History: Sunk Cost, or Widespread Externality?

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History: Sunk Cost, or Widespread Externality?*

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Abstract

In an intertemporal Arrow–Debreu economy with a continuum of agents, suppose that the auctioneer sets prices while the government institutes optimal lump-sum transfers period by period. An earlier paper showed how subgame imperfections arise because agents understand how their current decisions such as those determining investment will influence future lump-sum transfers. This observation undermines the second efficiency theorem of welfare economics and makes "history" a widespread externality. A two-period model is used to investigate the constrained efficiency properties of different kinds of equilibrium. Possibilities for remedial policy are also discussed.

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> The experience of the old Poor Law has made people very much afraid that any expectation of assistance from public funds will tempt the poor into idleness and thriftlessness. ...In reality different types of transference act in different ways .... The main lines of division are between transferences which differentiate against idleness and thriftlessness, transferences which are neutral, and transferences which differentiate in favour of idleness and thriftlessness.
>
> A.C. Pigou (1932, p. 720)

1 Introduction: Intertemporal Pareto Efficiency

1.1 The Efficiency Theorems of Intertemporal Welfare Economics

The two fundamental efficiency theorems of welfare economics form the basis of most economists' theoretical presumption that market forces should generally be encouraged wherever possible. The first of these theorems says that perfectly competitive and complete markets produce Pareto efficient allocations. This is true without any additional assumptions, provided that one uses a weak definition of Pareto efficiency — namely, that not all individuals in the economy can be made better off simultaneously by moving to some new feasible allocation. For the more usual definition of Pareto efficiency — namely, that no set of individuals in the economy can be made better off without making some others worse off — the theorem relies on the rather mild assumption that all individuals have locally non-satiated preferences.

By itself, this first efficiency theorem is not at all satisfying from an ethical point of view. It says only that "perfect" markets produce Pareto efficient outcomes, without any mention at all of distributive justice. Indeed, dictatorships and extreme inequality — even slavery (Bergstrom, 1971) or starvation (Coles and Hammond, 1991) — can all be Pareto efficient.

For this reason the second efficiency theorem is ethically much more satisfying, since it characterizes (virtually) *all* Pareto efficient allocations, both just and unjust. The only exceptions, typically, are those which violate a boundary assumption and so may give rise to Arrow's (1951) famous "exceptional case." One therefore expects that a truly optimal allocation, fully reflecting ethical views regarding distributive justice, should be among those allocations which are characterized by this second efficiency theorem. Indeed, the theorem tells us that virtually every Pareto efficient allocation could result from perfectly competitive and complete markets in general equilibrium, *provided* that purchasing power is redistributed by means of suitable "lump-sum" or "distortion free" taxes and transfers before markets are opened to trade. In addition, various well known convexity and continuity assumptions do have to be satisfied.

These two efficiency theorems were stated and proved, with increasing degrees of generality and mathematical rigour, by a series of writers — including Pareto (1906, 1909), Barone (1908), Lange (1942), Allais (1943), Lerner (1947), and Samuelson (1947) — before Arrow (1951) gave the first complete and definitive treatment. Koopmans

(1957) and Debreu (1959) also provide very elegant presentations of these two theorems.[1] Earlier Debreu (1954) had extended them significantly to the case of a commodity space which may not be finite dimensional.

Most of this work, however, considered only static or one period economies. Yet Fisher (1907, 1930) and Hicks (1939) were able to describe intertemporal allocations of resources by means of bundles of dated commodities. By using this framework, and extending it to allow uncertainty through the apparatus of state contingent commodities, Allais (1947, 1953), Arrow (1953), Malinvaud (1953), and Debreu (1959) were able simply to reinterpret the efficiency theorems in order to treat intertemporally Pareto efficient allocations and the efficiency properties of those allocations which are achieved by complete and perfectly competitive markets for dated (and state contingent) commodities.

1.2 Time Consistency and Subgame Perfection

As this work on static and then intertemporal efficiency theorems was being completed, however, Strotz (1956) started to investigate serious dynamic inconsistency issues which arise in intertemporal consumer theory. Strotz noticed how, after starting off down any optimal path, the relevant continuation of that path might become suboptimal later on, unless individual preferences met a specific consistency condition which he called "harmonious preferences." Later, this work was taken up most directly by Pollak (1968).

Strotz's paper was concerned with particular single person decision problems. More generally, in any n-person game, a solution or equilibrium path is said to be "time consistent" or "dynamically consistent" if the continuation of that solution within any subgame is actually a solution to the subgame. This property will be satisfied by *any* Nash equilibrium, provided that the players in the game all have dynamically consistent preferences.

Somewhat later, Schelling (1960), Farquharson (1969), Friedman (1971), Hammond (1975), and especially Selten (1965, 1973, 1975) noticed how related but slightly different problems could arise in game theory, even for Nash equilibrium in games whose players all have dynamically consistent preferences. Indeed, even if one has a time consistent Nash equilibrium, deviations from the equilibrium path could still benefit some players if the reactions by the other players have to be best responses in any subgame which is reached even *off* the equilibrium path. "Subgame perfect" equilibria are those in which no such deviations are worthwhile. Time consistency, on the other hand, merely requires the players' equilibrium strategies to be best responses within those subgames which are reached *on* the equilibrium path. This still allows Nash equilibrium strategies not to be best responses to deviations from the equilibrium path.

Considerations of this kind led Phelps and Pollak (1968) and Phelps (1975), followed eventually by Kydland and Prescott (1977), to begin considering subgame perfection issues in connection with first macroeconomic and later microeconomic policy. Just a few writings in a large and expanding field include Fischer (1980), Kotlikoff, Persson

[1] For an attempt to synthesize and extend these results, see Hammond (2007).

and Svensson (1987), Staiger and Tabellini (1987), Chari (1988), Maskin and Newbery (1990), Karp and Newbery (1993). In addition, Rogers (1986, 1987, 1991) and Judd (1985, 1987) have some especially clear discussions of how, although *ex ante* it is optimal to tax only wage income (in their models with inelastic labour supply), *ex post* it is optimal to impose redistributive taxation on capital as well. More recently, Bliss (1991) discusses how such concerns arise in practical policy issues connected with compensating workers who have been adversely affected by the abolition of obstacles to free trade within the European Community. Finally, Willmann (2004) shows how, in a two-period general equilibrium model with heterogeneous agents, Pareto gains from trade may be unachievable if the government uses lump-sum redistribution after trade liberalization without being able to commit to a particular redistributive policy beforehand. The reason is that agents anticipate the intervention and, by underinvesting strategically, remove all the gains from trade.

As is clear even from the title of some of these papers, it has been all too common for economists to write of the weak concept of "time consistency" when they really mean the stronger concept which game theorists call "subgame perfection." I shall use the latter term throughout the rest of this paper.

Almost all existing work on this subgame perfection problem has typically considered one or more forms of distortionary taxation or other departures from standard first best optimal government policy. Tesfatsion (1986), however, was able to show how exactly the same kind of subgame imperfection could also arise even in standard microeconomic models of intertemporal general equilibrium with first best optimal lump-sum redistribution. Unfortunately, however, her paper lacks a simple example, and this was partly remedied in Hammond (1999). In fact, imperfections of this kind appear seriously to undermine the relevance of the usual efficiency theorems (especially the second, which is in any case really the only ethically interesting one) if the government is unable to commit itself irrevocably to some "benevolent" policy of welfare maximizing income redistribution.

