# Essays on Fiscal Policy

by

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**Thesis**

Submitted to the University of Warwick

for the degree of

**Doctor of Philosophy**

**Department of Economics**

September 2015

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# Acknowledgments

I would like to thank my supervisors H. Polemarchakis and Thijs van Rens for their encouragement and guidance, Roberto Pancrazi for his comments and the University of Warwick for the financial support and the excellent research atmosphere .

# Abstract

High levels of either public debt or wealth inequality are detrimental to social and economic stability. At a time when reducing public debt and decreasing wealth inequality have become important policy priorities, the question arises about whether these two goals stand in conflict. With this in mind, Chapter 1 assesses the effects of public debt on wealth inequality based on an analytically tractable model of heterogeneous agents. Its scope, in particular, is to investigate whether a reduction of public debt or of budget deficits in general might amplify or not the levels of wealth inequality. In answering this question, we explore a novel channel where, for example, a reduction in budget deficits amplifies wealth inequality due to the change in factor prices, and in particular that of interest rates. Therefore, and besides that our research is the first to explore this type of question, our main contribution is that we show how a change in public debt can affect wealth inequality in an implicit way through the change in factor incomes - that is, the general equilibrium effects.

In Chapter 2, on the other hand, we study the design of policies within an endogenous growth model of incomplete markets and partial commitment. Markets are incomplete in two dimensions, the government cannot insure itself from the presence of aggregate risk, and the accumulation of human capital is subject to idiosyncratic risk. Our primary contribution highlights the importance of human capital to effectively manage the economy along the cycle. More specifically, we make a novel argument: taking short run risks are effective responses to a shock that might depress the economy. An investment in human capital which is subject to idiosyncratic risk, serves that purpose. Its returns however, must be protected over-time through an effective provision of liquidity and manipulation of taxes. In our case this policy requires to subsidise physical capital and tax human capital, while the government must own assets.

Finally, In Chapter 3 we estimate the fiscal multipliers for Greece. In particular, using the SVAR approach of Blanchard and Perotti we estimate the dynamic effects of government spending and tax revenues on output. The results over the available sample indicate some strong Keynesian effects. That is government spending multipliers are large while the tax multipliers are relatively small. However the conclusions are confined to the peculiarities of the available sample and are not easily exportable to alternative periods or allow any generalizations.

# Chapter 1

# Government debt and Wealth Inequality

## 1.1 Introduction

The aftermath of the financial crisis provoked fiscal consolidation plans with unknown distributive outcomes. Debt adjustments, in particular, become an important priority for some countries and this comes at a time where concerns over the rise in wealth inequality are growing. In parallel, the distributional aspects of debt policies have other prominent macroeconomic implications only recently discovered. For example, Bhandari et al. (2013) warn that the wealth inequality - or the distribution of public debt in particular - matters for the growth implications that high levels of public debt might have[1]. Besides, wealth inequality may, for example, be a source of inefficient fluctuations (Ghiglino and Venditti (2011)) or a factor affecting the risk premium (Gollier (2001)). Accordingly, debt policies might undermine or accelerate these risks through its effect on the distribution of wealth.

Motivated by such concerns, our main objective is to study the effects of government debt on the inequality of wealth. Thus our paper contributes into a particular aspect of fiscal policy that, surprisingly, remained relatively unexplored in the literature. There is a natural relationship linking debt with wealth and this comes from the savings behaviour of households. Individuals save in this asset and earn interest, but in the same time public debt becomes a burden that needs to be repaid (i.e. is a deferred tax)[2]. Therefore, the manipulation of government debt by fiscal authorities directly affects the accumulation of assets and by implication the inequality of wealth.

To keep things simple, we develop a “hybrid” version of Diamond (1965) model with altruistically linked families. However our assumption on altruism is that parents enjoy utility directly from bequests. The novelty of this approach compromises between the savings motive of a Ramsey and a Diamond economy. That is the altruism that exists

[1]The aftermath of the financial crisis triggered a fierce public and academic debate on the effects of debt on growth. The research of Reinhart and Rogoff (2010) further accelerated that particular debate. For a survey see Panizza and Presbitero (2013)

[2]In the US for example, currency and bonds holding amount to more than 200% of national income in the composition of household wealth[3].

in a Ramsey (infinite horizon) model is reconciled with the life-cycle consideration of a Diamond model. Finally, in order to facilitate comparisons, either for countries of different legal systems or across distinct tax instruments, we consider separately an extremely "Regressive", an extremely "Progressive" and an "Affine" tax system. The latter combines elements of the former two and is a good approximation of the US tax system[4].

Turning in our results, we consider as our measure of wealth inequality the coefficient of variation. Therefore, any change in debt induces a "mean" and a "variance" effect. At the macroeconomic level an increase in debt will crowd out capital and when the latter is sufficiently strong, *average* wealth will fall. For given savings this implies that inequality tends to *increase*. But, the increase in debt will also raise the tax burden and affect the interest rate. Savings responses then will determine the asset position among individuals and thus the change in variance. This in turn implies that inequality tends to *decrease*. In equilibrium, the drop in variance dominates the fall in the mean and therefore inequality is suppressed. Interestedly, this qualitative effect on inequality survived across all tax systems.

At the microeconomic level, as soon as debt is conceived as deferred taxation, different tax instruments coupled with the bequest transfer, constitute a composite transfers scheme with the usual income and substitution effects (Polemarchakis (1983); Galor and Polemarchakis (1987)). This insight affects the altruism of individuals and in consequence their saving behaviour. In this point, we analytically prove that savings behaviour of the "poor" is ambiguous. The latter property is what might make the effects of debt on wealth inequality also ambiguous. More specifically, the uncertain effects on the variability of savings ("variance effect"), might be reasonably attributed to the behaviour of that fraction of the population.

The significance of this particular result conveys an important message. In our model the source of the heterogeneity comes through the differences in the initial wealth, preferences are identical and market frictions are absent. Hence the possibility that individual might behave differently it suggests that ex-ante heterogeneity or more generally the distribution of wealth matters.

The rest of the chapter makes the previous points more rigorous and is organized as follows; In Section 1.3 the details of the model are set-up. In Section 1.4, the measure of wealth inequality is constructed and discussed. Section 1.5 focuses on the macroeconomic environment. Section 1.6 analyses the effects on wealth inequality, while Section 1.7 presents the main results.

## 1.2 Related Literature

Questions on the distributional role of public policies hitherto focused on the implications across generations, as in Romer (1988) or Altig and Davis (1989) among others, whereas

[4] See Bhandari et al. (2013).

this paper examines the effects across the population [5]. In general, and to the best of our knowledge our paper, if not the first, is among the very few that concerned with the implications of debt policies for the inequality of wealth. Closely related papers include the one of Floden (2001) and that of Röhrs and Winter (2013). Both papers are quantitatively orientated and rely on incomplete markets models (in the tradition of Aiyagari (1994)) to analyse the welfare properties of public debt. Therefore, their scope differs from ours significantly.

The first paper pins down specific combinations of optimal transfers and debt levels and examines, among other things, the change in wealth inequality when at a particular combination. But as in this paper, Floden (2001) also emphasizes the negative relationship between public debt and wealth inequality. Regarding the latter paper of Röhrs and Winter (2013), its primary concern is to weight the relative (welfare) merits of debt adjustments in cases where the distribution of wealth is highly unequal. In particular, they are evaluating how a reduction of debt affect (in terms of welfare) different groups of people along the wealth distribution.

In another related study, Heathcote (2005) examines the effects of fiscal policy (tax shocks) when agents face uninsurable idiosyncratic risk and compares his results against the first best[6]. An argument in his paper, highlights the "Ricardian" behaviour of the poor. As we will show in the main analysis, the model here can equally replicate this theoretical possibility (See Section 1.6.2 ). Finally, Azzimonti et al. (2012) address the role of financial liberation in the expansion of both public debt and income inequality.

## 1.3 The Model

The main environment modifies that of Diamond (1965) into two dimensions. First, we introduce heterogeneity in labour or "ability" endowments and second allow individuals to have a "joy-of-giving" bequest motive. Altonji et al. (1997) claim that the particular bequest motive might be more relevant in practice[7]. The present procedure also compromises, in a simple and tractable way, the two extremes between a Ramsey and a Diamond economy. In the first kind of economy, any life-cycle considerations are nullified while the latter abstracts from bequests which are clearly relevant in practise. Although the details of the wealth accumulation and of the main results will be discussed in later sections, it is instructive at this stage to point out that our findings can still go through even if one omits altruism. Bequests, however, allows a wealth accumulation process that is consistent with the literature and therefore we choose to follow this convention. In this respect, our framework follows the one developed by Bossmann et al. (2007), but with some variations.

[5] On the interaction of public debt with bequests, prominent examples are Barro (1974), Laitner (1979), Drazen (1978) or Burbidge (1983). However those papers do not discuss distributional issues.

[6] Within the general topic of Fiscal Policy, see also Garcia and Turnovsky (2007) or Alonso-Carrera et al. (2012) for the effects on income inequality.

[7] There is a consensus among the profession, over the presence of a bequest motive. For a survey see Laitner and Ohlsson (2001) or Light and McGarry (2003). For the "joy-of-giving" bequest motive as a reduced form specification of altruism see Abel and Warshawsky (1987).

The demand side of the economy consists of a large number of consumers born with different endowments. In particular, consumer $i$ of generation $t$ who lives in period $t$ maximizes lifetime utility by choosing consumption $c_{it}^t$ when young, consumption $c_{it+1}^t$ when old and bequests $x_{it+1}$ to transfer to his "son". In the notation, the superscript is the generation index and the subscript denotes calendar time. Total savings $a_{it+1}^t$ of the young, determined in period $t$, are allocated between government bonds and capital. Bonds to be purchased have to pay the same interest rate as capital, making the portfolio composition indetermined. Moreover, agents differ in labour endowments $l_i$, which supply inelastically. This is the first source of heterogeneity in this model and it is assumed to be exogenous. The second one, which is only implicit, is the bequest each "young" is endowed with[8]. Consumers then solve the following maximization problem, where it is assumed that preferences are time separable.

$$\max_{c_{it}^t, c_{it+1}^t, x_{it+1}} V_t^i = U(c_{it}^t) + \beta U(c_{it+1}^t) + \delta U(x_{it+1}) \tag{1.1}$$

s.t

$$c_{it}^t + a_{it+1}^t = D_t^i \tag{1.2}$$

$$c_{it+1}^t + (1+n)x_{it+1} = (1 + r_{t+1})a_{it+1}^t \tag{1.3}$$

$$c_{it}^t > 0, \;\; c_{it+1}^t > 0, \;\; x_{it+1} > 0$$

The utility function is standard homothetic with the usual neoclassical assumptions. Equations (1.2) and (1.3) are the budgets constraint in the two periods of life. In the notation, $D_t^i$ is the disposable income of the young and $n$ is the population growth. The components of disposable income include the after tax wage and the transfer received by the "parent". The degree of altruism, $\delta$ is assumed to be lower than the discount factor $\beta$, with $0 < \delta < \beta < 1$. Thus, the "parent" values his own consumption greater. The first order conditions, assuming bequests are operative, are[9]:

$$(1+n)\beta U'(c_{it+1}^t) = \delta U'(x_{it+1}) \tag{1.4}$$

$$U'(c_{it}^t) = \beta(1 + r_{t+1})U'(c_{it+1}^t) \tag{1.5}$$

For simplicity, we also assume the distribution of labour endowments to be constant over time with the mean normalized to 1 and some variance $\sigma^2$. Someone could justify this assumption based on recent research by Huggett et al. (2011). Their findings suggest that for life-time inequality in wealth and welfare, *ex-ante* heterogeneity is far more important than differences in luck (that is, *ex-post* heterogeneity). Alternatively, the assumption on the distribution of labour endowments, can also be thought as equivalent to an environment of a fixed intergenerational mobility, which it seems to

[8]It is also assumed the "initial old" differ in their assets. The maximization problem then will only involve the consumption and bequests choices.

[9]The Inada assumptions for preference ensure that bequest will always be operative

be consistent with the US data (See Chetty et al. (2014)).

The optimum plans, when the distribution of "skills" is constant and preferences are homothetic, admit solutions in "Gorman Form" (that is,"linear in wealth"). Accordingly, the optimum plans for savings and bequests are:

$$a^t_{it+1} = S_D D^i_t = S_D \left[ I^i_t + x^i_t \right] \equiv S_t(D^i_t, r_{t+1}) \tag{1.6}$$

$$x^i_{t+1} = X_S a^t_{it+1} \equiv X(r_{t+1}, a^t_{it+1}) \tag{1.7}$$

As usual, $r_{t+1}$ is the rate of interest in period $t+1$, $I^i_t$ is the after tax wage, while $S(D^i_t, r_{t+1})$ and $X(r_{t+1}, a^t_{it+1})$ are the optimal savings and bequests functions respectively. In their "Gorman form" counterparts shown in (1.6) and (1.7), $S_D$ denotes the marginal propensity to save (MPS) out of disposable income, while $X_s > 0$ is the marginal propensity to bequeath out of savings (MPB). In Section 1.5, we will use the inequalities $0 < S_D, \overline{X_S} < 1$, where $\overline{X_S} \equiv \frac{(1+n)X_s}{(1+r_{t+1})}$ describes the *modified* or the *dynamic efficiency adjusted* MPB[10].

On the supply side, markets are competitive and capital depreciates fully within a period. We also assume that a exists a representative firm, which uses capital and labour to produce output. The production function is standard neoclassical with common assumptions. From profits maximization, factor prices in their intensive form equal to:

$$w_t = f(k_t) - f'(k_t)k_t \tag{1.8}$$

$$1 + r_t = f'(k_t) > 0 \qquad \text{with} \qquad f(k_t)'' < 0 \tag{1.9}$$

Moreover it is assumed that exist a government which in period $t$ issues bonds, $B_{t+1}$, and collects taxes $\mathcal{T}_t$. The government uses its revenues to repay interest on the previously issued bonds, $B_t$. Thus, debt substitutes taxes to finance some given path of (unproductive) government expenses. In the text, we will analyse three different cases in terms of the available tax instrument; A flat (or "Regressive"), a proportional (or "Progressive") and an affine tax instrument. The latter, as Bhandari et al. (2013) argue, it approximates better the tax system in the US. In that case, the analysis on "Flat" or "Proportional" tax systems will become instrumental to the "Affine tax" economy for reasons we will discuss later. Nevertheless, flat or more progressive tax systems alone are also quite common in practice (See Keen et al. (2012) for countries with "Flat tax" systems). Finally, it is assumed that all taxes fall on "labour" or the "young" generation. For each case, total tax revenues equal:

$$\mathcal{T}_t = \left[ \tau^f_t N_t \ \text{ or } \ \tau^p_t w_t \sum_{i=1}^{N_t} l^i \ \text{ or } \ \tau^A_t w_t \sum_{i=1}^{N_t} l^i - T^A_t N_t \right]$$

where $N_t$ is the population size in period $t$, $\tau^F_t$ the lump-sum tax in "flat tax"

[10] To ease the notation, omit to make explicit the dependence of MPS and MPB on interest rates and hence on the time period. For the properties of the savings and bequest functions see On-line Appendix.

economy, $\tau_t^p$ the marginal tax rate in the "Progressive tax" system and $\tau_t^A$ and $T_t^A$ are the equivalent tax instruments for the "Affine tax" regime. In summary, government's budget constraint in absolute terms equals:

$$B_{t+1} + \mathcal{T}_t = (1 + r_t)B_t$$

Assuming constant debt per capita policy (and taking into consideration that for large economies averages converge to their means), the per capita cost for financing this policy in each case is:

$$\tau_t^F = (r_t - n)b \tag{1.10}$$

$$\tau_t^P = \frac{(r_t - n)b}{w_t} \tag{1.11}$$

$$T_t^A = (r_t - n)b - \tau_t^A w_t \tag{1.12}$$

where $b \equiv \frac{B_{t+1}}{N_{t+1}} = \frac{B_t}{N_t}$, is debt per capita. In all cases, the model assumes that debt generates positive savings. In the event of a debt shock and the subsequent permanent change in disposable income, we keep the terminology in the literature and refer to it as "wealth effects" (Baxter and King (1993)).

For the affine tax economies there are two degrees of freedom: The government to maintain its policy can either adjust its flat tax component or the marginal tax rate. The specification in (1.12) implies the first to occur. If instead, the option was on the marginal tax rate, results would follow that of the proportional tax system[11].

Another peculiarity that emerges in an "Affine" tax system is that individuals, for given endowments, are segregated between those who pay taxes and those who receive subsides. To see this, note that the after tax wages in this case are $w_t l^i - [(r_t - n)b + \tau_t^A w_t(l^i - 1)]$. Therefore, if the economy is dynamic inefficient, only those with endowment $l^i < 1$ are subsidised; Whether the rest will be paying taxes or not, depend on the degree of dynamic inefficiency and the marginal tax rate. On the other hand, if the economy is dynamic efficient, as we can see from equation (1.12), the $T^A$ term can be either positive or negative. This depends, in turn, on whether debt repayments are "high" or not relative to the revenues collected from wages. As previously, some "poor" ($l^i < 1 - \frac{(r_t - n)b}{\tau^A w_t}$) are subject to negative taxation (that is "implicit welfare benefits"). Nevertheless, none of those "discriminating" features will play any role for the qualitative pattern of the results.

## 1.4 Wealth Accumulation and Wealth Inequality

Bequests facilitate the intergenerational link for the transmission of wealth among families. Substituting (1.7) in (1.6), wealth accumulation takes the following form:

[11]See footnote 25 in Section 1.6 and the discussion therein.

$$a_{it+1}^{t} = S_D[I_t^i + X_s a_{it}^{t-1}] \Rightarrow a_{it+1}^{t} = S_D I_{it} + \underbrace{S_D X_s}_{=C_{4t}} a_{it}^{t-1} \tag{1.13}$$

Hence, the individual wealth (1.13) shows that is a first order, and non-homogeneous, difference equation in asset holdings, $a_{it}$. Its exact form, will depend on the three different tax structures, the after tax wage equals

$$I_t^i = \left[ w_t l_i - \tau_t^F, \quad (1-\tau_t^P) w_t l_i, \quad (1-\tau_t^A) w_t l_i - T_t^A \right]$$

.

Equation (1.13) describes the transmission process of wealth accumulation. Its intuition is very simple: The wealth of a family (or "Dynasty") is the sum of the own wealth plus any financial wealth left by the predecessor in form of transfers. In the previous period, part $(1 - S_D)$ of transfers were consumed and the rest $(S_D)$ were saved. From that financial stock, a certain amount was kept for consumption and the rest, $X_S$, passed on to the offspring. Thus, $S_D X_S$ reflects the fraction from total savings attributed to altruism. This component, along with own wealth, determines the available asset position in the current period.

It follows that in the absence of any willingness to bequeath, initial wealth levels persist forever, whereas some degree of altruism allows individual wealth to expand[12], but only under the assumption that debt cannot affect the coefficient of correlation between generations. Therefore, unless this degree is not too high (reflected in $X_S$), wealth accumulation might become explosive. In (1.13), a sufficient condition for mean-reversion is $C_{4t} \equiv S_D X_s < 1$. However, as soon as $X_S$ depends on the interest rate, debt policies become critical for the existence of a stationary state in the *microeconomic* level. Nonetheless, as we will show in Section 1.5, a more stringent condition is needed for the *macroeconomic or financial stability*[13]. In this respect, our paper is also related to Rankin and Roffia (2003) and it is interesting to extent the discussion, on the maximum level of debt that is possible to finance a given level of inequality. At present is it useful to clarify that although in our paper we do not explore sustainability issues, the implicit assumption is that the level of debt is such that a macroeconomic steady state always exists.

For later, is it useful to define the ratio $\frac{S_D}{1-S_D X_S}$. The numerator $(S_D)$ is the fraction of savings attributed to the *gross* life-cycle considerations; that is, to finance future expenditures. The denominator $(1 - S_D X_S)$ is the fraction of own investments allocated to personal consumption. Then, the ratio of these two can define the *net* life-cycle component of savings; that is, the part of total wealth invested to finance future consumption *net of bequests*. We call this ratio as the *egoistic MPS*[14]. In the subsequent analysis, this fraction will interact with the wage rate, giving a very natural meaning in the event of a shock. In fact, any perturbation on the $\frac{S_D w}{1-S_D X_S}$ term will reflect a

[12] Of course the ranking of individuals within the distribution remains the same. Nothing would have changed if we have allowed for a social mobility as in Bossmann et al. (2007)

[13] While $S_D$ might not depend on interest rates (e.g with log utility), the MPB always does.

[14] The term is inspired by Laitner and Ohlsson (2001).

"battle" between the own life-cycle and the altruistic considerations.

### 1.4.1 Stationary/Steady State wealth inequality

A tractable measure of inequality frequently used in the literature and the one also employed in Bossmann et al. (2007) is the coefficient of variation, $CV = \sqrt{\frac{Var(a_{it})}{E(a_{it})}}$. In this definition, $E(a_{it})$ is the mean wealth and $Var(a_{it})$ the variance. To construct the steady state measure of coefficient of variation, we use the law of motion in equation (1.13), and impose the steady state conditions - assuming $C_4 < 1$ - $\mathrm{E}(a_{it+1}) = \mathrm{E}(a_{it}) = \mathrm{E}(a_i)$, $\mathrm{Var}(a_{it+1}) = \mathrm{Var}(a_{it}) = \mathrm{Var}(a_i)$, and we substitute for taxes from the government budget constraint. Simple algebra then shows that average or mean wealth in the steady state is:

$$\mathrm{E}(a_i) = \frac{S_D[w-(r-n)b]}{1-S_D X_s} \tag{1.14}$$

Whereas, the steady state variance of wealth for each case of taxes is:

$$\mathrm{Var}^{\mathrm{P}}(a_i) = \frac{\Big[S_D[w-(r-n)b]\Big]^2 \sigma^2}{1-(S_D X_s)^2} \tag{1.15}$$

$$\mathrm{Var}^{\mathrm{F}}(a_i) = \frac{(S_D w)^2 \sigma^2}{1-(S_D X_s)^2} \tag{1.16}$$

$$\mathrm{Var}^{\mathrm{A}}(a_i) = (1-\tau^A)^2 \mathrm{Var}^{\mathrm{F}}(a) \tag{1.17}$$

where in the cases above, we made use of equations (1.10), (1.11) and (1.12) respectively. Note that in the "Progressive" tax system and in contrast to the other ones, debt policies affect the dispersion of wealth directly (equation (1.15)). Second, an "Affine" tax instrument only rescales the variance of a flat tax economy (equation (1.17)). Therefore, the choice for the marginal tax rate only parametrizes the level of inequality and it is irrelevant for the qualitative effects[15]. Third, since the mean for all these economies is the same by construction, to compare the levels of wealth inequality between them is just sufficient to compare the variances. It turns out that if the economy is dynamic inefficient ($r < n$), the "Progressive" tax system will produce more wealth inequality. This seems intuitive, since the rich are more heavily subsidised. Note also that the wealth dispersion in affine tax economy will be higher relative to "Progressive" one, *iff* the marginal tax rate is not "too low" relative to the interest rate[16].

Finally, from (1.14), (1.17), (1.16) and (1.15) the stationary coefficient of variation for each case equals:

[15] If instead, the free parameter was set to be the $T^A$, the respective measure would had followed that of the "Progressive" tax system.

[16] Comparing the variances someone has to sign the term $S_D(\tau^A w) - (r-n)b$, if positive this implies that the "progressive" tax system has higher inequalities than the "Affine" one, otherwise is the opposite.

$$\mathrm{CV}^{\mathrm{P}} = \sigma\left(\sqrt{\frac{S_D[w-(r-n)b]}{(1+S_D X_s)}}\right) \tag{1.18}$$

$$\mathrm{CV}^{\mathrm{F}} = \sigma\left(\frac{S_D w}{\sqrt{(1+S_D X_s)}\sqrt{S_D[w-(r-n)b]}}\right) \tag{1.19}$$

$$\mathrm{CV}^{\mathrm{A}} = \sigma(1-\tau)\mathrm{CV}^{\mathrm{F}} \tag{1.20}$$

Overall, inequality as captured by the coefficient of variation, is a function of wages, interest rates and taxes. The latter, however, and through the government's budget constraint it also depends on factor prices. In other words, we could assume that wealth inequality ultimately depends on wages and interest rates, i.e $CV = \mathcal{F}(w, r)$ where $\mathcal{F}(.)$ is some particular function describing the relationship - which in our case takes one of the three forms in equations (1.18)-(1.20). Of course, since factor prices are functions of the physical capital, we can also note that the level of inequality is ultimately an implicit function of capital only - the level of which will depend on the level of public debt, treated in our model as an exogenous parameter. Accordingly, "macroeconomic shocks" - like a permanent change in public debt - are of first-order importance in "shifting" the distribution of wealth (i.e. a change in inequality levels), through the change in factor prices.

As we will later prove, the source for the change in factor prices, and therefore the trigger for a "macroeconomic shock", in our exercise will be the change in the level of government debt. Furthermore, the coefficient of variation captures in a very simple term all the general equilibrium effects that can possibly affect *relative* wealth. As we will see on the one hand there is a permanent shift in the supply of assets - and thus average wealth is changing - but on the other hand, since the incomes of the households will also change, the is a respective effect on the variability of assets.

## 1.5 The Macroeconomic environment

The condition that characterizes the equilibrium of the model, comes through the financial markets clearing condition. The particular condition, requires capital and bonds to compete for the average savings of the "young". For the large economies, average savings equal the mean assets, which in turn equal the savings of the "representative-mean" individual. Therefore, the financial markets clearing condition in per capita terms is:

$$\mathrm{E}(a^i_{t+1}) \equiv (1+n)(k_{t+1}+b) = S_t(D_t, r_{t+1}) \tag{1.21}$$

where $S(D_t, r_{t+1})$ is the optimal mean saving function (See (1.6)), $D_t = w_t - (r - n)b + X(r_t, S_t)$ denotes the mean disposable income and $X(r_t, S_t))$ is the mean bequest function (See (1.7)), all in period $t$. For the macroeconomic state, only the mean taxes matter (i.e. the taxes the individual with the mean "skill" faces). As a consequence, for all of our three economies, the aggregate behaviour is governed by the same law of motion (1.21), of a first-order non-linear difference equation with respect to physical capital[17].

**Example 1** *Assuming that the preferences are logarithmic and taking the case of the flat tax regime, the optimal plans for individual savings and bequests are:*

$$a_{it+1} = \left(\frac{\beta+\delta}{1+\beta+\delta}\right)[w_t l^i - \tau_t^f + x_{it}] \tag{1.22}$$

$$x_{it+1} = \left(\frac{\delta}{(1+n)(\beta+\delta)}\right)(1+r_{t+1})a_{it+1} \tag{1.23}$$

*Substituting* (1.23) *in* (1.22)*, using the budget constraint of the government, imposing the market clearing condition* (1.21) *and taking the average across individuals, the law of motion for physical capital is:*

$$(1+n)(k_{t+1}+b) = \left(\frac{\beta+\delta}{1+\beta+\delta}\right)\left[w_t(k_t)-(r_t(k_t)-n)b+\left(\frac{\delta}{(1+n)(\beta+\delta)}\right)(1+r_t)(k_t+b)\right] \tag{1.24}$$

*Which clearly shows that the dynamics of the system are governed by a first-order difference equation.*

Obviously, (1.21) is the generic form that combines the sum of optimal savings and bequests plans of individual, i.e. equation (1.13), in per capita terms. For this specific law of motion, the stability condition is:

$$0 < \left.\frac{dk_{t+1}}{dk_t}\right|_{db=0} = \frac{S_D\Big[-(k+b)f'' + X_r f'' + (1+n)X_s\Big]}{[(1+n) - S_r f'']} < 1 \tag{1.25}$$

To gain some further insights from (1.25), this can alternatively be written as:

$$C_4 \equiv S_D X_s < 1 - \underbrace{\left(\frac{\Big[S_r + S_D[(\frac{1}{Z+1}-1)(k+b)\Big]}{1+n}\right) f''}_{\Pi} \tag{1.26}$$

where $X_r \equiv \frac{\partial X(r_t, S_{t-1})}{\partial r_{t+1}} = \frac{\frac{S(D,r)}{1+n}}{Z+1} = \frac{(k+b)}{Z+1}$, and $Z = \frac{\delta U''(x_{t+1})}{(1+n)^2 \beta U''(c_{t+1})} > 0 \Rightarrow \frac{1}{Z+1} < 1$. In the "normal" case where $\frac{\partial S(D,r)}{\partial r} \equiv S_r < 0$, the sign of $\Pi$ is positive for any (non-negative) debt level[18]. In consequence, for *macroeconomic or financial stability* a stronger restriction on $C_4 = S_D X_S$ is necessary. For example, in the special case of

[17] Recall, $w_t = w(k_t)$, $r_t = r(k_t)$ and as a consequence $X_t = X(k_t), D_t = D(k_t)$.

[18] $S_r < 0$ implies that any increase in the supply of assets will decrease interest rates for some given asset demand. In conventional models with CEIS (CRRA) utility function $S_r$ is negative when the inter-temporal elasticity of substitution is less than one. Nonetheless, the case of $S_r < 0$ does not rule out "sunspot" behaviour (i.e. multiple steady states). See Galor and Ryder (1989). Therefore, in the numerical exercises, we will assume local analysis.

a logarithmic utility function $S_r = 0$. In that particular case, the stability condition implies a more definite upper bound $\overline{C(b)}$ (controlled by debt policy) such as the *average* intergenerational wealth persistence which should not only be less than one but also less than a specific threshold, i.e. $C_4 < \overline{C(b)} < 1$.

The main message of the paragraph above, deems financial stability to be *jointly* determined by the amount of debt and the level of inequality. More specifically, in the most plausible case ($S_r < 0$), the microeconomic condition for stationary distribution is *necessary* but not *sufficient* for macroeconomic stability. Loosely put, "average *private* altruism" is detached from the "average *social* altruism" and debt policies can affect the distance between them, not necessarily directly but indirectly through its effect on marginal propensity to bequeath[19]. In what follows, we will assume that the steady state is stable and $S_r \leq 0$.

In what follows, we limit our analysis in the steady state. The equilibrium in financial markets then is:

$$\mathrm{E}(a) = \frac{S_D[w-(r-n)b]}{1-S_D X_s} \equiv (1+n)(k+b) = S(D,r) \tag{1.27}$$

Note from equation (1.27) that what matters for the supply of funds is the fraction of mean savings assigned to finance future consumption net of bequests (i.e. the egoistic MPS). When this feature is taken into consideration, then the model resembles that of Diamond (1965). By total differentiation of $(1+n)(k+b) = S(D,r)$, the change in capital due to a unit change in government debt is:

$$\frac{dk}{db} = \frac{S_D \overbrace{[\overline{X_S}-1]}^{(-)} f' + \overbrace{(S_D-1)}^{(-)}(1+n)}{\underbrace{(1+n-S_r f'')(1-\frac{dk_{t+1}}{dk_t}|_{db=0})}_{Q>0}} < 0 \tag{1.28}$$

where $Q = (1+n-S_r f'')(1-\frac{dk_{t+1}}{dk_t}|_{db=0}) > 0$ is positive from the stability condition and the numerator negative, since $0 < S_D, \overline{X_S} < 1$. Therefore, higher public debt always crowds out capital and increases the interest rates. Under the assumption of $S_r < 0$, the "income effect" dominates the "substitution effect" and in equilibrium, the rise in interest rate will depress *average* (mean) savings. The same occurs if $S_r = 0$ (i.e. when the utility is logarithmic)[20]. The result of the fall in average wealth, indicates that debt will crowd out capital by more than one. The implication of this statement is that average wealth falls. Thus, the direct macroeconomic effects of a positive debt shock tends to increase wealth inequality. However, the equilibrium effects on inequality also depend on the change in the variance. The next section takes this issue more closely.

[19]In completely different setting and concerns, Farhi and Werning (2007) show the existence of long-run (consumption and welfare) distributions when the social discounting (altruism) is different from the private.

[20]See Bertola et al. (2006, Chapter 5)

## 1.6 The Effects on Inequality

### 1.6.1 The "Mechanics" at the Macroeconomic level

To motivate the discussion on the mechanics of the model, the total change in inequality in the stationary state is decomposed as:

$$dCV(Var(a), E(a)) = W_1 dVar(a) + W_2(dE(a)) \Rightarrow \tag{1.29}$$

$$= W_1 dVar(a) + W_2(1+n)(dk+db) \Rightarrow \tag{1.30}$$

$$\frac{dCV}{db} = \underbrace{\underbrace{W_1}_{+} \frac{dVar(a)}{db}}_{\text{Variance effect}} + \underbrace{\underbrace{W_2(1+n)}_{-}(\frac{dk}{db}+1)}_{\text{Mean Effect}} \tag{1.31}$$

The formulation above reveals the "mean" and "variance" effects akin to the one described by Bossmann et al. (2007), where $W_1 = \frac{\partial CV}{\partial Var}$ and $W_2 = \frac{\partial CV}{\partial E(a)}$ are the relevant contributions of each separate change in the mean and the variance. The mean is affected by the extent of crowding out, while the variance -as we will discuss later- by the equilibrium changes in savings behaviour.

In equation (1.31), the "mean effect" is positive since debt crowds out capital by more than one (i.e. $\frac{dk}{db} + 1 < 0$ ). In this instance, average wealth falls and therefore inequality tends to increase for unchanged variance. Nevertheless, individual savings responses due to the perturbation in factor prices will also affect the variance. For example, when the variance drops, inequality tends to decrease for a given mean. Overall however, the equilibrium effect on wealth inequality will be ambiguous.

In this context, the debt shock will trigger factor price changes through the standard "OLG-Diamond model" mechanism. In short, when the *average* savings *net of government bonds* falls, the equilibrium interest rates will increase. In return, the change in factor prices will determine savings behaviour and thus the effect on the variance. The perturbation on the variance is crucially dependent on factor price changes. In fact, it is the silent features of those *general equilibrium effects* that could possibly reverse the effects on wealth inequality. If those are not in operation, any conclusions will be overturned.

