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# Single transit candidates from K2: detection and period estimation

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## ABSTRACT

Photometric surveys such as *Kepler* have the precision to identify exoplanet and eclipsing binary candidates from only a single transit. K2, with its 75 d campaign duration, is ideally suited to detect significant numbers of single-eclipsing objects. Here we develop a Bayesian transit-fitting tool (‘*Namaste*: An Mcmc Analysis of Single Transit Exoplanets’) to extract orbital information from single transit events. We achieve favourable results testing this technique on known *Kepler* planets, and apply the technique to seven candidates identified from a targeted search of K2 campaigns 1, 2 and 3. We find EPIC203311200 to host an excellent exoplanet candidate with a period, assuming zero eccentricity, of  $540^{+410}_{-230}$  d and a radius of  $0.51 \pm 0.05R_{\text{Jup}}$ . We also find six further transit candidates for which more follow-up is required to determine a planetary origin. Such a technique could be used in the future with *TESS*, *PLATO* and ground-based photometric surveys such as NGTS, potentially allowing the detection of planets in reach of confirmation by *Gaia*.

**Key words:** methods: analytical – techniques: photometric – planets and satellites: detection – stars, planets and satellites: general – planetary systems.

## 1 INTRODUCTION

Wide-field survey telescopes have for more than a decade searched for the repeated transits of exoplanets, detecting thousands of candidates and confirmed planets. With photometric precision orders of magnitudes better than on the ground, the *Kepler* mission (Borucki et al. 2010) has contributed most to this growing field, from sub-Earth-radius worlds (Barclay et al. 2013) to long-period gas giants (Wang et al. 2013).

In 2014, *Kepler* was repurposed as K2 (Howell et al. 2014) which, due to engineering constraints, observes multiple fields in the ecliptic on 75 d campaigns. Although reduced pointing stability limits the photometric precision of K2, many stars have been observed with precision on the order of 100 ppm per half hour cadence. More than 40 planet candidates have so far been detected from the first three campaigns of K2 (e.g. Foreman-Mackey et al. 2015), with orbital periods up to 50 d. More than a dozen of these systems have subsequently been validated (e.g. Armstrong et al. 2015b; Montet et al. 2015).

The occurrence rates of transiting planets with periods on the order of months (themselves derived by *Kepler*, e.g. Fressin et al. 2013) suggest that a handful of longer period planets should be

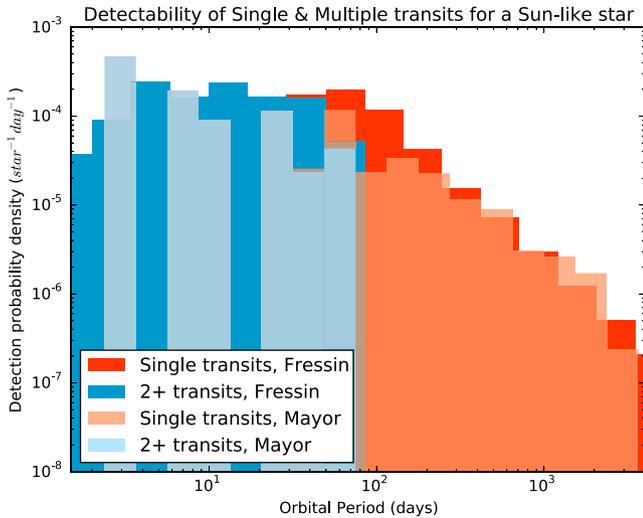
detected per K2 campaign. The reduced mission duration of 75 d (compared to 1400 in the primary mission) means that such planets are likely to only transit once. Planets with transit depths over  $\sim 1$  mmag offer the potential of discovery in just a single event. Such signals may then constitute strong planetary candidates with undefined orbital parameters, similar to those detected by microlensing surveys (e.g. Bennett & Rhie 1996).

One such planet has previously been detected and confirmed from K2. K2-1 b was initially spotted in a single transit during 6.5 d of engineering data (Vanderburg et al. 2015). Subsequent follow-up with both spectroscopy (HARPS) and photometry (MOST) determined that the planet was a  $2.5R_{\oplus}$  Earth on a 9.12 d orbit. However, in subsequent campaigns, planets capable of being detected in a single transit are likely to have orbital periods of 40 d or more, making follow-up more challenging.

The detection of single transits was speculated upon prior to the launch of *Kepler* (Yee & Gaudi 2008) and 17 long-period planet candidates were subsequently detected from single transits during the 4-yr primary mission (Wang et al. 2015). Orbital periods were estimated for these candidates using a non-specific transit model, and subsequent follow-up allowed the probabilistic validation of three systems.

Here we develop the Bayesian fitting tool *Namaste* to estimate orbital parameters specifically for single transiting planets. We show that, provided the host star can be characterized, the information

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**Figure 1.** Comparing the detectability of a planet around a sun-like star with K2. Blue: multiple transits detectable. Red: only single transits detectable.

contained within a single transit allows an accurate estimate of a planet's orbital period. We obtain period estimates for six *Kepler* planets as proof of concept and apply the technique to seven new candidate events detected in the first three full K2 campaigns.

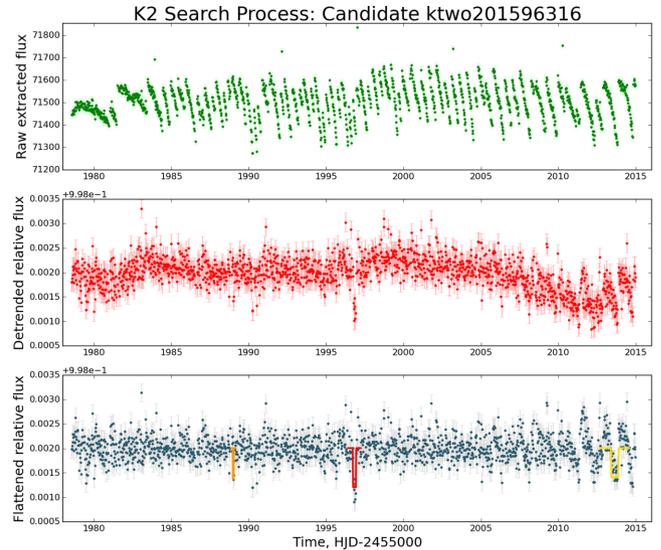
### 1.1 Single transit event occurrence rate

From its initial sampler of  $\sim 150\,000$  stars, *Kepler* detected 101 planets and 828 planet candidates with periods longer than 50 d. 53 per cent of these exhibit deep enough transits to allow their detection from a single transit ( $>5\sigma$ ). We would therefore expect, with  $\sim 40\,000$  stars now observed by K2, to detect substantial numbers of such planets.

A simple analysis, using *Kepler* occurrence rates from 0 to 85 d (Fressin et al. 2013) and assuming a flat distribution in  $\ln(P)$  beyond 85 d, suggests that 15 per cent of FGK stars should have a large Neptune or Jupiter on  $<3000$  d orbits. This is similar to the giant planet occurrence rates found by (Mayor et al. 2011), with  $14 \pm 2$  per cent of FGK stars with planets larger than  $>50M_{\oplus}$ . Accounting for transit probability (scaled with stellar radius and semimajor axis by  $\frac{R_s}{a}$ ) and timing probability (scaled with observation campaign duration and orbital period by  $\frac{t_{\text{obs}}}{P}$ ) gives detectable multi- and single-transiting planets around 0.15 and 0.03 per cent of stars from Fressin et al. (2013) and 0.09 and 0.02 per cent from Mayor et al. (2011) (Fig. 1). Hence, one in every few thousand FGK stars observed by K2 should have detectable single transits.

The decreasing probabilities with orbital distance also suggest that high-precision, shorter duration surveys such as TESS (Ricker et al. 2015) or NGTS (Wheatley et al. 2013) could detect monotransits in substantial numbers. The 28 d survey duration for the majority of the Transiting Exoplanet Survey Satellite (TESS) field, for example, would likely find monotransiting giant planets around  $>0.04$  per cent of FGK stars, potentially improving giant planet yield by as much as 50 per cent.

Occurrence rate estimations of this regime are, at present, extremely poorly constrained. Hence, the detection and subsequent follow-up of monotransiting planets in this regime will allow improved occurrence rates for this relatively unexplored parameter space.



**Figure 2.** Example of the detection process for validated, multi-transiting planet EPIC201596316 (Montet et al. 2015). (a) Raw aperture-extracted flux; (b) relative flux after de-trending for pixel motion; (c) transit-searched and trend-removed with detections in red/orange.

## 2 METHODS

### 2.1 Detrending

Three campaigns of K2 target pixel files were obtained from the Mikulski Archive for Space Telescopes (MAST). Limiting our analysis to objects classified as ‘STARS’ we performed aperture photometry on the target object. Aperture sizes were varied according to brightness, with radii of 3, 4, 5 and 12 pixels used for *Kepler* magnitudes bins with boundaries 16, 13 and 10. The extracted flux was then background-subtracted, with background RMS added in quadrature to the flux errors.

To remove the majority of systematics in K2 light curves, which are dominated by pointing drift, we independently developed a detrending method similar to Vanderburg & Johnson (2014) to remove all noise correlated with shifting target position regardless of its source. To do this, we compute centroid position for each timestamp, cutting the largest pixel shifts (eg. during thruster firings). We then create a 2D surface of raw flux against  $x$  and  $y$  position. By binning this to an evenly spaced  $10 \times 10$  grid (discarding bins with fewer than three points) and interpolating the median fluxes with SCIPY’s griddata function (Jones et al. 2001) we created a smooth surface map of the variation due to centroid shifts. This could then be divided out of the extracted light curve, with the result of significantly decreasing the RMS error of the light curve. Variations not related to pointing drift (eg. long-time-scale flux drifts) are not removed by this technique and we note that, in many cases, some systematic noise remains due to other instrumental effects. This method is described in detail in Armstrong et al. (2015a) and demonstrated in Fig. 2.<sup>1</sup>

### 2.2 Transit search

Long-duration variability was removed from the detrended light curves by fitting third-order polynomials to 2d windows either side

<sup>1</sup> Detrended light curves are publicly available on MAST at <https://archive.stsci.edu/prepds/k2varcat/>

of an untouched 4-h central window. The fit for each was iterated 20 times, with points  $>5\sigma$  from the best-fitting excluded each time. This method is further explained in Armstrong et al. (2014). The resulting polynomial fit was then applied to the central 4-h windows, thus avoiding artificially reducing transit depths.

