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The Logic of Measure in Hegel’s Science of Logic
Stephen Houlgate

Abstract

In his account of measure in the Logic Hegel discusses various natural phenomena, such as specific heat and specific gravity, and associates each one with a particular form of measure. Yet Hegel’s conception of measure is not guided by his understanding of nature or by modern science. His derivation of the forms of measure proceeds immanently by rendering explicit what is implicit in the concept of measure itself. Natural phenomena are then adduced as examples of the measures that have been derived logically. My aim in this essay is to explain how the distinctive logic of measure proceeds in its early stages.


I. Introduction

In the first Critique Kant counts among the conditions of the objects of experience the categories of “quantity” and of “quality”, but he does not derive one set of categories from the other logically.¹ In the Science of Logic, by contrast, Hegel argues that quantity is made necessary logically by quality, specifically by the qualitative category of the “one” (Eins) and the connected categories of “repulsion” and “attraction” (which, in quantity, become

¹ CPR B 106, 202-218. S. Houlgate 2014, p. 16-17. Note that, for Kant (in contrast to Hegel), “quantity” and “quality” are not themselves categories, but are rather the names of two sets or “classes” (Klassen) of categories (CPR B 110).
“discreteness” and “continuity”).² Hegel goes on to argue that quantity in turn makes quality necessary and thereby gives rise to the explicit unity of the two in the form of measure (Maß). Measure, he notes, was a central concept for the Greeks, who, indeed, maintained that “everything has a measure” (SL 329 / LS 367).³ It is, however, a concept that is lost on Kant, as it is lost on Spinoza.⁴ Hegel’s speculative logic thus restores the concept of measure, neglected by the moderns, to its rightful place in our understanding; and it does so by demonstrating that measure is made necessary logically by the very quantity and quality that for Kant — albeit in the guise of two “classes” of categories⁵ — are the indispensable conditions of objective cognition.

Now in his account of measure in the Logic Hegel discusses numerous natural phenomena and laws, many of which were unknown to the Greeks, and he associates each one with a particular form of measure. These phenomena and laws include specific heat, specific gravity and Kepler’s third law of planetary motion.⁶ Hegel’s interest in these aspects of nature is not accidental, but follows from the fact that “the different forms in which measure is realized belong also to different spheres of natural reality” (SL 331 / LS 369). It is important to emphasise, however, that Hegel’s conception of measure is not itself guided by his understanding of nature or by the findings of modern science. His derivation of the various forms of measure proceeds immanently by rendering explicit what is implicit in the concept of measure itself. (“The whole course of philosophising”, Hegel states in the Encyclopaedia, is, indeed, “nothing else but the mere positing [Setzen] of what is already contained in a concept” [EL § 88 R].) Natural phenomena and laws are then adduced as examples of the measures that have been derived logically. Speculative logic is understood by Hegel to be both a logic and an ontology or metaphysics: an account of the necessary categories of thought and of the fundamental ways of being.⁷ The examples from nature thus serve to confirm that the measures made necessary by logic belong just as much to the world.

³ S. also EL § 107 A, and Harris 1983, p. 146.
⁵ S. note 1 above.
Logic, then, does not follow nature or natural science, but nature exemplifies the structures derived a priori by logic. If we are to understand why there are measures in the world, therefore, it is crucial that we understand the distinctive logic of measure that makes its various forms necessary. My aim in this essay is to further such understanding by explaining, as clearly as I can, how that logic proceeds in its early stages.

II. The Specific Quantum

In the Logic a measure is initially some determinate amount, or “specific quantum”, that confers a determinate quality on a thing and without which the thing would lose that quality and cease to be what it is (SL 333 / LS 371). A mere quantum can be changed without destroying the thing concerned: the latter can become bigger or smaller and remain what it is. The quantum (or range of quanta) that constitutes a thing’s measure, however, cannot be changed without destroying the thing, because it is precisely what gives the thing, or enables it to have, its particular quality: it “belongs”, and is specific, to that quality. Water, for example, must be kept below 100°C or it turns into steam, and “a republican constitution like that of Athens, or an aristocratic constitution tempered by democracy, is suitable only for states of a certain size” (SL 332 / LS 370).

