Essays on Economics Of Information

by

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Contents

List of Tables

List of Figures

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Responsibility for the remaining errors is solely mine.

Declarations

I declare that the material contained in this thesis has not been used or published before. This thesis is my own work and it has not been submitted for another degree or at another university.

Abstract

This thesis consists of three chapters. In the first two chapters I study the optimal design of communication hierarchies between an uninformed decision maker and privately informed experts who have different preferences over decision maker's action.

The motivation for the model described in the first two chapters comes from the fact that in organizations, a central problem is that much of the information relevant for decision making is dispersed among employees who are biased and may lack the incentives to communicate their information to the management. This paper studies how a manager can elicit employees' information by designing a hierarchical communication network. The manager decides who communicates with whom, and in which order, where communication takes the form of cheap talk (Crawford and Sobel, 1982). I show that the optimal network is shaped by two competing forces: an *intermediation* force that calls for grouping employees together and an *uncertainty* force that favours separating them. The manager optimally divides employees into groups of similar bias. Under simple conditions, the optimal network features a single intermediary who communicates directly to the manager.

Third chapter is work in progress. Here, I study a situation in which a DM can commit to both communication networks and to the delegation of decision rights but not to transfers or arbitrary decision rules based on received information. I show a novel trade-off between centralization and decentralization with two experts and a decision maker, when experts receive noisy and complementary evidence. There is a single decision to be made, the decision maker can allocate the decision right to any

of the experts, and can commit to communication channels betwent the players. I study conditions under which delegation combined with decentralized communication outperforms centralization. This happens because *delegation encourages information sharing* between the experts. Two conditions have to be satisfied in order for a decision maker to benefit from decentralization. First, the expert who decides over policy has to be not too biased towards the decision maker. Second, he should have a smaller distance to the bias of the other expert compared to the DM. In this case, the other expert is willing to reveal more information to the first expert compared to the centralized case.

Chapter 1

Optimal Communication Networks: Basic Results

1.1 Introduction

"Information in an organization, particularly decision-related information, is rarely innocent, thus rarely as reliable as an innocent person would expect. Most information is subject to strategic misrepresentation..." James G. March, 1981.

Much of the information relevant for decision making in organizations is typically dispersed among employees. Due to time, location, and qualification constraints, management is unable to observe this information directly. Managers aim to collect decision-relevant information from their subordinates, but employees often have their own interests and hence communicate strategically to influence decision making in their favour. In this paper, I study how a manager optimally elicits information by designing a communication structure within the organization. The manager commits to a hierarchical network that specifies who communicates with whom, and in which order. Her objective is to maximize information transmission.[1]

My analysis shows that the optimal communication network is shaped by two

[1] Evidence suggests that the communication structure within an organization indeed affects employees' incentives to reveal their private information. See discussions in Schilling and Fang (2014) and Glaser et al. (2015).

competing forces: an *intermediation force* that calls for grouping employees together and an *uncertainty force* that favours separating them. The manager optimally divides employees into groups of similar bias. Each group has a group leader who collects information directly from the group members and communicates this information in a coarse way to either another group leader or the manager. If employees' biases are sufficiently close to one another and far away from the manager's, the optimal network consists of a single group. My results resonate with the classic studies of Dalton (1959), Crozier (1963), and Cyert and March (1963), who observe that groups — or "cliques" — collect decision-relevant information in organizations and distort this information before communicating it to organization members outside the group.

The model I present considers a decision maker (DM) and a set of employees whom I call experts. Each expert observes a noisy signal of a parameter that is relevant for a decision to be made by the DM. The DM and the experts have different preferences over this decision; specifically, the experts have biases of arbitrary sign and magnitude over the DM's choice. The DM does not observe any signal of the relevant parameter and relies on communication with the experts. As committing to transfers or to decisions as a function of the information transmitted is often difficult in an organizational context, I rule these out.[2] The DM instead commits to a communication network, which specifies who communicates with whom, and in which order.[3] Communication is direct and costless, i.e. it takes the form of "cheap-talk" as in Crawford and Sobel (1982). I focus on the best equilibrium payoffs for the DM in any given communication network and characterize the optimal network for the DM.

My model builds upon Galeotti et al. (2013) who study simultaneous communication in a similar setting. The crucial difference is that my model studies the optimal sequential structure from decision maker's perspective, where they focus on the properties of simultaneous communication in different network structures. In particular, I restrict attention to tree communication networks, or "hierarchies." This type of network is a natural starting point in the study of communication in organizations. In the theoretical literature, hierarchies are regarded as the optimal formal organization for reducing the costs of information processing (Sah and

[2] A manager cannot contract upon transfers or any information received in Dessein (2002), Alonso et al. (2008), Alonso et al. (2015), and Grenadier (2015). See also the literature discussion in Gibbons et al. (2013).

[3] The design of communication structures appears as a more natural form of commitment. For example, if a party commits not to communicate with an agent, she will ignore any reports from the agent so long as they are not informative, and the agent in turn will not send informative reports as he expects them to be dismissed.

Stiglitz, 1987; Radner, 1993; and Garicano, 2000) and for preventing conflicts between subordinates and their superiors (Friebel and Raith, 2004). In practice, hierarchies have been identified as a prominent communication structure in organizations, even in those that aim to have non-hierarchical communication and decision rights allocation (see Ahuja, 2000 and Oberg and Walgenbach, 2008).

I begin my analysis of optimal communication networks by identifying a trade-off between two competing forces. On the one hand, the intermediation force pushes in favour of grouping experts together, in order to enable them to pool privately held information and have more flexibility in communicating to the DM. On the other hand, the uncertainty force pushes in favour of separating the experts, in order to increase their uncertainty about the information held by other experts and relax their incentive constraints. As in other contexts, uncertainty allows to pool incentive constraints, so a less informed expert can be better incentivized because fewer constraints have to be satisfied compared to the case of a more informed expert.

Building upon the interaction between the intermediation force and the uncertainty force, I derive three main results. My first main result concerns star networks — those in which each expert communicates directly to the DM. Star networks are a simple and a prominent benchmark in the social network literature (see Jackson, 2008). My analysis shows, however, that a star communication network is always dominated by an optimally-designed sequential communication network. Sequential communication between the experts can generate as much information transmission to the DM as a star network, and sometimes strictly more. The improvement arises because coordination in reports gives experts the possibility to report pooled information in a coarse way. This is strictly beneficial for the DM whenever the experts would send a less informative report were they unable to coarsen information.

My second main result shows that an optimal communication network consists of "groups" of experts. In a group, a single expert — the group leader — receives direct reports from all other members of the group and then communicates the aggregated information in a coarse way either to another group leader or directly to the DM. The coarsening of information by a group leader is key to incentivize the experts to reveal their signals truthfully. As for the optimal composition of a group, I show that group members who only observe their own private signals have identical ranges of biases which support their equilibrium strategies; the reason is that they have the same expected uncertainty about the signals of other experts and their reports are treated symmetrically by their group leader. Consequently,

the DM benefits from grouping similarly biased experts together.

Finally, my third main result shows that if the experts' biases are sufficiently close to one another while large enough (relative to the DM's preferences), then the optimal network consists of a single group. The group leader acts as a single intermediary who aggregates all the information from the other experts and sends a coarse report to the DM. Aggregation of the entire information allows this intermediary to send a report with minimal information content. As a consequence, from the perspective of each expert, any deviation from truth-telling results in the largest possible shift in the DM's policy from the expected value of the state. This allows to incentivize highly-biased experts to reveal their private information truthfully.

As noted, my findings are in line with work on the modern theory of the firm, which emphasizes the importance of coordination between employees for intra-firm information transmission. Cyert and March (1963) observe that managerial decisions are lobbied by groups of employees which provide distorted information to the authority. Similarly, Dalton (1959) and Crozier (1963) view an organization as a collection of cliques that aim to conceal or distort information in order to reach their goals. Dalton claims that having cliques as producers and regulators of information is essential for the firm, and provides examples of how central management influences the composition of such groups through promotions and replacements.[4] Group leaders in my model also resemble the *internal communication stars* identified in the sociology and management literature. Allen (1977), Tuchman and Scanlan (1981), and Ahuja (2000) describe these stars as individuals who are highly connected and responsible for a large part of information transmission within an organization, often acting as informational bridges between different groups.

The next section discusses the related literature. Section 1.2 describes the model. Section 1.3 illustrates the main ideas with a simple example, provides a characterization of the intermediation and uncertainty forces, and derives the main results. Additional results of this model are provided in Chapter 2 where I study the optimal ordering of biases, the case of experts with opposing biases, the value of commitment, and the benefits and limitations of using non-hierarchical networks. Section 2.2 in Chapter 2 concludes.

[4]See p.65-67. Dalton describes a case in which the new members of a clique were instructed about the "distinction between their practices and *official misleading instructions*" (italics are from the original text).

1.1.1 Literature review

This paper relates to several literatures.

Communication Within Organizations: The importance of communication in organizations has been long recognized in economics (Gibbons et al. 2013). Early contributions that view an organization as an optimizer of communication structures do not model a conflict of interest between the players and focus on the costs of direct communication or information processing. Keren and Levhari (1979) look at the time required to prepare instructions for productive units. Marschak and Radner (1962) look at minimization of communication costs from a team-theoretical perspective. More recently, Bolton and Dewatripont (1994), and Radner and van Zandt (1992, 2001), study the processing of information and the costs of communication. The literature on knowledge-based hierarchies models organizational structures as a solution to the trade-off between the costly acquisition and the costly communication of knowledge (see Garicano, 2000, Garicano and Wu, 2012, and the literature review in Garicano and Rossi-Hansberg, 2014). Calvó-Armengol and di Marti (2007) analyse communication using a team-theoretic framework in the tradition of Radner (1962) and Marschak and Radner (1972). All those papers do not model strategic communication, which is different from my approach. Strategic communication is important, as shown in the motivating examples in the introduction. In the theoretical literature, Milgrom and Roberts (1988) emphasize that those who are endowed with knowledge but are excluded from the final decisions might have incentives to distort the information they forward to the management. Thus, the question of optimal communication design should account for the possibility that an expert might lie while reporting to the decision maker(s).

More recent contributions model strategic communication coupled with the possibility of delegating decision rights. Dessein (2002) shows conditions under which delegation is preferred to communication because the costs of strategic communication due to coarse reports are higher than the costs of a biased decision by a privately informed agent. Dessein and Santos (2006) use a team-theoretic approach to study how a task specialization is affected by the need for coordination while adapting decisions to local conditions. Alonso, Dessein and Matouschek (2008) and Rantakari (2008) introduce incentive conflicts combined with strategic communication. They model a central manager and two peripheral divisions and show when it is optimal to delegate authority to the divisions. Alonso, Dessein and Matouschek (2015) extend this framework to study the effect of complementarity of production decisions on the delegation of authority. These papers focus on the optimal alloca-

tion of authority; by contrast, I examine the optimal design of communication when the decision rights are exogenously assigned to a principal.

My focus on tree communication networks is motivated by a large literature showing the importance of hierarchies in organizations. For example, Bolton and Dewatripont (1994), van Zandt (1999a and 1999b) and Garicano (2000) show that communication hierarchies decrease the costs related to information processing. Friebel and Raith (2004) study hiring decisions and find that hierarchical communication prevents conflicts between supervisors and subordinates in organizations. Moreover, an empirical literature recognizes trees and hierarchies as the prevalent communication form in organizations. Ahuja and Carley (1998) show that virtual organizations adopt a hierarchical communication structure — contrary to a view that the use of information technology might result in non-hierarchical structures. Oberg and Walgenbach (2008) show that a firm with a goal of adopting a non-hierarchical communication, ended up having a communication hierarchy similar to a bureaucratic organization.

Sociological and managerial literature on communication in organizations is large.[5] Early studies of Cyert and March (1963), Dalton (1959) and Crozier (1963) identified intra-organizational groups that were lobbying decisions while distorting the information they provided to the management. Recent managerial literature emphasizes the effect of the topology of communication structures on information transfer in organizations.[6] Tuchman and Scalan (1981) and Allen (1977) provide evidence that internal communication stars — individuals who act as informational bridges between different groups — mediate a large portion of intra-organizational information. This finding is in line with my results on "group leaders" who collect information directly from members of their groups and communicate it in a distorted way either to other group leaders or directly to the management. My results are also in line with a recent empirical study of Wadell (2012) who found that internal communication stars control a considerable fraction of information, which they collect from groups and frequently share among other internal communication stars. Similarly, Dhanaraj and Parke (2006) and Schilling and Fang (2013) show that individuals who are influential in communication networks within organizations have incentives to systematically distort information.

Cheap Talk and Experts: Crawford and Sobel (1982) is the seminal framework of non-verifiable costless information transmission ("cheap talk"). They show

[5]See Argote for a literature review on knowledge transfer in organizations.

[6]See the literature reviewed in Glaser et al. (2015)

that once communication features an uninformed receiver and an informed and biased sender, credible information transmission involves noise. Using cheap talk as a communication device, Krishna and Morgan (2001a,b) and Battaglini (2002) study multiple experts who perfectly observe the state. They show conditions under which perfect revelation is possible. For example, with simultaneous communication, Krishna and Morgan (2001b) study a mechanism that achieves perfect revelation.[7] With sequential communication and two experts who perfectly observe the state, Krishna and Morgan (2001a) show that the DM always benefits from consulting both experts if they have opposing biases, but does not benefit from consulting a less biased expert if the biases are in the same direction. Battaglini (2002) shows that perfect revelation with multiple dimensions of the state is possible generically even when biases are large. All those papers achieve full revelation constructing an equilibrium in which an expert does not deviate from truth-telling given some "true" report of the other expert (For example, in Battaglini (2002) each of the experts reports to the DM according to a dimension in which their interests coincide. An expert has no incentives to deviate once the other expert reports truthfully according to his dimension). However, in my model it is not possible to play the experts against each other to always achieve full revelation. If an expert does not have incentives to report his signal truthfully in a tree, he has no incentives to always share his information with other experts in a complete network either.

Because the signals are correlated, the DM has no possibility to punish the experts due to "incompatibility" of their reports as in Ambrus and Lu (2014) or Mylovanov and Zapechelnyk (2013).

The case of imperfectly informed experts was first studied in Austen-Smith (1993) who compared simultaneous and sequential reporting with two biased experts[8]. Battaglini (2004) extends the analysis to many experts who communicate simultaneously to the DM. Thus, he does not ask the question of the optimal communication design. Further, for almost perfect revelation of information he puts restrictions on expert's preferences which is not the case in this paper.

My work closely relates to a paper by Wolinsky (2002) who studies a situation in which the DM consults a team of experts and cannot commit to a mechanism. The DM takes a binary action. The experts send their reports based on the expectations

[7]Battaglini (2002) defines a refinement where players might make mistakes that rules out the perfect revelation equilibria of Krishna and Morgan (2001b).

[8]Glazer and Rubinstein (1998) study a mechanism design approach in which they compare two different modes of communication with imperfectly informed experts: the one in which the experts' and social planner's preferences coincide, and the one in which the experts have private motives. The important difference to my approach is the lack of commitment to a mechanism conditional on reports in my model.

of whether their reports are pivotal for the final decision. Wolinsky compares two benchmarks: simultaneous communication versus allocation of experts into groups. He provides examples under which it is better to split experts into multiple groups rather than consulting each expert separately, and finds conditions on the lower bound of the group size. Wolinsky's approach differs from mine in three respects: 1) Wolinsky does not ask the question of an optimal communication design, and, thus, does not provide a general characterization of communication within groups.2) Wolinsky assumes the same preferences of the experts, and, therefore, abstracts from strategic information transmission motives between them. And, finally, 3) he assumes partial verifiability and non-correlated information.

My result that strategic uncertainty can relax incentive constraints for experts once they are partitioned into different groups relates to the risk effect studied in Moreno de Barreda (2013) and the information control effect studied in Ivanov (2010). In Moreno de Barreda (2013) the DM relies on communication with a privately informed agent and has access to an additional unbiased information source. Because the sender is risk-averse, uncertainty about DM's information is costly, in particular for less informative messages. Therefore, the risk effect favours communication. In Ivanov (2010) additional uncertainty of the sender about the state generates fewer deviation possibilities such that truth-telling constraints become easier to satisfy. In my model, separation of experts into groups creates uncertainty of an expert about the exact reports of other experts (as in Moreno de Barreda, 2013). Further, an expert has access to fewer signals, thus making his reporting incentives easier to satisfy (as in Ivanov, 2010). Crucially, I model multiple experts — an approach which introduces novel strategic considerations for information transmission.[9]

There is an emerging literature on cheap talk in networks. Galeotti et al. (2013) and Hagenbach and Koessler (2011) study how properties of networks affect truth-telling strategies of players in a network with multiple decision makers and a simultaneous one-round communication. I use a theoretical framework similar to that of Galeotti et al. (2013) which is also used in Morgan and Stocken (2008) and Argenziano et al. (2015). All these papers assume the communication structures as given; by contrast, I endogenize communication.

Generally, the question has not yet been studied regarding an optimal network design in the presence of cheap talk for a broad class of networks. However, the literature does provide valuable insights for complete networks (Galeotti et al., 2013;

[9]Other papers that introduce additional information sources for the DM include Lai (2010) and Ishida and Shimizu (2013) but none of them analyzes a setting with multiple senders.

Hagenbach and Koessler, 2011), sequential communication with a single intermediary (Li , 2007 and Ivanov 2010) and the case of multiple intermediaries (Ambrus, Azevedo, Kamado, 2011).

For reasons of tractability, I analyze a single round of communication. Multiple rounds enrich the equilibrium set in a non-trivial manner (see Aumann and Hart, 2003, or Morgan and Krishna, 2004).

Dewatripont and Tirole (1999) develop a model in which the players are partitioned into groups according to the objectives of the group. The agents collect information for the DM, and are only interested in monetary rewards associated with the direction of the final decision, or for career concerns. In particular, they are nonpartisan about the decision per se. Further, my framework is not directly comparable with theirs because Dewatripoint and Tirole (1999) study verifiable information transmission, abstracting from strategic transmission effects within groups. Nevertheless, both papers offer rationales for partitioning experts according to the direction in which they want to influence the final decision. Instead, in my model, all agents care about policy decisions and there are no monetary transfers. Recently, Gentzkow and Kamenica (2015) show conditions under which competition in persuasion benefits the decision maker. However, their focus is on information gathering, and thus their insights are complementary to this paper.

Finally, in my model the decision right is exogenously assigned to a single decision maker who optimizes over sequential communication structures within the space of communication hierarchies. Dewan et al. (2014) study the optimal assignment of decision rights in a framework with imperfectly informed experts (politicians) and information aggregation via cheap talk. In their model, the messages can be either private or public and thus they do not optimize over communication structures. They show that a centralized authority can outperform decentralization from the perspective of a social planner.

1.2 Model

Players and preferences: There are n experts and a single decision maker whom I denote by DM, $N = \{1, .., n, DM\}$. The payoff function of player $i \in N$ is

$$u_i = -(y - \theta - b_i)^2,$$

where $y \in \mathbb{R}$ denotes the choice of the DM, $b_i \in \mathbb{R}$ is the bias of player i and θ is the unknown state of the world with the common prior $\theta \sim U[0, 1]$. For simplicity, I normalize $b_{DM} = 0$.

Signals: Each expert receives a conditionally independent private signal $s_i \in \{0, 1\}$ upon realisation of θ with $Prob(s_i = 1) = \theta$. Bayesian updating follows the Beta-binomial model. As I show in the appendix, for n signals and k signals equal to 1, the expected value of the state is $\mathbb{E}(\theta|k, n) = \frac{k+1}{n+2}$. Further, the number of 1's is uniformly distributed over n signals: $Prob(k|n) = \frac{1}{n+1}$.

Communication: A communication network is $Q = (N, E)$: Q is a directed graph with the set of nodes N and the set of edges $E \subseteq N \times N$. For every pair of nodes (i, j), $e_{ij} \in \{0, 1\}$: $e_{ij} = 1$ means that there exists a directed link from i to j and $e_{ij} = 0$ means that no such link exists. I focus only on directed trees where the DM is located at the root of the tree. This means that every player has only one outgoing link but can have multiple incoming links, there are no cycles in the graph, and the DM has at least one incoming link, but no outgoing links.[10] The domain of all networks satisfying these conditions is denoted by $\mathcal{Q}$.

Each player can send non-verifiable messages only within the specified communication network, where a directed link from i to j means that i can send a message only to j and j cannot send a message to i. The message space for each player is an arbitrary large set M, where $m_i \in M$ denotes a message of expert $i \in \{1, .., n\}$. I assume that each player can send a message only once.

A path $H_{i_1 i_k}$ is a sequence $\{i_1, i_2\}, \{i_2, i_3\}, .., \{i_{k-1}, i_k\}$ such that the nodes in every pair $\{i_l, i_{l+1}\}$ are directly connected, $e_{i_l, i_{l+1}} = 1$.

Comparison between networks: Since every network features multiple equilibria, I focus on the best equilibrium for the DM for any given network. Network Q weakly dominates network Q' if for any biases of players the best equilibrium payoff for the DM in Q is at least as high as in Q', and for some biases it is strictly higher. For optimality, I use the criterion of a weak domination. Therefore, a network Q is optimal for given n and the biases of all players, if any other $Q' \in \mathcal{Q}$ is weakly dominated by Q.

Player types, partitional communication: Consider an equilibrium in which n experts communicate in some network Q. Since each expert receives a binary signal, the maximum information that a DM can receive is a sequence of 0's and 1's of length n which reflects how many $s = 0$ and how many $s = 1$ are received by all experts. A player might not know the exact sequence of private signals, but only a subset

[10] In the appendix, I give a formal definition of the communication network.

of $\{0,1\}^n$ that contains the true sequence. Player's private information constitutes her type, t_i, which is a probability measure on $\{0,1\}^n$, $t_i \in \Delta(\{0,1\}^n)$, where $\Delta(S)$ denotes a set of probability measures on a set S. A set of types of player i in a network Q is denoted by $T_i(Q)$.

A message strategy of expert i in a network Q is a partition on the set $\{0,1\}^n$ denoted by $P_i(Q)$. The set of strategies of all experts other than expert i for a given network Q is denoted by $P_{-i}(Q)$. An element of a partition is called a pool. For a given $t_i \in T_i(Q)$, a pool $p \in P_i(Q)$ which includes a subset of $\{0,1\}^n$ to which t_i assigns probability 1 is denoted by $p(t_i) \in P_i(Q)$. For tractability, I sometimes suppress the notation Q when referring to partitions.

Equilibrium: The solution concept is a pure strategy Perfect Bayesian Equilibrium. The following tuple is a strategy profile and a belief system for all players:

$$\Big(Q, \{T_i(Q)\}_{i=0,..,n'}, \{P_i(Q)\}_{i=0,..,n',DM}, y(P_{DM}(Q))\Big),$$

where Q is a communication network chosen by the DM, $n' \in \{0,..,n\}$ is the equilibrium number of experts participating in communication in Q, $\{T_i(Q)\}_{i=0,..,n',DM}$ is the set of experts' types which determines their beliefs, $\{P_i(Q)\}_{i=0,..,n',DM}$ is the set of experts' message strategies and the partition of the DM according to which she receives information, and $y(P_{DM}(Q))$ is the DM's action profile dependent on information received. The following conditions should be satisfied in equilibrium:

1. $y(P_{DM}(Q))$ is sequentially rational. For $k \in \{0,..,n'\}$ it means that if $p' \in P_{DM}(Q)$ is reported to the DM, she chooses y as follows:

$$y \in \operatorname*{argmin}_{y\in\mathbb{R}} \; -\int_0^1 (y-\theta)^2 f(\theta|k \in p', n')d\theta.$$

2. For every $t_i \in T_i(Q)$, the partition $P_i(Q)$ is incentive compatible. It means that for $k \in \{0,..,n'\}$ and $p' \in P_{DM}(Q)$ with $k \in p'$, and given the strategies of all other experts $P_{-i}(Q)$:

$$-\sum_{k\in\{0,..,n'\}} Pr(k|t_i)\int_0^1 (y(p'|P_{-i}(Q),p(t_i)) - \theta - b_i)^2 f(\theta|k,n')d\theta \geq$$
$$-\sum_{k\in\{0,..,n'\}} Pr(k|t_i)\int_0^1 (y(p''|P_{-i}(Q),\tilde{p}) - \theta - b_i)^2 f(\theta|k,n')d\theta,$$

for $p'' \in P_{DM}(Q),\ p'' \neq p'$ and $p(t_i), \tilde{p} \in P_i(Q)$, $p(t_i) \neq \tilde{p}$.

3. Finally, Q minimizes the expected losses of the DM:

$$Q \in \underset{Q \in \mathcal{Q}}{\operatorname{argmin}} - \sum_{p' \in P_{DM}(Q)} Pr(p' \in P_{DM}(Q)) \sum_{k \in p'} Pr(k) \int_0^1 (y(p') - \theta)^2 f(\theta | k \in p', n') d\theta.$$

In the analysis, I focus on the most informative pure strategy PBE. The reason is that as in Crawford and Sobel (1982) all equilibria can be Pareto-ranked[11] and more informative equilibria yield higher expected payoffs for every player.

Given the equilibrium conditions, notice that by backward induction the DM chooses $y(\cdot) = \mathbb{E}_{DM}(\theta | \hat{m})$, where $\hat{m}$ denotes the message profile of all experts which report directly to the DM.

1.3 Characterization of an optimal network

1.3.1 The main idea

In the following I focus on a simple case with three experts. I explain the main results and show how they are driven by the interplay of two competing forces: the intermediation force and the uncertainty force. To show the main results, it is not necessary to cover all tree networks that feature three experts. Therefore, I provide the complete characterization of the case with three experts in the later section 3.4.

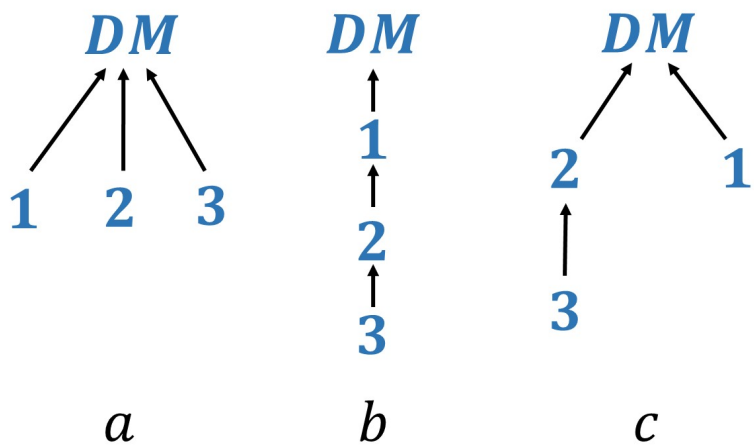
Consider three positively biased experts, labelled 1, 2 and 3. I, first, show that the star network (Figure 1.1a) is dominated by the line (Figure 1.1b). This is due to the intermediation force that favours grouping the experts together. Second, I show that for some parameter range the line is dominated by the network in Figure 1.1c which separates one of the experts from the remaining two. This is due to the uncertainty force which incentivizes experts to reveal their signals if they do not observe the messages of other experts.

Outcomes in the star network: Depending on the experts' biases, the star can generate one of four equilibrium outcomes. In the first case, all experts babble (send uninformative messages). The DM ignores their reports and chooses $y = \frac{1}{2}$ which is her optimal choice given the prior. This strategy profile is an equilibrium for any biases of the experts, and the resulting payoff for the DM is $\mathbb{E}U_{DM} = -\frac{1}{12}$. In the

[11] The difference to Crawford and Sobel (1982) is that their signal space is copntinuous and perfectly observed by the sender, wherear in my model the signals are discrete and the state is not perfectly observed by a sender.

second case, an expert $i \in \{1, 2, 3\}$ reveals his signal truthfully to the DM and the other two experts babble. The DM chooses y based on the report of expert i, m_i, as follows: $y(m_i = 0) = \frac{1}{3}$ and $y(m_i = 1) = \frac{2}{3}$. This strategy profile is an equilibrium for $b_i \leq \frac{1}{6}$ with the resulting $\mathbb{E}U_{DM} = -\frac{1}{18}$. In the third case, any two of the three experts reveal their signals truthfully, and the other expert babbles. The DM makes her choice dependent on the sum of the signals as follows: $y(0) = \frac{1}{4}$, $y(1) = \frac{1}{2}$ and $y(2) = \frac{3}{4}$. This strategy profile is an equilibrium if the biases of the two truthful experts are $b \leq \frac{1}{8}$ and it yields $\mathbb{E}U_{DM} = -\frac{1}{24}$. In the fourth case, all three experts reveal their signals truthfully. The DM chooses y depending on the sum of the signals as follows: $y(0) = \frac{1}{5}$, $y(1) = \frac{2}{5}$, $y(2) = \frac{3}{5}$ and $y(3) = \frac{4}{5}$. This strategy profile is an equilibrium for $b_i \leq \frac{1}{10}$, $i \in \{1, 2, 3\}$, and it yields $\mathbb{E}U_{DM} = -\frac{1}{30}$.

Figure 1.1: Illustration of the main idea



Outcomes in the line network: First, as shown in the appendix, the line can generate all the equilibrium outcomes of the star. In particular, consider an equilibrium in which the DM receives information about all signals. In this case, expert 3 reveals his signal truthfully to expert 2, and expert 2 reveals his signal and the message of expert 3 to expert 1. Expert 1 reveals all information communicated to him by expert 2, and his own private signal to the DM.

