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Jointly optimal regulation of bank capital and liquidity*

Ansgar Walther†

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Abstract

In an economy with financial frictions, banks endogenously choose excessive leverage and maturity mismatch in equilibrium, as they fail to internalize the risk of socially wasteful fire sales. Macroprudential regulators can achieve efficiency with simple linear constraints, which require less information than Pigouvian taxes. The Liquidity Coverage and Net Stable Funding ratios of Basel III can implement efficiency. Additional microprudential regulation of leverage is required when bank failures are socially costly. Micro- and macroprudential rules are imperfect substitutes. Optimally, macroprudential policy reacts to systematic risk and credit conditions over the cycle, while microprudential policy reacts to systematic and idiosyncratic risk.

Keywords: Banking, regulation, capital, liquidity, macroprudential, microprudential, fire sales.

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†ansgar.walther@economics.ox.ac.uk. Postdoctoral Research Fellow, Department of Economics, Nuffield College and INET, University of Oxford.

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1 INTRODUCTION

Financial policy is traditionally motivated by the 'microprudential' concern that bank failures are socially costly. Following the crisis of 2008, 'macroprudential' concerns have surfaced (Hanson, Kashyap and Stein 2011). Banks' individual choices can contribute to *systemic risk,* which causes inefficient fire sales and market freezes, thus harming the financial sector as a whole.

Systemic risk is not caused exclusively by insufficient bank capital. It is amplified if banks fund long-term investments with short-term debt and become illiquid in a crisis (Brunnermeier 2009). In response to this problem, the Basel III Accord (BIS 2010) introduces *Liquidity Coverage* and *Net Stable Funding* requirements, supplementing traditional capital requirements. These policies reflect macroprudential concerns and explicitly target bank liquidity.

This paper addresses two questions from a welfare-theoretic perspective:

1. How should capital and liquidity be regulated to deal with systemic risk?

2. How should this interact with microprudential regulation against costly bank failures?

I analyze an economy with three financial frictions. First, banks need equity downpayments, or '*skin in the game*', to raise funding. Second, banks can only sell assets to outside buyers at *fire sale prices.* Fire sales are socially wasteful because outside buyers cannot extract as much surplus from assets as banks. Third, banks' creditors have a preference for liquidity, so that long-term debt commands an interest premium.

The first two frictions are now standard in the literature on financial frictions, which I review in the next section. The liquidity premium is new, and allows me to study the interplay between endogenous maturity structure and fire sales.

Banks make long-term investments which are subject to aggregate and idiosyncratic shocks. They fund themselves with equity and debt, choosing the maturity of debt endogenously. Banks trade off the liquidity premium on long-term debt against rollover risk to choose their maturity structure.

Fire sales occur when a bad aggregate shock creates a liquidity shortfall in the banking sector. Then, banks struggle to roll over short-term debt due to the 'skin in the game' constraint, and have to sell assets to outside buyers in order to repay short-term creditors. Fire sales happen when banks are highly leveraged, or when their balance sheets feature strong maturity mismatch.

Macroprudential regulation is justified by a systemic risk externality – technically a pecuniary externality. Banks have weaker incentives to reduce leverage and maturity mismatch than a social planner. While a planner takes into account the effect of reduced leverage or maturity mismatch on prices and the likelihood of costly fire sales, competitive banks take prices as given. Therefore, the marginal private benefit of reduced leverage or mismatch is less than the social benefit. Due to the incompleteness of markets, the pecuniary externality renders the competitive equilibrium inefficient as in Geanakoplos and Polemarchakis (1985).

To answer my first question, I consider three modes of intervention: Centralization by a social planner, regulation using balance sheet constraints as in Basel III, and Pigouvian taxation. The planner's choices are efficient and maximize welfare subject to a 'no fire sale

condition'.

Constraint-based regulation can achieve efficiency by imposing the 'no fire sale' condition on banks. It only depends on systematic cash flow risk and the 'skin in the game' constraint, and does not require knowledge of other parameters such as funding costs, average returns to investment or idiosyncratic risk. The efficient Pigouvian tax, on the other hand, generally depends on all parameters. Thus, constraint-based regulation requires less information. This result is fairly robust to extensions of the baseline model.

For the second question, I consider an extended model where bank failures generate deadweight social costs. Systemic risk and default risk are not equivalent, and an additional microprudential constraint may be required to limit bank failures. I show that micro- and macroprudential regulation are imperfect substitutes from two perspectives. On the one hand, one regulation may be sufficient for efficiency, but this is not true for all parameter values. On the other hand, removing macroprudential regulation may increase the optimal toughness of microprudential regulation, and vice versa. Moreover, the informational requirements of macro- and microprudential regulation are similar, which suggests that delegating them to separate institutions may be suboptimal.

The optimal policy can be implemented within the Basel III framework. I show that the optimal macroprudential policy can be implemented using either the Liquidity Coverage Ratio (LCR) or Net Stable Funding Ratio (NSFR) requirements of Basel III. This result supports the ratios-based approach of Basel III. While the LCR and NSFR are equivalent in my three-period model, richer dynamics can justify the use of a short-term LCR and long-term NSFR, as is seen in practice.

My theory supports time-varying macroprudential regulation, because optimal rules should react to changes in systematic risk and credit conditions. However, microprudential rules should also be time-varying, and additionally keep track of cross-sectional idiosyncratic risk.

The next section puts the paper in the context of the related literature. Section 3 sets up the model. Section 4 analyzes the competitive equilibrium and welfare. Sections 5 and 6 consider optimal macro- and microprudential regulation. Section 7 discusses the reaction of optimal regulation over the business cycle. Section 8 concludes. All proofs are in Appendix A, and Appendix B explains some technical conditions.

2 RELATED LITERATURE

There is a rich recent literature on pecuniary externalities and fire sales. Shleifer and Vishny (2011) provide an excellent theoretical and empirical survey on fire sales. Lorenzoni (2008) demonstrates that fire sales generate excessive short-term borrowing, and Shleifer and Vishny (2010) further explore their destabilizing effects. Perotti and Suarez (2011) look at optimal ways of regulating maturity in a reduced-form model without a separate leverage choice. Korinek (2012) studies a three-period model of excessive leverage, whereas Bianchi and Mendoza (2012) and Jeanne and Korinek (2010) compute optimal leverage policies in calibrated infinite-horizon models. Stein (2012) considers regulating the choice between short-term and long-term debt through 'cap-and-trade' mechanisms. Gersbach and Rochet (2012) demonstrate a systemic externality in a model based on productivity shocks and moral hazard. Finally, Goodhart et al. (2013) study the impact of multiple regulations by simulating a complex general equilibrium model.[1]

In this context, this paper contributes by proposing optimal constraint-based regulation targeting both leverage and maturity mismatch, and by offering an explicit comparison with Pigouvian taxes. Moreover, I characterize the interplay of optimal macro- and microprudential regulation analytically in a model with several externalities.

In recent work, Farhi and Werning (2013) suggest 'aggregate demand externalities', driven by nominal rigidities, as a motivation for macroprudential policy. Korinek and Simsek (2014) show that such externalities can generate excessive leverage, because agents do not internalize the risk of Keynesian liquidity traps. My model does not have nominal rigidities, but the interactions capital and liquidity regulation would be similar in a setting with aggregate demand externalities.

The corporate finance literature emphasizes maturity choices based on asymmetric information. Flannery (1986) shows that firms can use short-term borrowing as a signal of quality. In Diamond (1991), firms choose debt maturity by trading off the risk of inefficient liquidation by short-term creditors against the possibility of improved credit-ratings and cheap refinancing. This paper proposes an alternative story specific to banks, where the trade-off is driven by possible fire sales and the liquidity preference of creditors. A similar mechanism is explored by Chen, Xu and Yang (2013), who model maturity choice as determined by rollover risk due to debt overhang and investors' exposure to liquidity needs.

Fire sales also arise in the macroeconomic literature on the 'financial accelerator' (Kiyotaki and Moore 1997, Bernanke, Gertler and Gilchrist 1999, Shin 2010 ,He and Krishnamurthy 2012). Additional sources of financial instability include 'margin spirals', when 'skin in the game' requirements rise in bad times (Geanakoplos 2009, Brunnermeier and Pedersen 2009),

uncertainty about complex financial networks (Caballero and Simsek 2009) and irrational expectations (Gennaioli et al. 2012).

Bank runs are another cause of financial fragility. Diamond and Dybvig (1983) sparked a vast literature on this subject based on multiple equilibria. More recently, Gertler and Kiyotaki (2013) analyse bank runs in a dynamic economy with endogenous liquidity mismatch. Bank runs do not occur in my model because bank debt is secured and there is no sequential service. If unsecured debt were introduced, runs on unsecured debt and 'skin in the game' constraints on secured debt would work together to exacerbate bank funding problems and fire sales.

3 THE MODEL

Agents and time. There are three types of agents: Banks, creditors and outside buyers. All agents are risk-neutral and nobody discounts the future. There is a unit measure of each type. The model has three dates $t \in \{0, 1, 2\}$ and one consumption good called cash, which can be stored costlessly over time by all agents.

Aggregate uncertainty. The aggregate state is good or bad: $s \in \{g, b\}$. The bad state occurs with probability $P[s = b] = \alpha$, and the good state occurs with probability $1-\alpha$. The aggregate state becomes public information at $t = 1$.

Individual uncertainty among banks. For bank $i \in [0, 1]$, the individual state is high or low: $z^i \in \{H, L\}$. The low individual state occurs with probability $P[z^i = L] = \beta$, and the high state occurs with probability $1-\beta$. The individual state is independent of the aggregate

state, and independent across banks. Individual states also become public information at $t = 1$.

Bank projects. Banks can invest in risky long-term projects at $t = 0$, which cost one unit of cash. At $t = 1$, each project yields a state-contingent cash flow $v_{1s} + \varepsilon_{z^i}$. v_{1s} is the aggregate cash flow, and $v_{1g} > v_{1b} \geq 0$. ε_{z^i} is the idiosyncratic shock, and $\varepsilon_H > \varepsilon_L$. Without loss of generality, assume that $E[\varepsilon_{z^i}] = (1 - \beta)\varepsilon_H + \beta\varepsilon_L = 0$.[2]

At $t = 2$, each project yields v_{2s}, where $v_{2b} < v_{2g} \leq 1$.[3] The $t = 2$ cash flow is a partial repayment of the initial investment, and the fraction repaid is higher in the good state. Let $R = E[v_{1s} + v_{2s}] - 1$, and assume that $R > 0$, so that projects have positive expected net present value.

After cash flows are received at $t = 1$, projects are traded in a competitive secondary market. The secondary market price for projects at $t = 1$ in aggregate state s is p_s.

Collateral constraint. Banks can raise debt from creditors, using projects as collateral. As in Shleifer and Vishny (2010), there is an exogenous 'marked-to-market' collateral constraint:

$$\text{loan} = (1 - h) \times \text{market value of collateral} \tag{1}$$

where $h \in (0, 1)$ is the 'haircut' on debt. Other things being equal, a larger haircut decreases the likelihood of default, since more collateral is pledged per unit borrowed. h is assumed to be large enough to rule out any bank default in the baseline model - an explicit condition is given below.[4]

I take the 'market value' of project collateral at time 0 to be the initial investment of 1.