A search for transit signals was then performed on each light curve. Least-square minimization was used to fit pre-generated transit models (developed from the Mandel & Agol 2002 small planet, quadratic limb-darkened model) to a window of the light curve 6 times the transit duration ( $T_D$ ). This was repeated for transit models with durations from 1.5 to 24 h in increasing  $T_D$  steps of 25 per cent, with each fitting window shifted by 25 per cent of the targeted  $T_D$  each time. Well-fitting models with depths greater than  $2.5\sigma$  from the out-of-transit RMS were recorded. A combination of highest SNR and lowest reduced  $\chi^2$  value was used to select the best transit fit when multiple durations and transit centres were flagged on the same region of light curve.

To reduce false positives from thruster firings, the SNR of detections with above-average numbers of events occurring concurrently were suppressed. Light curves were then sorted by the total SNR of detected signals and then manually ‘eyeballed’ by at least two independent observers, leaving only the best candidate events.

For Campaigns 2 and 3, we limited the search to stars brighter than 13th magnitude, a threshold beyond which RV follow-up is impractical.

### 2.3 Transit fitting – *Namaste*

Modelling transit light curves has been explored by numerous authors (e.g. Mandel & Agol 2002; Seager & Mallen-Ornelas 2003; Collier-Cameron et al. 2007), but the majority of full transit models rely on knowledge of the period (often scaled to transit duration) or semimajor axis (scaled with stellar radii). In the case of a single transit, these approximations cannot be used. Instead, we develop ‘*Namaste*: An Mcmc Analysis of Single Transiting Exoplanets’ (hereafter, *Namaste*<sup>2</sup>). This technique estimates a planetary velocity scaled to stellar radius ( $v'$ ) in place of a velocity calculated from the planetary period. This velocity can be geometrically defined from impact parameter ( $b$ ), planet-to-star ratio ( $R_p/R_*$ ) and transit duration ( $T_D$ ) (equation 1)

$$v' \equiv \frac{v_{pl}}{R_*} = \frac{2\sqrt{(1 + R_p/R_*)^2 - b^2}}{T_D} \quad (1)$$

The scaled velocity of a small planet crossing the centre of the stellar disc ( $b = 0$ ) is therefore twice the inverse of the transit duration ( $\sim 2/T_D$ ).

Velocity, impact parameter and radii ratio can be estimated geometrically from fitting the transit shape. We adapt the transit fitting regimes of Ian Crossfield<sup>3</sup> and the Monte Carlo Markov chain implementation *emcee* (Foreman-Mackey et al. 2013), to estimate posterior probability distributions for each monotransit signal. Quadratic limb darkening parameters adapted for the *Kepler* bandpass were interpolated from stellar temperature (Sing 2010). A Gaussian prior distribution was applied to these limb darkening parameters with values and errors set from the temperature probability distribution. Errors were set to the RMS of the four different colour estimates to a minimum of 150 K.

<sup>2</sup> Publicly available at <https://github.com/hposborn/namaste>.

<sup>3</sup> Accessed from <http://www.lpl.arizona.edu/ianc/python/>.

If eccentricity is assumed to be zero, a circular planetary period ( $P_{\text{circ}}$ ) can be estimated from the scaled transit velocity ( $v'$ ) and stellar density ( $\rho_*$ ) using Kepler’s laws:

$$P_{\text{circ}} = \frac{8\pi^2 G}{3} \frac{\rho_*}{v'^3} = 2\pi \frac{g}{R_* v'^3} \quad (2)$$

Longer period orbits (and hence lower velocity fits) are probabilistically less likely due to transit probability ( $p_{\text{tr}} \approx \frac{R_p}{a_{\text{pl}}} \approx \frac{v'}{R_*}$ ). We discourage longer period fits (and encourage faster velocity fits) with a linear prior on transit velocity. In the case of multi-planet systems, the probability of a further planet transiting does not simply scale with transit probability  $R_p/a$ , as co-planar orbits are favoured. Hence, in these cases, the forcing of fits to shorter orbits by a linear prior may not be valid. However, the increase in transit probability of a planet at distance  $x$  given transiting exoplanets on orbits  $y$ ,  $z$ , etc is a complex problem beyond the scope of this work. The non-detection of subsequent transits in the light curve can also be used to set a lower limit on the orbital period, and hence an upper limit on velocity. We do not apply this technique in order to produce transit fits fully independent of stellar parameters (e.g. density).

The impact parameter was limited to the range  $-1.2$  to  $1.2$  to avoid walkers building up at  $b = 0$ . Planet-to-star radius ratios were limited to 0.25, as above this the assumption that the transiting object is fully opaque and covering a uniform region of stellar surface breaks down.

Velocity, just like the transit duration scaled to period, is directly linked to stellar density (Seager & Mallen-Ornelas 2003). For the best constrained transit models, stellar density is likely to prove the largest uncertainty. Characterizing the star, therefore, is key to estimate orbital period. Such characterization is best performed with asteroseismology, or less accurately, with spectral fitting.

Large radii planets which spend longer crossing the rim of the stellar disc are most suitable to *Namaste* fitting as the impact parameter can be more easily distinguished. Price et al. (2015) showed that for smaller and lower signal-to-noise planet transits the uncertainty on impact parameter increases linearly, causing poor determination of perpendicular velocity and therefore eccentricity. For less well-defined fits, parameters such as impact parameter, planet-to-star ratio and velocity become correlated, as can be seen in their posterior distributions of Fig. 3. This is especially true for eclipses that cannot be constrained to an impact parameter less than 1.0. In these cases the fit cannot distinguish between planetary, high-velocity disc-crossing transits and lower velocity grazing eclipses. As impact parameter increases beyond 1.0, velocity stabilizes to a minimum value determined by transit duration. However, even for correlated and non-Gaussian parameters, *Namaste* allows us to put probabilistic constraints on the transit fit.

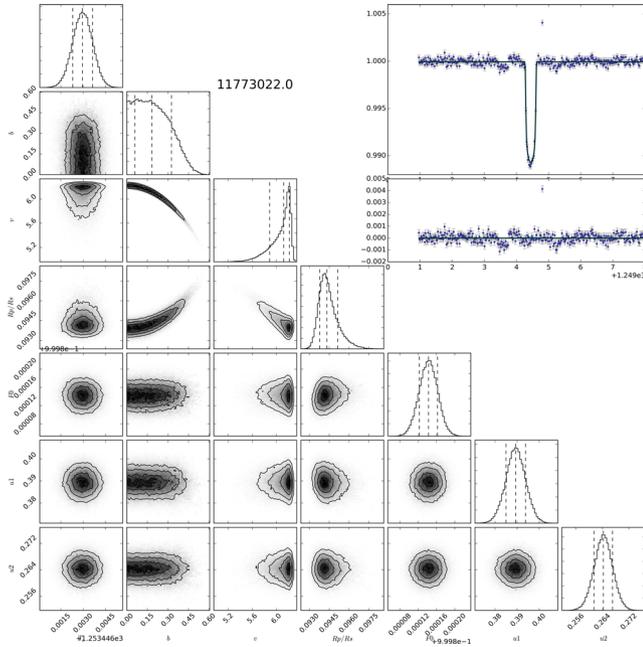
An example of the resulting posterior distributions between parameters (along with model fit and residuals) can be seen in Fig. 3

#### 2.3.1 Eccentricity

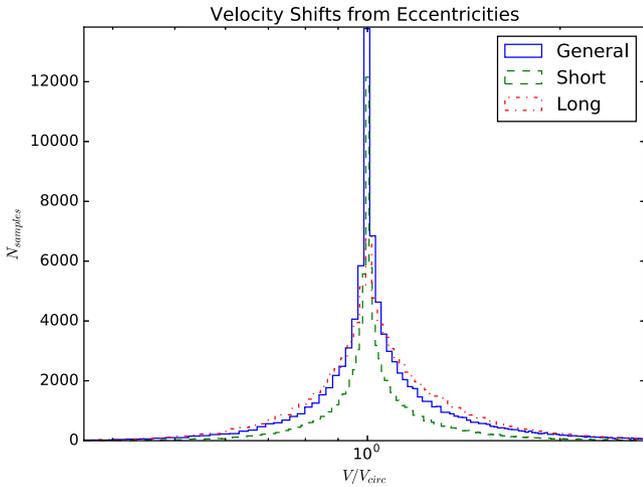
For exoplanets on non-circular orbits, the circular velocity estimated by *Namaste* ( $v_{\text{circ}}$ ) depends strongly on eccentricity and the argument of periastron (equation 3, Barnes 2007).

$$V_\theta = V_{\text{circ}} \frac{1 + e \cos \theta}{\sqrt{1 - e^2}} \quad (3)$$

As the solid angle swept out by an eccentric planet’s shadow is greater than that from an equivalent circular orbit, eccentric planets are also more likely to transit. Transit probability is especially raised near periastron, suggesting circular period estimates will on average



**Figure 3.** An example of the posterior distribution produced by *Namaste*. In this case for Kepler-51 d (KIC11773022). Triangle plots from all fits can be found in Appendix A.



**Figure 4.** Distribution of the ratio of true velocity and circular velocity for three different eccentricity distributions from Kipping (2013).

underestimate the true period. Kipping (2013) used RV planets to study the distribution of exoplanet eccentricities and showed that close-in planets (defined as  $P < 382$ d) have a more circular distribution than long-period planets. We use these distributions to study eccentricity’s effect on transit velocity (Fig. 4). We find that eccentricity increases the median velocity and its  $1 - \sigma$  confidence intervals by  $1.3^{+21}_{-7}$  per cent in the short case, and by  $3^{+35}_{-9}$  per cent in the general case.

