Optimisation of an acoustic resonator for particle manipulation in air

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Abstract

An acoustic resonator system has been investigated for the manipulation and entrapment of micron-sized particles in air. Careful consideration of the effect of the thickness and properties of the materials used in the design of the resonator was needed to ensure an optimised resonator. This was achieved using both analytical and finite-element modelling, as well as predictions of acoustic attenuation in air as a function of frequency over the 0.8 to 2.0 MHz frequency range. This resulted in a prediction of the likely operational frequency range to obtain particle manipulation. Experimental results are presented to demonstrate good capture of particles as small as 15 µm in diameter.

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1. Introduction

Biomedical applications increasingly use micro-electromechanical systems (MEMS) technology and small particle capture in their implementation. Examples include the analysis of airborne particles and pathogens. Microgrippers can be used to manipulate such particles in liquid [1], capture cells in suspension [2] and characterize mechanical properties of individual biological particles [3], and are usually designed to grip particles by frictional forces[1, 4-6] . However, these contact-based approaches can cause damage due to the mechanical forces involved [3]; they can also encounter problems with stiction, due to significant adhesive (capillary) forces that can be present at such small scales [7-9]. Stiction effects can be reduced by using surface coatings [10] and/or by minimising contact surface area by modifying the tip geometry of the gripper, for example [11]. However, a non-contact approach is required when handling delicate samples which may be damaged by frictional gripping methods.

A number of non-contact approaches to particle manipulation are available. These include optical traps [12] and electric field methods (dielectrophoresis or DEP) [13]. These methods have numerous applications, although miniaturisation and integration within MEMS-based devices is problematic. A high optical density is required for optical trapping, and the DEP method only operates over a limited spatial range. A third method, to be investigated here, uses forces generated by an ultrasonic field [14, 15]. This technique, also known as acoustophoresis, can be integrated into small-scale devices to manipulate particles and droplets in both liquids and gases [16, 17].

The acoustic method relies on differences in acoustic properties (i.e. acoustic impedance) between the particles and the medium, as this generates acoustic radiation forces (ARFs). These non-linear forces act directly on the suspended matter, causing a migration over multiple oscillation cycles [18]. Acoustic manipulation, using ARF, has been widely applied in microfluidic devices, due to good biocompatibility [19], relatively simple instrumentation, robust architectures and good on-chip integration possibilities. Capabilities such as positioning of particles in a single plane for filtration (acoustic filters) [20, 21], within a microfluidic channel [22] and within three dimensions [23] have been demonstrated. It can also be used for particle sorting and separation [24, 25], and for the production and manipulation of aqueous droplets in oil [26, 27].

While manipulation of particles in microfluidic liquid based systems is well established, there are only a handful of examples for manipulation of small particles in air. Acoustic levitation and transport of particles in air have been reported in the literature[17, 28-30], as has the ability to trap liquid droplets [31, 32]. Manipulation of particles as small as 500 µm in an acoustic resonator has been demonstrated [17]. However, most work with small particles has been performed in water within microfluidic resonators [20, 33, 34]. Here, we wish to manipulate very small particles in air, with diameters as small as 15 µm. Changing the dispersion medium to air at higher frequencies introduces losses due to attenuation [35], which are not a major factor in liquids. In addition, piezoelectric elements used for ultrasound generation are far less efficient in air, due to the large acoustic impedance mismatch between the two. This requires the use of a matching layer between the piezoelectric element and the air gap, which has to be optimised for use in a resonant air-filled cavity.

In this paper, both analytical and finite element (FE) models are used to examine the optimum criteria for an ultrasonic resonator that can be used for the manipulation of small particles in air, at frequencies of up to 2 MHz. Each layer within the design has to be chosen carefully in terms of thickness and material characteristics, for specific frequencies of operation. The modelling was then used to design and test a system which could be used with microparticle diameters as small as 15 µm experimentally.

2. Acoustic absorption in air

Models of acoustic resonators have been developed to predict the effects of acoustic impedance mismatch between the transducer and air, and subsequent attenuation in the air medium. FE modelling, using COMSOL Multiphysics™, has been used to predict the radiated pressure field within the air gap of the resonator within which the particles are held. This takes into account the frequency-dependent attenuation of ultrasound in air, allowing the selection of an optimum operating frequency for a particular application. Secondly, an analytical model has been developed to optimise the design of the transducer at the chosen frequency of operation, and considers the thickness and material properties of both the piezoelectric material and the impedance matching layer. Numerical evaluation of the analytical model has been conducted using MATLAB™. The system that has been modelled is shown in Fig. 1. The air-backed PZT (lead zirconate titanate) piezoelectric element generates ultrasound, which travels preferentially into the air-filled resonator due to the acoustic impedance matching layer (ML). The air gap itself, within which particle trapping should occur, forms a resonator by reflection from the reflector (i.e. glass slide) as shown in Fig. 1, assumed in this case to be a perfect reflector.