Speculative logic proves that being cannot just be indeterminate but must take the form of determinate, finite things. Such things must also have a certain quantity or “magnitude”, and they must have a certain measure: a specific quantum, thanks to which they are what they are. Hegel points out, however, that measure is in fact an ambiguous determination.

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8 On the relation between Hegel’s philosophy (including his philosophy of nature) and natural science, s. Houlgate 2005, p. 110-121. By contrast, Ruschig contends that, in the chapter on real measure at least, the logical transitions are actually determined by the scientific models that Hegel cites as mere ‘examples’; s. Ruschig 1997, p. 16, 28, 46, 233. Kruck argues, as I do in this essay, that Hegel’s account of measure proceeds logically and is intelligible “without the integrated, concrete material of intuition” supplied by science; s. Kruck 2014, p. 123-124.


11 S. also EL § 108 A, and Stace 1955, p. 169-170. Ruschig points out that water starts to evaporate below 100°C, but that at boiling point not only does the surface evaporate, but all the water turns to steam; s. Ruschig 1997, p. 287.

Note that a measure is not an ideal standard that a thing has to meet in order to be itself; it is the magnitude a thing actually has and to which it owes its distinctive quality. This magnitude must be (or fall within the range of) the thing’s measure, for if the thing has it, it obviously permits the thing to be what it is. The ambiguity of such a magnitude is this. As a simple quantum, it can change like any other: it is “an indifferent magnitude” that is “capable of increase and decrease” (SL 334 / LS 372). As a measure, however, it grounds, or makes possible, the quality of the thing. This in turn sets a limit to the extent to which it can change: for if it is to preserve the quality concerned, it can change only within a certain range. The measure is thus a thing’s magnitude or quantum that is “distinguished [verschieden] from itself as a quantum” and limits the latter (SL 334 / LS 372). The actual temperature of the water before us is a quantum that can vary from 1°C to 99°C. This temperature range enables the water to remain a liquid and so constitutes its measure. Yet that range is also just a range of quanta, and as such can be increased or decreased beyond the limits of the measure that it is. If this happens, however, water will change its quality and become ice or steam. The quantum (or range of quanta) that something actually has thus constitutes both the bare, changeable quantum of the thing and the measure that sets a limit to the changes that quantum can undergo.13

Note, though, that that measure cannot itself prevent the thing’s quantum from exceeding its limit: for that quantum as such is irreducibly changeable, and the measure cannot take this feature away from it. The measure does not, therefore, make it impossible for the quantum to go beyond the limit set for it, but simply determines that, if it does so, the quality disappears. In this sense, a thing’s magnitude, which allows the thing to be what it is and so is or belongs to its measure, is actually impotent as a measure in the face of its own quantitative nature.

It follows from the nature of measure, therefore, that a thing can change its quantity without altering its quality, but that it will (or may) reach a point at which that quality, and therewith the thing, ceases to be. This change in quality and demise of the thing, Hegel writes, will appear “unexpected”, if one is unaware of the thing’s measure, because it will seem that only a change in quantity is occurring. One can then be tempted to try to make such change in quality intelligible with the help of the idea of “gradualness” (Allmählichkeit): if one focuses one’s attention on the changes in quantity, one will be able to see — or so one might think — the qualitative change come about ‘gradually’ before one’s eyes. Yet Hegel

insists that thinking of qualitative change as merely ‘gradual’ actually reduces it to something purely quantitative and so makes it impossible to see any qualitative change, or to explain how the latter could occur (SL 335 / LS 373). The only thing that can explain how a change in quantity brings about a change of quality is the measure of a thing; and understanding the latter requires us to give up the desire to explain everything in quantitative terms.