1.3 Outline

This paper will therefore analyse some important features of the widespread externalities that private decisions can create in a sequence economy. In order to do so, Section 2 presents the simplest possible general model of a sequence economy with a continuum of agents and an arbitrary finite set of commodities. One reason to work with a continuum of agents is that aggregate preference sets and demand correspondences acquire suitable convexity properties. But more important is the fact that, as discussed in Hammond (1999), no one agent in a continuum economy has the power to influence equilibrium prices even in the future. Otherwise even the usual Walrasian model of perfect competition violates subgame perfection.

For simplicity, this paper considers an economy lasting only two periods. There is also complete certainty, as well as perfect and complete information. Each agent is assumed to have a feasible set of net trades which is separable into two history dependent feasible sets, one for each period. And to have preferences which can be represented by the sum of two separate history dependent single period utility functions, each defined on the relevant set of feasible net trades. Moreover, a Bergson social welfare function in the form of an integral over all agents' utilities is postulated.

Next, Section 3 briefly reviews work which shows how, under fairly standard assumptions, any intertemporal optimum can be decentralized through intertemporal competitive markets for dated commodities. Equivalently, it can be decentralized through competitive spot markets in each period together with a market for a single Arrow security in the form of a riskless bond. It also considers corresponding properties for all allocations satisfying the stronger "f-optimality" condition, requiring there to be no reallocation among a finite set of agents which increases their total utility.

Within this model, Section 4 finally brings to the fore the subgame perfection issues discussed by Tesfatsion (1986) and Hammond (1999). Of course, an intertemporal optimum prescribes an allocation for both periods together. Moreover, the standard decentralization through complete markets discussed in Section 2 involves lump-sum transfers in the first period only. Nevertheless, subgame perfection implies that lump-sum transfers transfers cannot be prevented from occurring in the second period as well, if individuals choose to depart from the choice of history prescribed by the intertemporal optimum. Indeed, a subgame perfect f-optimal policy in the second period will make the current allocation to each agent depend on that agent's personal history, as well as on the entire distribution of all agents' personal histories. Subgame perfect optimal lump-sum transfers in the second period exhibit exactly the same dependence. Section 4 shows how to describe the relevant history in formal terms. It also shows how policy feeds back from history created in the first period to optimal redistribution in the second period. This is precisely what makes history become a widespread externality in the first period.

Next, Section 5 introduces the notion of a history constraint. It also gives appropriate "history constrained" versions of the usual concepts involved in stating the fundamental efficiency theorems of welfare economics. These include, of course, feasible allocation rules, Pareto efficiency, systems of lump-sum transfers, and both Walrasian and compensated equilibrium relative to such transfer systems. In fact history constrained Pareto efficiency is very similar to the notion of externality constrained Pareto efficiency that was investigated in Hammond (1995). Thereafter, the concluding part of Section 5 presents history constrained versions of the efficiency theorems that apply to the model introduced in Section 2.

The notion of history that is used in Sections 4–5 is extremely broad, because it changes if any non-null set of agents alter their choices of personal history. The corresponding history constraints are therefore very severe. In fact, the choice of an appropriate history in these models is really a public good problem, which has to be made centrally in virtually any realistic economic environment. So it might be highly desirable to have more features of history determined through decentralized markets, if possible. For this reason, Section 6 considers the implications of assuming that history can be described adequately by a finite collection of real-valued sufficient statistics, each of which is the population mean of a suitably defined personal historical variable. An obvious example is when the only historical variables that matter arise from individual agents' decisions to accumulate a finite collection of different physical capital goods, and specifically, it is only aggregate stocks per head which affect what level of social welfare is possible in the second period. It is then shown that Pigovian taxes or subsidies on each separate historical variable allow the constrained efficiency theorems of Section 5 to be derived once more, subject to the obvious weaker history constraints.

Once the optimal choice of history is known, the form of remedial policy is easy to determine, as explained in Section 7. The widespread externality comes about because each agent's transfer is a predictable function of the choice of personal history in the first period, as well as of the distribution of personal histories. This is easily remedied by requiring all agents in the first period to pay the respective present values of any extra transfer which will be due to them individually in the second period because of their choices of personal history in the first period. Obviously, this internalizes all external effects. Putting such a scheme into effect, however, does require knowledge of what the second period transfer system will be, and how to convert the transfers into present values. Also, even in the special framework of Section 6, in general remedial policy cannot be based on linear Pigouvian prices.

Section 8 contains a few brief concluding remarks.

2 A Sequence Economy with a Continuum of Agents

2.1 A Continuum of Agents

Let $(A, \mathcal{B}, \alpha)$ be a measure space of economic agents, where A is a compact metric space, $\mathcal{B}$ is the Borel σ-field, and α is a non-atomic Borel measure with $\alpha(A) = 1$.

2.2 A Two-Period Commodity Space

The commodity space is assumed to be the finite-dimensional Euclidean product space $X \times Y$, where $X := \mathbb{R}^{G_1}$ and $Y := \mathbb{R}^{G_2}$. Each member $(x, y) \in X \times Y$ represents a pair of net trade vectors, with x as the net trade vector for period one, and y as the net trade vector for period two.

2.3 Personal Histories and Individually Feasible Net Trades

It is assumed that there is a set H of possible personal histories at the end of period one. For most of the paper H can be an entirely abstract metric space equipped with its Borel σ-field; only in Section 8 will it need a vector space structure.

Each agent $a \in A$ is assumed to have a set F_a of feasible net trades which can be decomposed into the form

$$F_a = \{ (x, y) \in X \times Y \mid \exists h \in H : (x, h) \in X_a, y \in Y_a(h) \}.$$

Thus, the set X_a represents the feasible choices of combinations (x, h) of a net trade vector with a personal history in period one. Also, for each $h \in H$, the set $Y_a(h)$

represents the feasible choices of net trade vector in period two, conditional on the choice of h as a personal history in period one. The implicit assumption is that personal history is always richly enough described so that it completely determines what is feasible for the agent in period two.

For the usual technical reasons, it will be assumed that the feasible set correspondences defined by $a \mapsto\!\!\!\to X_a$ and by $(a,h) \mapsto\!\!\!\to Y_a(h)$ both have measurable graphs.

Finally, to avoid some unimportant technical difficulties, I will assume *free disposal* of traded goods at the individual level. Hence, in the first period, if $(x,h) \in X_a$ and $x' \geqq x$, then $(x',h) \in X_a$. Similarly, in the second period, if $y \in Y_a(h)$ and $y' \geqq y$, then $y' \in Y_a(h)$.

2.4 Individually Feasible Allocations

An *individually feasible allocation* $(\mathbf{x}, \mathbf{h})$ in the first period economy will be a measurable mapping $a \mapsto (x_a, h_a)$ with the property that $(x_a, h_a) \in X_a$ (α-a.e. in A).

In the second period, even the definition of an individually feasible allocation will depend on the distribution of personal histories at the end of the first period. Given the space $A \times H$ equipped with its product topology, this distribution will be described by a member ν of the space $\Delta_\alpha(A \times H)$ of Borel measures over $A \times H$ satisfying the restriction that $\nu(A' \times H) = \alpha(A')$ for all Borel sets $A' \subset A$ — in other words, the marginal distribution on A induced by the joint distribution ν on $A \times H$ must equal α. Then, an *individually feasible allocation* $\mathbf{y}$ in the second period economy will be a measurable mapping $(a,h) \mapsto y_a(h)$ with the property that $y_a(h) \in Y_a(h)$ for ν-a.e. in $A \times H$.