Why is it the case that the expert 1's and 2's incentive constraints in the line are the same as in the star, although in the line they are more informed than in the star? To see this, consider, for example, the incentives of expert 1. If the sum of the signals is $k \in \{0, 1, 2\}$, then his smallest (and binding) upward deviation is to communicate the sum of the signals $k + 1$ to the DM. The true sum of the signals is uniformly distributed. Therefore, if expert 1 deviates from telling the true k to

communicating $k+1$ to the DM, the shift of the DM's policy from the expert 1's expected value of the state is the same for each $k \in \{0,1,2\}$. It follows that if expert 1 receives signal 0 in the star, the constraint which prevents him from reporting 1 is the same as the constraint which prevents him from deviating from k to $k+1$ in the line. By the same argument, it can be shown that the line can generate the equilibrium payoffs of a star network in which one or two of the three experts reveal their signals truthfully.

Second, there are additional equilibria in the line which cannot be replicated by the star. These equilibria feature one of the experts collecting information about the signal of the other experts, and transmitting the aggregated information to the DM in a coarse way. Consider the following strategy profile in the line in which expert 3 reveals his signal to expert 2. Expert 2 reveals his signal and the message of expert 3 to expert 1. Therefore, expert 1 observes both his private signal and the message of expert 2. Expert 1 reports to the DM as follows: if the sum of the signals which he observes is 0, he sends m_1. Otherwise he sends m_1'. This means that m_1' includes the sums of signals 1, 2 and 3. The DM's choices are: $y(m_1) = \frac{1}{5}$ and $y(m_1') = \frac{3}{5}$. As shown in the appendix, this strategy profile is an equilibrium for $b_i \leq \frac{1}{5}$, $i \in \{1,2,3\}$. It results in $\mathbb{E}U_{DM} = -\frac{4}{75}$. The DM's payoff is strictly higher compared to the equilibrium in the star in which either all experts babble or a single expert reports his signal truthfully. Therefore, for $b_i \in (\frac{1}{8}, \frac{1}{5}]$, $i \in \{1,2,3\}$, then the line performs strictly better than the star. Otherwise the line yields the same expected utility to the DM as the star. Therefore we have:

Result 1: The line network weakly dominates the star network.

Line can be dominated: Next, consider the network depicted in Figure 1.1c, denoted by Q'. It is crucial that experts 1 and 2 do not observe each other's messages. I focus on the following strategy profile: expert 1 reveals his signal to the DM — he sends m_1 if his signal is 0, and m_1' if his signal is 1. Expert 3 truthfully reveals his signal to expert 2. Expert 2 sends one of the two messages to the DM. If the sum of the signals which expert 2 observes is 0, then he sends m_2. If the sum of the signals is either 1 or 2, he sends m_2'. The DM's choices are:

$$y(m_1, m_2) = \frac{1}{5},\ y(m_1', m_2) = \frac{2}{5},\ y(m_1, m_2') = \frac{7}{15},\ y(m_1', m_2') = \frac{18}{25}.$$

I show in the appendix that this strategy profile is an equilibrium for $b_2, b_3 \leq$

$\frac{74}{525}$ and $b_1 \leq \frac{293}{2550}$. It yields $\mathbb{E}U_{DM} = -\frac{89}{2250}$ and therefore strictly outperforms the line for $b_2, b_3 \in (\frac{1}{8}, \frac{74}{525}]$ and $b_1 \in (\frac{1}{8}, \frac{293}{2550}]$.

Why is it important to separate the experts from one another? To answer this, I show that while the line can transmit the same amount of information as the above strategy profile in Q', but only for a strictly smaller range of experts' biases. Consider the following strategy profile: expert 3 reveals his signal truthfully to expert 2, and expert 2 sends one of the two messages to expert 1. If the sum of the signals which expert 2 observes is 0, he sends m_2. If the sum of the signals is 1 or 2, he sends m_2'. Expert 1 reveals to the DM his private signal s_1 *and* the message of expert 2. Clearly, the DM's choices are the same as in Q' yielding the same expected utility. The incentives of experts 2 and 3 are the same as in Q'. However, as shown in the appendix, expert 1 communicates according to the above strategy profile only for $b_1 \leq \frac{1}{30}$. The reason is that in the line expert 1 observes the message of expert 2. He, thus, has to satisfy more incentive constraints compared to Q' where he only observes his private signal. We have:

Result 2: There is a bias range for 3 experts in which the network Q' strictly dominates the line. In particular, the equilibrium in Q' is implementable in the line for a strictly smaller range of expert 1's bias, compared to Q'. This is because in the line expert 1 observes the message of expert 2.

The following sections 3.2 and 3.3 provide general analysis, whereas 3.4 fully characterizes the case with three positively biased experts.

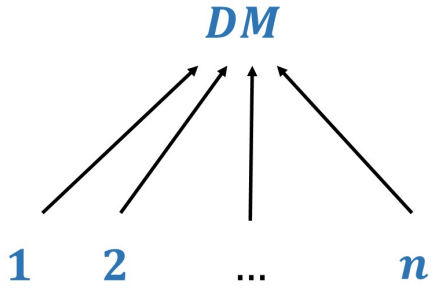
1.3.2 Simultaneous versus sequential communication

I start with the characterization of equilibria in a star network with the DM located at the center of the star as in Figure 1.2. This characterization is similar to Morgan and Stocken (2008), and Galeotti et al. (2013):

Proposition 1: *Take any number of experts, n, with arbitrary biases. An equilibrium in a star network in which $n' \leq n$ experts communicate their signals truthfully to the DM exists if for every expert $i \in \{1, .., n'\}$ the condition $|b_i| \leq \frac{1}{2(n'+2)}$ is satisfied.*

Proposition 1 shows that a smaller conflict of interest between the experts and the DM results in more experts revealing their signals. The equilibrium message strategy of an expert depends both on the distance between the biases of the expert

Figure 1.2: A star with n experts



and the DM, as well as on the message strategy of the other experts. In particular, a smaller number of equilibrium truthful messages increases the influence of each of these messages on the final decision. As a result, a deviation of an expert from his truthful message brings him further away from his expected ideal point. Therefore, an expert has fewer incentives to deviate. Conversely, if the number of equilibrium truthful messages increases, the influence of any given expert on the final decision of the DM decreases making a deviation more profitable. Therefore the range for experts' biases which supports equilibrium truth-telling decreases.

By definition, equilibria induced by simultaneous communication do not feature strategic intermediation because no expert observes the messages of other experts. In sequential communication intermediation is possible and is manifested in *strategic coarsening of information.*

The example in section 3.1 illustrated strategic coarsening of information by expert 1 in a line network. He either sent a message containing the sum of the signals 0, or a message containing the sums of signals 1, 2 and 3.

Proposition 2 considerably generalizes the example in section 3.1 and provides an important insight into the characterization of optimal information transmission hierarchies. It shows that an optimally designed sequential communication does weakly better than simultaneous communication. In particular, the former generates the same outcomes as the latter and sometimes generates additional outcomes which strictly dominate all equilibria of the simultaneous communication.

Proposition 2: *Take any number n of experts with arbitrary biases. Take any tree network Q that is not a star, such that if expert i communicates to expert j, $e_{ij} = 1$, then $|b_j| \leq |b_i|$. Then:*

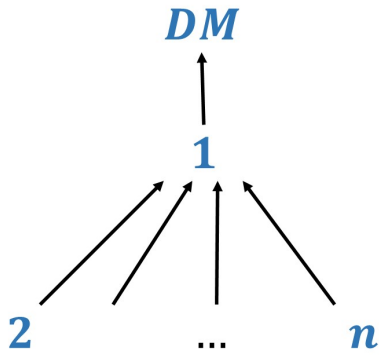
1. *Any equilibrium outcome in a star network is also implementable as an equilibrium outcome in network Q.*

2. *There is a range of biases for which the best equilibrium in Q strictly dominates the best equilibrium in a star. Further, this equilibrium in Q involves strategic coarsening of information.*

The intuition for the first part of Proposition 2 is that the deviation incentives of an expert in a separating equilibrium are independent of the expert's beliefs about the signals of other experts. Since the sum of the signals is uniformly distributed, the smallest upward (downward) deviation of any expert's type which determines his binding constraints always leads to the same upward (downward) shift in DM's policy. Therefore, the truth-telling incentives in a separating equilibrium depend only on the overall number of equilibrium truthful messages.

The second part of Proposition 2 states that there always exists a range of biases for which an optimally designed intermediation strictly dominates the star network. The intuition is that an expert with at least one predecessor can optimally coarsen pooled information. A deviation from the message containing the true sum of the signals brings him further away from his ideal point compared to a deviation in the star network. Thus, an optimal sequential communication supports information transmission for larger values of experts' biases compared to the star.

Figure 1.3: An example of an optimal network in Proposition 3



The next Proposition shows that if the biases of all experts are large enough, but do not exceed $\frac{1}{4}$, then the optimal network features a single expert directly connected to the DM. Further, each of the other experts should be able to communicate to some other expert. An example of such network is depicted in Figure 1.3.

Proposition 3: *Suppose that n experts are such that the biases of all experts are either within the interval $(\frac{n}{4(1+n)}, \frac{n+1}{4(n+2)}]$ or within the interval $[-\frac{n+1}{4(n+2)}, -\frac{n}{4(n+1)})$. Then:*

1. *In the optimal network the DM is connected to a single expert $i \in \{1, .., n\}$, $e_{iDM} = 1$, and each expert apart from i is connected to some expert $j \in \{1, .., n\}$.*

2. *Any other network does not transmit any information from the experts to the DM.*

Moreover, If the biases of all experts are above $\frac{n+1}{4(n+2)}$ or below $-\frac{n+1}{4(n+2)}$, then no information is transmitted in equilibrium.

The explanation for Proposition 3 is that a network which features a single expert communicating directly to the DM provides the maximum flexibility for this expert to send messages according to the coarsest possible profile. Consider the case in which all biases are positive. An expert connected to the DM sends either a message containing the sum of the signals 0, or a second message which pools together all other signals. In this case the incentive constraints of all experts participating in communication are identical. Moreover, an expert's deviation from communicating the sum of the signals 0 results in the largest possible shift from an expert's ideal point among all incentive compatible partitions of signals. It means that the described partition incentivizes information transmission by the experts with the largest possible biases who would not be reveal their information otherwise. If the biases of all experts are negative, the argument is similar: the equilibrium partition informs the DM that the sum of the signals is either n, or everything else.

As n approaches infinity, the upper bound for the positive and the lower bound for the negative biases above (below) which no information transmission is possible converges to $\frac{1}{4}$ $(-\frac{1}{4})$. These are the bounds for the sender's bias in Crawford and Sobel (1982) where above $\frac{1}{4}$ or below -$\frac{1}{4}$, no information can be credibly transmitted in equilibrium.

1.3.3 Uncertainty and incentives

Consider an expert who receives his private signal s_i. Before the communication within a chosen network takes place, an expert is uncertain about the signals of other experts. After the communication in a network takes place, those experts who receive messages from other experts have a more precise information about

the overall signals. In the previous section I showed that a network which enables aggregation of signals by some experts does better than the star because the experts can optimally coarsen pooled information. However, from the DM's perspective there is a downside of experts being informed about overall signals. The more information about other experts' signals that expert receives, the more incentive constraints have to be satisfied to incentivize the expert to send a message containing the true sum of the signals. Conversely, experts who have less information about the signals of other experts have to satisfy fewer incentive constraints for an informative communication.

With other words, from the perspective of a less informed expert, the incentive constraints are pooled together since they are formulated in expectation. Each of these constraints results generically in a different range of biases supporting the expert's equilibrium strategy. Since the constraints are pooled together, the resulting range of biases lies in-between the tightest and the weakest constraints. However, once an expert observes more information, the tightest incentive constraint becomes binding.

A similar idea arises in mechanism design when comparing the interim with the ex-post truth-telling constraints. In the interim stage when an agent only observes his type, but not the types of other players, the incentive constraints are formulated in expectation with respect to the types of the other players. However, once the types of the other players are known, it becomes harder to sustain truth-telling since more constraints have to be satisfied.

The next Lemma shows that in equilibrium which features strategic coarsening of information, if some expert j truthfully communicates all his signals, and some other expert i receives the entire information on j's signals, then in an optimal network a strategy profile of i is supported for a weakly smaller range of biases, compared to j. Define a set of experts who have directed links to an expert $i \in N$ in a network Q by $N_i(Q) := \{j \in N : \ e_{ji} = 1\}$. Define a partition according to which an expert $i \in N$ receives his signals in a network Q by $P_i^b(Q) := \prod_{i' \in N_i} P_{i'}(Q) \times \{0, 1\}$. Thus, $P_i^b(Q)$ includes the messages of all the experts directly connected to i together with the i's private signal.

Definition: Expert i receives full information from some other expert j if the partition P_i^b can be written as $P_i^b = P_j \times \{P_i^b \setminus P_j\}$. It means that the partition P_i^b is finer[12] than the partition P_j.

[12] A partition P is finer than a partition P' if every element in P' is a union of the elements of P.

Lemma 1: *Take any equilibrium in an optimal sequential communication tree that involves strategic coarsening of information. If some expert j truthfully communicates all his signals, $P_j(Q) = P_j^b(Q)$, and some other expert i receives full information from j, then the range of biases supporting the equilibrium strategy of i is weakly included in the range of biases supporting the equilibrium strategy of j.*

Intuitively, since expert i observes more signals compared to expert j, there are more incentive constraints to be satisfied for i compared to j. The incentive constraints for j are pooled together since they are formulated in expectations. Therefore, the binding constraint for j concerning the range of biases supporting his equilibrium strategy is in-between the tightest and the weakest constraints. For expert i, each of those constraints has to be satisfied individually, and therefore the tightest constraint becomes the binding one.

To understand the implication of Lemma 1 for optimal networks, consider again the strategy profiles of all players discussed in 1.3.1. I showed that both the line (Figure 1.1b) and the network Q' (Figure 1.1c) can implement the same payoff allocation in which the DM knows the signal of expert 1, and knows that experts 2 and 3 either have the sum of the signals 0, or 1 and 2. The difference between both networks is that in a line expert 1 observes the message of expert 2. As I showed in 3.1, this decreases expert 1's range of biases which supports his equilibrium strategy, compared to Q'.

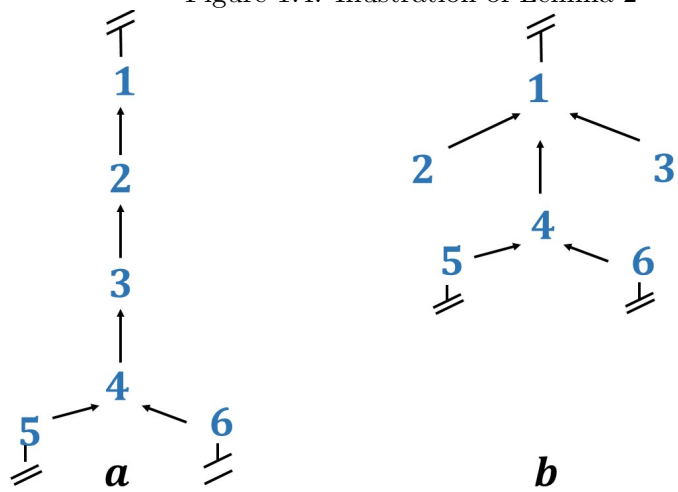
Therefore, in an optimal network, if a player i receives full information from some other player j, then j has to be directly connected to i. Otherwise, if there are some experts between j and i, then those experts face tighter constraints compared to the case in which j is directly connected to i, without changing the expected payoffs allocation. This finding is generalized in Lemma 2.

Lemma 2: *Take any number of experts with arbitrary biases. In the corresponding optimal network Q, if there is a player $i \in N$ who receives full information from some other expert j, then j has to be connected to i, $e_{ji} = 1$.*

Thus, if some expert receives full information from some other experts and coarsens the aggregated information, then he receives this information unmediated, it means from each of those experts directly. Figure 1.4 illustrates the argument. Suppose that expert 1 receives the entire information about the signals of experts 2, 3 and 4. Then, it is weakly optimal for the DM to arrange the experts as in Figure 1.4b instead of an alternative arrangement as in Figure 1.4a. This is because in

the network in Figure 1.4a the incentive constraints for experts 2 and 3 are weakly tighter, compared to the network in Figure 1.4b, without changing the expected utility of the DM.

Figure 1.4: Illustration of Lemma 2



Therefore, the optimal network consists of information coordination units in which some experts communicate their information directly to another expert, who in turn coarsens the aggregated information in his message. I call such an information coordination unit, a *group*, and an expert who collects information from other experts in order to strategically coarsen it, a *group leader*.

Definition: A group G is a non-empty subset of a communication network consisting of some expert $i \in N$ and a non-empty subset of experts $\tilde{N} \subset N \setminus \{i, DM\}$ such that each $j \in \tilde{N}$ is directly connected to i. Expert i is a group leader and I denote him by i^G.

Notice that I am not excluding the possibility that a group leader in one group can be directly connected to a leader of a different group. Thus, an optimal network can include multiple layers of groups.

Next, consider an optimal network which features a group G in which some of the experts who communicate to a group leader have no incoming links. The only information which these experts receive are their private signals. Lemma 3 shows that the equilibrium message strategy of each of those experts is supported for the same range of biases. This happens both because they face the same expected uncertainty about the overall signals and because their signals are treated in the same way by their group leader.

In equilibrium, a group leader has a better knowledge about the overall signals compared to those experts in his group who only observe their own signals. Therefore, according to the next Lemma, in an optimal network the range of biases supporting the equilibrium strategy of a group leader is weakly included into the range of biases supporting the equilibrium strategy of those experts.

Lemma 3: *Take any number n of experts with arbitrary biases. In the optimal network Q:*

1. *If two distinct experts j' and j belong to the same group G, none of them is a group leader of G and both j and j' have no incoming links, $\sum_{k \in N} e_{kj} = \sum_{k \in N} e_{kj'} = 0$, then they have identical ranges of biases which support their equilibrium strategies in which they reveal their signals truthfully, and*

2. *The equilibrium strategy of i^G is supported by a weakly smaller range of biases, compared to j and j'.*

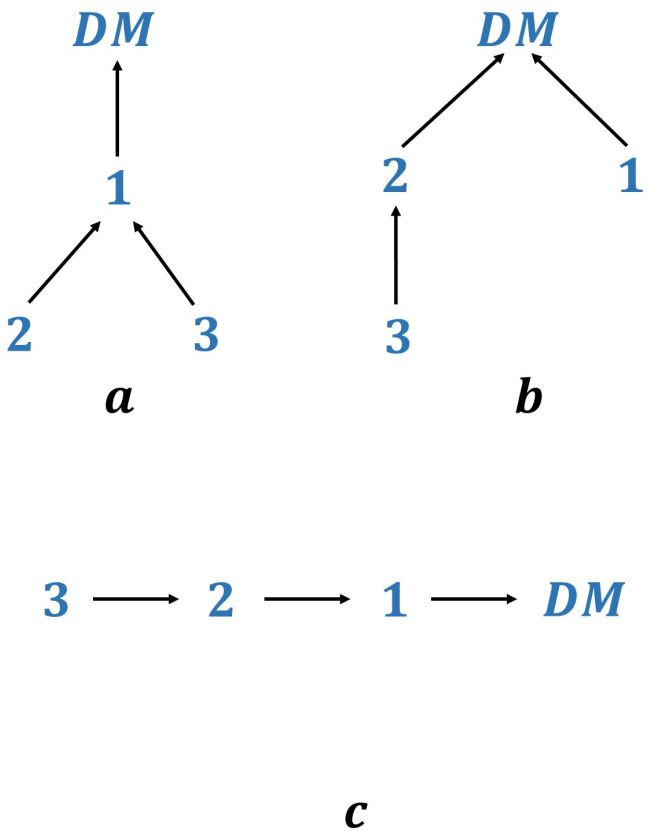
Summing up, the Lemmas 1-3 showed the implication of the uncertainty force for the optimal networks. An optimal network consists of groups of experts with similar biases. A group leader collects information from the other group members and communicates it in a coarse way either to another group leader or to the DM.

1.3.4 Optimal network

This section builds upon the results of the previous sections and fully characterizes the optimal networks and corresponding equilibria for three positively biased experts, labelled 1, 2, 3. All calculations are in the appendix. From Propositions 1 and 2 we know that for $b_i \leq 0.1$, $i = 1, 2, 3$, a star or any network which features an optimally designed intermediation implements the full separation of experts' signals. This results in $E_{DM} \simeq -0.033$. In the following, I assume $b_i > 0.1$ for at least one of the experts, $i \in \{1, 2, 3\}$. Since in this case the optimal network is not the star, it should be at least one of the three networks depicted in Figure 1.5. I denote these networks correspondingly by Q_a, Q_b and Q_c. Which network is the optimal one depends on the experts' biases.

1. *The optimal network is Q_a if the average bias is low.* In this case expert 1 is a single intermediary who directly receives messages from experts 2 and 3. The biases $b_1 \leq 0.1$ and $b_2, b_3 \leq 0.125$ support the following equilibrium strategy: Experts 2 and 3 communicate their signals truthfully to expert 1. Expert 1 sends one of the three messages: if the sum of the signals is 0, he sends m_1, if

Figure 1.5: Networks for 3 experts



the sum of the signals is 1, he sends m_1'. Otherwise, if the sum of the signals is either 2 or 3, he sends m_1''. DM's choices are $y(m_1) = \frac{1}{5}$, $y(m_1') = \frac{2}{5}$ and $y(m_1'') = \frac{7}{10}$. This strategy profile yields $\mathbb{E}U_{DM} \simeq -0.038$. As shown in the appendix, the same outcome can be implemented by the line (network Q_c) for a strictly smaller range of experts' biases.

2. *The optimal network is Q_a if the average bias is high.* This is the consequence of Proposition 3. If the biases are sufficiently high, the DM requires a single expert to collect all information. This expert has the maximum flexibility to send messages with minimal information content such that a deviation from truth-telling brings all experts to a maximum distance from their ideal points. The biases $0.115 \leq b_1 \leq 0.2$ and $0.141 \leq b_2, b_2 \leq 0.2$ support the following equilibrium strategy: experts 2 and 3 communicate their signals truthfully to expert 1. Expert 1 communicates to the DM as follows: if the sum of the signals is 0, he sends m_1, otherwise he sends m_1'. DM chooses $y(m_1) = \frac{1}{5}$ and $y(m_1') = \frac{3}{5}$ with the resulting $EU_{DM} \simeq -0.05$.

I show in the appendix, the same outcome can be implemented by Q_c for the same restrictions on the biases. This follows from Proposition 3.

3. *Finally, the optimal network is Q_b if the biases are in the intermediate range.* The biases $0.1 \leq b_1 \leq 0.115$ and $0.1 \leq b_2, b_3 \leq 0.14$ support the following equilibrium strategy: expert 3 communicates his signal truthfully to expert 2. Expert 2 communicates as follows: if both the private signal of expert 2 and the message of expert 3 are 0, expert 2 sends m_2 to the DM. Otherwise he sends m_2'. Further, expert 1 sends m_1 if his signal is 0, and m_1' otherwise. DM's choices are $y(m_1, m_2) = \frac{1}{5}$, $y(m_1', m_2) = \frac{2}{5}$, $y(m_1, m_2') = \frac{7}{15}$ and $y(m_1', m_2') = \frac{18}{25}$ resulting in $EU_{DM} \simeq 0.04$. We know from the section 3.1 that the same outcome can be implemented in a line, but only for a strictly smaller range of expert 1's bias.

   As shown in the appendix, it is possible to implement this equilibrium outcome in Q_c but only for a strictly smaller range of experts' biases.

To sum up the examples, the intermediation force dominates the uncertainty force if all biases are relatively small or relatively large. The uncertainty force dominates the intermediation force for the intermediate range of the biases. Further, it turns out that for 3 experts, the optimal line is never a strictly better network.

Chapter 2

Further Results on Optimal Communication Networks

2.0.5 Ordering of the biases

The next example shows that if the tree network in Figure 2.1 is optimal, it is feasible for a larger range of expert 2's biases compared to the range of expert 1's biases. Therefore, a smaller bias does not necessitate being closer to the DM.

Example 1: Consider the network with six positively biased experts depicted in Figure 2.1 and the following strategy profile: expert 4 (expert 6) reveals his signal truthfully to expert 3 (expert 5). The message strategies of experts 3 and 5 are identical: they either send a message informing expert 2 that the sum of the signals is 0, or a message that the sum of the signals is either 1 or 2. Expert 2 sends one of the two messages to expert 1: he sends m_2 if either both experts 3 and 5 communicated the sum of the signals 0, or expert 2's private signal is 0 and either expert 3 or expert 5 communicated the sum of the signals 0. Otherwise he sends m_2'. Expert 1 sends one of the two messages to the DM: he sends m_1 if his own private signal is 0 and expert 2 sent m_2 from expert 2. Otherwise he sends m_1'. The DM's choices are $y(m_1) = 0.195$ and $y(m_1') = 0.62$.

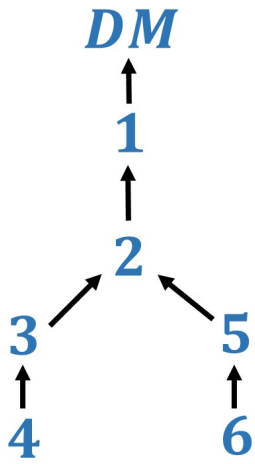
This strategy profile is an equilibrium if the biases of experts 1 and 2 satisfy $b_2 \leq 0.14$ and $b_1 \leq 0.21$. Therefore, expert 1, who is closer to the DM than expert 2, can have a larger bias compared to expert 2. The reason is that expert 2 is more informed about the overall signals compared to expert 1. This is because expert 1 receives only coarse information from expert 2.

To get a better intuition, think about $y(m_1)$ represented by the sum of the signals as in Table 2.1: $y(m_1)$ includes the sums of the signals 0, 1 and 2. The table

Table 2.1: Posteriors of experts over elements of $y(m_1)$ in Example 1

$y(m_1)$	0	1	2
Belief of expert 1	$\frac{30}{59}$	$\frac{25}{59}$	$\frac{4}{59}$
Belief of expert 2	0	$\frac{5}{6}$	$\frac{1}{6}$

Figure 2.1: Example with six experts



shows the beliefs of the most informed types of both experts. In particular, there is a type of expert 2 which attaches higher beliefs to higher integers, compared to the most informed type of expert 1. Thus, expert 2 communicates according to the specified strategy only for a strictly smaller bias range, compared to expert 1.

2.0.6 Different directions of biases

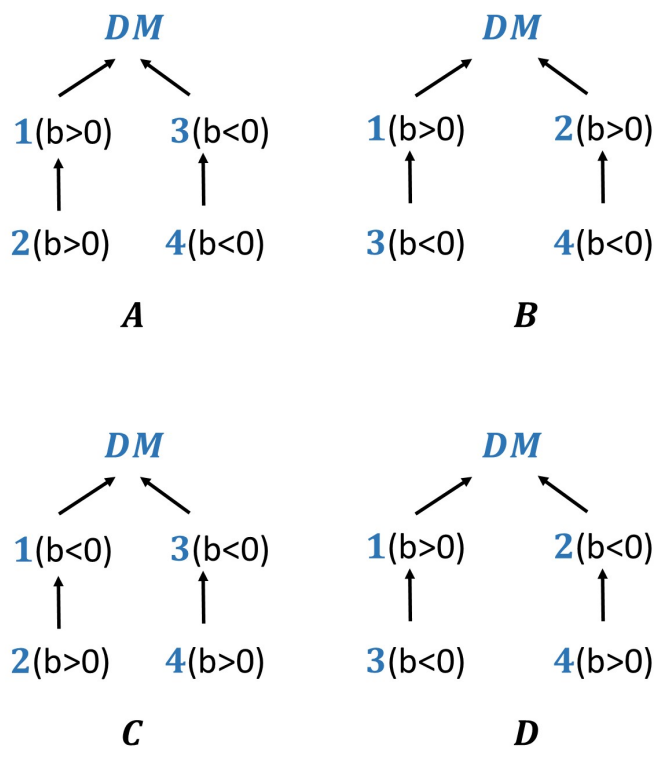
Suppose that the DM is designing communication between the experts with different directions of biases. This setting captures many real-life scenarios: In hospitals, the medical staff and the hospital administration might have a conflict of interest over the expenditure policy and in oil corporations, there might be a tension between a short-term oriented management and groups advocating sustainability and environmental concerns. How should a manager design communication between the experts with opposing interests? I show that if we restrict attention to networks with two groups, where each group leader communicates directly to the DM, it is beneficial for the DM to separate the experts according to the direction of their biases. Moreover, DM's expected utility is maximized in equilibrium in which the groups distort their messages in opposite directions.

Example 2: two teams with opposing preferences

Analytically I start with the simple case of four experts where two of the four experts have the same positive, and the remaining two the same negative bias. For tractability, suppose that the DM can only choose among networks which feature two equally sized groups. Thus, the DM is choosing one of the networks depicted in Figure 2.2. I show that the DM maximizes her expected utility by splitting the experts according to the direction of their biases because in this case she receives two messages that are biased in different directions.

Label the four experts with 1, 2, 3 and 4. Assume that $b_1 = b_2 = b^+ > 0$ and $b_3 = b_4 = b^- < 0$. Assume $max\{|b^-|, |b^+|\} > \frac{1}{12}$ as otherwise all experts communicate their signals truthfully as shown in Proposition 1.

Figure 2.2: Possible group arrangements



In case A the experts are partitioned according to the sign of their biases. In cases B, C and D the experts have mixed signs within the groups and differ by the signs of the communicating experts.