Van Eylen & Albrecht (2015) studied the discrepancy between *Kepler* planet densities determined by asteroseismology and those found from transit duration to determine their eccentricities. This method is analogous to the comparison of true period with those from single transit fits, and suggests *Namaste* could be useful for determining the eccentricities of polytransiting planets.

In this study, we note that eccentricity can add significant uncertainty to our results, but limit ourselves to estimations of circular periods ( $P_{\text{circ}}$ ) which are good approximations for the majority of cases. For the short-period regime, two-thirds of planets orbit with eccentricities less than 0.2, causing substantially lower increased uncertainties. Hence, in the majority of cases, the small increase in velocity uncertainty is negligible compared to the large uncertainties from stellar density.

## 2.4 Stellar parameter fits

Where stellar parameters from spectra are unavailable, we use photometric colours to approximate the temperatures, masses and radii of EPIC stars. The majority of K2 stars have broad-band photometric measurements in both the visible (e.g. Tycho B and V bands from Høg et al. 2000) and infrared [e.g. 2MASS J, H, K data from Skrutskie et al. (2006)]. From the four independent colours derived from these magnitudes, stellar temperatures can be estimated (Fitzgerald 1970). This relationship assumes that no bright companions are present, which can only be confirmed with more advanced follow-up. From this temperature, and making the assumption that the star is on the main sequence, stellar models allow a stellar radius and mass to be estimated (Torres, Andersen & Giménez 2010). Temperature uncertainty was estimated from the RMS of the temperature from each colour estimate, or 150 K if the RMS was smaller than this value. Mass and radius were taken from these temperature uncertainties down to a minimum relative error of 10 per cent.

## 3 RESULTS

### 3.1 Application to known *Kepler* systems

To test the fitting of *Namaste*, we applied it to single transits from the light curves of six long-period *Kepler* planets and KOI candidates. Fig. 5 shows the *Namaste* model fits to these transits and Table 1 gives the output parameters compared to their published values.

### 3.2 Application to K2 single transit candidates

We applied *Namaste* to seven potential single transit events found in K2. Fits are shown in Fig. 6, fit parameters in Table 2 and full posterior distributions in Appendix A2.

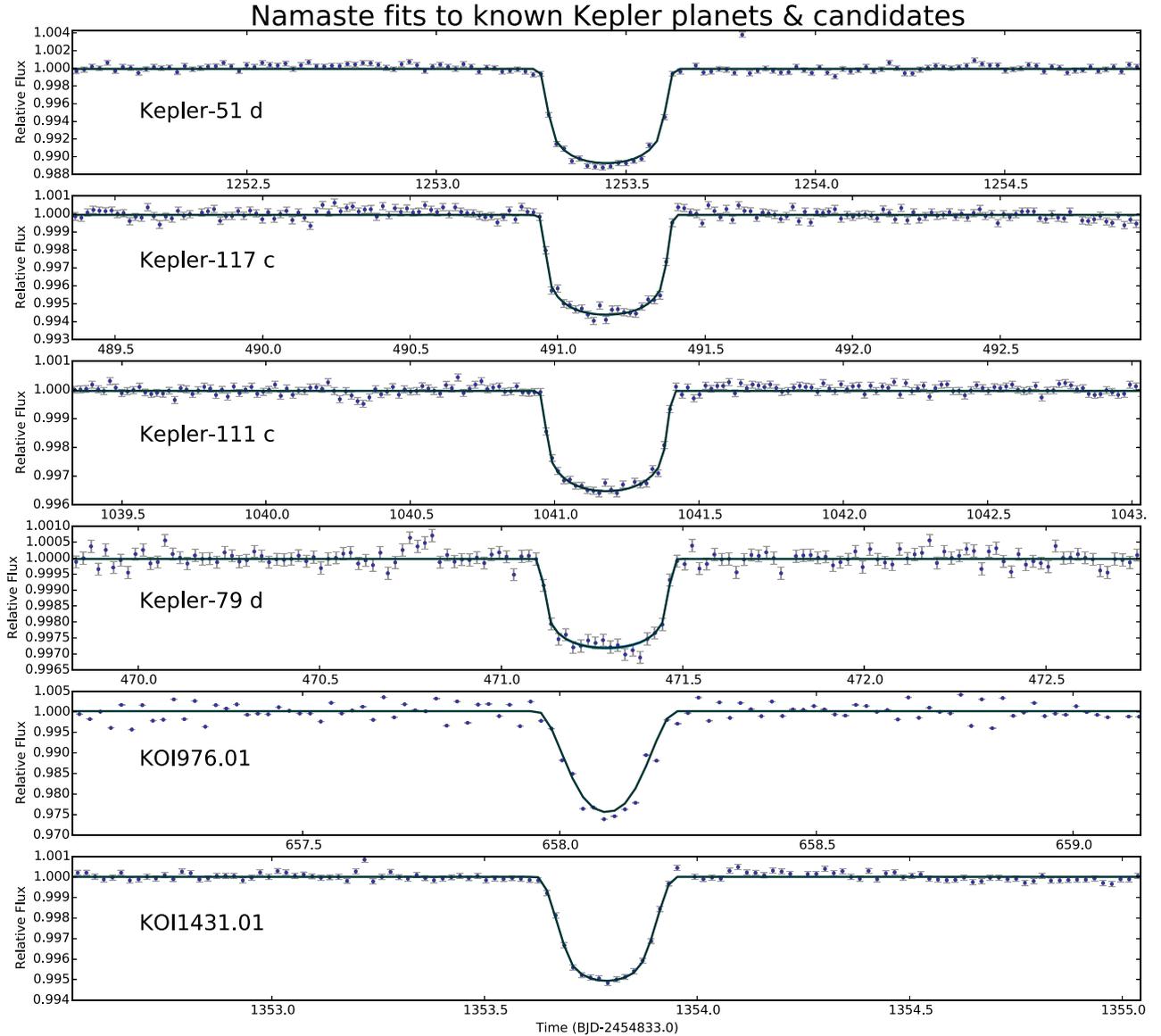
## 4 DISCUSSION

### 4.1 Known *Kepler* planets and candidates

(i) **Kepler-51 d**: Planetary velocity, planet-to-star ratio and impact parameter are all well-constrained by this fit. Well-derived stellar density from Masuda (2014) also gives period errors on the order of only 10 per cent, and within  $1\sigma$  of the true period of 130.2 d. The small discrepancy between true and estimated periods is likely from the overestimation of impact parameter ( $b_{\text{est}} = 0.18$ ;  $b_{\text{true}} = 0.094$ ). This further suggests Kepler-51 d is on a circular orbit.

(ii) **Kepler-117 c**: *Namaste* fits show good agreement to the published parameters, with all parameters within error bars. Well-constrained stellar parameters from Bruno et al. (2015) produce relatively small errors on the estimated period of 53 d, which is within 5 per cent of the true period of 50.7 d.

(iii) **Kepler-111 c**: This multiplanet system, validated by Rowe et al. (2014), has less well-constrained parameters than the other



**Figure 5.** Six long-period *Kepler* planets with Namaste fits. Best-fitting models are in black, while  $1 - \sigma$  error regions are in blue.  $x$ -axis is scaled to eight transit durations, whereas the  $y$ -axis is unconstrained. Full posterior distributions and fits in Figs A1–A6.

*Kepler* test cases, with a 20 per cent density error. This gives wide errors on the estimated period of  $240_{-90}^{+130}$  d, although the true period of 224.8 d sits well within this distribution, and suggests density uncertainties may be overestimated.

(iv) **Kepler-79 d**: An ultralow density gas giant, Kepler-79 is a well-constrained system with masses from TTVs (Jontof-Hutter et al. 2014). In this case, impact parameter is inaccurately fitted, leading to an overestimated period greater than  $1\sigma$  from the true value. The non-zero eccentricity suggested for this planet may also cause increased discrepancy.

(v) **KOI976.01**: Despite being on the *Kepler* candidate list, KOI976 is an eclipsing binary with a much larger radius than would be expected from an exoplanet ( $8_{-1}^{+7} R_{\text{Jup}}$ ). However, this is a good proof of concept for this technique on low-mass eclipsing stellar (or brown dwarf) companions, for which the period estimates are still valid. The long duration of the eclipse gives extremely well-fitting parameters, with output parameters such as velocity accurate to one part in 1000. However, these disagree with the published KOI cat-

alogue values in many cases, especially impact parameter, which may be due to the eclipsing body being self-luminous. This leads to an overestimated velocity and underestimated period of  $25_{-17}^{+100}$  d, although the true period is well within the error bars.

(vi) **KOI1431.01**: This candidate shows the most Gaussian and closest period approximation of all the fits, with a median period within 2 per cent of the true value. However, poor (and likely overestimated) stellar density constraints give large error bars of  $\sim \pm 100$  d