2.5 The Attainable Set

Free disposal at the aggregate level will not be assumed. It seems better to postulate instead that disposal is only possible if some individual agent can undertake it. It is also assumed that there is no production except by individual agents. Accordingly, an *attainable allocation* in the first (resp. second) period economy will be one that is individually feasible and also satisfies the first (resp. second) period *aggregate feasibility constraint* $\int_A x_a d\alpha = 0$ (resp. $\int_{A \times H} y_a(h) d\nu = 0$).

2.6 Additive Social Welfare

As has been usual in models with a continuum of agents, it is assumed that the Bergson social welfare function at the start of the first period takes the integral form

$$\Omega_1(\mathbf{x}, \mathbf{h}, \mathbf{y}) \equiv \int_{A \times H} U_a(x_a, h, y_a(h)) d\nu$$

for some suitable family of continuous cardinal utility functions U_a defined on agents' feasible sets F_a. In fact, for obvious technical reasons, it is also assumed that the mapping $(a, x, h, y) \mapsto U_a(x, h, y)$ is jointly measurable in all four variables.

In the sequence economy, subgame perfection forces us to consider optimal policies in each possible second period subeconomy, starting with a given history described by a distribution $\nu \in \Delta_\alpha(A \times H)$. It will be assumed that this history is sufficient to determine the second period social welfare function, and that this takes the integral form

$$\Omega_{2,\nu}(\mathbf{y}) \equiv \int_{A \times H} v_a(y_a(h); h) d\nu$$

for some suitable family of continuous cardinal utility functions $v_a(\cdot; h)$ defined on agents' conditionally feasible sets $Y_a(h)$. The notation is deliberately chosen to suggest that personal history h should be regarded as a parameter of the history-dependent utility function for the second period, rather than as an argument of that utility function.

2.7 Additive Utility

In order that both these additive separability assumptions be satisfied simultaneously, the most reasonable assumption is that in fact

$$\Omega_1(\mathbf{x}, \mathbf{h}, \mathbf{y}) \equiv \int_A [u_a(x_a, h_a) + v_a(y_a; h_a)] d\alpha$$

for a suitable family of continuous single period utility functions $(u_a, v_a)_{a \in A}$ with the indicated arguments. Note that h_a is regarded as an argument of $u_a(x_a, h_a)$, because this "myopic" utility function is intended to represent a's preferences for combinations of net trades and personal histories, while disregarding the effect of personal history on future utility. The obvious implication of this assumption is that, for almost all agents $a \in A$, the preferences of agent a can be represented by the additively separable utility function $U_a(x, h, y) \equiv u_a(x, h) + v_a(y; h)$. In fact, I shall assume that this is true for *every* agent $a \in A$, not just almost every agent. Moreover, for the usual technical reasons, it is now assumed that the mappings $(a, x, h) \mapsto u_a(x, h)$ and $(a, h, y) \mapsto v_a(y; h)$ are jointly measurable with respect to their respective sets of three variables.

2.8 Sequential Monotonicity

Local non-satiation, or a stronger sufficient condition for local non-satiation such as monotone preferences, plays a significant role in the usual fundamental efficiency theorems of welfare economics. The corresponding assumption here is *sequential monotonicity* requiring that, for each agent $a \in A$, both single period utility functions are monotone in that period's net trade vector — i.e., for all $h \in H$, the function $u_a(x, h)$ is monotone in x, whereas $v_a(y; h)$ is monotone as a function of y. In symbols, this means that in the first period, whenever $(x, h) \in X_a$ with $x' \geqq x$, then $u_a(x', h) \geq u_a(x, h)$ with strict inequality if $x' \gg x$. Similarly, in the second period, whenever $y \in Y_a(h)$ and $y' \geqq y$, then $v_a(y'; h) \geq v_a(y; h)$ with strict inequality if $y' \gg y$.

2.9 Example

A simple example of such a two-period economy is the following, whose properties will be discussed further in several of the later sections. Each agent $a \in A = [0, 1]$ has fixed endowments $e_a, f_a \in \mathbb{R}^G_{++}$ of all commodities in the set G within each of the two periods. Obviously, it is assumed that the two functions $e, f : A \to \mathbb{R}^G_{++}$ are both integrable. Then the mean endowment vectors $\bar{e} := \int_A e_a \, d\alpha$ and $\bar{f} := \int_A f_a \, d\alpha$ for both periods are well defined. For reasons to be explained in Section 3.1, it will be assumed that $\bar{e} \gg \bar{f}$.

A personal history $h_a \in \mathbb{R}^G_+$ will be a non-negative commodity vector representing what agent a stores at the end of the first period. Its effect is that the agent begins period two with the new endowment vector $f_a + h_a \in \mathbb{R}^G_{++}$. If agent a has the two-period net trade vector (x_a, y_a) and the personal history h_a, the resulting two-period consumption stream is

$$(c_a, d_a) := (x_a + e_a - h_a, y_a + f_a + h_a).$$

It is then assumed that individual feasibility requires $(c_a, d_a) \geqq (0, 0)$ to be satisfied. This implies that the individual feasible sets take the form:

$$X_a = \{ (x_a, h_a) \in \mathbb{R}^G \times \mathbb{R}^G_+ \mid x_a - h_a \geqq -e_a \}; \quad Y_a(h_a) = \{ y_a \in \mathbb{R}^G \mid y_a \geqq -f_a - h_a \}.$$

Finally, it is assumed that each agent in each period has the same single-period utility function $u(c)$ of consumption c, where u is twice continuously differentiable with gradient vector $u'(c) \gg 0$ and with its Hessian matrix $u''(c)$ negative definite for all $c \geqq 0$.

3 Intertemporal Optimality and Market Decentralization

3.1 Intertemporal Optimality

The usual concept of an *intertemporal welfare optimum* is evidently an attainable allocation $(\hat{\mathbf{x}}, \hat{\mathbf{h}}, \hat{\mathbf{y}})$ of net trade vectors in each period, and of personal histories at the end of the first period, which together maximize the welfare integral $\int_{A \times H} U_a(x_a, h_a, y_a) d\alpha$ subject to individual and aggregate feasibility constraints. Any such welfare optimum is evidently *intertemporally Pareto efficient* in the sense that any alternative attainable allocation $(\mathbf{x}, \mathbf{h}, \mathbf{y})$ for which

$$U_a(x_a, h_a, y_a) \geq U_a(\hat{x}_a, \hat{h}_a, \hat{y}_a) \text{ (α-a.e. in A)}$$

must in fact have

$$U_a(x_a, h_a, y_a) = U_a(\hat{x}_a, \hat{h}_a, \hat{y}_a) \text{ (α-a.e. in A).}$$

In fact, however, this paper will use a stronger concept of optimality, which pays attention even to null sets of agents. An *f-optimum*[2] is an intertemporal welfare opti-

[2] A term evoking the "f-core" of Kaneko and Wooders (1986, 1989) and of Hammond, Kaneko and Wooders (1989) has deliberately been chosen, since the f-core is also sensitive to the fates of finite sets of agents in a continuum economy.

mum with the additional property that, for all finite sets of agents $C \subset A$, the allocation $(\hat{x}_a, \hat{h}_a, \hat{y}_a)$ $(a \in C)$ to the members of C maximizes the utility sum $\sum_{a \in C} U_a(x_a, h_a, y_a)$ subject to the individual feasibility constraints $(x_a, h_a, y_a) \in F_a$ $(a \in C)$, as well as the aggregate feasibility constraints $\sum_{a \in C} x_a = \sum_{a \in C} \hat{x}_a$ and $\sum_{a \in C} y_a = \sum_{a \in C} \hat{y}_a$. That is, the resources that are made available to any finite set of agents C must also be distributed optimally among the members of C.