Think of the following message strategies in case A: experts 2 and 4 communicate their signals truthfully to experts 1 and 3. If the sum of the signals observed by expert 1 is 0, he sends m_1. Otherwise, if the sum of the signals which he observes is either 1 or 2, he sends m_1'. If the sum of the signals which expert 3 observes is either 0 or 1, he sends m_3, and if the sum of the signals which he observes is 2, he sends m_3'. Therefore, expert 1's communication strategy pools together the two of the largest sums of signals whereas expert 3's message strategy pools together the two of the lowest sums of signals. I refer to it as a communication biased in different directions. The DM's choices are:

$$y(m_1, m_3) = \frac{2}{9},\ y(m_1, m_3') = y(m_1', m_3) = \frac{1}{2},\ y(m_1', m_3') = \frac{7}{9}.$$

The expected utility of the DM is $\mathbb{E}U_{DM} \simeq -0.037$. This strategy profile is an equilibrium for $b^+ \leq 0.13$ and $b^- \geq -0.13$. As I show in the appendix, for $b^+ \in (0.1, 0.13]$ and $b^- \in [-0.13, -0.1)$ network A strictly dominates networks B, C and D. For these biases, the last three networks generate the same equilibrium outcome.

Does the DM specifically benefit from having communication biased in different directions? The answer is yes. Consider a strategy profile in which communication is biased in the same direction. In this case, expert 2 (expert 4) reveals his signals truthfully to expert 1 (expert 3). Both experts 1 and 3 either inform the DM that the sum of their signals is 0, or that the sum of their signals is either 1 or 2. This strategy profile is an equilibrium either in case of $0.1 < b^+ \leq 0.13$, and $b^- \geq -0.045$ or in case of $-0.13 \leq b^- < -0.1$ and $b^+ \leq 0.045$. Network A is still an optimal network but it only yields $\mathbb{E}U_{DM} \simeq -0.038$ which is strictly smaller than the equilibrium in which communication is biased in different directions.

Two bias case for a larger number of experts

In this section I extend the previous example to a larger number of players. I restrict attention to networks which feature two equally sized groups. The two respective group leaders communicate directly to the DM. I optimize over the partition according to which the DM receives her information. The optimization reveals that if the biases of experts are not too far away from the bias of the DM, it is beneficial for the DM to split the experts according to the direction of their biases.

Consider the network as in Figure 2.3, and denote it by Q'. I refer to the group on the left hand side of Q' as group A, and the group on the right hand side

as group B. Think about the strategy profile in which each expert in a group who is not a group leader, reveals his signal truthfully to a group leader. Each group leader communicates to the DM according to the partition which consists of two pools. The DM combines the messages of the two group leaders as follows:

$$P_{DM}(Q') = \{\{0,..,n-t\},\{n-t+1,..,2n\}\} \cup \{\{0,..,n-z\},\{n-z+1,..,2n\}\}, \quad (2.1)$$

where $2n$ denotes the size of each group. Previous example showed the optimality of partitioning the experts into groups according to the direction of their biases for $n = 1$. Here I show that for $n = 2, 3, 4, 5$ there are ranges of biases for which optimizing the partition $P_{DM}(Q')$ over t and z results in partitioning the experts into groups according to the direction of their biases. Denote the upper and lower bounds for the biases in groups A and B which implement the strategy profile specified above as a function of (t, z) by $b_A^-(t,z),\ b_A^+(t,z),\ b_B^-(t,z),\ b_B^+(t,z)$.

Figure 2.3: Network implementing $P_{DM}(Q')$

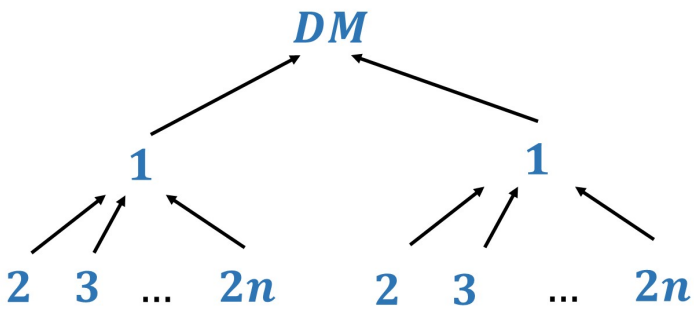


Table 2.2 shows the ranges of biases for different values of n and m if the expected utility of the DM is maximized jointly over t and z. In each group, all experts face the same constraint with respect to the range of biases which supports their equilibrium strategy captured by (1). Observe that for all n in Table 2.2, there is always a bias region for each group which is outside the intersection of bias regions of the both groups. If experts' biases in each group are within such a region, it is optimal for the DM to partition the experts according to the direction of their biases.

Next, I provide an example with unequal number of positively and negatively biased experts, and look at the situation in which it is optimal to allocate them ac-

Table 2.2: Different group sizes

n	Max EU_{DM}	$b_A^-(t,z)$	$b_A^+(t,z)$	$b_B^-(t,z)$	$b_B^+(t,z)$
2	$t=0,\ z=1$	-0.073	0.04	-0.04	0.073
3	$t=0,\ z=1$	-0.043	0.033	-0.033	0.043
4	$t=0,\ z=1$	-0.027	0.028	-0.028	0.027
5	$t=-1,\ z=2$	-0.036	0.026	-0.026	0.036

cording to the direction of their biases. I show that this is the case once the biases are sufficiently far apart from each other.

Example 3: Consider 8 experts: 2 are equally positively and 6 are equally negatively biased, with a positive bias denoted by b^+ and a negative bias denoted by b^-. Suppose that the DM has to design an optimal network consisting of 2 groups, but the groups can be unequal in size. I ask the question: For which biases is it optimal to divide the experts according to the direction of their biases?

Suppose, first, that the difference between experts' biases b^+ and b^- is relatively large: $b^+ \in (0.07, 0.18]$ and $b^- \geq -0.07$. In this case, the optimal network is shown in Figure 2.4a with positively biased experts labelled by 1 and 2 being separated from the negatively biased experts. The equilibrium partitions of the group leaders in a smaller and larger group are $\{\{0\},\{1,2\}\}$ and $\{\{0,1,2,3\},\{4,5,6\}\}$ resulting in $\mathbb{E}U_{DM} = -0.0283$.

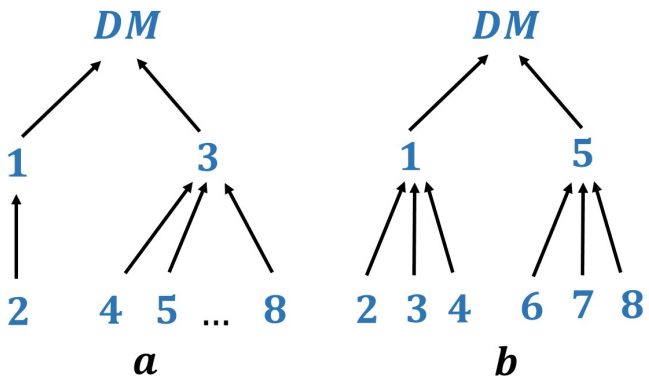
Next, suppose that the difference between experts' biases is not too large: $b^+ \leq 0.07$ and $b^- \geq -0.07$. In this case, the optimal network features two equally sized groups as in Figure 2.4b with an arbitrary assignment of experts into one of the two groups.

To summarize, it is optimal to divide the experts into groups according to their biases if the difference between b^+ and b^- is relatively large. Otherwise, it is optimal to have two equally sized groups of experts.

2.1 Benefits of an additional group leader

Consider two groups of experts with the group leaders reporting directly to the DM as in Figure 2.5. I show conditions under which this network is strictly dominated

Figure 2.4: Illustration of networks in example 5

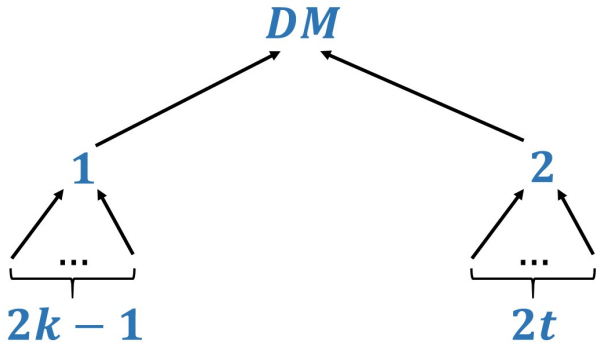


by another network in Figure 2.6. The latter network differs from the former one twofold. First, I decentralize information aggregation in one of the groups (with $2t+1$ experts) as I am splitting this group into two almost equal subgroups. Thus, I create uncertainty among members of each of the subgroups about the reports in the other subgroup. Second, I introduce stronger coordination on upper levels: group leaders of both subgroups report to another expert, who optimally coarsens information from both groups. Further, I introduce a single expert who is the only expert reporting directly to the DM. Thus, all information available to the DM is coordinated by a single expert which is not the case in Figure 2.5. Therefore, once decreasing coordination on lower levels, and increasing coordination on upper levels, the new network generates better outcomes for the DM.

In both networks, for each group I use a partition which consists of two pools and maximizes the distance between them in order to accommodate largest possible biases.

Case 1: Assume that the DM receives reports from 2 groups of experts: there are $2k$ experts in the first group, and $2t+1$ experts in the second group. Within each group, there is a group leader who receives direct reports from each group member before reporting the aggregate information in a coarse way to the DM. This is the consequence of Proposition 3 and Lemma 2 since such group communication structure maximizes uncertainty within a group. I assume that each group reports according to a partition which maximizes the positive distance between the pools, where the first pool includes the minimal amount of information. This implies that I look at cases of largest possible biases which a network can accommodate. The partitions for expert 1, 2 are as follows:

Figure 2.5: 2 groups



$$P_1 = \{\{0\}, \{1, .., 2k\}\},\ P_2 = \{\{0\}, \{1, .., 2t+1\}\},$$

such that the DM receives information according to a combined partition:

$$P_{DM} = \{\{0\}, \{1, .., 2k\}\} \times \{\{0\}, \{1, .., 2t+1\}\}.$$

The DM chooses her policies assuming sequential rationality.

Case 2: For the same number of experts, assume that the DM organizes them into a network depicted in Figure 2.6. There is a single expert 1 which collects information from two sources: first, he gets a report from an expert 2 who is a group leader and receives direct reports from $2k-1$ other experts, and second he gets a report from expert 3 who, in turn, is connected to two group leaders - expert 4 receives reports from $t-1$ experts and expert 5 receives reports from $t-2$ experts.

Experts 2, 4 and 5 report according to the following partitions:

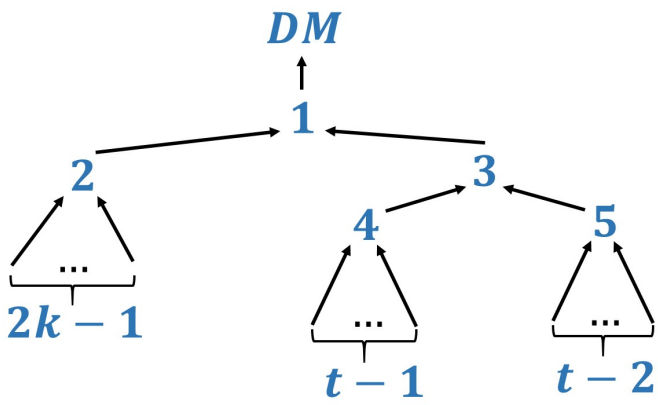
$$P_1 = \{\{0\}, \{1, .., 2k\}\},\ P_4 = \{\{0\}, \{1, .., t\}\},\ P_5 = \{\{0\}, \{1, .., t-1\}\}.$$

For a partition $P_i := \{p_i^1, p_i^2\}$, $i = 2, 4, 5$, denote the message the corresponding group leader send when reporting p_i^1 by m_i and when reporting p_i^2 by m_i'.

Expert 3 reports to expert 1 as follows:

- He reports m_3 if his private signal is 0 and he receives either (m_4, m_5), or (m_4', m_5), or (m_4, m_5'), or his private signal is 1 and he receives (m_4, m_5).
- Otherwise he reports m_3'.

Figure 2.6: 2 groups



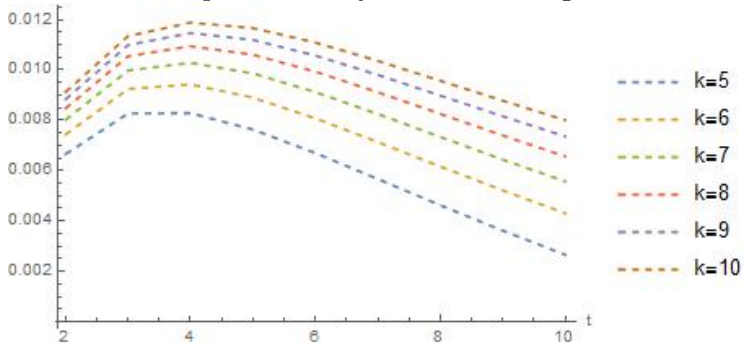
Expert 1 reports to the DM as follows:

- If his private signal is 0 and he receives (m_2, m_3), or (m_2', m_3), or when his private signal is 1 and he receives (m_2, m_3).

The DM chooses her policies assuming sequential rationality.

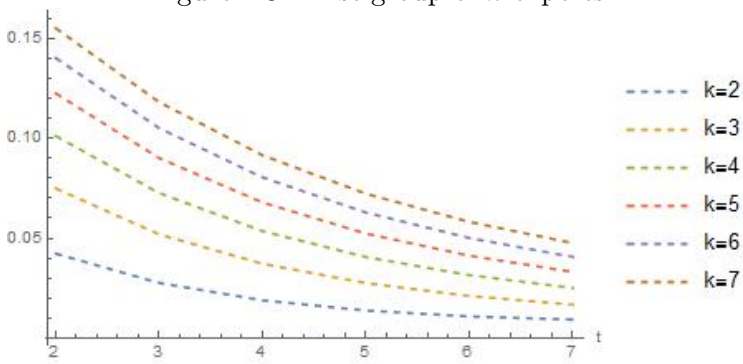
In the Figure 2.7 I plot the differences in welfare between two network and equilibrium configurations. Figure below shows the difference between the DM's expected utility in the second to the DM's expected utility in the first. The difference remain positive for high values of k and t, suggesting that DM benefits from partitioning $2t$ experts into two groups.

Figure 2.7: DM's expected utility is Case 2 is higher than in Case 1



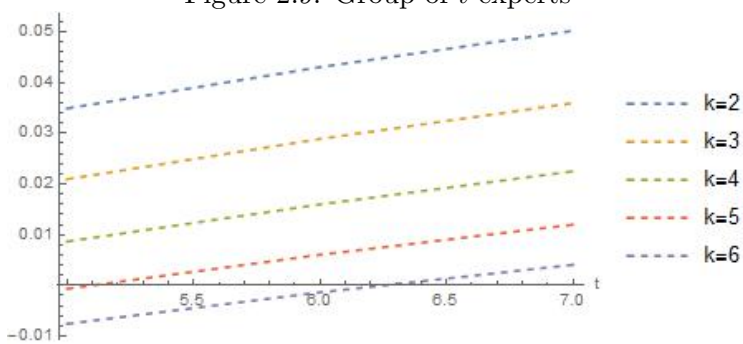
Next, consider the ICs of any expert in group with $2k$ experts. Denote the upper bound for the bias in the first case, b_k^+ and the second case, $b_k'^+$. In Figure 2.8 I am plotting $b_k'^+ - b_k^+$:

Figure 2.8: First group of k experts



Next, consider the group of $2t$ experts and define the upper bound for the bias in the first case, b_t^+ and the second case, $b_t'^+$ for the group with t experts and $b_t''^+$ for the group of $t-1$ experts. In the Figure below I am plotting $b_t'^+ - b_t^+$, and in Figure I am plotting $b_t''^+ - b_t^+$.

Figure 2.9: Group of t experts



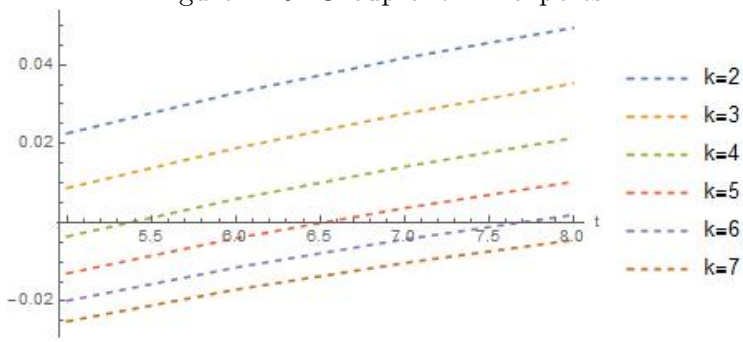
We see that a new network is implementable if $\frac{k}{t}$ is relatively small. We see that if k gets larger, there is a larger pressure on the ICs of the experts in both groups t and $t-1$.

In sum, we see that there exist multiple ranges of parameters for which splitting one of the groups into two (almost equal) groups benefits the DM and relaxes the reporting constraints of the experts in the smaller groups due to the introduced uncertainty.

2.1.1 Commitment

Suppose that the DM can commit to a mechanism that implements an allocation conditional on the information received from the experts. The revelation principle tells us that without loss of generality we can restrict attention to a direct mechanism

Figure 2.10: Group of $t-1$ experts



in which experts communicate their types. Formally, a mechanism is a rule q that maps experts' types to the final decision $y \in [0,1]$:

$$q : T \to [0,1].$$

I focus on incentive-compatible mechanisms: if expert i is of type $t_i \in \{0,1\}$ and all other experts communicate truthfully their types, then expert i has no incentive to communicate $1-t_i$, which formally means:

$$\mathbb{E}U_i(m_i = t_i, m_{-i}|t_i) \geq \mathbb{E}U_i(m_i = 1-t_i, m_{-i}|t_i).$$

Proposition 4: *Every equilibrium outcome generated by any tree communication network is also an equilibrium outcome of some direct mechanism q. The converse is false.*

The intuition for Proposition 4 is as follows. First, an equilibrium outcome of any communication network can be implemented by a direct mechanism. Each equilibrium outcome in a communication network can be summarized by a partition of experts' signals, which — given the standard definition of a type — is the same as a partition of experts' types. This partition defines the information sets of the DM who chooses the final policy in a sequentially rational way. Such a partition is incentive-compatible, and thus can be implemented by an incentive-compatible direct mechanism q.

Second, the converse is not true. The reason is that in a communication network an expert can observe the messages of some other experts. In a direct mechanism, no expert observes the messages of other experts. As Lemma 1 shows, if

an equilibrium involves strategic coarsening of information, then greater knowledge about other experts' messages leads to tighter incentives of an expert to stick to his equilibrium strategy. Thus, an equilibrium that involves strategic coarsening of information can be implemented in a direct mechanism for a weakly larger range of experts' biases than in an optimal communication network.

In equilibria that do not involve strategic coarsening of information, the incentives to communicate information are the same both in a direct mechanism and in a corresponding communication network. As Proposition 2 shows, this is because for a fixed number of equilibrium truthful messages, the deviation incentives of an expert do not depend on his beliefs over the types of other experts.

2.1.2 Beyond Tree Networks

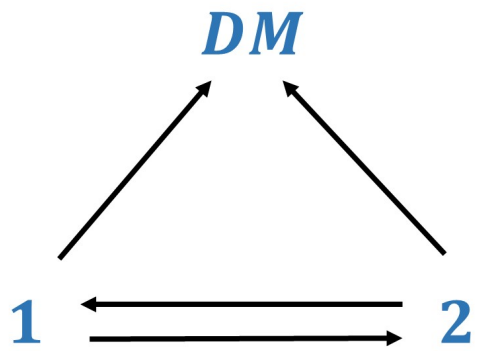
First, I look at a network with two experts and a DM which features the experts communicating with each other before talking to the DM. I show that this network does not necessarily induce the experts to reveal all of their private information. The reason is that if the experts do not have incentives to reveal their information in a tree network, they do not reveal it in any other network either, since the incentives in case of perfect revelation are the same in any network. Second, I look at a network which features multiple outgoing links for one of the experts. I show how this network generates a better outcome for the DM compared to the optimal tree.

Experts talk to each other before informing the DM: Consider a network in Figure 2.11 in which both experts exchange messages before communicating to the DM. Consider $b_1 \leq \frac{1}{8}$ and $\frac{1}{8} < b_2 \leq \frac{1}{6}$. As I show in the appendix, the optimal tree is a line. The line features expert 2 revealing his signal to expert 1, and expert 1 informing the DM that the sum of the signals is either 0, or 1 and 2. The payoff of the DM is strictly below the payoff in case of perfect revelation of signals. Can the network in Figure 2.11 generate a more informative equilibrium for the DM compared to the line? The answer is no for the following reason.

Consider the strategy profile in which expert 2 reveals his signal truthfully to expert 1 and expert 1 reveals both signals truthfully to the DM. In this case, the DM receives the most informative message from expert 1 independent of the message of expert 1 to expert 2, and of the message of expert 2 to the DM. However, this strategy profile cannot be an equilibrium since expert 2 has an incentive to deviate from communicating the true signal to expert 1 since $b_2 > \frac{1}{8}$. The DM cannot credibly commit to a punishment in case she receives non-matching messages from both experts, because her best strategy is to choose a policy based on those

messages which she regards as truthful. If expert 2 does not truthfully reveal his signal to expert 1, and expert 1 informs the DM about his own private signal s_1, the DM chooses her policy based on expert 1's message about s_1.

Figure 2.11: Both experts exchange messages before communicating to the DM

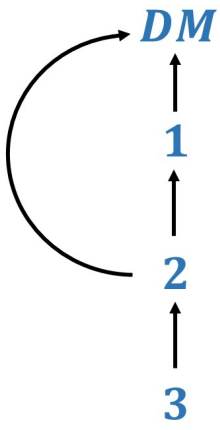


Example 4: Consider three positively biased experts organized in a network depicted in Figure 2.12 and the following strategy profile: expert 3 communicates his signal truthfully to expert 2, and expert 2 sends the same message to the DM and to expert 1: if both expert 3's message and his private signal are 0 he sends m_2; otherwise he sends m_2'. Expert 1 communicates as follows: if he receives m_2, he sends m_1 irrespective of his private signal. If he receives m_2', then he sends m_1' if his private signal is 0; finally, if he receives m_2' then he sends m_1'' if his private signal is 1. Thus, if the DM receives m_2 from expert 2, she disregards expert 1's message. Otherwise she can distinguish between different types of expert 1. Thus, in $\frac{1}{3}$ of cases (which is the case if m_2 is sent) the DM receives coarse information only from experts 2 and 3, and in $\frac{2}{3}$ of cases (which is the case if m_2' is sent) expert 1 truthfully communicates his signal to the DM. The expected utility of the DM is -0.044. It turns out that for $b_2, b_3 \leq 0.14$ and $0.115 < b_1 \leq 0.13$, the network depicted in Figure 2.12 dominates any tree network.

2.2 Conclusions

This paper has studied the optimal design of communication networks featuring multiple biased and imperfectly informed experts, and an uninformed DM. The DM adapts a tree communication network which specifies which of the players commu-

Figure 2.12: Expert 2 has two outgoing links



nicates with whom, and in which order. The DM's objective is to elicit maximum possible information from the experts.

I showed that the design of an optimal network is shaped by two competing forces. On the one hand, an intermediation force brings experts together and enables an optimal coarsening of pooled information. On the other hand, an uncertainty force separates them and relaxes their incentives to reveal their privately held information. I showed that simultaneous communication is dominated by optimally designed sequential communication. Optimal sequential communication, in general, separates the experts into groups of mostly similar bias. If the biases of experts are sufficiently close to one another, and sufficiently different to the bias of the DM, the optimal network features a single intermediary.

The model is easily computable. This makes it easier to bring the model to the data. To my knowledge, there is still no systematic empirical study looking at different organizational communication networks from the perspective of strategic communication.

The main motivation for this project came from the organizational economics literature. Communication within organizations takes a complex form. I argued in this paper that a tree network is a natural starting point when thinking about optimal hierarchies in organizations. It would be an important step to study other forms of networks. A richer set of communication networks can include experts talking to multiple audiences, cycles and a variation of noise in communication channels.

Finally, communication is a dynamic process which can feature multiple rounds of informational exchange. Literature on strategic communication shows

that two rounds of communication between an informed sender and an uninformed receiver can enlarge the set of equilibrium outcomes (Krishna and Morgan, 2004). It is interesting to see, how the equilibrium set changes once multiple rounds of communication are introduced into this model, and what is their implication for the optimal networks.

Chapter 3

Communication And Delegation In A Game With Evidence.

3.1 Introduction

> *The peculiar character of the problem of a rational economic order is determined precisely by the fact that the knowledge (...) never exists in concentrated or integrated form but solely as the dispersed bits of incomplete and frequently contradictory knowledge which all the separate individuals possess. The economic problem of society (...) is a problem of the utilization of knowledge which is not given to anyone in its totality.*
> Hayek (1945)

In organizations decision-relevant information is often dispersed among employees. This information can have two features. First, it is noisy in the sense that an employee receives only an imprecise signal about a decision-relevant parameter, and this signal depends on employee's personal characteristics such as formal qualification, experience or assigned task. Think, for example, of different market researchers who obtain information dependent on methods they use, or experience they bring in. Second, the information is verifiable. Think of the results of medical trials in pharmaceutical companies, evaluation tests for new technologies in industry or data on online surveys or field trials in market research.

A manager would like to make the best informed policy decision. Thus, she would like to know all information available to her employees. Suppose that the employees can conceal their information: Think of the possibility not to reveal part of test results on a new product. Suppose that the manager can design communication

channels and delegate the decision rights. Is it better for the manager to receive direct reports from both employees and keep the decision right to herself? Or is it better to let employees share decision-relevant information among themselves and let one of the employees choose the policy? Furthermore, if the manager decides to delegate authority and knows which of the employees is likely to generate which evidence, whom should she delegate the decision right to? This paper attempts to answer these questions.

Consider an uninformed decision maker (DM) and two experts. Each expert can be either uninformed or imperfectly informed about the state. Expert's types are private information. If imperfectly informed, the experts obtain different, and complementary, evidence about the state. The precision of the evidence depends on the state and differs between the experts. Each expert has an option to report to other players that he is uninformed. Thus, communication is strategic. The DM can choose between centralization and decentralization. Under centralization, both experts send direct reports to the DM who chooses a policy. Under decentralization, one of the experts communicates to another expert, and the latter expert chooses the policy.

I assume that the DM cannot commit to transfers or choice functions dependent on received information. She can only commit to the allocation of decision rights and to a communication mode. Thus, this paper follows the incomplete contracts approach forcefully argued by Grossman and Hart (1986) where the DM cannot specify all possible states of nature in advance and contract upon them. Therefore, once a state of nature is realized and communicated, the DM is not restricted in her decision by any contractual arrangement.

When is it optimal for the DM to decentralize? In general, two conditions have to be satisfied. First, the bias of an expert who decides over the policy should not be too large compared to the bias of the DM. Otherwise the costs of delegation are larger than the costs due to losses of information if an expert communicates directly to the DM. However, when making a decision over delegation the DM has also to consider the bias of the second expert who provides decision-relevant information to the first expert whom the decision is delegated to. Perhaps surprisingly, I show that the bias of the second expert does not have to be small compared to the bias of the DM or of the first expert. The range of biases for which the second expert reveals his evidence to the first expert depends on his signal structure. Suppose that under centralization one of the informed types of the second expert does not reveal his signal to the DM. If under decentralization the bias of the first expert is nearer to the bias of the second expert, compared to the bias of the DM, then the second

expert is more willing to reveal his signal to the first expert than to the DM.

In the next section I provide the literature review. Section 2 defines the model and section 3 provides the leading example. Section 4 characterizes equilibria under centralization, decentralization, and compares them. Section 5 concludes. All proofs are in the appendix.

3.1.1 Literature Review

Efficient aggregation of information, which is dispersed among multiple agents, goes back to the team theory proposed by Marschak and Rander (1972). They study optimal communication structures among the agents who face costs of communication and information processing. More recent contributions include Geanakoplos and Milgrom (1991), Radner (1993), Bolton and Dewatripont (1994), Van Zandt (1999b), and Garicano (2000). This literature assumes no conflict of interest between the agents. Thus it does not model strategic communication which is different to my model. Similar to my model, Marschak and Radner allow for different distributions of signals among the agents.

There is a rich literature on delegation. Much of it assumes that the DM is not able to commit to a decision both before and after receiving information from the agents. However, she can contract upon delegation of the decision right. It is assumed that this contract, if accepted by the agent, cannot be renegotiated. Delegation is considered to be useful in saving costs of information processing starting with Geanokoplos and Milgrom (1991) or Mount and Reiter (1990). However, this literature abstracts from incentive problems since the preferences of players are aligned. Incentive problems in delegation are studied in Holmström (1977, 1984) who shows the basic trade-off between eliciting information from an agent who might not have incentives to reveal everything he knows, and delegating him the decision right which results in a biased decision. More recently Dessein (2002) studies the choice between centralization and delegation where different to Holmström he assumes that the DM cannot limit agent's choices or write contracts to align the objectives between the DM and the agent. Dessein shows that delegation is optimal if the divergence in preferences between the agent and an uninformed decision maker is not too large relative to the agent's informativeness. Different to those papers I assume two agents who are imperfectly informed. This allows to study delegation as an instrument for information sharing between the agents which is not possible in the above literature.

Alonso et al. (2008) and Rantakari (2008) compare delegation with centralization when coordination between different tasks is important and when players

communicate in a costless and nonverifiable way as in cheap talk (Crawford and Sobel (1982)). In Alonso et al. (2008) there are two states which are perfectly observed by one of the division managers but not by the central manager. There are two decisions, and the central manager faces a trade-off between adaptation of a decision to one of the states, and coordination between two decisions. They show when coordination is important, centralization dominates decentralization. Different to their model, I do not model coordination of decisions since there is a single decision to be made. In my model the experts (similar to division managers in their model) are imperfectly informed and have different signal structures. Thus, different to those papers, I am able to link information structures of the agents to the decision over centralization versus delegation. Finally, I model communication with verifiable information such that equilibria of both models are not directly comparable.

