New Formula for Conversion Efficiency of RF EH and its Wireless Applications

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Abstract—Existing works on energy harvesting wireless systems often assume a constant conversion efficiency for the energy harvester. In practice, the conversion efficiency often varies with the input power. In this work, based on a review of existing energy harvesters in the literature, a heuristic expression for the conversion efficiency as a function of the input power is derived by curve fitting. Using this function, two example energy harvesters are used to analyze the realistic performances of wireless relaying and wireless energy transfer. Numerical results show that the realistic performances of the wireless systems could be considerably different from what predicted by the existing analysis.

Index Terms—Energy harvesting, relaying, throughput, wireless energy transfer.

I. INTRODUCTION

Energy efficiency is a long-standing problem in wireless communications [1], [2]. One of the most promising solutions is radio frequency (RF) energy harvesting (EH) [3]. The most important performance measure of the RF energy harvester is perhaps the conversion efficiency, defined as the ratio of the output power to the input power of the energy harvester. There have been quite a few different designs of RF energy harvester in the literature, such as [4] - [16], among others. A detailed discussion of these works will be presented in the next section, based on which a heuristic formula of the conversion efficiency will be obtained. In these designs, a common conclusion is that the conversion efficiency depends on the input power.

On the other hand, many researchers have studied the use of energy harvesting in wireless systems. For example, in [17], an energy-constrained wireless link was studied, where the receiver relies on harvesting the energy from the transmitter, by maximizing the throughput. In [18], an energy harvesting relaying system was studied. Two energy harvesting methods, time-switching (TS) and power-splitting (PS), were proposed. In all these works and most existing works, it was assumed that the conversion efficiency of the energy harvester is a fixed value that does not depend on the input power. However, this is not the case in reality. Thus, it is of great interest to study the realistic performances of the wireless systems by treating the conversion efficiency of the energy harvester as a function of the input power, as in practice.

In this paper, we study the realistic performances of the wireless systems under the assumption that the conversion efficiency of the energy harvester is a function of the input power. To do this, we first derive a heuristic model for the conversion efficiency as a function of the input power. Using this, the realistic throughputs of relaying in [18] and wireless energy transfer in [17] are analyzed. Numerical results show that the realistic throughput depends on the specific energy harvesters considered, and it varies significantly when the conversion efficiency changes.

This work aims to find a more practical expression for the efficiency of existing harvesters and use it to evaluate the realistic performances of systems using existing harvesters. It focuses on the theoretical aspect of existing harvesters. To the best of the authors’ knowledge, this has not been done before and thus, it represents contribution. To do this, it may be sufficient to use data from existing experiments in trusted sources. However, designing a new harvester or performing new experiments to collect new data could be an interesting future work when relevant laboratory resources are available.

II. RELATED WORK ON RF ENERGY HARVESTER

This section does not aim to provide a complete review of all works on RF energy harvester designs due to limited space. For such a review, the readers are referred to the survey in [19]. Rather, this section aims to provide a discussion of some representative works, based on which a heuristic model of the conversion efficiency can be derived. Thus, we focus on [4] - [16].

A. Low Input Power

It is desirable to have a RF energy harvester that can operate over a long distance at a low input power with high sensitivity. Reference [4] designed a RF energy harvester with a high sensitivity of -26.3 dBm. It works at a frequency of 868 MHz with a long distance of 25 meters, when the source transmits at 1.78 W. In this case, the peak efficiency of this harvester is 22%. In [5], further improvements were made. In particular, the sensitivity was increased to -27 dBm and hence the range was increased to 27 meters. The peak efficiency became 36%. Reference [6] designed a fully passive RFID tag with a sensitivity of -12 dBm. Further, the peak efficiency of the harvester implemented in this tag is 37%. In [7], a 953 MHz rectenna was designed. It allows a peak efficiency of 29% achieved at -9.9 dBm.

