the case where the marginal of Σ depends on e are minor, I leave it to the reader to explore this extension.

Σ has typical realization $\sigma \in [\underline{\sigma}, \overline{\sigma}]$, and follows a distribution with an arbitrary density $k(\sigma) > 0 \forall \sigma$ and cdf $K(\sigma)$. Since the distribution of Σ has full support, $K(\sigma)$, contains the same information as σ does itself, but is much more convenient to work with. So, I denote the random variable $S = K(\Sigma)$ as the signal. As is well known, S is distributed on a support $[\underline{s}, \overline{s}] = [0, 1]$ and follows a uniform distribution, *regardless of the function* $K(\cdot)$.[10]

I let $H(\beta | s, e)$ denote the resulting posterior cdf and let $h(\beta | s, e)$ denote the density of the posterior distribution, and assume that this density is differentiable in s and e to the order needed. Experiments can be ordered in the sense that high values of s indicate high costs in the sense of First Order Stochastic Dominance; for $\beta \in (\underline{\beta}, \overline{\beta})$ the posterior distribution satisfies

$$-\infty < H_s(\beta | s, e) < 0 \forall s, e. \tag{1}$$

For $\beta \in \{\underline{\beta}, \overline{\beta}\}$, $H_s(\beta | s, e) = 0$ for $\forall s, e$. (1) implies that $\int_{\underline{\beta}}^{\overline{\beta}} \beta dH(\beta | s, e)$ is increasing in s with a bounded rate of change. Below I will also introduce a precise sense in which higher effort corresponds to more informative experiments. For the time being this is not important and the only restriction I impose on the influence of effort on $H(\beta | s, e)$ is

$$H_e(\beta | \underline{s}, e) = H_e(\beta | \overline{s}, e) = 0 \forall \beta, e \tag{2}$$

(1) and (2) imply that there is a lowest and a highest estimate of costs conditional on the agent's information and these bounds are both independent of the level of effort the agent exerts. Formally, $\int_{\underline{\beta}}^{\overline{\beta}} \beta dH_e(\beta | \underline{s}, e) = \int_{\underline{\beta}}^{\overline{\beta}} \beta dH_e(\beta | \overline{s}, e) = 0$. This property is convenient because the relevant contracting variable will have a fixed support.

The cost of effort is $g(e)$, a strictly convex function, that satisfies $g_e(e) > 0$ for $e > 0$, $g_{ee}(e) > 0$ for all e, $g_e(0) = 0$, and $\lim_{e \to \overline{e}} g_e(e) = \infty$, where $\overline{e}$ is an upper bound on e that can be taken as infinite most of the time, except for some specific examples.

The game has the following time structure:

[10] This approach to model dependence among random variables is closely related to the notion of a *copula*, defined as the distribution function $C(P(\beta), K(\sigma); \cdot)$ on $[0, 1]^2$. The marginal distributions of P and K are uniform on $[0, 1]$, regardless of the functions $P(\cdot)$ and $K(\cdot)$ themselves. The function $C(\cdot)$ embodies the correlation structure between the random variables. In the present context, it is more convenient to specify the joint distribution over β and $K(\sigma)$. Otherwise the structure is the same.

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P offers a menu of contracts	A exerts effort e	s is realized and observed by A	A accepts a contract or refuses to participate	A produces and delivers and learns the true costs β only when producing

First, the principal offers a menu of contracts. Then the agent chooses an effort level, e, that determines the informativeness of the experiment. The experiment is realized and observed by the agent. Given this information he decides whether or not to participate, and, contingent on participating, also which contract to accept. If the agent refuses to participate the game ends. If the agent agreed to participate, production and transfers take place according to the contract the agent has chosen. Notice that the agent learns the true cost only at the time when he produces, not before. In particular, he does not know the true cost when he selects any of the offered contracts or his outside option. I assume that the agent's choice of effort is not observable to the principal and that the value of the signal is the agent's private knowledge.

3 Justifying a First Order Approach

As is customary, I will characterize solutions to the contracting problem taking as given that the principal whishes to implement a given level of effort, and will say very little about the optimal choice of effort to implement[11].

I think of contracting in terms of mechanism design. A mechanism is a tuple $\{q(\cdot), t(\cdot)\}$ which specifies quantities of production and transfers to the agent as a function of a (vector valued) message $\mathbf{m}$, the agent sends to the principal. Invoking the *Revelation Principle* I can restrict attention to direct, incentive compatible mechanisms, $\{q(\cdot), t(\cdot)\}$ that depend only on a reported tuple of signal realization and value of effort $(\hat{s}, \hat{e})$. Hence, one can write the principal's problem

[11] As is well known from Grossman and Hart's (1983) analysis of the problem of pure moral hazard, the principal's problem can be broken into two subproblems: a first problem which consists of finding the least costly way to implement a given effort choice, and a second one, building on the solution of the first, to select the optimal effort level for the agent. While the first problem provides rich insights into the structure of optimal contracts, the second one has very little structure; in particular, the principal's optimization problem with respect to the agent's effort is not generally concave in the choice variable. Due to this lack of regularity structure, the literature generally confines attention to the first part of the problem, and I follow this tradition here.

as follows:

$$\max_{q(\cdot,\cdot),\, t(\cdot,\cdot)} \int_{\underline{s}}^{\overline{s}} \left(V\left(q\left(s,e\right)\right) - t\left(s,e\right)\right) ds \tag{3}$$

s.t.

$$\forall s,e: \int_{\underline{\beta}}^{\overline{\beta}} \left(t\left(s,e\right) - \beta q\left(s,e\right)\right) dH\left(\beta\left|s,e\right.\right) \geq \int_{\underline{\beta}}^{\overline{\beta}} \left(t\left(\hat{s},\hat{e}\right) - \beta q\left(\hat{s},\hat{e}\right)\right) dH\left(\beta\left|s,e\right.\right) \forall \hat{s},\hat{e} \tag{4}$$

$$\int_{\underline{\beta}}^{\overline{\beta}} \left(t\left(s,e\right) - \beta q\left(s,e\right)\right) dH\left(\beta\left|s,e\right.\right) \geq 0\, \forall s,e \tag{5}$$

$$e \in \arg\max_{e} \left\{ \int_{\underline{s}}^{\overline{s}} \left(\int_{\underline{\beta}}^{\overline{\beta}} \left(t\left(s,e\right) - \beta q\left(s,e\right)\right) dH\left(\beta\left|s,e\right.\right) \right) ds - g\left(e\right) \right\} \tag{6}$$

(4) requires that the agent finds it optimal to report the true signal value and the true signal informativeness. (5) ensures that the agent finds it optimal to participate for all possible realizations of signal and informativeness. (6) imposes that the agent's choice of how much effort to acquire is optimal given the contract the principal offers. Observe that the agent's ex ante expected utility net of costs of information acquisition is always nonnegative. Notice that I impose (5) for all values of s and e, not only the equilibrium choice of effort. This involves no loss of generality under the non-moving support assumption. Extensions to the case of moving supports will be studied below.

The screening problem is multi-dimensional, and therefore potentially extremely complicated. However, due to the fact that the agent's utility is linear in β, and linearity is preserved under expectations, the agent's utility depends effectively only on the one-dimensional statistic $\int_{\underline{\beta}}^{\overline{\beta}} \beta dH\left(\beta\left|s,e\right.\right)$ (and the agent's reported type). For this reason, similar to Biais et al. (2000) in a different context, I can observe that non-stochastic mechanisms can only make use of this one dimensional statistic of the type instead of the two-dimensional type itself.[12] Since the agent's conditional expectation is the relevant contracting variable it is important to understand the properties of this variable. Denote the function

$$\pi\left(s,e\right) = \int_{\underline{\beta}}^{\overline{\beta}} \beta dH\left(\beta\left|s,e\right.\right)$$

Suppose that $\pi\left(s,e\right) = \theta$ for some real number θ. Given that $\pi\left(s,e\right)$ is increasing in s, the function is invertible and the signal that generated a value of the conditional expectation equal to θ satisfies $s = \pi^{-1}\left(\theta,e\right)$. Ex ante, i.e., before s is realized, the value of the conditional expectation is a random variable itself, Θ say. Using the fact that the distribution of s is uniform, the cdf of θ for

[12] Bergemann and Välimäki (2002) have noted that this is also the relevant contracting variable in ex post efficient mechanisms in the linear environment, since efficient mechanisms are non-stochastic.

given e is

$$F(\theta, e) = \begin{cases} 0 & for\, \theta < \pi(\underline{s}, e) \\ \pi^{-1}(\theta, e) & for\, \pi(\underline{s}, e) \leq \theta \leq \pi(\overline{s}, e) \\ 1 & \theta > \pi(\overline{s}, e). \end{cases} \tag{7}$$

Due to condition (2), the support of θ is the interval $\left[\underline{\theta}, \overline{\theta}\right]$, independent of effort. Formally, I have $\underline{\theta} = \pi(\underline{s}, e)$ for all e and $\overline{\theta} = \pi(\overline{s}, e)$ for all e. Together with the law of iterated expectations, the non-moving support property places some restrictions on the influence of e on $F(\theta, e)$. Define E_X as the expectation operator when the expectation is taken with respect to X. The law of iterated expectations requires that $E_S[E_\beta[\beta|\, s; e]] = E_\beta[\beta]$. Changing variables and integrating by parts, I can write

$$E_S[E_\beta[\beta|\, s; e]] = \overline{\theta} - \int_{\underline{\theta}}^{\overline{\theta}} F(\theta, e)\, d\theta.$$

This property must hold for any e. Since $E_\beta[\beta]$ is independent of e, it follows that

$$\int_{\underline{\theta}}^{\overline{\theta}} F_e(\theta, e)\, d\theta = 0. \tag{8}$$

(8) is a condition that any model with fixed supports must fulfil. If (8) fails to hold, then an increase in effort changes the ex ante mean of the distribution, which implies that effort is not purely a measure of informativeness but also of something else. It is obvious that the same conditions imply also that

$$\int_{\underline{\theta}}^{\overline{\theta}} F_{ee}(\theta, e)\, d\theta = 0. \tag{9}$$

Finally, notice that condition (1) implies that the distribution of θ has a density $f(\theta, e)$ which is strictly positive on $\left[\underline{\theta}, \overline{\theta}\right]$. To see this, differentiate (7), with respect to θ, using the inverse function theorem, to get

$$f(\theta, e) = \begin{cases} \frac{1}{\pi_s(\pi^{-1}(\theta, e), e)} > 0 & \text{for } \theta \in \left[\underline{\theta}, \overline{\theta}\right] \\ 0 & \text{otherwise.} \end{cases} \tag{10}$$

I now use this change of variables to state (3) s.t. (4), (5) and (6), equivalently as a message game with messages $\hat{\theta} \in [\underline{\theta}, \overline{\theta}]$ about "perceived costs". In this formulation, the principal's problem is

$$\max_{q(\theta),\, t(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \left(V\left(q\left(\theta\right)\right) - t\left(\theta\right)\right) f\left(\theta, e\right) d\theta$$

$$s.t.$$

$$t\left(\theta\right) - \theta q\left(\theta\right) \geq t\left(\hat{\theta}\right) - \theta q\left(\hat{\theta}\right) \;\forall \theta, \hat{\theta}$$

$$t\left(\theta\right) - \theta q\left(\theta\right) \geq 0 \,\forall \theta$$

$$e \in \arg\max_{e} \left\{ \int_{\underline{\theta}}^{\bar{\theta}} \left(t\left(\theta\right) - \theta q\left(\theta\right)\right) f\left(\theta, e\right) d\theta - g\left(e\right) \right\}$$

In order to solve this problem I need to be able to replace the final constraint by a first-order condition.

Proposition 1 *The principal's problem* (3) *s.t.* (4), (5) *and* (6) *is equivalent to the following problem*

$$\max_{q(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \left(V\left(q\left(\theta\right)\right) - \left(\theta + \frac{F\left(\theta, e\right)}{f\left(\theta, e\right)}\right) q\left(\theta\right)\right) f\left(\theta, e\right) d\theta \tag{11}$$

$$+\mu \left(\int_{\underline{\theta}}^{\bar{\theta}} F_e\left(\theta, e\right) q\left(\theta\right) d\theta - g_e\left(e\right) \right)$$

$$s.t. q\left(\theta\right) \text{ non-increasing}$$

for some Lagrange multiplier μ if an increase in e induces a mean preserving spread in the sense of Rothschild and Stiglitz (1970), at a decreasing rate, in the sense that

$$\int_{\underline{\theta}}^{y} F_e\left(\theta, e\right) d\theta \geq 0 \forall y \tag{12}$$

and (8), *and*

$$\int_{\underline{\theta}}^{y} F_{ee}\left(\theta, e\right) d\theta \leq 0 \forall y \tag{13}$$

and (9), *and in addition either $q\left(\theta\right)$ is continuously differentiable for all θ or the influence of the agent's effort on the density of θ is bounded below by the uniform density on $[\underline{\theta}, \bar{\theta}]$*

$$f_e\left(\theta, e\right) \geq -\frac{1}{\bar{\theta} - \underline{\theta}} \forall \theta. \tag{14}$$

It is well known[13] that the set of implementable contracts satisfies $t(\theta) = \theta q(\theta) + \int_{\theta}^{\overline{\theta}} q(\tau) d\tau$ and $q\left(\theta\right)$ non-increasing in θ. Substituting out transfers and integrating by parts one obtains

[13] For convenience of the reader the derivation is reproduced in the appendix. A more detailed treatment is found in Fudenberg and Tirole (1991), chap 7.

the principal's objective function: the principal maximizes expected surplus net of the agent's virtual surplus (Myerson (1981)). Substituting the same expression for transfers into the agent's objective function, the agent's expected utility, gross of costs of information acquisition, becomes $\int_{\underline{\theta}}^{\bar{\theta}} \int_{\theta}^{\overline{\theta}} q(\tau)d\tau f_e(\theta, e)\, d\theta$. It is easy to see that the function $\int_{\theta}^{\overline{\theta}} q(\tau)d\tau$ is decreasing and (weakly) convex. In other words, the agent is a quasi-risk lover because his indirect utility under any implementable contract is a convex function of θ (Rochet (1985)). Therefore he likes increases in risk in the distribution of types in the sense of Rothschild and Stiglitz (1970)[14], so given (12) and (8), $\int_{\underline{\theta}}^{\bar{\theta}} \int_{\theta}^{\overline{\theta}} q(\tau)d\tau f_e(\theta, e)\, d\theta > 0$ for all e. In the appendix, I demonstrate that - provided the technical condition (14) is satisfied - the Rothschild and Stiglitz (1970) result implies also that the agent's expected utility is concave in effort, so $\int_{\underline{\theta}}^{\bar{\theta}} \int_{\theta}^{\overline{\theta}} q(\tau)d\tau f_{ee}(\theta, e)\, d\theta < 0$ for all e. The technical restriction is needed to deal with issues of non-differentiability in $q(\theta)$, which may arise from bunching. However, to capture the intuition for the result, suppose these problems are absent and that $\int_{\theta}^{\overline{\theta}} q(\tau)d\tau$ is twice continuously differentiable so that I can integrate by parts twice. Proceeding like this, I can express the agent's first-order condition as

$$-\int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{\theta}}^{\theta} F_e(\tau, e)\, d\tau q_\theta(\theta)\, d\theta - g_e(e) = 0. \tag{15}$$

From (15) it is easy to see that (12) renders the agent's expected gross utility (gross of costs of information acquisition) non-decreasing in e for any non-increasing quantity schedule; (13) renders the agent's expected gross utility concave in e. The complete proof, which does not rely on the absence of bunching, is in the appendix.

