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Essays in Financial Economics

by

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Acknowledgments

My interest in financial economics started in the winter of 2008. I was a graduate student in political science, and I had neither the vocabulary nor the technical skills to follow the debate about the causes of the financial crisis that shooked the world at that time. However, as many other students at that time, I wanted to understand.

I begun studying a particular institution – the Credit Rating Agency – both as a vehicle of US soft power (with David Hellwood) and as an information provider (under the supervision of Paolo Manasse). I am especially indebted to Paolo for his advices, and for encouraging me to get a proper training in economics. I thank the faculty at Collegio Carlo Alberto for my initial training.

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To the memory of Gaetano Trigilia Caracciolo
Chapter 1

Introduction

This dissertation consists in three essays on hidden information and finance.\(^1\) The first two essays share a common underlying motivation, and I consider them as the starting point of a longer run reflection on financial contracts as institutions, defined as “the rules of the game in a society, [...] the humanly devised constraints that shape human interaction” as in North [1990]. The last chapter is an attempt to bridge micro hidden information issues and the macroeconomy; in particular, to provide a microfoundation of keynesian savings traps. In this introduction, I shall briefly review the objectives and results of these essays. Beforehand, I shortly discuss the methodology adopted.

Since the early 1970s, mainstream economic models did not allow financial contracts to play an active role in supporting resource allocations. Indeed, in the general equilibrium framework of Arrow and Debreu [1954], contracts are irrelevant; a result stressed by Modigliani and Miller [1958]. However, the asymmetric information paradigm developed in the early 1970s pave the way for contracts to acquire much more spotlight in subsequent decades. As Hurwicz [1973] put it: “This new approach refuses to accept the institutional status quo of a particular time and place as the only legitimate object of interest and yet recognises constraints that disqualify naive utopias.” [emphasis not in the original] To the contrary, an important objective of economic theory became precisely to analyse the optimal design of institutions, of which financial contracts are but one example. Institutions are studied as mechanisms, responsible for regulating (i) communication, and (ii) resource allocation across agents. Informational asymmetries came to play the role of those “constraints that disqualify naive utopias”.

\(^1\)The second essay is coauthored with Kostas Koufopulos and Roman Kozhan; the third is coauthored with Herakles Polemarchakis.
contracts, as Townsend [1979] exemplifies. The paper starts from acknowledging a striking ‘reality’: most financial contracts around the world (and throughout history) took the peculiar form of debt: one party promises to pay back a fixed amount to the other, in exchange for current capital lease; it is understood that, should he fail to repay this fixed amount, the creditor seizes all available assets, and perhaps some ‘non-pecuniary’ penalty ensues. Townsend provides a plausible explanation for why this is the case, based on asymmetric information. He argues that it is often costly to observe and verify the state of a debtor’s assets ex post. In such cases, debt is the contract that minimises the need for costly verification, requiring it only in bankruptcy states. All other financial contracts lead to greater deadweight losses and hence are suboptimal. The result became known as he costly-state-verification (CSV) argument for debt optimality, and was further developed by Gale and Hellwig [1985].

From Townsend onwards, the number of papers applying mechanism design to finance – so-called security design – grew steadily. Recent contributions include Antic [2014], Carroll [2015], Hebert [2015] and Farhi and Tirole [2015]. The first two essays of this dissertation (Trigilia [2015] and Koufopoulos et al. [2014]) belong to this literature.

In addition, asymmetric information is a natural source of endogenous incomplete asset markets; it is often impossible to issue securities conditional on privately held information. A growing literature examines the consequences of modelling explicitly endogenous incomplete markets in general equilibrium. The last chapter of this thesis belongs to this literature.

1.1 Optimal leverage and strategic disclosure

Firms seeking external financing jointly choose what securities to issue, and the extent of their disclosure commitments. The literature shows that enhanced disclosure reduces the cost of financing. This paper, in addition, analyses its effects on the composition of financing means. It considers a market where firms compete under costly-state-verification, but unlike the standard model assuming (i) that the degree of asymmetric information between firms and outside investors is variable, and (ii) that firms can affect it by committing to a disclosure policy, possibly incurring a cost. Two central predictions emerge.

On the positive side, disclosure and leverage are negatively correlated. Efficient equity financing requires a certain amount of disclosure, whereas debt does not; it is based on the threat of bankruptcy. Therefore, more transparent firms issue cheaper stocks and face a higher opportunity cost of leveraged financing. The prediction is shown to be consistent with the behaviour of US corporations since the 1980s.

On the normative side, disclosure externalities lead to under-disclosure and excessive leverage relative to the constrained best. Mandatory disclosures can be Pareto improving, when feasible. Otherwise, the mapping I derive from greater equity financing to voluntary higher transparency suggests that the regulator should tighten the capital requirements. According to the model, capital standards are especially useful when (i) firms performances are highly correlated, and (ii) disclosure requirements can be dodged to a large extent. Both conditions seem to apply to large financial firms.

1.2 Security design under asymmetric information and profit manipulation.

We consider a model of external financing in which entrepreneurs are privately informed about the quality of their projects and seek funds from competitive financiers. The literature restricts attention to monotonic or ‘manipulation proof’ securities and finds that straight debt is the uniquely optimal contract. Monotonicity is commonly justified arguing that it would arise endogenously if the entrepreneur can manipulate profits before contract’s maturity.

We characterize the optimal contract when entrepreneurs can misreport their earnings. We derive necessary and sufficient conditions for straight debt to be suboptimal, and we show that it is never uniquely optimal. Generically, the optimal contract is non-monotonic and involves profit manipulation in equilibrium. It can be implemented as debt with a strictly positive performance-based bonus. Importantly, our results suggest that ex ante asymmetric information does not suffice to theoretically justify the optimality of straight debt.

1.3 Credit Failures

Credit failures in financial markets subject to adverse selection account for competitive equilibrium allocations that are constrained suboptimal: Pareto improving intervention (i) balances the budget and (ii) does not require information finer than
that available to market participants. With multiple factors of production and endogenously determined relative prices, the constrained optimality of competitive allocations obtained in recent literature for simple economies fails. Successful intervention requires savings (or income) taxes and investment subsidies. In a macroeconomic context, this provides foundations for Keynesian phenomena and a role for policy.
Chapter 2

Optimal Leverage and Strategic Disclosure

2.1 Introduction

Firms seeking external financing face a multidimensional choice problem. On the one hand, they need to decide what securities to issue; whether to borrow or to issue stocks, for example. On the other hand, they choose the extent of their disclosure commitments; for instance, whether to go public or to keep private. The existing evidence suggests that greater disclosures tend to reduce the firm’s cost of financing, as the theory predicts, dampening the degree of asymmetric information in the market.\(^1\) However, the effect of disclosure on the composition of financing means has been largely overlooked by previous research. This paper aims to fill the gap, by modelling explicitly the interlinkage between disclosure and security design under asymmetric information. Two central predictions emerge from the analysis.

On the positive side, disclosure and leverage are negatively correlated. Enhanced disclosure leads to the possibility of issuing cheaper equity, increasing the opportunity cost of leveraged financing. Some firm-level evidence that supports this prediction is found analysing the behaviour of US corporates since the 1980s. Incidentally, one could also note that the results are consistent with the early development of modern stock markets, in the 19th century, that has been driven to a large extent by: (i) improvements in the information environment (e.g., the telegraph), and (ii) the growing financing needs of relatively more transparent industries such as the infrastructure sector (railways and canals, especially).\(^2\)


\(^{2}\)A prominent example is the London Stock Exchange (LSE). Prior to the 1840s, the LSE was
On the normative side, externalities in disclosure across firms lead to insufficient voluntary disclosures and excessive leverage relative to the constrained best. The inefficiency gets reduced if regulators can credibly mandate truthful disclosures, but this is often not the case.\textsuperscript{3} Modelling explicitly the interlinkage between disclosure and leverage suggests an alternative policy: setting capital requirements. Higher capital requirements encourage firms to be more transparent, in an effort to reduce the otherwise prohibitive costs of equity financing, and are especially useful when (i) profits are highly correlated across firms, and (ii) mandatory disclosures can be dodged. Both conditions seem to apply especially to financial firms, which – consistently with the model’s predictions – are both highly leveraged and opaque.\textsuperscript{4}

More specifically, I consider a financial market where firms seek financing from a competitive pool of investors under costly-state-verification (CSV). Firms and investors are symmetrically informed at the contracting stage, but acquire different information about the realised output ex post. Previous CSV models assumed an extreme type of hidden information: the entrepreneurs learn the output perfectly ex post; the investors learn nothing, but can verify the output reported by the entrepreneur at a cost.\textsuperscript{5} This paper relaxes the assumption, supposing that the investors learn the realised output with some probability $\pi \in [0, 1]$, and know nothing otherwise.\textsuperscript{6} Disclosure is privately costly and it affects the precision of the information revealed to investors. In addition, the private disclosure of a firm might convey information about its competitors.\textsuperscript{7}

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\textsuperscript{3} Two examples are particularly telling. First, Sloan [2007] documents that a typical RMBS (Residential Mortgage Backed Security) sold prior to 2008 had a disclosure prospectus of more than 300 pages. Though it complied with regulation, the prospectus hardly made such security transparent. Second, to show that banks balance sheet are a black box even for experienced investors, Partnoy and Eisinger [2013] quote Paul Singer (founder of Elliott Associates) writing to his partners that “There is no major financial institution today whose financial statements provide a meaningful clue [about its risks]”.

\textsuperscript{4} In the US, the median leverage ratio for financial firms after the 1980s ranges between 0.88 and 0.93 (Source: author’s calculation on Compustat data).

\textsuperscript{5} The results of a CSV model rely on the minimisation of expected bankruptcy costs. Recent evidence that these are substantial can be found in Molina [2005] and in Almeida and Philippon [2007].

\textsuperscript{6} The model generalises Townsend [1979] and Gale and Hellwig [1985], who restrict attention to $\pi = 0$. More general signal structures give rise to quite complex optimal contracts, but which maintain similar qualitative properties as those derived here. I refer the interested reader to Trigilia [2015].

\textsuperscript{7} Recent work of Badertscher et al. [2013], Shroff et al. [2013] and Durnev and Mangen [2009] identifies the presence of substantial information externalities across firms. See also the earlier
**Optimal securities.** The optimal capital structure is a mixture of debt and equity, and the amount of assets backed by debt (i.e., the leverage ratio) is monotonically decreasing in the probability that the investors are informed, denoted by $\pi$, which captures the degree of asymmetric information in the market. If $\pi = 0$, we have full leverage as in Gale and Hellwig [1985]. The intuition is as follows: (i) the financier must verify low messages to prevent cheating by the entrepreneur when output is higher; (ii) whenever there is verification, the optimal repayment equals the full realised output (this resembles bankruptcy, in which debt holders are senior claimants); finally, (iii) whenever there is no verification, the repayment is incentive compatible if and only if it equals a fixed constant (the face value of debt), regardless of the realised output.\(^8\)

Now consider $\pi > 0$. Property (iii) no longer holds: the highest incentive compatible repayment strictly increases with the output, because firms with higher output ex post have more to lose if caught cheating by the financiers (something that happens with probability $\pi > 0$). Moreover, it is always optimal to increase the repayments outside bankruptcy in order to minimise the ex ante need for costly verification. Therefore, the optimal contract has an equity component. Pure debt does not work because upon default the firm gets nothing, whereas if output is high it retains a needlessly large fraction of it. In other words, debt imposes an inefficient subsidy across states of nature ex post. Eventually, when $\pi$ is high enough, there is no need for verification on-the-equilibrium path and the optimal contract is pure equity.\(^9\)

Importantly, whenever there is verification on-the-equilibrium path the optimal capital structure is unique, for every $\pi$. Otherwise, though there may be multiple optimal securities, they are ex ante identical to issuing no debt, and selling a fraction $s\pi$ of shares, for some $s \in (0, 1)$ that is pinned down by the zero profit condition of the investors. As a result, the feasible strategies of a firm can be reduced to selecting the extent of its disclosure commitments, as this immediately maps into an optimal capital structure.

**Optimal disclosure.** The optimal amount of disclosure can be derived as a solution to the following trade-off: on the one hand, higher disclosure comes at a

\(^8\)More precisely, Townsend [1979] and Gale and Hellwig [1985] show that debt is the optimal contract among those that feature commitment to deterministic audits. The result does not hold if one allows for random audits (Border and Sobel [1987] and Mookherjee and Png [1989]) or lack of commitment (Gale and Hellwig [1989]). Krasa and Villamil [2000] argue that debt is optimal if both lack of commitment and random audits are assumed, see also Krasa and Villamil [2003].

\(^9\)Only in the limit, when $\pi = 1$, hidden information vanishes and Modigliani and Miller [1958] holds (i.e., the security design problem becomes irrelevant).
higher cost;\footnote{This is a central hypothesis of Admati and Pfleiderer [2000] and much of the subsequent disclosure literature. For evidence of the significant (direct and indirect) costs of disclosure see Bushee and Leuz [2005], Leuz et al. [2008], Iliev [2010], Ellis et al. [2012] and Alexander et al. [2013] and Dambra et al. [2015].} on the other hand, it decreases the degree of asymmetric information ex post, enabling the firm to issue cheaper equity – i.e., lower its leverage – and hence to reduce the expected bankruptcy costs. Each firm chooses its disclosure and capital structure as a solution to the aforementioned trade-off, best responding to its competitors who move simultaneously.

The disclosure game is potentially discontinuous, because the optimal leverage ratio might jump discretely for a marginal increase in disclosure, and it is not necessarily quasi-concave. Therefore, a Nash equilibrium is not guaranteed to exist in general. However, I present sufficient conditions for continuity and quasi-concavity, and show that the restrictions needed are relatively mild.\footnote{They require that the distribution of output satisfies two properties: (i) an increase in the interest rate at the optimal leverage ratio increases the expected profits of the investors (i.e., it more than compensates for the expected increase in verification costs); and (ii) the density function is continuously differentiable, and the first derivative is bounded below by some constant $z < 0$.} Under such restrictions, the set of Pure Strategy Nash Equilibria (PSNE) of the game is non-empty, and can be fully characterised.

**Comparative statics and the evidence.** Cæteris paribus, the model yields two main positive predictions, for which supporting empirical evidence on US data is found.

First, **leverage is monotonically decreasing in the degree of transparency.** The prediction is novel, to my knowledge, and indeed its empirical validity has not been much investigated.\footnote{A notable exception is Aggarwal and Kyaw [2009], who compare leverage and transparency across 14 EU countries and find a negative correlation. However, it seems that we still lack firm level evidence.} This paper takes a step toward filling this gap, by introducing a measure of transparency in an otherwise standard capital structure regression. In particular, I merge COMPUSTAT with IBES analysts’ forecast and CRSP prices.\footnote{COMPSTAT contains both balance sheet and cash flow (annual) information or the universe of US public firms. IBES (acronym for ‘Institutional Brokers’ Estimate System’) contains analysts’ estimates of earnings per share for several US corporations. Finally, CRSP (acronym for ‘Centre for Research in Security Prices’) offers equity prices used to calculate market-based equity measures.} I add to the standard variables considered in Frank and Goyal [2009] various market measures of transparency, such as the coefficient of variation of analysts’ Earnings Per Share (EPS) forecasts. The intuition behind this measure of transparency is that disagreement among analysts should decrease with the amount of public information about the firm (i.e., its transparency), and hence the variance of forecasts is likely to reflect – at least be correlated to – the degree of asymmetric information between
firm’s insiders and analysts. The regression analysis reveals that: (i) there exists a strong, statistically significant negative correlation between leverage and transparency; (ii) the correlation is robust to the inclusion of both standard control variables, and time-firm fixed effects. As a result, even if one restricts attention to variation within firm across time in leverage and transparency, the two remain reliably negatively correlated.

Second, consistently with the existing empirical evidence, leverage is monotonically decreasing in profitability. The intuition is that more profitable firms need to issue less shares (for a given price-per-share) to finance any given investment. Therefore, they have an easier chance of being able to issue incentive-compatible equity. The result is of interest from a theory perspective, as it reconciles the theory of optimal capital structure based on bankruptcy costs with the evidence. The negative relationship between leverage and profitability is further confirmed in my regression analysis.

**Mandatory capital and disclosure requirements.** Comparing the PSNE to the Socially Efficient (SE) disclosure levels, I show that whenever information is correlated across firms the private provision of information is excessively low, and hence leverage is excessively high. Firms under-disclose because they free ride on the information disclosed by their competitors, and they end up collectively stuck in a Pareto suboptimal equilibrium. The public good nature of information leads to the possibility of Pareto improving government interventions in financial markets.

A government that seeks to restore social optimality should consider two instruments. First, it could mandate a certain degree of disclosure. To the extent that this is feasible, and firms cannot dodge the disclosure requirements, then mandatory disclosures restore optimality. Indeed, we observe a wide range of disclosure requirements in every developed economy (Leuz [2010]).

However, as Ben-Shahar and Schneider [2011] document, disclosure regulation is not effective in many instances. In particular, ‘mandating transparency through disclosure’ proves harder (i) the more complex the underlying firm, and (ii)

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14 The idea of measuring transparency in this way is not new – see for instance Thomas [2002], Tong [2007], Chang et al. [2007]. Many other factors, such as herding or contrarianism – as well as personal opinions – enter the forecast process. Such factors are discussed in greater depth in Bernhardt et al. [2006]. I implicitly assume that these additional sources of disagreement are orthogonal to leverage. Bhat et al. [2006] show that analysts’ forecasts error and dispersion are strongly positively correlated with the country-level transparency measures of Bushman et al. [2004].

15 The negative correlation between leverage and profitability has been documented in several previous studies, such as Frank and Goyal [2009], Welch [2011] and Graham and Leary [2011].

16 Indeed, the static trade-off theory would suggest the exact opposite should hold (see, for instance, Kraus and Litzenberger [1973]). The interpretation of bankruptcy costs as the costs of verification is discussed in Gale and Hellwig [1985] and Tirole [2010].
the greater the opportunity cost of disclosure. As it is often argued (e.g., Partnoy and Eisinger [2013]), these conditions hold especially for large financial firms. So, are there indirect regulatory tools that could promote endogenously greater transparency of financial institutions?

The model I present suggests that capital requirements are a suitable instrument to this end. Through the lens of the model, even taking into account the costs of disclosure, firms who face stringent capital requirements are encouraged to disclose better information to the market in an effort to reduce the costs of equity financing.\textsuperscript{17} Although the argument is simple and plausible, it is strikingly absent from the current debate on capital standards, which I believe should not be as separated from that on information requirements as it is at present.\textsuperscript{18}

Consider the recent discussion around capital standards that is ongoing in the US. The Federal Reserve justifies its regulation as follows:

\textit{The primary function of capital is to support the bank’s operations, act as a cushion to absorb unanticipated losses and declines in asset values that could otherwise cause a bank to fail, and provide protection to uninsured depositors and debt holders in the event of liquidation.} [emphasis not in the original]

FED Supervisory Policy and Guidance Topics, as of 14.09.2015

The FED’s statement highlights three objectives. The first is to ‘support the bank’s operations’, a relatively vague proposition which is absent from much of the political and academic debate on the matter. The second objective is coherent with the position of many prominent economists, who emphasise the importance of requiring a sufficient ‘loss absorbing’ capital buffer, and is at the centre stage of both the public and the academic debate.\textsuperscript{19} However, it offers a natural counterargument to finance lobbyists and sceptics of regulation. Despite the virtue of capital buffers

\begin{itemize}
  \item [\textsuperscript{17}]Of course, the argument relies on the presumption that the government shares with the market a knowledge of individual firms covariates. Otherwise, the Pareto gains or losses in setting capital requirements depend on the average effect on firms, as in Admati and Pfleiderer [2000]. Though supposing that governments are well informed is empirically implausible in many instances, observe that at present Basel III does distinguish firms that are too-big-to-fail, and imposes a capital surcharge on them.
  \item [\textsuperscript{18}]The complementarities across different regulations are a generally under-researched and important area for future work, as emphasised in Leuz and Wysocki [2008]. This is but one instance of the more general phenomenon.
  \item [\textsuperscript{19}]See especially the Squam Lake Report (French et al. [2010]); recent influential books by Kotlikoff [2010], Sinn [2012], Admati and Hellwig [2014] and Stiglitz et al. [2015]; academic papers such as Admati et al. [2013], Chamley et al. [2012] and Miles et al. [2013]. The general discontent among academics (and a few politicians) with the outcome of Basel III, that sets capital requirements to less than 5%, shifted much of the debate at the national level.
\end{itemize}
ex-post, in crises times, they counterargue that stringent requirements tend to curb investment during booms, making it more expensive for firms to obtain external financing. So, from an ex ante perspective they are not necessarily desirable.\textsuperscript{20} Finally, the third argument surprised me at first sight, and can be considered as another subsidy to debt instruments relative to alternatives, in much the same spirit as the tax deduction of interest payments.\textsuperscript{21}

This paper wishes to shift spotlight toward the first goal, offering an argument that substantiates how capital requirements might ‘support the bank’s operations’. The mechanism I suggest starts with a coordination failure in information provision across banks, aggravated by (i) systemic risk and correlation of assets portfolios, and (ii) the easiness to dodge mandatory disclosures. The under-provision of information not only leads to opacity of financial intermediaries, evidently, but it also promotes an excessive reliance on debt instruments to get funding. Capital requirements force corporations to be more transparent, in order to obtain more favourable costs of equity financing, and this is unambiguously beneficial ex ante because it lowers the expected costs of distress, and the reduction in this deadweight loss more than compensates the increase in disclosure costs.

\textbf{Related Literature.} The paper wishes to contribute to the existing literature mainly pointing at the link between security design and disclosure.

On the security design side, it builds on Townsend \cite{townsend1979} and Gale and Hellwig \cite{gale1985} CSV framework. The idea that outside information leads to the optimality of issuing some equity in a CSV model dates back to Chang \cite{chang1999}, who considers a firm with two technologies: one subject to CSV and one observable and verifiable (for which Modigliani and Miller \cite{modigliani1958} holds). Although my interpretation in terms of signals is different, and in general it yields different conclusions from those in Chang (see Trigilia \cite{trigilia2015}), the intuition is similar: the presence of some \textit{reliable}

\textsuperscript{20}For instance, the former CEO of Deutsche Bank Josef Ackermann claimed that capital requirements ‘would restrict bank’s ability to provide loans to the rest of the economy’, which ‘reduces growth and has a negative effect for all’. The CEO of JP Morgan, Jamie Dimon, argued that capital requirements would ‘greatly diminish growth’, and a similar position has been expressed by the former CEO of Citigroup Vikram Pandit, as well as by the lobbying group \textit{Institute for International Finance} (see Admati and Hellwig \cite{admati2014}, pagg. 97, 232 (18) and 274 (60)). A few papers estimated the growth loss coming from capital requirements in a DSGE framework to be substantial, but crucially under the \textit{exogenous assumption} that equity is more costly for banks to issue (see for instance Van den Heuvel \cite{vandenheuvel2008}).

\textsuperscript{21}An often mentioned force pushing firms toward increasing their leverage is the tax deductibility of interest payments, but not of dividends. Observe, though, that such factor cannot account for the vast cross-sectional variation in leverage across firms in the US. It is therefore overlooked here. On the contradiction between capital requirements and tax advantages of debt, see especially De Mooij \cite{de2012} and Fleischer \cite{fleischer2013}. Both scholars promote the abolition of any tax advantage of debt.
information ex post leads to optimal contracts that cross debt from the right.

As such, the rationale for equity in the model I present is distinct from other stories that involve either risk-aversion and transaction costs (Cheung [1968]), costly-state-falsification (Lacker and Weinberg [1989] and Ellingsen and Kristiansen [2011]), double-sided moral hazard (Bhattacharyya and Lafontaine [1995]), control rights and infinite investment horizon (Fluck [1998]) or the combination of ambiguity and ex ante moral hazard (Carroll [2015] and Antic [2014]).

On the disclosure side, the model builds on Fishman and Hagerty [1989, 1990] and Admati and Pfleiderer [2000]. Like the aforementioned papers, disclosure is privately costly and it leads to an externality due to its public good nature. Namely, the disclosure made by one firm affects the optimal disclosure of its competitors, and this consideration feeds back into the initial optimal disclosure decision. In such a scenario, the private provision of information is likely to be socially inefficient, although as Fishman and Hagerty show inefficient does not necessarily mean too low.

As Leuz [2010] discusses at length, the presence of information externalities is a major justification for the existence of mandatory disclosure requirements in practice. This paper wishes to contribute by highlighting that a similar argument leads naturally to capital requirements as well.

### 2.2 Setup

There are two dates $t \in \{0, 1\}$, $N \geq 1$ identical firms and a large number of competitive investors. Both firms and investors are risk-neutral and maximise date one consumption.

Each firm is endowed with no initial wealth, and has access to an investment technology at $t = 0$ that requires a fixed input $K > 0$ and generates stochastic output $\tilde{x}$ at $t = 1$. I assume that $\tilde{x} \in X \equiv [0, \bar{x}]$, and denote by $F(x)$ the cumulative

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22 Explanations for optimal equity based on control rights face increasing difficulties in accounting for the empirical evidence that many corporations are adopting a two-tiered equity structure, whereby investors are offered non-voting stocks (e.g., Google and Facebook). On this point, see also Zingales [2000]. In contrast, explanations based on cash-flow rights used to require that the investors play an active role. Only recently, with Carroll [2015] and this paper, equity has been found optimal in models with relatively passive investors.

23 Recent papers on disclosure include Guttman et al. [2014] and Ben-Porath et al. [2014a]. Alvarez and Barlevy [2014] also emphasise externalities in information provision, though focusing more on contagion.

24 The argument for capital requirement presented in this paper differs markedly from the general equilibrium arguments based on pecuniary externalities (such as Korinek and Sinek [2014] and Geanakoplos and Kubler [2015]). It also differs from arguments based on excessive risk taking and ‘collective moral hazard’ (see Farhi and Tirole [2012] and Admati and Hellwig [2014]).
distribution of $\tilde{x}$, and by $f(x)$ its density. For simplicity let $f(x) > 0$ for all $x$ and suppose it is continuous. To make the problem interesting, Assumption 4.1 guarantees that the project has positive net present value (NPV) under full information.

**Assumption 2.1.** $K < \mathbb{E}_f[\tilde{x}]$. (Positive NPV)

In this paper, I overlook the presence of agency problems within the firm, and I refer to the owner/manager of each firm as the *entrepreneur*. I intend to explore the issue in future research.

The representative investor is endowed with large initial wealth and can either lend it to some firm, or invest it in a riskless bond with interest factor normalised to unity.

Investment occurs under symmetric information. Hidden information comes ex post, when the state of the project is privately observed by the entrepreneur. The investors observe the state with some probability $\pi \in [0, 1]$, which I will discuss in depth later on. If the investors do not observe the state, they still have the option of verifying it at a fixed cost $\mu \geq 0$. The entrepreneur can affect $\pi$ at $t = 0$ by committing to a disclosure policy – e.g., hiring an independent and trustworthy auditor or going public.

The timing of the game is as follows:

$t=0$ The entrepreneurs offer a contract (take-it-or-leave-it) to the investors. If the investors accept, $K$ is invested;

$t=1$ Nature determines the realised state $x \in X$. Then, in sequence:

1. Each entrepreneur privately observes $x$ and sends a public message $m \in M$ about it (e.g., a balance sheet statement);
2. Investors observe $x$ with probability $\pi$, and observe nothing otherwise;
3. If the investors did not observe the state, they can verify it at a cost $\mu$;
4. Transfers occur and the game ends.

I now describe the feasible portfolio of securities and disclosure policies.

