The Effect of Boundary Conditions on Resonant Ultrasonic Spherical Chains

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Abstract—The response of a resonant chain of spheres to changes in holder material and pre-compression is studied at ultrasonic frequencies. The system is found to be very sensitive to these parameters, with the creation of impulsive waveforms from a narrow bandwidth input seen only for certain chain lengths and holder materials. In addition, careful experiments were performed using known amounts of pre-compression force, using a calibrated stylus arrangement. At negligible pre-compression levels, impulses were generated within the chain, which were then suppressed by increased pre-compression. This was accompanied by large changes in propagation velocity as the system gradually changes from being strongly nonlinear to being more linear. Simulations using a discrete model for the motion of each sphere agree well with experimental data.

Index Terms — Granular media, non-linear, spherical chains, solitary waves.

I. Introduction

Acoustic propagation along granular chains has been the subject of increased interest, because of their non-linear acoustic properties caused by Hertzian contact between the spheres, and this allows such systems to support a range of properties, depending on the amount of non-linearity present [1-6]. Such behaviour has been observed in different types of material that can be used in granular chains [7,8]. One of the key parameters which affects the dynamics of the is the magnitude of the applied static compression force F_0 relative to the applied dynamic force F_m. Solitary wave propagation along an infinite chain of spheres has specific properties that can be predicted theoretically using a long-wave approximation, provided that there is negligible static pre-compression ($F_m >> F_0$). Under these conditions, propagation along the chain is in the form of a solitary wave with characteristics that depend upon the size and material of the spheres from which the chain is made. In fact, such solitary waves are predicted to have a constant wavelength which is a certain number of particles long [2]. As the levels of pre-compression increase, the nonlinearity reduces, and in the case of a strong static compression force ($F_m << F_0$), the system becomes only weakly nonlinear. The Hertzian contact can then be simplified to a reduced linearized spring connection, and the passage of a harmonic wave through the chain can be investigated by a linear model in which the system behaves as a low-pass filter.

The observations made in the above paragraph apply to an infinite chain. The situation starts to become more complicated once the chain is of finite length. Now, reflection within the chain is possible, complicating the response. One way is to study the Nonlinear Normal Modes (NNMs) of the system [9-12]. Under certain circumstances, separation between the spheres can occur, and the separation and collision of the spheres (in addition to their nonlinear Hertzian interaction) provides an added effect to the strong nonlinearity of the system. Although NNMs are usually defined as synchronous period particular solutions of the nonlinear equations of motion of dynamical systems, various authors [9-12] have identified a time-periodic oscillation where the bead oscillations possess identical frequencies but are not necessarily synchronous, leading to nonlinear and non-smooth features (such as the separation of spheres). Under the correct conditions, an in-phase NNM results, which can be considered as a traveling wave propagating along the chain (as the spheres displace from their centre positions in sequence). Sphere separation becomes less likely as pre-compression is applied, forcing the spheres closer together. Note that in all these theoretical discussions, it is the motion of individual spheres relative to each other that causes the effects. In practice, this means that the frequencies present have to remain below an upper cut-off frequency (f_c), which depends on the properties (size, material etc.) of the spheres used. The value of f_c can be approximately determined by a linearised discrete model.

Nonlinear systems have a distinct advantage in being able to transfer energy between frequencies, where sub-harmonic and super-harmonic frequencies may appear via sub-harmonic and super-harmonic bifurcations in energy transfer in the frequency domain, because of the presence of NNMs [11,12]. Previous work by the present authors [13,14] has demonstrated that a train of impulses can be generated within chains of spheres at ultrasonic frequencies because of this effect, provided a negligible amount of pre-compression force F_0 was applied. These studies used high amplitude, narrow-bandwidth inputs at 73 kHz from an ultrasonic horn, with a chain containing small spheres (typically 1 mm diameter) of different types of material held within an acrylic holder. Once the correct conditions needed to set up an in-phase NNM were

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established, the time-domain response was in the form of a periodic set of impulses, accompanied by frequency spectra containing regularly-spaced maxima which were separated in value as a whole fraction of the input frequency. The sub-harmonic and super-harmonic bifurcations in energy transfer in the frequency domain mentioned above were also exhibited in the experimental results. This response was able to be predicted by a theoretical model, the results from which were used to interpret the solitary wave propagation and the establishment of NNM behavior. This discrete model used a Velocity-Verlet algorithm [15] to solve the relevant equations numerically. Note also that sub-harmonic frequencies were also observed in the work of Lydon *et al*. [16] in shorter chains at lower frequencies**.**

It is known from previous work [17-19] that pre-compression can cause changes to the response of a granular chain. Our previous work verified that the solitary wave impulses can be generated using harmonic excitation and a negligible amount of pre-compression force in a finite-length chain [13,14]. In this paper, we will observe the effect of different holder materials and an increase in pre-compression on this response. It will be demonstrated that end-wall conditions are an important factor in dissipation and reflection at the boundaries, leading to changes in the characteristics of energy transfer to sub-harmonics and super-harmonics. The effect of pre-compression on the resultant waveform will also be investigated. It will be shown that an increase in pre-compression changes the wave regime from being strongly non-linear to weakly non-linear, accompanied by very sensitive change in propagation velocity.