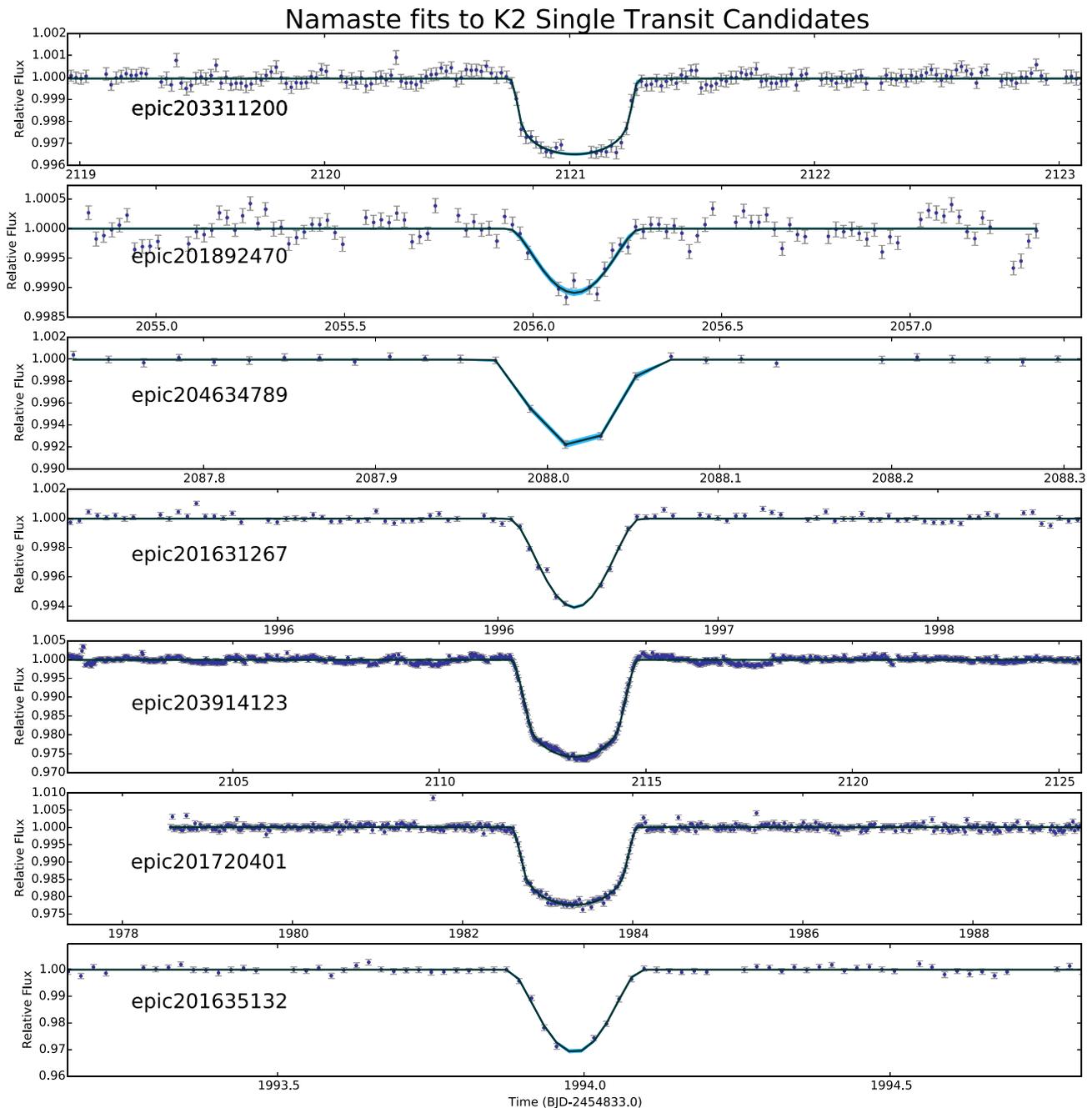
#### 4.2 Testing with published *Kepler* planet parameters

Initially, we used transit data for 102 confirmed *Kepler* planets with stellar radius and mass measurements available from the NASA Exoplanet Archive (Akeson et al. 2013).<sup>4</sup> From transit duration,

<sup>4</sup> <http://exoplanetarchive.ipac.caltech.edu/>

**Table 1.** Nmaaste results for four *Kepler* planets and two KOI candidates, compared with published parameters from traditional analyses. See Appendix A (Figs A1–A6) for posterior distributions.

	Kepler-51 d	Kepler-117 c	Kepler-111 c	Kepler-79 d	KOI976.01	KOI1431.01
KIC	11773022	10723750	8559644	8394721	3441784	11075279
$T_{\text{cent}}(\text{d})$	$1253.4490 \pm 0.0006$	$491.16605 \pm 0.0009$	$1041.17996 \pm 9 \times 10^{-4}$	$471.290 \pm 0.001$	$658.09145 \pm 2 \times 10^{-5}$	$1353.7881 \pm 6.6^{-4}$
$T_{\text{cent, true}}$	$1253.454 \pm 0.05$	$491.164 \pm 0.03$	$1041.210 \pm 0.07$	$471.290 \pm 0.04$	$658.093 \pm 0.05$	$1353.788 \pm 0.06$
$b$	$0.18^{+0.13}_{-0.11}$	$0.32^{+0.12}_{-0.19}$	$0.56^{+0.10}_{-0.29}$	$0.43 \pm 0.23$	$0.8420 \pm 7 \times 10^{-4}$	$0.872 \pm 0.006$
$b_{\text{true}}$	$0.094 \pm 0.08$	$0.20^{+0.05}_{-0.14}$	$0.787^{+0.007}_{-0.02}$	$0.02^{+0.15}_{-0.02}$	$1.29^{+0.34}_{-0.17}$	$0.869^{+0.005}_{-0.007}$
$v(R_{\star}, \text{d}^{-1})$	$6.1215^{+0.090}_{-0.21}$	$4.51^{+0.20}_{-0.25}$	$3.85^{+0.48}_{-0.34}$	$5.2^{+0.4}_{-0.9}$	$6.191 \pm 0.006$	$4.03^{+0.10}_{-0.09}$
$R_p/R_{\star}$	$0.0943^{+0.0007}_{-0.0005}$	$0.0690 \pm 6 \times 10^{-4}$	$0.0560 \pm 0.001$	$0.049^{+0.002}_{-0.001}$	$0.165 \pm 0.0002$	$0.0759 \pm 7 \times 10^{-4}$
$R_p/R_{\star, \text{true}}$	$0.11^{+0.015}_{-0.08}$	$0.057^{+0.003}_{-0.0015}$	$0.06^{+0.03}_{-0.02}$	$0.0372^{+0.02}_{-0.005}$	$0.5^{+0.1}_{-0.4}$	$0.076^{+0.04}_{-0.03}$
$F$	$0.99994 \pm 2 \times 10^{-5}$	$0.99994 \pm 1.3 \times 10^{-5}$	$0.9999617 \pm \times 10^{-6}$	$0.99997 \pm 1.0^{-5}$	$1.0001733 \pm 1.3 \times 10^{-6}$	$0.999958 \pm 8 \times 10^{-6}$
$u1$	$0.389 \pm 0.004$	$0.33^{+0.003}_{-0.004}$	$0.363 \pm 0.006$	$0.327 \pm 0.003$	$0.2616 \pm 0.003$	$0.421 \pm 0.006$
$u2$	$0.264 \pm 0.0025$	$0.297 \pm 0.002$	$0.280 \pm 0.003$	$0.298^{+0.002}_{-0.003}$	$0.3468 \pm 0.0015$	$0.245 \pm 0.004$
$R_p(R_{\text{Jup}})$	$0.84 \pm 0.41$	$0.99 \pm 0.17$	$0.63 \pm 0.07$	$0.7 \pm 0.1$	$2.7 \pm 1.1$	$0.70 \pm 0.06$
$R_p, \text{true}$	$0.96^{+0.39}_{-0.09}$	$0.8^{+0.5}_{-0.1}$	$0.67^{+0.15}_{-0.07}$	$0.51^{+0.2}_{-0.05}$	$8^{+7}_{-1}$	$0.71^{+0.11}_{-0.03}$
$P(\text{d})$	$138^{+20}_{-10}$	$53.0^{+5.3}_{-18.7}$	$240^{+130}_{-90}$	$55.3^{+71}_{-3.1}$	$25^{+100}_{-17}$	$340^{+110}_{-80}$
$P_{\text{true}}$	$130.1775 \pm 0.0001$	$50.79035 \pm 2 \times 10^{-5}$	$224.7782 \pm 0.0003$	$52.09082 \pm 4 \times 10^{-5}$	$52.56899 \pm 6 \times 10^{-5}$	$345.1599 \pm 4 \times 10^{-4}$
s.m.a.(au)	$0.5^{+1.4}_{-0.3}$	$0.36^{+0.17}_{-0.10}$	$0.80^{+0.27}_{-0.21}$	$0.3^{+0.2}_{-0.1}$	$0.20^{+0.42}_{-0.11}$	$0.98^{+0.20}_{-0.17}$
s.m.a.true	0.5121	0.2858	0.7469	0.2807	0.321	0.9809
$T_{\text{S}}(\text{K})$	$5800 \pm 110$	$6170 \pm 100$	$5952 \pm 74$	$6174^{+83}_{-117}$	$7200^{+310}_{-240}$	$5600^{+110}_{-90}$
$R_{\text{stat}}(R_{\odot})$	$0.91 \pm 0.5$	$1.61 \pm 0.05$	$1.16 \pm 0.14$	$1.302^{+0.026}_{-0.027}$	$1.66^{+0.2}_{-1.4}$	$0.95^{+0.05}_{-0.14}$
$M_{\star}(M_{\odot})$	$1.04 \pm 0.12$	$1.13^{+0.13}_{-0.02}$	$1.17 \pm 0.03$	$1.165^{+0.044}_{-0.045}$	$1.59^{+0.18}_{-0.4}$	$1.054^{+0.008}_{-0.006}$
$\rho_{\text{S}}(\rho_{\odot})$	$1.72 \pm 0.12$	$0.29^{+0.01}_{-0.02}$	$0.75 \pm 0.16$	$0.526^{+0.018}_{-0.024}$	$0.35^{+0.29}_{-0.30}$	$1.22^{+0.27}_{-0.47}$



**Figure 6.** Six new K2 planet-like signals with *Namaste* fits. Best-fitting models are in black with  $1 - \sigma$  error regions in light blue. *x*-axis is scaled to eight transit durations, whereas the *y*-axis is unconstrained. Full posterior distributions and fits in Figs A7–A13.

impact parameter, planetary radii and stellar radii, a circular transit period was estimated and compared to the known period. In the case of eccentric planets, as was detected by Van Eylen & Albrecht (2015), the period estimate is likely to be significantly offset from the circular period.