The above harvesters sacrificed efficiency for sensitivity and simplicity. In other works, the sensitivity was reduced or the complexity was increased to achieve higher efficiency. For example, in [8], an energy harvester with a peak efficiency of 45% but a reduced sensitivity of -14 dBm was designed at a frequency of 928 MHz. This design has an efficiency...
above 30% when the input power is between 0.1 mW and 1.3 mW. Further improvements were done in [9], where a peak efficiency of 60% with a sensitivity of -21 dBm was achieved at 868 MHz. The efficiency is above 30% when the input power is between 0.05 mW and 1.5 mW. Such a wide range of input power is very useful, as the input power in practice is often unpredictable. Reference [10] designed a dual-band harvester at the GSM1800 band and 3G band to harvest energies from multiple bands. Tests showed that it has a peak efficiency of 51%, and its normal operating efficiency is between 16% and 43%. In [11], a 906 MHz energy harvester was designed with a peak efficiency of 60% and a sensitivity of -22.5 dBm. It can operate at a distance of 42 meters from a 4 W power source. In [12], another GSM band harvester was designed that can achieve a peak efficiency of 67.5% using a differential drive technology. Further tests also showed that this harvester could be used for 500 MHz DTV band to achieve a peak efficiency of 80%.

All the above designs operate at low input power and are suitable for long-range harvesting applications. Their peak efficiencies are normally achieved at less than 0.2 mW.

B. High Input Power

In this subsection, several designs operate at high input power are discussed. Their peak efficiencies are achieved at more than 0.2 mW.

Reference [13] designed a 2.4 GHz energy harvester with a peak efficiency of 22.7% achieved at 0.5 dBm and a sensitivity of -10 dBm. In [14], another energy harvester at a frequency of 868 MHz was designed for RFID and remote powering applications. This design aimed to maximize the range of the input power that provides high efficiency, in particular, an efficiency higher than 40% for a range of 14 dB input power. This design has a peak efficiency of 60% achieved at 0.5 mW. Reference [15] proposed a dual-rectifier energy harvester such that the overall range of input power with high efficiency was considerably increased. The harvester can achieve an efficiency above 30% for up to 30 mW. It was tuned to 915 MHz but can be modified to other frequencies too. It achieves a peak efficiency of 72% at an input of 4 mW. Finally, in [16], a RF energy harvester operating at 2.45 GHz was designed. It can achieve an efficiency above 30% for up to 4 mW. Its peak efficiency of 70% can be obtained at an input power of 1 mW.

Table I shows the main parameters of the energy harvesters discussed above, where \( f_c \) is the operating frequency, \( \eta_{\text{max}} \) is the maximum achievable efficiency, \( P_{\text{in}} \) is the input power that achieves \( \eta_{\text{max}} \) and \( \epsilon \) is the sensitivity.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>( f_c ) (MHz)</th>
<th>( \eta_{\text{max}} )</th>
<th>( P_{\text{in}} ) (mW)</th>
<th>( \epsilon ) (dBm)</th>
<th>Fabrication</th>
</tr>
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<tbody>
<tr>
<td>[4]</td>
<td>868</td>
<td>22%</td>
<td>0.015</td>
<td>-26.3</td>
<td>90 nm CMOS</td>
</tr>
<tr>
<td>[5]</td>
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<td>0.018</td>
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</tr>
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<td>[6]</td>
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<td>37%</td>
<td>0.16</td>
<td>-12</td>
<td>130 nm CMOS</td>
</tr>
<tr>
<td>[7]</td>
<td>953</td>
<td>29%</td>
<td>0.1</td>
<td>unknown</td>
<td>350 nm CMOS</td>
</tr>
<tr>
<td>[8]</td>
<td>928</td>
<td>45%</td>
<td>0.16</td>
<td>-14</td>
<td>130 nm CMOS</td>
</tr>
<tr>
<td>[9]</td>
<td>868</td>
<td>60%</td>
<td>0.15</td>
<td>-21</td>
<td>130 nm CMOS</td>
</tr>
<tr>
<td>[10]</td>
<td>1800</td>
<td>51%</td>
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<td>substrate</td>
</tr>
<tr>
<td>[11]</td>
<td>906</td>
<td>60%</td>
<td>0.16</td>
<td>-22.6</td>
<td>250 nm CMOS</td>
</tr>
<tr>
<td>[12]</td>
<td>953</td>
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<tr>
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</tr>
<tr>
<td>[16]</td>
<td>2450</td>
<td>70%</td>
<td>1</td>
<td>unknown</td>
<td>substrate</td>
</tr>
</tbody>
</table>