II. Apparatus and Methods

The experiments used the apparatus shown in Fig 1. An exponential horn was used to provide high amplitude input signals with a narrow bandwidth. The output displacement of the horn tip was approximately 1 μm at a centre frequency of 73 kHz, when the horn/transducer assembly was driven with a tone-burst voltage signal. The frequency spectrum of the output had the expected main peak at 73 kHz, with only a small signal at the 2^{nd} harmonic at 146 kHz. The spheres were positioned within a cylindrical holder, and the output from the last sphere recorded using a Polytec OFV-505 vibrometer, whose output was a particle velocity waveform. The holders could be of different length to accommodate different chain lengths (from 3-10 mm) of 1 mm diameter chrome steel spheres. Three holder materials were investigated - an acrylic polymer (R11, as used in micro-stereolithography, a form of 3D-printing), aluminum and steel.

A force of known amplitude could be applied to the last sphere in the chain, using a calibrated 1.5 mm diameter stylus acting upon a pivot. The stylus itself was part of a surface profiler instrument, modified to act as a mechanism of applying controlled pre-compression force. The control system for the stylus consisted of three main parts: an electromagnetic force actuator, a differential capacitive sensor and a leaf spring suspension system. Further details of this design can be found in the paper by Chetwynd *et al.* [20]. To achieve the required function as a controlled mode of applying pre-compression, the current that passed through a coil/magnet assembly was varied, and careful design ensured that the force on the stylus along its axis was proportional to the current passing through the coil. Using a specially designed current drive, the force actuator was used to provide a calibrated static contact force at a neutral position. The stylus and the associated control system were mounted onto a Rank Taylor Hobson Talysurf 5 surface profiling instrument to ensure stability. Note that the force applied to the pivot was independent of small displacements; therefore, small vibrations within the system did not couple into the measurement loop. The Talysurf 5 instrument provided the means to carefully and accurately position the tip of the stylus onto a fulcrum or lever made of aluminum. The fulcrum was used to apply pre-compression directly on the chain of spheres as shown in Fig. 1, and was designed to be light and strong enough not to deform from the force exerted on it. The pre-compression system provided electrically selectable static force with a resolution of 1 mN. Note that in the absence of any applied pre-compression, a small force would still be exerted on the chain due to the need for the horn to touch the first sphere. This is discussed further below.

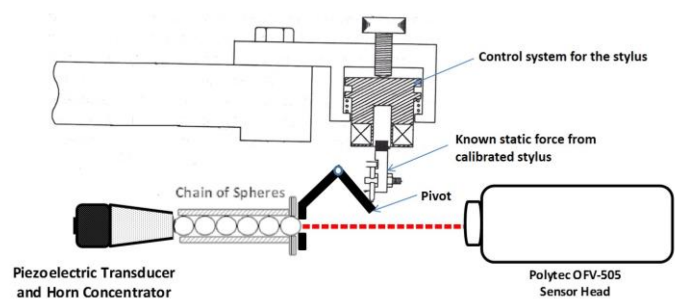


Fig. 1. Schematic diagram of the apparatus.

The work in this paper adds to our previous results [13,14] by considering both the effect of chain length for a fixed input frequency, the use of different holder materials (and hence end-wall materials), and investigates in detail the effect of pre-compression on these resonant systems, where the response varies from being highly non)linear in the case of negligible pre)compression (F_0) to being almost linear at higher values of F_0.

III. Theory

Use It is important for later discussions that the origin of the usual formula for the cut-off frequency for granular chains is given; in addition, it is shown why a more detailed analysis is required for finite chain lengths and varied pre-compression forces. For a chain consisting of the identical spheres, the equation of motion of the n^{th} sphere based on Hertzian contact has the general form [1]:

$$m\ddot{u}_n = A(\delta_0 + u_{n-1} - u_n)^{3/2} - A(\delta_0 + u_n - u_{n+1})^{3/2}, \quad (1)$$

where

$$A = \frac{E\sqrt{2R}}{3(1-\nu^2)} \quad (2)$$

and

$$m = \frac{4}{3}\pi\rho R^3 \qquad (3)$$

The spheres are assumed to be made of the same material with a radius R; m is mass; ρ is density; E is Young's modulus and ν is Poisson's ratio. This system is assumed to be in an equilibrium state under a static compression force F_0. At static equilibrium, δ_0 is the distance of approach of centres of spheres. u_n is the dynamic longitudinal displacement from the equilibrium position of the nth sphere. When $|u_{n-1} - u_n|/\delta_0 \ll 1$, (1) can be changed into a lattice model with quadratic nonlinearity, from which the nonlinear wave equation can be derived at the long wavelength limit. If the granular material is weakly compressed, so that $|u_{n-1} - u_n|/\delta_0 \gg 1$, the wave equation for a "Sonic Vacuum" in an infinite chain can be derived [2]. Strongly non-linear solitary waves then appear in the analytical solution.

It will be shown in this paper that use of single frequency (harmonic) excitation and different levels of pre-compression can result in the presence of both sub-harmonics and super-harmonics, which are related to the driving frequency used. It is thus of interest to study the effect of a cut-off frequency on this process. Here we firstly explore the dispersion relation in a simplified discrete linear model which represents an infinite chain consisting of the identical spheres without dissipation. Under the limits $|u_{n-1} - u_n| \ll \delta_0$ and $(\delta_0 + |u_{n-1} - u_n|) < \delta_{max}$, the wave propagates in a continuum and smooth system (no separation between spheres occurs). It is assumed that the connections between spheres are springs with an elastic coefficient μ, where δ_{max} represents the corresponding value of δ_0 in the situation of the maximum elastic deformation. Accordingly, the dynamic equation of the nth sphere of the chain is simplified as a linearized equation:

$$m\ddot{u}_n = \mu(u_{n-1} - u_n) - \mu(u_n - u_{n+1}) = \mu(u_{n-1} - 2u_n + u_{n+1}). \qquad (4)$$