To gain some further insights for the mechanics at the macro level, we totally differentiate $\hat{CV}^P \equiv \frac{(CV^P)^2}{\sigma^2}$ in the case of the "Progressive Tax" economy, and $\hat{CV}^F \equiv \frac{(CV^F)^2}{\sigma^2}$ for the "Flat" tax economy[21]. For the latter economy, equation (1.19) can be restated in a functional form as $\hat{CV}^F = \Phi^F(C_3, C4, C_5)$, where $C_3 = S_D w$, $C_4 = S_D X_S$, $C_5 = S_D(r-n)b$ and $\Phi^F = \frac{C_3^2}{(1+C_4)(C_3-C_5)}$. By total differentiation, someone gets:

[21] For the "Affine" tax economy, the equivalent expression is $\frac{(CV^A)^2}{((1-\tau^A)\sigma)^2}$. Since the qualitative effects are the same up to some constants, we will reserve the discussion for the "Flat tax" case only.

$$d\hat{CV}^F = \underbrace{\Phi_3^F}_{+} dC_3 + \underbrace{\Phi_4^F}_{-} dC_4 + \underbrace{\Phi_5^F}_{+} dC_5 \tag{1.32}$$

or equivalently,

$$\frac{d\hat{CV}^F}{db} = \underbrace{\left[ \underbrace{\Phi_3^F}_{+} \underbrace{[w\Delta - S_D k]}_{-} + \underbrace{\Phi_4^F}_{-} \underbrace{\Gamma}_{?} \right]}_{\substack{\text{Average change} \\ \text{in Life-Cycle considerations}}} \frac{dr}{db} + \underbrace{\underbrace{\Phi_5^F}_{+} \left[ \underbrace{E}_{+} b \frac{dr}{db} + S_D(r-n) \right]}_{\substack{\text{Average effect} \\ \text{of Taxation}}} \tag{1.33}$$

where the $phi's$ are $\Phi_j^F \equiv \frac{\partial \hat{CV}^F}{\partial C_j}$ for $j$=[3,4,5], and $\Gamma$, $\Delta$ and $E$ are terms defined in Appendix 1.9.4. In Table 1.1 below we can see the necessary and sufficient conditions to sign those. In the case of a logarithmic utility, the $\Gamma$ and $E$ terms are always positive and $\Delta = 0$ (See Appendix 1.9.4 for the proofs). For any other utility function, we will assume that $\Gamma < 0$ and $E > 0$. In other words, we implicitly assume that the level of debt is not "too high" and consumers have plausible marginal propensities to save [22].

**Conditions on MPC**

| | | |
|---|---|---|
| $\Gamma$ | (+) | $S_D > 0.5$ |
| | (-) | $S_D < 0.5$ |
| **E** | (+) | DI or $S_D \geq 0.5$ or $S_D^* < S_D < 0.5$ |
| | (-) | $S_D < S_D^* < 0.5$ |

DI=Dynamic Inefficiency. The special cases are: $S_D = \frac{1}{2} \Rightarrow \Gamma = 0$, $b = \frac{w}{2(r-n)} \Rightarrow \Phi_3^F = 0$ and $S_D = S^* < 0.5 \Rightarrow E = 0$. $S_D^*$ is a particular threshold which equals to $S^* = \frac{1}{2} - \frac{1}{2}\frac{1+n}{f'}$ and is assumed to be positive. In the case of logarithmic utility $\Gamma, E > 0$ always

Table 1.1: Main Conditions

From equation (1.32), the total change in wealth inequality is attributed to three main factors. First, it depends on how much individuals will save (or dissave) out of their new wage (the $dC_3$ term). As soon as the level of debt is not too high, to avoid any "immiseration" from very low wages, the "rich" can save more than the "poor" and therefore inequality tends to increase (the $\Phi_3^F > 0$ term). Second, a change in debt will affect the fraction of "parent's" savings that are "bequeathed" on top of "children's wealth (the $dC_4$ term). On average, leaving positive bequests tends to have an equalizing effect (the $\Phi_4^F$ term). Finally, a permanent increase in government borrowing will result to higher taxation (the $dC_3$ term), but as soon as taxes are of "regressive" nature they will tend to amplify the inequality levels (the $\Phi_5^F$ term).

In general, the effects on inequality are ambiguous. In particular, the source of ambiguity stems from the competing effects of the life-cycle and bequest motives (the $\Phi_3^F[w\Delta - S_D k] + \Phi_4^F\Gamma$ terms). Since these motives cannot be isolated at the individual

[22] See for example Jappelli and Pistaferri (2012) and the literature therein.

level, as a result they also appear at the macro level. In the subsequent sections, where the individual behaviour is analysed, the "rich" and the "poor" will have opposite qualitative responses on these two saving scopes.

Similarly, equation (1.18) can be restated as $C\hat{V}^P = \Phi^P(C_3, C_4, C_5)$ and by total differentiation the total change is:

$$d\hat{CV}^F = \underbrace{\Phi_3^P}_{+} dC_3 + \underbrace{\Phi_4^P}_{-} dC_4 + \underbrace{\Phi_5^P}_{-} dC_5 \tag{1.34}$$

or equivalently,

$$\frac{d\hat{CV}^P}{db} = \underbrace{\left[ \underbrace{\Phi_3^P}_{+} \underbrace{[w\Delta - S_D k]}_{-} + \underbrace{\Phi_4^P}_{-} \underbrace{\Gamma}_{?} \right] \frac{dr}{db}}_{\substack{\text{Average change} \\ \text{in Life-Cycle considerations}}} + \underbrace{\underbrace{\Phi_5^P}_{-} \left[ \underbrace{E}_{+} b \frac{dr}{db} + S_D(r-n) \right]}_{\substack{\text{Average effect} \\ \text{of Taxation}}} \tag{1.35}$$

where the $phi's$ are $\Phi_j^P \equiv \frac{\partial C\hat{V}^P}{\partial C_j}$ for $j$=[3,4,5]. The analysis is similar to the one before. The only caveat is that while wages before-tax have an unequalising effect (the $\Phi_3^P$ term), the "progressivism" of the tax system tends to offset it (the $\Phi_5^P$ term).

**Example:** Consider the case of the proportional tax system. If preferences are logarithmic ($\Gamma > 0$, $\Delta = 0$), it follows from equation (1.35) that higher public debt will decrease inequality. In the case of a flat tax system, this is only true if the economy is dynamic inefficient ($r < n$). If not, it requires the average change in life-cycle consideration to dominate the "wealth effects" from taxation.

Notice also that under a "partial equilibrium analysis" (captured in $\Phi_5^j S_D(r - n)$ terms for j=[P,F]), the effects on inequality will follow what the tax instrument is supposed to do by design. However, the general equilibrium "feedback", due to price changes, might overturn this and in fact the effect on wealth inequality can go in either direction. Therefore, it is more proper, not only to understand what will be observed at macro-level, but also what a debt shock will trigger at micro-level.

### 1.6.2 The "Mechanics" at the Individual level

The change in the variance, while it seems of quantitative nature, we can still filter out some interesting qualitative properties. In general, to calculate the variance of wealth, it is sufficient to know the asset position of each individual. In this simple model, in order to understand the change in the variance, we should know the equilibrium savings responses. In this context, where altruism is taken into account, individuals will program their savings responses on the basis of their "egoistic needs" and their "altruistic liabilities". These two scopes can further decompose the change in variance (and thus in total wealth) into the "egoistic" and "altruistic" part respectively.

To begin the exposition, we will first consider a "Flat" tax system. In the stationary state $x^i_{t+1} = x^i_t = x^i$ (and at the macro-level $k_{t+1} = k_t = k$) substituting (1.7) in (1.6) and using (1.10) in $D^i = w_t l^i - \tau^F + x^i$, individual savings are equal to:

$$a_i^F = \frac{S_D}{1 - S_D X_S} w l^i - \frac{S_D}{1 - S_D X_S}(r - n)b \Rightarrow \tag{1.36}$$

$$a_i^F = E(a^i) + \frac{S_D}{1 - S_D X_S} w(l^i - 1) \Rightarrow \tag{1.37}$$

Equation (1.37) confirms that better endowed individuals are wealthier. Taxes in this case are only implicit to the individual decision making and what in effect matters is the fraction of wages someone can save for his own consumption; the $\frac{S_D}{1-S_D X_S} w(l^i - 1)$ term[23]. In fact, this term will determine the change in the variance. To see this, restate the *egoistic* MPS times the wage in functional form as $\Psi(C_3, C_4) = \frac{C_3}{1-C_4}$, where $C_3 = S_D w$ and $C_4 = S_D X_S$ as defined before. From equation (1.37), the total change in individual wealth becomes:

$$da^i = dE(a^i) + \left[ \underbrace{\Psi_3}_{+} dC_3 + \underbrace{\Psi_4}_{+} dC_4 \right] (l^i - 1) \Rightarrow \tag{1.38}$$

or equivalently,

$$\frac{da^i}{db} = \underbrace{(1+n)(\frac{dk}{db} + 1)}_{\equiv \frac{dE(a^i)}{db}} + \left[ \underbrace{\underbrace{\Psi_3}_{+} \underbrace{(w\Delta - S_D k)}_{-} (l^i - 1)}_{N_1} + \underbrace{\underbrace{\Psi_4}_{+} \underbrace{\Gamma}_{-} (l^i - 1)}_{N_2} \right] \underbrace{\frac{dr}{db}}_{+} \tag{1.39}$$

where $C_3, C_4$, $\Delta < 0$ and $\Gamma$ are terms discussed before and $\Psi_i = \frac{\partial \Psi(C_3, C_4)}{\partial C_i}$ for $i = [3, 4]$. In the most plausible case where the MPS is $S_D < 0.5$, it implies $\Gamma < 0$ (See also Table 1.1). From equation (1.38), the perturbation in individual wealth is decomposed into two parts; an aggregate (which essentially comes from the change in factor prices) and an idiosyncratic one (which essentially comes from the different endowments). On the one hand, a debt shock will affect everyone equally (the $\frac{d(E(a))}{db}$ term). On the other hand, individuals will adjust their savings (an intertemporal choice) and in parallel decide how much to bequeath from their investments (an inratemporal choice). In other words, consumers will save a *different* fraction of their *new wage* (the $dC_3$ term), and at the same time will alter the fraction gone to bequests (the $dC_4$ term). By looking at equation (1.39), the "rich" consumers will save and bequeath less (the

[23] Note, the model can also be interpreted as one where individuals differ in their MPS, i.e. the $S_D l^i$ term, but the distribution of $l^i$ is such that the average MPS is $S_D$. For example, this is rationalized if agents differ in their discount factors and therefore the more impatient individuals end up having more wealth. In fact, the spirit of the model might be closer to this interpretation, however the convention in the literature considers the interpretation given in the main text as more appropriate.

$N_1 < 0$ and $N_2 < 0$ terms respectively) and the "poor" more. Thus, when a debt shock occurs, the "idiosyncratic" and "aggregate" components operate in the same direction for the "rich" but not for the "poor". In consequence, the wealth of the "rich" would fall but remain qualitatively uncertain for the other group.

Put simply, a rise in interest rates will induce the usual income and substitution effects. Nevertheless, those should be considered in terms of future expenditures (that is, future consumption and the bequest transfer). At the same time, the change in interest rate will affect the intratemporal choice. On the one hand, more bequests can be transferred due to the increase in $X_S$, whereas on the other hand, less can be bequeathed due to the effect in $S_D$. For people with different endowments, the intratemporal choice is not the same. But by looking in equation above it seems, the savings behaviour of the "rich" is aligned with the macroeconomic effect and their wealth falls. In this case, the extended to altruism "income effect" dominates the "substitution effect" and will reinforce the "wealth effect". For the "poor", the idiosyncratic component competes with the aggregate one, hence their equilibrium response is *ambiguous*[24].

However, if the utility is logarithmic ($\Delta = 0$ and $\Gamma > 0$), this implies that the change in savings will be ambiguous for either groups. In this case, the "rich" will now want to bequeath more and save less, whereas the "poor" will want to do the opposite. Nonetheless, as we show in Section 1.7, the relatively "rich" will still dis-save, which implies that the "egoistic" component (the $N_1$ term) will dominate the altruistic one ( the $N_2$ term). Possibly, this is because future consumption is valued more. In this case though, the fraction of own investments that go to bequests will (unambiguously) increase on average ($dC_4 \uparrow$). This average increase, in turn, boils down to the special case analysed by Bossmann et al. (2007), where higher bequests induce the aforementioned "mean" and "variance" effects on the macroeconomic level.

For the "Affine tax", the analysis is analogous. To see this, note that the stationary individual wealth can be written as:

$$a_i^A = a_i^F - \tau^A \left[ \frac{S_D}{1 - S_D X_S} w(l^i - 1) \right] \tag{1.40}$$

$$\frac{da_i^A}{db} = \frac{da_i^F}{db} - \tau^A \frac{\left[ \frac{S_D}{1 - S_D X_S} w(l^i - 1) \right]}{db} \tag{1.41}$$

Therefore, up to some constant determined by the marginal tax rate, the qualitative pattern remains identical[25].

For the "progressive" tax regime using equation (1.11), the disposable income is $D^i = (1 - \tau^p) w l^i + x^i$. As in the earlier procedure, the individual wealth equals:

[24] This property was also found in Dávila et al. (2012)

[25] If instead the free parameter for the "Affine tax" economy was $T^A$, stationery individual wealth would alter to $a_i^A = a_i^P - \left( \frac{S_d}{1 - S_D X_S} \right) T^A$. The total change in savings would be $\frac{da_i^A}{db} = \frac{a_i^P}{db} - \frac{T^A}{1 - S_D X_S} \left[ \Delta + \left( \frac{S_D}{1 - S_D X_S} \right) \Gamma \right] \frac{dr}{db}$. As soon as $0 < T^A < 1$, the qualitative pattern is similar to the one shown in Section 1.7 for the proportional tax.

$$a_i^P = \left(\frac{S_D(w-(r-n)b)}{1-S_D X_S}\right) l^i = E(a) l^i \Rightarrow \tag{1.42}$$

$$a_i^P - E(a) = E(a)(l^i - 1) \tag{1.43}$$

And the effect of debt on individual wealth is:

$$\frac{da^i}{db} = \frac{dE(a)}{db} l^i = \left[(1+n)(\frac{dk}{db}+1)\right] l^i \tag{1.44}$$

From equation (1.42), each individual holds a fraction of total wealth which is in proportion of his endowment. In consequence, the effect of debt on individual savings will also be in the same proportion (See equation (1.44)). From the previous analysis, we know that higher public debt will decrease average wealth. Therefore, equation (1.44) also implies a negative impact on individual wealth. In particular, the effect will be stronger for the relatively "rich" and milder for the relatively "poor". However, the change in variance is still ambiguous, but of quantitative nature, since all agents are dis-saving. Nonetheless, some hidden qualitative differences still exist.

As earlier, any change in the dispersion of asset holdings can be decomposed into the part coming from own savings and the part stemming from the altruism of parents. This decomposition is interesting on its own. For the "Flat" or "Affine" tax economy, the behavioural elements of those were clear. But even in this case, a similar decomposition is still possible. By adding and subtracting the relevant terms, the asset holdings in the "proportional" economy can be written in terms of the flat tax economy as:

$$a_i^P = a_i^F - \frac{C_5}{1-C_4}(l^i - 1) \tag{1.45}$$

where $C_5 = S_D(r-n)b$ and $C_4 = S_D X_S$. As expected, equation (1.45) confirms that the "rich" own less assets whereas the "poor" more, relative to a regressive tax system. Defining $\Psi^P = \frac{C_5}{1-C_4}$, using equation (1.39) and collecting terms, the effect of debt on asset holdings becomes:

$$\frac{da_i^P}{db} = \underbrace{\frac{dE(a^i)}{db}}_{-} + \left[\underbrace{\underbrace{\Psi_3}_{+}\underbrace{(w\Delta - S_D k)}_{-}}_{N_1^P} + \underbrace{\underbrace{\overline{\Psi}_4}_{+}\Gamma}_{N_2^P}\right] \underbrace{\frac{dr}{db}}_{+}(l^i - 1) - \underbrace{\Psi_5^P}_{+} \underbrace{\left[Eb\frac{dr}{db} + S_D(r-n)\right]}_{N_3^P > 0}(l^i - 1) \tag{1.46}$$

where $\overline{\Psi}_4 = \Psi_4 - \Psi_4^P = E(a^i)$, and $\Psi_i^P = \frac{\partial \Psi^P}{\partial C_i}$ for $i$=[4,5]. The $N_3^P$ term is positive because of the change in taxes[26]. Equation (1.46) is a simple reformulation

[26] We assume that this is also true even in the dynamic inefficient case. Under a particular bound on debt, this is always the case.

of (1.44), so all individual will have to dis-save in the end. Nonetheless, the different behavioural motives, as in the "Flat" tax economy, are still present ( the second term in (1.46), with the $N^P_{3,4}$ replacing the $N_{1,2}$ terms). However, the "progressivism" of the tax system modifies the personal "wealth effects" ( the third term in (1.46)) and thus the incentive to save or dis-save. For instance, the "rich" expecting to bear higher taxes have an incentive to save more, while the "poor" less.

In summary, when individuals have the particular bequest motive *ex-ante* heterogeneity itself becomes sufficient to generate different qualitative behaviour to a debt shock. The tax instrument then, will only determine the nature of the change in inequality (i.e. whether this will be of quantitative or qualitative nature).

## 1.7 Steady State Results

We restrict our analysis in the stationary state, where we also impose the condition $S_r \leq 0$. Debt is treated as an exogenous parameter, therefore it will determine the values of capital and the interest rates. We pick a number such as in our numerical exercises the stability and the condition for mean reversion wealth (See equation (1.25)) are satisfied. Moreover, to assess the effect of debt on capital we rely on equation (1.28), to assess the effects on wealth inequality we use equations (1.33) and (1.35), and finally to assess the individual savings behaviour, we rely on equations (1.44), (1.39) and (1.41) respectively. To figure out the effects on the mean and the variance, we evaluate the $(1+n)(\frac{dk}{db}+1)$, $\frac{dVar^P}{db}$, $\frac{dVar^F}{db}$ and $\frac{dVar^A}{db}$ derivatives.

We also assume a CEIS ("CRRA") utility function of the form: $V = \frac{c_t^{1-\theta}-1}{1-\theta} + \beta\left(\frac{c_{t+1}^{1-\theta}-1}{1-\theta}\right) + \delta\left(\frac{x_{t+1}^{1-\theta}-1}{1-\theta}\right)$, where the choices for the inverse of the intertemporal elasticity of substitution $\theta$ are 1.5 and $\theta = 1$ (i.e. logarithmic utility). The production function in its intensive form is assumed to be Cobb-Douglas, $y = Ak^\gamma$, with $A = 9.37$, $\gamma = 0.3$. Population growth, $n$, is set to 1.81 to match the average post war growth rate in the US. The choices for the discount factor, $\beta$ and the degree of altruism $\delta$, are set to 0.3 and 0.10 respectively, when the value for debt is 0.05. This is the case of a dynamic efficient economy where it ensures that all conditions of the model are satisfied. Similarly, we set $\beta = 0.4$ and $\delta = 0.15$ with $b = 0.11$ for the dynamic inefficient case.

The values for the discount factor are motivated by the RBC literature. There, $\beta = 0.99$ and each period represents a quarter. In our calibration, this is similar to consider the time period as 30 years in the first case and around 25 years in the second one. The choice for the degree of altruism is essentially arbitrary but closely follows that of Bossmann et al. (2007). In all cases, the MPS is restricted to be less than half. Finally, the choice for the marginal tax rate in the "Affine" tax economy is taken from Bhandari et al. (2013) and set to $\tau^A = 0.2$.

In Table 1.2 we can see the results obtained for our economy. At the macroeconomic level and in all cases, the mean and variance fall. This tends to move the inequality to opposite direction. Nevertheless, the "variance effect" dominates the "mean effect" and inequality is suppressed. But, the individual mechanics (between the tax systems) are different.

| | Dynamic Efficiency | | | Dynamic Inefficiency | | |
|---|---|---|---|---|---|---|
| **Endowment** | **Flat** | **Affine** | **Prop.** | **Flat** | **Affine** | **Prop.** |
| 0.2 | 3.20 | 2.14 | -0.42 | 2.97 | 2.01 | -0.37 |
| 0.3 | 2.54 | 1.61 | -0.63 | 2.37 | 1.53 | -0.55 |
| 0.4 | 1.88 | 1.08 | -0.84 | 1.77 | 1.05 | -0.73 |
| 0.7 | -0.11 | -0.51 | -1.47 | -0.03 | -0.39 | -1.28 |
| 1 | -2.10 | -2.10 | -2.10 | -1.83 | -1.83 | -1.83 |
| 2 | -8.73 | -7.40 | -4.20 | -7.84 | -6.64 | -3.67 |
| 5 | -28.62 | -23.32 | -10.50 | -25.87 | -21.06 | -9.17 |
| 10 | -61.77 | -49.84 | -21.00 | -55.92 | -45.10 | -18.35 |
| **Mean effect** | (+) | (+) | (+) | (+) | (+) | (+) |
| **Change in Variance** | (-) | (-) | (-) | (-) | (-) | (-) |
| **Inequality** | (-) | (-) | (-) | (-) | (-) | (-) |

Mean effect = $W_2(1+n)(\frac{dk}{db}+1)$, with $W_2 < 0$, Variance effect = $W_1 \frac{dVar(a^i)}{db}$, with $W_1 > 0$. Each column describes the savings response for the different taxes to a 1% increase in debt.

Table 1.2: Main Results

In the "progressive" tax regime, everyone's wealth falls proportionally, as discussed in Section 1.6.2. However, in the rest of the cases, there is a clear qualitative difference between different groups of people. In fact, the "poorer" someone is the more is willing to save. Thus, the "poor" in this context seem to behave as "Ricardians"[27]. The intuition for the behaviour of the poor is the following: The "poorest" individuals (who had already low wages prior to the shock) would see their real wage to decline further and their tax liability (regressive in nature) to increase. Unless they start saving more, by exploiting the rise in the rate of interest, they will not be in a position to finance their future consumption *net of bequests*.

In other words, the substitution effect is sufficiently strong to dominate both the income and the wealth effects. This is not the case for the "rich", where the income effect always dominates. In "Affine" tax system, as soon as the choice for the tax adjustment is on the common tax component, individual behaviour will follow that of a flat tax economy. However, since not everyone will pay the exact equal amounts in taxes, the quantitative response will be different. The same analysis goes through for the dynamic inefficient economies. Nevertheless, in this case and in contrast to the previous one, the "equity-efficiency" trade-off breaks down, which extends Diamond's original contribution.

In Table 1.3, we see the results obtained for the logarithmic utility. As someone can observe, the qualitative pattern is similar to the one described above and follows the discussion in Section 1.6.2

[27]See Laitner and Ohlsson (2001) for a clear exposition on the failure (at least on average) of the Ricardian equivalence under a "joy-of-giving" bequest motive. Moreover, the positive association between debt and the savings behaviour of the "poor" was also present in Heathcote (2005) .

| | Dynamic Efficiency | | | Dynamic Inefficiency | | |
|---|---|---|---|---|---|---|
| **Endowment** | **Flat** | **Affine** | **Prop.** | **Flat** | **Affine** | **Prop.** |
| 0.2 | 3.62 | 2.57 | -0.33 | 2.40 | 1.77 | -0.15 |
| 0.3 | 2.96 | 2.04 | -0.49 | 2.00 | 1.45 | -0.23 |
| 0.4 | 2.30 | 1.51 | -0.65 | 1.61 | 1.14 | -0.30 |
| 0.7 | 0.33 | -0.06 | -1.15 | 0.43 | 0.19 | -0.53 |
| 0.8 | -0.32 | -0.59 | -1.31 | 0.03 | -0.13 | -0.61 |
| 0.9 | -0.98 | -1.11 | -1.47 | -0.36 | -0.44 | -0.68 |
| 1 | -1.64 | -1.64 | -1.64 | -0.76 | -0.76 | -0.76 |
| 2 | -8.20 | -6.89 | -3.27 | -4.71 | -3.92 | -1.52 |
| 5 | -27.90 | -22.64 | -8.18 | -16.55 | -13.39 | -3.79 |
| 10 | -60.72 | -48.90 | -16.36 | -36.28 | -29.18 | -7.59 |
| **Mean effect** | (+) | (+) | (+) | (+) | (+) | (+) |
| **Change in Variance** | (-) | (-) | (-) | (-) | (-) | (-) |
| **Inequality** | (-) | (-) | (-) | (-) | (-) | (-) |

Mean effect $= W_2(1+n)(\frac{dk}{db}+1)$, with $W_2 < 0$, Variance effect $= W_1 \frac{dVar(a^i)}{db}$, with $W_1 > 0$.
Each column describes the saving response for the different taxes.

Table 1.3: Logarithmic Utility

To put it simple, the bottom line of the previous analyses is that public debt affects the accumulation of wealth, and by extension wealth inequality, through a dual channel. On the one hand, a permanent change in the supply of debt affects the average wealth of the economy, meaning that not only it affects the opportunity of someone to accrue interest, but also the level of assets available for each family.

This simple, yet plausible observation, it is mightily related to the standard neoclassical result of "crowing-out", which our paper also replicates. That is, when a unit increase of public debt, for example, causes a disproportional reduction in physical capital leading to a fall in average wealth, "poor" households are in disadvantage. This is because, the families that are "poor" are also the ones that save or invest less in assets. The crowing out of capital, therefore, tend to increase wealth inequality. It follows then that to the hypothetical case where bonds increase and are equally distributed across the population, this policy does not necessary improve the position of the poor compared to the rich, for the simple reason that the supply of other assets will disproportionally fall leaving, therefore, the "poor" with fewer assets than before.

On the other hand, the change in public debt it also affects incomes and this occurs in two ways. The one is due to the change in taxation, the other is due to the effect on prices. As soon as, however, taxes mostly fall on labour incomes - i.e the young - this is equivalent to a transfer scheme from young-to-old, as for example, would have occurred with unfunded social security. It follows then that older households got richer, and as a result would be able to leave more transfers to their kids. This helps the accumulation of wealth, and especially for the "poor" households this can even off-set the drop in wages and the increase in labour taxes. Bequest, therefore, and as Bossmann et al. (2007) have stated help households to accumulate more wealth on the one hand, but also to decrease the dispersion of wealth on the other. As we numerically showed, however, quantitatively the latter dominates; twisting therefore the implications that the crowd-out of capital could bring. As a result, we view bequests as important to the our findings.

To see this more clearly, consider that for *any given level of assets* the substitution effect - due to the change in the interest rate- for the rich is weaker relative to the “poor”, which implies that when interest rates rise, “poor” households will save proportionally more relative to the rich, allowing them to leave more in proportion to their wealth bequests.

## 1.8 Conclusions

To conclude, this paper designated a natural mechanism linking public debt and the inequality of wealth. Our primary concern was to evaluate the effects of the former to the latter macroeconomic variable. In our set-up we took into consideration the likely event that individuals across generations are linked through a particular form of altruism and analytically showed how a bequest motive affect the life-cycle considerations of the individuals.

Interestedly, we analytically proved that different classes or groups of people, the relatively “rich” and the “relatively” poor, do not in general respond in the same manner in the event of a “debt shock”. Therefore the main source of the ambiguous effects of debt on the inequality of wealth that in general exists, comes from that particular property. This feature it is consistent with the view that the distribution of wealth matters for aggregate outcomes.

Moreover and limiting our analysis to labour taxes, a set of our results showed that different tax regimes do not alter the observable outcome. Nevertheless, at microeconomic level the tax regime seems to matter. In the extreme case where the adjustment in debt comes mostly through a regressive tax instrument, the qualitative responses between the “rich” and the “poor” are not the same. This in turn implies that debt adjustments unless carefully designed might accelerate the inequality of wealth.

Finally, in some support of our results Vegh and Vuletin (2014) tested the effects of pro-cyclical fiscal policies (i.e. proxy to an exogenous debt shock) on income inequality (a measure which is highly correlated with wealth inequality (OECD (2009)) and documented that the first tend to exacerbate the latter. In addition, Laubach (2009) seems to confirm that budget deficits do affect the rate of interest, hence the general equilibrium effects of deficits (that our mechanism crucially rely on) seems to be a valid possibility. However, we mentioned in the main text the scale of the crowding out of capital might matter. In this respect, the assumption of a closed economy is important.

## 1.9 Appendix

### 1.9.1 Proof for $\Delta < 0$

**Proof.** From $dC_3 = S_D dw + w dS_D$. Noting that $S_D$ is a function of the interest rate and the MPB, then $dS_D = S_{Dr}dr + S_{Dx}dX_S$. Since $X_S$ is a function of the interest rate, we also have $dX_S = X_{Sr}dr$. Therefore, $dS_D = S_{Dr}dr + S_{Dx}X_{Sr}dr$. Define, $\Delta = S_{Dr} + S_{Dx}X_{Sr}$, substituting the expressions for all terms and using the fact that $(1+n)X_{Sr} = \overline{X_s}$, someone gets $\Delta = \frac{2S_{Dx}}{1+n}[\overline{X_s} - 1] < 0$ which is unambiguously negative. If the utility is logarithmic then $dS_D = 0 \Rightarrow \Delta = 0$. For the definitions of $S_{Dr}, S_{Dx}$ and $X_{Sr}$ see Appendix 1.9.4. For further details see On-line Appendix. ■

### 1.9.2 Proof for $\Gamma$

**Proof.** From $dC_4 = X_S dS_D + S_D dX_s$, using $dS_D = S_{Dr}dr + S_{Dx}dX_S$, then $dC_4 = [X_S\Delta + S_D X_{Sr}]dr$. Define $\Gamma = X_S\Delta + S_D X_{Sr}$. Using, $X_{Sr} = \frac{\overline{X_s}}{1+n}$ and the definition for $\overline{X_s}$, we have $\Gamma = X_S\left[\Delta + \frac{S_D}{f'}\right]$. So, if the utility is logarithmic $\Delta = 0 \Rightarrow \Gamma > 0$. Otherwise, $\Gamma$ can be of either sign. In particular, $\Gamma > 0$ *iff*:

$$\begin{aligned} \Delta + \frac{S_D}{f'} &> 0 \\ \text{Using the definition for } \Delta &= \frac{2S_{Dx}}{1+n}[\overline{X_s} - 1] \\ S_D &> \frac{2S_{Dx}}{1+n}[1 - \overline{X_s}]f' \\ 1 &> 2S_D Z_1 \end{aligned}$$

Where in the last steps we used $S_{Dx} = (S_D)^2\beta\frac{U''(C_{t+1})}{U''(C_t)}(1+n)f'$ and $S_D = \frac{1}{Z_1+1}$ with $Z_1 = \beta\frac{U''(c_{t+1})}{U''(C_t)}[1 - \overline{X_s}](f')^2$. Therefore, using $S_D = \frac{1}{Z_1+1}$, $1 > 2S_D Z_1 \Rightarrow S_D > \frac{1}{2}$. The conditions in the main text follow. ■

### 1.9.3 Proof for E

**Proof.** From $dC_5 = S_D d(r-n)b + (r-n)bdS_D$ and using $\Delta = -\frac{2}{f'}S_D(1-S_D)$, $dC_5 = S_D d(r-n)b + [(\frac{S_D}{f'} + \Delta)f' - (1+n)\Delta]bdr$. Define $E = (\frac{S_D}{f'} + \Delta)f' - (1+n)\Delta = S_D + (r-n)\Delta$. So, if the economy is dynamic inefficient this is always positive. Otherwise, some conditions must be put. With some algebra $E$ is also equals to $E = 2S_D[S_D - \frac{1}{2} + \frac{1+n}{f'}(1-S_D)]$. Define, $P = S_D - \frac{1}{2} + \frac{1+n}{f'}(1-S_D)$, thus if $S_D > \frac{1}{2}$ then $P > 0 \Rightarrow E > 0$. Hence more investigation is required for a dynamic efficient economy and $S_D < \frac{1}{2}$. Consider the case $S_D < \frac{1}{2}$ and $f' > 1+n$. Then, $P < 0$ *iff*, $S_D < 1 - \frac{0.5}{1-\frac{1+n}{f'}}$. Thus, if the MPS is less than $S_D < 1 - \frac{0.5}{1-\frac{1+n}{f'}} < 0.5$ then $P < 0 \Rightarrow E < 0$, otherwise, if $1 - \frac{0.5}{1-\frac{1+n}{f'}} < S_D < \frac{1}{2} \Rightarrow P > 0 \Rightarrow E > 0$. ■

### 1.9.4 Definitions and Notations

$$\mathbf{S}(\text{r,D}) = \text{saving function}$$
$$\mathbf{X}(r, \mathbf{S}) = \text{bequests function}$$
$$S_D = \frac{\partial \mathbf{S}(r, D)}{\partial D} < 1$$
$$= \frac{1}{Z_1 + 1}$$
$$\text{with } \; Z_1 = \beta(1 + r_{t+1})^2 \frac{U''(c_{t+1})}{U''(c_t)}(1 - \overline{X_s})$$
$$S_r = \frac{\partial \mathbf{S}(r, D)}{\partial r} < 1$$
$$X_S = \frac{\partial \mathbf{X}(r, \mathbf{S})}{\partial s}$$
$$\overline{X_S} \equiv \frac{(1+n)X_s}{f'} < 1$$
$$S_{Dr} = \frac{\partial S_D}{\partial r} < 0$$
$$= -(S_D)^2 \left[\beta \frac{U''(c_{t+1})}{U''(c_t)}(2 - \overline{X_s})(1 + r_{t+1})\right]$$
$$S_{Dx} = \frac{\partial S_D}{\partial X_S} > 0$$
$$= (S_D)^2 \left[\beta \frac{U''(c_{t+1})}{U''(c_t)}(1 + n)(1 + r_{t+1})\right]$$
$$X_{Sr} = \frac{\partial X_S}{\partial r} > 0$$
$$= \frac{\overline{X_s}}{1 + n}$$

$$\Delta = S_{Dr} + S_{Dx}X_{Sr} =$$
$$= \frac{2S_{Dx}}{1+n}[\overline{X_s} - 1] < 0$$
$$= -\frac{2}{f'}S_D(1 - S_D) < 0$$
$$E = S_D + (r - n)\Delta$$
$$\Gamma = X_s[\Delta + \frac{S_D}{f'}]$$
$$C_3 = S_D w$$
$$C_4 = S_D X_S$$
$$C_5 = S_D(r - n)b$$
$$\Phi_3^F = \hat{CV}^F \left[\frac{2}{C_3} - \frac{1}{C_3 - C_5}\right] > 0$$
$$\textit{iff} \;\; b < \frac{w}{2(r-n)} \;\; \text{or} \;\; r < n$$
$$\Phi_4^F = -\hat{CV}^F (\frac{1}{1 + C_4}) < 0$$
$$\Phi_5^F = -\hat{CV}^F (\frac{1}{C_3 - C_5}) > 0$$
$$\Phi_3^P = \frac{1}{1 + C_4} > 0$$
$$\Phi_4^P = -\frac{C_3 - C_5}{1 + C_4}(\frac{1}{1 + C_4}) < 0$$
$$\Phi_5^P = -\Phi_3^P < 0$$
$$dC_3 = [w\Delta - S_D k]dr$$
$$dC_4 = \Gamma dr$$
$$dC_5 = S_D(r - n)db + Ebdr$$