where \( x \) is the input power with a unit of mW, \( \eta \) is the efficiency as a percentage and the parameters of \( p_0, p_1, p_2, q_0, q_1, q_2, q_3 \) are different for different harvesters. This is achieved by testing the order of numerator from 0 to 5 and the order of denominator from 1 to 5 for the rational function in MATLAB curve-fitting and choosing the orders for best tradeoff between accuracy and complexity. All these observations are made heuristically from existing experiments without any systematic analysis, as it is impossible to perform such an analysis for these complicated circuits. However, we have done this for 36 different harvesters, almost all existing harvesters in the literature, and they all follow this model. Thus, (1) does have generality. Moreover, it is true that curve-fitting is limited by the range considered. However, most existing harvesters operate with an input power below 4 mW. In this case, our curve-fitting is useful for this small but practical range between 0 and 4 mW. Some insights can also be gained from (1). For example, when the input power \( x \) is small, the efficiency is mainly determined by \( p_0/q_0 \). Also, when the input power \( x \) is large, the efficiency decreases at a rate \( 1/x \) but is also determined by \( p_2/q_3 \). Figs. 1 and 2 compare the curve fitting results with the experimental results for the harvesters provided in [11] and [4], respectively. We did not reproduce the experimental results but only took them from the figures provided in the papers. One sees that they agree with each other reasonably well. Table II gives the fitting parameters for all the energy harvesters discussed above. The value of \( q_3 \) is normalized to 1 and therefore is not listed in Table II.

### III. Wireless Applications

In this section, we will use [17] and [18] as two examples to show the effect of varying efficiency. However, almost all existing energy harvesting wireless systems assume constant efficiency and therefore, will benefit from such an analysis. Due to the limited space of a correspondence item, other scenarios will not be discussed here but they will be considered in our future work, as some systems may be more sensitive to the varying efficiency than others and this difference could be quantized. In the following, the wireless channels are assumed to follow block Rayleigh fading, similar to what was assumed in [17] and [18].
Fig. 1. Comparison of the fitted curve and the experimental curve for [11].

Fig. 2. Comparison of the fitted curve and the experimental curve for [4].

A. Wireless relaying application

We consider the same system as [18] but with varying efficiency. In this case, a three-node relaying system is used, where the source sends information and energy to the relay and the relay uses the harvested energy to forward the information to the destination. There is no direct link. Each node has a single antenna and operates in half-duplex. The source-to-relay and relay-to-destination links are orthogonal in time. Assume that the total communication time is $T$.

In TS, a fraction of the total time $\alpha T$ is used for energy harvesting at the relay, followed by $(1 - \alpha)\frac{T}{2}$ for information reception at the relay and $(1 - \alpha)\frac{T}{2}$ for information reception at the destination, where $\alpha$ is the TS coefficient. In the existing analysis, $\eta$ is assumed constant and independent of the input power. Thus, one has $E_h = \eta P_s |h|^2 \alpha T$ as the harvested energy, where $P_s$ is the source transmission power and $h$ is the complex channel gain of the source-to-relay link. In this work and in reality, $\eta$ is a function of the input power. By replacing $\eta$ with (1) and following a similar analysis to [18], the outage probabilities can be derived as (2) for variable-gain relaying and (3) for fixed-gain relaying in the next page, where $\gamma_0 = 2^R - 1$ is the threshold SNR for outage, $R$ is the constant throughput required by the source, $\Gamma_1 = \frac{P_t E|h|^2}{\sigma^2_n + \sigma^2_d}$.

$\Gamma_2 = E[|g|^2]$, $g$ is the complex channel gain of the relay-to-destination link, $\sigma^2_n$ and $\sigma^2_d$ are the variances of the noise at the relay from the RF antenna and RF-baseband conversion, respectively, $\sigma^2_{da}$ and $\sigma^2_{dr}$ are the variances of the noise at the destination from the RF antenna and RF-baseband conversion, respectively.

