Essays in Empirical Finance

By

Qi Xu

Warwick Business School
University of Warwick

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To my parents

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Abstract

This thesis consists of three papers in the area of empirical finance. Chapter 2 investigates the role of realized jumps detected from high frequency data in predicting future volatility from both statistical and economic perspectives. We show that separating jumps from diffusion improves volatility forecasting both in-sample and out-of-sample. Moreover, we show that these statistical improvements can be translated into economic value. We find a risk-averse investor can significantly improve her portfolio performance by incorporating realized jumps into a volatility timing based portfolio strategy.

Chapter 3 investigates the use of high frequency data in large dimensional portfolio allocation. We consider the use of high frequency data beyond the estimation of the realized covariance matrix. Portfolio strategies using high frequency data in measuring and forecasting univariate realized volatility can generate statistically significant and economically tangible benefits compared to low frequency strategies. Moreover, using high frequency data to separate realized volatility into different components and construct realized higher moments can also enhance portfolio performance. Strategies using upside and downside volatility components and using realized skewness can deliver incremental economic benefits over the strategy using total realized volatility alone.

Chapter 4 investigates the pricing of volatility risks in currency markets. Firstly, we show that pricing volatility risk can be understood by pricing some of its components. We find that diffusive volatility dominates jump volatility in pricing carry trade returns, while jump volatility is important to explain the joint cross-section of carry trade and momentum returns. Both short run and long run components are priced, and the short run component is more important in general. Secondly, we suggest that factors similar to volatility in identifying bad states, i.e. volatility of volatility and cross sectional volatility are also priced in currency returns and they cannot be fully subsumed by conventional volatility risks.

Declaration

I declare that any material contained in this thesis has not been submitted for a degree to any other university. Chapter 2 and Chapter 3 are collaboration works with Ingmar Nolte (Lancaster University). Chapter 4 is a collaboration work with Ingmar Nolte and Mark Taylor (University of Warwick). I contribute by developing research ideas, conducting empirical analyses, and writing up. My co-authors contribute by providing constructive comments and improving the writing.

I also declare that one paper titled "The Economic Value of Volatility Timing with Realized Jumps ", which is dawn from Chapter 2, has been accepted for publication and is forthcoming in the *Journal of Empirical Finance*.

Chapter 1

Introduction

Volatility is the most conventional measure of risk. To make financial decisions, risk-averse investors need to accurately predict future risks and better understand current risks. This thesis consists of three papers analysing economic implications of using volatility measures. I focus on asset allocation and empirical asset pricing applications in equity and foreign exchange markets respectively.

Chapter 2 studies the role of realized jumps in improving portfolio allocation. An asset price process can be decomposed into jumps (large and discrete price movement) and the diffusion component (smooth and continuous component). The idea to incorporate jumps in portfolio allocation contexts is not new (Liu, Longstaff, and Pan 2003, Maheu, McCurdy, and Zhao 2012). Previous studies, however, mainly rely on parametric models to estimate jump parameters. In those contexts, jumps are simply treated as a mathematical device to generate return non-normality (Backus, Chernov, and Martin 2011). A different stream of literature focuses on using high frequency data and nonparametric tests to identify ex post, observable, and realized jumps (Barndorff-Nielsen and

Shephard 2006, Andersen, Bollerslev, and Dobrev 2007). Potential economic applications of detected jumps are, however, generally missing in the literature. Therefore, we are interested in whether realized jumps detected from high frequency data contain predictive information and whether they can contribute to portfolio allocation.

We firstly investigate the use of realized jumps in predicting future volatility. We identify realized jumps using high frequency data of market index and seven different nonparametric jump tests. We find that separating jumps from the diffusion component improves volatility forecasting performance both in-sample and out-of-sample.

We are also interested in whether such separation is economically important. Even though separating jumps from diffusion improves volatility forecasting performance, an investor may only be willing to access an expensive high frequency dataset and implement sophisticated nonparametric tests if the separation indeed delivers economically significant benefits. We address this issue by conducting a mean-variance portfolio allocation exercise in which a risk-averse investor allocates her wealth between one risky asset (market index) and one risk free rate. If separating jumps from diffusion is economically important, we should expect the investor to be better off using a strategy with realized jumps compared to a benchmark strategy without such decomposition. Our empirical evidence supports this conjecture and shows that separating jumps from diffusion can improve portfolio performance.

Chapter 3 analyses the use of high frequency data in relatively large dimensional portfolio allocation. A few existing studies (Fleming, Kirby, and Ostdiek 2003, Bandi, Russell, and Zhu 2008) highlight the importance of high frequency data in improving portfolio performance. However, almost all previous studies in this area focus on the use of high frequency data in estimating the whole realized variance-covariance matrix. Although this practice helps to capture asset return co-movements much faster than using low frequency data, there are two apparent drawbacks: Firstly, assets are traded at different time stamps, and hence asset returns are usually recorded non-synchronically when the sampling frequency is high. Such non-synchronicity generates non-negligible bias in estimating the realized covariance matrix, which will subsequently affect portfolio performance. Previous studies introduce different methods to mitigate the non-synchronicity problem, however, there is no consensus on it and these corrected methods may themselves introduce additional estimation errors. Therefore, we are interested in whether we can assess the use of high frequency data in portfolio allocation while avoiding the potential problem of non-synchronicity. The second disadvantage is that the use of a whole realized variance-covariance matrix almost fully overlooks the segmented but fast growing stream of the literature on alternative univariate realized measures such as realized volatility components and higher return moments (Andersen, Bollerslev, and Diebold 2007, Patton and Sheppard 2013, Amaya, Christoffersen, Jacobs, and Vasquez 2011). Previous studies fail to adapt these alternative realized risk measures to a large scale portfolio allocation framework, mainly because the technical difficulty of using these alternative realized measures and controlling for non-synchronicity at the same time. Hence, we are also interested in analysing the use of alternative realized measures in portfolio allocation.

Initially, we investigate whether using high frequency data to estimate and forecast univariate realized volatilities already provides a investor with sufficiently large economic benefits. We combine the high frequency based conditional volatilities with a low frequency based correlation structure to composite the conditional covariance matrix, and then solve the portfolio problem. We show that a risk-averse investor can significantly benefit from the strategy using high frequency data compared to a benchmark strategy purely using low frequency data.

Moreover, high frequency data also allow us to extract realized volatility components and construct realized higher return moments. We study the use of these alternative realized measures as volatility predictors to improve portfolio allocation. We find that both realized volatility components and realized higher moment strategies can also outperform the low frequency benchmark strategy. Strategies using upside and downside realized volatility components and realized skewness can further generate incremental improvements compared to the strategy using total realized volatility only.

Chapter 4 investigates the pricing of volatility risk in currency markets. Menkhoff, Sarno, Schmeling, and Schrimpf (2012a) already document that volatility risk is important for explaining currency carry trade returns. Two reasons motivate us to revisit the pricing of currency volatility risk. Firstly, volatility risk is especially important for understanding the risk return profile in currency markets. In equity markets, volatility risk is complementary to the equity market factor. In currency markets, due to the lack of a consensus global currency market factor, volatility risk is of first order importance. Previous studies on currency volatility risk mainly rely on a simplified proxy for volatility. Therefore, we aim to provide new insights on the pricing of volatility risk by better understanding the volatility process. Secondly, although volatility risk is priced in carry trade returns, previous studies (Burnside, Eichenbaum, and Eichenbaum 2011, Menkhoff, Sarno, Schmeling, and Schrimpf 2012b) also admit that volatility risk can hardly explain currency momentum returns or the joint cross-section of carry trade and momentum returns. We are interested in whether we can obtain new results with our volatility factors.

Firstly, we investigate whether pricing volatility risk can be explained by pricing some of its components. We decompose volatility into jump and diffusion components and into short and long run components. We find that the explanatory power of volatility risk in carry trade returns is almost exclusively coming from the diffusion component. However, neither total volatility nor the diffusive volatility is able to explain the joint cross-section of carry trade and momentum portfolios. Nevertheless, the jump component, which is less important for carry trade returns, contains unique pricing ability for the joint cross-section of carry trade and momentum portfolios. We also suggest that both short and long run volatility components are priced in carry trade portfolios. The short run component is slightly stronger than the long run component. However, neither of them can explain the joint cross-section. Therefore our findings support that the pricing ability of volatility risk is concentrated in some of its components.

Secondly, if the pricing of volatility risk is rationalized by a hedging argument from the Intertemporal Capital Asset Pricing Model (ICAPM), then intuitively other factors similar to volatility in characterizing bad states may also be priced in currency returns. We construct two alternative volatility factors: volatility of volatility and cross-sectional volatility, and find that they are also priced in carry trade returns. Moreover, we suggest that these alternative volatility factors contain unique explanatory powers for different test asset returns. Similar to jump volatility, volatility of volatility can also explain the joint cross-section, indicating the importance of the tail component of volatility. Cross-sectional volatility can explain carry trade portfolio returns when we controll for conventional volatility risk, and it also outperforms other measures in pricing individual currency returns. Our findings suggest that alternative volatility factors are also priced in currency returns and they are not subsumed by the conventional volatility risk.

To summarize, all three chapters present novel empirical evidence on the use of different volatility measures in important economic applications and contribute by bridging important literature gaps overlooked by previous studies.

Chapter 2

The Economic Value of Volatility Timing with Realized Jumps

2.1 Introduction

The importance of jumps in asset pricing, option pricing, and risk management is widely recognized (Ait-Sahalia 2004). Although, resorting on jumps as a modeling device is not new, realized jumps were generally overlooked until recently. In this paper, we comprehensively investigate the role of realized jumps detected from high frequency data for the prediction of future volatility. Different from previous studies with similar focus, we not only conduct an extensive statistical evaluation of volatility forecasting using all major jump tests, but also provide new economic insights in the form of whether a risk-averse investor can significantly benefit from considering realized jumps in volatility timing based portfolio allocation strategies.

The literature on this topic can be broadly categorized into two streams: The parametric literature starting with Merton (1976) includes jump-diffusion and stochastic-volatility with jumps (SVJ) models in continuous time (Eraker, Johannes, and Polson 2003, Eraker 2004, Chernov, Ronald Gallant, Ghysels, and Tauchen 2003) and GARCH-J models in discrete time (Maheu and McCurdy 2004, Duan, Ritchken, and Sun 2006, Christoffersen, Jacobs, and Ornthanalai 2012). These parametric models are widely used in portfolio choice, option pricing, and risk management applications and the jumps introduced in models are ex ante in nature. As Backus, Chernov, and Martin (2011) admit: "A jump component, in this context, is simply a mathematical device that produces non-normal distributions."

The second stream of the literature considers nonparametric approaches. Recently, many nonparametric jump tests (Barndorff-Nielsen and Shephard 2006, Andersen, Bollerslev, and Dobrev 2007, Ait-Sahalia and Jacod 2009) use high frequency data to estimate ex post realized jumps. This stream of the literature primarily focuses on issues such as why asset prices jump (e.g. macroeconomic new announcements) or how often asset prices jump (e.g. less than one per day). However, only a very few studies consider economic applications of realized jumps. We therefore aim to fill this gap between two related but different streams of literature by considering economic applications of realized jumps.

We focus on two research questions: Firstly, we are interested in whether realized jumps can forecast future volatility. We apply seven main stream nonparametric jump tests to identify realized jumps, decompose realized variance into jump and diffusion components, and then adapt them into a forecasting framework. Our findings suggest that realized jumps do contain predictive information for future volatility for the majority of jump tests both in-sample and out-of-sample. We find that jump models in general generate higher adjusted R^2s and lower Mean Squared Errors (MSE) compared to the benchmark model, which does not separate jumps from diffusion. Results hold true across the majority of jump specifications, and different forecasting horizons. Existing studies investigate similar issues. However, they mainly rely on one particular jump test and their results are mixed. For example, Andersen, Bollerslev, and Diebold (2007) find a negative (but insignificant) relationship between jumps and one period ahead volatility. Corsi, Pirino, and Reno (2010) on the contrary show statistically significant evidence to support a positive relationship if a modified jump test is applied. By using all major jump tests, our results contribute to the debate whether in general realized jumps help to forecast volatility.

Incorporating realized jumps into volatility forecasting require accessing intraday high frequency data and applying sophisticated nonparametric jump tests. Therefore, a natural question arises whether it is worth to estimate and use realized jumps. Even though separating jumps from diffusion improves volatility forecasting, it is interesting to know whether the improvement is large, and more importantly whether the improvement is economically valuable. Therefore, our second research question explicitly asks whether the potential statistical forecasting improvement obtained by separating jumps from diffusion can be translated into tangible economic benefits for a risk-averse investor. We construct a mean-variance portfolio strategy based on the predicted volatility obtained from the previous step. Our findings suggest that the statistical improvements are also economically significant. Under different risk aversion levels and jump specifications, jump strategies can in general generate positive and statistically significant performance fees relative to the benchmark strategy. A few existing papers also consider the role of jumps in asset allocations. For example, Liu, Longstaff, and Pan (2003) provide an analytical solution to the optimal portfolio choice problem when event risk or jumps are considered. They find that jumps play an important role in determining the optimal portfolio choice. Two recent studies by Chen, Hyde, and Poon (2010) and Maheu, McCurdy, and Zhao (2012) are also close to us in considering jumps in asset allocation. However, we differ from those studies on a few aspects. Firstly, our nonparametric framework enables us to exploit the information embedded in jump variations in a model free fashion while previous papers rely on a parametric specification. Secondly, we use high frequency data to separate the jumps and the diffusion component precisely, while they mainly rely on daily data to obtain relatively noisy proxies for jumps (i.e. large extraordinary movement or middle size jumps etc). The high frequency data we use also allows us to access intraday information, which is overlooked by previous studies.

We then conduct comprehensive robustness checks, and find that further controlling for market microstructure effects and transaction costs does not change our main results. We also investigate the predictive ability of realized jumps on alternative realized moments. We find that realized jumps can predict realized volatility and its signed components, but can hardly predict realized higher moments. We further show that a mean-variance portfolio strategy based on predicting positive and negative conditional volatilities separately can outperform the benchmark strategy based on predicting total volatility, and incorporating realized jumps can additionally improve economic benefits.

A few other studies are also related to ours. Firstly, our paper can be viewed as a natural extension of the stream of literature considering the economic value of volatility timing. Previous studies (Fleming, Kirby, and Ostdiek 2003, Bandi and Russell 2006, Bandi, Russell, and Zhu 2008, Liu 2009) already document that volatility timing performance can be improved by using high frequency data, optimal sampling, and optimal rebalancing frequencies. We extend the above studies by considering realized jumps. Secondly, our paper is also related to other uses of realized jumps or applications of jump tests. For example, Dumitru and Urga (2012) and Theodosiou and Zikes (2011) conduct comprehensive simulation studies to compare size and power of jump tests. Tauchen and Zhou (2011) and Jiang and Yao (2013) use detected realized jumps to predict bond risk premia and the cross-section of stock returns respectively. We distinguish ours from previous studies by focusing on the role of realized jumps in volatility timing.

The rest of the paper is structured as follows: Section 2.2 discusses the theoretical setup and the jump tests. Section 2.3 describes the data and methodology. Section 2.4 discusses empirical findings from both statistical and economic perspectives. Section 2.5 conducts comprehensive robustness checks. Section 2.6 concludes.

2.2 Jumps in Asset Prices

2.2.1 Theoretical Setup

Let p_t denote the logarithmic price which follows a jump diffusion process given by

$$dp_t = \mu_t dt + \sigma_t dW_t + dJ_t \tag{2.1}$$

where μ_t, σ_t, and dW_t are drift, diffusion parameter, and standard brownian motion respectively. J_t is a jump process, and $J_t = \sum_{j=1}^{N_t} c_{t_j}$, where c_{t_j} is the jump size and N_t a counting process. For simplicity, we only consider finite activity jumps and we assume that the jump and the diffusion components are independent. Let

$$r_{j,t} = p_{(t-1)h+\frac{hj}{M}} - p_{(t-1)h+\frac{h(j-1)}{M}}, \quad j = 1, \ldots, M,$$

where h is the length of the intraday sampling interval and M is the number of intraday returns during the day. Then the realized variance can be written as the sum of the squared intraday returns

$$RV_t = RV_{t,M} = \sum_{j=1}^{M} r_{j,t}^2$$

Given the price dynamics of the jump diffusion process, the realized variance as an approximation to the price's quadratic variance can be further written as follows

$$\lim_{M\to\infty} RV_{t,M} = \int_{t-1}^{t} \sigma_s^2 ds + \sum_{j=1}^{N_t} c_j^2 \tag{2.2}$$

Here $\int_{t-1}^{t} \sigma_s^2 ds$ is the integrated variance (IV_t), and $\sum_{j=1}^{N_t} c_j^2$ is the quadratic variation of the jump part (JV_t) over the period from $t-1$ to t (often a day). Jump tests are therefore designed to estimate JV_t using high frequency data.

2.2.2 Jump Tests

We consider the use of seven major jump tests developed in the literature, including Barndorff-Nielsen and Shephard (2006) (BNS), Ait-Sahalia and Jacod (2009)(AJ), Jiang and Oomen (2008)(JO), Andersen, Dobrev, and Schaumburg (2012) (Med, Min), Corsi, Pirino, and Reno (2010)(CPR), and Podolskij and Ziggel (2010) (PZ). In this part, we only describe the BNS jump test in detail. Specifications of all other six tests are described in Appendix 1. Andersen, Bollerslev, and Dobrev (2007) and Lee and Mykland (2008) are two other important tests that are however not studied in this paper.[1]

Barndorff-Nielsen and Shephard (2006) developed a bipower based jump estimator. The idea is using the calculated realized bipower variation to proxy the integrated variance. Since jumps are rare and are unlikely to occur for two consecutive intraday returns, when intervals are small enough, the realized bipower variation will converge in probability to the integrated variance. The difference between realized variance and realized bipower variation is then an estimator

[1] The reason is that both of them are intraday jump tests that average daily bipower variation to local volatility. Therefore, although they can identify which return contains a jump, they are not significantly different from BNS in volatility forecasting when only daily jump variation is considered.

of the jump variation. The realized bipower statistic is defined as

$$BV_{t,M} = \frac{\mu_1^{-2} M}{M-1} \sum_{j=2}^{M} |r_{t_{j-1}}||r_{t_j}|,$$

$$\lim_{M\to\infty} BV_{t,M} \longrightarrow IV = \int_0^t \sigma_s^2 ds.$$

Following Huang and Tauchen (2005), the standardized $BV_{t,M}/RV_{t,M}$ ratio converges to a standard normal distribution and the test statistic is given by

$$J_{t,M} = \frac{1 - \frac{BV_{t,M}}{RV_{t,M}}}{\sqrt{[(\frac{2}{\pi})^2 + \pi - 5]\frac{1}{M} max(1, \frac{TQ_{t,M}}{BV_{t,M}^2})}} \longrightarrow N(0,1), \tag{2.3}$$

where TQ_t refers to the tripower quarticity given by

$$TQ_{t,M} = M\mu_{4/3}^{-3}\frac{M}{M-2}\sum_{j=3}^{n} |r_{j-2}|^{4/3}|r_{j-1}|^{4/3}|r_j|^{4/3},$$

where $\mu_p = E(|U|)^p = \pi^{-1/2}2^{p/2}\Gamma(\frac{p+1}{2})$.

The jump variation can then be obtained as

$$JV_t = (RV_t - BV_t)I_{[J_{t,M} \geq \phi_\alpha^{-1}]}, \tag{2.4}$$

where the ϕ_α^{-1} is the α quantile of the normal distribution.

2.3 Data and Methodology

2.3.1 Realized Jumps and Volatility Forecasting

We first investigate the role of realized jumps in volatility forecasting. There are many approaches to forecast volatility using high frequency data. We use the Heterogeneous Autoregressive (HAR) model of Corsi (2009) because it can be implemented easily and it can capture the long memory property of volatility processes in a straightforward way. Our benchmark model considers the use of daily, weekly, and monthly lagged realized variances to forecast one step ahead realized variance. The HAR-RV specification is as follows:

$$RV_{t,t+h-1} = \beta_0 + \beta_{RVD} RV_{t-1} + \beta_{RVW} RV_{t-5,t-1} + \beta_{RVM} RV_{t-22,t-1} + \epsilon_{t,t+h-1}. \quad (2.5)$$

To assess the role of jumps, we consider the following HAR-RV-CJ specification:

$$RV_{t,t+h-1} = \beta_0 + \beta_{IVD} IV_{t-1} + \beta_{IVW} IV_{t-5,t-1} + \beta_{IVM} IV_{t-22,t-1} + \beta_{JVD} JV_{t-1} + \epsilon_{t,t+h-1} \quad (2.6)$$

where $RV_{t,t+h} = h^{-1}[RV_t + RV_{t+1} ... + RV_{t+h-1}]$ is the averaged h-periods realized variance. IV_{t-1}, $IV_{t-5,t-1}$, $IV_{t-22,t-1}$ are jump robust integrated variation estimators over a lagged daily, weekly, and monthly horizon, JV_{t-1} is the daily lagged jump variation detected using the jump tests introduced before.

2.3.2 Realized Jumps and Volatility Timing Based Portfolio Strategy

We then conduct our economic evaluations by constructing volatility timing based portfolio allocation strategies. We consider a risk-averse investor with mean-variance preferences, who allocates her wealth into one risky asset (a market index ETF) and one risk-free asset. Our one risky asset specification is similar to Marquering and Verbeek (2004). The economic intuition for this strategy is simple. Given the expected return, when the volatility level is high, the investor allocates more wealth into the risk-free asset, and when the volatility level is low, the investor allocates more wealth into the risky asset. If separating jumps from diffusion components lead to more accurate prediction of future volatility, then we should expect the investor to improve her portfolio performance by actively rebalancing the portfolio based on the signal of the predicted volatility.

Although more sophisticated utility functions can be used, we stick to mean-variance preferences because we are primarily interested in whether statistical improvements in volatility forecasting by separating jumps from diffusion can be translated into economic values. To concentrate on the impact of jumps, we consider only one market index as the risky asset. We choose this setting to avoid dealing with jump tests in multivariate settings and controlling for non-synchronicity of different assets, while we still maintain the generality of our empirical results. We also assume that the investor is myopic. Namely, the investor dynamically rebalances her portfolio period by period, and she does not consider the intertemporal hedging demand in the portfolio selection. We make this assumption both for simplicity to directly translate our volatility forecasting results into portfolio performance and for consistency with the existing volatility timing literature.

Hence, the investor solves the following optimization problem:

$$\underset{w_t}{\text{Max}} \quad U[E_t(r_{p,t+1}), Var_t(r_{p,t+1})],$$

where $E_t(r_{p,t+1})$ is the conditional expected portfolio return and $Var_t(r_{p,t+1})$ is the conditional variance of the portfolio return. The portfolio return is $E_t(r_{p,t+1}) = r_{f,t+1} + w_t(E_t(r_{m,t+1}) - r_{f,t+1})$, where w_t is the portfolio weight of the risky asset, $E_t(r_{m,t+1})$ is the conditional expected return of the risky asset and $r_{f,t+1}$ is the return for the risk free asset, which we know ex ante.

The mean-variance utility function is given by

$$U[E_t(r_{p,t+1}), Var_t(r_{p,t+1})] = E_t(r_{p,t+1}) - \frac{\gamma}{2} Var_t(r_{p,t+1}),$$

with γ the risk-aversion parameter. Hence, the optimal portfolio weight is given by

$$w_t = \frac{E_t(r_{m,t+1}) - r_{f,t+1}}{\gamma Var_t(r_{m,t+1})}. \tag{2.7}$$

We set the expected return for the risky asset equal to the in-sample mean [2], as over a short horizon, expected return changes are negligible and we are only interested in the volatility timing effect of realized jumps.

Fleming, Kirby, and Ostdiek (2003) show that using a dynamic volatility timing strategy relying solely on realized volatilities can outperform both static strategies and dynamic volatility timing strategies using lagged daily volatilities only. Therefore the question whether realized jumps are economically valuable can be translated into whether our jump augmented volatility timing strategy (HAR-

[2] We also tried using other specifications such as rolling windows, results are similar.

RV-CJ) can outperform the benchmark strategy (HAR-RV) which does not separate jumps from the diffusion component.

To implement the above strategy, we conduct two further adjustments. Firstly, we impose a short selling constraint. Following Marquering and Verbeek (2004), we restrict the negative portfolio weights to zero and the greater than one portfolio weights to one. Secondly, we also match the high frequency trading period (6.5 hour) to daily frequency (24 hour). Therefore, rather than directly plugging in the predicted realized variance, we adjust the predicted realized variance with a bias-correction factor. We follow existing studies (Fleming, Kirby, and Ostdiek 2003, Bandi and Russell 2006) and construct the bias-correction factor as follows:

$$BCF = \frac{1/n \sum_{t=1}^{n} r_t^2}{1/n \sum_{t=1}^{n} RV_t}, \tag{2.8}$$

where RV_t is the daily realized variance for 6.5 trading hours and r_t is the daily return for 24 hours. We construct the bias-correction factor using all data from the in-sample period. The conditional variance is estimated by predicted realized variance scaled by the bias correction factor, namely $Var_t(r_{m,t+1}) = BCF \cdot \widehat{RV}_{t+1}$, where $\widehat{RV}_{t+1}$ is the predicted realized variance obtained from the volatility forecasting part. We can then plug in the conditional variance into the optimal portfolio weight function to estimate the optimal portfolio weight.

2.3.3 Performance Evaluations

Following the volatility timing literature, we focus on the utility based performance evaluation measure to assess whether investors can benefit from including realized jumps into their information set. We follow Fleming, Kirby, and Ostdiek (2003) and Marquering and Verbeek (2004) and rely on averaged realized utility to compare jump strategies to the benchmark strategy.

The sample averaged realized utility for portfolio strategy p is given by

$$\bar{U}(R_p) = \frac{1}{T}\sum_{t=0}^{T-1}[r_{p,t+1} - \frac{\gamma}{2}Var_t(r_{p,t+1})]. \tag{2.9}$$

Given the optimal portfolio weights, we can compute daily time series of ex post portfolio returns $r_{p,t+1} = r_{f,t+1} + w_t(r_{m,t+1} - r_{f,t+1})$ and variances $Var_t(r_{p,t+1}) = (r_{p,t+1} - \bar{r}_p)^2$, and then plug that in to obtain the averaged realized utility.[3]

To quantify the economic benefit relative to the benchmark strategy, we use the performance fee Δ (in basis points), which is the fee that an investor is willing to pay to switch from the benchmark strategy (with portfolio return $r_{bm,t}$) to our strategy (e.g. strategy p with portfolio return $r_{p,t}$). In our analysis, we consider different risk aversion levels ($\gamma = 2, 6, 10$). The performance fee is computed as follows:

$$\frac{1}{T}\sum_{t=0}^{T-1}[(r_{p,t+1} - \Delta) - \frac{\gamma}{2}Var_t(r_{p,t+1})] = \frac{1}{T}\sum_{t=0}^{T-1}[r_{bm,t+1} - \frac{\gamma}{2}Var_t(r_{bm,t+1})]. \tag{2.10}$$

2.3.4 Statistical Significance of Economic Values

One of the major concerns of existing studies on the economic value of return/volatilty prediction is the statistical significance of the economic value obtained. The economic value computed is just a figure and usually not large (e.g in the unit of basis points), hence we don't know whether the value is significantly different from zero across different strategies. Therefore different meth-

[3]Alternatively, we can estimate the portfolio variance from the variance of the risky asset (we can use ex post realized variance scaled with the bias correction factor) scaled by the squared weight, $Var_t(r_{p,t+1}) = w_t^2 Var_t(r_{m,t+1}) = w_t^2 BCF \cdot RV_{t+1}$, results are similar.

ods have been used to investigate the statistical significance of economic values. Following Engle and Colacito (2006) and Bandi, Russell, and Zhu (2008), we address this concern by viewing the economic gains as loss differential in which we compare one portfolio to the benchmark portfolio. The approach is in the spirit of Diebold and Mariano (1995). The Diebold-Mariano (DM) test was designed to examine whether the loss differential of two forecasts is statistically significantly different from zero. The test can be used when the loss differential series is covariance stationary. Engle and Colacito (2006) and Bandi, Russell, and Zhu (2008) applied it to examine whether the ex post portfolio-volatility-difference between a candidate strategy and a benchmark strategy is statistically significantly different from zero. In our study, we investigate whether the performance fees (viewed as loss differential) are significantly different from zero. We first compute the time series of daily "spot" realized utilities and then the time series of daily performance fees for each strategy in comparison to the benchmark. Afterwards, we construct the DM statistics and test whether the alternative strategies do not outperform the benchmark (null hypothesis) using a one-sided t-test with a robust variance covariance estimator.[4]

2.3.5 Data Description

In our empirical analysis we use the S&P500 ETF or SPDR contract (SPY) as the risky asset, obtained from NYSE TAQ database. The contract tracks the S&P500 index and is very liquidly traded. Trading spans from 9:30 EST to 16:00 EST. Our sample spans from Jan 2nd 2001 to Dec 31st 2010. To start with we compute the realized volatility estimator based on equidistant observations

[4]An alternative way to assess the statistical significance of economic gains is to use bootrapping methods. A recent study by McCracken and Valente (2013) provides a formal test of economic values using the bootstrapping method.

sampled at the conventional five minute frequency in order to control for market microstructure noise. In the robustness checks section, we also report the results of a more advanced estimator to control for market microstructure noise – the average RV estimator of Andersen, Bollerslev, Christoffersen, and Diebold (2011). This estimator is a sub-sample estimator that can also be constructed easily. Starting from one minute regular spaced log-returns, we compute the average RV as an equally-weighted average of five overlapping five minute RV estimators. Andersen, Bollerslev, and Meddahi (2011) found that the average RV estimator can perform as well as more complex estimators (realized kernel, multiple time scale, pre-averaging etc) in volatility forecasting. We implement two further adjustments: Firstly, we remove the overnight periods. Secondly, we use linear interpolation to correct for different trading hours, especially in December 2008 and afterwards. We also collect S&P500 ETF or SPDR contract (SPY) daily data from CRSP in order to match intraday trading period to daily frequency. As the risk-free asset we use the daily average of the one month US Treasury bill series.

2.4 Empirical Findings

2.4.1 Statistical Findings

This part presents volatility forecasting results using realized jumps. Table 2.1 documents the descriptive statistics for the realized variance and realized jump variations using different jump tests. Although the statistics from different jump tests look different, they all share the same features, including high skewness and high kurtosis, supporting the asymmetric and rare event nature of jumps. We first conduct in-sample statistical evaluations by estimating our models with the whole sample data from 2001 and 2010.

Table 2.2 shows in-sample volatility forecasting results for the benchmark model and models using different jump tests. We follow Andersen, Bollerslev, and Diebold (2007) and use the Newey-West variance covariance matrix estimator with 5, 10 and 44 lags for daily, weekly, and monthly ahead forecasts. For the benchmark HAR-RV model, the one day ahead forecast shows that only the coefficient of weekly lagged realized variance is significant at the 5% level, while for one week and one month ahead forecasts all three lagged realized variance coefficients are significant. The adjusted R^2 takes values of 0.562, 0.682 and 0.644 for the different forecasting horizons. We then look at the HAR-RV-CJ models using different jump tests. At least four points are worth mentioning. Firstly, although at weekly and monthly horizons, coefficients for the integrated variances are all significant as in the benchmark model, the HAR-RV-CJ results differ from the HAR-RV model at daily horizon. The daily lagged integrated variances now become significant, indicating that jump robust integrated variation is more important than total realized variance in daily volatility forecasting. Secondly, the jump signs are almost all negative.[5] Our result is consistent with Andersen, Bollerslev, and Diebold (2007), but is different from Corsi, Pirino, and Reno (2010). Therefore, Corsi, Pirino, and Reno (2010)'s explanation that larger jumps lead to higher future volatility due to an increased level of disagreement may not hold in our setup. Instead, our findings are more consistent with Andersen, Bollerslev, and Diebold (2007)'s explanation that jumps are quickly mean reverting and hence can lead to a lower volatility rather than a higher one. Thirdly, jump coefficients differ in terms of the significance. Jump coefficients

[5] The exception of AJ and its later relative weak statistical performance can mainly be justified by its finite sample properties in a simulation analysis reported in the Appendix.

are significant across all horizons for BNS, JO and PZ, significant only at daily horizon for Med, Min, CPR, and insignificant for AJ across all horizons at the 5% level. Finally, we look at the goodness-of-fit of the models. We find that almost all models with different jump specifications can outperform the benchmark HAR-RV model at all forecasting horizons. At daily level, the highest adjusted R^2 is PZ of 0.604 and the lowest is AJ of 0.562. BNS has an adjusted R^2 of 0.592. Compared to BNS, CPR, JO, and Med have higher adjusted R^2s while Min has a lower adjusted R^2. For weekly and monthly ahead forecasts, adjusted R^2s are all close to 0.70 and 0.65 respectively. Although we observe a clear inverse U shape pattern of adjusted R^2s levels across forecasting horizons for all models, the improvements in adjusted R^2s compared to the benchmark model are diminishing from about 3% on daily horizon to about 1% on average on weekly and monthly horizons.

Although in-sample findings document a clear improvement in volatility forecasting by separating jumps from diffusion, we are also interested in whether results hold true out-of-sample. We first estimate model parameters using the first 1000 days of the whole sample as the in-sample period, and then use the rest of the sample from 2006 to 2010 as the out-of-sample period. Table 2.3 reports our out-of-sample volatility forecasting results. We report the Mean Squared Error (MSE) for the predicted value compared to the realized value. Similar to the in-sample analysis, we find that all HAR-RV-CJ models using different jump tests can outperform the benchmark HAR-RV in terms of lower MSEs. This finding holds true for all daily, weekly, and monthly horizons. Similar to the in-sample findings, we observe i) the largest statistical improvements at daily horizons and ii) improvements diminish when forecasting horizons increase. When we compare out-of-sample findings across different jump tests, we find that AJ has the lowest out-of-sample performance. Models using PZ, CPR, Med, or JO outperform BNS while the model using Min underperforms BNS. Results are consistent with in-sample findings and hold true across forecasting horizons.

2.4.2 Out-of-Sample Economic Findings

Given the significant statistical improvement by separating jump and diffusion components, we are now interested in whether such statistical accuracy can be translated into economic value for a risk-averse investor. We construct volatility timing strategies as discussed above for our out-of-sample period (2006 to 2010). The largest statistical forecasting improvement was observed for a daily horizon and given that the jump effect is quickly mean reverting we concentrate on volatility timing with daily re-balancing. To calculate the optimal portfolio weights we use the model-predicted volatility as a predictor for conditional volatility, and then adjust it with the bias-correction factor as illustrated in equation (2.8).

Table 2.4 reports the out-of-sample economic findings. Our main performance measure is performance fee, interpreted as the fee that an investor is willing to pay to switch from a benchmark strategy to a jump augmented strategy. We consider three risk aversion levels $\gamma = 2, 6, 10$. We show that all jump strategies generate positive performance fees in comparison to the benchmark strategy, and the economic values generated depend on different jump strategies and risk aversion levels. For the moderate risk aversion level of 6, we show that highest performance fees are 20 basis points for Med and Min, followed by 19 basis points for BNS and CPR, and 18 and 17 basis points for JO and PZ. AJ generates positive but very small performance fee; a result that is consistent with its negligible forecasting improvement in the statistical part. The economic magnitude is also affected by the change of the risk aversion level, ranging from 59 basis points ($\gamma = 2$) to 11 basis points ($\gamma = 10$), indicating that the strategy seems to work better for less risk averse investors. Around 0.6% annualized performance fee looks small in magnitude, and we therefore also assess the statistical significance of the economic value generated. We find that except for AJ, all jump strategies generate positive and statistically significant performance fees with DM t-statistics above 2. To summarize, we find that the separation of jumps from diffusion components improves volatility timing strategies for almost all jump tests. The out-of-sample economic findings are generally consistent with in-sample and out-of-sample statistical findings, although it does not necessarily match with the ranking of the in and out-of-sample volatility forecasting analyses. One possible explanation could be that jumps not only affect the volatility process, but also the return process, which is not captured by our volatility timing strategies.

2.5 Robustness Checks

In this section, we conduct comprehensive robustness checks. We focus on three issues: Firstly, our main results are based on the RV estimator sampled as the conventional five minutes sampling frequency. It is interesting to see whether our results still hold true under a more stringent control of market microstructure noise. Secondly, although we show that incorporating realized jumps in volatility timing generates economic value, we are also interested in whether it is feasible for an investor to exploit this in the presence of transaction costs. Thirdly, we also discuss whether realized jumps can help to predict realized higher moments and semi-variances, and whether performances can be improved using these in the portfolio allocation. Further extensions and robustness checks including simulation analysis, good and bad jumps, and sub-sample analysis can be found in the Appendix[6].

2.5.1 Market Microstructure Noises

We follow Andersen, Bollerslev, Christoffersen, and Diebold (2011) and Andersen, Bollerslev, and Meddahi (2011) and construct average realized variances and bipower variations. Table 2.5 reports the in-sample volatility forecasting results after further controlling for market microstructure noises. Although the jump coefficient is still negative and significant as shown in Section 2.3.2, the adjusted R^2 is different. For the one day ahead forecast, the adjusted R^2 for the benchmark model raises from 0.562 to 0.588 when using the average RV estimator. Similarly, the adjusted adjusted R^2 for the HAR-RV-CJ with BNS raises from 0.592 to 0.628. A similar statistical improvement is also found in the out-of-sample evaluation as shown in panel 1 of Table 2.6. Such statistical improvements by using subsample estimators also indicate potential economic improvements. The out-of-sample portfolio allocation results are shown in panel 2 of Table 2.6. We find that performance fees remain positive and statistically significant. Moreover, we show that economic magnitudes are larger using the microstructure noise robust estimators compared to the conventional five minutes estimator. A risk-averse investor is willing to pay performance fees ranging from 62 basis points ($\gamma = 2$) to 12 basis points ($\gamma = 10$) to use a jump

[6]In our earlier version of the paper, we also show the results of an alternative parametric portfolio allocation strategies using realized jumps, the economic values are positive but small in magnitude and in general statistically insignificant.

strategy. Our findings suggest that the statistical and economic improvements by separating jumps from diffusion are not likely to be driven by market microstructure noises. Instead, we show that controlling for market microstructure noises strengthens our findings. Our results are also consistent with previous studies (Bandi and Russell 2006, Bandi, Russell, and Zhu 2008, Liu 2009) that controlling for microstructure noises improve portfolio performances.

2.5.2 Transaction Costs

We then analyze the impact of transaction costs on our results. Different from existing studies comparing dynamic and static strategies (Fleming, Kirby, and Ostdiek 2001) or comparing two dynamic strategies which are based on high frequency and daily information respectively (Fleming, Kirby, and Ostdiek 2003), our analysis compares two dynamic strategies both using high frequency information. Therefore, we expect that the effect of transaction costs will not be as strong as documented in the existing literature. Following Bandi, Russell, and Zhu (2008), we define the transaction cost adjusted portfolio return in the following way:

$$\bar{r}_{p,t+1} = r_{p,t+1} - \rho(1 + r_{p,t+1})|\Delta w_{t+1}|, \tag{2.11}$$

where $\bar{r}_{p,t+1}$ is the transaction cost adjusted portfolio return, $r_{p,t+1}$ is pre-adjusted return, ρ is the transaction cost parameter, where we choose a high value of 0.0025, corresponding to a 2.5 cent half spread on a 10 dollar stock, Δw_{t+1} is the change of the weight from t to $t+1$, a proxy of trading turnover.

Table 2.7 presents the out-of-sample volatility timing results for all jump tests when transaction costs are taken into consideration. All specifications yield positive performance fees, implying that they can outperform the benchmark even when transaction costs are considered. Moreover, those performance fees are also statistically significant except for AJ. Jump strategies require incorporating recent information more quickly while the benchmark strategy is smoother, therefore we are not surprised by the higher turnover of the jump strategies compared to the benchmark strategy. Although the performance fees are slightly lower when controlling for transaction costs, we find that our results are generally consistent with our main findings in Section 2.4.2. Moreover, the relative performance of alternative jump specifications is also consistent with that in the main analysis, indicating that transaction costs have similar and only marginal effects for most of jump based volatility timing strategies.

2.5.3 Realized Jumps and Alternative Realized Moments

In practice, portfolio allocations are also subject to the impact of higher moments. Considering more general utility functions usually requires the prediction of higher moments. In addition to the sophistication of incorporating realized jumps into the portfolio allocation problem beyond mean-variance preferences, we present some statistical evidence of the predictive ability of realized jumps for alternative realized moments. We consider the use of realized jumps for predicting realized upside and downside volatilities, skewness, and kurtosis. We follow Barndorff-Nielsen, Kinnebrock, and Shephard (2008) and Amaya, Christoffersen, Jacobs, and Vasquez (2011), and construct alternative realized moments in the following way,

$$
\begin{aligned}
RV_{t,M}^{+} = \sum_{j=1}^{M} r_{j,t}^{2} 1_{r_{j,t}>0}, \quad RV_{t,M}^{-} = \sum_{j=1}^{M} r_{j,t}^{2} 1_{r_{j,t}<0} \\
RSK_{t,M} = \frac{\sqrt{M}\sum_{j=1}^{M} r_{j,t}^{3}}{(\sum_{j=1}^{M} r_{i,t}^{2})^{3/2}}, \quad RKU_{t,M} = \frac{M\sum_{j=1}^{M} r_{j,t}^{4}}{(\sum_{j=1}^{M} r_{i,t}^{2})^{2}}
\end{aligned}
$$

We first investigate the contemporaneous relationship between realized jumps and alternative realized moments using the following regression equation:

$$RM_t = \beta_0 + \beta_{RJ} RJ_t + \epsilon_t, \tag{2.12}$$

where RM_t is the realized moment including RV, RV^{+},RV^{-}, RSK, and RKU. Table 2.8 presents the contemporaneous regression results. We find that realized jump variation is a significant determinant of contemporaneous realized variance, positive and negative variances, and kurtosis, explaining 18% to 80% variation of realized moments respectively. A large jump variation is associated with a large variance, upside and downside variance, and kurtosis. Jump variation is also negatively related to realized skewness, however the relation is only marginally significant, and jumps can only explain about 1% variation in skewness.

We are more interested in the predictive relationship of realized jump variation and realized moments. Therefore we forecast realized moments using daily, weekly, and monthly lagged realized moments in the fashion of the HAR model, just as we did for realized variance in Section 2.4.1. We consider daily, weekly, and monthly ahead forecasting horizons. The models are specified as follows:

$$RM_{t,t+h-1} = \beta_0 + \beta_{RMD}RM_{t-1} + \beta_{RMW}RM_{t-5,t-1} + \beta_{RMM}RM_{t-22,t-1} + \epsilon_{t,t+h-1}. \tag{2.13}$$

To investigate the impact of jump, we then augment the model with realized jumps.

$$\begin{aligned} RM_{t,t+h-1} = &\beta_0 + \beta_{RMD}RM_{t-1} + \beta_{RMW}RM_{t-5,t-1} + \beta_{RMM}RM_{t-22,t-1} \\ &+ \beta_{JVD}JV_{t-1} + \epsilon_{t,t+h-1}. \end{aligned} \tag{2.14}$$

Table 2.9 reports in-sample forecasting results for realized moments. The results vary across different realized moments and forecasting horizons. Firstly, we find that realized jump helps to forecast realized variance, even though we do not use jump robust integrated variance as we did in the main part of the analysis. For all the forecasting horizons, we observe negative and statistically significant jump coefficients. Moreover, the adjusted R^2s of the models including jumps are higher than those of the models without jumps. This finding suggests that jumps do contain incremental information for predicting future volatility and the statistical and economic improvements we documented in the main analysis do not purely come from a better measurement of jump robust integrated variance. We now discuss the role of jumps in predicting alternative realized moments: We find that realized jumps have negative and statistically significant impacts on future downside volatility for all forecasting horizons and improvements in adjusted R^2s range from 1% to 3%. We also find that realized jumps help to predict upside volatility at the daily horizon and generate improvements in adjusted R^2 of about 0.2%. However, we show that realized jumps do not predict future realized skewness. Although realized jumps predict realized kurtosis at daily and weekly horizons, improvements in adjusted R^2s are almost negligible. These results, suggest that realized jumps may not contain predictive information beyond second moments at least in our empirical setups.