### 1.9.5 Optimal plans

Relative to the main paper we ignore heterogeneity and we switch the notation for savings from $a_{t+1}$ to $s_t^t$

$$\max_{c_t, c_{t+1}, x_{t+1}} V_t = U(c_t) + \beta U(c_{t+1}) + \delta U(x_{t+1}) \tag{1.47}$$

$$\text{s.t}$$

$$c_t + s_t^t = D_t \tag{1.48}$$

$$c_{t+1} + (1+n)x_{t+1} = (1+r_{t+1})s_t^t \tag{1.49}$$

$$c_t^t > 0, \;\; c_{t+1}^t > 0, \;\; x_{it+1} > 0 \;\text{ plus Inada Conditions}$$

where $D_t$ denotes disposable income, $x_{t+1}$ is the bequest, and $n$ population growth. The first order conditions are:

$$(1+n)\beta u'(c_{t+1}) = \delta u'(x_{t+1}) \tag{1.50}$$

$$U'(c_t) = \beta(1+r_{t+1})U'(c_{t+1}) \tag{1.51}$$

**The intertemporal elasticity of substitution**

Define $q_{t+1} \equiv \frac{c_{t+1}+(1+n)x_{t+1}}{c_t}$. From Euler equation (1.51)

$$U'(c_t) = \beta(1+r_{t+1})U'(c_{t+1}) \Rightarrow \tag{1.52}$$

$$U'(c_t) = \beta(1+r_{t+1})U'\left(\frac{\left[c_{t+1} + (1+n)x_{t+1} - (1+n)x_{t+1}\right]c_t}{c_t}\right) \Rightarrow \tag{1.53}$$

$$U'(c_t) = \beta(1+r_{t+1})U'\left(d_{t+1}c_t - (1+n)x_{t+1}\right) \Rightarrow \tag{1.54}$$

$$\frac{U'(c_t)}{U'\left(q_{t+1}c_t - (1+n)x_{t+1}\right)} = \beta(1+r_{t+1}) \Rightarrow \tag{1.55}$$

Using (1.51) and the definition for $q_{t+1}$ and rearranging

$$\frac{dq_{t+1}}{dr_{t+1}}\frac{r_{t+1}}{q_{t+1}} = -\frac{U'(c_{t+1})}{U''(c_{t+1})(c_{t+1} + (1+n)x_{t+1})} \tag{1.56}$$

where $\rho \equiv \frac{r_{t+1}}{q_{t+1}}$ is the IES in terms of future expenditures, in the main text $\theta \equiv \frac{1}{\rho}$. It is straight forward that:

$$\rho = -\frac{U'(c_{t+1})}{U''(c_{t+1})(c_{t+1} + (1+n)x_{t+1})} = -\frac{U'(c_{t+1})}{U''(c_{t+1})(c_{t+1})} = -\frac{U'(x_{t+1})}{U''(x_{t+1})(x_{t+1})}$$

### Properties of the Bequest function

from FOC define

$$\Gamma \equiv \delta U'(x_{t+1}) - (1+n)\beta U'(c_{t+1}) \tag{1.57}$$
$$\Gamma \equiv \delta U'(x_{t+1}) - (1+n)\beta U'\left[(1+r_{t+1})s_t - (1+n)x_{t+1}\right] \tag{1.58}$$

Therefore from (1.58) the optimal bequest is an implicit function of future interest rate and savings:

$$x^*_{t+1} = X(r_{t+1}, s_t)$$

The main scope is to figure out the marginal propensity to bequeath out of saving $X_s = \frac{\partial x*_{t+1}}{\partial s_t}$ and the effect of interest rates $X_r = \frac{\partial x*_{t+1}}{\partial r_{t+1}}$

Note that $\Gamma$ is an implicit function of $x_{t+1}, s_t,$ and $r_{t+1}$. Therefore we can write

$$\Gamma(x_{t+1}, s_t, r_{t+1}) = \delta U'(x_{t+1}) - (1+n)\beta U'\left[(1+r_{t+1})s_t - (1+n)x_{t+1}\right] \tag{1.59}$$

The partial effects of its main arguments are:

$$\Gamma_x \equiv \frac{\partial \Gamma(x_{t+1}, s_t, r_{t+1})}{\partial x_{t+1}} = \delta U''(x_{t+1}) + (1+n)^2 \beta U''(c_{t+1}) \ < 0 \tag{1.60}$$
$$\Gamma_s \equiv \frac{\partial \Gamma(x_{t+1}, s_t, r_{t+1})}{\partial s} = -(1+n)\beta U''(c_{t+1})(1+r_{t+1}) > 0 \tag{1.61}$$
$$\Gamma_r \equiv \frac{\partial \Gamma(x_{t+1}, s_t, r_{t+1})}{\partial r_{t+1}} = -(1+n)\beta U''(c_{t+1})s_t > 0 \tag{1.62}$$

By totally differentiating equation (1.59) someone gets:

$$d\Gamma(x_{t+1}, s_t, r_{t+1}) = \Gamma_x dx_{t+1} + \Gamma_r dr_{t+1} + \Gamma_s ds_t \tag{1.63}$$

Hence the marginal effects of savings and interests rates will be given by:

*Marginal Propensity to bequeath out of savings*

$$X_s = -\frac{\Gamma_s}{\Gamma_x}\bigg|_{dr=0} = \frac{(1+n)\beta U''(c_{t+1})(1+r_{t+1})}{\delta U''(x_{t+1}) + (1+n)^2\beta U''(c_{t+1})} > 0 \tag{1.64}$$

Note than in principle the marginal propensity to bequeath out of savings might exceed one. A sufficient condition for $X_s < 1$ is

$$0 < r_{t+1} - n < \frac{\delta}{\beta(1+n)} \frac{U''(x_{t+1})}{U''(c_{t+1})}$$

This condition imposes a bound on the *degree* of *dynamic efficiency*[28]. But, we can define the adjusted to dynamic efficiency marginal propensity to bequeath out of savings or the *modified marginal propensity to bequeath*

*modified marginal propensity to bequeath*

$$X_s = -\frac{\Gamma_s}{\Gamma_x} = \frac{(1+n)\beta U''(c_{t+1})(1+r_{t+1})}{\delta U''(x_{t+1}) + (1+n)^2\beta U''(c_{t+1})} \Rightarrow \tag{1.65}$$

$$\frac{1+n}{1+r_{t+1}} X_s = \frac{(1+n)^2\beta U''(c_{t+1})}{\delta U''(x_{t+1}) + (1+n)^2\beta U''(c_{t+1})} \Rightarrow \tag{1.66}$$

$$\frac{1+n}{1+r_{t+1}} X_s = \frac{1}{\underbrace{\left[\frac{\delta U''(x_{t+1})}{(1+n)^2\beta U''(c_{t+1})}\right]}_{+} + 1} < 1 \tag{1.67}$$

*the effects of interest rates on bequest*

$$X_r = -\frac{\Gamma_r}{\Gamma_x}\bigg|_{ds=0} = \frac{(1+n)\beta U''(c_{t+1})s_t}{\delta U''(x_{t+1}) + (1+n)^2\beta U''(c_{t+1})} > 0 \tag{1.68}$$

$$X_r = -\frac{\Gamma_r}{\Gamma_x}\bigg|_{ds=0} = \frac{s_t}{\frac{\delta U''(x_{t+1})}{(1+n)\beta U''(c_{t+1})} + (1+n)} \tag{1.69}$$

$$X_r = -\frac{\Gamma_r}{\Gamma_x}\bigg|_{ds=0} = \frac{(1+n)(k_{t+1}+b_{t+1})}{\frac{\delta U''(x_{t+1})}{(1+n)\beta U''(c_{t+1})} + (1+n)} \tag{1.70}$$

Thus, from (1.70) $\Rightarrow$

$$Xr = \frac{k_t + b}{Z+1} \Rightarrow (1+n)X_r = \frac{S(D_t, r_{t+1})}{Z+1} \tag{1.71}$$

with $Z = \frac{\delta U''(x_{t+1})}{(1+n)^2\beta U''(c_{t+1})}$. Also, from(1.64)

$$X_r = \frac{(1+n)(k_{t+1}+b_{t+1})}{1+r_{t+1}} X_s \equiv \frac{S(D_t, r_{t+1})}{1+r_{t+1}} X_s \tag{1.72}$$

where we used the financial markets clearing condition, $(1+n)(k_{t+1}+b_{t+1}) = S(D_t, r_{t+1})$. See next for $S(D_t, r_{t+1})$

**Properties of the Saving function**

From FOC define as:

[28] In the dynamic inefficient case this condition is *automatically* satisfied

$$\Phi \equiv \beta(1+r_{t+1})U'(c_{t+1}) - U'(c_t) \tag{1.73}$$
$$\Phi = \beta(1+r_{t+1})U'((1+r_{t+1})s_t - (1+n)x_t)) - U'(D_t - s_t) \tag{1.74}$$
$$\Phi = \beta(1+r_{t+1})U'\big[(1+r_{t+1})s_t - (1+n)\underbrace{X(r_{t+1}, s_t)}_{x^*_{t+1}}\big] - U'(D_t - s_t) \tag{1.75}$$

where in the last line we have substituted for the optimal bequest plan. From the last line, optimal savings is an implicit function of the disposable income and the interest rates:

$$s^*_t = S(D_t, r_{t+1})$$

Note that $\Phi$ is an implicit function of $s_t, D_t$and $r_{t+1}$. The scope is to find the marginal propensity to save out of disposable income $S_D = \frac{\partial s^*_{t+1}}{\partial D_t}$, and the effects of interest rates. As before, we can write:

$$\Phi(D_t, s_t, r_{t+1}) = \beta(1+r_{t+1})U'\left[(1+r_{t+1})s_t - (1+n)X(r_{t+1}, s_t)\right] - U'(D_t - s_t) \tag{1.76}$$

The partial effects of its main arguments are:

$$\Phi_D \equiv \frac{\Phi(D_t, s_t, r_{t+1})}{\partial D_t} = -U''(c_t) \tag{1.77}$$
$$\Phi_s \equiv \frac{\Phi(D_t, s_t, r_{t+1})}{\partial s_t} = \beta(1+r_{t+1})^2 U''(c_{t+1})\underbrace{\left[1 - \frac{1+n}{1+r_{t+1}}X_s\right]}_{(+)\ \text{from}(1.67)} + U''(C_t) < 0 \tag{1.78}$$

while

$$\Phi_r \equiv \frac{\partial\Phi(D_t, s_t, r_{t+1})}{\partial r_{t+1}} = \beta\big[(1+r_{t+1})U''(c_{t+1})[s_t - (1+n)X_r]) + U'(C_{t+1})\big] \tag{1.79}$$
$$= \beta U'(c_{t+1})\big[(1+r_{t+1})\frac{U''(c_{t+1})}{U'(c_{t+1})}[s_t - (1+n)X_r] + 1\big] \tag{1.80}$$

using (1.71)

$$= \beta U'(c_{t+1})\left[(1+r_{t+1})\frac{U''(c_{t+1})}{U'(c_{t+1})}[1 - \frac{1}{Z+1}]s_t + 1\right] \tag{1.81}$$
$$= \beta U'(c_{t+1})\left[(1+r_{t+1})\frac{U''(c_{t+1})}{U'(c_{t+1})}[1 - \frac{1}{Z+1}]s_t + 1\right] \tag{1.82}$$
$$= \beta U'(c_{t+1})\left[\frac{U''(c_{t+1})}{U'(c_{t+1})}(c_{t+1} + (1+n)x_{t+1})[1 - \frac{1}{Z+1}] + 1\right] \tag{1.83}$$
$$= \beta U'(c_{t+1})\left[1 - \theta\underbrace{[1 - \frac{1}{Z+1}]}_{+}\right] \tag{1.84}$$

where $-\theta = \frac{U''(c_{t+1})}{U'(c_{t+1})}(c_{t+1} + (1+n)x_{t+1})$ is the inverse of the IES,

The marginal propensity to save and the effects of interest rate is equal to

$$S_D = -\frac{\Phi_D}{\Phi_s}\Big|_{dr=0} = \frac{1}{\beta(1+r_{t+1})^2 \frac{U''(c_{t+1})}{U''(c_t)]} \underbrace{[1 - \frac{1+n}{1+r_{t+1}} X_s]}_{+} + 1} < 1 \tag{1.85}$$

$$S_r = -\frac{\Phi_r}{\Phi_s}\Big|_{dD=0} \qquad \text{can be either sign} \tag{1.86}$$

In the normal case, $S_r < 0$, which implies $\theta > 1 + Z$, with $Z > 0$. In this case, the income effect dominates the substitution effect when interest rates change.

### 1.9.6 Debt effects

Disposable income

$$D_t = I_t + x_t \tag{1.87}$$

$$I_t = I(r_t, \tau_t) \tag{1.88}$$

where $x_t$ is the bequest that the "young" receives and $I(r_t, \tau_t)$ is the after tax earnings function. Per capita taxes, $\tau_t$ are determined by the government's budget constraint. Note that the optimal plan for the bequest imply $x^*_{t+1} = X(r_{t+1}, s_t)$

Factor prices, from profit maximization (neoclassical production function)

$$w_t = f(k_t) - f'(k)k \tag{1.89}$$

$$1 + r_t = f' \tag{1.90}$$

The financial markets clearing condition is:

$$(1+n)(k_{t+1} + b) = S(D_t, r_{t+1}) \tag{1.91}$$

Total differentiation yields

$$(1+n)(dk_{t+1}+db) = S_D\Big[I_r dr_t + I_\tau d\tau_t + dX(r_t, s_{t-1})\Big] + S_r dr_{t+1} \tag{1.92}$$

$$(1+n)(dk_{t+1}+db) = S_D\Big[I_r dr_t + I_\tau d\tau_t + dX(r_t, s_{t-1})\Big] + S_r dr_{t+1} \tag{1.93}$$

$$(1+n)(dk_{t+1}+db) = S_D\Big[I_r dr_t + I_\tau d\tau_t + dX(r_t, s_{t-1})\Big] + S_r dr_{t+1} \tag{1.94}$$

$$(1+n)(dk_{t+1}+db) = S_D\Big[I_r dr_t + I_\tau d\tau_t + X_r dr_t + (1+n)X_s d(k_t+b)\Big] + S_r dr_{t+1} \tag{1.95}$$

$$I_t = w_t - \tau \Rightarrow \tag{1.96}$$

$$I_r \equiv \frac{\partial I_t}{\partial r_t} = -k_t \qquad I_t \equiv \frac{\partial I_t}{\partial \tau_t} = -1 \tag{1.97}$$

while government's budget constraint becomes $\tau_t = (r_t - n)b$. Hence

$$d\tau_t = (r_t - n)db + bdr \tag{1.98}$$

In consequence from (1.95):

$$\begin{aligned}(1+n)(dk_{t+1}+db) = S_D\Big[&-kdr_t - (r_t-n)db - bdr_t + \\ & X_r dr_t + (1+n)X_s d(k_t+b)\Big] + S_r dr_{t+1}\end{aligned} \tag{1.99}$$

$$\begin{aligned}(1+n)(dk_{t+1}+db) - S_r dr_{t+1} = S_D\Big[&-kdr_t - (r_t-n)db - bdr_t \\ &+ X_r dr_t + (1+n)X_s d(k_t+b)\Big]\end{aligned} \tag{1.100}$$

$$\begin{aligned}(1+n)(dk_{t+1}+db) - S_r dr_{t+1} &= S_D\Big[-kdr_t - bdr_t + X_r dr_t + (1+n)X_s d(k_t)\Big] \\ &+ S_D\big[(1+n)X_s - (r_t-n)\big]db\end{aligned} \tag{1.101}$$

$$\begin{aligned}[(1+n) - S_r f'']dk_{t+1} &= S_D\Big[-(k+b)f'' + X_r f'' + (1+n)X_s\Big]dk_t + \\ &+ \Big[S_D\big[(1+n)X_s - (r_t-n)\big] - (1+n)\Big]db\end{aligned} \tag{1.102}$$

<u>Stability condition</u>

$$\left.\frac{dk_{t+1}}{dk_t}\right|_{db=0} = \frac{S_D\Big[-(k+b)f'' + X_r f'' + (1+n)X_s\Big]}{[(1+n) - S_r f'']} < 1 \tag{1.103}$$

The effects of debt on capital

In the stationary state $k_t = k_{t+1} = k$. Therefore, the effect of debt on capital, from (1.102)

$$\frac{db}{dk} = \frac{\Big[S_D\big[(1+n)X_s - (r_t - n)\big] - (1+n)\Big]}{Q} \tag{1.104}$$

$$\frac{db}{dk} = \frac{S_D\Big[(1+n)X_s - \big(f' - (1+n)\big)\Big] - (1+n)}{Q} \tag{1.105}$$

$$\frac{db}{dk} = \frac{S_D[(1+n)X_s - f'] + S_D(1+n) - (1+n)}{Q} \tag{1.106}$$

Therefore

$$\frac{db}{dk} = \frac{\underbrace{S_D}_{+}\underbrace{[\frac{(1+n)X_s}{f'} - 1]}_{(-)\ from(1.67)} f' + \underbrace{(S_D - 1)}_{-}(1+n)}{Q} < 0 \tag{1.107}$$

where $Q > 0$ is the stability condition, assuming $S_r < 0$. Hence public debt crowds out capital. The above results extend Diamond's model with debt to account for the "joy-of-giving" bequest motive.

### 1.9.7 Welfare effects

In steady state $x_{t+1} = x_t = x$. The stationary problem using the life-time budget constraint is:

$$\max_{c_1, c_2, x} U = U(c_1, c_2, x) \tag{1.108}$$

s.t

$$c_1 + \frac{c_2}{1+r} + (n-r)\frac{x}{1+r} = I \tag{1.109}$$

where $c_1$ is the consumption of the young, and $c_2$ the consumption of the old. The optimal plans imply

$$U'(c_2) = \frac{U'(c_1)}{1+r} \tag{1.110}$$

$$U'(x) = \frac{U'(c_1)(n-r)}{1+r} \tag{1.111}$$

We now introduce heterogeneity and derive the welfare effects, for each tax system.

**Welfare Analysis: Lump-sum taxes**

The life-cycle budget constraint is:

$$c_1^i + \frac{c_2^i}{1+r} + (n-r)\frac{x^i}{1+r} = wl^i - \tau \tag{1.112}$$

the total change in budget constraint after some algebra becomes:

$$\begin{aligned} d(c_1^i) + \frac{1}{1+r}dc_2^i + \frac{n-r}{1+r}dx^i &= \\ -\frac{r-n}{1+r}(k+b)dr - (r-n)db &+ \left[ (\frac{s^i - S}{1+r}) + k(1-l^i) \right] dr \end{aligned} \tag{1.113}$$

The total change in utility for each individual is:

$$\begin{aligned} dU^i(c_1, c_2, x) = -u'(c_1^i)\Bigg[(r-n)(\frac{k+b}{1+r}dr + db) + \Bigg[ & \underbrace{[\frac{S}{1+r} - k]}_{\substack{\text{Aggregate change} \\ \text{in factor payments}}} \\ & - \underbrace{[\frac{s^i}{1+r} - kl^i]}_{\substack{\text{Individual change} \\ \text{in factor payments}}} \Bigg] dr \Bigg] \end{aligned} \tag{1.114}$$

$$\frac{dU^i(c_1, c_2, x)}{db} = -u'(c_1^i)\Bigg[(r-n)\underbrace{(\frac{k+b}{1+r}\frac{dr}{db} + 1)}_{+} + \Bigg[ \underbrace{[\frac{S}{1+r} - k]}_{\substack{\text{Aggregate change} \\ \text{in factor payments}}} - \tag{1.115}$$

$$\underbrace{[\frac{s^i}{1+r} - kl^i]}_{\substack{\text{Individual change} \\ \text{in factor payments}}} \Bigg] \frac{dr}{db} \Bigg] \tag{1.116}$$

The effect on the *average* welfare is given by:

$$\frac{dU(c_1,c_2,x)}{db} = -u'(c_1)\left[(r-n)\,\underbrace{(\frac{k+b}{1+r}\frac{dr}{db}+1)}_{+}\right] \tag{1.117}$$

The expression above replicates Diamond's results. In other words, in the dynamic inefficient case higher debt increases the *average family* utility. With similar steps we can arrive in the same expression and conclusion for the affine tax system.

**Welfare Analysis: proportional taxes**

Define $\tau' = w\tau l^i$ and note that $\tau = \frac{(r-n)b}{w}$ under similar steps as before. The total change in individual utility will be given by:

$$dU^i(c_1,c_2,x) = -u'(c_1^i)l^i\left[\left[(r-n)db + \left(\frac{(r-n)(k+b)}{1+r}\right)dr\right] - \left(\frac{\frac{s^i}{l^i}-S}{1+r}\right)dr\right] \tag{1.118}$$

The welfare effects on individual family becomes equal to:

$$\frac{dU^i(c_1,c_2,x)}{db} = -u'(c_1^i)l^i\left[\left[(r-n) + \left(\frac{(r-n)(k+b)}{1+r}\right)\frac{dr}{db}\right] - \left(\frac{\phi^i-S}{1+r}\right)\frac{dr}{db}\right] \tag{1.119}$$

where $\phi^i = \frac{s^i}{l^i}$. The effect on the *weighted* average welfare then is:

$$\frac{dU(c_1,c_2,x)}{db} = -u'(c_1)\left[(r-n) + \left(\frac{(r-n)(k+b)}{1+r}\right)\frac{dr}{db}\right] \tag{1.120}$$

Thus, the conclusions are the same as before.

# Chapter 2

# Optimal Taxation with Human Capital Risk and Aggregate Uncertainty

## 2.1 Introduction

During recessions governmental revenues usually collapse and the balance between rising taxes and restoring growth is hard. Effective labour, that is human capital, is essential component of growth, hence the stabilization of public finances is not orthogonal to the investment decisions in human capital. The latter type of asset is, however, a risky project from individual point of view and is subject to idiosyncratic risk. A decline in health, mortality risk or job-displacement risk are examples of negative human capital shocks. Improvements in the labour market, on the job training or internal promotions are examples of positive human capital shocks. Legal impediments restrict the possibility for risk-sharing and the inability to diversify idiosyncratic risk induces a precautionary savings motive.

If the government supplies the safe asset of the economy, and there exists another asset, such as physical capital, the idiosyncratic risk to human capital affects the composition of investments between, bonds, physical and human capital and by implication economic growth. The optimal manipulation of fiscal instruments then, will eventually determine the efficient path for growth. Therefore the main motivation and contribution of this paper is to study an optimal taxation problem within an endogenous growth model where the government in order to supply the safe asset (risk-free bond) taxes physical and human capital subject to idiosyncratic risk.

The share of human capital in individual portfolio is large, and the role of idiosyncratic human capital risk has being validated in the empirical literature only recently. For example, in Lustig and Nieuwerburgh (2008) the portfolio share of human capital wealth is between 0.77 %-0.8 % and similarly in Huggett and Kaplan (2015) is no less than 50 % over the life-cycle of a college graduate. The latter authors also document that a substantial amount of uninsurable human capital risk exists. Due to this riskiness the returns in human capital usually command a substantial premium over other assets (Palacios-Huerta (2003)). On the other hand, the assumption that the government bond is a risk-free asset, it also covers the equally likely empirical case. For instance, Marcet and Scott (2009) showed the behaviour of public debt inherit characteristics that are mostly attributed to a non-state contingent bond[1]. Accordingly, the spanning of the synthetic pay-off matrix of returns that is chosen as a basis for our analysis, has a reasonable degree of realism.

Our framework builds on Krebs (2006, 2003a) who extends the endogenous growth model of Jones and Manuelli (1990) to incomplete markets, on Toda (2014b) who generalizes these class of "Krebs" models in terms of preferences and the number of assets and on Gottardi et al. (2014b) who include a government sector. In our paper we further develop these class of models in two primary dimensions. The first one is methodological the second one quantitative. More specifically for the latter contribution, we show how to solve a Samuelson-Merton portfolio choice problem under borrowing constraints.

Gottardi et al. (2014b) were the first to study an optimal taxation and debt problem when human capital is subject to idiosyncratic risk but abstract from aggregate uncertainty. This assumption, however, we believe is restrictive in two important dimensions. Firstly, random disturbances do affect the economy, as for instance, the recent financial crisis proved. Secondly, it restricts the pay-off matrix of returns between government bonds and physical capital to be collinear. The latter simplification relegates government bonds to be no different from interest bearing money. Therefore the provision of such bonds is determined by a simple arbitrage argument between borrowing costs and the rate of economic growth resembling in this sense what Diamond (1965) first taught us.

In contrast in our framework the provision of non-state contingent bond and the idiosyncratic risk to human capital it creates an oxymoron trade-off similar to the one first Shin (2006) pointed. To the extent that the reference point is idiosyncratic uncertainty, higher insurance against that risk it implies that individuals want to lend to the government, but higher insurance against the aggregate risk it is the government that wants to lend to the consumers. There are three however fundamental differences between our paper and the exposition in Shin (2006). First, he focuses on endowment risk while our focus is on investment risk. Second, he abstracts from the role of prices and therefore from other forms of insurance while in our case we don't. Third, the distribution of assets in his paper matters but in our case does not. In this respect, we view our arguments here as complements. Nevertheless, our main argument highlights the pitfalls that the proposition of Shin (2006) might hide and in particular the role of prices as an insurance mechanism[2].

In our methodology, the taxes and allocations by the planner are engineered in

[1] This argument also extends to multiple maturities, see Faraglia et al. (2014a).

[2] Our communication with Shin revealed that the author proceeded to a major overhaul of his 2006 paper. Sadly, the new version is not yet in a state to be circulated. Therefore under this reservation our reflections is based on his 2006 manuscript.

way akin to a "Ramsey planner". That is, for the exogenously specified instruments, the planner calculates the "best" path of those instruments such as to maximize the welfare for each agent. For simplicity we assume a linear taxation problem, where the government taxes the returns of the physical assets. However, in contrast to the tradition in the optimal fiscal policy literature, we assume partial commitment. This type of equilibrium is by construction time-consistent although the implicit assumption here is that the commitment of each government does not extend beyond one period. This assumption serves two main purposes, first predetermined policies and in particular taxes restrict any government to complete the markets, and second it can be though as a reasonable approximation to the commitment technology that occurs in practice.

Furthermore, to retain tractability the model features three crucial assumptions: a) The investment in human capital is viewed as a cost rather than time investment, therefore we abstract from any labour-leisure trade-off and focus on effective labour hours, b) the constant returns to scale production function in physical and human capital and c) idiosyncratic risk is not a relevant state variable. The last assumption lead to a solution where aggregation is exact, but beside this abstraction idiosyncratic risk still matters. As we will show the presence of idiosyncratic risk to human capital affects agents' asset valuations and thus the price of aggregate risk. In consequence this type of risk does not lose its relevance for the design of policies. The assumption of unpredictable idiosyncratic risk in turn implies that the solution to the economy of heterogeneous agents is also the solution to the problem of one agent facing two types of *uninsurable* risks.

Turning on our main findings, the solution of the optimal plans is isomorphic to case where the planner becomes a "Wall Street" investor by taking the command of a large stock market. The planner then, assembles his portfolio by trading assets of different productivity and insurance properties. Limiting our analysis to the special case of logarithmic utility his objective becomes equivalent in maximizing the after tax portfolio returns. To achieve this objective, the planner replaces the conceptual single agent and tries to allocate his portfolio shares in way that is an optimal response against the effective uncertainty, that of uninsurable aggregate and idiosyncratic risk. Since these two types of risks determine the composition of the investments and the fluctuations of prices, the optimal allocation mimics the behaviour of agents who tries to protect his most risky investment. Our results are organized around this particular insight.

For the government it is optimal to accumulate assets. That is provide liquidity or be the net lender of the economy. This is optimal in our framework for two reasons a) due to the fiscal hedging needs to secure revenues against the aggregate uncertainty b) because idiosyncratic shocks are permanent events. For public policy this implies that government debt is an imperfect mean for self-insure. This particular proposition is strengthened under our special case of log utility. In this specific case, agents are approximately "risk neutral" and borrowing by the government in order to leverage human capital is an optimal response. The government in return will accommodate this through a reallocation of physical assets in way that protect its returns. The policy prescription then uncovers the first principles of public finances where less productive sources of income are subsidised and the risky ones insured. That is physical capital must be subsidised while human capital be taxed. Accordingly, the engine of growth comes through an efficient allocation of resources in the most productive asset. This particular insight determines the optimal path in the event of a negative productivity shock or a "stock Market crash". Front loading human capital in the short run and protecting its returns along the transition to the new growth path achieves an effective mean to promote welfare and stabilize the public finances.

## 2.2 Related Literature

A voluminous literature on the optimal taxes and government debt addressed similar concerns. An extensive survey goes beyond the scope of this project and therefore the focus will be directed to the most relevant ones. Chamley (1986) within a standard optimal growth model and its extension by Judd (1985) to heterogeneous agents, both showed that the tax on physical capital should be zero in the long-run. Jones et al. (1997) argue that similar result applies to human capital. Aiyagari (1995) was the first to study an optimal capital tax problem within an environment when markets are incomplete. Abstracting from any aggregate uncertainty and in contrast to earlier studies he finds that physical capital should be taxed in the long-run. As Reis and Panousi (2012) showed, the former result it might also survive in a different form of market incompleteness where entrepreneurs are subject to investment risk. Their result however depends on the degree of financial deepness.

Barro (1979) was the first to conjecture the property of public debt as shock absorber and its ability to smooth taxes over time. However, in a context where the government has instruments sufficient enough to insure against aggregate risk, Lucas and Stokey (1983) delimited the argument of Barro (1979), but later in Aiyagari et al. (2002) his insights were reinstated. Zhu (1992) extended the result in Chamley (1986) to the case of uncertainty, while Chari et al. (1994) quantitative assess those in a business cycle model. Similarly, Farhi (2010) extends the stochastic growth model to the case of non-state contingent bond, while Karantounias (2013) generalize it to recursive preferences although he maintains the assumption of complete markets.

Finally, Bassetto (2014) extends the environment of Lucas and Stokey (1983) to include heterogeneous agents and effectively studies how pareto weights influence the optimal labour tax responses of the planner to expenditures and other shocks. Werning (2007) also considers the complete markets economy of Lucas and Stokey (1983), but extends it to include heterogeneous agents, accumulation of physical assets and lump-sump taxes unrestricted in sign and characterizes the optimal allocations and distortions in this economy. Bhandari et al. (2013) retain an environment where agents have precautionary savings, but in contrast to this paper, they first abstract from any physical assets and second they focus on idiosyncratic labour endowments while in our case we focus on effective labour.

## 2.3 Model

The model considers an economy of aggregate and idiosyncratic shocks. Time is discrete and is indexed by $t = 0, 1, 2....$ The economy consists of three types of agents, a continuum of infinite lived consumers of measure one indexed by $i \in [0, 1]$, a single representative firm and a government. There is one (homogeneous and non-perishable) good in the economy produced by a neoclassical technology, $F$. The firm employs physical capital $K_t$ and human capital $H_t$ to produce $Y_t$ units of output. Human capital should be thought in terms of units per effective labour. Labour endowments are inelastically supplied and for simplicity are normalized to one. Therefore, the accumulation of efficiency units of labour and human capital is used interchangeably and it is the source of growth in this model[3]. The markets for hiring human and physical capital are competitive. The production function is further assumed to be of CRS with standard (neoclassical) assumptions. In total, the production function is calibrated as $Y_t = A_t K_t^{\alpha} H_t^{1-\alpha}$, where $A_t$ is the total factor productivity. Each period the firm hires human and physical capital up to point where profits are maximized. The optimality conditions for the firm are:

$$r_t = F_{k_t} \tag{2.1}$$

$$w_t = F_{h_t} \tag{2.2}$$

where $r_t$ and the $w_t$ are the returns for physical and human capital and $F_{k_t}, F_{h_t}$ their respective marginal products. $A_t$ is a source of aggregate uncertainty and is calibrated to follow an AR(1) process. TFP shocks rule out the returns of bonds and physical capital to be collinear. The law of motion for the productivity shocks is:

$$\log A_t = \rho \log A_{t-1} + (1-\rho) \log \bar{A} + e_t^A \qquad e_t^A \sim N(0, \sigma_A^2) \tag{2.3}$$

$\rho$ is the degree of persistence of the TFP shocks and $\bar{A}$ its mean. The government in order to finance a random path of (unproductive) government spending, raises revenues through taxes on physical and human capital income and issues the (short-lived) risk-free asset of the economy. We let $\tau_t^k$ and $\tau_t^w$ be the marginal tax rates of those incomes. Taxes on both capital goods are assumed to be announced one period in advance. Section 2.4 discusses in some detail the particular measurability assumption. Sluggishness in the legislation process could justify this inertia in fiscal policy.

Individuals on the other hand accumulate, physical and human capital according to the following laws of motion:

[3]The model, as first developed by Krebs (2006), is an Incomplete Markets version of the special class of convex growth models, as for instance in Alvarez and Stokey (1998), Jones and Manuelli (1990) or King and Rebelo (1990). See Acemoglu (2009, p.393) for the preliminaries.

$$\text{Phsyical capital:} \qquad k_{it+1} = I_{ik_t} + (1-\delta)k_{it} \tag{2.4}$$

$$\text{Human capital:} \qquad h_{it+1} = I_{ih_t} + (1-\delta_t^i)h_{it} \tag{2.5}$$

where $k_{it}$ and $h_{it}$ are the stocks of physical and human capital respectively, $I_{ik_t}, I_{ih_t}$ the equivalent investments in those assets, $\delta_t^i$ is the idiosyncratic depreciation shock to human capital and $\delta$ the depreciation rate of physical capital. For simplicity, as in Krebs (2003b) we assume that $\delta_t^i = \delta + \eta_{it} > -1$ where $\eta_{it}$ is a normal random variable identically and independently distributed across individuals and time with the mean normalized to zero and some variance $\sigma_\eta^2$. Idiosyncratic shocks is further assumed not to cause any fluctuations on the aggregate and hence are uncorrelated with the macroeconomic shocks. Individuals, use their human capital to work for the firm and lend their physical asset to earn interest. In dynamic programming form the maximization problem of the consumers is[4]:

$$V_t^i(x_{it}) = \max_{c_{it}, b_{it+1}, k_{it+1}, h_{it+1}} \left[ \exp\left( (1-\beta)\log c_{it} + \beta E_t \log\left[V_{t+1}^i(x_{it+1})\right] \right) \right] \tag{2.6}$$

s.t

$$c_{it} + b_{it+1} + k_{it+1} + h_{it+1} = \underbrace{R_{t-1}^f b_{it} + R_t^k k_{it} + R_t^i h_{it} + T_t^i}_{x_{it}} \tag{2.7}$$

$$x_{it} \geq 0 \tag{2.8}$$

$$k_{i0}, h_{i0}, b_{i0} \;\; \text{given}$$

$$R_{-1}^f = R_0^f, \tau_{-1}^k = \tau_0^k, \;\; \tau_{-1}^w = \tau_0^w \;\; \text{given} \tag{2.9}$$

where $R_t^k \equiv 1 + (1-\tau_t^k)r_t - \delta$ is the gross rate of return of physical capital net of taxes, $R_t^i \equiv 1 + (1-\tau_t^w)w_t - \delta + \eta_t^i$ is the equivalent idiosyncratic returns from human capital. Consumers also have the option to invest into a safe bond supplied by the government. One unit of bonds purchased in period $t$ delivers $R_t^f$ units next period. The time index follows the definition for the risk free asset when no re-selling (or pay-back) is possible [5]. In addition, agents might receive a lump-sum amount of transfers $T_t^i \equiv \tau_t b_{it} + \tau_t k_{it} + \tau_t h_{it}$ proportional to their investments. Their purpose will become clear in later sections. The variable $x_{it}$ is the returns inclusive wealth of the consumers which we assume is positive. This assumption ensures that the solution to the maximization problem exists. We can think of this condition as a natural borrowing limit that does not bind for all agents, periods and histories.