The upshot of proposition 1 is that one can complement the Mirrlees approach to reporting by a first-order approach to information acquisition, which yields a fairly easily tractable problem. Before I proceed to apply the approach to the specific context of procurement, I characterize sufficient conditions on the Bayesian updating process that induce Second Order Stochastic Dominance shifts in the distribution of Θ.

4 On the Informativeness of Experiments

In this section I study the properties of the distribution of the conditional expectation. I obtain sufficient conditions on the conditional distribution of β given s and e such that the distributions of the conditional expectation for different levels of e can be ordered by Second Order Stochastic

[14] See also Dai and Lewis (2005), who have observed this independently in a two experiment model.

Dominance, that the shift in the distribution due to an increase in effort satisfies the decreasing returns condition, and that the distribution has an increasing inverse hazard rate.

Recall that a high signal indicates a high β in the sense of (1). This sort of dependence arises naturally if, e.g., β and s are affiliated. Consider now the dependence on e. Let $\tilde{s} \equiv E_S S$ denote the expected value of the signal s. To obtain my first result, I impose a mean reversing FOSD condition, that I shall denote MRFOSD henceforth. For all e and all $\beta \in \left(\underline{\beta}, \overline{\beta}\right)$:

$$H_e\left(\beta\left|s,e\right.\right) > 0 \text{ for } s \in \left(\underline{s}, \tilde{s}\right) \text{ and } H_e\left(\beta\left|s,e\right.\right) < 0 \text{ for } s \in \left(\tilde{s}, \overline{s}\right). \tag{16}$$

The posterior distributions for different levels of e and for a given realization of s are ordered by First Order Stochastic Dominance. However, the direction into which higher effort shifts the posterior distribution depends on whether the signal realization is above or below its mean. More precisely, the sign of the influence of an increase in e on the posterior is reversed as s is increased above its mean value. The reason I impose this condition in a mean reversing rather than the usual global sense is because a global version of (16) would imply that for each s an increase in e increases the posterior. But since the distribution of the signal is fixed, this would imply an increase in the ex ante mean, which is inconsistent with the law of iterated expectations.

Let $\tilde{\theta} \equiv \pi\left(\tilde{s}, e\right)$, the conditional mean induced by the expected signal. Experiments that satisfy condition (16) induce the desired ordering in the distribution of the conditional expectation:

Proposition 2 *Assume that experiments satisfy* (1), (2), *and* (16). *Then,* $F\left(\theta, e\right)$ *satisfies*

$$\begin{aligned} F_e\left(\underline{\theta}, e\right) &= F_e\left(\tilde{\theta}, e\right) = F_e\left(\overline{\theta}, e\right) = 0, \\ F_e\left(\theta, e\right) &> 0\, for\, \theta \in \left(\underline{\theta}, \tilde{\theta}\right), \text{ and} \\ F_e\left(\theta, e\right) &< 0\, for\, \theta \in \left(\tilde{\theta}, \overline{\theta}\right) \end{aligned}$$

and thus condition (12).

We know from (7) that the properties of the distribution function $F\left(\theta, e\right)$ simply correspond to the properties of the inverse of the conditional expectation function. Relative to $\tilde{\theta}$ the agent revises his posterior expectation upwards if he receives a signal higher than ex ante expected, and downwards if he receives a downward surprise. If he receives the expected signal, the conditional expectation is equal to $\tilde{\theta}$. The upward (downward) revision for surprisingly high (low) signals is the larger the higher is e. As a consequence the conditional expectation functions for different e all cross three times, at $\underline{\theta} = \pi\left(\underline{s}, e\right)$, at $\overline{\theta} = \pi\left(\overline{s}, e\right)$, and at $\tilde{\theta}$, and the ex ante distributions of Θ

satisfy a triple crossing property. It s natural to think of $\tilde{\theta}$ as being equal to $E_\Theta \Theta$. This property necessarily holds if the signal contains no information for $e = 0$, because this affords that the distribution of Θ converges to a mass point around the prior mean when e goes to zero. I will assume this property holds henceforth, thus $E_\Theta \Theta = \tilde{\theta}$.

Before I illustrate these results with an example, I give an alternative sufficient condition on the posteriors that justify condition (12) in proposition 1. Although more restrictive, this condition may prove useful in other applications, because it implies more structure. In particular, one may impose a mean reversing version of the Monotone Likelihood Ratio Property:

$$\frac{\partial}{\partial \beta}\left(\frac{h_e(\beta\,|s,e)}{h(\beta\,|s,e)}\right) < 0 \text{ for } s \in (\underline{s}, \tilde{s}) \text{ and } \frac{\partial}{\partial \beta}\left(\frac{h_e(\beta\,|s,e)}{h(\beta\,|s,e)}\right) > 0 \text{ for } s \in (\tilde{s}, \overline{s}). \tag{17}$$

If the conditional distribution satisfies condition (17) and the agent receives a signal which is higher (lower) than ex ante expected, then it is relatively more likely that indeed the state is high (low) for a higher level of e. In this sense the signal is more informative when effort is higher. Non-moving supports and differentiability in s then require then that $\frac{\partial}{\partial \beta}\left(\frac{h_e(\beta|s,e)}{h(\beta|s,e)}\right) = 0$ for $s \in \{\underline{s}, \tilde{s}, \overline{s}\}$. Building on the proofs of Milgrom (1981) it is straightforward to show that this mean reversing version of the MLRP condition implies (16). Moreover, one can also show that under these assumptions the distribution of θ inherits the Mean Reversing Monotone Likelihood Ratio Property, i.e., one has $\frac{\partial}{\partial \theta}\left(\frac{f_e(\theta,e)}{f(\theta,e)}\right) \gtreqqless 0$ for $\theta \gtreqqless \tilde{\theta}$.[15] However, as is well known (Jewitt (1988)), joint conditions on the likelihood ratios and the convexity properties of the distribution function - which I will introduce shortly - are rather restrictive. Therefore, I use the weaker condition (16).

The following simple example illustrates the properties.

Example 1 *Let the marginal cost be* $\beta = B + \Delta\beta$ *for some* $B > 1$ *and let the marginal of* $\Delta\beta$ *be uniform on* $[-1, 1]$, *the marginal of* s *be uniform on* $[0, 1]$, *and the posterior density be* $h(\Delta\beta|\, s, e) = \frac{1+\Delta\beta y(s,e)}{2}$ *for* $\Delta\beta \in [-1, 1]$ *and zero otherwise. The function* $y(s, e)$ *is given by* $y(s, e) = s - \frac{1}{2} + k\left(s\left(s - \frac{1}{2}\right)(1 - s) + le^\alpha s^2 \left(s - \frac{1}{2}\right)^3 (1 - s)^2\right)$, *where* $\alpha \in (0, 1)$, *and* k, l, *and* $\overline{e}$ *(the upper bound on effort) are positive and sufficiently small. Then,* $h(\Delta\beta|\, s, e) > 0$ *for all* $\Delta\beta$, $\int_{-1}^{1} h(\Delta\beta|\, s, e)\, d\Delta\beta = 1$, *and the posterior expectation,* $\pi(s, e) = B + \frac{y(s,e)}{3}$, *satisfies the law of iterated expectations.*

With a slight departure from my notation, I take $\Delta\beta$ instead of β as the underlying random variable. The posterior cdf and its derivative with respect to effort can be computed as

[15] This last statement follows directly from Milgrom's (1981) proposition 3.

$H\left(\Delta\beta|\, s,e\right) = \frac{\Delta\beta+1}{2} + \frac{(\Delta\beta)^2-1}{4} y\left(s,e\right)$ and $H_e\left(\Delta\beta|\, s,e\right) = \frac{(\Delta\beta)^2-1}{4} y_e\left(s,e\right)$, respectively. The function $y_e\left(s,e\right)$ embodies the important assumptions I have made sofar. $y_e\left(s,e\right)$ takes a value of zero for $s \in \left\{0, \frac{1}{2}, 1\right\}$, a negative value for $s \in \left(0, \frac{1}{2}\right)$ and a positive value for $s \in \left(\frac{1}{2}, 1\right)$. Therefore, an increase in e increases the posterior cdf (and therefore decreases the conditional expectation) for low signal values and decreases the posterior cdf for high values, precisely as required in (16). In fact, the example also satisfies (17) as $\frac{\partial}{\partial \Delta\beta}\left(\frac{h_e(\Delta\beta|s,e)}{h(\Delta\beta|s,e)}\right) = \frac{y_e(s,e)}{(1+\Delta\beta y(s,e))^2}$.

To obtain the second result, I impose a mean reversing concavity/convexity condition of the distribution function, that I shall denote MRCDFC henceforth. For all $\beta \in \left(\underline{\beta}, \overline{\beta}\right)$:

$$H\left(\beta\,|s,e\right) \text{ is concave in } (s,e) \text{ for } s \in (\underline{s}, \tilde{s}) \text{ and convex in } (s,e) \text{ for } s \in (\tilde{s}, \overline{s}). \tag{18}$$

The reason to impose a mean reversing rather than a global concavity/convexity assumption is the same as for condition (16). The assumption that supports are non-moving, (2), directly implies that $H_{ee}\left(\beta|\, \underline{s}, e\right) = H_{ee}\left(\beta|\, \overline{s}, e\right) = 0$. Therefore, I have to assume that the distribution changes from concave to convex in effort as we increase the signal value above the prior expected value. Since a function is convex in two variables jointly only if it is convex in each of the variables alone, I cannot assume global convexity or concavity, but rather impose a mean reversing convexity condition.

Proposition 3 *Suppose experiments satisfy* (1), (2), *and* (18). *Then,* $F\left(\theta,e\right)$ *satisfies*

$$\begin{aligned} F_{ee}\left(\underline{\theta}, e\right) &= F_{ee}\left(\tilde{\theta}, e\right) = F_{ee}\left(\overline{\theta}, e\right) = 0, \\ F_{ee}\left(\theta, e\right) &\leq 0 \, for \, \theta \in \left(\underline{\theta}, \tilde{\theta}\right), \text{ and} \\ F_{ee}\left(\theta, e\right) &\geq 0 \, for \, \theta \in \left(\tilde{\theta}, \overline{\theta}\right) \end{aligned}$$

and thus condition (13).

Example 1 satisfies (18) if all the parameters sufficiently small. For all $\beta \in \left(\underline{\beta}, \overline{\beta}\right)$, I have $H_{ss}\left(\beta\,|s,e\right), H_{ee}\left(\beta\,|s,e\right) < (>)\, 0$ for $s \in (\underline{s}, \tilde{s})$ (for $s \in (\tilde{s}, \overline{s})$). Since $H\left(\beta\,|s,e\right)$ is twice continuously differentiable $H\left(\beta\,|s,e\right)$ is concave (convex) if and only its Hessian is negative semidefinite (positive semidefinite), which amounts to $H_{ss}\left(\beta\,|s,e\right) H_{ee}\left(\beta\,|s,e\right) - \left(H_{es}\left(\beta\,|s,e\right)\right)^2 \geq 0$ for all $s \notin \{\underline{s}, \tilde{s}, \overline{s}\}$. One can verify that this is the case.[16]

In addition to these conditions that ensure the regularity properties of my problem with respect to the agent's choice of effort[17], it will also be convenient to have conditions that guarantee that the

[16] Results are available from the author upon request.

[17] One can find similar conditions on the conditional expectation function to guarantee the condition $f_e\left(\theta, e\right) > -\frac{1}{\overline{\theta}-\underline{\theta}}$. However, whenever I make use of the agent's first-order condition with respect to e below, the quantity

monotonicity constraint in problem (11) is non binding at the optimum. Without such regularity conditions, one may encounter problems of bunching that are well known and do not add much to the present discussion.

Proposition 4 *The distribution of* θ *satisfies* $\frac{\partial}{\partial\theta}\frac{F(\theta,e)}{f(\theta,e)} \geq 0$ *if and only if*

$$\frac{s\pi_{ss}(s,e)}{\pi_s(s,e)} \geq -1\forall s. \tag{19}$$

In terms of the conditional distribution, condition (19) is equivalent to the condition $\frac{d}{ds}\left[s\int_{\underline{\beta}}^{\overline{\beta}} H_s(\beta\,|s,e)\,d\beta\right] \leq 0$, but that is hardly more informative than condition (19), which says that the distribution of θ has a non-decreasing inverse hazard rate if and only if the conditional expectation function is not too concave in the sense of a standard curvature measure. In terms of the distribution in Example 1, condition (19) is met if k is sufficiently small.

In the remainder of this paper I apply the first-order approach to study the specific problem of procurement. The first step is to sign the multiplier μ. The second is to characterize the structure of optimal contracts.

5 The Value of Information

In this section I establish two results. First, I show that it is optimal to implement a strictly positive amount of information acquisition, that is, that the optimal level of effort is strictly positive. I conclude from this result that the value of information to the principal is positive. Second, I show that the level of effort can be either too small or too large relative to the amount of effort that maximizes the expected surplus. In particular I will show that whether there is too much or too little information acquisition depends on the principal's quasi-attitudes towards risk, that is, on the shape of his indirect utility function.

Consider first the value of information to the principal, which I define as the difference in expected utility when he implements a positive amount of effort and zero effort. Implementing $e = 0$ requires that information has no value to the agent, neither for his decision what type to report conditional on participating, nor on his decision whether or not to participate. Building on Proposition 1 and the discussion that follows that Proposition, the first part of this statement implies that the interim expected utility function, $\int_{\theta}^{\overline{\theta}} q(\tau)\,d\tau$, cannot be strictly convex in θ, but

schedule $q(\theta)$ will be strictly monotonic. Given the remaining regularity conditions, this implies that $q(\theta)$ is continuously differentiable, which renders the boundedness assumption on $f_e(\theta,e)$ redundant.

must be linear. This means that production must be independent of the agent's announced type, so $q(\theta) = q$ for all θ. The participation constraint of the least efficient type, $\overline{\theta}$, is satisfied with equality (and by implication all types are willing to participate) if the principal pays the transfer $t = \theta q + (\overline{\theta} - \theta) q = \overline{\theta} q$ to all types. This is very expensive from the principal's perspective. To show this is suboptimal, I have to show that there exist contracts that give the principal a higher utility. It is hard to show this directly, because the level of the principal's utility depends on the shadow cost of implementing effort at the optimal level of effort. Therefore I establish my result in an indirect way, showing that there exist (possibly suboptimal) contracts that implement a positive level of effort at a zero shadow cost and that give the principal a higher utility than any contract that implements $e = 0$. Since the principal will be able to do *even better* if he is allowed to implement any level of effort, this argument shows that implementing $e = 0$ can't be optimal, or in other words, that information has a strictly positive value to the principal.