3.2 Model

There is a decision maker, DM, and two experts, labelled 1 and 2. The payoff function of player $i \in \{1, 2, DM\}$ is:

$$u_i = -(y - \theta - b_i)^2,$$

where $y \in \mathbb{R}$ denotes the policy decision, $\theta \in \mathbb{R}$ is the unobserved state of the world with the uniform common prior distribution over $[0, 1]$ and b_i is a bias of player i. I normalize $b_{DM} = 0$.

Signals: The DM does not receive any signal about the state. Each expert is either informed or uninformed, which is not observed by other players and happens with probability $\frac{1}{2}$ which is independent across experts. If being informed, each expert receives a signal in form of a strict subinterval of $[0, 1]$ where the probability of receiving a subinterval conditional on being informed coincides with the length (norm) of the subinterval due to the uniform distribution of the state. The subinterval constitutes the expert's *verifiable evidence* (which is equivalent to a standard notion of player's *type*). Verifiable evidence means that an expert can either show the piece of evidence which he got, or conceal it, claiming that he has not observed anything. Thus, an expert cannot show false evidence.

Denote by T_i the set of possible types of expert i:

$$T_1 = \Big\{[0, 1], [0, \frac{1}{2} - v], [\frac{1}{2} - v, 1]\Big\},\ v \in [0, \frac{1}{2})$$

$$T_2 = \Big\{[0,1], [0, \frac{1}{2}+d], [\frac{1}{2}+d, 1]\Big\},\ d \in [0, \frac{1}{2})$$

This means, for example, that expert 1 can either know nothing beyond the common prior (he receives an uninformative signal $[0,1]$), or he believes that the state is uniformly distributed either in $[0, \frac{1}{2}-v]$ or in $[\frac{1}{2}-v, 1]$.

For convenience I use the notations $t_1 = [0, \frac{1}{2}-v]$, $t_1' = [\frac{1}{2}-v, 1]$, $t_2 = [0, \frac{1}{2}+d]$ and $t_2' = [\frac{1}{2}+d, 1]$. Notice that, dependent on the state and on being informed, expert 1 is has a more precise knowledge about lower range of states, and expert 2 has a more precise knowledge about the upper range of states, compared to the other expert.

Communication and contracts: Following the incomplete contracts approach I assume that the DM cannot write a contract over decisions or over information received. However, she can commit to a tree communication network and ex ante allocation of a decision rights. A tree communication network $Q = (N, E)$ is a directed graph with a set of nodes N and the set of edges $E \subseteq N \times N$. For every pair of nodes (i, j), $e_{ij} \in \{0, 1\}$: $e_{ij} = 1$ means that there exists a directed link from i to j and $e_{ij} = 0$ means that no such link exists.

I focus on two communication and decision right architectures: *Centralization* and *Decentralization*. Under Centralization, each expert reports directly to the DM, which implies $e_{iDM} = 1$ for both $i = 1, 2$. Otherwise, there is no further information exchange between the players. After receiving expert's reports, the DM decides over y. Under Decentralization, expert i decides over y, and the other expert j reports to i, $e_{ji} = 1$. Otherwise, there is no further communication between the players. I denote the choice of the organizational architecture by $s \in \{C, D\}$ where C denotes Centralization and D denotes Decentralization.

The Game, Equilibrium: First, each type is drawn from a distribution $p_i \in \Delta(T_i)$, $i = 1, 2$, and each expert is privately informed about his type. The DM chooses either a decentralized or a centralized structure. Then, dependent on DM's choice, the messages are sent according to the specified communication. Thereafter, the player who has the decision right chooses y.

The solutions concept is a pure strategy Perfect Bayesian Equilibrium, where the DM chooses s such that $Max_s\ \mathbb{E}U_{DM}$ given the prior $\Delta(T_1)$ and $\Delta(T_2)$.

Thereafter, if C is chosen, each expert $i = 1, 2$ sends $m_i \in M_i$ to the DM which solves $Max_{m_i}\ \mathbb{E}U_i$ given t_i and the posterior over T_j conditional on t_i, $j \neq i$. Then, the DM chooses y which solves $Max_y\ \mathbb{E}U_{DM}$ given (m_1, m_2).

Alternatively, if D is chosen with expert i deciding over y, expert $j \neq i$ sends $m_j \in M_j$ to expert i which solves $Max_{m_j}\ \mathbb{E}U_j$ given t_j and the posterior over T_i conditional on t_j. Thereafter, i chooses y which solves $Max_y\ \mathbb{E}U_i$ given t_i and the posterior over T_j conditional on t_i and m_j.

3.3 Leading Example

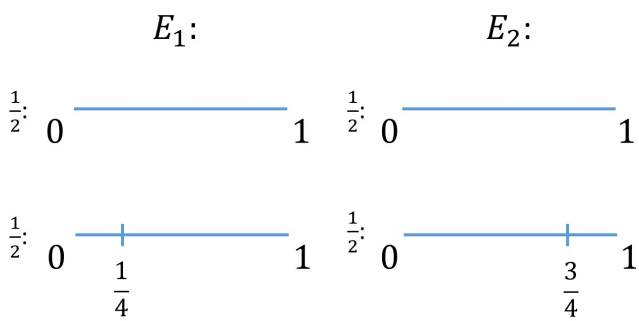
Consider $v = d = \frac{1}{4}$ such that the sets of possible evidences are:

$$T_1 = \{[0,1], [0, \frac{1}{4}], [\frac{1}{4}, 1]\},\ T_2 = \{[0,1], [0, \frac{3}{4}], [\frac{3}{4}, 1]\}.$$

Both sets of evidence are depicted in Figure 3.1. First, think about a strategy profile in which the DM chooses centralization and all informed types of both experts report their types truthfully. Then, the expected utility of the DM is -0.042 and the strategy profile is supported for $-0.0625 \leq b_1 \leq 0.1625,\ -0.1625 \leq b_2 \leq 0.0625$. Notice that the upper bound for incentive compatible biases of expert 1 is bigger than of expert 2 for the following reason. The upper bound for expert 1 is defined by deviation incentives of $t_1 = [0, \frac{1}{4}]$ to the report $[0, 1]$. The corresponding upper bound for expert 2 is defined by deviation incentives from $t_2 = [0, \frac{3}{4}]$ to $[0, 1]$. The deviation of t_1 to $[0, 1]$ results in a larger expected shift of DM's policy compared to the deviation of t_2 to the report $[0, 1]$, and thus brings expert 1 further away from his ideal point compared to a deviation of expert 2. Therefore, the truthtelling strategy of expert 1 can be supported by higher values of biases, compared to expert 2.

Same argument applies for expert 2 having a smaller lower range of biases supporting truthtelling, compared to expert 1.

Figure 3.1: Information sets for 2 experts



Now, suppose that the bias of second expert is higher compared to the per-

fect revelation range: $b_2 > 0.0625$. If b_1 is in the range defined above, the best equilibrium under centralization features t_1, t_1' and t_2' reporting truthfully to the DM, and t_2 reporting $[0, 1]$. This equilibrium yields the expected utility of the DM -0.045. Can the DM do better if the bias of expert 1 is not too large?

The answer is positive. Think about a strategy profile in which the DM chooses decentralization, assigns the decision right to expert 1, and expert 2 reports *all his types* to expert 1. This equilibrium requires $b_2 \leq 0.0625 + b_1$. What is the range of biases for which this equilibrium under decentralization dominates the best equilibrium under centralization defined above?

DM's expected utility under decentralization is $-0.042 - b_1^2$. Since the best centralized mechanism implements -0.045, the DM would benefit from decentralization if

$$-0.042 - b_1^2 > -0.045, \ \rightarrow \ b_1 \lesssim 0.055$$

Combining the constraints $b_2 \leq 0.0625 + b_1$ and $b_1 \lesssim 0.055$, we have:

$$D \succ_{DM} C \text{ if } |b_1| \leq 0.055, \ b_2 \leq 0.1.$$

Therefore, we see that for the range of expert's biases outside the perfect revelation range in a centralized architecture, if the bias of expert 1 is not too high compared to the bias of the DM and if the difference between expert's biases is not too large, then the DM benefits from the decentralized architecture.

3.4 Analysis

Player i who is assigned the decision right chooses:

$$y = \mathbb{E}U_i(\theta|\mathcal{I}_i) + b_i,$$

where $\mathcal{I}_i$ denotes the information set of player i once communication took place.

In the following I, first, calculate the equilibrium conditions for the first best outcome in a centralized mechanism. I, then, characterize equilibria in which 3 out of 4 informed types of experts reveal their signals truthfully under centralization and show conditions on the set of evidences under which these equilibria dominate all other equilibria under centralization. Given those conditions I show that there are equilibria under decentralization which strictly outperform best equilibria under centralization.

3.4.1 Centralization: first best

In a centralized mechanism both experts communicate directly to the DM without any prior informational exchange between each other. The DM decides over y based on experts' reports. I study the conditions on expert's biases for in which each type of each expert is incentivized to reveal his type in equilibrium. Obviously, in this case the DM has no interest to implement decentralization since it only adds on costs connected to biased policy decision of an expert responsible for a policy choice.

Using backward induction, the choices of the DM are:

$$y([0,1]) = \frac{1}{2}, \quad y([0, \frac{1}{2} - v]) = \frac{1-2v}{4}, \quad y([\frac{1}{2} - v, 1]) = \frac{3-2v}{4},$$

$$y([0, \frac{1}{2} + d] = \frac{1+2d}{4}, \; y([\frac{1}{2} - v, \frac{1}{2} + d]) = \frac{1-v+d}{2}, \quad y([\frac{1}{2} + d, 1]) = \frac{3+2d}{4}.$$

The corresponding expected utility of the DM is:

$$\mathbb{E}U^C_{DM} = -\frac{1}{4}\int_0^1 \Big(\frac{1}{2} - \theta\Big)^2 d\theta - \frac{1}{4}\Big(\int_0^{\frac{1}{2}-v} \Big(\frac{1}{4}(1-2v) - \theta\Big)^2 d\theta +$$

$$\int_{\frac{1}{2}-v}^1 \Big(\frac{1}{4}(3-2v) - \theta\Big)^2 d\theta\Big) - \frac{1}{4}\Big(\int_0^{\frac{1}{2}+d} \Big(\frac{1}{4}(1+2d) - \theta\Big)^2 d\theta +$$

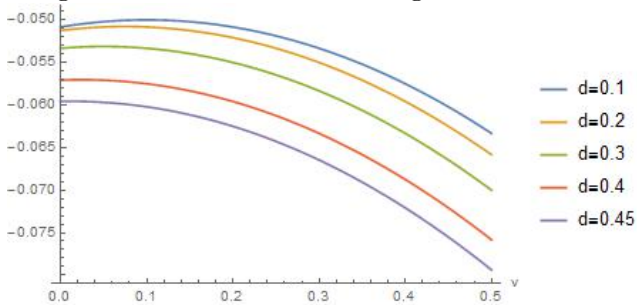
$$\int_{\frac{1}{2}+d}^1 \Big(\frac{1}{4}(3+2d) - \theta\Big)^2 d\theta\Big) - \frac{1}{4}\Big(\int_0^{\frac{1}{2}-v} \Big(\frac{1}{4}(1-2v) - \theta\Big)^2 d\theta +$$

$$\int_{\frac{1}{2}-v}^{\frac{1}{2}+d} \Big(\frac{1}{2}(1-v+d) - \theta\Big)^2 d\theta + \int_{\frac{1}{2}+d}^1 \Big(\frac{1}{4}(3+2d) - \theta\Big)^2 d\theta\Big)$$

In Figure 3.2 I show the variation of DM's expected utility as a function of v for some fixed values of d. Notice that the larger is d, the smaller is the optimal v. This is because the DM expects a very uninformative report from expert 2 as in case of $d = 0.45$, and would like to counterbalance it with a balanced report of expert 1 with v almost equal to 0. However, if the report of expert 2 is more informative - it means if d is relatively low - the DM benefits from a less balanced report of expert 1 with v between 0.1 and 0.2, since in this case combining such reports minimizes the expected residual variance.

Now, let us turn to the incentives of the experts. The derivation is relegated to the appendix and results in:

Figure 3.2: Variation of v for given values of d



$$\frac{2v-1}{8} \leq b_1 \leq \frac{1+4d^2+4v+8dv+8v^2}{8(1+2d+4v)}$$

$$\frac{-d(4+8v)-1-8d^2-4v^2}{8(1+4d+2v)} \leq b_2 \leq \frac{1-2d}{8}$$

Notice that $\frac{1+4d^2+4v+8dv+8v^2}{8(1+2d+4v)} > \frac{1-2d}{8}$ since $8(1+4d^2+4v+8dv+8v^2) - 8(1+2d+4v)(1-2d) = 64(d^2+2dv+v^2) > 0$. This implies that the upper bound for the range of biases supporting truthtelling in centralization is larger for expert 1 than for expert 2. The reason is that the deviation of expert 1 from $t_1 = [0, \frac{1}{2} - v]$ to $[0, 1]$ results in a larger expected shift of DM's policy compared to the deviation of expert 2 from $t_2 = [0, \frac{1}{2} + d]$ to $[0, 1]$. Therefore, the deviation brings expert 1 further away from his ideal point, compared to expert 2, and thus his strategy is incentive compatible for larger biases compared to expert 2.

3.4.2 Centralization: one expert's type deviates

Suppose that one type of one of the experts deviates whilst all other types fully report their information. Thus, there are four possible cases. Here, I cover the first case in which t_1, t_2 and t_2' report truthfully to the DM whilst t_1' always reports $[0, 1]$. All other cases are derived in a similar way and are relegated to the appendix.

If only t_1' deviates, and the DM observes the reports $[0, 1]$ from both experts, she chooses:

$$y' = \frac{1}{\left(1+\frac{1}{2}+v\right)2} + \frac{1}{4}\left(1 - \frac{1}{1+\frac{1}{2}+v}\right)(3-2v).$$

To derive y', notice, that it is sufficient for the DM to form posteriors, and apply them to corresponding expected values of the state, of the following two events: in the first one expert 1 is uninformed which happens with probability $\frac{1}{2}$, and in the

second one expert 1 is of type $[\frac{1}{2}-v,1]$ which happens with probability $\frac{1}{2}(\frac{1}{2}+v)$.

By a similar argument, if the DM receives report $[0,1]$ from the first expert and $m_2 = t_2$ from the second expert, Bayesian updating leads her to choose y as follows:

$$y'' = \frac{1+2d}{\left(1+\frac{2d+2v}{1+2d}\right)4} + \frac{1}{2}\left(1-\frac{1}{1+\frac{2d+2v}{1+2d}}\right)(1-v+d).$$

The expected utility of the DM is:

$$-\frac{1}{4}\int_0^1 (y'-\theta)^2\,d\theta - \frac{1}{4}\left(\int_0^{\frac{1}{2}+d}(y''-\theta)^2\,d\theta + \int_{\frac{1}{2}+d}^1\left(\frac{1}{4}(3+2d)-\theta\right)^2 d\theta\right) -$$

$$\frac{1}{4}\left(\int_0^{\frac{1}{2}-v}\left(\frac{1}{4}(1-2v)-\theta\right)^2 d\theta + \int_{\frac{1}{2}-v}^1 (y'-\theta)^2\,d\theta\right) -$$

$$\frac{1}{4}\left(\int_0^{\frac{1}{2}-v}\left(\frac{1}{4}(1-2v)-\theta\right)^2 d\theta + \int_{\frac{1}{2}-v}^{\frac{1}{2}+d}(y''-\theta)^2\,d\theta + \int_{\frac{1}{2}+d}^1\left(\frac{1}{4}(3+2d)-\theta\right)^2 d\theta\right).$$

The range of biases which supports the above strategy profile is derived in the appendix. Denote the upper bias supporting truthtelling for t_1 by $b_1'^{+}$. It can be shown that $b_1'^{+} > b_1^{+}$ where b_1^{+} is the upper bound of the t_1 in the first best scenario of perfect information revelation under centralization. The reason is that in case of t_1's deviation to the report $[0,1]$, the expected shift in DM's policy is higher compared to the first best. This happens because once the DM observes reports $[0,1]$ from each expert, she attaches some beliefs to type t_1' and therefore puts larger weight to higher states compared to the report $[0,1]$ in the first best. This result in generalized in Lemma 1.

Lemma 1: *In centralization, the strategy profile of experts in which both informed types of expert i and only one type of expert j report truthfully to the DM, results in a higher upper bound for the biases of expert j compared to the strategy profile in which all types of experts report truthfully to the DM.*

Proof: Notice that the upper bound is relevant only for the case in which the only type which deviates is either t_1' or t_2', since if only either t_1 or t_2 deviate, this defines a lower bound for the respective expert.

Think of a strategy profile in which the only informed type which deviates is

t_1': the argument for t_2' is similar. Once the DM observes $[0,1]$ from both experts, she chooses $y([0,1]) = \mathbb{E}_{DM}(\theta|[0,1])$ which is larger than $y([0,1])$ in case of perfect revelation in a centralized mechanism since uninformed types are pooled with t_1' which has higher expectation of the state than $\frac{1}{2}$: this is shown in the appendix.

This implies that for every type realization of expert 2, the deviation of t_1 leads to a higher shift in DM's choice. Therefore, the expected deviation leads to a higher expected shift of DM's policy resulting in a larger range of biases for t_1 supporting the specified strategy profile. Q.E.D.

3.4.3 When do such equilibria constitute the second best outcome?

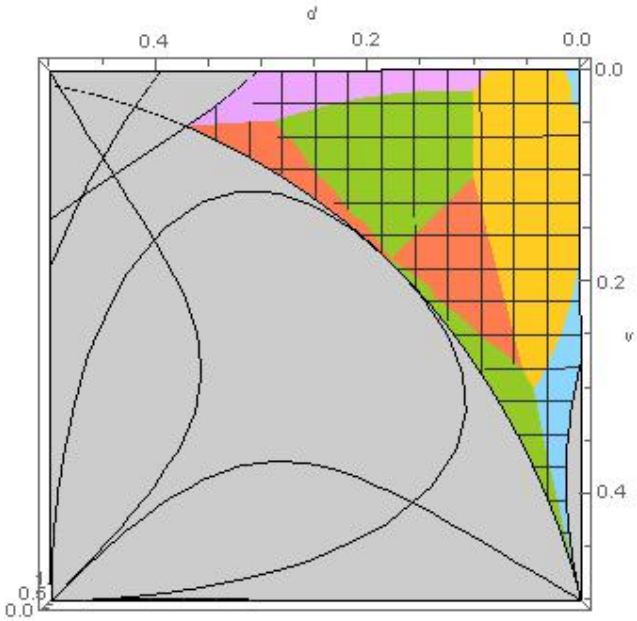
Here I show conditions on v and d under which, given that the first-best under centralization is not implementable, equilibria in which 3 out of 4 informed types of experts send truthful reports dominate other equilibria which arise under centralization.

Which other equilibria can there exist in a centralized architecture? The next lemma shows that for any biases, there is always an equilibrium in which at least one informed type of each expert reports truthfully. Furthermore, there is no equilibrium in which all types of any expert send the uninformative report $[0,1]$ because there is a profitable deviation for at least one informed type of that expert to report his piece of evidence. This happens because all players prefer more informative to less informative equilibria.

Lemma 2: *For any biases of experts 1 and 2, any equilibrium under centralization features at least one informed type of each expert who reports his signal truthfully.*

Therefore, under centralization, there is no equilibrium in which all informed types of an expert send an uninformative report $[0,1]$. As I show in the appendix, there are 8 conditions on v and d under which an equilibrium in which 3 out of 4 informed types of both experts report their signals truthfully, always dominates any other equilibrium in which only one type of each expert reports his type truthfully. The graphical intersection of these conditions is shown in Figure 3.3: equilibria in which 3 out of 4 informed types reveal their signals truthfully dominates the other equilibrium in a non-grey area for any values of v and d. In a non-grey region, if v is relatively high, then d is relatively low, and vice versa. This results in a larger amount of information transferred in expectation once 3 out of 4 types reveal their signals to the DM, compared to an equilibrium in which any 2 of the 4 types reveal their signals.

Figure 3.3: Truthtelling of 3 types dominates truthtelling of any 2 types in the non-grey region.



3.4.4 Decentralized mechanism with perfect information sharing

Suppose that the DM delegates authority to expert i and creates a directed communication link from j to i. Here I study conditions for equilibrium in which j reveals all his types to i.

The expected utility of the DM is:

$$\mathbb{E}U_{DM}^{D} = -\frac{1}{4}\int_0^1 \Big(\frac{1}{2} - \theta - b_i\Big)^2 d\theta - \frac{1}{4}\Big(\int_0^{\frac{1}{2}-v} \Big(\frac{1}{4}(1-2v) - \theta - b_i\Big)^2 d\theta +$$

$$\int_{\frac{1}{2}-v}^1 \Big(\frac{1}{4}(3-2v) - \theta - b_i\Big)^2 d\theta\Big) - \frac{1}{4}\Big(\int_0^{\frac{1}{2}+d} \Big(\frac{1}{4}(1+2d) - \theta - b_i\Big)^2 d\theta +$$

$$\int_{\frac{1}{2}+d}^1 \Big(\frac{1}{4}(3+2d) - \theta - b_i\Big)^2 d\theta\Big) - \frac{1}{4}\Big(\int_0^{\frac{1}{2}-v} \Big(\frac{1}{4}(1-2v) - \theta - b_i\Big)^2 d\theta +$$

$$\int_{\frac{1}{2}-v}^{\frac{1}{2}+d} \Big(\frac{1}{2}(1-v+d) - \theta - b_i\Big)^2 d\theta + \int_{\frac{1}{2}+d}^1 \Big(\frac{1}{4}(3+2d) - \theta - b_i\Big)^2 d\theta\Big) = \mathbb{E}U_{DM}^{C} - b_i^2.$$

Therefore, the losses of the DM compared to the best equilibrium in the centralized case are increasing quadratically in the bias difference between the DM and expert i. The incentives for expert j are derived in the appendix. If the DM

delegates authority to 1 then the incentives of expert 2 to report all his types to expert 1 are satisfied if:

$$\frac{-d(4+8v)-1-8d^2-4v^2}{1+4d+2v}+b_1 \leq b_2 \leq \frac{1-2d}{8}+b_1$$

If the DM delegates authority to expert 2, then the corresponding constraints for expert 1 are:

$$\frac{2v-1}{8}+b_2 \leq b_1 \leq \frac{1+4d^2+4v+8dv+8v^2}{8(1+2d+4v)}+b_2$$

3.5 Comparison between centralization and decentralization

Here I show that, for any value of v and d, there exists a range for the biases of both players such that decentralization dominates centralization. Denote the range of biases which support the first best in case of centralization by $B^F = B_1^F \times B_2^F \in \mathbb{R}^2$.

Proposition 1: *For given v and d assume that the range of biases for experts 1 and 2, B_1, B_2, is outside the range where both perfectly reveal information in the centralized case, $(B_1 \times B_2) \cap B^F = \emptyset$. If in the second best equilibrium only one informed type of expert j reveals his information, and both unformed types of i reveal their information, then there exists $(b_1, b_2) \in B_1 \times B_2$ where the DM delegates authority to i and creates a directed communication link from j to i such that the new equilibrium payoff dominates centralization.*

Proof: Denote the best equilibrium payoff for the DM for $(b_1, b_2) \in B_1 \times B_2$, if the decision and communication structure is centralized, by W^S. Denote by W^C the equilibrium payoff for the DM in a centralized communication if all types of both agents report their signal truthfully. Since in this equilibrium at least one informed type of one of the experts deviates in $[0, 1]$ in his report, $W^S < W^C$.

Suppose that the DM decided to delegate authority to expert 1 and creates a directed communication link from 2 to 1. The proof for delegation the authority to 2, and creating a directed communication link from 1 to 2 is proven analogously. If 2 always reveals his type to i, the DM benefits from decentralization if $W^C - b_1^2 > W^S$ or $b_1 < \sqrt{W^C - W^S}$.

The condition on the biases of expert 2 to report truthfully to 1 is: $\underline{b} + b_1 \leq b_2 \leq \overline{b} + b_1$ where $[\underline{b}, \overline{b}] = B_2^F$.

1. Suppose that in the second best equilibrium under centralization, both types of expert 1 report truthfully to the DM but only t_2 of expert 2 reports truthfully. For the DM to benefit from decentralization it should be true that $b_1 < \sqrt{W^C - W^S} = s_1$ In this case, this equilibrium is supported for $b_2 \leq k_1$ with $k_1 > \bar{b}$. The reason for $k_1 > \bar{b}$ is that if t_2 deviates in equilibrium in which t_2' pools with the type of 2 who reports $[0, 1]$, he expects a bigger shift in DM's choice compared to equilibrium in which t_2' reports truthfully, since the DM expects higher state upon observing $[0, 1]$ (I show it in the appendix for the derivation of the incentive constraints for this equilibrium). But then there exists a value for b_1 small enough such that two conditions are jointly satisfied: $b_2 \leq b_1 + \bar{b} \leq k_1$ and $b_1 \leq s_1$.

2. Suppose that in the second best equilibrium under centralization, both types of expert 1 report truthfully to the DM but only t_2' of expert 2 reports truthfully. For the DM to benefit from decentralization it should be true that $b_1 < \sqrt{W^C - W^S} = s_2$ In this case, this equilibrium is supported for $b_2 \geq k''$, with $k'' < 0$. But then the only 2 conditions which have to be satisfied for decentralization to yield a strictly better result is $b_2 \leq b_1 + \bar{b}$ and $b_1 < s_2$, which is true for values of b_1 within the constraints. Q.E.D.

3.6 Conclusions

This project studies the choice between centralization and decentralization with two experts who can be partially informed about the state, and an uninformed DM. The DM is able to commit to the allocation of decision rights and communication channels. I contrast two simple benchmarks: under centralization, the DM receives direct reports from both experts and decides over a single task; under decentralization, the DM delegates the decision right to one of the experts and creates a communication channel from the other expert to the first one. I show that decentralization can dominate centralization if the bias of an expert who decides over the task is not too large compared to the bias of the DM, and the bias of another expert is nearer to the bias of the first expert, compared to the bias of the DM.

Since this project is work in progress there are multiple steps I aim to do next. First, currently I study cases in which equilibria, which feature a pooling of an informed type of one of the experts with an uninformed type, dominate all other equilibria under centralization for given information structures and all possible biases. However, for different information structures, there might be some ranges of biases for which such equilibria still dominate all other equilibria under centraliza-

tion. Therefore, it can be that the range of parameters for which my findings apply is larger than the one stated in the paper.

Further, I would like to characterize cases in which equilibria featuring only a single type of each expert reporting truthfully to the DM are the best equilibria under centralization and compare them to the best equilibria under decentralization.

Second, I aim to look at a richer set of communication structures. For example, a centralized case can feature an expert reporting his signal to a different expert, who then reports his information to the DM, and the DM decides over the policy. This richer space of communication networks can be interesting combined with a richer set of signal structures, if, for example, each expert can receive signals according to more than two partitions of the state space.

Finally, I want to contrast the best outcome under decentralization with the best outcome in case the DM can commit to a mechanism which implements allocations conditional on reports from both experts. According to the revelation principle, it is sufficient to focus on direct mechanisms. I conjecture that the mechanism can implement any allocation of the game, but the reverse is not necessarily true. Therefore, it is interesting to study systematically the loss in welfare once moving from the environment with commitment to the non-commitment case.

Appendix A

Definitions and Proofs for Chapter 1

Remaining definitions from the model section:

A path $H_{i_1 i_k}$ is a sequence of edges $\{i_1, i_2\}, \{i_2, i_3\}, .., \{i_{k-1}, i_k\}$ such that the nodes in every pair $\{i_l, i_{l+1}\}$ are directly connected, $e_{i_l, i_{l+1}} = 1$.

In this paper I study directed graphs with the following properties:

(1) $\forall i \in N \setminus DM$, there is $j \in N$, $j \neq i$, such that $e_{ij} = 1$ and there is no other $j' \neq j$, $j' \in N$ connected to i, $e_{ij'} = 1$, which means that every expert has one outgoing link,

(2) $\forall i \in N$ there is no path H_{ij} with $j \neq i$ and $e_{ji} = 1$, which means that there are no cycles, and

(3) $\sum_{j \in N^e} e_{jDM} \geq 1$ and $\sum_{j \in N^e} e_{DMj} = 0$ which means that the DM has at least one incoming but no outgoing links.

Bayesian updating follows the Beta-binomial model: given k observations and k 1's the conditional pdf is:

$$f(k|\theta, n) = \frac{n!}{k!(n-k)!}\theta^k(1-\theta)^{n-k}.$$

Thus, when n observations are conducted, the distribution of "successes" is uniform as well:

$$Prob(k|n) = \int_0^1 Prob(k|\theta, n)d\theta = \int_0^1 \frac{n!}{k!(n-k)!}\theta^k(1-\theta)^{n-k}d\theta = \frac{1}{n+1}.$$

The posterior is

$$f(\theta|k,n) = \frac{(n+1)!}{k!(n-k)!}\theta^k(1-\theta)^{n-k},$$

Thus, $\mathbb{E}(\theta|k,n) = \frac{k+1}{n+2}$.