An additional remark is in order. Assuming that the interpretation of the id-

[4]we consider the Bellman equation a special case of the class of recursive (Epstein-Zin) preferences described in Toda (2014b). The coefficient, $1-\beta$ ensures the value function being homothetic.

[5]The standard relationship between the returns of the risk-free bond and its value is $R_t^f \equiv \frac{1}{Q_t}$Recall the definition of returns for any asset between $t-1$ and $t$, is $R_t = \frac{P_t + D_t}{P_{t-1}}$, where $P_t$ is the price of reselling that asset in the current period, and $D_t$ the dividend payment. For the risk free asset, the dividend payments is assumed to be constant and normalized to one.

iosyncratic shock is a particular loss of skills of a worker, then the wage rate paid in period $t$, we can think of it as the permanent wage differential *before* and *after* the event that led to that particular loss (e.g job-displacement). Empirically, this kind of wage differential is large (See Neal (1995), Topel (1991) or Jacobson et al. (1993)). However, the implicit assumption of this interpretation is that we effectively abstract for any wages forgone within the period of, for example, unemployment. It is instructive to derive the FOCs of the problem in (2.6). Those will only have an instrumental role for some of the proofs that follow. In particular the optimal conditions are:

$$(c_{it})^{-1} = \beta \mathrm{E}_t \left[ (c_{it+1})^{-1} R^k_{t+1} \right] \tag{2.10}$$

$$(c_{it})^{-1} = \beta \mathrm{E}_t \left[ (c_{it+1})^{-1} R^i_{t+1} \right] \tag{2.11}$$

$$(c_{it})^{-1} = \beta \mathrm{E}_t \left[ (c_{it+1})^{-1} \right] R^f_t \tag{2.12}$$

To solve the model, is more convenient to convert consumers' problem into a standard portfolio choice allocation in three assets. In the sequence, we will use the following definitions:

$$\text{Total Investments:} \quad s_{it+1} = b_{it+1} + k_{it+1} + h_{it+1} \tag{2.13}$$

$$\text{Share of debt:} \quad \phi_{it} = \frac{b_{it+1}}{s_{it+1}} \tag{2.14}$$

$$\text{Share of physical capital:} \quad \theta^k_{it+1} = \frac{k_{it+1}}{s_{it+1}} \tag{2.15}$$

$$\text{Share of human capital:} \quad \theta^h_{it+1} = \frac{h_{it+1}}{s_{it+1}} \tag{2.16}$$

$$\text{Accounting Balance:} \quad \phi_{it+1} + \theta^k_{it+1} + \theta^h_{it+1} = 1 \tag{2.17}$$

Using the above definitions, the budget constraint of the individuals can be converted to:

$$x_{it+1} = R_{ix_{t+1}}(x_{it} - c_{it}) \tag{2.18}$$

**Proof.** The returns inclusive wealth in period t by construction is $x_{it} \equiv R^f_{t-1} b_{it} + R^k_t k_{it} + R^i_t h_{it}$, substituting the definitions of the previous expression becomes $x_{it} = (R^f_{t-1}\phi_{it} + R^k_t \theta^k_{it} + R^i_t \theta^h_{it}) s_{it}$, thus the budget constraint can also be written as $c_{it} + s_{it+1} = x_{it}$ or $s_{it+1} = x_{it} - c_{it}$. Future wealth, by definition is $x_{it+1} = R_{x_{it+1}} s_{it+1}$, substituting for savings we get the equation in (2.18), where $R_{ix_{t+1}} \equiv R^f_t \phi_{it+1} + R^k_{t+1}\theta^k_{it+1} + R^i_{t+1}\theta^h_{it+1}$ are the idiosyncratic gross portfolio returns net of taxes. ■

In the transformed problem, agents maximize (2.6) subject to (2.18) and (2.17), where in the maximization problem asset levels are replaced by their respective shares and the maximization problem becomes a standard Samuelson (1969)-Merton (1969) portfolio choice problem in three assets. The solution to consumers problem is summarized by the lemma below:

**Lemma 2** *Assuming individual shocks do not enter the information set of the agents, a semi-closed solution with symmetric portfolio shares exist and is defined recursively as follows:*

$$\begin{aligned} \textit{Modified Euler / Welfare:} \quad & \log \nu_t = \kappa + \beta E_t[\log \nu_{t+1}] + \beta E_t[\log R_{x_{it+1}}] & (2.19) \\ \textit{Consumption:} \quad & c_{it} = (1-\beta) x_{it} & (2.20) \\ \textit{Total investments:} \quad & s_{it+1} = \beta x_{it} & (2.21) \end{aligned}$$

$$\begin{aligned} \textit{Plan for bonds:} \quad & b_{it+1} = \beta \phi_{t+1} x_{it} & (2.22) \\ \textit{Plan for Physical Capital:} \quad & k_{it+1} = \beta \theta^k_{t+1} x_{it} & (2.23) \\ \textit{Plan for Human Capital:} \quad & h_{it+1} = \beta \theta^h_{t+1} x_{it} & (2.24) \\ \textit{Growth of individual Wealth:} \quad & x_{it+1} = R_{ix_{t+1}} \beta x_{it} & (2.25) \end{aligned}$$

$$\begin{aligned} \textit{Euler (2.10):} \quad & 1 = E_t\left[R^{-1}_{x_{it+1}} R^k_{t+1}\right] & (2.26) \\ \textit{Euler (2.11):} \quad & 1 = E_t\left[R^{-1}_{x_{it+1}} R^i_{t+1}\right] & (2.27) \\ \textit{Euler (2.12):} \quad & 1 = E_t\left[R^{-1}_{x_{it+1}}\right] R^f_t & (2.28) \\ \textit{Market clearing:} \quad & \phi_{t+1} = 1 - \theta^k_{t+1} - \theta^h_{t+1} \qquad \forall t, S^t & (2.29) \end{aligned}$$

*where $\kappa$ in* (2.19) *is an unimportant constant.*

**Proof.** See Appendix 2.8.1. ■

Notice, in the above lemma we have dropped the individual index on the shares of assets. This is a direct consequence of the homothetic preferences and the assumption of unpredictable, and thus permanent, idiosyncratic shocks. The latter assumption it is consistent with a large body of empirical evidence found in the literature. For instance, in Deaton and Paxson (1994) consumption inequality, within a given cohort, increases over the life-cycle. This patter can be naturally supported by a permanent idiosyncratic component in individual consumption process. More recently Meghir and Pistaferri (2004) or Storesletten et al. (2007) among others, empirically document a similar permanent uninsurable risk to individual earnings. Accordingly, the assumption of unpredictable idiosyncratic risk although it seems strong, is not completely unrealistic.

On the other hand, the construction of symmetric portfolio shares has being analysed extensively in the literature and goes back to the "no trade" result of Constantinides and Duffie (1996). Intuitively, the marginal value of income is independent of individual wealth and in consequence every consumer, unable to extract information from idiosyncratic state, will choose the same portfolio share. For a recursive equilibrium to exist, it entails that each agent lives in autarky. It is immediate then that the model is equivalent to the solution of one agent facing *two different types* of shocks. Since the framework builds on Toda (2014b) and Krebs (2003a, 2006) rigorous proofs can be found therein.

**Idiosyncratic Risk Matters:** Equations (2.26)-(2.28) can be combined to get the standard arbitrage equations of CAPM models.

$$E_t\left[m^i_{t+1}\right]R^f_t = E_t\left[m^i_{t+1}R^k_{t+1}\right] \Rightarrow \tag{2.30}$$

$$E_t\left[R^k_{t+1}\right] - R^f_t = -\frac{\text{Cov}(m^i_{t+1}, R^k_{t+1})}{E_t[m^i_{t+1}]}$$

$$E_t\left[m^i_{t+1}R^i_{t+1}\right] = E_t\left[m^i_{t+1}R^k_{t+1}\right] \tag{2.31}$$

where $m_{it+1} \equiv R^{-1}_{x_{it+1}}$ is the stochastic discount factor (SDF) or the pricing kernel. This definition it implicitly assumes that the prices of assets rely on the *objective* probabilities of each state. An alternative definition which corrects for the risk-attitude of the consumers is to express the pricing of assets in terms of *risk-adjusted or risk-neutral* probabilities. This can happen in our example if we define the SDF to be $m^*_{it+1} \equiv \frac{R^{-1}_{x_{it+1}}}{E_t[R^{-1}_{x_{it+1}}]}$. Notice also that the discount factor depends only on $\eta_{it+1}$ and *not* on its value in the previous period. This property has two pleasant reverberations: a) the risk-free rate is identical across agents and b)if physical capital was owned by firms no disagreement would exist about the objective of the firm even if markets are incomplete. Jointly these two imply that for discounting we can use the pricing kernel of any agent.

Equation (2.30) proves that in general idiosyncratic risk matters. All else being equal, an increase in idiosyncratic risk will reduce investment in human capital, $\theta^h$, and increase $\theta^k$ (and $\phi$). This reallocation of portfolio choices affects the composition of investments and thus factor prices. Then, by the second line in (2.30) idiosyncratic risk affects the risk-premium (through $m^i$) and thus the price of assets. In Krueger and Lustig (2010) idiosyncratic risk seems irrelevant for the pricing of assets. However, their assumption of exchange economy seems to be strong. In fact Toda (2014a) showed that their result relies on the number of aggregate shocks. However, the latter is isomorphic to the case of endogenous factor prices. Since policies are in essence affected by the price of aggregate risk, idiosyncratic shocks indeed provide the additional bite for their design. The next proposition summarizes the main components of the preceding discussion.

**Proposition 1** *Individual consumption and labour income conditional on the aggregate state follow an approximate random walk, while agents under the assumption of logarithmic utility are approximately "risk neutral".*

**Proof.** From Lemma 2 individual consumption growth is $\frac{c_{it+1}}{c_{it}} = \frac{x_{it+1}}{x_{it}} = R_{x_{it+1}}$ taking logs and using the approximation $\log(R_{x_{it+1}}) \approx \mathcal{A}_{t+1} + \ \tilde{\eta}_{it+1}$ (2.29) we get

$$\log c_{it+1} \approx \log c_{it} + \mathcal{A}_{t+1} + \tilde{\eta}_{it+1} \tag{2.32}$$

Similarly, define $y_{it} \equiv \tilde{w}_t h_{it}$ as the after tax labour income so that the growth rate of the income process is $\frac{y_{it+1}}{y_{it}} = \frac{\theta_{t+1}}{\theta_t}\frac{x_{it+1}}{x_{it}} = \frac{\theta_{t+1}}{\theta_t}R_{x_{it+1}}$. Taking logs and using the same approximation for the gross portfolio returns, the labour income process is:

$$\log y_{it+1} \approx \log y_{it} + \mathcal{G}_{t+1} + \tilde{\eta}_{it+1} \tag{2.33}$$

where $\mathcal{A}_{t+1} \equiv (R_t^f - 1 - \delta)\phi_{t+1} + \tilde{w}_{t+1}\theta^h_{t+1} + \tilde{r}_{t+1}\theta^k_{t+1}$ and $\tilde{\eta}_{it+1} \equiv \eta_{it+1}\theta^h_{t+1}$, $\mathcal{G}_{t+1} = \mathcal{A}_{t+1} + \log(\frac{\tilde{w}_{t+1}\theta_{t+1}}{\tilde{w}_t\theta_t})$ and $\tilde{w}_{t+1}, \tilde{r}_{t+1}$ are the after tax prices. ■

Proposition 1 delivers some strong policy implications and will become the basis for our analysis and the interpretation given in the subsequent sections. First, under the special case of the log utility the marginal value of wealth (equivalent to the coefficient $\nu_t$) is independent from the individual consumption process. Comparing with the more conventional growth models, this case is similar but *not identical* to a risk-neutral agent. However since idiosyncratic risk still matters and it affects the composition of investments, policies then will determine the efficient path of growth. Therefore, consumption smoothing under the special case of log utility is replaced by an intertemporal problem between “allocating growth” across time and inratemporal one by allocating investments within each period.

Furthermore, the logarithmic random walk in the labour income it confirms that idiosyncratic shocks are permanent events. This implies that government debt although desirable is an *ineffective or imperfect* mean for self-insurance. In addition, the assumption of non-contingent bond implies that insurance against the aggregate state is also missing. Since idiosyncratic risk matters, fluctuations in prices (implicit in the term $\mathcal{G}_{t+1}$) they do have welfare effects. For instance, the real source for the important welfare effects of business cycles that Krebs (2003a) found in his paper, can be reasonably attributed to the inefficient price levels along the cycle.

### 2.3.1 Aggregate equilibrium

The assumptions of the model leads to solutions that are linear in wealth, therefore the distribution of wealth is not a relevant state variable and aggregation is exact. Market clearing conditions and lemma (2) imply:

$$\int_0^1 c_{it}\, d(i) = C_t,\ \int_0^1 k_{it}\, d(i) = K_t,\ \int_0^1 h_{it}\, d(i) = H_t,\ \int_0^1 b_{it}\, d(i) = B_t \Rightarrow \tag{2.34}$$

$$C_t = (1-\beta)X_t,\ \ B_{t+1} = \beta\phi_{t+1}X_t,\ \ H_{t+1} = \beta\theta^h_{t+1}X_t,\ \ K_{t+1} = \beta\theta^k_{t+1}X_t \tag{2.35}$$

Aggregate wealth then evolves as:

$$X_{t+1} \equiv E_t[x_{it+1}] = E_t[R_{x_{it+1}}]\beta X_t \tag{2.36}$$

Equation (2.36) is a novel application of the law of large number where for large economies averages converge to their means and $\beta E_t[R_{x_{it+1}}]$ is the endogenous growth rate. The budget constraint of the government is

$$B_{t+1} + \tau_t^k r_t K_t + \tau_t^w w_t H_t \geq R_{t-1}^f B_t + G_t \tag{2.37}$$

We also assume a simple rule for government spending of the form $G_t = g_t \beta X_{t-1}$, which states that the government is consuming a time varying fraction of aggregate wealth. The purpose of this rule is to render the solution well defined[6]. Beside the TFP shocks, the time-varying fraction $g_t \in (0, 1)$ is another source of aggregate uncertainty and follows the law of motion in equation (2.38) below.

$$\log g_t = \rho_g \log g_{t-1} + (1 - \rho_g) \log \bar{g} + e_t^g \qquad e_t^g \sim N(0, \sigma_g^2) \tag{2.38}$$

where $\rho_g$ is the degree of persistence and $\bar{g}$ is the mean of the process. As in Aiyagari et al. (2002) define the surplus (net of interest payments, $R^f \equiv 1 + r_t^f$) of the government as, $\mathcal{S}_t = \tau_t^k r_t K_t + \tau_t^w w_t H_t - G_t - r_t^f B_t$ so that the budget constraint can be also re-written as $B_t \leq \mathcal{S}_t + B_{t+1}$. If this constraint holds with inequality we let the difference be an equally shared between the investments of the household lump-sum amount $T_t \equiv \tau_t B_t + \tau_t H_t + \tau_t K_t$, returned back to them. That possibility might arise when the precautionary motive of the government is so strong that, the buffer stock of assets accumulated the period before, $B_t$, might be sufficiently high to accommodate any random disturbance[7]. The equality between transfers is done for simplicity and it guarantees to leave the Euler equations unaffected. As in the case of government expenses, proportionality to investments is essential for the existence of equilibria. Notice that the market clearing condition implies $\int_0^i T_t^i d(i) = T_t$.

From Lemma 2 and (2.35), (2.34) , the *implementatality* constraint for the government is:

$$\phi_{t+1}\beta R_{x_t} \geq R_{t-1}^f \phi_t + g_t - \tau_t^k r_t \theta_t^k - \tau_t^w w_t \theta_t^h \tag{2.39}$$

$$\underline{\mathcal{B}} \leq R_t^f \phi_{t+1} \leq \overline{\mathcal{B}}, \tag{2.40}$$

where $R_{x_t} \equiv E_{t-1}[R_{x_{it}}]$ is growth rate in the current period. Finally, I also assume that the government is subject to some *ad hoc* debt limits of the form $\underline{\mathcal{B}} \leq R_t^f \phi_{t+1} \leq \overline{\mathcal{B}}$ where $\underline{\mathcal{B}}, \overline{\mathcal{B}}$ are arbitrary constants[8]. This assumption closely follows Farhi

[6]For an equilibrium to exist in this framework it requires multiplicative shocks to investment decisions, unless this rule on government spending is not imposed no equilibrium exists. An alternative rule could be the more intuitive $G_t = g_t Y_t$. we chose the one in the main text for two main reasons, first to restrict the planner to directly affect his revenues through output and second for analytical convenience.

[7]In general this possibility of transfers cannot be ruled out whenever a non-state contingent bond is traded, see for example Ljungqvist and Sargent (2004, p.517-518) and the discussion there in.

[8]For an extensive discussion about the role of imposing similar constraint under incomplete markets,

(2010) or Faraglia et al. (2014b) who in the context of Incomplete markets -missing insurance for the aggregate risk- they impose similar debt limits. The motivation for the particular form of the debt limits is to ensure the growth of wealth to be positive. The scope of the lower bound is to restrict the government financing all of its expenses by accumulating assets and the upper bound rules out Ponzi-schemes[9].

The feasibility constraint then implies that the resources left from production are divided between consumption of the private sector, consumption for the government and investments in the capital goods.

$$C_t + K_{t+1} + H_{t+1} + G_t = Y_t + (1-\delta)K_t + (1-\delta)H_t \tag{2.41}$$

**Definition 3** *For given $x_{i0}$, policies $\{\tau_t^w, \tau_t^k\}_{t=0}^{\infty}$ and exogenous processes $\{g_t, A_t, \eta_{it}\}_{t=0}^{\infty}$, a (stationary) competitive equilibrium with unpredictable idiosyncratic risk is a collections of returns $\{r_t, w_t, R_t^f\}_{t=0}^{\infty}$, such as equations* (2.39), (2.40) *are satisfied, the financial market clearing condition* (2.29) *holds, asset shares are chosen optimally* (2.30), (2.31) *firms maximize profits $r_t = F_{k_t}(\theta_t^k, \theta_t^h)$, $w_t = F_{h_t}(\theta_t^k, \theta_t^h)$ $\forall$ $t$ and the individual welfare, allocations and growth of assets are determined by equations* (2.19)-(2.25).

## 2.4 Planner's Problem

We take a utilitarian social welfare function, where each agent is equally valued. More formally, the welfare function is:

$$\mathbb{E}_0\left[\int_0^1 V_0^i(x_{i0})\, d(i)\right] = \mathbb{E}_0\left[\nu_0 \int_0^1 x_{i0}\, d(i)\right] \tag{2.42}$$

in (2.42) we used the solution for the Bellman equation, $V(x_{it}) = \nu_t x_{it}$ (See Appendix 2.8.1 ). Recall that $x_{i0}$ is given. We also normalize $\int_0^1 \eta_{i0}h_{i0} = 0$ and assume $\tau_{-1}^k = \tau_0^k, \tau_{-1}^w = \tau_0^w, R_{-1}^f = R_0^f = 1$. The latter assumption is common in the literature and its purpose is to restrict the planner to use this period in his advantage by taxing assets that are inelastically supplied. From equation (2.42) it is clear that life time utility is maximized if the welfare coefficient $\nu_0$, common across agents, is maximized in every period and contingencies.

Since $\log \nu_t$ is a monotone transformation of $\nu_t$, the ordinal ranking of welfare is not affected. To determine a particular objective function for the planner we use the law of motion for the welfare coefficient, $\log \nu_t = \mathrm{k} + \beta \mathrm{E}_t[\log(R_{x_{it+1}})] + \beta \mathrm{E}_t[\log \nu_{t+1}]$, and solve forwards. Omitting the unimportant constant k, assuming that $\beta < 1$, $\lim_{t\to\infty} |\nu_t| < \infty$

see Faraglia et al. (2014a,b). In Krebs and Wilson (2004) the authors discuss that some borrowing constraints might be necessary for an equilibrium to exist. Toda (2014b) made that argument more formal and his semi-closed solutions that we used in Appendix for the consumers problem, allows *arbitrary* constraints in borrowing and lending. In practice no solution was possible without those constraints.

[9] Technically, no solution to the maximization problem can exist unless some borrowing constraints are imposed.

and imposing the transvesality condition $\lim_{T \to \infty} \beta^{t+T}$
$\mathrm{E}_t[\log \nu_{t+T+1}] = 0$, the solution to this stochastic difference equation implies the following objective for the planner:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^{t+1} E_t[\log R_{x_{it+1}}] \tag{2.43}$$

In other words, the life-time welfare of each agent is maximized when the *certainty equivalent* of the (log of the) gross portfolio returns, $E_t[\log R_{x_{it+1}}]$, is maximized in every period and up to the infinite future. Notice how under the special case of the log utility the results in Proposition 1 are passed in the objective of the planner. Managing an efficient path for portfolio returns is equivalent to allocate a rate of growth for individual wealth. Comparing with standard financial models, this is very similar to an agent that obtains utility by "consuming his wealth". The instruments of the government to manage the composition of investments are the risk-free rate and the taxes on physical and human capital.

In general and in any model involving policy decisions and rational expectations, *current equilibrium* is a function of the present state, the policy decisions in this state, and *future policy actions*. Since the competitive equilibrium is indexed by the chosen policies, consumers to evaluate the future need only to know a consistent rule for those policies. For analytical and computational tractability we focus on partial commitment equilibria which by construction are time-consistent. In particular we extend the approach taken in Klein and Rios-Rull (2003) where each government inherits the tax and policy rates of the previous one, it observes the current realization of shocks and announces the new policies for the next period. It is implicitly assumed next period's government does not renege on these policies. Individuals then take these announcements as given and simultaneously choose their investment plans. The minimal state vector that captures this type of institutional commitment mechanism then is $\mathcal{X}_t = [\theta_t, \tau_t, g_t, A_t]$ where $\theta = [\theta_t^k, \theta_t^h, \phi_t]$ and $\tau_t = [R_{t-1}^f, \tau_t^k, \tau_t^w]$. Therefore the vector of policies that the current government is choosing is $\pi_t = [R_t^f \ \tau_{t+1}^k \ \tau_{t+1}^w]$.

There is an additional reason for the tax decisions to be predetermined. In the optimal fiscal policy literature under full commitment, state contingent taxes on investment decisions allows the planner to complete the markets. This result was first established in Zhu (1992) and invoked in Chari et al. (1994). In our model where human capital is viewed as an additional investment, a state contingent tax for this plan can equally replicate the complete markets outcome. A formal proof of this claim can be found in Appendix 2.8.2. In these circumstance when a non-state contingent bond is traded none of the succeeding governments has the ability to use the fiscal instruments in order to insure against the aggregate fluctuations. The maximization problem of the planner can be formally stated as:

$$\max_{\Delta_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^{t+1} E_t[\log R_{x_{it+1}}] \tag{2.44}$$

s.t

$$\begin{aligned} &E_t\left[R^{-1}_{ix_{t+1}}(1+(1-\tau^k_{t+1})F_k(\theta^k_{t+1},\theta^h_{t+1})-\delta)\right] = \\ &E_t\left[R^{-1}_{ix_{t+1}}(1+(1-\tau^w_{t+1})F_h(\theta^k_{t+1},\theta^h_{t+1})-\delta+\eta^i_{t+1})\right] \qquad \forall t \end{aligned} \tag{2.45}$$

$$E_t\left[R^{-1}_{ix_{t+1}}(1+(1-\tau^k_{t+1})F_k(\theta^k_{t+1},\theta^h_{t+1})-\delta)\right] = E_t\left[R^{-1}_{ix_{t+1}}\right]R^f_t \qquad \forall t \tag{2.46}$$

$$\text{Financial markets clearing:} \qquad \phi_{t+1}+\theta^k_{t+1}+\theta^h_{t+1}=1 \qquad \forall t \tag{2.47}$$

$$\begin{aligned} \text{Gov. Budget Constraint:} \qquad & \phi_{t+1}\beta R_{x_t} \geq R^f_{t-1}\phi_t + g_t \\ & -\tau^k_t F_k\theta^k_t - \tau^w_t F_H \theta^h_t \end{aligned} \tag{2.48}$$

$$\textit{Ad hoc} \text{ borrowing constraint} \qquad \underline{\mathcal{B}} \leq R^f_t \phi_{t+1} \leq \overline{\mathcal{B}} \tag{2.49}$$

Due to Walras' law the feasibility constraint can be omitted[10]. Idiosyncratic portfolio returns in their respective transformations have a triple role, a) in their monotonic transformation, $\log R_{x_{it+1}}$, they affect welfare, b) in their transformation $R^{-1}_{x_{it+1}}$ they determine the stochastic discount factor and c) in $R_{x_t} \equiv E_{t-1}[R_{x_{it}}]$ they affect the growth rate. It is obvious then that the choice vector $\Delta_t = [\phi_{t+1}, \theta^k_{t+1}, \theta^h_{t+1}, R^f_t, \tau^k_{t+1}, \tau^w_{t+1}]$ directly affects all three simultaneously. That feature it essentially originates from the semi-closed solution that the model admits. For this reason and to avoid tedious repetition we put more effort to analyse some of the FOCs more thoroughly than others. In any case, most of them they have, if not identical, very similar interpretation. To reserve space for the main text the complete derivations are relegated to the Appendix.

The maximization problem in (2.44) can be solved by forming the Lagrangian[11]. We next attach the multipliers, $\beta^{t+1}\Psi_1, \beta^{t+1}\Psi_2, \beta^{t+1}\Lambda$ and $\beta^{t+1}\lambda_t$ for (2.45),(2.46), (2.47) and (2.48) respectively. Similarly, we attach $\beta^{t+1}\xi_L$ and $\beta^{t+1}\xi_U$ for the borrowing constraints in (2.49). The Lagrangian of the problem then is:

[10] Recall that individual consumption and the budget constraints have already being substituted in the original FOCs of the competitive economy to get the constraints (2.45)-(2.46).

[11] In Ramsey problems the timeless perspective usually arises when the constrains or the objective in the initial period problem are different from any $t \geq 1$. For example the marginal utility in a standard Euler equation is not defined at $t=-1$. In our case, the (endogenous) growth rate might not be defined. However, when $\phi_{-1}=\phi_0$, $\tau^k_{-1}=\tau^k_0, \tau^w_{-1}=\tau^w_0, R^f_{-1}=R^f_0$ then this distinction is not applicable and does not matter in practice, at least for the special case of logarithmic utility. Thus, we can replace the initial value of the growth rate the average gross portfolio returns at $t=0$.

$$\max_{\Delta_t} L = \mathbb{E}_0 \Bigg\{ \sum_{t=0}^{\infty} \Big[ \beta^{t+1} E_t [\log R_{x_{it+1}}] \tag{2.50}$$

$$-\beta^{t+1}\Psi_1 \underbrace{\left( R^{-1}_{ix_{t+1}} \left( (1-\tau^w_{t+1}) F_h(\theta^k_{t+1}, \theta^h_{t+1}) - (1-\tau^k_{t+1}) F_k(\theta^k_{t+1}, \theta^h_{t+1}) + \eta^i_{t+1} \right) \right)}_{\mathcal{M}_1}$$

$$-\beta^{t+1}\Psi_2 \underbrace{\left( R^{-1}_{ix_{t+1}} \left( (1 + (1-\tau^k_{t+1}) F_k(\theta^k_{t+1}, \theta^h_{t+1}) - \delta) - R^f_t \right) \right)}_{\mathcal{M}_2}$$

$$-\beta^{t+1}\Lambda(\phi_{t+1} + \theta^k_{t+1} + \theta^h_{t+1} - 1)$$

$$+\beta^{t+1}\lambda_t \underbrace{\left( R^f_{t-1}\phi_t + g_t - \tau^k_t F_k \theta^k_t - \tau^w_t F_H \theta^h_t - \phi_{t+1}\beta R_{x_t} \right)}_{\mathcal{M}_3}$$

$$+\beta^{t+1}\xi_L (R^f_t \phi_{t+1} - \underline{\mathcal{B}}) + \beta^{t+1}\xi_U (\overline{\mathcal{B}} - R^f_t \phi_{t+1}) \Big] \Bigg\} \tag{2.51}$$

$\mathcal{M}_1$ and $\mathcal{M}_2$ are functions of the control variables in period $t$, i.e $\mathcal{M}_j = \mathcal{M}_j(\Delta_t)$ for $j = 1, 2$ thus the multipliers attached on those constraints remain static (for convenience we suppressed the time-subscript for $M_1, M_2, M_3$ in period $t$). Put more simply, the objective and the static multipliers on investment decisions imply an *intra-temporal* decision problem that the planner faces. More specifically, it induces a trade off between taxing assets which are not only of different productivities but also of distinct insurance characteristics. To get some intuition on the mechanics of the model, consider for instance the optimal choice for $\tau^w_{t+1}$:

$$\tau^w_{t+1}: \quad \underbrace{\mathbb{E}_t \left[ \beta\lambda_{t+1} \left( \frac{\partial \mathcal{M}_{3_{t+1}}}{\partial \tau^w_{t+1}} \right) \right]}_{\text{MB}} = \underbrace{\underbrace{\Psi_1 \mathbb{E}_t \left[ \frac{\partial \mathcal{M}_1}{\partial \tau^w_{t+1}} \right] + \Psi_2 \mathbb{E}_t \left[ \frac{\partial \mathcal{M}_2}{\partial \tau^w_{t+1}} \right]}_{\text{Portfolio Reallocation Term}} - \underbrace{\mathbb{E}_t \left[ \frac{\partial \log R_{x_{it+1}}}{\partial \tau^w_{t+1}} \right]}_{\text{Welfare Term}}}_{\text{MC}} \tag{2.52}$$

The multiplier $\lambda_t$ represents the marginal need for funds by the government. Since taxes are predetermined, in this example, the need for funds arises for next period. Therefore, in (2.52) the LHS is the marginal benefit of raising one unit of extra revenue through taxes on human capital and the RHS is the respective total marginal cost, at the optimum the two must be equal. Marginal costs consists of two main components: a) a portfolio reallocation component and b) a welfare cost component. An increase in taxes causes a reallocation of investments for two main reasons. First, because the after tax returns for the particular investment changes and second because the pricing kernel changes. In either case the effective change is on the equilibrium price levels. Put simply, a change in tax will trigger a *portfolio rebalancing* between investment in assets and the demand for insurance[12].

The multipliers $\Psi_1$ and $\Psi_2$, play the role of specific weights (determined endogenously) that the planner uses for the reallocation of investments[13]. In equilibrium these weights are in principle implicit functions of the elasticities of the physical asset shares with respect to their taxes. Thus, depending on the responsiveness of the asset shares to their respective taxes, the optimal tax rate is affected in analogous way. This kind of responsiveness would be internalized in the equilibrium values of these multipliers.

Equation (2.52) can be written in a form that emphasizes the two competing motives, that of *fiscal hedging* and that of *self-insurance.* On one hand the Government tries to insure itself from the risk of aggregate shocks, on the other hand, and as long as the objective is the individual welfare maximization, the planner will also try to provide the necessary insurance that agents need due to idiosyncratic risk. This is equivalent to Shin (2006)'s point of a battle between tax and consumption smoothing. More specifically, equation (2.52) can be decomposed as follows:

$$\underbrace{\beta \mathrm{Cov}(\lambda_{t+1}, \frac{\partial M_{3t+1}}{\partial \tau^w_{t+1}})}_{\text{Fiscal Hedging}} \tag{2.53}$$

$$= \underbrace{\Psi_1 \mathrm{Cov}(m^i_{t+1}, \frac{\partial \mathcal{P}^H_{t+1}}{\partial \tau^w_{t+1}}) + \Psi_1 \mathrm{Cov}(\mathcal{P}^H_{t+1}, \frac{\partial m^i_{t+1}}{\partial \tau^w_{t+1}}) + \Psi_2 \mathrm{Cov}(\mathcal{P}^K_{t+1}, \frac{\partial m^i_{t+1}}{\partial \tau^w_{t+1}})}_{\text{Self-insurance}}$$

$$+ \underbrace{\tilde{\mathcal{MC}} \text{ - } \tilde{\mathcal{MB}}}_{\text{Planner's Term}} \tag{2.54}$$

Where

$$\tilde{\mathcal{P}}^H_{t+1} \equiv (1 - \tau^w_{t+1}) F_{h_{t+1}} - (1 - \tau^k_{t+1}) F_{k_{t+1}} + \eta^i_{t+1}$$
$$\tilde{\mathcal{P}}^K_{t+1} \equiv (1 + (1 - \tau^k_{t+1}) F_k - \delta - R^f_t)$$

$$\tilde{\mathcal{MC}} = \Psi_1 \mathrm{E_t}[m^i_{t+1}] \mathrm{E_t}[\frac{\mathcal{P}^H_{t+1}}{\partial \tau^w_{t+1}}] + \Psi_1 \mathrm{E_t}[\mathcal{P}^H_{t+1}] \mathrm{E_t}[\frac{\partial m^i_{t+1}}{\partial \tau^w_{t+1}}] +$$
$$\Psi_2 \mathrm{E_t}[\mathcal{P}^k_{t+1}] \mathrm{E_t}[\frac{\partial m^i_{t+1}}{\partial \tau^w_{t+1}}] - \underbrace{\mathbb{E}_t \left[\frac{\partial \log R_{x_{it+1}}}{\partial \tau^w_{t+1}}\right]}_{\tilde{\mathcal{W}}_{\tau^w_t}}, \quad \tilde{\mathcal{MB}} \equiv \beta E_t[\lambda_{t+1}] E_t[\frac{\partial M_{3t+1}}{\partial \tau^w_{t+1}}]$$

Or more compactly we re-write the previous equation as:

[12] In the absence of any Heterogeneity, the term fiscal hedging does not lose its meaning however the self-insurance term in that case can be thought as the indirect effects of taxes on the accumulation of physical assets. In that respect, the assumption of heterogeneous agents facing idiosyncratic risk is critical for the interpretation given in our analysis.