This can be motivated explicitly by introducing a competitive sector of entrepreneurs, who can convert cash into projects one-for-one and sell them to banks at $t = 0$. The zero-profit condition of an entrepreneur then states that the $t = 0$ market value of projects will be 1.[5]

Creditors and liquidity preference. Banks make 'take it or leave it' offers of short- and long-term debt contracts to creditors. Short-term debt contracts last one period and can be issued at $t = 0$ and $t = 1$. Long-term debt lasts two periods and is only available at $t = 0$. Each creditor has an endowment of Y at $t = 0$ and I assume that Y is large, so that the bank never exhausts creditors' endowment at any date.

Creditors have a preference for liquidity due to a potential investment opportunity at $t = 1$, which yields a net return of $q > 0$ at $t = 2$. Alternatively, one can think of creditors having a potential liquidity need at $t = 1$, and a marginal utility of $1 + q$ of servicing this need. Creditors are heterogeneous, and uniformly distributed on the unit interval. For creditor $k \in [0, 1]$, the investment opportunity arises with probability $\lambda(k)$, where $\lambda(0) = 0$ and $\lambda'(k) > 0$, independently across creditors.

Banks optimally offer interest payments to make creditors indifferent between debt contracts and holding cash. No interest payment is required on short-term debt, as long as Y is large enough so that banks can borrow freely from creditors without an investment opportunity at $t = 1$. The required long-term interest rate for creditor k is $\lambda(k)q > 0$, which compensates for the expected cost of forgone opportunities.

Note that long-term debt is not only costly for banks, but also reduces the social surplus. Long-term debt reduces socially valuable *maturity transformation* by intermediaries. I assume implicitly that creditors cannot trade long-term debt after liquidity shocks are revealed

at $t=1$. If this were relaxed, maturity transformation would be carried out in the market for long-term debt, and long-term debt would no longer be socially costly.

Bank investments at $t=0$. Each bank has a fixed equity endowment of e_0.[6] Since banks are ex ante identical, I suppress i-superscripts and analyse the decisions of the representative bank ('the bank'), contingent on aggregate and individual states.

The bank invests in n_0 projects funded by (and pledged as collateral against) *short-term debt*. By the collateral constraint (1), it raises $(1-h)\,n_0$ in short-term debt, and contributes hn_0 of its own equity.

The bank invests in $\bar{n}_0$ projects funded by *long-term debt*. It raises $(1-h)\,\bar{n}_0$ in long-term debt and contributes $h\bar{n}_0$ of its own equity. The bank is obliged to hold the collateral, $\bar{n}_0$ projects, until the debt matures.

The cheapest way to borrow $(1-h)\,\bar{n}_0$ units long-term is to go to the creditors who are least likely to have an investment opportunity. Thus, the bank borrows Y units from all creditors $k \in [0,(1-h)\,\bar{n}_0/Y]$, promising each of them an interest payment of $\lambda\,(k)\,qY$. This implies that the (minimized) interest payment at $t=2$ is equal to

$$\int_0^{\frac{(1-h)\bar{n}_0}{Y}} \lambda\,(k)\,qY\,dk \equiv r\,(\bar{n}_0) \tag{2}$$

It is easy to verify that the function r is strictly increasing, strictly convex and twice differentiable, with $r\,(0)=r'\,(0)=0$.

Any equity not spent on downpayments is held as *cash*, denoted c_0.[7] The bank's choices at $t=0$ are summarized by $\mathbf{x}_0=(c_0,n_0,\bar{n}_0)\in\mathbb{R}^3_+$, and its budget is

$$e_0 = h(n_0 + \bar{n}_0) + c_0 \tag{3}$$

Let $B_0 \subset \mathbb{R}^3_+$ be the set of bank choices $\mathbf{x}_0$ that satisfy the budget.

Diseconomies of scale. Investing in risky projects requires careful monitoring and risk management, which becomes increasingly costly as the scale of investment increases. This implies that there are diseconomies of scale to investment in risky projects. The bank incurs a disutility of monitoring effort at $t = 0$ which is equal to $g(n_0 + \bar{n}_0)$, where the function g is strictly increasing, strictly convex and twice differentiable.[8]

Bank cash flows at $t = 1$. After states s and z are revealed at $t = 1$, the bank receives the cash flow $v_{1s} + \varepsilon_z$ for the $n_0 + \bar{n}_0$ projects it invested in at $t = 0$. It also repays its short-term creditors $(1 - h)\, n_0$.

The bank's equity at $t = 1$, denoted e^z_{1s}, is defined as the sum of the net cash flow and the value of its marketable assets. Marketable assets exclude the $\bar{n}_0$ projects which are pledged as collateral against long-term debt. The remaining assets are cash c_0 and n_0 projects with a market value of p_s each. Hence, equity is

$$e^z_{1s} = c_0 + (n_0 + \bar{n}_0)(v_{1s} + \varepsilon_z) + n_0 p_s - (1 - h)\, n_0 \tag{4}$$

Bank investments at $t = 1$. After trading in the secondary market, the bank holds n^z_{1s} projects funded by new *short-term debt*. By the collateral constraint (1), it raises $(1 - h)\, p_s n^z_{1s}$ in debt and contributes $h p_s n^z_{1s}$ of its own equity. The bank holds c^z_{1s} units

of *cash*. Its choices at $t=1$ are $\mathbf{x}_{1s}^{z}=\left(c_{1s}^{z},n_{1s}^{z}\right)\in\mathbb{R}_{+}^{2}$, and its budget is

$$e_{1s}^{z}=hp_{s}n_{1s}^{z}+c_{1s}^{z} \tag{5}$$

where e_{1s}^{z} is its $t=1$ equity, which is related to its first-period choices $\mathbf{x}_0$ by equation (4). Let $B_{1s}^{z}\left(\mathbf{x}_0\right)\subset\mathbb{R}_{+}^{2}$ be the set of bank choices that satisfy the budget.

Bank cash flows at $t=2$ in state s. The bank receives the cash flow v_{2s} for the $\bar{n}_0+n_{1s}^{z}$ projects held at $t=1$. It repays short-term and long-term creditors. The bank's utility Π_{s}^{z} is given by the sum of the net cash flow and retained cash, minus monitoring costs incurred previously:

$$\Pi_{s}^{z}=c_{1s}^{z}+v_{2s}\left(n_{1s}^{z}+\bar{n}_0\right)-\left(1-h\right)p_{s}n_{1s}^{z}-\left(1-h\right)\bar{n}_0-r\left(\bar{n}_0\right)-g\left(n_0+\bar{n}_0\right) \tag{6}$$

Outside buyers. In addition to banks, *outside buyers* participate in the secondary market for projects at $t=1$. Outside buyers have less project management skills than banks, and can only extract a cash flow of $\underline{p}<v_{2b}$ per project at $t=2$.

Competitive equilibrium. Market clearing requires that the banking sector cannot be a net buyer of projects in the secondary market, since no projects are sold by anybody else. Moreover, if a strictly positive number of projects are sold by the banking sector, then their price must equal the marginal valuation of outside buyers $\underline{p}$. This value plays the role of a *fire sale price.*

Note that in aggregate state s, a measure β of banks have a low individual shock $z^{i}=L$, and sell $n_0-n_{1s}^{L}$ projects. A measure $1-\beta$ have a high shock and sell $n_0-n_{1s}^{H}$ projects.

This yields the following definition of equilibrium:

DEFINITION 1. *A competitive equilibrium is described by asset prices p_s for $s \in \{g, b\}$ and bank choices $\mathbf{x}_0 \in \mathbb{R}^3_+$ and $\mathbf{x}^z_{1s} \in \mathbb{R}^2_+$ for $s \in \{g, b\}$ and $z \in \{H, L\}$ satisfying*

1. *Optimality.* The bank's choices maximize $E[\Pi^z_s]$ subject to $\mathbf{x}_0 \in B_0$ and $\mathbf{x}^z_{1s} \in B^z_{1s}(\mathbf{x}_0)$, taking asset prices as given.

2. *Market clearing.* For $s \in \{g, b\}$, the bank's choices satisfy

$$n_0 - \left(\beta n^L_{1s} + (1-\beta)\, n^H_{1s}\right) \geq 0 \tag{7}$$

   Furthermore, $p_s \geq \underline{p}$, and if (7) is a strict inequality, then $p_s = \underline{p}$.

Parametric assumptions. The following assumption makes the analysis of fire sales interesting:

ASSUMPTION 1. *Aggregate cash flows at $t = 1$ satisfy*

$$v_{1g} > \left(1-\underline{p}\right)(1-h) \tag{8}$$

$$v_{1b} < (1-v_{2b})(1-h) \tag{9}$$

The first condition ensures that aggregate bank liquidity is high in the good state, so that banks never have to sell assets to outside buyers in equilibrium. The second condition ensures that aggregate liquidity is low enough in the bad state to make fire sales a possibility.

Moreover, I impose a condition guaranteeing that bank debt is risk free in the baseline model. As discussed above, this requires a lower bound on the haircut h.

ASSUMPTION 2. *For all $\bar{n}_0 \in [0, e_0/h]$, the haircut satisfies*

$$h > 1 - \underline{p} + r(\bar{n}_0) \tag{10}$$

This condition states that at $t = 1$, the bank can always sell its assets in the secondary market, service its outstanding debt and interest obligations, and have cash left over. Thus, it has positive continuation value at $t = 1$ and will not default.

4 COMPETITIVE EQUILIBRIUM

Bank profits. The bank earns a basic expected net return of R on each of the $(n_0 + \bar{n}_0)$ projects it invests in. It has to pay interest on long-term debt and incurs monitoring costs. Hence, its expected basic profits are

$$R(n_0 + \bar{n}_0) - r(\bar{n}_0) - g(n_0 + \bar{n}_0) \tag{11}$$

The bank may also make profits or losses from trading projects in the secondary market at $t = 1$. By virtue of Assumption 1, projects will be priced fairly in the good state in equilibrium, $p_g = v_{2g}$. There are two relevant scenarios in the bad state: Projects can be priced fairly ($p_b = v_{2b}$) or underpriced ($p_b < v_{2b}$). The proof of Lemma 1 verifies that these are the only possibilities.

If projects are priced fairly, then the bank cannot gain or lose from trading. If they are underpriced, the bank will optimally hold as many projects at as possible in the bad state. From the budget (5), it follows that this is achieved by holding no cash, i.e. setting $c^z_{1b} = 0$ and $n^z_{1b} = e^z_{1b}/p_b h$. Its net sale of assets is then

$$n_0 - \frac{e^z_{1b}}{p_b h} \tag{12}$$

On each asset sold, the bank loses $v_{2b} - p_b$ (it gains when the net sale is negative).

The bad state occurs with probability α, so that expected trading losses are $\alpha\left(v_{2b} - p_b\right)$ times the expected net sale. Using the definition of equity e^z_{1s} in (4), the budget constraint (3) and the fact that $E\left[\varepsilon_z\right] = 0$, expected trading losses are

$$\phi\left(p_b\right)\left[\left(n_0 + \bar{n}_0\right)\left(1 - p_b\left(1 - h\right) - v_{1b}\right) - \bar{n}_0\left(1 - p_b\right)\left(1 - h\right) - e_0\right] \tag{13}$$

where $\phi\left(p_b\right) = \alpha\left(v_{2b} - p_b\right)/p_b h$.