As stated above, a major difference between using acoustofluidic systems in water and in air is that attenuation is much greater in air at frequencies in the low MHz range. Hence, the effect of attenuation as a function of frequency in air needs to be incorporated into the modelling. Two main absorption mechanisms are present – classical and relaxation effects [35]. Classical losses are due to the change of kinetic energy of molecules into heat, caused mainly by viscous and heat conduction losses (sometimes referred to as viscous dissipation losses), and collectively known as the Stokes-Kirchhoff loss. Relaxation losses are associated with a change of kinetic translational energy of the molecules into internal energy within the molecules themselves. Relaxation losses have two main forms, namely rotational absorption, which consists of relaxation losses due to rotationally excited molecules, and vibrational absorption due to excited molecules of oxygen and nitrogen.

Since, both the classical losses and rotational absorption are a function of temperature (*T*), pressure (P) and frequency (*f*), they can be combined and described via a single absorption coefficient α_{cr}. There are also absorption coefficients that can be predicted for vibrational effects in oxygen ($\alpha_{vib,O}$) and nitrogen ($\alpha_{vib,N}$) [35], leading to absorption coefficient curves for each mechanism, as shown in Fig. 2, which has been generated using the following equations (expressed in units of dBm^{-1})

$$\alpha_{cr} = 15.895 \times 10^{-11} \frac{(T/T_0)^{1/2}}{(P/P_0)} f^2 \; ; \qquad (1(a))$$

$$\alpha_{vib,O} = 1.110 \times 10^{-1} \frac{e^{-2239.1/T}}{f_{r,O}+(f^2/f_{r,O})} (T_0/T)^{5/2} f^2; \qquad (1(b))$$

$$\alpha_{vib,N} = 9.480 \times 10^{-1} \frac{e^{-3352.0/T}}{f_{r,N}+(f^2/f_{r,N})} f^2. \qquad (1(c))$$

Here, T_0 and P_0 are reference values for temperature (293 K) and. atmospheric pressure (101.325 kPa) respectively. The terms $f_{r,O}$ and $f_{r,N}$ are the frequencies of maximum absorption by oxygen and nitrogen respectively, and are given by:

$$f_{r,O} = (P/P_0)\{24 + 4.41 \times 10^4 h[12.(0.05 + h)/(0.391 + h)]\} \qquad (2)$$

$$f_{r,N} = (P/P_0)(9 + 200h), \qquad (3)$$

where *h* is the molar concentration of water vapour. A value of *h* = 1.038 was used here, which corresponds to a relative humidity of approximately 45% at 20°C (conditions of the experimental setup and consistent with 293 K). The total acoustic absorption coefficient, α can then be found from

$$\alpha = \alpha_{cr} + \alpha_{vib,O} + \alpha_{vib,N}. \quad (4)$$

The resulting predictions are shown in Fig. 2 for the frequencies of interest. In particular, it can be seen that α increases by more than an order of magnitude between the frequencies of 100 kHz and 1 MHz (becoming increasingly dominated by classical and rotational losses) emphasizing the need to take attenuation into account in the model.

While such an attenuation coefficient is useful for analytical work, it is not so simple to apply to the FE models that formed part of the present work. To this end, Groschl [36] described a quality factor approach for describing energy loss within a liquid medium, by deriving a complex expression for the acoustic velocity in the presence of damping (c_{damped}). A similar approach can be used to account for attenuation in air. This involved a quality factor (Q_{air}) that modified the standard value for the speed of sound for air ($c = 343$ m/s at $20°$C), as follows:

$$c_{damped} = c \times \left(1 + i\frac{1}{Q_{air}}\right) \quad (5)$$

To show how this approach could be implemented, an FE model was created for a 10 mm long air-filled rectangular chamber of 5 mm width, with perfectly-reflecting side walls. A constant sinusoidal input pressure of 1 kPa was input at one end of chamber, and absorbed at the other end using a perfectly-matched impedance layer (see Fig. 3(a)). Suitable values for α could be estimated from Eqn. (4) for a particular frequency *f* within the range of interest (800 kHz - 2 MHz) of this study. The FE model was then run with an excitation at a given *f*, for different values of Q_{air}, until a value was found that corresponded to the level of attenuation that would have been predicted by Eqn. (4). An example of the FE output showing acoustic pressure decay at *f* = 2 MHz is shown in Figure 3(b) which resulted in a corresponding quality factor, Q_{air} of 482. These frequency-dependent values of Q_{air} have subsequently been used in the main finite element model which investigates the effect of attenuation on the choice of operating frequency.

3. Modelling of air-filled resonators

A. Details of the models

The modelling studies were designed to serve two purposes. The first aim was to be able to model the effect of acoustic propagation within an air-filled chamber, and to predict forces on particles at particular frequencies. This optimisation of the operational frequency, taking into account the role of attenuation, used a finite element (FE) model. The second approach was to use analytical modelling to optimise the design of the layered resonator, in terms of its main elements – the chamber, the piezoelectric element, and the acoustic matching layer needed for efficient operation in air. Taken together, they allowed the whole system to be designed for effective manipulation of small particles.