Intriguingly, stellar densities derived from stellar surface gravities for a wider range (660) of *Kepler* planets showed a correlation between estimated period and impact parameter for grazing transits ( $b > 0.6$ ). This effect is likely due to overestimation of the duration and impact parameter of the shortest transits, for example due to smeared TTVs, which would cause an increase in the estimated period. Such effects would only be present for phase-

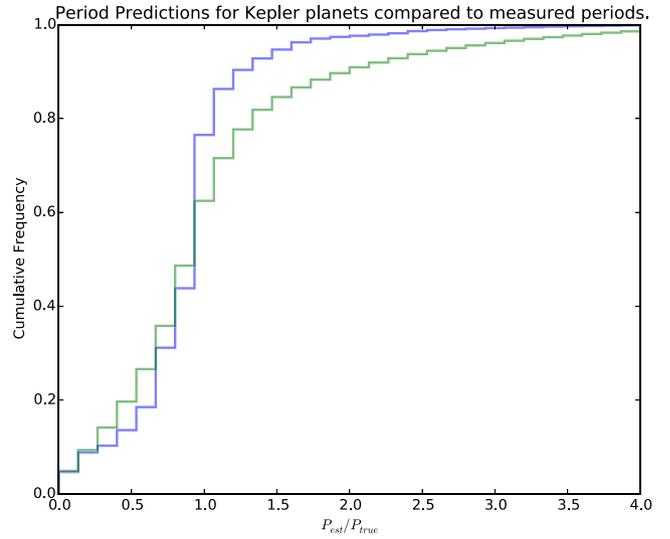
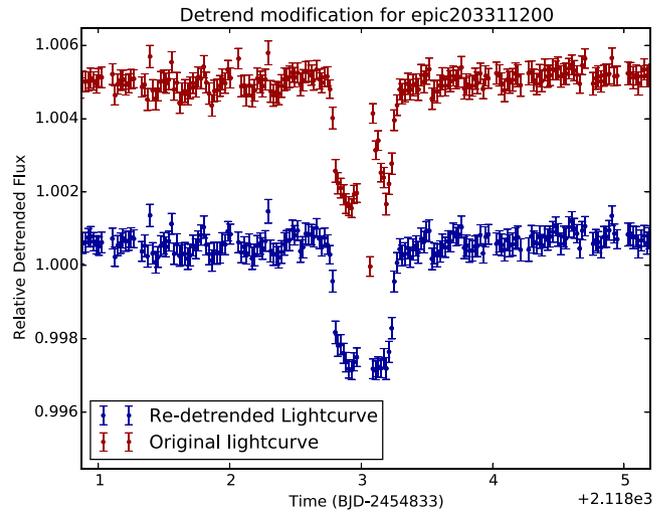
folded transits, hence unlikely to present problems for *Namaste* analysis.

To avoid this issue, we compared only the better constrained densities from  $R_*$  and  $M_*$  estimates, or directly from asteroseismology. Parameters from the 101 and 69 planets, respectively, were resampled 500 times from a Gaussian distribution derived from their uncertainties to produce a cumulative probability distribution of the estimated period compared to the true period can be seen in Fig. 7.

As can be seen, the less-precise density estimates from  $R_*$  and  $M_*$  give a shallower distribution of orbital period estimates. However both distributions peak at the true value, and suggest that the analytical technique is valid.

**Table 2.** Output median parameters for Namaste runs of seven K2 single transit candidates, with 16 per cent and 84 per cent percentile bounds. See Appendix A (Figs A7–A13) for posterior distributions.

	EPIC203311200	EPIC201892470	EPIC204634789	EPIC201631267	EPIC203914123	EPIC201720401	EPIC201635132
KepMag	11.896	11.587	12.292	12.78	9.039	14.74	15.143
$T_{\text{cent}}(\text{d})$	$2121.021^{+0.003}_{-0.002}$	$2056.11 \pm 0.005$	$2088.016 \pm 0.001$	$1996.674 \pm 0.001$	$2113.278 \pm 0.002$	$1983.314 \pm 0.002$	$1993.985 \pm 0.001$
$b$	$0.61^{+0.13}_{-0.03}$	$1.07 \pm 0.08$	$0.74^{+0.25}_{-0.43}$	$1.05^{+0.09}_{-0.08}$	$0.49 \pm 0.02$	$0.15^{+0.01}_{-0.09}$	$0.97^{+0.05}_{-0.07}$
$v(R_{\star} \text{d}^{-1})$	$3.3^{+0.7}_{-0.5}$	$1.9^{+0.2}_{-0.1}$	$21.7 \pm 8.1$	$3.7^{+0.2}_{-0.1}$	$0.679^{+0.006}_{-0.007}$	$1.51^{+0.01}_{-0.03}$	$7.5^{+0.3}_{-0.2}$
$R_p/R_{\star}$	$0.056^{+0.003}_{-0.002}$	$0.121^{+0.0073}_{-0.0062}$	$0.088^{0.038}_{-0.008}$	$0.176^{+0.074}_{-0.057}$	$0.149 \pm 0.001$	$0.133 \pm 0.001$	$0.24 \pm 0.038$
$F$	$0.999947 \pm 1.5 \times 10^{-5}$	$0.999998 \pm 1 \times 10^{-5}$	$0.999941 \pm 5 \times 10^{-5}$	$0.999965 \pm 1.3 \times 10^{-5}$	$0.999895 \pm 1.7 \times 10^{-5}$	$1.000087 \pm 3 \times 10^{-5}$	$0.999996 \pm 9.0 \times 10^{-5}$
$u1$	$0.499 \pm 0.009$	$0.524 \pm 0.015$	$0.603 \pm 0.002$	$0.627 \pm 0.007$	$0.591^{+0.023}_{-0.028}$	$0.589 \pm 0.016$	$0.039^{+0.038}_{-0.026}$
$u2$	$0.193 \pm 0.006$	$0.175 \pm 0.011$	$0.117 \pm 0.002$	$0.099 \pm 0.005$	$0.036^{+0.047}_{-0.025}$	$0.119 \pm 0.014$	$0.028^{+0.027}_{-0.018}$
$R_p(R_{\text{Jup}})$	$0.51 \pm 0.05$	$1.09^{+0.65}_{-0.56}$	$0.75^{0.32}_{-0.1}$	$1.45^{+0.61}_{-0.47}$	$1.07 \pm 0.1$	$1.14 \pm 0.1$	$2.03^{+0.83}_{-0.76}$
$P(\text{d})$	$540^{+410}_{-230}$	$2790^{+1270}_{-970}$	$2.2^{10}_{-1.4}$	$440^{+160}_{-120}$	$83\,830^{+316\,000}_{-223\,200}$	$6340^{+2230}_{-1530}$	$30^{+70}_{-20}$
s.m.a.(au)	$1.25^{+0.59}_{-0.39}$	$3.71^{+1.09}_{-0.94}$	$0.03^{0.04}_{-0.01}$	$1.03^{+0.25}_{-0.2}$	$31.17^{+8.35}_{-6.71}$	$6.17^{+1.46}_{-1.1}$	$0.13^{+0.19}_{-0.06}$
$T_S(\text{K})$	$5200 \pm 200$	$5100 \pm 200$	$4800 \pm 200$	$4600 \pm 200$	$4000 \pm 400$	$4800 \pm 200$	$3400 \pm 200$
$R_{\star}(R_{\odot})$	$0.94 \pm 0.09$	$0.93 \pm 0.09$	$0.87 \pm 0.09$	$0.85 \pm 0.08$	$0.74 \pm 0.07$	$0.88 \pm 0.09$	$0.89 \pm 0.36$
$M_{\star}(M_{\odot})$	$0.91 \pm 0.09$	$0.88 \pm 0.09$	$0.78 \pm 0.08$	$0.74 \pm 0.07$	$0.57 \pm 0.1$	$0.78 \pm 0.08$	$0.43 \pm 0.04$

**Figure 7.** Comparison of real period with true period for two density distributions:  $M_{\star}$  and  $R_{\star}$  estimates (green) and directly quotes density values.**Figure 8.** Change in light curve of EPIC203311200 after modifying detrending procedure.

### 4.3 K2 single transit events

#### 4.3.1 EPIC203311200

Unfortunately the centre of this single transit, which is the most obvious feature of the second half of the K2 light curve, is coincident with a global position shift at MJD 2121.05 of 1.23 pixels. This extreme pixel shift was poorly fitted by the interpolated flux map, causing a five-point bump in the data during the transit (see Fig. 8). To fix the poor detrending for these points, we extrapolated the well-fitted region of 3D fluxmap surface to this region of centroid shift and re-ran the detrending. This had the effect of reducing systematic noise across the light curve, including those in-transit. However, six of the largest pixel shifted points were still removed from central transit by this detrending procedure.

The Namaste fit for EPIC203311200 constrains the transit to being a disc-crossing (ie, non-grazing) eclipse, and can therefore put good constraints on the size of the eclipsing body. It estimates a size of  $0.51 \pm 0.05 R_{\text{Jup}}$ , putting it between the sizes of Uranus and Saturn. The posteriors show a two-peaked distribution in velocity

and radius ratio, which may be due to a build up of walkers at the maximum velocity threshold, or an excess of distribution at  $b \sim 0$ . We cannot currently separate the correct distribution, although transit probability favours the maximum velocity (with  $v' \sim 4R_* d^{-1}$  giving a potentially more reasonable period of  $P_{\text{circ}} \sim 310\text{d}$ ). Either period would make EPIC203311200 one of the longest period transiting exoplanets yet discovered, with the potential to exceed Kepler-421 b (704.2d, Kipping et al. (2014))

V-J, V-H and V-K show good agreement for a  $5200 \pm 200\text{K}$  star, however  $(B - V) = 0.19$  appears anomalously blue. A low-resolution spectrum taken with IDS on the 2.5-m Isaac Newton Telescope suggests it to be a relatively metal-poor star ( $[\text{Fe}/\text{H}] \sim -1.0$ ) with a temperature around  $5400 \pm 300\text{K}$ ; consistent with IR colours. The low resolution of the spectrum did not enable a log-g estimate. If no source of dilution is present, and the star is a main-sequence late-G star as suggested by photometric colours and spectra, then the source of this 3 mmag eclipse signal must be planetary.