[13] Alternatively, the multipliers can be interpreted in a more conventional "Lagrangian" fashion to reflect the marginal costs of relaxing the respective constraints. But by being static, the interpretation given in the text facilitates the intuition of the model better.

$$\mathcal{FH}_h = \mathcal{SL}_h + \underbrace{\tilde{\mathcal{MC}} \text{ - } \tilde{\mathcal{MB}}}_{\text{Planner's Term}} \tag{2.55}$$

The fiscal hedging term, in the example above, reflect the insurance that taxes provide for the planner against aggregate risk. Everything else being equal, when looking at the term $\frac{\partial M_{3t+1}}{\partial \tau^w_{t+1}}$, higher taxes increase the revenues in proportion to the tax base (here $F_{h_{t+1}} \theta^h_{t+1}$) and the need for funds tends to decrease. In the same time, higher taxes weakens the desire to accumulate assets, but is delimited by the negative effect on the (average) growth rate that the increase in taxes will bring.

The self-insurance term is related to the portfolio rebalancing decision discussed before and is determined by the respective covariances shown in (2.55). In the particular example, the amount of self-insurance will depend on the covariance of the human capital returns with the stochastic discount factor, and the covariances of the relevant risk-premiums with the new pricing kernel. The strength of the latter covariance terms constitute an effective "Self-insurance benefit". For example, a higher tax in the *most risky* asset allows the planner to gain a greater flexibility in supplying more of the *less risky* assets. Under the CRS production function, higher physical capital decreases the returns of this asset and increases the wages (returns) of human capital. This reallocation of the factors of production it effectively achieves an insurance benefit. This point is very crucial. In the competitive equilibrium without government physical capital was over-accumulated, but as Gottardi et al. (2014a) emphasize, and contrary to the general belief, over-accumulation of a physical asset *does not* justify a positive tax.

Therefore, the portfolio rebalancing decisions are pushed toward allocations in physical capital or public debt. In either case, consumers can earn more insurance. Both insurance items, either the fiscal hedging or the self-insurance one, are weighted differently with $\beta$, $\Psi_1$ and $\Psi_2$ being the respective weights. Finally, notice the appearance of the "Planner's term". This item has a very specific interpretation, if for example is positive, it implies that in order to retain *the same* amount of fiscal hedging the planner has to resort into higher (tax) distortions. Using (2.45)-(2.46) and manipulating the previous FOC, we can get a formula for the optimal taxation of Human Capital:

**Optimal Human Capital Tax:**

$$\tau^w_{t+1} = \overbrace{\left[\frac{1}{E_t[\frac{\partial m^i_{t+1}}{\partial \tau^w_{t+1}}]}\right] \left[\frac{(\mathcal{FH}_h - \mathcal{SL}_h) + \left(\tilde{\mathcal{MB}} + \tilde{\mathcal{W}}_{\tau^w_t}\right)}{\Psi_1(\epsilon^H + 1) + \Psi_2\left(\frac{Cov(m^*_{it+1}, \tilde{\mathcal{P}}^k_{t+1})}{Cov(m^*_{it+1}, \tilde{\mathcal{P}}^H_{t+1})}\right)}\right]}^{\text{Portfolio Allocation Term}} + \tag{2.56}$$

$$\underbrace{\underbrace{cov(m^*_{it+1}, F_{H_{t+1}}) + cov(m^*_{it+1}, (1-\tau^k_{t+1})F_{H_{t+1}})}_{\text{Insurance due to Prices}} + \underbrace{cov(m^*_{it+1}, \eta_{it+1})}_{\text{Insurance Due to Idiosyncratic Risk}}}_{\text{Planner's Insurance}} \tag{2.57}$$

The formula above summarizes all the elements that are affecting the decisions to tax. It consists of two main terms, a portfolio allocation term and an insurance item, labelled "Planner's Insurance". Looking at the first term, the planner will affect the portfolios decisions of each agents depending on two factors: a) the strength between the fiscal hedging needs and the demand for insurance net of the dead-weight costs, b) on the productivities of each asset. In the formula the term $\tilde{\epsilon}^H_{t+1} \equiv \left(E_t[\frac{\partial m^i_{t+1}}{\partial \tau^w_{t+1}}]/E_t[\frac{\partial \mathcal{P}^H_{t+1}}{\partial \tau^w_{t+1}}]\right) \times \frac{E_t[\mathcal{P}^H_{t+1}]}{E_t[m^i_{t+1}]}$ captures the elasticity of tax revenues with respect to the risk premium between the physical assets.

Looking at the second term, the theoretical formula reveals the *net* insurance that can be obtained by manipulating this instrument. In particular, the "Planner's insurance" term it consists of two main items. An item of insurance that can be obtained by the change in prices and a "pure" insurance item due to missing markets. If there was no aggregate uncertainty, the first covariance items would have being zero. Therefore, each tax instrument it also offers an insurance against fluctuating prices. This point would be crucial for the interpretation of the main results.

In the exposition above we focused only on one tax instrument. A similar intuition and manipulation of the FOC it also applies for $\tau^k_{t+1}$. However, in contrast to the previous case taxing physical capital it creates a form of distortion. More specifically, an increase in the tax rate of physical capital amounts to rebalance the portfolio allocations in a way that pushes towards the more risky asset (i.e. human capital), unless the consumer is not compensated with a *sufficient after tax risk-premium* in holding this asset, the amount of insurance might be inefficiently low.

**Optimal Capital taxation:** Suppose that assets shares and the risk-free rate obtain their *optimal* values and the tax on human capital is determined by equation (2.57). Manipulating constraint (2.45), the optimal tax on physical capital is:

$$\tau^k_{t+1} = \left[1 - \frac{E_t[m^*_{it+1}\eta_{it+1}]}{E_t[m^*_{it+1}F_{k_{t+1}}]}\right] + \frac{E_t[m^*_{it+1}F_{h_{t+1}}]}{E_t[m^*_{it+1}F_{k_{t+1}}]}(1 - \tau^w_{t+1}) \tag{2.58}$$

As in the case of human capital tax, the optimal tax rate on other physical asset is in general ambiguous. From the above formula however, what is noteworthy is that the correlation between the tax rates can be of either sign. Moreover, under our distributional assumptions for idiosyncratic risk, it implies that $E_t[m^*_{it+1}\eta_{it+1}] = Cov(m^*_{it+1}, \eta_{it+1})$. Then we can see that the partial effect of the insurance item due to idiosyncratic risk goes in the opposite direction relative to the human capital tax formula.

Finally, the planner beside the two fiscal instruments can also manipulate the risk-free rate. The FOC is very similar and can be found in Appendix 2.8.3. The only difference is that, away from the borrowing constraints, the manipulation of the rate of interest affects more directly the sources of public revenues (or interest expenses) as well as the price of aggregate risk. Moving to the optimal allocations of asset shares, the FOC with respect to the asset share of human capital is:

$$\mathbb{E}_t\left[\frac{\partial \log R_{x_{it+1}}}{\partial \theta^h_{t+1}}\right] = \Lambda + \overbrace{\underbrace{\Psi_1\mathbb{E}_t\left[\frac{\partial \mathcal{M}_1}{\partial \theta^h_{t+1}}\right] + \Psi_2\mathbb{E}_t\left[\frac{\partial \mathcal{M}_2}{\partial \theta^h_{t+1}}\right]}_{\text{Term I}} - \underbrace{\beta\mathbb{E}_t\left[\lambda_{t+1}\left(\frac{\partial \mathcal{M}_{3t+1}}{\partial \theta^h_{t+1}}\right)\right]}_{\text{Term II}}}^{}_{} \tag{2.59}$$

$$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}_{\text{Planner's term}}$$

The planner, in contrast to the agents, internalizes the effects of the "uncoordinated" actions of the consumers to factor prices, that is the "pecuniary externality". Absent of any government intervention, uninsured idiosyncratic risk leads to an over-accumulation of physical-to-human capital ratio which in equilibrium implies that the returns of investments are inefficiently lower relative to the first best[14].

It is instructive here, to clarify that when the ratio of physical-to-human capital is high relative to the complete market case, does *not* necessary justify a positive tax on physical capital, which might correct this over-accumulation. This point was originally made and formally proved in Gottardi et al. (2014a). The logic, that the over-accumulation of physical capital should be corrected, is based on an unwarranted comparison between different types of equilibria: the one in which markets are complete and in which the markets are incomplete. Using, Gottardi et al. (2014a) words

> ...the comparison between the level of capital accumulation with and without complete markets has no clear welfare implication. If there were a policy tool which could allow to attain the complete market allocation, there would be little doubt for the policy maker to adopt such a policy as far as attaining efficiency is concerned. Since a tax and subsidy scheme of the kind mentioned will not complete the markets, the aforementioned comparison tells little about the effectiveness of taxation, not to mention whether or not capital should be taxed. To properly assess whether or not positive taxes on capital are welfare improving when markets are incomplete, one should rather compare the competitive equilibria with and without taxes, keeping the other parts of the market structure, and in particular the set of available financial assets, fixed.

Indeed, as the same authors prove whether a planner corrects or not this over-accumulation - which is equivalent to the case whether physical capital should be taxed or not - it depends on the type of the shocks, and on the degree of heterogeneity of consumers.

Accordingly, in our case whether the optimal allocation of risk will feature higher or lower physical to human capital ratio will depend on the two types of risks - the aggregate and the idiosyncratic, both uninsurable. This is because, the insurance and

[14] In the competitive equilibrium without government the ratio of physical to human capital is higher relative to complete markets, which in turn implies that there is over-accumulation in physical capital. See Krebs (2003a,b), Gottardi et al. (2014a) or Toda (2014a) for the formal argument.

redistribution effects that were analysed in Gottardi et al. (2014a) manifest in our application to the fiscal hedging and self-insurance terms. These two motives that the planner tries to balance, are the decisive elements to characterize whether a welfare improvement is attained when there is over or under accumulation of physical-to-human capital ratio, when one keeps the same market structure fixed - that is, the instruments to complete the markets are missing[15].

Next, a similar equation with the respective interpretation holds for the investment in human capital. Finally, combining the FOCs for $\phi_{t+1}$ and $\theta^k_{t+1}$ to eliminate the multiplier $\Lambda$ we can get the Euler equation for the planner. This is:

$$\begin{aligned}
&\Psi_1 \mathbb{E}_t \left[ \frac{\partial \mathcal{M}_1}{\partial \theta^k_{t+1}} - \frac{\partial \mathcal{M}_1}{\partial \phi_{t+1}} \right] + \Psi_2 \mathbb{E}_t \left[ \frac{\partial \mathcal{M}_2}{\partial \theta^k_{t+1}} - \frac{\partial \mathcal{M}_2}{\partial \phi_{t+1}} \right] \\
&+ \mathbb{E}_t \left[ \beta \lambda_{t+1} \left( \frac{\partial \mathcal{M}_3}{\partial \phi_{t+1}} - \frac{\partial \mathcal{M}_3}{\partial \theta^k_{t+1}} \right) \right] \\
&= \lambda_t \beta R_{x_t} + \underbrace{(\xi_U - \xi_L) R^f_t}_{=0 \text{ if constraint does not bind}}
\end{aligned} \tag{2.60}$$

The optimal holding of the safe asset is determined by the equality between the marginal value of a unit increase in the supply of this bond this period and the effects that this change will bring on the reallocation of assets and on the revenues in the future. The former effect is absent in the competitive economy and it evidences the direct impact that the planner has on market prices. Pertaining to the latter effect, the budget constraint in the future will be affected in two ways: a) because the tax base will change and b) because the public finance discounting, that is the growth rate, will be affected. Implicit in the above equation is the precautionary savings motive for the planner adjusted for growth. The borrowing constraints in principle increase that motive, however the manipulation of the interest rate allows the planner to relax or make them more stringent. Next paragraphs introduce our main findings.

## 2.5 Results

To get the values for some of the parameters of the model, we assume that the planner takes those as given from the competitive economy. The main interest is to recover the values for the discount factor and the variance for idiosyncratic shocks. Although, the former parameter value except for some scaling effects on growth, it plays little role. We get those parameters by imposing certainty equivalence in the competitive economy. Due to idiosyncratic and aggregate shocks, the certainty equivalent should be understood only for the aggregate state. Following the approach in Krebs (2003b) the values obtained for the discount factor and idiosyncratic uncertainty are $\beta = 0.9458$ and $\sigma_\eta = 0.5384$ respectively[16]. The calibration strategy is discussed in detail in Appendix

[15] Other notable examples where under-accumulation of physical capital is welfare improving is Angeletos (2007), Reis and Panousi (2012) and Gottardi et al. (2014c,a)

[16] In practice, this number for the idiosyncratic volatility turn out to be quite high to obtain convergence. We therefore rescale it by the factor of 100.

2.8.5.

We also let $\rho_A = \rho_g = 0.95$ and approximated the volatility of government spending per wealth with that of per output found in the data (i.e $\log(G_t/Y_t)$). The upper bound was set to a number greater than 1 in order not to rule out short positions in physical assets. The numerical solution was insensitive to that bound but extremely sensitive to the lower one. The criterion to handle this issue was to choose a number sufficient to obtain convergence and in parallel be stringent enough to restrict the planner from financing all of his expenses by accumulating assets[17]. Table 2.1 summarizes main parameters.

| $\beta$ | $\alpha$ | $\delta$ | $\bar{A}$ | $\rho_A$ | $\overline{\mathcal{B}}$ |
|---|---|---|---|---|---|
| 0.9458 | 0.36 | 0.06 | 0.34 | 0.95 | $1.2/\beta$ |
| $\bar{g}$ | $\underline{\mathcal{B}}$ | $\sigma_g$ | $\sigma_\eta$ | $\sigma_A$ | |
| 0.04 | $-0.50/\beta$ | 0.09 | 0.0054 | 0.01 | |

Table 2.1: Benchmark Calibration

Before discussing the main results, a few more remarks are in order. In the representative agent literature, as for instance in Angeletos (2002) or Buera and Nicolini (2004) the government can complete the markets by trading its own assets. This however requires the number of assets to be equal to the number of states. In Farhi (2010) and in our case although the government is trading assets that option is not available and markets cannot be completed in the usual sense. Nevertheless between us and Farhi (2010) some subtle differences still exist. First, agents in our framework are heterogeneous second the government instead of managing its own liabilities trades, or dictates if you prefer, the investment decisions of the consumers. In this respect there are some crucial differences in the interpretation. Previously, we showed that the economy is equivalent to a single agent facing two types of *uninsurable* risks. The planner replaces that agent and trades assets to offer the necessary insurance against the **effective uncertainty**, that of aggregate and idiosyncratic risk. Therefore, the preferred interpretation here is that the planner tries to maintain the insurance mechanism that factor prices offer to the agents[18].

| | $\theta^k$ | $\theta^h$ | $\phi$ | $\tau^k$ | $\tau^w$ | $R^f$ |
|---|---|---|---|---|---|---|
| Means | 0.57% | 0.89% | -0.46% | -5.82% | 7.81% | 1.13% |
| Volatility | 0.42% | 0.69 % | 0.27% | 1.77% | 0.17 % | 0.67% |

*Note*: The statistics reflect sample moments from a 10,000 random draws where the first 10% disregarded.

Table 2.2: Properties of the Model

[17]We solved the model based on PEA. It is well known that the stability properties of such numerical method are extremely poor. To overcome this difficulty we borrowed some of the techniques in Judd et al. (2011). Nevertheless, the uncertainty parameter of the idiosyncratic shocks and the lower bound of the borrowing constraints, turned out to be extremely sensitive parameters for achieving convergence. A detailed description of the algorithm can be found in Appendix 2.8.4. In the future we hope to accommodate this issue much more robustly.

[18]For the insurance role of prices under incomplete markets see Dávila et al. (2012), Carvajal and Polemarchakis (2011), Gottardi et al. (2014a) among others

Turning on the analysis of the results, from Proposition 1 and the discussion therein, the (approximate) random walk hypothesis of labour income and the approximate risk-neutrality of the agents, jointly determine that government bonds are ineffective means for consumption smoothing, hence welfare is maximized whenever an efficient path for growth is chosen. Table 2.2 shows how this is achieved. The planner takes short positions in bonds in order to accumulate sufficient income while the stabilization of public finances mainly occurs through the manipulation of the interest rate. In addition, by taking short positions in debt and long in the other assets he balances between insurance and investment productivities. In particular, he takes (as expected) the largest position in human capital. Debt provides (*self-*)insurance against the macroeconomic shocks, thus by accumulating assets the planner is able to earn interest and secure a source of revenue. In consequence the safe asset is the least traded among all assets. Similarly physical capital it is optimally less traded than human capital. From the same table, the planner chose to intervene across states by manipulating the tax subsidies in physical capital (See Appendix 2.8.6 ), while labour (in efficiency units) taxes are kept relatively smooth. This result extends the standard labour tax smoothing to the broader category of effective labour.

Referring to the signs of the tax instruments, their justification follow the discussion in earlier sections. More specifically, a subsidy in physical capital increases the supply of this asset and depresses its price, this however helps the returns on human capital to *increase*. That is the insurance policy here is to protect the returns of the most risky asset. However, in order to discourage an excessive risk taking and thus an inefficiently low amount of returns, the planner will also like to impose a positive tax on that asset. This argument is in line with the first principles of public finances where welfare can be improved if risky sources of income are insured and encourage investments in less productive assets. Gottardi et al. (2014a) where the first to formalize this argument in a similar context and showed how the optimal tax on physical capital depends on the type of risk. In our case these two different types of risks are the aggregate and idiosyncratic.

In Table 2.3 we see the effects of the idiosyncratic risk to the portfolio rebalancing and the design of policies. The table shows the percentage changes relative to the benchmark values in Table 2.2. Higher idiosyncratic uncertainty has two important effects, first it dramatically increases the price of aggregate risk and secondly it rises the insurance (risk-premium) for the most risky asset. Relative to the benchmark case, the total effect is a decline in the growth rate and welfare. But how the planner would like to balance between the higher price of fiscal hedging and the demand for insurance by the individuals ? Or how precautionary savings against the effective uncertainty will be accommodated ?

From the results in Table 2.3 the planner will increase the tax on labour, cut the subsidies and will take longer positions in the two capital goods. The latter composition is assisted by making borrowing more cheap through the decline in the rate of interest. This allows to counter-balance the increase in tax and the cut in subsidies. However, to protect the returns in the most risky asset notice that quantitatively the increase in the share of the physical capital is larger from the increase in human even if the cut in subsidies is stronger.

Notice, that from this comparative static exercise we can see that the price of aggregate risk increases a lot more than the insurance for human capital. This might be indicative that the overall fiscal hedging dominates the demand for self-insurance by the individuals and accumulating more assets by the government is optimal.

| Porfolio Allocations | $\theta^k$ | $\theta^h$ | $\phi$ |
|---|---|---|---|
| | 0.087% | 0.056% | 0.0195 % |
| Policies | $\tau^k$ | $\tau^w$ | $R^f$ |
| % Changes | -2.86% | 1.84% | -0.0177% |
| Growth | | -0.093% | |
| Welfare loss | | 0.0024% | |
| Human Capital Premium | | 0.011 % | |
| Risk Premium | | 27.84 % | |

*Note*: The numbers denote percentage changes from the Benchmark values. The new value for the variance of idiosyncratic risk changed to $\sigma_\eta = 0.010$ from $\sigma_\eta = 0.0058$. As in Gottardi et al. (2014b) we define the Human capital Premium $F_h - F_k$ and the Risk Premium the difference between the market returns and the risk-free rate, $E[R_{x_{it}}] - R^f$

Table 2.3: Higher Idiosyncratic Risk

Finally, to show how the composition of investments in the portfolios of the agents affects economic growth and by implication welfare, we use a crude approximation of the US tax system taken from Chari et al. (1994)[19]. The average tax rates for both assets are positive in this economy. In Table 2.4 we have solved recursively the objective of the planner (i.e equation (2.19)) for two different economies. It is clear from Table 2.4 that the competitive economy produces an excessive risk taking and by implication welfare will be lower. Reallocating the factors of production in the way previously described and increasing the interest rate to off-set an inefficiently higher level of borrowing constitutes a welfare improvement.

| | $\theta^k$ | $\theta^h$ | $\phi$ | $R^f$ | Growth Rate | $Welfare$ |
|---|---|---|---|---|---|---|
| Optimal Plan | 0.57% | 0.89% | -0.46% | 1.13 | 7.91 % | -1.52 |
| US Economy | 0.52% | 0.96% | -0.49% | 1.09 | 2.97 % | -2.41 |

Table 2.4: Comparison with US policies

Putting altogether, the main intuition of the results can be expressed as follows. Begging with the properties explained in proposition (2), idiosyncratic risk although quantitatively important overall the assumption of uninsurable aggregate risk seems to dominate. Under our proposition where idiosyncratic risk is permanent event, public debt as a safe asset has little bearing for individual self-insurance. For example, a

[19]The time path or the fiscal rules for taxes that these authors estimated are $\tau_t^k\% = 27.1 - 0.71A_t + 0.52\frac{G_t}{Y_t}$ and $\tau_t^w\% = 23.8 - 0.027A_t + 0.11\frac{G_t}{Y_t}$ respectively. For convenience we assumed that taxes on labour are equivalent to those on effective labour.

shock such as one's leg is cut which changes his level of human capital and his income permanently, by running the buffer of savings down - accumulated through the safe asset - does little to cure the subsequent drop in his consumption. In this case, it is optimal not to issue debt, which would have implied much larger taxes - especially on labour - but rather accumulate assets. When this is the case, then the government has the option to finance part of its expenses by accruing interest and therefore set lower taxes.

Note that this logic is similar to the one found in standard models, such as in Aiyagari et al. (2002). Nevertheless, idiosyncratic uncertainty allows the potential twist of the result - which is a possibility that the standard models do not offer[20].

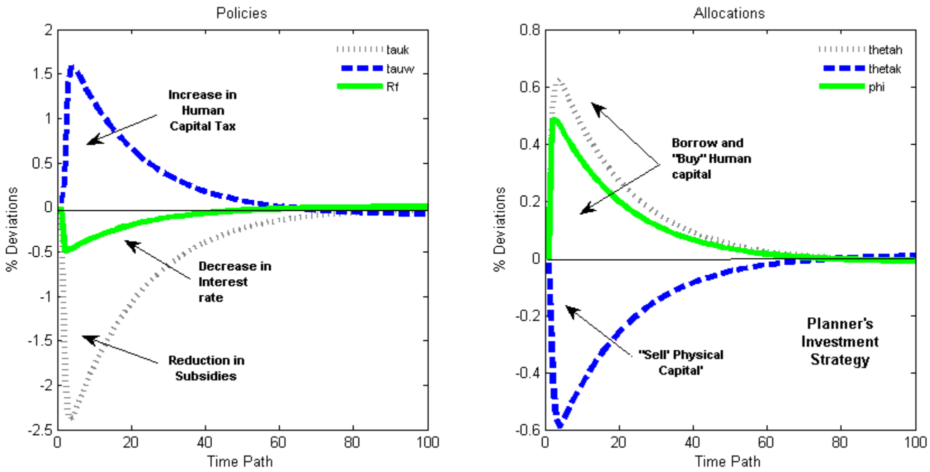
On the other hand, fluctuating prices - due to business cycles or aggregate uncertainty more generally - affect both, the level of government revenues as well as the income of the consumers. The planner, knowing that human capital is not only the most productive asset but it also consist the largest tax base, setting a positive though smooth tax rate, on the one hand can accumulate a sufficient amount of revenues and on the other reduce the variability of returns for this asset - which is a form of insurance against the effective uncertainty. To encourage however investment in other assets, the incentive that it provides is a tax subsidy. This policy, not only sets the "right" investment composition to maximize growth - which is the objective of planner in order to maximize welfare - but also prevents consumers to invest too much in human capital that would potentially diminish their returns due the subsequent over-supply. It is then possible, for the planner to stabilize public finances across different cycle periods by manipulating mostly the interest rate and the tax subsidy. The next section, provides an example of such policies over the cycle.

## 2.6 Managing A Stock Market Crash

Figure 2.1 below shows the response of the planner to a "Stock Market Crash" which in our case is equivalent to a negative TFP shock. The collapse in the stock market it implies that the growth rate, welfare and public revenues will all fall. From the previous discussion the objective of the planner is to set an efficient path for growth and stabilize the public finances. To do so, the planner will announce a tax hike for human capital and reduce the subsidies in physical capital. In the same time the planner will inject more liquidity to the economic system and facilitate borrowing by reducing the interest rate. The combination of these policies allow individuals to borrow more in the short-run and rebalance their portfolios towards the most productive asset. In other words, the spur in growth it comes from a decisive long-position in human capital financed through short positions in debt and short-selling physical capital.

Nevertheless there are some noteworthy subtleties. Allowing for a reduction in the rate of interest endangers the private sector to accumulate inefficient levels of leverage. To prevent this route, the planner after some time will gradually increase the borrowing costs and the subsidies in physical capital in a way that offset the steady

[20]We believe if preferences are other than logarithmic, and therefore agents might have a plausible risk aversion, idiosyncratic risk might play greater role. We aim to address this case in the near future.




A negative 1% productivity shock. The impulses are percentage deviations from the values in Table 2.2

Figure 2.1: Stock Market Crash

decline in labour taxes. Therefore, the share in human capital will start gradually to fall and increase that in physical capital until the new balanced growth path is reached. Under this policy it is evident that growth is restored mainly through the productivity (effective returns) of human capital.

Finally, once the effects of the shock cease the economy enters a new growth path where the interest rates, the share in physical capital and the assets for the government are higher relative to the "pre-crisis" levels, while the share in human capital will be permanently lower. This is again consistent with our previous analysis, where from the short-run to the medium or long-run (i.e transition to the new balanced growth path) the after tax returns on human capital (i.e the most risky-asset) are again protected.

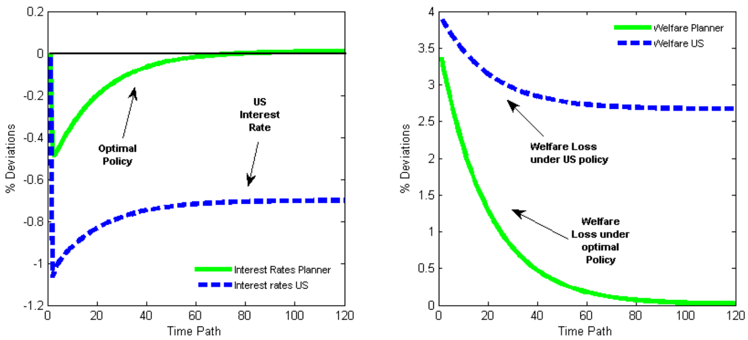



Figure 2.2: Time Paths for Risk-free Rate and Welfare

The previous paragraph conveyed a unique message over the optimal management of the economy. Simply put, when growth is endogenously determined and there exist different investment opportunities, the Key to optimal policy is: *let first the economy take excessive risks, discharge it later and protect its returns over-time.* To highlight that particular philosophy, we contrast the management of the stock market crash by the planner against the same "US fiscal rules" that we used for the results in Table 2.4.

The two figures below depict the time paths for the main variables of interest. Figure 2.2 displays the responses and the subsequent time path of for the interest rates (risk-free rate) and the welfare loss that occurs in each economy after the "stock market crash".

Equivalently, Figure 2.3 depicts the portfolio allocations. First, in the competitive economy the response to shock is accommodated with an inefficiently lower interest rate which results to an excessive borrowing. The investment strategy then of the "US economy" is to take larger than the optimal short positions in bonds and inefficiently lower long position in human capital. Relative to the optimal policy the competitive economy mis-allocates physical capital, in the sense that instead of selling stocks of physical capital the agent in the competitive economy inefficiently buys more stocks of that asset. Looking at Figure 2.3, in the new balanced growth path the *level* of shares in the portfolios of the agents will be higher than the pre-crisis levels, but its *composition* is inefficiently structured. In line with the earlier exposition, the inefficient portfolio composition amounts to improperly specified equilibrium (after tax) price levels. In consequence, the latter effect will not only amount to a permanent lower (relative to the pre-crisis level) welfare but also a steady increase of the welfare gap between the competitive and centralized economy, as we can see from Figure2.2 in the right panel.

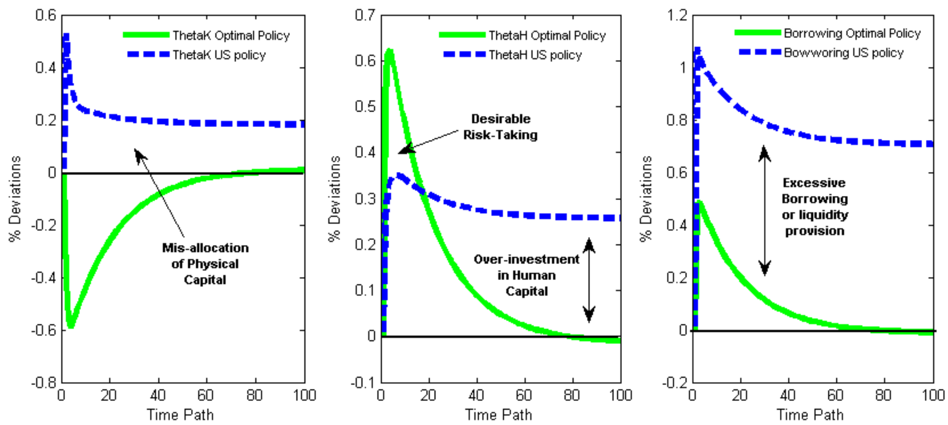



Figure 2.3: Portfolio Allocations: Planner vs US

## 2.7 Conclusions

To sum up, recessions are periods where people are frequently pursuing a human capital investment as for example through additional education. To some approximation anyone that invests in his education it essentially opts for a particular return. Macroeconomic policies rarely took that event into a serious consideration, although is crucial element for the short run and the determination of growth in the future. Even after the financial crisis and the frequent public debate about stagnated growth was rarely discussed let alone linked to the optimal government financing. Our paper addressed this concern and raised this issue implicitly.

The very large portfolio share that this asset has in the real world and the fact that is subject to idiosyncratic risk is not yet understood how it might affect the design of policies and by implication public finances. In principle, the presence of idiosyncratic risk to that asset is no different than other models of precautionary savings. However what is omitted is its productivity and the contribution to growth. Therefore an important element for the macroeconomic management in the short-run is silenced.

In a summary of our results, we essentially showed that what private equity might be for the monetary quantitative easing, human capital might be for fiscal policy. As a type of asset that is subject to idiosyncratic risk, in the short-run, can be a resources that fiscal policy can empower in order to efficiently promote growth. Nevertheless, this argument is strictly conditioned in the case where markets are incomplete both at the aggregate and the individual level.

Finally, our analysis was limited in the special case of the logarithmic utility. This had the advantage of being analytically tractable. Nevertheless, in this case we missed all the interesting interaction between asset prices and the marginal value of wealth. Effectively, this is equivalent to ignore the role of consumption smoothing. Secondly, we restricted ourselves to optimal policies with imperfect institution commitment and to the case where idiosyncratic risk does not covary along the cycle. All the above we conceive them as important limitations and therefore extending the model in this direction is promising.

## 2.8 Appendix

### 2.8.1 Agents Problem

To get the results for lemma 2 we use a guess and verify procedure based on the semi-closed solution offered by Toda (2014b) for these class of models. Notice that our distributional assumptions on aggregate and idiosyncratic risk satisfy his assumptions. The maximization problem is:

$$V(x_{it}) = \max_{c_{it},\phi_{t+1},\theta^k_{t+1},\theta^h_{t+1}} \left[ \exp\left( (1-\beta)\log c_{it} + \beta E_t \log\left[V(x_{it+1})\right] \right) \right] \tag{2.61}$$

s.t

$$x_{it+1} = R_{x_{it+1}}(x_{it} - c_{it}) \tag{2.62}$$

$$\phi_{t+1} + \theta^k_{t+1} + \theta^h_{t+1} = 1 \tag{2.63}$$

The FOC for consumption will satisfy:

$$(1-\beta)c_{it}^{-1} = \beta E_t\left[V^{-1}(x_{it+1})\frac{\partial V(x_{it+1})}{\partial x_{it+1}} R_{x_{it+1}}\right] \tag{2.64}$$

Guessing $V(x_{it}) = \nu_t x_{it}$, defining $u_t = \frac{c_{it}}{x_{it}}$ and substituting the budget constraint (2.62) in equation (2.64), gives $c_{it} = (1-\beta)x_{it}$ (with $u_t \equiv 1-\beta$) which is equation (2.20) in the lemma.

The FOCs for the portfolio shares give the standard arbitrage conditions:

$$E_t\left[R^{-1}_{x_{it+1}} R^k_{t+1}\right] = E_t\left[R^{-1}_{x_{it+1}} R^i_{t+1}\right] \tag{2.65}$$

$$E_t\left[R^{-1}_{x_{it+1}} R^k_{t+1}\right] = E_t\left[R^{-1}_{x_{it+1}}\right] R^f_t \tag{2.66}$$

These arbitrage conditions are the analogue of the Euler equations (2.10)-(2.12) once (2.20) and (2.62) are substituted in and deliver equations (2.26)-(2.28) in the lemma. Using the guess for the value function, the budget constraint and taking logarithms in the bellman equation, gives

$$\log \nu_t = \kappa + \beta E_t[\log \nu_{t+1}] + \beta E_t[\log R_{x_{it+1}}] \tag{2.67}$$

which is equation (2.19) in the main text and $\kappa$ is an unimportant constant equal to $\kappa = (1-\beta)\log(1-\beta) + \beta \log \beta$. It is straight forward to confirm that our guesses for the value function and the consumption rule satisfy the bellman equation. Finally, the rest of the equations in the lemma follow directly from the definitions in (2.13)-(2.16).

### 2.8.2 Proof for Tax Indeterminacy

In this part we formalize the discussion of Section 2.4. The proof strictly follows Ljungqvist and Sargent (2004) and Zhu (1992) and extends their economy by adding an additional capital good (Human Capital) to reflect our own framework. The main scope is to facilitate the exposition over the restrictions that need to be placed in the fiscal instruments in order to limit their scope to complete the market *on the aggregate*.

Since the solution of the model is equivalent to one agent problem facing two types of shocks, for simplification we drop all individual indexes. In the sequence we will also omit any formal reference in the description of the idiosyncratic risk. This is done for brevity and clarity as it is irrelevant for the argument. To begin with, we define the history of the economy over and up to period $t$ with the vector $s^t$ and let $s_t$ to describe a particular event. We also let $\Pi(s^t)$ to be the probability of a particular history. We assume the unfolding of history has a first order Markov structure, therefore the conditional probability of the next period event given the current state is $\Pi_{t+1}(s^{t+1}|s^t) = \frac{\Pi_{t+1}(s^{t+1})}{\Pi_t(s^t)}$. Conditional expectations are defined over this measure. One remark is in order, with idiosyncratic risk the measure over conditional expectations have to take into account the idiosyncratic event. However since this measure must be the same for all individuals and the risk-free rate is common, the thrust of our exposition here does not change. The assumptions of the model, as for instance stated in Krebs (2006), over the conditional probability is $\Pi_{t+1}(s^{t+1}, s^i_{t+1}|s^t)$ where $s^i_{t+1}$ is the idiosyncratic state. Finally, we let $r_t$ be the returns from physical capital and $w_t$ the returns on human capital, and ignore the depreciation rate in both.