### 2.2.1 Securities

For a given set of public messages $M$, the aggregate payout from firm $i$ to its investors can decomposed in three parts:

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25Investment is assumed to be an observable and verifiable action.
(i) The repayment function $s_i(m) : M \to \mathbb{R}$ specifies the payout when investors are uninformed about the state;

(ii) The clawback function $z_i(m, x) : M \times X \to \mathbb{R}$ specifies the payout when investors are informed about the state;

(iii) The verification function $\sigma_i(m) : M \to [0, 1]$ specifies the probability that the state is verified for every message, when the investors are uninformed otherwise.

I impose two restrictions on admissible securities: (i) limited liability; (ii) deterministic verification. Limited liability implies that repayments and clawbacks cannot be negative, and their upper bound depends on the verifiable output. Namely, if the investors are informed the upper bound is the realised output $x$, otherwise it is the message $m$. It is a standard assumption and it guarantees the existence of an optimal contract.  

Deterministic verification is commonly assumed in CSV models, but it is a restrictive assumption. Indeed, Border and Sobel [1987] and Mookherjee and Png [1989] show that the optimal random contract is not debt. I make the assumption for two reasons: (i) the optimal random contracts still exhibit the key features of interest here; and (ii) they cannot be fully characterised, because local incentive compatibility does not suffice for global (see Border and Sobel [1987]). Formally:

**Assumption 2.2.** A portfolio of securities is feasible only if, $\forall m, x$:

- Payments satisfy limited liability: $s_i(m) \in [0, m], z_i(m, x) \in [0, x]$
- Verification is deterministic: $\sigma_i(m) \in \{0, 1\}$

### 2.2.2 Disclosure policies

The disclosure policy of firm $i$ consists in the choice of a binary signal, which reveals with probability $p_i \in [0, 1]$ the state of nature ex-post to the investors public at a cost $c(p_i)$.

In the absence of correlation across firms, the probability that the investors observe $x$ for a given firm – denoted by $\pi_i$ – equals $p_i$. In contrast, when there is more than one firm and output is correlated across firms, we may have $p_i < \pi_i$.

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26What is important is that the verifiable output lies in a compact set for every state $x$. One could therefore easily accommodate the equivalent of a finite non-pecuniary penalty.

27Namely, lower messages are generally associated with higher verification, higher states are not verified and repay a flat rate (in the absence of signals).
Observing other firms' output might be informative about firm \( i \)'s realised output as well.

I assume that the correlation between firm \( i \) and firm \( j \) is captured by a parameter \( q_{i,j} \in [0,1] \), so that the probability that the signal sent by firm \( j \) is informative about firm \( i \) is \( q_{i,j}p_j \).\(^{28}\) In aggregate, the probability of having at least an informative signal out of \( N \) independent but not identically distributed Bernoulli trials is described by the inverse cdf of a Poisson Binomial distribution evaluated at zero successes, and it reads:

\[
\pi_i(p_i, p_{-i}, q_{-i}) = 1 - (1 - p_i) \prod_{j \neq i} (1 - q_{i,j}p_j) \tag{2.1}
\]

The formula captures a positive externality coming from each firm’s disclosure policy, because \( \partial \pi_i(p_i, p_{-i}, q_{-i}) / \partial p_j \geq 0 \) and \( \partial \pi_i(p_i, p_{-i}, q_{-i}) / \partial q_{i,j} \geq 0 \). However, one could envision the presence of negative externalities as well. For instance, in a model where the feasible aggregate media coverage is limited, the disclosures made by other firms may end up limiting the attention that firm \( i \) can attract, hence reducing the information that the investors can acquire about its output. The analysis of such scenarios, which may give rise to strategic complementarities across firms, is left for future research.

### 2.2.3 Equilibrium concept and preliminary lemmas

Before stating the equilibrium concept and the contracting problem, it is important to acknowledge that in the environment I described the revelation principle holds:

**Lemma 2.1.** Without loss of generality, we can restrict attention to direct revelation mechanisms.\(^{29}\)

As a result, from now onwards let \( M = X \) and focus on truthful implementation. A type of firm refers to the state \( x \) of the project that the entrepreneurs observe before sending their public messages. The driving force in deriving the optimal portfolio of securities for a firm is the continuum \([0, x]\) of incentive compatibility constraints for each ex-post type \( x \), which I now describe.

The expected payout from the firm to the investors when the realised state

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\(^{28}\)Evidently, it must be that \( q_{i,i} = 1 \) for every \( i \). Observe that \( q \) is not a statistical correlation coefficient, it just captures the presence of spillovers in information provision. Hence, it being positive is without loss of generality.

\(^{29}\)The validity of the revelation principle follows from the exact same logic as in Gale and Hellwig [1985]; the proof is omitted.
is $x$ and the message is $x'$ is denoted by:

$$r_i(x', x) \equiv [\pi_i + (1 - \pi_i)\sigma_i(x')]z_i(x', x) + (1 - \pi_i)(1 - \sigma_i(x'))s_i(x')$$

where recall that $\pi_i$ is a function of $p_i$, $p_{-i}$ and $q_{-i}$: $\pi_i(p_i, p_{-i}, q_{-i})$. To understand the above expression, observe that:

1. The payout equals $z_i(x', x)$ whenever: (i) there is verification, which happens with probability $(1 - \pi_i)\sigma_i(x')$; and (ii) whenever the investor is informed, which happens with probability $\pi_i$;

2. The payout is equal to $s_i(x')$ otherwise – i.e., when the signal is uninformative and no verification takes place. The probability of this event is equal to $(1 - \pi_i)(1 - \sigma_i(x'))$.

To simplify the notation, let $r_i(x, x) \equiv r_i(x)$.

As a consequence of Lemma 2.1, incentive compatibility requires that, for every $x$, at the optimal contract the expected payoff for the entrepreneur under truthful reporting (i.e. $x - r_i(x)$) is greater than the expected payoff by pretending to be any other type $x' \neq x$, i.e.:

$$x - r_i(x) \geq x - r_i(x, x'), \quad \forall (x, x') \in X^2 \quad (2.2)$$

It is useful to refer to the incentive compatibility constraint when (i) the true state is $x$ and (ii) the message sent is $x'$, as $IC(x', x)$.

Any contract that implements investment must also satisfy the participation constraint (PC) for the investor, which by Lemma 2.1 reads:

$$\int_X [r_i(x) - (1 - \pi_i)\sigma_i(x)\mu]dF(x) \geq K \quad (2.3)$$

I restrict attention to pure strategy Nash equilibria, defined as follows:

**Definition 2.1.** A Pure Strategy Nash Equilibrium (PSNE) of the game consists in a set of strategies $\{s_i^*, z_i^*, \sigma_i^*, p_i^*\}$ for all firms $i = 1, ..., N$ such that, for each firm $i$ and for a given vector $p_{-i}^*$, both the portfolio of securities issued and the disclosure policy are optimal:

$$\{s_i^*, z_i^*, \sigma_i^*, p_i^*\} \in \arg \max \int_X [x - r_i(x)]dF(x) - c(p_i) \quad (2.4)$$

s.t. $LL; \ DV; \ IC(x, x') \forall x, x'; \ PC.$
It is easy to see that PC must be binding at any optimal contract. This is because whenever a contract \( \{s, z, \sigma\} \) is feasible and incentive compatible, so is a contract \( \{s', z', \sigma'\} \) such that (i) \( \sigma' = \sigma \), (ii) \( s' = \alpha s \), and (iii) \( z' = \alpha z \) for some \( \alpha \in [0, 1) \). By substitution, the contracting problem can be rewritten as:

\[
\{s^*_i, z^*_i, \sigma^*_i, p^*_i\} \in \arg \max \int_X [x - (1 - \pi_i)\sigma_i(x)\mu]dF(x) - c(p_i) - K \\
\text{s.t. } LL; \ DV; \ IC(x, x') \forall x, x'
\]

The latter formulation highlights that the objective function is simply to minimise the expected deadweight costs of verification and disclosure. Two intuitive lemmas hold irrespective of \( p_i \), and prove useful in characterising the optimal contracts.

The first lemma deals with off-equilibrium clawback provisions, and shows that we can restrict attention to contracts that impose the harshest feasible clawbacks after cheating by the entrepreneur has been verified. Namely, optimal contracts are such that verification takes place when \( m < y \), which proves that the entrepreneur is cheating with certainty, and \( z(m, x) = x \) whenever \( m \neq x \).

**Lemma 2.2.** We can restrict attention to contracts such that:

(i) All assets are seized upon verified cheating: \( z^*(m, x) = x \) whenever \( m \neq x \);

(ii) Messages revealed to be false are verified.

**Proof.** See the Appendix. \( \square \)

Observe that we have one degree of freedom in setting \( s^*(m) \) whenever \( \sigma_i(m) = 1 \). As a consequence of Lemma 2.2, I let \( s^*(m) = z^*(m, x) = x \) in such events.

The second Lemma shows that we can restrict attention to securities such that both the aggregate payout and the repayment function are weakly increasing on \( X \). The intuition is that having a non-monotonic optimal contract implies that incentive compatibility is not binding in some states, and one can always construct a monotonic contract that replicates the same ex ante allocation satisfying all constraints.

**Lemma 2.3.** We can restrict attention to monotonic securities such that (i) \( r(x) \geq r(x') \), and (ii) \( s(x) \geq s(x') \) whenever \( x > x' \).

**Proof.** See the Appendix. \( \square \)

I now proceed to the characterisation of optimal contracts.
2.3 Privately Optimal Leverage and Disclosure

The results are presented according to the following roadmap. In section 2.3.1 I characterise the optimal portfolio of securities issued for a given $\pi_i$. In particular, I show that it is a mixture of debt and equity, with leverage decreasing monotonically with $\pi_i$. Moreover, $\pi_i$ is a sufficient statistic to fully characterise the optimal leverage ratio.

Next, in section 2.3.2, I characterise the set of Pure Strategy Nash Equilibria (PSNE) of the disclosure game, where the strategy set of each firm simply consists in choosing a $p_i \in [0, 1]$. Despite the simple structure of optimal contracts in the model, the game is generally discontinuous and not quasi-concave. I introduce two mild restrictions on the distribution of output $f(\cdot)$, and show that they are sufficient to obtain a well-behaved – i.e., continuous and quasi-concave – game, with a unique PSNE. Comparative static results are presented and discussed at the end of the analysis.

2.3.1 Optimal securities for a given disclosure policy

For this section, take $p_i$ as given for every $i$, and focus on the optimal associated portfolio of securities. The analysis is of independent interest because it generalises Gale and Hellwig [1985] – who restricted attention to the case of $\pi_i = 0$ for all $i$ – and it highlights the key driving forces behind optimal securities in a CSV model with signals. For easiness of notation, in this section I omit the subscript $i$ and any reference to the disclosure cost $c(\cdot)$.

To set a benchmark, consider the case of either $\pi = 1$ or $\mu = 0$. The participation constraint for investors in both cases reads $\int_X r(x)dF(x) \geq K$, and $IC(x,x')$ becomes $r(x) \leq x$. It follows that:

**Remark 2.1.** When either $\pi = 1$ or $\mu = 0$, Modigliani and Miller [1958] holds, and every feasible security that makes PC binding is optimal.

**Proof.** Immediate from the above reasoning.

From now onwards, I restrict attention to $\pi < 1$ and $\mu > 0$. Define the two securities that will be part of any optimal contract as follows:

**Definition 2.2.** A security is **debt** if and only if $s(m) = \min\{m,d\}$ for some $d \in X$.

**Definition 2.3.** A security is **equity** if and only if $s(m) = \alpha m$ for $\alpha \in [0, 1]$.
The two securities are depicted in Figure 4.1. It is important to stress that because investment is risky, any feasible debt contract that implements investment must be such that \( d > K \), as depicted in the left panel of the Figure. The following proposition characterises the optimal contract.

\[ D \equiv \{ m | \sigma(m) = 1 \}, \quad NV \equiv \{ m | \sigma(m) = 0 \} \]

Because \( X \) is bounded, there must exist \( x_{NV} \equiv \inf_{x \in NV} \{ x \} \) and \( x_V \equiv \sup_{x \in V} \{ x \} \). The first property implies that at the optimal contract \( x_{NV} > x_V \).

Property 2: whenever \( x_{NV} > 0 \), the optimal repayment function for every \( x \in NV \) is given by:

\[ s^*(x) = (1 - \pi_i)x_{NV} + \pi_i x \]

The expression follows from two considerations. First, \( r^*(x_{NV}) = x_{NV} \) by monotonicity (i.e., Lemma 2.3) and the fact that all states \( x < x_{NV} \) are verified and hence cannot be profitable deviations by Lemma 2.2. Second, it is optimal to extract the highest incentive compatible repayment in the no-verification region to push \( x_{NV} \)

---

**Proposition 2.1.** If \( \mathbb{E}_f[\pi \tilde{x}] \geq K \) equity is optimal and debt is suboptimal. If \( \mathbb{E}_f[\pi \tilde{x}] < K \) the uniquely optimal contract is a mixture of debt and equity.

**Proof.** See the Appendix. \( \square \)
to the minimum possible level that satisfies PC with equality. Under the given 
\( s^*(x) = (1 - \pi_i)x_{NV} + \pi_i x \), incentive compatibility binds for every \( x \in NV \) and hence it is optimal.

Otherwise, if \( x_{NV} = 0 \), there exist multiple optimal repayment functions. They only need to be such that the slope is less than or equal to \( \pi_i \) for every state in the no-verification region. Therefore, a pure equity contract with \( \alpha \leq \pi_i \) is optimal.

**Property 3:** for every \( x \in V \), \( z^*(x, x) = s^*(x) = x \). That is, investors are senior claimants in verification states (that are the model equivalent of bankruptcy). This holds because bankrupt firms have no feasible deviation such that they can repay less (in expectation) than their realised output. As a result, minimisation of bankruptcy costs requires them to payout all their output.

Figure 2.2, Panel (a), depicts the firm’s payout at the optimal mixture of debt and equity. Panel (b) sketches the characterisation of the optimal contract as a function of both transparency (measured by \( \pi \)) and profitability (measured as the ratio \( K/E_f[\tilde{x}] \)). Moving from the bottom-right corner – high profitability, high transparency – toward the top-left corner – low profitability, low transparency – the amount of debt in the optimal contract is increasing. The gray area denotes the parameter region where the first-best (no verification on-the-equilibrium path) can be implemented and firms have zero leverage at the optimal contract. In the upper-left triangle, instead, the solution is second-best and the amount of debt in the contract is increasing in \( K/E_f[\tilde{x}] \) and decreasing in \( \pi \).

The comparative static results behind the graph will be formally stated and proved in Corollary 2.3. First, observe that Proposition 2.1 implies that pure debt is optimal if and only if \( \pi_i = 0 \).

**Corollary 2.1.** Pure debt is optimal if and only if \( \pi = 0 \).

**Proof.** Immediate from Proposition 2.1, since whenever \( x_{NV} > 0 \) we must have \( \alpha = \pi \). \( \square \)

Notice that both Proposition 2.1 and Corollary 2.1 identify the shape of the optimal contract that implement investment, however they offer no guarantee that investment would take place. I turn to the question of whether investment occurs or not next.

The expected profits of the investors at a given mixture of debt and equity
are denoted by $R(x_{NV}) \equiv \mathbb{E}_f[r(x) - (1 - \pi)\sigma(x)\mu] - K$, or:

$$R(x_{NV}) = \int_0^{x_{NV}} [x - (1 - \pi)\mu]dF(x) + \int_{x_{NV}}^\pi \pi x dF(x) + (1 - F(x_{NV}))(1 - \pi)x_{NV} - K$$

(2.6)

$R(x_{NV})$ takes values on a compact subset of the real line, and the continuity of $f(\cdot)$ implies that it is continuous in $x$. As a result, there must exist (at least one) threshold $x^*$ that maximises $R(x_{NV})$. If there is more than one, pick the smallest. Formally

$$x^* \equiv \min \{x_{NV} \mid x_{NV} \in \arg \max \ R(x_{NV})\}$$

(2.7)

We obtain the following characterisation of the financing constraint coming from hidden information:

**Corollary 2.2.** *Investment takes place only if $R(x^*) \geq 0$.***

**Proof.** It follows from the above reasoning. □

In turn, the equilibrium face value of debt $d^*$ is given by:

$$d^* = \min \{x_{NV} \mid R(x_{NV}) = 0\}$$

(2.8)

Although the expected profits of the investors do not necessarily increase with the interest rate in a CSV model (due to the presence of verification costs), it must be that $R(d^*)$ is weakly increasing in its argument. Namely, that the expected *equilibrium* profits of the investors increase at the margin with the interest rate.
Lemma 2.4. \( R(d^*) \) is weakly increasing in \( d^* \).

Proof. See the Appendix. \( \square \)

Because of Lemma 2.4, the effect of transparency (\( \pi \)), profitability (lower \( K \) for a given \( \mathbb{E}_f[\tilde{x}] \)) and verification costs (\( \mu \)) on leverage (\( d^* \)) are monotonic and can be easily derived.

Corollary 2.3. Cæteris paribus, leverage (\( d^* \)) is monotonically increasing in profitability and decreasing in transparency. It also increases with the verification cost.

Proof. See the Appendix. \( \square \)

The effect of transparency and profitability on optimal leverage ratios is depicted in Figure 2.2, panel (b). More transparent firms can finance with equity projects of relatively lower profitability. As the converse, firms that are more opaque need to have highly profitable investment opportunities to issue equity, it is otherwise optimal for them to borrow (to some degree).

To provide some more intuition, I conclude the section solving an example.

**Example.** Suppose that \( \tilde{x} \) is distributed uniformly and \( X = [0, 10] \). If the verification cost is given by \( \mu = 1 \) and \( K = 4 \), the optimal leverage ratio (i.e. debt over total assets) is depicted in Figure 2.3, panel (a). Zero-leverage firms are such that \( \pi > 4/5 \), else some amount of debt will be issued, monotonically decreasing in transparency.

Consider now \( \pi \leq 4/5 \). The PC reads:

\[
\int_0^d [x - (1 - \pi)]dx + \int_d^{10} [\pi x + (1 - \pi) d]dx = 40
\]

which can be rewritten as: \( 0.5(1 - \pi)d^2 - 9(1 - \pi)d + 40 - 50\pi = 0 \). Of the two roots, it is easy to check that we should always pick the negative one. Therefore, we get:

\[
d^* = 9 - \frac{\sqrt{-19\pi^2 + 18\pi + 1}}{1 - \pi}
\]

Moreover, the derivative of the expression with respect to \( \pi \) reads:

\[
\frac{\partial d^*}{\partial \pi} = \frac{-10}{(1 - \pi)\sqrt{-19\pi^2 + 18\pi + 1}} < 0
\]

Panel (b) plots the firm’s profits as a function of both \( \pi \) and \( K \). In the purple region at the top-left corner, investment does not take place (in fact, firm’s profits
show to be negative in this region). Otherwise, investment takes place and profits are decreasing in $K$ and increasing in $\pi$. In particular, profits are strictly increasing in transparency when some debt is issued (i.e., $\pi < 0.8$), and constant otherwise.

Figure 2.3: Optimal Contract in the Example

(a) Leverage and Transparency   (b) Firm profits (gross of disclosure costs)

2.3.2 Optimal disclosure policies

The previous section offered a characterisation of the optimal contract as a function of $\pi_i$. The optimal contract is unique whenever verification takes place on-the-equilibrium path, and it can be implemented by pure equity otherwise. In this section, we take advantage of this characterisation to characterise the equilibria of the disclosure game.

To set a benchmark, consider what happens when disclosure is costless. From PC, it is obvious that the entrepreneur only gains from increasing $p_i$, as it prevents any need for ex post verification. Therefore, full disclosure is expected:

**Remark 2.2.** If disclosure is costless (i.e., if $c(p_i) = 0$, $\forall p_i$ and $\forall i$), optimal contracts are such that $p^*_i = p^*_j = 1$ for all $i, j$ and Modigliani and Miller [1958] holds.

**Proof.** Immediate from the above reasoning and Remark 2.1. ☐
A more interesting and realistic scenario occurs when disclosure is costly—e.g., the fee charged by an independent audit firm. Increasing the degree of disclosure on the one hand raises the disclosure cost \( c(p_i) \), on the other it lowers the costs of financing by enabling the entrepreneur to issue more (cheaper) equity, hence decreasing the face value of debt and the expected deadweight verification costs.

Observe that (2.8) allows us to express \( d^*_i \) as a function of \( p_i \) through its dependence on \( \pi(p_i, p_{-i}, q_{-i}) \), for any given strategy of the other \( N-1 \) firms. Moreover, we can disregard every \( p_i \) such that \( p_i > K/E_f[\tilde{x}] \) (regardless of strategy of the opponents), because it is dominated by \( p_i = K/E_f[\tilde{x}] \). To rule out uninteresting corner solution, suppose that the cost function satisfies the following Inada conditions:

**Assumption 2.3.** The cost function \( c(\cdot) \) is strictly increasing \((c' > 0)\), strictly convex \((c'' > 0)\) and it satisfies the following Inada conditions: \( c(0) = c'(0) = 0 \) and \( c'(1) \to +\infty \).

Because the optimal capital structure can be fully described by \( \pi_i \), Program (2.5) can be rewritten as follows:

\[
p_i^* \in \arg\max_{p_i \in [0, K/E_f[\tilde{x}]]} V(p_i, p_{-i}) \equiv E_f[\tilde{x}] - (1 - \pi_i(p_i, p_{-i}, q_{-i}))F(d^*(\pi_i(p_i, p_{-i}, q_{-i})))\mu - c(p_i) - K \tag{2.9}
\]

The objective function \( V(p_i, p_{-i}) \) need not be differentiable with respect to \( p_i \), because \( d^*(\pi_i(p_i, p_{-i}, q_{-i})) \) may jump as \( p_i \) changes infinitesimally. This phenomenon happens when the payout to investors does not increase with the face value of debt—that is, when \( (1 - F(d^*)) = f(d^*)\mu \) \(^{30}\) and such discontinuities are problematic for the existence of a solution to the program. However, if the set of points such that the equality holds is empty, then \( d^*(\pi_i(p_i, p_{-i}, q_{-i})) \) is differentiable and so is \( V(p_i, p_{-i}) \).

Define the following threshold, which corresponds to the equilibrium face value of debt of a standard CSV model with \( \pi_i = 0 \):

\[
\tilde{d} \equiv \min \left\{ x \in X \ \middle| \int_0^x [x - \mu]dF(x) + (1 - F(d))d = K \right\}
\]

A sufficient condition for differentiability of \( d^*(\pi_i(p_i, p_{-i}, q_{-i})) \) is the following:

**Lemma 2.5.** The objective function \( V(p_i, p_{-i}) \) is differentiable if the hazard rate

\(^{30}\)Recall that by Lemma 2.4 it can never be the case that \((1 - F(d^*)) < f(d^*)\mu\).
$h(x)$ is uniformly bounded so that:

$$h(x) = \frac{f(x)}{1 - F(x)} < \frac{1}{\mu}, \quad \forall x \leq \bar{d}$$  \hspace{1cm} (2.10)

**Proof.** See the Appendix.

The condition has a natural economic interpretation. It guarantees that the gains to the investors coming by an increase in the face value of debt (e.g., a marginally higher interest rate) more than compensate the losses due to verification. The bound becomes tighter when the verification cost $\mu$ increases, and/or profitability falls.

If (2.10) holds, Program (2.9) is guaranteed to have at least one solution by the theorem of the maximum. Moreover, totally differentiating (2.8) with respect to $x_{NV}$ and $p_i$, and evaluating at $x_{NV} = d_i^*$ yields:

$$\frac{d d_i^*}{d p_i} = \frac{d d_i^*}{d \pi_i} \cdot \frac{d \pi_i}{d p_i} = -\frac{d \pi_i}{d p_i} \cdot \frac{\mu F(d_i^*) + \int_{d_i^*}^{\bar{x}} [x - d_i^*]dF(x)}{(1 - \pi_i)[1 - F(d_i^*) - \mu f(d_i^*)]} < 0$$  \hspace{1cm} (2.11)

where the inequality follows from three observations: (i) $\pi_i$ is strictly increasing in $p_i$; (ii) $\mu F(d_i^*) + \int_{d_i^*}^{\bar{x}} [x - d_i^*]dF(x) > 0$ for every $d_i^* \in X$; and finally (iii) $(1 - \pi_i)[1 - F(d_i^*) - \mu f(d_i^*)] > 0$ by inequality (2.10) and Assumption 4.1.

As a result, the first derivative of the objective function $V(p_i, p_{-i})$ reads:

$$\frac{\partial V(p_i, p_{-i})}{\partial p_i} = \mu \frac{\partial \pi_i}{\partial p_i} \left[ F(d_i^*) + f(d_i^*) \cdot \frac{\mu F(d_i^*) + \int_{d_i^*}^{\bar{x}} [x - d_i^*]dF(x)}{1 - F(d_i^*) - \mu f(d_i^*)} \right] - c'(p_i)$$  \hspace{1cm} (2.12)

Equation (2.12) formalises the trade-off that underpins the choice of an optimal disclosure policy: on the one hand, greater disclosure comes at a higher marginal cost $c'$ (due to the strict convexity of the cost functional), on the other it pushes leverage down – enabling the firm to issue a larger fraction of incentive compatible equity – at a gain proportional to $\gamma > 0$.

The second derivatives with respect to $p_j$ for $j = 1, \ldots, N$ is a relatively long collection of terms, and therefore I leave its derivation and explanation to the Appendix (at the beginning of the proof of Lemma 2.6). It suffices to mention here that most of the terms can be signed to be negative, suggesting that the problem has a certain degree of concavity built in, and coming from the participation constraint.

---

31 Assumption 4.1 implies that $K/E_f[\bar{x}] < 1$, so it is never the case that $(1 - \pi_i) = 0$, irrespective of $q$. 

25
Lemma 2.6. A sufficient condition for $V(p_i, p_{-i})$ to be strictly concave is the following:

$$f'(x) > -\frac{1}{h(x) - 1 - \mu}, \quad \forall x \in [0, d] \quad (2.13)$$

Proof. See the Appendix.

The condition in Lemma 2.6 is not very restrictive if (2.10) holds, as $h(x)^{-1} > \mu$ and the lower bound is negative. Moreover, alike (2.10), it is a straightforward property to check. From now onwards, I assume that both restrictions on the distribution of output hold, so that the disclosure game is well behaved:

Assumption 2.4. Both (2.10) and (2.13) hold. Hence, $V(p_i, p_{-i})$ is $C^2$ and strictly concave.

Define strict submodularity and aggregativity of a game as follows:

Definition 2.4. A game is strictly submodular if $\partial V(p_i, p_{-i})/\partial p_i \partial p_{j \neq i} < 0$ for every $i$ and for every $j \neq i$.

Definition 2.5. A game is aggregative if there exists a continuous and additively separable function $g : [0, 1]^{N-1} \rightarrow [0, 1]$ (the aggregator) and functions $\tilde{V} : [0, 1]^2 \rightarrow \mathbb{R}$ (the reduced payoff functions) such that for each player $i$:32

$$V(p_i, p_{-i}) = \tilde{V}(p_i, g(p_{-i})), \quad \forall p \in [0, 1]^N$$

From these definitions, and from Assumption 2.4, it follows that:

Lemma 2.7. The disclosure game is aggregative and strictly submodular.

Proof. See the Appendix.

The aforementioned properties guarantee both the existence of a PSNE, and the presence of monotone comparative statics with respect to the correlation parameter $q$.