The chain then executes a motion of the form:

$$u_n(t) = e^{i(kna-\omega t)}. \qquad (5)$$

This is a harmonic wave of frequency ω and wavenumber k, and a is the distance between the centres of neighbouring spheres under static equilibrium, so that $a = 2R - \delta_0$. The cut-off frequency ω_c occurs when the wavelength ($2\pi/k$) equals $2a$. From Eqns. (4) and (5), we obtain the dispersion relation

$$\omega = \omega_c \sin\left(\frac{ka}{2}\right) = 2\sqrt{\frac{\mu}{m}}\sin\left(\frac{ka}{2}\right) \qquad (6)$$

where μ can be determined by Hertzian law, *i.e.*

$$F_0 = \frac{\sqrt{2R}E}{3(1-\nu^2)}{\delta_0}^{3/2},\ \mu = \frac{\partial F_0}{\partial \delta_0} = \sqrt{\frac{R}{2}}\frac{E}{1-\nu^2}{\delta_0}^{1/2} \qquad (7)$$

The cut off frequency f_c is then given by

$$f_c = \frac{\omega_c}{2\pi} = \frac{3}{4\pi^{3/2}}\frac{F_0^{1/6}}{\theta^{1/3}R^{4/3}\rho^{1/2}}, \qquad (8)$$

where $\theta = \frac{3(1-\nu^2)}{4E}$. This means that under strong pre-compression, the chain system operates as a low-pass filter and waves of frequency $f > f_c$ are strongly attenuated. However, this analysis is based on the assumed linear model; in fact, super-harmonic frequencies can appear in the discrete spherical chain due to the nonlinear Hertzian contact, which depends on the properties of the granular chain and the input conditions. Thus an accurate model to describe the experimental system is important, and the use of (8) to denote the expected upper frequency generated experimentally is not strictly correct.

In addition, in a finite length chain, reflection at the end-wall boundary and dissipation due to friction between the granular material and the holder materials are both essential factors which influence wave transmission in the chain. In our previous work [13,14], we presented dynamic equations of sphere motion which are described in (9a) – (9c), and investigated the characteristic of the solitary wave impulses using numerical calculations. These equations are still valid to model the motions of spheres based on the new experimental conditions in this paper. For the first sphere, positioned next to the horn, the equation is:

$$m\frac{d^2u_1}{dt^2} = \frac{2\sqrt{R}}{3}\left[2\theta_l(\delta_{0l} + u_0 - u_1)^{3/2} - \frac{\theta_m}{\sqrt{2}}(\delta_0 + u_1 - u_2)^{3/2}\right] + \lambda\left(\frac{du_0}{dt} - \frac{du_1}{dt}\right)H(\delta_{0l} + u_0 - u_1) - \lambda\left(\frac{du_1}{dt} - \frac{du_2}{dt}\right)H(\delta_0 + u_1 - u_2), \qquad (9a)$$

where u_0 is the displacement of the input signal, and λ is the damping coefficient. For the second sphere to the penultimate one, the equivalent equation of motion is:

$$m\frac{d^2u_n}{dt^2} = \frac{\sqrt{2R}}{3}\theta_m\left[(\delta_0 + u_{n-1} - u_i)^{3/2} - (\delta_0 + u_n - u_{n+1})^{3/2}\right] + \lambda\left(\frac{du_{n-1}}{dt} - \frac{du_n}{dt}\right)H(\delta_0 + u_{n-1} - u_n) - \lambda\left(\frac{du_n}{dt} - \frac{du_{n+1}}{dt}\right)H(\delta_0 + u_n - u_{n+1}). \qquad (9b)$$

Finally, for the last (output) sphere, the relevant equation is:

$$m\frac{d^2u_n}{dt^2} = \frac{2\sqrt{R}}{3}\left[\frac{\theta_m}{\sqrt{2}}(\delta_0 + u_{n-1} - u_n)^{3/2} - 2\theta_r(\delta_{0r} + u_n)^{3/2}\right] + \lambda\left(\frac{du_{n-1}}{dt} - \frac{du_n}{dt}\right)H(\delta_0 + u_{n-1} - u_n) - \lambda\frac{du_n}{dt}H(\delta_{0r} + u_n). \qquad (9c)$$

Here, $\frac{1}{\theta_l} = \frac{1-\nu_l^2}{E_l} + \frac{1-\nu_s^2}{E_s}$, $\theta_m = \frac{E_s}{1-\nu_s^2}$ and $\frac{1}{\theta_r} = \frac{1-\nu_r^2}{E_r} + \frac{1-\nu_s^2}{E_s}$. (10)

In (10), E_l and ν_l are the Young's modulus and Poisson ratio of the horn, E_r and ν_r are that of the holder, and E_s and ν_s those of the spheres themselves. Here, δ_{0l}, δ_0 and δ_{0r} denote the mutual approach caused by the static force between the horn and the first sphere, between intermediate spheres, and between the last sphere and the end wall respectively. Note that the equations relating to the first and last sphere of the chain assume that these are in contact with spheres of infinite radius, thus in effect modelling a wall, and a Heaviside function H is incorporated to judge if the spheres are in contact.