We then consider a simple portfolio allocation within the mean variance framework to quantify the predictive ability of realized jumps on alternative realized moments. We focus on upside and downside volatilities. Since the optimal portfolio weight is a function of the conditional variance of the risky asset, we can decompose it, as:

$$Var_t(r_{m,t+1}) = Var_t(r_{m,t+1})^+ + Var_t(r_{m,t+1})^- = BCF(\hat{RV}^+_{t,t+1} + \hat{RV}^-_{t,t+1}). \tag{2.15}$$

Since jumps have different predictive abilities to forecast upside and downside volatilities, an investor may improve portfolio performances by forecasting $RV^+_{t,t+1}$ and $RV^-_{t,t+1}$ separately and combining and scaling them by the Bias Correction Factor to obtain the total conditional variance $Var_t(r_{m,t+1})$, which can then be plugged into the portfolio weights function as shown in equation (2.7). We construct two portfolio strategies: The first strategy is based on predicting upside and downside volatilities with their lagged values in the HAR fashion. To improve forecasting performance, we include both upside and downside components at daily, weekly, and monthly lagged levels to predict each component one day ahead.

$$\begin{aligned} RV^{+}_{t,t+h-1} &= \beta_0 + \beta_{RVPD} RV^{+}_{t-1} + \beta_{RVMD} RV^{-}_{t-1} + \beta_{RVPW} RV^{+}_{t-5,t-1} \\ &\quad + \beta_{RVMW} RV^{-}_{t-5,t-1} + \beta_{RVPM} RV^{+}_{t-22,t-1} + \beta_{RVMM} RV^{-}_{t-22,t-1} + \epsilon_{t,t+h-1}, \end{aligned} \tag{2.16}$$

$$\begin{aligned} RV^{-}_{t,t+h-1} &= \beta_0 + \beta_{RVPD} RV^{+}_{t-1} + \beta_{RVMD} RV^{-}_{t-1} + \beta_{RVPW} RV^{+}_{t-5,t-1} \\ &\quad + \beta_{RVMW} RV^{-}_{t-5,t-1} + \beta_{RVPM} RV^{+}_{t-22,t-1} + \beta_{RVMM} RV^{-}_{t-22,t-1} + \epsilon_{t,t+h-1}. \end{aligned} \tag{2.17}$$

The second strategy augments the first strategy with daily lagged realized jump variation as additional regressor.

$$\begin{aligned} RV^{+}_{t,t+h-1} &= \beta_0 + \beta_{RVPD} RV^{+}_{t-1} + \beta_{RVMD} RV^{-}_{t-1} + \beta_{RVPW} RV^{+}_{t-5,t-1} \\ &\quad + \beta_{RVMW} RV^{-}_{t-5,t-1} + \beta_{RVPM} RV^{+}_{t-22,t-1} + \beta_{RVMM} RV^{-}_{t-22,t-1} \\ &\quad + \beta_{JVD} JV_{t-1} + \epsilon_{t,t+h-1}, \end{aligned} \tag{2.18}$$

$$\begin{aligned} RV^{-}_{t,t+h-1} &= \beta_0 + \beta_{RVPD} RV^{+}_{t-1} + \beta_{RVMD} RV^{-}_{t-1} + \beta_{RVPW} RV^{+}_{t-5,t-1} \\ &\quad + \beta_{RVMW} RV^{-}_{t-5,t-1} + \beta_{RVPM} RV^{+}_{t-22,t-1} + \beta_{RVMM} RV^{-}_{t-22,t-1} \\ &\quad + \beta_{JVD} JV_{t-1} + \epsilon_{t,t+h-1}. \end{aligned} \tag{2.19}$$

We focus on out-of-sample performances and consider three comparisons. The first comparison is between the first strategy based on equations (2.16) and (2.17) and the benchmark strategy in the main analysis, which predicts the total variance using HAR-RV in equation (2.5). The purpose is to assess whether predicting each volatility component separately can be economically valuable in comparison to predicting the total volatility. The second comparison is between the second strategy using jumps in equations (2.18) and (2.19) and the benchmark strategy in equation (2.5). The third comparison is between the first and second strategies, showing whether jumps convey incremental economic improvements. Table 2.10 reports out-of-sample portfolio performance fees for those three cases. We find that strategies based on predicting each volatility component separately outperform the benchmark strategy based on predicting total volatility, and can generate positive and statistically significant economic values. To be specific, in the first comparison, if we only use lagged upside and downside volatilities, we can generate annualized performance fees ranging from 13 basis points ($\gamma = 2$) to 2 basis points ($\gamma = 10$). If we include realized jumps, then the performance fees increase to range from 45 basis points ($\gamma = 2$) to 8 basis points ($\gamma = 10$). The third comparison suggests that including jump is important and can generate incremental economic improvements from 31 basis points ($\gamma = 2$) to 6 basis points ($\gamma = 10$) .

To summarize, we show that jumps do contain incremental predictive information for future volatility and its signed components, however realized jumps can hardly predict future realized higher moments. Therefore, the results suggest that realized jumps do not contribute much to moment timing based portfolio strategies beyond mean-variance approaches. If we remain in the mean-variance framework, predicting positive and negative volatility components separately can generate tangible economic improvements compared to predicting total volatility, and incorporating jumps can further improve the magnitude of the economic value.

2.6 Conclusion

Although a number of different nonparametric jump tests were developed in the literature, only very few studies analyze the potential use of realized jumps. Using high frequency data and seven major nonparametric jump tests, this paper investigates the predictive information content of realized jumps on volatility timing from both statistical and economic perspectives.

Covering all major jump tests, we confirm that separating jumps from the diffusion component does improve volatility forecasting in general. The result holds true both in-sample and out-of-sample. Moreover, we show that using a simple volatility timing strategy, a risk-averse investor can generate a significant economic value by separating jumps from the diffusion component. We conduct comprehensive robustness checks. We show that after controlling for microstructure noise and transaction costs, our main results still hold. We also find that realized jumps can predict realized volatility and its signed up and down components, and portfolio performance can be improved by separately predicting each component.

Our paper contributes to the field on a few aspects: Firstly, we contribute to the existing literature on the role of jumps in volatility forecasting. By using seven different jump tests, we show that jumps in general help to forecast volatility. Secondly, we show that the statistical improvement can also be exploited in a mean-variance portfolio allocation strategy. Hence, we also contribute to the literature on economic value of volatility timing. Thirdly, we contribute to the literature on the use of high frequency data and nonparametric jump tests. Our study can be viewed as evaluations of alternative jump tests using real world data while most previous studies focus on simulations. Further extensions include dealing with multivariate jumps (co-jumps), using more sophisticated utility functions, and considering alternative economic applications. They are beyond the purpose of our paper, and we leave them to future studies.

2.7 Tables

Table 2.1: Descriptive Statistics

	RV	JV_{BNS}	JV_{AJ}	JV_{JO}	JV_{Med}	JV_{Min}	JV_{CPR}	JV_{PZ}
$Mean$	1.324E-4	4.693E-6	1.313E-6	4.919E-6	6.377E-6	5.906E-6	8.223E-6	6.855E-6
Std	3.109E-4	7.692E-5	1.748E-5	7.689E-5	9.836E-5	9.805E-5	1.042E-4	7.836E-5
$Skew$	10.321	43.829	40.756	43.881	42.242	43.188	40.479	41.555
$Kurt$	158.187	2.056E+3	1.867E+3	2.060E+3	1.945E+3	2.040E+3	1.827E+3	1.906E+3
Min	3.468E-6	0	0	0	0	0	0	0
Max	0.0065	0.0037	8.131E-4	0.0037	0.0046	0.0046	0.0048	0.0037

Notes: The table summarizes the main descriptive statistics. We report realized variance (RV), and realized jump variations for seven different jump tests (BNS, AJ, JO, Med, Min, CPR, PZ). The sample period spans from Jan 1st 2001 to Dec 31st 2010. Both realized variance and realized jump variations are computed as shown in Section 2.2 using the 5 minutes high frequency Spyder contract (SPY) tracking the S&P500 index.

Table 2.2: In-Sample Volatility Forecasting Results

	β_0	β_{RVD}	β_{RVW}	β_{RVM}		$adjR^2$
BM						
$h=1$	1.436E-5	0.278	0.425	0.187		0.562
	(1.953)	(1.887)	(4.115)	(1.767)		
$h=5$	2.204E-5	0.180	0.370	0.282		0.682
	(2.779)	(2.029)	(4.643)	(2.568)		
$h=22$	4.103E-5	0.092	0.290	0.303		0.644
	(4.249)	(2.211)	(2.951)	(3.401)		
	β_0	β_{IVD}	β_{IVW}	β_{IVM}	β_{JVD}	$adjR^2$
BNS						
$h=1$	1.601E-5	0.467	0.325	0.132	-0.359	0.592
	(2.548)	(3.897)	(3.745)	(1.372)	(-4.121)	
$h=5$	2.354E-5	0.279	0.335	0.242	-0.158	0.699
	(3.130)	(3.197)	(3.840)	(2.143)	(-2.451)	
$h=22$	4.223E-5	0.145	0.284	0.275	-0.097	0.654
	(4.256)	(4.390)	(3.840)	(2.855)	(-2.001)	
AJ						
$h=1$	1.362E-5	0.280	0.419	0.188	2.931	0.562
	(1.845)	(1.883)	(4.061)	(1.791)	(1.641)	
$h=5$	1.991E-5	0.177	0.369	0.276	6.662	0.693
	(2.436)	(2.038)	(4.544)	(2.471)	(1.608)	
$h=22$	3.996E-5	0.091	0.290	0.299	3.599	0.648
	(4.261)	(2.213)	(2.944)	(3.306)	(1.647)	
JO						
$h=1$	1.532E-5	0.466	0.338	0.128	-0.366	0.593
	(2.503)	(3.848)	(3.958)	(1.349)	(-4.093)	
$h=5$	2.306E-5	0.287	0.337	0.239	-0.195	0.702
	(3.173)	(3.161)	(4.039)	(2.175)	(-2.936)	
$h=22$	4.187E-5	0.147	0.281	0.279	-0.099	0.654
	(4.219)	(4.418)	(2.806)	(2.920)	(-2.258)	
Med						
$h=1$	1.594E-5	0.489	0.321	0.124	-0.214	0.597
	(2.576)	(4.040)	(1.283)	(1.283)	(-5.021)	
$h=5$	2.368E-5	0.297	0.313	0.254	-0.062	0.697
	(3.181)	(3.022)	(2.358)	(2.358)	(-1.712)	
$h=22$	4.224E-5	0.146	0.280	0.285	-0.018	0.652
	(4.253)	(4.365)	(2.962)	(2.962)	(-0.362)	
Min						
$h=1$	1.670E-5	0.464	0.337	0.121	-0.162	0.592
	(2.664)	(3.887)	(3.894)	(1.283)	(-3.046)	
$h=5$	2.431E-5	0.275	0.352	0.228	-0.049	0.699
	(3.262)	(3.174)	(3.971)	(2.010)	(-1.355)	
$h=22$	4.307E-5	0.143	0.294	0.265	-0.024	0.653
	(4.261)	(4.569)	(2.817)	(2.748)	(-0.617)	
CPR						
$h=1$	1.725E-5	0.509	0.319	0.108	-0.188	0.598
	(2.838)	(4.019)	(3.402)	(1.133)	(-3.836)	
$h=5$	2.490E-5	0.311	0.324	0.232	-0.070	0.702
	(3.334)	(3.327)	(3.580)	(2.057)	(-1.849)	
$h=22$	4.333E-5	0.154	0.285	0.272	-0.021	0.653

	β_0	β_{IVD}	β_{IVW}	β_{IVM}	β_{JVD}	$adjR^2$
	(4.276)	(4.932)	(2.807)	(2.787)	(-0.505)	
PZ						
$h = 1$	1.665E-5	0.537	0.316	0.095	-0.498	0.604
	(2.884)	(4.311)	(3.500)	(1.052)	(-4.035)	
$h = 5$	2.406E-5	0.320	0.336	0.219	-0.248	0.705
	(3.319)	(3.306)	(3.717)	(2.001)	(-2.481)	
$h = 22$	4.268E-5	0.165	0.288	0.263	-0.133	0.655
	(4.261	(4.781)	(2.776)	(2.621)	(-3.403)	

Notes: The table reports in-sample volatility forecasting results for the SPY contract from 2001 to 2010 using different jump test. The HAR-RV model is used as the benchmark model. The HAR-RV-CJ models use realized jump variation detected using different jump tests: BNS, AJ, JO, Med, Min, CPR, and PZ. We forecast one day, one week, and one month ahead realized variances. The figures in parentheses are t-statistics computed using Newey-West corrected standard error for autocorrelation orders 5, 10, and 44 respectively. The $adjR^2$ is the adjusted R square.

Table 2.3: Out-of-Sample Volatility Forecasting Results

MSE	h=1	h=5	h=22
BM	5.657E-8	3.532E-8	3.504E-8
BNS	4.830E-8	3.195E-8	3.268E-8
AJ	5.656E-8	3.517E-8	3.483E-8
JO	4.794E-8	3.150E-8	3.273E-8
Med	4.733E-8	3.219E-8	3.258E-8
Min	4.881E-8	3.179E-8	3.238E-8
CPR	4.653E-8	3.107E-8	3.222E-8
PZ	4.523E-8	3.017E-8	3.214E-8

Notes: The table reports out-of-sample volatility forecasting results for the SPY contract for 2006 to 2010 using alternative jump tests. The HAR-RV model is used as the benchmark model. The HAR-RV-CJ model use realized jump variation detected using different jump tests: BNS, AJ, JO, Med, Min, CPR, and PZ. The out-of-sample period ranges from 2006 to 2010. We report the Mean Squared Error (MSE) for predicted volatility over one day, one week, and one month forecasting horizons.

Table 2.4: Out-of-Sample Portfolio Performance: Daily Rebalancing

	Volatility Timing: Performance Fees		
Strategies	$\gamma = 2$	$\gamma = 6$	$\gamma = 10$
BNS	0.0056	0.0019	0.0011
	(2.5684)	(2.5332)	(2.4980)
AJ	3.2391E-5	1.0797E-5	6.4781E-6
	(0.2951)	(0.2895)	(0.2839)
JO	0.0055	0.0018	0.0011
	(2.5355)	(2.5003)	(2.4655)
Med	0.0059	0.0020	0.0012
	(2.6870)	(2.6521)	(2.6171)
Min	0.0059	0.0020	0.0012
	(2.5301)	(2.4944)	(2.4586)
CPR	0.0056	0.0019	0.0011
	(2.6071)	(2.5713)	(2.5355)
PZ	0.0046	0.0017	9.5992E-4
	(2.2351)	(2.2001)	(2.1652)

Notes: The table reports out-of-sample portfolio allocation results for the SPY contract for 2006 to 2010 using alternative jump tests. The benchmark strategy uses the HAR-RV model to predict one day ahead volatility, other strategies using HAR-RV-CJ models with realized jump variation detected from the respective jump tests. We report performance fees relative to the benchmark strategy under risk aversion levels of 2, 6, and 10. Figures in parentheses are t-statistics for the DM test, where under the null hypothesis it is assumed that the (mean) performance fee equals zero.

Table 2.5: In-Sample Volatility Forecasting: Average RV

	β_0	β_{RVD}	β_{RVW}	β_{RVM}		$adjR^2$
BM						
$h = 1$	1.348E-5	0.330	0.369	0.197		0.588
	(1.897)	(1.887)	(2.984)	(1.936)		
$h = 5$	2.150E-5	0.207	0.331	0.294		0.685
	(2.834)	(1.971)	(3.869)	(2.647)		
$h = 22$	3.960E-5	0.108	0.268	0.313		0.651
	(4.202)	(1.997)	(2.738)	(3.311)		
	β_0	β_{IVD}	β_{IVW}	β_{IVM}	β_{JVD}	$adjR^2$
BNS						
$h = 1$	1.066E-5	0.669	0.195	0.097	-0.176	0.628
	(1.834)	(5.556)	(1.891)	(1.098)	(-6.362)	
$h = 5$	1.966E-5	0.372	0.292	0.225	-0.054	0.714
	(2.645)	(4.442)	(2.916)	(1.911)	(-2.624)	
$h = 22$	3.915E-5	0.195	0.274	0.262	-0.051	0.664
	(4.031)	(5.958)	(2.420)	(2.368)	(-3.238)	

Notes: The table reports in-sample volatility forecasting results for the SPY contract from 2001 to 2010 controlling for market microstructure noises. The HAR-RV model and the HAR-RV-CJ model using a BNS jump test are applied to forecast one day, one week, and one month ahead realized variances. The average realized variance is used in order to control for market microstructure noise. The figures in parentheses are t-statistics using Newey-West standard errors for autocorrelation orders of 5, 10, and 44. $adjR^2$ is the adjusted R square.

Table 2.6: Out-of-Sample Statistical and Economic Performances: Average RV

	Panel 1: Volatility Forecasting		
MSE	h=1	h=5	h=22
BM	4.501E-8	3.246E-8	3.231E-8
BNS	3.262E-8	2.764E-8	2.846E-8
	Panel 2: Volatility Timing: Daily		
Strategies	$\gamma = 2$	$\gamma = 6$	$\gamma = 10$
BNS	0.0062	0.0021	0.0012
	(2.7019)	(2.6660)	(2.6301)

Notes: The table reports out-of-sample statistical and economic performances from 2006 to 2010 after controlling for market microstructure noises. Average realized variance, integrated variance, and jump variation are used to control for market microstructure noise. Panel 1 reports out-of-sample volatility forecasting results. The HAR-RV model is used as the benchmark model. The HAR-RV-CJ model using a BNS jumps is also used. We report the Mean Squared Error for predicted volatility over one day, one week, and one month forecasting horizons. Panel 2 reports out-of-sample volatility timing results using the average RV estimator. Parameters are all estimated in-sample (2001-2005) and out-of-sample volatility forecasting and volatility timing are conducted out-of-sample (2006-2010). We report performance fees relative to the benchmark strategy under risk aversion levels of 2, 6, and 10. Figures in parentheses are t-statistics for the DM test, where under the null hypothesis it is assumed that the (mean) performance fee equals zero.

Table 2.7: Out-of-Sample Portfolio Allocation with Transaction Costs

	$\gamma = 2$		$\gamma = 6$		$\gamma = 10$	
Strategies	PF	TO	PF	TO	PF	TO
BM		0.0040		0.0013		7.9190E-4
BNS	0.0054	0.0267	0.0018	0.0089	0.0011	0.0053
	(2.5088)		(2.4736)		(2.4385)	
AJ	3.1067E-5	0.0044	1.0356E-5	0.0015	6.2134E-6	8.8352E-4
	(0.2827)		(0.2772)		(0.2716)	
JO	0.0053	0.0251	0.0018	0.0084	0.0011	0.0050
	(2.4767)		(2.4415)		(2.4064)	
Med	0.0058	0.0263	0.0019	0.0088	0.0012	0.0053
	(2.6438)		(2.6109)		(2.5760)	
Min	0.0054	0.0284	0.0018	0.0095	0.0011	0.0057
	(2.5083)		(2.4732)		(2.4381)	
CPR	0.0055	0.0254	0.0018	0.0085	0.0011	0.0051
	(2.5537)		(2.5180)		(2.4823)	
PZ	0.0045	0.0267	0.0015	0.0088	9.1207E-4	0.0053
	(2.1772)		(2.1427)		(2.1081)	

Notes: The table reports out-of-sample portfolio allocation results for the SPY contract for 2006 to 2010 controlling for transaction costs. The parameters are estimated in-sample from 2001 to 2005 and the ex post portfolio returns are obtained out-of-sample (2006 to 2010). We report performance fees (PF) and turnovers (TO) for risk aversion level of 2, 6, and 10. Figures in parentheses are t-statistics for the DM test. The test has the null hypothesis that the (mean) performance fee equals to zero. TO is the mean value of the absolute change of portfolio weight.

Table 2.8: Contemporaneous Regressions of Realized Jumps

RM	β_0	β_{JVD}	$adjR^2$
RV	1.2392E-4	1.7351	0.3135
	(10.5855)	(124.7009)	
RV^+	6.4057E-5	0.3005	0.1831
	(10.8169)	(2.6581)	
RV^-	5.9861E-5	1.4346	0.4437
	(10.2246)	(11.9167)	
RSK	0.0310	-1.1907E+3	0.0145
	(2.1479)	(-1.6645)	
RKU	4.1987	1.7231E+4	0.8055
	(88.9212)	(3.7758)	

Notes: This table reports the coefficients of the contemporaneous regressions of realized moments including RV, RV^+, RV^-, RSK, RKU on realized jumps for the SPY contract for 2001 to 2010 using the BNS jump test. $adjR^2$ is the adjusted R square

Table 2.9: Realized Moments Forecasting

	β_0	β_{RMD}	β_{RMW}	β_{RMM}	β_{JVD}	$adjR^2$
RV						
$h=1$	1.436E-5	0.278	0.425	0.187		0.562
	(1.923)	(1.887)	(4.115)	(1.767)		
	1.518E-5	0.485	0.286	0.147	-0.914	0.591
	(2.4670)	(4.012)	(3.549)	(1.659)	(-4.687)	
$h=5$	2.204E-5	0.180	0.370	0.282		0.682
	(2.568)	(1.872)	(4.137)	(2.413)		
	2.252E-5	0.301	0.289	0.258	-0.536	0.697
	(2.760)	(2.966)	(3.247)	(2.340)	(-3.292)	
$h=22$	4.103E-5	0.092	0.290	0.303		0.644
	(6.329)	(1.675)	(2.903)	(2.898)		
	4.133E-5	0.167	0.239	0.288	-0.333	0.651
	(6.318)	(2.784)	(2.383)	(2.765)	(-3.086)	
RV^+						
$h=1$	6.326E-6	0.289	0.454	0.159		0.604
	(1.923)	(2.628)	(3.714)	(1.420)		
	6.634E-6	0.308	0.442	0.156	-.0.108	0.606
	(2.047)	(2.716)	(3.589)	(1.399)	(-2.545)	
$h=5$	9.825E-6	0.186	0.411	0.252		0.717
	(2.481)	(2.336)	(4.089)	(2.087)		
	9.914E-6	0.191	0.408	0.251	-0.031	0.717
	(2.511)	(2.329)	(4.028)	(2.078)	(-0.913)	
$h=22$	1.873E-5	0.098	0.301	0.310		0.668
	(4.129)	(4.149)	(2.896)	(3.059)		
	1.879E-5	0.102	0.299	0.309	-0.023	0.668
	(4.139)	(4.228)	(2.903)	(3.039)	(-1.209)	
RV^-						
$h=1$	1.030E-6	0.134	0.411	0.299		0.387
	(2.072)	(0.984)	(3.493)	(2.133)		
	1.035E-5	0.401	0.268	0.221	-0.647	0.419
	(2.616)	(3.299)	(2.814)	(2.352)	(-3.501)	
$h=5$	1.402E-5	0.103	0.321	0.365		0.583
	(2.959)	(1.253)	(3.007)	(2.932)		
	1.405E-5	0.286	0.222	0.311	-0.445	0.609
	(3.378)	(3.062)	(2.050)	(3.233)	(-3.110)	
$h=22$	2.362E-5	0.053	0.252	0.335		0.594
	(4.113)	(1.287)	(2.307)	(3.845)		
	2.364E-5	0.172	0.188	0.301	-0.289	0.609
	(4.212)	(2.638)	(1.602)	(4.224)	(-2.725)	
RSK						
$h=1$	0.027	-0.029	-0.087	0.050		0.003
	(1.697)	(-1.424)	(-1.601)	(0.422)		
	0.028	-0.030	-0.086	0.049	-102.129	0.003
	(1.739)	(-1.442)	(-1.59)	(0.413)	(-1.188)	
$h=5$	0.027	-0.004	-0.100	0.079		0.013
	(1.908)	(-0.457)	(-2.283)	(0.746)		
	0.027	-0.004	-0.099	0.078	-42.374	0.013
	(1.927)	(-0.519)	(-2.287)	(0.742)	(-0.950)	
$h=22$	0.026	-0.002	-0.010	0.023		0.027
	(1.924)	(-0.843)	(-0.496)	(0.225)		

Table 2.9: Realized Moments Forecasting

	β_0	β_{RMD}	β_{RMW}	β_{RMM}	β_{JVD}	$adjR^2$
	0.026	-0.002	-0.010	0.023	3.862	0.027
	(1.918)	(-0.825)	(-0.497)	(0.225)	(0.429)	
RKU						
$h = 1$	3.333	-0.002	0.071	0.153		0.735
	(7.858)	(-0.120)	(1.289)	(1.533)		
	3.290	0.012	0.071	0.150	-925.924	0.735
	(7.608)	(0.550)	(1.300)	(1.513)	(-2.190)	
$h = 5$	3.245	0.007	0.021	0.214		0.926
	(9.464)	(0.602)	(0.335)	(2.382)		
	3.213	0.018	0.021	0.217	-686.931	0.926
	(9.351)	(1.448)	(0.337)	(2.353)	(-2.516)	
$h = 22$	3.378	-0.002	0.013	0.201		0.980
	(12.221)	(-0.345)	(0.519)	(2.842)		
	3.374	-1.081E-4	0.013	0.201	-88.993	0.980
	(12.073)	(-0.020)	(0.519)	(2.849)	(-0.558)	

Notes: The table reports in-sample RM forecasting results for the SPY contract from 2001 to 2010. RV, RV^+, RV^-, RSK, RKU are RMs used. For each RM forecasting, the HAR-RM model using its own daily, weekly, and monthly lagged RM, and the HAR-RM-J model using also the jump variation from the BNS jump test are applied to obtain one day, one week, and one month ahead forecasts. The figures in parentheses are t-statistics using Newey-West standard errors for autocorrelation orders of 5, 10, and 44 respectively. The $adjR^2$ is adjusted R square.

Table 2.10: Out-of-Sample Portfolio Performance: Alternative Realized Moments

	Moment Timing: Performance Fees		
Strategies	$\gamma = 2$	$\gamma = 6$	$\gamma = 10$
RM	0.0013	4.4101E-4	2.6401E-4
	(2.0816)	(2.0566)	(2.0316)
$RM + JV$	0.0045	0.0015	8.9427E-4
	(2.1240)	(2.0899)	(2.0588)
JV	0.0031	0.0010	6.2967E-4
	(1.7982)	(1.7688)	(1.7355)

Notes: The table reports out-of-sample portfolio allocation results for the SPY contract for 2006 to 2010 by predicting upside and downside volatilities. The first case compares portfolio performances between predicting upside and downside volatilities and predicting total realized volatility. The second case compares portfolio performances between predicting upside and downside volatilities with their past values and jumps and predicting total realized volatility. The third case compares predicting upside and downside volatilities with their past values and jumps. We report performance fees relative to the benchmark strategy under risk aversion levels of 2, 6, and 10. Figures in parentheses are t-statistics for DM test. The test has the null hypothesis that the (mean) performance fee equals to zero.

Online Appendix

2.8 Appendix 1: Jump Tests

In this part, we discuss specifications of six alternative jump tests, including Ait-Sahalia and Jacod (2009)(AJ), Jiang and Oomen (2008)(JO), Andersen, Dobrev, and Schaumburg (2012) (Med, Min), Corsi, Pirino, and Reno (2010)(CPR), and Podolskij and Ziggel (2010) (PZ).

2.8.1 Ait-Sahalia and Jacod (2009)

Ait-Sahalia and Jacod (2009) find that realized power variation is invariant to different sampling scales when jumps are present. Therefore the AJ test detects the presence of jumps using the ratio of realized power variation sampled from two scales. For the realized power variation for the sampling scale h and kh with scalar $k > 0$ we have

$$\begin{aligned} PV_{t,M}(p,h) &= \sum_{j=1}^{M/h} |r_j|^p, \text{ and} \\ \hat{S}_t(p,k,h) &= \frac{\hat{PV}_{t,M}(p,kh)}{\hat{PV}_{t,M}(p,h)}. \end{aligned}$$

Then the test statistic is given by

$$AJ_{t,M} = \frac{\hat{S}_t(p,k,h) - k^{p/2-1}}{\sqrt{\hat{V}_{t,M}}} \longrightarrow N(0,1), \tag{2.20}$$

where $\hat{V}_{t,M}$ is the asymptotic variance of $S_t(p,k,h)$.

$$\hat{V}_{t,M} = \frac{N(p,k)\hat{A}_{t,M(2p)}}{M\hat{A}_{t,M(p)}},$$

$$N(p,k) = (1/\mu_p^2)(k^{p-2}(1+k))\mu_{2p} + k^{p-2}(k-1)\mu_p^2 - 2k^{p/2-1}\mu_{k,p},$$

$$\mu_{k,p} = E(|U|^p|U + \sqrt{k-1}V|^p),$$

$$\hat{A}_{t,M(2p)} = \frac{(1/M)^{1-p/2}}{\mu_p}\sum_{j=1}^{M}|r_{t_j}|^p I_{|r_{t_j}|<\alpha(1/M)^{\omega}}.$$

U, V are random variables that $U \sim N(0,1)$ and $V \sim N(0,1)$ and p, k, α, ω are parameters.

2.8.2 Jiang and Oomen (2008)

The swap variance test developed by Jiang and Oomen (2008) is in the spirit of Neuberger (1994)'s replicating strategy on the payoff of the variance swap contract. Different from the other tests, which generate jump robust estimators, the JO test constructs a jump sensitive estimator. The difference between simple returns R_{t_j} and logarithmic returns r_{t_j} can capture one half of the integrated variance if there is no jump in the underlying sample path. Therefore the difference between twice the return difference and the realized variance can capture the jump component if a jump occurs.

$$SwV_{t,M} = 2\sum_{j=1}^{M}(R_{t_j} - r_{t_j}).$$

Hence the test statistic using the ratio of $SwV_{t,M}$ and $RV_{t,M}$ is

$$JO_{t,M} = \frac{MBV_{t,M}}{\sqrt{\Omega_{SwV}}}(1 - \frac{RV_{t,M}}{SwV_{t,M}}) \longrightarrow N(0,1), \tag{2.21}$$

where Ω_{SwV} is the asymptotic variance using estimated integrated sixicity.

$$\Omega_{SwV} = \frac{\mu_6}{9} \frac{M^3 \mu_{6/p}^{-p}}{(M-p-1)} \sum_{j=0}^{M-p} \prod_{k=1}^{p} |r_{t_j}|^{6/p}.$$

$p = 4$ or 6 are suggested parameters.

2.8.3 Andersen, Dobrev, and Schaumburg (2012)

Andersen, Dobrev, and Schaumburg (2012) introduced two simple to implement but powerful estimators for integrated variance, which use nearest neighbour truncation to control for the impact of market microstructure noise and zero returns.

$$MinRV_t = \frac{\pi}{\pi - 2} \frac{m}{m-1} \sum_{j=2}^{m} min(|r_j|, |r_{j-1}|)^2, \tag{2.22}$$

$$MedRV_t = \frac{\pi}{6 - 4\sqrt{3} + \pi} \frac{m}{m-2} \sum_{j=3}^{m} med(|r_j|, |r_{j-1}|, |r_{j-2}|)^2 \tag{2.23}$$

The statistics are as follows,

$$\frac{1 - \frac{MinRV_t}{RV_t}}{1.81\delta max(1, \frac{MinRQ_t}{MinRV_t^2})} \longrightarrow N(0,1), \tag{2.24}$$

$$\frac{1 - \frac{MedRV_t}{RV_t}}{0.96\delta max(1, \frac{MedRQ_t}{MedRV_t^2})} \longrightarrow N(0,1), \tag{2.25}$$

where $MinRQ_t, MedRQ_t$ are minimum and median realized quarticity to estimate the integrated quarticity. The specifications are as follow

$$MinRQ_t = \frac{\pi m}{3\pi - 8} \frac{m}{m-1} \sum_{j=2}^{m} min(|r_j|, |r_{j-1}|)^4, \tag{2.26}$$

$$MedRQ_t = \frac{3\pi m}{9\pi + 72 - 52\sqrt{3}} \frac{m}{m-2} \sum_{j=3}^{m} med(|r_j||r_{j-1}||r_{j-2}|)^4. \tag{2.27}$$

2.8.4 Corsi, Pirino,and Reno (2010)

A recent study by Corsi, Pirino, and Reno (2010) extended the multipower variation in the spirit of BNS by incorporating a threshold. The idea is that large returns can result in an underrejection of jumps using multipower variation based tests. Therefore by introducing a local variance based threshold to filter out large returns, the test expects to reduce the bias. The corrected realized threshold bipower variation and corrected threshold tripower quarticity are as follows,

$$ctBV_{t,M} = \frac{\pi}{2} \sum_{j=2}^{M} Z1(r_{t_j}, \theta_{t_j}) Z1(r_{t_{j-1}}, \theta_{t_{j-1}}),$$

$$ctTQ_{t,M} = \mu_{4/3}^{-3} \sum_{j=3}^{M} Z1(r_{t_j}, \theta_{t_j}) Z1(r_{t_{j-1}}, \theta_{t_{j-1}}) Z1(r_{t_{j-2}}, \theta_{t_{j-2}}),$$

where $\mu_{4/3} = E(|U|)^{4/3}$, $U \sim N(0,1)$, and $\theta_{t_j} = c_\theta^2 \hat{V}_{t_j}$, c_θ is constant, and $\hat{V}_{t_j}$ is local volatility. $Z1(r_{t_j}, \theta_{t_j})$ is the threshold function given by,

$$Z1(r_{t_j}, \theta_{t_j}) = \begin{cases} |r_{t_j}| & r_{t_j}^2 < \theta_{t_j}, \\ 1.094\theta_{t_j}^{1/2} & r_{t_j}^2 > \theta_{t_j} \end{cases}$$

The statistic is as follows

$$\frac{1 - ctBV_{t,M}/RV_t}{\sqrt{(\pi^2/4 + \pi - 5)\delta max(1, ctBV_{t,M}/ctTQ_{t,M})}} \longrightarrow N(0,1). \tag{2.28}$$

2.8.5 Podolskij and Ziggel (2010)

Similar to CPR, Podolskij and Ziggel (2010)'s test is also inspired by the Mancini (2009)'s threshold estimator. However, different from the CPR, PZ constructs the statistic based on random perturbed intraday returns. The difference of realized power variation and the truncated estimator captures the jump component. The estimator is as follows,

$$tMV_{t,M}(p) = \frac{1}{M^{1-p/2}} \sum_{j=1}^{M} |r_{t_j}|^p (1 - \eta_j I_{|r_{t_j}| \leq ch^{\omega}}),$$

and the statistic is given by,

$$PZ_{t,M}(p) = \frac{tMV_{t,M}(p)}{\sqrt{Var(\eta_j) MV_{t,M}(2p)}} \longrightarrow N(0,1), \tag{2.29}$$

where the perturbing variable η_j is drawn from the distribution $P^{\eta} = 1/2(\delta_{1-\tau} + \delta_{1+\tau})$, δ is the Dirac measure and $\tau = 0.1$ or 0.05.

2.9 Appendix 2: Additional Robustness Checks

In this part, we document additional robustness check results, which have not been included in the main analysis. We first conduct a Monte Carlo simulation experiment to understand finite sample properties of different jump tests. Then, we consider decomposing jumps into positive and negative components and investigate their statistical and economic performances. Finally we conduct sub-sample analyses.

2.9.1 Simulations

In this section, we conduct a comprehensive Monte Carlo simulation analysis to understand the finite sample properties of different jump estimators. The purpose is to justify different statistical and economic performance of different jump estimators documented in the main part of the paper. Following Barndorff-Nielsen and Shephard (2004) and Huang and Tauchen (2005), we generate sample paths through a one factor stochastic volatility model under the null hypothesis of no jumps. The one factor stochastic volatility model is given by

$$dp_t = \mu dt + exp[\beta_0 + \beta_1 v_t] dW_{pt}, \tag{2.30}$$
$$dv_t = \alpha_v v_t dt + dW_{vt}, \quad corr(dW_{pr}, dW_{vt}) = \rho.$$

The one factor model shows that the price dynamic is driven by the price diffusion term dW_{pt} and the volatility diffusion term dW_{vt}, and these two terms are correlated in order to allow for a leverage effect. Under the alternative hypothesis a jump component is added to the price process, which is a compounded Poisson process with intensity λ and sizes drawn from $N(0, \sigma^2_{jump})$. To implement the simulation, we use the Euler scheme with increments of 1 second. We generate 10000 trading days each with 6.5 hours. Then we sample with a frequency of 1, 2, 5, 15 minutes.

Table 2.11: Simulation Results: Size

Median Mean reversion $av = -0.100$				
h	1	2	5	15
BNS	0.0525	0.0485	0.0330	0.0405
AJ	0.0780	0.0700	0.0775	0.0890
JO	0.0705	0.0745	0.0840	0.1255
Med	0.0470	0.0495	0.0475	0.0480
Min	0.0500	0.0450	0.0380	0.0375
CPR	0.0465	0.0470	0.0485	0.0535
PZ	0.0650	0.0810	0.0820	0.0910

Notes: The table reports the simulation size for all seven jump tests we used, including BNS, AJ, JO, Med, Min, CPR, and PZ. We report the type I errors for 1, 2, 5, and 15 minutes frequency compared to a significant level of 5%. The values are obtained through simulations with 10,000 trading days under the null hypothesis that no jumps occur.

Table 2.11 shows the empirical sizes (α) for a 5% level of the different jump tests. We follow Huang and Tauchen (2005) and Dumitru and Urga (2012) to choose parameters and set $av = -0.1$ to represent a moderate level of mean reversion. The simulated data does not include a noise component, therefore the sizes for the different frequencies should only reflect the sizes of the tests and should not be affected by market microstructure noise as in real world data. For the 1 minute frequency, Med RV, Min RV, and CPR have a better size (closer to 5%) in comparison to the other jump tests, while AJ and JO seem to be a bit oversized. The relative merits of Med RV, Min RV, and CPR generally hold true throughout the other frequencies. The bipower variation test in general performs well across different frequencies, but is slightly oversized at a 1 minute frequency.

Table 2.12: Simulation Results: Size Corrected Power with Varying Intensity

Median Mean reversion $av = -0.100$, Median Jump size $\sigma = 1.5$.

h	1	2	5	15
$\lambda = 2$				
BNS	0.8554	0.8213	0.7627	0.6107
AJ	0.2467	0.1532	0.0575	0.0346
JO	0.8897	0.8671	0.8270	0.7776
Med	0.8437	0.8206	0.7606	0.6155
Min	0.8400	0.7963	0.7256	0.5397
CPR	0.8584	0.8484	0.8056	0.6857
PZ	0.8898	0.9064	0.8769	0.7943
$\lambda = 1.5$				
BNS	0.7673	0.7257	0.6567	0.5367
AJ	0.2674	0.1688	0.0656	0.0456
JO	0.7999	0.7736	0.7320	0.6975
Med	0.7576	0.7322	0.6688	0.5541
Min	0.7532	0.7105	0.6393	0.4779
CPR	0.7740	0.7450	0.6942	0.5980
PZ	0.8080	0.8107	0.7941	0.7178
$\lambda = 1.0$				
BNS	0.5995	0.5812	0.5253	0.4252
AJ	0.2614	0.1667	0.0699	0.0587
JO	0.6299	0.6256	0.6075	0.5975
Med	0.5971	0.5850	0.5391	0.4627
Min	0.5832	0.5649	0.5005	0.3844
CPR	0.6051	0.6028	0.5665	0.4849
PZ	0.6508	0.6551	0.6220	0.5545
$\lambda = 0.5$				
BNS	0.3794	0.3305	0.3268	0.2465
AJ	0.2034	0.1403	0.0759	0.0840
JO	0.4024	0.3663	0.3936	0.4105
Med	0.3820	0.3372	0.3381	0.2757
Min	0.3668	0.3215	0.3160	0.2286
CPR	0.3907	0.3452	0.3505	0.2932
PZ	0.4214	0.4353	0.4101	0.3768

Notes: The table reports the simulation power for all seven jump tests we used, including BNS, AJ, JO, Med, Min, CPR, and PZ. We report one minus type II errors for 1, 2, 5, and 15 minutes frequency compared to a significant level of 5%. The values are obtained through simulations with 10,000 trading days under the alternative hypothesis with jumps. We report how the power of test varies when the jump intensity λ changes.

Tables 2.12 and 2.13 report the empirical power $(1 - \beta)$ for the different jump tests when the jump intensity and/or the jump size is varying. Similar to the size part, we fix the mean reversion parameter to a median level of $av = -0.1$. Firstly, we allow for a changing jump intensity while keeping the jump size fixed equal to its median level $\sigma = 1.5$ as shown in Table 2.12. When the jump intensity decreases, the power for all tests also decreases as there are less jumps in the data. Moreover, when the sampling frequency is reduced, the power of the tests monotonically decreases in general. One exception is the PZ test. Here the highest power is found for the 2 min sampling frequency. The highest powers are found for PZ, JO, and CPR, while Med RV and Min RV have powers close to the baseline BNS test. AJ is found to have the lowest power for all frequencies and levels of intensities.

Table 2.13: Simulation Results: Size Corrected Power with Varying Size

Median Mean reversion $av = -0.100$, Median Jump Intensity $\lambda = 1$.

h	1	2	5	15
$\sigma = 2$				
BNS	0.6327	0.5949	0.5724	0.4815
AJ	0.2614	0.1769	0.0656	0.0582
JO	0.6573	0.6451	0.6496	0.6209
Med	0.6270	0.5986	0.5869	0.5016
Min	0.6232	0.5775	0.5593	0.4426
CPR	0.6424	0.6107	0.6143	0.5357
PZ	0.6535	0.6708	0.6552	0.6568
$\sigma = 1.5$				
BNS	0.6137	0.5770	0.5109	0.4070
AJ	0.2473	0.1726	0.0721	0.0615
JO	0.6514	0.6294	0.5944	0.5603
Med	0.6133	0.5813	0.5249	0.4307
Min	0.6053	0.5602	0.4922	0.3714
CPR	0.6214	0.5965	0.5518	0.4638
PZ	0.6567	0.6649	0.6285	0.5825
$\sigma = 1.0$				
BNS	0.5520	0.5255	0.4390	0.3007
AJ	0.2218	0.1683	0.0900	0.0703
JO	0.5933	0.5808	0.5393	0.4923
Med	0.5514	0.5397	0.4572	0.3451
Min	0.5326	0.5079	0.4210	0.2540
CPR	0.5721	0.5509	0.4924	0.3693
PZ	0.6134	0.6126	0.5861	0.4895
$\sigma = 0.5$				
BNS	0.4084	0.3594	0.2404	0.1042
AJ	0.1773	0.1065	0.0726	0.0873
JO	0.4825	0.4392	0.3652	0.2613
Med	0.4171	0.3730	0.2724	0.1308
Min	0.3800	0.3115	0.2095	0.0774
CPR	0.4457	0.3977	0.3079	0.1490
PZ	0.5947	0.5212	0.4112	0.2503

Notes: The table reports the simulation power for all seven jump tests we used, including BNS, AJ, JO, Med, Min, CPR, and PZ. We report one minus type II errors for 1, 2, 5, and 15 minutes frequency compared to a significant level of 5%. The values are obtained through simulations with 10,000 trading days under the alternative hypothesis with jumps. We report how the power of test varying when jump intensity σ changes.

Secondly, we fix the jump intensity and allow for the value of the jump size to vary. Table 2.13 reports that the power of the tests decreases when jump size decreases. This result holds true for all jump tests, indicating that all existing jump tests have difficulties to detect small jumps or to distinguish volatility bursts from jumps. As before, the power decreases with lower sampling frequency. Similar to the size part, we also find a peak of power for the PZ test for a 2 min frequency, but only when the jump size is large $\sigma = 2$ or $\sigma = 1.5$. The highest power is again found for the tests: PZ, JO, and CPR. Moreover, both PZ and JO seem to have high power even when the sampling frequency is relative low (15 min).

The relative ranking is similar to the case when intensity is varying. Med RV and Min RV have similar power in comparison to the baseline bipower test. To summarize, we show that the finite sample properties of different jump tests are varying across sampling frequencies and jump characteristics. Med RV, Min RV, and CPR have the best size properties while PZ, JO, and CPR have the best power properties. In general, the performance of the different tests in the simulation study reflects very well the trade-off between size and power.

2.9.2 Good and Bad Jumps

As reported in the descriptive statistics in Table 2.1, the jump densities are significantly skewed. The conventional view is that negative jumps should matter more than positive jumps. Therefore, we analyze in this section whether further decomposing jump variations into positive and negative jumps can improve statistical and economic performances. We follow Barndorff-Nielsen, Kinnebrock, and Shephard (2008) and Patton and Sheppard (2013) and use realized semivariances for this purpose.

Following Patton and Sheppard (2013), we define realized semivariances as follows:

$$RV^- = \sum_{i=1}^{n} r_i^2 I_{[r_i<0]}, \tag{2.31}$$

$$RV^+ = \sum_{i=1}^{n} r_i^2 I_{[r_i>0]}. \tag{2.32}$$

Realized semivariances converge to the sum of half the integrated variance and jump variation for positive and negative returns respectively:

$$RV^- \to \frac{1}{2}\int_{t-1}^{t} \sigma_s^2 ds + \sum_{j=1}^{N_t} c_j^2 I_{[c_j<0]}, \tag{2.33}$$

$$RV^+ \to \frac{1}{2}\int_{t-1}^{t} \sigma_s^2 ds + \sum_{j=1}^{N_t} c_j^2 I_{[c_j>0]}. \tag{2.34}$$

Therefore, the difference of the two realized semivariances ensures that the half integrated variation terms vanishes, so that the signed jump variation is given by

$$\Delta J^2 = RV^+ - RV^- \to \sum_{j=1}^{N_t} c_j^2 I_{[c_j>0]} - \sum_{j=1}^{N_t} c_j^2 I_{[c_j<0]}. \tag{2.35}$$

The negative and positive jump components are given by

$$\Delta J^{2+} = (RV^+ - RV^-) I_{[(RV^+ - RV^-)>0]}, \tag{2.36}$$

$$\Delta J^{2-} = (RV^+ - RV^-) I_{[(RV^+ - RV^-)<0]}. \tag{2.37}$$

Using these measures we discuss four alternative HAR-RV-CJ specifications to evaluate volatility timing statistically and economically.

$$
\begin{aligned}
RV_{t,t+h-1} &= \beta_0 + \beta_{IVD} IV_{t-1} + \beta_{IDW} IV_{t-5,t-1} + \beta_{IVM} IV_{t-22,t-1} + \beta_{JSD} JSD_{t-1} + \epsilon_{t,t+h-1}, \\
RV_{t,t+h-1} &= \beta_0 + \beta_{IVD} IV_{t-1} + \beta_{IDW} IV_{t-5,t-1} + \beta_{IVM} IV_{t-22,t-1} + \beta_{JND} JND_{t-1} + \epsilon_{t,t+h-1}, \\
RV_{t,t+h-1} &= \beta_0 + \beta_{IVD} IV_{t-1} + \beta_{IDW} IV_{t-5,t-1} + \beta_{IVM} IV_{t-22,t-1} + \beta_{JPD} JPD_{t-1} + \epsilon_{t,t+h-1}, \\
RV_{t,t+h-1} &= \beta_0 + \beta_{IVD} IV_{t-1} + \beta_{IDW} IV_{t-5,t-1} + \beta_{IVM} IV_{t-22,t-1} + \beta_{JND} JND_{t-1} \\
&\quad + \beta_{JPD} JPD_{t-1} + \epsilon_{t,t+h-1}.
\end{aligned}
\tag{2.38}
$$

where JSD is the daily lagged signed jump variation, JND and JPD are the negative and positive jump components, respectively.