The FE model considered a half wavelength ($\lambda_{air}/2$) air gap (meaning the air-gap thickness was matched to the frequency of operation) within a chamber with parallel opposing edges (see Fig. 4(a)). The aim was to find the conditions for optimal capture and positioning of small particles. When absorption is present, attenuation will increase with frequency, hence, our investigation focussed on determining the optimum frequency of operation to trap a given particle in air with a fixed excitation energy.

A complete three-dimensional model could not be developed due to high memory requirements and processing power restrictions. However, an axisymmetric model, with the full air chamber formed by rotation about the central axis (as shown on the left hand side of Fig. 4(a)), was used to provide a modelling comparison to the experimental results from a similarly-sized chamber, to be described in the next section. Note that the chamber side was assumed to have the same acoustic impedance as air, thus effectively modelling a non-reflecting boundary; this simulated a chamber with no sides. The input kinetic energy density into the system was held constant across the range of frequencies examined, as a basis for comparison, by determining the required acceleration, $a_{boundary}$, for different angular frequencies (ω). This was achieved by altering the displacement ζ of the input surface using

$$a_{boundary} = \omega^2 \times \zeta \tag{6}$$

$$\zeta = \frac{\sqrt{E}}{f} \quad . \tag{7}$$

Here, E is the input kinetic energy. A value of ζ = 1 nm was assumed for f = 1 MHz, and the displacement scaled accordingly for the other frequencies considered. This displacement and the appropriate quality factors for air, Q_{air}, were then implemented in the FE model of the air chamber to obtain the resultant maximum absolute pressure within the air gap. Based on these pressures, the resultant ARF, $\langle F \rangle$ on a compressible sphere was calculated, based on Yosioka's formula [37, 38]. This can be stated as:

$$\langle F \rangle = \rho_f \pi \Phi^2 (k_F r_S)^3 F_Y sin(2k_F x_S) \tag{8}$$

with the density compressibility factor F_Y given by:

$$F_Y = \frac{\lambda + \frac{2}{3}(\lambda - 1)}{1 + 2\lambda} - \frac{1}{3\lambda\sigma^2}\,. \tag{9}$$

Here, $\lambda = \rho_S/\rho_F$ and $\sigma = c_S/c_F$, in which ρ_S, ρ_F, c_S and c_F are the density and speed of sound of the solid and fluid respectively. The wave number k_F is given by $k_F = 2\pi f/c_F$, r_S is the radius of the particle, and x_S is the position of the sphere. The velocity potential amplitude in a harmonic system, Φ is given in terms of the density, ρ, the pressure, P and the angular frequency ω by

$$\Phi = \frac{P}{i\omega\rho_F}\,. \tag{10}$$

Evaluation of the forces via Eqn. (8) allowed the FE model to be used for optimising the value of f for particle manipulation in air.

The next step was to use analytical modelling to determine the optimum design parameters for the construction of an air-filled chamber for a given value of f. The parameters to be investigated were the thickness of the main components within the resonator (the PZT piezoelectric element, the matching layer and the air gap) and the optimum acoustic properties of the matching layer. The latter was needed due to the large acoustic impedance differences between different components in an air-based system, a far more significant issue than for operation in liquid. A one-dimensional numerical analysis was carried out using fundamental equations to represent the chamber. This approach was similar to that used by Haake [38], although no assumptions were made concerning the phase of the output wave from the piezoelectric substrate. The effect of using a matching layer and air gap on the system as a whole was considered, the coupling between the matching layer and air gap being of especial interest due to the large acoustic impedance mismatch. It is worth noting that a layered resonator design for operation in liquid systems has been developed [20, 33, 34] previously and our results are compared within this study.

The model assumed an acoustic source in the form of a piezoelectric element, attached to which was an impedance matching layer in contact with the air. A standing wave is then established by transmission across the impedance boundary between the matching layer and air, with perfect reflection from the boundary at the far end of the air cavity, this set up is depicted in Fig 4 (b and c). In contrast to Hill [20], who modelled a resonator for liquid use, this model includes the piezoelectric element and the matching layer, in order to investigate the role of material selection in energy transfer across the high acoustic contrast boundary between the matching layer material and air.

The analytical model started by considering the conversion of the electrical input to an acoustic output from the piezoelectric layer. Nowotny *et* al[39] analysed a general one dimensional propagation through a two-electrode arbitrarily oriented layered piezoelectric substrate. As the propagation direction is dictated by the orientation of our system, we use the simpler approach outlined in [40]. This involves forces: F_1 and F_2 and particle velocities: v_1 and v_2 as shown in Fig. 4(b), voltage potential (U), current (I), piezoelectric element thickness (l) and angular frequency (ω). The relevant expression is

$$\begin{bmatrix} F_1 \\ F_2 \\ U \end{bmatrix} = -i \begin{bmatrix} Z_{pz}cot(\beta_{pz}l) & Z_{pz}csc(\beta_{pz}l) & h/\omega \\ Z_{pz}csc(\beta_{pz}l) & Z_{pz}cot(\beta_{pz}l) & h/\omega \\ h/\omega & h/\omega & 1/\omega C_o \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ I \end{bmatrix}. \tag{11}$$