Finally, suppose we have n trials with k 1's. What is the probability of having j 1's given additional m trials?

$$P(j|m,n,k) = \int_0^1 P(j|m,\theta)P(\theta|k,n)d\theta =$$

$$\int_0^1 \frac{m!}{j!(m-j)!}\theta^j(1-\theta)^{m-j}\frac{(n+1)!}{k!(n-k)!}\theta^k(1-\theta)^{n-k}d\theta =$$

$$\int_0^1 \frac{m!}{j!(m-j)!}\frac{(n+1)!}{k!(n-k)!}\theta^{k+j}(1-\theta)^{m-j+n-k}d\theta =$$

$$\frac{m!}{j!(m-j)!}\frac{(n+1)!}{k!(n-k)!}\frac{(k+j)!(m-j+n-k)!}{(n+m+1)!}.$$

Calculations for 1.3.1:

<u>First I analyze equilibria in a star network.</u>

Consider a single expert i who reveals his signals truthfully in a star. DM's choices are

$$y(0) = \mathbb{E}_{DM}(\theta|0,1) = \frac{1}{3},\ y(1) = \mathbb{E}_{DM}(\theta|1,1) = \frac{2}{3},$$

where $\mathbb{E}_{DM}(\theta|k,n)$ denotes the expected value of the state by the DM given n equilibrium messages and k number of 1's. Therefore, the expert is incentivized to communicate truthfully if:

$$-\int_0^1 (\frac{1}{3} - \theta - b_i)^2 f(\theta|0,1)d\theta \geq -\int_0^1 (\frac{2}{3} - \theta - b_i)^2 f(\theta|0,1)d\theta,$$

and

$$-\int_0^1 (\frac{2}{3} - \theta - b_i)^2 f(\theta|1,1)d\theta \geq -\int_0^1 (\frac{1}{3} - \theta - b_i)^2 f(\theta|1,1)d\theta,$$

where $f(\theta|k,n)$ is a posterior with n experts and the sum of the signals k. The above inequalities imply $b_i \leq \frac{1}{6}$.

Consider two experts revealing their signals truthfully. Then, DM receives messages according to the partition $\{\{0\},\{1\},\{2\}\}$ of the sum of the signals. Her choices are

$$y(0) = \frac{1}{4},\ y(1) = \frac{1}{2},\ y(2) = \frac{3}{4}.$$

If expert i receives $s_i = 0 = k$, he expects that the other expert j who sends truthful message to the DM has $s_j = k$ with prob $\frac{2}{3}$ and $s_j = 1-k$ with prob $\frac{1}{3}$.

Therefore, the incentive constraint preventing the upward deviation of i is:

$$-\frac{2}{3}\int_0^1 (\frac{1}{4} - \theta - b_i)^2 f(\theta|0,2)d\theta - \frac{1}{3}\int_0^1 (\frac{1}{2} - \theta - b_i)^2 f(\theta|1,2)d\theta \geq$$

$$-\frac{2}{3}\int_0^1 (\frac{1}{2} - \theta - b_i)^2 f(\theta|0,2)d\theta - \frac{1}{3}\int_0^1 (\frac{3}{4} - \theta - b_i)^2 f(\theta|1,2)d\theta.$$

The incentive constraint preventing the downward deviation for i is:

$$-\frac{2}{3}\int_0^1 (\frac{3}{4} - \theta - b_i)^2 f(\theta|2,2)d\theta - \frac{1}{3}\int_0^1 (\frac{1}{2} - \theta - b_i)^2 f(\theta|1,2)d\theta \geq$$

$$-\frac{2}{3}\int_0^1 (\frac{1}{2} - \theta - b_i)^2 f(\theta|2,2)d\theta - \frac{1}{3}\int_0^1 (\frac{1}{4} - \theta - b_i)^2 f(\theta|1,2)d\theta.$$

Both constraints result in $|b_i| \leq \frac{1}{8}$. Due to symmetry in strategies and payoffs between both experts which communicate truthfully in equilibrium, expert j's constraint is the same: $b_i \leq \frac{1}{8}$.

Finally, consider the strategy profile in which all three experts communicate truthfully to the DM. DM's choices dependent on the sum of the signals are:

$$y(0) = \frac{1}{5},\ y(1) = \frac{2}{5},\ y(2) = \frac{3}{5},\ y(3) = \frac{4}{5}.$$

Take any expert $i = 1, 2, 3$ who receives $s_i = 0$. He expects that the sum of the signals of the other two experts is 0 with prob $\frac{1}{2}$, is 1 with prob $\frac{1}{3}$ and is 2 with prob $\frac{1}{6}$. If i receives $s_i = 1$, he expects the sum of the other experts' signals to be 3 with prob $\frac{1}{2}$, 2 with prob $\frac{1}{3}$ and 1 with prob $\frac{1}{6}$.

Therefore, the incentive constraint of i which prevents his upward deviation is:

$$-\frac{1}{2}\int_0^1 (\frac{1}{5} - \theta - b_i)^2 f(\theta|0,3)d\theta - \frac{1}{3}\int_0^1 (\frac{2}{5} - \theta - b_i)^2 f(\theta|1,3)d\theta -$$

$$\frac{1}{6}\int_0^1 (\frac{3}{5} - \theta - b_i)^2 f(\theta|2,3)d\theta \geq$$

$$-\frac{1}{2}\int_0^1 (\frac{2}{5} - \theta - b_i)^2 f(\theta|0,3)d\theta - \frac{1}{3}\int_0^1 (\frac{3}{5} - \theta - b_i)^2 f(\theta|1,3)d\theta -$$

$$\frac{1}{6}\int_0^1 (\frac{4}{5} - \theta - b_i)^2 f(\theta|2,3)d\theta.$$

The incentive constraint for the downward deviation of i is:

$$-\frac{1}{2}\int_0^1 (\frac{4}{5} - \theta - b_i)^2 f(\theta|3,3)d\theta - \frac{1}{3}\int_0^1 (\frac{3}{5} - \theta - b_i)^2 f(\theta|2,3)d\theta -$$

$$\frac{1}{6}\int_0^1 (\frac{2}{5} - \theta - b_i)^2 f(\theta|1,3)d\theta \geq$$

$$-\frac{1}{2}\int_0^1 (\frac{3}{5} - \theta - b_i)^2 f(\theta|3,3)d\theta - \frac{1}{3}\int_0^1 (\frac{2}{5} - \theta - b_i)^2 f(\theta|2,3)d\theta -$$

$$\frac{1}{6}\int_0^1 (\frac{1}{5} - \theta - b_i)^2 f(\theta|1,3)d\theta.$$

Notice that the constraints for each expert are symmetric. Therefore, the incentives for truth-telling for each $i = 1, 2, 3$ are satisfied for $b_i \leq \frac{1}{10}$.

<u>Second, I show that, for the same biases, the line is able to induce same payoff allocations as the star.</u>

It is straightforward to show that it is true if only a single expert reveals his signal truthfully. Fix the expert who reveals his signal truthfully in a star and place him on the top of the line. The bias of this expert satisfies $|b_i| \leq \frac{1}{6}$. Then, there is

an equilibrium in which a single expert reveals his signal truthfully to the DM.

Consider, next, the equilibrium in a star in which only two experts reveal their signal truthfully to the DM. Label those experts by 1 and 2. Design a line network such that 1 and 2 are the experts closest to the DM (the third expert is at the bottom of the line). The order of 1 and 2 does not matter. Assume, for example, that expert 1 is connected to the DM and expert 2 communicates to expert 1. Consider the strategy profile in which expert 3 babbles (sends an uninformative message to expert 2), expert 2 reveals his private signal to expert 1, and expert 1 reveals if the sum of the signals is either 0, 1 or 2 to the DM. Incentives of expert 2 are the same as in the star. The choices of the DM are the same as in equilibrium in the star, in which two experts communicate truthfully. Now I show that the incentives of expert 1 remain the same.

Expert 1 observes the sum of the signals 0, 1 or 2. Suppose that after he receives his signal and the message of expert 2, his type is $t_1 = 0$. If he deviates to message 1, his deviation implies:

$$-\int_0^1 (\frac{1}{4} - \theta - b_i)^2 f(\theta|0,2)d\theta \geq -\int_0^1 (\frac{1}{2} - \theta - b_i)^2 f(\theta|0,2)d\theta,$$

resulting in $b_1 \leq \frac{1}{8}$.
If he deviates to message 2, then his constraint is:

$$-\int_0^1 (\frac{1}{4} - \theta - b_i)^2 f(\theta|0,2)d\theta \geq -\int_0^1 (\frac{3}{4} - \theta - b_i)^2 f(\theta|0,2)d\theta,$$

resulting in $b_1 \leq \frac{1}{4}$.
If $t_1 = 1$ then his upward deviation to message 2 is

$$-\int_0^1 (\frac{1}{2} - \theta - b_i)^2 f(\theta|1,2)d\theta \geq -\int_0^1 (\frac{3}{4} - \theta - b_i)^2 f(\theta|1,2)d\theta,$$

resulting in $b_i \leq \frac{1}{8}$.
Therefore, $b_i \leq \frac{1}{8}$ is the binding constraint preventing upward deviation.
If expert 1 is $t_1 = 2$, then his downward deviation to message 1 is:

$$-\int_0^1 (\frac{3}{4} - \theta - b_i)^2 f(\theta|2,2)d\theta \geq -\int_0^1 (\frac{1}{2} - \theta - b_i)^2 f(\theta|2,2)d\theta,$$

resulting in $b_i \geq -\frac{1}{8}$.

If $t_1 = 2$, his downward deviation to message 0 implies:

$$-\int_0^1 (\frac{3}{4} - \theta - b_i)^2 f(\theta|2,2)d\theta \geq -\int_0^1 (\frac{1}{4} - \theta - b_i)^2 f(\theta|2,2)d\theta,$$

resulting in $b_i \geq -\frac{1}{4}$.

Finally, if $t_1 = 1$, then his downward deviation to message 0 implies:

$$-\int_0^1 (\frac{1}{2} - \theta - b_i)^2 f(\theta|1,2)d\theta \geq -\int_0^1 (\frac{1}{4} - \theta - b_i)^2 f(\theta|1,2)d\theta,$$

resulting in $b_i \geq -\frac{1}{4}$.

Therefore, the binding constraint in case of the downward deviation is $b_i \geq -\frac{1}{8}$, which is satisfied since we assumed that each expert has a positive bias.

In a similar way, it can be shown that is all three experts in a line reveal all their information to the player next in the line, the binding constraints result in $b_i \leq \frac{1}{10}$ for $i = 1, 2, 3$.

Third, I show that the line generates an additional equilibrium which is strictly better for the DM.

Think about a strategy profile in which experts 2 and 3 forward all available information to the expert next in the line. Therefore, expert 1 receives information about the sum of all signals available to three experts. Consider the strategy profile in which expert 1 sends m_1 is the sum of the signals is 0, and m_1' otherwise. DM's choices are: $y(m_1) = \frac{1}{5}$ and $y(m_1') = \frac{3}{5}$. This strategy profile results in $\mathbb{E}U_{DM} = -\frac{4}{75}$.

To calculate the constraint preventing the upward deviation of an expert, notice, that all three incentive constraints are the same because each of the experts conditions the deviation on the sum of the signals 0.

Therefore, for any $i = 1, 2, 3$, the upward deviation results in:

$$-\int_0^1 (\frac{1}{5} - \theta - b_i)^2 f(\theta|0,3)d\theta \geq -\int_0^1 (\frac{3}{5} - \theta - b_i)^2 f(\theta|0,3)d\theta,$$

resulting in $b_i \leq \frac{1}{5}$.

The smallest downward deviation occurs if either expert 1 observes the sum of the signals 1, or any of the remaining experts expects expert 1 to observe the sum of the signals 1:

$$-\int_0^1 (\frac{3}{5} - \theta - b_i)^2 f(\theta|1,3)d\theta \geq -\int_0^1 (\frac{1}{5} - \theta - b_i)^2 f(\theta|1,3)d\theta,$$

resulting in $b_i \geq 0$, $i = 1,2,3$, which is satisfied since we assumed that all experts are positively biased.

<u>Equilibrium in network Q'</u>:

The strategy profile specified in 3.1 implies that the message strategies of experts can be represented by the following partitions: $P_2 = \{\{0\},\{1,2\}\}$, $P_1 = \{\{0\},\{1\}\}$ and $P_3 = \{\{0\},\{1\}\}$. The corresponding decisions of the DM are

$$y(m_1,m_2) = \frac{1}{5},\ y(m_1',m_2) = \frac{2}{5},\ y(m_1,m_2') = \frac{7}{15},\ y(m_1',m_2') = \frac{18}{25}.$$

We start with the ICs for both types of expert 2 assuming that all other players choose their strategies according to the fixed profile. $t_2 = 0$ assigns the posteriors $\frac{3}{4}$ to $t_1 = 0$ and $\frac{1}{4}$ to $t_1 = 1$. Thus, the IC for $t_2 = 0$ is:

$$-\frac{3}{4}\int_0^1 (\frac{1}{5} - \theta - b_2)^2 f(\theta|0,3)d\theta - \frac{1}{4}\int_0^1 (\frac{2}{5} - \theta - b_2)^2 f(\theta|1,3)d\theta \geq$$

$$-\frac{3}{4}\int_0^1 (\frac{7}{15} - \theta - b_2)^2 f(\theta|0,3)d\theta - \frac{1}{4}\int_0^1 (\frac{18}{25} - \theta - b_2)^2 f(\theta|1,3)d\theta,$$

so that $b_2 \leq \frac{74}{525} \simeq 0.140952$.

$t_2 = 1$ assigns the posteriors $\frac{1}{2}$ to $t_1 = 0$ and $\frac{1}{2}$ to $t_1 = 1$. The ICs of $t_2 = 1$ is:

$$-\frac{1}{2}\int_0^1 (\frac{7}{15} - \theta - b_2)^2 f(\theta|1,3)d\theta - \frac{1}{2}\int_0^1 (\frac{18}{25} - \theta - b_2)^2 f(\theta|2,3)d\theta \geq$$

$$-\frac{1}{2}\int_0^1 (\frac{1}{5} - \theta - b_2)^2 f(\theta|1,3)d\theta - \frac{1}{2}\int_0^1 (\frac{2}{5} - \theta - b_2)^2 f(\theta|2,3)d\theta,$$

so that $b_2 \geq -0.00521$ which is satisfied since we assumed that all experts are poisitively biased.

In a similar way it is easy to show that the ICs for expert 3 are the same: $b_3 \leq \frac{74}{525}$.

Next, $t_1 = 0$ assigns the posteriors $\frac{1}{2}$ to $t_2 = 0$, $\frac{1}{3}$ to $t_2 = 1$ and $\frac{1}{6}$ to $t_2 = 2$. The corresponding IC for $t_1 = 0$, assuming that other players play according to specified strategy profile, is:

$$-\frac{1}{2}\int_0^1 (\frac{1}{5} - \theta - b_1)^2 f(\theta|0,3)d\theta - \frac{1}{3}\int_0^1 (\frac{7}{15} - \theta - b_1)^2 f(\theta|1,3)d\theta -$$

$$\frac{1}{6}\int_0^1 (\frac{7}{15} - \theta - b_1)^2 f(\theta|2,3)d\theta \geq$$

$$-\frac{1}{2}\int_0^1 (\frac{2}{5} - \theta - b_1)^2 f(\theta|0,3)d\theta - \frac{1}{3}\int_0^1 (\frac{18}{25} - \theta - b_1)^2 f(\theta|1,3)d\theta -$$

$$\frac{1}{6}\int_0^1 (\frac{18}{25} - \theta - b_1)^2 f(\theta|2,3)d\theta,$$

so that $b_1 \leq \frac{293}{2550}$.

Finally, $t_1 = 1$ assigns the posteriors $\frac{1}{6}$ to $t_2 = 0$, $\frac{1}{3}$ to $t_2 = 1$ and $\frac{1}{2}$ to $t_2 = 2$. The corresponding IC for t_1, assuming that other players play according to specified strategy profile, is:

$$-\frac{1}{6}\int_0^1 (\frac{2}{5} - \theta - b_1)^2 f(\theta|1,3)d\theta - \frac{1}{3}\int_0^1 (\frac{18}{25} - \theta - b_1)^2 f(\theta|2,3)d\theta -$$

$$\frac{1}{2}\int_0^1 (\frac{18}{25} - \theta - b_1)^2 f(\theta|3,3)d\theta \geq$$

$$-\frac{1}{6}\int_0^1 (\frac{1}{5} - \theta - b_1)^2 f(\theta|1,3)d\theta - \frac{1}{3}\int_0^1 (\frac{7}{15} - \theta - b_1)^2 f(\theta|2,3)d\theta -$$

$$\frac{1}{2}\int_0^1 (\frac{7}{15} - \theta - b_1)^2 f(\theta|3,3)d\theta,$$

resulting in $b_1 \geq -0.12303$ which is satisfied per assumption.

To summarize, the above strategy profile constitutes an equilibrium for $b_2, b_3 \leq 0.14095$, and for $b_1 \leq 0.1149$. The expected utility of the DM is $= -0.039556$.

<u>Implementation of the equilibrium outcome of Q' in the line</u>:

Consider the following strategy profile. Expert 3 communicates his signals to expert 2, and expert sends either m_2 to expert 1 if his private information is summarized by the sufficient statistic 0, and m_2' otherwise. Expert 1 sends 4 messages to the DM: m_1 if $(s_1 = 0, m_2)$, m_1' if $(s_1 = 0, m_2')$, m_1'' if $(s_1 = 1, m_2)$ and m_1''' if $(s_1 = 1, m_2')$. DM chooses:

$$y(m_1) = \frac{1}{5},\ y(m_1') = \frac{7}{15},\ y(m_1'') = \frac{2}{5} \text{ and } y(m_1''') = \frac{18}{25}.$$

The incentive constraints for experts 3 and 2 are the same as in the equilibrium in network B above, since nothing changes for them in terms of their information, the expected information sets of the DM and how their signals enter the information sets of the DM. However, the constraints for expert 1 get tighter. In particular, his tightest constraint for the upward deviation is defined for $t_1 = (s_1 = 1, m_2)$, when he deviates to the message $(s_1 = 0, m_2')$:

$$-\int_0^1 (\frac{2}{5} - \theta - b_1)^2 f(\theta|1,3)d\theta \geq -\int_0^1 (\frac{7}{15} - \theta - b_1)^2 f(\theta|1,3)d\theta,$$

or:

$$\frac{2}{5} + \frac{7}{15} - 2\frac{2}{5} - 2b_1 \geq 0,$$

such that $b_1 \leq 0.033$.

The tightest constraint for the downward deviation of expert 1 is defined for $t_1 = (s_1 = 0, m_2')$, when he deviates to $(s_1 = 1, m_2)$:

$$-\frac{2}{3}\int_0^1 (\frac{7}{15} - \theta - b_1)^2 f(\theta|1,3)d\theta - \frac{1}{3}\int_0^1 (\frac{7}{15} - \theta - b_1)^2 f(\theta|2,3)d\theta \geq$$

$$-\frac{2}{3}\int_0^1 (\frac{2}{5} - \theta - b_1)^2 f(\theta|1,3)d\theta - \frac{1}{3}\int_0^1 (\frac{2}{5} - \theta - b_1)^2 f(\theta|2,3)d\theta,$$

which can be written as:

$$\frac{2}{3}\Big(\frac{7}{15} + \frac{2}{5} - 2\frac{2}{5} - 2b_1\Big) + \frac{1}{3}\Big(\frac{7}{15} + \frac{2}{5} - 2\frac{2}{5} - 2b_1\Big) \geq 0,$$

or:

$b_1 \geq -0.033$, which is satisfied per assumption. Therefore, the binding constraint for expert 1 shows that he is incentivized to communicate according to the

specified strategy profile in a line for a strictly smaller range of biases, compared to communication in network Q'.

Proof of Proposition 1:

The construction is the following: I fix a separating equilibrium in a star network Q in which n experts communicate truthfully. First, I show the optimal decision of the DM. Second, I show that if $|b_i| \leq \frac{1}{2(n+2)}$ then none of n experts has an incentive to deviate from his equilibrium strategy given that all other experts adhere to their equilibrium strategies.

Fix a separating equilibrium in which n experts report truthfully. The corresponding equilibrium partition of $\{0,1\}^n$ according to which the DM receives her information is $P_{DM} = \{\{0\}, ..., \{n\}\}$. The prior of the DM on k which is the sum of the signals is

$$\Pr(k) = \int_0^1 \Pr(k|\theta)d\theta = \int_0^1 \frac{n!}{l!(n-k)!}\theta^k(1-\theta)^{n-k}d\theta =$$
$$\frac{n!}{l!(n-k)!}\frac{l!(n-k)!}{(n+1)!} = \frac{1}{n+1}.$$

Thus, after the DM receives $p \in P_{DM}$, where the pool p simply stands for the sum of the signals (so I use them interchangeably here), and she believes that all experts adhere to their strategy profiles, she chooses:

$$y(k) = \mathbb{E}_{DM}(\theta|k,n) = \frac{k+1}{n+2}.$$

Denote by t_{-i} the vector of types of all n experts rather than expert i. If $t_i = 0$, then expert i's incentive constraints to report his signal truthfully imply:

$$-\Big(\Pr(t_{-i}=0|t_i)\int_0^1 (\frac{1}{n+2} - \theta - b_i)^2 f(\theta|k=0,n)d\theta + ...$$
$$+\Pr(t_{-i}=n-1|t_i)\int_0^1 (\frac{n}{n+2} - \theta - b_i)^2 f(\theta|k=n-1,n)d\theta\Big) \geq$$
$$-\Big(\Pr(t_{-i}=0|t_i)\int_0^1 (\frac{2}{n+2} - \theta - b_i)^2 f(\theta|k=0,n)d\theta + ...$$
$$+\Pr(t_{-i}=n-1|t_i)\int_0^1 (\frac{n+1}{n+2} - \theta - b_i)^2 f(\theta|k=n-1,n)d\theta\Big).$$

This can be reformulated as:

$$\sum_{k=0}^{n-1} \Pr(t_{-i} = k|t_i) \frac{1}{n+2} \left(\frac{k+1}{n+2} + \frac{k+2}{n+2} - 2\frac{k+1}{n+2} - 2b_i\right) \geq 0$$

or:

$$b_i \leq \frac{1}{2(n+2)}.$$

Similar, if $t_i = 1$ then the incentive constraints for expert i to report his signal truthfully imply:

$$-\Big(\Pr(t_{-i} = 0|t_i) \int_0^1 (\frac{2}{n+2} - \theta - b_i)^2 f(\theta|l = 1, n) d\theta + ...$$

$$+\Pr(t_{-i} = n-1|t_i) \int_0^1 (\frac{n+1}{n+2} - \theta - b_i)^2 f(\theta|l = n, n) d\theta\Big) \geq$$

$$-\Big(\Pr(t_{-i} = 0|t_i) \int_0^1 (\frac{1}{n+1} - \theta - b_i)^2 f(\theta|l = 1, n) d\theta + ...$$

$$+\Pr(t_{-i} = n-1|t_i) \int_0^1 (\frac{n}{n+2} - \theta - b_i)^2 f(\theta|l = n, n) d\theta\Big).$$

This can be reformulated as:

$$\sum_{k=0}^{n-1} \Pr(t_{-i} = (k, n)|t_i) \frac{1}{n+2} \left(\frac{k+1}{n+2} + \frac{k+2}{n+2} - 2\frac{k+2}{n+2} - 2b_i\right) \geq 0$$

or:

$$b_i \geq -\frac{1}{2(n+2)}.$$

Summing up, the separating equilibrium with n experts exists if

$$|b_i| \leq \frac{1}{2(n+2)}.$$

Finally, notice that the condition for the case where $n' \leq n$ experts report truthfully is $|b_i| \leq \frac{1}{2(n'+2)}$. Q.E.D.

Proof of Proposition 2:

Here is the proof of the first part.

Take any tuple $(n, b(n))$, where $b(n)$ is the bias profile of n experts, $b(n) = (b_1, .., b_n)$, and fix any equilibrium outcome in a star network in which there is a perfect separation involving n' experts with $n' \in \{1, .., n\}$. According to Proposition 1 the incentive constraints for the truth-telling experts result in: $|b_i| \leq \frac{1}{2(n'+2)}$ for all $i \in \{1, .., n'\}$. By backward induction, the DM chooses $y(k) = \frac{k+1}{n'+2}$ where k is the reported summary statistic.

Fix any tree network Q in which experts are ordered monotonically according to the absolute value of their biases such that if expert j reports to expert i, then $|b_j| \geq |b_i|$. In the following I show that for a given $b(n)$, Q has the same equilibrium outcome as the star.

Take any expert i in Q. If his private information is summarized by $t_i = 0$, then his incentive constraints for the upward deviation is:

$$\sum_{j=0}^{n'} \Pr(k = j|t_i)\Big(y(k = j+1) - y(k = j)\Big)\Big(y(k = j+1) + y(k = j) - 2\mathbb{E}_i(\theta|j, n') - 2b_i)\Big) \geq 0$$

Notice that

$$y(k = j + 1) - y(k = j) = \frac{j + 2}{n' + 2} - \frac{j + 1}{n' + 2} = \frac{1}{n' + 2}$$

and

$$y(k = j + 1) + y(k = j) - 2\mathbb{E}_i(\theta|j, n') = \frac{j + 2}{n' + 2} + \frac{j + 1}{n' + 2} - 2\frac{j + 1}{n' + 2} = \frac{1}{n' + 1}.$$

Given that $\sum_{j=0}^{n'} \Pr(k = j|t_i) = 1$, the incentive constraint can written as

$$b_i < \frac{1}{2(n' + 2)}$$

Similarly, the incentive constraint for the downward deviation of $t_i = 1$ is:

$$\sum_{j=0}^{n'} \Pr(k = j|t_i)\Big(y(k = j-1) - y(k = j)\Big)\Big(y(k = j-1) + y(k = j) - 2\mathbb{E}_i(\theta|j, n') - 2b_i)\Big) \geq 0$$

Since

$$y(k=j-1)-y(k=j)=\frac{j}{n'+2}-\frac{j+1}{n'+2}=-\frac{1}{n'+2}$$

and

$$y(k=j+1)+y(k=j)-2\mathbb{E}_i(\theta|j,n')=\frac{j}{n'+2}+\frac{j+1}{n'+2}-2\frac{j+1}{n'+2}=-\frac{1}{n'+1}.$$

Given that $\sum_{j=0}^{n'}\Pr(k=j|t_i)=1$, the incentive constraint can written as

$$b_i>\frac{1}{2(n'+2)},$$

which proves the first part of the proposition.

Here is the proof of the second part.

Example 1 already provided an example in which, for some range of biases, sequential communication dominated simultaneous communication. For $(n, b(n))$, where $b(n)$ denotes the vector of biases of all n experts, it is possible to construct many equilibria which dominate the corresponding equilibria in a star. Here I provide one of such constructions (related to the Example 1)

Take any network Q which satisfies the conditions in Proposition 2 and is not a star, and assume that all biases of the experts are strictly bigger than $\frac{1}{6}$. Then, Proposition 1 tells us that the only equilibrium in a star is a babbling equilibrium which results in $\mathbb{E}U_{DM}=-\int_0^1(\frac{1}{2}-\theta)^2d\theta=-\frac{1}{12}$.

Denote the number of experts which belong to the longest path in Q, which starts with from some expert $j\in N$ and ends by the DM, H_{jDM}, by r. Since I assumed that Q is not a star, it follows that $r\geq 2$. Then, if in equilibrium expert $i'\in H_{jDM}$ who is directly connected to the DM receives full information about the signals of $r-1$ experts, he can report his signals according to the partition $P_{i'}(Q)=\{\{0\},\{1,..,r\}\}$. In the following, I show that $P_{i'}(Q)$ is implementable as an equilibrium partition for biases strictly higher than $\frac{1}{6}$, where the exact range of biases depends on r, and the DM's expected utility resulting from $P_{i'}(Q)$ is higher than the expected utility in the case of no information transmission, $-\frac{1}{12}$.

First, notice that once the DM receives reports according to $P_{i'}(Q)$, she chooses her policies as follows:

$$y(0) = \frac{1}{r+2},\ y(1,..,r) = \Big(\frac{2}{r+2} + .. + \frac{r+1}{r+2}\Big)\frac{1}{r} = \frac{r+3}{2(r+2)}.$$

Take any expert $i \in H_{jDM}$. The upward deviation incentives which depend on i's private information and his beliefs over the signal distribution, given the strategy profile of other experts and the optimal response of the DM, are:

$$-\Pr(k=0|t_i)\int_0^1 (y(0)-\theta-b_i)^2 f(\theta|k,r)d\theta -$$

$$\sum_{j=1}^{r}\Pr(k=j|t_i)\int_0^1 (y(1,..,r)-\theta-b_i)^2 f(\theta|j,r)d\theta \geq$$

$$-\Pr(k=0|t_i)\int_0^1 (y(1,..,r)-\theta-b_i)^2 f(\theta|k,r)d\theta -$$

$$\sum_{j=1}^{r}\Pr(k=j|t_i)\int_0^1 (y(1,..,r)-\theta-b_i)^2 f(\theta|j,r)d\theta,$$

which imply $b_i \leq \frac{r+1}{4(r+2)}$, $i = 0,..,r$. Notice that:

$$\frac{r+1}{4(r+2)} > \frac{1}{6} \text{ if } r > 1,$$

Therefore, $P_{i'}(Q)$ can be supported for biases strictly larger than $\frac{1}{6}$.

In a similar way it can be calculated that the downward deviation for every expert $i \in H_{jDM}$ implies $b_i \geq \frac{r-3}{4(r+2)}$ which is strictly smaller than $\frac{r+1}{4(r+2)}$.

It remains to show that this partition results in a higher expected utility for the DM compared to the case in which every expert babbles. The expected utility of the DM is:

$$\mathbb{E}U_{DM}(P_{i'}(Q)) = -\frac{1}{3} + \frac{1}{(r+1)(n+2)^2}\Big(1 + \frac{(2+..+(r+1))^2}{r}\Big) = -\frac{4+r+r^2}{12(2+r)^2}.$$

Therefore, $-\frac{4+r+r^2}{12(2+r)^2} > -\frac{1}{12}$ is true for $r > 0$.