Proposition 2.2. The set of Pure Strategy Nash Equilibrium (PSNE) is non-empty, and each firm $i = 1, \ldots, N$ chooses a disclosure policy $p_i^*$ such that:

1. If $V_1(K/E_f[\tilde{x}], p_{-i}^*) > 0$, $p_i^* = K/E_f[\tilde{x}];$

32Both definitions are the analog of those in Acemoglu and Jensen [2013], for instance, for the case of one-dimensional strategy sets.
2. Otherwise \( V_1(p_i^*, p_{-i}^*) = 0 \) and \( p_i^* \in [0, K/E_f(x)] \).\(^{33}\)

Moreover, the smallest and the largest equilibria, denoted by \( Q_*(q) \) and \( Q^*(q) \) respectively, are such that: \( Q_* : [0, 1]^{N(N-1)} \rightarrow \mathbb{R} \) is lower-semicontinuous and \( Q^* : [0, 1]^{N(N-1)} \rightarrow \mathbb{R} \) is upper semi-continuous.

**Proof.** See the Appendix. \( \square \)

Existence of a PSNE follows from three properties of the game: (i) the convexity and compactness of the strategy set \([0, 1]\), for all \( i \); (ii) the continuity of \( V(p_i, p_{-i}) \) in all arguments; and (iii) the quasi-concavity of \( V(p_i, p_{-i}) \) in \( p_i \). Aggregativity and submodularity also imply that monotone comparative statics with respect to the correlation vector \( q_{-i} \) can be derived.\(^{34}\)

**Corollary 2.4.** Cæteris paribus, the equilibrium disclosure \( p_i^* \) decreases with \( q_{i,j} \), for every \( i, j \). The equilibrium leverage might decrease or increase with \( q_{i,j} \).

**Proof.** See the Appendix. \( \square \)

Summing up, the equilibrium disclosure policies are a function of the correlation vector \( q \), and the higher the correlation the lower the disclosure of each firm, because the larger the gains from free riding on the information produced by competitors. It remains to consider the efficiency properties of the private disclosure and leverage policies, which is the subject of the next section.

### 2.4 Socially Optimal Leverage and Disclosure

The set of Socially Efficient (SE) disclosure policies can be simply defined as the set of disclosure vectors of length \( N \) that maximise the aggregate surplus:

\[
SE \equiv \left\{ p^e \in [0, 1]^N \left| p^e \in \arg \max_{p \in [0, 1]^N} \sum_{i=1}^{N} V(p_i, p_{-i}) \right. \right\}
\]

The set \( SE \) is non-empty, and can be characterised as follows:

**Proposition 2.3.** There exists a non-empty set of Socially Efficient (SE) disclosure policy vectors such that \( p^e > p^* \), where \( p^* \) belong to the largest Nash equilibrium \( Q^*(q) \). In addition, \( p^e >> p^* \) whenever \( q_{-i} > 0 \).

**Proof.** See the Appendix. \( \square \)

\(^{33}\)As standard, \( V_1 \) denotes the derivative of \( V \) with respect to the first argument.

\(^{34}\)See Acemoglu and Jensen [2013] for general results, of which my results are a special case.
Proposition 2.3 shows that the presence of disclosure spillovers across firms leads to an inefficiently low private provision of information, and consequently inefficiently high leverage ratios. A social planner could increase the aggregate welfare by promoting higher disclosure and lower borrowing. How could the result be achieved?

A first policy would focus on mandatory disclosures, and mandate that firms disclose according to the vector $p^e$. However, there may be limits in the efficacy of mandatory requirements, especially when dealing with firms that are naturally opaque (such as banks or insurance companies).

In fact, opaque sectors such as the financial industry are regulated according to different principles. In particular, they tend to be subject to mandatory capital requirements – that is, a certain amount of their assets must be backed by equity claims. At present, Basel III confirms capital requirements in the range of 4% of the risk weighted assets for banks.\textsuperscript{35} This paper shows that mandatory capital requirements may well be welfare increasing, and can be an alternative to disclosure regulation in those instances where reaching effective disclosures may prove daunting. The result is summarised in the following proposition.

**Proposition 2.4.** When $q_{-i} > 0$ for some $i$, the SE can be implemented as a PSNE either mandating a certain amount of disclosure $p^e_i$, or mandating capital requirements of size $l^e_i$. Transfers across firms guarantee that all firms who were investing in the absence of regulation continue to invest.

**Proof.** The case for mandatory disclosure is straightforward: simply solve for the SE, and set $p^e_i$ equal to the disclosure at the unique SE.

If disclosure cannot be mandated effectively, consider the leverage at the SE: it would be $\alpha^e_i = p^e_i$ by Proposition 2.1. Then, compute the corresponding $d(p^e_i)$, and set:

$$l^e_i = \frac{d(p^e_i)}{d(p^e_i)(1 - p^e_i) + p^e_i E_f[\tilde{x}]}$$

Note that: $l^e_i = 0$ if $d(p^e_i) = 0$, and $l^e_i = 1$ if $d(p^e_i) = \bar{d}$ (which implies that $\alpha^e_i = 0$).

Although very interconnected firms may find it too costly to invest under capital standards, transfers across firms exist such that they all continue to invest. Existence of such transfers is guaranteed by the definition of SE, and in particular the fact that any SE maximises the aggregate market surplus.

An important remark on the implementation of socially efficient outcomes concerns the assumption that the regulator knows the degree of connectedness of

\textsuperscript{35} However, many policy makers and academics called for substantially higher requirements. For instance, Calomiris called for 10%, Admati and Hellwig 20-30%, and Kotlikoff 100%.
individual firms. Although we implicitly assumed that the market knows such information, and price it correctly, it could be that a regulator does not know it. In such a scenario, it cannot rely on firms disclosing truthfully their systemic risk: all firms have strong incentives to underreport so they can avoid the regulatory requirements. Similar problems arise in most models of disclosure under externalities, such as Admati and Pfleiderer [2000].

Though this limitation is likely to be relevant in practice, observe that current US regulation is implicitly following the approach sketched here, when it imposes additional capital requirement to the too-big-too-fail institutions. Effectively, the regulator is using a measure of the size of firms to capture their potential interconnectedness, and requires better capitalisation precisely when the model I presented suggests it to be necessary. Better measures are currently studied by academics and policy makers.

I conclude the section by returning to our example, and solving for the privately and socially optimal disclosure policies.

Example (cont’d). Recall from the previous analysis that:

\[
\bar{d}^* = \begin{cases} 
9 - \frac{\sqrt{-19\pi_i^2 + 18\pi_i + 1}}{1 - \pi_i} & \text{if } \pi \leq 4/5 \\
0 & \text{otherwise}
\end{cases}
\]

The function is continuous, and inequality (2.10) holds because: (i) \( \bar{d} = 8 \); and (ii) the hazard rate reads: \( 1/(10 - x) \), which is strictly less than \( 1/\mu = 1 \) for every \( x \in [0, 9] \). Moreover, \( f' = 0 \) implies that (2.13) holds.

Suppose that \( N = 2 \), \( \pi_i = p_i + q(1 - p_i)p_{j\neq i} \) for both firms, and \( c = |1 - 0.8(0.8 - p_i)^{-1}|/100 \), and focus without loss of generality on symmetric equilibria. Program (2.5) can be written as:

\[
\max_{p_i \in [0,0.8]} V(p_i, p_{-i}) = -\frac{1 - \pi_i}{10} \left[ 9 - \frac{\sqrt{-19\pi_i^2 + 18\pi_i + 1}}{1 - \pi_i} \right] - \frac{|1 - 0.8(0.8 - p_i)^{-1}|}{100} + 1
\]

(2.14)

It is easy to check that \( \partial^2 V(p_i, p_{-i})/\partial p_i^2 < 0 \) and \( \partial^2 V(p_i, p_{-i})/\partial p_i \partial p_{-i} < 0 \). As a result, there exists a unique interior maximum, fully characterised by the first order
condition: \( \partial V(p^*_i, p_{-i})/\partial p_i = 0. \)

In contrast, exploiting symmetry, the socially optimal disclosure can be derived as the solution of a planner’s problem, who maximises aggregate welfare with \( p_i = p_{-i} = p \):

\[
\max_{p \in [0, 0.8]} W(p) = \frac{(1 - p)(1 - qp)}{5} \left[ 9 - \sqrt{-19(p + q(1 - p)p)^2 + 18(p + q(1 - p)p + 1)} \right]
- \frac{1 - 0.8(0.8 - p_i)^{-1}}{50} + 2
\]

Again, it is easy to check that the planner’s objective function is strictly concave in \( p \). Hence, the socially optimal disclosure level satisfies: \( \partial W(p^*)/\partial p = 0 \).

The SNE and the planner’s solution are plotted in Figure 2.4. As expected, in the absence of externalities (i.e., when \( q = 0 \)) the private and social optimum coincide. However, for every \( q > 0 \) the SNE displays an inefficiently low level of disclosure, relative to the social optimum. Moreover, the divergence between private and social optimum increases with the externality parameter \( q \).\(^{37}\) From Proposition 2.1, it follows that leverage is inefficiently high whenever \( q > 0 \), and the inefficiency is increasing in \( q \).

\(^{37}\)This is an instance of the monotone comparative static derived in Acemoglu and Jensen [2013] for more general (though still aggregative) submodular games.
2.5 Empirical analysis

In order to check whether the predictions of the model appear consistent with the empirical evidence, I first build a firm-level panel of the universe of US public firms, then I construct various measures of transparency and leverage (as well as other standard controls), and finally I run a series of regressions.

To construct my data, I combine two sources: (i) the CRSP/COMPUSTAT merged dataset; and (ii) the IBES analysts’ forecast dataset. To do so, I follow the path described below.

I first collect the raw CRSP/Compustat merged dataset, which contains balance sheet information about the universe of US public corporations, as well as the prices of their securities for the period 1979-2014. From the original file, I drop the observations that satisfy at least one of the following requirements: (i) total assets (AT) are missing or negative; (ii) the firm is not US based (i.e. FIC ≠ USA); (iii) total liabilities (LT) are missing or negative; (iv) total liabilities exceed total assets (LT > AT); (v) either the equity price (PRCC) or the market capitalisation (CSHO) are missing.

Then I collect the detail IBES dataset (adjusted for stock splits), which contains individual analysts’ forecasts for US corporates EPS (Earnings per share). For any given firm-year pair, I generate the following summary statistics: (i) NUMEST – the number of analysts’ estimates of expected EPS; and (ii) CV – the coefficient of variation of analysts’ forecasts (i.e. their standard deviation normalised by the mean). I drop those firm-date pairs for which there are less than five analysts’ forecast, and I collapse the data at the firm-year level.\(^38\)

The procedure ends with 32,361 matched firm-year pairs such that (i) both Compustat and IBES data is successfully merged, and (ii) more than five forecasts are available.

The descriptive statistics for the variables of interest are reported in Table 2.1. The definition I adopt of leverage includes both financial and non-financial liabilities (as suggested in Welch [2011]), and it is easily computed as the ratio of total liabilities (LT) over total assets (AT).\(^39\) The Book-to-Market ratio is computed as

\(^38\)In the empirical Appendix, I conduct robustness exercises where I let the cutoff run from 1 to 4, and show the results are unchanged. Moreover, I consider alternatives to CV such as the median absolute deviation from the mean (both normalised and not). Again, the results do not change.

\(^39\)Other definitions I consider in the Appendix are: (i) LT/AM, where AM stands for market value of assets; (ii) DT/AT, where DT = DLC + DLTT refers to the aggregate financial liabilities (debt); and finally (iii) DT/AM. Overall, the qualitative results are not very sensitive to the leverage measure chosen, although they are more statistically significant when book values are considered rather than market values.
Table 2.1: Summary statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min.</th>
<th>Max.</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>LT/AT</td>
<td>0.56</td>
<td>0.24</td>
<td>0</td>
<td>1</td>
<td>32361</td>
</tr>
<tr>
<td>CV forecasts</td>
<td>0.07</td>
<td>0.15</td>
<td>0</td>
<td>7.92</td>
<td>32361</td>
</tr>
<tr>
<td>Estimates</td>
<td>13.12</td>
<td>8.19</td>
<td>5</td>
<td>59</td>
<td>32361</td>
</tr>
<tr>
<td>Total Assets</td>
<td>7.29</td>
<td>1.87</td>
<td>-0.03</td>
<td>14.7</td>
<td>32361</td>
</tr>
<tr>
<td>Profitability</td>
<td>0.01</td>
<td>0.18</td>
<td>-5.88</td>
<td>4.1</td>
<td>32361</td>
</tr>
<tr>
<td>Book-to-Market</td>
<td>0.59</td>
<td>0.56</td>
<td>0</td>
<td>21.26</td>
<td>32361</td>
</tr>
<tr>
<td>Intangibles</td>
<td>0.13</td>
<td>0.18</td>
<td>0</td>
<td>0.92</td>
<td>28949</td>
</tr>
<tr>
<td>Industry Leverage</td>
<td>0.57</td>
<td>0.18</td>
<td>0.17</td>
<td>0.94</td>
<td>32361</td>
</tr>
</tbody>
</table>

the book value of a share (PRCCF) multiplied times the total number of outstanding shares (CSHO), and then divided by the market value of equity (MEQ). Intangibles consists in the fraction of intangible assets, defined as INTAN/AT. Finally, Total Assets are reported as the natural logarithm of AT, hence the negative minimum numbers which obtain for AT ∈ (0, 1).

I now proceed to the regression analysis. I follow the procedure of gradually introducing independent variables, to check how the sensitivity and significance of the coefficients of interest evolve. The general linear regression that I estimate takes the following form (where \( i \) indexes firms and \( t \) years):

\[
\text{Leverage}_{i,t} = \alpha + \beta X_{i,t-1} + \gamma_i + \gamma_t + \epsilon_{i,t}
\]

where the matrix \( X_{i,t} \) includes various covariates of a firm-date pair, among which the main regressor of interest (i.e. CV – the coefficient of variation of analysts’ forecasts).

The regression results are reported in Table 2.2. I first regress leverage on CV, a constant and the time dummies (column (1)). Then, in column (2) I add the set of controls that the previous papers (e.g. Frank and Goyal [2009]) identified as reliable predictors of the leverage of a firm. In column (3) I regress leverage on CV, a constant, the time dummies and firms fixed effects. Column (4) adds the controls to the fixed-effect regression of column (3). Next, I present two robustness checks: in column (5) I restrict attention to non-financial firms; in column (6) I increase the cutoff on the number of forecasts to ten. In both cases, the coefficient of interest remains significant, and it even marginally increases in magnitude relative to that of column (4).\(^{40}\)

Importantly, the sign of most other controls is consistent with previous stud-

\(^{40}\)In the Appendix, I run additional robustness exercises and show that the results are qualitatively similar throughout various specifications.
### Table 2.2: Regression table

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LT/AT</td>
<td>LT/AT</td>
<td>LT/AT</td>
<td>LT/AT</td>
<td>LT/AT</td>
<td>LT/AT</td>
</tr>
<tr>
<td>L.CV forecasts</td>
<td>0.0983</td>
<td>0.0427</td>
<td>0.0432</td>
<td>0.0217</td>
<td>0.0342</td>
<td>0.0354</td>
</tr>
<tr>
<td></td>
<td>(5.11)</td>
<td>(2.90)</td>
<td>(4.18)</td>
<td>(2.53)</td>
<td>(2.99)</td>
<td>(2.72)</td>
</tr>
<tr>
<td>L.Total Assets</td>
<td>0.0589</td>
<td>0.0093</td>
<td>0.0059</td>
<td>0.0053</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(31.28)</td>
<td>(1.73)</td>
<td>(1.06)</td>
<td></td>
<td>(0.82)</td>
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<td>L.Profitability</td>
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<td>-0.160</td>
<td>-0.216</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-6.28)</td>
<td>(-7.96)</td>
<td>(-7.33)</td>
<td></td>
<td>(-7.79)</td>
<td></td>
</tr>
<tr>
<td>L.Book-to-Market</td>
<td>-0.000492</td>
<td>0.00132</td>
<td>-0.00218</td>
<td>-0.0143</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-0.10)</td>
<td>(0.33)</td>
<td>(-0.50)</td>
<td></td>
<td>(-2.75)</td>
<td></td>
</tr>
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<td>L.Intangibles</td>
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<td>0.0303</td>
<td>0.00306</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-0.05)</td>
<td>(1.06)</td>
<td>(1.19)</td>
<td>(0.10)</td>
<td></td>
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</tr>
<tr>
<td>L.Industry Leverage</td>
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<td>0.146</td>
<td>0.135</td>
<td>0.123</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(22.09)</td>
<td>(2.67)</td>
<td>(2.27)</td>
<td>(1.77)</td>
<td></td>
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</tr>
<tr>
<td>Constant</td>
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<td>-0.111</td>
<td>0.592</td>
<td>0.424</td>
<td>0.420</td>
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<tr>
<td></td>
<td>(49.22)</td>
<td>(-8.19)</td>
<td>(133.26)</td>
<td>(7.55)</td>
<td>(7.42)</td>
<td>(6.49)</td>
</tr>
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<td>Time FE</td>
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<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Firm FE</td>
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<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<td>Exclude Finance</td>
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<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>10 forecasts or more</td>
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<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
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<td>23499</td>
<td>26337</td>
<td>23499</td>
<td>19121</td>
<td>13395</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.010</td>
<td>0.479</td>
<td>0.847</td>
<td>0.846</td>
<td>0.778</td>
<td>0.856</td>
</tr>
</tbody>
</table>

$t$ statistics in parentheses. $^* p < 0.05$, $^{**} p < 0.01$, $^{***} p < 0.001$

Sources: Compustat merged with CRSP (annual), IBES (detail, adjusted for stock splits).
Notes: all independent variables are lagged by one year. Stnd. errors clustered at the firm level.
ies. Profitability is strongly negatively correlated with leverage. Average industry leverage is strongly positively correlated with leverage. Total assets (i.e., size) is positively correlated with leverage, though the correlation vanishes after the inclusion of firms fixed effects. Both the Book-to-Market ratio and the fraction of intangible assets are not robustly signed. Finally, the inclusion of firms fixed effects explains about 50% of the observed variation in leverage, consistently with other studies such as Lemmon et al. [2008].

Overall, the results support the predictions of the model I propose, although a validation of the my hypotheses with statistical causality is left for future research.

### 2.6 Conclusions

This paper analyses the effect of disclosure on the composition of financing means for firms. In a novel costly-state-verification setting with variable and endogenous degrees of asymmetric information between firms and investors, the paper highlights that disclosure and leverage should be negative correlated. Higher disclosure leads to the possibility of issuing cheaper incentive-compatible stocks, hence increasing the opportunity of leveraged financing and its bankruptcy costs.

I find this prediction consistent with the empirical evidence for US public firms after the 1980s, to the extent that effective transparency is correlated to the dispersion in analysts’ EPS forecasts. Of course, the dispersion in analysts forecasts appears a noisy proxy of transparency, and one needs to confirm that the results are robust across various alternative measures in future work. Nevertheless, the validity of the correlation derived in the paper hinges on the observing that most factors that influence the dispersion of forecasts, such as herding or contrarianism, and not linked unambiguously to leverage ratios by any existing theory.

The presence of disclosure externalities across firms yields insufficient voluntary disclosure and excessive leverage, relative to the constrained best. Therefore, it brings about the question of regulation. If regulators can effectively mandate truthful disclosures, then social efficiency can be restored. However, the explicit treatment of the interlinkage between disclosure and financing policies suggests an additional tool that regulators should explore when truthful disclosures prove hard to implement: capital requirements. By setting higher capital requirements, regulators promote endogenously enhanced transparency and can restore social efficiency.

The argument for mandatory capital standards I put forward relies on two pillars: (i) firms’ output should be sufficiently correlated (e.g., in the presence of high systemic risk); and (ii) mandatory disclosures are hard to translate into greater
transparency, because they can be dodged to a large extent. Both conditions plausibly apply to financial firms, and indeed they are the subject of regulatory capital requirements.

Moreover, the argument is immune from the most common critique of the existing, alternative, story based on the absorbing of losses in crises (e.g., Admati and Hellwig [2014]). Banking lobbyists commonly counter argue that, although ex-post desirable in crises times, capital requirement are ex-ante detrimental to credit extension and would dampen growth during boom times, because they increase the cost of funding for banks. Indeed, if this was not the case we would observe already much higher equity financing in the financial industry. The model I present is immune from this critique, because capital requirements are efficient ex ante, and solve a coordination failure in information provisions across firms.

As always with regulation, the devil lies in the details. Moreover, important aspects such as agency problems within firms and on the government side have been ignored here, for the sake of simplicity. Any regulatory effort must confront such issues convincingly in order to be credible. All this paper wishes to achieve is to raise awareness that the debates around capital requirements and mandatory disclosures for financial firms should be more closely connected both in the policy and in the academic arenas.
2.7 Appendices

2.7.1 Proofs

Lemma 2.2

Proof. Claim (i). Suppose there exists an optimal contract \( \{s, z, \sigma, p\} \) such that:

\[
\{x \mid z(m, x) < x, \text{ for some } m \neq x\} \neq \emptyset
\]

Consider replacing it with another contract \( \{s', z', \sigma', p'\} \) such that \( \sigma = \sigma', s = s', p = p' \) and:

\[
z'(m, x) = \begin{cases} x & \text{if } m \neq x \\ z(m, x) & \text{otherwise} \end{cases}
\]

Clearly, the new contract is feasible because when \( \sigma = 1 \) the maximum feasible clawback equals \( x \). To see that it is incentive compatible, observe that because \( \{s, z, \sigma, p\} \) is optimal, we know that \( r(x) \leq r(x, x') \) for every pair \( x, x' \). We also know that (i) \( r(x) = r'(x) \) for every \( x \), and (ii) \( r'(x, x') \leq r(x, x') \) for every \( x, x' \) by construction. Hence, \( \{s', z', \sigma', p'\} \) is incentive compatible. The participation constraint remains binding because \( E[r'(x)] = E[r(x)] \), and the deadweight loss due to verification and disclosure do not change. Therefore, the entrepreneur is indifferent between \( \{s, z, \sigma, p\} \) and \( \{s', z', \sigma', p'\} \), proving our claim.

Claim (ii). It mirrors the proof of claim (i): start with an optimal \( \{s, z, \sigma, p\} \) that does not satisfy the property (i.e., \( \sigma(m) = 0 \) for some \( m < y \)). For all such cases, replace \( \sigma \) with \( \sigma' = 1 \). Otherwise, set \( \sigma = \sigma', z = z' \) and \( s = s' \) and \( p = p' \). Because the change occurs only off-equilibrium path, the participation constraint remains unchanged. Furthermore, incentive compatibility and feasibility are trivially satisfied, proving the claim. \( \square \)

Lemma 2.3

Proof. Claim (i). First, we know that when \( \pi = 0 \) the optimal contract is debt, and it is monotonic (Gale and Hellwig [1985]). Therefore, we can restrict attention to \( \pi > 0 \) and consider an optimal contract \( \{s, z, \sigma, p\} \). Suppose that under \( \{s, z, \sigma, p\} \) there exists a set \( A \subset X \) and an \( \hat{x} \) such that \( A \equiv \{x > \hat{x} \mid r(\hat{x}) > r(x)\} \). Evidently, the contract is not monotonic. Without loss of generality, suppose there only exists one such \( \hat{x} \) (if there was more than one, the same reasoning could be iterated).

Consider another contract \( \{s', z', \sigma', p'\} \) such that \( \sigma = \sigma', p = p' \), \( s'(x) \in [s(x), x] \), \( z'(x, x) \in [z(x, x), x] \) and:

\[
r'(x) = \begin{cases} r(x) & \text{if } x \notin A \\ r(\hat{x}) & \text{otherwise} \end{cases}
\]

The new contract is feasible because \( r(\hat{x}) \leq \hat{x} < x \) for every \( x \in A \). To show that it is also incentive compatible, I partition the state according to whether they belong to \( A \) or not.

First, consider \( x \notin A \). By construction (i) \( r'(x) = r(x) \), and (ii) \( r'(x', x) \geq r(x', x) \) for every \( x' \). Hence, because \( \{s, z, \sigma, p\} \) was incentive compatible, incentive compatibility holds also under \( \{s', z', \sigma', p'\} \).

Second, consider \( x \in A \). From the way I constructed \( r' \), I know that \( r'(x') = r(x') \). First,
The new repayment function is monotonic. To see that the prime contract is feasible, notice that:

\[ s = \text{compatible and: (i)} \]

\[ \pi > 0. \text{ Under the new contract, by construction we have: } \sigma'(x') = \sigma(x') ; \ s'(x') = s(x') \text{ and } r'(x) = r(\hat{x}), \text{ so we can write:} \]

\[ \hat{x} \geq \frac{r(\hat{x}) - (1 - \pi)(1 - \sigma(x'))s(x')}{\pi + (1 - \pi)\sigma(x')} \]

The ratio is well defined because \( \pi > 0 \). Under the new contract, by construction we have:

\[ \pi > 0. \text{ Under the new contract, by construction we have: } \sigma'(x') = \sigma(x') ; \ s'(x') = s(x') \text{ and } r'(x) = r(\hat{x}), \text{ so we can write:} \]

\[ \hat{x} \geq \frac{r(\hat{x}) - (1 - \pi)(1 - \sigma(x'))s(x')}{\pi + (1 - \pi)\sigma(x')} \]

Observe that:

\[ x \geq \frac{r'(x) - (1 - \pi)(1 - \sigma'(x'))s'(x')}{\pi + (1 - \pi)\sigma'(x')} = \frac{r'(\hat{x}) - (1 - \pi)(1 - \sigma'(x'))s'(x')}{\pi + (1 - \pi)\sigma'(x')} \]

where the last equality holds by construction of the new contract \( \{ s', z', \sigma', p' \} \) – i.e., the fact that, for every \( x \in A \), \( r'(x) = r(\hat{x}) \). Since \( x > \hat{x} \) the prime contract is incentive compatible as well.

Now consider the participation constraint. Regardless of the measure of the set \( A \), at the prime contract the investors make strictly positive profits. Define a third contract \( \{ s'', z'', \sigma'', p'' \} \) such that \( p'' = p' = p, \sigma'' = \sigma' = \sigma, z'' = \alpha z' \) and \( s'' = \alpha s' \) for some \( \alpha \in [0,1] \) such that:

\[ \mathbb{E}_f[\sigma''(x) - (1 - \pi)\sigma''(x)\mu] = \mathbb{E}_f[\alpha\sigma'(x) - (1 - \pi)\sigma''(x)\mu] = K \]

We know that such an \( \alpha \) exists because: (i) when \( \alpha = 1 \) we have \( \mathbb{E}_f[\sigma''(x) - (1 - \pi)\sigma''(x)\mu] \geq K \); (ii) when \( \alpha = 0 \) we have \( \mathbb{E}_f[-(1 - \pi)\sigma''(x)\mu] < 0 \); and (iii) the left hand side of the equation is continuous in \( \alpha \). The new (double-prime) contract is feasible because \( \alpha \in (0,1) \), and it is trivially incentive compatible.

Because the deadweight loss does not change and the investors make zero profits, the firm must be indifferent between \( \{ s, z, \sigma, p \} \) and \( \{ s', z', \sigma', p' \} \), proving the claim.