Here, Z_{pz} is the acoustic impedance of the piezoelectric element, given by $Z_{pz} = A\sqrt{\rho_{pz}E^D}$, where ρ_{pz} is the density of the PZT transducer, and E^D is the stiffened elastic constant (the "stiffening" occurring due to piezoelectric effects [40]) defined by $E^D = E^E + (e^2/\varepsilon^s)$, e being the piezoelectric constant, E^E the elastic constant and $\varepsilon^s$the permittivity at constant strain in the piezoelectric material. The wavenumber β_{pz} in the piezoelectric is $\beta_{pz} = \omega/c_L^D$, where c_L^D is the stiffened longitudinal velocity defined by $c_L^D = \sqrt{E^D/\rho_{pz}}$, and $h = e/\varepsilon^s$. C_0 is the clamped (zero strain) capacitance of the piezoelectric element, the measured capacitance at a frequency well above any pronounced resonance.

Boundary conditions were implemented to link the piezoelectric layer shown in Fig. 4(b) with the matching layer and air gap as shown in Fig. 4(c) (the axes *l* and *x*, respectively, are aligned). These boundary conditions are applied to a general velocity potential equation representing waves propagating in each of the media considered. In general, the velocity potential equation is defined as in Equation 12 and the subscript, n can be replaced by *A*, *B*, *C* or *D* as defined in Fig. 4(c):

$$\emptyset_n = \Phi_n e^{i(\omega t \pm kx + \vartheta_n)} \tag{12}$$

where, $\emptyset_n$ is the velocity potential, Φ_n is the velocity potential amplitude (see Eqn. (10)) at some position *x*. ϑ_n is the phase of the wave at *x* = 0, and the sign of the wavenumber *k* in the medium denotes the direction of propagation. From this definition of the velocity potential functions, the particle velocity v_n and pressure P_n can be derived and implemented as boundary conditions using, respectively:

$$v_n = -\nabla \emptyset_n \tag{13}$$

$$P_n = \rho \frac{\partial \emptyset_n}{\partial t} \tag{14}$$

For example, we arrive with an expression for the velocity and pressure in the air gap, v_{air} and P_{air}

$$v_{air} = \mathrm{i}k_{air}e^{i\omega t}(\Phi_B e^{i(-k_{air}x+\vartheta_B)} - \Phi_A e^{i(k_{air}x+\vartheta_A)}) \tag{15}$$

$$P_{air} = \mathrm{i}\omega\rho_{air}e^{i\omega t}(\Phi_B e^{i(-k_{air}x+\vartheta_B)} + \Phi_A e^{i(k_{air}x+\vartheta_A)}) \tag{16}$$

By imposing multiples of half-wavelength air gap and a perfect reflector (v_{air} = 0m/s) boundary condition at the right end of the air gap as shown in Fig 4 (c), a relationship between $\emptyset_A$and $\emptyset_B$ can be found using Eqn. (15) using simultaneous equations comparing the real and imaginary components:

$$\Phi_B = \Phi_A \tag{17}$$

and

$$\vartheta_B = \vartheta_A + 2(k_{air})\left(\frac{n\lambda}{2}\right) \tag{18}$$

here, the wavenumber has the subscript *air* to indicate it is in relation to the wave in the air layer. Consequently, a relationship between $\emptyset_D$ and $\emptyset_C$ can be obtained by implementing pressure and velocity equilibrium boundary conditions at the matching layer-air gap interface.

By equating pressure and velocity at the matching layer-air gap (x = 0) interface using Eqns. (13) and (14), we obtain

$$\mathrm{i}\omega\rho_{ML}e^{i\omega t}(\Phi_D e^{i(\vartheta_D)} + \Phi_C e^{i(\vartheta_C)}) = \mathrm{i}\omega\rho_{air}e^{i\omega t}(\Phi_A e^{i\left(2k_{air}\left(\frac{n\lambda}{2}\right)+\vartheta_A\right)} + \Phi_A e^{i(\vartheta_A)}) \tag{19}$$

and,

$$\mathrm{i}k_{ML}e^{i\omega t}(\Phi_D e^{i(\vartheta_D)} - \Phi_C e^{i(\vartheta_C)}) = \mathrm{i}k_{air}e^{i\omega t}(\Phi_A e^{i\left(2k_{air}\left(\frac{n\lambda}{2}\right)+\vartheta_A\right)} - \Phi_A e^{i(\vartheta_A)}) \tag{20}$$

where Eqns. (15) to (18) have been used to eliminate terms Φ_B and ϑ_B.

We can solve these two simultaneous equations to obtain an expression for $\emptyset_C$ in terms of $\emptyset_D$ noting that $\emptyset_C$ at x = 0 is given by $\Phi_C e^{i(\vartheta_C)}$ (likewise $\emptyset_D$=$\Phi_D e^{i(\vartheta_D)}$) . We define an acoustic reflection term R that occurs at the interface between the matching layer and the air gap such that $\emptyset_C = R \times \emptyset_D$. The R can be written as:

$$R = \left[\frac{D-1}{D+1}\right] \tag{21}$$

where,

$$D = \left(\frac{\rho_{air}k_{ML}}{\rho_{ML}k_{air}}\right) \times \left[\frac{e^{i(2k_{air}d)}+1}{e^{i(2k_{air}d)}-1}\right]. \tag{22}$$

Here, ρ_{air} and ρ_{ML} are the densities and k_{air} and k_{ML} are the wave numbers of the air and matching layers respectively, and d is the thickness of the air gap. Information obtained from this analysis allows for a relationship between $\emptyset_C$ and $\emptyset_D$ which simplifies the subsequent analysis steps described below.