Thus, two conditions $r > 1$ and $r > 0$ imply that for $r > 1$, if all experts' biases are within $[\frac{r-3}{4(r+2)}, \frac{r+1}{4(r+2)}] \cap (\frac{1}{6}, \infty]$, then Q has at least one equilibrium which strictly dominates the (uninformative) equilibrium in a corresponding star network. Q.E.D.

Proof of Proposition 3:

Proof strategy: I start with the partition which features the largest possible shift from experts' expected values of the state if one of the experts deviates from truth-telling. I obtain the range of biases which support this partition as an equilibrium. Thus, I find for an upper bound for biases which can be accommodated by any possible network. I then describe a network which implements such partition: such network features a single intermediary communicating to the DM. I show that a network which features at least two experts communicating to the DM cannot implement this partition. Next, I characterize the partition which features the second largest shift from experts' expected values of the state if one of the experts deviates from truth-telling. I find the upper bound on the biases which make this partition implementable in some network. If all biases are in between the two bounds, then only the network which implements the partition with the largest possible shift can induce some information revelation. Otherwise, babbling is the only equilibrium. Finally, I show that any other partition which features $n' < n$ experts is supported by the biases which are weakly smaller than the interval between the two bounds derived earlier. Therefore, I exclude the lower bound from this interval. The resulting interval specifies the biases which support the coarsest possible information transmission in a network which features a single intermediary. Therefore, if the biases are above the upper bound, no information transmission is possible in any tree network.

Consider a partition of the entire signals according to which the DM receives her information and which consists of two pools, $P'_{DM} = \{\{p_1\}, \{p_2\}\}$. Notice that a partition with 3 or more pools cannot implement a larger shift from the expert's ideal point compared to the largest possible shift in a partition with 2 pools. This is because for a partition with 3 or more pools it is always possible to merge all but the first pool and therefore implement a larger shift from the expert's ideal point compared to the original partition.

Denote DM's choices as a function of a reported pool of P'_{DM} by $y(p_1)$ and $y(p_2)$. If there exists a tree network which implements P'_{DM}, then the incentive constraints of a type t_i communicating in such network can be formulated as:

$$-Pr(k \in p_1|t_i) \int_0^1 (y(p_1) - \theta - b_i)^2 f(\theta|k,n) d\theta \geq$$

$$-Pr(k \in p_1|t_i) \int_0^1 (y(p_2) - \theta - b_i)^2 f(\theta|k,n) d\theta.$$

This can be rewritten as:

$$y(p_1) + y(p_2) - 2\sum_{k\in p_1} Pr(k|t_i)\mathbb{E}(\theta|k,n) \leq 2b_i.$$

Therefore, maximizing $y(p_1) + y(p_2) - 2\sum_{k\in p_1} Pr(k)\mathbb{E}(\theta|k,n)$ supports the largest possible bias preventing a deviation of t_i. I show that this is the case for $p_1 = \{0\}$ and $p_2 = \{1,..,n\}$.

Take any $p_1 = \{0,..,k\}$ and $p_2 = \{k+1-z,..,n\}$ with $k < n$ and $z \leq k$ implementable in some network Q and fix any t_i of an expert i who is communicating in Q. Notice that in general the same sum of the signals, apart from 0 and n, can appear in both pools. Consider $p_1' = \{0,..,k-z\}$ and $p_2' = \{k-z+1,..,n\}$. Denote the belief of an expert i over an element $k \in p_1'$ by $Pr'(k)$. My objective is to show that:

$$y(p_1')+y(p_2')-2\sum_{k\in p_1'} Pr'(k)\mathbb{E}(\theta|k,n) \geq y(p_1)+y(p_2)-2\sum_{k\in p_1} Pr(k|t_i)\mathbb{E}(\theta|k,n). \quad \text{(A.1)}$$

Assume that for each $k \in p_1'$, $Pr'(k)$ is such that $\mathbb{E}_i(k|p_1') = \mathbb{E}_{DM}(k|p_1')$. Therefore, $\sum_{k\in p_1'} Pr'(k)\mathbb{E}(\theta|k,n) = y(p_1')$. This assumption implies a lower bound on $\sum_{k\in p_1'} Pr'(k)\mathbb{E}(\theta|k,n)$ for the following reason. Since an expert has a better observation of the state compared to the DM, there exists a type of an expert i, t_i, who's beliefs over the elements of p_1' assign weakly higher values to higher sums of signals compared to DM's belief once she receives a report p_1'. It means, $\sum_{k\in p_1'} Pr(k|t_i)\mathbb{E}(\theta|k,n) \geq \sum_{k\in p_1'} Pr(k)\mathbb{E}(\theta|k,n)$. Thus, if:

$$y(p_1') + y(p_2') - 2y(p_1') \geq y(p_1) + y(p_2) - 2\sum_{k\in p_1} Pr(k|t_i)\mathbb{E}(\theta|k,n) \quad \text{(A.2)}$$

is satisfied, then (2) is also satisfied. First, notice that:

$$y(p_1') - y(p_1') \geq y(p_1) - \sum_{k\in p_1} Pr(k|t_i)\mathbb{E}(\theta|k,n),$$

because $\sum_{k\in p_1} Pr(k|t_i)\mathbb{E}(\theta|k,n) \geq y(p_1)$, because there is a type t_i who assigns weakly higher beliefs to higher sums of signals, compared to the DM upon receiving the report p_1. This is because the expert has a more precise information about signal realizations conditional on p_1.

It remains to show that

$$\sum_{k \in p_1} Pr(k|t_i)\mathbb{E}(\theta|k,n) - y(p_2) \geq y(p_1') - y(p_2').$$

By the same logic as before, the above inequality is satisfied if:

$$y(p_1) - y(p_2) \geq y(p_1') - y(p_2'). \tag{A.3}$$

is satisfied.

Consider, first, the case of $z = 0$. In this case, $p_1 = \{0, .., k\}$ and $p_2 = \{k+1, .., n\}$. Then,

$$y(0, .., k) = \frac{\sum_{i=1}^{k+1} i}{(n+2)(k+1)} = \frac{k+2}{2(n+2)},$$

$$y(k+1, .., n) = \frac{\sum_{i=k+2}^{n+1} i}{(n+2)(n-k)} = \frac{n+k+3}{2(n+2)}.$$

Then, $y(p_1') - y(p_2') = y(p_1) - y(p_2) = \frac{n+1}{2(n+2)}$ which satisfies (2).

Consider, next, that $z = 1$ such that there is an overlap of a single sum of the signals. Therefore, $p_1 = \{0, .., k\}$ and $p_2 = \{k, .., n\}$. Notice that the sum of the signals k can be formed in $\binom{n}{k}$ different ways (with other words, there are $\binom{n}{k}$ different members of $\{0,1\}^n$ which generate the sum of the signals k). Suppose that k appears in the first pool a times, and in the second pool $\binom{n}{k} - a$ times, with $a \in \{1, .., \binom{n}{k} - 1\}$ (since $a = 0$ or $a = \binom{n}{k}$ are covered by the previous case).

Since each sum of the signals has equal probability:

$$Pr(k) = \int_0^1 Pr(k|\theta)d\theta = \int_0^1 \frac{n!}{k!(n-k)!}\theta^k(1-\theta)^{n-k}d\theta =$$

$$\frac{n!}{k!(n-k)!}\frac{k!(n-k)!}{(n+1)!} = \frac{1}{n+1},$$

If the DM receives the report p_1, she assigns posterior $\frac{1}{k+\frac{a}{\binom{n}{k}}}$ to each of the elements $0, .., k-1$, and the posterior $\frac{\frac{a}{\binom{n}{k}}}{k+\frac{a}{\binom{n}{k}}}$ to the element k. Therefore, her choice is:

$$y(p_1) = \frac{1}{(n+2)(k+\frac{a}{\binom{n}{k}})}(1 + .. + k) + \frac{\frac{a}{\binom{n}{k}}}{(n+2)(k+\frac{a}{\binom{n}{k}})}(k+1) =$$

$$\frac{1}{(2(n+2))(k+\frac{a}{\binom{n}{k}})}k(k+1)+\frac{\frac{a}{\binom{n}{k}}}{(n+2)(k+\frac{a}{\binom{n}{k}})}(k+1)=$$

$$\frac{(k+1)(k+2\frac{a}{\binom{n}{k}})}{(2(n+2))(k+\frac{a}{\binom{n}{k}})}.$$

Similarly, her choice upon receiving p_2 is:

$$y(p_2)=\frac{\frac{\binom{n}{k}-a}{\binom{n}{k}}}{(n+2)(n-k+\frac{\binom{n}{k}-a}{\binom{n}{k}})}(k+1)+\frac{1}{(n+2)(n-k+\frac{\binom{n}{k}-a}{\binom{n}{k}})}((k+2)+..+(n+1))=$$

$$\frac{\frac{\binom{n}{k}-a}{\binom{n}{k}}}{(n+2)(n-k+\frac{\binom{n}{k}-a}{\binom{n}{k}})}(k+1)+\frac{1}{(2(n+2))(n-k+\frac{\binom{n}{k}-a}{\binom{n}{k}})}(k-n)(k+n+3)=$$

$$\frac{2\frac{\binom{n}{k}-a}{\binom{n}{k}}(k+1)+(k-n)(k+n+3)}{(2(n+2))(n-k+\frac{\binom{n}{k}-a}{\binom{n}{k}})}.$$

Then, given that $y(p_1')=\frac{k+1}{2(n+2)}$ and $y(p_2')=\frac{k+n+2}{2(n+2)}$, the expression (4) can be rewritten as:

$$\frac{2\frac{\binom{n}{k}-a}{\binom{n}{k}}(k+1)+(k-n)(k+n+3)}{(2(n+2))(n-k+\frac{\binom{n}{k}-a}{\binom{n}{k}})}-\frac{(k+1)(k+2\frac{a}{\binom{n}{k}})}{(2(n+2))(k+\frac{a}{\binom{n}{k}})}-\frac{-n-1}{2(n+2)}\geq 0,$$

or

$$-a^2(n+1)+a(1-6k-2k^2+7n+2n^2)\binom{n}{k}-2k(3k+k^2-3n-n^2)\binom{n}{k}^2\geq 0,$$

which is true for $k<n$. Therefore, (2) is satisfied.

Finally, consider $z=2$. Thus, $p_1=\{0,..,k\}$ and $p_2=\{k-1,..,n\}$, and $p_1'=\{0,..,k-2\}$ and $p_2'=\{k-1,..,n\}$. To prove that (4) is true for $z=2$ I proceed in two steps. First, I define the following partitions:

- Define a partition $P''=\{p_1'',p_2''\}$ which differs to $\{p_1,p_2\}$ only in that the sum

of the signals k is located entirely in p_2'',

- define a partition $\hat{P} = \{\hat{p}_1, \hat{p}_2\}$ which differs to $\{p_1, p_2\}$ only in that the sum of the signals $k-1$ is located entirely in $\hat{p}_1$, and finally
- define a partition $\hat{P}' = \{\hat{p}_1', \hat{p}_2'\}$ which differs to $\hat{P}'$ only in that the sum of the signals k is located entirely in $\hat{p}_2'$.

From the previous step which covered the case $z = 1$ we know that:

$$y(\hat{p}_1) - y(\hat{p}_1') \geq y(\hat{p}_2) - y(\hat{p}_2'). \tag{A.4}$$

Notice that because in $y(p_1)$ the sum of the signals k has a larger posterior compared to $y(\hat{p}_1)$, the downward shift from $y(p_1)$ to $y(p_1'')$ is larger compared to the downward shift from $y(\hat{p}_1)$ to $y(\hat{p}_1')$. Therefore

$$y(p_1) - y(p_1'') \geq y(\hat{p}_1) - y(\hat{p}_1').$$

Similarly, it is true that

$$y(p_2) - y(p_2'') \leq y(\hat{p}_2) - y(\hat{p}_2'),$$

such that $y(p_1) - y(p_1'') \geq y(p_2) - y(p_2'')$ is satisfied. But then, given the results on $z = 1$ it is also true that

$$y(p_1'') - y(p_1') \geq y(p_2'') - y(p_2').$$

Summing up the last two inequalities we obtain:

$$y(p_1) - y(p_1') \geq y(p_2) - y(p_2').$$

The case $z > 2$ can be solved in a similar way.

Next, I characterize the equilibrium strategies and the corresponding biases which support $P_{DM} = \{\{0\}, \{1, .., n\}\}$ as an equilibrium. By backward induction, I start with the decisions of the DM conditional on messages $m_1 = 0$ or $m_2 = (1, .., n)$.

$$y(0) = \frac{1}{n+2}$$

For $y(1, .., n)$ notice that each sufficient statistic has equal probability because the probability of having k 1's in n experiments is:

$$Pr(k) = \int_0^1 Pr(k|\theta)d\theta = \int_0^1 \frac{n!}{k!(n-k)!}\theta^k(1-\theta)^{n-k}d\theta =$$

$$\frac{n!}{k!(n-k)!}\frac{k!(n-k)!}{(n+1)!} = \frac{1}{n+1}.$$

Thus, the decision of the DM is:

$$y(1,..,n) = \frac{\sum_{i=1}^{n}(i+1)}{(n+2)}\frac{1}{n} = \frac{n+3}{2(n+2)}.$$

The upward deviation incentive of any expert $i \in N$ is determined by:

$$-\int_0^1 (\frac{1}{n+2} - \theta - b_i)^2 f(\theta|0,n)d\theta \geq -\int_0^1 (\frac{n+3}{2(n+2)} - \theta - b_i)^2 f(\theta|0,n)d\theta,$$

or

$$\frac{1}{n+2} + \frac{n+3}{2(n+2} - 2\frac{2}{2(n+2)} - 2b_i \geq 0,$$

which results in:

$$b_i \leq \frac{n+1}{4(n+2)}.$$

Similarly, the downward deviation of any expert $i \in N$ is determined by:

$$\frac{1}{n+2} + \frac{n+3}{2(n+2} - 2\frac{4}{2(n+2)} - 2b_i \geq 0,$$

which results in:

$$b_i \geq \frac{n-3}{4(n+2)}.$$

Next I show that the partition $\{0\}, \{1,..,n\}$ cannot be implemented in a network which features at least two intermediaries communicating directly to the DM. Notice that informative communication requires an intermediary to send at least two messages in equilibrium. Otherwise his message is ignored by the DM. In this case, $r > 1$ intermediaries feature at least $2r$ pools according to which the DM receives her information. But I showed that this cannot be the case which features the largest possible deviation from experts' expected values of the states by deviation. If all but the first pools are merged together, then the deviation from the first pool results in a larger shift from experts' expected values of the state, compared to the

case of multiple experts communicating to the DM.

Next, I characterize the partition which implements the second largest shift from experts' ideal points if one of the experts deviates from truth-telling. Given the considerations above, such partition should have the form $\{\{0,1\},\{1,..,n\}\}$, where the sum of the signals 1 featured in the first pool has the lowest possible posterior. Notice that there are $\binom{n}{1}$ ways for the elements of $\{0,1\}^n$ to result in sum of the signals 1. Therefore, the case of the lowest possible posterior of the sum of the signals 1 in the first pool features a single element of $\{0,1\}^n$ with the sum of the signals 1 being part of the first pool. Therefore, the second pool includes $\binom{n}{1}-1$ sequences which have the sum of the signals 1.

In this case, if the DM receives the message $m_1=(0,1)$, she attaches posterior $\frac{n}{n+1}$ to signal 0 and the posterior to $\frac{1}{n+1}$ to signal 1. Notice that the posterior or 1 is much smaller because I assumed that the first pool includes only one sequence with the summary statistic 1. Thus,

$$y(0,1)=\frac{n}{n+1}\frac{1}{n+2}+\frac{1}{n+1}\frac{2}{n+2}=\frac{1}{1+n}.$$

Upon receiving $m_2=(1,..,n)$ the DM assigns the posterior $\frac{\frac{n-1}{n}}{\frac{n-1}{n}+(n-1)}=\frac{1}{n+1}$ to 1 and an equal posterior of $\frac{1}{\frac{n-1}{n}+(n-1)}=\frac{n}{(n-1)(n+1)}$ to sufficient statistic 2,3,..,n. Thus, the DM chooses:

$$y(1,..,n)=\frac{1}{n+1}\frac{2}{n+2}+\frac{n}{(n-1)(n+1)(n+2)}(3+..+(n+1))=\frac{2+n}{2+2n}.$$

To check the deviation incentives of an expert, notice, that depending on network and information received he can assign different beliefs to the sum of the signals 0 or 1. The larger the posterior attached to 0, the larger the upper bound for biases supporting the equilibrium strategy specified above. I focus on the largest possible upper bound for the biases, and therefore assign the lowest possible posterior to sum of the signals 1. This happens if - in the case the expert reporting to the DM observes the pool $\{0,1\}$ - he assigns probability $\frac{1}{n+1}\frac{1}{n}$ to the sum of the signals 1, which is the lowest possible posterior, and the probability $\frac{1}{n+1}$ to the sum of the signals 0, which is the largest possible posterior given the observation $\{0,1\}$. Therefore, the posterior assigned to 1 is $\frac{1}{n+1}$ and the posterior assigned to 0 is $\frac{n}{n+1}$. The following incentive constraint has to hold:

$$-\frac{n}{n+1}\int_0^1(\frac{1}{1+n}-\theta-b_i)^2 f(\theta|0,n)d\theta-\frac{1}{n+1}\int_0^1(\frac{1}{1+n}-\theta-b_i)^2 f(\theta|1,n)d\theta \geq$$

$$-\frac{n}{n+1}\int_0^1(\frac{2+n}{2(1+n)}-\theta-b_i)^2 f(\theta|0,n)d\theta-\frac{1}{n+1}\int_0^1(\frac{2+n}{2(1+n)}-\theta-b_i)^2 f(\theta|1,n)d\theta.$$

This results in $b_i \leq \frac{n}{4(1+n)}$.

Notice that $\frac{n-3}{4(n+2)} < \frac{n}{4(1+n)} < \frac{n+1}{4(n+2)}$ for $n \geq 2$ which implies that there is a range for biases $[\frac{n}{4(1+n)}, \frac{n+1}{4(n+2)}]$ for which P_{DM} is the only implementable partition for given n.

Now, suppose that only $n' < n$ experts report truthfully, and $n - n'$ experts babble. Partition which accommodates largest possible biases is $P''_{DM} = \{\{0\},\{1,..,n'\}\}$ because this partition has the maximum heterogeneity between any two pools. The range of biases which support P''_{DM} as an equilibrium partition is $[\frac{n'-3}{4(n'+2)}, \frac{n'+1}{4(n'+2)}]$

Is and yef yes, when, is $\frac{n'+1}{4(n'+2)}$ smaller than $\frac{n}{4(1+n)}$?

$$\frac{n'+1}{4(n'+2)} < \frac{n}{4(1+n)}, \text{ if } n > n'+1.$$

However, the biggest possible $n' = n-1$, such that in this case both bounds coincide. Therefore, only if the biases of all experts is in the interval $(\frac{n}{4(1+n)}, \frac{n+1}{4(n+2)}]$, then the optimal network is the one which features a single group and a single expert reporting directly to the DM.

To prove the statement regarding the range $(-\frac{n+1}{4(n+2)}, -\frac{n}{4(n+1)}]$, notice that for n experts and for the partition $P_{DM} = \{\{0,..,n-1\},\{n\}\}$, the deviation of an expert from reporting n to the DM maximizes the distance between the DM's policy and the expected value of the expert, leading to the largest possible upper bound for biases. The way to prove it is similar to the mirror case of the partition $\{\{0\},\{1,..,n\}\}$ which was proven in the first part of the Lemma.

To derive the lower bound by deviation of type n for $P_{DM} = \{\{0,..,n-1\},\{n\}\}$, I, first, calculate the corresponding choices of the DM:

$$y(0,..,n-1) = \frac{1}{n}\sum_{i=0}^{n-1}\frac{i+1}{n+2} = \frac{n+1}{2(n+2)}, \quad y(n) = \frac{n+1}{n+2}.$$

Therefore, the downward deviation of type n of any expert $i \in N$ implies:

$$\frac{n+1}{2(n+2)} + \frac{n+1}{n+2} - 2\frac{n+1}{n+2} - 2b_i \leq 0,$$

which results in $b_i \geq -\frac{n+1}{4(n+2)}$.

To calculate the binding upward deviation, notice, that for a given change in DM's policy, the range of expert's biases decreases with higher expectation over the state. Therefore, the binding deviation from the first pool occurs when the type of an expert i is $n-1$. In this case, the following incentive constraint has to hold:

$$\frac{n+1}{2(n+2)} + \frac{n+1}{n+2} - 2\frac{n}{n+2} - 2b_i \geq 0,$$

which results in $b_i \leq -\frac{n-3}{4(n+2)}$.

The second largest downward deviation assumes $P_{DM} = \{\{0,..,n-1\},\{n-1,n\}\}$ with only a single sequence having the sum of the signals $n-1$ included in the second pool, and the posterior belief of an expert which assigns maximum probability to n if the expert beliefs that the state is within the second pool.

If the DM receives the report $\{0,..,n-1\}$, she assigns the posterior $\frac{n}{(n-1)(n+1)}$ to each element $0,..,n-2$, and the posterior $\frac{n-1}{(n-1)(n+1)}$ to $n-1$. If the DM receives the report $\{n-1,n\}$, she assigns the posterior $\frac{1}{n+1}$ to the element $n-1$ and the posterior $\frac{n}{n+1}$ to the element n.

The corresponding choices of the DM are:

$$y(0,..,n-1) = \frac{n}{(n-1)(n+1)}\frac{(1+..+n-1)}{n+2} + \frac{n-1}{(n-1)(n+1)}\frac{n}{n+2} = \frac{n}{2(n+1)},$$

$$y(n-1,n) = \frac{1}{n+1}\frac{n}{n+2} + \frac{n}{n+1}\frac{n+1}{n+2} = \frac{n}{n+1}.$$

An expert i, when being informed that the true state is in $\{n-1,n\}$, can at most assign posterior belief $\frac{1}{n+1}$ to the element $n-1$, and the posterior belief $\frac{n}{n+1}$ to the element n. The following incentive constraint has to hold for a downward deviation:

$$-\frac{n}{n+1}\int_0^1 (\frac{n}{n+1} - \theta - b_i)^2 f(\theta|n-1,n)d\theta - \frac{n}{n+1}\int_0^1 (\frac{1}{n+1} - \theta - b_i)^2 f(\theta|n,n)d\theta \geq$$

$$-\frac{n}{n+1}\int_0^1 (\frac{n}{2(n+1)} - \theta - b_i)^2 f(\theta|n-1,n)d\theta - \frac{1}{n+1}\int_0^1 (\frac{n}{2(n+1)} - \theta - b_i)^2 f(\theta|n,n)d\theta.$$

This results in: $b_i \leq -\frac{n}{4(n+1)}$.

Finally, suppose that there are $n' < n$ reporting in equilibrium. The partition which implements the largest possible downward shift of DM's policy from the expected value of the state by an expert is $P'_{DM} = \{\{0, .., n-2\}, \{n-1\}\}$. The corresponding choices of the DM are:

$$y(0, .., n-2) = \frac{n}{2(n+1)}, y(n-1) = \frac{n}{n+1}.$$

The incentive constraint preventing the downward deviation of any expert from the second pool implies:

$$\frac{n}{2(n+1)} + \frac{n}{n+1} - 2\frac{n}{n+1} - 2b \leq 0.$$

This implies $b_i \geq -\frac{n}{4(n+1)}$.

Therefore, the range of biases which supports communication to the DM in a single group is $[-\frac{n+1}{4(n+2)}, -\frac{n}{4(n+1)})$.

Finally, since I have shown that for positive biases the partition $\{\{0\}, \{1, .., n\}\}$ implements the largest possible upward shift from the experts' expected values of the state due to the coarsest possible communication to the DM. Therefore, any biases beyond $\frac{n+1}{4(n+2)}$ cannot lead to any information transmission. Similarly, for negative biases, the partition $\{\{0, .., n-1\}, \{n\}\}$ implements the largest possible downward shift from the experts' expected values of the state. Therefore, any biases below $-\frac{n+1}{4(n+2)}$ cannot lead to any information transmission. Q.E.D.

Proof of Lemma 1:

The idea of the proof goes as follows. The beliefs of expert j about overall signals can be written as the sum of beliefs conditional on the realizations of i's types. Since expert i is informed about all signals available to expert j, there is a type of expert i which results in a weakly smaller range of biases supporting his equilibrium strategy compared to expert j.

1. Suppose that an equilibrium features $n' \geq 2$ non-babbling experts. A type of j is a probability distribution over the set of all possible signals $\{0, 1\}^{n'}$, conditional on an element of a partition $P_j^b(Q)$ according to which j receives his signals.

Suppose that j receives $\tilde{p} \in P_j^b$. Given his type, j forms beliefs over different sums of signals, $k \in \{0, .., n\}$ conditional on his expectations of the elements of DM's partition, $p \in P_{DM}(Q)$ with $k \in p$ given the strategy profile of all other experts denoted by $P_{-j}(Q)$. The following incentive constraints give a condition on the bias of j to prevent a deviation from truthful reporting $\tilde{p}$ to some other report $\tilde{p}' \in P_j^b(Q)$. For notational convenience, in the following I supress Q:

$$-\sum_{t_i|t_j} \sum_{k\in\{0,..,n'\}} Pr(t_i|t_j) \times Pr(k|t_i) \int_0^1 (y(p|P_{-j}, \tilde{p})) - \theta - b_j)^2 f(\theta|k, n) d\theta \geq$$

$$-\sum_{t_i|t_j} \sum_{k\in\{0,..,n'\}} Pr(t_i|t_j) \times Pr(k|t_i) \int_0^1 (y(p|P_{-j}, \tilde{p}') - \theta - b_j)^2 f(\theta|k, n) d\theta.$$

The above system of inequalities can be rewritten as:

$$\sum_{t_i|t_j} \sum_{k\in\{0,..,n'\}} Pr(t_i|t_j) \times Pr(k|t_i) \times \Big(y(p|P_{-j}, \tilde{p}') - y(p|P_{-i}, \tilde{p})\Big) \times$$

$$\Big(y(p|P_{-j}, \tilde{p}') + y(p|P_{-i}, \tilde{p}) - 2\mathbb{E}(\theta|k) - 2b_j\Big) \geq 0.$$

2. First, let us look at the upward deviation of t_j. Since it is the smallest upward deviation of t_j which is binding for the reporting incentives, we look at the deviation resulting in $y(p|P_{-j}, \tilde{p}') > y(p|P_{-i}, \tilde{p})$ such that there is no $\tilde{p}'' \in P_j$ such that $y(p|P_{-j}, \tilde{p}') > y(p|P_{-j}, \tilde{p}'') > y(p|P_{-i}, \tilde{p})$.

   Since

$$\sum_{t_i|t_j} \sum_{k\in\{0,..,n'\}} Pr(t_i|t_j) \times Pr(k|t_i) \times y(p|P_{-j}, \tilde{p}') >$$

$$\sum_{t_i|t_j} \sum_{k\in\{0,..,n'\}} Pr(t_i|t_j) \times Pr(k|t_i) \times y(p|P_{-j}, \tilde{p}),$$

   there is a value $\overline{b}(t_j) \in \mathcal{R}$ which solves:

$$\sum_{t_i|t_j} \sum_{k\in\{0,..,n'\}} Pr(t_i|t_j) \times Pr(k|t_i) \times \Big(y(p|P_{-j}, \tilde{p}') - y(p|P_{-i}, \tilde{p})\Big) \times$$

$$\Big(y(p|P_{-j},\tilde{p}') + y(p|P_{-i},\tilde{p}) - 2\mathbb{E}(\theta|k) - 2\bar{b}(t_j)\Big) = 0.$$

The incentive constraints of a type t_i can be written as:

$$-\sum_{k\in\{0,..,n'\}} Pr(k|t_i)\int_0^1 (y(p|P_{-i},\hat{p})) - \theta - b_j)^2 f(\theta|k,n)d\theta \geq$$

$$-\sum_{k\in\{0,..,n'\}} Pr(k|t_i)\int_0^1 (y(p|P_{-i},\hat{p}') - \theta - b_j)^2 f(\theta|k,n)d\theta,$$

where $\hat{p} \in P_i$ is a signal which t_i receives, and $\hat{p}' \in P_i$ is a different element of i's partition, $\hat{p} \neq \hat{p}'$. Since I assumed that t_j reports truthfully, $y(p|P_{-i},\hat{p})) = y(p|P_{-i},\tilde{p})|t_i$. Further, since we are looking at the smallest upward deviation of t_j in terms of DM's policy, conditional on types of i, the smallest upward deviation of t_i results in the same policy of the DM: $y(p|P_{-j},\tilde{p}')|t_i = y(p|P_{-i},\hat{p}')$. But then the IC for t_i can be written as:

$$\sum_{k\in\{0,..,n'\}} Pr(k|t_i) \times \Big(y(p|P_{-j},\tilde{p}') - y(p|P_{-i},\tilde{p})\Big)\times$$

$$\Big(y(p|P_{-j},\tilde{p}') + y(p|P_{-i},\tilde{p}) - 2\mathbb{E}(\theta|k) - 2b_j\Big) \geq 0.$$

There exists some t_i for which $Pr(t_i|t_j) > 0$, where $\bar{b}(t_i)$ solves:

$$\sum_{k\in\{0,..,n'\}} Pr(k|t_i) \times \Big(y(p|P_{-j},\tilde{p}') - y(p|P_{-i},\tilde{p})\Big)\times$$

$$\Big(y(p|P_{-j},\tilde{p}') + y(p|P_{-i},\tilde{p}) - 2\mathbb{E}(\theta|k) - 2\bar{b}(t_i)\Big) = 0.$$

with $\bar{b}(t_i) \leq \bar{b}(t_j)$.