Consider now the effect of damping. The dissipation term λ in (8) is determined in our work from experimental results of each chain structure, and differs slightly in each case. In addition, the initial positions of spheres are determined in turn for a particular pre-compression force, starting from the last sphere. Here a boundary constraint is assumed in that the position of the contact point between the last sphere and the wall at the end of the chain is fixed. The properties of this end wall can be changed in the model via its effective Young's modulus θ_r which is associated with contact interactions between the last sphere and the holder. The effects of dissipation for harmonic excitation of a statically-compressed chain have also been investigated by Lydon *et al*. [16], but in their dynamic model the boundary conditions were not explicitly stated for a finite length chain with different end-wall conditions. They used a spring with force F_0 (the initial compression force) to describe the experimental boundary condition. In our model, the $\frac{4\sqrt{R}}{3}\theta_r(\delta_{0r}+u_n)^{3/2}$ term in (9c) allows the reflection process due to the different holder materials to be modified via the value of the effective Young's modulus θ_r. Hence the two models differ in this respect. Experimentally, the end-wall material has a big effect on response, as will be shown later in this paper.

IV. Results

A. The Effect of Changing the Holder Material

It was found experimentally that the resonant chains were very sensitive to the physical conditions in which they were held. One other factor that had a measurable effect was the material from which the cylindrical holder of the spheres was made. The input sphere was touching the horn tip, whereas the output sphere protruded through the annular end of the holder. To investigate this, a chain of six chrome-steel spheres of 1 mm diameter were tested when contained within holders of three different materials: R11 photo-reactive acrylic polymer (as used for additive manufacture via micro-stereolithography (MSL)), steel and aluminum. Table 1 gives the relevant properties of these materials.

TABLE I
PROPERTIES OF END-WALL MATERIALS

End-wall material	Poisson's ratio	Young's Modulus (GPa)
Acrylic polymer (R11)	0.35	2.45
Aluminum	0.33	75
Steel	0.3	200

The results of these experiments are shown in Fig. 2. When the metallic steel and aluminum holders were used, the dominant feature was the creation of harmonics of the input frequency, a feature of nonlinear behaviour; however, no subharmonics were present. It was only in the R11 acrylic material that the system truly exhibited strongly non-linear behaviour, with the creation of sub-harmonics and a series of super-harmonics of the lowest sub-harmonic peak (and not just harmonics of the input frequency as in the case of steel and aluminium holders). Note that Lydon *et al.* [16] also observed the presence of frequencies which were subharmonics of the input frequency in their experiments; however, multiple harmonics of the lowest subharmonic peak were not present, as is the case here. It is this difference that is particularly striking in the present measurements, with strong solitary-wave generation as impulses in time. This is thought to be partly due to both the increased input energy levels which result from the use of an ultrasonic horn at the input stage, but also the fact that the final sphere interacted directly with the end-wall, and not via a spring. The measurements demonstrate just how sensitive the system is to the end-wall material.

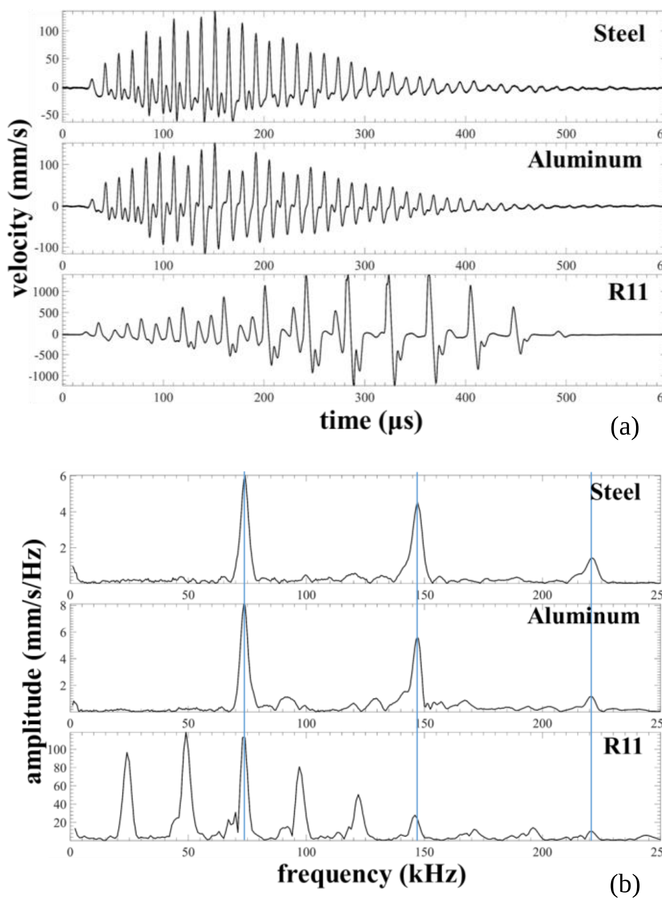


Fig.2. (a) Experimental waveforms and (b) spectra obtained from a chain containing six chrome-steel spheres of 1 mm diameter, when enclosed within holders of various types of material.

The predicted results from the theoretical model, using the known physical properties of the three different holder materials (Table 1) are shown in Fig. 3. The predicted waveforms and spectra correlate well with those observed in the experiments - the R11 holder material was the only one predicted to lead to a set of periodic impulses in the time domain, with a regular set of more closely-spaced frequency peaks. It was found that the damping coefficient used in the theoretical model needed to be adjusted between each material for the end wall to get suitable correlation with the experimental results in each case. This is reasonable: each wall material will have its own damping properties, as well as its own elastic modulus. Both effects feed into the damping coefficient (λ) used theoretically for the best correlation between theoretical prediction and experiment. The presence of subharmonics is predicted theoretically by our models, but only once the correct end-wall conditions have been met. The subharmonics are some of the allowed nonlinear normal modes of a system which is highly nonlinear. It is difficult to derive a definitive value for λ based on physical properties of the steel spheres and the holder material – in practice, we have had to adjust the value for λ to obtain the best fit of theory with experiment. Note that the main features (such as subharmonics and multiples thereof) exist for a range of values of λ for each holder material, although subharmonics are never predicted for steel and aluminium, no matter what value of λ is chosen.