Table 2.14 presents in-sample volatility forecasting results. The sign of the jump component is positive for negative jumps and negative for positive jumps, just as suggested in Patton and Sheppard (2013). Therefore, a large price drop is likely to increase the volatility in the future while a price increase tends to reduce volatility. Moreover, the coefficients for signed jumps are positive, implying that negative jumps play a dominant role. However, we also find that for a daily horizon, the coefficients for jump specifications are insignificant, which is different from our main findings using jump variations. Then the coefficients turn significant for weekly and monthly horizons. Similar to before, we observe that all four models can lead to an improvement in adjusted R^2s compared to the benchmark HAR-RV model. For the one day ahead horizon, the adjusted R^2 is on average about 2% higher than that of the HAR-RV model. The adjusted R^2s are close to each other for all four specifications, however, the adjusted R^2 for the negative jump model is slightly higher than that for the positive jump model, which is consistent with Patton and Sheppard (2013).

The out-of-sample volatility forecasting results are reported in Table 2.15 panel

Table 2.14: In-Sample Volatility Forecasting Results

	β_0	β_{IVD}	β_{IVW}	β_{IVM}	β_{JSD}	β_{JND}	β_{JPD}	$adjR^2$
JS								
$h = 1$	1.527E-5	0.435	0.350	0.133	0.089			0.587
	(2.358)	(3.850)	(4.023)	(1.351)	(0.545)			
$h = 5$	2.316E-5	0.260	0.349	0.245	0.009			0.697
	(3.048)	(4.643)	(4.019)	(2.148)	(0.095)			
$h = 22$	4.198E-5	0.131	0.294	0.277	-0.010			0.653
	(4.257)	(4.418)	(2.858)	(2.862)	(-0.120)			
JN								
$h = 1$	1.544E-5	0.461	0.341	0.133		0.181		0.589
	(2.430)	(3.778)	(3.852)	(1.381)		(1.275)		
$h = 5$	2.333E-5	0.282	0.340	0.242		0.105		0.699
	(3.100)	(3.334)	(3.832)	(2.141)		(2.052)		
$h = 22$	4.206E-5	0.142	0.289	0.275		0.042		0.654
	(4.254)	(4.450)	(2.776)	(2.858)		(0.662)		
JP								
$h = 1$	1.515E-5	0.433	0.359	0.145			-0.194	0.587
	(2.352)	(3.440)	(3.871)	(1.476)			(-0.698)	
$h = 5$	2.318E-5	0.282	0.351	0.258			-0.364	0.701
	(3.102)	(3.357)	(4.055)	(2.295)			(-2.720)	
$h = 22$	4.201E-5	0.147	0.294	0.284			-0.218	0.655
	(4.260)	(4.421)	(2.831)	(2.980)			(-2.189)	
JN, JP								
$h = 1$	1.551E-5	0.487	0.339	0.142		0.212	-0.285	0.590
	(2.469)	(3.629)	(3.798)	(1.453)		(1.618)	(-1.061)	
$h = 5$	2.344E-5	0.321	0.336	0.256		0.153	-0.429	0.704
	(3.173)	(3.586)	(3.828)	(2.275)		(2.934)	(-2.907)	
$h = 22$	4.213E-5	0.164	0.287	0.283		0.069	-0.247	0.656
	(4.255)	(4.637)	(2.754)	(2.961)		(1.321)	(-2.426)	

Notes: The table reports in-sample volatility forecasting results for the SPY contract from 2001 to 2010 using good and bad jumps. The HAR-RV-CJ model is applied to forecast one day, one week, and one month ahead realized variance. We consider four specifications: realized semivariance, negative jumps, positive jumps, and both negative and positive jumps. The figures in parentheses are t-statistics using Newey-West standard errors for autocorrelation order 5, 10, and 44 respectively. $adjR^2$ is the adjusted R square.

1. For the daily horizon, all four specifications lead to better forecasts than the HAR-RV model. The negative jump model generates lower MSEs than the positive jump model, which is in line with the in-sample findings. However, we show that using signed jumps without separating them into positive and negative could generate lower MSEs. Moreover, all MSEs are close to the MSEs of the baseline HAR-RV-CJ models using the BNS test. Nevertheless, although coefficients become significant for weekly and monthly horizons, the out-of-sample performance deteriorates when the forecasting horizon is extended.

Table 2.15 panel 2 presents volatility timing performances for these models. We find that all those four specifications can outperform the benchmark HAR-RV model, and can generate positive and statistically significant performance fees at the 5% significance level. For the signed jump model, we show that annualized performance fees range from 40 ($\gamma = 2$) to 8 ($\gamma = 10$) basis points. We also find

Table 2.15: Out-of-Sample Statistical and Economic Performances: Good and Bad Jumps

	Panel 1: Volatility Forecasting		
MSE	h=1	h=5	h=22
BM	5.657E-8	3.532E-8	3.504E-8
JS	4.719E-8	3.100E-8	3.194E-8
JN	4.778E-8	4.700E-8	5.516E-8
JP	4.791E-8	4.881E-8	5.834E-8
JN, JP	4.811E-8	4.613E-8	5.348E-8
	Panel 2: Volatility Timing: Daily		
Strategies	$\gamma = 2$	$\gamma = 6$	$\gamma = 10$
JS	(0.0040)	0.0013	8.2063E-4
	(2.6780)	(2.6450)	(2.6100)
JN	(0.0047)	0.0016	9.4140E-4
	(2.6512)	(2.6171)	(2.6512)
JP	(0.0037)	0.0012	7.4159E-4
	(3.3700)	(3.3357)	(3.3015)
JN, JP	(0.0052)	0.0017	0.0010
	(2.7244)	(2.6901)	(2.6558)

Notes: The table reports out-of-sample statistical and economic performances for 2001 to 2010 using good and bad jumps. Panel 1 reports out-of-sample volatility forecasting results. The HAR-RV model is used as the benchmark model. The HAR-RV-CJ model with semivariance (JS), negative jumps (JN), positive jumps(JP), and both negative and positive jumps (JN, JP). Panel 2 reports out-of-sample volatility timing results. Parameters are estimated in-sample (from 2001 to 2005). The volatility forecasting and volatility timing results are obtained out-of-sample (from 2006 to 2010). We report performance fees relative to benchmark for risk aversion levels of 2, 6, and 10. Figures in parentheses are t-statistics for DM test. The test has the null hypothesis that the (mean) performance fee equals to zero.

that the negative jump model (from 47 basis points for $\gamma = 2$ to 9 basis points for $\gamma = 10$) outperforms its positive counterpart (37 basis points for $\gamma = 2$ to 7 basis points for $\gamma = 10$) slightly, which is consistent with both our statistical findings and the existing literature. Including both positive and negative jumps can further improve portfolio performances, and generate economic value from 52 basis points ($\gamma = 2$) to 10 basis points ($\gamma = 10$). Our findings suggest that separating jumps from diffusion improve volatility timing performances even without the use of nonparametric jump tests. And decomposing jumps into positive and negative components can further improve economic performances.

2.9.3 Sub-Sample Analysis

We then conduct a sub-sample analysis in order to understand whether the results hold true for different time periods. We split the whole data sample into period 1 (2001 to 2005) and period 2 (2006 to 2010). In each period, we use the first two years as the in-sample period and the remaining years as the out-of-sample period. Table 2.16 summarizes the out-of-sample volatility forecasting results for different jump tests. Initially, we find that the MSE in period 1 is significantly lower than in period 2. This finding can be attributed to the financial crisis period covered in period 2, which yields time varying parameters (especially for the in-sample period for 2006 to 2008). In period 1, the estimated in-sample parameters are more stable across the out-of-sample period and hence produce better forecast. Secondly, almost all HAR-RV-CJ models can outperform the benchmark HAR-RV in period 1 for all forecasting horizons. In period 2, only CPR and Med can outperform the benchmark at daily and weekly horizons, but not at a monthly horizon. During the financial crisis, local volatility is higher, hence jumps are more difficult to detect. When the forecasting horizon increases the reduction in forecasting accuracy is expected during the financial crisis period as the parameters are not immediately updated with the changing high local volatility information.

Table 2.16: Out-of-Sample Volatility Forecasting HAR-RV-CJ: Sub Samples

$$RV_{t,t+h-1} = \beta_0 + \beta_{IVD} IV_{t-1} + \beta_{IDW} IV_{t-5,t-1} + \beta_{IVM} IV_{t-22,t-1} + \beta_{JVD} JV_{t-1} + \epsilon_{t,t+h-1}$$

	Period 1			Period 2		
MSE	h=1	h=5	h=22	h=1	h=5	h=22
BM	1.528E-9	2.071E-9	4.511E-9	3.385E-8	1.356E-8	7.585E-9
BNS	6.698E-10	1.118E-9	3.280E-9	3.393E-8	1.374E-8	8.370E-9
AJ	1.522E-9	2.080E-9	4.496E-9	3.502E-8	1.892E-8	3.483E-8
JO	6.684E-10	1.120E-9	3.275E-9	3.388E-8	1.464E-8	8.576E-9
Med	6.686E-10	1.090E-9	3.194E-9	3.833E-8	1.397E-8	7.590E-9
Min	6.381E-10	1.063E-9	3.199E-9	3.396E-8	1.367E-8	8.508E-9
CPR	6.616E-10	1.085E-9	3.197E-9	3.124E-8	1.345E-8	1.578E-8
PZ	7.108E-10	1.193E-9	3.350E-9	3.386E-8	1.122E-8	8.601E-9

Notes: The table reports out-of-sample volatility forecasting results for the SPY contract for period 1 (2001 to 2005) and period 2 (2006 to 2010). HAR-RV model is used as the benchmark model.HAR-RV-CJ model with different jumps are used. The parameters are estimated in-sample from 2001 to 2003 for period 1 and 2006 to 2008 for period 2. The forecasting studies are implemented in the out-of-sample period 2004 to 2005 for period 1 and 2009 to 2010 for period 2. We report the Mean Squared Error (MSE) for predicted volatility over one day, one week, and one month forecasting horizons.

Table 2.17 illustrates out-of-sample volatility timing results for period 1 and 2. In period 1, the realized utility criteria show that all tests but AJ can outperform the benchmark. This result is consistent with the out-of-sample volatility forecasting results shown in Table 2.16, and with the whole in and out-of-sample volatility forecasting results. The performance fees are also statistically significant for almost all models. We find that all but the AJ jump strategies can generate positive and statistically significant economic values. Moreover, the economic magnitudes are higher than those using the whole sample in the main part of the paper. In period 2, only AJ can outperform the benchmark. The t-statistics are all lower than the single tail 5% critical value. The results show that economic values in period 2 are either negative or insignificant.

Table 2.17: Out-of-Sample Volatility Timing: Sub-Samples

	Period 1			Period 2		
Strategies	$\gamma = 2$	$\gamma = 6$	$\gamma = 10$	$\gamma = 2$	$\gamma = 62$	$\gamma = 10$
BNS	0.0299	0.0273	0.0247	-1.7027E-5	-7.6982E-5	-1.3654E-4
	(2.5814)	(2.3650)	(2.1464)	(-0.161)	(-0.0724)	(-0.1279)
AJ	-1.0276E-4	-1.1435E-4	-1.2586E-4	0.0015	0.0015	0.00015
	(-0.3415)	(-0.3812)	(-0.4212)	(0.2168)	(0.2131)	(0.2093)
JO	0.0279	0.0252	0.0226	-3.2155E-4	-4.0598E-4	-4.8986E-4
	(2.4002)	(2.1824)	(1.9622)	(-0.1845)	(-0.2317)	(-0.2776)
Med	0.0307	0.0280	0.0254	-0.0105	-0.0099	-0.0093
	(2.6058)	(2.3884)	(2.1687)	(-1.1615)	(-1.0998)	(-1.0316)
Min	0.0322	0.0293	0.0263	-2.3896E-4	-4.0598E-4	-5.6343E-4
	(2.5149)	(2.2936)	(2.0699)	(-0.0865)	(-0.1418)	(-0.1940)
CPR	0.0289	0.0262	0.0236	-0.0021	-0.0028	-0.0035
	(2.4808)	(2.2623)	(2.0416)	(-0.3326)	(-0.4455)	(-0.5537)
PZ	0.0256	0.0233	0.0209	-0.0706	-0.0723	-0.0741
	(2.3422)	(2.1933)	(1.9149)	(-1.5079)	(-1.1793)	(-2.4810)

Notes: The table reports out-of-sample volatility timing results for the SPY contract for period 1 (2001 to 2005) and period 2 (2006 to 2008). The benchmark strategy uses the HAR-RV to predict one day ahead volatility, and other strategies using the HAR-RV-CJ to predict one day ahead volatility with realized jump variation detected from respective jump test. The parameters are estimated in-sample (2001 to 2003 for period 1 and 2006 to 2008 for period 2) , the predicted volatility and the ex post portfolio returns are obtained out-of-sample (2004 to 2005 for period 1 and 2009 to 2010 for period 2). We report performance fee relative to benchmark strategy under risk aversion of 2, 6, and 10. Figures in parentheses are t-statistics for DM test. The test has the null hypothesis that the (mean) performance fee equals to zero.

To summarize, due to the short sample period of 10 years, our sub-sample periods covered the recent financial crisis, which leads to unstable parameter estimation and high local volatility. We suggest that separating the integrated variance from the jump components precisely is more difficult during a high local volatility regime and models have more difficulties to update time varying parameters. Hence, we find that our main result is stronger in the first sub-sample period, where the economic environment is relatively stable. Nevertheless, our main findings still holds true in general for both sub sample periods: The separation of jumps from diffusion component rather than jump itself is responsible for delivering statistically significant economic values.

Chapter 3

Uncovering the Benefit of High Frequency Data in Portfolio Allocation

3.1 Introduction

Asset return volatility is a measure of risk, which has direct economic implications in option pricing, risk management, and portfolio allocation. The volatility process is unobservable; hence it needs to be estimated using a parametric approach when data is available at low frequency (i.e. daily, weekly, monthly). The availability of high frequency data not only allows us to better understand market microstructure but also enables us to measure the return volatility process more precisely. Andersen, Bollerslev, Diebold, and Labys (2001) and Barndorff-Nielsen and Shephard (2002) initiate the literature of realized volatility. Realized volatility can be constructed by aggregating return variations of small intervals over a constant period (e.g. a day). This realized measure of volatility is now observable and model free. Theoretically, in the absence of microstructure noise, realized volatility is an accurate approximation of the true latent integrated volatility process when the sampling frequency is high enough. Using high frequency data to measure and forecast realized volatility enables us to better understand future risk, and hence may improve portfolio allocation performance for a risk-averse investor.

There is a long tradition in the literature using high frequency data to measure and forecast realized volatility. There are also many studies using high frequency data to improve portfolio allocation. We observe, however, a clear segmentation of the literature: One stream of the literature focuses on univariate realized volatility. Previous studies investigate issues, such as understanding distributional return properties (Andersen, Bollerslev, Diebold, and Labys 2001), market microstructure noise (Bandi and Russell 2006, Andersen, Bollerslev, and Meddahi 2011), and volatility forecasting (Andersen, Bollerslev, and Diebold 2007, Andersen, Bollerslev, and Huang 2011). This stream of the literature is still growing rapidly and recently also expanding from realized volatility to the consideration of other important risk measures. Barndorff-Nielsen, Kinnebrock, and Shephard (2008) introduce realized semvariances, which measure upside and downside volatility components. Barndorff-Nielsen and Shephard (2004) develop the realized bipower variation, which enables us to decompose the total realized variance into jump and diffusion components. Amaya, Christoffersen, Jacobs, and Vasquez (2011) construct realized measures for skewness and kurtosis. Those alternative measures play important roles in improving volatility forecasting performances (Andersen, Bollerslev, and Diebold 2007, Patton and Sheppard 2013) and recently also in predicting the cross-section of expected stock returns (Amaya, Christoffersen, Jacobs, and Vasquez 2011). However, the use of alternative realized measures are mainly assessed statistically, and only very few studies consider the economic benefits of employing alternative realized risk measures.

Another stream of the literature considers multivariate realized volatility. Despite the commonality between the two streams of the literature in using high frequency data to improve forecasting performance, the second stream also concentrates on economic applications, especially mean variance portfolio allocation. Fleming, Kirby, and Ostdiek (2003) initiate this area of the literature on the economic value of volatility timing using realized covariance matrices. Bandi, Russell, and Zhu (2008) investigate the benefit of an optimal sampling frequency to mitigate microstructure noise in portfolio allocation. Liu (2009) and Chiriac and Voev (2011) study the economic value of volatility timing by evaluating a set of low frequency and high frequency multivariate forecasting models. Hautsch, Kyj, and Malec (2013) analyse large scale portfolio allocation with high frequency data and emphasize the importance of controlling for non-synchronicity. However, all those studies use the total realized covariance matrix, and other economically meaningful alternative realized measures discussed above are overlooked both for forecasting and more importantly for portfolio allocation.

In this paper, we aim to bridge this gap and investigate the use of high frequency data in portfolio allocation from a different angle. We address four main research questions: Firstly, we are interested in the volatility forecasting performance of realized volatility and alternative realized measures. Since we concentrate on volatility timing based portfolio allocation, the precise forecasting of future volatility is a building block for the success of portfolio performance. We conduct comprehensive in-sample and out-of-sample forecasting studies. We show that a model using realized volatility already performs well in forecasting. When we then decompose realized volatility fully into upside and downside components or jump and diffusive components, we observe a clear improvement in forecasting performance compared to the model using realized volatility only. Models using realized skewness and kurtosis as additional volatility predictors are also able to improve the forecasting performance, however, improvements are smaller in magnitudes and results are more mixed across different assets. From all specifications considered, the model using upside and downside volatilities can consistently outperform the model using realized volatility only and generate the largest statistical improvement.

Our next question is whether the precise measurement and forecasting of univariate volatility significantly contributes to the benefit of high frequency data in portfolio allocation. Almost all previous studies in portfolio allocation with high frequency data focus only on the realized covariance matrix. Fleming, Kirby, and Ostdiek (2003) suggest that realized correlation is more important than realized volatility in their portfolio allocation setup. Although, we acknowledge that using realized covariance (and especially realized correlation) enables investors to benefit from capturing asset return co-movements much faster than using covariance based on daily data, we also notice that realized covariance has its own potential drawbacks of non-synchronicity and associated estimation errors. These drawbacks motivate the research question whether simply using high frequency data to estimate and forecast univariate realized volatilities only already provides a investor with sufficiently large economic benefits over low frequency strategies. This question can also be viewed as an attempt to understand the source of the benefit of high frequency data in portfolio allocation indirectly. To address this question, we apply a simple approach for the estimation of a composite covariance matrix in a mixed frequency fashion. This approach allows us to combine the high frequency based univariate realized volatilities with a low frequency based correlation structure. Hence, we can directly assess the benefit of high frequency data over low frequency data with different univariate volatility forecasts using the same correlation structure. This approach therefore also allows us to avoid the potential issue of non-synchronicity, and any subsequent adjustment to correct for non-synchronicity, which may affect the result. Our findings suggest that investors can generate statistically significant and economically tangible benefits in comparison to strategies that are based on low frequency data. Although, we agree that realized correlation is important, our empirical results support our conjecture that the precise measurement and forecasting of univariate volatility with high frequency data already significantly contributes to better portfolio allocation.

High frequency data also allows us to extract different components of realized volatility in a model free fashion. Hence, we further ask whether investors can obtain additional economic benefits in portfolio performance when realized volatility components are used as volatility predictors. A volatility process can be described by different components. Using different volatility components is not only important for statistical purposes, but also matters for our understanding of different types of risks embedded in the volatility process, such as downside risk, upside uncertainty, jump risk, and diffusive risks. Existing studies already document that modelling volatility with different components can improve the in-sample fit and out-of-sample forecasting performance (Andersen, Bollerslev, and Diebold 2007, Patton and Sheppard 2013). However, the question whether those separations do economically matter remains unanswered. Previous studies fail to address the question in portfolio allocation contexts. A reason might be the challenge to decompose the full covariance matrix into different economically meaningful components while controlling for non-synchronicity at the same time. Using the simple and flexible mixed frequency framework discussed above, our paper can directly assess the economic benefit of using volatility components and avoid any problems related to potential non-synchronicity. We find that strategies using either upside and downside volatilities or jump and diffusion components can outperform respective low frequency strategies. The strategy using upside and downside volatilities can generate economic values, which are over 60% larger than those obtained by strategy using the conventional realized volatility as discussed above. The incremental benefit for the upside and downside volatility strategy compared to the conventional realized volatility strategy is also statistically significant and economically large.

In addition to realized volatility and its components, high frequency data further enables us to construct realized measures of higher return moments. Therefore, we are interested in whether realized higher moments can also contribute to the portfolio performance. To be consistent with existing studies in the volatility timing literature and for simplicity, this paper only focuses on mean variance portfolio allocation, which does not explicitly deal with investor's higher order preferences. However, we include realized higher moments as additional volatility predictors to assess their economic benefits for portfolio allocation. In the volatility forecasting part, we show both measures can significantly predict future volatility but the economic improvements are small and results are more mixed. In the portfolio allocation analysis, we find that higher moment strategies generally generate large economic benefits over low frequency and high frequency benchmarks. However these benefits are also more unstable. Nevertheless, we find that the skewness based strategy can generate statistically and economically significant additional improvements under some circumstances.

We then conduct comprehensive robustness checks. We show that high frequency strategies can outperform well known low frequency benchmark strategies used in the literature. High frequency data remains important for portfolio allocation when different correlation structures are used. Our main results still hold after controlling for market microstructure noise and transaction costs.

The paper is organized as follows: Section 3.2 discusses different realized measures constructed from high frequency data. Section 3.3 introduces data and methodologies used in this paper. Section 3.4 reports the main empirical findings. Section 3.5 conducts comprehensive robustness checks. Section 3.6 concludes.

3.2 Realized Measures from High Frequency Data

3.2.1 Realized Volatility

Since the seminal works of Andersen, Bollerslev, Diebold, and Labys (2001) and Barndorff-Nielsen and Shephard (2002), daily realized measures constructed from high frequency data have become popular in financial econometrics and finance to measure risks, i.e. variance, covariance, higher moments, and betas. The general setup is outlined below. We consider a log price series p_s that follows a jump-diffusion process in continuous time:

$$dp_s = \mu_s d_s + \sigma_s dW_s + dJ_s \tag{3.1}$$

where μ_s is the drift, σ_s is the diffusive volatility, W_s is the standard Brownian motion, J_s is a pure jump process.

The daily quadratic variation of this process on day t is

$$QV_t = [p,p]_t = \int_{t-1}^{t} \sigma_s^2 ds + \sum_{t-1<s\le t} (\Delta p_s)^2 \tag{3.2}$$

where $\Delta p_s = p_s - p_{s-}$ is the jump component.

The simplest realized measure is the realized variance, which can be constructed as the sum of squared intraday returns.

$$RV_t = \sum_{j=1}^{m} r_{j,t}^2 \xrightarrow{p} QV_t \tag{3.3}$$

where $r_{j,t} = p_{j,t} - p_{j-1,t}$ and m is the number of intraday intervals. When the sampling frequency is high and the intraday interval is very small, RV_t converges to the daily quadratic variation QV_t in the absence of microstructure noise .

3.2.2 Realized Volatility Components

The availability of high frequency data allows us not only to measure volatility more precisely, but also to extract different components of total volatility in a model free way. In this section, we consider two prevalent decompositions of realized volatility, namely decomposing realized volatility into upside and downside components and into jump and diffusion components.

Barndorff-Nielsen, Kinnebrock, and Shephard (2008) introduced the terms realized semivariances in order to capture upward and downward components of daily quadratic variations. Realized semivariances can be constructed using the sum of squared intraday returns conditional on whether the intraday return is positive or negative.

$$RS_t^- = \sum_{j=1}^{m} r_j^2 I_{(r_j<0)} \xrightarrow{p} \frac{1}{2}\int_{t-1}^{t} \sigma_s ds + \sum_{t-1<s\leq t} [\Delta p_s^2 I_{(\Delta p_s<0)}]; \tag{3.4}$$

$$RS_t^+ = \sum_{j=1}^{m} r_j^2 I_{(r_j>0)} \xrightarrow{p} \frac{1}{2}\int_{t-1}^{t} \sigma_s ds + \sum_{t-1<s\leq t} [\Delta p_s^2 I_{(\Delta p_s>0)}]; \tag{3.5}$$

where I is an indicator function for intradaily returns being negative ($I_{(r_j<0)}$) or positive ($I_{(r_j>0)}$). By construction, the sum of upside and downside realized semivariances equals the realized variance. If the asset price simply follows a pure diffusion, then we should expect both realized semivariancs to converge to half of the integrated variance over a long enough horizon. Splitting the realized variance into two equal parts may therefore not reveal much information. However, if the price process contains jumps or asymmetries, then the upside and downside realized semivariances may not be equal.[1] Hence, splitting realized variance into two parts may reveal incremental information that is not captured by the realized variance. Exploiting this potentially incremental information appropriately may generate economic benefits.

Another important decomposition of realize volatility is into jump and diffusion components. Barndorff-Nielsen and Shephard (2004) introduced bipower variation to isolate diffusion variation (integrated variance) from jump variation. Assuming jumps to be rare and unlikely to occur for two consecutive intraday returns, when intervals are small enough, the realized bipower variation $BV_{m,t}$ will converge in probability to the jump-robust integrated variance IV_t. The realized bipower statistic is defined as

$$BV_t = \frac{\mu_1^{-2} m}{m-1} \sum_{j=2}^{m} |r_{t_{j-1}}||r_{t_j}| \tag{3.6}$$

$$BV_t \xrightarrow{p} IV_t = \int_{t-1}^{t} \sigma_s^2 ds$$

where $\mu_1 = \sqrt{2/\pi}$. The difference between realized variance and realized bipower variation is then an estimator of the jump variation component JV_t.

Since our purpose is to predict volatility rather than identify exactly the jump returns, we do not follow the procedure to test for jumps and then use indicator functions to compute JV_t. Instead, we directly use the difference of RV_t and

[1] See Barndorff-Nielsen, Kinnebrock, and Shephard (2008) and Patton and Sheppard (2013) for more details and asymptotic properties.

BV_t to compute JV_t. This method is also supported by Corradi, Distaso, and Fernandes (2013), who argued that avoiding the use of indicator functions to detect jumps can improve the efficiency of the estimator:

$$JV_t = RV_t - BV_t \xrightarrow{p} \sum_{t-1<s\leq t} (\Delta p_s)^2 \tag{3.7}$$

3.2.3 Realized Higher Moments

Besides realized volatility and its components, we can also construct realized higher moments using high frequency data. Skewness and kurtosis can be related to downside risk and the jumps introduced above. However, the realized measures for skewness and kurtosis constructed using high frequency data have not been studied until recently. Motivated by Barndorff-Nielsen, Kinnebrock, and Shephard (2008), Amaya, Christoffersen, Jacobs, and Vasquez (2011) suggest that the realized third (RTM) and fourth moments (RFM) can be written as follows:

$$RTM_t = \sum_{j=1}^{m} (r_{j,t})^3 \xrightarrow{p} \sum_{t-1<s\leq t} (\Delta p_s)^3 \tag{3.8}$$

$$RFM_t = \sum_{j=1}^{m} (r_{j,t})^4 \xrightarrow{p} \sum_{t-1<s\leq t} (\Delta p_s)^4 \tag{3.9}$$

To be consistent with conventional concepts about skewness and kurtosis risks, we follow Amaya, Christoffersen, Jacobs, and Vasquez (2011) to construct daily realized skewness (RSK) and realized kurtosis (RKU) as follows:

$$RSK_t = \frac{\sqrt{m}\sum_{j=1}^{m}(r_{j,t})^3}{[\sum_{j=1}^{m}(r_{j,t})^2]^{3/2}} \tag{3.10}$$

$$RKU_t = \frac{m\sum_{j=1}^{m}(r_{j,t})^4}{[\sum_{j=1}^{m}(r_{j,t})^2]^2} \tag{3.11}$$

3.3 Data and Methodology

3.3.1 Data

Our main empirical analysis uses 30 assets including 29 stocks from historical constituencies of the Dow Jones 30 and 1 index ETF Spyder Contract (SPY) tracking the S&P500 market index. Our data ranges from Jan 2nd 2001 to Sep 30th 2009 covering the recent financial crisis. The tick by tick data is collected from the NYSE Trade and Quote Database (TAQ). The daily trading period is from 9:30 EST to 16:00 EST. Table 3.1 documents the list of 30 assets. We follow usual procedures to clean the data and construct equal distance 5 minutes interval intraday returns for each series to control for market microstructure noise in the construction of realized measures. Hence we obtain 78 intraday observations per day. In the robustness check section, we further control for microstructure noise using a sub-sampled estimator using 5 overlapping 5 min returns. We use open to close prices to construct daily returns and therefore avoid the impact of overnight returns and the need to use bias correction factors to reconcile daily and intra-daily information.

3.3.2 Realized Volatility Forecasting

To assess the use of high frequency data in volatility timing based portfolio allocation, we need to forecast the future conditional covariance matrix, and then use it to compute optimal portfolio weights. Different from almost all previous studies in this area, we try to avoid estimating and forecasting the realized covariance matrix directly. Instead, we apply a simple mixed frequency framework, in which we explicitly construct a conditional covariance matrix by forecasting conditional volatility and conditional correlation parts separately.

Therefore, our first step is to forecast future univariate realized volatility asset by asset using high frequency data.

We follow Corsi (2009) and use the Heterogeneous Autoregressive model (HAR) because it allows to approximate long memory properties in a straightforward way:

$$RV_{t,t+1} = \beta_0 + \beta_{RVD} RV_t + \beta_{RVW} RV_{t,t-4} + \beta_{RVM} RV_{t,t-21} + \epsilon_{t+1} \tag{3.12}$$

where $RV_{t,t+1}$ is one step ahead RV, $RV_{t,t-4}$ and $RV_{t,t-21}$ are lagged 5 days (weekly) and 22 days (monthly) averaged RVs that are included to capture long memory properties. The model predicts one step ahead RV using daily, weekly, and monthly lagged RVs.

The HAR model is a flexible framework, hence we can also accommodate it with alternative realized measures. We therefore consider four alternative specifications using realized volatility components and higher moments as volatility predictors.

To investigate the effect of volatility components on volatility forecasting and consequentially portfolio performance, we employ models using upside and downside volatility components and jump and diffusion components as follows:[2]

$$\begin{aligned} RV_{t,t+1} = \beta_0 &+ \beta_{RSmD} RS_t^- + \beta_{RSpD} RS_t^+ + \beta_{RSmW} RS_{t,t-4}^- + \beta_{RSpW} RS_{t,t-4}^+ \\ &+ \beta_{RSmM} RS_{t,t-21}^- + \beta_{RSpM} RS_{t,t-21}^+ + \epsilon_{t+1} \end{aligned} \tag{3.13}$$

[2]Similar models have also been used by Andersen, Bollerslev, and Diebold (2007), Corsi, Pirino, and Reno (2010), and Patton and Sheppard (2013).

$$RV_{t,t+1} = \beta_0 + \beta_{BVD} BV_t + \beta_{JVD} JV_t + \beta_{BVW} BV_{t,t-4} + \beta_{JVW} JV_{t,t-4} + \beta_{BVM} BV_{t,t-21} + \beta_{JVM} JV_{t,t-21} + \epsilon_{t+1} \quad (3.14)$$

where $RS^-_{t,t-4}$, $RS^+_{t,t-4}$, $RS^-_{t,t-22}$, $RS^+_{t,t-22}$ are lagged 5 days (weekly) and 22 days (monthly) averaged RS^-s and RS^+s; and $BV_{t,t-4}$, $JV_{t,t-4}$, $BV_{t,t-22}$, $JV_{t,t-22}$ are lagged 5 days (weekly) and 22 days (monthly) averaged BVs and JVs.

To understand whether realized skewness and kurtosis, which have already been found to improve return predictability, may also improve one step ahead realized volatility forecasting, we also incorporate realized skewness and kurtosis as predictors into the model with realized volatility only.

$$RV_{t,t+1} = \beta_0 + \beta_{RVD} RV_t + \beta_{RSKD} RSK_t + \beta_{RVW} RV_{t,t-4} + \beta_{RSKW} RSK_{t,t-4} + \beta_{RVM} RV_{t,t-21} + \beta_{RSKM} RSK_{t,t-21} + \epsilon_{t+1} \quad (3.15)$$

$$RV_{t,t+1} = \beta_0 + \beta_{RVD} RV_t + \beta_{RKUD} RKU_t + \beta_{RVW} RV_{t,t-4} + \beta_{RKUW} RKU_{t,t-4} + \beta_{RVM} RV_{t,t-21} + \beta_{RKUM} RKU_{t,t-21} + \epsilon_{t+1} \quad (3.16)$$

where $RSK_{t,t-4}$, $RSK_{t,t-22}$are lagged 5 days (weekly) and 22 days (monthly) averaged RSKs . And $RKU_{t,t-4}$, $RKU_{t,t-22}$ are lagged 5 days (weekly) and 22 days (monthly) averaged RKUs.

In this part, we primarily focus on one day ahead forecasts in order to incorporate lagged realized risk information immediately. In the portfolio allocation part, we also consider different rebalancing frequencies. We could also follow Patton and Sheppard (2013) and use a panel regression framework. However, our primary focus lies on out-of-sample prediction and due to the large heterogeneity across assets, using equation by equation estimation without imposing panel restrictions may lead to better individual volatility forecasts.

3.3.3 Correlation Structures

Our second step is to construct a covariance matrix, which will then be used to derive the optimal portfolio weights in the asset allocation problem part. Due to the relative large dimension of assets, the impact of non-synchronicity on realized covariance estimation is non-negligible. While previous studies have developed different methods to mitigate this problem, there is no consensus on it, and those sophisticated techniques may incur additional estimation errors. Therefore, in the mixed frequency framework we discussed above, we only use high frequency data to measure and forecast the volatility part, and then use low frequency data to construct the correlation matrix. We use two different correlation structures: Firstly, we consider a simple identity matrix, which we call zero correlation or Zero. The matrix consists of zeros for the off-diagonal elements and ones for the diagonal elements; hence we assume that assets are not correlated. Therefore, we can construct the conditional covariance matrix using the high frequency based univariate conditional volatilities we obtained in the previous step and the zero correlation matrix. Although, this assumption looks oversimplified, it allows us to directly assess the potential improvement of univariate volatility forecasting of high frequency data. If high frequency data can be used for estimating realized correlation without the problem of non-synchronicity, we should expect a further improvement in the economic benefit of high frequency data. Therefore, the use of zero correlation is expected to underestimate the true benefit of high frequency data, and hence our results in this context can be viewed as conservative estimates of the true benefits of high frequency data. The zero correlation is also used by DeMiguel, Plyakha, Uppal, and Vilkov (2013) when assessing the use of option implied volatility relative to historical volatility in portfolio allocation.

Our second correlation structure is dynamic and time varying. Using daily data, we estimate a dynamic conditional correlation model (DCC) by Engle (2002). The DCC model is specified as follows:

$$Q_t = (1 - \theta_1 - \theta_2)\bar{Q} + \theta_1 u_{t-1}u'_{t-1} + \theta_2 Q_{t-1} \tag{3.17}$$

$$R_t = diag(Q_t)^{-1/2} Q_t diag(Q_t)^{-1/2} \tag{3.18}$$

where Q_t is the conditional covariance matrix and R_t is the conditional correlation matrix. u_t is the demeaned daily return vector in which each component is divided by conditional volatility from a Generalized Autoregressive Conditional Heteroskedasticity or GARCH (1,1) model. We then combine the DCC based conditional correlation stated above with high frequency based conditional volatility obtained in the previous section to construct the conditional covariance matrix. A similar approach is also used by Halbleib and Voev (2012). They find that the mixed frequency approach combining high frequency based volatility and low frequency DCC based correlations can outperform pure low frequency models and perform as well as models using pure high frequency based variance covariance matrix in multivariate volatility forecasting exercises. In the robustness checks part, we also consider alternative correlation structures.

3.3.4 Asset Allocation Problem

In this section, we consider the asset allocation problem for a risk-averse investor. Following the stream of literature on volatility timing (e.g. Fleming, Kirby, and Ostdiek (2001) and Marquering and Verbeek (2004)), we assume that the investor has mean variance preferences, and rebalances the portfolio regularly according to the predicted conditional covariance matrix. One of the major problems of the mean variance based portfolio allocation is the difficulty to accurately predict the conditional mean, which can be very noisy when the number of assets is very large. Therefore, we follow Hautsch, Kyj, and Malec (2013) and consider the global minimum variance portfolio with the constraint that sum of weights equals to one.

$$\underset{w_{t,t+1}}{\text{Min}} \quad w'_{t,t+1}\hat{\Sigma}_{t,t+1}w_{t,t+1}, \quad s.t. \quad w'_{t,t+1}\mathbf{1} = 1$$

where $w_{t,t+1}$ is the vector of portfolio weights, $\mathbf{1}$ is the vector of ones, $\hat{\Sigma}_{t,t+1}$ is the predicted conditional covariance matrix obtained as described in the previous section.

The optimal weights can be solved for as

$$w_{t,t+1} = \frac{\hat{\Sigma}^{-1}_{t,t+1}\mathbf{1}}{\mathbf{1}'\hat{\Sigma}^{-1}_{t,t+1}\mathbf{1}} \tag{3.19}$$

Since the optimal portfolio weight is only the function of the conditional covariance matrix, we avoid estimation error due to forecasting the conditional mean. We obtain the vector of optimal portfolio weights at each point in time. We also impose short selling constraint, hence weights are within the range from 0 to 1. Then we get the time series of daily ex post portfolio returns. To reflect real world investment problems, we also consider weekly and monthly rebalancing portfolios. We first obtain the daily portfolio weights, and then hold the portfolio for one week and one month.[3] The holding period portfolio return is $r^p_{t,t+h} = w'_{t,t+1} r_{t,t+h}$, where $r_{t,t+h}$ is the vector of returns from t to $t+h$ and $r^p_{t,t+h}$ is the ex post portfolio return for the holding period h.

3.3.5 Performance Evaluations

Before we start to introduce the performance metrics, we first discuss our benchmark strategies. In this paper, we consider two types of benchmark strategies, which directly relate to our research questions mentioned in the introduction part. The first benchmark strategy is a low frequency strategy. We employ the conventional GARCH (1,1) model of Bollerslev (1986). We use it to assess the potential benefit of high frequency data over low frequency data. The second benchmark strategy is a high frequency strategy. We employ the HAR-RV model by Corsi (2009) (RV only). We use it to assess the potential incremental benefit of decomposing realized volatility into components and using realized higher moments. For each of the low frequency and high frequency benchmark strategies, we consider two specifications, i.e. zero correlations and DCC correlations. To answer our main research questions, we fix the correlation structure for the benchmark strategy and the candidate strategy, for example we compare RV+zero with GARCH+zero or compare RV+DCC with GARCH+DCC to ensure fair comparisons and directly assess whether high frequency data contributes to portfolio performance. We follow the same procedures to assess the incremental improvement of realized volatility components and higher moments

[3]An alternative way is to conduct multi-horizon volatility forecasts and use the conditional covariance matrix for that horizon (e.g. daily, weekly, or monthly) to construct respective portfolio weights and ex post portfolio returns. We find the method we used is straightforward to implement and is also economic intuitive.

over total realized volatility. In the robustness checks part, we also compare high frequency strategies with alternative low frequency benchmarks.

We consider three performance metrics: Firstly, we use a conventional ex-post Sharp Ratio:

$$SR^p = \frac{\bar{r}^p}{\sigma_p} \tag{3.20}$$

where $\bar{r}^p$ and σ_p are average portfolio return and portfolio volatility respectively over the out-of-sample period.

Secondly, we use the turnover rate.

$$TO_t^P = \sum_i^n \left| w_{i,t+1} - w_{i,t} \frac{1 + r_{i,t}}{1 + r^{p,t}} \right| \tag{3.21}$$

Liu (2009) and Hautsch, Kyj, and Malec (2013) also use turnover to assess portfolio performance. Before the rebalancing, the portfolio weights change to $w_{i,t}\frac{1+r_{i,t}}{1+r^{p,t}}$, therefore the absolute difference between it and the new weight can measure the portfolio turnover caused by rebalancing.

Thirdly, to quantify the potential economic improvements relative to benchmark strategies, we also use utility based criteria following Fleming, Kirby, and Ostdiek (2001) and Marquering and Verbeek (2004). Similar to Hautsch, Kyj, and Malec (2013), we assume that the investor has quadratic preferences. The quadratic utility and the performance fees between two strategies are as follows,

$$U(r^p_{t,t+h}) = 1 + r^p_{t,t+h} - \frac{\gamma}{2(1+\gamma)}(1 + r^p_{t,t+h})^2 \tag{3.22}$$

$$\frac{1}{T-h}\sum_{t=0}^{T-h}[U(r^p_{t,t+h} - \Delta)] = \frac{1}{T-h}\sum_{t=0}^{T-h}[U(r^{bm}_{t,t+h})] \tag{3.23}$$

γ is the risk aversion parameter, and we consider different levels of 2, 7, and 10. Δ refers to the performance fee investor willing to pay to switch from the benchmark strategy to the candidate strategy.

We further investigate the statistical significance of our performance fee measure. Motivated by Engle and Colacito (2006) and Bandi, Russell, and Zhu (2008), we use the Diebold and Mariano (1995) test for it. We view performance fee as the loss differential of two alternative but nested forecasts. We compute daily spot realized utility for main and benchmark strategies and then compute daily spot performance fees. We then project the time series of performance fees on a vector of ones using Newey-West standard errors; the resulting t-statistic allows us to make inference about the statistical significance of performance fees with the null hypothesis of a zero performance fees

3.4 Empirical Findings

3.4.1 Realized Volatility Forecasting

In this section, we discuss in and out-of-sample volatility forecasting results. Table 3.2 documents the summary statistics of different realized measures for the cross-section of 30 assets from 2001 to 2009. We only report mean and standard deviations for each realized measure to save space. A few observations should be noted. Firstly, SPY has a much lower mean and standard deviation compared to individual stocks, which can be explained by the diversification feature of the index, as the individual stocks contain idiosyncratic components. Secondly, although there is no clear cut pattern whether upside volatilities or downside volatilities are larger; it seems that the downside ones are less volatile across time for almost all assets. Moreover, jump variation is much smaller than the bipower variation, confirming jumps are rare. Finally, we find positive skewness and high excess kurtosis for almost all assets. The non-normality in the data suggests that realized measures beyond variances may play an important role in our following empirical analysis.

Table 3.3 reports the one day ahead in-sample volatility forecasting results using models with different realized measures. Due to the relative large number of assets, we focus exemplary on the index ETF SPY. Main findings can be summarized as follows. Firstly, the RV only model already fits data well with adjusted R^2 of 57.73%. While both the weekly and monthly lagged RV coefficients are statistically significant, we show the daily lagged RV coefficient is insignificant. Secondly, when we decompose RV into upside and downside components, we observe a clear improvement in model fit, namely the adjusted R^2 increases to 65.15%. Although the coefficient of the daily upside volatility component remains insignificant, the coefficient of daily downside volatility component becomes highly significant, suggesting that the downside part is more important. We also show that weekly and monthly upside and downside components are insignificant, implying recent downside information is more important for forecasting short horizon volatility. Similarly, when we decompose RV into jump and diffusive components, we observe an increase in the adjusted

R^2 to 60.58%, however the magnitude of improvement is smaller compared to that for upside and downside decomposition. We also show that the coefficient for the daily diffusive component is significant while the one for the daily jump component is insignificant, suggesting the diffusion component is more important. Different from results in upside and downside volatilities, we also suggest that weekly and monthly volatility components are also important. Our findings regarding volatility forecasting using different volatility components are generally consistent with previous studies (Patton and Sheppard 2013, Andersen, Bollerslev, and Diebold 2007). We show that separating total realized volatility into different components improves volatility forecasting performance, and hence may also potentially improve volatility timing based portfolio allocation strategies. Thirdly, we assess the usefulness of realized higher moments as additional volatility predictors. We show that realized skewness is negative and significant at daily level while realized kurtosis is negative and significant at daily and weekly levels. However, improvements in adjusted R^2 are less than 1%, indicating they may possess only limited incremental information for future volatility prediction.