For the special example described in Section 2.9, the intertemporal f-welfare optimum not only exists, but is also unique and easy to describe. It is simply the consumption allocation satisfying $c_a = d_a = \frac{1}{2}(\bar{e} + \bar{f})$ for *all* $a \in A$, and not just for almost all $a \in A$. The reason for assuming that $\bar{e} \gg \bar{f}$ was precisely to allow the f-optimum to be characterized this simply. Because each agent in the first period consumes less than the existing mean endowment of each commodity, there is no need to consider any possible corner solution.

This example also illustrates how, when marginal utilities of income are well defined, an f-optimum involves choosing income levels which equate those marginal utilities for *all* individuals, not just almost all individuals. Usually there is no problem in doing this.

3.2 Compensated Equilibrium

In order to show that an f-optimum can be decentralized in the usual way as a Walrasian equilibrium relative to a system of lump-sum transfers, the following constructions are useful. First, define the new feasible set

$$F_a^* := \mathrm{proj}_{X \times Y}\, F_a := \{\, (x, y) \in X \times Y \mid \exists h \in H : (x, h, y) \in F_a \,\}$$

of possible net trade vectors, and the new utility function

$$u_a^*(x, y) := \max_{h \in H} \{\, u_a(x, h) + v_a(y; h) \mid (x, h, y) \in F_a \,\}$$

on this domain. It is assumed, of course, that the relevant maximum is always achieved; a sufficient condition for this is that the set

$$H(x, y) := \mathrm{proj}_H\, F_a := \{\, h \in H \mid (x, h, y) \in F_a \,\}$$

of histories that are compatible with (x, y) is compact. By Berge's maximum theorem, the function $u_a^*(x, y)$ must be continuous. Denote the corresponding weak preference relation on F_a^* by $\succsim_a$, and the corresponding strict preference relation by $\succ_a$. Let

$$\begin{aligned} R_a(x, y) &:= \{\, (x', y') \in F_a^* \mid (x', y') \succsim_a (x, y) \,\} \\ P_a(x, y) &:= \{\, (x', y') \in F_a^* \mid (x', y') \succ_a (x, y) \,\} \end{aligned}$$

denote the corresponding upper contour and strict preference sets, respectively.

Now, arguing as in Hildenbrand (1974), given the optimal allocation $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ of net trade vectors, which must also be Pareto efficient, the set $P := \int_A P_a(\hat{x}_a, \hat{y}_a) d\alpha$ cannot contain the origin of the product space $X \times Y$. Moreover, because the measure α is non-atomic, the set P is convex. By local non-satiation, however, the origin must be a boundary point of P. Hence there exists a hyperplane $p\,x + q\,y = 0$ through the origin of $X \times Y$ which has the set P all on one side. That is, there exists a price vector $(p, q) \neq (0, 0)$ such that $(x, y) \in P \implies p\,x + q\,y \geq 0$. Using local non-satiation once again, it follows that for almost all $a \in A$ one has $(x_a, y_a) \in P_a(\hat{x}_a, \hat{y}_a)$ implies $p\,x_a + q\,y_a \geq p\,\hat{x}_a + q\,\hat{y}_a$. But then, because each agent's utility function is continuous, it also follows that for almost all $a \in A$ one has

$$(x_a, y_a) \in R_a(\hat{x}_a, \hat{y}_a) \implies p\,x_a + q\,y_a \geq p\,\hat{x}_a + q\,\hat{y}_a.$$

In this sense, the optimal allocation $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ of net trade vectors is a *compensated equilibrium* at prices (p, q) relative to the wealth distribution given by $w_a(p, q) := p\,\hat{x}_a + q\,\hat{y}_a$ for all $a \in A$. Moreover, it is a compensated equilibrium in which individuals are free to choose their own personal histories.

3.3 Uncompensated Equilibrium

Under additional assumptions such as those discussed in Hammond (2004), one can show that $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ is an *uncompensated* or Walrasian equilibrium at prices (p, q) relative to the wealth distribution $w_a(p, q)$, in the sense that, for almost all $a \in A$, one has

$$(x_a, y_a) \in P_a(\hat{x}_a, \hat{y}_a) \implies p\,x_a + q\,y_a > p\,\hat{x}_a + q\,\hat{y}_a =: w_a(p, q).$$

Note in particular that, because of sequential monotonicity, a Walrasian equilibrium price vector satisfies both $p > 0$ and $q > 0$.

3.4 Full Walrasian Equilibrium

One question that naturally arises is whether an f-optimum $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ might also be a *full* Walrasian equilibrium in the sense that, for *all* $a \in A$ without exception, one has

$$(x_a, y_a) \in P_a(\hat{x}_a, \hat{y}_a) \implies p\,x_a + q\,y_a > p\,\hat{x}_a + q\,\hat{y}_a = w_a(p, q).$$

In fact, it is rather evident that this need not be true in general economies without smooth preferences, especially if there may be individual non-convexities. Nevertheless, if there is some (null) set of agents C who are not in Walrasian equilibrium at the optimum $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$, it is obviously possible to find a new allocation $(\mathbf{x}^*, \mathbf{y}^*)$ which coincides with $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ for all $a \in A \setminus C$, and which is a Walrasian equilibrium for all agents because, even for $a \in C$, it is true that (x_a^*, y_a^*) together maximize $u_a^*(x_a, y_a)$ subject to $(x_a, y_a) \in F_a^*$ and $p\,x_a + q\,y_a \leq p\,\hat{x}_a + q\,\hat{y}_a = w_a(p, q)$. Since the allocation $(\hat{\mathbf{x}}, \hat{\mathbf{y}})$ has been changed for only a null set of agents, (x_a^*, y_a^*) is obviously feasible, so it is a Walrasian f-equilibrium at prices (p, q) given the transfer system $w_a(p, q)$. Moreover, the welfare integral cannot have been changed. Therefore, (x_a^*, y_a^*) is also a welfare optimum. Furthermore, since it is a Walrasian f-equilibrium, a standard proof of the first efficiency theorem of welfare economics shows that it must be an f-optimum.

Hence, when an optimal allocation is a Walrasian equilibrium, as discussed in Section 3.3, then there is no loss of generality in considering f-optimal allocations that are full Walrasian equilibria relative to appropriate rules for lump-sum wealth redistribution.