We now have one equation ($\emptyset_C = R \times \emptyset_D$) with two unknowns ($\emptyset_C$, $\emptyset_D$). In order to be able to evaluate these unknowns and ultimately find the pressure profile in the air gap, we utilise the conversion matrix (Eqn (11)) for the piezoelectric element, as this yields a second equation relating $\emptyset_D$ and $\emptyset_C$, thusproviding a solution for $\emptyset_D$ in terms of the voltage input, U. Using Equations 12-14 and the boundary conditions of $F_1 = Z_{BL}v_1$, $F_2 = -A\omega\rho_{ML}[\emptyset_D + \emptyset_C]$ and $v_2 = -ik_{ML}e^{i\omega t}[\emptyset_D - \emptyset_C]$, yields:

$$\emptyset_D = UX_T + \emptyset_C R_T \tag{23}$$

where,

$$U = \frac{-2h^2 Z_{pz}}{\omega^2}\left[\frac{cos(\beta_{pz}l)-1}{sin(\beta_{pz}l)}\right] - \frac{iZ_{pz}cot(\beta_{pz}l)}{\omega C_0}[Z_{ML} + Z_{aBL}] - \frac{[Z_{ML}Z_{BL}+Z_{pz}^2]}{\omega C_0} + \frac{ih^2}{\omega^2}[Z_{ML} + Z_{BL}] \tag{24}$$

$$R_T = \frac{\left(\frac{-2h^2 Z_{pz}}{\omega^2}\left[\frac{cos(\beta_{pz}l)-1}{sin(\beta_{pz}l)}\right] + \frac{iZ_{pz}cot(\beta_{pz}l)}{\omega C_0}[Z_{ML}-Z_{BL}] + \frac{[Z_{ML}Z_{BL}-Z_{pz}^2]}{\omega C_0} - \frac{ih^2}{\omega^2}[Z_{ML}-Z_{BL}]\right)}{Y} \tag{25}$$

$$X_T = \left(\frac{h}{\omega k_{ML}}\right)\frac{\left[\frac{Z_{pz}(cos(\beta_{pz}l)-1)}{sin(\beta_{pz}l)} - iZ_{BL}\right]}{Y} \tag{26}$$

and where Z_{ML} and Z_{BL} are the acoustics impedances of the matching layer and the backing layer (air) respectively. As $\emptyset_C = R \times \emptyset_D$, with R defined above, then combining eqns. (23)-(26)produces an expression for $\emptyset_D$ in terms of the voltage input and piezoelectric properties as below:

$$\emptyset_D = \frac{UX_T}{(1-RR_T)}\,. \tag{27}$$

It is thus possible to determine the pressure in the air gap as a result of applied voltage potential, the material properties and layer dimensions. Eqn. (27) gives an expression for $\emptyset_D$;using this and the relation $\emptyset_C = R \times \emptyset_D$, Eqns. (19) or (20) will give $\emptyset_A$. This allows a description of the pressure field in the air gap, bearing in mind that P_{air} is given by Eqn. (16) and that Φ_A, Φ_B, ϑ_A and ϑ_B have been related above in Eqns. (17) and (18).

B. Predictions of the models

The models were used to find an optimum design for a complete resonant chamber, which could then be evaluated experimentally. The variables under consideration for the layered resonator design included the material chosen for the matching layer, and individual layer thicknesses of the piezoelectric material, matching layer and air gap. A numerical simulation was carried out by varying the density, ρ_{ML} and speed of sound, c_{ML} of the matching layer (the product of which results in the specific acoustic impedance, Z_{ML}). The aim was to study the combined effects of optimising geometry and materials to maximise the pressure (in the air gap) which in turn gives us the force (which acts on the particles) and consider how the acoustic attenuation limits operation in air at higher frequencies.

In order to investigate the optimum operating frequency, two sets of data are presented from the FE model for a fixed input energy. The first uses a constant value for Q_{air} (taken as that at a frequency of 0.95 MHz (Q_{air}=1013). This is to illustrate the expected rise in force with frequency. The second set studies the change in Q_{air} with frequency so as to investigate attenuation within the air layer. In this second data set, we expect to see a maximum force to occur at a particular frequency due to the trade-off between the increase in force with frequency (as shown in Eqn. (8) where <F> scales with k_F) and the loss of energy with attenuation at higher frequencies. The results are shown in Fig. 5 for path lengths that represent odd multiples of $\lambda_{air}/2$, e each being resonances within the air gap. For a constant energy input condition (scaled to 1 nm input displacement at 1MHz using Eqn. (7)), and a constant Q_{air}, it is observed in Fig. 5(a) that the resultant absolute pressure, $\langle P \rangle$ experiences a maximum in amplitude at ~1.5 MHz for the $\lambda_{air}/2$ case (Fig. 5(a)). The curves for $3\lambda_{air}/2$ and

$5\lambda_{air}/2$ show a less pronounced maximum at the same value. For the dry PMMA particles used in the experiments, the radiation force per unit particle mass ($F/mass$) calculated using Eqn. (8) increases with frequency for the entire range considered (Fig. 5(b)). $F/mass$ demonstrates the ratio of radiation force exerted upon a particle relative to the particle's mass (capability of levitation) and its dependency on the frequency of operation.