Since we are interested in portfolio allocation with a relative large number of assets, we then report the volatility forecasting performance for the cross-section of 30 assets. Table 3.4 reports in-sample adjusted R^2s for the 30 assets. Our findings are generally consistent with the results for SPY discussed above. Initially, the RV only model generates adjusted R^2s of over 50% on average. Secondly, both the upside and downside components model and the jump and diffusive components model yield higher adjusted R^2s in general, however the improvements are generally small compared to results for SPY. The model using upside and downside components can still generate the highest improvements in adjusted R^2s for both the median and the mean value. Thirdly, the models using realized higher moments can still outperform the RV only model, however the improvements are small.

We then report the out-of-sample volatility forecasting results, which will then be used to construct our out-of-sample portfolio allocation analysis. We use the first 1000 days as in-sample period to estimate the models, and then use the rest of the observations to conduct out-of-sample forecasts. Table 3.5 reports out-of-sample mean squared errors (MSEs) for the set of 30 assets. We find that MSEs for SPY are generally smaller than for individual stocks. Models using upside and downside components, jump and diffusive components, and skewness can generate smaller MSEs than the model using RV only. For individual stocks, both mean and median values of MSEs suggest that the model using upside and downside components can have lower MSEs than the model using RV only. However, for models using jump and diffusive components and higher moments, there are more cross sectional variations, and results are more mixed. The different empirical results for index and individual stocks may again be explained by higher microstructure noise and higher idiosyncratic risks for individual stocks compared to the more liquidly traded index ETF SPY. To summarize, both in-sample and out-of-sample results suggest that by using high frequency data, one can measure and forecast volatility precisely, and using realized volatility components and higher moments can further improve forecasting performance.

3.4.2 Portfolio Allocation with Realized Volatility

In this section, we discuss whether high frequency data improves portfolio allocation by measuring and forecasting realized volatility. As mentioned before, we use the predicted volatility combined with the respective correlation matrix to construct the corresponding conditional covariance matrix. We then use it to derive the optimal portfolio weights and yield ex-post portfolio returns.

Table 3.6 reports the out-of-sample portfolio allocation results using realized volatility. Our main findings can be summarized as follows: Firstly, we present clear evidence that strategies using RV always generate higher Sharp Ratios (SRs) compared to the respective low frequency strategies. With a daily rebalancing frequency, the RV strategy has a SR of 0.50 compared to 0.37 for the GARCH strategy with zero correlations and 0.41 compared to -0.08 with DCC correlations. When we rebalance the portfolio at the weekly frequency, the SR of RV rises to 0.61 compared to 0.46 for GARCH with zero correlation and 0.63 compared to 0.00 with the DCC correlations. When we hold the portfolio for one month, we observe a slightly decline of SRs to 0.57 and 0.41 for the zero correlation with 0.44 and -0.08 for the DCC correlation. Although DCC correlation captures the time varying correlation dynamics across different assets better, the associated portfolio allocation strategies are more unstable. Hence GARCH strategies with DCC correlations even generate negative or close to zero SRs, and RV strategies with DCC correlations underperform the RV strategies with zero correlations for daily and monthly rebalancing. Nevertheless, the relative superiority of RV over GARCH remains strong under different correlation structures and different rebalancing frequencies.

Secondly, we show that high frequency strategies generally have higher turnover rates compared with low frequency strategies. For example, at the daily frequency, we observe a 0.96% turnover per day (21.12% per month) for RV compared to 0.94% (20.68% per month) for GARCH with zero correlations and 1.33% (29.26% per month) compared to 1.19% (26.18% per month) with DCC correlations. Similarly, at the weekly frequency, RV strategies also have higher turnover statistics with both correlation structures. For the monthly frequency, the RV strategy has higher turnover than the GARCH strategy for zero correlations but it has lower turnover than the GARCH strategy for DCC correlations. The slightly higher turnover rates in general for high frequency strategies compared to low frequency strategies are not surprising, since high frequency strategies need to incorporate recent information more rapidly. The differences of turnover statistics between high frequency and low frequency strategies are small in magnitudes, hence we expect the impact of transaction costs to be small. In the robustness checks, we formally investigate the impact of transaction costs on portfolio performance.

Thirdly, we quantify the magnitude of potential economic improvements using utility based criteria. We report annualized performance fees relative to the low frequency benchmark strategies for different risk aversion levels. We find that strategies using RV can always outperform the respective low frequency strategies. At the daily rebalancing frequency, the RV strategy with zero correlations can generate an economic value of 46 basis points for the moderate risk aversion level ($\gamma = 7$), while the RV strategy with DCC correlations can generate an economic value of 272 basis points. Performance fees are also varying across different risk aversion levels from 33 ($\gamma = 10$) to 124 ($\gamma = 2$) basis points for zero correlations, and from 217 ($\gamma = 10$) to 605 ($\gamma = 2$) basis points for DCC correlations. Although we showed above that SRs and TOs (level based performance metrics) are different across different rebalancing frequencies, we find that performance fees (relative performance) are generally similar in magnitude across different rebalancing frequencies. The large performance fees with DCC correlation are mainly due to the poor performance of low frequency benchmark strategies (GARCH-DCC), which have negative or close to zero SRs as we discussed above. In the robustness checks part, we also compare high frequency strategies with other low frequency benchmarks. Both the zero and the DCC based RV strategies can generate positive and statistically significant performance fees relative to the respective low frequency strategies. T-statistics are all above 2.00 at the daily frequency. At the weekly frequency, t-statistics become slightly smaller; however performance fees are still all positive and statistically significant. At the monthly frequency, performance fees remain positive but generally become statistically insignificant. To summarize, despite the slightly higher turnover, using high frequency information to improve portfolio allocation can lead to higher Sharp ratios and generate positive and statistically significant economic values compared to low frequency strategies. Our main findings are robust across different correlation matrices, rebalancing frequency, and risk aversion levels.

3.4.3 Portfolio Allocation with Realized Volatility Components

High frequency data also allows us to extract different components of total realized volatility. In this section, we investigate whether decomposing realized volatility into different components can improve portfolio allocation. The purpose is twofold: Firstly, we investigate whether strategies using realized volatility components can outperform low frequency strategies. Secondly, we also want to assess whether the decomposition leads to significant incremental improvement over the high frequency benchmark strategy, i.e. the RV strategy we used in the previous section.

Table 3.7 documents the out-of-sample portfolio allocation results using realized volatility components. Initially, we show that both the upside and downside volatility strategy (RS) and the jump and diffusive volatility strategy (RJ) can lead to higher SRs relative to the low frequency benchmark and high frequency benchmark strategies. The result holds true for both zero and DCC correlations and for different rebalancing frequencies. The upside and downside volatility strategy performs better than the jump and diffusive volatility strategy, which is consistent with its statistical performance. At the daily rebalancing frequency, the upside and downside volatility strategy can generate SRs of 0.59 and 0.60 for zero and DCC correlations respectively. We observe improvements of SRs to 0.75 and 1.01 at the weekly frequency. At the monthly frequency, SRs are 0.63 and 0.56. Different from strategies using realized volatility alone, strategies using realized volatility components with DCC perform better than with zero correlations when we rebalance portfolios at daily and weekly frequencies.

We also find that strategies using upside and downside volatility componnets have lower turnover rates than the total realized volatility strategy. Although, the strategy using total realized volatility has higher turnover than the low frequency benchmark as we documented in the previous section, we find that the strategy using upside and downside volatility components has even lower turnover than the low frequency benchmark under zero correlation for different rebalancing frequencies. For instance, at a daily level, the turnover statistics of the upside and downside volatility strategy under zero correlation is 0.93% per day, which is lower than 0.96% for the high frequency benchmark and 0.94% for the low frequency benchmark. With DCC correlation, the strategy has lower turnover than the high frequency benchmark but not the low frequency benchmark. Although upside and downside volatility component strategies need to incorporate recent downside risks information quickly, they can still have lower turnover, supporting that further decomposing realized volatility into different component can be beneficial for portfolio allocation.

Moreover, we show that strategies using realized volatility components can outperform the low frequency benchmark strategies and deliver larger economic improvements compared to high frequency benchmark strategies. At the daily frequency, the upside and downside volatility strategy can generate performance fees from 60 ($\gamma = 10$) to 208 ($\gamma = 2$) basis points using zero correlations, representing 81% and 67% increases respectively compared with ones we obtained from realized volatility strategy discussed above. Using DCC correlations again increases the economic values. However, since the SRs are also higher with DCC, these larger economic values do not entirely reflect the poor performance of the low frequency benchmark strategy, but also reflect the benefit of modelling the correlation dynamics. Upside and downside volatility strategies and jump and diffusive volatility strategies have positive and statistically significant performance fees with t-statistics above 3.00 and 2.00 respectively. The performance fees remain positive and statistically significant when we rebalance the portfolio at the weekly frequency. In the last section, we show that RV strategies become insignificant at the monthly frequency. In this section, however, we show that both the realized volatility component strategies are significant for zero correlation and the jump and diffusive strategy is still significant for DCC correlations. The larger economic magnitudes and the success even in longer horizons imply that decomposing total realized volatility into different components improves portfolio allocation.

Strategies using realized volatility components can also generate incremental economic benefits over the high frequency benchmark strategies using realized volatility alone. We compute the performance fees of realized volatility component strategies relative to high frequency benchmarks. The economic magnitudes of incremental benefits are smaller due to the use of high frequency benchmarks. However, all performance fees relative to high frequency benchmarks are positive. Moreover, at the daily frequency, the strategy using upside and downside volatility components can generate positive and statistically significant performance fees relative to the high frequency benchmarks with zero correlations. The incremental benefits range from 27 to 83 basis points. At the weekly frequency, upside and downside volatility strategies under both zero and DCC correlations can generate positive and statistically significant incremental economic values relative to high frequency benchmarks, ranging from 37 to 109 (zero) and 109 to 386 (DCC). The large incremental benefit for DCC implies its benefits for modelling correlations, while its statistical insignificance at the daily frequency suggests that it is still unstable. To summarize, our findings suggest that the previous documented statistical success of decomposing realized volatility into different components is also economically significant. We show that separating total realized volatility into different components can lead to higher SRs, lower TOs, large economic improvements to low frequency benchmarks, and positive and statistically significant incremental improvements over high frequency benchmarks.

3.4.4 Portfolio Allocation with Realized Higher Moments

High frequency data further contributes to the construction of realized higher moments. In this section, we investigate whether including realized higher moments as additional volatility predictors can improve portfolio allocation.

Table 3.8 reports out-of-sample portfolio allocation results using realized higher moments. Although models using realized higher moments have mixed volatility forecasting performance as we documented before, we find that strategies using realized skewness and kurtosis can generally generate higher SRs compared to both low frequency and high frequency benchmarks. At the daily frequency, SRs are 0.59 (zero) and 0.72 (DCC) for skewness strategies (RSK) and 0.72 (zero) and 0.78 (DCC) for kurtosis strategies (RKU). SRs for skewness strategies strengthen at the weekly frequency to 0.89 and 1.14, while SRs for kurtosis strategies become 0.63 and 0.79. The skewness strategy also performs better at the monthly frequency. However both higher moment strategies have higher turnover compared to the low frequency benchmarks, the high frequency benchmarks, and the realized volatility component strategies. For example, at the daily frequency, the TOs are 0.99% (zero) and 1.43% (DCC) for skewness strategies, and 1.02% (zero) and 1.54% (DCC) for kurtosis strategies. The higher SRs and TOs of higher moment strategies suggest that these strategies maybe more volatile compared to the realized volatility and volatility component strategies.

Similar to volatility component strategies, higher moment strategies also generate larger economic benefits relative to low frequency benchmarks. At the daily frequency, performance fees are 56 to 200 basis points for the skewness strategy and 102 to 324 basis points for the kurtosis strategy with zero correlations. Both higher moments strategies can generate positive and statistically significant economic values, although the kurtosis strategy with zero correlations has slightly weaker statistical significance. At the weekly frequency, while the skewness strategy remains positive and significant, the kurtosis strategy becomes insignificant. At the monthly frequency, both higher moment strategies are generally insignificant for zero correlations but the kurtosis strategy is significant for DCC correlations. Different from all high frequency strategies we considered before, the kurtosis strategy fails to generate significant benefit relative to the low frequency benchmark with zero correlations in short horizons, implying that including kurtosis as an additional volatility predictor actually introduces more noise.

We further investigate the incremental benefit of using realized higher moment information. At the daily frequency, the skewness strategies with DCC are marginally significant with incremental values of 87 to 338 basis points. At the weekly frequency, the skewness strategies generate positive and statistically significant performance fees of 61 to 233 basis points (zero) and 134 to 538 basis points (DCC). Kurtosis strategies, however, generate insignificant performance fees at daily and weekly frequencies, and even underperform the high frequency benchmark at the monthly frequency with zero correlation. To summarize, realized higher moments, measuring asymmetry and tail events, contain important information to improve portfolio performance. Compared with realized volatility and volatility component strategies, higher moment strategies generate larger economic benefits, however their improvements are generally more unstable.

3.5 Robustness Checks

In this section, we conduct comprehensive robustness checks. We compare high frequency strategies with different low frequency benchmarks and consider different correlation structures. We also assess the impact of market microstructure noise and transaction costs on portfolio performance.

3.5.1 Alternative Benchmarks and Correlation Structures

Our main findings provide clear evidence that high frequency data improves portfolio allocation. However, we do observe that the magnitudes of economic improvements are affected by the choice of the benchmark strategies and the selection of correlation structures. In this section, we analyse whether high frequency data can still improve portfolio allocation when alternative benchmarks and correlation structures are employed.

Rather than attempting to develop the optimal portfolio strategy, our paper aims for a better understanding of the use of high frequency data in portfolio allocation. Hence, the benchmark strategies in the main analysis are selected to facilitate answering our main research questions, but they do not necessarily reflect the real world investment problems, e.g. the poor performance of GARCH-DCC as a low frequency benchmark tends to overestimate the true benefit of high frequency data in real world portfolio allocation exercises. Hence, we are interested in how high frequency strategies perform in comparison to low frequency strategies commonly used in the literature. We use two alternative low frequency benchmark strategies: the 1/N strategy and the RiskMetrics (RM) strategy. The naive 1/N strategy assigns static and equal weights to all assets, which is proportional to the number of assets (hence 1/N). Previous studies (DeMiguel, DeMiguel, and Uppal 2009, DeMiguel, Plyakha, Uppal, and Vilkov 2013, Jacobs, Muller, and Weber 2014) document that the 1/N strategy performs very well compared to a few more sophisticated and dynamic strategies, despite its simplicity. The RiskMetrics 1994 (RM) is a standard approach following a simple exponentially weighted moving average rule used in the industry to forecast covariance matrix. We follow J.P.Morgan/Reuters (1996) and Hautsch, Kyj, and Malec (2013), and specify the model as follows:

$$\hat{\Sigma}_{t+1} = \frac{1-\lambda}{1-\lambda^{L-1}} \sum_{l=1}^{L} \lambda^{l-1} u_{t-l+1} u'_{t-l+1} \tag{3.24}$$

We follow J.P.Morgan/Reuters (1996) to select the smoothing parameter $\lambda = 0.94$, and the rolling window length $L = 250$. In this section, we are interested in how high frequency based strategies perform relative to these alternative low frequency benchmark strategies.

In the main analysis, we observe a large difference in magnitudes of economic values due to the use of different correlations structures. In this section, we consider two additional correlation structures. Firstly, we extract the correlation matrix from the RM model we employed above and combine it with high frequency based volatilities. Secondly, we use the sample correlation (SC) computed using in-sample data.[4] We investigate the performance of high frequency strategies with four different correlation structures (zero, DCC, RM, SC) relative to two low frequency benchmark strategies (1/N and RM). Table 3.9 reports out-of-sample portfolio performance results for these different strategies. We focus on the annualized Sharp Ratios for different rebalancing frequencies.

We first examine how our existing strategies perform relative to these two benchmark strategies. For the zero correlations, the RV strategy cannot outperform either benchmark strategy at any frequency, suggesting that these two low frequency benchmarks (e.g. SRs are 0.61 and 0.57 for 1/N and RM respectively with daily rebalancing) are higher than ones used in the main analysis. However, decomposing volatility into components and using higher moments can outperform these benchmarks at daily and weekly frequencies. Strategies using upside and downside volatility components (RS) can generate higher SRs than the RM strategy at the daily frequency and than the 1/N at the weekly frequency. Strategies using skewness (RSK) can beat the RM at the daily frequency and both benchmarks at the weekly frequency. Jump strategies (RJ) fail to outperform either benchmark while Kurtosis strategies (RKU) outperform both benchmark at the daily frequency but underperform both at the weekly frequency. Our findings confirm that high frequency data is important for portfolio allocation even if we do not model the correlation dynamics.

For the DCC correlations, we show that the RV strategy still fails to outperform either benchmark. However, strategies using volatility components and

[4]In an unreported analysis, we also consider two additional correlation structures based on smoothing the DCC correlations: the in-sample mean of DCC based correlation and 250 days rolling window mean of DCC based correlation, we found results are similar to the raw DCC based strategies.

higher moments generally perform better at daily and weekly frequencies. The RS strategy beats the RM at the daily and both benchmarks at the weekly frequency. The RJ strategy can now generate higher SRs relative to both benchmarks at the weekly frequency. Higher moment strategies can outperform both benchmarks at weekly and monthly frequencies. Therefore, the large economic values with DCC correlations, especially incremental improvements relative to high frequency benchmarks documented in the main analysis are not entirely due to the selection of benchmarks. Instead, modelling time varying correlations does lead to further portfolio performance improvements.

We then discuss portfolio performance with alternative correlation structures. Despite the success of zero and DCC based strategies, the relative unstable performance for the DCC correlations motivates us to consider more stable correlation structures. The smoothed (RM) and static (SC) correlations are less volatile than DCC. Hence their associated portfolios are also expected to perform better. We find that all high frequency strategies with RM correlations can outperform both low frequency benchmarks. Different from results before, RV strategies can outperform both 1/N and RM benchmarks at all frequencies, when RM correlations are used. For instance, it has a SR of 0.86 at the daily frequency, which is also higher than RV strategies with zero (0.50) and DCC (0.41) correlations. Moreover, high frequency strategies with RM can even outperform benchmarks at the monthly frequency, which is not observed when other correlation structures are used. However, we find that incremental benefits for volatility components and higher moments relative to RV are reduced or even negative, which is different from strategies using other correlations. For example, RS (SRs are 0.77 and 1.15 for daily and weekly) and RSK

(0.80 and 1.28) underperform RV (0.86 and 1.13) at the daily frequency and outperform slightly at the weekly frequency. Due to the smoothed correlations from RM, the drop of incremental benefit of using more rapidly changing alternative realized measures are as expected. Our findings imply that combining high frequency based volatilities with a slow moving correlation structure seems to be promising for portfolio allocation.

For SC correlations, RV strategies again fail to outperform the benchmarks. However, portfolio performance improves when we decompose volatility into different components. The RS strategy outperforms benchmarks for all frequencies (SRs are 0.75, 1.09, and 0.88 respectively). The RJ strategies outperform benchmarks at weekly and monthly frequencies. Higher moment strategies, which performed well with other correlation structures, generally perform poorly. Results with sample correlations (SC) are more consistent with statistical performance, which may be largely due to the use of static correlation structures.

Although the zero correlation based strategies look oversimplified while the DCC based strategies seem unstable, we find that these high frequency strategies can outperform well known low frequency strategies used in the literature under some circumstances. Using alternative correlation structures even strengthens our main results. We also show that the additional benefits from volatility components and higher moments depend on the choice of the correlation structures. To summarize, our main arguments remain hold when different low frequency benchmarks and different correlation structures are used.

3.5.2 Microstructure Noise

Since sparsely sampled five minutes high frequency returns may still contain microstructure noise, especially for relatively illiquid traded individual stocks, we also use more sophisticated microstructure noise robust estimators. Following Andersen, Bollerslev, and Meddahi (2011), we start from one minute equal-distance intraday prices and construct five overlapping five minutes returns and then average the five estimators to obtain a sub-sample estimator. The average realized estimator can control for market microstructure noises, but may also have adverse effects on alternative realized measures, especially for signed measures.

Table 3.10 reports out-of-sample portfolio allocation results using realized volatility when controlling for microstructure noise. We find that controlling for microstructure noise improves the RV strategies. For instance, compared with results in Table 3.6, using average RV measure increases SRs from 0.50 to 0.53 (zero) and from 0.41 to 0.62 (DCC), and reduces TOs from 0.96% to 0.95% (zero) and from 1.33% to 1.30% (DCC), at the daily frequency. The performance fees relative to the low frequency benchmark strategies are positive and statistically significant. Economic magnitudes range from 44 to 146 basis points for zero correlations and from 291 to 836 basis points for DCC correlations, which are all larger than respective values without controlling for market microstructure noise in Table 3.6. The economic improvements also hold true for different rebalancing frequencies. The enhanced portfolio performance results of using microstructure noise robust RV estimators are also consistent with previous studies. Bandi, Russell, and Zhu (2008), for example, suggest that using an optimally sampled RV estimator controlling for microstructure noises can improve portfolio performance compared to the conventional five minutes RV estimator.

Table 3.11 and Table 3.12 document portfolio performance using realized volatility components and realized higher moments respectively when controlling for microstructure noise. We focus on the performance fees. The economic benefits relative to low frequency benchmarks are consistent with our main results. Namely, both volatility component strategies can generate positive and statistically significant performance fees relative to low frequency benchmarks. The skewness strategy generates positive and significant performance fees at the daily frequency, and the kurtosis strategy is significant with DCC correlations at the weekly frequency. However, the incremental benefits relative to high frequency benchmarks become either negative or statistically insignificant, which is different from the main results. Nevertheless, at the weekly horizon, the upside and downside volatility strategy with zero correlations can still generate positive and statistically significant incremental benefits from 17 to 62 basis points. The potential adverse effect of average realized estimators on alternative realized measures, especially on signed components, jumps, and higher moments, and the rise of high frequency benchmarks (more difficulty to beat) may jointly explain the insignificant or even negative incremental economic values for different high frequency strategies. To summarize, controlling for market microstructure noise strengthens our main results that using realized volatility improves portfolio allocation. Results for volatility components and higher moment strategies are more mixed. However, the strategy using upside and downside volatility component can still deliver incremental improvements, suggesting these alternative realized measures remain important for portfolio allocation even after controlling for microstructure noise.

3.5.3 Transaction Costs

We then assess whether our empirical results are feasible in practice by introducing transaction costs. In the main analysis, we already documented that high frequency strategies generally have a slightly higher turnover than respective low frequency benchmarks, hence high frequency strategies may incur higher transaction costs. In this section, we formally construct transaction cost adjusted portfolio returns subject to the change of portfolio weights following Bandi and Russell (2006) and Bandi, Russell, and Zhu (2008). The transaction cost adjusted portfolio returns can be defined in the following way,

$$\bar{r}_{p,t+1} = r_{p,t+1} - \sum_{i=1}^{n} \rho(1 + r_{i,t+1})|\Delta w^i_{t+1}| \tag{3.25}$$

where $\bar{r}_{p,t+1}$ is the transaction cost adjusted portfolio return, $r_{p,t+1}$ and $r_{i,t+1}$ are pre-adjusted portfolio and individual asset returns, ρ is the transaction cost parameter, where we choose the highest transaction level of 0.0025, corresponding to a 2.5 cent half spread on a 10 dollar stock. Δw_{t+1} is the change of the weight from t to $t + 1$. Since our main candidate strategies and benchmark strategies are both dynamically rebalanced portfolios, we expect the impact of transaction cost to be small.

Table 3.13 presents out-of-sample portfolio allocation findings using realized volatility and controlling for transaction costs. We find that both Sharp ratios and the performance fees are lower compared to the main results without controlling for transaction costs. For instance, at the daily frequency, SRs for high frequency strategies controlling for transaction costs are 0.39 (zero) and 0.35 (DCC) while SRs without controlling for transaction costs are 0.50 (zero) and 0.41 (DCC) as shown in Table 3.6. Performance fees are now 30 to 112 basis points for zero correlations and 204 to 550 basis points for DCC correlations, compared with 33 to 124 basis points and 217 to 605 basis points as shown in Table 3.6. The larger drops in performance fees for DCC after controlling for transaction costs are associated with the higher turnover rates for DCC correlations as shown in the main analysis. Similar to our main results, economic values are statistically significant for daily and weekly frequencies.

Table 3.14 and Table 3.15 report empirical findings using realized volatility components and higher moments controlling for transaction costs. For daily and weekly frequencies, separating realized volatility into different components generate positive and statistically significant performance fees relative to low frequency benchmarks. Skewness strategies are positive and statistically significant while kurtosis strategies are positive and statistically significant with DCC correlations. Despite the slightly smaller magnitudes in economic values, results are consistent with the main results without controlling for transaction costs as shown in Table 3.7 and Table 3.8. Both the upside and downside volatility strategies and the realized skewness strategies can also generate statistically significant incremental economic values relative to the high frequency benchmark strategies as shown in our main analysis. Moreover, since the upside and downside volatility strategies have lower turnover statistics than high frequency benchmarks as discussed in the main analysis, impact of transaction costs are smaller, and hence incremental economic values even increase slightly. For example, at the daily frequency, incremental performance fees for upside and downside volatility strategies relative to high frequency benchmarks range between 28 to 90 basis points (zero) and 49 to 231 basis points (DCC), compared with 27 to 83 basis points (zero) and 43 to 207 basis points (DCC) as shown in Table 3.7. To summarize, after controlling for transaction costs, our main results remain valid.

3.6 Conclusion

This paper investigates the use of high frequency data in volatility timing based portfolio allocation. While a few recent studies use high frequency data to extract realized volatility components and construct realized higher moments, economic benefits of using these measures, especially in portfolio allocation contexts remain unanswered. Meanwhile, there is a long tradition in the literature to use high frequency data in improving portfolio allocation. Previous studies mainly use high frequency data to construct realized covariance matrices, and focus on the statistical refinement issues such as mitigating microstructure noise and controlling for non-synchronicity. Our paper bridges the gap between the two streams of the literature and studies the roles of realized volatility and alternative realized measures in portfolio allocation.

Firstly, we show that using high frequency data can predict future volatility reasonably well both in-sample and out-of-sample. High frequency data also allows us to separate total volatility into different components and to construct realized higher moments, which can further improve volatility forecasting performance. Among different model specifications, the model using upside and downside volatility components can consistently outperform the model using total realized volatility only.

Secondly, we demonstrate that realized volatility based strategies can outperform respective low frequency benchmark strategies across different specifications. Previous studies mainly focus on estimating the whole realized variance-covariance matrix. We acknowledge that using high frequency data to measure realized correlations may further improve portfolio allocations, however the non-synchronicity problem when estimating correlations using high frequency data with large dimensions and potential estimation errors due to correcting for non-synchronicity may make the strategy practically unattractive. Instead, we provide clear empirical evidence that strategies using high frequency data to measure and forecast univariate realized volatilities can already generate statistically significant and economically tangible benefits over low frequency strategies.

Moreover, we find that separating total realized volatility into different components can further improve portfolio performance. We show that both types of volatility component strategies can outperform the low frequency benchmark strategies, and generate economic values larger than those obtained using realized volatility only, hence further strengthening the importance of precisely measuring and forecasting univariate realized volatility in portfolio allocation. We also show that the upside and downside volatility components strategy can further generate positive and statistically significant incremental benefits relative to the high frequency benchmark strategy, which is consistent with its superior volatility forecasting performance. Previous studies already highlight that importance of separating volatility into different components in volatility forecasting. Our paper provides empirical evidence that such statistical improvements are also economically significant.

Furthermore, we suggest that realized higher moments can also contribute to portfolio allocation. Although realized higher moments can improve volatility forecasting performance with small magnitudes, we find that strategies using realized higher moments can also outperform low frequency benchmark better than the realized volatility strategy. The strategy using realized skewness can further generate statistically significant incremental benefits. However, both the realized higher moments strategies are more unstable compared to the realized volatility and the volatility components strategies.

We conduct comprehensive robustness checks. We show that our main results remain hold after using different benchmark strategies, applying different correlation structures, controlling for market microstructure noise, and taking transaction costs into account.

To conclude, we show that high frequency data can improve portfolio allocation beyond the conventional use of estimating the realized variance-covariance matrix. Measuring and forecasting univariate realized volatility, separating volatility into different components, and constructing realized higher moments can all play important roles in portfolio allocation. Our paper can be extended in a few directions: Firstly, for simplicity and consistency with existing studies, this paper concentrates on the mean variance utility framework; however realized volatility components and realized higher moments may play important roles in portfolio allocation when the investor has a more complex utility function. Secondly, high frequency data may also contribute to alternative economic applications beyond portfolio allocation, such as risk management and option pricing. Thirdly, finding the best way to combine high frequency based volatility with low frequency based correlations may further improve portfolio performance. These extensions are beyond the scope of this paper, and we leave them for future studies.

3.7 Tables

Table 3.1: List of 30 Assets

Ticker	Name	Dates		Days
SPY	S&P500 ETF	02/01/2001	30/09/2009	2196
ABT	Abbott Laboratories	02/01/2001	30/09/2009	2196
BA	Boeing Co	02/01/2001	30/09/2009	2196
CL	Colgate-Palmolive Co	02/01/2001	30/09/2009	2196
DD	E. I. du Pont de Nemours and Company	02/01/2001	30/09/2009	2196
DIS	Walt Disney Co	02/01/2001	30/09/2009	2196
EMC	EMC Corp	02/01/2001	30/09/2009	2196
EMR	Emerson Electric Co	02/01/2001	30/09/2009	2196
FDX	FedEx Corp	02/01/2001	30/09/2009	2196
GD	General Dynamics	02/01/2001	30/09/2009	2196
HD	Home Depot Inc	02/01/2001	30/09/2009	2196
IBM	Intl Business Machines Corp	02/01/2001	30/09/2009	2196
JNJ	Johnson & Johnson	02/01/2001	30/09/2009	2196
KO	Coca-Cola Co	02/01/2001	30/09/2009	2196
LLY	Lilly Eli & Co	02/01/2001	30/09/2009	2196
LMT	Lockheed Martin	02/01/2001	30/09/2009	2196
MCD	McDonald's Corp	02/01/2001	30/09/2009	2196
MMM	3M Co	02/01/2001	30/09/2009	2196
MO	Altria Group Inc	02/01/2001	30/09/2009	2196
NKE	NIKE Inc B	02/01/2001	30/09/2009	2196
PFE	Pfizer Inc	02/01/2001	30/09/2009	2196
PG	Procter & Gamble	02/01/2001	30/09/2009	2196
SO	Southern Co	02/01/2001	30/09/2009	2196
TXN	Texas Instruments Inc	02/01/2001	30/09/2009	2196
UNP	Union Pacific Corp	02/01/2001	30/09/2009	2196
UPS	United Parcel Service Inc B	02/01/2001	30/09/2009	2196
UTX	United Technologies Corp	02/01/2001	30/09/2009	2196
VZ	Verizon Communications Inc	02/01/2001	30/09/2009	2196
WAG	Walgreen Co	02/01/2001	30/09/2009	2196
WMT	Wal-Mart Stores	02/01/2001	30/09/2009	2196

Table 3.2: Summary Statistics of Realized Measures

Ticker	RV		RS^{-}		RS^{+}		BV		JV		RSK		RKU	
	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ
SPY	1.3386	2.7754	0.6518	1.2493	0.6868	1.6394	1.2656	2.7006	0.0730	0.3822	0.0315	0.7899	4.2176	2.4931
ABT	2.4807	3.1989	1.2381	1.6444	1.2426	1.7469	2.3257	3.1730	0.1550	0.6237	0.0087	0.9732	5.5769	3.7267
BA	3.4676	4.6363	1.7419	2.3707	1.7257	2.4628	3.2876	4.5243	0.1799	0.6372	0.0269	0.9918	5.5749	3.4735
CL	1.7306	3.1257	0.8617	1.7452	0.8689	1.4620	1.6001	2.9751	0.1305	0.3513	0.0091	0.9039	5.3525	3.0753
DD	3.1533	4.7093	1.5568	2.2869	1.5965	2.5966	2.9795	4.6860	0.1738	0.6437	0.0183	0.9047	5.2551	3.0401
DIS	3.9030	6.2259	1.9358	3.5405	1.9672	3.1802	3.6197	5.6479	0.2833	1.1880	0.0104	0.9112	5.2296	3.1087
EMC	8.2595	11.4143	3.9984	5.2252	4.2611	7.2554	7.6884	9.9401	0.5711	4.1997	0.0811	0.8760	5.1698	3.0886
EMR	3.2901	5.2132	1.6300	2.5398	1.6601	2.8565	3.1175	5.1431	0.1726	0.6301	0.0252	0.9193	5.2499	3.1481
FDX	3.0873	4.4030	1.4991	2.2020	1.5882	2.4308	2.8518	4.1833	0.2355	0.7501	0.0349	0.9778	5.6035	3.1294
GD	2.5235	3.6060	1.2621	1.8723	1.2614	1.8748	2.3537	3.5662	0.1698	0.5274	0.0101	0.9974	5.7497	3.4214
HD	4.0403	5.7904	1.9585	2.6318	2.0818	3.4296	3.8325	5.6682	0.2078	0.8898	0.0358	0.8867	5.2533	2.8755
IBM	2.4769	4.5658	1.1939	2.1437	1.2829	2.7060	2.3348	4.2379	0.1420	1.0614	0.1537	0.8618	4.9968	3.1898
JNJ	1.6236	2.6798	0.7953	1.2704	0.8283	1.5031	1.5330	2.6374	0.0905	0.4771	0.0606	0.9465	5.5252	3.5452
KO	1.7679	2.6837	0.8803	1.4213	0.8876	1.4000	1.6333	2.3963	0.1346	0.5283	0.0274	0.8365	5.0232	2.9187
LLY	2.4272	3.7167	1.2080	1.8753	1.2192	2.0496	2.2632	3.4905	0.1640	0.8283	-0.0191	0.9772	5.5991	3.7867
LMT	2.8002	3.8854	1.3937	1.9119	1.4065	2.1522	2.6125	3.8100	0.1877	0.5842	0.0176	0.9887	5.7628	3.3767
MCD	2.7404	4.3138	1.3361	1.9592	1.4043	2.5070	2.5851	4.3404	0.1554	0.6183	0.0640	0.9262	5.4581	3.3089
MMM	2.2342	3.8141	1.0945	1.7720	1.1396	2.1715	2.1239	3.8768	0.1103	0.5441	0.0394	0.8877	5.1357	3.0302
MO	2.2706	3.8821	1.1242	2.1617	1.1464	2.1035	2.1136	3.6883	0.1570	0.8625	0.0420	1.0079	5.7543	4.2672
NKE	3.0143	4.0113	1.4669	1.9046	1.5474	2.3254	2.7943	3.8440	0.2200	0.7276	0.0490	1.0287	5.9182	3.7720
PFE	2.6272	4.0454	1.3052	2.0518	1.3220	2.1224	2.4593	3.9239	0.1679	0.5731	-0.0015	0.8543	5.0300	2.9872
PG	1.6529	3.2278	0.8154	1.6689	0.8375	1.6311	1.5603	3.2627	0.0926	0.3884	0.0692	0.8433	5.0901	3.0687
SO	2.0771	3.2118	1.0425	1.6708	1.0347	1.6650	1.9147	2.8325	0.1624	0.5926	0.0188	0.8436	5.3577	3.0084
TXN	6.7538	7.9454	3.3088	3.8715	3.4450	4.5032	6.3022	7.4285	0.4516	1.5119	0.0524	0.9231	5.3489	3.1058
UNP	3.2204	5.8788	1.5622	2.7954	1.6582	3.3128	3.0222	5.6595	0.1982	0.9997	0.0569	1.0202	5.7259	3.7175
UPS	1.8000	3.4251	0.8867	1.7611	0.9133	1.7613	1.6689	3.0923	0.1311	0.6652	0.0077	0.9135	5.3723	3.3068
UTX	2.6649	4.3806	1.3388	2.2088	1.3261	2.4195	2.5137	4.2619	0.1512	0.7674	-0.0390	0.9352	5.3034	3.3553
VZ	3.0425	4.7649	1.5113	2.2912	1.5312	2.6743	2.8309	4.5103	0.2117	0.7996	-0.0219	0.9380	5.3935	3.4387
WAG	2.9931	4.6758	1.4606	2.1225	1.5325	2.7489	2.7616	4.1432	0.2315	0.9551	0.0749	0.9730	5.6646	3.5640
WMT	2.3427	3.4450	1.1443	1.5849	1.1985	1.9810	2.2107	3.2603	0.1321	0.5413	0.0187	0.8414	4.9642	3.0478

Table 3.3: One Day Ahead In-Sample Volatility Forecasting: SPY

			HAR-RV (RV)				
β_0	β_{RVD}	β_{RVW}	β_{RVM}				$adjR^2$
0.1144	0.2298	0.5344	0.1491				0.5773
(2.3654)	(1.5934)	(2.8195)	(1.7511)				
			HAR-RS (RS)				
β_0	β_{RSmD}	β_{RSpD}	β_{RSmW}	β_{RSpW}	β_{RSmM}	β_{RSpM}	$adjR^2$
-0.0167	1.0687	-0.3004	1.6741	0.5412	1.7670	-1.4860	0.6515
(-0.2546)	(2.8703)	(-0.8354)	(1.4017)	(-0.8134)	(1.1985)	(-1.2584)	
			HAR-BV-JV (RJ)				
β_0	β_{BVD}	β_{JVD}	β_{BVW}	β_{JVW}	β_{BVM}	β_{JVM}	$adjR^2$
0.0697	0.2572	-0.3994	0.6021	-1.5619	-0.0802	4.7084	0.6058
(1.3235)	(1.8942)	(-0.7980)	(2.3850)	(-1.0481)	(-0.5271)	(1.8439)	
			HAR-RV-RSK (RSK)				
β_0	β_{RVD}	β_{RSKD}	β_{RVW}	β_{RSKW}	β_{RVM}	β_{RSKM}	$adjR^2$
0.1292	0.2272	-0.0994	0.5398	-0.3249	0.1488	-0.1397	0.5806
(2.5957)	(1.5590)	(-2.7160)	(2.7494)	(-1.3418)	(1.7302)	(-0.4604)	
			HAR-RV-RKU (RKU)				
β_0	β_{RVD}	β_{RKUD}	β_{RVW}	β_{RKUW}	β_{RVM}	β_{RKUM}	$adjR^2$
0.4893	0.2355	-0.0334	0.5291	-0.0948	0.1420	0.0460	0.5798
(2.0347)	(1.5807)	(-2.3266)	(2.7242)	(-2.0665)	(1.6708)	(0.8746)	

Notes: The table reports in-sample volatility forecasting results for SPY from 2001 to 2009. HAR models with different realized measures are used and the parameters are estimated using OLS. The figures in parentheses are t-statistics with Newey-West corrected standard errors for autocorrelation order 5 for one day ahead forecasts. The $adjR^2$ is adjusted R square.

Table 3.4: One Day Ahead In-Sample Volatility Forecasting: 30 Assets

$adjR^2$	RV	RS	RJ	RSK	RKU
SPY	0.5773	0.6515	0.6058	0.5806	0.5798
ABT	0.4874	0.5244	0.4996	0.4879	0.4899
BA	0.6583	0.6731	0.6681	0.6593	0.6607
CL	0.2594	0.2608	0.2613	0.2606	0.2607
DD	0.6223	0.6479	0.6325	0.6230	0.6254
DIS	0.5400	0.5550	0.5470	0.5400	0.5442
EMC	0.6402	0.6448	0.6423	0.6410	0.6405
EMR	0.6460	0.6566	0.6452	0.6461	0.6471
FDX	0.5847	0.5872	0.5884	0.5842	0.5871
GD	0.5417	0.5584	0.5465	0.5419	0.5447
HD	0.5779	0.5913	0.5739	0.5784	0.5802
IBM	0.5153	0.5615	0.5302	0.5196	0.5190
JN	0.5222	0.5408	0.5198	0.5234	0.5240
KO	0.5661	0.5719	0.5808	0.5660	0.5692
LLY	0.4855	0.5098	0.4933	0.4868	0.4871
LMT	0.5037	0.5233	0.5074	0.5039	0.5068
MCD	0.2983	0.3164	0.2965	0.2986	0.3027
MMM	0.5427	0.6002	0.5435	0.5453	0.5430
MO	0.2759	0.2785	0.2832	0.2771	0.2845
NKE	0.5925	0.6170	0.5923	0.5938	0.5946
PFE	0.3810	0.3812	0.3885	0.3802	0.3808
PG	0.3417	0.3487	0.3466	0.3414	0.3436
SO	0.4428	0.4454	0.4474	0.4423	0.4439
TXN	0.6132	0.6236	0.6167	0.6144	0.6165
UNP	0.6711	0.6835	0.6843	0.6712	0.6725
UPS	0.5011	0.5065	0.5340	0.5005	0.5014
UTX	0.5383	0.5557	0.5514	0.5393	0.5403
VZ	0.5374	0.5564	0.5379	0.5369	0.5390
WAG	0.3749	0.3903	0.4103	0.3753	0.3785
WMT	0.5170	0.5324	0.5186	0.5170	0.5182
Mean	0.5119	0.5298	0.5198	0.5125	0.5142
Median	0.5378	0.5560	0.5407	0.5381	0.5397

Notes: The table reports in-sample volatility forecasting adjusted R squares for all 30 assets from 2001 to 2009. HAR models with different realized measures are used. We also report cross-sectional means and medians for adjusted R squares.

Table 3.5: One Day Ahead Out-of-Sample Volatility Forecasting: 30 Assets

MSE	RV	RS	RJ	RSK	RKU
SPY	5.3712	4.6473	5.1559	5.3409	5.5303
ABT	6.3253	6.0646	6.2934	6.3428	6.3030
BA	8.6799	8.3524	8.5626	8.6852	8.7401
CL	13.0937	13.5176	13.0353	13.0757	13.1101
DD	12.8119	12.1801	12.9707	12.7917	12.7534
DIS	10.9629	10.9541	11.4425	11.0086	10.8800
EMC	16.6419	16.6314	16.7546	17.0079	16.6490
EMR	15.7960	15.5913	16.0614	15.7805	15.7884
FDX	10.7298	10.7200	10.5636	10.7306	10.6485
GD	8.1022	7.9614	8.1151	8.1066	8.0488
HD	17.4639	17.7222	17.9684	17.4701	17.3863
IBM	13.8818	13.1292	14.2826	13.7399	13.9642
JNJ	3.2832	3.5026	3.5709	3.3051	3.3070
KO	4.3363	4.6718	4.3788	4.3724	4.3602
LLY	8.8280	8.9919	8.9335	8.7923	8.8472
LMT	9.2266	9.0713	9.1588	9.2557	9.1746
MCD	17.1308	16.8563	17.0390	17.1987	17.2907
MMM	10.8271	9.6268	10.9407	10.7690	10.9349
MO	10.1738	10.7356	10.0739	10.1699	10.0201
NKE	8.4793	8.0326	8.5776	8.4744	8.4590
PFE	15.2722	15.2625	14.9619	15.2960	15.3915
PG	12.1610	12.0661	12.6440	12.1306	12.2184
SO	7.2265	7.2083	7.1878	7.2754	7.2352
TXN	17.5027	17.3411	17.2365	17.6539	17.4309
UNP	20.6139	20.4589	21.4678	20.6511	20.5910
UPS	10.8176	10.5934	11.2086	10.8155	10.8493
UTX	11.7843	12.2728	13.4061	12.0673	11.7555
VZ	15.4667	15.0239	15.4504	15.4783	15.5027
WAG	20.4593	20.0970	19.1480	20.4717	20.3584
WMT	8.1344	7.9819	9.1344	8.1899	8.5309
Mean	11.7195	11.5756	11.8575	11.7483	11.7353
Median	10.8950	10.8448	11.3256	10.9121	10.9074

Notes: The table reports out-of-sample volatility forecasting Mean Squared Errors (MSEs) for SPY and individual stocks for 2001 to 2009. Parameters are estimated in-sample (2001-2004) and forecasting is conducted out-of-sample (2005-2009). We also report cross-sectional medians and means for the MSEs.

Table 3.6: Out-of-Sample Portfolio Performances Using Realized Volatility

	SR	TO	Δ_{10}^{LF}	Δ_{7}^{LF}	Δ_{2}^{LF}
	Daily Rebalancing				
$GARCH + Zero$	0.3689	0.9390			
$RV + Zero$	0.5034	0.9581	0.0033	0.0046	0.0124
			(2.0043)	(2.0537)	(2.1246)
$GARCH + DCC$	-0.0825	1.1853			
$RV + DCC$	0.4134	1.3320	0.0217	0.0272	0.0605
			(2.7695)	(2.5964)	(2.2453)
	Weekly Rebalancing				
$GARCH + Zero$	0.4585	2.0634			
$RV + Zero$	0.6095	2.0822	0.0030	0.0043	0.0124
			(1.7909)	(1.8873)	(2.0070)
$GARCH + DCC$	0.0016	2.6677			
$RV + DCC$	0.6332	2.7134	0.0229	0.0294	0.0695
			(2.6896)	(2.6790)	(2.5183)
	Monthly Rebalancing				
$GARCH + Zero$	0.4124	4.7065			
$RV + Zero$	0.5652	4.7480	0.0029	0.0044	0.0132
			(1.0221)	(1.2328)	(1.6332)
$GARCH + DCC$	-0.0815	6.2385			
$RV + DCC$	0.4425	6.1498	0.0187	0.0248	0.0618
			(1.3227)	(1.4868)	(1.8523)

Notes: The table reports out-of-sample volatility timing strategies using realized volatility from 2001 to 2009. Parameters are all estimated in-sample (2001-2004) and volatility timing results are obtained out-of-sample (2005-2009). We report Sharp ratio, turnover rate, and annualized performance fee relative to low frequency benchmarks of GARCH (1, 1) model with respective correlation structure. Risk aversion parameter ranges from 2, 7, and 10. Figures in parentheses are t-statistics for DM test. The test has null hypothesis that (mean) performance fee equal to zero.