The term in square brackets is the bank's *expected liquidity shortfall,* which measures the difference between its $t = 1$ equity and the amount required to hold on to n_0 assets. The factor $\phi\left(p_b\right)$ is the *marginal value of bank liquidity* at $t = 1$. It is the product of the likelihood of the bad state, the undervaluation of projects, and the number of assets that can be bought with one unit of liquidity, which is $1/p_b h$. Lemma 1 summarizes this characterization of bank profits.

LEMMA 1. *In any competitive equilibrium, asset prices satisfy* $p_g = v_{2g}$ *and* $p_b \in \left[\underline{p}, v_{2b}\right]$. *The bank's expected profits satisfy* $E\left[\Pi_s\right] = V\left(\mathbf{x}_0, p_b\right)$*, and its choices solve the problem*

$\max_{\mathbf{x}_0 \in B_0} V(\mathbf{x}_0, p_b)$, *where*

$$\begin{aligned} V(\mathbf{x}_0, p_b) &= R(n_0 + \bar{n}_0) - r(\bar{n}_0) - g(n_0 + \bar{n}_0) \\ &- \phi(p_b)\left[(n_0 + \bar{n}_0)(1 - p_b(1-h) - v_{1b}) - \bar{n}_0(1-p_b)(1-h) - e_0\right] \end{aligned} \tag{14}$$

Optimality conditions. Assuming that the solution to the bank's maximization problem is interior, its choices are characterized by two marginal conditions:

$$r'(\bar{n}_0^\star) = \phi(p_b)(1-p_b)(1-h) \tag{15}$$

$$R - g'(n_0^\star + \bar{n}_0^\star) = \phi(p_b)\left[1 - p_b(1-h) - v_{1b}\right] \tag{16}$$

Equation (15) determines optimal maturity structure. The left-hand side is the marginal cost of long-term funding, driven by creditors' liquidity preference. The right-hand side is the marginal liquidity cost of short-term funding. It is equal to the marginal value of liquidity times the liquidity injection required to roll over short-term debt.

Equation (16) determines optimal investment. The left-hand side is the marginal basic profit of investment in projects, net of monitoring costs. The right-hand side is the marginal benefit of cash, driven by the fact that cash offers liquidity in the bad state. It is equal to the marginal value of liquidity times the decrease in the expected shortfall when cash is substituted for projects.

Appendix B derives parametric conditions that guarantee an interior solution.

4.1 Fire sales

When there is an *aggregate liquidity shortfall* at $t = 1$, banks must sell assets to outside buyers. By the law of large numbers, the aggregate liquidity shortfall is the same as the expected liquidity shortfall of an individual bank, i.e. the term in square brackets in (13). If this is positive, the banking sector is an aggregate net seller of projects in the bad state. Consequently, there is a *fire sale* and the price drops to the valuation of outside buyers $\underline{p}$.

This paper focuses on situations where fire sales happen in competitive equilibrium. This occurs when investment in projects is sufficiently attractive, and when long-term debt is sufficiently costly. In that case, banks find it worthwhile to invest a large proportion of their initial equity in projects funded by short-term debt, and hold few cash reserves, which naturally creates an aggregate liquidity shortfall.

Projects are attractive when the net present value R is high. Long-term debt is costly when creditors have a strong liquidity preference, which corresponds to a high return q on their potential investments. Proposition 1 demonstrates that high values of R and q indeed lead to fire sales in equilibrium.

PROPOSITION 1. *There are two thresholds $R_f > 0$ and $q_f > 0$, which are functions of the parameters other than R and q, such that when $R > R_f$, and $q > q_f$, the unique competitive equilibrium has a fire sale, with $p_b = \underline{p}$.*

In the remainder of the paper, I assume that there is a fire sale in equilibrium. Thus, I will focus on the parametric region where $R > R_f$ and $q > q_f$.

I have assumed that there is some systematic risk in the economy ($v_{1g} > v_{1b}$), or equivalently,

that there is positive correlation in cash flow risk across banks. If cash flow risk were entirely idiosyncratic ($v_{1b} = v_{1g}$), fire sales would not arise, as banks with high cash flows would buy assets from banks with low cash flows, and outside buyers would not need to get involved. This is analogous to the point on aggregate liquidity shortages in Holmström and Tirole (1998). In their model, there is no role for public liquidity provision when shocks are purely idiosyncratic, since firms can insure each other. Here, there is no role for regulation when cash flow shocks are purely idiosyncratic.

4.2 The cost of fire sales and systematic risk

Fire sales are a systematic phenomenon, as they are driven by aggregate illiquidity. I now analyze how the cost of fire sales is affected by increases in systematic risk. Increased systematic risk is captured by mean-preserving spreads in the aggregate cash flows v_{1s} and v_{2s}.

Fire sales are costly because projects are sold to outside buyers, which destroys value. Outside buyers do not make profits in a fire sale, since asset prices reflect their valuation $\underline{p}$. Therefore, the social cost of fire sales coincides with banks' losses in equilibrium. These losses are equal to (13), evaluated at the equilibrium price $p_b = \underline{p}$ and the optimal choices $n_0^\star$ and $\bar{n}_0^\star$.

Fire sale costs do not depend on the individual shock ε_z, and are unaffected by idiosyncratic risk. They do, however, respond to changes in systematic risk. Proposition 2 explains their response in detail.

PROPOSITION 2. *A mean-preserving spread in v_{1s} increases the equilibrium cost of fire*

sales if and only if

$$g''\left(n_0^\star+\bar{n}_0^\star\right)>g_1\equiv\frac{\phi\left(\underline{p}\right)\left(1-\underline{p}\left(1-h\right)-v_{1b}\right)}{\left(n_0^\star+\bar{n}_0^\star\right)} \tag{17}$$

Suppose the fire sale price (in the bad state) is proportional to v_{2b}, with $\underline{p}=\eta v_{2b}$, $0<\eta<1$. Then there exists a parameter-dependent threshold $g_2>g_1$, such that a mean-preserving spread in v_{2s} increases the cost of fire sales if and only if $g''\left(n_0^\star+\bar{n}_0^\star\right)>g_2$.

Suppose the fire sale price $\underline{p}$ does not depend on v_{2s}. Then there exists a parameter-dependent threshold $G_2>g_1$ such that a mean-preserving spread in v_{2s} increases the cost of fire sales if and only if $g''\left(n_0^\star+\bar{n}_0^\star\right)<G_2$.

To understand this result, note there are direct and indirect effects of increased systematic risk. A mean preserving spread in v_{1s} decreases v_{1b}, which directly increases fire sale costs by reducing cash flow liquidity. However, banks optimally react by investing less, which reduces their liquidity shortfall and the cost of fire sales.[9] Costs increase on balance when the first effect dominates. This occurs when banks' demand for investment is inelastic, i.e. when the curvature of monitoring costs g'' is big enough.

A mean-preserving spread in v_{2s} decreases v_{2b}. The effect depends on whether the fire sale price $\underline{p}$ responds. Suppose first that it responds proportionally, with $\underline{p}=\eta v_{2b}$. Then the value of liquidity ϕ in case of a fire sale is not affected, since

$$\phi\left(\underline{p}\right)=\frac{\alpha}{h}\frac{1-\eta}{\eta} \tag{18}$$

However, the fall in $\underline{p}$ directly increases fire sales costs by reducing the market value of projects at time 1 and tightening the borrowing constraint. Banks react optimally by investing less and issuing more long-term debt, which reduces their liquidity shortfall. Costs increase when the first effect dominates, which again happens when g'' is big enough.

This reasoning is reversed when $\underline{p}$ is constant. In this case, the fall in v_{2b} directly lowers cost by diminishing the fire sales discount and lowering the marginal value of liquidity ϕ. However, banks optimally respond to a lower ϕ by investing more and issuing less long-term debt, which increases their liquidity shortfall. Costs increase when the second effect dominates. This occurs when banks' demand for investment is not too inelastic, i.e. when g'' is not too big.

An alternative interpretation of these results is in terms of the correlation between banks. The correlation between the $t = 1$ cash flows of bank i and j is

$$Corr\left(v_{1s} + \varepsilon_{z^i}, v_{1s} + \varepsilon_{z^j}\right) = \frac{1}{1 + Var\left(\varepsilon_z\right)/Var\left(v_{1s}\right)} \tag{19}$$

A change in correlation affects the cost of fire sales if and only if it is driven by an increase in systematic risk through $Var\left(v_{1s}\right)$. If correlation changes due to changes in systematic risk through $Var\left(\varepsilon_z\right)$, the costs are unaffected.

4.3 The social planner's choice

I now study the choices of a benevolent planner who dictates $t = 0$ bank choices $\mathbf{x}_0 = (c_0, n_0, \bar{n}_0)$ but has to satisfy the bank's budget constraint (3). He leaves banks to optimize

given $\mathbf{x}_0$ and market prices at $t = 1$. His choices induce a *planned equilibrium.*

There are three types of agents: Banks, creditors and outside buyers. Creditors and outside buyers are always indifferent between dealing with the bank and consuming their exogenous endowment, so that their utility is the same in any planned equilibrium. Hence, the social planner seeks to maximize expected bank profits in planned equilibrium. His optimal choices are called *constrained efficient.* Constrained efficiency differs from full Pareto efficiency in that my social planner cannot freely transfer wealth between groups of agents, e.g. between creditors and banks.

One vector of social planner's choices $\mathbf{x}_0$ may induce multiple planned equilibria. This is due to a self-fulfilling debt-deflation spiral, which implies that both high and low asset prices may clear the market: When asset prices are high, banks have no funding problems and do not sell assets. When asset prices are low, banks are forced to sell. To facilitate the analysis, I assume that the market selects the 'better' equilibrium without a fire sale.

ASSUMPTION 3. *If the social planner's choices induce multiple planned equilibria, the one with the highest equilibrium price p_b is selected with probability 1.*

It is helpful to examine what the social planner needs to do to avoid a fire sale. He has to choose $t = 0$ investments such that fair pricing in all states ($p_s = v_{2s}$) is an equilibrium. This can only be the case if, given fair prices, there is no aggregate liquidity shortfall.

LEMMA 2. *In planned equilibrium prices satisfy $p_g = v_{2g}$. Furthermore, $p_b = v_{2b}$ if the planner's choice $\mathbf{x}_0$ satisfies the 'no fire sale' condition*

$$(n_0 + \bar{n}_0)(1 - v_{2b}(1-h) - v_{1b}) - \bar{n}_0 (1 - v_{2b})(1-h) \leq e_0 \tag{20}$$

and $p_b = \underline{p}$ otherwise.[10]

This characterization illustrates the *two basic trade-offs* the planner faces if he wishes to avoid a fire sale. First, he trades off investment in projects $n_0 + \bar{n}_0$ against holding cash reserves. The no fire sale condition is more likely to hold when cash reserves are high and $n_0 + \bar{n}_0$ is low, because cash provides a liquidity cushion. The downside is that projects have a higher net present value than cash.

Second, the planner trades off short-term against long-term borrowing. The no fire sale condition (20) is more likely to hold when long-term debt is used, since it alleviates the rollover problem. However, long-term debt is expensive due to the social value of bank maturity transformation.