3.5 Sequential Decentralization with an Arrow Security Market

The fundamental idea of Arrow (1953, 1964) is easy to apply to this intertemporal welfare optimum. Since there is no uncertainty, and only two periods, complete markets for Arrow securities in fact require only a market for a single riskless Arrow security that pays one unit of the *numéraire* for sure in period two. With such a security, the Walrasian equilibrium $(\hat{\mathbf{x}}, \hat{\mathbf{y}}, p, q)$ relative to the wealth distribution $w_a(p, q)$ corresponds to an equilibrium $(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{b}}, p, q, \rho)$ in spot markets each period, as well as in the market for the single Arrow security whose price in the first period is ρ, which must be positive because $q \neq 0$. Also, b_a denotes agent a's net purchase of this security. However, each agent's single budget constraint

$$p\, x_a + q\, y_a \leq w_a(p, q)$$

must be replaced by an equivalent pair of budget constraints, one for each separate period, of the particular form

$$p\, x_a + \rho\, b_a \leq w_a(p, q); \quad (q/\rho)\, y_a \leq b_a.$$

Note that the equilibrium value of ρ is actually completely indeterminate; if ρ is multiplied by any positive scalar λ, there is still essentially the same Walrasian equilibrium after each agent's bondholding b_a has been divided by λ. For this reason, it is harmless to normalize and take $\rho = 1$.

3.6 Example

In the example that was discussed in Sections 2.9 and 3.1, the intertemporal f-welfare optimum involves the consumption allocation with $c_a = d_a = \frac{1}{2}(\bar{e} + \bar{f})$ for all $a \in A$. Given any storage allocation described by an integrable mapping $a \mapsto h_a \in \mathbb{R}^G_+$ satisfying $\int_A h_a d\alpha = \frac{1}{2}(\bar{e} - \bar{f})$, there is an associated net trade allocation given by

$$x_a = \frac{1}{2}(\bar{e} + \bar{f}) - e_a + h_a \text{ and } y_a = \frac{1}{2}(\bar{e} + \bar{f}) - f_a - h_a \text{ for all } a \in A.$$

In the special case when $h_a = \frac{1}{2}(\bar{e} - \bar{f})$ for all $a \in A$, the net trade allocation simplifies to $x_a = \bar{e} - e_a$ and $y_a = \bar{f} - f_a$ for all $a \in A$.

No matter what the storage allocation $a \mapsto h_a \in \mathbb{R}^G_+$ satisfying $\int_A h_a d\alpha = \frac{1}{2}(\bar{e} - \bar{f})$ may be, the f-optimal allocation can be decentralized by the single budget constraint

$$p\, x_a + p\, y_a \leq p\,(\bar{e} + \bar{f} - e_a - f_a),$$

where $p \in \mathbb{R}^G_{++}$ is any vector proportional to the gradient vector $u'(\frac{1}{2}(\bar{e} + \bar{f}))$. Alternatively, if a market for Arrow securities is introduced, the f-welfare optimum can be decentralized by the pair of budget constraints

$$p\, x_a + b_a \leq p\,(\bar{e} + \bar{f} - e_a - f_a) \text{ and } p\, y_a \leq b_a.$$

4 Subgame Imperfection

4.1 Second Period f-Optimality

Suppose that the first period of the economy is already over and that the different agents' allocations of (x_a, h_a) (all $a \in A$) have given rise to the joint distribution $\nu \in \Delta_\alpha(A \times H)$ of agents' labels and personal histories. Then there is a well-defined second period objective given by the welfare integral $\int_A v_a(y_a; h_a) d\alpha$. This should be maximized subject to the remaining individual feasibility constraints $y_a \in Y_a(h_a)$ (all $a \in A$) and the aggregate feasibility constraint $\int_A y_a d\alpha = 0$.

In the particular case when the first period allocations are all equal to those for the intertemporal welfare optimum $(\hat{\mathbf{x}}, \hat{\mathbf{h}}, \hat{\mathbf{y}})$, namely $(\hat{x}_a, \hat{h}_a)$ (all $a \in A$), the resulting history $\hat{\nu}$ is optimal, and a welfare maximizing continuation is indeed to proceed with $\mathbf{y}$, as originally planned. There is no subgame imperfection here, of course. Moreover, exactly the same policy appears at first to remain optimal even if a null set of agents deviate from their optimal allocations, so that $(x_a, h_a) = (\hat{x}_a, \hat{h}_a)$ only for almost all $a \in A$, and not for all $a \in A$. After all, such a deviation makes no difference to the distribution $\hat{\nu}$, nor to the maximized value $\int_A [u_a(\hat{x}_a; \hat{h}_a) + v_a(\hat{y}_a; \hat{h}_a)] d\alpha$ of the welfare integral that results from continuing with the second-period plan $\mathbf{y}$. Indeed, since the overall allocation is effectively the same even if a null set of agents have different allocations, one could well argue that such deviations should be ignored. If they are, the intertemporal welfare optimum $(\hat{\mathbf{x}}, \hat{\mathbf{h}}, \hat{\mathbf{y}})$ is subgame perfect, and there is nothing more to write about.

Neglecting null sets of agents in this way, however, poses fundamental difficulties. For one thing, there can be no corresponding limit result for large finite economies; in any finite economy, no matter how large, if any one agent were to have a different history at the start of period two, the second period welfare optimal allocation to that agent would be affected. Furthermore, if individual feasibility constraints are to be taken seriously, what are we to do about the (null) set of agents for whom $\hat{y}_a \notin Y_a(h_a)$ after a deviation from $\hat{h}_a$ to h_a? Such agents, for instance, played an important role in creating a need for additional incentive constraints in the two-period economy considered by Hammond (1992). And the blocking powers of finite sets of agents in a continuum economy were used to prove an f-core equivalence theorem in Hammond, Kaneko and Wooders (1989).

Accordingly, as with the f-optimum considered in Section 3.1, I am going to require that the second-period allocation be an f-optimum in each subeconomy. That is, not only should it maximize $\int_A v_a(y_a; h_a) d\alpha$ subject to $y_a \in Y_a(h_a)$ (almost all $a \in A$) and $\int_A y_a d\alpha = 0$. In addition, for any finite set of agents $C \subset A$, it should also maximize the sum $\sum_{a \in C} v_a(y_a; h_a)$ subject to $y_a \in Y_a(h_a)$ (all $a \in C$) for the given level of total net demands $\sum_{a \in C} y_a$ made available to the members of C. It is important to notice that, as with the intertemporal f-optimum considered in Section 3.1, such an optimum does generally exist.

4.2 Example

For the example discussed in Sections 2.9, 3.1 and 3.6, history is represented by the mean consumption vector $\bar{c}$ — or equivalently, by the mean storage vector $\bar{h} = \bar{e} - \bar{c}$. So $\bar{f} + \bar{h}$ is the mean endowment vector available for distribution in the second period. Given this history, the obvious f-optimum in the second-period subeconomy satisfies $d_a = \bar{h} + \bar{f}$ for all $a \in A$. The associated net trade vectors then satisfy $y_a = d_a - h_a - f_a = \bar{h} + \bar{f} - h_a - f_a$ for all $a \in A$.

Let $q := u'(\bar{h}+\bar{f})$ be the gradient vector of the common utility function at this second-period optimal allocation. Then the appropriate second-period budget constraint for each agent $a \in A$ becomes $q\, y_a \leq q\,(\bar{h}+\bar{f}-h_a-f_a)$, which is equivalent to $q\, d_a \leq q\,(\bar{h}+\bar{f})$. Thus, no agent gains anything extra to spend in the second period as a reward for undertaking private storage in the first period.