To make this argument I denote $q(\theta; e)$ an optimal quantity schedule contingent on the effort level e. Suppose the principal offers a contract that implements a level of effort e at zero shadow cost; that is the value of the multiplier μ, associated to the problem of implementing effort e is zero. Then, we know from Baron and Myerson (1982) that the optimal quantity schedule satisfies the condition

$$V_q\left(q^{BM}(\theta; e)\right) = \theta + \frac{F(\theta, e)}{f(\theta, e)}. \tag{20}$$

To see this, maximize (11) point-wise with respect to q for $\mu = 0$. Conversely, consistency with $\mu = 0$ requires that the agent be willing to choose the effort level e that the quantity schedule $q^{BM}(\theta; e)$ is conditioned on. Let $\hat{e}$ denote the level of effort that the agent finds optimal to exert when he is offered a contract with associated quantity schedule $q^{BM}(\theta; e)$. $\hat{e}$ satisfies the first-order condition

$$\int_{\underline{\theta}}^{\overline{\theta}} F_e(\theta, \hat{e})\, q^{BM}(\theta; e)\, d\theta - g_e(\hat{e}) = 0. \tag{21}$$

The solution of (21), when viewed as a function of e, defines a best reply for the agent, $\hat{e} = r\left(q^{BM}(\theta, e)\right)$. Contract offer and effort choice are in simultaneous equilibrium if

$$e = r\left(q^{BM}(\theta, e)\right). \tag{22}$$

Let $\mathbf{e}$ denote the (possibly empty) set of solutions to (22). If $\mathbf{e}$ is non-empty, then the principal can implement any effort level in $\mathbf{e}$ by offering the associated Baron-Myerson quantity schedule defined by (20). Offering such a contract, the principal extracts some rent, and therefore he does better than under the contract where the agent is always paid as if he had costs equal to $\overline{\theta}$.

Proposition 5 *It is optimal to implement a positive level of effort. Formally, the set* $\mathbf{e}$*, defined by* (20), (21)*, and* (22), *is non-empty.*

To ease notation again in what follows I will drop the dependence of the optimal quantity schedule on e where this can be done without creating confusion. Consider a locally optimal choice of effort to implement, and denote such a locally optimal value of e by e^*, and the associated multiplier by μ^*. Such a choice satisfies the first-order condition

$$\int_{\underline{\theta}}^{\bar{\theta}} \left(\left(V\left(q\left(\theta\right)\right) - \theta q\left(\theta\right)\right) f_e\left(\theta, e^*\right) - F_e\left(\theta, e^*\right) q\left(\theta\right)\right) d\theta + \mu^* \left(\int_{\underline{\theta}}^{\bar{\theta}} F_{ee}\left(\theta, e^*\right) q\left(\theta\right) d\theta - g_{ee}\left(e^*\right)\right) = 0,$$

where I have used the envelope theorem to conclude that all indirect effects through e and μ on $q\left(\theta; e\right)$ are zero around an optimum. Rearranging the first-order condition, and substituting from the first-order condition with respect to the agent's effort choice, I can write

$$\mu^* = \frac{\int_{\underline{\theta}}^{\bar{\theta}} \left(V\left(q\left(\theta\right)\right) - \theta q\left(\theta\right) f_e\left(\theta, e^*\right)\right) d\theta - g_e\left(e^*\right)}{-\left(\int_{\underline{\theta}}^{\bar{\theta}} F_{ee}\left(\theta, e^*\right) q\left(\theta\right) d\theta - g_{ee}\left(e^*\right)\right)}.$$

The term inside the brackets of the denominator is the second-order condition of the agent's effort choice. Hence, the sign of μ^* is equal to the sign of the numerator. If the increase in the social surplus due to an increase in e exceeds the marginal cost of acquiring information, then μ^* is positive; if the two terms are just equal then μ^* is zero; otherwise the multiplier is negative at the optimum. I will now argue that μ^* can be of either sign at a stationary point of the principal's problem, and will give sufficient conditions for each case to occur.

I use the following chain of reasoning. Let $\tilde{e}$ denote an element of $\mathbf{e}$, defined by (22), and let $\underline{\tilde{e}}$ denote the smallest element in $\mathbf{e}$ and let $\bar{\tilde{e}}$ denote the largest element in $\mathbf{e}$. By definition $\mu\left(\underline{\tilde{e}}\right) = \mu\left(\bar{\tilde{e}}\right) = 0$. Since an increase in μ makes contracts more risky and the agent is a quasi-risk lover - because incentive compatible indirect utility profiles are convex - we must have $\mu < 0$ for any $e < \underline{\tilde{e}}$ and $\mu > 0$ for any $e > \bar{\tilde{e}}$. To establish my result, it suffices to give sufficient conditions that render the principal's utility i) locally decreasing around $e = \tilde{e}$ and ii) locally increasing around $e = \tilde{e}$. By implication the principal's utility will be locally decreasing around $\underline{\tilde{e}}$ in the former case and will be locally increasing around $\bar{\tilde{e}}$ in the latter case, which implies the desired result.

An increase in the agent's effort increases the likelihood of more extreme cost perceptions. The principal benefits ex post if the agent's signal is better than expected but is harmed if the agent perceives his cost as being higher. Whether the principal likes to consume such a lottery depends on the shape of his indirect utility function. In turn the shape of the indirect utility function depends on the curvature of the direct utility function and on properties of the family of distributions $\{F(\theta, e)\}_{e\geq 0}$. Define

$$\rho(q) = \frac{-V''(q)}{V'(q)}.$$

$\rho(q)$ is the Arrow-Pratt measure of absolute risk aversion with respect to production shocks in the function $V(q)$. I will make my point by means of the example, where the posterior density is given by $h(\Delta\beta| s, e) = \frac{1+\Delta\beta y(s,e)}{2}$ for $\Delta\beta \in [-1, 1]$ and zero otherwise and $y(s, e)$ is equal to $s - \frac{1}{2}$ plus k times a function of s and e. In the limit as k tends to zero, the conditional expectation function tends to $\pi(s, e) = B + \frac{s-\frac{1}{2}}{3}$, a linear function of s. Hence, in the limit as k tends to zero, the distribution of θ tends to a uniform distribution on $\left[B - \frac{1}{6}, B + \frac{1}{6}\right]$.

Proposition 6 *Consider the posterior as in Example 1 as a function of k. In the limit as k tends to zero,* $\rho_q(q) < (>) 0$ *for all q implies that there exists a stationary point to the principal's problem of choosing e where* $\mu > (<) 0$.

If $V(q)$ features decreasing absolute risk aversion and the distribution of θ tends to the uniform, then the principal benefits from a marginal increase in effort. These conditions render the Principal's indirect utility function convex in θ, so he behaves as a *quasi-risk-lover*. If $V(q)$ features increasing absolute risk aversion, then the converse result obtains in the limit as the distribution of θ tends to the uniform.

These arguments show the existence of local maximizers with the property that μ is positive or negative, respectively; of course, these results doe not say anything about the optimal level of effort to implement, but as I have explained above (see footnote 11), this was not to be expected. Since both constellations with μ positive and μ negative are possible, I will now proceed to characterize optimal contracts for both constellations where the shadow cost of effort is positive or negative.

6 The Structure of Contracts

Let $\{q^*(\theta), t^*(\theta)\}\,\forall\theta$ denote a menu of contracts that optimally implements a given amount of effort in a truth-telling equilibrium. I shall characterize such contracts, taking their existence for granted.[18] The main obstacle to this analysis is that the value of the multiplier μ is unknown. A global treatment necessitates the use of dynamic optimization and delivers little additional insights. Therefore it is useful to characterize the solution for effort levels that are easy to implement in the

[18] Conditions for existence of solutions for exogenous type distributions can be found in Guesnerie and Laffont (1984). With a suitable adjustment for the endogeneity of information their results could be carried over.

following sense. Define, for any e, the statistics $\overline{\kappa}(e) \equiv \frac{\sqrt{Var\left(\theta+\frac{F(\theta,e)}{f(\theta,e)}\middle|\theta\leq\tilde{\theta}\right)}}{\sqrt{Var\left(\frac{F_e(\theta,e)}{f(\theta,e)}\middle|\theta\leq\tilde{\theta}\right)}} + \frac{E_\Theta\left[\theta+\frac{F(\theta,e)}{f(\theta,e)}\middle|\theta\leq\tilde{\theta}\right]}{E_\Theta\left[\frac{F_e(\theta,e)}{f(\theta,e)}\middle|\theta\leq\tilde{\theta}\right]}$ and $\underline{\kappa}(e) \equiv \frac{\sqrt{Var\left(\theta+\frac{F(\theta,e)}{f(\theta,e)}\middle|\theta\geq\tilde{\theta}\right)}}{\sqrt{Var\left(\frac{F_e(\theta,e)}{f(\theta,e)}\middle|\theta\geq\tilde{\theta}\right)}}$. Heuristically, the smaller is $\overline{\kappa}(e)$, the "easier" is the inference about the unobserved effort from observing θ relative the variation of the agent's virtual surplus when values of θ below the mean are observed. $\underline{\kappa}(e)$ is a similar ratio when only the subinterval of θ above the mean is considered.

Lemma 1 *Suppose that for a given e*

$$\frac{\partial}{\partial\theta}\left(\theta+\frac{F(\theta,e)}{f(\theta,e)}-\overline{\kappa}(e)\frac{F_e(\theta,e)}{f(\theta,e)}\right)\geq 0 \text{ and } \frac{\partial}{\partial\theta}\left(\theta+\frac{F(\theta,e)}{f(\theta,e)}+\underline{\kappa}(e)\frac{F_e(\theta,e)}{f(\theta,e)}\right)\geq 0 \text{ forall } \theta. \tag{23}$$

Then, the multiplier satisfies $-\underline{\kappa}(e)\leq\mu(e)\leq\overline{\kappa}(e)$.

$|\mu|$ measures the utility loss due to the need to give extra (less) incentives for information gathering when marginal costs of information gathering, evaluated at a given effort level, increase by a small amount. One way to place a bound on this loss is to find a simple contract that continues to implement a given level of effort when marginal cost of effort increase (decrease) by a small amount. One difficulty is again to avoid the need to invoke control theory to make this point. The monotonicity conditions in the statement of the lemma are imposed to this end. Then, starting from a strictly monotonic contract, the principal can shock the amount of production by adding $\varepsilon\frac{F_e(\theta,e)}{f(\theta,e)}$ to the original quantity schedule for θ below $\tilde{\theta}$. Since $F_e(\theta,e)$ is non-negative for these values of θ, the agent has higher incentives to acquire information. The ε that is needed to compensate for a given increase in marginal costs of effort is inversely proportional to $E_\Theta\left[\left(\frac{F_e(\theta,e)}{f(\theta,e)}\right)^2\middle|\theta\leq\tilde{\theta}\right]$. On the other hand, the expected cost in terms of higher payments to the agent is proportional to $E_\Theta\left[\left(\theta+\frac{F(\theta,e)}{f(\theta,e)}\right)\frac{F_e(\theta,e)}{f(\theta,e)}\middle|\theta\leq\tilde{\theta}\right]$. $\overline{\kappa}(e)$ is an upper bound on the ratio of these two expectations. $\underline{\kappa}(e)$ is derived from an analogous procedure when the principal shocks production by an amount $\varepsilon\frac{F_e(\theta,e)}{f(\theta,e)}$ for $\theta\geq\tilde{\theta}$ and $\varepsilon<0$, which reduces the agent's incentive to acquire information.

The posterior in Example 1 meets the conditions in Lemma 1 if both k and $\overline{e}$ are sufficiently small. If e is small, then the expectation and the variance of $\theta+\frac{F(\theta,e)}{f(\theta,e)}$ are largely exogenous, whereas the expectation and the variance of $\frac{F_e(\theta,e)}{f(\theta,e)}=\pi_e^{-1}(\theta;e)\,\pi_s\left(\pi^{-1}(\theta;e);e\right)$ become larger as e becomes smaller. The smaller is e, the larger is the marginal effect of an increase in e on the conditional expectation for given s, so $|\pi_e(s;e)|$ is higher for all $s\notin\left\{0,\frac{1}{2},1\right\}$. Hence $\left|\pi_e^{-1}(\theta;e)\right|$ is higher for all $\theta\notin\left\{\underline{\theta},\tilde{\theta},\overline{\theta}\right\}$. Hence, the given variation in s induces a larger variation in $\frac{F_e(\theta,e)}{f(\theta,e)}$ the smaller is e. As a result $\underline{\kappa}(e)$ and $\overline{\kappa}(e)$ tend to be small. Changing variables from θ to s, condition

(23) can be written as $2+\frac{s\pi_{ss}(s,e)}{\pi_s(s,e)}+\overline{\kappa}(e)\frac{\pi_{es}(s,e)}{\pi_s(s,e)}\geq 0$ and $2+\frac{s\pi_{ss}(s,e)}{\pi_s(s,e)}-\underline{\kappa}(e)\frac{\pi_{es}(s,e)}{\pi_s(s,e)}\geq 0$, which is met for k small.

I abstain from a discussion of the case where these conditions are violated, because the advantage of bounding the absolute value of the multiplier is that one can characterize the solution to the contracting problem without recourse to control techniques:

Proposition 7 *Suppose that $F(\theta,e)$ satisfies condition* (23) *for a given level of e. Then the optimal quantity schedule that implements e is characterized by*

$$V_q(q^*(\theta))=\theta+\frac{F(\theta,e)}{f(\theta,e)}-\mu\frac{F_e(\theta,e)}{f(\theta,e)}. \tag{24}$$

The formal proof of this proposition is omitted, since it follows straightforwardly from the previous results. The production schedule coincides with the Baron Myerson schedule at the top, at the prior mean, and at the bottom. Otherwise, there is an additional distortion. The direction of the extra distortion depends on whether the principal wants to give the agent more or less of an incentive to acquire information relative to the Baron Myerson contract. In the former case production is increased for surprisingly low cost perceptions and decreased for surprisingly bad cost assessments. The sensitivity of the production scheme with respect to the agent's information is increased to provide extra incentives for information acquisition. In the latter case, the reverse happens and production is more equalized in order to dampen the agent's interest in additional information. The size of the additional distortion depends on how informative a given message is about the agent's unobserved effort choice.[19]

In the remainder of this article I study how these results are affected by changes in the underlying structure of experiments.

7 Alternative Experiment Structures

7.1 Moving Supports and Distortions at the top

Sofar, I have characterized solutions to the contracting problem when the support of the agent's conditional expectation is fixed. This is analytically very convenient, but moving supports may

[19]The term $\frac{F_e(\theta,e)}{f(\theta,e)}$ has an interpretation in terms of hypothesis testing. Write $\frac{F_e(\theta,e)}{F(\theta,e)}/\frac{f(\theta,e)}{F(\theta,e)}$. $\frac{F_e(\theta,e)}{F(\theta,e)}$ is the derivative of the log-likelihood if the statistician observes only if the values in a sample are smaller than θ and wants to compute the optimal value of e. This statistic is important in the contract because the production at θ changes the rent of all types who are at least as efficient as θ. Division by $\frac{f(\theta,e)}{F(\theta,e)}$ normalizes by the conditional density.

easily arise. To see this, modify example 1[20] to the case where $y(e,s) = e\left(s-\frac{1}{2}\right)$, so that the posterior density becomes $h(\Delta\beta|\, s,e) = \frac{1+\Delta\beta e\left(s-\frac{1}{2}\right)}{2}$ for $\Delta\beta \in [-1,1]$ and zero otherwise. For an upper bound of e equal to $\overline{e} = 2$ I have $h(\Delta\beta|\, s,e) \geq 0$ for all $\Delta\beta$, s and all $e \leq \overline{e}$. Again this posterior satisfies (16) and (17).[21] One verifies that $\pi(s,e) = B + \frac{e\left(s-\frac{1}{2}\right)}{3}$. The bounds of the support are $\pi(\underline{s},e) = B - \frac{e}{6}$ and $\pi(\overline{s},e) = B + \frac{e}{6}$. Observe that the support of θ is a subset of the support of β and the upper bound is increasing in e and the lower bound is decreasing in e. For all values of e, the distribution of θ is uniform. Thus it is natural to wonder how the analysis is affected by the possibility of moving supports.