**Claim (ii)** Consider an optimal contract \( \{ s, z, \sigma, p \} \) that satisfies Claim (i). Suppose there exists an interval \( A \subset X \), such that \( s(x) < s(\hat{x}) \) for every \( x \in A \) and some \( \hat{x} < \inf\{ x \mid x \in A \} \). The repayment function is not monotonic. Introduce another contract \( \{ s', z', \sigma', p' \} \) such that: \( p = p' \), \( \sigma = \sigma' \), \( r = r' \) but:

\[ s'(m) = \begin{cases} s(m) & \text{if } m \notin A \\ s(\hat{x}) & \text{otherwise} \end{cases} \]

Of course, for all \( x \in A \) the fact that \( r = r' \) and the shape of \( s' \) imply that:

\[ z'(x,x) = z(x,x) - \frac{(1 - \pi)}{\pi} [s(\hat{x}) - s(x)] < z(x,x) \]

The new repayment function is monotonic. To see that the prime contract is feasible, notice that:

(i) the original contract was feasible; (ii) \( s(\hat{x}) < z(\hat{x}) \) and (iii) by the monotonicity of \( r \) we have:

\[ r(x) \geq r(\hat{x}) \geq (1 - \pi)s(\hat{x}) \Rightarrow z'(x,x) = z(x,x) - \frac{(1 - \pi)}{\pi} [s(\hat{x}) - s(x)] \geq 0, \forall x \in A \]

To show it is also incentive compatible, partition the incentive constraints in the following categories:

First, consider \( x \notin A \). All incentive constraints hold because \( \{ s, z, \sigma, p \} \) was incentive compatible and:

(i) \( s(x) = s'(x) \) for all \( x' \notin A \); (ii) \( s(x) < s(\hat{x}) = s'(x') \) for all \( x' \in A \).

Second, consider \( x \in A \) if message \( x' \neq x \) is such that \( \sigma'(x') = 1 \) incentive compatibility trivially holds. Moreover, if \( x \) is such that \( \sigma'(x) = 1 \) incentive compatibility holds because \( s'(x) \) is
irrelevant (i.e., \( r(x) = z(x, x) \)). Finally, if \( x, x' \) are such that \( \sigma'(x) = 0 = \sigma'(x) \), we have:

\[
\pi z'(x, x) + (1 - \pi) s'(x) \leq \pi x + (1 - \pi) s'(x')
\]

If \( x' \in A \), then \( s'(x) = s'(x) = s(x) \) by construction and since \( z'(x, x) \leq x \) by limited liability incentive compatibility holds. If \( x' \not\in A \) and \( x' > x \), incentive compatibility follows from \( s'(x') = s(x') \geq s(x) = s(x) \), by definition of the set \( A \).

Finally, if \( x' \not\in A \) and \( x' < x \), incentive compatibility follows from \( r = r' \) and \( s'(x') = s(x') \). So, the prime contract is incentive compatible.

In conclusion, observe that: (i) because \( \sigma = \sigma' \) the deadweight verification cost does not change; and (ii) because \( r = r' \) the investors revenues do not change. As a result, the two contracts are equivalent from the firm’s perspective and because \( \{s, z, \sigma, p\} \) is optimal, so is \( \{s', z', \sigma', p'\} \).

**Proposition 2.1**

**Proof.** **Case 1:** \( \mathbb{E}_f[\pi \bar{x}] \geq K \). The contract with minimum possible verification on-the-equilibrium path is such that \( \sigma(m) = 0 \) for every \( m \). Because of Lemmas 2.2-2.3, when \( \sigma(m) = 0 \) for every \( m \) there is at most one binding incentive constraint for each type \( x \in X, IC(0, x) : x \sim r(x) \geq (1 - \pi)x, \) or equivalently: \( r(x) \leq \pi x - \delta \) where I substituted \( s(0) = 0 \) by limited liability. In addition, evidently one can set \( s(x) = r(x) \) for every \( x \). If \( \sigma(m) = 0 \) for every \( m \) and incentive compatibility holds, the fraction of equity that needs to be sold is \( \alpha = K/\mathbb{E}_f[\pi \bar{x}] \), and because \( \alpha \leq \pi \) equity is optimal.\(^{41}\)

Debt is suboptimal because the incentive constraint for a type \( x \leq d \) reads \( x \leq \pi x \), which is never satisfied because \( \pi < 1 \). Moreover, \( d > K \) because investment is risky, and hence the set of \( x < d \) is nonempty.

**Case 2:** \( \mathbb{E}_f[\pi \bar{x}] < K \). The proof proceeds in three steps:

**Step 1:** Any optimal contract is such that \( x_V < x_{N^V} \).

**Proof.** Divide \( X \) into intervals \( X_1, X_2, ..., X_n \) such that (i) \( \min X_1 = 0, \max X_n = \bar{x}, \cup_{i=1}^n X_i = X \), and (ii) for every \( i \) and for every pair \( x, x', x' \in X_i^2, \sigma(x) = \sigma(x') \). By contradiction, suppose that at the optimal contract \( \{s, z, \} \) we have \( x_V > x_{N^V} \). Without loss of generality, suppose that \( X_1 \subseteq N^V \), so that (i) \( X_2 \not= \emptyset \) and \( X_2 \subseteq V \), (ii) \( X_3 \not= \emptyset \) and \( X_3 \subseteq N^V \), and so on. For \( x \in X_3 \), incentive compatibility of \( \{s, z, \sigma, p\} \) requires that (i) for every \( x' \in X_1 \) we have \( r(x) \leq \pi x + (1 - \pi) s(x') \); and (ii) for every \( x'' \in X_2 \) we have \( r(x) \leq x \).

Consider another contract \( \{s', z', \sigma'\} \) such that \( s' = s, z' = z, p = p' \) and:

\[
\sigma'(m, 0) = \begin{cases} 
\sigma(m) & \text{if } m \not\in X_2 \\
0 & \text{otherwise}
\end{cases}
\]

By Lemma 2.1 the new contract is feasible, because \( \max \{m'(x), y\} = x \) for every \( x \). Now I prove it is incentive compatible.

If \( x \in X_2 \), incentive compatibility of \( \{s, z, \sigma, p\} \), \( s = s' \) and \( z = z' \) jointly imply that \( IC(x, x') \) is satisfied at the prime contract for every \( x' \in X \). If \( x \in X_1 \), incentive compatibility follows from the monotonicity of \( s(m) \) – by Lemma 2.3. If \( x \in X_3 \) we have two cases: (i) if \( x' \in X_1 \) or \( x' \in X_2 \) and \( i \geq 3 \) then we have \( r'(x) \leq r'(x, x') \) because \( r = r' \) and \( \sigma'(x) = \sigma(x') \); (ii) if instead \( x' \in X_3 \) incentive compatibility reads: \( \pi z'(x, x) + (1 - \pi)s'(x) \leq \pi x + (1 - \pi) s'(x') \). Because \( x' > x'' \) for every \( x'' \in X_1 \), and since \( X_1 \subseteq V \), we also have: \( \pi x + (1 - \pi) s'(x') \geq \)

\(^{41}\)In the limit, when \( \mathbb{E}_f[\pi \bar{x}] = K \), pure equity is the uniquely optimal contract.
\[\pi x + (1 - \pi)s(x') = \pi x + (1 - \pi)s'(x''),\]\nwhere the inequality follows from incentive compatibility of \(\{s, z, \sigma, p\}\). Similar arguments can be used for \(x \in X_i\) and \(i > 3\), proving the claim. \qed

\textbf{Step 2: For every} \(x \geq x_{NV}\), \(z^*(x, x) = s^*(x) = (1 - \pi)x_{NV} + \pi x\).

\textbf{Proof.} First I show that \(s(x_{NV}) = x_{NV}\). Suppose not, i.e. there exists an optimal contract \(\{s, z, \sigma, p\}\) such that \(x_{NV} > r(x_{NV})\) (the case of the opposite inequality is prevented by limited liability). Define the set \(B \equiv \{x \in NV \mid r(x) < x_{NV}\}\). Design a new contract \(\{s', z', \sigma', p'\}\) such that \(z = z', \sigma = \sigma', p = p'\) and:

\[s'(m) = \begin{cases} s(m) & \text{if } x \notin A \\ x_{NV} & \text{otherwise} \end{cases}\]

Clearly, the prime contract is feasible. It is also incentive compatible because \(\{s, z, \sigma, p\}\) is incentive compatible. It remains to show that from the optimality of \(\{s, z, \sigma, p\}\) it follows that \(B\) is of zero measure, hence PC remains binding. By contradiction, suppose not. Define the following threshold:

\[\hat{x} \equiv \left\{ x \in X \mid \int_0^{\hat{x}} [x - (1 - \pi)\mu]dF(x) + \int_{\hat{x}}^x \min\{s'(x), x\}dF(x) = K \right\}\]

We know that \(\hat{x}\) exists and \(0 < \hat{x} < x_{NV}\) because if \(\hat{x} = 1\) we have:

\[\int_0^{\hat{x}} [x - (1 - \pi)\mu]dF(x) + \int_{\hat{x}}^x \min\{s'(x), x\}dF(x) = \int_0^{x_{NV}} [x - (1 - \pi)\mu]dF(x) + \int_{x_{NV}}^{\hat{x}} s'(x)dF(x) > K\]

if, instead, \(\hat{x} = 0\) we have \(\int_0^0 \min\{s'(x), x\}dF(x) < K\), where the inequality follows from the fact that \(\mathbb{E}[\pi\hat{x}] < K\). Observe that a contract \(\{s'', z'', \sigma'', p''\}\) such that \(z'' = z' = z, p'' = p' = p, s'' = \min\{s', x\}\) and:

\[\sigma''(m) = \begin{cases} \sigma(m) & \text{if } m \notin [\hat{x}, x_{NV}] \\ 0 & \text{otherwise} \end{cases}\]

would be both feasible and incentive compatible. Moreover, it would make the participation constraint for the investors binding, strictly reducing the expected verification costs relative to \(\{s, z, \sigma, p\}\). As a result, \(\{s, z, \sigma, p\}\) cannot be optimal, proving our claim.

That \(s(x) = (1 - \pi)x_{NV} + \pi x\) follows from three observations. First, incentive compatibility for \(x, x' \in NV^2\) reads:

\[s(x) \leq \pi x + (1 - \pi)s(x')\]

Second, because \(r(x_{NV}) = x_{NV}\) and by Lemma 2.3 (i.e., monotonicity of \(s(\cdot)\)) we have: \(\min\{s(m)\mid m \in NV\} = x_{NV}\). Third, incentive compatibility must be binding almost surely for every \(x \in NV\) (that is, up to sets of zero measure). To see the latter observation must hold, simply observe that if there is a set of strictly positive measure where incentive compatibility does not hold at any candidate optimal contract, one can repeat the argument given for the previous claim (i.e., \(r(x_{NV}) = x_{NV}\)) and show that the candidate contract cannot be optimal. \qed

\textbf{Step 3: For every} \(x\) \textbf{such that} \(\sigma(x) = 1\), \textbf{we have} \(z^*(x, x) = s^*(x) = x\).

\textbf{Proof.} The proof is identical to that of Step 2. It consists in showing that if a contract is such that \(z^*(x, x) < x\) for a set of states of strictly positive measure, such contract cannot be optimal

\footnote{Strictly because we supposed that \(B\) had a strictly positive measure.}
because the deadweight verification costs can be reduced moving to \( z^*(x, x) = x \) for every \( x \in V \) with another feasible, incentive compatible contract that makes PC binding.

Summing up, steps 1-3 imply that the optimal contract is a mixture of debt and equity with \( \alpha^* = \pi \) and \( d^* = \min\{x_{NV} | \text{PC binds} \} \).

**Lemma 2.4**

*Proof.* First notice that the repayment to investors when \( x^* = 0 \) is equal to \( E_f[\pi \tilde{x}] \), and it must be strictly less than \( K \) when \( x^* > 0 \) by Proposition 2.1. Suppose that – by contradiction – the derivative at \( x^* \) of the objective function in (2.7) is strictly negative, i.e.: \( (1 - F(x^*)) < f(x^*)\mu \). Because the function is continuous, and it starts at a positive value below strictly below \( K \), then whenever the derivative is negative it must be that there exists an \( x' < x^* \) such that the repayment to investors equals \( K \). But this contradicts the definition of \( x^* \), proving our claim. \( \square \)

**Corollary 2.3**

*Proof.* Consider profitability first. We have two cases: \( d = 0 \) and \( d > 0 \). If \( d = 0 \), it means that \( K/E_f[\tilde{x}] = \alpha \leq \pi \). If \( K' < K \) I have \( K'/E_f[\tilde{x}] = \alpha' < K/E_f[\tilde{x}] = \alpha \leq \pi \) and \( d' = d = 0 \). Now consider the case of \( d > 0 \). At any optimal contract that sustains investment where \( d > 0 \), (2.2) holds with equality at \( x^* = d \). We can rewrite (2.2) at the optimum as:

\[
[E_f[\tilde{x}] - K] - (1 - \pi)\mu F(x^*) - \int_{x^*}^{\pi} (1 - \pi) xdF(x) + (1 - F(x^*)) (1 - \pi)x^* = 0
\]

Suppose that \( K \) increases for a given \( E_f[\tilde{x}] \). By Lemma 2.4 I know that \( (1 - F(x^*)) \geq f(x^*)\mu \). If the inequality is strict, totally differentiating the expression with respect to \( K \) and \( x^* \) I get:

\[
-dK + dx^*(1 - \pi)(1 - F(x^*) - f(x^*)\mu) = 0
\]

and \( dx^*/dK > 0 \) implies that either \( d \) increases as profitability falls, or at the new \( K \) there is no investment. If, instead, \( (1 - F(x^*)) = f(x^*)\mu \), then \( d \) must jump to the right and again either there exists a higher \( d \) that satisfies PC, or there is no investment.

As for transparency, suppose it decreases to \( \pi' < \pi \). If \( \pi' \geq K/E_f[\tilde{x}] \), then \( d' = d = 0 \). If \( \pi' < K/E_f[\tilde{x}] \leq \pi \), then either at \( \pi' \) there is no investment or it must be that \( d' > d \). Finally, if \( \pi' < \pi < K/E_f[\tilde{x}] \), I must have that again either at \( \pi' \) there is no investment or \( d' > d \) because the derivative of (2.2) with respect to \( \pi \) is equal to \( \mu F(x^*) + \int_{x^*}^{\pi} (x - x^*) f(x) dx > 0 \).

Finally, that \( x^* \) increases with \( \mu \) is immediate from inspection. \( \square \)

**Lemma 2.5**

*Proof.* First, recall that by Lemma 2.3 the equilibrium face value of debt is monotonically decreasing with \( p_i \). Therefore, we must have \( d^* \leq \tilde{d} \).

Second, observe that the derivative of (2.8) (conditional on \( E_f[\pi, \tilde{x}] \leq K \)) with respect to \( x_{NV} \) is given by \( (1 - \pi_i)[(1 - F(x_{NV})) - \mu f(x_{NV})] \), and it is strictly positive when (i) \( h(x) < 1/\mu \) for every \( x \leq \tilde{d} \) and (ii) \( \pi_i \leq K/E_f[\tilde{x}] \)

As a result, the change in \( d^* \) as \( p_i \) increases infinitesimally can be computed simply total differentiating (2.8) with respect to \( x_{NV} \) and \( p_i \), and evaluating at \( x_{NV} = d \). \( \square \)
Lemma 2.6

Proof. The second derivative of $V(p_i, p_{-i})$ with respect to $p_i$ reads:

$$
\frac{\partial^2 V(p_i, p_{-i})}{\partial p_i^2} = \mu \frac{\partial^2 \pi_i}{\partial p_i^2} \left[ F(d_i^*) + f(d_i^*) \cdot \frac{\mu F(d_i^*) + \int_{d_i^*}^{x} [x - d_i^*]dF(x)}{1 - F(d_i^*) - \mu f(d_i^*)} \right] + 
\mu \left( \frac{\partial \pi_i}{\partial p_i} \right)^2 \frac{\partial F}{\partial \pi_i} \left[ f(d_i^*) \cdot \frac{\mu F(d_i^*) + \int_{d_i^*}^{x} [x - d_i^*]dF(x)}{1 - F(d_i^*) - \mu f(d_i^*)} \frac{\partial f(d_i^*)}{\partial d_i^*} \right] + 
\mu F(d_i^*) + \int_{d_i^*}^{x} [x - d_i^*]dF(x) \frac{\left( \frac{\partial f(d_i^*)}{\partial d_i^*} \right)^2}{1 - F(d_i^*) - \mu f(d_i^*)} \left\{ \frac{\partial f(d_i^*)}{\partial d_i^*} \frac{\mu f(d_i^*)}{\partial d_i^*} \right\} \right] > 0 \text{ because } \frac{\partial^2 \pi_i}{\partial p_i^2} = 0
$$

(2.17)

Though the expression looks frightening, observe that we can sign all terms but those that involve the derivative of the density function $f(x)$. Moreover, all terms are negative, suggesting that the problem has a certain degree of concavity built in from the zero profit condition for investors.

Strict concavity requires $\frac{\partial^2 V(p_i, p_{-i})}{\partial p_i^2} < 0$. From (2.17):

$$
f'(x) > -\frac{f(x)}{1 - F(x) - \mu f(x)} \quad \forall x \in [0, \bar{d}] \Rightarrow \frac{\partial^2 V(p_i, p_{-i})}{\partial p_i^2} < 0
$$

dividing through the fraction in the right hand side by $1 - F(x) > 0$ and applying the definition of $h(x)$ yields the result.

Lemma 2.7

Proof. Strict Concavity: The second cross derivative of $V(p_i, p_{-i})$ with respect to $p_j, j \neq i$, for every such $j$, reads:

$$
\frac{\partial^2 V(p_i, p_{-i})}{\partial p_i \partial p_j} = \mu \frac{\partial^2 \pi_i}{\partial p_i \partial p_j} \left[ F(d_i^*) + f(d_i^*) \cdot \frac{\mu F(d_i^*) + \int_{d_i^*}^{x} [x - d_i^*]dF(x)}{1 - F(d_i^*) - \mu f(d_i^*)} \right] + 
\mu \left( \frac{\partial \pi_i}{\partial p_i} \right)^2 \frac{\partial F}{\partial \pi_i} \left[ f(d_i^*) \cdot \frac{\mu F(d_i^*) + \int_{d_i^*}^{x} [x - d_i^*]dF(x)}{1 - F(d_i^*) - \mu f(d_i^*)} \frac{\partial f(d_i^*)}{\partial d_i^*} \right] + 
\mu F(d_i^*) + \int_{d_i^*}^{x} [x - d_i^*]dF(x) \frac{\left( \frac{\partial f(d_i^*)}{\partial d_i^*} \right)^2}{1 - F(d_i^*) - \mu f(d_i^*)} \left\{ \frac{\partial f(d_i^*)}{\partial d_i^*} \frac{\mu f(d_i^*)}{\partial d_i^*} \right\} \right] - c'(p_i)
$$

(2.18)

The expression in curly brackets is the same that we found in (2.17), hence it is strictly positive under Assumption 2.4. As a result, the game is strictly concave.

Aggregativity: It follows immediately from the definition of $\pi_i(p_i, p_{-i}, q_i)$ (i.e., equation (2.1)).
Proposition 2.2

Proof. Define the best response correspondence for firm \( i \) as follows:

\[
b_i(p_{-i}) \equiv \arg\max_{p_i \in [0, K/E\tilde{x}]} V(p_i, p_{-i})
\]

We know \( b_i(p_{-i}) \) is nonempty by the theorem of the maximum because \( V(p_i, p_{-i}) \) is continuous and the set \([0, K/E\tilde{x}]\) is compact. Moreover, \( b_i(p_{-i}) \) is a singleton because \( V(p_i, p_{-i}) \) is strictly concave. Hence, \( b_i(p_{-i}) \) is convex and upper hemicontinuous. It follows by Kakutani fixed point theorem that a PSNE exists.

As for the properties of \( Q_* \) and \( Q^* \), they follow from Lemma 2.7, which guarantees that my game is a special case of those to which Theorem 1 in Acemoglu and Jensen [2013] applies.

Corollary 2.4

Proof. Observe first that the FOC can be written as:

\[
\mu \frac{\partial \pi_i}{\partial p_i} \bigg|_{p_i = p^*} \cdot \left[ F(d(p^*)) + f(d(p^*)) \cdot \frac{\mu F(d(p^*)) + f(d(p^*))[x - d(p^*)]dF(x)}{1 - F(d(p^*))} \right] = c'(p^*)
\]

The right hand side is not a function of \( q_{i,j} \). In contrast, the left hand side is a function of \( q_{i,j} \), through its effect on \( \pi_i \). Moreover, the sign of the derivative of the left hand side with respect to \( q_{i,j} \) is the same as that in (2.18), hence it is strictly positive. Evidently, \( p^*_i \) must decrease for the equation to keep holding, proving that equilibrium disclosure decreases with \( q_{i,j} \).

As a shock to \( q \) hits the aggregator, in the sense of Acemoglu and Jensen [2013], both \( Q_* \) and – more importantly – \( Q^* \) decrease with it. Coming to leverage, from Proposition 2.1 we know that leverage increases with \( q_{i,j} \) if and only if \( \partial \pi^*/\partial p_{i,j} < 0 \). However, this derivative embeds two effects: on the one hand, a higher correlation directly increases \( \pi^*_i \). On the other hand, it lowers the equilibrium disclosure which in turns lowers \( \pi^*_i \). The elasticities cannot be signed a priori.

Proposition 2.3

Proof. Existence is immediate from continuity. Moreover, \( \frac{\partial V(p_i, p_{-i})}{\partial p_{i,j}} > 0 \) whenever \( q_{i,j} > 0 \) implies that the private disclosure is inefficiently lower than that at the SE.

2.7.2 Empirics: Robustness Checks

In this appendix, I present and discuss additional empirical exercises to confirm that the correlations presented in the paper are robust.

The first exercise pertain the cutoff in the number of analysts’ forecast required for an observation to be included in the data. In the man text, I consider a cutoff of 5, but I claim this choice does not affect the results. To show that this is the case, Table 2.3 presents the fixed effect regression results for cutoffs ranging from 2 to 7.\(^{43}\)

\[^{43}\]Evidently, two is the minimum number of forecasts needed to be able to actually compute a coefficient of variation. Robustness to even higher cutoffs (in particular, ten) is presented in Table

42
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<td>(2.50)</td>
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<td>0.340***</td>
<td>0.339***</td>
<td>0.316***</td>
<td>0.322***</td>
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<td>(5.86)</td>
<td>(5.36)</td>
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$t$ statistics in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Sources: Compustat merged with CRSP (annual), IBES (detail, adjusted for stock splits).

Notes: all independent variables are lagged by one year. Stnd. errors clustered at the firm level.
From now onwards, by ‘Usual Controls’ I shall refer to those included in the regressions of Table 2.3.

The second set of robustness checks, presented in Table 2.4, studies how the results change with different measures of analysts’ forecast dispersion. Column (1) reports the benchmark estimate using the coefficient of variation (it is equivalent to column (4) of Table 2.2). Column (2) clarifies the importance of normalising the standard deviation by the mean: without the normalisation the significance is lost. Column (3) and (4) do the same replacing CV with MAD (the median absolute deviation from the mean forecast). Similar results attain. Finally, column (5) shows that one could also use directly the number of analysts following the firm in a given year. As expected, the number is negatively correlated with leverage, suggesting that the higher the number of analysts following a firm, the lower its subsequent leverage ratio.

<table>
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$t$ statistics in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Sources: Compustat merged with CRSP (annual), IBES (detail, adjusted for stock splits).
Notes: all independent variables are lagged by one year.
Standard errors are clustered at the firm level.

2.2 in the main text.
The third series of robustness checks is presented in Table 2.5. It considers the effects on the estimates of changing the definition of leverage. In particular, column (1) presents again the estimates shown in the main text, where leverage is defined as in Welch [2011], to equal the ratio of Total Liabilities (LT) over Total Assets (AT). Column (2) replaces AT with the market value of assets (AM = MEQ + LT). The coefficient of interest is positive but loses a one degree of significance. Column (3) shows what happens when leverage is defined as the ratio of Total debt (DT) – defined as the sum of Debt in Current Liabilities (DLC) and Long Term Debt (DLTT) – over the book value of assets. The result is similar to that of column (2). Finally, column (4) shows what happens when leverage is defined as DT/AM. The coefficient loses significance altogether. Columns (5)-(7) repeat the exercise of substituting LT/AT with alternative measures of leverage for the independent variable MAD. Similar results attain.

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Sources: Compustat merged with CRSP (annual), IBES (detail, adjusted for stock splits).
Notes: all independent variables are lagged by one year. Stnd. errors clustered at the firm level.

Finally, Table 2.6 explores the leads and lags structure of the data. Although CV is serially correlated, the Table shows that the results are stronger when CV is assumed to precede leverage than the other way around. Of course, the results do not rule out reverse causality, and a statistically causal analysis is still required in future work.
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$t$ statistics in parentheses. $^*$ $p < 0.05$, $^{**} p < 0.01$, $^{***} p < 0.001$

Sources: Compustat merged with CRSP (annual), IBES (detail, adjusted for stock splits).
Notes: all independent variables are lagged by one year. Stnd. errors clustered at the firm level.
Chapter 3

Optimal Security Design under Asymmetric Information and Profit Manipulation

3.1 Introduction

Since Myers and Majluf [1984], asymmetric information has been used to explain the prevalence of simple debt contracts as a means of external financing. Debt emerges from the lack of credible signaling opportunities which leads to pooling equilibria. However, the impossibility of signaling is due to the assumption that the payoff of admissible securities must be non decreasing in the project’s earnings.¹

The justification for this monotonicity assumption is that the entrepreneur can observe the realized earnings before outside financiers do, and can manipulate them. If the payoff of a security has a decreasing segment, an entrepreneur would borrow secretly and report higher earnings in order to repay less. Without explicitly modelling profit manipulation, existing papers argue that this cannot be an equilibrium phenomenon and rule it out by exogenously restricting attention to monotonic or ‘manipulation-proof’ securities.²

In this paper, we explicitly model both ex ante asymmetric information and ex post profit manipulation and we solve the security design problem without exogenously restricting the admissible contracts to be monotonic.³ More specifically,

¹Technically, the assumption is sufficient to prevent signaling if the earnings distribution is assumed to satisfy the hazard rate ordering, a stronger property than first-order stochastic dominance.
²This argument has been widely used in the literature. See, for instance, Nachman and Noe [1994]; DeMarzo and Duffie [1999]; DeMarzo et al. [2005].
³The only restriction imposed on admissible contracts is limited liability, as appropriately rede-
our baseline framework features two types of entrepreneurs, both endowed with a positive net present value project. A type corresponds to a distribution over future earnings, and the distributions are ordered according to the monotone likelihood ratio property. Entrepreneurs have no initial wealth, and seek funding from competitive financiers. They privately know their type (ex ante asymmetric information) and can misreport the realized earnings (ex post profit manipulation). The admissible set of reported earnings is possibly a function of the realized output and it always includes the true realization (or, equivalently, the true state of the world).

The objective is twofold: (i) to check whether the conventional justification for restricting attention to monotone securities is sound; and (ii) to uncover the joint effect of ex ante asymmetric information and profit manipulation on the optimal portfolio of securities issued by a firm. Our results can be summarized as follows:

First, any optimal contract induces profit manipulation (either output diversion or window dressing) on-the-equilibrium path. This is fully anticipated by outside financiers and hence properly priced. The result is novel, as it depends on modelling profit manipulation explicitly, and it highlights the key flaw in the story that was supposed to justify the monotonicity constraint: when diversion and window dressing are feasible, the revelation principle does not generically hold (see Green and Laffont [1986]), and hence profit manipulation may well be part of an optimal contract. The optimal portfolio of securities does not prevent profit manipulation. In contrast, it trades them off against the asymmetric information costs of financing. Our result shows that it is indeed suboptimal to issue manipulation-proof securities.

Second, non-monotonic (or bonus) contracts are always optimal. A bonus contract requires that the financier receives the full realized earnings up to a prespecified threshold, and beyond the threshold the repayment falls. This contract can be thought of as a standard debt contract with a strictly positive performance-based bonus for the entrepreneur, that is included in any optimal contract because it minimizes the mispricing of the securities issued by high quality entrepreneurs. The intuition for the result is that bonus contracts impose the maximum expected repayment when realized earnings are low, hence minimizing it when earnings are high. Because lower quality types are more likely to obtain low earnings, bonus contracts maximize the cost for them to mimic the high types.