4.3 Distortion

With this extra requirement on the second-period optimum, subgame perfection becomes a serious issue once again, and in fact the intertemporal f-optimum is generally subgame imperfect. Indeed, notice that each agent a's second-period f-optimal allocation y_a is no longer a function only of the distribution $\nu \in \Delta_\alpha(A \times H)$, but also of the agent's own history h_a. Let $\eta_a(\nu, h_a)$ denote this function. It must solve the problem of maximizing $\int_A v_a(y_a; h_a) d\alpha$ subject to $y_a \in Y_a(h_a)$ (for *all* $a \in A$) and $\int_A y_a d\alpha = 0$. And, for any finite set of agents $C \subset A$, it should also maximize the sum $\sum_{a \in C} v_a(y_a; h_a)$ subject to $y_a \in Y_a(h_a)$ (all $a \in C$) and $\sum_{a \in C} y_a = \sum_{a \in C} \eta_a(\nu, h_a)$.

In view of the second efficiency theorem, we assume that for each fixed $\nu \in \Delta_\alpha(A \times H)$, $a \in A$ and $h \in H$, the net trade vector $y = \eta_a(\nu, h)$ maximizes $v_a(y; h)$ subject to $y \in Y_a(h)$ and $q(\nu) y \leq n_a(\nu, h) := q(\nu)\eta_a(\nu, h)$.

As will now be shown, the fact that $\eta_a(\nu, h_a)$ depends directly on h_a induces inefficient distortions in the first-period equilibrium allocation.

4.4 The First-Period Subeconomy

Given both the second-period allocation rule $\eta_a(\nu, h)$ and the specific statistical history ν, define each agent a's *anticipated utility function* in the first-period subeconomy by

$$U_a^*(x, h; \nu) := u_a(x, h) + v_a(\eta_a(\nu, h); h).$$

The dependence of anticipated utility on the distribution ν of all personal histories is evidence of a widespread externality. It arises becaue each agent's welfare can be directly affected by the statistical distribution of personal histories among all agents.

There would be no problem here if in the first period some central planner could simply choose $(\mathbf{x}, \mathbf{h})$ and the associated distribution ν in order to maximize the social welfare integral $\int_A U_a^*(x_a, h_a; \nu) d\alpha$ subject to $(x_a, h_a) \in X_a$ (all $a \in A$) and $\int_A x_a d\alpha = 0$. The usual principle of optimality ensures that the first-period part $(\hat{\mathbf{x}}, \hat{\mathbf{h}})$ of an intertemporal welfare optimum will be chosen, which will then be extended to the entire path of such an optimum. The trouble comes from trying to decentralize this optimum through competitive markets in the first-period economy.

Indeed, suppose that each agent is faced with the first of the two budget constraints in the previous decentralization through spot markets with a market for an Arrow security. That is, each agent $a \in A$ faces the constraint $p\, x_a + \rho\, b_a \leq w_a(p, q)$, where $\rho > 0$. Then, by choosing $b_a = [w_a(p, q) - p\, x_a]/\rho$, agent a can fund any desired x_a; since monotonicty of preferences in x_a evidently implies global non-satiation, there cannot be any utility maximum. *A fortiori*, there can be no equilibrium. The same argument works for any other price vector, provided that there is a market for Arrow securities.

Closing down the market for Arrow securities by imposing the constraint $b_a \geq 0$ on each agent will generally overcome the non-existence problem. Yet it still does not decentralize the intertemporal optimum, in general, or produce any very satisfactory first period allocation. For suppose an attempt is made to decentralize the first-period part $(\hat{\mathbf{x}}, \hat{\mathbf{h}})$ of an intertemporal optimum by confronting each agent $a \in A$ with the budget constraint $p\, x_a \leq p\, \hat{x}_a$. Then a will maximize $U_a^*(x_a, h_a; \nu) = u_a(x_a, h_a) + v_a(\eta_a(\nu, h_a); h_a)$ subject to the individual feasibility constraint $(x_a, h_a) \in X_a$ and this budget constraint. Whereas agent a should be maximizing $u_a(x_a, h_a) + v_a(\hat{y}_a; h_a)$ instead. The dependence of the future net trade vector y_a on h_a through the policy feedback function $\eta_a(\nu, h_a)$ is liable to distort the agent's choice of personal history h_a.

4.5 Example, Continued

In the specific example discussed in Sections 2.9, 3.1, 3.6 and 4.2, each agent $a \in A$ has anticipated utility given by

$$U_a^*(x_a, h_a; \bar{h}) = u(x_a + e_a - h_a) + u(\bar{h} + \bar{f}).$$

This depends directly on $\bar{h}$, but second-period utility $u(\bar{h} + \bar{f})$ is independent of h_a. This implies that each agent $a \in A$ will choose $h_a = 0$ in any first-period equilibrium, unless restrictions are imposed to prevent this. Instead of the intertemporal optimum with an equal distribution $c_a = d_a = \frac{1}{2}(\bar{e} + \bar{f})$ of the total endowment over both periods, the best that is possible is the constrained optimal allocation with $c_a = \bar{e} \gg d_a = \bar{f}$ for all $a \in A$.

This example illustrates how it is rather easy for the optimal feedback function $\eta_a(\nu, h_a)$ to make the second-period utility $v_a(\eta_a(\nu, h_a); h_a)$ of each agent $a \in A$ entirely independent of h_a. Then each agent will make an entirely myopic choice of h_a, so that (x_a, h_a) together maximize $u_a(x_a, h_a)$ subject to $p\, x_a \leq p\, \hat{x}_a$ and $(x_a, h_a) \in X_a$. In this case nobody will give any thought to the future in choosing their personal history, because all incentives to do so have been completely destroyed. Only first-period utility remains under the agent's personal control.

5 Characterizing History Constrained Efficiency

5.1 History Constrained Allocations

Let $\hat{\nu} \in \Delta_\alpha(A \times H)$ denote any fixed "statistical history". Given this $\hat{\nu}$, a *history-constrained allocation rule* in the first period is a pair of measurable functions $\mathbf{x} : A \to X$ and $\mathbf{h} : A \to H$ satisfying the restrictions that $(x_a, h_a) \in X_a$ (α-a.e. in A), while $\int_A x_a d\alpha = 0$ and $\alpha(\{ a \in A \mid (a, h_a) \in B \}) = \hat{\nu}(B)$ for each Borel set $B \subset A \times H$. That is, $\hat{\nu}$ is the distribution on $A \times H$ that results when $\mathbf{h}$ is the function describing each agent's choice of personal history.

5.2 History Constrained Efficiency

Given any fixed $\hat{\nu}$, a history-constrained allocation $(\hat{\mathbf{x}}, \hat{\mathbf{h}})$ in the first period is said to be *history-constrained Pareto efficient* if, given any alternative (feasible) history-constrained allocation $(\mathbf{x}, \mathbf{h})$, one has $U_a^*(x_a, h_a; \hat{\nu}) \geq U_a^*(\hat{x}_a, \hat{h}_a; \hat{\nu})$ for all $a \in A$ only if $U_a^*(x_a, h_a; \hat{\nu}) = U_a^*(\hat{x}_a, \hat{h}_a; \hat{\nu})$ for α-a.e. $a \in A$.