4.4 Inefficiency of equilibrium

Recall that in competitive equilibrium, the bank invests much of its equity in projects funded by short-term debt, which leads to a fire sale. I show that the competitive equilibrium is potentially inefficient. For a range of parameter values, the social planner chooses not to replicate banks' choices in competitive equilibrium.

PROPOSITION 3. *There exists an open set of values $R > R_f$ and $q > q_f$ for which the competitive equilibrium is constrained inefficient. Then the unique constrained efficient choice, denoted $\mathbf{x}_0^E$, solves the problem*

$$\max_{\mathbf{x}_0 \in B_0} R\left(n_0 + \bar{n}_0\right) - r\left(\bar{n}_0\right) - g\left(n_0 + \bar{n}_0\right)$$

$$\textit{subject to} \qquad \left(n_0 + \bar{n}_0\right)\left(1 - v_{2b}\left(1 - h\right) - v_{1b}\right) - \bar{n}_0\left(1 - v_{2b}\right)\left(1 - h\right) \leq e_0 \tag{21}$$

Aggregate actions in competitive equilibrium lead to a situation with fire sales and reduced individual profitability. This creates a 'systemic risk externality': The choices of individual banks, by contributing to aggregate illiquidity and low equilibrium prices, reduce the profitability of others. Thus, banks may be better off if they are forced to coordinate on lower aggregate risk-taking by a social planner.

Systemic externalities are *pecuniary externalities* because they work through equilibrium prices. Pecuniary externalities affect welfare when financial markets are incomplete (Geanakoplos and Polemarchakis 1985). Here, markets are incomplete due to binding borrowing constraints.[11] Raising the equilibrium price relaxes this constraint, which benefits banks more than it harms buyers. The argument of Greenwald and Stiglitz (1986) on the neutrality of pecuniary externalities breaks down.

To understand the characterization of the efficient choice, note that when the competitive equilibrium is constrained inefficient, the social planner will choose to prevent a fire sale in the bad state by satisfying the no fire sales condition.Indeed, any choice that induces a fire sale is dominated by a replication of the competitive equilibrium. Thus the planner wishes to maximize profits subject to the no fire sale condition, as stated in the second part of proposition 3.

4.5 A graphical illustration

Figure 1 illustrates the welfare analysis. The social planner dictates $t = 0$ bank investments $\mathbf{x}_0 = (c_0, n_0, \bar{n}_0)$. Given the project investments n_0 and $\bar{n}_0$, cash holdings are determined by the budget constraint in equation (3). There are two free choice variables, n_0 and $\bar{n}_0$. The feasible set is the area under the solid budget line, which has slope -1.

Leverage and maturity mismatch can be visualized in Figure 1. Leverage corresponds to a high total number of projects $n_0 + \bar{n}_0$. Leverage is high in the north-east of the figure, when the choice is close to the budget line. Maturity mismatch corresponds to a high ratio of short-term debt to total debt $n_0/(n_0 + \bar{n}_0)$. Maturity mismatch is high in the north-west of the figure.

The dashed line illustrates the no fire sale condition (20). The social planner needs to choose a point on or below the line to avoid a fire sale. The slope of the line is flatter than the budget line, because substituting long-term debt for short-term debt alleviates liquidity issues.

The planner maximizes the objective in (21) subject to the no fire sales condition. The isoprofit contours are level curves of the objective function, and are ellipses around the unconstrained optimum at point A. Point A would be the choice of unregulated banks given $p_b = v_{2b}$. Since A lies above the no fire sale condition, $p_b = v_{2b}$ is not a competitive equilibrium.

The constrained efficient point is at point B, where the isoprofit curves are tangent to the no fire sale condition. This point features both reduced leverage and reduced maturity mismatch compared to the unconstrained choice A.

5 OPTIMAL MACROPRUDENTIAL REGULATION

This section discusses how financial regulation can be used to avoid inefficient fire sales in a decentralized equilibrium. In the language of Hanson, Kashyap and Stein (2011), regulation against fire sales is *macroprudential* regulation, as it is concerned with aggregate behaviour and systematic risk.

I consider two macroprudential tools: Constraint-based regulation and Pigouvian taxation. Constraint-based regulation requires that bank choices $\mathbf{x}_0$ lie in some non-empty set $Q_0 \subset B_0$. Pigouvian taxation requires banks to pay the regulator τ per unit of investment, and receive a rebate $\bar{\rho}$ per unit of investment which is funded by long-term debt. Each constraint-based or Pigouvian regulatory regime induces a *regulated equilibrium.* Unlike in previous studies of Pigouvian taxes against systemic risk, I model a two-tier tax regime. A simple tax would not be able to achieve efficiency, as it cannot give nuanced incentives for reduced leverage and maturity mismatch.

As in the previous section, I impose an assumption to deal with multiple regulated equilibria:

ASSUMPTION 4. *If a regulatory regime induces multiple regulated equilibria, the one with the highest equilibrium price p_b is selected with probability 1.*

5.1 Constraint-based regulation

By definition, a regulator can never do better than the social planner in Section 4.3. Hence, if a regulated equilibrium is constrained efficient (i.e. if it replicates the planner's choice), then it must be optimal. Trivially, dictating that banks must replicate the planner's choice ($Q_0 =$

$\left\{\mathbf{x}_0^E\right\}$) is an optimal regulatory constraint. However, this regime offers no informational advantage over outright centralization.

As an alternative, the regulator can impose a constraint which mimics the no fire sale condition (20). Under this constraint, the bank's problem becomes to maximize profits subject to 'no fire sale'. But Proposition 3 shows that this is equivalent to finding the constrained efficient investment. Hence, imposing the no fire sale condition leads to a constrained efficient allocation in equilibrium.

PROPOSITION 4. *The following regulatory constraint induces a constrained efficient regulated equilibrium:*

$$Q_0^{macro} = \{\mathbf{x}_0 \in B_0 | \, (n_0 + \bar{n}_0)(1 - v_{2b}(1-h) - v_{1b}) - \bar{n}_0 (1 - v_{2b})(1-h) \leq e_0\} \tag{22}$$

In terms of the graphical analysis in Figure 1, suppose that the no fire sale condition is imposed on banks as a constraint. Then their privately optimal choice will be where isoprofit contours are tangent to the regulatory constraint. This point coincide with the constrained efficient point B, yielding an efficient allocation in regulated equilibrium.

The optimal rule (22) is remarkably simple in two ways. First, if she can observe the bank's balance sheet, the regulator only needs to know three parameters to impose Q_0^{macro}: Aggregate cash flows in the bad state v_{1b} and v_{2b}, and the haircut h. Second, it is *linear* in banks' investment choices.

It is easy to see why a linear constraint is optimal in the current model. The optimal rule mimics the no fire sale condition (20). This reduces to the requirement that bank equity at

$t = 1$, which is a linear function of investment choices, must be above a certain threshold to avoid an aggregate liquidity shortfall. Therefore, even though the marginal value of bank liquidity ϕ is non-linear, the optimal regulation to ensure sufficient bank liquidity is linear.[12]

This is a key result of this paper. A simple linear rule is fully efficient, in the sense that it achieves the same level of welfare as a benevolent planner. The generality of this conclusion is discussed at the end of this section.

In contrast, a central planner generally needs to know all parameters of the model to solve the full optimization problem (21). This comparison illustrates the considerable informational advantage of constraint-based regulation over centralization. Limited knowledge of the value of investment R, monitoring costs g or funding costs r, for instance, does not render this policy ineffective.[13] Intuitively, the constraint Q_0^{macro} gives banks the incentive to use their information efficiently, without the regulator having to obtain it.

5.2 Pigouvian taxation

The purpose of taxes is to ensure that banks' optimal choices when there is no fire sale ($p_b = v_{2b}$) coincide with the choices of the social planner ($\mathbf{x}_0^E$). Under this condition, the regulated equilibrium with taxes will be constrained efficient.

The representative bank's problem when $p_b = v_{2b}$ is to maximize $V(\mathbf{x}_0, v_{2b})$ net of taxes. Its first-order conditions are now

$$R - g'(n_0^\star + \bar{n}_0^\star) = \tau \tag{23}$$

$$r'(\bar{n}_0^\star) = \bar{\rho} \tag{24}$$

An efficient tax regime must ensure that these choices coincide with the social planner's. By taking the first-order condition of the planner's program (21), it is easy obtain the following characterization.

PROPOSITION 5. *The unique tax regime inducing a constrained efficient regulated equilibrium is*

$$\tau = \lambda \left[1 - v_{2b}(1-h) - v_{1b}\right] \quad (25)$$

$$\bar{\rho} = \lambda (1 - v_{2b})(1-h) \quad (26)$$

where λ is the Lagrange multiplier associated with the no fire sale condition at the solution of the social planner's problem (21).

This proposition shows that a two-tier Pigouvian tax can achieve efficiency. However, it does not compare favourably with constraint-based regulation when it comes to informational requirements. In order to pick the optimal tax regime τ and $\bar{\rho}$, the regulator needs to know the cash flow parameters v_{1b}, v_{2b} and the haircut h. In addition, she needs to know the social planner's Lagrange multiplier λ, which measures the social value of aggregate bank liquidity in the bad state.

In particular, one can use the planner's first-order condition to solve for λ, which yields

$$\tau = R - g'\left(n_0^E + \bar{n}_0^E\right) \quad (27)$$

$$\bar{\rho} = r'\left(\bar{n}_0^E\right) \quad (28)$$

The efficient tax rates are functions of the planner's optimal choices n_0^E and $\bar{n}_0^E$. In general, these choices depend on all parameters of the model. Consequently, it appears that Pigouvian taxes are efficient, but more complex to implement than constraint-based regulation.

5.3 Implementation and Basel III

Capital requirements are insufficient to implement the optimal macroprudential regulation. They treat all debt equally, and cannot provide the differential treatment of long-term and short-term borrowing that is prescribed by Q_0^{macro}. To see this, note that traditional capital regulation would bound the ratio of equity to risky investment, yielding the constraint $e_0/(n_0 + \bar{n}_0) \leq \kappa$ for some κ. In Figure 1, this constraint would run parallel to the bank's budget constraint. It is easy to see that bank choices subject to this constraint would not, in general, coincide with the constrained efficient allocation at point B. I will return to the merits of capital regulation when discussing microprudential concerns in Section 6.

The Basel III Accord (BIS 2010) introduces two new tools to target liquidity: Net Stable Funding Ratio (NSFR) and Liquidity Coverage Ratio (LCR) requirements. Both can be used to implement optimal regulation in my model.

Net Stable Funding Ratio. The NSFR requirement works as follows:

1. *Available stable funding* (ASF) is a weighted sum of bank liabilities. Liabilities which lead to liquidity shortfalls have low weights.

2. *Required stable funding* (RSF) is a weighted sum of bank assets. Illiquid assets which cannot be sold easily have high weights.

3. The *NSFR* is calculated as the ratio of available to required stable funding. The regulatory constraint is $ASF/RSF \geq 1$.