We shall take the extent of profit manipulation possibilities as exogenously given, and do comparative statics with respect to it. The presence of profit manipulation possibilities may reflect the imperfect quality of the legal system and/or corporate governance issues (e.g. LaPorta et al. [1997]).
Innes [1993] suggests that non-monotonic contracts would be optimal in the absence of profit manipulation. His findings motivated the introduction of the exogenous monotonicity constraint. In contrast, we explicitly model profit manipulation and characterize necessary and sufficient conditions under which the optimal securities are monotonic.

Coming to the optimality of debt contracts, we show that: (i) straight debt is suboptimal when profit manipulation and/or adverse selection are not severe. In these cases, non-monotonic securities are uniquely optimal and they may implement separating equilibria which are absent in the previous models because of monotonicity; and (ii) straight debt is never optimal if the distribution of earnings is unbounded above, as long as profit manipulation possibilities are bounded. The result implies that debt is never the equilibrium contract in models assuming, for instance, exponentially or (log)-normally distributed earnings.

The latter observation leads us to our final result: a characterization of the necessary and sufficient conditions under which straight debt is optimal, and monotone securities may arise in equilibrium. Such conditions are restrictive, and whenever debt is optimal there exists a bonus contract that is ex post equivalent to it. That is, debt is never uniquely optimal.

Overall, our findings have the following implications: (i) they show that the assumption that admissible securities are monotonic leads to sub-optimal securities and must be reconsidered; (ii) they provide a set of tools that can be used to construct models in which profit manipulation occurs on-the-equilibrium path; (iii) they make clear that ex ante asymmetric information does not suffice to theoretically justify the optimality of debt. One needs to introduce yet additional frictions, for example ambiguity and ambiguity aversion (see Antic [2014]), to prevent non-monotonic contracts from being optimal and dominate debt.

The paper is structured as follows: Section 3.2 briefly reviews the literature; Section 3.3 describes the model; Sections 3.4 and 3.5 introduce the relevant securities, and discuss when and how they induce profit manipulation; Section 3.6 derives the main results; Section 3.7 illustrates how the general results apply in specific settings; Section 3.8 discusses extensions including allowing for ex ante moral hazard and increasing the cardinality of the type space; Section 3.9 concludes.

### 3.2 Literature Review

Our paper is closely related to the literature on security design under asymmetric information. Myers and Majluf [1984] developed the ‘pecking order’ theory of
debtf optimality under asymmetric information in a setup where only debt and (inside or outside) equity contracts were allowed. Noe [1988] showed that the Myers and Majluf theory requires somewhat restrictive assumptions on the distributions of earnings. Innes [1993] and Nachman and Noe [1994] revisited the theoretical argument allowing for a broader set of contracts than debt and equity. These papers found that to obtain debt as the optimal security some monotonicity constraint has to be *exogenously* imposed. Such a constraint restricts the feasible contracts to be ‘manipulation proof’.

Since then, the monotonicity constraint has been widely used. Prominent examples include DeMarzo and Duffie [1999]; DeMarzo et al. [2005]; Inderst and Mueller [2006]; Axelson [2007], Axelson et al. [2009]; Gorbenko and Malenko [2011]; Philippou and Skreta [2012]; Scheuer [2013]; Vanasco [2014]. Our contribution is to derive necessary and sufficient conditions for monotonicity to be without loss of generality, and to argue that ex ante adverse selection and profit manipulation can explain performance-sensitive debt (Manso et al. [2010]), but generically not straight debt as previously argued.

Furthermore, our paper is related to the literature on optimal contracting under profit manipulation. The existing papers can be separated along two dimensions: (i) whether manipulations are assumed to be bounded (and a function of types) or not; and (ii) whether repayments can be extracted via additional tools such as verification, termination or liquidation of the firm.

A literature originating from Townsend [1979] and Gale and Hellwig [1985] modeled unbounded manipulation with the possibility of verifying the earnings at a cost (the so-called ‘costly state verification’ - CSV - models). Bolton and Scharfstein [1990] and Hart and Moore [1998] study similar models where verification is substituted with the threats of termination and liquidation.⁵

In contrast, Green and Laffont [1986] considered a setup with bounded manipulation possibilities but no verification. They provided a necessary and sufficient condition for the revelation principle to hold - the so-called *nested range condition*. The condition fails naturally in financial contracting models where the set of feasible manipulations is likely to be convex and depends on the type.⁶

⁵See Ben-Porath and Lipman [2012] and Ben-Porath et al. [2014b] for a version of CSV without transfers.

⁶See Koessler and Perez-Richet [2014] and references therein for recent models along these lines.
3.3 The Economy

There are two dates $t \in \{0, 1\}$, an entrepreneur and a competitive financier. Both agents are risk-neutral and maximise date one consumption. The entrepreneur has a project that generates stochastic date one earnings $x \in X$ and requires a fixed input of $I$ at date zero. The financier has wealth $W$, and can either lend it to the entrepreneur or store it without depreciation.

The set of possible earnings realizations is $X \equiv [0, K]$. When we allow for unbounded future earnings, we let $K$ approach infinity. There are two types of projects (entrepreneurs), $t \in \mathcal{T} \equiv \{l, h\}$. Types differ according to their distribution of earnings.

The cumulative distribution function (cdf) over $X$ for a type $t$ project is $F_t(x)$. The project’s type is private information of the entrepreneur. Outside financiers only know that a fraction $\lambda_l \in (0, 1)$ are type $l$ projects, and a fraction $\lambda_h = (1 - \lambda_l)$ are type $h$ projects. All projects have positive net present value, and the firm’s assets in place are assumed to be zero. Denote by $E_t(x) = \int_0^K x dF_t(x)$ the full information expected value of a type $t$ project. We make the following assumption:

**Assumption 3.1.** $\min\{W, E_t(x)\} \geq I > 0$ for every $t \in \mathcal{T}$. \hfill (A1)

**A1** guarantees that: (i) the financier can finance a project; (ii) all projects have positive net present value; and (iii) investment is risky, because $I > 0$ and strict positivity of $f_t(x)$ for every $x$ imply that $F_t(I - \epsilon) > 0$ for all $t \in \mathcal{T}$, for $\epsilon > 0$.

In addition, we make the following standard assumptions on the distributions of earnings:

**Assumption 3.2.** The cumulative distribution functions are mutually absolutely continuous, and satisfy the Strict Monotone Likelihood Ratio Property (MLRP): $\frac{\partial}{\partial x} (f_h(x)/f_l(x)) > 0$ for every $x$. \hfill (A2)

Continuity is standard: it simplifies the analysis and it prevents contracts that penalise realisations that have strictly positive probability only for one type. Strict MLRP implies that $E_h(x) > E_l(x)$, and it is a standard assumption in the literature (see for instance DeMarzo et al. [2005]).

The timing of the game is as follows:

**date 0:** The entrepreneur of type $t$ issues publicly a portfolio of securities (financial contract) denoted by $s$. Each financier simultaneously quotes a price $P(s)$ at which he is willing to buy the securities. If a contract is signed (securities
are sold), the entrepreneur collects $P(s)$. Subsequent investment is observable and verifiable;

date 1: Realized earnings $x \in X$ are perfectly but privately observed by the entrepreneur. He can costlessly manipulate reported earnings, reporting them to be $m \in M(x)$. The profit manipulation technology $M(x)$ for a generic ex post type $x$ is fully characterised by two functions: $\eta(x)$ and $\overline{\eta}(x)$. In particular, the entrepreneur can divert resources up to $x - \eta(x)$ and window dress (or overstate) the earnings up to $x + \overline{\eta}(x)$. Hence, the convex set of feasible messages when the realised earnings are $x$ is given by $M(x) \equiv [x - \eta(x), x + \overline{\eta}(x)]$. The manipulation technology is common knowledge at $t = 0$;

date 2: Claims are settled on the basis of the borrower’s self reported earning and the game ends.

Timeline

| $t = 0$ | $t = 1$ | $t = 2$ |
| Contracting stage | Entrepreneur observes $x$ and reports $m(x|s)$ | Claims are settled |

The novel ingredient that differentiates our findings from existing results is the possibility of ex post profit manipulation. We summarize the restrictions imposed on the manipulation technology in the following assumption:

Assumption 3.3. The set of admissible profit manipulation technologies for any realised earnings $x \in X$ is given by:

$$M(x) \equiv [x - \eta(x), x + \overline{\eta}(x)]$$  \hspace{1cm} (A3)

where $\eta(x)$ and $\overline{\eta}(x)$ are $C^1$ functions such that for every $x$: (i) $\eta(x) \leq x$ and $\eta'(x) \in [0,1)$. (ii) $\eta(x) \leq K - x$ and $\overline{\eta}'(x) \geq -1$.\(^7\)

We impose $\eta'(x) \in [0,1)$ for tractability, and it will become clear that the assumption does not drive our qualitative results. Moreover, $\overline{\eta}'(x) \geq -1$ guarantees that the higher the realised output, the higher the output that can be reported. Overall, the following comments are due:

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\(^7\)The bounds on the $\eta$ functions just guarantee that both the upper and the lower bound of the set $M(x)$ belong to $X$ for every $x \in X$. 

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1. Although we allow for unbounded window dressing (to nest Nachman and Noe [1994] results as a special case of our framework), we rule out unbounded diversion which would trivially lead to no financing;

2. An equivalent way of modelling profit manipulation would be to allow for secret borrowing from ‘friends’. Our results do not depend on the reason why imperfect verification comes about, but on its extent;

3. We could have modelled diversion as output destruction, in which case the entrepreneur could not put the diverted amount in his pocket. However, in such a scenario the entrepreneur would be indifferent between diverting and not in equilibrium, making such possibilities useless;

4. Finally, we assume that $M(x)$ is a convex set. This is not without loss of generality, however we find it natural because if the entrepreneur can divert $k$ dollars from the project to his own accounts, we believe he should be able to divert also $k - \epsilon$ for $\epsilon > 0$. Further, the assumption greatly simplifies the analysis.

The possibility of earnings misreporting means that a security $s(\cdot)$ cannot be a function of $x$ as in the previous literature. Instead, it is a function of reported earnings $m(x|s)$. Because $M(x)$ is a compact set for every $x \in X$, we know that for every security $s$ and every $x$ there exists a best message $m^*(x|s)$ defined so that:

$$m^*(x|s) = \arg\min_{m \in M(x)} \{s(m)\}$$

Further, because $M(x) \subset X$, we know that $m^*(x|s) \in X$. Hence, the expected repayment of a security (or its real payoff) is a function $s(m^*(x|s)) : X \to \mathbb{R}$. It should be noticed that the ex post verification problem created by the possibility of profit manipulation prevents the application of the revelation principle, since $M(x)$ does not need to satisfy the nested-range-condition of Green and Laffont [1986].

The only restriction we impose on the contract space is that each security must satisfy limited liability, as appropriately redefined in terms of messages:

**Assumption 3.4.** The set of admissible securities is given by:

$$S \equiv \{s(m) \mid 0 \leq s(m) \leq m, \ \forall m\}$$

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8Our formulation restricts attention to direct mechanisms, where the set of messages is a subset of the set of states $X$. It is easy to prove that the restriction is without loss of generality.
If the borrower declares $m$ and cannot repay $s(m)$ to his financier, then the financier becomes the legitimate owner of borrower’s assets.\footnote{Since the limited liability constraint must be defined in terms of messages rather than realized output, we should consider the case in which the entrepreneur declares earnings that exceed true earnings, and he does not have the resources to repay the contractual obligation. In this case, the fraud becomes observable and verifiable: it is revealed that he is either lying about $x$ or refusing to make the payment he committed to make. We implicitly assume that when the fraud is revealed, the agent receives a punishment large enough to prevent such behavior.}

Denote by $V_t$ the profits of an entrepreneur of type $t$ whose offered security $s$ has been priced at $P$ by the financier, and by $V_f$ the financier’s profits. Then we can write:

\[
V_t = P - I + \mathbb{E}_t [x - s(m^*(x|s))] \\
V_f = \mathbb{E} \lambda(s) [s(m^*(x|s))] - P(s)
\]

The expectation in (4.2) is given by the sum across types (weighted by the posterior belief $\lambda(t|s)$ that type $t$ is issuing the contract $s$) of the final payoff of the security after manipulation takes place:

\[
\mathbb{E} \lambda(s) [s(m^*(x|s))] \equiv \sum_{t \in T} \lambda(t|s) \left[ \int_{x \in X} s(m^*(x|s)) dF_t(x) \right]
\]

Notice that we can write $m_t^*(x|s) = m^*(x|s)$ because the cost and benefits of committing accounting fraud ex post are not type-dependent.

Here we adopt the concept of Perfect Bayesian Equilibrium (PBE):

**PBE:** A strategy profile $(s_t^*, m^*(x|s), P^*(s))$ and a common posterior belief $\lambda^*(t|s)$ for $t \in T$ form a PBE of the game if the following conditions are satisfied:

1. For every $x \in X$ and for $s \in S$: $m^*(x|s) = \arg \min_{x \in M(x)} \{s(m)\}$;
2. For every $t \in T$, $s_t^*$ maximizes $V_t(s_t, P^*(s_t), m^*)$ subject to the limited liability constraint ($s_t \in S$);
3. The posterior belief $\lambda^*(t|s_t)$ is obtained from Bayes’ Rule whenever possible;
4. Competitive Rationality: for $s_t \in S$, $P^*(s_t) = \mathbb{E} \lambda^*(t|s_t)(s_t)$

As standard, a PBE is said to be separating if $s^*_h \neq s^*_l$, and pooling otherwise. Notice that, because investment is observable and verifiable, in every equilibrium it must be the case that either $P^*(s_t) = 0$ (no investment), or $P^*(s_t) \geq I$ (investment takes place).
To rule out ‘unreasonable’ equilibria, we refine the off-equilibrium-path beliefs using the Intuitive Criterion by Cho and Kreps [1987]. Denote by $V_t(s_t^*, e^*)$ the expected utility of type $t$ entrepreneur issuing $s_t^*$ at the equilibrium $e^*$, and by $\Pi^*(s|T)$ the set of all possible Nash Equilibria of the game played by financiers given an observed $s \in S$.

The Intuitive Criterion: A PBE is not reasonable if there exist an out-of-equilibrium security $s' \in S$ such that only one type may benefit from deviating to $s'$:

$$V_t(s_t^*, e^*) \leq \max_{P^* \in \Pi^*(s'|T)} V_t(s', e^*)$$

$$V_{-t}(s_t^*, e^*) > \max_{P^* \in \Pi^*(s'|T)} V_{-t}(s', e^*)$$

In words, suppose there are two types. Consider a pooling equilibrium, and a deviant security that could only benefit the high type (if accepted), for some off-equilibrium beliefs, and never the low type. The Intuitive Criterion prevents equilibria that are sustained by the off-equilibrium belief that such a security would be offered by a low type with positive probability.

The next two sections introduce the key properties of the two contracts which are relevant in this framework: debt and bonus contracts.

### 3.4 Debt Contracts

It is useful to stress again the distinction that arises in this model (unlike the existing literature) between the promised expected payoff and the real expected payoff of a security. The promised expected payoff is given by $E_{X^*(s)}(s(m = x))$, where each type $x$ is assumed to report truthfully his type. In contrast, the real expected payoff is given by $E_{X^*(s)}(s(m^*(x|s)))$, where $m^*(x|s)$ solves condition (1) of a PBE, i.e. it maximizes the type $x$ entrepreneur’s ex post payoff.

The characteristic features of a debt contract are: (i) the fixed repayment in non-bankruptcy states; and (ii) seniority in bankruptcy states. If we denote the face value of debt by $d$, then whenever $m \geq d$, the debt security specifies $s = d$. If, instead, $m < d$, a bankruptcy state, the debt holder is a senior claimant on the assets, obtaining repayment $s(m) = m$.

---

10 Each element of the set can be parameterized by a posterior belief $\lambda(s) \in \Delta_T$, where we adopt the convention that bold symbols represent vectors.

11 We provide a definition of the Intuitive Criterion for a generic set of types $T$ because we extend our main result to the case of $|T| > 2$. 

55
Definition 3.1. A security \( s \in S \) is a debt contract if and only if \( s = \min\{m, d\} \) for some \( d \in M \).

Figure 3.1: The Promised Payoff of Debt

Like any other security with positive expected value, a debt contract provides incentives to divert output for some \( x \in X \). In order to characterize the real payoff of a standard debt contract, we need define the introduce some additional notation. In particular, consider a debt contract with a face value \( d \) and suppose that \( K - \eta(K) > d \). Define as \( \tilde{m}(d) \) the highest threshold such that diversion can be profitable, i.e.:

\[
\tilde{m}(d) \equiv \min_{x \in X} \{x \mid x - \eta(x) = d\}.
\]

Such point exists and is unique by the intermediate value theorem due to continuity and monotonicity of function \( x - \eta(x) \) (see Assumption 3) and the fact that \( K - \eta(K) > d \) and \( -\eta(0) < d \). If \( K - \eta(K) \leq d \), instead, then simply set \( \tilde{m}(d) \equiv K \).

Lemma 1 provides a characterization of the real payoff of a standard debt contract.

Lemma 3.1. (The Real Payoff of Debt) Given any debt security \( s \) with fixed repayment \( d \), the entrepreneur optimally reports:

\[
m^*(x|s) = \begin{cases} 
x - \eta(x) & \text{if } x \leq \tilde{m}(d) \\
x & \text{otherwise}
\end{cases}
\]

Proof. Notice first that because a debt contract is monotonic there cannot be any benefit from overstating earnings. The Lemma follows then trivially from
the definition of $\tilde{m}(d)$, which is guaranteed to exist and be unique by Assumption 3. Q.E.D.

The dashed curve in Figure 3.2 depicts the real payoff of a standard debt contract.

Figure 3.2: The Real Payoff of Debt

![Figure 3.2: The Real Payoff of Debt](image)

3.5 Bonus Contracts

The contract that turns out to be generically optimal takes the following form:

**Definition 3.2.** A security $s \in S$ is a bonus contract if and only if, for some $(m, d) \in X^2$,

$$s(m) = \begin{cases} m & \text{if } m < m \\ d & \text{otherwise} \end{cases} \quad (3.3)$$

Notice that because any admissible security satisfies limited liability ($s \in S$), we must have $d \in [0, m]$. Figure 3.3 depicts the promised payoff of contracts as defined in (3.3).

We next characterize the optimal ex post accounting fraud under bonus contracts. In order to do so we need to introduce a final piece of notation. In particular, define a function $\bar{\eta}(x) \equiv x + \eta(x)$. This function is continuous and strictly increasing on $[0, K]$. Consider a bonus contract $(d, m)$ such that $\bar{\eta}(0) < m$. Given that $\bar{\eta}(m) > m$ by definition, the intermediate value theorem implies that there

---

12Standard debt contracts are special cases of (3.3) where $d = m$. For this reason we shall always make explicit whether the contracts we discuss must feature a strictly positive bonus or not.
exists $\tilde{x}(m) \in (0, m]$ such that $\bar{\eta}(\tilde{x}) = \tilde{x} + \bar{\eta}(\tilde{x}) = m$. Monotonicity of the function $\bar{\eta}(x)$ ensures the uniqueness of such point. Hence, we can define a function $\tilde{x}: [d, K] \rightarrow [0, K]$ as follows:

$$
\tilde{x}(m) = \begin{cases} 
0 & \text{if } \bar{\eta}(0) > m, \\
\tilde{x} & \text{otherwise}.
\end{cases}
$$

Moreover, the function $\tilde{x}(m)$ is strictly monotonic on the interval $(\bar{\eta}(0), K)$, because of the implicit function theorem and Assumption 3.

**Lemma 3.2. (The Real Payoff of a Bonus Contract)** Given any bonus contract $s$ with fixed repayment $d$ and threshold $m$, we have two cases:

1. If $m - d > \bar{\eta}(m)$, then the entrepreneur optimally reports,

   $$m^*(x|s) = \begin{cases} 
x - \bar{\eta}(x) & \text{if } x < \max\{\tilde{x}(m), \tilde{m}(d)\} \\
\tilde{m}(d) & \text{if } x \in [\max\{\tilde{x}(m), \tilde{m}(d)\}, m] \\
x & \text{otherwise}
\end{cases}$$

2. If $m - d \leq \bar{\eta}(m)$, then the real payoff of a bonus contract is equivalent to that of a debt contract (see Lemma 1).

**Proof.** 1. Suppose that $m - d > \bar{\eta}(m)$ and $\tilde{x}(m) \leq \tilde{m}(d)$. In this case, for any $x < \tilde{m}$ it is not optimal to window dress as the entrepreneur is better off with output diversion. For any $x \in [\tilde{m}(d), m]$ we have that $\bar{\eta}(x) \geq \bar{\eta}(\tilde{x}(m)) \geq m$ and therefore the entrepreneur can report $m$ which makes him better off than diverting.
the output. Finally, for any \( x > m \) neither output diversion nor windows dressing benefits the entrepreneur and he truthfully reports \( x \).

Suppose now that \( \tilde{x}(m) > \tilde{m}(d) \). In this case the entrepreneur diverts output for any \( x < \tilde{x}(m) \) since it is impossible to reach the bonus region \( \{ x : x \geq m \} \) by means of window dressing. For any \( x \in [\tilde{x}(m), m) \) window dressing is beneficial since \( d = \tilde{m}(d) - \eta(\tilde{m}(d)) < x - \eta(x) \). Finally, for any \( x > m \), as above, the entrepreneur truthfully reports \( x \).

2. Assume now that \( m - d \leq \eta(m) \). Note that in this case \( \tilde{m}(d) > m \). For any \( x < \tilde{m}(d) \) it is not optimal to window dress as the entrepreneur is better off with output diversion. For any \( x > \tilde{m}(d) > m \) the entrepreneur truthfully reports \( x \). Q.E.D.

Figure 3.4 depicts the real payoff of a bonus contract for the cases of two different levels of profit manipulation. In Panel A of Figure 3.4, \( \tilde{x}(m) > \tilde{m}(d) \) and the real payoff is not ex post monotonic. In Panels B and C of Figure 3.4, \( \tilde{x}(m) < \tilde{m}(d) \) and the real payoff is ex post equivalent to that of a debt contract with face value \( d \).

### 3.6 Optimal Security Design

In existing work, \( A1 \) was enough to guarantee that all projects would get financing in equilibrium. However, the possibility of profit manipulation changes this conclusion. It is instructive to begin our analysis by deriving conditions under which financing occurs. To do so, we simply need to consider the contract in which the financier receives all reported earnings: \( s = m, \forall m \). If financing does not take place with such a contract it cannot take place with any other contract that satisfies \( A3 \).

Clearly, under the \( s = m \) contract, it is never optimal to window dress, because the contract is strictly increasing in \( m \). Hence, one should only worry about diversion. Further, it is always optimal to divert as much as feasible, for every ex post realised earnings \( x \).

Suppose first that there is no ex ante asymmetric information. The condition for type \( t \) to get financing is \( \mathbb{E}_t(x - \eta(x)) \geq I \). Since \( \mathbb{E}_l(x - \eta(x)) < \mathbb{E}_h(x - \eta(x)) \), we conclude that whenever the low type (type \( l \)) gets financing, the high type (type \( h \)) does as well. Hence, if \( \mathbb{E}_l(x - \eta(x)) < I \leq \mathbb{E}_h(x - \eta(x)) \), only type \( h \) receives financing. If \( \mathbb{E}_h(x - \eta(x)) < I \) no one gets funding, and finally if \( \mathbb{E}_l(x - \eta(x)) \geq I \) every type is financed.

\[ \text{Notice that: (i) the } s = m \text{ } \forall m \text{ contract may be thought of as a bonus contract where } m = K; \text{ (ii) a borrower still prefers to offer such a contract than to get no financing, as ex post he can divert a positive amount of output.} \]
Once we introduce asymmetric information, if
\[ \mathbb{E}_l(x - \eta(x)) < I \leq \mathbb{E}_h(x - \eta(x)) \]
there can only exist pooling equilibria. Hence, the boundaries of the region in which
financing takes place depends on the pooling zero profit condition, which is:
\[ \lambda_l \mathbb{E}_l(x - \eta(x)) + (1 - \lambda_l) \mathbb{E}_h(x - \eta(x)) \geq I \]

### 3.6.1 Separating Equilibria

In this section we characterize the set of Separating Perfect Bayesian Equilibria (SPBE). Such equilibria never arise with the exogenous monotonicity constraint. The intuition behind the SPBE is the following: the most productive type tries to distinguish himself from the less productive one by offering securities with high downside protection and a low upside payoff for the financier (such as bonus contracts). By doing so, high types impose a relatively higher cost on low types should they try to mimic.

In a SPBE, \( s_l \neq s_h \). Moreover, given the offered security \( s_l \), the posterior
belief that it is offered by type \( t \) is one, i.e. \( \lambda(t|s_t) = 1 \) for every \( t \in T \). Incentive compatibility for type \( t \) reads:

\[
\mathbb{E}_t(x - s_t(m^*(x|s))) \geq \mathbb{E}_t(x - s_{t'\neq t}(m^*(x|s)))
\]

or, equivalently: \( \mathbb{E}_t(s_t(m^*(x|s))) \leq \mathbb{E}_t(s_{t'\neq t}(m^*(x|s))) \). Further, at any SPBE it must be that \( \mathbb{E}_t(s_t(m^*(x|s))) = I \), for every \( t \in T \). Hence we can rewrite the incentive constraint as:

\[
\mathbb{E}_{t'}(s_{t'}(m^*(x|s))) - \mathbb{E}_t(s_t(m^*(x|s))) \leq 0
\]

Finally, it is trivial to show that the only incentive constraint that may be binding is that for the \( l \) type not to mimic the \( h \) type, i.e: \( \mathbb{E}_h(s_h(m^*(x|s))) - \mathbb{E}_l(s_h(m^*(x|s))) \leq 0 \). To formulate the incentive constraint in this way allows us to proceed and solve for the optimal contract as will become clear below.

To simplify the notation, we sometimes write \( \tilde{m}_h \) instead of \( m_h(d_h) \) and \( \tilde{x}_h \) instead of \( x_h(m_h) \). Suppose that \( s_h \) has the shape given in (3.3). It would read:

\[
\int_0^{\max\{\tilde{m}_h, \tilde{x}_h\}} (x - \eta(x)) \left[ dF_h(x) - dF_l(x) \right] + \left[ F_l(\max\{\tilde{m}_h, \tilde{x}_h\}) - F_h(\max\{\tilde{m}_h, \tilde{x}_h\}) \right] d_h \leq 0.
\]

Notice that if \( \max\{\tilde{m}_h, \tilde{x}_h\} = \tilde{m}_h \), the incentive constraint can never be satisfied because the real payoff of the bonus contract is monotonic. To see this, integrate the above expression by parts and get:\[14\]

\[
\int_0^{\tilde{m}_h} (1 - \eta(x)) \left[ F_l(x) - F_h(x) \right] dx + \left[ F_l(\tilde{m}_h) - F_h(\tilde{m}_h) \right] [d_h - \tilde{m}_h + \eta(\tilde{m}_h)] > 0
\]

\[= 0 \text{ by the definition of } \tilde{m}_h\]

Hence, for the rest of this section suppose that \( \max\{\tilde{m}_h, \tilde{x}_h\} = \tilde{x}_h \), and that \( m_h - d_h > \eta(\tilde{m}_h) \). Rewrite the incentive compatibility constraint as:

\[
\int_0^{\tilde{x}_h(m_h)} (x - \eta(x)) \left[ dF_h(x) - dF_l(x) \right] + \left[ F_l(\tilde{x}_h(m_h)) - F_h(\tilde{x}_h(m_h)) \right] d_h \leq 0 \quad (3.4)
\]

In a SPBE, competitive financing yields \( \mathbb{E}_h(s_h) = I \). Substituting this into (4.4) we

---

\[14\] As standard, MLRP is the acronym for Monotone Likelihood Ratio Property, and FOSD for First Order Stochastic Dominance.
\[ \int_{0}^{\bar{x}_h(m_h)} (x - \eta(x)) \left[ dF_h(x) - dF_l(x) \right] + \left[ F_l(\bar{x}_h(m_h)) - F_h(\bar{x}_h(m_h)) \right] \left( I - \int_{0}^{\bar{x}_h(m_h)} (x - \eta(x))dF_h(x) \right) \leq 0 \]

Integrating by parts and rearranging yields:

\[ \text{IC} \equiv \int_{0}^{\bar{x}_h(m_h)} \left[ F_l(x) (1 - F_h(\bar{x}_h(m_h))) - F_h(x) (1 - F_l(\bar{x}_h(m_h))) \right] (1 - \eta'(x))dx \]

\[ \geq 0 \text{ by FOSD and by Assumption 3} \]

\[ + \left[ F_l(\bar{x}_h(m_h)) - F_h(\bar{x}_h(m_h)) \right] \left[ I - \bar{x}_h(m_h) + \eta(\bar{x}_h(m_h)) \right] \leq 0 \]

\[ \geq 0 \text{ by FOSD} \]

\[ \text{sign?} \]

\[ (3.5) \]

Recall that \( \bar{x}_h(m_h) \) is defined as the threshold such that for every \( x \leq \bar{x}_h(m_h) \) diversion is weakly preferred to window dressing, and vice versa for \( x > \bar{x}_h(m_h) \). Hence, inequality (4.6) highlights the key mechanism that underlies separation: setting a threshold \( m_h \) that makes \( \bar{x}_h \) high enough so that the last bracket becomes not just negative, but low enough that the second line counterbalances the first.