Consider now the case where Q_{air} is adjusted as a function of frequency (i.e. the "corrected damping") case. Now it is observed (see Fig. 5) that the maximum values of <P> and $F/mass$ occur at 1MHz and 1.1 MHz respectively for the $\lambda_{air}/2$ resonance, with a significant decrease in amplitude at higher frequencies. Similar trends are seen for the $3\lambda_{air}/2$ and 5 $\lambda_{air}/2$ cases. This indicates the effect of frequency-dependant attenuation on the resonant characteristics of the chamber. It is worth noting that the relationship of both <P> and $F/mass$ with frequency is dependent on the size of the chamber itself. The FE model assumes side walls through which energy can be dissipated. Thus, if the diameter of the chamber was to be increased for the same thickness of air gap, a shift in the maxima to lower frequencies would be observed. This is illustrated in Fig. 6. This is due to the fact that for smaller systems more energy is lost due to diffraction; thus, to compensate for this, a higher frequency is required to reduce diffractive effects. This trend is similar for the $3\lambda_{air}/2$ and 5 $\lambda_{air}/2$ cases. However, it should be noted that at larger air gap thicknesses, values of both <P> and $F/mass$ decrease significantly as shown in Fig. 5(a) and (b). As was demonstrated in Fig. 5(b), the optimum frequency to operate a parallel acoustic trapping mode in air for r = 3.18 mm (equal perimeter to a 5 x 5 mm square chamber as used in experiments) is predicted to be 1.1MHz.

The value of the best frequency of operation could now be used within the analytical model, the aim being to optimise the physical design of the resonant acoustic chamber design. This was achieved by determining the optimum layer thickness of each layer in the system. The model was constructed with three layers, as shown in Fig. 7(a): a piezoelectric layer (backed by air), a matching layer and an air gap (enclosed by a rigid reflector). The air gap was chosen to be 0.5001 λ_{air} (*i.e.* very close to 0.5 λ_{air} but avoiding a numerical error in the Matlab$^{\mathrm{TM}}$ code when calculating D as shown in Eqn. (22) within the numerical analysis). The optimum thickness values of the other two layers, and their material parameters, could then be investigated, together with the role of damping. A three-stage approach was adopted. Firstly, the material and damping factor of the piezoelectric layer was set to that given by the manufacturer, PZT, manufactured by Ferroperm Ltd [41]). The role of the thickness of the piezoelectric layer (t_{PZT}) was examined by finding the velocity potential amplitude (Φ_D) in an aluminium matching layer across a range of values. Fig. 7(a) shows the velocity potential for a range of thicknesses (normalised by piezoelectric wavelength, *i.e.* t_{PZT}/λ_{PZT}). A peak value occurs at t_{PZT}/λ_{PZT} = 0.254. Such data was collected for a range of matching layer acoustic impedances, and in each case the thickness at which a maximum velocity potential occurs (see circle in Fig. 7(a)) has been identified. The result is shown in Fig. 7(b) for the case in which the matching layer has a Q factor (Q_{ML}) of 400 which corresponds to that of typical solids. In both cases it can be seen that there is no significant (3.6% variation) effect on the optimum piezoelectric thickness for a matching- layer impedance Z_{ML} above 15MRayls.

I. Layered resonator design optimisation

The second stage was to investigate the best matching layer thickness (t_{ML}) for the optimum thickness of piezoelectric material identified above, with the air gap remaining fixed at 0.5001 λ_{air}. The value of t_{ML} would be expected to depend on Z_{ML} (and hence λ_{ML}).The dependence of absolute pressure, <P> in the aluminium matching layer on t_{ML}/λ_{ML} is shown in Fig. 8(a). This allowed the best normalised thickness (t_{ML}/λ_{ML}) to be determined as a function of impedance Z_{ML}, and the results are shown in Fig. 8(b). This is for fixed values of Q_{air}=965, Q_{PZT}=1000 (based on supplier's (Ferroperm) minimal value) and Q_{ML}=400. It is interesting to note that the optimum thickness only differs from a steady value of $t_{ML}/\lambda_{ML} \approx 0.5$ at impedance values which are below 10 MRayls. This then predicts that in a chamber such as that assumed in this work, there is very little sensitivity to the optimum value of t_{ML}/λ_{ML}, and that for the situation where $z_{ML} \gg z_{air}$, the matching-layer to air interface acts as a pressure-release boundary. This is consistent with acoustic wave reflection theory [42] where a reflection coefficient, R of -1 or a phase shift of π radians with a magnitude of 1 would be expected on reflection at the matching layer-air interface. Based on this observation, a value of $t_{ML} \approx n\lambda_{ML}/2$ should hold for any solid material, making the design of the matching layer very simple. Note that this finding differs from the requirement in a travelling wave airborne transducer system (due to the presence of the reflected wave in our system altering the boundary condition), as used for applications such as non-destructive testing, where a $\lambda_{ML}/4$ matching layer thickness is used.