The same allocation is said to be *history-constrained f-Pareto efficient* if in addition, given any finite $C \subset A$ and any alternative (feasible) allocation $(\mathbf{x}, \mathbf{h})$ with $\sum_{a \in C} x_a = \sum_{a \in C} \hat{x}_a$, one has $U_a^*(x_a, h_a; \hat{\nu}) \geq U_a^*(\hat{x}_a, \hat{h}_a; \hat{\nu})$ for all $a \in C$ only if $U_a^*(x_a, h_a; \hat{\nu}) = U_a^*(\hat{x}_a, \hat{h}_a; \hat{\nu})$ for all $a \in C$.

5.3 History Constrained Equilibrium

A *first-period transfer system* is a function $(a, p) \mapsto m_a(p)$ with the property that for every fixed price vector $p > 0$ the mapping $a \mapsto m_a(p)$ is integrable, and $\int_A m_a(p) d\alpha = 0$.

Given $\hat{\nu}$, a *history-constrained Walrasian equilibrium relative to a transfer system* $m_a(p)$ is a history-constrained first-period allocation $(\hat{\mathbf{x}}, \hat{\mathbf{h}})$ and a price vector $p > 0$ such that:

(i) for all $a \in A$ one has $p\,\hat{x}_a \leq m_a(p)$;

(ii) for α-a.e. $a \in A$ one has $p\,x > m_a(p)$ whenever $(x, h_a) \in X_a$ with $U_a^*(x_a, h_a; \hat{\nu}) > U_a^*(\hat{x}_a, \hat{h}_a; \hat{\nu})$.

The same pair $(\hat{\mathbf{x}}, p)$ is described as a *full* equilibrium if (ii) holds for *all* $a \in A$ without exception.

By contrast, a *history-constrained compensated equilibrium relative to a transfer system* $m_a(p)$ is defined with weak inequalities in (ii) above. That is, it consists of a history-constrained allocation $(\hat{\mathbf{x}}, \hat{\mathbf{h}})$ and a price vector $p > 0$ such that (i) is satisfied, and also:

(ii′) for α-a.e. $a \in A$ one has $p\,x \geq m_a(p)$ whenever $(x, h_a) \in X_a$ with $U_a^*(x_a, h_a; \hat{\nu}) \geq U_a^*(\hat{x}_a, \hat{h}_a; \hat{\nu})$.

5.4 Nash–Walrasian (full) equilibrium

A *Nash–Walrasian equilibrium relative to a transfer system* $m_a(p)$ is any first-period allocation $(\hat{\mathbf{x}}, \hat{\mathbf{h}})$ and a price vector $p > 0$ such that:

(i) for all $a \in A$ one has $p\,\hat{x}_a \leq m_a(p)$;

(ii) for α-a.e. $a \in A$ one has $p\,x > m_a(p)$ whenever $(x, \hat{h}_a) \in X_a$ with $U_a^*(x_a, h_a; \hat{\nu}) \geq U_a^*(\hat{x}_a, \hat{h}_a; \hat{\nu})$, where $\hat{\nu}$ is the *equilibrium measure* on $A \times H$ induced by the mapping $a \mapsto \hat{h}_a$.

The same pair $(\hat{\mathbf{x}}, p)$ is described as a *full* equilibrium if (ii) holds for *all* $a \in A$ without exception.

5.5 History Constrained Efficiency Theorems

Here are history constrained versions of the efficiency theorems that apply to the first-period allocations in the model introduced in Section 2.

Theorem 1 (First Constrained Efficiency Theorem) *Assuming preferences in the first period are monotone, any history-constrained Walrasian equilibrium relative to a transfer system must be history-constrained Pareto efficient. And any history-constrained full Walrasian equilibrium must be history-constrained f-Pareto efficient. Moreover, any Nash–Walrasian (full) equilibrium has the same (respective) properties.*

Theorem 2 (Second Constrained Efficiency Theorem) *Assuming preferences in the first period are monotone, given any history-constrained Pareto efficient allocation, there must exist a price vector at which the allocation forms a history-constrained compensated equilibrium relative to a suitable transfer system.*

Neither of the two main results requires a proof here. They are direct applications of the usual efficiency theorems for a continuum economy — see Hildenbrand (1974) or Hammond (2004) — to the history constrained first-period economy in which the distribution $\hat{\nu}$ is fixed. Then the additional fact that a history-constrained full Walrasian equilibrium must be history-constrained f-Pareto efficient is proved in the same way as the usual first efficiency theorem for any finite set of agents. Finally, the definitions immediately imply that any Nash–Walrasian (full) equilibrium is a history-constrained (full) Walrasian equilibrium, given the statistical history described by the equilibrium measure.

6 Historical Aggregates

6.1 Pigou–Walrasian Equilibrium

In this section, suppose that the set H of personal histories can be taken as a subset of some finite-dimensional Euclidean space, and that the population mean $\bar{h} := \int_A h\,d\alpha$ is a sufficient statistic for the distribution $\nu \in \Delta_\alpha(A \times H)$ in determining the second-period f-optimum. Assume, moreover, that the feasible set F_a of each agent $a \in A$ is a convex subset of $X \times H \times Y$.

Under these assumptions, given the mean $\bar{\hat{h}}$, let $(\hat{\mathbf{x}}, \hat{\mathbf{h}})$ be a history-constrained Pareto efficient allocation. For all $a \in A$, define the set

$$P_a := \{\, (x, h) \in X \times H \mid U_a^*(x, h; \hat{\nu}) > U_a^*(\hat{x}_a, \hat{h}_a; \hat{\nu}) \,\}$$

of first-period net trade vectors x and personal histories h that are jointly preferred to $(\hat{x}_a, \hat{h}_a)$. Define $P := \int_A P_a d\alpha - \{(0, \bar{\hat{h}})\}$. Note that $0 \notin P$ because $(\hat{\mathbf{x}}, \hat{\mathbf{h}})$ is history-constrained Pareto efficient. Moreover, P is a convex set because α is a non-atomic measure. So there exists a first-period price vector $(p, t) \neq (0, 0)$ and a corresponding hyperplane $p\,x + t\,z = 0$ through the origin such that $(x, z) \in P$ implies $p\,x + t\,z \geq 0$. Because of sequential local non-satiation, $(\hat{x}_a, \hat{h}_a)$ is a boundary point of P_a for all $a \in A$. Because $\int_A (\hat{x}_a, \hat{h}_a)\, d\alpha = (0, \bar{\hat{h}})$, it follows that $(x_a, h_a) \in P_a$ implies $p\,x_a + t\,h_a \geq p\,\hat{x}_a + t\,\hat{h}_a$ (α-a.e. in A).

This proves that the decentralization $p\,x_a \leq m_a(p) := p\,\hat{x}_a$ derived in Sections 5.3 and 5.4 can now be replaced with $p\,x_a + t\,h_a \leq m_a(p, t) := p\,\hat{x}_a + t\,\hat{h}_a$ for a suitable vector t of net Pigou taxes. Thus, there is a decentralization by means of a Pigou–Walrasian equilibrium (or compensated equilibrium) in the sense of Hammond (1995). For a suitable value of t, it should even be possible to decentralize an intertemporal f-optimal allocation.