I will show in this section that there are some problems with the first-order approach; it is not possible to justify such an approach in general. However, whenever such an approach is valid, then the main qualitative features of contracts remain unchanged. One notable exception is that there is now a distortion at the top.

There are some essential differences in the agent's problem. I will stick to the following notation in this section. I let $\overline{\theta}(e)$ and $\underline{\theta}(e)$ denote the upper and the lower bound of the support of the conditional mean, respectively. I assume that the upper bound is increasing in e and that the lower bound is decreasing in e. In addition, I let $\overline{\theta}$ and $\underline{\theta}$ denote the bounds of the support associated to the effort level that the principal wishes to implement. Notice that these are independent of the agent's actual actions. Obviously the principal's contract offer satisfies the participation constraint of type $\overline{\theta}$ with equality. Suppose the agent chooses an effort level that is higher than the one the principal wishes to implement. If the agent receives a high signal, then his participation constraint is violated for all $\theta \in \left(\overline{\theta}, \overline{\theta}(e)\right]$. So, the agent refuses to participate and obtains zero rent in this case[22]. Suppose after choosing an effort level that is too high, the agent receives a very low signal. In that case, for $\theta \in \left[\underline{\theta}(e), \underline{\theta}\right]$ the agent will be treated the same way as an agent with expected marginal costs equal to $\underline{\theta}$[23]. Suppose on the other hand, that the agent chooses an effort level which is too low. In that case we have $\overline{\theta}(e) < \overline{\theta}$, which implies that type $\overline{\theta}(e)$ receives a strictly positive rent equal to $\int_{\overline{\theta}(e)}^{\overline{\theta}} q(\tau)\, d\tau$. It follows from these considerations that I can always write the agent's

[20] This specification of the example is adapted from Ottaviani and Sorensen (2001).

[21] The example does not satisfy (18), but this is inessential for the point I am making here. As an example that sastisfies all three conditions, take $h(\Delta\beta|\, s,e) = \frac{1+\Delta\beta y(s,e)}{2}$ for $\Delta\beta \in [-1,1]$ and zero otherwise, where $y(s,e) = s - \frac{1}{2} + k\left(s\left(s-\frac{1}{2}\right)(1-s) + le^{\alpha}\left(s-\frac{1}{2}\right)^3\right)$.

[22] Of course, contracts that are declined can be represented by null-contracts which are always acceptable to the agent. I discuss the relationship between these two notions of contracts in the proof of Result 1 in the appendix.

[23] See the proof of Result 1 in the appendix for details.

indirect utility, $u(\theta)$, for any given effort choice and any effort (and support) that the principal wishes to implement as

$$u(\theta) = \max\left\{0, \int_{\underline{\theta}}^{\overline{\theta}} q(\tau)\, d\tau\right\}. \tag{25}$$

Finally, consider the probability distribution. It has the properties that $F(\underline{\theta}(e), e) = 0$ and $F(\overline{\theta}(e), e) = 1$. Moreover, it satisfies

$$\frac{dF(\theta, e)}{d\theta} = \begin{cases} 0 & \text{for } \theta < \underline{\theta}(e) \\ f(\theta, e) > 0 & \text{for } \theta \in [\underline{\theta}(e), \overline{\theta}(e)] \\ 0 & \theta > \overline{\theta}(e). \end{cases} \tag{26}$$

I can now derive the agent's ex ante expected utility from (25) and (26). This is somewhat tedious but straightforward, so I relegate the derivation of the following result to the appendix.

Result 1 *With moving supports the agent's ex ante expected indirect utility satisfies*

$$E_\Theta[u(\theta)] = q(\underline{\theta}) \int_{\underline{\theta}(e)}^{\underline{\theta}} F(\theta, e)\, d\theta + \int_{\underline{\theta}}^{\overline{\theta}} q(\theta) F(\theta, e)\, d\theta.$$

Notice that the equilibrium expected utility boils down to $E_\Theta[u(\theta)] = \int_{\underline{\theta}}^{\overline{\theta}} q(\theta) F(\theta, e)\, d\theta$. But there is a crucial difference at the ex ante stage when the agent chooses the level of effort. An incentive compatible choice of effort must satisfy the condition

$$e = \arg\max_{\hat{e}} \left\{ q(\underline{\theta}) \int_{\underline{\theta}(e)}^{\underline{\theta}} F(\theta, \hat{e})\, d\theta + \int_{\underline{\theta}}^{\overline{\theta}} q(\theta) F(\theta, \hat{e})\, d\theta - g(\hat{e}) \right\}. \tag{27}$$

Even if at the optimum one obviously has $\underline{\theta}(e) = \underline{\theta}$, (27) cannot simply be replaced by the first-order condition

$$\int_{\underline{\theta}}^{\overline{\theta}} q(\theta) F_e(\theta, e)\, d\theta - g_e(e) = 0$$

for any arbitrary, incentive compatible quantity schedule $q(\theta)$. Even if I impose the same conditions as before, namely that the law of iterated expectations holds, and that an increase in effort induces a mean reversing first order stochastic dominance shift, and that the distribution satisfies the mean reversing concavity/convexity conditions, it is no longer true that the agent always prefers to have more information (at the same cost). To see this, assume for simplicity that $q(\theta)$ is continuously differentiable for $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$[24], and integrate by parts to obtain

$$\int_{\underline{\theta}}^{\overline{\theta}} q(\theta) F_e(\theta, e)\, d\theta = q\left(\overline{\theta}\right) F_e\left(\overline{\theta}, e\right) - q\left(\underline{\theta}\right) F_e\left(\underline{\theta}, e\right) - \int_{\underline{\theta}}^{\overline{\theta}} q_\theta(\theta) \int_{\underline{\theta}}^{\theta} F_e(\tau, e)\, d\tau d\theta.$$

Under my assumptions $q\left(\overline{\theta}\right) F_e\left(\overline{\theta}, e\right) - q\left(\underline{\theta}\right) F_e\left(\underline{\theta}, e\right) \leq 0$, and this inequality is strict for the case where $\overline{\theta} < \overline{\theta}(e)$ and $\underline{\theta} > \underline{\theta}(e)$. Hence, one can find monotonic quantity schedules where the agent does not value additional information. It is also no longer true that the agent's expected indirect utility (gross of effort costs) is concave in effort, since

$$\int_{\underline{\theta}}^{\overline{\theta}} q(\theta) F_{ee}(\theta, e)\, d\theta = q\left(\overline{\theta}\right) F_{ee}\left(\overline{\theta}, e\right) - q\left(\underline{\theta}\right) F_{ee}\left(\underline{\theta}, e\right) - \int_{\underline{\theta}}^{\overline{\theta}} q_\theta(\theta) \int_{\underline{\theta}}^{\theta} F_{ee}(\tau, e)\, d\tau d\theta$$

and $q\left(\overline{\theta}\right) F_{ee}\left(\overline{\theta}, e\right) - q\left(\underline{\theta}\right) F_{ee}\left(\underline{\theta}, e\right) \geq 0$ with a strict inequality when $\overline{\theta} < \overline{\theta}(e)$ and $\underline{\theta} > \underline{\theta}(e)$. Hence, the same caveat applies here. However, whenever the first-order condition adequately describes the solution to the agent's problem, I have the following result.

Proposition 8 *If the first-order approach is valid, and condition* (23) *holds, then an optimal quantity schedule satisfies the condition*

$$V_q(q^*(\theta)) = \theta + \frac{F(\theta, e)}{f(\theta, e)} - \mu \frac{F_e(\theta, e)}{f(\theta, e)}.$$

For the case where $\mu > 0$ ($\mu < 0$) the level of production at the top is higher (smaller) than the Baron Myerson quantity at $\theta = \underline{\theta}$; the level of production at $\theta = \overline{\theta}$ is lower (higher) than the Baron Myerson quantity.

The rationale for this result is simple. With moving supports, an increase of the agent's effort does have an impact on the mass at the bounds of the support that the principal wishes to implement; at the lower bound the agent's effort increases the value of the distribution function at the margin, at the upper bound of the support his effort decreases the mass at the margin. Hence, there are additional distortions to consider relative to the case with a fixed distribution of types.

7.2 Stochastic Experiments

I end this article with a discussion of a class of updating processes that gives rise to a particularly tractable model. Suppose effort does not influence the posterior distribution directly, but rather

[24] As I have demonstrated in the proof of Proposition 1, the argument can be generalized to the case where $q(\theta)$ is only piecewise differentiable.

influences only the likelihood of obtaining different posteriors that are independent of effort. I show in this section that the first-order approach is rather easy to justify in that case. In addition, all the qualitative insights developed for the more general model are still valid.

Suppose an experiment is the realization of *two* random variables, S and I, and a resulting posterior with cdf $H(\beta|\, s, i)$. The variable S is still the signal, I is an informativeness parameter. Typical realizations of these variables are $s \in [\underline{s}, \overline{s}] = [0, 1]$ and $i \in [0, 1]$, respectively. The marginal distributions of s and i are *independent* of each other and fully supported with densities $k(s) = 1$ for $s \in [0, 1]$ (and zero otherwise) and $l(i, e)$, respectively. Let $L(i, e)$ denote the cdf of the random variable i. Assume that $l(i, e) > 0$ for all i and all e. Denote the conditional expectation function as $\pi(s, i) = \int_{\underline{\beta}}^{\overline{\beta}} \beta dH(\beta\, |s, i)$. The interpretation of the random variable θ is unchanged. Provided that $\pi(s, i)$ is strictly increasing in s for all i, the function is invertible and we can write $s = \pi^{-1}(\theta, i)$ for the value of s that generates the conditional expected value θ. The cdf of θ conditional on i is

$$F^i(\theta, i) = \begin{cases} 0 & for\, \theta < \pi(\underline{s}, i) \\ \pi^{-1}(\theta, i) & for\, \pi(\underline{s}, i) \leq \theta \leq \pi(\overline{s}, i) \\ 1 & \theta > \pi(\overline{s}, i) \end{cases}$$

Let $F(\theta, e)$ denote the unconditional cdf of θ. I have

$$F(\theta, e) = \int_0^1 F^i(\theta, i)\, dL(i, e) \tag{28}$$

By construction, θ is independent of effort and its distribution is fully supported on an interval $[\underline{\theta}, \overline{\theta}]$, independent of effort where $\underline{\theta} = \min_i \pi(\underline{s}, i)$ and $\overline{\theta} = \max_i \pi(\overline{s}, i)$.

To order experiments, I assume that the posterior density satisfies the mean reversing monotone likelihood ratio property, formally, I assume that

$$\frac{\partial}{\partial \beta}\left(\frac{h_i(\beta\, |s, i)}{h(\beta\, |s, i)}\right) < 0 \text{ for } s \in (\underline{s}, \tilde{s}) \text{ and } \frac{\partial}{\partial \beta}\left(\frac{h_i(\beta\, |s, i)}{h(\beta\, |s, i)}\right) > 0 \text{ for } s \in (\tilde{s}, \overline{s}) \tag{29}$$

and

$$\frac{\partial}{\partial \beta}\left(\frac{h_i(\beta\, |s, i)}{h(\beta\, |s, i)}\right) = 0 \text{ for } s \in \{\underline{s}, \tilde{s}, \overline{s}\}. \tag{30}$$

As I have explained in section 4, (29) implies that higher values of i correspond to more informative experiments. In particular, this implies again that conditional on a signal above (below) the mean, the posterior distribution conditional on a given informativeness i is the higher (lower) in the sense of FOSD the higher is i. In addition let

$$L_e(i, e) \leq 0 \text{ and } L_{ee}(i, e) \geq 0. \tag{31}$$

Then, an increase in effort makes it more likely to perform a more informative experiment; and the marginal impact of effort on the distribution of experiments is decreasing in e. Within this structure, I have the following result:

Proposition 9 *Given conditions* (29) , (30) , *and* (31) , *the distribution* $F(\theta, e)$ *satisfies conditions* (8) , (9) , (12) , *and* (13) , *and hence the first-order approach is valid. Under the monotonicity condition* (23)*, the optimal quantity schedule satisfies the condition*

$$V_q(q^*(\theta)) = \theta + \frac{F(\theta, e)}{f(\theta, e)} - \mu \frac{F_e(\theta, e)}{f(\theta, e)}.$$

Thus, it is easy to justify a first-order approach if we think of the agent's effort as of "spanning" the possible posteriors. Moreover, this model is appealing because it comprises much of the existing literature and therefore generalizes the findings of this literature. All-or-nothing information acquisition corresponds to the case where there are just two distributions of the conditional expectation conditional on i, $F^0(\theta, 0)$ and $F^1(\theta, 1)$; the distribution $F^0(\theta, 0)$ has mass one at $E_\Theta \Theta = E_\beta \beta$ and the distribution $F^1(\theta, 1)$ corresponds to the distribution $P(\beta)$. In the current setup I assume that the distribution $F^0(\theta, 0)$ has no atoms, but of course it can be close to a mass-point at $E_\beta \beta$. This assumption eliminates the discontinuities found in the earlier literature. Moreover, I allow for a continuum of levels of informativeness, i, that are (heuristically) ordered the way that the distributions $F^i(\theta, i)$ are the closer to $P(\beta)$ the higher is i[25]. Since this model is particularly easy to handle, it should prove useful in further applications.

8 Conclusion

The main result of the paper is that information and risk are equivalent in a wide class of reporting games with endogenous information. It is justified to describe the amount of information acquisition by the solution of a first-order condition for any incentive compatible contract, if the agent's information gathering increases risk in the ex ante distribution of the conditional expectation in the sense of Rothschild and Stiglitz (1970). Sufficient conditions on experiment structures are provided that generate such an ordering. The robust results that follow from the approach are that contracts that provide the agent with extra incentives for information acquisition are more sensitive to the agent's information relative to their fixed information counterparts. The reverse is true when incentives for information acquisition are reduced. Results beyond these depend on the specific information structure and are therefore not robust.

[25] I thank an anonymous referee for suggesting this interpretation.

The paper has derived a tractable modeling of information acquisition and a reduced form which is relatively easy to handle. It can be used to address any problem of mechanism design in the single agent case and extends easily to multi-agent mechanism design problems in the linear, private values environment.