The exposition proceeds in two steps. In the first, we assume that the aggregate state can be insured. That is we assume the existence of Arrow securities (i.e state-contingent bonds) and show that state contingent taxes when public debt is state contingent result to indeterminacy. In the second step we show how the complete market outcome *can be attained* by restricting *some* of the instruments. Doing so, becomes immediate that for the *incomplete markets* outcome it is necessary *all* of the instruments to be restricted.

**Complete Markets**

Let $b_{t+1}(s_{t+1}|s^t)$ be the government debt at the beginning in period $t+1$ if the event $s_{t+1}$ occurs, and let $p_t(s_{t+1}|s^t)$ its Arrow price. The first order conditions and budget constraints are:

$$c_t(s^t) + k_{t+1}(s^t) + h_{t+1}(s^t) + \sum_{s_{t+1}} p_t(s_{t+1}|s^t) b_{t+1}(s_{t+1}|s^t) = \tag{2.68}$$

$$b_t(s_t|s^{t-1}) + (1 - \tau_t^k(s^t)) r_t(s^t) k_t(s^{t-1}) + (1 - \tau_t^w(s^t)) w_t(s^t) h_t(s^{t-1})$$

$$p_t(s_{t+1}|s^t) = \beta \Pi_{t+1}(s^{t+1}|s^t) \frac{U_{c_{t+1}}}{U_{c_t}}$$

$$\Rightarrow \sum_{s_{t+1}|s^t} p_t(s_{t+1}|s^t) = E_t[\beta \frac{U_{c_{t+1}}}{U_{c_t}}] \tag{2.69}$$

$$U_{c_t} = \beta \mathrm{E}_t \left[ U_{c_{t+1}} (1 - \tau_{t+1}^k(s^{t+1})) r_{t+1}(s^{t+1}) \right] \tag{2.70}$$

$$U_{c_t} = \beta \mathrm{E}_t \left[ U_{c_{t+1}} (1 - \tau_{t+1}^w(s^{t+1})) w_{t+1}(s^{t+1}) \right] \tag{2.71}$$

$$G(s^t) = \tau_t^k(s^t) r_t(s^t) k_t(s^{t-1}) + \tau_t^w(s^t) w_t(s^t) h_t(s^{t-1})$$

$$+ \sum_{s_{t+1}} p_t(s_{t+1}|s^t) b_{t+1}(s_{t+1}|s^t) - b_t(s_t|s^{t-1}) \tag{2.72}$$

where $U_{c_t}$ is the marginal utility of consumption in period $t$. Obviously the logarithmic utility function that we used in the main text is a special case. Let $\{\epsilon_t^k(s^t)\}$ and $\{\epsilon_t^w(s^t)\}$ be two random processes with the following two properties:

$$E_t[U_{c_{t+1}} \epsilon_{t+1}^k r_{t+1}(s^{t+1})] = 0 \tag{2.73}$$

$$E_t[U_{c_{t+1}} \epsilon_{t+1}^w w_{t+1}(s^{t+1})] = 0 \tag{2.74}$$

Consider the following alternative policies which are also feasible:

$$\hat{\tau}_{t+1}^k = \tau_{t+1}^k + \epsilon_{t+1}^k(s^{t+1}) \tag{2.75}$$

$$\hat{\tau}_{t+1}^w = \tau_{t+1}^w + \epsilon_{t+1}^w(s^{t+1}) \tag{2.76}$$

$$\hat{b}(s_{t+1}|s^t) = b(s_{t+1}|s^t) + \epsilon_{t+1}^k(s^{t+1}) r_{t+1}(s^{t+1}) k_{t+1}(s^t) \tag{2.77}$$

$$+ \epsilon_{t+1}^w(s^{t+1}) w_{t+1}(s^t) h_{t+1}(s^t) \tag{2.78}$$

and the plans:

$$U_{c_t} = \beta \mathrm{E}_t \left[ U_{c_{t+1}} (1 - \hat{\tau}_{t+1}^k) r_{t+1}(s^{t+1}) \right] \tag{2.79}$$

$$U_{c_t} = \beta \mathrm{E}_t \left[ U_{c_{t+1}} (1 - \hat{\tau}_{t+1}^w) w_{t+1}(s^{t+1}) \right] \tag{2.80}$$

$$G(s^t) = \tau_t^k(s^t) r_t(s^t) k_t(s^{t-1}) + \tau_t^w(s^t) w_t(s^t) h_t(s^{t-1}) +$$

$$\sum_{s_{t+1}} p_t(s_{t+1}|s^t) \hat{b}(s_{t+1}|s^t) - b_t(s_t|s^{t-1}) \tag{2.81}$$

Substituting the tax policies in the Euler equations and invoking (2.73) and (2.74), it leaves the Euler equations in the competitive system unaltered. This in turn implies that the Arrow prices are not affected either. Therefore, substituting the plans for the alternative bond holding in the budget constraint of the government, the revenues remain the same. Consequently, since there are infinite ways of generating random variables that satisfy the properties in (2.73) and (2.74) state-contingent taxes will be indetermined.

**Completing the Markets through instruments**

Suppose now that we restrict the taxes to be non-state contingent. Choose numbers $\bar{\tau}^k_{t+1}(s^t)$ and $\bar{\tau}^w_{t+1}(s^t)$ such as:

$$\mathrm{E}_t\left[U_{c_{t+1}}(1-\tau^k_{t+1}(s^{t+1}))r_{t+1}(s^{t+1})\right] = \mathrm{E}_t\left[U_{c_{t+1}}(1-\bar{\tau}^k_{t+1}(s^t))r_{t+1}(s^{t+1})\right] \tag{2.82}$$

$$\mathrm{E}_t\left[U_{c_{t+1}}(1-\tau^w_{t+1}(s^{t+1}))w_{t+1}(s^{t+1})\right] = \mathrm{E}_t\left[U_{c_{t+1}}(1-\bar{\tau}^w_{t+1}(s^t))w_{t+1}(s^{t+1})\right] \tag{2.83}$$

which implies:

$$\bar{\tau}^k_{t+1}(s^t) = \frac{\mathrm{E}_t\left[U_{c_{t+1}}(1-\tau^k_{t+1}(s^{t+1}))r_{t+1}(s^{t+1})\right]}{\mathrm{E}_t\left[U_{c_{t+1}}r_{t+1}(s^{t+1})\right]} \tag{2.84}$$

$$\bar{\tau}^w_{t+1}(s^t) = \frac{\mathrm{E}_t\left[U_{c_{t+1}}(1-\tau^w_{t+1}(s^{t+1}))w_{t+1}(s^{t+1})\right]}{\mathrm{E}_t\left[U_{c_{t+1}}w_{t+1}(s^{t+1})\right]} \tag{2.85}$$

Then invoking (2.75) and (2.76), two particular random sequences consistent with the complete markets outcome are:

$$\epsilon^k_{t+1}(s^{t+1}) = \bar{\tau}^k_{t+1}(s^t) - \tau^k_{t+1} \tag{2.86}$$

$$\epsilon^w_{t+1}(s^{t+1}) = \bar{\tau}^w_{t+1}(s^t) - \tau^w_{t+1} \tag{2.87}$$

Therefore we can construct any tax policy based on the previous procedure and be able to complete the markets. Next, suppose that debt is safe (risk-free) and in addition assume that the taxes on human capital are known in advance (non-state contingent). This help us to construct a particular sequence for $\epsilon^w_{t+1}(s^{t+1})$ as before. It remains to show what kind of particular sequence for $\epsilon^k_{t+1}(s^{t+1})$ can be constructed in order to support the complete markets outcome. This can be done as follows:

Suppose that $\sum_{s_{t+1}} p_t(s_{t+1}|s^t)b_{t+1}(s_{t+1}|s^t) = \sum_{s_{t+1}} p_t(s_{t+1}|s^t)\bar{b}_{t+1}(s^t)$, substitute in this expression the equilibrium Arrow price, to get:

$$\bar{b}_{t+1}(s^t) = \frac{E_t U_{c_{t+1}} b_{t+1}(s_{t+1}|s^t)}{E_t U_{c_{t+1}}} \tag{2.88}$$

Then by (2.78) the change in taxes needed to support the complete markets under those restrictions is: $\epsilon^k_{t+1}(s^{t+1}) = \frac{\bar{b}_{t+1}(s^t) - b(s_{t+1}|s^t) - \epsilon^w_{t+1}(s^{t+1})w(s^{t+1})h_{t+1}(s^t)}{r_{t+1}(s^{t+1})k_{t+1}(s^t)}$. It is possible to show through similar steps that the complete markets outcome is attainable if instead we have assumed that $\tau_t^k$ are non-state contingent. In consequence, to restrict the planner to use any of the fiscal instruments to complete the markets on the aggregate, it is necessary all of the instruments to be simultaneously restricted.

### 2.8.3 FOCs and Computational Methodology

The FOCs for the allocations are:

$$\begin{aligned}\theta_{t+1}^k\colon \qquad & \beta^{t+1}\mathbb{E}_t\left[\frac{\partial \log R_{x_{it+1}}}{\partial \theta_{t+1}^k}\right] - \beta^{t+1}\Psi_1\mathbb{E}_t\left[\frac{\partial \mathcal{M}_1}{\partial \theta_{t+1}^k}\right] + ... \\ & -\beta^{t+1}\Psi_2\mathbb{E}_t\left[\frac{\partial \mathcal{M}_2}{\partial \theta_{t+1}^k}\right] + \beta^{t+2}\mathbb{E}_t\left[\lambda_{t+1}\left(\frac{\partial \mathcal{M}_{3t+1}}{\partial \theta_{t+1}^k}\right)\right] = \beta^{t+1}\Lambda \end{aligned} \tag{2.89}$$

$$\begin{aligned}\theta_{t+1}^h\colon \qquad & \beta^{t+1}\mathbb{E}_t\left[\frac{\partial \log R_{x_{it+1}}}{\partial \theta_{t+1}^h}\right] - \beta^{t+1}\Psi_1\mathbb{E}_t\left[\frac{\partial \mathcal{M}_1}{\partial \theta_{t+1}^h}\right] + ... \\ & -\beta^{t+1}\Psi_2\mathbb{E}_t\left[\frac{\partial \mathcal{M}_2}{\partial \theta_{t+1}^h}\right] + \beta^{t+2}\mathbb{E}_t\left[\lambda_{t+1}\left(\frac{\partial \mathcal{M}_{3t+1}}{\partial \theta_{t+1}^h}\right)\right] = \beta^{t+1}\Lambda \end{aligned} \tag{2.90}$$

$$\begin{aligned}\phi_{t+1}\colon \qquad & \beta^{t+1}\mathbb{E}_t\left[\frac{\partial \log R_{x_{it+1}}}{\partial \phi_{t+1}}\right] - \beta^{t+1}\Psi_1\mathbb{E}_t\left[\frac{\partial \mathcal{M}_1}{\partial \phi_{t+1}}\right] + ... \\ & -\beta^{t+1}\Psi_2\mathbb{E}_t\left[\frac{\partial \mathcal{M}_2}{\partial \phi_{t+1}}\right] .... \\ -\beta^{t+1}\lambda_t\beta R_{x_t} + {} & \beta^{t+2}\mathbb{E}_t\left[\lambda_{t+1}\left(\frac{\partial \mathcal{M}_{3t+1}}{\partial \phi_{t+1}}\right)\right] = \beta^{t+1}\Lambda + \beta^{t+1}(\xi_U - \xi_L)R_t^f \end{aligned} \tag{2.91}$$

The FOCs for the instruments are:

$$\tau^k_{t+1}: \qquad \beta^{t+1}\mathbb{E}_t\left[\frac{\partial \log R_{x_{it+1}}}{\partial \tau^k_{t+1}}\right] - \beta^{t+1}\Psi_1\mathbb{E}_t\left[\frac{\partial \mathcal{M}_1}{\partial \tau^k_{t+1}}\right] + ...$$

$$-\beta^{t+1}\Psi_2\mathbb{E}_t\left[\frac{\partial \mathcal{M}_2}{\partial \tau^k_{t+1}}\right] + \beta^{t+2}\mathbb{E}_t\left[\lambda_{t+1}\left(\frac{\partial \mathcal{M}_{3t+1}}{\partial \tau^k_{t+1}}\right)\right] = 0 \tag{2.92}$$

$$\tau^w_{t+1}: \qquad \beta^{t+1}\mathbb{E}_t\left[\frac{\partial \log R_{x_{it+1}}}{\partial \tau^w_{t+1}}\right] - \beta^{t+1}\Psi_1\mathbb{E}_t\left[\frac{\partial \mathcal{M}_1}{\partial \tau^w_{t+1}}\right] + ...$$

$$-\beta^{t+1}\Psi_2\mathbb{E}_t\left[\frac{\partial \mathcal{M}_2}{\partial \tau^w_{t+1}}\right] + \beta^{t+2}\mathbb{E}_t\left[\lambda_{t+1}\left(\frac{\partial \mathcal{M}_{3t+1}}{\partial \tau^w_{t+1}}\right)\right] = 0 \tag{2.93}$$

$$R^f_t: \qquad \beta^{t+1}\mathbb{E}_t\left[\frac{\partial \log R_{ix_{t+1}}}{\partial R^f_t}\right] - \beta^{t+1}\Psi_1\mathbb{E}_t\left[\frac{\partial \mathcal{M}_1}{\partial R^f_t}\right] - \beta^{t+1}\Psi_2\beta\mathbb{E}_t\left[\frac{\partial \mathcal{M}_2}{\partial R^f_t}\right] + ...$$

$$+\beta^{t+2}\mathbb{E}_t\left[\lambda_{t+1}\left(\frac{\partial \mathcal{M}_{3t+1}}{\partial R^f_t}\right)\right] = (\xi_U - \xi_L)\phi_{t+1} \tag{2.94}$$

The system of equations can be organized and simplified by eliminating the multiplier, $\Lambda$, as follows:

$$\Psi_1\mathbb{E}_t\left[\frac{\partial \mathcal{M}_1}{\partial \theta^h_{t+1}} - \frac{\partial \mathcal{M}_1}{\partial \theta^k_{t+1}}\right] + \Psi_2\mathbb{E}_t\left[\frac{\partial \mathcal{M}_2}{\partial \theta^h_{t+1}} - \frac{\partial \mathcal{M}_2}{\partial \theta^k_{t+1}}\right] = \mathbb{E}_t\left[\beta\lambda_{t+1}\left(\frac{\partial \mathcal{M}_3}{\partial \theta^h_{t+1}} - \frac{\partial \mathcal{M}_3}{\partial \theta^k_{t+1}}\right)\right] \tag{2.95}$$

$$\begin{aligned}\Psi_1\mathbb{E}_t\left[\frac{\partial \mathcal{M}_1}{\partial \theta^k_{t+1}} - \frac{\partial \mathcal{M}_1}{\partial \phi_{t+1}}\right] &+ \Psi_2\mathbb{E}_t\left[\frac{\partial \mathcal{M}_2}{\partial \theta^k_{t+1}} - \frac{\partial \mathcal{M}_2}{\partial \phi_{t+1}}\right]\\ - \mathbb{E}_t&\left[\beta\lambda_{t+1}\left(\frac{\partial \mathcal{M}_3}{\partial \theta^k_{t+1}} - \frac{\partial \mathcal{M}_3}{\partial \phi_{t+1}}\right)\right]\\ - \lambda_t\beta R_{x_t} = &\underbrace{(\xi_U - \xi_L)R^f_t}_{=0 \text{ if constraint does not bind}}\end{aligned} \tag{2.96}$$

$$\begin{aligned}\mathbb{E}_t\left[\frac{\partial \log R_{x_{it+1}}}{\partial \tau^k_{t+1}} - \frac{\partial \log R_{x_{it+1}}}{\partial \tau^w_{t+1}}\right] + \Psi_1\mathbb{E}_t\left[\frac{\partial \mathcal{M}_1}{\partial \tau^w_{t+1}} - \frac{\partial \mathcal{M}_1}{\partial \tau^k_{t+1}}\right]\\ + \Psi_2\mathbb{E}_t\left[\frac{\partial \mathcal{M}_2}{\partial \tau^w_{t+1}} - \frac{\partial \mathcal{M}_2}{\partial \tau^k_{t+1}}\right] = \mathbb{E}_t\left[\beta\lambda_{t+1}\left(\frac{\partial \mathcal{M}_3}{\partial \tau^w_{t+1}} - \frac{\partial \mathcal{M}_3}{\partial \tau^k_{t+1}}\right)\right]\end{aligned} \tag{2.97}$$

$$
\begin{aligned}
&\mathbb{E}_t\left[\frac{\partial \log R_{x_{it+1}}}{\partial R^f_{t+1}} - \frac{\partial \log R_{x_{it+1}}}{\partial \tau^k_{t+1}}\right] + \Psi_1 \mathbb{E}_t\left[\frac{\partial \mathcal{M}_1}{\partial \tau^k_{t+1}} - \frac{\partial \mathcal{M}_1}{\partial R^f_{t+1}}\right] \\
&+ \Psi_2 \mathbb{E}_t\left[\frac{\partial \mathcal{M}_2}{\partial \tau^k_{t+1}} - \frac{\partial \mathcal{M}_1}{\partial R^f_{t+1}}\right] \\
&- \mathbb{E}_t\left[\beta\lambda_{t+1}\left(\frac{\partial \mathcal{M}_3}{\partial \tau^k_{t+1}} - \frac{\partial \mathcal{M}_3}{\partial R^f_{t+1}}\right)\right] = \underbrace{(\xi_U - \xi_L)\phi_{t+1}}_{=0 \text{ if constraint does not bind}}
\end{aligned}
\tag{2.98}
$$

In the above system, we combine (2.89) with (2.90) to get (2.95), (2.89) with (2.91) to get (2.96), (2.92) with (2.93) to get (2.97) and (2.93) with (2.94) to get (2.98). Solving for this system together with the constraints of the planner determines the equilibrium of the model. Notice that the multipliers Ψ1 and Ψ2 can be recovered in multiple ways. Thus the system is non-invertible. That feature imposes a significant problem in solving for the equilibrium. we next describe the computational methodology and the algorithm that the solution was based.

### 2.8.4 Computational Algorithm

Solving for the equilibrium of the Ramsay problem is computationally challenging. First the relatively large state space and second the nature of some of the state variables increases the perplexity of the calibration for the grid choices. For example, negative asset shares on physical assets cannot be ruled out, i.e short sales. To add more to the complexity, asset shares must always add up to 1 which creates an infinite combination of grid points that could potentially satisfy this relationship. Similarly, characterizing a grid for the tax rates adds to the computational challenge since subsidies cannot be ruled out either. A potential resolution was to adapt the "one period" dynamic programming suggested in Kydland and Prescott (1980) and as applied in Farhi (2010) where some of the state variables can be treated as controls. This approach however, relies on the assumption that the value function (in our case the equivalent will be on the coefficient $\nu_t$) is differentiable which is hard to prove in practice due to the non-convex constraint set and is uncertain whether would be satisfied numerically. Beside this, a standard dynamic programming could run at a very expensive computational cost given our relatively large state space.

In the traditional projection methods the state vector is usually fixed relying on some polynomials to distribute the collocation points, due to the concerns outlined before, an alternative is to rely on simulation methods. This approach has advantages and disadvantages. The advantage is that the equilibrium is computed at points of the ergodic set, that is the part of the state space which is visited in equilibrium. The disadvantage is the poor numerical stability properties that exhibit. The approach we choose mixes simulations and projection methods as developed by Judd et al. (2011) or Maliar and Maliar (2015) and uses their numerical routines. However, the nature of our problem exhibits characteristics rarely found in the literate and so we discuss elements of the numerical methodology that we used to solve the model that might be also useful for other applications. The philosophy of the algorithm tries to balance between stability, accuracy and speed.

The equilibrium of the model is found by solving the FOCs in the system (2.95)-(2.98) together with the constraints of the planner. We repeat the constraints for convenience:

$$E_t\left[R^{-1}_{ix_{t+1}}((1-\tau^w_{t+1})F_h(\theta^k_{t+1},\theta^h_{t+1})+\eta^i_{t+1})\right]=E_t\left[R^{-1}_{ix_{t+1}}((1-\tau^k_{t+1})F_k(\theta^k_{t+1},\theta^h_{t+1}))\right] \tag{2.99}$$

$$E_t\left[R^{-1}_{ix_{t+1}}(1+(1-\tau^k_{t+1})F_k(\theta^k_{t+1},\theta^h_{t+1})-\delta)\right]=E_t\left[R^{-1}_{ix_{t+1}}\right]R^f_t \tag{2.100}$$

$$\phi_{t+1}+\theta^k_{t+1}+\theta^h_{t+1}=1 \tag{2.101}$$

$$\phi_{t+1}\beta R_{x_t}=R^f_{t-1}\phi_t+g_t-\tau^k_tF_k\theta^k_t-\tau^w_tF_H\theta^h_t \tag{2.102}$$

First notice that if we know the policy functions for the three instruments and the assets shares we can compute certain expectations. In particular all expectations that do not involve the Lagrangian $\lambda_{t+1}$ can be computed using Quadrature. We use a two node integration for all cases. Define the state vector described in the text as $\mathcal{X}_t$. The algorithm relies on parametrizing expectations and using them to define relevant policy functions for the variables of interest. In particular, the algorithm proceeds as follows

STEP 1: calibrate the model and draw random variables in accordance with the distributional assumptions about the shocks. We use T=20000 random draws.

STEP 2: parametrize the following expectations

$$\mathbb{E}_t\left[\beta\lambda_{t+1}\left(\frac{\partial\mathcal{M}_3}{\partial\theta^h_{t+1}}-\frac{\partial\mathcal{M}_3}{\partial\theta^k_{t+1}}\right)\right]\approx\Phi_1(\mathcal{X}_t,b_1) \tag{2.103}$$

$$\mathbb{E}_t\left[\beta\lambda_{t+1}\left(\frac{\partial\mathcal{M}_3}{\partial\tau^w_{t+1}}-\frac{\partial\mathcal{M}_3}{\partial\tau^k_{t+1}}\right)\right]\approx\Phi_2(\mathcal{X}_t,b_2) \tag{2.104}$$

$$\mathbb{E}_t\left[\beta\lambda_{t+1}\left(\frac{\partial\mathcal{M}_3}{\partial\theta^k_{t+1}}-\frac{\partial\mathcal{M}_3}{\partial\phi_{t+1}}\right)\right]\approx\Phi_3(\mathcal{X}_t,b_3) \tag{2.105}$$

$$\mathbb{E}_t\left[\beta\lambda_{t+1}\left(\frac{\partial\mathcal{M}_3}{\partial\tau^k_{t+1}}-\frac{\partial\mathcal{M}_3}{\partial R^f_{t+1}}\right)\right]\approx\Phi_4(\mathcal{X}_t,b_4) \tag{2.106}$$

where $\Phi(\mathcal{X}_t,b)$ is a linear polynomial in the levels of the state variables and $b$ the respective polynomial coefficients. In fact, for all approximations we used linear polynomials.

STEP 3: Form the policy functions below, by parametrizing the following expectations:

$$\tilde{k}_{t+1} \approx \mathcal{K}(\mathcal{X}_t) \equiv \frac{\mathrm{E}_t\left[R^{-1}_{ix_{t+1}}\left((1-\tau^k_{t+1})F_k(\theta^k_{t+1},\theta^h_{t+1})\tilde{k}_{t+1}\right)\right] \approx \Phi_5(\mathcal{X}_t, b_5)}{\mathrm{E}_t\left[R^{-1}_{ix_{t+1}}\left((1-\tau^w_{t+1})(F_h(\theta^k_{t+1},\theta^h_{t+1})+\eta_{it+1})\right)\right] \approx \Phi_6(\mathcal{X}_t, b_6)} \tag{2.107}$$

$$R^f_t \approx \mathcal{R}^f(\mathcal{X}_t) \equiv \frac{\mathrm{E}_t\left[R^{-1}_{ix_{t+1}}\left(1+(1-\tau^k_{t+1})F_k(\theta^k_{t+1},\theta^h_{t+1})-\delta\right)\right] \approx \Phi_7(\mathcal{X}_t, b_7)}{\mathrm{E}_t\left[R^{-1}_{ix_{t+1}}\right] \approx \Phi_8(\mathcal{X}_t, b_8)} \tag{2.108}$$

$$\tau^k_{t+1} \approx \mathcal{T}^k(\mathcal{X}_t) \equiv \frac{\mathrm{E}_t\left[R^{-1}_{ix_{t+1}}\left((1-\tau^k_{t+1})F_k(\theta^k_{t+1},\theta^h_{t+1})\tau^k_{t+1}\right)\Phi_3(\mathcal{X}_t, b_3)\right] \approx \Phi_9(\mathcal{X}_t, b_9)}{\Phi_3(\mathcal{X}_t, b_3)} \tag{2.109}$$

$$\tau^w_{t+1} \approx \mathcal{T}^w(\mathcal{X}_t) \equiv \frac{\mathrm{E}_t\left[R^{-1}_{ix_{t+1}}\left((1-\tau^w_{t+1})(F_h(\theta^k_{t+1},\theta^h_{t+1})+\eta_{it+1})\right)\tau^w_{t+1}\right] \approx \Phi_{10}(\mathcal{X}_t, b_{10})}{\mathrm{E}_t\left[R^{-1}_{ix_{t+1}}\left((1-\tau^k_{t+1})F_k(\theta^k_{t+1},\theta^h_{t+1})\right)\right] \approx \Phi_{11}(\mathcal{X}_t, g_{11})} \tag{2.110}$$

STEP 4: Calculate the asset shares.

In this step, calculate the growth rate $E_{t-1} \equiv R_{x_t}$ analytically and use equation (2.102) to get $\phi_{t+1}$. Check whether the constraint binds or not and assign the relevant value. From this point and until step 7, the algorithm is split between cases about the binding constraint.

- From the financial market clearing condition, that is equation (2.101), write it as: $\phi_{t+1} + (\underbrace{\frac{\theta^k_{t+1}}{\theta^h_{t+1}}}_{\tilde{k}_{t+1}} + 1)\theta^h_{t+1} = 1$ and solve for the share of human capital, $\theta^h_{t+1}$. Recover then the share of physical capital as, $\theta^k_{t+1} = \tilde{k}_{t+1}\theta^h_{t+1}$

STEP 5 Given the steps in 2 and 3, and having computed the asset shares we can compute the remaining expectations that do not involve the future value of the Lagrangian $\lambda_{t+1}$. We did this using Quadrature with two nodes for each shocks.

STEP 6: Given step 5 use the FOCs, (2.95) and (2.97), to recover the values for $\Psi_1$ and $\Psi_2$ by solving the linear system of equations with respect to these two variables.

Step 7: Given steps 5 and 6 recover $\lambda_t$ by combining the other two FOCs, that is equations (2.96) and (2.98). Notice, that if the constraints bind, these two equations can still be combined by eliminating the Lagrangian (either $\xi_U$ or $\xi_L$) and so do not have to be stored.

STEP 8: Given the series of the variables obtained in the previous steps, calculate the *realized* expectations in the tradition of PEA and update coefficients through a linear regression. In the regression method we used regularization techniques described in Judd et al. (2011) and in particular the RLS-Tikhonov with normalized data and penalty -7. The regressions are run by the routines provided by those authors.

We will shortly discuss the formulation and justification of the policy functions formed in step 3. In forming the policy function for $\tilde{k}_t$, $\tau^w_{t+1}$ or $R^f_{t+1}$ we utilized the predetermined nature of these variables ( i.e. all are $t$ measurable) and reformulated the constraints (2.99) and (2.100) in the way shown. This is a standard artifice either to break the non-singularity of portfolio choice problems (See for instance Marcet and Lorenzoni (1998)) or to perform a derivative-free fixed point iteration (See for example Maliar and Maliar (2015), or Judd et al. (2014)). We could have applied the same approach and reformulate the FOC's of the planner in order to back up some of the same variables. However in practice the process failed to convergence. Thus I used a simple idea. Since constraint (2.45) must also hold and is a reformulation of the portfolio (Euler) conditions, we used the already parametrized expectation $\mathbb{E}_t\left[\beta\lambda_{t+1}\left(\frac{\partial\mathcal{M}_3}{\partial\theta^k_{t+1}}-\frac{\partial\mathcal{M}_3}{\partial\phi_{t+1}}\right)\right]\approx \Phi_3(\mathcal{X}_t, b_3)$ that shows up in the Euler equation for planner (i.e (2.96)) and together with the predetermined nature of $\tau^k_{t+1}$ we reformulated constraint (2.45) in accordance with (2.109). Nothing in the properties of the equilibrium changed if the same approach would have being used for any other candidate variable. The criterion for performing this strategy to construct the policy function for $\tau^k_{t+1}$ was solely for computational convenience, since with the initializations on the coefficient, the speed of convergence was faster. A similar idea, although not identical, is the "Forward State PAE" modification proposed by Faraglia et al. (2014b), where future values inside the expectations are treated as policy functions and by the law of iterated expectation it is possible to break the non-singularity of the system.

**Practical Issues:** The guess about the initial conditions for the polynomial coefficients, in our context, was hard to find. Traditional methods suggested in the literature (den Haan and Marcet (1990), Marcet and Lorenzoni (1998)) did not work. The strategy we developed then was, to first solve the model assuming a state space that *jointly* encompasses some of the state variables and initialized the polynomial coefficients by some arbitrary guesses. That is we reduced the state space through some educated procedure. For example, in this model the asset shares, taxes and the TFP shock all are entering the average gross portfolio returns and define a particular function that its value is guaranteed to be *different from zero*, i.e $E_t[R_{x_{it}}] \equiv \bar{R}_{x_t} = R^f_{t-1}\phi_t + (1 + (1-\tau^k)A_tF_{h_t} - \delta)\theta^h_t + (1 + (1-\tau^k)A_tF_{k_t} - \delta)\theta^k_t$. Hence we started with the reduced space $[\bar{R}_{x_t}, g_t]$, and with the help of moving bounds (see Maliar and Maliar (2003)) the algorithm in most of the cases was converging. However this was possible only when we used the regression routines provided by Judd et al. (2011). After obtaining convergence, we used the solution for the polynomial coefficients as initial guesses and add each time autonomously a state variable dictated by the model until we reach the original state space. The dumping parameter was set to 0.2. Moreover, the algorithm turned out to be quite sensitive to the variance of idiosyncratic shocks, $\sigma^2_\eta$. In that case we were loosening the convergence criteria and set the dumping parameter to a lower value.

### 2.8.5 Competitive Equilibrium: Steady State

In this section we describe in some detail the calibration process used to get the results. Notice that due to endogenous growth the economy is non-stationary. We thus convert the model in stationary form, by dividing each variable with the returns inclusive wealth, $X_t$. In this case the economy is on its balanced growth path. Second, in the macro-economic state debt and capital are collinear assets. By a standard arbitrage argument their returns must be equal, so we also set $R^f = R^k$ where we also normalized the price of bonds to one. We re-define the asset shares as $\theta^h_t = \frac{H_t}{S_t}$ and $1-\theta^h_t = \frac{K_t+B_t}{S_t}$. Finally, we omit any borrowing constraints. The system of equations to be solved is:

$$\begin{aligned}
\text{Aggregate Portfolio returns :} \quad & R_x = R^k(1-\theta^h) + \theta^h R^h && (2.111)\\
\text{Growth rate :} \quad & 1+gr = R_x\beta && (2.112)\\
\text{Consumption :} \quad & \tilde{c} = (1-\beta) && (2.113)\\
\text{Human Capital :} \quad & \tilde{h} = \beta\theta^h && (2.114)\\
\text{Physical Capital :} \quad & \tilde{k} = \beta(1-\theta^h) - \tilde{b} && (2.115)\\
\text{Governemnt debt :} \quad & \tilde{b} = \frac{\bar{g} - \tau^k r\tilde{k} - \tau^w w\tilde{h}}{1+gr-R^k} && (2.116)\\
\text{FOC for } \theta^h \text{ :} \quad & E\left[\frac{R^k - R^i}{R_{ix}}\right] = 0 && (2.117)\\
\text{Distribution of idiosyncratic shocks :} \quad & \eta^i_t \sim \text{iid } N(0,\sigma^2_\eta) && (2.118)\\
\text{Microeconomic state :} \quad & h^i_{it} = \beta R_{x_{it}} h_{it-1} && (2.119)
\end{aligned}$$

To keep things simple, the distributional assumptions about the idiosyncratic shock allows to manipulate the FOC for the optimal share in a way that we can use the following approximation:

$$\theta^h \approx \frac{\tilde{r}_h - \tilde{r}_k}{\sigma^2} \tag{2.120}$$

The steady state then becomes a non-linear system of equations to be solved for. The calibration process then is similar to Krebs (2003b) and Gottardi et al. (2014b) which we described in the main text. We next show how to arrive to the previous approximation and get the value for the parameter $\sigma^2_\eta$.

In the balanced growth path, the market clearing condition for the share of human capital comes from the respective FOC. This is:

$$E[R^{-1}_{x_{it+1}}\eta^i_{t+1}] = (\tilde{r}_k - \tilde{r}_h)E[R^{-1}_{x_{it+1}}] \tag{2.121}$$

where $\tilde{r}_k = (1-\tau^k)f_k$, $\tilde{r}_h = (1-\tau^w)f_h$ are the after tax returns of physical and human capital. To find the previous approximation we utilize the Gaussian formula. More specifically notice that the gross portfolio returns can be also be written as,

$\exp(\log(Rx_i^{-1})) = \exp(-\log(R)) \approx \exp[-(\tilde{r}_k(1-\theta^h) + \tilde{r}_h^i\theta^h)]$. The stochastic part, is equal to $\exp[-(\eta_{t+1}^i\theta^h)]$. Manipulating (2.121), someone can get the following equation:

$$E[\exp[-(\eta_{t+1}^i\theta^h)]\eta_{t+1}^i] = (\tilde{r}_k - \tilde{r}_h)E[\exp[-(\eta_{t+1}^i\theta^h)]] \tag{2.122}$$

$$E[\Big(\exp[-(\sigma\theta^h Z)]\Big)\,\sigma Z] = (\tilde{r}_k - \tilde{r}_h)E[\exp[-(\eta_{t+1}^i\theta^h)]] \tag{2.123}$$

$$E[\Big(\exp[-(\sigma\theta^h Z)]\Big)\,Z] = \frac{(\tilde{r}_k - \tilde{r}_h)}{\sigma}E[\exp[-(\eta_{t+1}^i\theta^h)]] \tag{2.124}$$

Where $Z$ is the standard normal, then by the Gaussian formula (i.e $-\sigma\theta^h = \frac{(\tilde{r}_k - \tilde{r}_h)}{\sigma}$) the above equation implies:

$$\theta^h = \frac{\tilde{r}_h(\tilde{k}) - \tilde{r}_k(\tilde{k})}{\sigma^2} \tag{2.125}$$

Using the definition of $\theta^h \equiv (\frac{1}{1+\tilde{k}+\tilde{b}})$ we can solve the non-linear equations in (2.121) by replacing the FOC for the human capital share with the closed form counterpart derived before. The subsequent steps show how to get the number for the idiosyncratic volatility.