3. Next, let us look at the downward deviation of t_j. Since the smallest downward deviation is binding for the reporting incentives, we look at the deviation resulting in $y(p|P_{-j},\tilde{p}') < y(p|P_{-i},\tilde{p})$ such that there is no $\tilde{p}'' \in P_j$ such that $y(p|P_{-j},\tilde{p}') < y(p|P_{-j},\tilde{p}'') < y(p|P_{-i},\tilde{p})$.

   Since

$$\sum_{t_i|t_j}\sum_{k\in\{0,..,n'\}} Pr(t_i|t_j)\times Pr(k|t_i)\times y(p|P_{-j},\tilde{p}') <$$

$$\sum_{t_i|t_j}\sum_{k\in\{0,..,n'\}} Pr(t_i|t_j)\times Pr(k|t_i)\times y(p|P_{-j},\tilde{p}),$$

there is a value $\underline{b}(t_j)\in\mathcal{R}$ which solves:

$$\sum_{t_i|t_j}\sum_{k\in\{0,..,n'\}} Pr(t_i|t_j)\times Pr(k|t_i)\times \Big(y(p|P_{-j},\tilde{p}')-y(p|P_{-i},\tilde{p})\Big)\times$$

$$\Big(y(p|P_{-j},\tilde{p}')+y(p|P_{-i},\tilde{p})-2\mathbb{E}(\theta|k)-2\underline{b}(t_j)\Big)=0.$$

The incentive constraints of a type t_i can be written as:

$$-\sum_{k\in\{0,..,n'\}} Pr(k|t_i)\int_0^1 (y(p|P_{-i},\hat{p}))-\theta-b_j)^2 f(\theta|k,n)d\theta \geq$$

$$-\sum_{k\in\{0,..,n'\}} Pr(k|t_i)\int_0^1 (y(p|P_{-i},\hat{p}')-\theta-b_j)^2 f(\theta|k,n)d\theta,$$

where $\hat{p}\in P_i$ is i's signal and $\hat{p}'\in P_i$ is a different element of i's partition, $\hat{p}\neq\hat{p}'$. Similar as above, assuming that t_j adheres to his strategy P_j, we have $y(p|P_{-i},\hat{p}))=y(p|P_{-i},\tilde{p})|t_i$. Since we are looking at the smallest upward deviation of t_j in terms of DM's policy, conditional on types of i, the smallest upward deviation of t_i results in the same policy of the DM: $y(p|P_{-j},\tilde{p}')|t_i = y(p|P_{-i},\hat{p}')$. But then the IC for t_i can be written as:

$$\sum_{k\in\{0,..,n'\}} Pr(k|t_i)\times \Big(y(p|P_{-j},\tilde{p}')-y(p|P_{-i},\tilde{p})\Big)\times$$

$$\Big(y(p|P_{-j},\tilde{p}')+y(p|P_{-i},\tilde{p})-2\mathbb{E}(\theta|k)-2b_j\Big)\leq 0.$$

There exists some t_i for which $Pr(t_i|t_j)>0$, where $\underline{b}(t_i)$ solves:

$$\sum_{k\in\{0,..,n'\}} Pr(k|t_i)\times \Big(y(p|P_{-j},\tilde{p}')-y(p|P_{-i},\tilde{p})\Big)\times$$

$$\Big(y(p|P_{-j},\tilde{p}')+y(p|P_{-i},\tilde{p})-2\mathbb{E}(\theta|k)-2\underline{b}(t_i)\Big)=0.$$

with $\underline{b}(t_i) \geq \underline{b}(t_j)$. Q.E.D.

Proof of Lemma 2:

Fix an equilibrium strategy profile in Q such that there is an expert i who receives full statistical information from another expert j, $j \in H_{ji}$. If $e_{ji} = 1$, then the Lemma is satisfied. Therefore, assume that $|H_{ji}| > 2$ such that there is at least one other expert on the path H_{ji}, strictly between j and i. Denote such expert by j' and his successor in the network by j'', $e_{j'j''} = 1$. Notice that the partition according to which j'' receives information from j' can be written as $P_j \times \{P_{j'} \setminus P_j\}$ because if expert i receives full statistical information from j, then any expert along the path H_{ji} receives the full statistical information from j.

Next, delete a link from j to another expert to whom he is connected in Q, and add a link from j to i, such that $e_{ji} = 1$. Denote the new network by Q'. Assume that every expert j' on the path H_{ji} other than j and i sends his reports according to the partition $\{P_{j'} \setminus P_j\}$, and any other expert has the same strategy profile as in above equilibrium in Q.

Notice that the incentive constraints of j do not change since he has the same beliefs about the set of all signal realizations and his signals are treated in the same way by expert i. Similarly, the incentive constraints of i do not change since he receives the same statistical information as before and nothing has changed for his successors in the new network Q' compared to Q.

However, the incentive constraints of all experts in H_{ji} which are not i and j are relaxed due to Lemma 1 since they have strictly more uncertainty compared to their beliefs in network Q. Therefore, this rearrangement of links from Q to Q' contributes to a bigger slack in the incentive constraints of experts on the path H_{ji} other than j and i and possibly contributes to a larger transfer of information to the DM in the best equilibrium. Q.E.D.

Proof of Lemma 3:

Suppose $G = 3$ such that there are no other experts apart from j, j' reporting to i^G in group G. Denote by m_{i^G} the equilibrium message strategy of i^G in an optimal network Q. Notice that in an optimal network both j and j' do not babble, otherwise they would not be part of an optimal network. If $s_j = k \in \{0, 1\}$ and

$s_{j'} = k' \in \{0,1\}$, then $m_{i^G}(s_{i^G}, m_j = k, m_{j'} = k') = m_{i^G}(s_{i^G}, m_j = k', m_{j'} = k)$. Otherwise the type of i^G with private information summarized by $(s_{i^G}, m_j = k, m_{j'} = k')$ takes a different equilibrium action than type with private information summarized by $(s_{i^G}, m_j = k', m_{j'} = k)$. But this contradicts equilibrium conditions since both types have the same beliefs over the signals summarized by the summary statistic.

Since j and j' have same ex ante beliefs over the signals of all experts, and their reports are treated symmetrically by i^G, their strategies are symmetric and supported by the same range of biases.

Suppose $|G| > 3$, and denote by P'_G the message strategy of group members other than j, j' and i^G, and by m_{i^G} the equilibrium message strategy of i^G in an optimal network Q. As above, in an optimal network both j and j' do not babble, otherwise they would not be part of an optimal network. Notice that it should be true that for any report of all group members rather than j, j' to the DM, $p' \in P'_G$, if $s_j = k \in \{0,1\}$ and $s_{j'} = k' \in \{0,1\}$, then $m_{i^G}(s_{i^G}, p', m_j = k, m_{j'} = k') = m_{i^G}(s_{i^G}, p', m_j = k', m_{j'} = k)$. Otherwise the type of i^G with private information summarized by $(s_{i^G}, p', m_j = k, m_{j'} = k')$ takes a different equilibrium action than type with private information summarized by $(s_{i^G}, p', m_j = k', m_{j'} = k)$. But this contradicts equilibrium conditions since both types have the same beliefs over the signals summarized by the summary statistic.

Since j and j' have same ex ante beliefs over the signals of all experts, and their reports are treated symmetrically by i^G, their strategies are symmetric and supported by the same range of biases.

Further, since i^G has strictly more information than j or j', $P_{i^G} > P_j = P_{j'}$, and there are two paths H_{ji^G} and $H_{j'i^G}$, conditions of Lemma 1 are satisfied and expert i^G has a weakly lower range of biases which support his equilibrium strategy in an optimal network Q, compared to the corresponding ranges of biases for j or j' (which are the same). Q.E.D.

Example for the section 1.3.4

<u>First equilibrium in network Q_a</u>:

The strategy profile of the experts leads to a partition $P_{DM} = \{\{0\}, \{1\}, \{2,3\}\}$ according to which the DM receives her information.

This is the same equilibrium as in Example 2 analyzed above leading to

$$b_i \leq 0.125, \ i = 2,3, \quad b_1 \leq 0.1.$$

DM's expected utility is:

$$-\frac{1}{3} + \frac{1}{4}\Big[\Big(\frac{1}{5}\Big)^2 + \Big(\frac{2}{5}\Big)^2 + 2\Big(\frac{7}{10}\Big)^2\Big] \simeq 0.038$$

<u>Implementation of the same equilibrium outcome ($P_{DM} = \{\{0\}, \{1\}, \{2,3\}\}$) in an optimal line</u>:

Experts 3 reports his signal truthfully to expert 2, and expert 2 reports truthfully both the message of expert 3 and his signal to expert 1. Expert 1 partitions the information in the same way as in network Q_a: if the sufficient statistic of all 3 signals is 0, report m_1 is sent. If the sufficient statistic of all 3 reports is 1, m_1' is sent. Otherwise m_1'' is sent. The subsequent choices of the DM are $y(m_1) = \frac{1}{5}, \ y(m_1') = \frac{2}{5}, \ y(m_1'') = \frac{7}{10}$. This strategy profile is an equilibrium for $b_1 \leq 0.1, \ b_2 \leq 0.117, \ b_3 \leq 0.125$.

<u>Second equilibrium in network Q_a</u>: Think about a strategy profile in which expert 2 and 3 always report their signals truthfully to expert 1, and expert 1 sends either m_1 to the DM if the summary statistic of information he receives is 0, or m_1' otherwise. Thus, the DM received information according to the partition $P_{DM} = \{\{0\}, \{1,2,3\}\}$. The subsequential decisions of the DM are a function of the reported pool:

$$y(0) = \frac{1}{5}, \ y(1,2,3) = \frac{3}{5}.$$

The upward deviation of any of the experts is determined by:

$$-\int_0^1 (\frac{1}{5} - \theta - b_i)^2 f(\theta|0,3)d\theta \geq -\int_0^1 (\frac{3}{5} - \theta - b_i)^2 f(\theta|0,3)d\theta, \ i = 1,2,3$$

which implies:

$b_i \leq 0.2$ for $i = 1,2,3$.

Similarly, the downward deviation of any of the experts is determined by:

$$-\int_0^1 (\frac{3}{5} - \theta - b_i)^2 f(\theta|1,3)d\theta \geq -\int_0^1 (\frac{1}{5} - \theta - b_i)^2 f(\theta|1,3)d\theta, \ i = 1,2,3,$$

which implies:

$b_i \geq 0$.

DM's expected utility is:

$$-\frac{1}{3} + \frac{1}{4}\Big[\Big(\frac{1}{5}\Big)^2 + 3\Big(\frac{3}{5}\Big)^2\Big] \simeq -0.053.$$

<u>Implementation of the same equilibrium outcome in an optimal line</u>:

Notice that experts 3 and 1 has the same amount of uncertainty and are pivotal for the same signals as in the equilibrium in network A, thus, nothing changes in terms of ICs for them. Expert 2 observes the signal of expert 3, and is pivotal only both message of expert 3 and his private signal are 0. His incentive constraints for the smallest upward and downward deviation are:

$$-\int_0^1 (\frac{1}{5} - \theta - b_2)^2 f(\theta|0,3)d\theta \geq -\int_0^1 (\frac{3}{5} - \theta - b_2)^2 f(\theta|0,3)d\theta,$$

and

$$-\int_0^1 (\frac{3}{5} - \theta - b_2)^2 f(\theta|1,3)d\theta \geq -\int_0^1 (\frac{1}{5} - \theta - b_2)^2 f(\theta|1,3)d\theta,$$

implying: $0 \leq b_2 \leq 0.2$, which is the same is above.

<u>Equilibrium in network Q_b vs implementation in a line</u>:

These calculation have been covered in part 1.3.1.

Appendix B

Definitions and Proofs for Chapter 2

Rest of the equilibria from section 2.0.6, on the case of two bias groups: Networks B, C and D generate same best equilibrium outcomes for given biases as follows. If $b^+ \leq 0.1$ and $b^- \geq 0.1$ then the best equilibrium involves perfect separation of signals of any 3 out of the 4 experts yielding $\mathbb{E}U_{DM} \simeq -0.033$. If either $(0.1 < b^+ \leq 0.13,\ b^- \geq -0.045)$ or $(-0.13 \leq b^- < -0.1,\ b^+ \leq 0.045)$ then the best equilibrium involves each group leader reporting sufficient statistic 0 in one message and all other signals in the other message yielding $\mathbb{E}U_{DM} \simeq -0.038$. If $(0.1 < b^+ \leq 0.115,\ -0.052 \leq b^- < -0.045)$ or $(0.045 < b^+ \leq 0.052,\ -0.115 \leq b^- < 0.1)$ then the best equilibrium involves one of the group leaders reporting either sufficient statistic 0 from both experts in a group in the first message, or all other signals in the second message, and the other group leader reporting truthfully only his private signal assuming that the other expert in a group babbles. This equilibrium yields $\mathbb{E}U_{DM} \simeq -0.04$ (see Example 2). Otherwise any of other equilibria is discussed in the leading example and yields an expected utility for the DM strictly lower than any other equilibria discussed above.

Calculations for partitions (2.1), p.29

I looks at the partition $P_{DM}(Q')$.
The DM receives her signals according to:

$$P_{DM}(Q') := \{\{0,..,n-t\},\{n-t+1,..,2n\}\} \times \{\{0,..,n-z\},\{m-z+1,..,2n\}\},$$

because she combines the report from the $2n$ group in which the group leader informs the DM according to the partition $\{\{0,..,n-t\},\{n-t+1,..,2n\}\}$, and the report from the other $2n$ group in which the group leader informs the DM according to the partition $\{\{0,..,n-z\},\{n-z+1,..,2n\}\}$.

The corresponding choices of the DM, conditional on the reported pools are:

$$y(\{0,..,n-t\},\{0,..,n-z\}) = \frac{\sum_{i=0}^{n-t}\sum_{j=0}^{m-z}\frac{\binom{2n}{i}\binom{2n}{j}\frac{i+j+1}{4n+2}}{\binom{4n}{i+j}}}{\sum_{i=0}^{n-t}\sum_{j=0}^{n-z}\frac{\binom{2n}{i}\binom{2n}{j}}{\binom{4n}{i+j}}}$$

$$y(\{0,..,n-t\},\{n-z+1,..,2n\}) = \frac{\sum_{i=0}^{n-t}\sum_{j=n-z+1}^{2m}\frac{\binom{2n}{i}\binom{2n}{j}\frac{i+j+1}{4n+2}}{\binom{2n+2n}{i+j}}}{\sum_{i=0}^{n-t}\sum_{j=n-z+1}^{2n}\frac{\binom{2n}{i}\binom{2n}{j}}{\binom{4n}{i+j}}}$$

$$y(\{n-t+1,..,2n\},\{0,..,n-z\}) = \frac{\sum_{i=n-t+1}^{2n}\sum_{j=0}^{n-z}\frac{\binom{2n}{i}\binom{2m}{j}\frac{i+j+1}{4n+2}}{\binom{4n}{i+j}}}{\sum_{i=n-t+1}^{2n}\sum_{j=0}^{n-z}\frac{\binom{2n}{i}\binom{2n}{j}}{\binom{4n}{i+j}}}$$

$$y(\{n-t+1,..,2n\},\{n-z+1,..,2n\}) = \frac{\sum_{i=n-t+1}^{2n}\sum_{j=n-z+1}^{2n}\frac{\binom{2n}{i}\binom{2n}{j}\frac{i+j+1}{4n+2}}{\binom{2n+2n}{i+j}}}{\sum_{i=n-t+1}^{2n}\sum_{j=n-z+1}^{2m}\frac{\binom{2n}{i}\binom{2n}{j}}{\binom{4n}{i+j}}}$$

DM's expected utility is:

$$-\frac{1}{3}+\frac{1}{4n+1}\sum_{y}(Pr(y)\cdot y)) =$$

$$-\frac{1}{3}+\frac{1}{4n+1}\Big(\frac{\left[\sum_{i=0}^{n-t}\sum_{j=0}^{m-z}\frac{\binom{2n}{i}\binom{2n}{j}\frac{i+j+1}{4n+2}}{\binom{4n}{i+j}}\right]^2}{\sum_{i=0}^{n-t}\sum_{j=0}^{m-z}\frac{\binom{2n}{i}\binom{2n}{j}}{\binom{4n}{}i+j}}+\frac{\left[\sum_{i=0}^{n-t}\sum_{j=m-z+1}^{2n}\frac{\binom{2n}{i}\binom{2n}{j}\frac{}{i+j+1}4n+2}{\binom{4n}{i+j}}\right]^2}{\sum_{i=0}^{n-t}\sum_{j=m-z+1}^{2n}\frac{\binom{2n}{i}\binom{2n}{j}}{\binom{}{4n}i+j}}+$$

$$\frac{\left[\sum_{i=n-t+1}^{2n}\sum_{j=0}^{m-z}\frac{\binom{2n}{i}\binom{2n}{j}\frac{i+j+1}{4n+2}}{\binom{4n}{i+j}}\right]^2}{\sum_{i=n-t+1}^{2n}\sum_{j=0}^{m-z}\frac{\binom{2n}{i}\binom{2n}{j}}{\binom{4n}{i+j}}}+\frac{\left[\sum_{i=n-t+1}^{2n}\sum_{j=m-z+1}^{2n}\frac{\binom{2n}{i}\binom{2n}{j}\frac{i+j+1}{4n+2}}{\binom{4n}{i+j}}\right]^2}{\sum_{i=n-t+1}^{2n}\sum_{j=m-z+1}^{2n}\frac{\binom{2n}{i}\binom{2n}{j}}{\binom{4n}{i+j}}}\Bigg)$$

Next I write down the incentive constraints for expert 1 in Q'. It can be shown that all other experts in the group with $2n$ experts face the same constraints. The incentive constraints for expert $2n+1$, which are the same as the incentive constraints for all other experts in group with $2m$ experts can be calculated similarly.

The incentive constraint for the upward deviation of expert 1 in Q' are:

$$\sum_{i=0}^{m-z} Pr(k'=i|k=n-t)\Big(y(\{n-t+1,..,2n\},\{0,..,n-z\})-y(\{0,..,n-t\},\{0,..,n-z\})\Big)$$

$$\Big(y(\{n-t+1,..,2n\},\{0,..,n-z\})+y(\{0,..,n-t\},\{0,..,n-z\})-2\frac{n-t+i+1}{4n+2}-2b^{+}\Big)+$$

$$\sum_{i=n-z+1}^{2m} Pr(k'=i|k=n-t)\Big(y(\{n-t+1,..,2n\},\{n-z+1,..,2m\})-y(\{0,..,n-t\},\{n-z+1,..,2n\})\Big)$$

$$\Big(y(\{n-t+1,2n\},\{n-z+1,..,2n\})+y(\{0,..,n-t\},\{n-z+1,..,2n\})-2\frac{n-t+i+1}{4n+2}-2b^{+}\Big)=0$$

The incentive constraints for the downward deviation of expert 1 in Q' are:

$$\sum_{i=0}^{n-z} Pr(k'=i|k=n-t+1)\Big(-y(\{n-t+1,2n\},\{0,..,n-z\})+y(\{0,..,n-t\},\{0,..,n-z\})\Big)$$

$$\Big(y(\{n-t+1,2n\},\{0,..,n-z\})+y(\{0,..,n-t\},\{0,..,n-z\})-2\frac{n-t+i+2}{4n+2}-2b^{-}\Big)+$$

$$\sum_{i=n-z+1}^{2n} Pr(k'=i|k=n-t+1)\Big(-y(\{n-t+1,2n\},\{n-z+1,..,2n\})+y(\{0,..,n-t\},\{n-z+1,..,2n\})\Big)$$

$$\Big(y(\{n-t+1,2n\},\{n-z+1,..,2n\})+y(\{0,..,n-t\},\{n-z+1,..,2n\})-2\frac{n-t+i+2}{4n+2}-2b^{-}\Big)=0$$

Proof of Proposition 4:

Fix any network $Q \in \mathcal{Q}$. First, I show that if P_{DM} is implementable in Q, then it is also implementable in a direct mechanism. Notice that per definition the DM can commit to implement any partition P_{DM} of the space of $\{0,1\}^n$, which is the space of all possible signal realizations. Each pool of the partition consists of a subset of $\{0,1\}^n$, where each digit of the element of $\{0,1\}^n$ within a pool can be unambiguously matched to a particular expert. Thus, if every expert sends either message 0 or 1, then the set of all messages belongs to a unique element in some pool.

Moreover, in a direct mechanism each expert observes only his own private signal. In a network Q, however, he observes at least his own private signal. Therefore, according to Lemma 1, the bias range which supports the equilibrium strategy of an expert in a direct mechanism is weakly larger compared to the bias range supporting his equilibrium strategy in a network Q. Therefore, the outcome of a network Q is implementable in a direct mechanism.

Second, I show that if partition P_{DM} is implementable in a direct mechanism, it is not necessarily implementable in any network Q. The reason is that if the message profile of experts resulting in P_{DM} involves strategic coarsening of information, then there is at least one expert in Q who observes the signal of at least one other expert. This is not the case in a direct mechanism in which each expert only observes his own private signal. But then, according to Lemma 1, the range of biases which support the equilibrium strategy of an expert in Q is weakly smaller compared to the direct mechanism. Therefore, P_{DM} is not necessarily incentive compatible for at least one of the experts in Q. Q.E.D.

Characterizing equilibria in a tree network with 2 experts: From Proposition 1 we know the star network generates equilibria in which n' experts fully reveal their signals to the DM if $b_i \leq \frac{1}{2(n'+2)}$, $i \in \{1,2\}$. From Proposition 2 we know that an optimally designed line can transmit the same amount of information as the star network.

Line network gives rise to additional equilibria which strictly dominate DM's equilibrium payoffs in the star.

One of those equilibria is characterized by the partition $P_{DM} = \{\{0\},\{1,2\}\}$. The decisions of DM are $y(m_1) = \frac{1}{4}$ an $y(m_1') = \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{3}{4} = \frac{5}{8}$.

Denote the expert who is located at the bottom of the line by 2. Assume that expert 2 communicates truthfully to expert 1. The incentive constraint of expert 1 if $t_1 = 0$ is:

$$-\int_0^1 (\frac{1}{4} - \theta - b_1)^2 f(\theta|0,2)d\theta \geq -\int_0^1 (\frac{5}{8} - \theta - b_1)^2 f(\theta|0,2)d\theta.$$

If $t_1 = 1$, the incentive constraint for expert 1 implies:

$$-\int_0^1 (\frac{5}{8} - \theta - b_1)^2 f(\theta|1,2)d\theta \geq -\int_0^1 (\frac{1}{4} - \theta - b_1)^2 f(\theta|1,2)d\theta,$$

and if $t_1 = 2$:

$$-\int_0^1 (\frac{5}{8} - \theta - b_1)^2 f(\theta|2,2)d\theta \geq -\int_0^1 (\frac{1}{4} - \theta - b_1)^2 f(\theta|2,2)d\theta.$$

The above incentive constraints hold for

$$-\frac{1}{16} \leq b_1 \leq \frac{3}{16}.$$

Finally, assuming that expert 1 communicates according to the partition $P_1 = P_{DM}$, the incentive constraints for $t_2 = 0$ is determined by:

$$-\{\frac{2}{3}\int_0^1 (\frac{1}{4} - \theta - b_2)^2 f(\theta|0,2)d\theta + \frac{1}{3}\int_0^1 (\frac{5}{8} - \theta - b_2)^2 f(\theta|1,2)d\theta\} \geq$$

$$-\{\frac{2}{3}\int_0^1 (\frac{5}{8} - \theta - b_2)^2 f(\theta|0,2)d\theta + \frac{1}{3}\int_0^1 (\frac{5}{8} - \theta - b_2)^2 f(\theta|1,2)d\theta\}$$

and the incentive constraint for $t_2 = 1$ is determined by:

$$-\{\frac{2}{3}\int_0^1 (\frac{5}{8} - \theta - b_1)^2 f(\theta|1,2)d\theta + \frac{1}{3}\int_0^1 (\frac{5}{8} - \theta - b_1)^2 f(\theta|2,2)d\theta\} \geq$$

$$-\{\frac{2}{3}\int_0^1 (\frac{5}{8} - \theta - b_1)^2 f(\theta|1,2)d\theta + \frac{1}{3}\int_0^1 (\frac{5}{8} - \theta - b_1)^2 f(\theta|2,2)d\theta\}.$$

The above incentive constraints hold for

$$-\frac{1}{16} \leq b_1 \leq \frac{3}{16}.$$

The corresponding welfare for the DM is $\mathbb{E}U_{DM}(P_{DM}) = -0.052$.

Appendix C

Calculations for Chapter 3

C.0.1 Only one type of an expert reports truthfully

Only t_1 reports truthfully

If the DM observes $[0,1]$ reported from both experts, she assigns some belief that expert 1 is of t_1'. In this case she chooses

$$y' = \frac{1}{\left(1+\frac{1}{2}+v\right)2} + \frac{1}{4}\left(1 - \frac{1}{1+\frac{1}{2}+v}\right)(3-2v).$$

If she observes $m_1 = (0,1)$ and $m_2 = t_2$ then her policy choice is

$$y'' = \frac{1+2d}{\left(1+\frac{2d+2v}{1+2d}\right)4} + \frac{1}{2}\left(1 - \frac{1}{1+\frac{2d+2v}{1+2d}}\right)(1-v+d).$$

Thus, the expected utility of the DM is:

$$\mathbb{E}U_{DM}^{A} = -\frac{1}{4}\int_0^1 (y'-\theta)^2\,d\theta - \frac{1}{4}\left(\int_0^{\frac{1}{2}+d}(y''-\theta)^2\,d\theta + \int_{\frac{1}{2}+d}^1\left(\frac{1}{4}(3+2d)-\theta\right)^2 d\theta\right) -$$

$$\frac{1}{4}\left(\int_0^{\frac{1}{2}-v}\left(\frac{1}{4}(1-2v)-\theta\right)^2 d\theta + \int_{\frac{1}{2}-v}^1 (y'-\theta)^2\,d\theta\right) -$$

$$\frac{1}{4}\left(\int_0^{\frac{1}{2}-v}\left(\frac{1}{4}(1-2v)-\theta\right)^2 d\theta + \int_{\frac{1}{2}-v}^{\frac{1}{2}+d}(y''-\theta)^2\,d\theta + \int_{\frac{1}{2}+d}^1\left(\frac{1}{4}(3+2d)-x\right)^2 d\theta\right)$$

The incentives for all experts are derived in the appendix.

Only t_1' reports truthfully

If the DM receives (0,1) from both experts, she chooses:

$$y' = \frac{1}{\left(1+\frac{1}{2}-v\right)2} + \frac{1}{4}\left(1 - \frac{1}{1+\frac{1}{2}-v}\right)(1-2v),$$

and if she observes $m_1 = (0,1)$ and $m_2 = (0, \frac{1}{2}+d$, she chooses:

$$y' = \frac{1+2d}{\left(1+\frac{1-2v}{1+2d}\right)4} + \frac{1}{4}\left(1 - \frac{1}{1+\frac{1-2v}{1+2d}}\right)(1-2v)$$

The expected utility of the DM is:

$$\mathbb{E}U_{DM}^{B} = \frac{1}{4}\left(-\int_0^1 (y'-\theta)^2\,d\theta\right) - \frac{1}{4}\left(\int_0^{\frac{1}{2}+d} (y''-x)^2\,d\theta + \int_{\frac{1}{2}+d}^1 \left(\frac{1}{4}(3+2d)-\theta\right)^2 d\theta\right) -$$

$$\frac{1}{4}\left(\int_0^{\frac{1}{2}-v} (y'-\theta)^2\,dx + \int_{\frac{1}{2}-v}^1 \left(\frac{1}{4}(3-2v)-\theta\right)^2 d\theta\right) -$$

$$\frac{1}{4}\left(\int_0^{\frac{1}{2}-v} (y''-\theta)^2\,d\theta + \int_{\frac{1}{2}-v}^{\frac{1}{2}+d} \left(\frac{1}{2}(1+d-v)-\theta\right)^2 d\theta + \int_{\frac{1}{2}+d}^1 \left(\frac{1}{4}(3+2d)-\theta\right)^2 d\theta\right)$$

The incentives for the experts which report their signals truthfully are derived in the appendix.

Only t_2 reports truthfully

Of the DM receives reports $(0,1)$ from both experts, she chooses:

$$y' = \frac{1}{\left(1+\frac{1}{2}-d\right)2} + \frac{1}{4}\left(1 - \frac{1}{1+\frac{1}{2}-d}\right)(3+2d),$$

and if she receives $m_1 = (\frac{1}{2}-v, 1)$ and $m_2 = (0,1)$ then she chooses:

$$y'' = \frac{3-2v}{\left(1+\frac{1-2d}{1+2v}\right)4} + \frac{1}{4}\left(1 - \frac{1}{1+\frac{1-2d}{1+2v}}\right)(3+2d).$$

The expected utility of the DM is:

$$\mathbb{E}U^C_{DM} = -\frac{1}{4}\int_0^1 (y'-\theta)^2\,d\theta - \frac{1}{4}\left(\int_0^{\frac{1}{2}+d}\left(\frac{1}{4}(1+2d)-\theta\right)^2 d\theta + \int_{\frac{1}{2}+d}^1 (y'-\theta)^2\,d\theta\right) -$$

$$\frac{1}{4}\left(\int_0^{\frac{1}{2}-v}\left(\frac{1}{4}(1-2v)-\theta\right)^2 d\theta + \int_{\frac{1}{2}-v}^1 (y''-\theta)^2\,d\theta\right) -$$

$$\frac{1}{4}\left(\int_0^{\frac{1}{2}-v}\left(\frac{1}{4}(1-2v)-\theta\right)^2 d\theta + \int_{\frac{1}{2}-v}^{\frac{1}{2}+d}\left(\frac{1}{2}(1+d-v)-\theta\right)^2 d\theta + \int_{\frac{1}{2}+d}^1 (y''-\theta)^2\,d\theta\right)$$

The incentives for experts are relegated to the appendix.