Using the budget constraint (3), the optimal constraint Q_0^{macro} can be written as

$$\frac{ASF}{RSF} = \frac{e_0 + (1 - v_{2b})(1 - h)\,\bar{n}_0}{[1 - v_{2b}(1 - h) - v_{1b}](n_0 + \bar{n}_0)} \geq 1 \tag{29}$$

This formulation allows me to provide practical guidance for the calibration of weights in the NSFR requirement. In the numerator, equity receives a weight of 100%, whereas long-term debt $(1 - h)\,\bar{n}_0$ receives a lower weight of $(1 - v_{2b}) < 1$. Long-term debt is not quite as 'stable' as equity, because it obliges the bank to pledge assets as collateral, reducing its liquidity in a crisis as these assets cannot be pledged against new debt.

In the denominator, liquid cash receives zero weight. Projects receive a weight equal to $1 - v_{2b}(1 - h) - v_{1b} \in (0, 1)$. Their 'stability weight' is lower when systematic cash flow risk is high and when funding conditions are tight (high h), since both factors exacerbate rollover problems.

Liquidity Coverage Ratio. The LCR requirement works as follows:

1. *Net cash outflows* (NCO) are a weighted sum of bank liabilities. Liabilities which are withdrawn by creditors in a 30-day stress test scenario (determined by the regulator) have high weights.

2. *High quality liquid assets* (HQLA) are a weighted sum of bank assets. Illiquid assets have low weights.

3. The LCR is calculated as the ratio of high quality liquid assets to net cash outflows. The regulatory constraint is $HQLA/NCO \geq 1$.

The optimal constraint Q_0^{macro} can also be written as

$$\frac{HQLA}{NCO} = \frac{c_0 + (n_0 + \bar{n}_0)\, v_{1b}}{(1 - v_{2b})\,(1 - h)\, n_0} \geq 1 \tag{30}$$

This yields practical insights for the design of 'liquidity weights' and the stress test scenario underlying the NCO. In the numerator, liquid cash receives a 100% weight, and projects receive a weight that is equal to their liquidity contribution in the bad state, $v_{1b} < 1$. In the denominator, representing the *optimal stress-test scenario*, long-term debt and equity are not withdrawn, whereas a proportion $(1 - v_{2b})$ of short-term debt $(1 - h)\, n_0$ is withdrawn. The withdrawal rate on short-term debt is lower when systematic long-term risk is high, since this erodes the value of projects as collateral in a crisis, again exacerbating rollover problems.

In sum, my model supports the application of new macroprudential ratios which jointly target capital and liquidity. I remain agnostic about which of the new tools in Basel III is preferable; either tool can achieve efficiency. The Basel committee's proposals include both due to worries about systemic risk at different time horizons. The LCR's weights are designed with short-term risk in mind, while the NSFR focuses on longer-term stability. This is not captured by my three-period model, and richer dynamics justify the use of several constraints, as is shown in the next subsection. However, the equivalence demonstrated here is general to an extent: For example, two LCR's, with different weights, can achieve the same

constraint as a combination of LCR and NSFR, which would perhaps be more parsimonious than the current Basel framework.

5.4 Robustness of the optimal regulation

I examine whether the linearity and simplicity of Q_0^{macro} is an artifact of my simple model economy. An obvious first observation is that if project cash flows did not exhibit constant returns to scale, the no fire sale condition would not be linear. One response is that a first-order approximation of the condition would still be linear, and that this may offer a useful regulatory benchmark. Another response is that the assumption of constant returns, a standard in the banking literature, is quite reasonable. Financial assets and loans to households and businesses with similar characteristics have a natural constant-returns property.

A more interesting question is whether the static nature of the model or the binary nature of fire sales (outside buyers demand any quantity at price $\underline{p}$, but nothing at any higher price) are driving the result. Moreover, it is interesting to examine the effect of allowing banks to issue equity at $t = 1$. The following extensions address these concerns.

Dynamic extension. Consider an extension of the model until time $T > 2$, where the individual and aggregate states, $s_t \in \{g, b\}$ and $z_t^i \in \{H, L\}$ for $t \geq 1$, evolve according to independent Markov chains. For simplicity, I continue to assume that project investment and long-term borrowing only takes place at $t = 0$. For period t, let v_t denote the per-unit cash flow of projects, c_{t-1} the aggregate cash holdings, and n_{t-1} the amount of projects held by banks and pledged against short-term debt between $t - 1$ and t.[14] Denote the fair value of projects by $\hat{p}_t = E_t\left[\sum_{s=t+1}^{T} v_t\right]$ for $t \geq 1$. It is then possible to repeat the steps above to

show that the condition for no fire sale in period t is

$$n_{t-1}\left(p_{t-1}-\hat{p}_{t}\right)(1-h)-\left(n_{t-1}+\bar{n}_{0}\right)v_{t}-c_{t-1}\leq 0 \tag{31}$$

where $p_0 \equiv 1$. If a social planner wants to prevent fire sales, he will maximize bank profits subject to a series of such linear conditions. Thus, imposing linear constraints on banks period-by-period would again sufficient to induce efficiency.

Partial fire sales. In my model, outside buyers demand any quantity of projects at the fixed fire sale price $\underline{p}$. An alternative assumption is that they have a general downward-sloping inverse demand function for projects. Their demand for projects in state $s \in \{g, b\}$ is $\max\{0, D(p_s)\}$, where $D'(p_s) < 0$ and $D(v_{2b}) \leq 0$. Recall that the number of assets sold by banks in the bad aggregate state is equal to $1/p_b h$ times the aggregate liquidity shortfall. Market clearing now requires that[15]

$$D\left(p_{b}\right)=\frac{\left(n_{0}+\bar{n}_{0}\right)\left(1-p_{b}(1-h)-v_{1b}\right)-\bar{n}_{0}\left(1-p_{b}\right)(1-h)-e_{0}}{p_{b}h} \tag{32}$$

In this extension, the social planner might settle for a 'partial fire sale', aiming for an intermediate value $p_b \in \left[\underline{p}, v_{2b}\right]$ and adjusting bank investments to satisfy (32). Let p_b^E denote the price in the optimal planned equilibrium. As noted by Lorenzoni (2008), calculating p_b^E is difficult because the planner's problem is not concave in general. However, in any case, the planner's choice of bank investment $\mathbf{x}_0^E$ will be most profitable choice that satisfies (32)

for $p_b = p_b^E$. Therefore, the efficient choice solves the problem

$$\begin{aligned}
&\max_{\mathbf{x}_0 \in B_0} V\left(\mathbf{x}_0, p_b^E\right) \\
\text{subject to } D\left(p_b^E\right) \times p_b^E h \quad &= \quad (n_0 + \bar{n}_0)\left(1 - p_b^E (1-h) - v_{1b}\right) - \bar{n}_0 \left(1 - p_b^E\right)(1-h) - e_0
\end{aligned} \tag{33}$$

The planner's choice still maximizes bank profits subject to a linear condition, which means that it can be decentralized with a linear regulatory constraint. However, the informational efficiency of constraint-based regulation is compromised. Solving this problem requires knowledge of p_b^E, the socially optimal price level, which generally depends on all parameters of the model.

In conclusion, it appears that the optimality of a linear regulatory rule is fairly general, while its informational efficiency partly depends on the baseline model's assumption that fire sale prices are fixed.

However, I would argue that the informational efficiency is preserved in a heuristic sense. Even with partial fire sales, a regulator can simplify her decision by solving a realistic two-step problem. First, she can decide how much of a drop in asset prices she is willing to tolerate. Conditional on this decision, she can implement the optimal policy with a linear constraint which, as in the baseline model, requires little information.

Equity issuance at $t = 1$. Fire sales occur when banks are unable to raise sufficient funding in the bad state at $t = 1$. Suppose that banks can raise new equity $\hat{e}$ in this state, but that investors require an excess return on equity of $\chi \hat{e}$, where $\chi > 0$. This excess return is a cost from the bank's perspective.[16] For simplicity only, suppose that banks choose $\hat{e}$

before observing their idiosyncratic shock z^i. An extra unit of equity reduces a bank's net sale in (12) by $1/p_b h$ units. Each unit sold loses the bank $v_{2b} - p_b$, so that the marginal value of equity is $(v_{2b} - p_b)/p_b h$. Therefore, banks will issue equity $\hat{e} > 0$ only if the value of equity exceeds its cost, or

$$\chi \leq \frac{v_{2b} - p_b}{p_b h} \tag{34}$$

If assets are priced fairly ($p_b = v_{2b}$), then (34) does not hold and banks choose $\hat{e} = 0$. Banks' optimal choices when $p_b = v_{2b}$ are therefore the same as in the baseline model without equity issuance. Thus, if these is a fire sale in the baseline model (e.g. if R and q are large in the sense of Proposition 1), then prices must also fall below v_{2b} in the model with equity issuance.

The structure of equilibrium depends on the cost of equity χ. If equity is costly enough to satisfy $\chi > (v_{2b} - \underline{p})/\underline{p}h$, then banks never issue equity. In this case, the equilibrium exhibits a fire sale ($p_b = \underline{p}$) as in the baseline model, and the analysis of welfare and macroprudential regulation is unchanged.

If equity costs are intermediate with $\chi \leq (v_{2b} - \underline{p})/\underline{p}h$, the equilibrium has $\underline{p} \leq p_b < v_{2b}$, and banks raise just enough equity to avoid selling assets. The welfare analysis in this case depends on whether χ is interpreted as a private or social cost of equity. If it is a purely private cost, such as a transfer from banks to investors, then the case for macroprudential regulation is weakened. If it is a social deadweight cost, macroprudential regulation is as valuable as in the baseline model, and the optimal constraint Q_0^{macro} is unchanged.

In sum, macroprudential regulation can be motivated by the desire to avoid the social costs of emergency equity issuance, as well as by the desire to avoid fire sales. In either case,

optimal regulation takes the shape of the linear rule Q_0^{macro}.

6 OPTIMAL MICROPRUDENTIAL REGULATION

This section drops Assumption 2, which ensured that all bank debt was risk-free in the baseline model. A bank defaults at $t = 1$ if bad news have driven its continuation value below zero.Since all uncertainty is resolved at $t = 1$, there is no loss of generality in assuming that all default decisions are made at this date.

Bank resolution causes a deadweight social cost of $\kappa \geq 0$ times the measure of defaulting banks. The parameter κ captures administrative costs of resolution and the wider social costs of credit crunches and disruptions to intermediation and payment systems. I impose a parametric assumption which ensures that default is possible:

ASSUMPTION 5. *The individual shock* $|\varepsilon_L|$ *satisfies*

$$|\varepsilon_L| \leq v_{1b} + v_{2b} - (1 - h) + \frac{h}{e_0} r \left(\frac{e_0}{h} \right) \tag{35}$$

In this setting, additional regulation may be required to curb the social cost of bank default. Hanson, Kashyap and Stein (2011) define this as *microprudential* regulation.

Agents' payoffs upon default depend on the precise mechanism of bank resolution. I make two assumptions. First, short-term debt enjoys seniority over long-term debt, and the bank's choices are safe enough to keep short-term debt risk-free. Second, the bank's asset goes to a receiver upon default. The receiver pays back short-term creditors at $t = 1$, runs the bank

as efficiently as a banker, and then passes the (maximized) proceeds to long-term creditors at $t = 2$. These conditions simplify the analysis by ensuring that the constraint Q_0^{macro} continues to rule out fire sales. If short-term debt were risky or the receiver inefficient, bank default would exacerbate the fire sale, creating a need for stricter macroprudential regulation as well as an additional microprudential constraint. I also continue to assume that we are in the parametric region where it is efficient to prevent a fire sale.