The key properties of (4.6) that are useful in the analysis are derived in Lemma 3:

**Lemma 3.3.** If the set of \( m_h \) that satisfies (4.6) is non-empty, then:

1. There is a unique \( m_h \) at which the inequality binds. We denote it by \( m_h^{IC} \);
2. For every \( m_h < m_h^{IC} \) the inequality is violated;
3. For every \( m_h \geq m_h^{IC} \) the inequality is satisfied.

**Proof.** See the Appendix.

The argument to prove Lemma 3 is not immediate, as inequality (4.6) is a non monotonic function of \( m_h \). The proof relies on the fact that if the inequality is satisfied for some \( m_h < K \), one can show that the set of \( m_h \) such that the inequality is binding is a singleton, and the inequality is always satisfied for values \( m_h \geq m_h^{IC} \), and never otherwise.

Notice now that if (4.6) is satisfied, then a contract is incentive compatible and leaves the financier at his participation constraint. However, it remains to
guarantee that the underlying contract belongs to the set of admissible securities, i.e. that \( d_h \geq 0 \).

Denote by \( \tilde{x}_h^{\max} \) the solution to the zero profit condition in a SPBE for type \( h \) when the face value of debt \( d_h = 0 \), and by \( m_h^{\max} \) the corresponding contractual threshold such that \( \tilde{x}_h^{\max} = \tilde{x}_h(m_h^{\max}) \) (which exists and is unique by the monotonicity of \( \tilde{x}_h \)). We have:

\[
\int_0^{\tilde{x}_h^{\max}(m_h^{\max})} (x - \eta(x)) f_h(x) dx = I. \tag{3.6}
\]

Notice that in the financing region equation (3.6) is guaranteed to have a solution. Also, equation (3.6) implies that all feasible thresholds are such that \( m_h \leq m_h^{\max} \). Given that, we can state the following proposition:

**Proposition 3.1. (SPBE)** If \( m_h^{IC} \) exists and \( m_h^{IC} \leq m_h^{\max} \) then:

1. There exists a separating equilibrium \( e^*_s \) in which a type \( h \) entrepreneur issues a contract as in (3.3) such that the financiers make zero profits, and \( m_h^* \in [m_h^{IC}, m_h^{\max}] \);

2. Type \( l \) entrepreneurs are indifferent between any contract such that \( E_l(s) = I \), as long as it is not a bonus contract with \( d_l^* \leq d_h^* \);

3. No pooling equilibrium satisfies the Intuitive Criterion.

**Proof.** See the Appendix.

Intuitively, when \( m_h^{IC} \leq m_h^{\max} \) separation may be achieved because MLRP implies that the low type (\( t = l \)) expects to repay relatively more than the high type. Thus, by choosing a sufficiently high threshold for the bonus contract (and a sufficiently low face value of debt) the high type can make the cost of mimicking for the low type excessively high, and credibly signal his type to the uninformed financiers.

Nevertheless, our choice of focusing on bonus contracts still needs to be justified. Does a separating equilibrium exist outside the region covered by Theorem 1? And if so, what contracts support it? The answers to these questions are negative: if separation is not implementable through bonus contracts, then credible signaling cannot happen under any other security that satisfies limited liability. This occurs because under a bonus contract the full reported earnings are transferred to the financier if they lie between zero and the threshold \( m \). Because the probability that the low type reports earnings below \( m \) is higher, the bonus contract maximizes the cost of mimicking for the \( l \) type. Given limited liability, no other contract can achieve
a higher expected repayment for the low type in this region. In other words, the conditions in Theorem 1 are both necessary and sufficient for separating equilibria to exist:

**Corollary 3.1.** If \( m_{IC}^{L} \) does not exist or \( m_{IC}^{L} > m_{IC}^{max} \), then any PBE of the game must be pooling.

**Proof.** See the Appendix.

The intuition for this result is as follows. Because a higher threshold for the bonus contract (and a lower face value of debt) increases the cost of mimicking for the low type, this cost is maximized when \( d_h = 0 \) and the threshold is \( m_{IC}^{max} \). If the distributions are such that the incentive constraint for the low type is violated at this contract, then a separating equilibrium cannot exist and the only possible equilibria are pooling. The argument is sometimes referred to in the literature as showing that ‘no security in \( S \) crosses the repayment function of a bonus contract from the left’. We characterize the pooling equilibria next.

### 3.6.2 Pooling Equilibria

Since Nachman and Noe [1994] seminal paper, the literature has adopted a stronger refinement than the intuitive criterion to deal with pooling equilibria: the D1 criterion. As is well known, the intuitive criterion does not bind in the pooling region of such models. The reason is that both types may benefit from any deviation depending on the posterior belief of the financier. D1 allow us to refine the equilibrium set and obtain a unique equilibrium because it is a condition on the range of beliefs for which a deviation is profitable.

Denote by \( V_t'(s') \) the utility of type \( t \) entrepreneur at the deviant contract, and by \( V_t^* \) the utility of type \( t \) entrepreneur at the equilibrium contract. Moreover, denote by \( D(t|s') \) the set of responses of the financier that would deliver strictly higher utility to type \( t \) entrepreneurs than the utility he would obtain at the equilibrium contract. Formally:

\[
D(t|s') \equiv \{ P^*(s') \geq I : V_t' > V_t^* \}
\]

where by competitive rationality, \( P^*(s') = E_{\lambda^*(s')}(s') \) for all \( \lambda^*(s') \in \Delta_T \), as beliefs off-the-equilibrium path are arbitrary.

Finally, define the indifference set \( D^0(t|s') \):

\[
D^0(t|s') \equiv \{ P^*(s') \geq I : V_t' = V_t^* \}.
\]
The D1 restriction can be defined as follows\(^{15}\):

**D1:** Suppose \( s' \in S \) is observed off-the-equilibrium path. Then for all \( t \in T \):

\[
\lambda^*_t(s') = \begin{cases} 
0 & \text{if } \exists t' \in T \text{ s.t. } t' \neq t, \text{ and } D(t|s') \cup D^0(t|s') \subset D(t'|s') \\
1 & \text{if } D(t'|s') \cup D^0(t'|s') \subset D(t|s'), \forall t' \neq t \in T \\
1 - \lambda_{t' \neq t} & \text{otherwise}
\end{cases}
\]

The pooling zero profit condition at a contract such that \( d = 0 \) is given by:

\[
\lambda_h \left[ \int_0^{\tilde{x}_\lambda} (x - \eta(x)) f_h(x) dx \right] + (1 - \lambda_h) \left[ \int_0^{\tilde{x}_\lambda} (x - \eta(x)) f_l(x) dx \right] = I. \quad (3.7)
\]

where we denote by \( \tilde{x}_\lambda \) the threshold that solves the equation, and by \( m_\lambda \) the corresponding contractual threshold. Applying D1 yields:

**Proposition 3.2.** *(PPBE, part (a))*  If \( m_{IC}^h > m_{max}^h \) (or \( m_{IC}^l \) does not exist) and \( m_\lambda < K - \eta(m_\lambda) \), then there is a unique pooling equilibrium \( e_p^* \) which satisfies D1. At \( e_p^* \), all types issue a contract as defined in (3.3) with \( d_p^* = 0 \).

*Proof.* See the Appendix.

Theorem 2 characterizes the set of equilibria that satisfy D1 when separation is not feasible (\( m_{IC}^h > m_{max}^h \)), but the earnings manipulation possibilities are low (\( m_\lambda < K - \eta(m_\lambda) \)). In this region, the uniquely optimal contract is a bonus contract with zero face value of debt. The intuition for the result is similar to that of Theorem 1: bonus contracts minimize the mispricing of securities issued, even though they do not reduce it to zero.

To conclude the characterization, we consider two final cases:

**Proposition 3.3.** *(PPBE, part (b))* If \( m_{IC}^l > m_{max}^l \) (or \( m_{IC}^l \) does not exist) and \( m_\lambda \geq K - \eta(m_\lambda) \), then either there is a unique pooling equilibrium \( e_p^* \) that satisfies D1, at which all types issue a contract as defined in (3.3) with \( d_p^* > 0 \); or there is no financing.

*Proof.* See the Appendix.

Contrary to Theorems 1 and 2, the result in Theorem 3 allows debt contracts in equilibrium because we have \( d_p^* > 0 \). To be precise:

**Corollary 3.2.** The optimal contract is ex post equivalent to straight debt if and only if \( m_\lambda - d_p < \eta(m_\lambda) \).

\(^{15}\)The D1 restriction is stronger than the intuitive criterion, hence Theorem 1 goes through unchanged if D1 is imposed.
The Corollary follows immediately from two observations. First, straight debt is the optimal monotonic contract (Nachman and Noe [1994]). Second, if \( m_\lambda - d_p < \eta(m_\lambda) \), there does not exist a feasible non-monotonic contract that is ex post distinguishable from debt. Importantly, debt is never uniquely optimal because its payoff can always be replicated by a bonus contract with a higher threshold and identical face value \( d \). The Corollary gives us a necessary and sufficient condition for the optimality of debt.

Finally, observe that both Theorem 3 and its corollary rely on the distribution of earnings being bounded above. For this reason, whenever the earnings distribution is not bounded above they describe empty sets. Such result would hold, for instance, whenever earnings belong to the normal or exponential family.

**Proposition 3.4.** If the distribution of earnings is unbounded above, i.e. \( K \to \infty \), then debt contracts are never issued in equilibrium, regardless of parameter values.

**Proof.** See the Appendix.

In this section we characterized the set of equilibria of the model. Bonus contracts are always optimal, and they provide necessary and sufficient conditions to characterize the unique equilibrium allocation of the game. Debt contracts only arise as a corner solution, when limited liability is binding and the earnings distribution is bounded above.

A question that may be asked concerns the likelihood of separating and pooling equilibria in specific settings. For instance, are the conditions necessary for separation very restrictive? Clearly, such questions require a quantitative answer, and the answer depends on the earnings distribution that is assumed. In the coming section we characterize the full set of equilibrium allocations for the exponential and the (truncated and log) normal case. Moreover, because these distributions have a support that is unbounded above, we also solve an example with linear densities that have a finite upper bound.

### 3.7 Examples

We now show how our results translate both for some families of widely used distributions that satisfy \( A2 \) (the exponential and normal families) and for a distribution with bounded support. Table 1 summarizes the parameter values that we assume\(^{16}\). The examples are solved under the assumption that \( \eta(x) = \eta(x) = \eta \) for every \( x \).

\(^{16}\)The examples include all projects such that \( \$1 = I \leq E_d(x) < E_h(x) \) for a given \( E_h(x) = \$4 \).
for \( \eta \in [0, 4] \).

Figure 3.5 shows the characterization of equilibria for the examples. The gray region is where a separating equilibrium exists (and it is unique, in terms of allocations). The black region is where a pooling equilibrium in bonus contracts exists, and it is unique. Finally, the white region is where no financing occurs.

Figure 3.5 show that the regions described in Theorems 1 and 2 are non-empty. Because of the unboundedness of the earnings distributions support, the pooling region does not admit any monotonic security (including debt, of course) in both the exponential and normal (and lognormal) cases.

The unboundedness of the support is not a necessary condition for non-monotonic contracts to be the unique equilibrium securities. To show this is the case, we introduce a family of distribution functions that has bounded support (i.e. \( K \) is finite), and satisfies MLRP. The pdfs for this family are linear and given by:

\[
f_t(x) = \frac{1}{K} \left[ \frac{K - 2x}{(\mu_t + 1)K + 1} \right]
\]

with \( \mu_{t=1} > 0 \) and \( \mu_{t=t} < \mu_{t=h} \). This family satisfies strict MLRP, because for any \( (t, t') \in T^2 \) such that \( t' < t \) we have:

\[
\frac{\partial}{\partial x} \left( \frac{f_t(x)}{f_{t'}(x)} \right) = \frac{2K(1 + \mu_t)}{(K(2 + \mu_{t'}) - 2x)^2(1 + \mu_t)} [\mu_t - \mu_{t'}] > 0
\]

and \( \mu_{t'} < \mu_t \) whenever \( t' < t \).

The bottom row of Table 3.1 summarizes the parameter values that we assume for this case, and the upper left panel of Figure 3.5 characterizes the set of equilibria. It should be clear from our results that the region where the unique equilibrium is separating admits only non-monotonic securities. However, contrary to the previous examples, the pooling region may include some monotonic securities, close to the no financing region.\(^{18}\)

---

\(^{17}\)More precisely, at the boundaries of the set \( X \) we assume that: (i) whenever \( x - \eta < 0 \), then \( \eta(x) = x \); and (ii) whenever \( x + \eta > K \), then \( \eta(x) = K - x \).

\(^{18}\)The reader may notice two features of this final example: (i) The division between the separating and the pooling regions seems not to depend on \( t_l \). We explore this fact in the next Section. (ii) The division between the pooling and the no financing regions seems to depend (very loosely) on \( t_l \). This is driven by the zero-profit pooling condition: as we increase \( t_l \) the no financing region shrinks, albeit by a very small amount.
### Table 3.1: Parameter Assumptions

<table>
<thead>
<tr>
<th>Family</th>
<th>Type</th>
<th>Parameter</th>
<th>CDF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential</td>
<td>l</td>
<td>$t_l \equiv \gamma_l^{-1} \in [1,4]$</td>
<td>$1 - e^{-(\gamma_l)^{-1}x}$</td>
</tr>
<tr>
<td></td>
<td>h</td>
<td>$t_h \equiv \gamma_h^{-1} = 4$</td>
<td>$1 - e^{-4x}$</td>
</tr>
<tr>
<td>Truncated Normal</td>
<td>l</td>
<td>$t_l \equiv \gamma_l \in [1,4]$</td>
<td>$0.5 \left(1 + \text{erf} \left( \frac{x - \gamma_l}{\sqrt{2}} \right) \right)$</td>
</tr>
<tr>
<td></td>
<td>h</td>
<td>$t_h \equiv \gamma_h = 4$</td>
<td>$0.5 \left(1 + \text{erf} \left( \frac{x - 4}{\sqrt{2}} \right) \right)$</td>
</tr>
<tr>
<td>Lognormal</td>
<td>l</td>
<td>$t_l \equiv e^{\gamma_l+0.5} \in [1,4]$</td>
<td>$0.5 \left(1 + \text{erf} \left( \frac{\ln x - \gamma_l}{\sqrt{2}} \right) \right)$</td>
</tr>
<tr>
<td></td>
<td>h</td>
<td>$t_h \equiv e^{\gamma_h+0.5} = 4$</td>
<td>$0.5 \left(1 + \text{erf} \left( \frac{\ln x - 0.85}{\sqrt{2}} \right) \right)$</td>
</tr>
<tr>
<td>Linear</td>
<td>l</td>
<td>$t_l \equiv \mu_l \in [1,4]$</td>
<td>$(K(2 + \mu_l)x - x^2)/(K^2(1 + \mu_l))$</td>
</tr>
<tr>
<td></td>
<td>h</td>
<td>$t_h \equiv \mu_h = 4$</td>
<td>$(K(2 + \mu_h)x - x^2)/(K^2(1 + \mu_h))$</td>
</tr>
</tbody>
</table>

### 3.8 Extensions

Our model is deliberately stylized in many respects. We now discuss some extensions and we show that the main insights of our analysis do not depend on (i) the type of asymmetric information assumed; (ii) the cardinality of the type space.

**Moral hazard.** Suppose that in addition to (or instead of) adverse selection, the capital market is subject to moral hazard: borrowers can increase the expected value of their projects by exerting costly (unobservable) effort. As Innes [1990] has shown, as long as the effort decision generates a family of distributions which satisfy the MLRP ordering, non-monotonic contracts dominate debt.

Innes [1990] assumes that output is perfectly verifiable, hence his conclusions do not directly apply here. However, it is clear what the driving force of the result is: by choosing a non-monotonic contract borrowers have incentives to exert higher effort, because their payoff is zero unless they obtain high earnings. The optimal contract display a pay-for-performance payoff.

In our setup, where output is only coarsely verifiable, optimal contracts are constrained by the profit manipulation possibilities, which reduce the effort exerted by borrowers relative to that in Innes [1990]. However, the results would be similar.
Beyond the two-type case. Consider our assumption that there are just two types. One wonders how the results depend on it. To show that their qualitative properties of the optimal contracts we derive extend to richer type spaces, we characterize the set of separating equilibria for an example that admits a closed form solution.\footnote{As we mentioned at the beginning of Section VI, the existence of separating equilibria is what contrasts the most with the existing results. The solutions for the other, less interesting, cases are available upon request.}

In particular, consider the family of linear density functions described by (3.8). This family has a convenient property: the densities all cross at the same
point. Indeed, it is easy to verify that for any pair of types \((t, t') \in T\), \(f_t(x) = f_{t'}(x)\) if and only if \(x = K/2\), in which case \(f_t(x) = 1/K\) for every \(t\).

Consider now the incentive compatibility constraint (4.6) at the limit bonus contract given by the solution to (3.6). Differentiating the LHS with respect to the type \(t'\) yields\(^{20}\):

\[
\frac{(\mu_{t'} - \mu_t)(2\eta + 3K - 4m_{t'}^{\text{max}})(m_t^{\text{max}} - 2\eta)^2}{6K(1 + \mu_{t'})(1 + \mu_n)} \tag{3.9}
\]

To achieve separation we must have that \(m_t^{\text{max}} > 2\eta\), so the expression is negative if and only if:

\[
m_t^{\text{max}} < \frac{\eta}{2} + \frac{3K}{4} \tag{3.10}
\]

We know that the two inequalities describe a non-empty set of earnings realizations if \(K > 2\eta\). Using (3.10), we can sign the derivative of the incentive constraint given by (3.9) with respect to type \(t'\), which is always negative in the relevant range, i.e. for every \(x \leq K/2\).

The result has an immediate economic interpretation. It tells us that if a type \(t\) can separate from a type \(t' < t\), then it can separate from any other \(t'' \in (t', t)\).

We can now restate our Theorem 1 for this case:

**Proposition 3.5.** Suppose that the pdfs are described by (3.8) for every \(t \in T\), and suppose that \(T = \{t_1, t_2, ..., t_N\}\). If there exists a bonus contract with threshold \(m_2 \leq m_t^{\text{max}}\) that satisfies the incentive constraint for the pair \((t_2, t_1)\), then:

1. There exists a fully separating equilibrium \(e^*_s\) in which every \(t \in T \setminus \{t_1\}\) issue a contract as in (3.3) such that the financiers make zero profits. The contracts are such that \(d_n < ... < d_2\);

2. Type \(t_1\) is indifferent between any contract such that \(E_1(s) = I\). If it is a bonus contract, though, it must be such that \(d_1 > d_2\);

3. No pooling equilibrium satisfies the Intuitive Criterion.

**Proof.** From our previous analysis we know that if the incentive constraint for the pair \((t, t_1)\) is satisfied, then the one for any pair \((t, t')\) such that \(t' \in T \setminus \{t_1\}\) also holds. Because of our special distributitional assumption, when the condition holds for \(t_1\), then it holds for all \(t \in T \setminus \{t_1, t_2\}\) and a fully separating equilibrium in which financiers make zero profits exists. That no pooling equilibrium is reasonable can be proved as in the two-type case. Q.E.D.

\(^{20}\)Technically, we can take such a derivative only if we assume a continuum type space. We suppose so, and later we shall draw a finite set of types from such a continuum.
Theorem 5 is given for a specific distribution, as the general case is difficult to analyze\textsuperscript{21}. However, it clearly shows that our main result on the signaling property of capital structure does not depend on the two-type assumption.

3.9 Conclusion

We have shown that the optimal financial contract under ex ante asymmetric information, limited liability and ex post profit manipulation has the following features: (i) it is non-monotonic in earnings; (ii) it exhibits profit manipulation on-the-equilibrium path. That is, the standard justification for restricting attention to monotonic, manipulation-proof securities is not sound. The results suggest that ex ante asymmetric information is not sufficient to theoretically justify the optimality and the widespread use of debt contracts.

We derive necessary and sufficient conditions for monotonic securities to arise in equilibrium. Monotonic securities are never uniquely optimal, and they may prevail only if both earnings are bounded, and feasibility is binding.

\textsuperscript{21}To prove that the incentive constraints are ordered in the type space one deals with two countervailing forces: on the one hand, lower quality types have more to gain by mimicking higher ones. But, on the other hand, they are the ones for whom the costs of mimicking are the highest. Which of these two forces prevails is clear with the analytically tractable linear densities, but not for general distributions. One can prove graphically that the order of incentive constraints holds also for the exponential and truncated normal distributions. Such results are available from the authors upon request.
Appendix for: Optimal Security Design under Asymmetric Information and Profit Manipulation

Proof of Lemma 3

Proof. From the definition of $m^{IC}_h$ we know that, if $m^{IC}_h$ exists, it must generate a threshold $\tilde{x}^{IC}_h$ such that:

$$\int_0^{\tilde{x}^{IC}_h} \left( F_l(x) (1 - F_h(\tilde{x}^{IC}_h)) - F_l(x) (1 - F_h(\tilde{x}^{IC}_h)) \right) \left( 1 - \eta'(x) \right) dx$$

$$+ \left[ F_l(\tilde{x}^{IC}_h) - F_h(\tilde{x}^{IC}_h) \right] \left[ I - \tilde{x}^{IC}_h + \eta(\tilde{x}^{IC}_h) \right] = 0.$$ 

The proof consists on showing that the derivative of the incentive constraint with respect to $\tilde{x}_h$ evaluated at $\tilde{x}^{IC}_h$ is strictly negative.

Differentiating the incentive constraint (4.6) with respect to $\tilde{x}_h$ yields:

$$\frac{\partial IC}{\partial \tilde{x}_h} = \left( f_l(\tilde{x}_h) - f_h(\tilde{x}_h) \right) [I - \tilde{x}_h + \eta(\tilde{x}_h)] - \left( F_l(\tilde{x}_h) - F_h(\tilde{x}_h) \right) *$$

$$* [1 - \eta'(x)] + \left( F_l(\tilde{x}_h) - F_h(\tilde{x}_h) \right) [1 - F_l(\tilde{x}_h)] \left( 1 - \eta(\tilde{x}_h) \right) +$$

$$- f_h(\tilde{x}_h) \left[ \int_0^{\tilde{x}_h} F_l(x) [1 - \eta'(x)] dx + f_l(\tilde{x}_h) \left[ \int_0^{\tilde{x}_h} F_h(x) [1 - \eta'(x)] dx \right] +$$

$$+ \left\{ f_l(\tilde{x}_h) \left[ \int_0^{\tilde{x}_h} F_l(x) [1 - \eta'(x)] dx - f_l(\tilde{x}_h) \left[ \int_0^{\tilde{x}_h} F_l(x) [1 - \eta'(x)] dx \right] \right\}$$

where the last row is obtained adding and subtracting the same expression to the derivative, and it is introduced so that the derivative simplifies to:

$$\frac{\partial IC}{\partial \tilde{x}_h} = \left( f_l(\tilde{x}_h) - f_h(\tilde{x}_h) \right) [I - \tilde{x}_h + \eta(\tilde{x}_h)] + \int_0^{\tilde{x}_h} F_l(x) [1 - \eta'(x)] dx$$

Evaluating the derivative at $\tilde{x}^{IC}_h$ yields:

$$\left( f_l(\tilde{x}_h) - f_h(\tilde{x}_h) \right) \left[ \int_0^{\tilde{x}^{IC}_h} \left( F_l(x) (1 - F_h(\tilde{x}^{IC}_h)) - F_l(x) (1 - F_h(\tilde{x}^{IC}_h)) \right) \right] *$$

$$* [1 - \eta'(x)] dx + \left( f_l(\tilde{x}_h) - f_h(\tilde{x}_h) \right) \left[ \int_0^{\tilde{x}_h} F_l(x) [1 - \eta'(x)] dx \right] -$$

$$- f_l(\tilde{x}_h) \left[ \int_0^{\tilde{x}_h} (F_l(x) - F_h(x)) [1 - \eta'(x)] dx \right]$$

which can be rewritten as:

$$\frac{\int_0^{\tilde{x}^{IC}_h} \left( F_l(x) - F_h(x) \right) [1 - \eta'(x)] dx}{\left( f_l(\tilde{x}_h) - f_h(\tilde{x}_h) \right)} > 0 \text{ by POSD and because } \eta'(x) < 1$$

$$\left[ f_l(\tilde{x}_h) \left[ 1 - F_h(\tilde{x}_h) \right] - f_h(\tilde{x}_h) \left[ 1 - F_l(\tilde{x}_h) \right] \right] < 0 \text{ because MLRP implies HRO}$$

The fraction is strictly positive whenever $\tilde{x}_h > 0$, which is clearly satisfied at every contract that implements investment. The second bracket is negative because we assumed strict MLRP, and it
is well known that strict MLRP implies the strict HRO - hazard rate ordering - which in turns guarantees that the bracket is strictly negative.

An immediate consequence of the strict inequality is that if the incentive constraint crosses zero, it must do so only once. Lemma 3 follows. Q.E.D.

Proof of Theorem 1

Proof. Claims 1 and 2: Follow from the previous discussion, and the fact that if \( t_1 \) issues a bonus contract with \( d_1^t \leq d_h^t \) that breaks even on his type, the good type would mimic him and he would end up with a rate of repayment higher than one.

Claim 3: Suppose that all agents are in the pooling equilibrium \( \hat{e} \) of the game. Type \( t = h \) (the better type) is certainly paying a strictly positive net rate of return to the investors. No type other than \( t = h \) is in the set \( \Theta \) for a security \( s' \) that satisfies (4.6). Hence, the Intuitive Criterion implies that the investor must believe that the deviation comes from type \( h \) with probability one. If this is so, the deviation is profitable and the pooling equilibrium does not satisfy the Intuitive Criterion. Q.E.D.

Proof of Corollary 1

Proof. To establish this result, some preliminary steps are required.