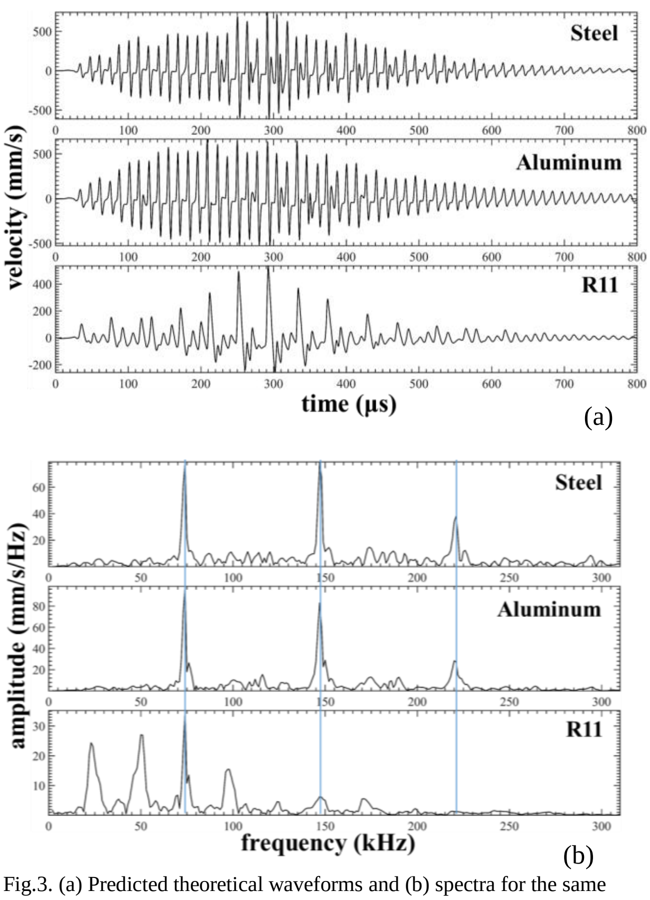


Fig.3. (a) Predicted theoretical waveforms and (b) spectra for the same materials and excitation conditions as used in Fig. 2.

The results thus indicate that the material from which the end wall is made has a direct influence on the output from the spherical chain. This is likely to be due to the reflection properties of the end wall. In particular, this creates new frequency components as a result of bifurcations of energy in the frequency domain. For a holder material with a high elastic modulus, *e.g.* steel, the smaller damping coefficient with a strong reflection from the boundary promotes the rapid propagation of the reflected signal returning to the source along the chain. This makes the period of the impulse close to the period of the input sinusoidal wave. Correspondingly, for the softer R11 acrylic material, the period of the impulses is three times that of the input sinusoidal wave, and the lowest peak in the spectrum is at one third of the frequency of the input. This is because of the larger damping effect of the viscoelastic R11 acrylic material, and the fact that its lower elastic modulus delays the transmission of the reflection wave. Both factors encourage the formation of a set of impulses, whose width in time effectively sets the envelope of the bandwidth in the frequency spectrum. The repetition rate of these impulses then fixes the frequency separation in the frequency domain.

As observed in a previous publication by the authors [13], the length of the chain (as defined by the number of 1 mm diameter spheres present) determines the number and spacing of both the impulses in time, and the frequency peaks in the corresponding spectrum. Longer chains produce a set of impulses which are farther apart in time, as might be expected. This is accompanied by a greater number of equally-spaced frequency peaks, starting at a lower frequency (always a whole fraction of the input frequency of 73 kHz), and being closer together in frequency in longer chains. This is illustrated by results obtained experimentally using the R11 acrylic holder, as presented in Fig. 4, which should be compared to those for the 6-sphere chain (Fig. 2).

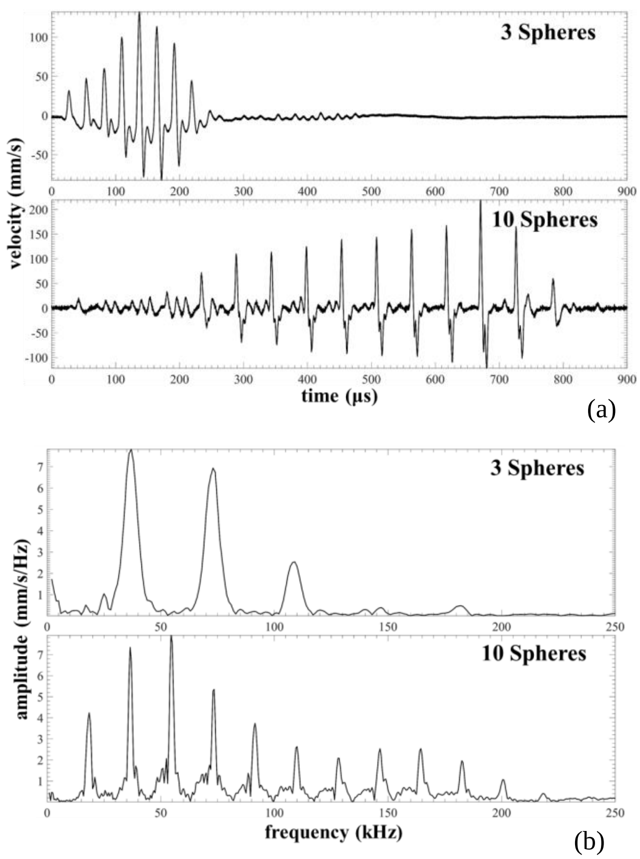


Fig.4. (a) Experimental waveforms and (b) spectra obtained for chains of 3 and 10 chrome-steel spheres of 1 mm diameter, when enclosed within an R11 acrylic holder.