PROPOSITION 6. *When $\kappa = 0$, the constraint Q_0^{macro} as defined in* (22) *induces a constrained efficient regulated equilibrium. When κ is sufficiently large, the constraint $Q_0^{macro} \cap Q_0^{micro}$ induces a constrained efficient equilibrium, where*

$$Q_0^{micro} = \{\mathbf{x}_0 \in B_0 |\, (n_0 + \bar{n}_0)\,(1 - v_{1b} - \varepsilon_L - v_{2b}) + r\,(\bar{n}_0) \leq e_0\} \tag{36}$$

Without deadweight costs, the macroprudential constraint Q_0^{macro} remains sufficient for efficiency. Long-term interest rates reflect default risk as well as the liquidity premium. This leads the bank to internalize the losses of long-term creditors in default, and there is no additional externality. With $\kappa > 0$, nobody will internalize the social deadweight cost, and the regulated equilibrium may be inefficient.

When κ is large, the planner, who maximizes bank profits net of deadweight costs, chooses to rule out bank default. A necessary and sufficient condition is that a bank with a low individual shock ($z = L$) in the bad aggregate state ($s = b$) has a non-negative continuation value.

This translates to the requirement that bank choices lie in the set Q_0^{micro}. Proceeding as in

the analysis of fire sales, the planner's choices maximize bank profits subject to the no fire sale condition *and* Q_0^{micro}. Consequently, it is efficient to impose both constraints on banks.

Interaction between macro- and microprudential regulation. Micro- and macroprudential regulation both constrain the risk-taking of banks. Thus, one would expect them to be substitutes. However, they are not perfect substitutes, because they target different types of risk. In particular, macroprudential regulation is more concerned with maturity mismatch than microprudential regulation.

I analyze substitutability from two angles. First, I show that one regulation can substitute for the other and achieve full efficiency, but that this substitutability is imperfect in that it depends on parameter values. Second, I consider the 'partial efficiency' goals of ruling out default (for microprudential regulation) and preventing fire sales (for macroprudential regulation), and show that each instrument may need to become tougher to achieve its partial efficiency goal when the other instrument is removed.

If the regulator has perfect information, efficiency can always be achieved with one constraint. I focus on a more nuanced case where the regulator observes the parameters of the constraints Q_0^{macro} and Q_0^{micro}, but need not know the remaining parameters of the model, such as expected returns R or the cost of investment $g(n_0 + \bar{n}_0)$.

PROPOSITION 7. *If the individual shock satisfies* $|\varepsilon_L| \geq hv_{2b}$*, then microprudential regulation is sufficient for efficiency.*

If $|\varepsilon_L| < hv_{2b}$*, then macroprudential regulation is sufficient for efficiency if the solution to*

$$\max_{\mathbf{x}_0 \in Q_0^{macro}} R\left(n_0 + \bar{n}_0\right) - r\left(\bar{n}_0\right) - g\left(n_0 + \bar{n}_0\right) \tag{37}$$

lies in the set Q_0^{micro} for all possible parameter values.

If bad individual shocks are sufficiently severe (i.e. if $|\varepsilon_L|$ is large), then microprudential regulation needs to be very tough to rule out default. The microprudential constraint on bank leverage is so tight that, regardless of banks' maturity mismatch, their choices satisfy the no fire sale condition. An additional macroprudential constraint is not required.

If individual shocks are not severe, the macroprudential constraint on bank leverage prevents the default of banks with adverse individual shocks. However, macroprudential regulation can never rule out default *in general*. For instance, macroprudential regulation would allow banks to invest the maximal amount and fund it entirely with long-term debt (the point $\bar{n}_0 = e_0/h$ and $n_0 = c_0 = 0$ always lies in the set Q_0^{macro}). Banks with adverse shocks may still default given this choice, as they are weighed down by large outstanding interest. Therefore, the second part of the proposition appropriately qualifies the sufficiency of macroprudential regulation.

Proposition 7 suggests imperfect substitutability of micro- and macroprudential regulation. In certain circumstances, but not generally, one instrument can be sufficient for efficiency and substitute for the other. Because there are only two aggregate and two individual states, policy decisions are binary here: It is either optimal to rule out fire sales and defaults, or to allow them. As a result, substitutability is also binary. When one regulation is introduced, the other regulation either becomes redundant or remains fully optimal. In a richer setting with continuous policy decisions, substitutability would also be more gradual.

So far I have held the constraints Q_0^{micro} and Q_0^{macro} constant and considered full efficiency. There is another type of substitutability in terms of 'partial efficiency' goals.

On the one hand, when macroprudential regulation is removed, the microprudential constraint may need to become tougher to rule out defaults. The absence of macroprudential regulation may trigger a fire sale in the bad aggregate state. Then, the continuation value of a bank with a low individual shock drops, as it makes losses on forced asset sales. It is easy to show that the microprudential constraint required to rule out default in this case is strictly tighter than in (36).

On the other hand, when microprudential regulation is removed, the macroprudential constraint does not change, since aggregate liquidity shortfalls are unaffected by individual defaults. Macroprudential policy is indifferent to the removal of microprudential rules, but the indifference hinges on the assumption that the receiver of defaulting banks is efficient. Consider the case of an inefficient receiver who sells defaulting banks' assets into the secondary market immediately at $t = 1$. In this case, the net supply of assets in the secondary market strictly increases when there is default, and the macroprudential constraint required to rule out fire sales becomes strictly tighter than (22).

Informational requirements. The macroprudential part of the optimal regulation requires the same information as before: v_{1b}, v_{2b} and h. The microprudential part requires these three parameters plus the idiosyncratic shock ε_L and the interest payment on long-term debt $r(\bar{n}_0)$. Regulating against default is more informationally costly than regulating against fire sales, as both 'micro' and 'macro' information is necessary.

Note that there is considerable overlap in the information required for micro- and macroprudential regulation. This exposes a potential weakness of recent reforms. Many governments have established macroprudential regulators which are institutionally separate from existing

microprudential agencies. This dichotomy may inhibit information sharing between agencies. My model shows that information sharing is indeed valuable, as optimal micro- and macroprudential regulations depend on many common factors.

Implementation. In the Basel regulatory framework, the optimal microprudential constraint Q_0^{micro} can be implemented by a tool that resembles a capital requirement. Rearranging (36) yields

$$\frac{e_0}{n_0 + \bar{n}_0} \geq 1 - v_{1b} - \varepsilon_b - v_{2b} + \frac{r\left(\bar{n}_0\right)}{n_0 + \bar{n}_0} \tag{38}$$

The left-hand side is just as in a traditional capital requirement - the ratio of equity capital to risky assets. Unlike in a traditional capital requirement, which constrains this ratio to lie above a constant value, the right-hand side is modified to account for interest payments. This is because banks may default due to high outstanding interest payments, not just due to a high face value of debt. However, this term may be negligible if interest rates are close to zero, so that a capital requirement would be approximately optimal.

7 TIME-VARYING REGULATION, RISK MEASURES AND PERSISTENCE

Many commentators have isolated the lack of *time-variation* in financial policy as one of its key flaws prior to 2008 (Brunnermeier et al. 2009, Hanson, Kashyap and Stein 2011). They emphasize that new macroprudential tools should be applied in a time-varying fashion,

with the stringency of rules adapting to the business cycle, credit conditions and fluctuating systematic risk. My model supports this point, as the optimal macroprudential regulation depends on the economy's risk profile.

The optimal macroprudential constraint Q_0^{macro} in (22) does not depend on the individual shock z, and changes in idiosyncratic risk do not affect macroprudential regulation. Changes in systematic risk do matter. Their precise implications depend whether long-term or short-term cash flows are affected.

A mean-preserving spread in v_{1s} decreases v_{1b}, so that the (positive) coefficient on total investment $(n_0 + \bar{n}_0)$ in Q_0^{macro} increases. Risky investments now provide less liquidity in the bad state, and regulation must be tougher on leverage. A mean preserving spread in v_{2s} decreases v_{2b}, so that the coefficient on total investment increases and the coefficient on investment funded by long-term debt $\bar{n}_0$ decreases (becomes more negative). With a decrease in future asset values, borrowing constraints at $t = 1$ are tighter, rollover problems are more severe, and regulation must be tougher on both leverage and maturity mismatch.

Moreover, one can interpret a mean-preserving spread in v_{2s} as an increase in the *persistence* of macroeconomic shocks. Consider a model in which the aggregate $t = 1$ cash flows are described by $v_{1s} = \bar{v}_1 + u_s$, where $\bar{v}_1$ is a basic cash flow and u_s is a macroeconomic shock satisfying $E[u_s] = 0$. Suppose further that $t = 2$ cash flows are $v_{2s} = \bar{v}_2 + \varphi u_s$, where $\varphi > 0$ measures the persistence of the shock u_s. An increase in the persistence φ corresponds to a mean-preserving spread in v_{2s}. From the preceding analysis, it follows that regulation should respond to increased persistence by being tougher on both leverage and maturity mismatch.

Another risk measure is the tightness of funding conditions, captured by the haircut h. When

h falls, the coefficient on total investment increases, and the coefficient on $\bar{n}_0$ decreases (becomes more negative). With relaxed funding conditions, banks lever up more, exacerbating rollover problems, and regulation must again be tougher on leverage and maturity mismatch.

Finally, Section 6 shows that microprudential regulation should also adapt to changes in risk. The optimal microprudential constraint Q_0^{micro} in (36) depends on both systematic and idiosyncratic risk. A mean-preserving spread in either v_{1s}, v_{2s} or ε_z raises the coefficient on total investment $(n_0 + \bar{n}_0)$. All increased risk implies default risk, regardless of its source, and regulation must become stricter on leverage.

This leads to another critique of the dichotomy between macro- and microprudential regulation. Neither time-variation nor a concern with systematic risk are exclusively macroprudential issues. They should be part of the micro- and the macroprudential toolkit.

8 CONCLUSION

This paper focuses on an economy where banks have incentives to create excessive systemic risk through leverage and maturity mismatch. I show that efficiency can be achieved through a relatively simple linear macroprudential constraint on banks' balance sheets, which requires less information than central planning or Pigouvian taxation. The Liquidity Coverage and Net Stable Funding ratios of Basel III are both capable of implementing the optimal policy. Macroprudential regulation should react to changes in systematic risk in the economy and credit conditions over the business cycle, but not to changes in the idiosyncratic risk of individual institutions.

In an extended model with potential socially costly bank failures, additional microprudential regulation against default is needed. This can be implemented with a constraint resembling a capital adequacy requirement, and needs to track changes in both systematic and idiosyncratic risk.

While my results offer support for recent regulatory reforms, they shed a critical light on the emerging view of macro- and microprudential regulation as separate tasks. Micro- and macroprudential regulators are substitutes and benefit from the exchange of information, which speaks against institutional separation. Moreover, my analysis demonstrates that both types of regulation ought to be time-varying and react to changes in systematic risk.

APPENDIX A: PROOFS

Proof of Lemma 1

Given the argument in the text, it remains to be verified that in any competitive equilibrium, asset prices satisfy (i) $\underline{p} \leq p_s \leq v_{2s}$ and (ii) $p_g = v_{2g}$.