More specifically, the individual law of motion for the human capital accumulation is:

$$h_{it} = \beta R_t^i h_{it-1} \tag{2.126}$$

Next define the *after tax* individual labour income, as follows:

$$y_{it} = \tilde{f}_h(\tilde{k}_t)h_{it} \Rightarrow \tag{2.127}$$

$$y_{it} = \tilde{f}_h(k_{ss})h_{it} \tag{2.128}$$

hence, the log difference of labour income, will be equal to:

$$\log(y_{it+1}) - \log(y_{it}) = \log(\tilde{f}_h(k_{ss})) + \log(h_{it}) - \log(\tilde{f}_h(k_{ss})) + \log(h_{it-1}) \tag{2.129}$$

$$= \log(h_{it}) - \log(h_{it-1}) \tag{2.130}$$

$$= \log(\beta) + \log(R_{x_{it}}) \tag{2.131}$$

$$\approx \log(\beta) + r_k + \theta[r_h - r_k + \eta^i] \tag{2.132}$$

$$= \beta_0 + \underbrace{\theta\eta^i}_{e^i} \tag{2.133}$$

and we can get:

$$\text{Var}(\Delta \log(y_{it})) \equiv \sigma_y^2 = \theta_h^2 \sigma^2 \Rightarrow \tag{2.134}$$

$$\sigma_y = \theta^h \sigma \tag{2.135}$$

$$\sigma_y = (\frac{1}{1 + \tilde{k} + \tilde{b}})\sigma \tag{2.136}$$

$\sigma_y$ is the standard deviation of labour income that can be matched from the data. According to Meghir and Pistaferri (2004) the permanent component of the income process conditional to aggregate shocks has a standard deviation $\sigma_y \equiv E[\frac{y_{it+1}}{y_{it}}|S^t] = 0.19$, while Storesletten et al. (2007) find $\sigma_y = 0.25$. We choose a compromise between the two by setting $\sigma_y = 0.215$.

We next calibrate, the discount factor $\beta$ to match the average consumption growth, $\frac{C_{t+1}}{C_t} = E_t[\frac{c_{it+1}}{c_{it}}] = \beta E_t[R_{x_{it+1}}]$ which in our case is equal to the growth rate of the economy. In US data the average annual rate of growth for output, 1.6%, thus in our steady state we set $1 + g = 1.016$, which implies that $\beta = \frac{1.016}{R_x}$. Along the balanced growth we follow Gottardi et al. (2014b) and assume $\tau^w = \tau^k = \tau$. Moreover as in Chari et al. (1994) we choose $\frac{G_t}{Y_t} = 0.18$, $\frac{K_{t-1}}{Y_t} = 2.71$, $\frac{B_{t-1}}{Y_t} = \frac{B_t}{Y_{t+1}} = 0.51 \Rightarrow \tilde{b} = 0.51A\tilde{k}^{\alpha}$. These numbers in the balanced growth path can pin down taxes, $\tau^k = \tau^w = 0.196$. The normalization for TFP is obtained by matching the saving rate, $s_x$, in the US economy (defined as the investment, $I_k$ of physical capital to output) which is 0.25. Using the law of motion for capital accumulation $A = \left(\frac{g+\delta_k}{s_x}\right)\tilde{k}^{1-\alpha}$. Solving the system the values obtained, are $\beta = 0.9458, \tau^w = \tau^k = 0.196, \bar{A} = 0.36$ and $\sigma = 0.5384$. The remaining parameters are chosen according to the RBC literature and in particular $\sigma_e = 0.01$, $\rho = 0.95$ and $\delta = 0.06$ (From Krebs (2003b)).

### 2.8.6 List of Figures

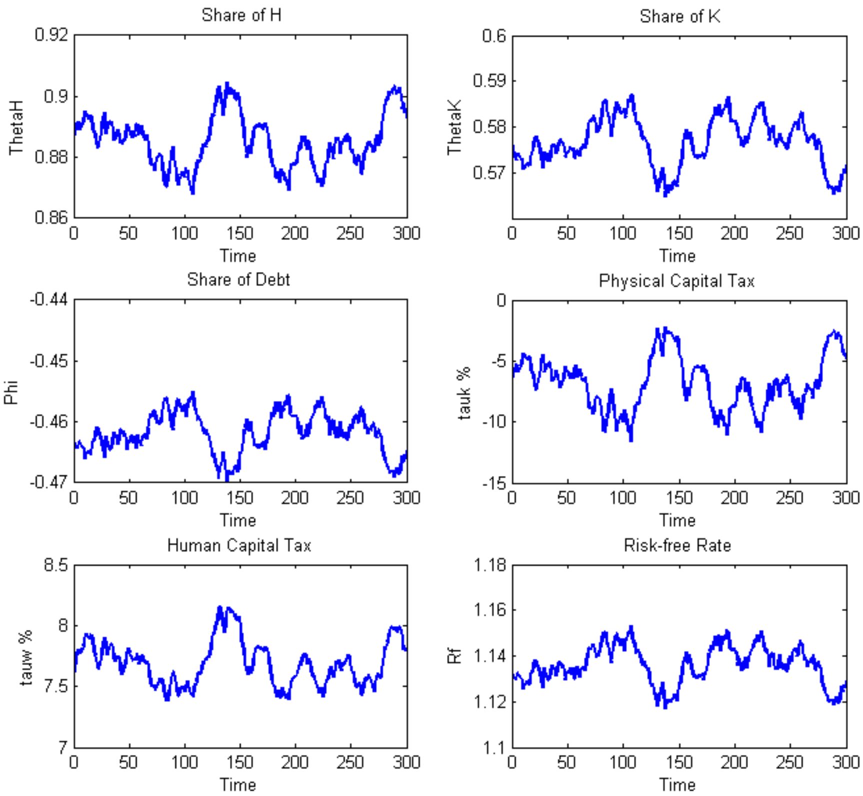



Figure 2.4: Simulated Series Benchmark case

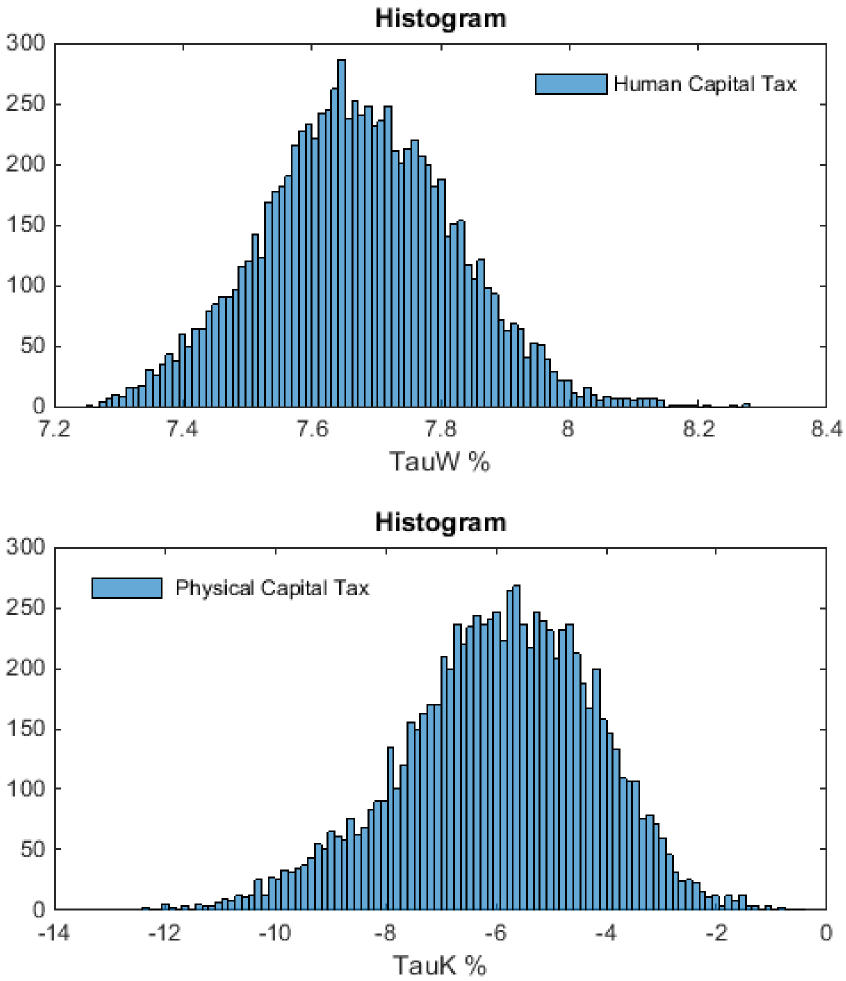



Figure 2.5: Histograms for Taxes

### 2.8.7 Definitions and Notation

$$
\begin{aligned}
\tilde{\mathcal{W}}_{\tau_t^w} &\equiv \mathbb{E}_t\left[\frac{\partial \log R_{x_{it+1}}}{\partial \tau_{t+1}^w}\right] \\
m_{t+1}^i &\equiv R_{ix_{t+1}}^{-1} \\
m_{it+1}^* &\equiv \frac{R_{ix_{t+1}}^{-1}}{E_t[R_{ix_{t+1}}^{-1}]} \\
\tilde{\mathcal{P}}_{t+1}^H &\equiv (1-\tau_{t+1}^w)F_h(\theta_{t+1}^k, \theta_{t+1}^h) - (1-\tau_{t+1}^k)F_k(\theta_{t+1}^k, \theta_{t+1}^h) + \eta_{t+1}^i \\
\tilde{\mathcal{P}}_{t+1}^K &\equiv 1 + (1-\tau_{t+1}^w)F_k(\theta_{t+1}^k, \theta_{t+1}^h) - \delta - R_t^f
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_1 &\equiv \left(m_{t+1}^i \times \tilde{\mathcal{P}}_{t+1}^H\right) \\
\mathcal{M}_2 &\equiv \left(m_{t+1}^i \times \tilde{\mathcal{P}}_{t+1}^k\right) \\
R_{x_t} &\equiv E_{t-1}[R_{x_{it}}] \\
R_{x_{t+1}} &\equiv E_t[R_{x_{it+1}}] \\
\mathcal{M}_3 &\equiv \left(R_{t-1}^f\phi_t + g_t - \tau_t^k F_k\theta_t^k - \tau_t^w F_H\theta_t^h - \phi_{t+1}\beta R_{x_t}\right) \\
\mathcal{M}_{3_{t+1}} &\equiv \left(R_t^f\phi_{t+1} + g_{t+1} - \tau_{t+1}^k F_{k_{t+1}}\theta_{t+1}^k - \tau_{t+1}^w F_{H_{t+1}}\theta_{t+1}^h - \phi_{t+2}\beta R_{x_{t+1}}\right)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{DSF}_{t+1} &\equiv R_{ix_{t+1}}^{-2} \times \tilde{\mathcal{P}}_{t+1}^H \\
\mathrm{DSF2}_{t+1} &\equiv R_{ix_{t+1}}^{-2} \times \tilde{\mathcal{P}}_{t+1}^K
\end{aligned}
$$

**Block 1**

$$\frac{\partial \mathcal{M}_1}{\partial \theta^k_{t+1}} \equiv R^{-1}_{ix_{t+1}}\left[(1-\tau^w_{t+1})\underbrace{F_{HK}}_{=-\frac{F_{KK}\theta^k_{t+1}}{\theta^h_{t+1}}} -(1-\tau^k_{t+1})\underbrace{F_{KK}}_{-F_K(1-\alpha)(\theta^k_{t+1})^{-1}}\right] - \mathcal{DSF}_{t+1}\frac{\partial R_{ix_{t+1}}}{\partial \theta^k_{t+1}}$$

$$= R^{-1}_{ix_{t+1}}\left[(1-\tau^w_{t+1})(\theta^h_{t+1})^{-1} + (1-\tau^k_{t+1})(\theta^k_{t+1})^{-1}\right](1-\alpha)F_k - \mathcal{DSF}_{t+1}\frac{\partial R_{ix_{t+1}}}{\partial \theta^k_{t+1}}$$

$$\frac{\partial \mathcal{M}_1}{\partial \theta^h_{t+1}} \equiv R^{-1}_{ix_{t+1}}\left[(1-\tau^w_{t+1})\underbrace{F_{HH}}_{-\alpha F_H(\theta^h_{t+1})^{-1}} -(1-\tau^k_{t+1})\underbrace{F_{KH}}_{-\frac{F_{HH}\theta^h_{t+1}}{\theta^k_{t+1}}}\right] - \mathcal{DSF}_{t+1}\frac{\partial R_{ix_{t+1}}}{\partial \theta^h_{t+1}}$$

$$\equiv -R^{-1}_{ix_{t+1}}\left[(1-\tau^w_{t+1})(\theta^h_{t+1})^{-1} + (1-\tau^k_{t+1})(\theta^k_{t+1})^{-1}\right]\alpha F_H - \mathcal{DSF}_{t+1}\frac{\partial R_{ix_{t+1}}}{\partial \theta^h_{t+1}}$$

$$\frac{\partial \mathcal{M}_1}{\partial \tau^k_{t+1}} \equiv R^{-1}_{ix_{t+1}}F_K - \mathcal{DSF}_{t+1}\frac{\partial R_{ix_{t+1}}}{\partial \tau^k_{t+1}}$$

$$\frac{\partial \mathcal{M}_1}{\partial \tau^h_{t+1}} \equiv -R^{-1}_{ix_{t+1}}F_H - \mathcal{DSF}_{t+1}\frac{\partial R_{ix_{t+1}}}{\partial \tau^h_{t+1}}$$

$$\frac{\partial \mathcal{M}_1}{\partial \phi_{t+1}} \equiv -\mathcal{DSF}_{t+1}\frac{\partial R_{ix_{t+1}}}{\partial \phi_{t+1}}$$

$$\frac{\partial \mathcal{M}_1}{\partial R^f_t} \equiv -\mathcal{DSF}_{t+1}\frac{\partial R_{ix_{t+1}}}{\partial R^f_t}$$

**Block 2**

$$\begin{aligned}
\frac{\partial \mathcal{M}_2}{\partial \theta^k_{t+1}} &\equiv R^{-1}_{ix_{t+1}}(1-\tau^k_{t+1})F_{KK} - \mathrm{DSF2}_{t+1}\frac{\partial R_{ix_{t+1}}}{\partial \theta^k_{t+1}} \\
&= -R^{-1}_{ix_{t+1}}(1-\tau^k_{t+1})F_K(1-\alpha)(\theta^k_{t+1})^{-1} - \mathrm{DSF2}_{t+1}\frac{\partial R_{ix_{t+1}}}{\partial \theta^k_{t+1}}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \mathcal{M}_2}{\partial \theta^h_{t+1}} &\equiv R^{-1}_{ix_{t+1}}(1-\tau^k_{t+1})F_{KH} - \mathrm{DSF2}_{t+1}\frac{\partial R_{ix_{t+1}}}{\partial \theta^h_{t+1}} \\
&= R^{-1}_{ix_{t+1}}(1-\tau^k_{t+1})F_K(1-\alpha)(\theta^h_{t+1})^{-1} - \mathrm{DSF2}_{t+1}\frac{\partial R_{ix_{t+1}}}{\partial \theta^h_{t+1}}
\end{aligned}$$

$$\frac{\partial \mathcal{M}_2}{\partial \tau^k_{t+1}} \equiv -R^{-1}_{ix_{t+1}}F_K - \mathrm{DSF2}_{t+1}\frac{\partial R_{ix_{t+1}}}{\partial \tau^k_{t+1}}$$

$$\frac{\partial \mathcal{M}_2}{\partial \tau^h_{t+1}} \equiv -\mathrm{DSF2}_{t+1}\frac{\partial R_{ix_{t+1}}}{\partial \tau^h_{t+1}}$$

$$\frac{\partial \mathcal{M}_2}{\partial \phi_{t+1}} \equiv -\mathrm{DSF2}_{t+1}\frac{\partial R_{ix_{t+1}}}{\partial \phi_{t+1}}$$

$$\frac{\partial \mathcal{M}_2}{\partial R^f_t} \equiv -\mathrm{DSF2}_{t+1}\frac{\partial R_{ix_{t+1}}}{\partial R^f_t}$$

**Block 3**

$$\frac{\partial \mathcal{M}_3}{\partial \theta^k_{t+1}} \equiv -\tau^k_t[\underbrace{F_{KK}\theta^k_{t+1}}_{F_K(\alpha-1)} + F_K] - \tau^w_t \underbrace{F_{HK}\theta^h_{t+1}}_{-F_{KK}\theta^k} - \phi_{t+2}\beta \mathrm{E}_t\left[\frac{\partial R_{ix_{t+1}}}{\partial \theta^k_{t+1}}\right]$$

$$= -\underbrace{[\tau^w_{t+1}(1-\alpha) + \tau^k_{t+1}\alpha]}_{\tau_2} F_K - \phi_{t+2}\beta \mathrm{E}_t\left[\frac{\partial R_{ix_{t+1}}}{\partial \theta^k_{t+1}}\right]$$

$$\frac{\partial \mathcal{M}_3}{\partial \theta^h_{t+1}} \equiv -\tau^w_t[\underbrace{F_{HH}\theta^h_{t+1}}_{-\alpha F_H} + F_H] - \tau^k_t \underbrace{F_{KH}\theta^k_{t+1}}_{=-F_{HH}\theta^h} - \phi_{t+2}\beta \mathrm{E}_t\left[\frac{\partial R_{ix_{t+1}}}{\partial \theta^h_{t+1}}\right]$$

$$= -[\tau^w_{t+1}(1-\alpha) + \tau^k_{t+1}\alpha] F_H - \phi_{t+2}\beta \mathrm{E}_t\left[\frac{\partial R_{ix_{t+1}}}{\partial \theta^h_{t+1}}\right]$$

$$\frac{\partial \mathcal{M}_3}{\partial \tau^k_{t+1}} \equiv -F_K\theta^k_{t+1} - \phi_{t+2}\beta \mathrm{E}_t\left[\frac{\partial R_{ix_{t+1}}}{\partial \tau^k_{t+1}}\right]$$

$$\frac{\partial \mathcal{M}_3}{\partial \tau^h_{t+1}} \equiv -F_H\theta^h_{t+1} - \phi_{t+2}\beta \mathrm{E}_t\left[\frac{\partial R_{ix_{t+1}}}{\partial \tau^w_{t+1}}\right]$$

$$\frac{\partial \mathcal{M}_3}{\partial \phi_{t+1}} \equiv R^f_t - \phi_{t+2}\beta \mathrm{E}_t\left[\frac{\partial R_{ix_{t+1}}}{\partial \phi_{t+1}}\right]$$

$$\frac{\partial \mathcal{M}_3}{\partial R^f_t} \equiv \phi_{t+1} - \phi_{t+2}\beta \mathrm{E}_t\left[\frac{\partial R_{ix_{t+1}}}{\partial R^f_t}\right]$$

$$\frac{\partial R_{ix_{t+1}}}{\partial \theta^k_{t+1}} \equiv 1 + (1-\tau^k_{t+1})F_k - \delta + (1-\tau^k_{t+1})\underbrace{\theta^k_{t+1}F_{KK}}_{F_K(\alpha-1)} + (1-\tau^w_{t+1})\underbrace{\theta^h_{t+1}\underbrace{F_{HK}}_{=F_{KH}}}_{-F_{KK}\theta^k}$$

$$= 1 - \delta + (1-\tau^k_{t+1})F_k + (1-\tau^k_{t+1})F_k(\alpha-1) - (1-\tau^w_{t+1})F_k(\alpha-1)$$

$$= 1 - \delta + \Big[(1-\tau^k_{t+1})\alpha + (1-\tau^w_{t+1})(1-\alpha)\Big]F_k$$

$$\frac{\partial R_{ix_{t+1}}}{\partial \theta^h_{t+1}} \equiv 1 + (1-\tau^w_{t+1})F_H - \delta + \eta^i_{t+1} + (1-\tau^k_{t+1})\underbrace{F_{KH}\theta^k_{t+1}}_{\alpha F_H} + (1-\tau^w_{t+1})\underbrace{F_{HH}\theta^h_{t+1}}_{-\alpha F_H}$$

$$= 1 - \delta + \eta^i_{t+1} + \Big[(1-\tau^k_{t+1})\alpha + (1-\tau^w_{t+1})(1-\alpha)\Big]F_H$$

$$\frac{\partial R_{ix_{t+1}}}{\partial \tau^k_{t+1}} \equiv -F_K\theta^k_{t+1}$$

$$\frac{\partial R_{ix_{t+1}}}{\partial \tau^h_{t+1}} \equiv -F_H\theta^h_{t+1}$$

$$\frac{\partial R_{ix_{t+1}}}{\partial \phi_{t+1}} \equiv R^f_t$$

$$\frac{\partial R_{ix_{t+1}}}{\partial R^f_t} \equiv \phi_{t+1}$$

$$\frac{\partial \mathcal{M}_1}{\partial \theta^h_{t+1}} - \frac{\partial \mathcal{M}_1}{\partial \theta^k_{t+1}} = -R^{-1}_{ix_{t+1}} F_K (1-\alpha)(\tilde{k}_{t+1}+1)\tau_1$$
$$- \mathcal{DSF}_{t+1} \left( \frac{\partial R_{ix_{t+1}}}{\partial \theta^h_{t+1}} - \frac{\partial R_{ix_{t+1}}}{\partial \theta^k_{t+1}} \right)$$
$$\frac{\partial \mathcal{M}_2}{\partial \theta^h_{t+1}} - \frac{\partial \mathcal{M}_2}{\partial \theta^k_{t+1}} =$$
$$R^{-1}_{ix_{t+1}} F_K (1-\alpha)[(\theta^k_{t+1})^{-1} + (\theta^h_{t+1})^{-1}]$$
$$- \mathrm{DSF2}_{t+1} \left( \frac{\partial R_{ix_{t+1}}}{\partial \theta^h_{t+1}} - \frac{\partial R_{ix_{t+1}}}{\partial \theta^k_{t+1}} \right)$$
$$\frac{\partial \mathcal{M}_3}{\partial \theta^h_{t+1}} - \frac{\partial \mathcal{M}_3}{\partial \theta^k_{t+1}} = \tau_2 \left( 1 - \frac{(1-\alpha)\tilde{k}_{t+1}}{\alpha} \right) F_K$$
$$- \phi_{t+2}\beta E_t \left[ \frac{\partial R_{ix_{t+1}}}{\partial \theta^h_{t+1}} - \frac{\partial R_{ix_{t+1}}}{\partial \theta^k_{t+1}} \right]$$

$$\frac{\partial \mathcal{M}_1}{\partial \tau^w_{t+1}} - \frac{\partial \mathcal{M}_1}{\partial \tau^k_{t+1}} =$$
$$- R^{-1}_{ix_{t+1}} F_K \left( 1 + \frac{(1-\alpha)\tilde{k}_{t+1}}{\alpha} \right) -$$
$$\mathcal{DSF}_{t+1} \left( \frac{\partial R_{ix_{t+1}}}{\partial \tau^w_{t+1}} - \frac{\partial R_{ix_{t+1}}}{\partial \tau^k_{t+1}} \right)$$
$$\frac{\partial \mathcal{M}_2}{\partial \tau^w_{t+1}} - \frac{\partial \mathcal{M}_2}{\partial \tau^k_{t+1}} = R^{-1}_{ix_{t+1}} F_K - \mathrm{DSF2}_{t+1} \left( \frac{\partial R_{ix_{t+1}}}{\partial \tau^w_{t+1}} - \frac{\partial R_{ix_{t+1}}}{\partial \tau^k_{t+1}} \right)$$
$$\frac{\partial \mathcal{M}_3}{\partial \tau^w_{t+1}} - \frac{\partial \mathcal{M}_3}{\partial \tau^k_{t+1}} = -\frac{F_H \theta^h_{t+1}}{1-\alpha} - \phi_{t+2}\beta E_t \left[ \frac{\partial R_{ix_{t+1}}}{\partial \tau^w_{t+1}} - \frac{\partial R_{ix_{t+1}}}{\partial \tau^k_{t+1}} \right]$$

$$\frac{\partial R_{ix_{t+1}}}{\partial \theta^h_{t+1}} - \frac{\partial R_{ix_{t+1}}}{\partial \theta^k_{t+1}} = \eta^i_{t+1} + \tau_2 \left( \frac{(1-\alpha)\tilde{k}_{t+1}}{\alpha} - 1 \right) F_K$$
$$\frac{\partial R_{ix_{t+1}}}{\partial \tau^w_{t+1}} - \frac{\partial R_{ix_{t+1}}}{\partial \tau^k_{t+1}} = -\frac{F_H \theta^h_{t+1}}{1-\alpha}$$
$$\frac{\partial \log R_{ix_{t+1}}}{\partial \tau^w_{t+1}} - \frac{\partial \log R_{ix_{t+1}}}{\partial \tau^k_{t+1}} = -R^{-1}_{ix_{t+1}} F_K \left( \frac{(1-\alpha)\tilde{k}_{t+1}}{\alpha} + \theta^k_{t+1} \right)$$

In some expressions we also used the property of Cobb-Douglas production function, $F_{k_{t+1}}\tilde{k}_{t+1} = \left(\frac{\alpha}{1-\alpha}\right) F_{H_{t+1}}$ where $\tilde{k}_{t+1} \equiv \frac{\theta_{t+1}}{\theta^h_{t+1}}$

# Chapter 3

# Fiscal Multipliers: The case of Greece

## 3.1 Introduction

Standard neoclassical growth models of fiscal policy usually predict private consumption and real wages to fall, output and labour supply to increase and investment can take either sign depending on the persistence of the fiscal shock. In contrast New Keynesian models are able to generate an increase in real wages and private consumption, an otherwise formidable challenge for RBC models. Although, those differences should be thought as of short-run, in the long run the qualitative features seem to be very similar. On the empirical side, the dispute is also thriving providing support for either paradigms. For a survey see Perotti (2007) who critically reviews the literature.

The main purpose of this chapter is to address the case of Greece and implement some empirical estimates of the tax and spending multipliers. Guided by the empirical results, someone can also deduce at which side of the literature the Greek economy possibly falls. Moreover, is a natural first step for analysing the effects of fiscal consolidations, and the short to long-run effects that a fiscal adjustment can bring on growth, fiscal sustainability and fiscal solvency. However and after being data constrained, the main focus will be reserved for the effects on national output. To best of our knowledge this study is the first application of the structural VAR to Greece.

The recent financial crisis and its conversion to a debt crisis within the Eurozone, resurrected a widespread interest on fiscal policy. The interest was greatly extended on the empirical side for its obvious policy implications, while the ongoing Greek Great Depression feeds an equal academic interest on its own. From the point of view of Greece, the empirical challenges faced were manifold while the implications for policy analysis become more relevant than ever before. As such, the fundamental intention of this research is to advance the debate over the empirical fiscal policy and draw some relevant to Greece policy conclusions.

## 3.2 Literature Review

Studies on fiscal policy for Greece are limited to a few applications of the neoclassical growth model. A leading example is Papageorgiou (2012). In his study, simulation results do not report the size fiscal multipliers with respect to output, although is implicitly assumed to be less than one in absolute value. Given the lack of available literature for Greece which can be directly comparable with this research it could be useful to refer to other studies and leading research on empirical fiscal policy.

As mentioned above, the disagreement between the RBC and New Keynesian predictions subsequently led to a voluminous, but interesting, literature on empirical fiscal policy. A reflection on the different empirical methodologies, include among others, the pioneer works of Ramey and Shapiro (1998) with their dummy variable approach, Blanchard and Perotti (2002) with their SVAR approach, while more recently Mountford and Uhlig (2009) with their sign restriction approach, all of which constitute classic references on the fiscal policy empirical methodologies. Moreover, the narrative approach of Romer and Romer (2010) advanced the debate over the size of tax multipliers by providing support of large in size tax multipliers. The first study, gives support to the RBC predictions while the SVAR approach of Blanchard and Perotti (2002) is more favourable to the New Keynesian paradigm. On the other hand, Mountford and Uhlig (2009) circumvents some of the limitations of Blanchard and Perotti (2002) methodology and highlights the significance of tax rebates. The merits and distinct features of the different methodologies are unified and critically assessed by Caldara and Kamps (2008)

## 3.3 The SVAR methodology and the Econometric Model

The empirically challenging task for assessing the effects of fiscal policy on the economy, lies on the identification of exogenous fiscal shocks. In other words, the difficulty lies in finding directly observable responses of fiscal variables that can be characterized as exogenous which then can constitute the basis for analysing the effects on the economy. Loosely, speaking the curious task is to demonstrate discretionary changes in spending or taxes.

After Sim's pioneering article on VARs, fiscal shocks where 'mechanically' constructed utilizing a Cholesky decomposition [1]. In principle, this is what the SVAR approach of Blanchard and Perotti does, although under more plausible strategy than an *atheoretic* Cholesky decomposition[2].

In fact, the distinctive feature of the VARs against the other approaches is that deduces the fiscal shocks from the data, in contrast with the narrative or dummy variable approaches which regard specific external data as exogenous. In the language of Ramey and Shapiro, military build-ups are the identified data - 'events' - of exogenous fiscal shocks. In contrast, in Romer and Romer (2010) data is collected from Presidential

[1] See for instance Fatás and Mihov (2001)

[2] In the empirical fiscal policy literature, VARs that use the Cholesky decomposition as a basis of the identification problem is usually referred as the "Recursive approach".

speeches and congressional reports that are reasoning the scope of tax changes. Full explanations on the merits and disadvantages of each approach is out of the scope of this paper[3].

Apparently, the main need to identify exogenous movements of fiscal variables stems from the influence that automatic stabilizers can exert over the business cycle on public finances or endogenous policy responses from the government. Indeed, the process of identification aims at isolating this effect from changes in the fiscal variables which could be assumed as independent to the cycle. The econometrician ought to disentangle these effects, otherwise he risks losing the true causation between fiscal variables and output. In this study we employ the methodology of Blanchard and Perotti. We ruled out the dummy or narrative approach on the basis of data availability, while Mountford and Uhlig (2009) method imposes *ad hoc* sign restrictions on impulse functions, a feature that might be too restrictive for Greece given that output responses might positive after a tax shock and negative after a government shock for economies carrying high debt or high deficits (See, for example Ilzetzki et al. (2011).)

### 3.3.1 The Blanchard-Perotti Approach

Our benchmark econometric model follows BP specification of three variables ($n = 3$) using taxes, government spending and output as the variables in the VAR system. In other words, we decompose the budget constraint of the government in its broadest sense, having public spending on the one side and tax revenues on the other side.

For government spending we use total government consumption of goods and services. For taxes, we use total tax revenues net of transfer and interest payments (See Appendix for further details.) Although, marginal tax rates would have being the ideal instrument to test our question, time-series data for such fiscal variables - to the best of our knowledge - do not exist for Greece. Nevertheless, in our identification problem and as we will later explain, the tax elasticities used to identify fiscal shocks they implicitly contain the effects of marginal tax rates through the constructed estimate of the aggregate tax elasticity (See also the discussion in Blanchard and Perotti (2002)).

Also, and because the identification strategy relies on quarterly data, inclusion of more macroeconomic variables in the system - for the relatively short sample period - might bias our estimates. Therefore, we chose to take a basic analysis and use only three variables with the sole aim to first study the effectiveness of fiscal policy in general. That is, the three variable approach tries to answer the general question of which between the two fiscal variables - government spending in general and taxes - have the greatest impact on output. This type of analyses compares against an alternative - though equally important - approach in studying the impact of specific fiscal plans. However, the former approach, and although general enough, is a natural first step in assessing the effectiveness of fiscal policy - especially if one considers the lack in consensus of the theoretical and empirical literature on the effects of fiscal policy.

[3]The interested reader can advice Kilian (2011) and Caldara and Kamps (2008)

$$X_t = c + \Phi(L)X_{t-\tau} + u_t \tag{3.1}$$

where

- $X_t = (\tau_t, g_t, y_t)$ is the vector of endogenous variables.
- c is the constant term.
- $\Phi(L)$ is an $3 \times 3$ matrix of the lag polynomial having as elements the estimated coefficients of the lagged variables.
- And finally $u_t = (u_t^\tau, u_t^g, u_t^y)$ represent the reduced form residuals vector

More specifically, $\tau_t$ represents the log of real net revenues per capita, $g_t$ represents the log of real net government expenses per capita, and $y_t$ the log of real output per capita[4].

### 3.3.2 The Identification Problem

Estimating (3.1) will generate a variance - covariance matrix ($\Sigma = E(u_t u_t')$) of residuals that is not diagonal, implying that the reduced form residuals are correlated with each other. This undesirable feature restricts any identification, since for example a shock (or impulse) of one of the fiscal variables will have a direct effect on the other. The target is to obtain orthogonal to each other shocks. From the econometrics point of view this is equivalent of orthogonalizing the variance - covariance matrix. In practise, the target is to find a relationship between the structural errors and the reduced form residuals. To get such a relationship, if someone pre-multiplies (3.1) with a matrix $A$ of $n \times n$ obtains the structural form of the system:

$$AX_t = Ac + A\Phi(L)X_{t-1} + Au_t \tag{3.2}$$

$$AX_t = c^* + \Phi^*(L)X_{t-1} + Be_t \tag{3.3}$$

where $A\Phi = \Phi^*$, $Ac = c^*$ and $Au_t = Be_t$ with $e_t = (e_t^\tau, e_t^g, e_t^y)$. The last of this equation lies at the heart of the identification problem and is usually referred as the AB model. In simple terms, what essentially means is to find matrices A and B such as to be able to establish the relationship between the reduced form residuals, $u_t$ and the structural ones, $e_t$ and therefore recover the second from the first.