The third stage (Fig. 9) investigated the signal in the air gap and its dependency on material properties, specifically the acoustic impedance, Z_{ML}. The maximum pressure in the air gap is shown for a range of impedances of the matching layer (Z_{ML}) in Fig 9(a), for fixed values of PZT thickness (t_{PZT} = 0.254 λ_{PZT}), air gap ($t_{air} \approx 0.5\ \lambda_{air}$) and the matching layer thickness ($t_{ML} \approx 0.5\ \lambda_{ML}$). It can be seen that there are multiple peaks in amplitude, and that these are more closely-spaced at lower values of Z_{ML}. However, when damping is added to the matching layer and piezoelectric layer, these oscillations are less pronounced, as would be expected (Fig. 9(b)). Here, Q_{ML} was fixed at 400 (typical for aluminium) and Q_{PZT} varied. Similarly, when Q_{PZT} was set to 1000 whilst Q_{ML} was varied, Fig. 9(c), the oscillations were again much less pronounced. It is also worth noting that the Q_{ML} values used play a minimal role at realistic acoustic impedances as there is not much variation in the resultant pressure for the range of Q_{ML} values considered.

It can be concluded from the above analysis that the choice of matching layer material is not nearly as significant as the thickness. These numerical findings are consistent with the experimental observations by Hill [20] who used three different matching layer materials (i.e. brass, aluminium and macor) where minimal variations in the transmitted acoustic energy was observed when constructing a layered resonator in a liquid based resonator. In the above, it has been demonstrated theoretically that an air-filled cavity behaves in a similar fashion. These predictions can be used to design effective experimental resonators for particle manipulation in air, as will now be described.

4. Experimental Fabrication and Testing

To validate and demonstrate the findings of the models, an acoustic resonator was fabricated to show that micron-sized particles could be positioned and held stationary experimentally within an air-filled chamber. The design was shown schematically in Fig. 1(a). The actual chamber consisted of a 5 mm square, 0.5 mm thick PZT transducer element (Ferroperm PZ-26; with diminishingly thin electrodes on either side of the piezoelectric element) adhered to a 3 mm thick aluminium matching layer using a thin layer of epoxy resin. It is worth noting that the epoxy resin is not considered in the numerical analysis. As a result an additional loss of energy transmitted into the matching layer may be experienced; consequently leading to a reduced absolute pressure within the air gap should be experienced. However, as a very thin uniform layer of adhesive material is used, a relatively small effect should be expected as shown in the modelling study carried out by Hill et al[34]. The transducer was excited by an alternating voltage at the bottom surface of the piezoelectric transducer and electrically grounded on the top surface (*i.e*. at the PZT/aluminium interface). Electrical connections were made using insulated copper wires and silver conductive paint. Acoustic resonances within the air cavity were achieved using a glass reflector, attached rigidly to the moving stage of a micro- positioner, so as to allow precise control over the air gap. This allowed the optimum conditions predicted by the modelling to be achieved. The PZT transducer was driven at the chosen frequency using a Stanford Research Systems Model No.DS345 waveform generator and an Amplifier Research Model No.25A1250A power amplifier. This was used to trap dry PMMA particles (with properties ρ_s=1190kg/m^3 and c_s=2350 m/s) at specified locations within the air cavity. Imaging was carried out using an optical microscope (Infinity Photo-optical Company) fitted with a Hitachi KPD20AU CCD camera.

The existing literature [17, 28-30] is concerned with significantly larger objects compared to those chosen for this study. Experiments have confirmed that the design needed for the efficient manipulation of micro-particles in air can be achieved using acoustic resonators, provided the conditions outlined in the modelling above are met. To this end an acoustic resonator has been fabricated to operate at 1.1 MHz. The transducer was excited at a 2V_{pp} waveform from the signal generator at 1.0025 MHz (i.e. resonant frequency of the transducer) with a $3/2\,\lambda_{air}$ (≈500µm) gap size. Note that this was used to aid particle visualisation (instead of $1/2\,\lambda_{air}$). Therefore to achieve sufficient force to levitate a PMMA particle, the input excitation amplitude/displacement, ζ_i should be scaled accordingly ($F/mass \propto \left(\frac{\zeta_i}{\zeta_{@1nm}}\right)^2$). The PMMA microspheres (Bangs Laboratories Inc.; $\rho_{particle} = 1{,}190\,\mathrm{kg/m^3}$) of sizes 83µm and 15 µm respectively were introduced at the bottom surface of the air gap (i.e. on top of the matching layer). As shown in Figs 10(a) and 10(b), dry particles were successfully trapped in air. As shown in Fig. 10(a), the 83µm particles were individually trapped at the pressure nodes (*i.e.* at the desired collection location). A range of particles including dry expanded hollow polystyrene microspheres (Expancel Microspheres 461 DET 40 d25; 35-55 µm and 25 kg/m^3) were also used. The system was able to trap the above mentioned Expancel microspheres at resonator air gap distances of up to $9/2\,\lambda_{air}$. It was thus demonstrated that the use of the models led to the design of an efficient resonator for particle collection in air, which, for the first time, was able to operate at the small particle sizes used.