6.2 Example

In the example that has been discussed in Sections 2.9, 3.1, 3.6, 4.2 and 4.5, the mean storage vector $\bar{h} \in \mathbb{R}^G$ is a historical aggregate. The budget constraint $p\,x_a + t\,h_a \leq m_a(p, t)$ is equivalent to

$$p\,(c_a - e_a) + (p + t)\,h_a \leq m_a(p, t).$$

Equilibrium exists only if $p + t \geqq 0$, otherwise if $p_g + t_g < 0$ for any good g, agents could profit indefinitely by offering to store arbitrarily large quantities of that good. But if $p_g + t_g > 0$ for any good g, then agents will obviously not want to store any of that good. Hence, the obvious price vector to choose in order to subsidize storage is $t = -p$.

Recall that any intertemporal f-optimum takes the form $c_a = d_a = \frac{1}{2}(\bar{e} + \bar{f})$ for all $a \in A$, with $a \mapsto h_a$ any integrable mapping from A to $\mathbb{R}^G_+$ that satisfies $\int_A h_a d\alpha = \bar{e} - \bar{f}$. Such an allocation can obviously be decentralized by taking $p = -t = u'(\frac{1}{2}(\bar{e} + \bar{f}))$ as well as $m_a(p, t) = p\,(\frac{1}{2}(\bar{e} + \bar{f}) - e_a)$ for all $a \in A$.

7 Remedial Policy

7.1 Nonlinear Pricing of History

Let $(\hat{\mathbf{x}}, \hat{\mathbf{h}}, \hat{\mathbf{y}})$ be any intertemporal f-welfare optimum, as considered in Section 3.1. As explained in Sections 3.2–3.4, it can be decentralized α-a.e. in A by budget constraints of the form

$$p\,x_a + q\,y_a \leq w_a(p,q) := p\,\hat{x}_a + q\,\hat{y}_a$$

— at least as a full compensated equilibria, if not always as a full Walrasian equilibrium. Let $\hat{\nu} \in \Delta_\alpha(A \times H)$ denote the history distribution that results when each agent $a \in A$ is assigned personal history $\hat{h}_a$.

So far, there is no inducement for agents to choose $h_a = \hat{h}_a$ α-a.e. in A, as required by the history constraint. Remedying this serious defect requires modifying the kind of first-period budget constraint $p\,x_a \leq m_a(p) := p\,\hat{x}_a$ described in Sections 5.3 and 5.4 so that $m_a(p)$ also depends on the choice of personal history. In fact, given the f-optimal allocation rule specified by a function $\eta_a(\nu, h)$ of a, ν, and the personal history $h \in H$, the new budget constraint to be considered takes the form $p\,x_a \leq m_a(p,q,h)$ where

$$m_a(p,q,h) := p\,\hat{x}_a + q\,\hat{y}_a - q\,\eta_a(\hat{\nu}, h) = w_a(p,q) - q\,\eta_a(\hat{\nu}, h).$$

Note that this depends also on q, the second-period price vector. The effect of the change is to make each agent $a \in A$ pay a net tax $q\,\eta_a(\hat{\nu}, h) - q\,\hat{y}_a$ for choosing personal history $h \in H$. This net tax is the present discounted value of the increased net subsidy that agent a will receive in period 2 as a result of choosing h instead of $\hat{h}_a$ in period 1.

Formally, this modified first-period decentralization works because $\eta_a(\hat{\nu}, \hat{h}) = \hat{y}_a$ and so $m_a(p,q,\hat{h}_a) = p\,\hat{x}_a$, while for any pair $(x_a, h_a) \in X_a$ with $U_a^*(x_a, h_a; \hat{\nu}) \geq U_a^*(\hat{x}_a, \hat{h}_a; \hat{\nu})$ it must be true that $p\,x_a + q\,\hat{y}_a(h_a) \geq w_a(p,q) = p\,\hat{x}_a + q\,\hat{y}_a$ and so $p\,x_a \geq m_a(p,q,h_a)$. Thus, if $(\hat{\mathbf{x}}, \hat{\mathbf{h}}, \hat{\mathbf{y}}, p, q)$ is an intertemporal compensated equilibrium relative to the transfer system specified by $w_a(p,q)$ $(a \in A)$, then $(\hat{\mathbf{x}}, \hat{\mathbf{h}}, p)$ is a first-period compensated equilibrium relative to the transfer system specified by $m_a(p,q,h)$ $(a \in A,\, h \in H)$. Similarly, if $(\hat{\mathbf{x}}, \hat{\mathbf{h}}, \hat{\mathbf{y}}, p, q)$ is an intertemporal Walrasian equilibrium relative to $w_a(p,q)$ $(a \in A)$, then $(\hat{\mathbf{x}}, \hat{\mathbf{h}}, p)$ is a first-period Walrasian equilibrium relative to $m_a(p,q,h)$ $(a \in A,\, h \in H)$.

Informally, the decentralization works because it forces each agent to internalize entirely the effect that the choice of personal history has on the widespread externality caused by the transfer system. This is only possible, however, if each agent's choice of personal history $h \in H$ can be observed in time to enforce the appropriate budget constraint $p\,x_a \leq m_a(p,q,h)$; otherwise, the economy may be restricted to some second-best allocation of the kind that arises when there are binding moral hazard constraints. Indeed, if no observations are possible in time to enforce this first period budget constraint, then m_a must remain independent of personal history. In this case there can be no escape from the inferior allocation discussed at the end of Section 4.5, or even from the kind of disastrous allocation that arose in Section 5.2 of Hammond (1999).

Note that even in the special framework of Section 6, there seems little use for linear Pigouvian prices in decentralizing the optimum. Instead, the price system presented in this section uses what are in effect non-linear prices for personal history.

7.2 Example

In the example that has been discussed in Sections 2.9, 3.1, 3.6, 4.2, 4.5 and 6.2, the appropriate transfer system is given by

$$\begin{aligned} m_a(p,q,h) &= p\,\hat{x}_a + q\,\hat{y}_a - q\,\eta_a(\hat{\nu},h) \\ &= p\,(\hat{c}_a - e_a + \hat{h}_a) + q\,(\hat{d}_a - f_a - \hat{h}_a) - q\,(\hat{d}_a - f_a - h). \end{aligned}$$

But $\hat{c}_a = \hat{d}_a = \frac{1}{2}(\bar{e} + \bar{f})$ and $p = q = u'(\frac{1}{2}(\bar{e} + \bar{f}))$, so this reduces to

$$m_a(p,q,h) = p\,(\frac{1}{2}(\bar{e} + \bar{f}) - e_a + h).$$

8 Concluding Remarks

The introduction offered a reminder of how markets would be very unlikely to produce optimal allocations in the real world economy, even if every agent, including those in the governments of the world, were planning everything perfectly in advance. This paper has largely been concerned with additional reasons for doubting the efficacy of markets when it is recognized that prices and fiscal systems adjust period by period. The subgame imperfections which can arise in intertemporal economies make the usual textbook defence of perfectly competitive markets, based on the second fundamental efficiency theorem of welfare economics, even more suspect than in static economies. In fact these subgame imperfections make statistical history into a widespread externality, characterized by constrained efficiency properties like those in Hammond (1999).

For simplicity this paper has concentrated on the two-period case with no uncertainty. Extensions to allow more periods and uncertain states of the world are conceptually fairly straightforward, even if the technical details and notation become significantly more of a challenge for the reader.

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