Consider now the cut-off frequency (f_c) expected for this system as a function of pre-compression force (F_0). The predictions of (8) are shown in Fig. 5, where the input frequency of 73 kHz is also indicated. Assuming that the experiments are taking place at a value of $F_0 \leq 0.1$ mN [13], all three end-wall materials give outputs that exceed the predicted value of f_c from Fig. 5 of 180 kHz. This is quite reasonable, because (8) was based on a linear assumption. In fact, the highest frequency generated by the system depends on the nonlinear Hertzian contact and any dissipation. The low-pass filter analysis provided by a linear discrete lattice model does not apply to the current spherical chain system of finite length.

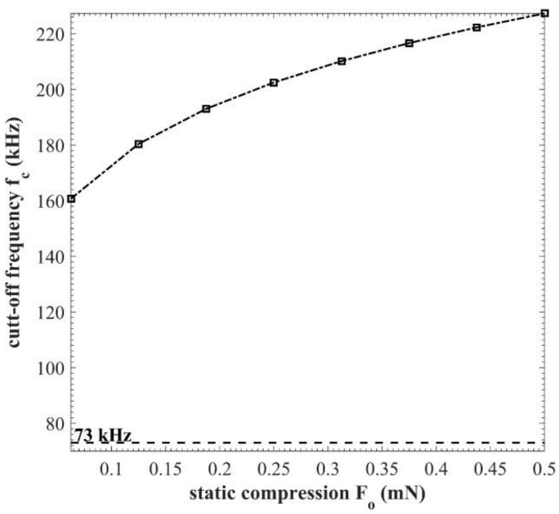


Fig. 5. Predictions of eq. (7) for the expected cut-off frequency for solitary waves in chains of 1 mm diameter chrome steel spheres

It is clear from the results of Figs. 2 and 3 that the properties of the end-wall material have a significant effect on the resulting response of the chain. It is thus also interesting to examine the motion of the end wall. This has been measured experimentally for the R11 material, and the result is shown in Fig. 6. Note that there is some coupling of energy into the end wall, with two main features. The first is a resonance of the holder system at ~ 1.5 kHz. The second is higher frequency coupling of the ultrasonic signal from the sphere motion. Note that, compared to the motion of the steel spheres (Fig. 2), the end-wall moves with an amplitude of approximately 3% of that of the sphere itself. This would help to explain some of the results, in that the end-wall of the holder does move, and this will affect the form of the energy reflected back along the chain, in terms of the amplitude and phase of the reflected impulses, as has already been observed. It also means that some energy is lost to the holder material, which will affect the estimate of the damping coefficient. This will be different for each holder/end-wall material, as already discussed.

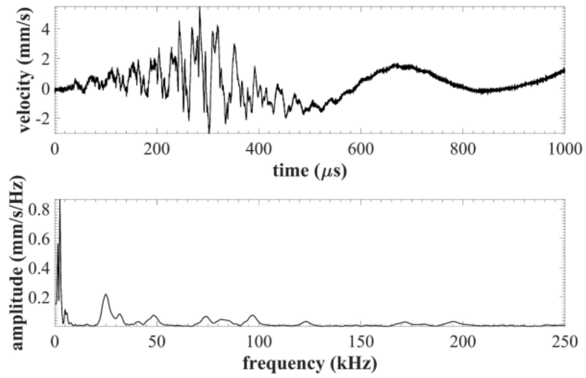
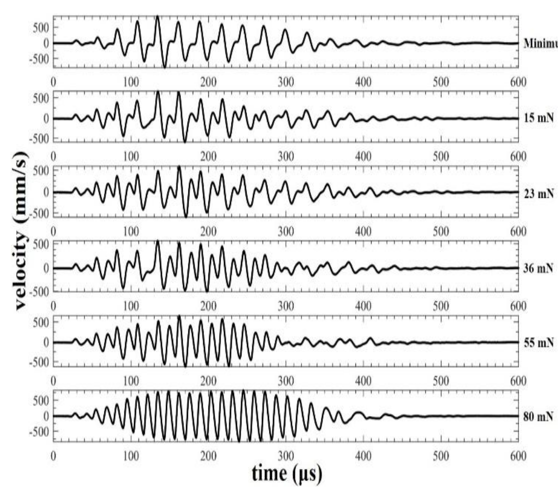


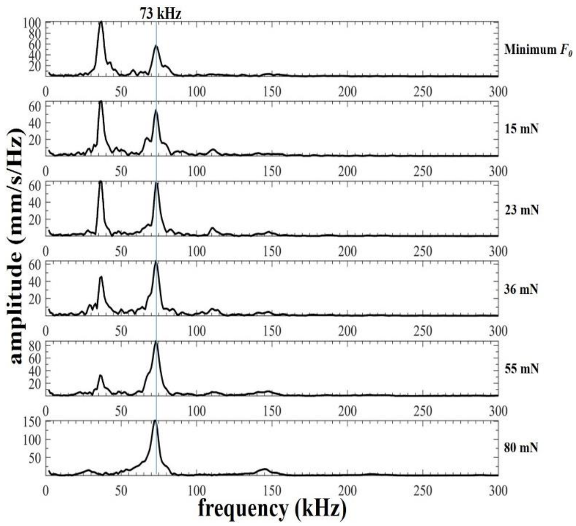
Fig. 6. Measurement of the motion of the end wall of the R11 acrylic holder using the vibrometer. (top)Time waveform; (bottom) corresponding spectrum.