Table 3.7: Out-of-Sample Portfolio Performances Using Realized Volatility Components

	SR	TO	Δ_{10}^{LF}	Δ_{7}^{LF}	Δ_{2}^{LF}	Δ_{10}^{HF}	Δ_{7}^{HF}	Δ_{2}^{HF}
				Daily Rebalancing				
$RS + Zero$	0.5938	0.9265	0.0060	0.0081	0.0208	0.0027	0.0035	0.0083
			(3.4062)	(3.3517)	(3.2454)	(2.0778)	(1.9674)	(1.7729)
$RJ + Zero$	0.5190	0.9552	0.0042	0.0055	0.0139	0.0008	0.0009	0.0015
			(2.2270)	(2.7198)	(2.0867)	(0.6711)	(0.5422)	(0.3231)
$RS + DCC$	0.6013	1.2620	0.0261	0.0338	0.0813	0.0043	0.0066	0.0207
			(3.5010)	(3.3422)	(3.0404)	(0.7697)	(0.9033)	(1.1247)
$RJ + DCC$	0.4792	1.3698	0.0231	0.0293	0.0677	0.0013	0.0021	0.0071
			(2.9800)	(2.8337)	(2.5281)	(0.3305)	(0.3889)	(0.4814)
				Weekly Rebalancing				
$RS + Zero$	0.7453	2.0244	0.0067	0.0090	0.0233	0.0037	0.0047	0.0109
			(3.7077)	(3.6213)	(3.4197)	(2.9694)	(2.7646)	(2.3972)
$RJ + Zero$	0.6533	2.0698	0.0045	0.0061	0.0159	0.0015	0.0018	0.0034
			(2.7147)	(2.6816)	(2.5615)	(1.6388)	(1.4290)	(1.0552)
$RS + DCC$	1.0069	2.6069	0.0338	0.0442	0.1081	0.0109	0.0148	0.0386
			(3.4810)	(3.4656)	(3.2883)	(2.0770)	(2.0728)	(2.0205)
$RJ + DCC$	0.8435	2.7689	0.0292	0.0379	0.0912	0.0063	0.0085	0.0218
			(3.1985)	(3.2203)	(3.0973)	(1.7651)	(1.7388)	(1.6612)
				Monthly Rebalancing				
$RS + Zero$	0.6276	4.6747	0.0055	0.0073	0.0184	0.0026	0.0030	0.0052
			(1.8058)	(1.9032)	(2.0344)	(1.8986)	(1.5126)	(0.8992)
$RJ + Zero$	0.5989	4.7125	0.0046	0.0062	0.0161	0.0017	0.0018	0.0028
			(1.7259)	(1.8842)	(2.1842)	(1.5975)	(1.3372)	(0.8022)
$RS + DCC$	0.5552	6.0449	0.0225	0.0298	0.0747	0.0038	0.0051	0.0128
			(1.3010)	(1.4395)	(1.7175)	(0.5906)	(0.5910)	(0.5566)
$RJ + DCC$	0.6350	6.1335	0.0272	0.0351	0.0838	0.0085	0.0104	0.0219
			(1.8830)	(2.0641)	(2.4357)	(2.0381)	(1.8886)	(1.5460)

Notes: The table reports out-of-sample volatility timing strategies using realized volatility components from 2001 to 2009. Parameters are all estimated in-sample (2001-2004) and volatility timing results are obtained out-of-sample (2005-2009). We report Sharp ratio, turnover rate, and annualized performance fee relative to low frequency benchmark of GARCH(1,1) model and high frequency benchmark of realized volatility with respective correlation structure. Risk aversion parameter ranges from 2,7, and 10. Figures in parentheses are t-statistics for DM test. The test has null hypothesis that (mean) performance fee equal to zero.

Table 3.8: Out-of-Sample Portfolio Performances Using Realized Higher Moments

	SR	TO	Δ_{10}^{LF}	Δ_{7}^{LF}	Δ_{2}^{LF}	Δ_{10}^{HF}	Δ_{7}^{HF}	Δ_{2}^{HF}
				Daily Rebalancing				
$RSK + Zero$	0.5850	0.9862	0.0056	0.0076	0.0200	0.0022	0.0030	0.0075
			(2.0744)	(2.0710)	(2.0596)	(1.0280)	(0.9963)	(0.9429)
$RKU + Zero$	0.7238	1.0167	0.0102	0.0134	0.0324	0.0069	0.0088	0.0124
			(1.8680)	(1.7817)	(1.6446)	(1.2838)	(1.1915)	(1.0333)
$RSK + DCC$	0.7170	1.4312	0.0305	0.0394	0.0943	0.0087	0.0122	0.0338
			(3.2824)	(3.1504)	(2.8854)	(1.6182)	(1.6508)	(1.7026)
$RKU + DCC$	0.7753	1.5435	0.0344	0.0437	0.1004	0.0127	0.0165	0.0399
			(2.9760)	(2.7965)	(2.4637)	(1.3466)	(1.2847)	(1.1771)
				Weekly Rebalancing				
$RSK + Zero$	0.8902	2.0913	0.0091	0.0129	0.0358	0.0061	0.0085	0.0233
			(2.7727)	(2.7694)	(2.7427)	(2.1306)	(2.0882)	(2.0180)
$RKU + Zero$	0.6209	1.9722	0.0052	0.0064	0.0134	0.0022	0.0020	0.0009
			(0.8874)	(0.7815)	(0.6056)	(0.3817)	(0.2519)	(0.0425)
$RSK + DCC$	1.1422	2.8258	0.0362	0.0485	0.1233	0.0134	0.0191	0.0538
			(3.5515)	(3.5797)	(3.4973)	(2.2172)	(2.2642)	(2.3291)
$RKU + DCC$	0.7890	2.7236	0.0307	0.0383	0.0847	0.0079	0.0098	0.0152
			(2.5061)	(2.2537)	(1.9419)	(0.8299)	(0.6748)	(0.4212)
				Monthly Rebalancing				
$RSK + Zero$	0.6385	4.7395	0.0045	0.0066	0.0197	0.0016	0.0023	0.0064
			(1.3303)	(1.4987)	(1.7589)	(0.7427)	(0.7271)	(0.7026)
$RKU + Zero$	0.5489	4.4622	0.0031	0.0043	0.0180	0.0002	-0.0001	-0.0015
			(0.7301)	(0.7552)	(0.7803)	(0.0537)	(-0.0097)	(-0.1109)
$RSK + DCC$	0.7264	6.3733	0.0227	0.0327	0.0941	0.0040	0.0080	0.0323
			(1.4553)	(1.7354)	(2.3046)	(0.6534)	(0.9219)	(1.2957)
$RKU + DCC$	0.6534	5.9385	0.0248	0.0334	0.0857	0.0061	0.0086	0.0239
			(1.7367)	(1.9028)	(2.1659)	(0.7699)	(0.8082)	(0.8507)

Notes: The table reports out-of-sample volatility timing strategies using realized higher moments from 2001 to 2009. Parameters are all estimated in-sample (2001-2004) and volatility timing results are obtained out-of-sample (2005-2009). We report Sharp ratio, turnover rate, and annualized performance fee relative to low frequency benchmark of GARCH(1,1) model and high frequency benchmark of realized volatility with respective correlation structure. Risk aversion parameter ranges from 2,7, and 10. Figures in parentheses are t-statistics for DM test. The test has null hypothesis that (mean) performance fee equal to zero.

Table 3.9: Alternative Benchmarks and Correlation Structures

	Zero Correlations						
	$1/N$	RM	$RV + Zero$	$RS + Zero$	$RJ + Zero$	$RSK + Zero$	$RKU + Zero$
SR_{Daily}	0.6135	0.5699	0.5034	0.5938	0.5190	0.5850	0.7238
SR_{Weekly}	0.7074	0.7480	0.6095	0.7453	0.6533	0.8902	0.6269
$SR_{Monthly}$	0.7362	0.8627	0.5652	0.6276	0.5989	0.6385	0.5489
	DCC Correlations						
	$1/N$	RM	$RV + DCC$	$RS + DCC$	$RJ + DCC$	$RSK + DCC$	$RKU + DCC$
SR_{Daily}	0.6135	0.5699	0.4134	0.6013	0.4792	0.7170	0.7713
SR_{Weekly}	0.7044	0.7480	0.6332	1.0069	0.8435	1.1422	0.7890
$SR_{Monthly}$	0.7362	0.8627	0.4452	0.5552	0.6350	0.7264	0.6534
	RM Correlations						
	$1/N$	RM	$RV + RM$	$RS + RM$	$RJ + RM$	$RSK + RM$	$RKU + RM$
SR_{Daily}	0.6135	0.5699	0.8624	0.7679	0.7509	0.8504	1.1076
SR_{Weekly}	0.7074	0.7480	1.1268	1.1490	1.0313	1.2787	1.0877
$SR_{Monthly}$	0.7362	0.8627	1.0075	1.0992	1.1492	1.1231	0.9835
	Sample Correlations						
	$1/N$	RM	$RV + SC$	$RS + SC$	$RJ + SC$	$RSK + SC$	$RKU + SC$
SR_{Daily}	0.6135	0.5699	0.4099	0.7458	0.4792	0.3280	0.5778
SR_{Weekly}	0.7974	0.7480	0.7306	1.0886	1.0657	0.7824	0.5524
$SR_{Monthly}$	0.7362	0.8627	0.6270	0.8812	0.9274	0.5708	0.7523

Notes: The table reports out-of-sample volatility timing strategies from 2001 to 2009 using all realized measures with four correlation structures and two low frequency benchmarks. Parameters are all estimated in-sample (2001-2004) and volatility timing results are obtained out-of-sample (2005-2009). We report annualized Sharp ratios with different portfolio rebalancing frequencies.

Table 3.10: Out-of-Sample Portfolio Performances Using Realized Volatility under MMS

	SR	TO	Δ_{10}^{LF}	Δ_{7}^{LF}	Δ_{2}^{LF}
			Daily Rebalancing		
$GARCH + Zero$	0.3689	0.9390			
$RV + Zero$	0.5272	0.9486	0.0044	0.0059	0.0146
			(2.8706)	(2.7679)	(2.5635)
$GARCH + DCC$	-0.0825	1.1853			
$RV + DCC$	0.6209	1.3017	0.0291	0.0368	0.0836
			(3.9064)	(3.6818)	(3.2220)
			Weekly Rebalancing		
$GARCH + Zero$	0.4585	2.0634			
$RV + Zero$	0.6475	2.0690	0.0044	0.0059	0.0154
			(3.0010)	(2.9459)	(2.8022)
$GARCH + DCC$	0.0016	2.6677			
$RV + DCC$	0.8029	2.6913	0.0288	0.0370	0.0865
			(3.5384)	(3.5096)	(3.2685)
			Monthly Rebalancing		
$GARCH + Zero$	0.4124	4.7065			
$RV + Zero$	0.5752	4.7051	0.0041	0.0055	0.0140
			(1.4771)	(1.6008)	(1.8465)
$GARCH + DCC$	-0.0815	6.2385			
$RV + DCC$	0.6111	6.0058	0.0275	0.0351	0.0812
			(1.9639)	(2.1363)	(2.5202)

Notes: The table reports out-of-sample volatility timing strategies using realized volatility controlling for microstructure noises from 2001 to 2009. Parameters are all estimated in-sample (2001-2004) and volatility timing results are obtained out-of-sample (2005-2009). We report Sharp ratio, turnover rate, and annualized performance fee relative to low frequency benchmarks of GARCH (1, 1) model with respective correlation structure. Risk aversion parameter ranges from 2, 7, and 10. Figures in parentheses are t-statistics for DM test. The test has null hypothesis that (mean) performance fee equal to zero.

Table 3.11: Out-of-Sample Portfolio Performances Using Realized Volatility Components under MMS

	SR	TO	Δ_{10}^{LF}	Δ_{7}^{LF}	Δ_{2}^{LF}	Δ_{10}^{HF}	Δ_{7}^{HF}	Δ_{2}^{HF}
Daily Rebalancing								
$RS + Zero$	0.5771	0.9253	0.0057	0.0076	0.0192	0.0012	0.0017	0.0046
			(3.2698)	(3.1609)	(2.9579)	(1.3416)	(1.3414)	(1.3348)
$RJ + Zero$	0.5126	0.9478	0.00042	0.0055	0.0133	-0.0003	-0.0004	-0.0013
			(2.6384)	(2.4992)	(2.2382)	(-0.5462)	(-0.6244)	(-0.7517)
$RS + DCC$	0.4614	1.2370	0.0234	0.0293	0.0659	-0.0058	-0.0075	-0.0177
			(3.1242)	(2.8522)	(2.3681)	(-1.5315)	(-1.4719)	(-1.3508)
$RJ + DCC$	0.5377	1.3134	0.0271	0.0338	0.0746	0.0021	-0.0030	-0.0090
			(3.4462)	(3.1915)	(2.6919)	(-0.8058)	(-0.8770)	(-0.9908)
Weekly Rebalancing								
$RS + Zero$	0.7240	2.0329	0.0061	0.0083	0.0217	0.0017	0.0023	0.0062
			(3.6031)	(3.5462)	(3.4036)	(1.9329)	(1.9572)	(1.9867)
$RJ + Zero$	0.6460	2.0672	0.0042	0.0058	0.0153	-0.0002	-0.0002	-0.0001
			(2.8255)	(2.8005)	(2.7098)	(-0.3938)	(-0.2630)	(-0.0459)
$RS + DCC$	0.7790	2.6065	0.0268	0.0349	0.0847	-0.0020	-0.0020	-0.0021
			(3.2465)	(3.1735)	(2.9153)	(-0.5427)	(-0.3968)	(-0.1595)
$RJ + DCC$	0.7994	2.6978	0.0285	0.0367	0.0865	-0.0003	-0.0003	-0.0003
			(3.2465)	(3.4419)	(3.2489)	(-0.1294)	(-0.0952)	(-0.0344)
Monthly Rebalancing								
$RS + Zero$	0.5684	4.6559	0.0041	0.0054	0.0134	-0.0001	-0.0001	-0.0006
			(1.3280)	(1.4085)	(1.5418)	(-0.0454)	(-0.1091)	(-0.2115)
$RJ + Zero$	0.5496	4.6920	0.0036	0.0047	0.0118	-0.0006	-0.0008	-0.0022
			(1.2477)	(1.3422)	(1.5289)	(-0.9843)	(-1.0546)	(-1.1608)
$RS + DCC$	0.3782	5.8951	0.0184	0.0236	0.0551	-0.00091	-0.0115	-0.0261
			(1.0839)	(1.1703)	(1.3474)	(-1.6541)	(-1.6771)	(-1.6482)
$RJ + DCC$	0.4949	5.9617	0.0250	0.0311	0.0688	-0.0025	-0.0039	-0.0124
			(1.6857)	(1.7932)	(2.0283)	(-0.6218)	(-0.7527)	(-0.9944)

Notes: The table reports out-of-sample volatility timing strategies using realized volatility components controlling for microstructure noises from 2001 to 2009. Parameters are all estimated in-sample (2001-2004) and volatility timing results are obtained out-of-sample (2005-2009). We report Sharp ratio, turnover rate, and annualized performance fee relative to low frequency benchmark of GARCH(1,1) model and high frequency benchmark of realized volatility with respective correlation structure. Risk aversion parameter ranges from 2,7, and 10. Figures in parentheses are t-statistics for DM test. The test has null hypothesis that (mean) performance fee equal to zero.

Table 3.12: Out-of-Sample Portfolio Performances Using Realized Higher Moments under MMS

	SR	TO	Δ_{10}^{LF}	Δ_{7}^{LF}	Δ_{2}^{LF}	Δ_{10}^{HF}	Δ_{7}^{HF}	Δ_{2}^{HF}
				Daily Rebalancing				
$RSK + Zero$	0.6504	0.9744	0.0074	0.0100	0.0260	0.0029	0.0041	0.0113
			(2.9593)	(2.9173)	(2.8334)	(1.5228)	(1.5535)	(1.6035)
$RKU + Zero$	0.5226	1.0201	0.0050	0.0063	0.0142	0.0005	0.0004	-0.0004
			(0.9872)	(0.9087)	(0.7760)	(0.1052)	(0.0573)	(-0.0228)
$RSK + DCC$	0.7622	1.3980	0.0332	0.0425	0.0993	0.0041	0.0057	0.0155
			(3.6722)	(3.4711)	(3.0784)	(0.7781)	(0.7894)	(0.8052)
$RKU + DCC$	0.3810	1.5625	0.0228	0.0277	0.0576	-0.0064	-0.0091	-0.0260
			(2.0849)	(1.8612)	(1.4640)	(-0.7857)	(-0.8173)	(-0.8676)
				Weekly Rebalancing				
$RSK + Zero$	0.6172	2.0722	0.0032	0.0046	0.0131	-0.0012	-0.0014	-0.0023
			(1.5728)	(1.6398)	(1.7308)	(-0.8593)	(-0.7105)	(-0.4593)
$RKU + Zero$	0.5661	2.0266	0.0029	0.0037	0.0087	-0.0015	-0.0022	-0.0067
			(1.7500)	(0.6338)	(0.5582)	(-0.3566)	(-0.3897)	(-0.4469)
$RSK + DCC$	0.6011	2.7278	0.0216	0.0279	0.0661	-0.0072	-0.0091	-0.0207
			(2.5595)	(2.4957)	(2.2878)	(-1.5376)	(-1.4275)	(-1.2352)
$RKU + DCC$	0.5821	2.8063	0.0231	0.0289	0.0642	-0.0057	-0.0080	-0.0226
			(2.1803)	(2.0465)	(1.7540)	(-0.6980)	(-0.7262)	(-0.7693)
				Monthly Rebalancing				
$RSK + Zero$	0.5600	4.7024	0.0028	0.0042	0.0128	-0.0013	-0.0013	-0.0012
			(0.9017)	(1.0886)	(1.4602)	(-0.9931)	(-0.7488)	(-0.2741)
$RKU + Zero$	0.4047	4.4021	-0.0008	-0.0007	-0.0008	-0.0048	-0.0062	-0.0148
			(-0.1554)	(-0.1182)	(-0.0491)	(-1.2797)	(-1.1960)	(-1.0446)
$RSK + DCC$	0.3328	6.0587	0.0152	0.0200	0.0493	0.0123	-0.0151	-0.0319
			(0.9834)	(1.0839)	(1.2964)	(-1.8685)	(-1.7839)	(-1.5603)
$RKU + DCC$	0.1230	5.7759	0.0068	0.0093	0.0241	-0.0207	-0.0258	-0.0571
			(0.4122)	(0.4613)	(0.5595)	(-1.8176)	(-1.7632)	(-1.6542)

Notes: The table reports out-of-sample volatility timing strategies using realized higher moments controlling for microstructure noises from 2001 to 2009. Parameters are all estimated in-sample (2001-2004) and volatility timing results are obtained out-of-sample (2005-2009). We report Sharp ratio, turnover rate, and annualized performance fee relative to low frequency benchmark of GARCH(1,1) model and high frequency benchmark of realized volatility with respective correlation structure. Risk aversion parameter ranges from 2,7, and 10. Figures in parentheses are t-statistics for DM test. The test has null hypothesis that (mean) performance fee equal to zero.

Table 3.13: Out-of-Sample Portfolio Performances Using Realized Volatility Controlling for Transaction Costs

	SR	TO	Δ_{10}^{LF}	Δ_{7}^{LF}	Δ_{2}^{LF}
	Daily Rebalancing				
$GARCH + Zero$	0.3673	0.9390			
$RV + Zero$	0.3883	0.9581	0.0030	0.0042	0.0112
			(1.8115)	(1.8524)	(1.9009)
$GARCH + DCC$	-0.0913	1.1853			
$RV + DCC$	0.3542	1.3320	0.0204	0.0252	0.0550
			(2.5879)	(2.4039)	(2.0365)
	Weekly Rebalancing				
$GARCH + Zero$	0.4394	2.0634			
$RV + Zero$	0.5946	2.0822	0.0031	0.0045	0.0128
			(1.8494)	(1.9452)	(2.0621)
$GARCH + DCC$	-0.0372	2.6677			
$RV + DCC$	0.6092	2.7134	0.0234	0.0301	0.0713
			(2.7454)	(2.7389)	(2.5818)
	Monthly Rebalancing				
$GARCH + Zero$	0.3279	4.7065			
$RV + Zero$	0.4920	4.7480	0.0031	0.0047	0.0142
			(1.0796)	(1.3040)	(1.7337)
$GARCH + DCC$	-0.1839	6.2385			
$RV + DCC$	0.3773	6.1498	0.0197	0.0263	0.0664
			(1.3775)	(1.5601)	(1.9754)

Notes: The table reports out-of-sample volatility timing strategies using realized volatility controlling for transaction costs from 2001 to 2009. Parameters are all estimated in-sample (2001-2004) and volatility timing results are obtained out-of-sample (2005-2009). We report Sharp ratio, turnover rate, and annualized performance fee relative to low frequency benchmarks of GARCH (1, 1) model with respective correlation structure. Risk aversion parameter ranges from 2, 7, and 10. Figures in parentheses are t-statistics for DM test. The test has null hypothesis that (mean) performance fee equal to zero.

Table 3.14: Out-of-Sample Portfolio Performances Using Realized Volatility Components Controlling for Transaction Costs

	SR	TO	Δ_{10}^{LF}	Δ_{7}^{LF}	Δ_{2}^{LF}	Δ_{10}^{HF}	Δ_{7}^{HF}	Δ_{2}^{HF}
				Daily Rebalancing				
$RS + Zero$	0.5853	0.9265	0.0058	0.0079	0.0201	0.0028	0.0037	0.0090
			(3.3138)	(3.2569)	(3.1468)	(2.2006)	(2.0953)	(1.9085)
$RJ + Zero$	0.5104	0.9552	0.0040	0.0053	0.0133	0.0010	0.0011	0.0021
			(2.1377)	(2.0874)	(1.9869)	(0.7941)	(0.6703)	(0.4595)
$RS + DCC$	0.5645	1.2620	0.0253	0.0327	0.0781	0.0049	0.0075	0.0231
			(3.3904)	(3.2281)	(2.9215)	(0.9076)	(1.0147)	(1.1843)
$RJ + DCC$	0.4430	1.3698	0.0223	0.0282	0.0646	0.0019	0.0030	0.0096
			(2.8764)	(2.7249)	(2.4119)	(0.4811)	(1.0147)	(0.6483)
				Weekly Rebalancing				
$RS + Zero$	0.7279	2.0244	0.0068	0.0091	0.0235	0.0036	0.0046	0.0107
			(3.7284)	(3.6410)	(3.4368)	(2.9266)	(2.7225)	(2.3560)
$RJ + Zero$	0.6370	2.0698	0.0046	0.0062	0.0162	0.0015	0.0017	0.0034
			(2.7516)	(2.7184)	(2.3566)	(1.6666)	(1.3982)	(2.5968)
$RS + DCC$	0.9684	2.6069	0.0339	0.0444	0.1084	0.0105	0.0143	0.0371
			(3.4814)	(3.4652)	(3.2868)	(2.0002)	(1.9950)	(1.9428)
$RJ + DCC$	0.8165	2.7689	0.0296	0.0385	0.0927	0.0062	0.0083	0.0215
			(3.2372)	(3.2624)	(3.1434)	(1.7311)	(1.7076)	(1.6354)
				Monthly Rebalancing				
$RS + Zero$	0.5540	4.6747	0.0058	0.0077	0.0196	0.0027	0.0030	0.0054
			(1.8682)	(1.9796)	(2.1378)	(1.9082)	(1.5636)	(0.9122)
$RJ + Zero$	0.5277	4.7125	0.0049	0.0066	0.0173	0.0018	0.0020	0.0031
			(1.8160)	(1.9985)	(2.3538)	(1.6536)	(1.4030)	(0.8824)
$RS + DCC$	0.4804	6.0449	0.0233	0.0311	0.0789	0.0036	0.0047	0.0119
			(1.3299)	(1.4776)	(1.7775)	(0.5503)	(0.5461)	(0.5069)
$RJ + DCC$	0.5792	6.1335	0.0286	0.0372	0.0898	0.0088	0.0109	0.0234
			(1.9567)	(2.1644)	(2.6077)	(2.0972)	(1.9575)	(1.6295)

Notes: The table reports out-of-sample volatility timing strategies using realized volatility components controlling for transaction costs from 2001 to 2009. Parameters are all estimated in-sample (2001-2004) and volatility timing results are obtained out-of-sample (2005-2009). We report Sharp ratio, turnover rate, and annualized performance fee relative to low frequency benchmark of GARCH(1,1) model and high frequency benchmark of realized volatility with respective correlation structure. Risk aversion parameter ranges from 2,7, and 10. Figures in parentheses are t-statistics for DM test. The test has null hypothesis that (mean) performance fee equal to zero.

Table 3.15: Out-of-Sample Portfolio Performances Using Realized Higher Moments Controlling for Transaction Costs

	SR	TO	Δ_{10}^{LF}	Δ_{7}^{LF}	Δ_{2}^{LF}	Δ_{10}^{HF}	Δ_{7}^{HF}	Δ_{2}^{HF}
Daily Rebalancing								
$RSK + Zero$	0.5682	0.9862	0.0052	0.0071	0.0186	0.0022	0.0029	0.0074
			(1.9362)	(1.9302)	(1.9147)	(1.0046)	(0.9746)	(0.9248)
$RKU + Zero$	0.6990	1.0167	0.0097	0.0126	0.0303	0.0067	0.0084	0.0193
			(1.7584)	(1.6772)	(1.5366)	(1.2393)	(1.1472)	(0.9893)
$RSK + DCC$	0.6694	1.4312	0.0293	0.0379	0.0900	0.0090	0.0126	0.0350
			(3.1605)	(3.0238)	(2.7519)	(1.6668)	(1.7040)	(1.7634)
$RKU + DCC$	0.7170	1.5435	0.0331	0.0418	0.0951	0.0127	0.0165	0.0401
			(2.8538)	(2.6701)	(2.3312)	(1.3456)	(1.2860)	(1.2860)
Weekly Rebalancing								
$RSK + Zero$	0.8710	2.0913	0.0091	0.0129	0.0358	0.0060	0.0084	0.0230
			(2.7694)	(2.7657)	(2.7385)	(2.0940)	(2.0532)	(1.9856)
$RKU + Zero$	0.6057	1.9722	0.0052	0.0063	0.0133	0.0021	0.0018	0.0005
			(0.8810)	(0.7748)	(0.5986)	(0.3597)	(0.2319)	(0.0227)
$RSK + DCC$	1.1063	2.8258	0.0364	0.0487	0.1238	0.0130	0.0186	0.0528
			(3.5589)	(3.5866)	(3.5031)	(2.1528)	(2.2009)	(2.2678)
$RKU + DCC$	0.7574	2.7236	0.0311	0.0388	0.0859	0.0077	0.0086	0.0146
			(2.5275)	(2.3472)	(1.9640)	(0.8077)	(0.6540)	(0.4028)
Monthly Rebalancing								
$RSK + Zero$	0.5613	4.7395	0.0046	0.0069	0.0203	0.0015	0.0022	0.0061
			(1.3475)	(1.5189)	(1.7840)	(0.6958)	(0.6790)	(0.6535)
$RKU + Zero$	0.4709	4.4622	0.0032	0.0045	0.0124	0.0001	-0.0002	-0.0018
			(0.7490)	(0.7769)	(0.8062)	(0.0323)	(-0.0321)	(-0.1344)
$RSK + DCC$	0.6424	6.3733	0.0232	0.0335	0.0963	0.0035	0.0072	0.0299
			(1.4655)	(1.7475)	(2.3209)	(0.5577)	(0.8169)	(1.1801)
$RKU + DCC$	0.5670	5.9385	0.0254	0.0342	0.0881	0.0057	0.0079	0.0236
			(1.7519)	(1.9210)	(2.1901)	(0.7029)	(0.7310)	(0.7619)

Notes: The table reports out-of-sample volatility timing strategies using realized higher moments controlling for transaction costs from 2001 to 2009. Parameters are all estimated in-sample (2001-2004) and volatility timing results are obtained out-of-sample (2005-2009). We report Sharp ratio, turnover rate, and annualized performance fee relative to low frequency benchmark of GARCH(1,1) model and high frequency benchmark of realized volatility with respective correlation structure. Risk aversion parameter ranges from 2,7, and 10. Figures in parentheses are t-statistics for DM test. The test has null hypothesis that (mean) performance fee equal to zero.

Chapter 4

Dissecting Volatility Risks in Currency Markets

4.1 Introduction

Uncovered interest rate parity (UIP) is a simple proposition that the expected rate of appreciation of one currency relative to another should be just offset by the interest rate differential between them, so that the return from exploiting the interest differential is just offset by the expected rate of depreciation, as one would expect under risk neutrality, rational expectations and zero net commitment of funds. However, the empirical violation of UIP is arguably one of the most robust stylized facts in international finance.[1] Based on its violation, a carry trade investor, who invests in relatively high-interest-rate currencies, funding the investment by borrowing low-interest-rate currencies, can generate on average positive excess returns. The failure of UIP (or the success of the carry trade strategy) may therefore reflect compensation for risk or departures

[1]For surveys of this literature, see, e.g. Hodrick (1987), Froot and Thaler (1990), Taylor (1995), Sarno and Taylor (2002).

from rational expectations, or both. This paper investigates the risk-return profile of currency returns using a set of risk factors related to aggregate exchange rate volatility.

The Intertemporal Capital Asset Pricing Model (ICAPM) developed by Merton (1973) predicts that volatility is a state variable. Volatility deteriorates investors' future investment opportunity sets, and assets highly correlated with volatility provide a hedge for such deterioration. Those hedging assets are traded at higher prices and hence are expected to earn lower returns, implying volatility risk is negatively priced in the cross-section of asset returns (Campbell 1993, Chen 2003). Based on that intuition, Ang, Hodrick, Xing, and Zhang (2006) empirically verify that volatility risk is priced in the cross-section of US stock returns. Using the monthly average of daily absolute returns as a simple proxy of volatility, Menkhoff, Sarno, Schmeling, and Schrimpf (2012a) extend the above analysis into currency markets and show that global exchange rate volatility risk helps to explain the cross-section of carry trade returns.[2]

Unlike equity volatility risk, which is complementary to the market factor, currency volatility risk is crucial for understanding the risk-return profile in currency markets, given the absence of a commonly agreed global currency market factor. Therefore, a deep understanding of volatility risk is perhaps more important in currency markets relative to other asset classes. Besides carry trade portfolios, which is our main test asset in this context, we are also interested in

[2]Previous studies also introduce a few other currency risk factors to explain the cross-section of carry trade returns, including carry trade high minus low (Lustig, Roussanov, and Verdelhan 2011), skewness (Rafferty 2012), liquidity (Mancini, Ranaldo, and Wrampelmeyer 2013), global equity market downside risk (Lettau, Maggiori, and Weber 2014, Dobrynskaya 2014). In this paper, we only focus on the currency volatility risk.

whether volatility risk can explain other currency portfolios (e.g. currency momentum portfolios). Menkhoff, Sarno, Schmeling, and Schrimpf (2012b) suggest that volatility risk is not likely to explain currency momentum returns, especially for a short formation period. Burnside, Eichenbaum, and Eichenbaum (2011) also find that volatility risk cannot jointly explain both carry trade and momentum returns. In this paper, we are interested in whether our volatility factors may provide new insights into understanding currency carry trade and momentum returns.

In this connection, we investigate two aspects of the pricing of volatility risk in currency returns. First, we ask whether the pricing ability of currency volatility is concentrated in some of its components. In financial econometrics, there is a long tradition of modeling volatility processes using different components ranging from high-frequency data (Corsi 2009) to low-frequency data (Engle, Ghysels, and Sohn 2013), and from discrete-time models (Engle and Lee 1999, Adrian and Rosenberg 2008) to continuous-time models (Christoffersen, Jacobs, Ornthanalai, and Wang 2008). If the underlying exchange rate volatility process consists of different components, then these components may represent different sources of risk and hence are likely to be priced separately in currency returns. Second, if currency volatility risk is priced due to identifying bad states and hedging argument, as suggested by the ICAPM, then, intuitively, other factors resembling volatility in identifying bad states may also contribute to explaining currency returns.

We first investigate the pricing of currency volatility risk by understanding the fine structure of volatility. In particular, we decompose currency volatility into jump and diffusive and into short and long-run components. A common parameterization of volatility process consists of a continuous component (diffusion) and a discrete component (jump). A few studies already investigate the separate pricing of diffusive and jump risks in equity and option markets (Merton 1976, Yan 2011, Cremers, Halling, and Weinbaum 2015). In currency markets, several previous studies (Chernov, Graveline, and Zviadadze 2015, Farhi, Fraiberger, Gabaix, Ranciere, and Verdelhan 2015, Jurek 2014, Jurek and Xu 2014) also incorporate jumps in their models to investigate crash-risk-based explanations of currency returns. Those studies mainly rely on option prices or parametric models and mainly focus on individual or a small set of exchange rates. In the present analysis, we focus on the role of diffusive and jump components of aggregate exchange rate volatility. We apply a nonparametric approach to explicitly separate ex post measures of diffusive and jump volatilities, and investigate whether they help to explain currency returns for a broad set of exchange rates. We find that the diffusive volatility component dominates the jump component in pricing carry trade portfolio returns, while the jump component can contribute to explaining the joint cross-section of carry trade and momentum portfolios.

Volatility can also be disaggregated into short-run and long-run components according to the level of persistence.[3] The short-run volatility risk is related more to transitory shocks, including liquidity or financial stress, while long-run volatility risk captures more persistent shocks, which are related to low-frequency, slow moving business cycle fluctuations. Adrian and Rosenberg

[3]Universally accepted definitions of short-run and long-run volatilities are still missing in the literature. In this paper we treat daily fluctuations as the short run and lower frequency movements (i.e. monthly, quarterly) as the long run.

(2008) find both types of volatility risks are priced in the cross-section of stock returns. Dew-Becker, Giglio, Le, and Rodriguez (2013) suggest that only transitory volatility risk should be priced in option markets. Sohn (2014), however, finds that long-run volatility should matter more in equity markets. These mixed results suggest that the relative importance of short and long-run volatilities can vary across methods used, asset classes studied, and samples selected. In this paper, we find both components are priced in currency excess returns, with the short run component more important in general, while the long run component performs slightly better for a sample of developed country currencies.[4]

A second focus of this paper is to investigate whether alternative factors related to volatility in identifying bad states can also explain currency returns. In bad times, the economy is more uncertain, and therefore we consider two empirical proxies for economic uncertainty: volatility of volatility and cross-sectional volatility. These measures are related to but are different from the conventionally defined volatility, and we view them as alternative volatility factors. In bad times, not only may volatility be high, there may also be a tendency that volatility itself can fluctuate, and hence volatility of volatility can also be high.

[4] Ahmed and Valente (2015) also investigate short and long-run volatilities in carry trade returns, using component GARCH models on averaged exchange rate returns. Our research differs from them in at least four aspects: First, we aim to investigate the pricing of volatility risks in currency markets as comprehensively as possible. Therefore, we provide new insights on other volatility components, alternative volatility factors, and other currency portfolios. Second, we rely on nonparametric or model-free methods to construct our volatility measures, avoiding potential model misspecification. Third, we initially construct our volatility measures at individual currency levels and then aggregate them to the global currency level, since there is no consensus on global currency market portfolio in the literature. While these authors suggest that only long-run volatility is priced, we find that both short and long-run volatilities are priced and short run volatility performs better as a pricing factor in general. The difference in our results may be explained by the different empirical methods used, although our results are shown to be robust even if we use a parametric volatility model.

There are a number of reasons why one might expect volatility of volatility to help explain currency returns. Song and Xiu (2014) and Song (2012) suggest that the higher moments of volatility should be considered when investigating volatility risks, so that volatility of volatility can in some sense be interpreted as providing a more comprehensive characterization of "volatility risk". In addition, volatility of volatility is related to tail risk: Park (2013) demonstrates that volatility of volatility is proportional to skewness and kurtosis. Volatility of volatility in currency markets, for example, may capture exposure to rare events. A number of previous studies have also used volatility of volatility to proxy economic uncertainty: both Bollerslev, Tauchen, and Zhou (2009) and Barndorff-Nielsen and Veraart (2013) argue that volatility of volatility is the key driver of the variance risk premium, a measure of economic uncertainty, whilst Baltussen, Bekkum, and Grient (2014) interpret volatility of volatility as a measure of uncertainty and investigate its cross-sectional predictability in equity markets. A few studies (Wang, Kirby, and Clark 2013, Chen, Chung, and Lin 2014, Huang and Shaliastovich 2014) also suggest that volatility of volatility helps to explain equity or option returns. We find that volatility of volatility is priced in carry trade returns. Although volatility is more important than volatility of volatility for carry trade portfolios, volatility of volatility has the ability to price the joint cross-section of carry trade and momentum portfolios.

Another economic uncertainty measure we consider is cross-sectional volatility (or returns dispersion). In contrast to other volatility factors, which mainly capture the second moment of the *time-series* return distributions for particular currencies, cross-sectional volatility is constructed from the *cross-sectional* distribution of returns across currencies. Hence, this factor naturally captures market dispersion of currency returns, which is not considered by other factors. In bad times, not only can volatility be high, returns can also be more dispersed than in normal periods, for example if there is a 'flight to safety' (or 'flight from risk') in some currencies. Therefore, cross-sectional volatility should also affect investors' future investment opportunity sets, and hence it should also be priced in asset returns. Garcia, Mantilla-Garcia, and Martellini (2014) show that the cross-sectional variance is a consistent and asymptotic efficient estimator for aggregate idiosyncratic volatility, and that it is countercyclical. A few studies (Garcia, Mantilla-Garcia, and Martellini 2014, Angelidis, Sakkas, and Tessaromatis 2015, Maio 2015) suggest that cross-sectional volatility is a strong predictor for stock returns both at market and portfolio levels. In this paper, we find that cross-sectional volatility is also priced in currency excess returns and remains significant in explaining carry trade returns even after controlling for conventional volatility. Moreover, cross-sectional volatility outperforms other volatility factors in explaining the cross-section of individual currency returns. Our findings suggest that these alternative volatility factors contain incremental explanatory power for currency returns, and they are not fully subsumed by conventional measures of volatility risks.

The remainder of the paper is organized as follows. In Section 4.2 we discuss different currency volatility factors. In Section 4.3 we describe the dataset and the test assets. Section 4.4 presents our main empirical findings while in Section 4.5 we report the results of a number of robustness checks. Finally, in Section 4.6 we make some concluding remarks.

4.2 Volatility Risks in Currency Markets

In this section we discuss the different volatility measures that we will utilize in our asset pricing analysis. To avoid making assumptions of return distributions or imposing statistical restrictions on the return-volatility relationship, all volatility measures are constructed nonparametrically or model-free fashion. Hence, our volatility measures are immune to potential model misspecification issues. In Section 4.5, as part of our battery of robustness checks, we consider alternative volatility measures. Due to the absence of a commonly agreed global currency market factor, we do not compute global currency market volatility measures directly. Instead, we first construct each volatility measure at the individual exchange rate level and then we compute global currency market volatility measures and pricing factors from those individual-level volatility measures.

4.2.1 Realized Currency Volatility

First, we measure conventional currency volatility using realized volatility. We compute individual currency level realized volatility as follows:

$$RV_{k,t} = \sqrt{\frac{1}{T_t}\sum_{\tau=1}^{T_t} r_{k,\tau}^2} \tag{4.1}$$

where $r_{k,\tau} = \ln S_{k,\tau} - \ln S_{k,\tau-1}$ is the daily spot exchange rate log return for exchange rate k on day τ, T_t is the number of days in month t. $RV_{k,t}$ is realized volatility for exchange rate k in month t. The currency realized volatility is very closely related to the absolute return based volatility measure used in the literature. Although Menkhoff, Sarno, Schmeling, and Schrimpf (2012a) suggest that the absolute return based volatility measure can avoid outliers, we prefer to use the realized volatility measure in our paper for two reasons. First, realized variance, which aggregates intra-period squared returns over a period, is asymptotically convergent to quadratic variations over that period. On the other hand, the asymptotic properties of aggregated intra-period absolute returns over a period are not clear. Second, our paper focuses on understanding different components of volatility and using realized volatility can facilitate the various decompositions in which we are interested.

4.2.2 Jump and Diffusive Volatility Components

We now discuss the jump and diffusive component of exchange rate volatility. If the underlying exchange rate dynamics follow a jump and diffusion process, then the total quadratic variation measured by the realized variance can be decomposed into a diffusive part and a jump component. A number of studies introduce different nonparametric methods to detect realized jumps and separate jumps from diffusion components. In this paper, we use the bipower variation method introduced by Barndorff-Nielsen and Shephard (2006) for its simplicity.[5] The currency level diffusive and jump volatility components are constructed as follows:

$$BV_{k,t} = \sqrt{\frac{1}{T_t}\frac{\mu_1^{-2}T_t}{T_t-1}\sum_{\tau=2}^{T_t}|r_{k,\tau}||r_{k,\tau-1}|} \tag{4.2}$$

$$JV_{k,t} = Max(RV_{k,t} - BV_{k,t}, 0) \tag{4.3}$$

where $\mu_1 = \sqrt{\frac{2}{\pi}}$, $BV_{k,t}$ and $JV_{k,t}$ are monthly bipower volatility (diffusive volatility), and jump volatility for currency k in month t respectively.

[5]Nonparametric jump tests are usually used to construct daily realized measure using intraday data. In this paper, we apply it to construct monthly realize measures using daily data. Previous studies (Jiang and Yao 2013, Pukthuanthong and Roll 2014) also apply bipower approach to daily data to construct low frequency realized measures.

4.2.3 Short and Long Run Volatility Components

Volatility processes can also be decomposed into transitory and persistent factors and existing studies have developed different parametric models which model a volatility process with these two factors. In this paper, we use the Hodrick and Prescott (1997) filter instead, which does not require the volatility process to be modelled parametrically.[6] We apply the filter on daily squared exchange rate returns $V_{k,\tau} = r^2_{k,\tau}$. In the Hodrick-Prescott framework, the long-run component $V^L_{k,\tau}$ can be extracted by solving the following minimization problem:

$$\min_{V^L_{k,\tau}} \sum_{\tau=2}^{T} (V_{k,\tau} - V^L_{k,\tau})^2 + \theta[(V^L_{k,\tau+1} - V^L_{k,\tau}) - (V^L_{k,\tau} - V^L_{k,\tau-1})]^2 \tag{4.4}$$

where θ is the smoothing parameter.[7] The short-run component of daily squared returns, $V^S_{k,\tau}$ can then be extracted as the complement, $V^S_{k,\tau} = V_{k,\tau} - V^L_{k,\tau}$. We then take the monthly average and the square root to obtain monthly long-run and short-run volatility measures, $L_{k,t}$ and $S_{k,t}$, respectively, for currency k in month t.[8]

[6]Other studies such as Adrian and Rosenberg (2008), Cao and Xu (2010), and Harris, Stoja, and Yilmaz (2011) also apply the Hodrick and Prescott (1997) filter to extract short and long run volatility components.

[7]Following Harris, Stoja, and Yilmaz (2011), we set it as 5,760,000 for daily data.

[8]Note that $L_{k,t}$ and $S_{k,t}$ do not add up to root mean squared daily returns, i.e. realized volatility $RV_{k,t}$. This would only be true if we applied the Hodrick-Prescott filter to realized volatility to decompose its long-run and short-run components directly. This, however, would discard the information available at the daily level in terms of transitory and persistent components. Effectively, we are decomposing daily squared returns into short and long-run components and then constructing a realized volatility measure of each component independently at the monthly level.

4.2.4 Volatility of Volatility

We then construct a model free measure of volatility of volatility as follows,

$$VoV_{k,t} = \sqrt{Var_t(\sigma_{k,\tau})} = \sqrt{\frac{1}{T_t}\sum_{\tau=1}^{T_t}\sigma_{k,\tau}^2 - (\frac{1}{T_t}\sum_{\tau=1}^{T_t}\sigma_{k,\tau})^2} \tag{4.5}$$

where $\sigma_{k,\tau} = |r_{k,\tau}|$. We use the daily absolute returns to measure the daily volatility for exchange rate k on day τ. There are a few other ways to construct volatility of volatility as suggested in Wang, Kirby, and Clark (2013), we find results are similar.