Finally, the throughput in this case is given by [18]

$$\beta_{TSVG}^{New} = \frac{R}{2} \left(1 - \alpha \right) \left(1 - P_{out-TSVG}^{New} \right)$$

(4)

$$\beta_{TSFG}^{New} = \frac{R}{2} \left(1 - \alpha \right) \left(1 - P_{out-TSFG}^{New} \right).$$

(5)

If PS is used, a fraction of the received signal is harvested without any dedicated harvesting time. In this case, the transmission from the source to the relay takes $\frac{T}{2}$ seconds for both harvesting and reception and the relay takes another $\frac{T}{2}$ seconds to use the harvested energy to transmit the signal to the destination. Using the varying efficiency in (1) and following a similar analysis to [18], the outage probabilities are (6) for variable-gain relaying and (7) for fixed-gain relaying in the next page, where $\Gamma_3 = \frac{P_t E|h|^2}{\sigma^2_n + \sigma^2_d}$.

Then, the throughput is given by [18]

$$\beta_{PSVG}^{New} = \frac{R}{2} \left(1 - \alpha \right) \left(1 - P_{out-PSVG}^{New} \right)$$

(8)

$$\beta_{PSFG}^{New} = \frac{R}{2} \left(1 - \alpha \right) \left(1 - P_{out-PSFG}^{New} \right).$$

(9)

Note that the results for fixed-gain relaying are new, as [18] did not consider fixed-gain.

B. Wireless energy transfer

In [17], the authors proposed a wireless energy transfer network. Using the same system model, we consider a network where the access point transmits energy to the nodes in the downlink for $\gamma_0 T$ seconds, and then the nodes use the harvested energy to transmit information to the access point in the uplink for $\gamma_1 T$ seconds, $\gamma_2 T$ seconds, and so on, in a time division multiple access (TDMA) way, where $T$ is the total transmission time and $\sum \gamma_i = 1$. In the existing analysis, one has $E_i = \eta P_A |h_i| \gamma_0 T$ as the harvested energy, where $\eta$ is the constant conversion efficiency assumed in [17], $P_A$ is the transmitted power of the access point, $h_i$ is the fading transmission power. Thus, one has

\[
\begin{align*}
\text{Table II:}
\quad & \text{Fitting parameters of different energy harvesters.} \\
\quad & \text{Ref.} \quad p_2 \quad p_1 \quad p_0 \quad q_2 \quad q_1 \quad q_0 \quad q_{tsv} \\
[4] & 1.14 \quad 5.36 \quad 1.61 \quad 0.0547 \quad -0.000318 \quad 2.8 \quad 6 \\
[5] & -5.16 \quad 1.16 \quad -1.125 \quad 5.35 \quad 9.39 \quad 16.1 \\
[6] & 1.23 \quad 10.5 \quad -0.238 \quad -0.125 \quad 0.24 \quad 0.00045 \\
[7] & 14.1 \quad 0.171 \quad -0.00284 \quad 0.956 \quad -0.119 \quad 0.0069 \\
[8] & 108 \quad 1.47 \quad -0.11 \quad 2.35 \quad -0.0852 \quad 0.00223 \\
[9] & 83.3 \quad 19.4 \quad -0.313 \quad 7.28 \quad 0.315 \quad 0.00023 \\
[10] & 4.32 \quad 7.96 \quad -0.18 \quad 1.18 \quad 1.36 \quad 7.4 \\
[11] & 78.5 \quad -2.34 \quad 1.62 \quad 2.43 \quad -0.482 \quad 0.0658 \\
[12] & 413 \quad 1160 \quad 704 \quad 15.1 \quad 25.4 \quad 12.5 \\
[13] & 230 \quad -20.5 \quad 0.623 \quad 9.24 \quad -0.77 \quad 0.0808 \\
[14] & 300 \quad -12.7 \quad 0.135 \quad 4.4 \quad -0.00104 \quad -0.00166 \\
[15] & 0 \quad 7.34 \quad 0.394 \quad 0.332 \quad 0.951 \quad 98 \\
[16] & 99.9 \quad 140 \quad -0.059 \quad -0.295 \quad 2.71 \quad 0.039
\end{align*}
\]
power in the downlink from the access point to the node $i$, $\tau_0 T$ is the harvesting time. In this work and in practice, the conversion efficiency is a function of the input power such that $E_i = \eta[P_A h_i] P_A h_i \tau_0 T$, where (1) has been used. Following a similar analysis to [17] but using a varying efficiency, the average throughput becomes

$$\bar{R}_i = -\frac{\tau_i}{\Gamma \ln 2} \int_0^\infty e^{-\frac{t}{\Gamma}} \frac{\sigma_i^2}{\Gamma \eta (\rho + \eta)} E_i(\frac{\sigma_i^2}{\Gamma \eta (\rho + \eta)}) dy$$

(10)

where $\sigma^2$ is the variance of the additive white Gaussian noise, $\Gamma$ is the average fading power, $E_i(\cdot)$ is the exponential integral defined in [20, eq. (8.211)] and the relationship in [20, eq. (4.337.2)] has been used.