9 Appendix

Proof of proposition 1. Truth-telling: For convenience I summarize the known features of the contract. For a more extensive treatment, see Fudenberg and Tirole (1991) or Laffont and Tirole (1993). Consider first monotonicity of $q(\theta)$. From incentive compatibility of reports, we know that type θ must not have any incentive to mimic type $\hat{\theta}$, and vice-versa; formally

$$\begin{aligned} t(\theta) - \theta q(\theta) &\geq t\left(\hat{\theta}\right) - \theta q\left(\hat{\theta}\right), \text{ and} \\ t\left(\hat{\theta}\right) - \hat{\theta} q\left(\hat{\theta}\right) &\geq t(\theta) - \hat{\theta} q(\theta). \end{aligned}$$

Adding these inequalities, I obtain

$$\left(\theta - \hat{\theta}\right)\left(q\left(\hat{\theta}\right) - q(\theta)\right) \geq 0.$$

Hence, the pair of schedules $(t(\theta), q(\theta))$ is incentive compatible only if $q(\theta)$ is non-increasing in θ. Moreover, since $q(\theta)$ is non-increasing in θ, it is differentiable almost everywhere. Hence, $q(\theta)$ satisfies almost everywhere $q_\theta(\theta) \leq 0$. Let $u\left(\theta, \hat{\theta}\right) = t\left(\hat{\theta}\right) - \theta q\left(\hat{\theta}\right)$ and

$$u(\theta) \equiv \max_{\hat{\theta}} \left\{ t\left(\hat{\theta}\right) - \theta q\left(\hat{\theta}\right) \right\}. \tag{32}$$

By the envelope theorem, $u_\theta(\theta) = -q(\theta)$ a.e.. Moreover, the least efficient type $\overline{\theta}$, is indifferent between participating and not, $u\left(\overline{\theta}\right) = 0$. Hence $u(\theta) = -\int_\theta^{\overline{\theta}} u_\theta(\tau)\, d\tau = \int_\theta^{\overline{\theta}} q(\tau) d\tau$. Since $q(\theta)$ is non-increasing $u(\theta)$ is convex. Finally, monotonicity makes the local first-order condition with respect to $\hat{\theta}$ sufficient for a global optimum in truth-telling. Substituting $t(\theta) = \theta q(\theta) + \int_\theta^{\overline{\theta}} q(\tau) d\tau$ into the objective one has

$$\int_{\underline{\theta}}^{\overline{\theta}} \left(V(q(\theta)) - \left(\theta q(\theta) + \int_\theta^{\overline{\theta}} q(\tau) d\tau \right) \right) f(\theta, e)\, d\theta. \tag{33}$$

Integration by parts delivers the representation in terms of expected surplus net of the agent's expected virtual surplus (Myerson (1981)), $\int_{\underline{\theta}}^{\overline{\theta}} \left(V(q(\theta)) - \left(\theta + \frac{F(\theta,e)}{f(\theta,e)} \right) q(\theta) \right) f(\theta, e)\, d\theta$.

The effort constraint: The effort constraint can be written as

$$e \in \arg\max_e \left\{ \int_{\underline{\theta}}^{\overline{\theta}} u(\theta) f(\theta, e)\, d\theta - g(e) \right\},$$

where $u(\theta) = \int_{\theta}^{\bar{\theta}} q(\tau)d\tau$ is the convex function defined in (32). Consider the integral $\int_{\underline{\theta}}^{\bar{\theta}} u(\theta) f(\theta, e) d\theta$. By conditions (8) and (12), an increase in e induces a mean preserving spread in the distribution of θ. Since $u(\theta)$ is convex, the equivalence results of Rothschild and Stiglitz (1970) imply that $\int_{\underline{\theta}}^{\bar{\theta}} u(\theta) f(\theta, e'') d\theta \geq \int_{\underline{\theta}}^{\bar{\theta}} u(\theta) f(\theta, e') d\theta$ for any $e'' > e'$, so $\int_{\underline{\theta}}^{\bar{\theta}} u(\theta) f(\theta, e) d\theta$ is non-decreasing in e. Consider now the derivative of $\int_{\underline{\theta}}^{\bar{\theta}} u(\theta) f(\theta, e) d\theta$ with respect to e, $\int_{\underline{\theta}}^{\bar{\theta}} u(\theta) f_e(\theta, e) d\theta$. Obviously, I can write

$$\int_{\underline{\theta}}^{\bar{\theta}} u(\theta) f_e(\theta, e) d\theta = -\int_{\underline{\theta}}^{\bar{\theta}} u(\theta) \frac{1}{\bar{\theta} - \underline{\theta}} d\theta + \int_{\underline{\theta}}^{\bar{\theta}} u(\theta) \left[f_e(\theta, e) + \frac{1}{\bar{\theta} - \underline{\theta}} \right] d\theta.$$

Notice that $f_e(\theta, e) + \frac{1}{\bar{\theta}-\underline{\theta}}$ is a density. To see this, observe that $f_e(\theta, e) + \frac{1}{\bar{\theta}-\underline{\theta}} \geq 0 \forall \theta$ by assumption and

$$\int_{\underline{\theta}}^{\bar{\theta}} \left[f_e(\theta, e) + \frac{1}{\bar{\theta} - \underline{\theta}} \right] d\theta = 1,$$

because $f(\theta, e)$ being a density for all e requires that $\int_{\underline{\theta}}^{\bar{\theta}} f_e(\theta, e) d\theta = 0$, and $\int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\bar{\theta}-\underline{\theta}} d\theta = 1$, $\frac{1}{\bar{\theta}-\underline{\theta}}$ is the density of the uniform distribution on $\left[\underline{\theta}, \bar{\theta}\right]$. Then, we can apply again the Rothschild and Stiglitz (1970) equivalence result, but this time to the density $\hat{f}(\theta, e) \equiv f_e(\theta, e) + \frac{1}{\bar{\theta}-\underline{\theta}}$. Let $\hat{F}(\theta, e) \equiv \int_{\underline{\theta}}^{\theta} \hat{f}(\tau, e) d\tau$ denote the cdf of the new distribution. We have,

$$\hat{F}(\theta, e) = F_e(\theta, e) + \frac{\theta - \underline{\theta}}{\bar{\theta} - \underline{\theta}}$$

and hence

$$\int_{\underline{\theta}}^{\theta} \hat{F}_e(\tau, e) d\tau = \int_{\underline{\theta}}^{\theta} F_{ee}(\tau, e) d\tau.$$

It follows from conditions (9) and (13) that a reduction in e induces a mean preserving spread in the distribution of θ under $\hat{F}(\theta, e)$. So, by the equivalence result of Rothschild and Stiglitz (1970) a decision-maker with a convex utility function dislikes an increase in e, which implies that for any $e'' > e'$

$$\int_{\underline{\theta}}^{\bar{\theta}} u(\theta) \hat{f}_e(\theta, e'') d\theta \leq \int_{\underline{\theta}}^{\bar{\theta}} u(\theta) \hat{f}(\theta, e') d\theta.$$

Adding $-\int_{\underline{\theta}}^{\bar{\theta}} u(\theta) \frac{1}{\bar{\theta}-\underline{\theta}} d\theta$ on both sides and eliminating the redundant terms, I have shown that $\int_{\underline{\theta}}^{\bar{\theta}} u(\theta) f_e(\theta, e) d\theta$ is non-increasing in e. Hence, $\int_{\underline{\theta}}^{\bar{\theta}} u(\theta) f(\theta, e) d\theta$ is non-decreasing and concave in e, which implies that the first-order condition,

$$\int_{\underline{\theta}}^{\bar{\theta}} u(\theta) f_e(\theta, e) d\theta - g_e(e) = 0,$$

is necessary and sufficient for an optimal level of e. Substituting for $u(\theta) = \int_{\theta}^{\bar{\theta}} q(\tau)d\tau$, integrating by parts, and using the assumption of non-moving supports, I can write

$$\int_{\underline{\theta}}^{\bar{\theta}} \int_{\theta}^{\bar{\theta}} q(\tau)d\tau f_e(\theta, e) d\theta = \int_{\underline{\theta}}^{\bar{\theta}} q(\theta) F_e(\theta, e) d\theta.$$

Hence, the agent's optimal choice of effort is the solution to the first-order condition

$$\int_{\underline{\theta}}^{\bar{\theta}} q(\theta) F_e(\theta, e)\, d\theta - g_e(e) = 0.$$

■

Proof of Proposition 2. Differentiating (7) with respect to e, I have

$$F_e(\theta, e) = \pi_e^{-1}(\theta; e).$$

The proof is given in two parts. In the first part I establish the properties of the conditional expectation function that follow from the assumptions; in the second part I use these characteristics to establish the properties of the distribution of the conditional expectation.

Part I: After an integration by parts, I can write the conditional expectation as

$$\pi(s, e) = \overline{\beta} - \int_{\underline{\beta}}^{\overline{\beta}} H(\beta | s, e)\, d\beta.$$

Differentiating with respect to s, I have

$$\pi_s(s, e) = -\int_{\underline{\beta}}^{\overline{\beta}} H_s(\beta | s, e)\, d\beta;$$

due to assumption (1), I have $\pi_s(s, e) > 0$ for all s.

Differentiating with respect to e I have

$$\pi_e(s, e) = -\int_{\underline{\beta}}^{\overline{\beta}} H_e(\beta | s, e)\, d\beta;$$

due to assumption (16), I have $\pi_e(s, e) < 0$ for $s \in (\underline{s}, \tilde{s})$, $\pi_e(s, e) > 0$ for $s \in (\tilde{s}, \overline{s})$, and $\pi_e(s, e) = 0$ for $s \in \{\underline{s}, \tilde{s}, \overline{s}\}$.

Part II: Differentiating the conditional expectation function totally, I obtain

$$d\theta = \pi_s(s, e)\, ds + \pi_e(s, e)\, de.$$

For a constant θ, I have

$$\frac{ds}{de} = -\frac{\pi_e(s, e)}{\pi_s(s, e)},$$

where $s = \pi^{-1}(\theta, e)$. Note that the inverse is well defined due to the property that $\pi_s(s, e) > 0$. Substituting for s and ds, I have

$$\pi_e^{-1}(\theta, e) = \frac{d\pi^{-1}(\theta, e)}{de} = -\frac{\pi_e\left(\pi^{-1}(\theta, e), e\right)}{\pi_s\left(\pi^{-1}(\theta, e), e\right)}. \tag{34}$$

Since $\theta \in \left\{\underline{\theta}, \tilde{\theta}, \overline{\theta}\right\}$ iff $s \in \{\underline{s}, \tilde{s}, \overline{s}\}$, $\theta \in \left(\underline{\theta}, \tilde{\theta}\right)$ iff $s \in (\underline{s}, \tilde{s})$, and $\theta \in \left(\tilde{\theta}, \overline{\theta}\right)$ iff $s \in (\tilde{s}, \overline{s})$, it follows that

$$F_e(\theta, e) = 0 \text{ for } \theta \in \left\{\underline{\theta}, \tilde{\theta}, \overline{\theta}\right\} \text{ and } F_e(\theta, e) \gtrless 0 \text{ for } \theta \lessgtr \tilde{\theta}.$$

■

Proof of Proposition 3. Differentiating (7) another time with respect to e, I have

$$F_{ee}(\theta, e) = \pi_{ee}^{-1}(\theta, e).$$

Differentiating (34) once more with respect to e, I obtain

$$\begin{aligned}\pi_{ee}^{-1}(\theta, e) &= -\frac{\pi_{ee}(\cdot, e)\,\pi_s(\cdot, e) + \pi_{es}\left(\pi^{-1}(\theta, e), e\right)\frac{ds}{de}\pi_s(\cdot, e) - \left(\pi_{ss}(\cdot, e)\frac{ds}{de} + \pi_{se}(\cdot, e)\right)\pi_e(\cdot, e)}{(\pi_s(\cdot, e))^2} \\ &= \frac{-1}{\pi_s(\cdot, e)}\left(\pi_{ee}(\cdot, e) + 2\frac{\pi_{es}(\cdot, e)}{\pi_s(\cdot, e)}\frac{ds}{de} + \frac{\pi_{ss}(\cdot, e)}{\pi_s(\cdot, e)}\left(\frac{ds}{de}\right)^2\right),\end{aligned}$$

where I have used (34), to obtain the second equality. Define

$$\Pi(\theta, e) \equiv \pi_{ee}(\cdot, e) + 2\frac{\pi_{es}(\cdot, e)}{\pi_s(\cdot, e)}\frac{ds}{de} + \frac{\pi_{ss}(\cdot, e)}{\pi_s(\cdot, e)}\left(\frac{ds}{de}\right)^2$$

The conditions in the proposition are satisfied iff

$$\pi_{ee}^{-1}(\theta, e) \lesseqgtr 0 \; for \; \theta \lesseqgtr \tilde{\theta}.$$

Since $\frac{-1}{\pi_s(\cdot, e)} < 0$, this is equivalent to

$$\Pi(\theta, e) \equiv \pi_{ee}(\cdot, e) + 2\frac{\pi_{es}(\cdot, e)}{\pi_s(\cdot, e)}\frac{ds}{de} + \frac{\pi_{ss}(\cdot, e)}{\pi_s(\cdot, e)}\left(\frac{ds}{de}\right)^2 \geq (\leq)\, 0 \; for \; \theta \leq (\geq)\, \tilde{\theta}. \tag{35}$$

I now prove that condition (18) implies that the function $\pi(s, e)$ satisfies condition (35). To show this, I will treat $\frac{ds}{de}$ as a free variable despite the fact that $\frac{ds}{de} = -\frac{\pi_e\left(\pi^{-1}(\theta, e), e\right)}{\pi_s\left(\pi^{-1}(\theta, e), e\right)}$ is determined by the function π. Letting $\Pi\left(\frac{ds}{de}, \theta, e\right)$ denote the function $\Pi(\theta, e)$ when $\frac{ds}{de}$ is a free variable, I choose $\frac{ds}{de}$ to minimize $\Pi\left(\frac{ds}{de}, \theta, e\right)$ for $\theta \leq \tilde{\theta}$ and choose $\frac{ds}{de}$ to maximize $\Pi\left(\frac{ds}{de}, \theta, e\right)$ for $\theta \geq \tilde{\theta}$. Since the procedures are identical in both cases, I treat only the first case.

For the case $\theta \leq \tilde{\theta}$, let $\frac{ds}{de}^*$ denote a minimizer of $\Pi\left(\frac{ds}{de}, \theta, e\right)$. By definition, $\Pi(\theta, e) \geq \Pi\left(\frac{ds}{de}^*, \theta, e\right)$. Hence, if $\Pi\left(\frac{ds}{de}^*, \theta, e\right) \geq 0$ for all $\theta \leq \tilde{\theta}$, then $\Pi(\theta, e) \geq 0$ for all $\theta \leq \tilde{\theta}$ and the proof is complete.

First, I show that $\Pi\left(\frac{ds}{de}, \theta, e\right)$ is convex in $\frac{ds}{de}$ for $\theta \leq \tilde{\theta}$. To see this, note that since $H(\beta\,|s, e)$ is concave in (s, e) for $s \in (\underline{s}, \tilde{s})$, $-H(\beta\,|s, e)$ is convex in (s, e) for $s \in (\underline{s}, \tilde{s})$. Moreover, convexity is preserved under summation and integration. Hence, $\pi(s, e) = \overline{\beta} - \int_{\underline{\beta}}^{\overline{\beta}} H(\beta|\, s, e)\, d\beta$ is convex in (s, e)

for $s \in (\underline{s}, \tilde{s})$. Hence, I have $\pi_{ss}(s,e) > 0$, $\pi_{ee}(s,e) > 0$, and $\pi_{ss}(s,e)\pi_{ee}(s,e) - (\pi_{se}(s,e))^2 \geq 0$ for all e and $s \in (\underline{s}, \tilde{s})$. Differentiating $\Pi\left(\frac{ds}{de}, \theta, e\right)$ twice with respect to $\frac{ds}{de}$ I get

$$\frac{\partial^2 \Pi\left(\frac{ds}{de}, \theta, e\right)}{\partial\left(\frac{ds}{de}\right)^2} = 2\frac{\pi_{ss}(\cdot,e)}{\pi_s(\cdot,e)} > 0 \text{ for } \theta \in \left(\underline{\theta}, \tilde{\theta}\right).$$

Hence, $\Pi\left(\frac{ds}{de}, \theta, e\right)$ has a unique minimizer, which satisfies the first-order condition

$$\frac{ds}{de}^* = -\frac{\pi_{es}(\cdot,e)}{\pi_{ss}(\cdot,e)},$$

and the minimum satisfies

$$\Pi\left(\frac{ds}{de}^*, \theta, e\right) = \pi_{ee}(\cdot,e) - \frac{(\pi_{es}(\cdot,e))^2}{\pi_s(\cdot,e)\pi_{ss}(\cdot,e)}.$$

Convexity of $\pi(s,e)$ implies that $\Pi\left(\frac{ds}{de}^*, \theta, e\right) \geq 0$. Since $\Pi(\theta,e) \geq \Pi\left(\frac{ds}{de}^*, \theta, e\right)$ by definition, I have shown that $\Pi(\theta,e) \geq 0$.