#### 4.3.2 EPIC201892470

This shallow but V-shaped eclipse occurs only 1.2 d before the end of the K2 light curve. This allows us to place a high lower limit ( $>77\text{d}$ ) on the orbital period. A grazing eclipse is favoured by *Namaste* ( $b = 1.07$ ). The long circular period of  $2790_{-970}^{+1270}$  estimated by *Namaste* is likely unphysical, and could be an indication that this is a giant rather than a main-sequence star. However, it should be noted that the more planetary posterior regions – those with smallest  $R_p/R_*$ , lowest  $b$  and hence larger  $v'$  – give the shortest and therefore more likely periods.

#### 4.3.3 EPIC204634789

With only four in-transit data points, the fit is poorly constrained and is unable to distinguish between any impact parameters. The velocity posterior is again double peaked because of walker build-ups at maximum velocity ( $b = 0$ ) and minimum velocity where the distribution goes flat ( $b > 1.0$ ). Either hypothesis gives extremely fast velocities, and therefore circular periods far shorter than can be constrained by the non-detection of other events in the light curve. Hence, an eclipse at the perihelion of an eccentric object (either a planet or grazing EB) is more likely. Alternatively, the signal could simply be unexplained red noise or even an outburst of evaporation from an otherwise undetectable planet.

#### 4.3.4 EPIC201631267

*Namaste* suggests this shallow but V-shaped transit is most likely a grazing eclipse, although the distributions build up at the  $R_p/R_*$  and  $b$  limits, suggesting the true eclipsing body could be larger than our model can fit. The period of  $440_{-120}^{+160}$  d is well-constrained, however, as the velocity of the eclipsing body becomes constant for extremely grazing eclipses.

#### 4.3.5 EPIC203914123

The deepest and longest eclipse detected in K2, this star undergoes nearly 3 d of 2.5 per cent dimming. *Namaste*, using a main-sequence approximation from photometric colours, estimates a period of 84 000 d (230 yr). The length of the eclipse means fitting was extremely precise and the posterior distributions are extremely

well-constrained, although some red noise (seen in the form of a V-shape during the base of the transit) remains. However, the likelihood of seeing such an eclipse around a main-sequence star is around  $10^{-7}$ . Hence, our assumptions are likely wrong, and this is probably a low-mass star eclipsing a sub-giant or giant star. Indeed, this hypothesis also explains the ‘ramp’ seen up to and away from the eclipse as gravity darkening or ellipsoidal variation in a tidally locked binary system. As period is directly proportional to stellar density, a main-sequence star passing in front of a  $10R_\odot$ , stellar mass giant would produce a similar eclipse on only an 80 d, rather than 80 000 d, orbit. Such an orbit would be easily distinguished by RV follow-up. Asteroseismology could be used with the K2 light curve to achieve better density (and therefore period) constraints for giant star eclipses such as this. Such discoveries are important finds on their own, allowing models of stellar evolution to be tested (Gaulme et al. 2013).

#### 4.3.6 EPIC201720401

Similar to EPIC203914123, *Namaste* fits this eclipse extremely well but gives an unfeasibly long period for EPIC201720401 of  $6300_{-1500}^{+2200}$  d. Hence a giant/M-dwarf eclipsing binary is the more likely scenario, but we cannot rule out either scenario without further follow-up, such as radial velocities. With good estimates of stellar density, however, the period could be estimated accurately and subsequent eclipses followed up.

#### 4.3.7 EPIC201635132

Similarly deep to the previous eclipses, this eclipse is also short and V-shaped, which *Namaste* identifies as a grazing eclipse with minimum impact parameter of 0.85 and minimum radius of  $0.17R_*$ , constrained only by our imposed limits on  $b$  and  $R_p/R_*$ . With a minimum period of 63 d from the light curve, the velocity is likely to be around half of the expected value, suggesting a much more grazing eclipse than possible to detect with *Namaste*.

### 4.4 Source of uncertainty

For our best-constrained K2 candidates, the dominant source of error is stellar density. Even for a main-sequence star with a well-constrained stellar temperature, density uncertainties can be on the order of 50 per cent.

For less well-constrained probability distributions, for example from noisy light curves, the uncertainty in planetary velocity can be important. This is especially true for small planets for which impact parameter becomes more degenerate.

As shown in Section 2.4, eccentricity also has a role in increasing period uncertainties. However, the majority of exoplanets are expected to be on circular or near-circular orbits for which the correction is minimal (Kipping 2013). If density can be constrained by follow-up observations, however, eccentricity could become an important factor when searching for a subsequent transit.

One interesting exception may be known multiplanet systems, which improve the single transit method in three ways: the low mutual inclination in such systems increases the transit probability of exterior planets; stability constraints and formation pathways also limit long-period planets to more circular orbits; and stellar parameters such as density are significantly improved due to the information gathered by interior planets (for example densities through

transit durations). As such, searches for single transits in known multi-planet systems could have valuable results.

#### 4.5 Potential for follow-up observations

Single spectra of candidates could immediately rule out large binary companions from cross-correlation functions. Spectral fitting would also allow better estimates of stellar parameters, including density from  $\log-g$ . This would also rule out giant stars from the analysis. Lucky or AO imaging could rule out close companions that could either be false positives, or diluting the transit depth.

Estimated planetary radius and orbital period can give likely RV amplitudes and determine the observability of individual targets. Radial velocities can be tailored to the likely period to give best observing strategy. For objects with poorly constrained parameters (e.g. grazing transits), RVs could be used to reduce the uncertainty on planet-to-star radius ratios. This could then be used as a prior to recalculate the planetary velocity, and hence improve the period estimate.

Good period estimates from single transit candidates could then be used to search for repeat transits. Such analysis would ideally utilize observers at different longitudes to cover a likely transit range to be many days. Long-period (and low impact parameter) planets may have a better probability of reobservation due to their longer transit duration. Precise transit survey telescopes such as M<sub>Earth</sub> (Irwin et al. 2008) or NGTS (Wheatley et al. 2013), in combination with amateur observer programmes, could be used most effectively to this goal.

For the smallest transit depths, small space-based telescopes such as MOST (Rucinski et al. 2003) or Cheops (Bruno et al. 2013) may be the only method of confirming the orbital period.

#### 4.6 Validation

We have shown that, for favourable single transits, the orbit and size of a transiting exoplanet can be accurately determined, especially for giant planets. However, alternative sources for such transits (e.g. from background eclipsing binaries) remain. As has been shown (Morton 2012), the source of such false positives can be probabilistically excluded with follow-up data. For example sub-arcsec imaging can rule out stellar companions that could be producing spurious signals. Spectral fitting to high-SNR spectra can be used to rule out closer binary companions. Radial velocity measurements (even where the detection of the signal from planet is impossible) could be used to place limits on the size of any companion. In those cases without close companions, the planetary radius of the eclipsing object can be probabilistically limited to a planetary, rather than stellar, origin.

Such follow-up could, in a similar way to the validation of other single transiting systems such as Kepler-452 (Jenkins et al. 2015), constitute a probabilistic validation of the planet without observing subsequent transits. Similarly, three of the 17 single transiting exoplanets detected by Wang et al. (2015) were probabilistically validated using such methods.

#### 4.7 Application to future missions

Single transits could be used by many future surveys to detect long-period transiting planets around bright stars.

The TESS (Ricker et al. (2015)) will monitor 200 000 stars on 2-min cadence over 28 d observing windows. Studies of TESS planet yield suggest more than 100 single transits could be detected above

a noise threshold of  $7.3\sigma$  (Sullivan et al. 2015). Namaste could be an important tool in the follow-up of these planets.

Initial observing plans for the PLANetary Transits and Oscillation of Stars (PLATO) mission suggest six fields could be observed on 2–5 month campaigns, yielding 60 000 bright stars ( $V_{\text{mag}} < 12$ ) with 30-s cadence and hundreds of thousands of fainter stars ( $12 < V_{\text{mag}} < 16$ ) with 10-min cadence. The potential combination of asteroseismology-derived densities (accurate to 10 per cent) with high-cadence, high-precision photometric data could produce dozens of validated long-period planets (Osborn, Brown & Pollacco, in preparation).

A large sample of single transiting exoplanets may also begin to reveal the effect of outer giant planet companions on close-in terrestrial exoplanets, which could have a significant effect on the occurrence rates of Earth-like planets.

Ground-based surveys such as M<sub>Earth</sub> (Irwin et al. 2008) already search for the single transits of stars with an innovative detection strategy (Nutzman & Charbonneau 2008). This method could also be applied to more traditional single-field photometric surveys such as NGTS (Wheatley et al. 2013).

*Gaia* has the capability to detect tens of thousands of giant planets on 1–4 yr orbits (Dzigan & Zucker 2012), hence any transiting gas giants on these orbits found by TESS, PLATO or from the ground stand a good chance of being confirmed by *Gaia* astrometry. Hence this method could lead to the first overlap between the realms of transiting and astrometric exoplanet astronomy.

The long transit duration of planets detected from a single transit also makes them amenable to transmission spectroscopy, for example by JWST (Belu et al. 2011). With a dearth of transiting warm and cold Jupiters known around bright stars, such objects could prove a vital link between the atmospheres of hot Jupiters and those of Solar system gas giants.

## 5 CONCLUSION

We have developed Namaste, a method of combining stellar parameters with the light curve of a single transit to estimate orbital parameters. We have tested this analysis on published transit parameters for a large sample of *Kepler* multiplanets, showing close agreement. A test of the full fitting method on the light curves of four known *Kepler* planets and two KOI candidates showed extremely good agreement, with the periods of Kepler-51 d, Kepler-117 c, Kepler-111 c and KOI1431.01 all estimated to within 10 per cent.

We performed an iterative search on three campaigns of K2 data and identified seven preliminary single transit events. One of these, EPIC203311200, is an extremely good planet candidate with a Namaste-estimated period of  $540_{-230}^{+410}$  d and a size of  $0.51 \pm 0.05 R_{\text{Jup}}$ . We also detect three single eclipses from ambiguous but potentially planetary bodies, and three from likely eclipsing binaries. For all candidates, future follow-up campaigns are vital to determine the source of the eclipse and better constrain the orbital parameters.

Namaste is therefore a useful tool, allowing more targeted follow-up observations with radial velocity measurements and the search for additional transits. For giant planets around bright stars, subsequent RV follow-up could lead to their confirmation. Smaller planets around fainter stars could still be validated with follow-up techniques such as diffraction-limited imaging and high-resolution spectroscopy.