4.2.5 Cross-Sectional Volatility

Finally, we construct a measure of cross-sectional volatility. Garcia, Mantilla-Garcia, and Martellini (2014) suggest that the cross-sectional variance is a consistent and efficient estimator of aggregate idiosyncratic volatility. In this paper, we focus on the cross-sectional volatility rather than the variance, in line with other volatility factors. In contrast to other volatility factors at the individual currency level, the cross-sectional volatility which we employ is by construction available at the aggregate level. The measure is constructed as follows:

$$CSV_{m,t} = \sqrt{\frac{1}{T_t}\sum_{\tau=1}^{T_t}\sum_{k=1}^{K}(r_{k,\tau} - \bar{r}_{m,\tau})^2} \tag{4.6}$$

Effectively, we first obtain the cross-sectional variance based on daily exchange rate returns and then we compute the monthly average and take the square root to obtain the monthly cross-sectional volatility measure $CSV_{m,t}$ at the aggregate level in month t.

4.2.6 Global Currency Volatility and Volatility Innovations

Theory predicts that an increase in unexpected volatility deteriorates investors' opportunity sets and that only the unexpected part of volatility (i.e. volatility innovations) should be priced. Our volatility-based pricing factors are constructed in three steps. First, we construct monthly individual currency level volatility measures ($VF_{k,t}$, $VF = RV, BV, JV, L, S, VoV, CSV$) from daily data as introduced above (except for CSV, which is already at the market level). Second, we compute corresponding global currency market volatility measures ($VF_{m,t}$) from cross-sectional averages of the monthly individual currency level volatility measures. Third, we compute volatility innovations ($\Delta VF_{m,t}$) by fitting an $AR(1)$ model and retrieving the residuals.[9]

4.3 Data and Currency Portfolios

4.3.1 Data

We collected daily data on spot exchange rates and one-month forward exchange rates with respect to the US dollar from November 1983 to November 2014. The data is collected from Datastream (BBI and Reuters). We use daily observations to construct our monthly volatility factors and end-of-month observations to construct monthly currency excess returns and forward discounts, which we use to construct carry trade portfolios. The whole sample covers 48 countries (or currency regions), similar to Menkhoff, Sarno, Schmeling, and Schrimpf (2012a), and includes Australia, Austria, Belgium, Brazil, Bulgaria,

[9] We also consider other ways to compute volatility innovations using first difference or ARMA(1,1) residuals, and our results are qualitatively unchanged.

Canada, Croatia, Cyprus, Czech Republic, Denmark, Egypt, Euro Zone, Finland, France, Germany, Greece, Hong Kong, Hungary, Iceland, India, Indonesia, Ireland, Israel, Italy, Japan, Kuwait, Malaysia, Mexico, Netherlands, New Zealand, Norway, Philippines, Poland, Portugal, Russia, Saudi Arabia, Singapore, Slovakia, Slovenia, South Africa, South Korea, Spain, Sweden, Switzerland, Taiwan, Thailand, Ukraine, and the United Kingdom. We apply two filters following Lustig, Roussanov, and Verdelhan (2011) to remove Eurozone currencies after the introduction of the euro and to remove currencies which violate covered interest rate parity (CIP).[10]

As robustness checks, we also consider two alternative samples: The first one is a developed countries sample, including 15 countries (or currency regions), also from November 1983 to November 2014.[11] The second is a sub-sample, which includes 22 countries (or currency regions) with a balanced panel structure from January 1999 to November 2014.[12] Along with currency portfolios, the second sample is also used to analyze individual currency returns.

4.3.2 Currency Portfolios

Our spot rates S_t and forward rates F_t represent units of foreign currency per one unit of US dollar (USD/FCU), i.e. the foreign currency price of one US dollar. Therefore, an increase in S_t is associated with an appreciation of US

[10] CIP violation observations were removed for the South African Rand (Jul 31st 1985 to Aug 30th 1985), the Indonesia Rupiah (Dec 29th 2000 to May 31st 2007), and the Malaysia Ringgit (Aug 31st 1998 to Jun 30th 2005).

[11] The 15 developed countries (or currency regions) are Australia, Belgium, Canada, Denmark, the Eurozone, France, Germany, Italy, Japan, Netherlands, New Zealand, Norway, Sweden, Switzerland and the United Kingdom.

[12] The 22 alternative sample countries (or currency regions) are Australia, Canada, Hong Kong, Czech Republic, Denmark, the Eurozone, Hungary, India, Japan, Kuwait, Mexico, New Zealand, Norway, Philippines, Saudi Arabia, Singapore, South Africa, Sweden, Switzerland, Taiwan, Thailand and the United Kingdom.

dollar and a depreciation of foreign currency, relative to one another. The currency excess returns RX_{t+1} is the return from taking an open forward position and immediately liquidating it in the spot market when the forward contract matures. Similar to Lustig, Roussanov, and Verdelhan (2011) and Menkhoff, Sarno, Schmeling, and Schrimpf (2012a), we run our asset pricing tests on levels of currency excess returns rather than log returns, to avoid assuming the joint normality of returns and the pricing kernel.

$$RX_{t+1} = (F_t - S_{t+1})/S_t \tag{4.7}$$

If Covered Interest Rate Parity (CIP) holds, then the interest differential $i_t^* - i_t$ (i_t and i_t^* are domestic and foreign interest rates respectively) will be approximately equal to the forward premium $(F_t - S_t)/S_t$, so that RX_{t+1} can be alternatively expressed as

$$RX_{t+1} \simeq i_t^* - i_t - (S_{t+1} - S_t)/S_t \tag{4.8}$$

which can be expressed as interest rate differential minus the percentage rate of appreciation of the USD, and can be interpreted as the return from attempting to exploit an interest differential by borrowing the low-interest rate currency and lending the high-interest rate currency.

We also compute transaction costs-adjusted currency excess returns, $RX_{t+1}^L = (F_t^B - S_{t+1}^A)/S_t^B$ and $RX_{t+1}^S = -(F_t^A - S_{t+1}^B)/S_t^A$, using bid (superscript B) and ask (superscript A) quotations, depending on whether an investor goes long (superscript L) or short(superscript S) the foreign currency.

We then constructed the carry trade (i.e. interest rate differential-sorted) portfolios. At the beginning of each month, we sort all currencies into five portfolios according to their forward discounts (or interest rate differentials under the assumption that CIP holds). The first portfolio includes 20% of equally weighted currencies with the lowest interest rates relative to USD, and the fifth portfolio includes 20% of equally weighted currencies with the highest interest rates relative to USD. We hold those portfolios for one month, and rebalance portfolios on a monthly basis according to the latest forward discount information. We go short portfolio 1 and long the rest of portfolios, and then use transaction costs adjusted returns accordingly.

One of the main concerns of the currency asset pricing literature is the high cross-sectional R^2 (sometimes as high as 90%), which is largely due to the low dimension of the asset space (usually five or so portfolios). The problem can be more severe when we try to decompose the existing volatility factor into two components or include additional factors. In an attempt to mitigate this problem, we expand our set of test assets by considering two additional sets of assets.

First, rather than focusing on the five carry trade portfolios, we also consider sorting currencies into ten carry trade portfolios, sorted by interest differentials against the US, as in the five portfolio case. Using ten portfolios increases the degree of freedom, and hence we should expect that pricing performances will be weaker. Using ten carry trade portfolios also reduces the number of currencies within each portfolio and so returns are less diversified, noisier, and may be subject to currency specific idiosyncratic risks unrelated to the interest rate differentials. Nevertheless, we are still interested in whether our volatility factors are priced in those carry trade portfolios.

Second, we go beyond carry trade portfolios and consider currency momentum portfolios with a one-month formation period and a one-month holding period. More particularly, at the beginning of the month, we sort currencies into five portfolios according to their currency excess returns in the past month. The first portfolio contains 20% of currencies with lowest excess returns in the past month, and the fifth portfolio contains 20% of currencies with highest excess returns in the past month. We implement a strategy of holding those portfolios for one month and record their ex post returns. We then focus on the joint cross-section of the five carry trade and five momentum portfolios.

These two additional sets of test assets are more difficult to price than the conventional five carry trade portfolios, and we are interested in whether our volatility factors can explain currency returns under these more stringent conditions and provide new insights on the pricing of volatility risks.

4.4 Empirical Findings

4.4.1 Asset Pricing Tests and Descriptive Statistics

In this section, we briefly set out our asset pricing test procedures and discuss descriptive statistics for pricing factors and currency portfolios constructed using our data.

We follow the recent carry trade asset pricing literature (Burnside 2012, Menkhoff, Sarno, Schmeling, and Schrimpf 2012a), and consider a parsimonious two-factor stochastic discount factor model. This can be developed straightforwardly from a no-arbitrage condition from which we obtain the Euler equation for currency excess returns RX^i_{t+1} for portfolio i at time t. We assume a linear stochastic discount factor M_{t+1}. Since the carry trade is a zero-net investment strategy, the expected value of $M_{t+1}RX^i_{t+1}$ must be zero, giving the basic Euler equation:

$$E_t[M_{t+1}RX^i_{t+1}] = 0 \tag{4.9}$$

If we assume a model for the stochastic discount factor that is linear in a US dollar factor, DOL_{t+1}, measured as the average of five (ten) portfolio returns, and a volatility innovations factor, ΔVF_{t+1}, measured as discussed in Section 4.2 (ΔVF includes ΔRV, ΔBV, ΔJV, ΔL, ΔS, ΔVoV, and ΔCSV). The two-factor linear model may be expressed:

$$M_{t+1} = 1 - b_{DOL}DOL_{t+1} - b_{VF}\Delta VF_{t+1} \tag{4.10}$$

where b_{DOL} and b_{VF} are factor loadings and where the intercept is normalized to unity to identify the model. Equations (4.9) and (4.10) imply a beta pricing model where the expected excess returns depend on a vector of factor risk prices, λ, and the vector of regression betas of portfolio excess returns on the risk factors for each portfolio i, β^i (see, e.g. Cochrane (2005) or, specifically in the context of the currency carry trade, Burnside (2012)):

$$E[RX^i] = \lambda'\beta^i \tag{4.11}$$

We estimate the beta pricing model using two-stage cross-sectional regressions. In the first stage, we run time-series regressions of portfolio returns on risk factors to obtain the time-series estimates of the regression betas of portfolio returns on the risk factors for the five (or ten) portfolios. In the second stage, we then run cross-sectional regressions of portfolio returns on the estimated regression betas obtained from the first stage, to estimate the vector of factor risk prices λ. We use both Newey and West (1987) and Shanken (1992) heteroskedastic and autocorrelation-consistent standard errors to construct t-statistics. To assess the in-sample model fit, we report cross-sectional R^2, root mean squared pricing errors, and χ^2 test statistics under the null hypothesis of joint zero pricing errors.

Table 4.1 reports descriptive statistics for the pricing factors. While the conventional volatility factor (ΔRV) is highly correlated to some volatility factors (over 0.90 for ΔBV and ΔS), it is less correlated with others (0.30 for ΔJV, 0.25 for ΔL, and 0.55 for ΔVoV). The low correlations (-0.03 between ΔBV and ΔJV, and 0.19 between ΔL and ΔS) between different volatility components suggest that volatility components are likely to contain different sets of information, and hence may have different implications in pricing currency returns. Although constructed differently, the cross-sectional volatility ΔCSV has a high correlation of 0.81 with ΔRV, indicating that ΔCSV may also perform well in explaining currency returns. Figure 4.1 plots levels and innovations of the different volatility measures. The patterns are related but not identical. In particular, the long-run volatility factor is very different from the remaining factors.

Table 4.2 reports descriptive statistics for currency portfolios. Panel A documents the conventional five carry trade (interest rate differential sorted) portfolios C1 to C5, sorted such that C1 is the portfolio of currencies with the lowest interest differential relative to USD and C5 the highest. As expected, we observe a monotonically increasing pattern in mean excess returns from C1 to C5. A similar pattern in cumulative returns is also observed in Figure 4.2 (upper panel). A long-short strategy (high minus low, HML) generates high mean excess return (around 7% annually) and Sharpe Ratio (0.73). Panel B reports the ten carry trade portfolios case. Similar to the five carry trade portfolios case, the ten carry trade portfolios also have an increasing pattern in mean excess returns from C1 (low interest rate differential relative to USD) to C10 (highest interest rate differential). As explained in Section 4.3, currencies are less diversified within each portfolio and subject to currency specific idiosyncratic risks unrelated to forward discounts when we sort them into ten portfolios. Therefore, the mean excess returns for the ten carry trade portfolios are not as smoothly increasing as we observe in the case of five carry trade portfolios. The non-monotonic return patterns and the possibility of undiversified idiosyncratic risks imply that the subsequent asset pricing performances may be weaker compared to that for five carry trade portfolios. Nevertheless, the long-short strategy generates a high mean excess return of around 10% annually and a Sharpe Ratio of 0.80. The generally increasing pattern across portfolios and the large spread between extreme portfolios are also illustrated in the cumulative returns plots in Figure 4.2 (middle panel). Panel C includes the five currency momentum portfolios based on a one-month formation period and one-month holding period. Similar to the five carry trade portfolios, the five momentum portfolios also have a monotonically increasing pattern in mean excess returns from the low past excess returns portfolio (M1) to the high past excess returns portfolio (M5). In contrast to the carry trade portfolios, which have positive skewness in funding currency (C1) and negative skewness in investment currencies (C2 to C5) and the long short strategy (HML), currency momentum portfolios have negative skewness in all portfolios (M1 to M5) and positive skewness in the long-short strategy (HML). Moreover, the momentum portfolios are also more volatile than carry trade portfolios. The correlation between carry trade HML and momentum HML is small and negative (-0.08), indicating that factors explaining carry trade portfolios may have little explanatory power for momentum portfolios. The monotonically increasing returns and different return patterns compared to carry trade portfolios, especially in the crisis period are also demonstrated in Figure 4.2 (lower panel).

4.4.2 Volatility Components and Currency Returns

If volatility risk is priced in currency returns, we are interested in where the potential explanatory power of volatility originates. In this section, we investigate whether pricing aggregate volatility risk can be understood by pricing some of its components. Table 4.3 summarizes our main empirical results about factor prices of risk for models in explaining three currency test assets, i.e. five carry trade portfolios (Panel A), ten carry trade portfolios (Panel B), and five carry trade and five momentum portfolios (Panel C).

Firstly, we test the explanatory power of volatility risk from the model with conventional realized volatility ($DOL + \Delta RV$). For the five carry trade portfolios (Panel A), while the estimated price of risk (λ) for DOL is positive and insignificant, the estimated λ for ΔRV is negative and statistically significant. The price of risk is -0.07% per month (-0.84% per year) and t-statistics (NW and SH) are over 2. We also show that the model has a high cross-sectional R^2

(89%), a low root mean squared pricing error (0.04%), and a χ^2 statistic, which fails to reject the null hypothesis of zero pricing errors. Our results therefore support that volatility risk is negatively and significantly priced in carry trade returns in our sample. Our results are consistent with the hedging intuition from the ICAPM and previous studies based on a different volatility measures, such as Menkhoff, Sarno, Schmeling, and Schrimpf (2012a).

For the ten carry trade portfolios case (Panel B), we observe that the cross-sectional R^2 drops to 43% and the pricing errors rise to 0.12%. The increase in the degrees of freedom, the idiosyncratic risks due to less diversified returns within each portfolio, and the non-monotonic return pattern may jointly explain the reduction in model fit as discussed earlier (Section 4.3.2 and Section 4.4.1). Nevertheless, the χ^2 statistic still cannot reject the null hypothesis of zero pricing errors, the estimated price of risk (λ) only reduces slightly to -0.06% per month, and the t-statistics remain highly significant (over 2). Hence, although pricing performances become weaker, volatility risk is still significantly priced in the ten carry trade portfolios case.

For the joint cross-section of carry trade and momentum portfolios (Panel C), we find very different results. The price of risk remains negative but is small in magnitude (-0.02% per month) and statistically insignificant (t-statistics are around 0.95). The cross-sectional R^2 drops to only 9%, the pricing error is 0.13%, and the χ^2 statistic suggests that the null hypothesis of zero pricing errors is rejected at the 1% level. Despite its success in explaining carry trade portfolios, the model with risk factor ΔRV performs poorly in explaining the joint cross-section. The failure of total volatility to explain the joint cross-section is perhaps not surprising, however. Earlier, we showed that carry trade and momentum strategies are almost uncorrelated. Hence the risk factor explaining carry trade returns would be expected to have low explanatory power for currency momentum returns. Similar findings are also suggested by previous studies (Burnside, Eichenbaum, and Eichenbaum 2011, Menkhoff, Sarno, Schmeling, and Schrimpf 2012b). The failure of ΔRV to explain the joint cross-section further motivates us to investigate whether decomposing total volatility into different components or considering alternative volatility factors can make a difference.

We now turn to models with jump and diffusive volatility components (i.e. $DOL + \Delta BV$, $DOL + \Delta JV$, and $DOL + \Delta BV + \Delta JV$). For both the five and ten carry trade portfolios (Panel A and B) cases, diffusive volatility (ΔBV) is negatively and significantly priced with similar significance and magnitude to the price of risk for realized volatility (ΔRV). The model with ΔBV also has similar goodness of fit (cross-sectional R^2s are 90% and 42%) to the model with ΔRV. In contrast, jump volatility ΔJV has a negative but insignificant estimated price of risk, and the cross-sectional R^2s are small (24% and 15%). In the joint model with both jump and diffusive volatilities, we find that diffusive volatility remains negative and significant while the jump volatility is insignificant. Our evidence clearly suggests that diffusive volatility dominates jump volatility in explaining carry trade returns. The explanatory power of volatility risk for carry trade returns presented above and documented in the literature is almost exclusively driven by the diffusive volatility component. Cremers, Halling, and Weinbaum (2015) also suggest that volatility risk is more important than jump risk in equity markets.[13]

[13] Our results, however, do not conclude that jump risks are not important in currency returns.

For Panel C, ΔBV, similar to ΔRV, cannot explain the joint cross-section. However, we find that ΔJV is negatively and significantly priced in the joint cross-section of carry trade and momentum portfolios. The model with ΔJV has a cross-sectional R^2 of 28%, much higher than the 9% found for ΔRV and the 6% for ΔBV. Including both ΔBV and ΔJV further strengthens the significance of the price of risk for ΔJV and increases the cross-sectional R^2 further, up to 33%. In contrast to the findings from previous models, we show that models with jump volatility and both jump and diffusive volatilities are not rejected for the null hypothesis of zero pricing errors at the 5% and 15% levels. Although previous studies (Burnside, Eichenbaum, and Eichenbaum 2011, Menkhoff, Sarno, Schmeling, and Schrimpf 2012b) suggest that total volatility has difficulty in explaining currency momentum returns or the joint cross-section between carry trade and momentum, our findings clearly show that the jump component of volatility contains important explanatory power for currency momentum returns. Therefore, our findings also support that decomposing total volatility into jump and diffusion components is economically important for explaining excess returns of different currency portfolios.

We now consider models with short and long-run volatilities ($DOL + \Delta L$, $DOL + \Delta S$, and $DOL + \Delta L + \Delta S$). For carry trade portfolios (Panel A and B), both long-run volatility (ΔL) and short-run volatility (ΔS) are negatively and significantly priced. The price of risk for ΔS are highly significant for both

Instead, we suggest that as a component of total volatility, diffusive volatility dominates jump volatility in pricing carry trade returns. Jump risk may still affect currency returns through higher return moments. Hence our findings are not inconsistent with existing studies suggesting jump risks are priced in currency returns (Chernov, Graveline, and Zviadadze 2015, Farhi, Fraiberger, Gabaix, Ranciere, and Verdelhan 2015, Jurek 2014, Jurek and Xu 2014).

panels and large in magnitude. However, the significance for ΔL declines for the ten carry trade portfolios case. The model with ΔS also fits the data better than the model with ΔL, with higher cross-sectional R^2 (88% and 39% for ΔS and 62% and 33% for ΔL) and lower pricing errors. The model with ΔS cannot be rejected for the null hypothesis of zero pricing errors for both panels at the 1% level. The model with ΔL cannot be rejected for five carry trade portfolios, but can be rejected at the 5% significance level for ten carry trade portfolios. The joint model with both components also suggests that short-run volatility is more important in general, although its statistical significance also declines in the ten carry trade portfolio case. Our evidence suggests that both short and long-run volatilities are priced in carry trade portfolios and that short-run volatility performs better than long-run volatility in general.

From Panel C, neither ΔS nor ΔL is able to explain the joint cross-section. Using the joint model with both ΔS and ΔL does not improve the performance. Cross-sectional R^2s are all around 8% to 9%. The models are rejected at the 1% level for the null hypothesis of zero pricing errors. Our results imply that short and long-run volatility decompositions are different from the jump and diffusive volatility decomposition discussed above. Although both short and long-run volatilities are priced in carry trade portfolios, they can hardly explain the joint cross-section, similarly to total realized volatility.

Table 4.4 reports factor betas for each model from the time series regressions. Factor betas vary across portfolios and across factors. If volatility risks are priced then, intuitively, the lowest interest rate carry trade portfolio (C1) should earn the lowest mean return, since it has the highest co-movement (large beta) with volatility risks. Therefore, C1 will serve as a hedging asset, traded with a premia, and expected to earn lower returns in the future. On the contrary, high interest rate carry trade portfolios should earn higher expected returns because they are less correlated (small betas) with volatility risks. In Panel A, we show that betas are generally decreasing from C1 to C5, supporting the above explanation. We also find that betas for ΔRV, ΔBV, and ΔS are monotonically decreasing, in line with their relative superior pricing performances documented above. Betas for extreme portfolios (C1 and C5) are highly significant for all priced volatility factors, indicating that the hedging portfolio (C1) is significantly sensitive to volatility risks. In Panel B, we do not observe strict monotonicity for betas, which is also consistent with non-monotonic return patterns in the ten carry trade portfolios case and relative weak pricing performances for all models with volatility factors. Nevertheless, volatility betas are still in a generally decreasing pattern from C1 to C10. Betas for priced factors in extreme portfolios are also statistically significant. In Panel C, we do not observe clear decreasing or increasing patterns. Betas are largely increasing from M1 to M3, but decreasing from M3 to M5. Betas for extreme portfolios are almost all insignificant. The nonlinear beta patterns and insignificant extreme betas may jointly explain why volatility risks in our linear stochastic discount factor and linear beta pricing model generally fail to explain the joint cross-section and momentum portfolios. However, we find that ΔJV has a generally decreasing pattern in betas and its beta in M1 is positive and statistically significant, supporting its strong pricing ability compared with other factors for the joint cross-section.

Figures 4.3 to 4.5 plot the realized mean excess returns against model predicted mean excess returns for each portfolio using models with different volatility factors. If models price currency returns correctly, we should expect portfolio scatter plots to lie on the 45 degree line. A few observations are worth mentioning. First, five carry trade portfolios (left panel for each figure) are located close to the 45 degree line across all models. Compared to models using ΔJV and ΔL, models using ΔRV, ΔBV, and ΔS move the portfolio scatter plots closer to the 45 degree line. Those models perform better mainly because they can fit the extreme portfolios well, which are almost on the line. Using joint models with two components does not move the scatter closer to the line, indicating that the pricing abilities of volatility risk are concentrated in some of its components but not in others. Second, although these results mainly continue to hold, the scatter plots generally deviate more from the 45 degree line when we consider ten carry trade portfolios (middle panel for each figure). Models which price carry trade portfolios better can still yield scatter plots closer to the line than other models. They perform especially well in fitting C1 (funding currencies) compared to other portfolios (investment currencies), suggesting that volatility risks seem to be more important for low interest rate currencies (hedging assets) than high interest rate currencies. Thirdly, although carry trade portfolio scatters are close to the 45 degree line, momentum portfolio scatters are more dispersed from the lines (right panel for each figure). Momentum portfolios are almost flat and extreme portfolios are far from the 45 degree line, suggesting that volatility risks have very limited explanatory power. Nevertheless, compared to ΔRV or ΔBV, ΔJV moves the extreme portfolio M1 much closer to the 45 degree line, in accordance with its explanatory power for the joint cross-section discussed above. In short, therefore, these graphical illustrations are consistent with our main findings reported above.

4.4.3 Alternative Volatility Factors and Currency Returns

If volatility risk is priced in currency returns due to characterizing bad states, other factors identifying bad states may also help to explain currency returns. In this section, we investigate whether alternative volatility factors, volatility of volatility and cross-sectional volatility, can explain currency returns. Table 4.5 reports factor prices of risks for models using alternative volatility factors in explaining three currency test assets.

We first investigate the role of volatility of volatility ($DOL+\Delta VoV$ and $DOL+\Delta RV+\Delta VoV$). In carry trade portfolios (Panel A and B), volatility of volatility (ΔVoV) is negatively and significantly priced. Besides the similar magnitude and significances in price of risk, ΔVoV fits the data slightly worse compared with ΔRV. The cross-sectional R^2s are 75% and 27%, smaller than those of ΔRV. The pricing errors are also slightly larger. Nevertheless, the models with ΔVoV are not rejected for the null hypothesis of zero pricing errors in both panels. When controlling for ΔRV, we show that ΔVoV has an insignificant estimated price of risk. Our findings thus suggest that volatility is more important than volatility of volatility for pricing carry trade portfolios. This finding is not surprising since it is likely that investors care more about risk than "risk of risk". In the joint cross-section of carry trade and momentum portfolios (Panel C), ΔRV is marginally significant at the 10% level. The cross-sectional R^2 is 17%, higher than 9% obtained for ΔRV. Controlling for ΔRV in the joint model makes ΔVoV even more significant. Although slightly weaker in statistical significance, ΔVoV seems to be similar to ΔJV, which also contains pricing ability for the joint cross-section. In Table 4.1, we have already shown that ΔVoV has a high correlation of 0.73 with ΔJV, higher than with other volatility factors. The high correlation and similar performances in the joint cross-section imply that ΔVoV and ΔJV may contain some common information. One major commonality of jump volatility and volatility of volatility is that they are both related to the higher moments of returns.[14]. Therefore, the fact that conventional volatility risks fail to explain returns but jump volatility and volatility of volatility do have explanatory power may suggest that only the non-normal (tails or extreme returns) component of volatility are priced in the joint cross-section of carry trade and momentum portfolios.

We now turn to the role of cross-sectional volatility ($DOL+\Delta CSV$ and $DOL+\Delta RV+\Delta CSV$). For carry trade portfolios (Panel A and B), we find that the price of risk, cross-sectional R^2, and pricing errors for ΔCSV are similar to those of ΔRV. Although constructed differently, the cross-sectional volatility (ΔCSV) closely mimics the performance of the time-series based volatility measure (ΔRV). Different from the results for ΔVoV discussed above, ΔCSV still has a negative and statistically significant estimated price of risk when we control for ΔRV. Our findings therefore suggest that the information content of cross-sectional volatility is not fully subsumed by conventional volatility risk. In Panel C, ΔCSV also fails to explain the joint cross-section of carry trade and momentum portfolios, similar to ΔRV. This result is as expected, given their high correlation (0.81 shown in Table 4.1) and similar pricing performances in carry trade portfolios. When we include both ΔCSV and ΔRV into the model, ΔCSV is still insignificant but ΔRV becomes significant. The cross-sectional

[14]Backus, Chernov, and Martin (2011) describe that jumps are often used as a mathematical device to generate return non-normality. Amaya, Christoffersen, Jacobs, and Vasquez (2011) show that realized skewness is driven by jumps. Park (2013) suggests that volatility of volatility is proportional to skewness and kurtosis.

R^2 also rises to 28%, higher than the 9% obtained for ΔRV and the 5% for ΔCSV. If our previous explanations for ΔJV and ΔVoV are valid, then the significant estimated price of risk for ΔRV in the joint model may suggest that ΔCSV controls for the unpriced normal return component, and the remaining component of ΔRV reflecting the non-normal or tail component is priced. In an unreported analysis, we consider the model $DOL + \Delta RV + \Delta BV$ and find that, controlling for the normal return component measured by the diffusive volatility ΔBV, ΔRV is also negatively and significantly priced in the joint cross-section of carry trade and momentum portfolios, which seems to support our conjecture.

Table 4.6 reports factors betas for alternative volatility factors. For carry trade portfolios (Panel A and B), we show that betas of alternative volatility factors are decreasing from low interest rate currencies (C1) to high interest rate currencies (C5 or C10) and betas for extreme portfolios are significant, just as we observed in Table 4.4 for betas of volatility and volatility components. For the five carry trade portfolios, we also show that betas for ΔCSV are monotonically decreasing, consistent with its better pricing performance compared to ΔVoV. When controlling for ΔRV, we find that extreme portfolio betas for ΔCSV remain highly significant, supporting the view that ΔCSV is not fully subsumed by ΔRV even though they are correlated. For the joint cross-section (Panel C), we find that the estimated betas of ΔVoV are generally decreasing, and that the beta for M1 is positive but not significant. When we control for ΔRV, we find that the estimated beta for M1 becomes positive and significant. We also show that the estimated betas of ΔCSV are not significant, similar to the betas for ΔRV. After including both ΔRV and ΔCSV into the model, the estimated beta of ΔRV becomes positive and significant for M1. To summarize, therefore, the factor beta results support our findings for the factor price of risk discussed above.

Figure 4.6 and 4.7 plot the realized mean excess returns against model predicted mean excess returns for each portfolio using models with alternative volatility factors. Similar to the case for ΔRV, both ΔVoV and ΔCSV can price carry trade portfolios (left and middle panels for each figure) around the 45 degree lines. However, the portfolio scatters deviate slightly more from the line compared with scatters using conventional volatility. For the joint cross-section (right panel), models using ΔRV with each alternative volatility factor or using ΔVoV alone can both move the scatters closer to the line compared to model using ΔRV only. Hence, graphical illustrations also confirm that alternative volatility factors are priced and cannot be subsumed by conventional volatility.

4.4.4 Factor Mimicking Portfolios

Our volatility factors are constructed using volatility innovations. In this section, we consider an alternative approach to constructing volatility factors using factor-mimicking portfolios (Ang, Hodrick, Xing, and Zhang 2006, Menkhoff, Sarno, Schmeling, and Schrimpf 2012a). The factor mimicking portfolios are constructed by regressing volatility innovations on a constant and our test portfolios. The regression coefficients are then used as weights to construct a weighted average portfolio. Hence, those new factors are returns-based, investable, and mimicking volatility innovations in terms of exposures in different portfolios. For simplicity, we focus on the conventional five carry trade portfolios as our test asset and only include the volatility factors one by one into the models.

Table 4.7 summarizes the results for the factor-mimicking portfolios. Panel A reports the estimated factor prices of risks for the models when volatility innovations are replaced with their corresponding factor-mimicking portfolios. We find that our main conclusions are supported and the results are very close to the results obtained for models using volatility innovations, shown in Table 4.3 and Table 4.5: volatility risks are negatively priced, diffusive volatility dominates jump volatility in explaining carry trade returns, both short and long-run volatilities are priced, and alternative volatility factors are also priced.

Panel B reports some characteristics of the factor-mimicking portfolios. W1 to W5 are regression factor loadings, which are used as weights to construct those factors. W1 is positive for all models, supporting the hedging argument that the low interest rate currency portfolio (C1) earns low expected returns because it has the highest co-movement with volatility innovations. We also show that the weights are generally decreasing from C1 to C5 for all specifications, but monotonically decreasing only for the model using ΔBV, consistent with its strong pricing performance.

Another focus is to understand the economic significance through the magnitude of the estimated price of risk. From a no-arbitrage condition, the price of risk for return-based factors should equal to its unconditional average return. Panel B documents that the unconditional monthly average returns of factor-mimicking portfolios ($\bar{RX}_{FMP}$) are all close to their prices of risk (λ) estimated for different specifications. The estimated price of risk varies from -0.004%

to -0.095% per month across different models and the majority of statistically significant estimated prices of risks are above 0.060% per month. Our results therefore show that prices of risks make economic sense and satisfy the no-arbitrage condition.

4.4.5 Beta Sorted Portfolios

In this section, we investigate the pricing of volatility factors from a different perspective. We construct volatility beta-sorted portfolios. At the beginning of each month, we regress individual currency excess returns on a constant and a volatility factor using a rolling window including the past 36 months. We sort currencies into five portfolios according to their volatility betas, hold portfolios for one month, and record their returns. After that, we move the estimation window one month forward and continue with the same procedure for the next month.

Table 4.8 reports the results obtained for the volatility beta-sorted portfolios. The pre-formation betas are monotonically increasing by construction. Hence V1 (V5) represents currencies with the lowest (highest) co-movements (betas) with volatility risks. The post-formation betas are also increasing but not monotonic. If volatility risks are negatively priced as we documented earlier, then V1 (V5) should earn higher (lower) mean excess returns, and we should observe a decreasing pattern in mean returns. We do indeed find that that mean returns are all decreasing from V1 to V5, although the decrements are not strictly monotonic. Currencies with higher co-movements with volatility factors (V5) act as hedging portfolios, and hence are traded at a premium and are expected to earn lower returns, while currencies with lower co-movements with volatility factors (V1) are compensated for higher expected returns. If risks of volatility, volatility components, and alternative volatility factors are priced, then we should also expect their respective beta-sorted portfolios to have significant return spreads. We show that a strategy investing in V1 and shorting V5 generates returns of up to 6% annually. We also find that models using ΔRV, ΔBV, ΔS, ΔVoV, and ΔCSV, which perform relatively well in the asset pricing tests discussed in previous sections, can also generate statistically significant return spreads. Models using ΔJV and ΔL generate insignificant or marginally significant spreads, which is consistent with their relatively weak asset pricing performances. To summarize, our results from an analysis using beta-sorted portfolios further support the view that volatility factors are negatively priced in currency returns.

4.5 Robustness Checks

In this section, we conduct a battery of robustness checks. We focus on two aspects. First, we assess whether our results hold true for alternative samples. Second, we investigate whether our results are affected by using alternative volatility measures.

4.5.1 Alternative Samples

Table 4.9 and 4.10 summarize currency asset pricing results obtained using just the developed countries sample of 15 countries or currency regions (Australia, Belgium, Canada, Denmark, the Eurozone, France, Germany, Italy, Japan, Netherlands, New Zealand, Norway, Sweden, Switzerland and the United Kingdom). We focus on the same three test assets as in the main analysis.[15]

The developed countries sample results are similar to those obtained using our full-sample results. Diffusive volatility dominates jump volatility in carry trade portfolios, while jump volatility is negatively and significantly priced in the joint cross-section of carry trade and momentum portfolios. Both short run and long-run volatilities are priced. However, we show that long-run volatility fits the data slightly better in the five carry trade portfolios case for developed countries alone. This benefit deteriorates in the ten carry trade portfolios case, however. In contrast to the full-sample evidence, we also find that long-run volatility is marginally priced in the joint cross-section. Our results strengthen when we control for short-run volatility, and imply that the low-frequency business cycle fluctuations captured by long-run volatility seem to be more important than the transitory financial stress driven by short-run volatility in explaining currency returns in the developed countries sample. Volatility of volatility is also negatively priced. Similar to our main findings, volatility is more important for the carry trade portfolios, while volatility of volatility is more important for the joint cross-section. We further show that cross-sectional volatility is priced and that it dominates conventional volatility in explaining the five carry trade portfolios. In the joint cross-section, conventional volatility is priced after we control for cross-sectional volatility, as mentioned in the main analysis.

Table 4.11 and 4.12 report currency asset pricing results for portfolios and individual currency returns based on an alternative sample of balanced panel

[15] Due to the statistically insignificant and economically small return spreads for momentum portfolios in the developed countries, the joint cross-section (Panel C) includes five carry trade portfolios from the developed countries and five momentum portfolios from the full sample.

of 22 countries (or currency regions), namely Australia, Canada, Hong Kong, Czech Republic, Denmark, the Eurozone, Hungary, India, Japan, Kuwait, Mexico, New Zealand, Norway, Philippines, Saudi Arabia, Singapore, South Africa, Sweden, Switzerland, Taiwan, Thailand and the United Kingdom.

Panel A in both tables report the results for the five carry trade portfolio case, which are largely similar to results using the full sample. Panel B in both tables document results using individual currency returns. While results are again mainly consistent with our main results using the full sample, two points need to be mentioned here. First, we find that the estimated price of risk of long-run volatility is insignificant for individual currency returns. In previous sections, we also observed that the price of risk for long-run volatility becomes less significant in the ten carry trade portfolios case compared with five carry trade portfolios case, both for the full sample and the developed countries samples. This finding seems to suggest that the effect of long-run volatility weakens when currencies within each portfolio become less diversified and are more subject to currency-specific idiosyncratic risk. Instead, short-run volatility is still important for those assets. Second, we show that cross-sectional volatility dominates conventional volatility. The model using cross-sectional volatility also outperforms other models in explaining individual currency returns. The strong performance of cross-sectional volatility again supports that view that alternative volatility factors contain incremental information beyond conventional volatility risk.

4.5.2 Alternative Measures

In this section we investigate whether our main results are affected when alternative volatility measures are used. We consider alternative measures for jump and diffusive volatilities, short and long-run volatilities, volatility of volatility, and cross-sectional volatilities respectively.

We primarily consider alternative measures of the jump and diffusive volatilities. In the main analysis reported above, we rely on the most commonly used bipower variation to measure diffusive volatility. In this part, we consider two alternative but nevertheless easily constructed nonparametric estimators. In particular, we follow Andersen, Dobrev, and Schaumburg (2012) in constructing Median and Minimum Realized Volatility ($MedRV$, $MinRV$) based on nearest-neighbor truncation to control for noise. $MedJV$ and $MinJV$ are jump volatilities respectively. The specifications of estimators can be found in the Appendix. Table 4.13 reports the currency asset pricing results obtained using these alternative measures of jump and diffusive volatilities. For carry trade portfolios (Panel 1.A, 1.B, 2.A, 2.B), we find that $\Delta MedRV$ and $\Delta MinRV$ are negatively and significantly priced. $\Delta MedJV$ and $\Delta MinJV$, in contrast, have insignificant estimated prices of risk. Diffusive volatility dominates jump volatility in explaining carry trade portfolio returns, in line with our results in the main analysis. In the joint cross-section (Panel 1.C, 2.C), we show that diffusive volatility risks are not priced but the jump volatility risks are negatively and significantly priced, which is again consistent with our previous findings. Therefore, using alternative measures of jump and diffusive volatilities does not appear to affect our main findings.

We then construct alternative measures of short and long-run volatilities. Besides the Hodrick-Prescott filter we used in main analysis, it is difficult to decompose volatility into short run and long-run components without explicitly modelling the volatility dynamics. In this section, we consider a parametric approach to model the short and long run volatility components. In particular, we use the GARCH-MIDAS model introduced by Engle, Ghysels, and Sohn (2013). The model uses a mean-reverting generalized autoregressive conditional heteroskedacity (GARCH) process to model the short-run volatility component and apply a mixed data sampling (MIDAS) filter on monthly realized variance with rolling window to model the long-run component. Exact details are given in the Appendix. Table 4.14 document results using these alternative measures of short and long-run volatilities. These results reveal that both short-run and long-run volatilities are negatively and significantly priced in carry trade portfolios (Panel A, B). Short-run volatility fits the data better in general and remains significant when we consider both short-run and long-run components in the model. In the joint model, the long-run component is insignificant for the five carry trade portfolios, but is marginally significant for the ten carry trade portfolios. In the joint cross-section of carry trade and momentum portfolios (Panel C), neither the short-run nor the long-run component is priced. Our main results therefore continue to hold when alternative measures of the short-run and long-run volatility components are used.

We also discuss two alternative approaches to constructing volatility of volatility. First, we use a GARCH (1, 1) model to obtain daily conditional volatility, and then compute the volatility of the estimated conditional volatility. Secondly, we use the daily Chicago Board Options Exchange (CBOE) VIX index to measure volatility, and then construct a monthly volatility of VIX. Table 4.15 reports results using these alternative measures of volatility of volatility. ΔVoV is negatively and significantly priced under both measures for carry trade portfolios (Panel 1.A, 1.B, 2.A, 2.B). Results are a little more mixed when we compare the relative importance of volatility and volatility of volatility. Results obtained using the GARCH-based measures suggest that volatility of volatility is more important for the five carry trade portfolios (Panel 1.A) and that both of them are important for the ten carry trade portfolios (Panel 1.B). Results obtained using the VIX-based measures suggest that volatility is more important. For the joint cross-section, we find that GARCH-based volatility and volatility of volatility measures (Panel 1.C) are not priced, while VIX-based volatility is priced (Panel 2.C). These findings, therefore, suggest that the relative importance of volatility versus volatility of volatility can vary across samples and measures. Moreover, the correlation between VIX-based volatility and volatility of volatility is only 0.30, lower than what we observed in the main analysis. The significant pricing of volatility of volatility and the low correlation between volatility and volatility of volatility further support our argument that the effect of volatility of volatility may not be fully captured by that of volatility.

Finally, we construct an alternative measure of cross-sectional volatility. Cross-sectional volatility, by definitions, can be observed at any frequency. Rather than using daily exchange rate returns, we therefore construct a measure based on monthly currency excess returns. Table 4.16 report results using this alternative measure of cross-sectional volatility. For the five carry trade portfolios (Panel A), ΔCSV is negatively and significantly priced as in the main analysis; however, the cross-sectional R^2 is smaller, and it becomes insignificant when we control for ΔRV or when we consider the ten carry trade portfolios case (Panel B). In the joint cross-section (Panel C), ΔRV is significantly priced after controlling for ΔCSV as in the main analysis. In the main analysis, the correlation between ΔRV and ΔCSV was found to be over 0.80. In this section, we find that the correlation is only 0.36 when the monthly-based cross-sectional volatility is used. Similar to volatility of volatility, the significant pricing of cross-sectional volatility and the low correlation between cross-sectional volatility and conventional volatility again imply that the effect of cross-sectional volatility may also not be fully captured by that of conventional volatility.

4.6 Conclusion

The empirical violation of uncovered interest rate parity and the documented high excess returns of currency carry trade strategies motivate risk-based explanations of the carry trade. In this paper, we revisit the pricing of volatility risk in currency markets. The research reported in this paper contributes to the literature by providing new empirical results on currency volatility components and alternative volatility factors.

First, we find that pricing currency volatility risk can be understood by pricing some of its components. We show that volatility risk is overall negatively priced in carry trade returns for all specifications considered. When we decompose volatility into jump and diffusion components, we find that the diffusion component dominates the jump component in explaining carry trade portfolio returns. The pricing ability of volatility risk in carry trade portfolios is almost entirely due to the diffusive volatility component. Despite its marginal explanatory power in carry trade portfolios, the jump volatility component plays a unique role in explaining the joint cross-section of carry trade and momentum portfolios, which neither total volatility nor diffusive volatility is able to explain. When we decompose volatility into short-run and long-run components, we show that both components are negatively priced. The transitory shock-driven short-run volatility is generally more important than the low frequency fluctuations-driven long-run volatility.

Second, we suggest that other factors similar to volatility in characterizing bad states are priced in currency returns. We find that two alternative volatility factors, volatility of volatility and cross-sectional volatility are also priced in currency returns. Although conventional volatility seems to perform slightly better in carry trade portfolios, volatility of volatility and cross-sectional volatility contain unique explanatory power for the joint cross-section of carry trade and momentum portfolio and the cross-section of individual currency excess returns respectively. Hence, alternative volatility factors are not fully subsumed by conventional volatility risk.

Our research can be extended in a number of dimensions. First, although we show which part of volatility matters most for explaining carry trade returns; it would also be interesting to derive the underlying economic fundamental drivers of different types of volatility risk. Second, it would also be interesting to construct a structural model which incorporates different types of volatility risk. We leave these extensions on the agenda for future studies.

4.7 Tables and Figures

Table 4.1: Descriptive Statistics: Volatility Factors

This table reports descriptive statistics of different volatility factors from 1984 to 2014. We report mean (%), standard deviation (%), skewness, kurtosis, and cross correlations among different factors.

	ΔRV	ΔBV	ΔJV	ΔL	ΔS	ΔVoV	ΔCSV
Mean	0.02	0.02	0.01	0.00	0.03	0.03	0.02
Std	0.14	0.14	0.04	0.03	0.16	0.10	0.16
Skew	0.95	0.84	1.44	0.44	0.97	0.88	1.88
Kurt	9.33	8.19	12.97	7.01	9.29	9.57	20.35
Corr							
ΔRV	1.00	0.94	0.30	0.25	0.99	0.55	0.81
ΔBV		1.00	-0.03	0.24	0.89	0.31	0.80
ΔJV			1.00	0.07	0.39	0.73	0.15
ΔL				1.00	0.19	0.12	0.21
ΔS					1.00	0.64	0.80
ΔVoV						1.00	0.42
ΔCSV							1.00

Table 4.2: Descriptive Statistics: Currency Portfolios

This table reports descriptive statistics of currency portfolios from 1984 to 2014. We report annualized mean (%), annualized standard deviation (%), skewness, kurtosis, and annualized Sharp Ratios.