(i) Suppose $p_s > v_{2s}$ for some s. Then the bank optimally sells all projects at $t = 1$ in state s, $n_{1s}^z = 0$. By market clearing, it follows that $n_0 = 0$. Now consider the other state $s' \neq s$. We have $p_{s'} \geq v_{2s'}$, because otherwise, the bank would choose $n_{1s'} > 0$ and the market would not clear. Hence $p_s \geq v_{2s}$ for all $s \in \{g, b\}$. But then it is easy to show that the bank would optimally set $n_0 > 0$, a contradiction.

(ii) Suppose that $v_{2g} > p_g \geq \underline{p}$. Then it is optimal for the bank to buy as many projects as possible, setting $n_{1g}^z = e_{1g}^z / p_g h$. Using (4) for e_{1g}^z and $E[\varepsilon_z] = 0$, this yields

$$n_0 - \left(\beta n_{1s}^L + (1-\beta)\, n_{1s}^H\right) = \frac{n_0\left[(1-p_g)(1-h) - v_{1g}\right] - \bar{n}_0 v_{1g} - c_0}{p_g h} < 0$$

where the inequality follows from Assumption 1 and the fact that one of n_0, c_0 and $\bar{n}_0$ is strictly positive by (3). This contradicts market clearing.

Proof of Proposition 1

The aggregate liquidity shortfall chosen by the bank given the price p_b is

$$\left(n_0^\star + \bar{n}_0^\star\right)\left[1 - p_b(1-h) - v_{1b}\right] - (1-p_b)(1-h)\,\bar{n}_0^\star - e_0$$

where $n_0^\star$ and $\bar{n}_0^\star$ are defined by (15) and (16). We have $d\left(n_0^\star + \bar{n}_0^\star\right)/dR > 0$ and, since $r'\left(\bar{n}_0\right)$ is increasing in q for all $\bar{n}_0$, $d\bar{n}_0^\star/dq < 0$. Thus, the aggregate liquidity shortfall is increasing in R and q. As R and q become large, holding other parameters fixed, optimal choices converge to the corner solution $\bar{n}_0^\star = 0$, $n_0^\star = \frac{e_0}{h}$ and $c_0^\star = 0$, for all $p_b \in \left[\underline{p}, v_{2b}\right]$. At this limit, the aggregate liquidity shortfall is strictly positive by Assumption 1.

Therefore, we can find two numbers $R_f > 0$ and $q_f > 0$ such that $R = R_f$ and $q = q_f$ implies

$$\min_{p_b \in \left[\underline{p}, v_{2b}\right]} \left(n_0^\star + \bar{n}_0^\star\right)\left[1 - p_b\left(1-h\right) - v_{1b}\right] - \left(1 - p_b\right)\left(1-h\right)\bar{n}_0^\star - e_0 = 0$$

When $R > R_f$ and $q > q_f$, the aggregate liquidity shortfall is strictly positive for all possible p_b. Therefore, the unique equilibrium price is $p_b = \underline{p}$ by Definition 1.

Proof of Proposition 2

Let $k = \left(1 - \underline{p}\left(1-h\right) - v_{1b}\right)$, $\bar{k} = \left(1 - \underline{p}\right)\left(1-h\right)$, $\underline{\phi} = \phi\left(\underline{p}\right)$, $y_0^\star = \left(n_0^\star + \bar{n}_0^\star\right)$. The cost of fire sales in equilibrium is $\underline{\phi}\left[k y_0^\star - \bar{k}\bar{n}_0^\star - e_0\right]$.

(i) Mean-preserving spread in v_{1s}. The cost does not depend on v_{1g} and by an application of the implicit function theorem to (15) and (16), its derivative with respect to v_{1b} is $\underline{\phi}$ times

$$k\frac{dy_0^\star}{dv_{1b}} - y_0^\star = k\frac{\underline{\phi}}{g''\left(y_0^\star\right)} - y_0^\star$$

This is negative, implying that the mean-preserving spread increases cost, if and only if $g'' > g_1$ as required.

(ii) Mean-preserving spread in v_{2s} with $\underline{p} = \eta v_{2b}$. The cost does not depend on v_{2g}, nor on v_{2b} directly, since v_{2b} cancels out in the expression for $\underline{\phi}$. However, it depends on $\underline{p}$, and $d\underline{p}/dv_{2b} = \eta$. It follows that its total derivative with respect to v_{2b} is $\underline{\phi}\eta$ times

$$k\frac{dy_0^\star}{d\underline{p}} - \bar{k}\frac{d\bar{n}_0^\star}{d\underline{p}} - (1-h)(y_0^\star - \bar{n}_0^\star) = (1-h)\left[\frac{k\underline{\phi}}{g''(y_0^\star)} - \frac{\bar{k}\underline{\phi}}{r''(\bar{n}_0^\star)} - (y_0^\star - \bar{n}_0^\star)\right]$$

When $g'' = g_1$, the terms involving $y_0^\star$ cancel and the expression is strictly positive. As $g'' \to \infty$, the expression becomes strictly negative. Hence it is negative if g'' is sufficiently far above g_1 as required.

(iii) Mean-preserving spread in v_{2b} with $\underline{p}$ fixed. The cost (13) does not depend on v_{2g} and it only depends on v_{2b} through $\underline{\phi}$, which is strictly increasing in v_{2b}. Its derivative with respect to $\underline{\phi}$ is

$$\left[ky_0^\star - \bar{k}\bar{n}_0^\star - e_0\right] + \underline{\phi}\left[k\frac{dy_0^\star}{d\underline{\phi}} - \bar{k}\frac{d\bar{n}_0^\star}{d\underline{\phi}}\right] = \left[ky_0^\star - \bar{k}\bar{n}_0^\star - e_0\right] - \underline{\phi}\left[\frac{k^2}{g''\left(y_0^\star\right)} + \frac{\bar{k}^2}{r''\left(\bar{n}_0^\star\right)}\right]$$

When $g'' = g_1$, the terms involving $y_0^\star$ cancel and the expression is strictly negative. By continuity, it is negative when when g'' is not too far above g_1, as required.

Proof of Lemma 2

By a parallel argument to Lemma 1, prices in planned equilibrium satisfy $\underline{p} \leq p_b \leq v_{2b}$ and $p_g = v_{2g}$ (as long as $n_0 > 0$). First, suppose the planner's choices satisfy (20). If $p_b = v_{2b}$ then banks are indifferent between all feasible $t = 1$ investments, the aggregate liquidity

shortfall is non-positive by (20), and it is feasible for banks to set $\beta n_{1s}^{L}+(1-\beta)\,n_{1s}^{H}=n_0$. Thus $p_b=v_{2b}$ is an equilibrium price, and will be selected by Assumption 3. Second, suppose the planner's choices do not satisfy (20). Then for all $\underline{p}\leq p_b\leq v_{2b}$, the aggregate liquidity shortfall is positive, because it is positive for $p_b=v_{2b}$ and decreasing in p_b. All feasible bank choices then have $\beta n_{1s}^{L}+(1-\beta)\,n_{1s}^{H}<n_0$. Thus $p_b=\underline{p}$ is the only equilibrium price.

Proof of Proposition 3

(i) Inefficiency. I show that for R and δ sufficiently close to R_f and q_f, there exists a choice $\mathbf{x}_0^A\in B_0$ such that the planned equilibrium induced by $\mathbf{x}_0^A$ yields higher profits for the banks than the competitive equilibrium. Define

$$\hat{\sigma}=\min_{p_b\in\left[\underline{p},v_{2b}\right]}\left(n_0^{\star}+\bar{n}_0^{\star}\right)\left[1-p_b\left(1-h\right)-v_{1b}\right]-\left(1-p_b\right)\left(1-h\right)\bar{n}_0^{\star}-e_0$$

and let $\hat{p}$ denote any minimizer of the right-hand side. From the definition of R_f and q_f (see the proof of Proposition 1) and Berge's maximum theorem, it follows that $\hat{\sigma}\downarrow 0$ as $(R,q)\downarrow(R_f,q_f)$, and that $\hat{p}$ converges to some $\hat{p}_f\in\left[\underline{p},v_{2b}\right]$. I prove that there is a welfare-improving choice in two steps.

First, suppose $\hat{p}_f>\underline{p}$. Making the dependence of optimal choices on prices explicit, consider the choice $\mathbf{x}_0^A$, defined by $n_0^A=n_0^{\star}(\hat{p})-\epsilon$ and $\bar{n}_0^A=\bar{n}_0^{\star}(\hat{p})$, where $\epsilon\geq 0$ is chosen to ensure that the choices satisfy the no fire sale condition with equality if necessary. Define the bank's indirect utility as $W(p_b)=\max_{\mathbf{x}_0\in B_0}V(\mathbf{x}_0,p_b)$. I show that as $(R,q)\downarrow(R_f,q_f)$, the planned equilibrium yields higher limiting profits than the competitive equilibrium. It is easy to see

that $\epsilon \downarrow 0$, so that the planned equilibrium induced by $\mathbf{x}_0^A$ yields limiting profits equal to $W(\hat{p}_f)$, whereas the competitive equilibrium yields $W\left(\underline{p}\right)$. It remains to be shown that $W(\hat{p}_f) > W\left(\underline{p}\right)$ in the limit. By the envelope theorem and the fact that $\hat{\sigma} \downarrow 0$, the limiting case satisfies $W'(p_b) > 0$ for all $p_b < \hat{p}_f$, which implies $W(\hat{p}_f) > W\left(\underline{p}\right)$ as required.

Second, suppose $\hat{p}_f = \underline{p}$. Suppose the social planner picks $n_0^A = n_0^\star(\hat{p})$ and $\bar{n}_0^A = \bar{n}_0^\star(\hat{p})$. For R and q sufficiently close to R_f and q_f, these choices satisfy the no fire sales condition, so that the planned equilibrium yields profits $V\left(\mathbf{x}_0^A, v_{2b}\right) > W(\hat{p})$. The competitive equilibrium yields $W\left(\underline{p}\right)$. It remains to be shown that $W(\hat{p}) \geq W\left(\underline{p}\right)$. By the envelope theorem, we have $W'(p_b) > 0$ for all $p_b > \underline{p}$, which implies $W(\hat{p}) \geq W\left(\underline{p}\right)$ as required.

(ii) Characterization of optimum. By Lemma 2, the planner knows his choices will induce $p_b \in \left\{\underline{p}, v_{2b}\right\}$. For $p_b = \underline{p}$, the highest welfare possible is $W\left(\underline{p}\right)$, which is equal to welfare in competitive equilibrium. This is inefficient by assumption, so a choice inducing $p_b = v_{2b}$ must be optimal. The planner then maximizes $V(\mathbf{x}_0, v_{2b})$ subject to (20) as required.

Proof of Proposition 6

(i) $\kappa = 0$. By assumption, the efficient choice maximizes bank profits, which are zero in states where the bank defaults, subject to (20), which continues to guarantee no fire sale due to the assumptions about resolution. This again coincides with the bank's objective when constraint Q_0^{macro} is imposed.