Denote the bonus contract with \( m_h = m_h^{max} \) and \( d_h = 0 \) as \( s^* \), and compare it with another generic security \( s \) such that \( \mathbb{E}_h(s^*) = \mathbb{E}_h(s) = I \). Define the following sets:

\[
\Pi_+ (s) \equiv \{ m | s^* (m = x) > s(m = x) \}
\]

\[
\Pi_- (s) \equiv \{ m | s^* (m = x) < s(m = x) \}
\]

Lemma 4. For every pair \((m_l = x_l, m_h = x_h)\) in \( X^2 \) such that \( m_l \in \Pi_+ \) and \( m_h \in \Pi_- \) we have \( m_h > m_l \). Moreover, \( m^\ast (x_l | s^*) \geq m^\ast (x_l | s) \) and \( m^\ast (x_h | s^*) \leq m^\ast (x_h | s) \).

Proof. First notice that \( \Pi_+ (s) = \emptyset \) if and only if \( \Pi_- (s) = \emptyset \), because \( f_t(x) > 0 \) for every \( x \in [0, K] \), for every \( t \in T \). In this case the lemma is not very useful, but it is still satisfied. Suppose \( \Pi_+ (s) \) is non-empty. Because of limited liability, it must be the case that \( m_h > m_h^{max} - \eta(m_h^{max}) \) for every \( m_h \in \Pi_- \), and \( m_l < m_l^{max} - \eta(m_l^{max}) \) for every \( m_l \in \Pi_+ \). As for the claim about the real payoff, it follows directly from the shape of \( s^* \). Q.E.D.

Lemma 5. Denote the bonus contract with \( m_h = m_h^{max} \) and \( d_h = 0 \) as \( s^* \). For any generic security \( s \) such that \( \mathbb{E}_h(s^*) = \mathbb{E}_h(s) = I \), we have that \( \mathbb{E}_s (s^*) > \mathbb{E}_s (s) \).

Proof. The only interesting case is, again, when \( \Pi_+ (s) \) is non-empty (else the lemma holds trivially). Suppose so. Furthermore, suppose we move from \( s^* \) toward \( s \) through a series of steps such that in each step we create a security \( s' \) such that \( \mathbb{E}_s (s') = I \), but there exists a small interval \( dx_a \in \Pi_+ (s) \) such that \( s'(m^\ast(dx_a)) < s^*(m^\ast(dx_a)) \) and this change is compensated by inducing a change in the real payoff for another small interval \( dx_b \in \Pi_- (s) \) so that \( s'(m^\ast(dx_b)) > s^*(m^\ast(dx_b)) \). Then,

\(^{22}\)In both cases, construct the interval such that it is of equal length as the pdf centered at the two points: \( f(x_a), f(x_b) \)
\[ E_t(s^*) - E_t(s') = f_t(x_a)[s^*(m^*(dx_a)) - s'(m^*(dx_a))] + f_t(x_b)[s^*(m^*(dx_b)) - s'(m^*(dx_b))] \]

\[ < 0 \text{ by construction} \]

\[ < 0 \text{ by MLRP} \]

where the second equality comes from \( E_h(s^*) = E_h(s') \). The iteration of this procedure one step at a time concludes the proof. \( Q.E.D. \)

Because of Lemma 5 we know that \( E_h(s^*) - E_h(s^*) < E_h(s) - E_t(s) \), for every \( E_h(s^*) = E_h(s) = I \). The Corollary follows. \( Q.E.D. \)

**Proof of Theorem 2**

**Proof.** \textbf{Existence:} Suppose there exists an \( m_a \) that satisfies the pooling zero profit condition. Define the security \( s_p \) so that: \( d_p = 0 \) and \( m_a \) solves the pooling zero profit condition. Moreover, suppose that the market posterior is equal to the prior at \( s_p \), and it is \( \lambda_h = 0 \) at any other \( s' \neq s_p \) such that \( s' \in S \). Then, all types issuing \( s_p \) is an equilibrium. It remains to show that it satisfies D1. In particular, we need to prove that \( D(1|s') \cup D'(1|s') \not\subseteq D(2|s') \) for every \( s' \neq s_p \) such that \( s' \in S \). There are two cases:

1. If \( E_t(s') < E_t(s_p) \), then \( D(1|s') = [I, \infty) \). Hence it must be that \( D(2|s') \subseteq D(1|s') \cup D'(1|s') \);

2. If \( E_t(s') \geq E_t(s_p) \), Lemma 5 implies \( E_h(s') \geq E_h(s_p) \) as well. But we can say more:

   Suppose we move from \( s_p \) to \( s' \) through a series of consecutive steps (i.e. interim contracts \( s'' \)) such that in each step we induce an increase in the real payoff of \( s_p \) by raising \( s''(m_a = x_k) \) for some \( x_k \in X \). Clearly, it must be that \( x_k \geq m_a \). Notice that because \( s_p \) is a pooling equilibrium, it must be that it does not satisfy (4.6). Hence, because of MLRP, at \( x_k \) we must have \( f_t(x_k) < f_h(x_k) \) - i.e. \( x_k \) must exceed the (unique) crossing point of the two densities. Therefore:

\[ E_t(s') - E_t(s_p) = f_t(x_a)[s''(m^*(x_k|s'')) - s_p(m^*(x_k|s_p))] \]

\[ = f_t(x_a)(s''(m^*(x_k|s''))) < f_h(x_k)(s''(m^*(x_k|s''))) = E_h(s'') - E_h(s_p) \]

Iterating the same logic we conclude that \( E_h(s') - E_h(s_p) > E_t(s') - E_t(s_p) \). It follows that at \( c_p^* \) it must be the case that, for all \( P^* \geq I \):

\[ (V_{c_p}^* - V_{c_p}^*) - (V_{c_p}^* - V_{c_p}^*) = (E_t(s') - E_t(s_p)) - (E_h(s') - E_h(s_p)) < 0 \]

which implies that \( D(2|s') \subseteq D(1|s') \cup D'(1|s') \) again.

\textbf{Uniqueness:} From Corollary 1 we know that there can only exist other pooling equilibria if the conditions required for Theorem 1 to apply do not hold. We now show that if there exists an \( m_{a} \in (m_{a}^{max}, K - \eta(K)) \) such that (3.7) is satisfied, then every pooling equilibrium \( e' \) of the game such that \( e' \neq e_p^* \) does not satisfy D1.

Consider a generic \( e' \neq e_p^* \). From the analysis above and Lemma 5, we know that there exists at least an \( s' \) such that \( E_t(s') \geq E_t(s_p) \) but \( E_h(s') < E_h(s_p) \). Then the logic of the previous proof (point 2 above) is reversed. We conclude that such an equilibrium does not satisfy D1. \( Q.E.D. \)

**Proof of Theorem 3**
**Proof.** Part (1) can be proved in the same fashion as Theorem 2, with a twist: now it must be the case that a bonus contract with \( d = 0 \) cannot satisfy the pooling zero profit condition. Hence, we start by finding the minimum \( d > 0 \) such that the condition can be satisfied. Then, the result follows from the logic of the previous proof.

Part (2) follows from the fact that with a contract as in (1) we are hitting the upper bound of the distribution of earnings. If such a contract does not exist, then any other security could not break even for the financier. *Q.E.D.*

**Proof of Theorem 4**

**Proof.** When \( K \to \infty \) there always exists an \( m_{\max} \) such that the pooling zero profit condition is satisfied for a face value of debt of \( d_p = 0 \), because \( f_t(x) > 0 \) for every \( x \in X \) and every \( t \in T \). Moreover, regardless of the extent profit manipulation, as long as it is bounded, the pooling contract with \( d_p = 0 \) has a real payoff which is non-monotonic. As a result, any contract with a monotonic real payoff cannot be part of an equilibrium that satisfies D1. *Q.E.D.*
Chapter 4

Credit Failures

4.1 Introduction

In the aftermath of the crisis of 2008, government intervention in financial markets has been widespread\(^1\), and it has prompted economists to focus, primarily, on two questions. First, how should a bailout be designed? In particular, should governments inject equity capital into troubled institutions, as happened in the UK, or should they favour asset purchases and debt guarantees, as the US government did\(^2\)? Second, assuming that government interventions are inherently advantageous relative to market solutions, what size and type of intervention can better leverage on government capabilities\(^3\)?

With only few who would openly advocate that governments let financial institutions go under, a debate on intervention in financial markets is again taking place. More broadly, Keynesian claims that market economies may suffer low or fluctuating output and employment and that active monetary and fiscal policy is both effective and desirable have gained interest. This is an important debate, to which we wish to contribute. In particular, we want to emphasize that an argument for intervention requires consideration of both sides of the balance sheet of the government, and explicitly accounting for any opportunity cost. Whether or not balanced interventions are desirable naturally feeds back into more practical matters

\(^1\)In the US, as of March 2012, the net government bailout outlays amounted to $3.3 trillions. These include the TARP (Troubled Asset Relief Program), Treasury outlays and Federal Reserve outlays. The gross bailout is estimated at $4.6 trillion. In Europe, the scope and size of intervention has differed substantially across countries. In the UK, according to the National Audit Office, in 2011 the net outlays to the financial sector amounted to £512 billion. Smaller bailouts packages have been issued in France and other continental European countries.

\(^2\)An issue of the *American Economic Review*, 102(1) of 2012, was dedicated to this question; the contributions by Philippon and Skreta [2012] and Tirole [2012] are particularly relevant.

\(^3\)Gertler and Kiyotaki [2010] and Holmstrom and Tirole [2011] address this question.
such as the optimal design of bailout (or stimulus) packages that we clarify.

Chamley [2013] sets up the question in an ingenious and methodologically sound framework: (i) matching frictions \(^4\) prevent the economy from operating at first best, and (ii) the government does not have inherent advantages (nor disadvantages) relative to market participants. Not all dynamic equilibria are Pareto optimal, and dynamic adjustment may lead to “savings traps.” Suboptimality derives from failures of \textit{effective demand} that was introduced by Say [1803], and it plays a dominant role in Keynesian arguments. It refers to the demand that is actually satisfied at equilibrium and may reflect multiple constraints; in contrast, \textit{notional demand} which refers to demand subject only to the budget constraint. The concept was first formalized in the context of equilibria with price rigidities by Drèze [1975] and Benassy [1975], and gave rise to extensive work that provides microeconomic foundations for Keynesian arguments, such as Barro and Grossman [1971], Benassy [1975] and Malinvaud [1977].

The work on fix-price equilibria, appropriately understood, reveals an evident but important intuition: the theorems of classical welfare economics require that prices are the only signals that serve to coordinate economic activity; if, alternatively, other parameters such as perceived constraints on trades are allowed to play a role, than even simple Edgeworth-box economies can generate multiple equilibria where, typically, effective and notional demand diverge, and, evidently, Pareto optimality fails. This is indeed true in the construction of Chamley [2013], where a Pareto optimal dynamic equilibrium exists and is selected exactly by not allowing individuals to consider constraints on their choices other than the budget constraint. As a consequence, it suffice for the government to select the efficient equilibrium, as is the case in Cooper and John [1988] where Keynesian effects derive from strategic complementarities.

Other recent work relaxes an alternative underpinning for the classical theorems of welfare economics to hold: the completeness of the asset market. For example, Lorenzoni [2008] shows that inefficient credit booms may arise when asset markets are incomplete. Similar arguments can generate inefficient credit busts, for that matter, because it is well known since Greenwald and Stiglitz [1986] and Geanakoplos and Polemarchakis [1986] that markets are constrained inefficient in economies with exogenously incomplete asset markets.\(^5\) The incomplete markets

\(^4\)Similar frictions were considered in Diamond [1982] and Green and Zhou [2002].

\(^5\)Although in these papers government interventions were modelled as reallocations of asset portfolios, the result can be implemented via transfers and subsidies as well (See Citanna et al. [2006]). Further, it applies to economies with (exogenously) uninsurable idiosyncratic risk, as shown in Carvajal and Polemarchakis [2011].
approach does provide a stronger reason for the government to intervene, as all competitive equilibria are (generically) constrained Pareto suboptimal. However, it relies on the assumption that the asset market is exogenously incomplete, begging the question as to why this is the case.

In summary, the former approach that lead to Chamley’s results rests on modeling a friction explicitly, and obtains multiple Pareto ranked equilibria, whereas the latter approach we described relies on exogenously assuming that markets are incomplete and obtains a set of constrained suboptimal competitive equilibria. It remains a challenge to construct a model in which

1. markets are as complete as the fundamentals of the economy allow,
2. competitive equilibria are not optimal, and
3. interventions compatible with the fundamentals allow for superior allocations.

In this paper, we construct an economy with hidden information where all three properties hold. First, hidden information leads to endogenously incomplete asset markets because individuals cannot trade assets contingent on their (unobservable) idiosyncratic risk. Second, severe hidden information problems result in all competitive equilibria being constrained suboptimal. Third, balanced-budget government interventions improve upon the market outcome even if the government does not have better information relative to market participants. We deliberately consider a simple framework where the Pareto improving government interventions can be fully characterised analytically, in order to clarify how modeling explicitly the revenues side of a government’s balance sheet feeds back into the optimal design of its spending programs.

More specifically, we introduce a labour market in an economy otherwise similar to Philippon and Skreta [2012]. There is a continuum of measure one two types of agents: savers-workers and entrepreneurs; and two dates. The workers are endowed with labour at both dates, but only want to consume at the end and hence they save all their labour income in the first period. At each date, they can either enter the labour market (i.e. work for some entrepreneur), or be self employed. We assume that under full information labour is more productive if the worker enters the labour market. The entrepreneurs are endowed with a technology that, if productive, can employ future labour to produce consumption good at the end period. The entrepreneurs are of heterogenous, privately known, types, and the type

\[ More precisely, fundamentals are such that separating equilibria cannot exist and, as a result, all assets payoffs cannot be contingent on agents’ private information. \]
affects the probability that the technology will be productive. All types can increase their chances of being able to produce by employing a fixed amount of labour at the initial date, and investment is efficient under full information.\(^7\) Entrepreneurs are initially cashless, but can borrow from a bank in order to pay the workers’ wages at the initial date. The wages are then deposited in the bank until the final date, at which (i) the productive entrepreneurs pay back the bank, and (ii) workers withdraw from the bank and consume the proceeds of their work.

In this setup, and despite the presence of a non-convexity in the investment technology, competitive equilibria always exist. However, whenever at a competitive equilibrium some entrepreneur is not investing, the equilibrium is \textit{constrained inefficient}: there exist balanced-budget government interventions that improve upon the market outcome. Furthermore, for some parameter values - namely, when hidden information problems are sufficiently severe - there only exist inefficient equilibria.\(^8\)

Our results are driven by the interaction of two factors. First, markets are endogenously incomplete: the entrepreneurs cannot trade commodities contingent on their (privately known) type. Second, a labour (spot) market opens after the investment decisions have been made. When asymmetric information in financial markets becomes more severe, high quality entrepreneurs prefer not to borrow and cut down on their investment plans. As a result, the equilibrium wage decreases. The government does not have better (or worse) information than market participants, and it is assumed not to face internal agency costs. It can introduce taxes and subsidies in markets, but is constrained to adopt balanced-budget interventions. We show that the relative price effect introduced by the presence of both credit and labour markets can be exploited by the government to improve upon the market allocation. The optimal interventions can be implemented combining a savings (or income) tax with two types of subsidies: a direct - lump-sum - subsidy to investment and an interest rate subsidy. The workers are weakly better off after the intervention because they benefit from the increase in future wages, whereas the entrepreneurs are able to invest at a reduced informational cost.

The results differ from those derived in Bisin and Gottardi [2006], Dubey and Geanakoplos [2002] and Rustichini and Siconolfi [2008] mainly because they

\(^7\)This formulation allows us to work with a state space that contains only two states (productive; unproductive) and with a fixed investment level. Both ingredients are essential in ruling out any signaling possibility.

\(^8\)Contrary to Philippon and Skreta [2012], we abstract from the security design question by assuming that there are only two states of the world, and in one state entrepreneurs have no resources available to pay back their creditors. However, notice that even in models with multiple states it is not obvious that signalling can always be ruled out, as emphasised by Koufopoulos et al. [2014].
study economies where re-trading is not allowed, i.e. no spot market opens after the realisation of uncertainty. The implication of allowing for spot markets in more general environments subject to adverse selection is left for future research.

4.2 The Economy

4.2.1 Primitives

We consider an economy which lasts two periods $t = \{0, 1\}$. All agents are risk-neutral and consume only at $t = 1$.

There is a continuum $[0, 1]$ of two kind of agents: savers-workers and entrepreneurs.

The savers-workers are endowed with $l_0 = \bar{x}$ units of time at date zero, and $l_1 = 1$ at date one. At each date, they can either enter the labour force, or remain self employed. Self employment is a constant return to scale technology: every unit of labour supplied yields one unit of date one consumption good.

The entrepreneurs privately know their type, indexed by $\theta$, and it is common knowledge that types are drawn from a compact set $\Theta \equiv [\underline{\theta}, \overline{\theta}]$ with cumulative distribution function $F(\theta)$. An entrepreneur of type $\theta$ has potential access to a technology that produces date one consumption good by means of labour at time one $f(l_{1,\theta})$, where $l_{t,\theta}$ denotes the labour demand of an entrepreneur of type $\theta$ at date $t$. The probability that the entrepreneur can use his technology depends on (i) his type, (ii) an investment undertaken at date zero, and it is denoted by $p(\theta, l_{0,\theta})$. In particular:

$$p(\theta, l_{0,\theta}) = \begin{cases} p_{\theta} & \text{if } l_{0,\theta} \geq x > 0 \\ p_{\theta} & \text{if } l_{0,\theta} < x > 0 \end{cases}$$

The technology is common for all entrepreneurs, and it exhibits decreasing returns to scale: $f(l_{1,\theta}) = \xi l_{1,\theta}^{\alpha}$ for some $\alpha \in (0, 1)$ and $\xi > 0$. Also, suppose that $p_{\theta} > p_{\theta'}$; $x < \bar{x}$ and, without loss of generality, let $p_{\theta} > p_{\theta'}$ whenever $\theta > \theta'$.

All agents are risk-neutral and do not derive any utility from leisure. As a result, the workers save all their $t = 0$ income (both the labour income, and the income derived from self employment). It is useful to think that they deposit it into a bank, and have in mind the following circuit: (i) the bank lends to the entrepreneurs the amount they need to pay the workers at date zero; (ii) then, it gets the worker’s savings as a deposit; and (iii) finally, at $t = 1$, it collects repayments from productive entrepreneurs and uses it to pay the workers.
The timing of the economy is as follows:

- **date 0**: The (privately informed) entrepreneurs decide whether to borrow from a bank and employ labour $x$ or not. The workers decide whether to enter the labour force or remain self employed;

- **date 1**: labour is employed (for the technologies that are operative) and output is realised; obligations are repaid (whenever possible); consumption takes place.

Denote the wage at date $t$ by $w_t$ and the (gross) loan rate by $r$. The $t = 1$ consumption good is taken as the numéraire, and its price is normalised to one. Further, we can normalise $w_0 = 1$. To simplify the notation, let $w_1 = w$.

**Separating contracts.** Under full information the credit market would be segmented: each type of entrepreneur would face a loan rate $r(\theta) = \frac{p - 1}{\theta}$. The loan rate is easily pinned down by a no arbitrage condition: the return to storage (equal to one) must be equated to the expected return from lending to entrepreneur ($r \frac{p - 1}{\theta}$).

When information is asymmetric, the loan rate $r$ might depend on $\theta$ only at a (possibly partially) separating equilibrium. However, such equilibrium only obtains under *random contracts*, i.e. contracts that consist in both a loan rate, and a probability with which the loan is granted strictly between zero and one for all $\theta \in \Theta \setminus \{\bar{\theta}\}$. If contract are not random, we cannot have $r(\theta)$ such that $r(\theta) \neq r(\theta')$. To see why, notice that the expected repayment of a type $\theta$ agent who declares to be $\theta'$ and invests is $r(\theta')x$. All agents will choose $\theta'$ so that $\theta' = \arg \min \{r(\theta)\}$, and separation cannot occur.

We do not consider such contracts in the analysis for the following reasons:

- First, our qualitative results do not change if we allow for such contracts. In particular, random contracts never implement full investment because all types higher than $\bar{\theta}$ have to choose a probability of investment strictly less than one. In contrast, we shall prove that full investment is implementable as a competitive equilibrium with government intervention;

- Second, they would require us to introduce incentive compatibility constraints in the definition of a competitive equilibrium;

- Finally, random contracts of this type are not observed empirically. Although this is not a valid critique, there may be a theoretical reason: these contracts

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9 We can normalise an additional price because we are left with three goods and four prices.
are not renegotiation proof. Suppose that an entrepreneur offers a random contract. After the contract is signed, there exists a Pareto improvement available for both parties: increase the probability of investment to one and split the surplus. Since renegotiation would be anticipated, incentive compatibility would not be satisfied hence and no separation could take place.

As a result, in our analysis the loan rate \( r \) cannot depend on \( \theta \), and competitive equilibria are defined as standard. It is useful to present the three maximisation problems that agents solve at equilibrium:

At date zero, the bank chooses whether to lend to the entrepreneurs or not. Each entrepreneur who decides to invest needs to borrow \( x \), because the wage at \( t = 0 \) has been normalised to one and it is never optimal to employ more that \( x \). The bank chooses optimally lends an amount \( b^* \) that solves:

\[
b^* = \arg \max_{b \in \mathbb{R}} b [r^* \mathbb{E}[p_\theta | r^*] - 1]
\]

Clearly, the interior solution \( b^* = x \) requires the no-arbitrage condition \( r^* \mathbb{E}[p_\theta | r] = 1 \) to hold.

The entrepreneurs have two decisions: at date one – if they are productive – they decide how much labour to employ, and \( l^*_{1,\theta} \) solves \( f'(l^*_{1,\theta}) = w \).

At date zero, they choose whether to employ \( x \) units of labour or not correctly forecasting their future \( l^*_{1,\theta} \), and for a given vector of prices:

\[
l^*_{0,\theta} = \arg \max_{l_{0,\theta} \in \{0,x\}} l_{0,\theta} \{ x^{-1}[f(l^*_{1,\theta}) - w l^*_{1,\theta}] - r \} + (x - l^*_{0,\theta}) x^{-1} p_\theta [f(l^*_{1,\theta}) - w l^*_{1,\theta}]
\]

Before proceeding to the analysis, it is useful to recall the standard definition of a competitive equilibrium:

**Definition 1:** A competitive equilibrium (CE) consists of a vector of prices \( (r^*, w^*) \) such that: (i) all agents maximise expected utility subject to budget constraints; (ii) markets clear.

Define the effective interest rate for type \( \theta \) as \( r_\theta \equiv \bar{p}_\theta r \). It is individually rational for an entrepreneur of type \( \theta \) to invest if and only if the expected increase in his profits from production (i.e. \( (\bar{p}_\theta - p_\theta) \xi (1 - \alpha) l^*_{0,\theta} \)) more than offsets the cost
of investing (given by $r_\theta x$):

$$
(\bar{p}_\theta - \underline{p}_\theta)\xi(1 - \alpha)l_\theta^a \geq r_\theta x
$$

(4.1)

Furthermore, in equilibrium, each productive entrepreneur demands a quantity of labour such that its marginal productivity equals the wage rate $w$, i.e. $l_\theta = (w^*/\alpha \xi)^{1/(\alpha - 1)}$. The aggregate labour supply is inelastic: $L_S = 1$, whereas the aggregate labour demand is denoted by $L_D \equiv \int_\Theta l_\theta dF(\theta)$ and it is maximal when all types invest. In this case, we would have:

$$
L_D \big|_{\text{Full Investment (FI)}} = \left(\frac{w^*}{\alpha \xi}\right)^{\frac{1}{\alpha - 1}} \left(\int_\Theta \bar{p}_\theta dF(\theta)\right) = 1 = L_S
$$

As a consequence, under full investment we observe the highest feasible equilibrium wage: $w^*_{\text{FI}} = \alpha \xi (\int_\Theta \bar{p}_\theta d\theta)^{1-\alpha}$; and hence the lowest individual labour demand by any type $\theta$ entrepreneur who is productive: $l^*_{\theta, \text{FI}} = (\int_\Theta \bar{p}_\theta dF(\theta))^{-1}$.

We make the following assumptions on parameters:

**Assumption 4.1.** Investment has positive net present value for all types under full information:

For every $\theta$: $$(\bar{p}_\theta - \underline{p}_\theta)\xi(1 - \alpha)\left(\int_\Theta \bar{p}_\theta dF(\theta)\right)^{-\alpha} \geq x$$

where we substituted in (4.1): (i) the full information equilibrium loan rate for type $\theta$: $r_\theta = \bar{p}_\theta^{-1}$; and (ii) the lowest possible equilibrium labour demand $l^*_{\theta, \text{FI}}$.

**Assumption 4.2.** Type-independent benefits from investment:

For every $\theta$: $$(\bar{p}_\theta - \underline{p}_\theta) = q$$

Assumption 4.2 is not needed to derive our results, however it greatly simplifies the notation and makes our results immediately comparable to those of other related models that make the same assumption such as Philippon and Skreta [2012].

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10\textsuperscript{To obtain the expression for the expected increase in his profits from production notice that, since $f'(l_\theta) = w$:}

$$(\bar{p}_\theta - \underline{p}_\theta)[\xi l_\theta^a - wl_\theta] = (\bar{p}_\theta - \underline{p}_\theta)[\xi l_\theta^a - \alpha \xi l_\theta^{a-1} l_\theta] = (\bar{p}_\theta - \underline{p}_\theta)\xi(1 - \alpha)l_\theta^a
$$

11\textsuperscript{Evidently, the condition guarantees that for every equilibrium labour demand $l_\theta > l^*_{\theta, \text{FI}}$ investment is of positive net present value under full information.}
Assumption 4.3. The distribution function $F(\theta)$ is continuous over $\Theta$ and $F(\theta) = 0$. Both $\underline{p}_\theta$ and $\underline{p}_\theta$ are continuous in $\Theta$ under the integral sign.

Assumption 4.3 is made for technical reasons, and it simplifies the arguments.

4.2.2 Properties of equilibria

Before we proceed to the results, it is useful to highlight two key properties of competitive equilibria in our setup.

First property of equilibria: whenever type $\theta$ invests in equilibrium, so do all types $\theta' < \theta$.

Proof. To see this, consider condition (4.1). In equilibrium we always have $l^*_{\theta} = l$, the same constant for every $\theta$ - regardless of whether they invested or not. Hence, the left hand side does not depend on $\theta$. As for the right hand side, its derivative with respect to $\theta$ is equal to $r^*x > 0$. As a result, whenever type $\theta$ invests, all types $\theta' < \theta$ do as well. \qed

From now onwards, define $\hat{\theta} \equiv \max_{\theta \in \Theta} \{\theta \mid (4.1) \text{ holds}\}$.

Second property of equilibria: an equilibrium can be fully characterised by a threshold $\hat{\theta}$.

Proof. To see that this is the case, we show that both the loan rate $r$ and the wage $w$ in equilibrium only depend on $\hat{\theta}$. From (4.1) it then follows that all endogenous quantities only depend on the threshold for investing agents $\hat{\theta}$.

The equilibrium loan rate $r^*$ is pinned down by a no arbitrage condition: workers must be indifferent between using the storage technology and lending to the entrepreneurs. Because consumption at date one is the numéraire, the return of the storage technology is one. No arbitrage and the first property of equilibria we just derived yield: \footnote{Notice that only the limit of the expression is well defined at $\theta = \hat{\theta}$, in which case by de l’Hôpital rule we get: $r^* \to \underline{p}_{\hat{\theta}}^{-1}$. Because this limit is equivalent to the full information loan rate for the lowest productivity types, as one would have expected, we define $r^*(\hat{\theta}) = \underline{p}_{\hat{\theta}}^{-1}$.}

\begin{equation}
    r^*(\hat{\theta}) = \frac{F(\hat{\theta})}{\int_0^{\hat{\theta}} \underline{p}_\theta dF(\theta)} \tag{4.2}
\end{equation}
It is easy to see that the equilibrium loan rate \( w^* \) is given by:

\[
 w^*(\hat{\theta}) = \alpha \xi \left( \int_{\theta}^{\hat{\theta}} P_\theta dF(\theta) + \int_{\hat{\theta}}^{\bar{\theta}} P_\theta dF(\theta) \right)^{1-\alpha} 
\]

and as we claimed both prices only depend on \( \hat{\theta} \).