4. Conclusions

An acoustic modelling approach has been used to help identify design parameters of an optimised robust design of an acoustic resonator in air. A method to obtain an optimum frequency of operation was demonstrated and found to be dependent on the size of the system. However, an optimum operational frequency of 1.1MHz was determined using finite element analysis when frequency dependent attenuation factors were considered for the system studied. In addition, the thickness of the piezoelectric element considered (Ferroperm Pz26) should be $\approx 0.254\lambda_{PZT}$, together with a matching layer thickness of $\approx 0.5\lambda_{ML}$ for relatively high acoustic impedance properties. When suitable quality factors are considered, the selection of matching layer material becomes less important as the resultant pressure field amplitudes do not vary significantly with material selection. However, individual layer thicknesses play a significant role. The knowledge from the theoretical analysis was used to design an experimental resonator which successfully levitated and trapped solid particles of micron-sizes (83 µm and 15 µm). Particle levitation and trapping at this size scale has not been reported in the literature to date which demonstrates the success of using modelling to elucidate the correct experimental conditions.

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Figure Captions

Fig. 1: Device assembly and terminal configuration (intended standing wave profile and particle collection location depicted in white).

Fig. 2: Plot of absorption coefficient, α (dB/m) against frequency, f (Hz) for air

Fig. 3: (a)) FE (COMSOL) model depicting boundary conditions and pressure amplitude variations that result, and (b) total acoustic pressure decay over the air path considered (10 mm at 2MHz; Q_{air}=482).

Fig. 4: (a) An axisymmetric FE model depicting boundary conditions and absolute pressure distribution. The half-wavelength standing wave profile and particle collection position are depicted in white. (b) Force and velocity direction along with the positive z direction used in the piezoelectric numerical analysis. (c) Boundary conditions implemented in the matching layer-air gap system.

Fig. 5: Results of FE modelling for acoustic transmission through the resonator air gap in terms of (a) absolute pressure <*P*> and (b) force per unit mass (*F/mass*) as a function of frequency. The results are shown over odd multiples of $\lambda_{air}/2$. Two sets of data are presented in each plot for a 3.18 mm diameter chamber (see Fig. 4(a)) – one for a constant value of Q_{air}=1013 ("Constant Damping"), and a second with Q_{air} calculated for each frequency ("Corrected Damping").

Fig. 6: Effect of chamber radius, r on (a) absolute pressure, <*P*> vs frequency, f , (b) *F/mass* vs frequency, f , and (c) the frequency at which the peak force occurs.

Fig. 7: Investigation of PZT parameters. (a) Arrangement of PZT element, matching layer and a fixed air gap of $\approx 0.5\lambda_{air}$. (b) Dependence of Φ_D on the normalised piezoelectric element thickness (t_{PZT}/λ_{PZT}). (c) Dependence of (t_{PZT}/λ_{PZT})on the specific acoustic impedance, Z_{ML} (Rayls) of the matching layer, for Q_{air}=965, Q_{PZT}=1000 and Q_{ML}=400. The dotted line depicts the specific acoustic impedance of aluminium (Z_{ML}=17.28 MRayls).

Fig. 8: Effect of variations in thickness of the matching layer (t_{ML}) for a constant PZT thickness ($0.254\lambda_{PZT}$) and air gap ($\approx 0.5\ \lambda_{air}$). (a) Dependence of <*P*> in the matching layer medium (Aluminium) on the matching layer thickness (normalised by λ_{ML}) (b) Matching layer thickness (normalised by λ_{ML}) dependence on specific acoustic impedance, Z_{ML} [Rayls], for values of Q_{air}=965, Q_{PZT}=1000 and Q_{ML}=400. The dotted line depicts the specific acoustic impedance of aluminium (Z_{ML}=17.28 MRayls).

Fig. 9: Variation of <*P*> with specific acoustic impedance of the matching layer (Z_{ML}). (a) Result for fixed values of PZT thickness ($t_{PZ} = 0.254\ \lambda_{PZT}$), air gap ($t_{air} \approx 0.5\ \lambda_{air}$) and the matching layer thickness ($t_{ML} \approx 0.5\ \lambda_{ML}$), with Q_{air}= 965. (b) Conditions as in (a) but with Q_{ML}=400 and Q_{PZT} varied. (c) Conditions as in (a), but with Q_{PZT} =1000 and Q_{ML} varied.

Fig. 10: Optical photographs showing levitation of dry PMMA particles of various diameters in an air-filled cavity of 3/2 λ_{air} thickness. (a) 83μm diameter particles (circled) that were trapped at the pressure nodes of the resonant cavity, as shown to the right of the image. (b) Entrapment of a very small 15 μm diameter PMMA particle.