Panel A: 5 Carry Trade Portfolios											
	C1	C2	C3	C4	C5	HML					
Mean	-1.52	-0.08	2.05	2.59	5.06	6.58					
Std	7.39	7.53	8.09	8.56	10.40	9.05					
Skew	0.00	-0.53	-0.47	-1.05	-0.80	-0.89					
Kurt	3.71	5.38	5.36	7.05	5.32	4.88					
SR	-0.21	-0.01	0.25	0.30	0.49	0.73					
Panel B: 10 Carry Trade Portfolios											
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	HML
Mean	-2.15	-0.42	-1.05	1.37	2.98	1.62	2.29	2.43	2.28	7.77	9.92
Std	8.29	7.91	7.64	8.71	8.12	8.96	8.92	10.64	12.08	12.39	12.34
Skew	0.07	-0.22	-0.57	-0.40	-0.16	-0.86	-0.69	-3.59	-0.81	-1.14	-0.89
Kurt	4.33	4.75	4.95	5.88	4.24	7.36	6.07	35.91	6.92	8.20	6.11
SR	-0.26	-0.05	-0.14	0.16	0.37	0.18	0.26	0.23	0.19	0.63	0.80
Panel C: 5 Currency Momentum Portfolios											
	M1	M2	M3	M4	M5	HML					
Mean	-2.58	0.71	2.16	3.32	4.60	7.18					
Std	9.93	8.28	8.33	8.11	8.73	10.19					
Skew	-1.27	-1.26	-0.44	-0.36	-0.14	0.48					
Kurt	9.26	10.06	4.66	4.96	3.41	7.11					
SR	-0.26	0.09	0.26	0.41	0.53	0.71					

Table 4.3: Currency Asset Pricing with Volatility Components: Factor Prices

This table reports cross-sectional asset pricing results for carry trade returns from 1984 to 2014 using currency volatility and volatility components. We consider five carry trade portfolios, ten carry trade portfolios, and five carry and five momentum portfolios. We report factor price and t-statistics based on Newey West(in parentheses) and Shanken Correction (in brackets) standard errors. We also report cross-sectional R square, root mean squared error, and χ^2 statistics (with p-values in parentheses).

	Panel A: Carry 5						Panel B: Carry 10						Panel C: Carry 5&Mom 5					
	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2
$DOL + \Delta RV$	0.08	-0.07		0.89	0.04	1.31	0.07	-0.06		0.43	0.12	9.69	0.09	-0.02		0.09	0.13	30.33
	(0.57)	(-2.47)				[0.73]	(0.54)	(-2.30)				[0.29]	(0.64)	(-0.98)				[0.00]
	[0.70]	[-2.36]					[0.66]	[-2.28]					[0.78]	[-0.95]				
$DOL + \Delta BV$	0.08	-0.06		0.90	0.04	1.23	0.07	-0.05		0.42	0.12	9.75	0.09	-0.01		0.06	0.13	32.31
	(0.57)	(-2.46)				[0.75]	(0.53)	(-2.24)				[0.28]	(0.65)	(-0.61)				[0.00]
	[0.70]	[-2.35]					[0.65]	[-2.23]					[0.79]	[-0.58]				
$DOL + \Delta JV$	0.09		-0.02	0.24	0.11	3.16	0.08		-0.02	0.15	0.15	6.95	0.09		-0.02	0.28	0.11	14.92
	(0.65)		(-1.36)			[0.37]	(0.62)		(-1.74)			[0.54]	(0.67)		(-2.43)			[0.06]
	[0.80]		[-1.15]				[0.76]		[-1.52]				[0.82]		[-1.91]			
$DOL + \Delta BV + \Delta JV$	0.08	-0.06	0.01	0.90	0.04	1.19	0.07	-0.05	-0.01	0.43	0.12	9.61	0.09	-0.02	-0.03	0.33	0.11	10.80
	(0.57)	(-2.23)	(0.04)			[0.55]	(0.54)	(-1.90)	(-0.66)			[0.21]	(0.65)	(-0.86)	(-2.69)			[0.15]
	[0.70]	[-2.07]	[0.03]				[0.66]	[-1.83]	[-0.60]				[0.80]	[-0.68]	[-2.11]			
$DOL + \Delta L$	0.08	-0.02		0.62	0.08	5.33	0.08	-0.01		0.33	0.13	17.32	0.09	-0.01		0.09	0.13	33.15
	(0.60)	(-2.01)				[0.15]	(0.62)	(-1.80)				[0.04]	(0.64)	(-0.99)				[0.01]
	[0.73]	[-1.71]					[0.75]	[-1.87]					[0.78]	[-0.97]				
$DOL + \Delta S$	0.08		-0.08	0.88	0.04	1.18	0.07		-0.06	0.39	0.13	8.64	0.09		-0.02	0.08	0.13	31.47
	(0.57)		(-2.46)			[0.75]	(0.55)		(-2.20)			[0.37]	(0.64)		(-0.89)			[0.01]
	[0.70]		[-2.31]				[0.67]		[-2.20]				[0.79]		[-0.86]			
$DOL + \Delta L + \Delta S$	0.08	0.02	-0.12	0.95	0.03	0.23	0.08	-0.01	-0.05	0.41	0.12	9.11	0.09	-0.01	-0.01	0.09	0.13	32.49
	(0.57)	(0.91)	(-2.37)			[0.89]	(0.56)	(-0.69)	(-1.53)			[0.24]	(0.64)	(-0.86)	(-0.30)			[0.00]
	[0.70]	[0.71]	[-1.84]				[0.69]	[-0.79]	[-1.58]				[0.78]	[-0.78]	[-0.27]			

Table 4.4: Currency Asset Pricing with Volatility Components: Factor Betas

This table reports cross-sectional asset pricing results for carry trade returns from 1984 to 2014 using currency volatility and volatility components. We consider five carry trade portfolios, ten carry trade portfolios, and five carry and five momentum portfolios. We report factor betas and t-statistics (in parentheses) based on Newey West standard errors.

		Carry 5					Carry 10										Carry 5 & Mom 5				
		C1	C2	C3	C4	C5	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	M1	M2	M3	M4	M5
$DOL + \Delta RV$	$\beta_{\Delta RV}$	2.52	0.79	-0.22	-0.55	-2.53	3.24	1.37	0.26	1.71	0.55	-0.95	0.06	-1.54	-2.17	-2.65	-1.29	-0.95	1.25	0.82	-0.02
		(4.25)	(2.32)	(-0.55)	(-1.05)	(-3.98)	(3.12)	(2.11)	(0.55)	(2.69)	(1.05)	(-1.60)	(0.07)	(-1.29)	(-2.19)	(-2.68)	(-1.24)	(-1.21)	(2.80)	(1.35)	(-0.03)
$DOL + \Delta BV$	$\beta_{\Delta BV}$	2.47	0.91	-0.03	-0.58	-2.77	3.13	1.39	0.53	1.70	0.92	-0.89	-0.07	-1.50	-2.56	-2.79	-2.01	-0.65	1.41	0.82	0.20
		(4.20)	(2.67)	(-0.07)	(-1.05)	(-4.10)	(3.13)	(2.25)	(1.02)	(2.86)	(1.66)	(-1.36)	(-0.08)	(-1.07)	(-2.19)	(-2.69)	(-1.70)	(-0.86)	(2.88)	(1.29)	(0.29)
$DOL + \Delta JV$	$\beta_{\Delta JV}$	2.76	-0.48	-2.05	-1.04	0.80	3.73	1.57	-2.21	1.61	-3.16	-1.41	-0.40	-2.18	2.09	-0.08	6.29	-2.97	-1.21	0.52	-1.92
		(1.55)	(-0.23)	(-1.56)	(-0.68)	(0.35)	(1.39)	(0.57)	(-0.93)	(0.57)	(-1.50)	(-0.88)	(-0.15)	(-0.90)	(0.68)	(-0.02)	(2.45)	(-1.24)	(-0.93)	(0.27)	(-0.96)
$DOL + \Delta BV + \Delta JV$	$\beta_{\Delta BV}$	2.51	0.91	-0.05	-0.60	-2.76	3.18	1.41	0.50	1.72	0.89	-0.90	-0.07	-1.52	-2.54	-2.79	-1.95	-0.68	1.40	0.83	0.18
		(4.29)	(2.66)	(-0.12)	(-1.01)	(-4.10)	(3.19)	(2.23)	(0.97	(2.90)	(1.59)	(-1.39)	(-0.09)	(-1.10)	(-2.73)	(-2.70)	(-1.67)	(-0.93)	(2.90)	(1.31)	(0.26)
	$\beta_{\Delta JV}$	3.14	-0.33	-2.05	-1.13	0.39	4.21	1.78	-2.13	1.87	-3.02	-0.07	-0.41	-2.41	1.71	-0.50	6.00	-3.08	-1.00	0.18	-1.90
		(1.70)	(-0.18)	(-1.59)	(-0.77)	(-0.19)	(1.37)	(0.68)	(-0.93)	(0.76)	(-1.51)	(-1.06)	(-0.15)	(-0.99)	(-0.58)	(-0.19)	(2.41)	(-1.27)	(-0.74)	(0.33)	(-0.96)
$DOL + \Delta L$	$\beta_{\Delta L}$	6.65	-0.72	0.23	1.20	-7.35	8.63	3.41	-2.25	0.13	6.56	-4.81	6.80	-2.67	0.58	-13.02	-1.14	-2.72	0.86	1.30	1.12
		(2.25)	(-0.33)	(0.11)	(0.52)	(-1.98)	(2.31)	(0.99)	(-0.82)	(0.04)	(2.13)	(1.59)	(2.04)	(-0.88)	(0.09)	(-2.28)	(-0.42)	(-1.00)	(0.57)	(0.62)	(0.41)
$DOL + \Delta S$	$\beta_{\Delta S}$	2.16	0.58	-0.23	-0.46	-2.05	2.85	1.12	0.09	1.42	0.44	-0.88	0.24	-1.48	-1.77	-2.09	-1.06	-0.92	1.13	0.79	-0.04
		(3.96)	(1.80)	(-0.66)	(-0.95)	(-3.60)	(2.92)	(1.85)	(0.22)	(2.45)	(0.96)	(1.72)	(0.31)	(-1.30)	(-1.94)	(-2.41)	(-1.11)	(-1.26)	(2.78)	(1.45)	(-0.06)
$DOL + \Delta L + \Delta S$	$\beta_{\Delta L}$	4.70	-1.31	0.46	1.68	-5.54	6.07	2.39	-2.40	-1.25	6.29	-4.07	6.73	-1.30	2.33	-11.3	-0.13	-1.89	-0.22	0.56	1.19
		(1.79)	(-0.59)	(0.21)	(0.72)	(-1.57)	(1.86)	(0.70)	(-0.88)	(-0.42)	(1.99)	(-1.35)	(2.12)	(-0.40)	(0.39)	(-1.99)	(-0.05)	(-0.78)	(-0.14)	(0.26)	(0.46)
	$\beta_{\Delta S}$	2.04	0.61	-0.24	-0.50	-1.91	2.69	1.07	0.16	1.46	0.28	-0.78	0.07	-1.44	-1.83	-1.80	-1.05	-0.87	1.14	0.77	-0.07
		(4.01)	(1.95)	(-0.68)	(-1.04)	(-3.38)	(2.88)	(1.73)	(0.38)	(2.45)	(0.59)	(-1.56)	(0.10)	(-1.24)	(-2.12)	(-1.99)	(-1.09)	(-1.22)	(2.72)	(1.39)	(-0.11)

Table 4.5: Currency Asset Pricing with Alternative Volatility Factors: Factor Prices

This table reports cross-sectional asset pricing results for carry trade returns from 1984 to 2014 using currency volatility and alternative volatility factors. We consider five carry trade portfolios, ten carry trade portfolios, and five carry and five momentum portfolios. We report factor price and t-statistics based on Newey West(in parentheses) and Shanken Correction (in brackets) standard errors. We also report cross-sectional R square, root mean squared error, and χ^2 statistics (with p-values in parentheses).

	Panel A: Carry 5						Panel B: Carry 10						Panel C: Carry 5 &Mom 5					
	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2
$DOL + \Delta RV$	0.08	-0.07		0.89	0.04	1.31	0.07	-0.06		0.43	0.12	9.69	0.09	-0.02		0.09	0.13	30.33
	(0.57)	(-2.47)				[0.73]	(0.54)	(-2.30)				[0.29]	(0.64)	(-0.98)				[0.00]
	[0.70]	[-2.36]					[0.66]	[-2.28]					[0.78]	[-0.95]				
$DOL + \Delta VoV$	0.08		-0.08	0.75	0.06	0.83	0.08		-0.05	0.27	0.14	6.84	0.09		-0.04	0.17	0.12	20.09
	(0.59)		(-2.27)			[0.84]	(0.60)		(-2.01)			[0.55]	(0.65)		(-1.89)			[0.01]
	[0.72]		[-1.74]				[0.73]		[-1.81]				[0.79]		[-1.80]			
$DOL + \Delta RV + \Delta VoV$	0.08	-0.05	0.04	0.93	0.03	0.40	0.07	-0.05	0.02	0.48	0.12	4.08	0.09	-0.03	-0.05	0.17	0.12	12.67
	(0.57)	(-1.81)	(0.51)			[0.82]	(0.50)	(-2.21)	(0.81)			[0.77]	(0.65)	(-1.45)	(-2.07)			[0.08]
	[0.70]	[-1.44]	[0.42]				[0.61]	[-1.98]	[0.62]				[0.80]	[-1.34]	[-1.77]			
$DOL + \Delta CSV$	0.08		-0.06	0.86	0.05	2.66	0.07		-0.04	0.39	0.13	12.98	0.09		-0.01	0.05	0.13	35.88
	(0.57)		(-2.43)			[0.45]	(0.55)		(-2.13)			[0.11]	(0.65)		(-0.33)			[0.00]
	[0.70]		[-2.40]				[0.67]		[-2.23]				[0.80]		[-0.31]			
$DOL + \Delta RV + \Delta CSV$	0.08	-0.06	-0.06	0.89	0.04	1.36	0.07	-0.06	-0.05	0.43	0.12	9.66	0.08	-0.07	-0.03	0.28	0.11	11.29
	(0.57)	(-2.21)	(-2.54)			[0.51]	(0.54)	(-2.19)	(-2.42)			[0.21]	(0.63)	(-2.67)	(-1.51)			[0.12]
	[0.70]	[-1.90]	[-2.42]				[0.66]	[-1.92]	[-2.34]				[0.77]	[-2.78]	[-1.51]			

Table 4.6: Currency Asset Pricing with Alternative Volatility Factors: Factor Betas

This table reports cross-sectional asset pricing results for carry trade returns from 1984 to 2014 using currency volatility and alternative volatility factors. We consider five carry trade portfolios, ten carry trade portfolios, and five carry and five momentum portfolios.We report factor betas and t-statistics (in parentheses) based on Newey West standard errors.

		Panel A. Carry 5					Panel B. Carry 10										Panel C. Carry 5 & Mom 5				
		C1	C2	C3	C4	C5	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	M1	M2	M3	M4	M5
$DOL + \Delta RV$	$\beta_{\Delta RV}$	2.52	0.79	-0.22	-0.55	-2.53	3.24	1.37	0.26	1.71	0.55	-0.95	0.06	-1.54	-2.17	-2.65	-1.29	-0.95	1.25	0.82	-0.02
		(4.25)	(2.32)	(-0.55)	(-1.05)	(-3.98)	(3.12)	(2.11)	(0.55)	(2.69)	(1.05)	(-1.60)	(0.07)	(-1.29)	(-2.19)	(-2.68)	(-1.24)	(-1.21)	(2.80)	(1.35)	(-0.03)
$DOL + \Delta VoV$	$\beta_{\Delta VoV}$	1.67	0.17	-0.16	0.20	-1.87	2.99	-0.05	-0.59	1.62	0.31	-0.83	1.06	-0.63	-1.29	-2.07	1.00	-1.84	0.58	0.98	-0.45
		(2.15)	(0.23)	(-0.42)	(0.35)	(-1.84)	(2.02)	(-0.05)	(-0.69)	(1.73)	(0.50)	(-1.31)	(0.98)	(-1.29)	(-0.79)	(-1.57)	(0.99)	(-1.64)	(1.09)	(1.21)	(-0.49)
$DOL + \Delta RV + \Delta VoV$	$\beta_{\Delta RV}$	2.70	1.03	-0.23	-0.88	-2.62	3.04	1.99	0.67	1.59	0.62	-0.90	-0.47	-1.86	-2.40	-2.68	-2.35	-0.39	1.48	0.66	0.21
		(4.36)	(2.51)	(-0.44)	(-1.20)	(-3.70)	(3.18)	(3.40)	(1.15)	(2.23)	(0.95)	(-1.26)	(-0.51)	(-1.05)	(-2.68)	(-2.34)	(-1.85)	(-0.58)	(2.73)	(0.87)	(0.30)
	$\beta_{\Delta VoV}$	-0.50	-0.66	0.02	0.91	0.23	0.55	-1.65	-1.14	0.34	-0.19	-0.11	1.44	0.87	0.65	0.09	2.89	-1.53	-0.61	0.45	-0.62
		(-0.75)	(-0.81)	(0.04)	(1.04)	(0.23)	(0.45)	(-1.52)	(-1.12)	(0.35)	(-0.26)	(-0.15)	(1.54)	(0.48)	(0.45)	(0.06)	(2.01)	(-1.62)	(-0.84)	(0.46)	(-0.73)
$DOL + \Delta CSV$	$\beta_{\Delta CSV}$	2.60	0.99	0.55	-1.01	-3.12	3.41	1.46	0.69	1.65	1.25	-0.16	1.02	-3.43	-2.34	-3.50	-2.69	-0.44	1.55	0.95	0.54
		(6.65)	(2.70)	(0.95)	(-1.12)	(-5.93)	(4.41)	(2.35)	(1.18)	(3.52)	(2.72)	(-0.18)	(1.74)	(-1.42)	(-3.01)	(-3.59)	(-1.79)	(-0.43)	(2.86)	(2.05)	(0.88)
$DOL + \Delta RV + \Delta CSV$	$\beta_{\Delta RV}$	0.47	-0.31	-2.04	1.04	0.83	0.43	0.15	-1.05	0.62	-1.66	-2.29	-2.45	4.45	-0.13	-2.68	3.27	-1.55	-0.44	-0.10	-1.45
		(0.72)	(-0.41)	(-3.23)	(0.73)	(0.75)	(0.47)	(0.17)	(-1.27)	(0.52)	(-1.90)	(-2.82)	(-1.65)	(1.40)	(-0.09)	(0.93)	(1.90)	(-1.28)	(-0.69)	(-0.12)	(-1.42)
	$\beta_{\Delta CSV}$	2.26	1.21	2.01	-1.76	-3.72	3.10	1.35	1.44	1.21	2.44	1.48	2.79	-6.63	-2.25	-4.54	-5.04	0.67	1.87	1.02	1.59
		(4.06)	(1.72)	(2.73)	(-1.03)	(-3.37)	(3.29)	(1.57)	(1.61)	(1.49)	(3.25)	(1.37)	(2.35)	(-1.55)	(-1.86)	(-2.69)	(-2.29)	(0.44)	(2.47)	(1.62)	(1.69)

Table 4.7: Currency Asset Pricing with Volatility: Factor Mimicking Portfolios

This table reports cross-sectional asset pricing results for carry trade returns from 1999 to 2014 using factor mimicking portfolios. We consider carry trade portfolios returns. We report factor price and t-statistics based on Newey West(in parentheses) and Shanken Correction (in brackets) standard errors. We also report cross-sectional R square, root mean squared error, and χ^2 statistics (with p-values in parenthese). We additional report portfolio weights and monthly average portfolio return for factor mimicking portfolios.

	Panel A: Carry 5						Panel B: FMP Weights and Returns						
	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2	$W1$	$W2$	$W3$	$W4$	$W5$	RX_{FMP}	λ
$DOL + \Delta RV$	0.08	-0.07		0.89	0.04	2.39	0.229	-0.032	-0.080	-0.053	-0.128	-0.062%	-0.069%
	(0.57)	(-2.14)				[0.50]							
	[0.69]	[-2.53]											
$DOL + \Delta BV$	0.08	-0.07		0.90	0.04	2.27	0.205	-0.017	-0.044	-0.063	-0.143	-0.063%	-0.067%
	(0.57)	(-2.10)				[0.52]							
	[0.69]	[-2.52]											
$DOL + \Delta JV$	0.09		-0.00	0.24	0.11	7.30	0.029	-0.012	-0.027	-0.004	0.008	-0.003%	-0.004%
	(0.65)		(-1.55)			[0.06]							
	[0.79]		[-1.53]										
$DOL + \Delta L$	0.08	-0.01		0.62	0.08	5.90	0.022	-0.023	-0.042	0.002	-0.017	-0.004%	-0.007%
	(0.60)	(-1.56)				[0.12]							
	(0.73)	(-1.98)											
$DOL + \Delta S$	0.08		-0.07	0.88	0.04	2.71	0.247	-0.051	-0.089	-0.052	-0.123	-0.062%	-0.071%
	(0.57)		(-2.14)			[0.44]							
	[0.70]		[-2.53]										
$DOL + \Delta VoV$	0.08		-0.02	0.75	0.06	4.18	0.075	-0.041	-0.032	0.015	-0.050	-0.017%	-0.023%
	(0.59)		(-1.90)			[0.24]							
	[0.72]		[-2.30]										
$DOL + \Delta CSV$	0.08		-0.10	0.86	0.05	3.84	0.256	-0.048	0.101	-0.179	-0.215	-0.082%	-0.095%
	(0.57)		(-2.08)			[0.28]							
	[0.70]		[-2.49]										

Table 4.8: Volatility Beta Sorted Portfolios

This table reports volatility beta sorted portfolios. We report mean of portfolio returns, high minus low portfolio returns and t-statistics (in parentheses) based on Newey West standard errors, pre formation betas, and post-formation betas.

		$V1$	$V2$	$V3$	$V4$	$V5$	H/L
ΔRV	β_{pre}	-5.11	-2.82	-1.89	-0.64	0.69	
	β_{post}	-5.02	-3.38	-2.97	-2.64	-0.86	
	$mean$	4.42	1.67	0.92	2.18	-1.21	-5.63
							(-2.60)
ΔBV	β_{pre}	-5.23	-2.79	-1.88	-0.65	0.66	
	β_{post}	-4.45	-2.93	-2.68	-2.55	-1.02	
	$mean$	4.20	1.71	1.19	2.31	-1.36	-5.56
							(-2.55)
ΔJV	β_{pre}	-37.22	-26.21	-18.84	-6.47	9.30	
	β_{post}	-5.01	-4.94	-5.80	-4.48	-3.95	
	$mean$	3.28	1.22	0.66	0.41	2.11	-1.17
							(-0.62)
ΔL	β_{pre}	-46.44	-41.52	25.64	-15.16	-3.50	
	β_{post}	-22.62	-11.45	-11.38	-9.17	-2.10	
	$mean$	3.89	2.43	0.74	0.99	-0.18	-4.07
							(-1.72)
ΔS	β_{pre}	-4.93	-2.76	-1.86	-0.64	0.66	
	β_{post}	-4.36	-2.97	-2.74	-2.57	-0.71	
	$mean$	4.47	2.68	3.17	2.24	0.39	-5.72
							(-2.59)
ΔVoV	β_{pre}	-15.33	-9.08	-6.53	- 3.24	0.44	
	β_{post}	-5.39	-2.35	-2.19	-2.02	-1.37	
	$mean$	4.08	2.27	0.85	1.60	-0.92	-5.00
							(-2.53)
ΔCSV	β_{pre}	-14.60	-8.93	-6.49	-3.41	-0.29	
	β_{post}	-4.00	-2.15	-1.95	-1.41	-0.67	
	$mean$	3.63	2.33	1.18	1.72	-0.69	-4.32
							(-2.18)

Table 4.9: Currency Asset Pricing with Volatility Components: Developed Countries

This table reports cross-sectional asset pricing results for carry trade returns from 1984 to 2014 for developed countries of 15 currencies using volatility components. We consider five carry trade portfolios, ten carry trade portfolios, and five carry and five momentum portfolios. We report factor price and t-statistics based on Newey West(in parentheses) and Shanken Correction (in brackets) standard errors. We also report cross-sectional R square, root mean squared error, and χ^2 statistics (with p-values in parentheses).

	Panel A: Carry 5						Panel B: Carry 10						Panel C: Carry 5&Mom 5					
	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2
$DOL + \Delta RV$	0.15	-0.09		0.85	0.06	1.44	0.16	-0.08		0.74	0.09	2.98	0.13	-0.02		0.05	0.18	32.20
	(1.10)	(-2.81)				[0.70]	(1.17)	(-2.70)				[0.94]	(0.97)	(-0.93)				[0.00]
	[1.31]	[-2.54]					[1.40]	[-2.48]					[0.97]	[-0.93]				
$DOL + \Delta BV$	0.17	-0.09		0.91	0.05	0.87	0.18	-0.08		0.79	0.08	2.48	0.13	-0.01		0.02	0.18	35.49
	(1.20)	(-2.89)				[0.83]	(1.30)	(-2.82)				[0.96]	(0.96)	(-0.37)				[0.00]
	[1.43]	[-2.60]					[1.54]	[-2.55]					[1.17]	[-0.37]				
$DOL + \Delta JV$	0.12		-0.01	0.14	0.15	8.12	0.13		-0.01	0.13	0.16	13.88	0.11		-0.04	0.27	0.16	8.09
	(0.89)		(-0.28)			[0.04]	(0.97)		(-0.32)			[0.08]	(0.84)		(-2.51)			[0.42]
	[1.08]		[-0.27]				[1.17]		[-0.32]				[1.02]		[-2.40]			
$DOL + \Delta BV + \Delta JV$	0.19	-0.10	0.02	0.98	0.02	0.13	0.20	-0.09	0.01	0.82	0.07	1.80	0.12	-0.02	-0.03	0.29	0.15	7.36
	(1.35)	(-3.04)	(1.32)			[0.94]	(1.42)	(-3.14)	(1.06)			[0.97]	(0.86)	(-0.51)	(-2.57)			[0.39]
	[1.56]	[-2.44]	[0.99]				[1.65]	[-2.56]	[0.83]				[1.05]	[-0.52]	[-2.49]			
$DOL + \Delta L$	0.15	-0.02		0.84	0.06	2.99	0.16	-0.02		0.63	0.10	9.41	0.14	-0.12		0.15	0.17	27.34
	(1.12)	(-2.60)				[0.39]	(1.18)	(-2.34)				[0.31]	(1.01)	(-1.85)				[0.00]
	[1.35]	[-2.15]					[1.40]	[-2.16]					[1.23]	[-1.81]				
$DOL + \Delta S$	0.15		-0.10	0.82	0.07	1.31	0.16		-0.09	0.70	0.09	2.78	0.13		-0.02	0.04	0.18	31.82
	(1.07)		(-2.78)			[0.73]	(1.14)		(-2.61)			[0.95]	(0.96)		(-0.86)			[0.00]
	[1.27]		[-2.48]				[1.36]		[-2.40]				[1.18]		[-0.85]			
$DOL + \Delta L + \Delta S$	0.15	-0.02	-0.04	0.85	0.06	2.46	0.16	-0.01	-0.07	0.70	0.09	3.00	0.14	-0.05	0.10	0.32	0.15	3.71
	(1.11)	(-0.97)	(-0.72)			[0.29]	(1.15)	(-0.75)	(-1.85)			[0.89]	(1.05)	(-3.76)	(3.02)			[0.81]
	[1.31]	[-0.82]	[-0.56]				[1.37]	[-0.67]	[-1.71]				[1.21]	[-1.66]	[1.11]			

Table 4.10: Currency Asset Pricing with Alternative Volatility Factors: Developed Countries

This table reports cross-sectional asset pricing results for carry trade returns from 1984 to 2014 for developed countries of 15 currencies using alternative volatility factors. We consider five carry trade portfolios, ten carry trade portfolios, and five carry and five momentum portfolios. We report factor price and t-statistics based on Newey West(in parentheses) and Shanken Correction (in brackets) standard errors. We also report cross-sectional R square, root mean squared error, and χ^2 statistics (with p-values in parentheses).

	Panel A: Carry 5						Panel B: Carry 10						Panel C: Carry 5 &Mom 5					
	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2
$DOL + \Delta RV$	0.15	-0.09		0.85	0.06	1.44	0.16	-0.08		0.74	0.09	2.98	0.13	-0.02		0.05	0.18	32.20
	(1.10)	(-2.81)				[0.70]	(1.17)	(-2.70)				[0.94]	(0.97)	(-0.93)				[0.00]
	[1.31]	[-2.54]					[1.40]	[-2.48]					[0.97]	[-0.93]				
$DOL + \Delta VoV$	0.12		-0.09	0.54	0.11	1.18	0.13		-0.04	0.28	0.14	7.53	0.13		-0.06	0.15	0.17	9.84
	(0.90)		(-2.37)			[0.76]	(0.97)		(-1.63)			[0.48]	(0.95)		(-2.57)			[0.28]
	[1.06]		[-1.74]				[1.17]		[-1.59]				[1.15]		[-2.19]			
$DOL + \Delta RV + \Delta VoV$	0.18	-0.06	0.07	0.98	0.02	0.07	0.17	-0.07	-0.01	0.79	0.08	2.47	0.13	-0.04	-0.08	0.17	0.17	6.80
	(1.30)	(-2.11)	(1.35)			[0.96]	(1.24)	(-2.52)	(-0.31)			[0.93]	(0.93)	(-1.81)	(-2.93)			[0.45]
	[1.45]	[-1.30]	[0.80]				[1.47]	[-2.32]	[0.26]				[1.12]	[-1.48]	[-2.24]			
$DOL + \Delta CSV$	0.21		-0.09	0.84	0.06	1.71	0.21		-0.07	0.67	0.10	4.37	0.12		-0.01	0.03	0.18	35.33
	(1.46)		(-2.85)			[0.64]	(1.47)		(-2.47)			[0.82]	(0.91)		(0.51)			[0.00]
	[1.72]		[-2.62]				[1.74]		[-2.45]				[1.10]		[0.48]			
$DOL + \Delta RV + \Delta CSV$	0.15	-0.09	-0.08	0.85	0.06	1.46	0.12	-0.09	-0.06	0.76	0.08	1.83	0.07	-0.10	-0.02	0.45	0.14	5.94
	(0.83)	(-1.44)	(-2.36)			[0.48]	(0.72)	(-2.82)	(-1.91)			[0.97]	(0.51)	(-3.15)	(-0.98)			[0.55]
	[0.87]	[-1.33]	[-1.98]				[0.82]	[-2.18]	[-1.83]				[0.62]	[-2.34]	[-0.72]			

Table 4.11: Currency Asset Pricing with Volatility Components: Alternative Samples and Individual Currencies

This table reports cross-sectional asset pricing results for carry trade returns from 1999 to 2014 for an alternative sample of 22 currencies using volatility components. We consider carry trade portfolios and individual currency returns. We report factor price and t-statistics based on Newey West(in parentheses) and Shanken Correction (in brackets) standard errors. We also report cross-sectional R square, root mean squared error, and χ^2 statistics (with p-values in parentheses).

	Panel A: Carry 5						Panel B: Individual Currencies					
	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2
$DOL + \Delta RV$	0.16	-0.06		0.87	0.08	3.09	0.16	-0.05		0.75	0.12	30.15
	(1.01)	(-3.04)				[0.38]	(0.96)	(-2.26)				[0.07]
	[1.01]	[-2.71]					[1.12]	[-2.33]				
$DOL + \Delta BV$	0.16	-0.06		0.87	0.08	3.07	0.15	-0.05		0.78	0.11	27.93
	(1.02)	(-3.02)				[0.38]	(0.96)	(-2.27)				[0.11]
	[1.19]	[-2.93]					[1.12]	[-2.34]				
$DOL + \Delta JV$	0.19		-0.05	0.53	0.16	0.62	0.17		-0.01	0.33	0.19	32.18
	(1.18)		(-2.57)			[0.89]	(1.03)		(-1.26)			[0.04]
	[1.37]		[-1.39]				[1.21]		[-1.15]			
$DOL + \Delta BV + \Delta JV$	0.16	-0.05	-0.02	0.89	0.08	1.81	0.15	-0.06	0.01	0.81	0.10	20.62
	(1.01)	(-2.42)	(-0.95)			[0.40]	(0.95)	(-2.34)	(1.38)			[0.36]
	[1.18]	[-2.03]	[-0.69]				[1.11]	[-2.30]	[1.10]			
$DOL + \Delta L$	0.18	-0.02		0.64	0.14	7.18	0.17	-0.01		0.36	0.19	34.81
	(1.11)	(-2.68)				[0.07]	(1.04)	(-1.06)				[0.02]
	[1.30]	[-2.40]					[-1.22]	[-1.17]				
$DOL + \Delta S$	0.16		-0.07	0.87	0.08	2.50	0.15		-0.05	0.75	0.12	30.04
	(1.02)		(-3.05)			[0.48]	(0.96)		(-2.22)			[0.07]
	[1.19]		[-2.91]				[1.12]		[-2.31]			
$DOL + \Delta L + \Delta S$	0.16	-0.01	-0.07	0.87	0.08	1.99	0.15	0.01	-0.06	0.81	0.10	25.08
	(1.01)	(-0.53)	(-2.81)			[0.37]	(0.92)	(0.52)	(-1.88)			[0.16]
	[1.19]	[-0.47]	[-2.64]				[1.08]	[0.66]	[-2.15]			

Table 4.12: Currency Asset Pricing with Alternative Volatility Factors: Alternative Samples and Individual Currencies

This table reports cross-sectional asset pricing results for carry trade returns from 1999 to 2014 for an alternative sample of 22 currencies using alternative volatility factors. We consider carry trade portfolios and individual currency returns. We report factor price and t-statistics based on Newey West(in parentheses) and Shanken Correction (in brackets) standard errors. We also report cross-sectional R square, root mean squared error, and χ^2 statistics (with p-values in parentheses).

	Panel A: Carry 5						Panel B: Individual Currencies					
	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2
$DOL + \Delta RV$	0.16	-0.06		0.87	0.08	3.09	0.16	-0.05		0.75	0.12	30.15
	(1.01)	(-3.04)				[0.38]	(0.96)	(-2.26)				[0.07]
	[1.01]	[-2.71]					[1.12]	[-2.33]				
$DOL + \Delta VoV$	0.16		-0.08	0.86	0.09	0.89	0.16		-0.04	0.65	0.14	29.45
	(1.02)		(-3.04)			[0.83]	(0.98)		(-1.90)			[0.08]
	[1.19]		[-2.26]				[1.15]		[-1.99]			
$DOL + \Delta RV + \Delta VoV$	0.16	-0.06	-0.03	0.87	0.08	2.87	0.16	-0.05	-0.01	0.75	0.12	29.65
	(1.01)	(-3.06)	(-0.54)			[0.24]	(0.96)	(-2.50)	(-0.67)			[0.06]
	[1.19]	[-2.95]	[-0.45]				[1.12]	[-2.35]	[-0.84]			
$DOL + \Delta CSV$	0.16		-0.06	0.87	0.08	3.15	0.15		-0.05	0.84	0.09	29.83
	(1.02)		(-3.00)			[0.37]	(0.96)		(-2.06)			[0.07]
	[1.17]		[-2.97]				[1.12]		[-2.38]			
$DOL + \Delta RV + \Delta CSV$	0.16	-0.06	-0.06	0.87	0.08	3.00	0.16	-0.03	-0.04	0.88	0.08	29.13
	(1.02)	(-2.61)	(-2.89)			[0.22]	(0.97)	(-1.44)	(-2.26)			[0.06]
	[1.19]	[-2.38]	[-2.81]				[1.13]	[-1.37]	[-2.35]			

Table 4.13: Currency Asset Pricing with Jump and Diffusive Volatilities: Alternative Measures

This table reports cross-sectional asset pricing results for carry trade returns from 1984 to 2014 for currencies using alternative measures of jump and diffusive volatility components. We consider five carry trade portfolios, ten carry trade portfolios, and five carry and five momentum portfolios. We report factor price and t-statistics based on Newey West(in parentheses) and Shanken Correction (in brackets) standard errors. We also report cross-sectional R square, root mean squared error, and χ^2 statistics (with p-values in parentheses).

Panel 1: Minimum RV

Panel 1.A: Carry 5	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2
$DOL + \Delta MinRV$	0.08	-0.07		0.89	0.04	1.28
	(0.58)	(-2.40)				[0.73]
	[0.71]	[-2.29]				
$DOL + \Delta MinJV$	0.09		-0.03	0.20	0.11	2.59
	(0.66)		(-1.13)			[0.46]
	[0.91]		[-0.98]			
$DOL + \Delta MinRV + \Delta MinJV$	0.08	-0.06	-0.01	0.90	0.04	1.08
	(0.58)	(-1.94)	(-0.50)			[0.58]
	[0.71]	[-1.78]	[-0.43]			

Panel 1.B: Carry 10	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2
$DOL + \Delta MinRV$	0.07	-0.06		0.37	0.13	9.65
	(0.55)	(-1.96)				[0.29]
	[0.67]	[-2.10]				
$DOL + \Delta MinJV$	0.08		-0.03	0.12	0.15	6.94
	(0.62)		(-1.24)			[0.54]
	[0.76]		[-1.17]			
$DOL + \Delta MinRV + \Delta MinJV$	0.07	-0.05	-0.02	0.40	0.13	8.23
	(0.55)	(-1.67)	(-0.81)			[0.31]
	[0.67]	[-1.57]	[-0.75]			

Panel 1.C: Carry 5 &Mom 5	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2
$DOL + \Delta MinRV$	0.09	-0.01		0.05	0.13	34.30
	(0.65)	(-0.33)				[0.00]
	[0.80]	[-0.31]				
$DOL + \Delta MinJV$	0.09		-0.04	0.29	0.11	8.85
	(0.67)		(-2.48)			[0.35]
	[0.82]		[-1.92]			
$DOL + \Delta MinRV + \Delta MinJV$	0.09	-0.01	-0.05	0.35	0.11	6.08
	(0.65)	(-0.36)	(-2.83)			[0.53]
	[0.79]	[-0.28]	[-2.19]			

Panel 2: Median RV

Panel 2.A: Carry 5	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2
$DOL + \Delta MedRV$	0.08	-0.07		0.89	0.04	1.20
	(0.58)	(-2.41)				[0.75]
	[0.70]	[-2.32]				
$DOL + \Delta MedJV$	0.09		0.04	0.25	0.11	1.57
	(0.64)		(1.23)			[0.66]
	[0.79]		[0.96]			
$DOL + \Delta MedRV + \Delta MedJV$	0.08	-0.07	-0.02	0.90	0.04	1.21
	(0.58)	(-2.26)	(-0.06)			[0.55]
	[0.70]	[-2.13]	[-0.05]			

Panel 2.B: Carry 10	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2
$DOL + \Delta MedRV$	0.07	-0.06		0.41	0.12	9.55
	(0.55)	(-2.18)				[0.30]
	[0.67]	[-2.19]				
$DOL + \Delta MedJV$	0.09		0.03	0.13	0.15	4.41
	(0.64)		(1.22)			[0.82]
	[0.78]		[1.05]			
$DOL + \Delta MedRV + \Delta MedJV$	0.08	-0.06	0.01	0.42	0.12	6.26
	(0.56)	(-2.12)	(0.61)			[0.51]
	[0.68]	[-2.05]	[0.55]			

Panel 2.C: Carry 5 &Mom 5	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2
$DOL + \Delta MedRV$	0.09	-0.01		0.05	0.13	34.21
	(0.65)	(-0.39)				[0.00]
	[0.80]	[-0.37]				
$DOL + \Delta MedJV$	0.09		-0.03	0.19	0.12	13.09
	(0.68)		(-1.83)			[0.11]
	[0.83]		[-1.48]			
$DOL + \Delta MedRV + \Delta MedJV$	0.09	-0.02	-0.04	0.28	0.11	6.45
	(0.66)	(-0.72)	(-2.57)			[0.49]
	[0.81]	[-0.58]	[-2.00]			

Table 4.14: Currency Asset Pricing with Short and Long Run Volatilities: Alternative Measures

This table reports cross-sectional asset pricing results for carry trade returns from 1999 to 2014 for currencies using alternative measures of short and long run volatility components. We consider five carry trade portfolios, ten carry trade portfolios, and five carry and five momentum portfolios. We report factor price and t-statistics based on Newey West(in parentheses) and Shanken Correction (in brackets) standard errors. We also report cross-sectional R square, root mean squared error, and χ^2 statistics (with p-values in parentheses).

	GARCH-MIDAS Volatilities																	
	Panel A: Carry 5						Panel B: Carry 10						Panel C: Carry 5 &Mom 5					
	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2
$DOL+\Delta VOL$	0.18	-0.04		0.74	0.11	4.90	0.20	-0.03		0.31	0.26	10.58	0.19	-0.01		0.32	0.15	18.01
	(1.11)	(-2.63)				[0.18]	(1.19)	(-2.25)				[0.23]	(1.15)	(-0.58)				[0.02]
	[1.31]	[-2.52]					[1.40]	[-2.13]					[1.35]	[-0.50]				
$DOL+\Delta\tau$	0.20	-0.48		0.58	0.14	5.94	0.20	-0.44		0.32	0.26	9.81	0.19	-0.21		0.34	0.15	19.04
	(1.19)	(-2.52)				[0.11]	(1.21)	(-2.52)				[0.28]	(1.15)	(-1.23)				[0.01]
	[1.40]	[-2.17]					[1.42]	[-2.62]					[1.34]	[-1.11]				
$DOL+\Delta g$	0.14		-7.04	0.85	0.09	1.88	0.19		-4.84	0.34	0.25	6.60	0.19		-1.73	0.34	0.15	18.25
	(1.08)		(-2.70)			[0.60]	(1.17)		(-2.47)			[0.58]	(1.13)		(-0.96)			[0.02]
	[1.26]		[-2.40]				[1.38]		[-2.21]				[1.33]		[-0.79]			
$DOL+\Delta\tau+\Delta g$	0.18	0.01	-6.99	0.85	0.09	1.69	0.19	-0.31	-3.74	0.40	0.09	7.31	0.19	-0.15	-1.12	0.36	0.15	16.83
	(1.08)	(0.05)	(-2.30)			[0.43]	(1.12)	(-2.04)	(-2.11)			[0.39]	(1.13)	(-0.85)	(-0.58)			[0.02]
	[1.26]	[0.04]	[-2.00]				[1.31]	[-1.83]	[-1.74]				[1.33]	[-0.82]	[-0.48]			

Table 4.15: Currency Asset Pricing with Volatility of Volatility: Alternative Measures

This table reports cross-sectional asset pricing results for carry trade returns from 1984 to 2014 for currencies using alternative measures of volatility of volatility. We consider five carry trade portfolios, ten carry trade portfolios, and five carry and five momentum portfolios. We report factor price and t-statistics based on Newey West(in parentheses) and Shanken Correction (in brackets) standard errors. We also report cross-sectional R square, root mean squared error, and χ^2 statistics (with p-values in parentheses).