The proof for $s \in (\tilde{s}, \overline{s})$ is analogous and therefore omitted. It follows that $F_{ee}(\theta,e) = \pi_{ee}^{-1}(\theta,e) \lesseqgtr 0$ for $\theta \lesseqgtr \tilde{\theta}$. ■

Proof of Proposition 4. Using (7) and (10), the inverse hazard rate is

$$\frac{F(\theta,e)}{f(\theta,e)} = \pi^{-1}(\theta;e)\pi_s\left(\pi^{-1}(\theta;e), e\right).$$

Differentiating with respect to θ I obtain

$$\begin{aligned}\frac{\partial}{\partial\theta}\frac{F(\theta,e)}{f(\theta,e)} &= \frac{\pi_s\left(\pi^{-1}(\theta;e),e\right)}{\pi_s\left(\pi^{-1}(\theta;e),e\right)} + \pi^{-1}(\theta;e)\frac{\pi_{ss}\left(\pi^{-1}(\theta;e),e\right)}{\pi_s\left(\pi^{-1}(\theta;e),e\right)} \\ &= 1 + \pi^{-1}(\theta;e)\frac{\pi_{ss}\left(\pi^{-1}(\theta;e),e\right)}{\pi_s\left(\pi^{-1}(\theta;e),e\right)}.\end{aligned}$$

Thus,

$$\frac{\partial}{\partial\theta}\frac{F(\theta,e)}{f(\theta,e)} \geq 0 \Leftrightarrow 1 + \frac{s\pi_{ss}(s,e)}{\pi_s(s,e)} \geq 0.$$

■

Proof of Proposition 5. $e = 0$ is optimal for the agent if and only if $q(\theta) = q$ for all θ and $t - \overline{\theta}q \geq 0$ for all θ. The best such contract from the principal's perspective solves

$$\max_{q,t} \int_{\underline{\theta}}^{\overline{\theta}} (V(q) - t)\, dF(\theta,e)$$

$$s.t.\ t - \overline{\theta}q \geq 0.$$

The optimal contract in this class satisfies

$$V_q(q)|_{q=\hat{q}} = \overline{\theta}$$

and $\hat{t} = \overline{\theta}\hat{q}$. This contract is very costly to the principal, because he pays the agent always as if this one had the highest possible cost. Suppose instead the principal offers the contract

$$q^{BM}(\theta, e) = V_q^{-1}\left(\theta + \frac{F(\theta, e)}{f(\theta, e)}\right). \tag{36}$$

This contract corresponds to the case where the principal neglects his influence on the agent's effort choice but offers a contract which elicits information truthfully. Notice that due to condition (19) in Proposition 4 $q^{BM}(\theta, e)$ is strictly monotonic in θ. Since the principal extracts some rents offering quantity schedule $q^{BM}(\theta, e)$ with the associated transfer schedule to the agent, this contract dominates the contract $\{\hat{t}, \hat{q}\}$.

I now prove that there exist effort levels such that the principal's contract offer is a best reply to the agent's choice of effort and the agent's choice of effort is consistent with the contract offered; that is, in addition to (36), it must also be true that

$$\left.\int_{\underline{\theta}}^{\overline{\theta}} F_e(\theta, \hat{e})\, q^{BM}(\theta, e)\, d\theta - g_e(\hat{e})\right|_{\hat{e}=e} = 0. \tag{37}$$

Consider the agent's utility as a function of $\hat{e}$ and e :

$$\int_{\underline{\theta}}^{\overline{\theta}} F(\theta, \hat{e})\, q^{BM}(\theta, e)\, d\theta - g(\hat{e}).$$

Under our assumptions, $q^{BM}(\theta, e)$ is differentiable in e. Hence, the agent's utility is continuous in e and $\hat{e}$ and strictly concave in $\hat{e}$. By the theorem of the maximum, the maximizer correspondence of the agent's utility function with respect to $\hat{e}$ is upper hemicontinuous. By strict concavity in $\hat{e}$, the maximizer correspondence is in fact a function. Since a single valued correspondence is upper hemicontinuous if and only if it is continuous as a function, it follows that the maximizer of the agent's utility function is a continuous function of the principal's conjectured effort level. Let $\hat{e} = r\left(q^{BM}(\theta, e)\right)$ denote the agent's optimal choice of effort when the principal offers contract $q^{BM}(\theta, e)$. Define

$$\Gamma(e) \equiv \int_{\underline{\theta}}^{\overline{\theta}} F_e\left(\theta, r\left(q^{BM}(\theta, e)\right)\right) V_q^{-1}\left(\theta + \frac{F(\theta, e)}{f(\theta, e)}\right) d\theta - g_e\left(r\left(q^{BM}(\theta, e)\right)\right). \tag{38}$$

An equilibrium effort (that satisfies both (36) and (37)) is then defined as a solution to the equation $\Gamma(e) = 0$, or, equivalently, as fixed point satisfying $e = r\left(q^{BM}(\theta, e)\right)$.

Such a fixed point must exist, because I have $r\left(q^{BM}(\theta, e)\right)\big|_{e=0} > 0$ and $r\left(q^{BM}(\theta, e)\right)\big|_{e=\overline{e}} < \overline{e}$. To see the first point, notice that the family of distributions has a monotone hazard rate for all e. Therefore, $q^{BM}(\theta, 0)$ is a strictly monotonic contract, and the agent has a strictly positive incentive to acquire information. To see the second point, notice that the marginal cost of effort goes to infinity as e approaches $\overline{e}$. Since $r\left(q^{BM}(\theta, e)\right)$ is a continuous function, it must have a fixed point by Brouwer's fixed point theorem. ■

Proof of Proposition 6. The proof is split into two parts. In the first part, I show that the multiplier μ is negative for $e < \underline{\tilde{e}}$ and that μ is positive for $e > \overline{\tilde{e}}$. In the second part, I give sufficient conditions for a small increase in the effort level to be beneficial to the principal around $\mu = 0$.

Part i) If $e < \underline{\tilde{e}}$ then $\mu < 0$; if $e > \overline{\tilde{e}}$ then $\mu > 0$.

By the definition of the smallest fixed point, we know that $r\left(q^{BM}(\theta, e)\right) > e$ for $e < \underline{\tilde{e}}$. To make sure that the agent indeed chooses e, the principal must reduce the agent's incentive to acquire information. This is achieved by reducing production for $\theta \leq \tilde{\theta}$ and increasing production for $\theta \geq \tilde{\theta}$. From the condition of optimality,

$$V_q(q(\theta)) = \theta + \frac{F(\theta, e)}{f(\theta, e)} - \mu \frac{F_e(\theta, e)}{f(\theta, e)}$$

we conclude that $\mu < 0$ since $F_e(\theta, e) \geq 0$ for $\theta \leq \tilde{\theta}$ and $F_e(\theta, e) \leq 0$ for $\theta > \tilde{\theta}$. The proof for $e > \overline{\tilde{e}}$ is analogous and therefore omitted.

Part ii) The marginal effect of a small increase in e around a point where $\mu = 0:$

Let

$$W(e) \equiv \max_{q(\theta)} \left\{ \begin{array}{c} \int_{\underline{\theta}}^{\bar{\theta}} \left(V(q(\theta)) - \left(\theta + \frac{F(\theta,e)}{f(\theta,e)} \right) q(\theta) \right) f(\theta, e)\, d\theta \\ + \mu \left(\int_{\underline{\theta}}^{\bar{\theta}} F_e(\theta, e)\, q(\theta)\, d\theta - g_e(e) \right) \end{array} \right\}$$

Invoking the envelope theorem I have around a point where $\mu = 0$

$$W_e(e) = \int_{\underline{\theta}}^{\bar{\theta}} \left(V(q(\theta)) - \left(\theta q(\theta) + \int_{\theta}^{\bar{\theta}} q(\tau) d\tau \right) \right) f_e(\theta, e)\, d\theta.$$

Integrating by parts, and noting that $F_e(\underline{\theta}, e) = F_e(\bar{\theta}, e) = 0$, I can write

$$W_e(e) = -\int_{\underline{\theta}}^{\bar{\theta}} (V_q(q(\theta)) - \theta)\, q_\theta(\theta)\, F_e(\theta, e)\, d\theta.$$

Substituting for $q_\theta(\theta) = \frac{\frac{\partial}{\partial \theta}\left(\theta + \frac{F(\theta,e)}{f(\theta,e)}\right)}{V_{qq}(q(\theta))}$, for $\frac{F(\theta,e)}{f(\theta,e)} = V_q(q(\theta)) - \theta$, and multiplying by $\frac{V_q(q(\theta))}{\theta + \frac{F(\theta,e)}{f(\theta,e)}} = 1$ I obtain

$$W_e(e) = -\int_{\underline{\theta}}^{\bar{\theta}} \frac{\frac{F(\theta,e)}{f(\theta,e)}}{V_{qq}(q(\theta))} \frac{V_q(q(\theta))}{\theta + \frac{F(\theta,e)}{f(\theta,e)}} \frac{\partial}{\partial \theta} \left(\theta + \frac{F(\theta, e)}{f(\theta, e)} \right) F_e(\theta, e)\, d\theta$$

Recall that $\rho(q) = \frac{-V_{qq}(q)}{V_q(q)}$ and let $\phi(q) \equiv \frac{V_q(q(\theta))}{V_{qq}(q(\theta))} = -\frac{1}{\rho(q)}$. Then, recollecting terms, I can write

$$W_e(e) = -\int_{\underline{\theta}}^{\bar{\theta}} \left(\phi(q(\theta)) \frac{\frac{F(\theta,e)}{f(\theta,e)}}{\theta + \frac{F(\theta,e)}{f(\theta,e)}} \frac{\partial}{\partial \theta} \left(\theta + \frac{F(\theta, e)}{f(\theta, e)} \right) \right) F_e(\theta, e)\, d\theta.$$

After another integration by parts, using the fact that $\int_{\underline{\theta}}^{\theta} F_e(\tau, e)\, d\tau = 0$ for $\theta = \underline{\theta}$ and for $\theta = \bar{\theta}$, I have

$$W_e(e) = \int_{\underline{\theta}}^{\bar{\theta}} \left(\int_{\underline{\theta}}^{\theta} F_e(\tau, e)\, d\tau \frac{\partial}{\partial \theta} \left[\phi(q(\theta)) \frac{\frac{F(\theta,e)}{f(\theta,e)}}{\theta + \frac{F(\theta,e)}{f(\theta,e)}} \frac{\partial}{\partial \theta} \left(\theta + \frac{F(\theta, e)}{f(\theta, e)} \right) \right] \right) d\theta.$$

Notice that $\int_{\underline{\theta}}^{\theta} F_e(\tau, e)\, d\tau \geq 0$ by proposition 2. Thus, to prove the result, it suffices to sign the expression $\frac{\partial}{\partial \theta}[\cdot]$. Define

$$X(\theta) \equiv \phi(q(\theta)) \frac{\frac{F(\theta,e)}{f(\theta,e)}}{\theta + \frac{F(\theta,e)}{f(\theta,e)}} \frac{\partial}{\partial \theta} \left(\theta + \frac{F(\theta, e)}{f(\theta, e)} \right).$$

Performing the differentiation, I have

$$\begin{aligned} X_\theta(\theta) &= \phi_q(q(\theta))\, q_\theta(\theta) \frac{\frac{F(\theta,e)}{f(\theta,e)}}{\theta + \frac{F(\theta,e)}{f(\theta,e)}} \frac{\partial}{\partial \theta} \left(\theta + \frac{F(\theta, e)}{f(\theta, e)} \right) \\ &\quad + \phi(q(\theta)) \frac{\theta \frac{\partial}{\partial \theta} \frac{F(\theta,e)}{f(\theta,e)} - \frac{F(\theta,e)}{f(\theta,e)}}{\left(\theta + \frac{F(\theta,e)}{f(\theta,e)} \right)^2} \frac{\partial}{\partial \theta} \left(\theta + \frac{F(\theta, e)}{f(\theta, e)} \right) \\ &\quad + \phi(q(\theta)) \frac{\frac{F(\theta,e)}{f(\theta,e)}}{\theta + \frac{F(\theta,e)}{f(\theta,e)}} \frac{\partial^2}{\partial \theta^2} \left(\theta + \frac{F(\theta, e)}{f(\theta, e)} \right). \end{aligned}$$

To sign, these expressions, notice that $\theta \frac{\partial}{\partial \theta} \frac{F(\theta,e)}{f(\theta,e)} - \frac{F(\theta,e)}{f(\theta,e)}$ and $\frac{\partial^2}{\partial \theta^2} \left(\theta + \frac{F(\theta,e)}{f(\theta,e)} \right)$ both go to zero when $F(\theta, e)$ converges to a uniform distribution. Hence the sign of $X_\theta(\theta)$ is the sign of $-\phi_q(q(\theta))$ (since $q_\theta(\theta) < 0$). Since $sign(\phi_q(q)) = sign(\rho_q(q))$, it follows that $\rho_q(q) < 0$ implies $W_e(e) > 0$ and $\rho_q(q) > 0$ implies that $W_e(e) < 0$. ■

Proof of Lemma 1. Suppose the cost function is changed to $\hat{g}(e) = g(e) + \alpha g(e)$ where α is a parameter that takes values in the interval $[-\overline{\alpha}, \overline{\alpha}]$, and where $\overline{\alpha} < 1$. Notice that the function $\hat{g}(e)$ is an Inada cost function for any such α, and an interior solution is guaranteed. The marginal cost to the agent of exerting effort e is now $\hat{g}_e(e) = g_e(e) + \alpha g_e(e)$. The multiplier μ is equal to the change in the principal's utility due to a change in $\alpha g_e(e)$. Since e is a constant, I can define $c(\alpha) \equiv \alpha g_e(e)$. Let $W(c)$ denote the welfare of the principal as a function of c

$$\begin{aligned} W(c) = \max_{q(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} &\left(V(q^*(\theta)) - \left(\theta + \frac{F(\theta, e)}{f(\theta, e)} \right) q^*(\theta) \right) f(\theta, e)\, d\theta \\ &+ \mu \left(\int_{\underline{\theta}}^{\bar{\theta}} F_e(\theta, e)\, q^*(\theta)\, d\theta - g_e(e) - c \right) \end{aligned}$$

and let $q^*(\theta)$ denote the optimal quantity schedule for $c = 0$. Finally, let $W(0)$ denote the value of welfare for $c = 0$ (that is, $\alpha = 0$). From the envelope theorem, I have

$$W_c(c) = -\mu.$$

I now provide bounds on the multiplier. I distinguish two cases, a) $\alpha > 0$ and b) $\alpha < 0$. Since $c(\alpha) \gtrless 0$ iff $\alpha \gtrless 0$ I directly state my results in terms of c.