B. The Effect of Pre-compression

Having characterised the response of the system without pre-compression, the stylus and fulcrum were now attached as shown in Fig. 1. The response of a chain of three chrome steel spheres of 1 mm diameter to increased levels of F_0 is shown in Figs. 7(a) and 7(b) for waveforms and spectra respectively. It will be seen from the time-domain data of Fig. 7(a) that the periodic impulses created using the minimum F_0 (estimated at 10 mN) start to become less distinct as the pre-compression force increases. By 55 mN pre-compression, the impulses are barely visible, and by 80 mN the signal is dominated by the periodicity expected from the input signal at 73 kHz. Thus, it can be concluded that with an additional 80 mN of pre-compression, the impulses have been damped out by the increasing stiffness of the chain. The spectra of Fig. 7(b) reinforce this view – increased pre-compression is associated with supression of the sub-harmonics seen when a strong NNM is present, and a corresponding reduction in the presence of harmonics. The system has thus become weakly non-linear, decreasing non-linearity, and suppressing the presence of the solitary wave impulses.



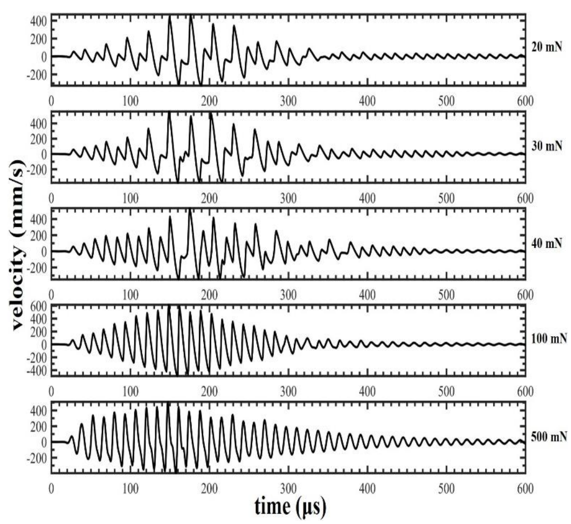
(a)



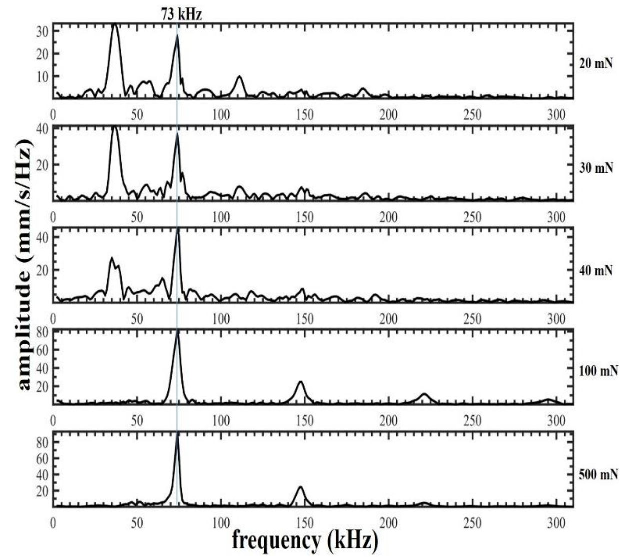
(b)

Fig. 7. Output waveforms for a chain of 3 spheres excited at 73 kHz and using the fulcrum to vary the pre-compression (a) time waveform, and (b) corresponding frequency spectra obtained via an FFT.

The model described by (9) allows both the pre-compression force, and the amount of viscous damping present, to be varied, and has been used to predict the expected outputs for the chain of 3 spheres. The results of Fig. 8 were obtained using a viscous damping coefficient of 0.23 Nsm^{-1} for the theoretical model.



(a)



(b)

Fig.8. Theoretical predictions for a 3-sphere chain using a tone-burst input of from an ultrasonic horn at 73 kHz. Waveforms (left) and Spectra (right) at various levels of pre-compression force F_0, and assuming the maximum input amplitude from the ultrasonic horn (Fig. 1(c)). Results are shown in terms of (a) the time waveform, and (b) corresponding frequency spectra obtained via an FFT.

Comparison of Figs. 7 and 8 indicates that the theoretical predictions have many features in common with the experimental results. The waveforms obtained when the chain was subject to a pre-compression force from 20 – 40 mN contained increasing amounts of the input frequency. This was accompanied by a reduction in the relative amplitude of the sub-harmonic frequency peak at 37 kHz as the pre-compression was increased further. The theory also explores the response at relatively high pre-compression levels where, as expected, the response is dominated by the input frequency. There is also some first harmonic signal still present, due to the existence of limited non-linearity in the stiffened, weakly-non-linear chain.

Pre-compression is known to affect propagation delays along granular chains [17,18]. It was thus also interesting to establish a method whereby the time of flight of the signals along the chain could be measured, and hence the velocity of the signal along the chain determined. Wavelet decomposition provided the ability to decompose a waveform into multiple levels with precise information in the time and frequency domains [21], and allowed the time delays along the chain to be determined. This could be carried out for both the experimental data and theoretical predictions. Consider then the propagation velocity of the solitary wave pulses along the three-sphere chain, excited by a tone-burst at 73 kHz at the maximum input amplitude available (a particle velocity amplitude of 600 mm/s in this case). The results are presented in Fig. 9. It is interesting to correlate Fig. 9 with the waveforms and spectra of Figs. 7 and 8. At levels of pre-compression below 50 mN, the spectra that resulted were characteristic of NNMs being present, with strong subharmonics (as in Figs. 7 and 8 at low pre-compression amplitudes). Thereafter, a transition occurred, and the sub-harmonics started to be supressed, with the input signal becoming more dominant, indicative of weakly nonlinear behaviour.