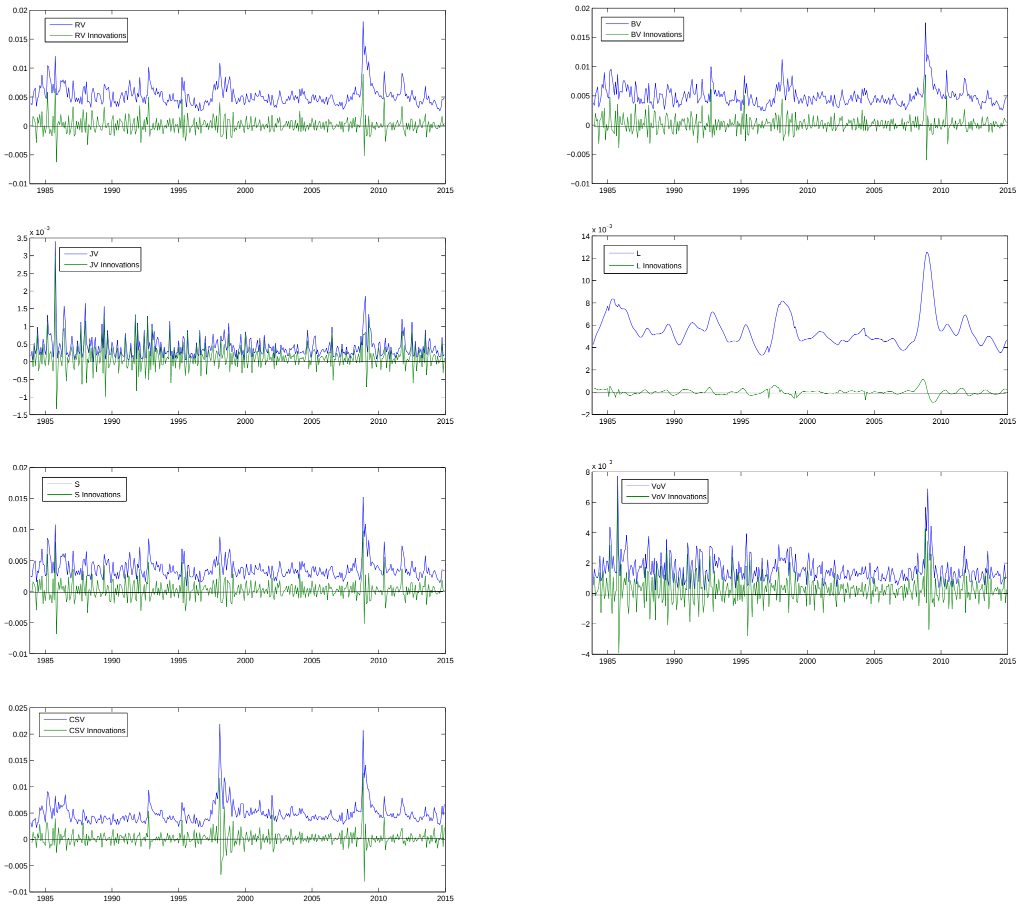
	Panel 1: GARCH Based Volatility of Volatility																	
	Panel 1.A: Carry 5						Panel 1.B: Carry 10						Panel 1.C: Carry 5 &Mom 5					
	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2
$DOL + \Delta VOL_{GARCH}$	0.17	-0.03		0.75	0.11	5.69	0.18	-0.03		0.38	0.25	11.70	0.18	-0.01		0.30	0.16	19.29
	(1.06)	(-2.97)				[0.13]	(1.11)	(-2.79)				[0.17]	(1.10)	(-0.95)				[0.01]
	[1.24]	[-2.78]					[1.30]	[-2.69]					[1.29]	[-0.84]				
$DOL + \Delta VoV_{GARCH}$	0.17		-0.01	0.82	0.10	4.28	0.18		-0.01	0.36	0.26	11.16	0.18		-0.01	0.30	0.16	19.78
	(1.03)		(-3.01)			[0.23]	(1.13)		(-2.75)			[0.19]	(1.10)		(-0.96)			[0.01]
	[1.21]		[-2.88]				[1.33]		[-2.61]				[1.29]		[-0.82]			
$DOL + \Delta VOL_{GARCH} + \Delta VoV_{GARCH}$	0.16	-0.01	-0.01	0.86	0.09	1.16	0.18	-0.03	-0.01	0.38	0.25	11.32	0.18	-0.01	-0.01	0.30	0.16	19.17
	(1.03)	(-1.17)	(-2.98)			[0.56]	(1.11)	(-2.67)	(-2.63)			[0.13]	(1.10)	(-0.94)	(-0.95)			[0.01]
	[1.20]	[-0.88]	[-2.54]				[1.30]	[-2.61]	[-2.38]				[1.29]	[-0.93]	[-0.81]			
	Panel 2: VIX Based Volatility of Volatility																	
	Panel 2.A: Carry 5						Panel 2.B: Carry 10						Panel 2.C: Carry 5 &Mom 5					
	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2
$DOL + \Delta VOL_{VIX}$	0.16	-0.09		0.89	0.08	2.76	0.18	-0.07		0.39	0.25	10.33	0.16	-0.06		0.47	0.14	13.59
	(1.00)	(-3.20)				[0.43]	(1.10)	(-2.97)				[0.24]	(1.03)	(-2.40)				[0.09]
	[1.18]	[-2.98]					[1.29]	[-2.76]					[1.20]	[-2.08]				
$DOL + \Delta VoV_{VIX}$	0.16		-0.03	0.83	0.10	1.95	0.17		-0.03	0.40	0.25	6.42	0.17		-0.02	0.38	0.15	14.80
	(1.03)		(-3.07)			[0.58]	(1.08)		(-2.89)			[0.60]	(1.06)		(-1.76)			[0.06]
	(1.20)		(-2.56)				[1.27]		[-2.54]				[1.24]		[-1.40]			
$DOL + \Delta VOL_{VIX} + \Delta VoV_{VIX}$	0.16	-0.15	0.03	0.92	0.07	0.24	0.17	-0.05	-0.02	0.40	0.25	9.47	0.16	-0.13	0.04	0.59	0.12	1.06
	(1.00)	(-2.27)	(1.06)			[0.88]	(1.09)	(-1.95)	(-1.47)			[0.22]	(1.01)	(-2.18)	(1.33)			[0.99]
	[1.17]	[-1.18]	[0.52]				[1.27]	[-1.55]	[-1.17]				[1.18]	[-1.60]	[0.88]			

Table 4.16: Currency Asset Pricing with Cross-Sectional Volatility: Alternative Measures

This table reports cross-sectional asset pricing results for carry trade returns from 1984 to 2014 for currencies using alternative measures of cross-sectional volatility. We consider five carry trade portfolios, ten carry trade portfolios, and five carry and five momentum portfolios. We report factor price and t-statistics based on Newey West(in parentheses) and Shanken Correction (in brackets) standard errors. We also report cross-sectional R square, root mean squared error, and χ^2 statistics (with p-values in parentheses).

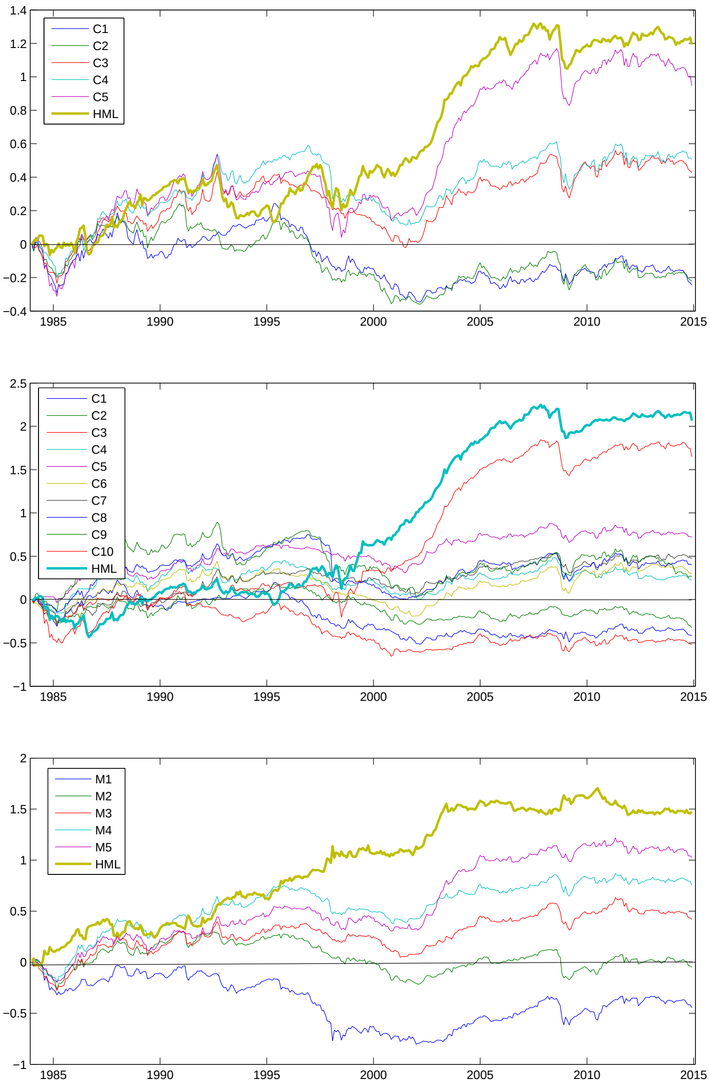
	Monthly Based Cross-Sectional Volatility																	
	Panel A: Carry 5						Panel B: Carry 10						Panel C: Carry 5 &Mom 5					
	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2	λ_{DOL}	λ_{VF1}	λ_{VF2}	R^2	$RMSE$	χ^2
$DOL + \Delta CSV$	0.08		-0.55	0.62	0.08	3.90	0.08		-0.35	0.22	0.14	12.58	0.09		0.15	0.09	0.13	34.35
	(0.60)		(-2.01)			[0.27]	(0.58)		(-1.48)			[0.13]	(0.78)		(0.88)			[0.00]
	[0.74]		[-1.97]				[0.70]		[-1.61]				[0.83]		[0.80]			
$DOL + \Delta RV + \Delta CSV$	0.08	-0.08	-0.06	0.91	0.04	0.84	0.07	-0.08	0.07	0.48	0.12	5.41	0.08	-0.06	0.32	0.40	0.10	11.16
	(0.57)	(-2.75)	(-0.24)			[0.65]	(0.54)	(-3.01)	(0.31)			[0.61]	(0.65)	(-2.49)	(1.87)			[0.13]
	[0.70]	[-2.48]	[-0.21]				[0.66]	[-2.60]	[0.30]				[0.80]	[-2.31]	[1.46]			

Figure 4.1: Currency Volatility and Volatility Innovations



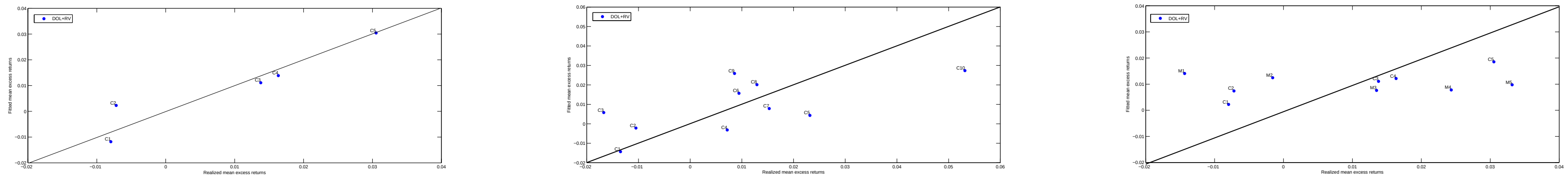
The figure illustrates time series plots of monthly levels and innovations of different currency volatility factors including RV, BV, JV, L, S, VoV, and CSV from 1984 to 2014.

Figure 4.2: Cumulative Currency Portfolio Returns



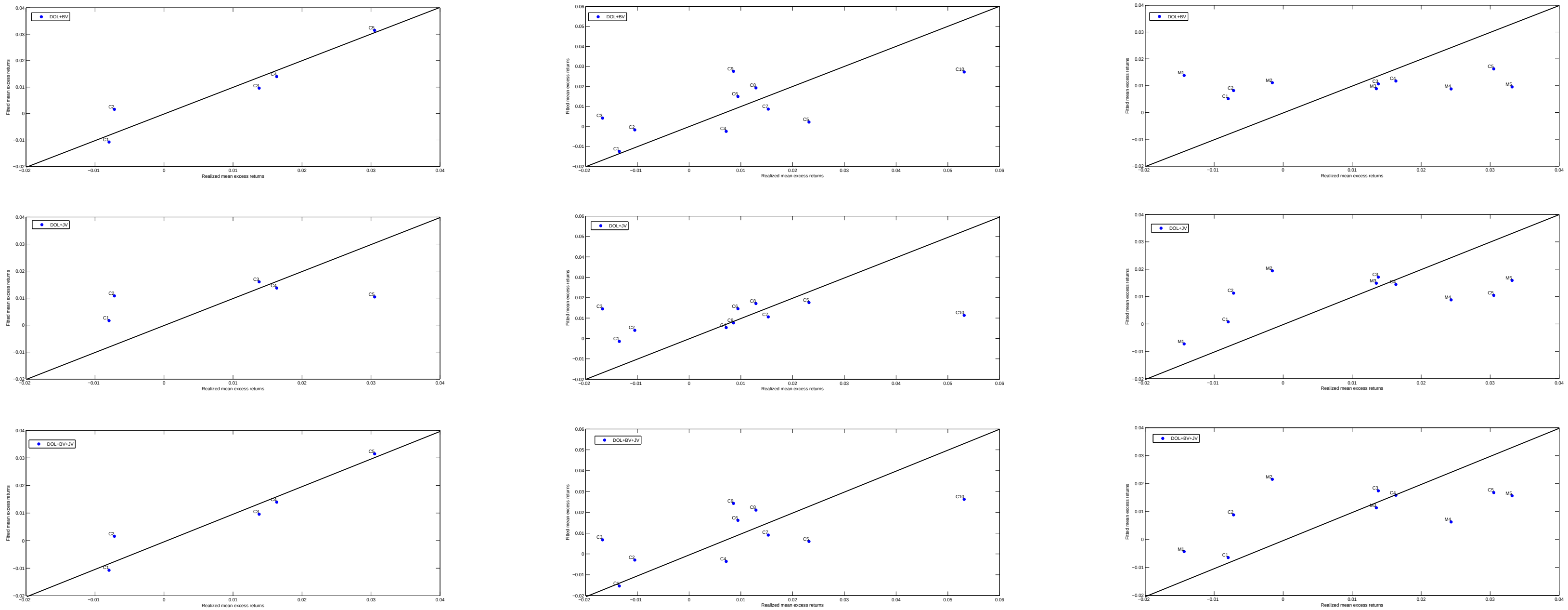
The figure shows time series plots of cumulative returns of five carry trade portfolios, ten carry trade portfolios, and five currency momentum portfolios from 1984 to 2014.

Figure 4.3: Currency Asset Pricing with Realized Volatility: Pricing Errors



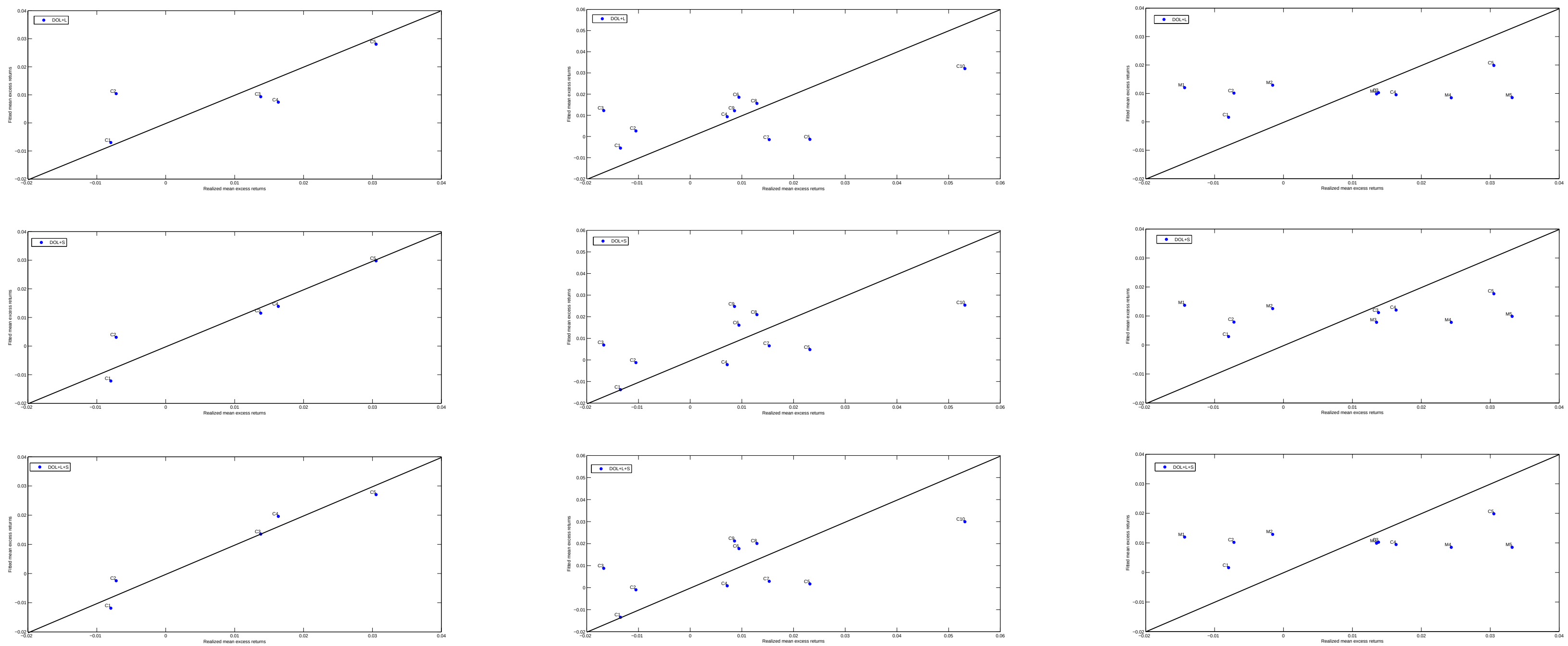
The figure illustrates pricing error plots for five carry trade portfolios (left), ten carry trade portfolios (middle), and five carry and five currency momentum portfolios (right) using $DOL + \Delta RV$ from 1984 to 2014.

Figure 4.4: Currency Asset Pricing with Jump and Diffusive Volatilities: Pricing Errors



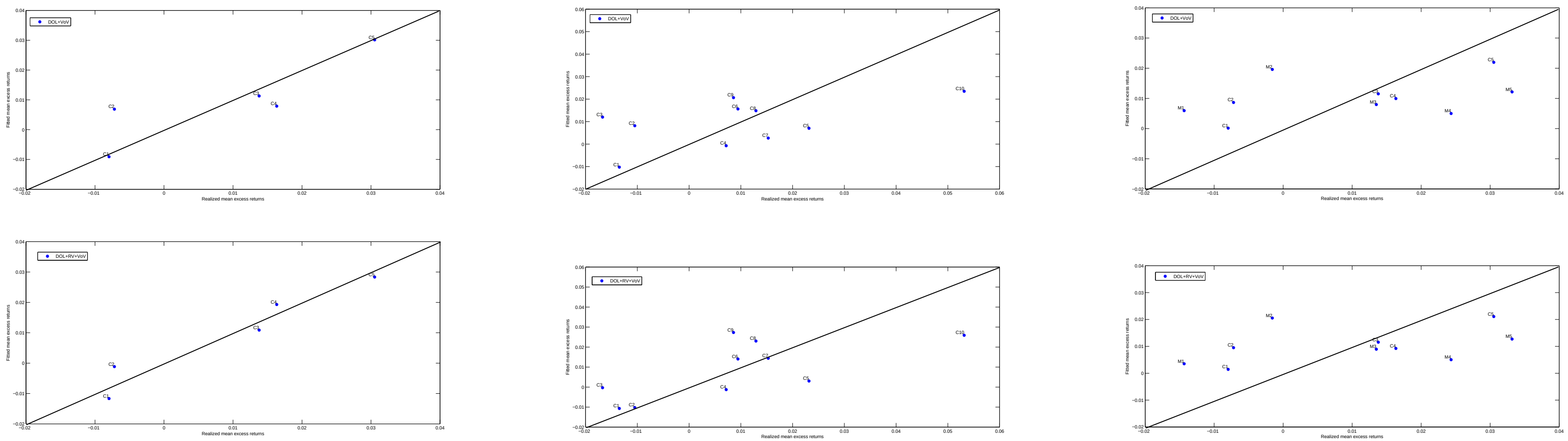
The figure illustrates pricing error plots for five carry trade portfolios (left), ten carry trade portfolios (middle), and five carry and five currency momentum portfolios (right) using $DOL + \Delta BV$ (upper), $DOL + \Delta JV$ (middle), and $DOL + \Delta BV + \Delta JV$ (lower) from 1984 to 2014.

Figure 4.5: Currency Asset Pricing with Short and Long Run Volatilities: Pricing Errors



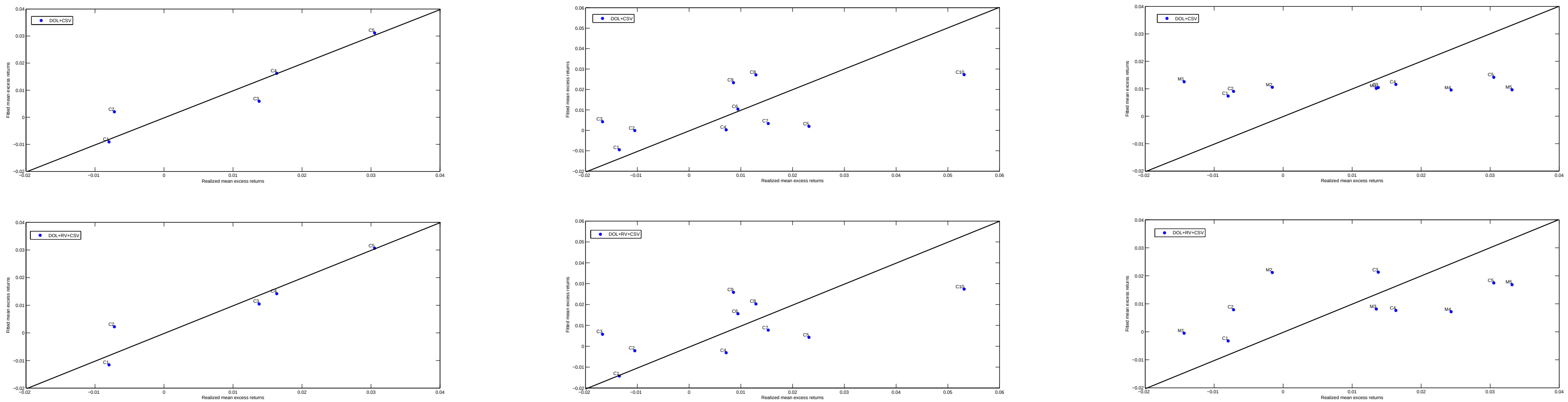
The figure illustrates pricing error plots for five carry trade portfolios (left), ten carry trade portfolios (middle), and five carry and five currency momentum portfolios (right) using $DOL + \Delta L$ (upper), $DOL + \Delta S$ (middle), and $DOL + \Delta L + \Delta S$ (lower) from 1984 to 2014.

Figure 4.6: Currency Asset Pricing with Volatility of Volatility: Pricing Errors



The figure illustrates pricing error plots for five carry trade portfolios (left), ten carry trade portfolios (middle), and five carry and five currency momentum portfolios (right) using $DOL + \Delta VoV$ (upper), and $DOL + \Delta RV + \Delta VoV$ (lower) from 1984 to 2014.

Figure 4.7: Currency Asset Pricing with Cross-Sectional Volatility: Pricing Errors



The figure illustrates pricing error plots for five carry trade portfolios (left), ten carry trade portfolios (middle), and five carry and five currency momentum portfolios (right) using $DOL + \Delta CSV$ (upper), and $DOL + \Delta RV + \Delta CSV$ (lower) from 1984 to 2014.

Online Appendix

4.8 Appendix: Alternative Volatility Measures

In this section, we discuss the specifications of alternative volatility measures used in Section 4.5.2. We describe MinRV and MedRV for jump and diffusive volatilities, GARCH-MIDAS model for short and long run volatilities, GARCH and VIX measures for volatility of volatility, and a monthly based measure of cross-sectional volatility.

4.8.1 Alternative Jump and Diffusive Volatilities

Andersen, Dobrev, and Schaumburg (2012) propose two simple and powerful estimators for integrated variance, as alternatives to commonly used bipower variation. Different from bipower variation, which uses the sum of the product of two consecutive returns to capture diffusion part, the minimum and median realized variances use nearest neighbour truncation based on the minimum value of two consecutive returns or the median value of three consecutive returns in order to control for zero returns and noise. The measures are constructed as follow,

$$MinRV_{k,t} = \sqrt{\frac{1}{T}\frac{\pi}{\pi-2}\frac{T_t}{T_t-1}\sum_{\tau=2}^{T_t} Min(|r_{k,\tau}|, |r_{k,\tau-1}|)^2} \quad (4.12)$$

$$MedRV_{k,t} = \sqrt{\frac{1}{T}\frac{\pi}{6-4\sqrt{3}+\pi}\frac{T_t}{T_t-2}\sum_{\tau=3}^{T_t} Med(|r_{k,\tau}|, |r_{k,\tau-1}|, |r_{k,\tau-2}|)^2} \quad (4.13)$$

$$MinJV_{k,t} = Max(RV_{k,t} - MinRV_{k,t}, 0) \quad (4.14)$$

$$MedJV_{k,t} = Max(RV_{k,t} - MedRV_{k,t}, 0) \quad (4.15)$$

where $MinRV_{k,t}$ and $MedRV_{k,t}$ are monthly minimum and median realized volatilities standardized by the number of intra month observations, which are our proxies for diffusive volatilities. $MinJV_{k,t}$ and $MedJV_{k,t}$ are respective measures of jump volatilities. Similar to measures in Section 4.3.2, all measures are constructed at individual currency level for currency k. We then compute the cross-sectional averages to get the global currency market level volatility measures.

4.8.2 Alternative Short and Long Run Volatilities

Engle, Ghysels, and Sohn (2013) introduce the GARCH-MIDAS model. The model suggests that the total variance can be described as the product of a short run component using GARCH(1,1) process and a long run component using a MIDAS filter on realized variance. The model is described as follow,

$$r_{k,\delta,t} = \mu + \sqrt{\tau_{k,\delta,t} g_{k,\delta,t}} \epsilon_{k,\delta,t}, \epsilon_{k,\delta,t} \sim N(0,1) \tag{4.16}$$

$$g_{k,\delta,t} = (1 - \alpha - \beta) + \alpha \frac{(r_{k,\delta-1,t} - \mu)^2}{\tau_{k,\delta-1,t}} + \beta g_{k,\delta-1,t} \tag{4.17}$$

$$\tau_{k,\delta,t} = m + \theta \sum_{j=1}^{J} \phi_j(w) RV_{k,\delta-j,t} \tag{4.18}$$

$$RV_{k,\delta,t} = \sum_{n=1}^{N} r^2_{k,\delta-n,t} \tag{4.19}$$

$$\phi_j(w) = \frac{(1 - \frac{j}{J})^{w-1}}{\sum_{j=1}^{j} (1 - \frac{j}{J})^{w-1}} \tag{4.20}$$

where $r_{k,\delta,t}$ refers to a day δ exchange rate return for exchange rate k in month t, g is the short run volatility component described by a GARCH(1,1), τ is the long run component modelled by rolling window RV with MIDAS filter, we choose $N = 22$ to reflect monthly RV and $J = 125$ for the MIDAS lags, $\phi_j(w)$ is weighting function, we follow the literature and use the beta weighting function. $\Theta = [\mu, \alpha, \beta, \theta, w, m]$ are parameters that to be estimated. We estimate the model on individual exchange rate daily returns, and then compute the cross-sectional averages of τ and g to obtain the global currency market short and long run volatilities.

4.8.3 Alternative Volatility of Volatility

We construct volatility of volatility using both the GARCH and VIX approaches. We first estimate GARCH(1,1) of Bollerslev (1986) on daily individual exchange rate returns.

$$r_{k,\tau} = \mu + \sqrt{h_{k,\tau}}\epsilon_{k,\tau}, \epsilon_{k,\tau} \sim N(0,1) \tag{4.21}$$

$$h_{k,\tau} = \omega + \beta h_{k,\tau-1} + \alpha\epsilon_{k,\tau-1}^2 \tag{4.22}$$

Then we compute monthly volatility and volatility of volatility based on the daily GARCH fitted conditional variance for each currency k.

$$Vol_{k,t}^{GARCH} = \frac{1}{T_t}\sum_{\tau=1}^{T_t}\sigma_{k,\tau}^{GARCH} \tag{4.23}$$

$$VoV_{k,t}^{GARCH} = \sqrt{\frac{1}{T_t}\sum_{\tau=1}^{T_t}(\sigma_{k,\tau}^{GARCH})^2 - (\frac{1}{T}\sum_{\tau=1}^{T_t}\sigma_{k,\tau}^{GARCH})^2} \tag{4.24}$$

where $\sigma_{k,\tau}^{GARCH} = \sqrt{h_{k,\tau}}$, and $h_{k,\tau}$ is GARCH fitted conditional variance for exchange rate k in day τ. We then compute the cross-sectional average for all currencies to get the global currency market volatility and volatility of volatility. We then compute VIX based volatility of volatility. VIX is CBOE volatility index implied from SPX option prices with 1 month maturities constructed in a model free way. Given the daily VIX as a measure of volatility $Vol^{VIX} = VIX$, we can compute volatility of volatility as follows,

$$VoV_t^{VIX} = \sqrt{\frac{1}{T_t}\sum_{\tau=1}^{T_t}(Vol_\tau^{VIX})^2 - (\frac{1}{T_t}\sum_{\tau=1}^{T_t}Vol_\tau^{VIX})^2} \tag{4.25}$$

where VoV_t^{VIX} is month t volatility of volatility based on VIX index.

4.8.4 Alternative Cross-Sectional Volatility

The cross-sectional volatility measure we used in the main analysis is based on daily exchange rate returns. In this part, we construct another measure of cross-sectional volatility based on monthly currency excess returns.

$$CSV_t = \sqrt{\sum_{k=1}^{K}(RX_{k,t} - \bar{RX}_t)^2} \tag{4.26}$$

where $RX_{k,t}$ is month t excess return of currency k, and $\bar{RX}_t$ is the cross-sectional mean of currency excess returns.

Bibliography

Adrian, T., and J. Rosenberg (2008): "Stock Returns and Volatility: Pricing the Short-Run and Long-Run Components of Market Risk," *Journal of Finance*, 63(6), 2997–3030.

Ahmed, S., and G. Valente (2015): "Understanding the Price of Volatility Risk in Carry Trades," *Journal of Banking and Finance*, 57, 118–129.

Ait-Sahalia, Y. (2004): "Disentangling diffusion from jumps," *Journal of Financial Economics*, 74(3), 487–528.

Ait-Sahalia, Y., and J. Jacod (2009): "Testing for Jumps in a Discretely Observed Process," *Annal of Statistics*, 37(1), 184–222.

Amaya, D., P. Christoffersen, K. Jacobs, and A. Vasquez (2011): "Do Realized Skewness and Kurtosis Predict the Cross-Section of Equity Returns?," working paper, School of Economics and Management, University of Aarhus.

Andersen, T., T. Bollerslev, F. Diebold, and P. Labys (2001): "The Distribution of Realized Exchange Rate Volatility," *Journal of the American Statistical Association*, 96, 42–55.

Andersen, T. G., T. Bollerslev, P. F. Christoffersen, and F. X. Diebold (2011): "Financial Risk Measurement for Financial Risk Management," Creates research papers, School of Economics and Management, University of Aarhus.

Andersen, T. G., T. Bollerslev, and F. X. Diebold (2007): "Roughing It Up: Including Jump Components in the Measurement, Modeling, and Forecasting of Return Volatility," *The Review of Economics and Statistics*, 89(4), 701–720.

Andersen, T. G., T. Bollerslev, and D. Dobrev (2007): "No-arbitrage semi-martingale restrictions for continuous-time volatility models subject to leverage effects, jumps and i.i.d. noise: Theory and testable distributional implications," *Journal of Econometrics*, 138(1), 125–180.

Andersen, T. G., T. Bollerslev, and X. Huang (2011): "A reduced form framework for modeling volatility of speculative prices based on realized variation measures," *Journal of Econometrics*, 160(1), 176–189.

Andersen, T. G., T. Bollerslev, and N. Meddahi (2011): "Realized volatility forecasting and market microstructure noise," *Journal of Econometrics*, 160(1), 220–234.

Andersen, T. G., D. Dobrev, and E. Schaumburg (2012): "Jump-Robust Volatility Estimation using Nearest Neighbor Truncation," *Journal of Econometrics*, 169(1), 75–93.

Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang (2006): "The Cross-Section of Volatility and Expected Returns," *Journal of Finance*, 61(1), 259–299.

ANGELIDIS, T., A. SAKKAS, AND N. TESSAROMATIS (2015): "Stock Market Dispersion, the Business Cycle and Expected Factor Returns," *Journal of Banking and Finance*, 57, 118–129.

BACKUS, D., M. CHERNOV, AND I. MARTIN (2011): "Disasters Implied by Equity Index Options," *Journal of Finance*, 66(6), 1969–2012.

BALTUSSEN, G., S. V. BEKKUM, AND B. V. D. GRIENT (2014): "Unknown Unknowns: Uncertainty About Risk and Stock Returns," Discussion paper.

BANDI, F., J. RUSSELL, AND Y. ZHU (2008): "Using High-Frequency Data in Dynamic Portfolio Choice," *Econometric Reviews*, 27(1-3), 163–198.

BANDI, F. M., AND J. R. RUSSELL (2006): "Separating microstructure noise from volatility," *Journal of Financial Economics*, 79(3), 655–692.

BARNDORFF-NIELSEN, O. E., S. KINNEBROCK, AND N. SHEPHARD (2008): "Measuring downside risk-realised semivariance," Creates research papers, School of Economics and Management, University of Aarhus.

BARNDORFF-NIELSEN, O. E., AND N. SHEPHARD (2002): "Estimating quadratic variation using realized variance," *Journal of Applied Econometrics*, 17(5), 457–477.

——— (2004): "Power and Bipower Variation with Stochastic Volatility and Jumps," *Journal of Financial Econometrics*, 2(1), 1–37.

——— (2006): "Impact of jumps on returns and realised variances: econometric analysis of time-deformed Levy processes," *Journal of Econometrics*, 131(1-2), 217–252.

Barndorff-Nielsen, O. E., and A. E. D. Veraart (2013): "Stochastic Volatility of Volatility and Variance Risk Premia," *Journal of Financial Econometrics*, 11, 1–46.

Bollerslev, T. (1986): "Generalized autoregressive conditional heteroskedasticity," *Journal of Econometrics*, 31(3), 307–327.

Bollerslev, T., G. Tauchen, and H. Zhou (2009): "Expected Stock Returns and Variance Risk Premia," *Review of Financial Studies*, 22(11), 4463–4492.

Burnside, C. (2012): "Carry Trade and Risk," in *Handbook of Exchange Rates*, ed. by J. James, I. W. Marsh, and L. Sarno. John Wiley & Sons.

Burnside, C., M. Eichenbaum, and S. Eichenbaum (2011): "Carry Trade and Momentum in Currency Markets," *Annual Review of Financial Economics*, 3(1), 511–535.

Campbell, J. (1993): "Intertemporal Asset Pricing without Consumption Data," *American Economic Review*, 83(3), 487–512.

Cao, X., and Y. Xu (2010): "Long-run Idiosyncratic Volatilities and Cross-sectional Stock Returns," Working papers series, University of Texas at Dallas.

Chen, J. (2003): "Intertemporal CAPM and the Cross Section of Stock Returns," Working papers series, University of California Davis.

Chen, K., S. Hyde, and S.-H. Poon (2010): "Economic Implications of Extraordinary Movements in Stock Markets," Working paper, University of Manchester.

Chen, T.-F., S.-L. Chung, and J.-C. Lin (2014): "Volatility-of-Volatility Risks and Asset Prices," Working papers series, University of Taiwan.

Chernov, M., J. Graveline, and I. Zviadadze (2015): "Crash Risks in Currency Returns," Working papers series, UCLA.

Chernov, M., A. Ronald Gallant, E. Ghysels, and G. Tauchen (2003): "Alternative models for stock price dynamics," *Journal of Econometrics*, 116(1-2), 225–257.

Chiriac, R., and V. Voev (2011): "Modelling and forecasting multivariate realized volatility," *Journal of Applied Econometrics*, 26(6), 922–947.

Christoffersen, P., K. Jacobs, and C. Ornthanalai (2012): "Dynamicjumpintensities and risk premiums: Evidence from S&P500 returns and options," *Journal of Financial Economics*, 106(3), 447–472.

Christoffersen, P., K. Jacobs, C. Ornthanalai, and Y. Wang (2008): "Option valuation with long-run and short-run volatility components," *Journal of Financial Economics*, 90(3), 272–297.

Cochrane, J. (2005): *Asset Pricing*. Princeton University Press, Princeton, NJ.

Corradi, V., W. Distaso, and M. Fernandes (2013): "Testing for jump spillovers without testing for jumps," Discussion paper, Imperial College London.

Corsi, F. (2009): "A Simple Approximate Long-Memory Model of Realized Volatility," *Journal of Financial Econometrics*, 7(2), 174–196.

Corsi, F., D. Pirino, and R. Reno (2010): "Threshold bipower variation and the impact of jumps on volatility forecasting," *Journal of Econometrics*, 159(2), 276–288.

Cremers, M., M. Halling, and D. Weinbaum (2015): "Aggregate Jump and Volatility Risks in the Cross Section of Stock Returns," *Journal of Finance*, 70(2), 577–614.

DeMiguel, V., L. DeMiguel, and R. Uppal (2009): "Optimal Versus Naive Diversification: How Inefficient is the 1-N Portfolio Strategy?," *Review of Financial Studies*, 22(5), 1915–1953.

DeMiguel, V., Y. Plyakha, R. Uppal, and G. Vilkov (2013): "Improving Portfolio Selection Using Option-Implied Volatility and Skewness," *Journal of Financial and Quantitative Analysis*, forthcoming.

Dew-Becker, I., S. Giglio, A. Le, and M. Rodriguez (2013): "The Price of Variance Risk," Working papers series, University of Chicago.

Diebold, F. X., and R. S. Mariano (1995): "Comparing Predictive Accuracy," *Journal of Business & Economic Statistics*, 13(3), 253–63.

Dobrynskaya, V. (2014): "Downside Market Risk of Carry Trades," *Review of Finance*, 18(5), 1885–1913.

Duan, J.-C., P. Ritchken, and Z. Sun (2006): "Approximating Garch-Jump Models, Jump-Diffusion Processes, And Option Pricing," *Mathematical Finance*, 16(1), 21–52.

Dumitru, A., and G. Urga (2012): "Identifying Jumps in Financial Assets: a Comparison between Nonparametric Jump Tests," *Journal of Business and Economic Statistics*, 30(02), 242–255.

ENGLE, R. (2002): "Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models," *Journal of Business & Economic Statistics*, 20(3), 339–50.

ENGLE, R., AND G. LEE (1999): *A Permanent and Transitory Component Model of Stock Return Volatility*pp. 475–497, Cointegration, Causality, and Forecasting: A Festschrift in Honor of Clive W. J.Granger. Oxford University Press.

ENGLE, R. F., AND R. COLACITO (2006): "Testing and Valuing Dynamic Correlations for Asset Allocation," *Journal of Business and Economic Statistics*, 24, 238–253.

ENGLE, R. F., E. GHYSELS, AND B. SOHN (2013): "Stock Market Volatility and Macroeconomic Fundamentals," *The Review of Economics and Statistics*, 95(3), 776–797.

ERAKER, B. (2004): "Do Stock Prices and Volatility Jump? Reconciling Evidence from Spot and Option Prices," *Journal of Finance*, 59(3), 1367–1404.

ERAKER, B., M. JOHANNES, AND N. POLSON (2003): "The Impact of Jumps in Volatility and Returns," *Journal of Finance*, 58(3), 1269–1300.

FARHI, E., S. FRAIBERGER, X. GABAIX, R. RANCIERE, AND A. VERDELHAN (2015): "Crash Risks in Currency Markets," Working papers series, Harvard University.

FLEMING, J., C. KIRBY, AND B. OSTDIEK (2001): "The Economic Value of Volatility Timing," *Journal of Finance*, 56(1), 329–352.

——— (2003): “The economic value of volatility timing using "realized" volatility,” *Journal of Financial Economics*, 67(3), 473–509.

Froot, K. A., and R. H. Thaler (1990): “Foreign Exchange,” *Journal of Economic Perspectives*, 4(3), 179–92.

Garcia, R., D. Mantilla-Garcia, and L. Martellini (2014): “A Model Free Measure of Aggregate Idiosycratic Volatility and the Prediction of Market Returns,” *Journal of Financial and Quantitative Analysis*, forthcoming.

Halbleib, R., and V. Voev (2012): “Forecasting Covariance Matrices: A Mixed Frequency Approach,” Discussion paper, Department of Economics, University of Konstanz.

Harris, R. D., E. Stoja, and F. Yilmaz (2011): “A cyclical model of exchange rate volatility,” *Journal of Banking and Finance*, 35(11), 3055–3064.

Hautsch, N., L. Kyj, and P. Malec (2013): “Do High-Frequency Data Improve High-Dimensional Portfolio Allocations,” *Journal of Applied Econometrics*, forthcoming.

Hodrick, R. (1987): “The Empirical Evidence on the Efficiency of Forward and Futures Foreign Exchange Markets,” in *Fundamentals of Pure and Applied Economics*, ed. by J. Lesourne, and H. Sonnenschein. Harwood Academic Publishers.

Hodrick, R. J., and E. C. Prescott (1997): “Postwar U.S. Business Cycles: An Empirical Investigation,” *Journal of Money, Credit and Banking*, 29(1), 1–16.

Huang, D., and I. Shaliastovich (2014): “Volatility-of-Volatility Risks,” Working papers series, University of Pennsylvania.

Huang, X., and G. Tauchen (2005): “The Relative Contribution of Jumps to Total Price Variance,” *Journal of Financial Econometrics*, 3(4), 456–499.

Jacobs, H., S. Muller, and M. Weber (2014): “How should individual investors diversify? An empirical evaluation of alternative asset allocation policies ,” *Journal of Financial Markets*, 19, 62–85.

Jiang, G., and T. Yao (2013): “Stock Price Jumps and Cross-Sectional Return Predictability,” *Journal of Financial and Quantitative Analysis*, 48(5), 1519–1544.

Jiang, G. J., and R. C. Oomen (2008): “Testing for jumps when asset prices are observed with noise-a "swap variance" approach,” *Journal of Econometrics*, 144(2), 352–370.

J.P.Morgan/Reuters (1996): “RiskMetrics-Technical Document,” .

Jurek, J. (2014): “Crash-neutral currency carry trades,” *Journal of Financial Economics*, 113(1), 325–347.

Jurek, J., and Z. Xu (2014): “Option-Implied Currency Risk Premia,” Working papers series, University of Pennsylvania.

Lee, S. S., and P. A. Mykland (2008): “Jumps in Financial Markets: A New Nonparametric Test and Jump Dynamics,” *Review of Financial Studies*, 21(6), 2535–2563.

Lettau, M., M. Maggiori, and M. Weber (2014): "Conditional Risk Premia in Currency Markets and Other Asset Classes," *Journal of Financial Economics*, 114(2), 197–225.

Liu, J., F. A. Longstaff, and J. Pan (2003): "Dynamic Asset Allocation with Event Risk," *Journal of Finance*, 58(1), 231–259.

Liu, Q. (2009): "On portfolio optimization: How and when do we benefit from high-frequency data?," *Journal of Applied Econometrics*, 24(4), 560–582.

Lustig, H., N. Roussanov, and A. Verdelhan (2011): "Common Risk Factors in Currency Markets," *Review of Financial Studies*, 24(11), 3731–3777.

Maheu, J. M., and T. H. McCurdy (2004): "News Arrival, Jump Dynamics, and Volatility Components for Individual Stock Returns," *Journal of Finance*, 59(2), 755–793.

Maheu, J. M., T. H. McCurdy, and X. Zhao (2012): "Do Jumps Contribute to the Dynamics of the Equity Premium?," Working paper, University of Toronto.

Maio, P. (2015): "Return dispersion and the predictability of stock Returns," Working papers series, Hanken School of Economics.

Mancini, C. (2009): "Non-parametric Threshold Estimation for Models with Stochastic Diffusion Coefficient and Jumps," *Scandinavian Journal of Statistics*, 36(2), 270–296.

Mancini, L., A. Ranaldo, and J. Wrampelmeyer (2013): "Liquidity in the Foreign Exchange Market: Measurement, Commonality, and Risk Premiums," *Journal of Finance*, 68(5), 1805–1841.

Marquering, W., and M. Verbeek (2004): "The Economic Value of Predicting Stock Index Returns and Volatility," *Journal of Financial and Quantitative Analysis*, 39(02), 407–429.

McCracken, M., and G. Valente (2013): "Testing Economic Value of Asset Return Predictability," working paper, Federal Reserve Bank of St. Louis.

Menkhoff, L., L. Sarno, M. Schmeling, and A. Schrimpf (2012a): "Carry Trades and Global Foreign Exchange Volatility," *Journal of Finance*, 67(2), 681–718.

——— (2012b): "Currency Momentum Strategies," *Journal of Financial Economics*, 106(3), 660–684.

Merton, R. C. (1973): "An Intertemporal Capital Asset Pricing Model," *Econometrica*, 41(5), 867–87.

Merton, R. C. (1976): "Option pricing when underlying stock returns are discontinuous," *Journal of Financial Economics*, 3(1-2), 125–144.

Neuberger, A. (1994): "The Log Contract," *Journal of Portfolio Management*, 20(2), 74–80.

Newey, W. K., and K. D. West (1987): "A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica*, 55(3), 703–08.

Park, Y.-H. (2013): "Volatility of Volatility and Tail Risk Premiums," Working papers series, Federal Reserve Board.

Patton, A., and K. Sheppard (2013): “Good Volatility, Bad Volatility: Signed Jumps and the Persistence of Volatility,” working paper, Duke University.

Podolskij, M., and D. Ziggel (2010): “New tests for jumps in semimartingale models,” *Statistical Inference for Stochastic Processes*, 13(1), 15–41.

Pukthuanthong, K., and R. Roll (2014): “International Correlated Jumps,” *Review of Asset Pricing Studies*, forthcoming.

Rafferty, B. (2012): “Currency Returns, Skewness and Crash Risk,” Working papers series, Duke University.

Sarno, L., and M. P. Taylor (2002): *The Economics of Exchange Rates*. Cambridge University Press, Cambridge, United Kingdom.

Shanken, J. (1992): “On the Estimation of Beta-Pricing Models,” *Review of Financial Studies*, 5(1), 1–33.

Sohn, B. (2014): “Stock Market Volatility and Trading Strategy Based Factors,” Working papers series, Korea University.

Song, Z. (2012): “Expected VIX Option Returns,” Working papers series, Federal Reserve Board.

Song, Z., and D. Xiu (2014): “A Tale of Two Option Markets: Pricing Kernels and Volatility Risk,” Working papers series, University of Chicago.

Tauchen, G., and H. Zhou (2011): “Realized jumps on financial markets and predicting credit spreads,” *Journal of Econometrics*, 160(1), 102–118.

Taylor, M. P. (1995): “The Economics of Exchange Rates,” *Journal of Economic Literature*, 33(1), 13–47.

Theodosiou, M., and F. Zikes (2011): "A Comprehensive Comparison of Nonparametric Tests for Jumps in Asset Prices," Working paper, Imerial College London.

Wang, R., C. Kirby, and S. Clark (2013): "Volatility of Volatility, Expected Stock Return and Variance Risk Premium," Working papers series, University of North Carolina at Charlotte.

Yan, S. (2011): "Jump risk, stock returns, and slope of implied volatility smile," *Journal of Financial Economics*, 99(1), 216–233.