(ii) κ large. The efficient choice maximizes bank profits minus κ times the measure of defaulting banks, subject to (20). It is easy to see that as $\kappa \to \infty$, since profits are bounded above, this converges to a choice where default is ruled out. This choice must maximize bank

profits subject to (20) and the requirement that the bank's continuation value is positive in every state at $t = 1$. Since $v_{tg} > v_{tb}$ and $\varepsilon_H > \varepsilon_L$, it is sufficient that the continuation value is positive in state (b, L). Given that (20) is satisfied, we have $p_b = v_{2b}$, and positive continuation value is equivalent to

$$(v_{1b} + \varepsilon_L + v_{2b})(n_0 + \bar{n}_0) + c_0 - (1-h)(n_0 + \bar{n}_0) - r(\bar{n}_0) \geq 0$$

By the budget (3), this is in turn equivalent to the proposed requirement Q_0^{micro}.

Proof of Proposition 7

(i) Sufficiency of microprudential regulation. Microprudential regulation is sufficient for efficiency if $Q_0^{micro} \subset Q_0^{macro}$. Suppose $|\varepsilon_L| \geq h v_{2b}$ and $\mathbf{x}_0 \in Q_0^{micro}$. Then we have

$$\begin{aligned}(n_0 + \bar{n}_0)(1 - v_{2b}(1-h) - v_{1b}) - \bar{n}_0(1 - v_{2b})(1-h) &\leq (n_0 + \bar{n}_0)(1 - v_{2b}(1-h) - v_{1b}) \\ = (n_0 + \bar{n}_0)\left[(1 - v_{1b} - \varepsilon_L - v_{2b}) - (|\varepsilon_L| - h v_{2b})\right] &\leq e_0\end{aligned}$$

where the last inequality follows from the definition of Q_0^{micro} and $r(\bar{n}_0) \geq 0$. Thus, $\mathbf{x}_0 \in Q_0^{macro}$, implying $Q_0^{micro} \subset Q_0^{macro}$ as required.

(ii) Sufficiency of macroprudential regulation. Given the stated condition, banks' choices in equilibrium, subject to macroprudential regulation, lie within the set Q_0^{micro}. Hence, when Q_0^{macro} is imposed on banks, there is no fire sale and no default, which is sufficient for efficiency.

APPENDIX B: INTERIOR SOLUTIONS

This appendix derives parametric restrictions under which the bank's first-order conditions (15) and (16) are necessary and sufficient for optimality. By the concavity of the bank's problem, this is the case as long as the choices they imply are feasible. The feasibility conditions, implied by the budget constraint (3) and non-negativity of the bank's choices, are

$$\begin{aligned} 0 &\leq n_0^\star + \bar{n}_0^\star \leq \frac{e_0}{h} \\ 0 &\leq \bar{n}_0^\star \leq n_0^\star + \bar{n}_0^\star \end{aligned}$$

For the total investment in (16) to satisfy the first condition for all $p_b \in \left[\underline{p}, v_{2b}\right]$, we need

$$\begin{aligned} R - \phi\left(\underline{p}\right)\left[1 - \underline{p}\left(1-h\right) - v_{1b}\right] &\geq 0 \\ R &\leq g'\left(\frac{e_0}{h}\right) \end{aligned}$$

The investment funded short-term debt satisfies $\bar{n}_0^\star \geq 0$ since $r'(0) = 0$. Thus, to obtain $\bar{n}_0^\star \leq n_0^\star + \bar{n}_0^\star$ for all $p_b \in \left[\underline{p}, v_{2b}\right]$, we need

$$\left(r'\right)^{-1}\left(\phi\left(\underline{p}\right)\left(1-\underline{p}\right)\left(1-h\right)\right) \leq \left(g'\right)^{-1}\left(R - \phi\left(\underline{p}\right)\left[1 - \underline{p}\left(1-h\right) - v_{1b}\right]\right)$$

Since r and g are strictly convex, r', g' and their inverses are strictly increasing functions.

It is then easy to see that the above can be guaranteed by a suitable upper bound on $\phi\left(\underline{p}\right) = \alpha\frac{v_{2b}-\underline{p}}{\underline{p}h}$.

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FOOTNOTES

1. Farhi and Tirole (2012), Jeanne and Korinek (2013) and Benigno et al. (2013) address the issue of ex post policies such as bailouts, central bank loans and quantitative easing. Their analysis complements the ex ante perspective of this paper.

2. If $E[\varepsilon_{z^i}] = \bar{\varepsilon} \neq 0$, one could simply redefine $\hat{v}_{1s} = v_{1s} + \bar{\varepsilon}$ for all s, and $\hat{\varepsilon}_{z^i} = \varepsilon_{z^i} - \bar{\varepsilon}$ for all z^i.

3. I assume away idiosyncratic risk at $t = 2$. This is for clarity of exposition and does not affect the qualitative results.

4. Similar borrowing constraints can be derived as part of optimal contracts when there are inalienable returns (Hart and Moore 1994) or moral hazard (Holmström and Tirole 1997).

5. This convention is for simplicity only. Using a different $t = 0$ market value does not change my qualitative results, but introduces an additional constant into banks' $t = 0$ budget constraint. However, the $t = 1$ budget constraint and the analysis of fire sales would remain the same.

6. The assumption of fixed equity is fairly standard in the related literature, and represents a parsimonious approach to modelling costs of or limits to outside equity issues, for instance due to informational asymmetries (Myers and Majluf 1984). Additionally, I assume that no additional equity can be raised at $t = 1$. Relaxing this would only affect my results if banks were able to raise outside equity in times of crisis, i.e. in the bad state at $t = 1$. It is generally accepted that doing so is costly due to time pressure or debt overhang problems (see Hanson, Kashyap and Stein 2011). In Section 5, I consider an extension with costly equity issuance at $t = 1$.

7. The bank does not invest in unlevered projects that are funded in full by equity. It can be shown that this assumption is without loss of generality, since the cash flow from an unlevered project can be replicated by a combination of short-term debt and cash.

8. The assumption of convex disutility (e.g. Allen and Gale 2000) ensures that there is an interior solution to the bank's maximization problem, and a meaningful trade-off between investment in projects and holding

cash. There are other ways to achieve this, such as limited risk tolerance or decreasing monetary returns to investment, but this formulation makes for a particularly neat exposition.

9. This indirect effect is closely linked to the point on maturity and systematic risk in Chen, Xu and Yang (2013). In their dynamic model, firms with high systematic risk exposure choose a longer debt maturity. Condition (15) shows that the optimal maturity of debt also lengthens with an increase in systematic risk in my model.

10. I assume that the social planner always invests in some projects funded by short-term debt, or $n_0 > 0$. For $n_0 = 0$, then there would be no projects for sale at $t = 1$ and the secondary market would shut down. $n_0 = 0$ would never be optimal since projects have positive NPV.

11. Other applications to finance include Kehoe and Levine (1993), Gromb and Vayanos (2002) and Caballero and Krishnamurthy (2003).

12. Bolton, Chen and Wang (2011) provide a general characterization of the (highly non-linear) marginal value of firm liquidity in a dynamic model.

13. Even though both centralization $Q_0 = \{\mathbf{x}_0^E\}$ and Q_0^{macro} are technically efficient in this model, it is easy to show that for any constraint $Q_0' \neq Q_0^{macro}$, there exist values of the parameters other than v_{1b}, v_{2b} and h such that Q_0' does not induce a constrained efficient equilibrium. In this sense, Q_0^{macro} is uniquely optimal.

14. These variables of course depend on the time-varying states, but I suppress the additional subscript to save notation.

15. As in Lorenzoni (2008), a bound on the slope of D would be required to guarantee uniqueness of equilibrium.

16. The source of this cost could be information asymmetries as in Myers and Majluf (1984), debt overhang (Hanson, Kashyap and Stein 2011), or indeed any violation of the Modigliani-Miller conditions in times of crisis. Acharya and Steffen (2012) provide evidence that investors required significant discounts on bank equity following the recent crisis.

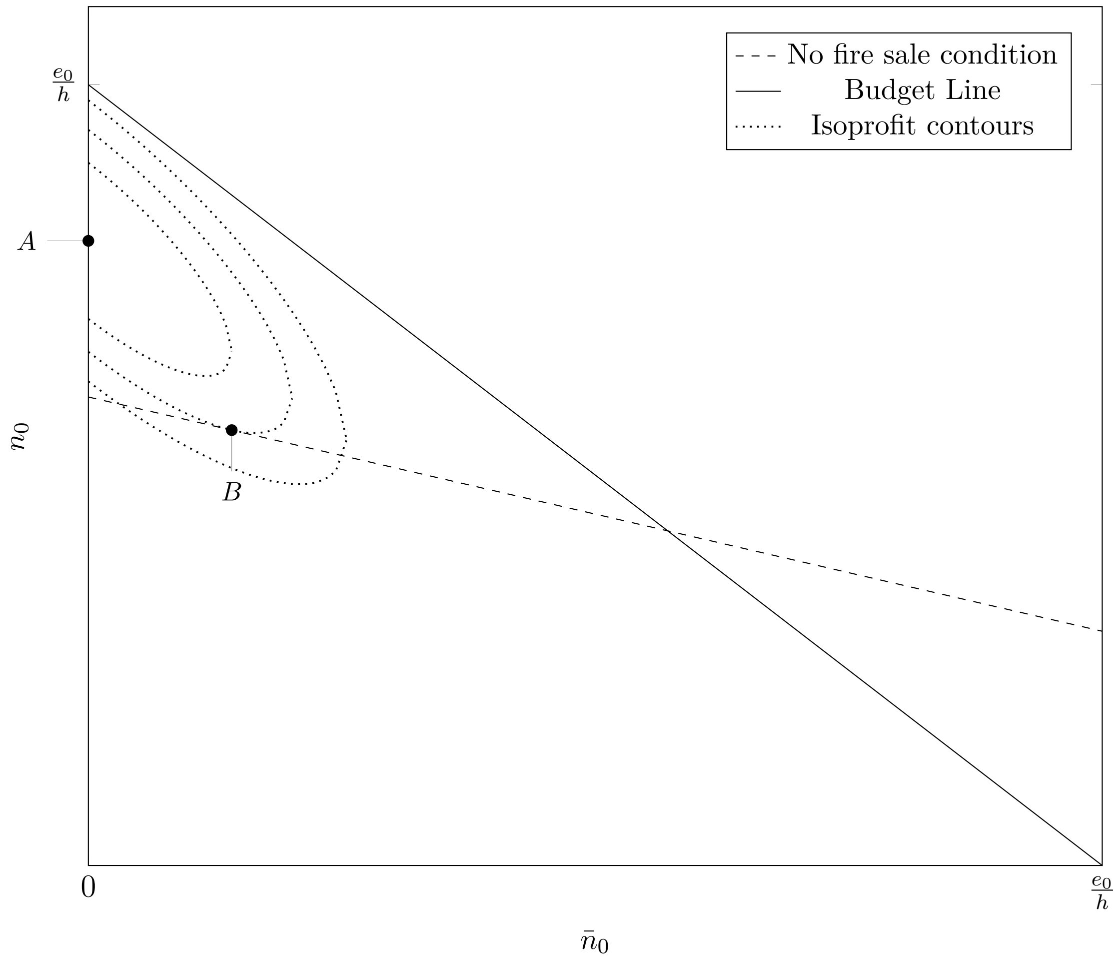

Figure 1: Welfare analysis