Case a): If $c > 0$, then the principal must do at least as well as when he offers a contract with production schedule

$$\hat{q}(\theta) = \begin{cases} q^*(\theta) + \varepsilon \frac{F_e(\theta,e)}{f(\theta,e)} & \text{for } \theta \leq \tilde{\theta} \\ q^*(\theta) & \text{for } \theta > \tilde{\theta}. \end{cases}$$

From the agent's first-order conditions with respect to e, for $c = 0$ and for $c > 0$, respectively, I have

$$\begin{aligned} \int_{\underline{\theta}}^{\bar{\theta}} F_e(\theta, e)\, q^*(\theta)\, d\theta &= g_e(e) \text{ and} \\ \int_{\underline{\theta}}^{\bar{\theta}} F_e(\theta, e)\, \hat{q}(\theta)\, d\theta &= g_e(e) + c. \end{aligned}$$

Hence, ε and c are related by the condition

$$\varepsilon = \frac{c}{\int_{\underline{\theta}}^{\tilde{\theta}} F_e(\theta, e) \frac{F_e(\theta,e)}{f(\theta,e)} d\theta}.$$

Define

$$Y^+ \equiv \int_{\underline{\theta}}^{\tilde{\theta}} F_e(\theta, e) \frac{F_e(\theta, e)}{f(\theta, e)} d\theta.$$

Notice that

$$\begin{aligned} Y^+ &= F\left(\tilde{\theta}, e\right) \int_{\underline{\theta}}^{\tilde{\theta}} \left(\frac{F_e(\theta, e)}{f(\theta, e)}\right)^2 \frac{f(\theta, e)}{F\left(\tilde{\theta}, e\right)} d\theta \\ &= F\left(\tilde{\theta}, e\right) \left(Var\left(\frac{F_e(\theta, e)}{f(\theta, e)} \middle| \theta \leq \tilde{\theta} \right) + \left(E_\Theta \left[\frac{F_e(\theta, e)}{f(\theta, e)} \middle| \theta \leq \tilde{\theta} \right] \right)^2 \right), \end{aligned}$$

where the second equality follows from completing the square by $E_\Theta \left[\frac{F_e(\theta,e)}{f(\theta,e)} \middle| \theta \leq \tilde{\theta} \right] - E_\Theta \left[\frac{F_e(\theta,e)}{f(\theta,e)} \middle| \theta \leq \tilde{\theta} \right]$.

For the welfare of the principal, I have

$$\begin{aligned} W(c) &\geq \int_{\underline{\theta}}^{\bar{\theta}} \left(V\left(q^*(\theta) + 1_{\theta \leq \tilde{\theta}} \varepsilon \frac{F_e(\theta, e)}{f(\theta, e)} \right) - \left(\theta + \frac{F(\theta, e)}{f(\theta, e)} \right) \left(q^*(\theta) + 1_{\theta \leq \tilde{\theta}} \varepsilon \frac{F_e(\theta, e)}{f(\theta, e)} \right) \right) f(\theta, e)\, d\theta \\ &\geq \int_{\underline{\theta}}^{\bar{\theta}} \left(V(q^*(\theta)) - \left(\theta + \frac{F(\theta, e)}{f(\theta, e)} \right) \left(q^*(\theta) + 1_{\theta \leq \tilde{\theta}} \varepsilon \frac{F_e(\theta, e)}{f(\theta, e)} \right) \right) f(\theta, e)\, d\theta \\ &= W(0) - \varepsilon \int_{\underline{\theta}}^{\tilde{\theta}} \left(\theta + \frac{F(\theta, e)}{f(\theta, e)} \right) \frac{F_e(\theta, e)}{f(\theta, e)} f(\theta, e)\, d\theta, \end{aligned}$$

where the first inequality follows from the definition of $W(c)$ and the second uses the fact that $\varepsilon \frac{F_e(\theta,e)}{f(\theta,e)} \geq 0$ for all $\theta \leq \tilde{\theta}$, so that the principal must do at least as well as by simply not consuming the extra quantity that is produced. Define

$$Z^{+} \equiv \int_{\underline{\theta}}^{\tilde{\theta}} \left(\theta + \frac{F(\theta,e)}{f(\theta,e)}\right) \frac{F_e(\theta,e)}{f(\theta,e)} f(\theta,e)\, d\theta$$

and notice that, again by completing the square,

$$\begin{aligned} Z^{+} &= F\left(\tilde{\theta},e\right) \int_{\underline{\theta}}^{\tilde{\theta}} \left(\theta + \frac{F(\theta,e)}{f(\theta,e)}\right) \frac{F_e(\theta,e)}{f(\theta,e)} \frac{f(\theta,e)}{F\left(\tilde{\theta},e\right)} d\theta \\ &= F\left(\tilde{\theta},e\right) \left(\begin{array}{c} Cov\left(\theta + \frac{F(\theta,e)}{f(\theta,e)}, \frac{F_e(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right) \\ +E_{\Theta}\left[\theta + \frac{F(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right] E_{\Theta}\left[\frac{F_e(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right] \end{array} \right). \end{aligned}$$

Combining these results, I can write

$$W(c) - W(0) \ge -c\frac{Z^{+}}{Y^{+}}.$$

Dividing by $c > 0$ and taking limits as $c \to 0$ I obtain

$$\lim_{c \to 0} \frac{W(c) - W(0)}{c} = -\mu \ge -\frac{Z^{+}}{Y^{+}},$$

or

$$\mu \le \frac{Z^{+}}{Y^{+}}.$$

Substituting for Y^{+} and Z^{+}, I have

$$\begin{aligned} \frac{Z^{+}}{Y^{+}} &= \frac{Cov\left(\theta + \frac{F(\theta,e)}{f(\theta,e)}, \frac{F_e(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right)}{Var\left(\frac{F_e(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right) + \left(E_{\Theta}\left[\frac{F_e(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right]\right)^2} + \frac{E_{\Theta}\left[\theta + \frac{F(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right] E_{\Theta}\left[\frac{F_e(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right]}{Var\left(\frac{F_e(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right) + \left(E_{\Theta}\left[\frac{F_e(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right]\right)^2} \\ &\le \frac{\sqrt{Var\left(\theta + \frac{F(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right)}\sqrt{Var\left(\frac{F_e(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right)}}{Var\left(\frac{F_e(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right) + \left(E_{\Theta}\left[\frac{F_e(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right]\right)^2} + \frac{E_{\Theta}\left[\theta + \frac{F(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right] E_{\Theta}\left[\frac{F_e(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right]}{Var\left(\frac{F_e(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right) + \left(E_{\Theta}\left[\frac{F_e(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right]\right)^2} \\ &\le \frac{\sqrt{Var\left(\theta + \frac{F(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right)}\sqrt{Var\left(\frac{F_e(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right)}}{Var\left(\frac{F_e(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right)} + \frac{E_{\Theta}\left[\theta + \frac{F(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right] E_{\Theta}\left[\frac{F_e(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right]}{\left(E_{\Theta}\left[\frac{F_e(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right]\right)^2} \\ &= \frac{\sqrt{Var\left(\theta + \frac{F(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right)}}{\sqrt{Var\left(\frac{F_e(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right)}} + \frac{E_{\Theta}\left[\theta + \frac{F(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right]}{E_{\Theta}\left[\frac{F_e(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right]}, \end{aligned}$$

where the first inequality follows from the fact that for any two random variables A and B, $Cov(A,B) \le \sqrt{Var(A)}\sqrt{Var(B)}$ and the second from the fact that the denominators become smaller. Hence, I have shown that

$$\mu \le \frac{\sqrt{Var\left(\theta + \frac{F(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right)}}{\sqrt{Var\left(\frac{F_e(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right)}} + \frac{E_{\Theta}\left[\theta + \frac{F(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right]}{E_{\Theta}\left[\frac{F_e(\theta,e)}{f(\theta,e)} \middle| \theta \le \tilde{\theta}\right]}. \tag{39}$$

Case b) $c < 0$. In this case, the principal can do at least as well as by offering the contract

$$\hat{q}(\theta) = \begin{cases} q^*(\theta) & \text{for } \theta < \tilde{\theta} \\ q^*(\theta) + \varepsilon \frac{F_e(\theta,e)}{f(\theta,e)} & \text{for } \theta \geq \tilde{\theta}. \end{cases}$$

ε is again defined by the first-order condition for effort

$$\varepsilon = -\frac{c}{Y^-},$$

where

$$Y^- \equiv \int_{\tilde{\theta}}^{\overline{\theta}} F_e(\theta,e) \frac{F_e(\theta,e)}{f(\theta,e)} d\theta.$$

Notice that $\varepsilon < 0$. I have

$$\begin{aligned} W(c) &\geq \int_{\underline{\theta}}^{\overline{\theta}} \left(V\left(q^*(\theta) + 1_{\theta \geq \tilde{\theta}} \varepsilon \frac{F_e(\theta,e)}{f(\theta,e)} \right) - \left(\theta + \frac{F(\theta,e)}{f(\theta,e)} \right) \left(q^*(\theta) + 1_{\theta \geq \tilde{\theta}} \varepsilon \frac{F_e(\theta,e)}{f(\theta,e)} \right) \right) f(\theta,e) \, d\theta \\ &\geq \int_{\underline{\theta}}^{\overline{\theta}} \left(V(q^*(\theta)) - \left(\theta + \frac{F(\theta,e)}{f(\theta,e)} \right) \left(q^*(\theta) + 1_{\theta \geq \tilde{\theta}} \varepsilon \frac{F_e(\theta,e)}{f(\theta,e)} \right) \right) f(\theta,e) \, d\theta \\ &= W(0) - \varepsilon \int_{\tilde{\theta}}^{\overline{\theta}} \left(\theta + \frac{F(\theta,e)}{f(\theta,e)} \right) \frac{F_e(\theta,e)}{f(\theta,e)} f(\theta,e) \, d\theta, \end{aligned}$$

where the first inequality uses the definition of $W(c)$, the second uses the fact that $\varepsilon \frac{F_e(\theta,e)}{f(\theta,e)}$ is non-negative for $\theta \geq \tilde{\theta}$, so the principal's utility is at least as high as when he does not consume the additional quantity at all. Define

$$Z^- \equiv \int_{\tilde{\theta}}^{\overline{\theta}} \left(\theta + \frac{F(\theta,e)}{f(\theta,e)} \right) \frac{F_e(\theta,e)}{f(\theta,e)} f(\theta,e) \, d\theta.$$

Substituting from the agent's first-order condition for ε, and taking limits as c goes to zero, I can write

$$\lim_{c \to 0} \frac{W(0) - W(c)}{-c} = -\mu \leq \frac{Z^-}{Y^-},$$

since the left-hand side is the left-side differential of W with respect to c.

Performing the same operations as in part a) I find that

$$\begin{aligned} \frac{Z^-}{Y^-} &\leq \frac{\sqrt{Var\left(\frac{F_e(\theta,e)}{f(\theta,e)} \middle| \theta \geq \tilde{\theta} \right)} \sqrt{Var\left(\theta + \frac{F(\theta,e)}{f(\theta,e)} \middle| \theta \geq \tilde{\theta} \right)} + E_\Theta\left(\theta + \frac{F(\theta,e)}{f(\theta,e)} \middle| \theta \geq \tilde{\theta} \right) E_\Theta\left(\frac{F_e(\theta,e)}{f(\theta,e)} \middle| \theta \geq \tilde{\theta} \right)}{Var\left(\frac{F_e(\theta,e)}{f(\theta,e)} \middle| \theta \geq \tilde{\theta} \right) + \left(E_\Theta\left(\frac{F_e(\theta,e)}{f(\theta,e)} \middle| \theta \geq \tilde{\theta} \right) \right)^2} \\ &\leq \frac{\sqrt{Var\left(\theta + \frac{F(\theta,e)}{f(\theta,e)} \middle| \theta \geq \tilde{\theta} \right)}}{\sqrt{Var\left(\frac{F_e(\theta,e)}{f(\theta,e)} \middle| \theta \geq \tilde{\theta} \right)}}, \end{aligned}$$

where the first inequality is again based on the observation that for any two random variables A and B, $Cov(A,B) \leq \sqrt{Var(A)}\sqrt{Var(B)}$, and the second inequality uses the fact that $E_\Theta\left(\frac{F_e(\theta,e)}{f(\theta,e)}\middle|\theta \geq \tilde{\theta}\right) < 0$. Hence, I have shown that

$$\mu \geq -\frac{\sqrt{Var\left(\theta + \frac{F(\theta,e)}{f(\theta,e)}\middle|\theta \geq \tilde{\theta}\right)}}{\sqrt{Var\left(\frac{F_e(\theta,e)}{f(\theta,e)}\middle|\theta \geq \tilde{\theta}\right)}}.$$

■

Proof of Result 1. Suppose the principal wishes to implement a level of effort e such that the agent's equilibrium type distribution is supported on $\left[\underline{\theta}, \overline{\theta}\right]$. In equilibrium, the effort level e is implemented by a pair of schedules $(q(\theta), t(\theta))$ defined on $\left[\underline{\theta}, \overline{\theta}\right]$. However, since the agent can deviate to higher effort levels, the message space needs to be extended to the set $\left[\underline{\theta}(\overline{e}), \overline{\theta}(\overline{e})\right]$, so that the agent can still be induced to reveal his true type, regardless of what his effort choice is. To deal with this issue, I extend the domain of definition of the schedules $(q(\theta), t(\theta))$ to $\left[\underline{\theta}(\overline{e}), \overline{\theta}(\overline{e})\right]$ as follows. I let

$$\hat{q}(\theta) \equiv \begin{cases} q(\underline{\theta}) & \text{for } \theta \leq \underline{\theta} \\ q(\theta) & \text{for } \underline{\theta} \leq \theta \leq \overline{\theta} \\ 0 & \text{for } \theta > \overline{\theta}. \end{cases}$$

$\hat{t}(\theta)$ is defined as $\hat{t}(\theta) = \theta\hat{q}(\theta) + \int_{\theta}^{\overline{\theta}(\overline{e})} \hat{q}(\tau)\,d\tau$. Notice that the agent participates under the menu $\left(\hat{q}(\theta), \hat{t}(\theta)\right)$ for all θ, whereas under $(q(\theta), t(\theta))$ the agent rejects if $\theta > \overline{\theta}$. Since under $\left(\hat{q}(\theta), \hat{t}(\theta)\right)$ the principal offers the null-contract for $\theta > \overline{\theta}$, the two formulations are equivalent in the sense that they induce the same expected utilities. Finally, under $\left(\hat{q}(\theta), \hat{t}(\theta)\right)$, the principal offers contract $q(\underline{\theta}), t(\underline{\theta})$ to all types with $\theta \leq \underline{\theta}$. Since the principal wishes to implement a distribution of types which is supported only on the interval $\left[\underline{\theta}, \overline{\theta}\right]$, any pair of quantity and payment schedules for $\theta < \underline{\theta}$ is optimal as long as it is incentive compatible. Since $\hat{q}(\theta)$ is monotonic $\left(\hat{q}(\theta), \hat{t}(\theta)\right)$ is indeed incentive compatible.