IV. NUMERICAL RESULTS AND DISCUSSION

In this section, numerical examples are presented to show the performances of the wireless systems examined using the realistic assumption that the conversion efficiency is dependent of the input power. To do this, we set $P_s = 1$, $R = 3$, $E\{|h|^2\} = E\{|g|^2\} = 1$ and $\sigma_{rc}^2 = \sigma_{da}^2 = \sigma_{do}^2 = \sigma^2$ in the wireless relaying in [18], $P_A = 1$, $\Gamma = 1$, $f_m T = 1$ in the wireless energy transfer in [17]. Also, for the existing analysis using constant conversion efficiency, we set $\eta = 0.5$. For our new analysis using varying conversion efficiency, we use the harvesters in [4] and [11]. Other constants and other harvesters can be examined in a similar way.

Fig. 3 shows the throughput for AF relaying using TS. Several observations can be made. First, there exists a maximum throughput in all the curves shown. However, the optimal TS coefficient is considerably different for different curves. This means that one cannot use the performances predicted by the analysis based on the assumption of constant conversion efficiency to set up the optimal TS coefficient in practice. This must be done by using the realistic assumption of varying conversion efficiency. Second, fixed-gain relaying and variable-gain relaying have different performances. In particular, from Fig. 3, the maximum throughput for fixed-gain relaying is smaller than that for variable-gain relaying. Third, the harvester in [4] has a peak efficiency of 22%. Thus, the curves using [4] have a very small throughput and therefore require a larger value of the optimal TS coefficient in order to harvest more energies. In all the considered cases, the constant conversion efficiency has larger throughput or overestimates the realistic throughput.

Fig. 4 shows the throughput for AF relaying using PS. In this case, the throughput for [4] is close to zero for all values of $\rho$. Again, there exists a maximum throughput in the curves. The optimal value of the PS factor for the harvester in [11] is considerably different from that assuming a constant conversion efficiency. In this case, fixed-gain relaying has a larger throughput than variable-gain relaying. Moreover, the
the peak efficiency by finding the maximum of (1) using the following methods in [17] and [18]. Also, although the varying efficiency in the analysis is quite straightforward by comparing the curve with independent links when $\sigma^2 = 0.01$. 

![Fig. 5. Throughput vs. $\tau_1$ with independent links when $\sigma^2 = 0.01$.]


Fig. 5 shows the average throughput for wireless energy transfer in [17]. Only one node is considered such that $\tau_0 = 1 - \tau_1$, where $\tau_1$ is the transmission time of node 1. One sees that a maximum throughput exists in all curves, implying that it is necessary to choose the optimal value of the transmission time to achieve the highest throughput. Comparing the curve using the constant conversion efficiency with those using [11] and [4], one sees that they are significantly different. More specifically, the constant conversion efficiency always predicts a overly larger throughput.

V. CONCLUSION

In this paper, we have discussed several RF energy harvester designs. Based on this discussion, a heuristic model that describes the conversion efficiency as a function of the input power has been derived. Using this model, two example harvesters have been used to analyze the realistic performances of wireless relaying and wireless energy transfer. Numerical results have shown that the realistic performances of wireless systems could be considerably different from what was predicted by the existing analysis. The novelty of this work lies in the derived heuristic model of efficiency and the realistic performances of the systems. However, using the varying efficiency in the analysis is quite straightforward by following the methods in [17] and [18]. Also, although the heuristic model is obtained by curve-fitting, it still provides some analytical insights. For example, it predicts the efficiency when the input power is very large or very small as well as the peak efficiency by finding the maximum of (1) using the first-order derivative.

REFERENCES