In future the detection and analysis of single transits in this way could lead K2, TESS, PLATO and ground-based photometric surveys to detect transiting exoplanets on orbits longer than their

observing campaigns would traditionally allow. Such long-period planets, especially if found around bright stars, could pave the way for a new regime of exoplanetary science. This includes the detection of planets within the astrometric sensitivity of *Gaia*, and cold Jupiters with atmospheres observable in transmission spectroscopy by JWST. EPIC203311200 b, if proved to be planetary by ongoing follow-up work, could fit both roles.

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## APPENDIX A: MCMC POSTERIOR DISTRIBUTIONS

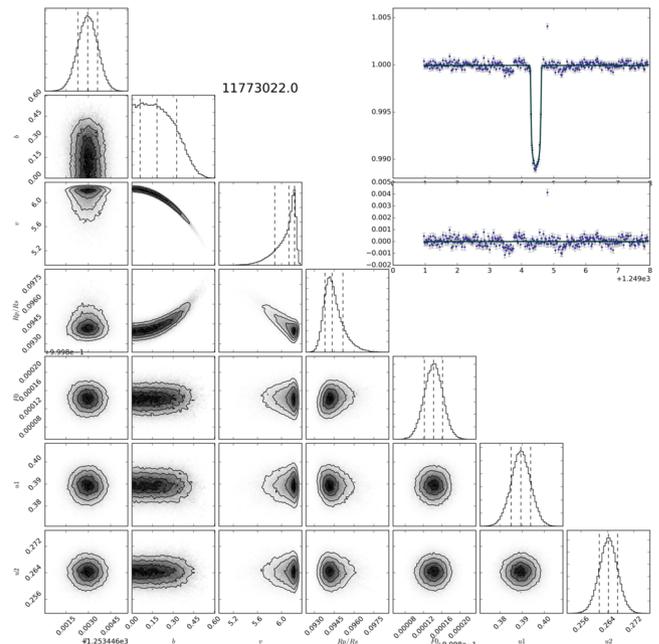


Figure A1. Posterior distribution for Kepler-51 d (KIC11773022).

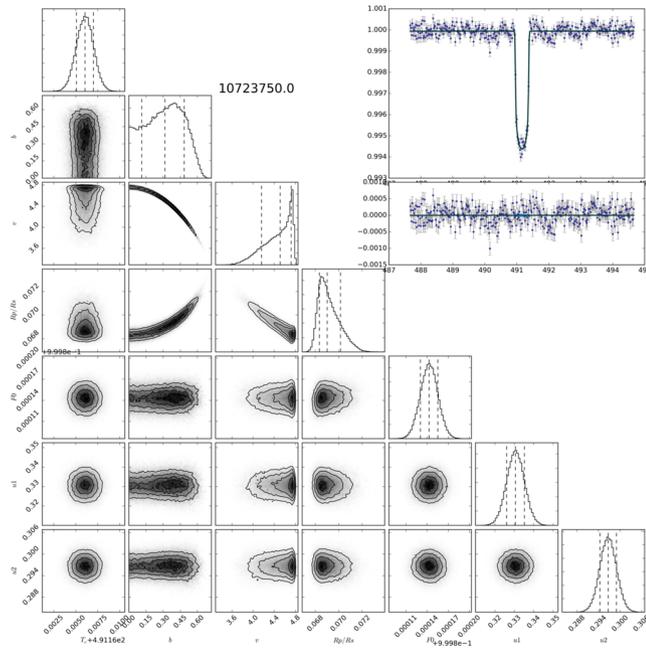


Figure A2. Posterior distribution for Kepler-117 c (KIC10723750).

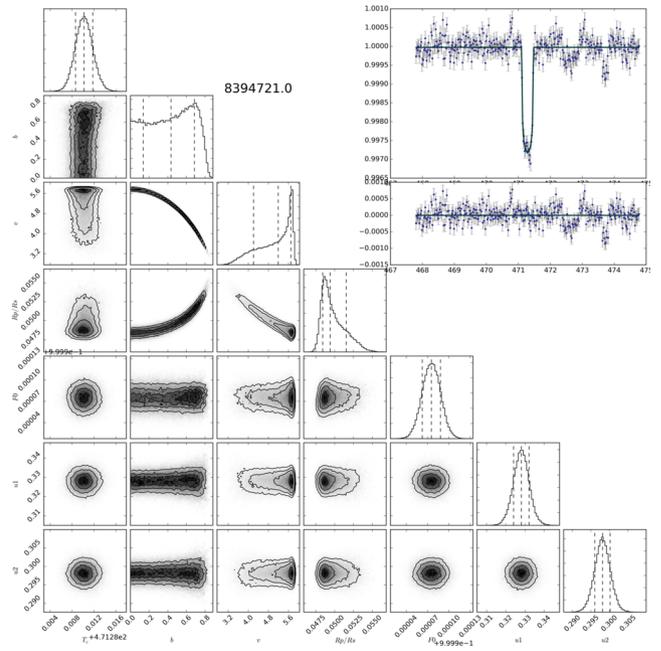


Figure A4. Posterior distribution for Kepler-79 d (KIC8394721).

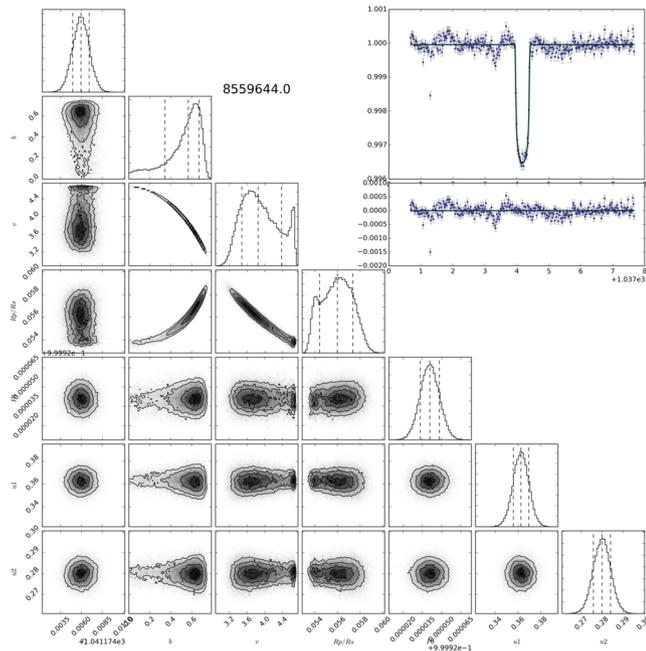


Figure A3. Posterior distribution for Kepler-111 c (KIC8559644).

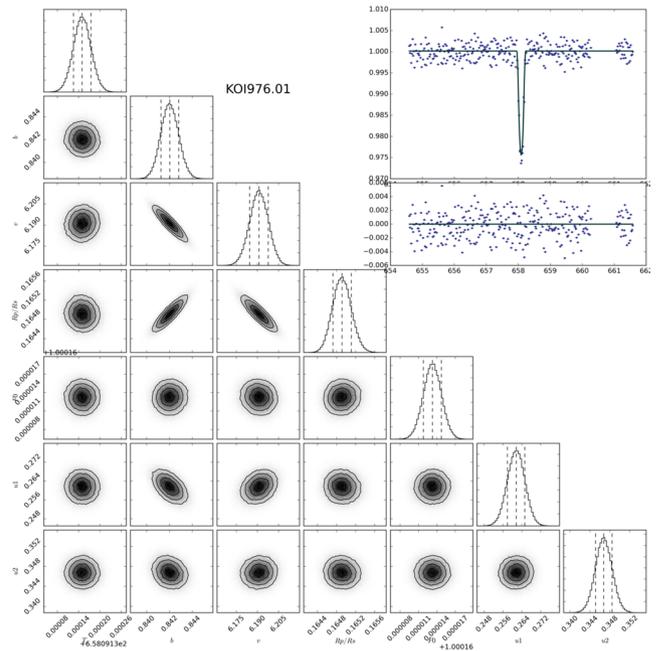


Figure A5. Posterior distribution for KOI 976.01 (KIC34417842).

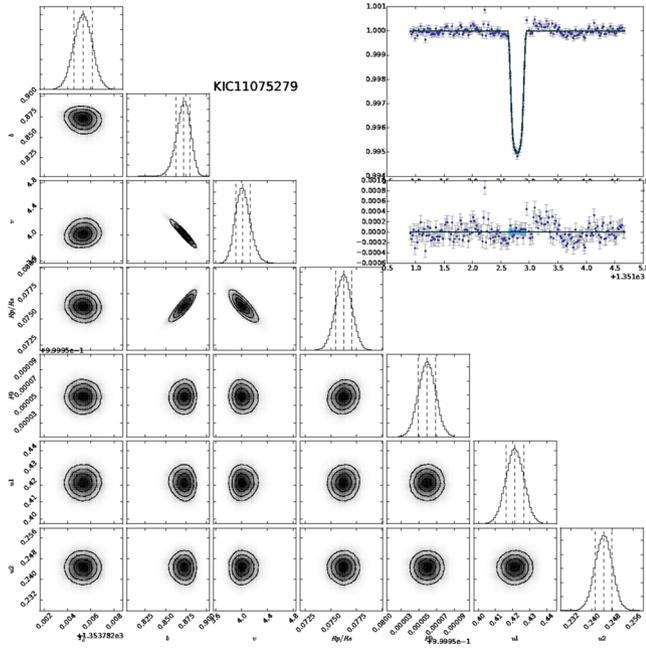


Figure A6. Posterior distribution for KOI 1431.01 (KIC11075279).