The agent's ex ante expected utility (gross of costs of information acquisition) can now be written as

$$E_\Theta[u(\theta)] = \int_{\underline{\theta}(\overline{e})}^{\overline{\theta}(\overline{e})} \int_{\theta}^{\overline{\theta}(\overline{e})} \hat{q}(\tau)\,d\tau dF(\theta,e).$$

Three cases can arise. In case i) the agent chooses the equilibrium level of effort in which case $\overline{\theta}(e) = \overline{\theta}$ and $\underline{\theta}(e) = \underline{\theta}$. This case corresponds to the case discussed in the main model. In case

ii) the agent deviates from the equilibrium effort to a higher level, in which case $\overline{\theta}(e) > \overline{\theta}$ and $\underline{\theta}(e) < \underline{\theta}$. In case iii) the agent deviates to a lower level of effort which implies that $\overline{\theta}(e) < \overline{\theta}$ and $\underline{\theta}(e) > \underline{\theta}$. I now show that in all three cases, I can write

$$E_\Theta [u(\theta)] = \int_{\underline{\theta}(e)}^{\overline{\theta}(e)} \max \left\{ 0, \int_{\theta}^{\overline{\theta}} \hat{q}(\tau)\, d\tau \right\} dF(\theta, e). \tag{40}$$

In case i) $\overline{\theta}(e) = \overline{\theta}$ and $\underline{\theta}(e) = \underline{\theta}$ and $dF(\theta, e) = 0$ for $\theta \in [\underline{\theta}(\overline{e}), \underline{\theta})$ or $\theta \in (\overline{\theta}, \overline{\theta}(\overline{e})]$. Moreover, $\hat{q}(\theta) \equiv q(\theta)$ for $\theta \in [\underline{\theta}, \overline{\theta}]$. Hence, $E_\Theta [u(\theta)] = \int_{\underline{\theta}}^{\overline{\theta}} \int_{\theta}^{\overline{\theta}} q(\tau)\, d\tau dF(\theta, e)$, a special case of (40). In case ii) $dF(\theta, e) = 0$ for $\theta \in [\underline{\theta}(\overline{e}), \underline{\theta}(e))$ or $\theta \in (\overline{\theta}(e), \overline{\theta}(\overline{e})]$. Moreover, under the pair of schedules $(q(\theta), t(\theta))$ the agent rejects in case $\theta \in (\overline{\theta}, \overline{\theta}(e)]$ and obtains no rent. Since $\int_{\theta}^{\overline{\theta}} \hat{q}(\tau)\, d\tau < 0$ for $\theta \in (\overline{\theta}, \overline{\theta}(e)]$, $\max \left\{ 0, \int_{\theta}^{\overline{\theta}} \hat{q}(\tau)\, d\tau \right\} = 0$ in this case. So, expected utility can be written as in (40) again. Finally, in case iii) $dF(\theta, e) = 0$ for $\theta \in [\underline{\theta}(\overline{e}), \underline{\theta}(e))$ and $\theta \in (\overline{\theta}(e), \overline{\theta}(\overline{e})]$, and moreover $\int_{\theta}^{\overline{\theta}} \hat{q}(\tau)\, d\tau \geq 0$ for all θ such that $dF(\theta, e) > 0$. Hence, (40) applies.

I now prove that (40) is equivalent to

$$E_\Theta [u(\theta)] = q(\underline{\theta}) \int_{\underline{\theta}(e)}^{\underline{\theta}} F(\theta, e)\, d\theta + \int_{\underline{\theta}}^{\overline{\theta}} q(\theta) F(\theta, e)\, d\theta.$$

The proof is trivial for case i) since $\underline{\theta}(e) = \underline{\theta}$ in this case, which implies that $E_\Theta [u(\theta)] = \int_{\underline{\theta}}^{\overline{\theta}} q(\theta) F(\theta, e)\, d\theta$ as in the main model.

Case ii) In this case, I can write

$$\int_{\underline{\theta}(e)}^{\overline{\theta}(e)} \max \left\{ 0, \int_{\theta}^{\overline{\theta}} \hat{q}(\tau)\, d\tau \right\} dF(\theta, e) = \int_{\underline{\theta}(e)}^{\underline{\theta}} \int_{\theta}^{\overline{\theta}} \hat{q}(\tau)\, d\tau dF(\theta, e) + \int_{\underline{\theta}}^{\overline{\theta}} \int_{\theta}^{\overline{\theta}} \hat{q}(\tau)\, d\tau dF(\theta, e),$$

since $\int_{\overline{\theta}}^{\overline{\theta}(e)} \max \left\{ 0, \int_{\theta}^{\overline{\theta}} \hat{q}(\tau)\, d\tau \right\} dF(\theta, e) = 0$. Switching from $\hat{q}(\theta)$ to $q(\theta)$, I can write

$$\int_{\underline{\theta}(e)}^{\overline{\theta}(e)} \max \left\{ 0, \int_{\theta}^{\overline{\theta}} \hat{q}(\tau)\, d\tau \right\} dF(\theta, e) = \int_{\underline{\theta}(e)}^{\underline{\theta}} \left((\underline{\theta} - \theta) q(\underline{\theta}) + \int_{\underline{\theta}}^{\overline{\theta}} q(\tau)\, d\tau \right) dF(\theta, e) + \int_{\underline{\theta}}^{\overline{\theta}} \int_{\theta}^{\overline{\theta}} q(\tau)\, d\tau dF(\theta, e).$$

Noting that $\int_{\underline{\theta}(e)}^{\underline{\theta}} dF(\theta, e) = F(\underline{\theta}, e)$, and that $\int_{\underline{\theta}(e)}^{\underline{\theta}} (\underline{\theta} - \theta) q(\underline{\theta})\, dF(\theta, e) = q(\underline{\theta}) \int_{\underline{\theta}(e)}^{\underline{\theta}} F(\theta, e)\, d\theta$ by an integration by parts, I can write

$$\int_{\underline{\theta}(e)}^{\overline{\theta}(e)} \max\left\{0, \int_{\theta}^{\overline{\theta}} \hat{q}(\tau)\,d\tau\right\} dF(\theta,e) = F(\underline{\theta},e)\int_{\underline{\theta}}^{\overline{\theta}} q(\tau)\,d\tau + q(\underline{\theta})\int_{\underline{\theta}(e)}^{\underline{\theta}} F(\theta,e)\,d\theta + \int_{\underline{\theta}}^{\overline{\theta}}\int_{\theta}^{\overline{\theta}} q(\tau)\,d\tau dF(\theta,e).$$

Integrating the last term on the right-hand side of this expression by parts and simplifying, I obtain

$$\begin{aligned} & F(\underline{\theta},e)\int_{\underline{\theta}}^{\overline{\theta}} q(\tau)\,d\tau + q(\underline{\theta})\int_{\underline{\theta}(e)}^{\underline{\theta}} F(\theta,e)\,d\theta + F(\overline{\theta},e)\int_{\overline{\theta}}^{\overline{\theta}} q(\tau)\,d\tau - F(\underline{\theta},e)\int_{\underline{\theta}}^{\overline{\theta}} q(\tau)\,d\tau + \int_{\underline{\theta}}^{\overline{\theta}} q(\theta)\,F(\theta,e)\,d\theta \\ = \; & q(\underline{\theta})\int_{\underline{\theta}(e)}^{\underline{\theta}} F(\theta,e)\,d\theta + \int_{\underline{\theta}}^{\overline{\theta}} q(\theta)\,F(\theta,e)\,d\theta, \end{aligned}$$

which is exactly (40).

iii) In this case, $\int_{\theta}^{\overline{\theta}} \hat{q}(\tau)\,d\tau \geq 0$ for all $\underline{\theta}(e) \leq \theta \leq \overline{\theta}(e)$ and therefore I have

$$\int_{\underline{\theta}(e)}^{\overline{\theta}(e)} \max\left\{0, \int_{\theta}^{\overline{\theta}} \hat{q}(\tau)\,d\tau\right\} dF(\theta,e) = \int_{\underline{\theta}(e)}^{\overline{\theta}(e)}\int_{\theta}^{\overline{\theta}} q(\tau)\,d\tau dF(\theta,e). \tag{41}$$

Integrating by parts, I obtain

$$\int_{\underline{\theta}(e)}^{\overline{\theta}(e)}\int_{\theta}^{\overline{\theta}} q(\tau)\,d\tau dF(\theta,e) = \int_{\overline{\theta}(e)}^{\overline{\theta}} q(\theta)\,d\theta F(\overline{\theta}(e),e) - \int_{\underline{\theta}(e)}^{\overline{\theta}} q(\theta)\,d\theta F(\underline{\theta}(e),e) + \int_{\underline{\theta}(e)}^{\overline{\theta}(e)} q(\theta)\,F(\theta,e)\,d\theta \tag{42}$$

Since $F(\theta,e) = 1$ for $\theta \in \left[\overline{\theta}(e), \overline{\theta}\right]$, I can write $\int_{\overline{\theta}(e)}^{\overline{\theta}} q(\theta)\,d\theta F(\overline{\theta}(e),e) = \int_{\overline{\theta}(e)}^{\overline{\theta}} q(\theta)\,F(\theta,e)\,d\theta$. Since $F(\theta,e) = 0$ for $\theta \in [\underline{\theta}, \underline{\theta}(e)]$, I have $\int_{\underline{\theta}}^{\underline{\theta}(e)} q(\theta)\,F(\theta,e)\,d\theta = 0$. Hence, I can write (42) as

$$\int_{\underline{\theta}(e)}^{\overline{\theta}(e)}\int_{\theta}^{\overline{\theta}} q(\tau)\,d\tau dF(\theta,e) = \int_{\underline{\theta}}^{\overline{\theta}} q(\theta)\,F(\theta,e)\,d\theta.$$

Again because $F(\theta,e) = 0$ for $\theta \in [\underline{\theta}, \underline{\theta}(e)]$, I have

$$\int_{\underline{\theta}}^{\underline{\theta}(e)} F(\theta,e)\,d\theta = -\int_{\underline{\theta}(e)}^{\underline{\theta}} F(\theta,e)\,d\theta = \int_{\underline{\theta}(e)}^{\underline{\theta}} F(\theta,e)\,d\theta = 0,$$

so I can write

$$E_{\Theta}[u(\theta)] = q(\underline{\theta})\int_{\underline{\theta}(e)}^{\underline{\theta}} F(\theta,e)\,d\theta + \int_{\underline{\theta}}^{\overline{\theta}} q(\theta)\,F(\theta,e)\,d\theta.$$

■

Proof of Proposition 8. Since $F(\underline{\theta}(e), e) = 0$ for all e, I can differentiate totally and have $f(\underline{\theta}(e), e)\underline{\theta}_e(e) + F_e(\underline{\theta}(e), e) = 0$. At $\underline{\theta}(e) = \underline{\theta}$, I have $F_e(\underline{\theta}, e) = -f(\underline{\theta}, e)\underline{\theta}_e(e) > 0$ since $\underline{\theta}_e(e) < 0$. Therefore, for a contract that implements a high effort level ($\mu > 0$), production at the top is going to be unusually high. A similar argument can be used to show that production at the bottom is smaller than the Baron Myerson quantity for the case where $\mu > 0$. ■

Proof of Proposition 9. The proof is split into two parts. In part i I derive the properties of the conditional expectation function. In part ii I use these properties to derive those of the ex ante distribution of θ.

Part i: Properties of the conditional expectation function

From Milgrom (1981) it follows directly that $\frac{\partial}{\partial\beta}\frac{h_i(\beta|s,i)}{h(\beta|s,i)} > 0$ for $s \in (\tilde{s}, \overline{s})$ implies $H_i(\beta|\, s, i) < 0$ for $s \in (\tilde{s}, \overline{s})$. Likewise, $\frac{\partial}{\partial\beta}\frac{h_i(\beta|s,i)}{h(\beta|s,i)} < 0$ for $s \in (\underline{s}, \tilde{s})$ implies $H_i(\beta|\, s, i) > 0$ for $s \in (\underline{s}, \tilde{s})$. Since

$$\pi_i(s, i) = -\int_{\underline{\beta}}^{\overline{\beta}} H_i(\beta|\, s, i)\, d\beta$$

this proves that

$$\pi_i(s, i) < 0 \text{ for } s \in (\underline{s}, \tilde{s}) \text{ and } \pi_i(s, i) > 0 \text{ for } s \in (\tilde{s}, \overline{s}).$$

Finally, I show that $\pi_i(s, i) = 0$ for $s \in \{\underline{s}, \tilde{s}, \overline{s}\}$. To see this, note that one can write for $s \in \{\underline{s}, \tilde{s}, \overline{s}\}$

$$\frac{\partial}{\partial\beta}\frac{h_i(\beta|\, s, i)}{h(\beta|\, s, i)} H(\beta|\, s, i) = 0.$$

Integrating I have

$$\int_{\underline{\beta}}^{\overline{\beta}} \frac{\partial}{\partial\beta}\frac{h_i(\beta|\, s, i)}{h(\beta|\, s, i)} H(\beta|\, s, i)\, d\beta = 0.$$

Integrating by parts, I obtain

$$\frac{h_e(\overline{\beta}|\, s, i)}{h(\overline{\beta}|\, s, i)} - \int_{\underline{\beta}}^{\overline{\beta}} \frac{h_i(\beta|\, s, i)}{h(\beta|\, s, i)} h(\beta|\, s, i)\, d\beta = 0.$$

Since $h(\beta|\, s, i)$ is a density for all i, I have $\int_{\underline{\beta}}^{\overline{\beta}} h_i(\beta|\, s, i)\, d\beta = 0$. It follows that $\frac{h_i(\overline{\beta}|s,i)}{h(\overline{\beta}|s,i)} = 0$. From $\frac{\partial}{\partial\beta}\frac{h_i(\beta|s,i)}{h(\beta|s,i)} = 0$, it follows that $\frac{h_i(\beta|s,i)}{h(\beta|s,i)} = 0$. Finally, from the fact that $h(\beta|\, s, i) > 0$ for all β it follows that $h_i(\beta|\, s, i) = 0$ for all β. Hence, for $s \in \{\underline{s}, \tilde{s}, \overline{s}\}$ $\pi(s, i)$ is independent of i.

Part ii: Properties of $F(\theta, e)$:

Since $l(i,e)$ has full support for all e, the distribution of θ has a nonmoving support $F(\underline{\theta}, e) = 0 \forall e$ and $F(\overline{\theta}, e) = 1 \forall e$. Hence $F_e(\underline{\theta}, e) = F_e(\overline{\theta}, e) = 0$. By the law of iterated expectations $E_\theta \theta = E_\beta \beta$ for all e. Since $\int_{\underline{\theta}}^{\overline{\theta}} \theta dF(\theta, e) = \overline{\theta} - \int_{\underline{\theta}}^{\overline{\theta}} F(\theta, e) d\theta$, this is equivalent to $\int_{\underline{\theta}}^{\overline{\theta}} F_e(\tau, e) d\tau = 0 \forall e$.

By an integration by parts

$$\begin{aligned} F(\theta, e) &= \int_0^1 F_i(\theta, i) \, dL(i,e) \\ &= F_i(\theta, i) L(i,e)|_0^1 - \int_0^1 \pi_i^{-1}(\theta, i) L(i,e) \, di, \end{aligned}$$

since $F_i(\theta, i)$ is locally constant for $\theta \notin [\pi(\underline{s}, i), \pi(\overline{s}, i)]$. Taking derivatives with respect to e, since $L(1,e) = 1 \forall e$, I have

$$F_e(\theta, e) = -\int_0^1 \pi_i^{-1}(\theta, i) L_e(i,e) \, di$$

From part i, I have

$$\pi_i(\theta, i) \gtreqless 0 \Leftrightarrow \theta \lesseqgtr \tilde{\theta}$$

and hence

$$\begin{aligned} F_e(\theta, e) &> 0 \, for \, \theta \in \left(\underline{\theta}, \tilde{\theta}\right) \\ F_e(\theta, e) &< 0 \, for \, \theta \in \left(\tilde{\theta}, \overline{\theta}\right). \end{aligned}$$

Since $L_{ee}(i,e)$ and $L_e(i,e)$ have opposing signs for all i, I have also

$$\begin{aligned} F_{ee}(\theta, e) &< 0 \, for \, \theta \in \left(\underline{\theta}, \tilde{\theta}\right) \\ F_{ee}(\theta, e) &> 0 \, for \, \theta \in \left(\tilde{\theta}, \overline{\theta}\right). \end{aligned}$$

■

10 References

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