Only t_2' reports truthfully

If the DM receives $(0,1)$ from both experts, she chooses

$$y' = \frac{3-2v}{\left(1+\frac{2d+2v}{1+2v}\right)4} + \frac{1}{2}\left(1-\frac{1}{1+\frac{2d+2v}{1+2v}}\right)(1+d-v),$$

and if she observes $m_1 = (\frac{1}{2}+v)$ and $m_2 = (0,1)$ then she chooses:

$$y'' = \frac{3-2v}{\left(1+\frac{2d+2v}{1+2v}\right)4} + \frac{1}{2}\left(1-\frac{1}{1+\frac{2d+2v}{1+2v}}\right)(1+d-v)$$

The expected utility of the DM is:

$$\mathbb{E}U^D_{DM} = -\frac{1}{4}\int_0^1 (y'-\theta)^2\,d\theta - \frac{1}{4}\left(\int_0^{\frac{1}{2}+d}(y'-\theta)^2\,d\theta + \int_{\frac{1}{2}+d}^1\left(\frac{1}{4}(3+2d)-\theta\right)^2 d\theta\right) -$$

$$\frac{1}{4}\left(\int_0^{\frac{1}{2}-v}\left(\frac{1}{4}(1-2v)-\theta\right)^2 d\theta + \int_{\frac{1}{2}-v}^1 (y''-\theta)^2\,d\theta\right) -$$

$$\frac{1}{4}\left(\int_0^{\frac{1}{2}-v}\left(\frac{1}{4}(1-2v)-\theta\right)^2 d\theta + \int_{\frac{1}{2}-v}^{\frac{1}{2}+d}(y''-\theta)^2\,d\theta + \int_{\frac{1}{2}+d}^1\left(\frac{1}{4}(3+2d)-\theta\right)^2 d\theta\right)$$

C.0.2 Centralization

Derivation of ICs if both experts report their signals truthfully

Assume $t_1 = [0, \frac{1}{2} - v]$. This type has the following posterior over the types of E_2: prob $\frac{1}{2}$ over $t_2 = [0, 1]$ and prob $\frac{1}{2}$ over $t_2 = [0, \frac{1}{2} + d]$. IC:

$$-\int_0^{\frac{1}{2}-v} (\frac{1-2v}{4} - \theta - b_1)^2 d\theta - \int_0^{\frac{1}{2}-v} (\frac{1-2v}{4} - \theta - b_1)^2 d\theta \geq$$

$$-\int_0^{\frac{1}{2}-v} (\frac{1}{2} - \theta - b_1)^2 d\theta - \int_0^{\frac{1}{2}-v} (\frac{1+2d}{4} - \theta - b_1)^2 d\theta.$$

It can be rewritten as:

$$\begin{aligned} &(\frac{1}{2} - \frac{1-2v}{4})(\frac{1}{2} + \frac{1-2v}{4} - 2\mathbb{E}_1(\theta|t_1, t_2) - 2b_1) \\ +((\frac{1+2d}{4} - \frac{1-2v}{4})((\frac{1+2d}{4} &+ \frac{1-2v}{4} - 2\mathbb{E}_1(\theta|t_1, t_2) - 2b_1) \geq 0 \end{aligned} \tag{C.1}$$

Equation (1) can be rewritten as:

$$1 + 4d^2 + 4v + 8dv + 8v^2 - 8b_1(1 + 2d + 4v) \geq 0 \tag{C.2}$$

or:

$$b_1 \leq \frac{1 + 4d^2 + 4v + 8dv + 8v^2}{8(1 + 2d + 4v)} \tag{C.3}$$

Notice that if $v = d = 0$ then $b_1 \leq \frac{1}{8}$.

Second, we look at ICs of $t_1 = [\frac{1}{2} - v, 1]$. The posterior beliefs over the types of E_2 are: prob $\frac{1}{2}$ over $t_2 = [0, 1]$ and prob $\frac{1}{2}(\frac{2d+2v}{1+2v})$ over $t_2 = [0, \frac{1}{2} + d]$, and $\frac{1}{2}(\frac{1-2d}{1+2v})$ over $t_2 = [\frac{1}{2} + d, 1]$. IC:

$$-\int_{\frac{1}{2}-v}^{1} (\frac{3-2v}{4} - \theta - b_1)^2 d\theta - (\frac{2d+2v}{1+2v}) \int_{\frac{1}{2}-v}^{\frac{1}{2}+d} (\frac{1-v+d}{2} - \theta - b_1)^2 d\theta \geq$$

$$-\int_{\frac{1}{2}-v}^{1} (\frac{1}{2} - \theta - b_1)^2 d\theta - (\frac{2d+2v}{1+2v}) \int_{\frac{1}{2}-v}^{\frac{1}{2}+d} (\frac{1+2d}{4} - \theta - b_1)^2 d\theta$$

The IC constraints can be rewritten as:

$$\Big(\frac{1}{2}-\frac{3-2v}{4}\Big)\Big(\frac{1}{2}+\frac{3-2v}{4}-2\mathbb{E}_1(\theta|t_1,t_2)-2b_1\Big)+$$
$$\frac{1-2d}{1+2v}\Big(\frac{1+2d}{4}-\frac{1-v+d}{2}\Big)\Big(\frac{1+2d}{4}+\frac{1-v+d}{2}-2\mathbb{E}_1(\theta|t_1,t_2)-2b_1\Big)\geq 0, \tag{C.4}$$

which can be rewritten as:

$$(1+8b_1-2v)(d-v-1)(2v-1)\geq 0. \tag{C.5}$$

Since $2v-1<0$ and $d-v-1<0$, the above condition can be rewritten as:

$$b_1\geq\frac{2v-1}{8}.$$

Now, we are studying the ICs of 2. We, first, look at $t_2=[0,\frac{1}{2}+d]$. The posterior beliefs over the types of E_1 are: prob $\frac{1}{2}$ over $t_1=[0,1]$ and prob $\frac{1}{2}(\frac{1-2v}{1+2d})$ over $t_1=[0,\frac{1}{2}-v]$ and prob $\frac{1}{2}(\frac{2d+2v}{1+2d})$ over $t_2=[\frac{1}{2}-v,1]$. IC:

$$-\int_0^{\frac{1}{2}+d}(\frac{1+2d}{4}-\theta-b_2)^2d\theta-(\frac{2d+2v}{1+2d})\int_{\frac{1}{2}-v}^{\frac{1}{2}+d}(\frac{1-v+d}{2}-\theta-b_2)^2d\theta\geq$$

$$-\int_0^{\frac{1}{2}+d}(\frac{1}{2}-\theta-b_2)^2d\theta-(\frac{2d+2v}{1+2d})\int_{\frac{1}{2}-v}^{\frac{1}{2}+d}(\frac{3-2v}{4}-\theta-b_2)^2d\theta$$

The IC can be rewritten as:

$$\Big(\frac{1}{2}-\frac{1+2d}{4}\Big)\Big(\frac{1}{2}+\frac{1+2d}{4}-2\mathbb{E}_2(\theta|t_1,t_2)-2b_2\Big)+$$
$$\frac{2d+2vn}{1+2d}\Big(\frac{3-2v}{4}-\frac{1-v+d}{2}\Big)\Big(\frac{3-2v}{4}+\frac{1-v+d}{2}-2\mathbb{E}_2(\theta|t_1,t_2)-2b_2\Big)\geq 0, \tag{C.6}$$

or

$$(2d-1)(8b_2+2d-1)(1+4d+2v)\geq 0. \tag{C.7}$$

Since $2d-1\leq 0$, the IC can be rewritten as:

$$b_2 \leq \frac{1-2d}{8}. \tag{C.8}$$

Finally, we look at $t_2 = [\frac{1}{2}+d, 1]$. The posterior beliefs over types of E_1 are: prob $\frac{1}{2}$ over $t_1 = [0,1]$ and prob $\frac{1}{2}$ over $t_1 = [\frac{1}{2}-v, 1]$. IC:

$$-\int_{\frac{1}{2}+d}^{1}(\frac{3+2d}{4}-\theta-b_2)^2 d\theta - \int_{\frac{1}{2}+d}^{1}(\frac{3+2d}{4}-\theta-b_2)^2 d\theta \geq$$
$$-\int_{\frac{1}{2}+d}^{1}(\frac{1}{2}-\theta-b_2)^2 d\theta - \int_{\frac{1}{2}+d}^{1}(\frac{3-2v}{4}-\theta-b_2)^2 d\theta$$

The IC can be rewritten as:

$$\begin{gathered}\Big(\frac{1}{2}-\frac{3+2d}{4}\Big)\Big(\frac{1}{2}+\frac{3+2d}{4}-2\mathbb{E}_2(\theta|t_1,t_2)-2b_2\Big)+\\ \Big(\frac{3-2v}{4}-\frac{3+2d}{4}\Big)\Big(\frac{3-2v}{4}+\frac{3+2d}{4}-2\mathbb{E}_2(\theta|t_1,t_2)-2b_2\Big)\geq 0\end{gathered} \tag{C.9}$$

or:

$$b_2 \geq \frac{-d(4+8v)-1-8d^2-4v^2}{1+4d+2v}.$$

One expert reports truthfully, and the other babbles

If only t_1 reports truthfully:

The IC for t_1 is:

$$\left(\text{y11}-\frac{1}{4}(1-2v)\right)\left(\text{y11}+\frac{1}{4}(1-2v)-\frac{2}{4}(1-2v)-2b\right)+$$
$$\left(\text{y21}-\frac{1}{4}(1-2v)\right)\left(\text{y21}+\frac{1}{4}(1-2v)-\frac{2}{4}(1-2v)-2b\right)=0,$$

for t_2 is:

$$(\text{y11}-\text{y21})\left(\text{y11}+\text{y21}-\frac{2}{4}(1+2d)-2b\right)+$$
$$\frac{(2d+2v)(\text{y11}-\text{y21})\left(\text{y11}+\text{y21}-\frac{2}{2}(1-v+d)-2b\right)}{1+2d}=0,$$

and for t_2' solves:

$$\text{y1} - \frac{1}{4}(3+2d) - 2b = 0.$$

If only t_1' reports truthfully:
IC for t_1':

$$\left(\text{y11a} - \frac{1}{4}(3-2v)\right)\left(\text{y11a} - \frac{1}{4}(3-2v) - 2b\right) +$$
$$\frac{(2d+2v)\left(\text{y21a} - \frac{1}{2}(1+d-v)\right)\left(\text{y21a} - \frac{1}{2}(1+d-v) - 2b\right)}{1+2v} = 0,$$

for t_2:

$$\left(1 + \frac{1-2v}{1+2d}\right)(\text{y11a} - \text{y21a})\left(\text{y11a} + \text{y21a} - \frac{2}{4}(1+2d) - 2b\right) +$$
$$\frac{(2d+2v)\left(\frac{1}{4}(3-2v) - \frac{1}{2}(1+d-v)\right)\left(\frac{1}{4}(3-2v) - \frac{1}{2}(1+d-v) - 2b\right)}{1+2d} = 0,$$

and for t_2':

$$\frac{1}{2}\left(\text{y11a} - \frac{1}{4}(3+2d)\right)\left(\text{y11a} - \frac{1}{4}(3+2d) - 2b\right) +$$
$$\frac{1}{2}\left(\frac{1}{4}(3-2v) - \frac{1}{4}(3+2d)\right)\left(\frac{1}{4}(3+2d) + \frac{1}{4}(3-2v) - \frac{2}{4}(3+2d) - 2b\right) = 0$$

If only t_2 reports truthfully:
The IC for t_1 solves:

$$\left(\text{y1b} - \frac{1}{4}(1-2v)\right)\left(\text{y1b} - \frac{1}{4}(1-2v) - 2b\right) +$$
$$\left(\frac{1}{4}(1+2d) - \frac{1}{4}(1-2v)\right)\left(\frac{1}{4}(1+2d) - \frac{1}{4}(1-2v) - 2b\right) = 0,$$

the IC for t_1' solves:

$$\left(1 + \frac{1-2d}{1+2v}\right)(\text{y1b} - \text{y2b})\left(\text{y1b} + \text{y2b} - \frac{2}{4}(3-2v) - 2b\right) +$$
$$\frac{(2d+2v)\left(\frac{1}{4}(1+2d) - \frac{1}{2}(1-v+d)\right)\left(\frac{1}{4}(1+2d) - \frac{1}{2}(1-v+d) - 2b\right)}{1+2v} = 0,$$

and the IC for t_2:

$$\left(\text{y1b} - \frac{1}{4}(1+2d)\right)\left(\text{y1b} - \frac{1}{4}(1+2d) - 2b\right) +$$

$$\frac{(2d+2v)\left(\text{y2b} - \frac{1}{2}(1-v+d)\right)\left(\text{y2b} - \frac{1}{2}(1-v+d) - 2b\right)}{1+2d} = 0$$

If only t_2' reports truthfully:

The IC for t_1 is:

$$y2a - (1-2v)/4 - 2b = 0,$$

the IC for ? is:

$$(\text{y2a} - \text{y2ab})\left(\text{y2a} + \text{y2ab} - \frac{2}{4}(3-2v) - 2b\right) +$$

$$\frac{(2d+2v)(\text{y2a} - \text{y2ab})\left(\text{y2a} + \text{y2ab} - \frac{2}{2}(1-v+d) - 2b\right)}{1+2v} = 0.$$

Lemma 1b: For every bias of expert j, there exists an equilibrium in which at least one informed type of j reports his signal truthfully.

Proof of Lemma 1b: Take an informed type of expert 1, t_1. The argument is similar for t_2. I show that for any number of informed types of expert 2, there exists an equilibrium in which t_1 reports truthfully if b_1 is below some positive upper bound, $b_1 < a$, $a > 0$.

Assume that $k \in \{0, 1, 2\}$ informed types of expert 2 report truthfully. The incentive constraints of t_1 to report his type truthfully to the DM are as follows:

$$-\frac{1}{2}\Big(\int_0^1 (y_1-\theta-b_1)^2 f(\theta|t_1, t_2 = [0,1])d\theta\Big) - \frac{1}{2}\Big(\int_0^{\frac{1}{2}+d} (y_2-\theta-b_1)^2 f(\theta|t_1, t_2 = [0, \frac{1}{2}+d])d\theta +$$

$$\int_{\frac{1}{2}+d}^1 (y_3-\theta-b_1)^2 f(\theta|t_1, t_2 = [\frac{1}{2}+d, 1])d\theta\Big) \geq$$

$$-\frac{1}{2}\Big(\int_0^1 (y_1'-\theta-b_1)^2 f(\theta|t_1, t_2 = [0,1])d\theta\Big) - \frac{1}{2}\Big(\int_0^{\frac{1}{2}+d} (y_2'-\theta-b_1)^2 f(\theta|t_1, t_2 = [0, \frac{1}{2}+d])d\theta +$$

$$\int_{\frac{1}{2}+d}^{1} (y_3' - \theta - b_1)^2 f(\theta|t_1, t_2 = [\frac{1}{2} + d, 1])d\theta\Big),$$

where y' stands for DM's policy once t_1 deviates to $[0, 1]$. Notice that y_1, y_2, y_3 are not necessarily distinct because in equilibrium there might be a pooling between the uninformed and an informed type of expert 2.

The above conditions can be rewritten as:

$$\sum_{t_2 \in T_2} (y_i' - y_i)(y_i' + y_i - 2\mathbb{E}_1(\theta|t_1, t_2) - b_1) \geq 0, \ i = 1, 2, 3.$$

Notice that $y_i' > y_i$ for any $i = 1, 2, 3$ because of t_1 deviates, then the DM puts less weight on lower states while forming expectations about θ. Furthermore, $y_i' > \mathbb{E}_1(\theta|t_1, t_2)$ because by deviation the expectation of θ by the DM assigns more weight to higher values of θ than by t_1 since the DM assigns positive beliefs to types going outside the range of $[0, \frac{1}{2} - v]$. Moreover, if t_1 reports truthfully, then the expectation of θ by the DM and t_1 coincide. Therefore, there exists $b' > 0$ such that:

$$\sum_{t_2 \in T_2} (y_i' - y_i)(y_i' + y_i - 2\mathbb{E}_1(\theta|t_1, t_2) - b') = 0, \ i = 1, 2, 3.$$

This b' is the upper bound for the range of biases of expert 1 which support truthtelling of t_1 in an equilibrium in which k informed types of expert 2, $k \in \{0, 1, 2\}$, report their signal truthfully.

Similarly it can be shown that independent of the number of informed types of player 2, equilibrium in which t_1' reports his signal truthfully is supported for a range of biases which are larger than some lower bound $b'' < 0$, where b'' depends on which types of expert 2 report their signals truthfully. The argument is symmetric for expert 2.

Combining those insights, we see that for any bias there always exists an equilibrium in which k informed types of expert j and at least one informed type of expert i reports his signal truthfully.

Proof of Lemma 2: Suppose not. Suppose that there is an equilibrium in which $k \in \{0, 1, 2\}$ informed types of expert i and no informed types of expert j report their signals truthfully. I show that at least one type of expert j has a profitable and incentive-compatible deviation.

First, notice that according to Lemma 1b (see above in the Appendix), for

any bias of expert j there exists an equilibrium in which at least one informed type of j reports his signal truthfully. Denote this type by $\hat{t}_j$. If $\hat{t}_j$ does not reveal his signal to the DM in equilibrium, then he expects $\sum_{t_i \in T_i} \mathbb{E}_j[(y_p - \theta - b_j)^2 | P(t_i, k)]$, where $P(t_i, k)$ is the partition of the interval $[0, 1]$ conditional on type t_i given that k informed types of expert i report their signal truthfully, and p is an element of the partition, $p \in P(t_i, k)$. If $\hat{t}_j$ deviates and reports his type truthfully, then the DM assigns belief 1 to $\hat{t}_j$ once she observes the signal $\hat{t}_j$ because information is verifiable. In this case, the expected utility of $\hat{t}_j$ is $\sum_{t_i \in T_i} \mathbb{E}_j[(y_{p'} - \theta - b_j)^2 | P'(t_i k)]$, where $P'(t_i, k)$ is a finer expected partition of $[0, 1]$ compared to $P(k)$ since the DM receives strictly more information once $\hat{t}_j$ reports his signal truthfully.

Since any expected utility for any given partition P can be rewritten as $\mathbb{E}_j[(y_p - \theta - b)^2 | P] = \mathbb{E}_j[(y_p - \theta)^2 | P] - b^2$, the comnparison between the two above expected utilities reveals that $\hat{t}_j$ benefits from deviating and reporting his type.

Conditions on v and d under which an equilibrium in which 3 out of 4 informed types of experts report truthfully dominates any other equilibrium under centralization:

Fix any equilibrium in which 3 out of 4 informed types of experts report their signals truthfully. Obviously, if a different equilibrium 2 informed types report truthfully, and those 2 types are a strict subset of 3 types which reported truthfully in the equilibrium mentioned above, then the new equilibrium is less informative. Thus, we have to check conditions under which an equilibrium with 2 truthtelling types dominates the equilibrium with 3 truthtelling types, where there is one types among 2 truthtelling types in the first equilibrium which is different from 3 truthtelling types in another equilibrium. The following cases are to consider:

1. Among all informed types, only t_1 and t_2' report truthfully. If the DM observes reports $[0, 1]$ from both experts, she chooses:

$$\frac{1}{\left(1 + \left(\frac{1}{2} + d\right) + \left(\frac{1}{2} + v\right) + \frac{\left(\frac{1}{2}+v\right)(2d+2v)}{1+2v}\right) 2} +$$

$$\frac{\left(\frac{1}{2} + v\right)(3 - 2v)\left(\frac{1}{2} + d\right)(1 + 2d)}{\left(1 + \left(\frac{1}{2} + d\right) + \left(\frac{1}{2} + v\right) + \frac{\left(\frac{1}{2}+v\right)(2d+2v)}{1+2v}\right) 4 \left(1 + \left(\frac{1}{2} + d\right) + \left(\frac{1}{2} + v\right) + \frac{\left(\frac{1}{2}+v\right)(2d+2v)}{1+2v}\right) 4} +$$

$$\frac{\left(\left(\frac{1}{2} + v\right)(2d + 2v)\right)(1 - v + d)}{\left((1 + 2v)\left(1 + \left(\frac{1}{2} + d\right) + 1 + \frac{2d+2v}{1+2v}\right)\right) 2}$$

The expected utility is in this case is:

$$\mathbb{E}U^I_{DM} = -\frac{1}{4}\int_0^1 (y-x)^2\,dx - \frac{1}{4}\left(\int_0^{\frac{1}{2}-v}\left(\frac{1}{4}(1-2v)-x\right)^2 dx + \int_{\frac{1}{2}-v}^1 (y-x)^2\,dx\right) -$$

$$\frac{1}{4}\left(\int_0^{\frac{1}{2}+d}(y-x)^2\,dx + \int_{\frac{1}{2}+d}^1\left(\frac{1}{4}(3+2d)-x\right)^2 dx\right) -$$

$$\frac{1}{4}\left(\int_0^{\frac{1}{2}-v}\left(\frac{1}{4}(1-2v)-x\right)^2 dx + \int_{\frac{1}{2}-v}^{\frac{1}{2}+d}(y-x)^2\,dx + \int_{\frac{1}{2}+d}^1\left(\frac{1}{4}(3+2d)-x\right)^2 dx\right)$$

Condition 1: is strictly lower than the expected utility in equilibrium in which t_1, t_1' and t_2' report truthfully:

$$\mathbb{E}U^I_{DM} < \mathbb{E}U^D_{DM}.$$

Condition 2: is strictly lower than the expected utility in equilibrium in which t_1', t_2 and t_2' report truthfully:

$$\mathbb{E}U^I_{DM} < \mathbb{E}U^B_{DM}.$$

2. Among all informed types, only t_1 and t_2 report truthfully. If the DM observes reports $[0,1]$ from both experts, she chooses:

$$y = \frac{1}{\left(1+\left(\frac{1}{2}-d\right)+\left(\frac{1}{2}+v\right)+\left(\frac{1}{2}-d\right)\right)2} +$$

$$\frac{\left(\frac{1}{2}-d\right)(3+2d)}{\left(1+\left(\frac{1}{2}-d\right)+\left(\frac{1}{2}+v\right)+\left(\frac{1}{2}-d\right)\right)4} +$$

$$\frac{\left(\frac{1}{2}+v\right)(3-2v)}{\left(1+\left(\frac{1}{2}-d\right)+\left(\frac{1}{2}+v\right)+\left(\frac{1}{2}-d\right)\right)4} + \frac{\left(\frac{1}{2}-d\right)(3+2d)}{\left(1+\left(\frac{1}{2}-d\right)+\left(\frac{1}{2}+v\right)+\left(\frac{1}{2}-d\right)\right)4}$$

If the DM observes $[0,1]$ from expert 1 and $[0,\frac{1}{2}+d]$ from expert 2, she chooses:

$$y' = \frac{1+2d}{\left(1+\frac{2d+2v}{1+2d}\right)4} + \frac{(2d+2v)(1-v+d)}{\left((1+2d)\left(1+\frac{2d+2v}{1+2d}\right)\right)2}$$

The expected utility is in this case is:

$$\mathbb{E}U_{DM}^{II} = -\frac{1}{4}\int_0^1 (y-x)^2\,dx - \frac{1}{4}\left(\int_0^{\frac{1}{2}-v}\left(\frac{1}{4}(1-2v)-x\right)^2 dx + \int_{\frac{1}{2}-v}^1 (y-x)^2\,dx\right) -$$

$$\frac{1}{4}\left(\int_0^{\frac{1}{2}+d}(y'-x)^2\,dx + \int_{\frac{1}{2}+d}^1 (y-x)^2\,dx\right) -$$

$$\frac{1}{4}\left(\int_0^{\frac{1}{2}-v}\left(\frac{1}{4}(1-2v)-x\right)^2 dx + \int_{\frac{1}{2}-v}^{\frac{1}{2}+d}(y'-x)^2\,dx + \int_{\frac{1}{2}+d}^1 (y-x)^2\,dx\right)$$

Condition 3: is strictly lower than the expected utility in equilibrium in which t_1', t_2 and t_2' report truthfully:

$$\mathbb{E}U_{DM}^{II} < \mathbb{E}U_{DM}^{B}.$$

Condition 4: is strictly lower than the expected utility in equilibrium in which t_1, t_1' and t_2' report truthfully:

$$\mathbb{E}U_{DM}^{II} < \mathbb{E}U_{DM}^{D}.$$

3. Among all informed types, only t_1' and t_2' report truthfully. If the DM observes reports $[0,1]$ from both experts, she chooses:

$$y = \frac{1}{\left(1+\left(\frac{1}{2}+d\right)+\left(\frac{1}{2}-v\right)+\left(\frac{1}{2}-v\right)\right)2} +$$

$$\frac{\left(\frac{1}{2}+d\right)(1+2d)}{\left(1+\left(\frac{1}{2}+d\right)+\left(\frac{1}{2}-v\right)+\left(\frac{1}{2}-v\right)\right)4} +$$

$$\frac{\left(\frac{1}{2}-v\right)(1-2v)}{\left(1+\left(\frac{1}{2}+d\right)+\left(\frac{1}{2}-v\right)+\left(\frac{1}{2}-v\right)\right)4} + \frac{\left(\frac{1}{2}-v\right)(1-2v)}{\left(1+\left(\frac{1}{2}+d\right)+\left(\frac{1}{2}-v\right)+\left(\frac{1}{2}-v\right)\right)4}$$

If she observes $[\frac{1}{2}-v, 1]$ from expert 1 and $[0,1]$ from expert 2 then she chooses:

$$y' = \frac{1}{\left(1+\frac{2d+2v}{1+2v}\right)2} + \frac{1}{2}\left(1 - \frac{1}{1+\frac{2d+2v}{1+2v}}\right)(1-v+d)$$

The expected utility is in this case is:

$$\mathbb{E}U_{DM}^{III} = -\frac{1}{4}\int_0^1 (y-x)^2\,dx - \frac{1}{4}\left(\int_0^{\frac{1}{2}-v} (\text{ya}-x)^2\,dx + \int_{\frac{1}{2}-v}^1 (y'-x)^2\,dx\right) -$$

$$\frac{1}{4}\left(\int_0^{\frac{1}{2}+d} (y-x)^2\,dx + \int_{\frac{1}{2}+d}^1 \left(\frac{1}{4}(3+2d)-x\right)^2\,dx\right) -$$

$$\frac{1}{4}\left(\int_0^{\frac{1}{2}-v} (y-x)^2\,dx + \int_{\frac{1}{2}-v}^{\frac{1}{2}+d} (y'-x)^2\,dx + \int_{\frac{1}{2}+d}^1 \left(\frac{1}{4}(3+2d)-x\right)^2\,dx\right)$$

Condition 5: is strictly lower than the expected utility in equilibrium in which t_1, t_2 and t_2' report truthfully:

$$\mathbb{E}U_{DM}^{III} < \mathbb{E}U_{DM}^{A}.$$

Condition 6: is strictly lower than the expected utility in equilibrium in which t_1, t_1' and t_2' report truthfully:

$$\mathbb{E}U_{DM}^{III} < \mathbb{E}U_{DM}^{D}.$$

4. Among all informed types, only t_1' and t_2 report truthfully. If the DM observes reports $[0,1]$ from both experts, she chooses:

$$y = \frac{1}{\left(1+\left(\frac{1}{2}-d\right)+\left(\frac{1}{2}-v\right)\right)2} + \frac{\left(\frac{1}{2}-d\right)(3+2d)}{\left(1+\left(\frac{1}{2}-d\right)+\left(\frac{1}{2}-v\right)\right)4} +$$

$$\frac{\left(\frac{1}{2}-v\right)(1-2v)}{\left(1+\left(\frac{1}{2}-d\right)+\left(\frac{1}{2}-v\right)\right)4}$$

If she observes $[0,1]$ from expert 1 and $[0,\frac{1}{2}+d]$ from expert 2 then she chooses:

$$y' = \frac{1+2d}{\left(1+\frac{1-2v}{1+2d}\right)4} + \frac{(1-2v)(1-2v)}{\left((1+2d)\left(1+\frac{1-2v}{1+2d}\right)\right)4}$$

The expected utility is in this case is:

$$\mathbb{E}U_{DM}^{IV} = -\frac{1}{4}\int_0^1 (\text{y11}-x)^2\,dx - \frac{1}{4}\left(\int_0^{\frac{1}{2}-v} (\text{y11}-x)^2\,dx + \int_{\frac{1}{2}-v}^1 (\text{y13}-x)^2\,dx\right) -$$

$$\frac{1}{4}\left(\int_0^{\frac{1}{2}+d}(\text{y12}-x)^2\,dx+\int_{\frac{1}{2}+d}^{1}(\text{y11}-x)^2\,dx\right)-$$

$$\frac{1}{4}\left(\int_0^{\frac{1}{2}-v}(\text{y12}-x)^2\,dx+\int_{\frac{1}{2}-v}^{\frac{1}{2}+d}\left(\frac{1}{2}(1-v+d)-x\right)^2\,dx+\int_{\frac{1}{2}+d}^{1}(\text{y13}-x)^2\,dx\right)$$

Condition 7: is strictly lower than the expected utility in equilibrium in which t_1, t_2 and t_2' report truthfully:

$$\mathbb{E}U_{DM}^{IV} < \mathbb{E}U_{DM}^{A}.$$

Condition 8: is strictly lower than the expected utility in equilibrium in which t_1, t_1' and t_2' report truthfully:

$$\mathbb{E}U_{DM}^{IV} < \mathbb{E}U_{DM}^{D}.$$

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