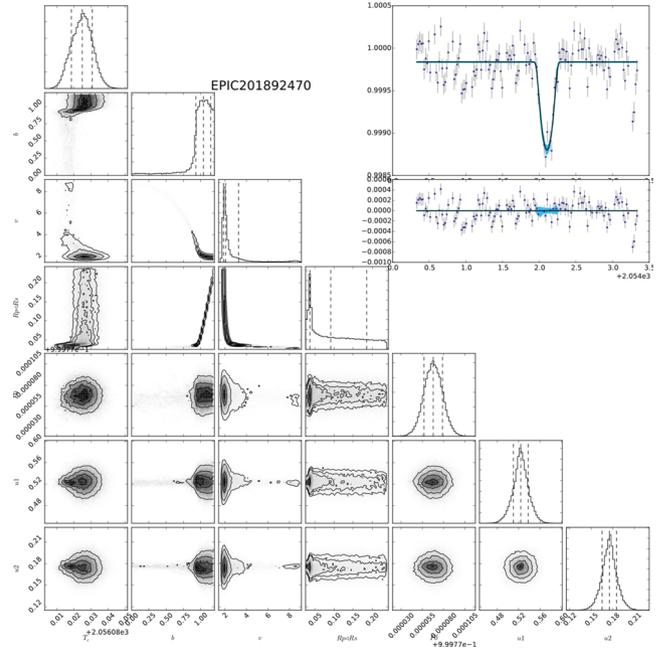


Figure A8. Posterior distribution for EPIC201892470.

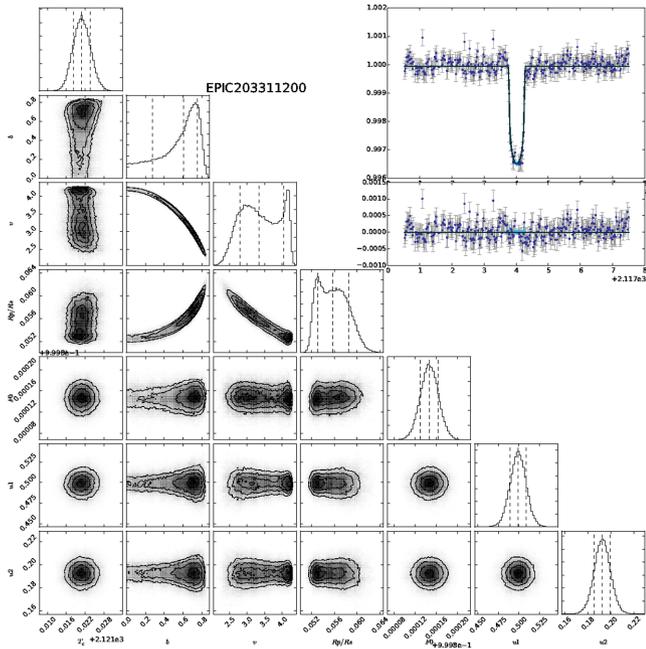


Figure A7. Posterior distribution for EPIC203311200.

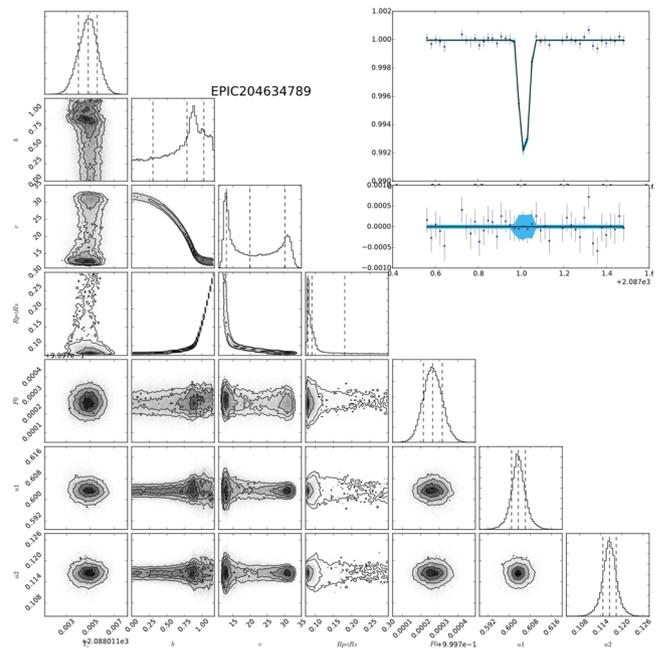


Figure A9. Posterior distribution for EPIC204634789.

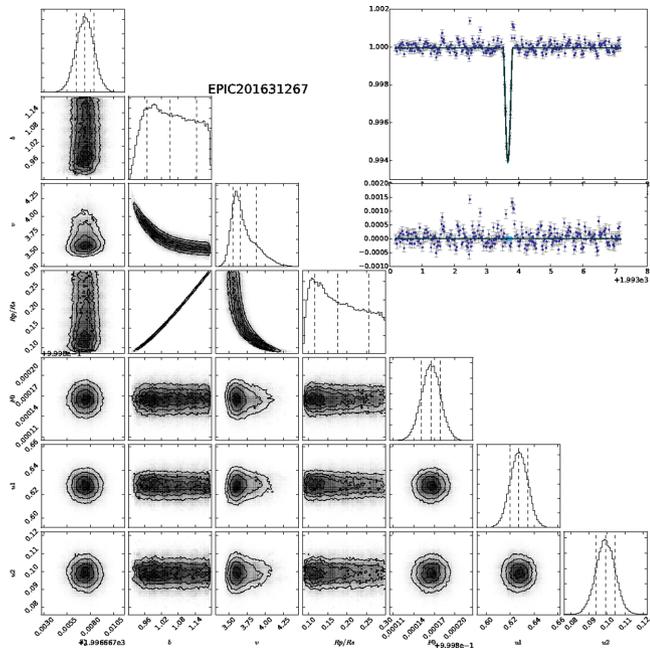


Figure A10. Posterior distribution for EPIC201631267.

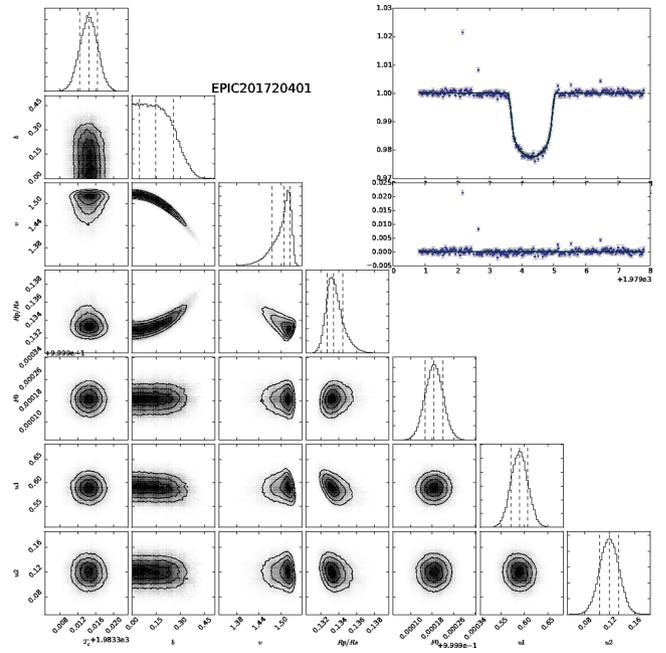


Figure A12. Posterior distribution for EPIC201720401.

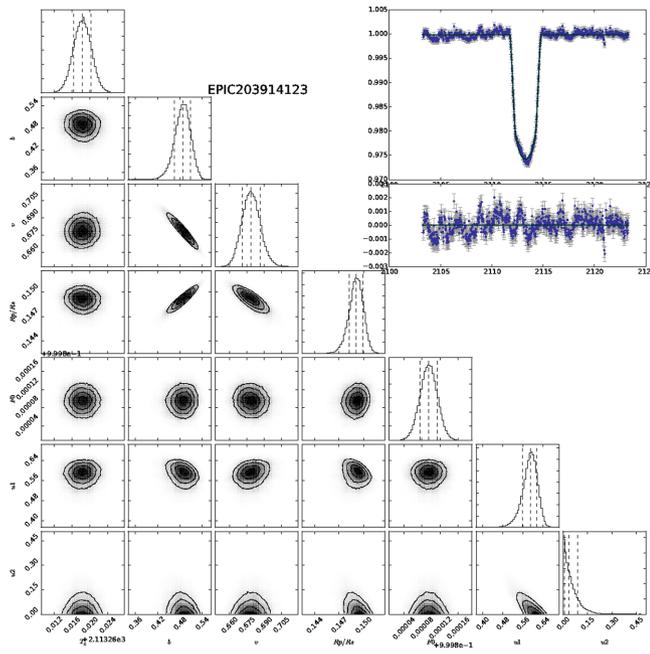


Figure A11. Posterior distribution for EPIC203914123.

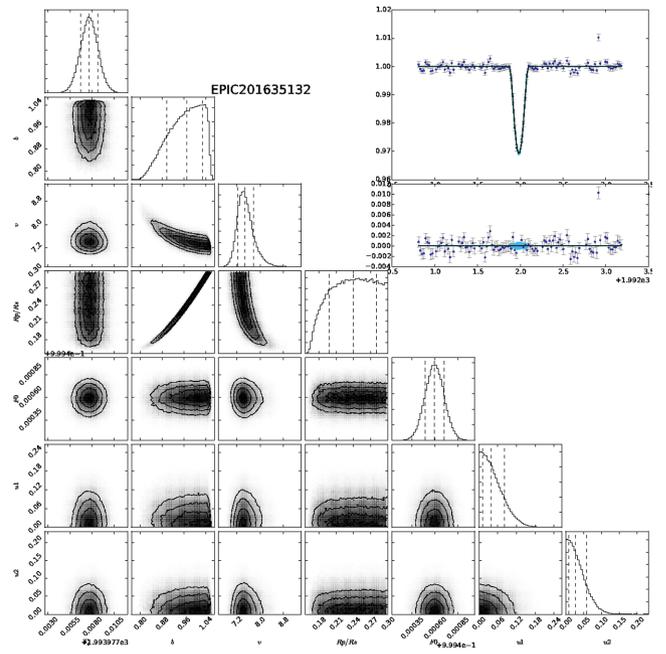


Figure A13. Posterior distribution for EPIC201635132.

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