This preliminary analysis allows us to translate the problem of finding the set of equilibrium prices and allocations into the equivalent problem of finding the equilibrium \( \hat{\theta} \). In the next section we derive existence of equilibria, and constrained suboptimality of those competitive equilibria such that \( \hat{\theta} < \bar{\theta} \).

### 4.3 Existence and optimality of competitive equilibria

The key condition that determines the set of competitive equilibria in our setup is inequality (4.1), which identifies the set of types for whom it is individually rational to invest at any given price vector \((r, w)\). From the properties of equilibria we derived, (4.1) can be written only as a function of \( \hat{\theta} \) as follows:

\[
 \beta \equiv q(1 - \alpha) \xi \int_{\theta}^{\hat{\theta}} P_\theta dF(\theta) \left( \int_{\theta}^{\hat{\theta}} P_\theta dF(\theta) + \int_{\hat{\theta}}^{\bar{\theta}} P_\theta dF(\theta) \right)^{\alpha} \beta \equiv h(\theta, \hat{\theta}) 
\]

The left hand side is just a strictly positive constant \( \beta > 0 \), whereas the right hand side is a strictly increasing function of the first argument (i.e. \( \theta \)), and a continuous function of the second argument (i.e. the threshold \( \hat{\theta} \)).

In particular, \( h(\theta, \theta) = \beta \equiv h(\theta, \hat{\theta}) = q(1 - \alpha) \xi \int_{\theta}^{\hat{\theta}} P_\theta dF(\theta) \left( \int_{\theta}^{\hat{\theta}} P_\theta dF(\theta) + \int_{\hat{\theta}}^{\bar{\theta}} P_\theta dF(\theta) \right)^{\alpha} \beta > 0 \), and it is finite.\(^{13}\) Moreover, \( h(\theta, \bar{\theta}) = \beta \equiv h(\theta, \hat{\theta}) = q(1 - \alpha) \xi \int_{\theta}^{\hat{\theta}} P_\theta dF(\theta) \left( \int_{\theta}^{\hat{\theta}} P_\theta dF(\theta) + \int_{\hat{\theta}}^{\bar{\theta}} P_\theta dF(\theta) \right)^{\alpha-1} > 0 \) and finite.

Although evidently the derivative of \( h(\theta, \hat{\theta}) \) with respect to \( \theta \) is strictly positive (i.e. \( h_1(\theta, \hat{\theta}) > 0 \)), that with respect to \( \hat{\theta} \) has an indeterminate sign. The latter observation potentially leads to multiple equilibria for a range of parameter values. Nevertheless, we show that there exist parameters for which the equilibrium is unique.

Notice that \( h(\theta, \hat{\theta}) \) is a continuous function of the threshold \( \hat{\theta} \) which takes values over a compact set. By the extreme value theorem we then know that there exist at least a maximal and a minimal element for every \( \theta \). We are especially interested in the extreme points for \( \hat{\theta} = \bar{\theta} \), for reasons that will become clear below.

\(^{13}\)The equality - instead of the limit operator - is a consequence of our definition \( r^*(\hat{\theta}) = \beta \).

It is useful to define them as follows:

\[ \bar{y} \equiv \max_{\hat{\theta} \in \Theta} h(\bar{\theta}, \hat{\theta}) \]

\[ y \equiv \min_{\hat{\theta} \in \Theta} h(\bar{\theta}, \hat{\theta}) \]

Our first result characterises the set of competitive equilibria of the economy.

**Proposition 4.1.** A Competitive Equilibrium (CE) always exists and:

1. If \( \beta > \bar{y} \), at the unique CE every type invests \( (\hat{\theta} = \bar{\theta}) \);
2. \( \beta < y \), at every CE some type is not investing \( (\hat{\theta} < \bar{\theta}) \);
3. If \( \beta \in [y, \bar{y}] \), there may be multiple CE, possibly with full investment.

**Proof.**

We shall prove existence and (where applicable) uniqueness case by case.

**Case (1).** First, notice that an equilibrium with \( \hat{\theta} = \bar{\theta} \) exists because \( \beta > \bar{y} \geq h(\bar{\theta}, \bar{\theta}) \). Further, the equilibrium with \( \hat{\theta} = \bar{\theta} \) is unique because \( \beta > \bar{y} \geq h(\bar{\theta}, \hat{\theta}) \) for every \( \hat{\theta} \in \Theta \).

**Case (2).** It is obvious that we cannot have an equilibrium with \( \hat{\theta} = \bar{\theta} \) because \( \beta < y \leq h(\bar{\theta}, \hat{\theta}) \) for every \( \hat{\theta} \in \Theta \). It remains to show that an equilibrium with \( \hat{\theta} < \bar{\theta} \) exists.

Notice that Assumption 4.1 can be rewritten as: \( \beta \geq h(\bar{\theta}, \bar{\theta}) \). We know that \( h(\bar{\theta}, \bar{\theta}) \leq \beta < y \leq h(\bar{\theta}, \hat{\theta}) \). Notice that the function \( g(\hat{\theta}) \equiv h(\bar{\theta}, \hat{\theta}) \) is continuous in \( \hat{\theta} \), and a CE is characterised by a threshold \( \hat{\theta} \) such that \( g(\hat{\theta}) = \beta \). The existence of a CE then follows immediately from the intermediate value theorem. Figure 1 shows an example of such case, where there exist three equilibria \((\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3)\).

**Case (3).** Recall from Case (2) that Assumption 4.1 can be written as: \( \beta \geq h(\bar{\theta}, \bar{\theta}) \). We know that \( h(\bar{\theta}, \bar{\theta}) \leq y \leq \beta \leq \bar{y} \). We need to study again the function \( g(\hat{\theta}) \equiv h(\bar{\theta}, \hat{\theta}) \), which is continuous in \( \hat{\theta} \). In particular, we have two cases: (i) if \( \max_{\theta \in \Theta} g(\hat{\theta}) \geq \beta \), then a CE exists by the intermediate value theorem (like in Case (2)); (ii) if instead \( \max_{\theta \in \Theta} g(\hat{\theta}) < \beta \), then there exists a unique equilibrium with \( \hat{\theta} = \bar{\theta} \) (like in Case (1)).

Notice that in Cases (2) and (3) there may well be multiple CE. However, in Case (2) we know that any CE must be such that \( \hat{\theta} < \bar{\theta} \); in Case (3) an equilibrium with full investment may well exist.
Note that Proposition 4.1 identifies a non-empty, dense set of parameter values for which asymmetric information leads to allocations that are all less efficient than the first best - full information - benchmark. The competitive equilibrium allocations are inefficient because they induce agents to leave profitable investment opportunities on the table, and hence to misallocate their savings (so-called savings traps). However, can something be done about it?

To answer this question, it is misleading to focus on the first best because the comparison in terms of efficiency is unfair: to the extent that asymmetric information is a binding constraint, clearly being able to relax it would be Pareto improving. In this respect, it is no different than asking whether in a standard Edgeworth box economy adding additional endowment of some desirable good leads to a Pareto improvement. The correct benchmark is not the first best, but the constrained best, derived by taking into account all the relevant constraints among which the informational asymmetry.

Operationally, one can address the question by introducing a benevolent government in the economy. The government is constrained to balance its budget, and it has no informational advantages (nor disadvantages) relative to market participants. It can raise resources through taxes and redistribute them as subsidies. We shall consider one of possibly many government intervention, and show that it leads to a Pareto improvement relative to any CE with $\hat{\theta} < \bar{\theta}$.

To be specific, suppose that the government can subsidise interest payments at a rate $\tau_r$; it can tax savings at a rate $\tau_s$; and finally it can subsidise investment via a lump-sum transfer $\phi \in [0, x]$. Moreover, for simplicity of exposition, assume
that the savings tax is progressive:

\[
\tau_s = \begin{cases} 
\tau & \text{for the excess savings } (\bar{x} - x) \\
0 & \text{otherwise}
\end{cases}
\]  

(4.5)

Given any CE with threshold for investing entrepreneurs equal to \( \hat{\theta} \in \Theta \) if the government’s intervention pushes the economy to full investment, the balanced budget condition requires that:

\[
\tau(\bar{x} - x) = \phi + \tau_r(x - \phi)
\]  

(4.6)

We are concerned with Pareto improvements: all agents must be at least as well off as at the laissez-faire market allocation, and some agents must be strictly better off.

To make the notation less cumbersome, we need to introduce a final bit of notation:

\[
z(\hat{\theta}) \equiv \int_\theta^{\hat{\theta}} p_\theta dF(\theta) + \int_{\hat{\theta}}^\infty p_\theta dF(\theta)
\]

which denotes the fraction of productive entrepreneurs at \( t = 1 \) if the threshold for investment is \( \hat{\theta} \).

Consider the workers first. Without a government intervention their indirect utility would be:

\[
V_{s,NG} = \alpha \xi z(\hat{\theta})^{1-\alpha} + \bar{x}
\]

where the first term denotes the labour income, the second the expected return on savings.

If the government does intervene and manages to push investment to \( \overline{\theta} > \hat{\theta} \), they would get:

\[
V_{s,G} = \alpha \xi z(\overline{\theta})^{1-\alpha} + x + (1 - \tau)(\bar{x} - x)
\]

They are weakly better off after the government’s intervention if and only if \( V_{s,NG} \leq V_{s,G} \), or:

\[
\alpha \xi \left[ z(\overline{\theta})^{1-\alpha} - z(\hat{\theta})^{1-\alpha} \right] \geq \tau(\bar{x} - x)
\]  

(4.7)

That is, workers are better off only if the amount they gain in the labour market weakly offsets the savings tax they are required to pay.

Now consider the entrepreneurs who did not invest before the government’s intervention, i.e. the pool of higher types \( \theta \in \Theta \setminus \{ \theta' | \theta' \leq \hat{\theta} \} \). After the government
intervenes, they invest if and only if:

\[
q(1 - \alpha)\xi z(\bar{\theta})^{-\alpha} \geq (x - \phi) \left[ \bar{p}_\theta \left( \int_{\theta}^{\bar{\theta}} p_\theta dF(\theta) \right)^{-1} - \tau_r \right]
\]  \hspace{1cm} (4.8)

whereas their utility constraint \( V_{\theta > \hat{\theta}, NG} \leq V_{\theta > \hat{\theta}, G} \) is:

\[
(1 - \alpha)\xi \left[ \bar{p}_\theta z(\bar{\theta})^{-\alpha} - \bar{p}_\hat{\theta} z(\hat{\theta})^{-\alpha} \right] \geq (x - \phi) \left[ \bar{p}_\theta \left( \int_{\theta}^{\bar{\theta}} p_\theta dF(\theta) \right)^{-1} - \tau_r \right]
\]  \hspace{1cm} (4.9)

Inequalities (4.8) and (4.9) differ only in the left hand side. Moreover, \( \bar{\theta} > \hat{\theta} \) implies that (4.8) is always slack. Intuitively, the difference between the inequalities (4.8) and (4.9) is that in (4.8) a high type entrepreneur who invests gets a low return from production. The fact that all other agents are investing leads to high labour costs. On the contrary, in (4.9) all entrepreneurs of high quality are not investing, and hence the wage rate is lower. This increases the outside option of not investing for each of them.

Furthermore, it is easily seen that (4.9) is easier the be satisfied the higher the type \( \theta \). In fact, it can be rewritten as follows:

\[
(1 - \alpha)\xi q z(\bar{\theta})^{-\alpha} \geq (x - \phi) \left[ \bar{p}_\theta \left( \int_{\theta}^{\bar{\theta}} p_\theta dF(\theta) \right)^{-1} - \tau_r \right] + (1 - \alpha)\xi \bar{p}_\theta \left[ z(\bar{\theta})^{-\alpha} - z(\hat{\theta})^{-\alpha} \right]
\]

The left hand side is a strictly positive constant, whereas the right hand side is an increasing function of \( \theta \).\(^{14}\) Hence, we can restrict attention to the highest type that is supposed to switch from investing to not investing, i.e. \( \bar{\theta} \).

Finally, consider the entrepreneurs who did invest before the government’s intervention. We have that \( V_{\theta \leq \hat{\theta}, NG} \leq V_{\theta \leq \hat{\theta}, G} \) if and only if:

\[
\bar{p}_\theta (1 - \alpha)\xi \left[ z(\bar{\theta})^{-\alpha} - z(\hat{\theta})^{-\alpha} \right] \geq (x - \phi) \left[ \bar{p}_\theta \left( \int_{\theta}^{\bar{\theta}} p_\theta dF(\theta) \right)^{-1} - \tau_r \right]
\]

\[
- x F(\hat{\theta}) \bar{p}_\theta \left( \int_{\theta}^{\hat{\theta}} p_\theta dF(\theta) \right)^{-1}
\]

\(^{14}\)This holds provided that \( \phi \leq x \), which we conjecture and we confirm at the end of the proof of Proposition 4.2.
Diving both sides by $\mathcal{p}_\theta$ and rearranging we get:

$$
(1 - \alpha)\xi \left[ z(\hat{\theta})^{-\alpha} - z(\bar{\theta})^{-\alpha} \right] \leq (x - \phi) \left( \frac{\tau_r}{\mathcal{p}_\theta} \right) + \phi \left( \int_{\theta}^{\bar{\theta}} \mathcal{p} \mathcal{dF}(\theta) \right)^{-1} 
+ x \left[ r(\hat{\theta}) - r(\bar{\theta}) \right]
$$

(4.10)

The left hand side of (4.10) is a positive constant independent of $\theta$, whereas the right hand side is decreasing with $\theta$.\(^{15}\) Hence, the inequality is binding at most for type $\hat{\theta}$ himself.

The next result considers any equilibrium with less than full investment, and it shows that there always exist a Pareto improving government intervention.

**Proposition 4.2.** All equilibria such that $\hat{\theta} < \bar{\theta}$ are constrained suboptimal.

**Proof.** Consider any equilibrium with $\hat{\theta} < \bar{\theta}$. First we restate the conditions we derived above which must be satisfied for the government intervention we sketched to support an equilibrium with full investment; second we check that there exist feasible values for the government’s parameters such that the system has a solution.

The condition for *workers* reads:

$$
\alpha \xi \left[ z(\bar{\theta})^{1-\alpha} - z(\hat{\theta})^{1-\alpha} \right] \geq \tau (\bar{x} - x)
$$

(4.11)

From an earlier argument, we know that we can restrict attention to the entrepreneurs of type $\bar{\theta}$ and $\hat{\theta}$, and if they are weakly better off after the government’s intervention all other entrepreneurs will be as well.

The condition for the *entrepreneurs of highest quality* such that $\bar{\theta} > \hat{\theta}$ reads:

$$
(1 - \alpha)\xi \left[ \mathcal{p}_\bar{\theta} z(\bar{\theta})^{-\alpha} - \mathcal{p}_\hat{\theta} z(\bar{\theta})^{-\alpha} \right] \geq (x - \phi) \left[ \mathcal{p}_\bar{\theta} z(\bar{\theta})^{-1} - \tau_r \right]
$$

(4.12)

and recall that it guarantees that their investment constraint holds as well.

Finally, the condition for the *entrepreneurs of highest quality among those that were investing absent the government’s intervention*, i.e. $\hat{\theta}$, reads:

$$
\mathcal{p}_\hat{\theta}(1 - \alpha)\xi \left[ z(\bar{\theta})^{-\alpha} - z(\hat{\theta})^{-\alpha} \right] \geq (x - \phi) \left[ \mathcal{p}_\hat{\theta} z(\hat{\theta})^{-1} - \tau_r \right] - x \mathcal{F}(\hat{\theta}) \mathcal{p}_\hat{\theta} z(\hat{\theta})^{-1}
$$

(4.13)

The analysis above left us with a system of one equation (given by (4.6))\(^{15}\) This holds provided that $\tau_r \geq 0$, which we conjecture and we confirm at the end of the proof of Proposition 4.2.
and three inequalities (that are: (4.11), (4.12) and (4.13)).

Take inequality (4.11), and set \( \tau^* \) such that it binds. We get:

\[
\alpha \xi \left[ z(\theta)^{1-\alpha} - z(\hat{\theta})^{1-\alpha} \right] = \tau^*(\bar{x} - x) \tag{4.14}
\]

Plugging (4.14) into the balanced budget constraint we get:

\[
\alpha \xi \left[ z(\theta)^{1-\alpha} - z(\hat{\theta})^{1-\alpha} \right] = \phi + \tau_r(x - \phi) \tag{4.15}
\]

Plugging (4.15) into (4.12) we get:

\[
\phi \geq \left( \frac{p_\theta z(\theta)}{(1 - \alpha)p_\theta} - 1 \right)^{-1} \left[ x \frac{p_\theta z(\theta)}{(1 - \alpha)p_\theta} - s_1(\hat{\theta}) \right] = z_1 \tag{4.16}
\]

where we define:

\[
s_1(\hat{\theta}) \equiv \xi \left\{ (1 - \alpha) \left[ p_\theta z(\theta)^{1-\alpha} - p_\theta z(\hat{\theta})^{1-\alpha} \right] + \alpha \left[ z(\theta)^{1-\alpha} - z(\hat{\theta})^{1-\alpha} \right] \right\}
\]

Recall from Assumptions 4.1 and 4.2 that \( q(1 - \alpha) \xi z(\hat{\theta})^{-\alpha} \geq x \) for every \( \hat{\theta} \in \Theta \). As a consequence, we can rewrite \( s_1(\hat{\theta}) \) as follows:

\[
s_1(\hat{\theta}) = \xi \left\{ (1 - \alpha) \left[ q z(\hat{\theta})^{-\alpha} + p_\theta [z(\theta)^{-\alpha} - z(\hat{\theta})^{-\alpha}] \right] + \alpha \left[ z(\theta)^{1-\alpha} - z(\hat{\theta})^{1-\alpha} \right] \right\}
\]

\[
\geq x + \xi \left\{ (1 - \alpha) p_\theta [z(\theta)^{-\alpha} - z(\hat{\theta})^{-\alpha}] + \alpha [z(\theta)^{1-\alpha} - z(\hat{\theta})^{1-\alpha}] \right\}
\]

\[
\equiv x + \xi \left[ (1 - \alpha) p_\theta s_1 + \alpha \hat{s}_2 \right]
\]

Now consider the lower types. We have to deal with three cases, depending on whether the threshold type \( \hat{\theta} \) is paying or receiving a subsidy under full investment, i.e. depending on the sign of the sign of \( p_\theta r(\theta) - 1 \).

**Case 1:** \( p_\theta r(\theta) < 1 \).

Plugging (4.15) into (4.13) we get:

\[
\phi \leq (1 - p_\theta z(\theta)^{-1})^{-1} \left[ s_2(\hat{\theta}) - x p_\theta [z(\theta)^{-1} - F(\hat{\theta}) z(\hat{\theta})^{-1}] \right] \equiv z_2 \tag{4.17}
\]
where we define:

\[ s_2(\hat{\theta}) \equiv \xi \left\{ p_{\hat{\theta}}(1 - \alpha) \left[ z(\theta)^{-\alpha} - z(\hat{\theta})^{-\alpha} \right] + \alpha \left[ z(\theta)^{1-\alpha} - z(\hat{\theta})^{1-\alpha} \right] \right\} \]

\[ = \xi \left\{ p_{\hat{\theta}}(1 - \alpha) \delta_1 + \alpha \delta_2 \right\} \]

We need to show that (4.16) and (4.17) can be jointly satisfied, i.e. that

\[ z_2 \geq z_1. \]

The condition can be rewritten as

\[ x[p_{\hat{\theta}} F(\hat{\theta}) z(\hat{\theta})^{-1} - 1] \geq - \frac{(p_{\hat{\theta}} - p_{\hat{\theta}})[(1 - \alpha)\delta_1 + \alpha \delta_2 z(\hat{\theta})^{-1}]}{p_{\hat{\theta}} z(\hat{\theta})^{-1} - 1} \]  

(4.18)

where the left hand side of (4.18) is weakly positive because of cross type subsidisation. Namely, the fact that if type \( \hat{\theta} \) is the highest type who is investing, he must be paying a gross real interest rate greater than the full information rate (which is one, as we discussed earlier). As for the right hand side of (4.18), notice that \( (p_{\hat{\theta}} - p_{\hat{\theta}}) > 0 \) because \( \hat{\theta} < \theta \). Moreover, the denominator is strictly positive again because of cross subsidisation: \( p_{\hat{\theta}} z(\hat{\theta})^{-1} > 1 \). It remains to study the sign of the square bracket in the numerator, that is the expression \( (1 - \alpha)\delta_1 + \alpha \delta_2 z(\hat{\theta})^{-1} \). We can rewrite it as follows:

\[ \frac{(1 - \alpha)z(\theta)\delta_1 + \alpha \delta_2}{z(\theta)} = \frac{(1 - \alpha)z(\theta)\left[ z(\theta)^{-\alpha} - z(\hat{\theta})^{-\alpha} \right] + \alpha \left[ z(\theta)^{1-\alpha} - z(\hat{\theta})^{1-\alpha} \right]}{z(\theta)} \]

\[ = (1 - \alpha)\left[ z(\theta)^{1-\alpha} - z(\hat{\theta}) \right] + \alpha \left[ z(\theta)^{1-\alpha} - z(\hat{\theta})^{1-\alpha} \right] \]

\[ \frac{z(\theta)^{1-\alpha} - \left[ (1 - \alpha)z(\theta) z(\hat{\theta})^{-\alpha} + \alpha z(\hat{\theta})^{1-\alpha} \right]}{z(\theta)} > 0 \]

The strict inequality follows from \( \hat{\theta} < \theta \) (and of course \( z(\theta) > 0 \)). In particular, taking the derivative of the right hand side with respect to \( \hat{\theta} \) yields

\[ \frac{\partial \left[ (1 - \alpha)z(\theta) z(\hat{\theta})^{-\alpha} + \alpha z(\hat{\theta})^{1-\alpha} \right]}{\partial \theta} = \alpha(1 - \alpha)gz(\hat{\theta})^{-\alpha} z(\theta) z(\hat{\theta})^{-1} > 0 \]

and when \( \hat{\theta} \to \theta \) the expression tends to zero. As a result, inequality (4.18) holds for every parameter configuration, and hence the equilibria with \( \hat{\theta} < \theta \) are constrained suboptimal.

\[ ^{16}\text{Notice that it is enough to prove a weak inequality to ensure that at least some agents is strictly better off after the government’s intervention. This follows from the presence of types different from } \theta \text{ and } \theta, \text{ for whom the inequality is strict by necessity.} \]

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In addition, notice that \( z_2 \leq x \) if and only if
\[
x [p^\theta F(\hat{\theta}) z(\hat{\theta})^{-1} - 1] \geq -s_2(\hat{\theta})
\]
which is always satisfied because we proved \( s_2(\hat{\theta}) \geq 0 \).

As a result, \( \phi \leq x \) and \( \tau_r > 0 \) as we conjectured.

**Case 2:** \( \bar{p}^\theta r(\bar{\theta}) > 1 \).

Plugging (4.15) into (4.13) we get:
\[
\phi \geq \left( \bar{p}^\theta z(\bar{\theta})^{-1} \right)^{-1} \left[ \xi \left( \bar{p}^\theta (1 - \alpha) \delta_1 + \alpha \delta_2 \right) - x \bar{p}^\theta (r(\hat{\theta}) - r(\bar{\theta})) \right] \equiv z_3 \quad (4.19)
\]
To get a Pareto improvement we can simply set \( \phi \geq \max \{z_1, z_3\} \).

Again, it is easy to verify that \( \phi \leq x \) and \( \tau_r > 0 \).

**Case 3:** \( \bar{p}^\theta r(\bar{\theta}) = 1 \).

In this final case, we can set \( \phi \geq z_1 \) as the condition for type \( \hat{\theta} \) does not depend on \( \phi \). However, we still need to show that type \( \hat{\theta} \) is weakly better off, which is the case if and only if:
\[
x (p^\theta r(\hat{\theta}) - 1) \geq -\xi \left[ (1 - \alpha) p^\theta \delta_1 + \alpha \delta_2 \right] \quad (4.20)
\]
From an earlier argument, we now that the left hand side is strictly positive. Notice that in Case 3 we have \( \bar{p}^\theta = r(\bar{\theta})^{-1} \), hence the right hand side can be simplified along the lines of Case 1 and shown to be strictly negative whenever \( \hat{\theta} < \bar{\theta} \).

Again, it is easy to verify that \( \phi \leq x \) and \( \tau_r > 0 \).

It is important to notice that in setting up the intervention we implicitly considered a case where \( \bar{x} > x \), and the difference between the two is ‘sufficiently large’. However, it could be that the borrowing need wipes out most (or even all) of available savings. In such case, the Pareto improvement that we characterised does not work. Nevertheless, we can always replace the savings tax with an income tax of identical size, and the reasoning goes through unchanged. The proof of this equivalence is straightforward and hence we omit it.

### 4.4 Information crises and government interventions

So far, we argued that competitive equilibria are constrained suboptimal when hidden information problems are severe. Namely, there exist Pareto improving government intervention that (i) balance the government’s budget; and (ii) do not require
more or less information than the one market participants have.

To conclude our analysis, we come back to the question of studying interventions in times of crises. Asymmetric information is key in most arguments used by politicians and academics to justify emergency interventions. They roughly go as follows: although crises may depend on solvency and not on liquidity issues (e.g. Lehman was bust in September 2008; it did not suffer from a self fulfilling run), they originate panic in financial markets. Suddenly, investors run on many other institutions because they fear they would also go under. Why do they fear so? Because they do not have accurate information about their portfolio holdings. Because subsequent runs are inefficient, namely they trigger defaults of solvent institutions that run out of liquid assets, governments ought to restore confidence and intervene.

To capture this scenario - i.e. a purely informational crisis - one needs to be careful that, playing with the information structure, he does not affect the real resources in the economy. For instance, shifting the probability distribution over the type space in the sense of first order stochastic dominance does not work, as we would fall back into an output crisis. In this section, we proceed as follows: (i) first, we introduce a parameter that captures the measure of entrepreneurs that cannot costlessly certify their type.

In other words, we segment the market into the fraction subject to hidden information, and the fraction fully transparent; (ii) we solve for the competitive equilibria and study whether increasing the we also increase the need for governments to intervene.

Observe that the previous analysis covered the case of , whereas the case of can be easily described. It would imply that, at the unique competitive equilibrium: (i) all types invest (because of Assumption 4.1); (ii) the wage equals and the equilibrium is Pareto optimal by the First Welfare Theorem.

It remains to consider the case of . For the fraction of agents not subject to hidden information, the equilibrium behaviour is simple to characterise: all agents invest, and they face type-specific interest rates . As for the others, and assuming that each type has the same probability of being subject to hidden information, the equilibrium is exactly the same as in the previous section.

As a result, aggregate investment as a function reads . Evidently, the higher the degree of asymmetric information, the lower the investment at equilibrium and hence the greater the need for government’s intervention.

\footnote{It is trivial to show that if a type is known to posses private information which he can certify, in equilibrium there must be unraveling of this information. In other words, all types known to be able to certify who they are optimally reveal their private information in equilibrium. The result is reminiscent of Dye [1985].}
Bibliography