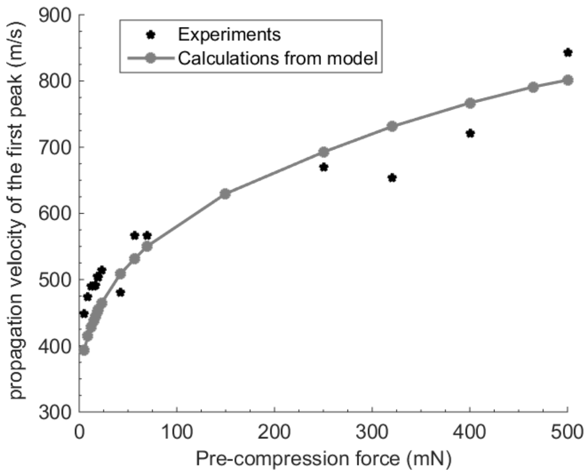


Fig. 9. Changes in propagation velocity for a three-sphere chain excited by a tone burst at 73 kHz at various levels of pre-compression force F_0. Results are shown for the maximum input amplitude, with the time delay along the chain estimated using wavelet decomposition methods. Results are shown after processing waveforms from both experimental data and theoretical predictions for an R11 end-wall of the holder.

It was observed that there was a general increase in velocity along the chain with increased pre-compression force, with the value increasing from 503 m/s to 843 m/s as the pre-compression force increased from 20 to 500 mN. Note that the experimental and theoretical curves exhibit similar trends. In the strongly nonlinear regime, the velocity changes greatly with small changes in pre-compression force. This is expected, in that the system will be very sensitive to the initial conditions. With increased pre-compression, the system changes to being weakly nonlinear, and the change in velocity will be less marked.

Similar trends were seen in different lengths of chains using the 1 mm diameter steel ball-bearings. This included the gradual suppression of the impulses seen at minimal pre-compression, and the increasing dominance of the input frequency of 73 kHz in the signal. Fig. 10 shows an example for a chain containing 10 spheres. It can be seen that similar trends, namely that the velocity along the chain increased steadily with additional pre-compression. There was a period of instability in the experimental velocity values at low levels of F_0 in the experimental data, not replicated theoretically. The reasons for this are not clear. As for the case of the three-sphere chain, this region exhibited characteristics expected from the presence of NNMs, with strong subharmonics in the spectrum indicating a strongly-nonlinear response. At pre-compression values above 100 mN, sub-harmonics were suppressed, with changing amounts of harmonics of the input frequency, indicative of weakly nonlinear behaviour.

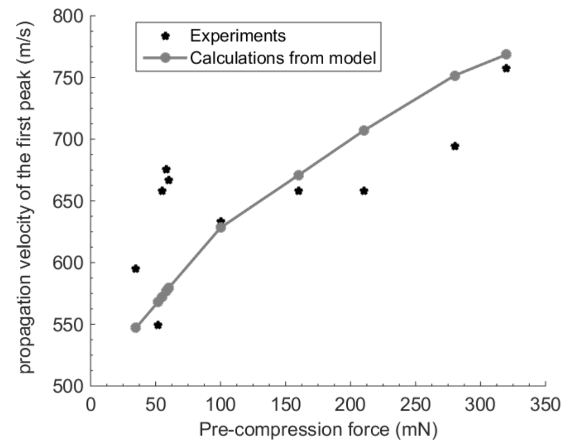


Fig.10. As Fig. 9, but for a 10-sphere chain.

V. Discussion and Conclusions

The work described in this paper demonstrates that resonant granular chains can exhibit distinct properties. The chains themselves are able to generate an impulse train at high amplitude. This is achieved via the creation of a resonant chain of spheres, within which Nonlinear Normal Modes (NNMs) can exist at ultrasonic frequencies. However, this requires careful design. The spheres have to be small enough that the spheres can exhibit individual motion relative to each other at the frequencies of interest, and this has to be below a certain cut-off frequency. If this condition holds, then relative motion between spheres leads to a strongly nonlinear interaction at the interface between them due to Hertzian mechanics. Provided the input signal (in terms of a force F_m) is much greater than any static force holding them together (F_0), then the spheres can even lose contact, a feature of a strong NNM. However, the pre-compression forces needed to achieve this are very small (10 mN in the present case). This requires care over the design of the experimental system.

A highly non-linear system would be expected to be very sensitive to boundary conditions, and this is the case here. Thus, for example, changing the end-wall material in contact with the final sphere of the chain has a large effect on the response. In the present study, only an acrylic polymer led to the creation of the interesting impulse trains in the time domain. This indicates how sensitive such systems are to boundary conditions. The effect of pre-compression force was also studied, and it was demonstrated that an additional 100 mN of pre-compression was enough to damp out the subharmonics in the frequency response of the chains. This was due to characteristics of the system gradually changing from being highly-nonlinear to being only weakly-nonlinear. This was accompanied by a gradual increase in propagation velocity along the chain. In addition to the above, the effect of the damping coefficient is of interest, and perhaps should be examined further. It has been known to be a sensitive parameter in theoretical modelling predictions, and it is likely that experimentally it is a factor in determining the nature of the response of the chain. This and other factors could perhaps be explored further, for example by using Finite element (FE) models of inter-granular interactions, and the effects of end-wall conditions [22].

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