#### The AB model

For our case the AB model in matrix form can be written as:

[4]For the construction of data, see Appendix

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{23} & a_{33} \end{pmatrix} \begin{pmatrix} u_t^{\tau} \\ u_t^{g} \\ u_t^{y} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{23} & b_{33} \end{pmatrix} \begin{pmatrix} e_t^{\tau} \\ e_t^{g} \\ e_t^{y} \end{pmatrix}$$

Now in principle to solve the system of the unknown parameters, it is required $\frac{n^2-n}{2}$ number of restrictions to exactly identify the system, given that we normalize to unity the diagonal elements of B matrix. In our case this implies that we need at least 3 restrictions to impose on the system. The strategy of BP approach is to impose such restrictions by looking at institutional information and make specific assumptions regarding some parameters. To make the point more clear rewrite the AB model with the diagonals normalized to unity and $a_{12} = a_{21} = b_{13} = b_{23} = b_{31} = b_{32} = 0$ since we wish the $cov(u_t^{\tau}, u_t^{g}) = 0$. Following BP we ultimately get the following AB model

$$\begin{pmatrix} 1 & 0 & -a_{13} \\ 0 & 1 & -a_{23} \\ -a_{31} & -a_{23} & 1 \end{pmatrix} \begin{pmatrix} u_t^{\tau} \\ u_t^{g} \\ u_t^{y} \end{pmatrix} = \begin{pmatrix} 1 & b_{12} & 0 \\ b_{21} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e_t^{\tau} \\ e_t^{g} \\ e_t^{y} \end{pmatrix}$$

The above matrix form, and especially the minus signs, comes from the equivalent relationship of $u_t = A^{-1}Be_t$ and the assumed BP reduced-form errors relationship with the structural errors of the following type:

$$u_t^{\tau} = a_{13}u_t^{y} + b_{12}e_t^{g} + e_t^{\tau} \tag{3.4}$$

$$u_t^{g} = a_{23}u_t^{y} + b_{22}e_t^{t} + e_t^{g} \tag{3.5}$$

$$u_t^{y} = a_{31}u_t^{\tau} + a_{32}u_t^{g} + e_t^{y} \tag{3.6}$$

Equation (3.4) says that any unexpected movements in revenues (taxes) can be attributed into three factors. First, due to unexpected movements in output or if you prefer the response of the automatic stabilizers, second due to unexpected but exogenous movements in government spending (structural shocks in spending) and finally due to unexpected structural shocks to taxes. Equation (3.5) and (3.6) have similar interpretations.

The fundamental innovation of the BP methodology is to use institutional and extraneous information on the tax and transfer system so as to impose the three additional restrictions that needed. This is achieved by constructing the value for $a_{13}$ and imposing it as a restriction in the above system. In our sample the estimate for this particular value was 2.43[5]. The second, comes from the assumption that within a quarter implementation and decision lags on the part of the government eliminates any discretionary action in this particular quarter. The plausibility of this assumption, lies on the intuitive observation of public governance where a government in order to respond and

[5] See appendix for the details

realize the shock, either needs to collect information (akin to the assumption of *imperfect knowledge*) or the legislation that is needed for a policy to be implemented requires time, possibly of more than a quarter (*legislative rigidities*). In this sense, quarterly data becomes crucial for making the particular assumption. For instance, with annual data, a government certainly has the time and speed to take discretionary actions. Accordingly, the parameter that captures such channel goes to zero, i.e. $a_{2,3} = 0$. Finally the third restriction comes from the assumption on 'policy ranking' of the fiscal variables. For example, do we suppose that authorities first observe the tax revenues and then 'announce' their spending decisions, or is exactly the opposite? On this, we follow BP and assume that taxes come first, which is equivalent of imposing the restriction on $b_{12}$ to be zero[6].

$$u_t^{\tau} = 2.43 u_t^{y} + e_t^{\tau} \tag{3.7}$$
$$u_t^{g} = b_{22} e_t^{\tau} + e_t^{g} \tag{3.8}$$
$$u_t^{y} = a_{31} u_t^{\tau} + a_{32} u_t^{g} + u_t^{y} \tag{3.9}$$

These three additional assumptions completes the process of identification, whereas the rest of the parameters are left unrestricted and following BP are estimated according to the following steps.

First, we compute the cyclically adjusted taxes and spendings. In particular, we have

$$u_{tCA}^{\tau} = u_t^{\tau} - 2.43 u_t^{y} = e_t^{\tau}$$
$$u_{tCA}^{g} = u_t^{g} - a_{23} u_t^{y} = u_t^{g} = b_{22} e_t^{\tau} + e_t^{g}$$

Next, we regress $u_{tCA}^{\tau}$ on $u_{tCA}^{g}$ estimate $b_{22}$ and extract the residuals of the regression, which are equal to $e_t^{g}$ and extract the $e_t^{x}$ residuals. Therefore, the system is exactly identified and can recover the variance-covariance matrix of the structural errors. Moreover, these estimates characterize the contemporaneous - within the quarter- effects of shocks to our endogenous variables. Then, after the decomposition of structural shocks we can estimate the impulse response functions and trace the *short-run* or *long-run fiscal multipliers*.

## 3.4 Model Specification

In addition to the model specification outlined in the previous section, we decided to include quarterly dummies, mainly to account for the seasonal collection of tax receipts, and two distinct dummies to capture the effects of the financial crisis and the 2008-2009 jump in spending, respectively. The first dummy -for short the 'crisis dummy'- takes the values of 1 from 2010q1-2012q1 and zero everywhere else, while the second - for short the 'spending dummy' - is switching on between 2008q4-2009q3. Figure 3.8

[6]Results do not change if the opposite holds. Indeed, BP also found that their own result were robust to this assumption

in the Appendix clearly depicts the structural break from the crisis, and the jump in government expenses that occurred prior to the 2009 elections.

In the regression we also added a linear trend and estimated a VAR of order one. In choosing the lag order, we balanced between the suggestions of lag tests, the VAR stability condition, as well as we tried to save on data observations. In practice, it turned out that our VAR was extremely sensitive to different lag orders, therefore we decided to use the most consistent among all lag tests criteria, which in our case was the SIC test indicating a lag order one[7]. The other available tests, were either providing conflicting results or were not consistent. In addition, for a lag order higher than 5 the system found to be unstable. Finally, the tables below report the estimations that are based on the levels of the data.[8].

## 3.5 Results and Analysis

| | tax | spending | $b_{22}$ |
|---|---|---|---|
| Multiplier | -0.36 | 1.73 | 0.01 |
| t-statistic | -1.82 | 1.90 | 0.38 |
| p-value | 0.08 | 0.06 | 0.70 |

Figure 3.1: Contemporaneous Multipliers

Figure (3.1) above reports our estimates for the contemporaneous multipliers in each quarter[9]. The estimated tax multiplier shows that a Euro increase in net taxes drops economic activity by 0.36 cents, while a Euro increase in spending increases output by 1.73 Euros. Interestingly, both estimates have their expected sign, invalidating the theory of expansionary fiscal consolidations. Our results, also show that the spending multiplier, is larger in absolute value than the tax multiplier which is consistent with the Keynesian model. The size of the tax multiplier, however, is lower than one but consistent with a variety of other studies[10]. The spendings multiplier on the other hand, and in line with the Keynesian view, was found to be larger than one. This places Greece as the country with the second largest government spending multiplier behind France (See Table 3.2 )

Table (3.2), in particular, summarizes some empirical results found in the literature for other EU countries. From this table, the picture that emerges shows that the spending multiplier for Greece is well above the EU average, in contrast with the

[7]Ilzetzki et al. (2011) also found that for highly indebted countries the lag order criteria to be 1.

[8]We also, conducted the estimations based on first-differences and on de-trended data using HP filter to account for the non-stationarity. The results, showed only minor changes in the magnitudes. In order to save data observations and keep measurement errors into minimum we decided to rely on the levels

[9]Estimating the original coefficients obviously have the interpretation of elasticities. To convert to a unit change, here Euro change, we simply used the formula of elasticity, i.e $e = \dfrac{\vartheta y}{\vartheta x}\dfrac{x}{y} \Rightarrow \dfrac{\vartheta y}{\vartheta x} = e \times \dfrac{x}{y}$ where $\dfrac{x}{y}$ is the point mean

[10]See a summary table of various research in Figure 3.11

| | EXPENDITURE MULTIPLIERS | | TAX MULTIPLIERS | |
|---|---|---|---|---|
| | Short-term (1 year) | Medium-Term (3 years ) | Short-term (1 year) | Medium-Term (3 years ) |
| Greece | 1.81 | 2.12 | -0.62 | -0.55 |
| France | 1.9 | 1.5 | -0.5 | -0.8 |
| Italy | 1.2 | 1.7 | 0.16 | - |
| Spain | [0.94 , 1.54] | [0.55 , 1.04] | 0.05 | 0.39 |
| Portugal | 1.32 | 1.07 | - | - |
| Germany | [0.2 , 0.7] | [-0.23 , 1.2] | [0.29 , -1.17] | [0.59, -1.08 ] |
| UK | [0.27 , 0.48] | [-0.6 , 0.27 ] | [-0.23 , -0.1] | [0.7, -2.5 ] |
| EU area | 0.87 | 0.85 | -0.63 | -0.49 |

*Table Based on Boussard et al (2012). For Greece our estimtes

**For Greece and Portugal the sample Period covers 11 and 9 years respectively

***The table shows the cumulative multipliers

Figure 3.2: Fiscal Multipliers for EU countries: Summary of Figures (3.10) and (3.11)

tax multiplier which is within the EU average. Moreover, while the tax multiplier is similar to the one found in Blanchard and Perotti (2002) (i.e. -0.46 to -0.36 in their paper), the spending multiplier is almost three times larger. However, this result is consistent with other empirical studies, such as Ilzetzki et al. (2011) that account for the exchange rate regimes, open economy issues and the level of public debt. Meanwhile, and in a study that received widespread media attention, Blanchard and Leigh (2013) documented that fiscal multipliers might have being underestimated in the initial phase of the Greek crisis[11], a statement which is in line with our findings.

**RESPONSE OF GDP**

| QUARTER | Q1 | Q4 | Q8 | Q12 | Q16 | Q20 | (Peak) |
|---|---|---|---|---|---|---|---|
| TAX SHOCK | -0.39* | -0.46* | 0.02 | 0.02 | 0.01 | 0.01 | -0.46* (Q4) |
| GOV SHOCK | 1.55* | 1.34* | 0.86* | 0.50* | 0.280 | 0.150 | 1.55* (Q1) |

*Statistically significant

Figure 3.3: Short-run Multipliers

Turning on the dynamic responses, Figure (3.3) summarizes the on average dynamic effects of shocks in spendings and taxes on economic activity. In particular, it summarizes a permanent but unexpected unit shock, equivalent to a Euro increase in taxes (first row) and spendings (second row), and the equivalent per Euro response of output. Together with Figure (3.6) in the Appendix, the diagrams reveal that fiscal shocks have only short-lived influence on GDP, persisting in most of the cases for 4 quarters with the exception of the effects of spendings where the effects last for around 2 years. Typically, the effect on output is quite large on impact following a government

[11]The Troika program in performing some of its forecasts, assumed that government spending multiplier was around 0.50.

spendings shock but it declines quickly. In contrast, a tax shock is significant for one year only and low in magnitude. Overall, the main conclusion that emerges is that spendings channel is more potent relative to a tax rebate in stimulating output in the short-run.

To assess this point further, figure (3.4) below shows the cumulative fiscal multipliers. A cumulative multiplier is defined as the ratio of the cumulative change in GDP to cumulative change in fiscal variables, i.e $\frac{\Sigma_{t=0}^{T} \Delta y_t}{\Sigma_{t=0}^{T} \Delta g_t}$ and as long as $T \to \infty$ it reverts the *Long-run multiplier*. This measurement might be a better indicator in assessing any governmental stimulus for the reason that it contains the lagged responses and therefore can account for the diffusion of stimulus over time.

**CUMULATIVE MULTIPLIERS**

| Years | SPENDING | TAX |
|---|---|---|
| 1 | 1.81* | -0.62 |
| 2 | 2.01* | -0.64 |
| 3 | 2.12* | -0.55 |
| 4 | 2.17* | -0.48 |
| 5 | 2.20* | -0.44 |

* Statistically significant

Figure 3.4: Cumulative Multipliers

From the above figure it becomes more than evident that government spendings is more effective in simulating output. The cumulative multipliers are quite large and always statistically significant in the medium run, in contrast with the tax cumulative multipliers which again have lower impact on output than spendings, decline over time and are always statistically insignificant. It appears, that the spendings instrument has ample effects on output with quite large relative to the literature multiplier. So, for instance in the medium run a multiplier of 2.20 indicates that output more than doubled its size relative to the increase in government spendings. An almost identical response found in Italy by Giordano et al. (2008), a country of similar fiscal culture as in Greece. It is noteworthy that the multiplier is constantly increases over time and stabilizes only after 6 years - hence, the 2.23 estimate (See Appendix mid graph of third row of Figure (3.7)) can also be thought as the long-run multiplier for Greece.

The rest of the impulse responses in figure (3.6) depict a negligible short run effect on the remaining variables. Interestingly, taxes are not persistent on its own shock and seems to have some minor but positive influence on spendings. In contrast spendings shocks seem very persistent featuring somehow a well known behaviour of the Greek public finances, although for our sample this can be explained by the increase in public infrastructure and investments (e.g. for Olympic Games). Also is in line with the original BP study for US or with (Giordano et al. (2007), p.575) for Italy. Moreover, as Burriel et al. (2009) claim military expenses can also affect the degree of persistence. The authors for instance, once they re-estimate the US model net of military purchases the degree of persistence fell significantly. For Greece is it a well known fact that has the highest military expenses to GDP ratio among the EU countries. This might explain the differences in the degree of persistence of the spendings shocks in Greece and the rest of EU countries (See Boussard et al. (2012) or Burriel et al. (2009)). Altogether, If one combines this with the nullified effects from and on taxes, it approximates reasonably well some of the pathologies of the Greek economy on the sustainability of public finances[12].

## 3.6 Determining the size of Government Spending Multiplier

To determine the size of spendings multipliers and provide some possible explanations for its magnitude, we necessary have to take into account the sample period in which this study focused on.

First, from 1999 until 2002 the final stage of the Euro entrance adjustment occurred, where prior to Euro entrance a prolonged period of some fiscal retrenchment and an initial currency devaluation followed by a specific peg of Drachma on Euro took place. Part of this period is covered by our 1999q1-2001q4 observations (22% of the total observations). Next, the Euro-euphoria phase and the 2004 Olympic games had the lions share on the 5% average annual growth (highest in EU) between 2002-2008 period. Together with the aforementioned spendings bonanza of 2009 prior to elections, it facilitates a period of high government spendings, high private consumption, easy credit and clearly a regime shift in expectations. This sustained period of high economic growth lies at the core of our sample accounting for nearly 60 % of our data. In addition, the global environment accommodated perfectly the reception of Euro with a decrease in risk premia and interest rates, (See Burriel et al. (2009)). Thereafter, the effects of the global financial and the Greek debt crisis start to appear (approximately equal to 17 % of our data).

Overall the previous paragraph lines point out that any off setting mechanism of fiscal expansions probably were not at work. Indeed, for Greece a triplet of major off-setting mechanisms did not operate. First, possibly no crowding out of investment might have occurred due to a decrease in risk premia and interest rates[13]. Also, no monetary policy could accommodate any radical expansion in the economy neither exchange rates could adjust for the inflows or outflows of capital. From this point of view, it resembles a textbook Mundell-Fleming model of fixed exchange rates, which predicts strong output effects from changes in government expenditures. Finally, tax revenues were not responding neither to a government expenditure shock nor to the increase in GDP. Putting all these together, and in combination with the persistence of the govern-

[12] Cointegration tests between taxes and spendings (an implicit test for the sustainability of public finances) confirmed that there is no cointegrated relationship between these two variables. This implies that spendings and revenues were not co-moving during the particular period

[13] We will address this in the feature

ment expenses[14], the insignificance of tax shocks and the ‘catch up’ effect on growth - which implies that the economy was possibly not at full employment - then we view out results as reasonable first approximation.

Furthermore our findings, are consistent with a recent study by Ilzetzki et al. (2011) who found that a) for high income countries government purchases persist, b) fixed exchange rate regime have higher on average impact and long-run multipliers (especially if monetary does not accommodate any response to output) and c) these multipliers can increase in value the more closed the economy is - which, paradoxically, is also the case of Greece(See Ilzetzki et al. (2011)).

Nevertheless, from the econometrics point of view the spendings multiplier might be biased upwards for three main reasons. Aside the small sample bias, anticipated effects that the BP approach cannot capture, might play some role. For instance, the decisions for organizing the Olympic games was held in 1997. Agents knew that a major boost of government spendings will occur in order to improve the current infrastructure or invest in new one. Second, changes in expectations before and after the Euro entrance could have made the agents ‘*less Ricardian*’. Third, and of particular importance for Greece, any debt feedback is completely missing in our analysis. As pointed out by Favero and Giavazzi (2011) debt dynamics are crucial for determining the size of fiscal multipliers. Finally, our finding on the tax effects might be downwards biased mainly due to enforceability problems on collecting tax revenues. Also, as advocated by Crichton et al. (2012) a tax variable under the direct control of the government is better data for empirical analysis. We intend in the future to address all those issues of fiscal foresight and the presence of Ricardian agents more convincingly.

Finally, in comparing the methodology of Blanchard and Perotti (2002) with the one in Alesina et al. (2015) which addresses a related though not identical question, there are two main differences. First, Alesina et al. (2015) use a narrative method similar to the one in Romer and Romer (2010), where their identification of shocks is based on the information that the publicly available documents provide on the reasons for tax changes. In contrast, the approach taken by the SVAR relies heavily on a mechanical construction of the fiscal shocks, and on some a priori assumption that theory might impose. Second, in their paper their focus is explicitly on fiscal consolidation episodes. But this then, ignores by construction the Keynesian prescription of optimal policy and says little about period of fiscal expansions. In addition, in their methodology is hard to tackle the fiscal actions in neighbouring countries which in the context of their analysis is likely to influence the design of optimal fiscal plans.

## 3.7 Conclusions

Obviously, the major constraint of this paper is the limited data availability for performing a VAR analysis in sufficient depth and scale. Nevertheless, to conclude this research monograph on Greek fiscal policy we wrap up our main findings in Figure 3.5. During

[14] Since government purchases are part of total GDP makes the second depend on the persistence of the first

**SUMMARY OF FINDINGS**

| | CONTEMPORANEOUS | IMPACT | SHORT-RUN | MEDIUM-RUN | LONG-RUN |
|---|---|---|---|---|---|
| TAX MULTIPLIER | -0.36** | -0.39* | -0.62 | -0.55 | - |
| SPENDING MULTIPLIER | 1.73** | 1.55* | 1.81* | 2.12* | 2.23* |

*Statistically Significant at 5% Confidence interval

**Statistically Significant at 10% Confidence interval

Figure 3.5: Summary of Main Findings

the underlined sample government purchases were very potent on stimulating output, not only in the short-run but also in the long-run. In absolute value, the spending multiplier is well above the tax multiplier suggesting some Keynesian effects on aggregate demand. The dynamic response of output after a tax shock last for around one year in contrast with a spending shock which lasts significantly more.

On the other hand, the size of spending multiplier is statistically significant and above one even in the long-run making this type of fiscal instrument much more effective. Although, the results are empirically likely and meaningful from the theoretical point of view, are not easily exportable or allow any generalizations for two main reasons. First, the results might not be robust due to the possible anticipated effects, and therefore the results might be upward biased because of the non-inclusion of debt dynamics. Second, external factors that accompany our data sample, and in particular the non-existence of mechanisms that can off-set government spending shocks, might also have biased our results upwards.

## 3.8 Appendix

### 3.8.1 Data Appendix

All data except otherwise described are extracted from Eurostat, in quarterly government finance statistics from the quarterly non-financial account for general government. The available data sample covers 1999q1:2012q1 period.

**Government Expenditures**

Government spending = Total general government expenditures - total transfers - interest payable

Total transfers = Social benefits + Subsidies (payable)
+ Capital transfers(payable) +Other current transfers(payable)+Social transfers in kind

BP regard transfers as negative taxes. Moreover, someone can claim that transfers only serve distributional concerns and have zero net impact. In addition particular transfers, like unemployment benefits co-move with the business cycle, as such eliminating this effect can also reduce the influence of fluctuations. On the other hand, interest rate payments are excluded because it is assumed that are largely out of government's control. See Giordano et al. (2007, p. 566-567)

**Net Taxes**

Government revenues = Current taxes on income and wealth + Taxes on imports and production(indirect taxes) + social contributions (receivable) + capital taxes receivable - total transfers

**Output**

Gross National Product at current market prices (in Euros). Data prior to 2000 was not available from Eurostat but provided by ELSTAT, with the note that the methodologies differ after the national account revisions.

**Consumer Price Index**

Monthly series as provided from ELSTAT. I converted the monthly series to quarterly using quarter averages

**Population**

Used the quarterly series for Population (in thousands) of 15 years and over by employment status: 1998-2012, by quarter (Greece, total), Grand Total measurement, as provided by ELSTAT.

**Seasonal Adjustment**

All series were seasonally adjusted by Eviews 7 routine TRAMO / SEATS

**Construction of Elasticities**

On the construction of the elasticity of net taxes to output,we followed Perotti (2004) approach with the only exception that we did not updated the values of elasticities but used instead the one calculated in den Noord (2000).Data for some particular variables were not covering the sample under study. However, we found that the results were insensitive to the choice of the elasticity of taxes w.r.t output.

### 3.8.2 Main figures and diagrams

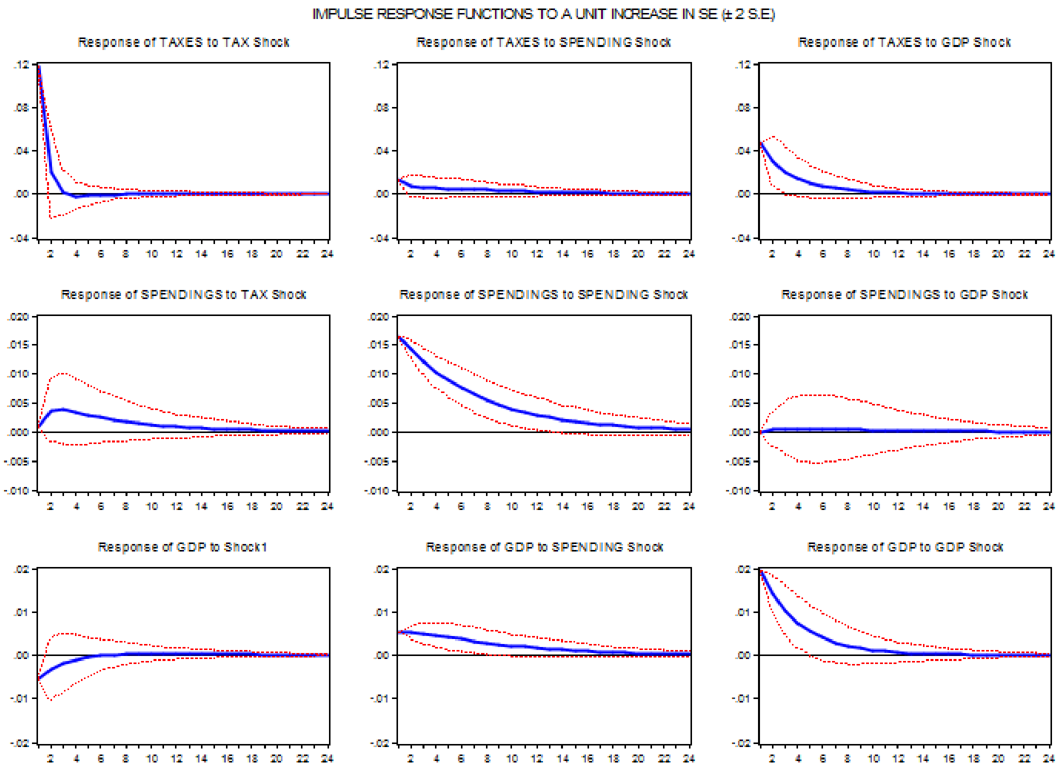



Figure 3.6: Impulse responses to 1% standard deviation shock

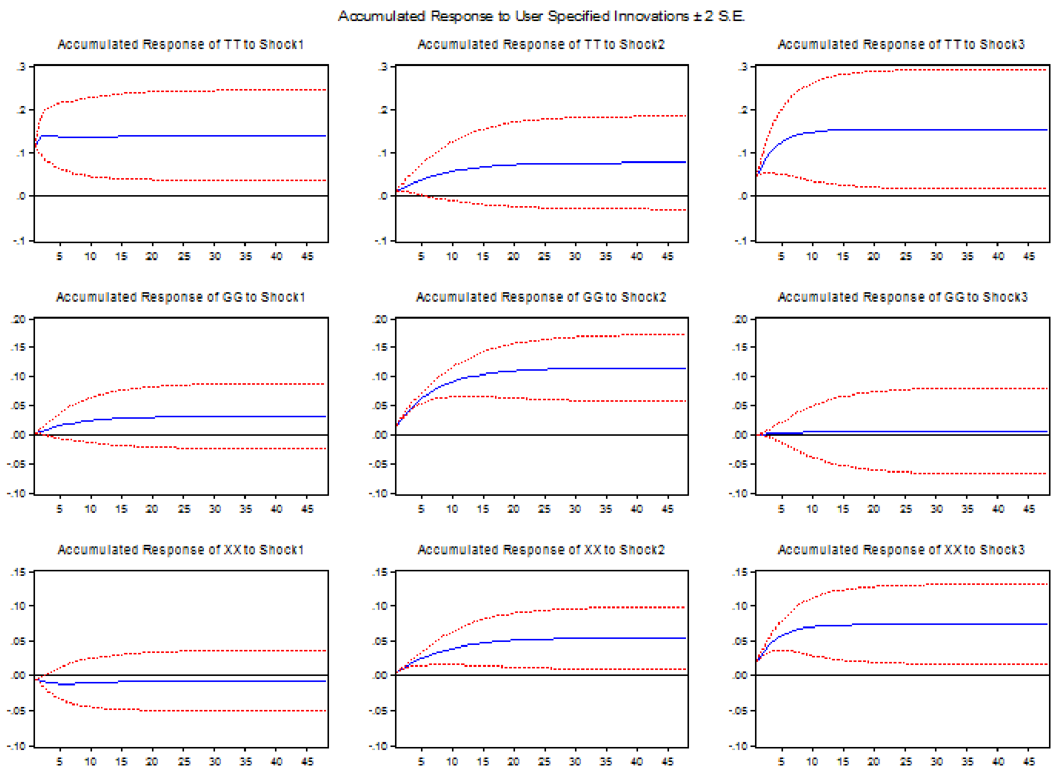



Figure 3.7: Accumulated Impulse responses to 1 % standard deviation shock

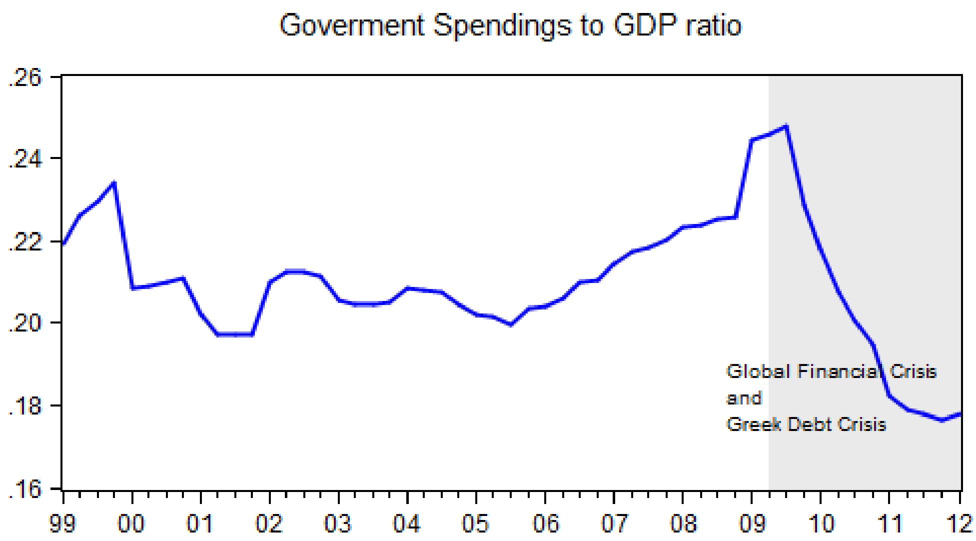



Figure 3.8: Government Spending over time as part of GDP

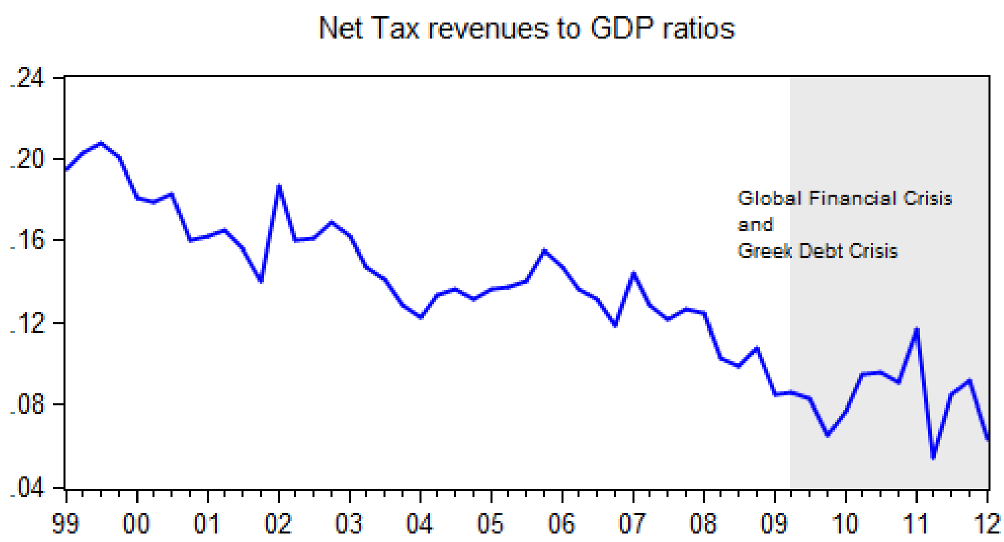



Figure 3.9: Net Tax revenues over time as part of GDP

| Studies | Sample | Short-term multiplier[1] | Medium-term Multiplier [2] | Identification strategy[3] |
|---|---|---|---|---|
| Blanchard and Perotti (2002) | US (1947:1-1997:4) | -0.69 | 0.5[4] | Decision lags in policy making and imposition of contemporaneous GDP elasticities |
| Perotti (2004) | US (1960:1-1979:4)<br>US (1980:1-2001:4) | 1.29<br>0.36 | 1.4<br>0.28 | Blanchard-Perotti |
| Galí et al. (2007) | US (1954:1-2003:4) | 0.7 | 1.74 | Cholesky decomposition |
| Ramey (2011) | US (1939:1-2008:4) | 0.6 to 1.2 | No estimate | Narrative approach |
| Mountford and Uhlig (2009) | US (1955:1-2000:4) | 0.65[5]; 0.46; 0.28[6] | -0.22 | Sign restrictions on impulse responses |
| Fatas and Mihov (2001) | US (1960:1 - 1996:4) | Similar to Galí et al. (2007) | Similar to Galí et al (2007) | Cholesky decomposition |
| Perotti (2004) | Germany (1960:1-1974:4)<br>Germany (1975:1-1989:4) | 0.36 | 0.28 | Blanchard-Perotti |
| Heppke - Falk et al. (2006) | Germany (1974:1-2004:4) | 0.62 | 1.27 | Blanchard-Perotti |
| Baum and Koester (2011) | Germany (1976:1-2009:4) | 0.7 | 0.69 | Blanchard-Perotti and Threshold VAR |
| Benassy-Quere and Cimadomo (2006) | Germany (1971:1-2004:4) | 0.23 | -0.23 | FVAR and Blanchard-Perotti |
| Biau and Girard (2005) | France (1978:1-2003:4) | 1.9 | 1.5 | Blanchard-Perotti |
| Giordano et al. (2007) | Italy (1982:1-2004:4) | 1.2 | 1.7 | Blanchard-Perotti |
| De Castro (2006) | Spain (1980:1-2001:2) | 1.14-1.54 | 0.58-1.04 | Cholesky decomposition |
| De Castro and Hernández de Cos (2008) | Spain (1980:1-2004:4) | 1.3 | 1 | Blanchard-Perotti |
| De Castro and Fernández (2011) | Spain (1981:1-2008:4) | 0.94 | 0.55 | Blanchard-Perotti |
| IMF (2005) | Portugal (1995:3-2004:4) | 1.32 | 1.07 | Blanchard-Perotti |
| Perotti (2004) | UK (1963:1-1979:4)<br>UK (1980:1-2001:2) | 0.48<br>-0.27 | 0.27<br>-0.6 | Blanchard-Perotti |
| Benassy-Quere and Cimadomo (2006) | UK (1971:1-2004:4) | 0.12 | -0.3 | FVAR and Blanchard-Perotti |
| Burriel et al. (2010) | Euro Area (1981:1-2007:4) | 0.87 | 0.85 | Blanchard-Perotti |

Figure 3.10: Estimated Expenditure Multipliers from Literature. Table from Boussard et al (2012)

| Studies | Sample | Short-term multiplier | Medium-term multiplier | Identification strategy |
|---|---|---|---|---|
| Blanchard and Perotti (2002) | US (1947:1-1997:4) | Within range -0.7 and -1.3 | Within range -0.4 and -1.3 | Decision lags in policy making and imposition of contemporaneous GDP elasticities |
| Perotti (2004) | US (1960:1-1979:4)<br>US (1980:1-2001:4) | -1.41<br>0.7 | -23.87<br>1.55 | Blanchard-Perotti |
| Favero and Giavazzi (2007) | US (1980:1-2006:4) | 0.29 | 0.65 | Narrative approach |
| Mountford and Uhlig (2009) | US (1955:1-2000:4) | -0.16 | -2.35 | Sign restrictions on impulse responses |
| Romer and Romer (2010) | US (1945:1-2007:4) | | -3 | Narrative approach |
| Perotti (2004) | Germany (1960:1-1974:4)<br>Germany (1975:1-1989:4) | 0.29<br>-0.04 | -0.05<br>0.59 | Blanchard-Perotti |
| Baum and Koester (2011) | Germany (1976:1-2009:4) | -0.66 | -0.53 | Blanchard-Perotti and TVAR |
| Benassy-Quere and Cimadomo (2006) | Germany (1971:1-2004:4) | -1.17 | -1.08 | FVAR and Blanchard-Perotti |
| Biau and Girard (2005) | France (1978:1-2003:4) | -0.5 | -0.8 | Blanchard-Perotti |
| Giordano et al. (2007) | Italy (1982:1-2004:4) | 0.16 | | Blanchard-Perotti |
| De Castro (2006) | Spain (1980:1-2001:2) | 0.05 | 0.39 | Cholesky decomposition |
| Afonso and Sousa (2009) | Portugal (1979:1-2007:4) | + | + | Blanchard-Perotti |
| Perotti (2004) | UK (1963:1-1979:4)<br>UK (1980:1-2001:2) | -0.23<br>0.43 | -0.21<br>0.7 | Blanchard-Perotti |
| Benassy-Quere and Cimadomo (2006) | UK (1971:1-2004:4) | -0.23 | -0.07 | FVAR and Blanchard-Perotti |
| Cloyne (2011) | UK (1945-2010) | Between -0.5 and -1.0 | -2.5 | Narrative approach |
| Burriel et al. (2010) | Euro Area (1981-2007) | -0.63 | -0.49 | Blanchard-Perotti |

Figure 3.11: Estimated Net Tax Multipliers from Literature. Table from Boussard et al (2012)

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