In this context Hegel briefly discusses the ancient Greek “sorites” paradoxes (from the Greek for heap, soros), attributed to Eubulides of Miletus.14 “The question was asked”, Hegel writes, “does the pulling out of a single hair from the head or from a horse’s tail produce baldness, or does a heap cease to be a heap if a grain is removed?” (SL 335 / LS 373). The answer, surely, is no, and it continues to be no when one pulls out a second and then a third hair, or removes a second and then a third grain. Indeed, formal logic tells us that the answer should remain no, as long as one continues to remove just one item at a time. Yet, of course, we eventually reach a point at which we say that the head is bald or there is no more heap; so pulling out a single hair would appear to make us bald after all, leaving us with a paradox.

Hegel insists that such paradoxes are neither “an empty or pedantic joke”, nor merely sophistical as if the contradiction they contain were a “sham”, but that they are “in themselves correct” (SL 336 / LS 374, translation altered). They are, in other words, not just puzzles to be solved or dissolved with the resources of formal logic, but paradoxes that disclose a fundamental truth. This truth is the truth of measure, namely, that quantitative differences are not merely quantitative, but at some point — or within a certain range of points — make a qualitative difference. The value of the paradoxes, therefore, is that they expose the “mistake” of “assuming a quantity to be only an indifferent limit”. As Hegel notes, those who think that repeatedly removing just one grain should not eliminate the heap forget that “the individually insignificant quantities […] add up” and that the sum constitutes a “qualitative whole” (SL 335 / LS 374). Similarly, those to whom the steady increase in their wealth “appears at first to be their good fortune” overlook the fact that such an increase may well at some point lead to their misfortune (SL 336 / LS 374).15

14 On the early history of these paradoxes, s. Moline 1969.

15 From the Hegelian point of view, the phenomenon exposed by sorites paradoxes is thus not principally the ‘vagueness’ of concepts such as ‘heap’, but (in Harris’s words) “the interdependence of the moments of measure” (Harris 1983, p. 147). For an account of the relation between the paradoxes and the topic of vagueness, s. Hyde 2011. On Hegel’s discussion of the paradoxes, s. also Johnson 1988, p. 70, and Winfield 2012, p. 145.
III. The Specifying Measure

A measure is a quantum that constitutes (or enables there to be) a certain quality. We know from the first section of the doctrine of being, however, that quality does not stand alone, but is the quality of something (Etwas).\(^\text{16}\) A measure is thus not just an abstraction, but a quantum constitutive of a thing with a certain quality, and it is as such that it differs from a “mere” quantum in the way we have seen.\(^\text{17}\) Following the logic of “something”, however, that mere quantum should itself be something other than the measure. The measure is immediately itself and must, therefore, be immediately different from a quantum that is in turn immediately itself. When this thought is rendered explicit, a new form of measure emerges. This new measure does not simply differ from the mere quantum that it is, but both sides now have “a distinct existence” (eine verschiedene Existenz) (SL 336 / LS 375).

Yet, as we know, a measure is not indifferent to the mere quantum, but sets a limit to it and in that sense ‘negates’ it. This continues to be true of the new measure: it, too, limits the mere quantum that lies outside it. It does so on the basis of its own specific determinacy and so proves to be the activity of specifying that external quantum. Measure has thus now to be understood, not just by itself, but in relation to an “alterable, external” quantum, which it specifies (SL 336 / LS 375).\(^\text{18}\)

It should be stressed that what drives the logic of measure forward here is the double character of the measure itself. On the one hand, a measure is the unity of quantity and quality: it is a quantum that constitutes and sustains a quality. On the other hand, quantity and quality remain different in the measure, since the latter contains quantity in two forms: once as constituting quality and thus as the measure, and once as a mere quantum. Moreover, the quantum as qualitative — as the measure — ‘negates’ the mere quantum that the measure also is by setting a limit to it: this limit is one that that quantum cannot exceed without undermining the quality attached to the measure. This difference between the quantum as measure and as mere quantum initially falls within the measure itself: the measure sets a limit to the changes that it, as mere quantum, can undergo. As a self-relating something, however,


\(^{17}\) Hegel employs the phrase “[the] mere quantum” (das bloße Quantum) on SL 336 / LS 375.

\(^{18}\) Kruck appears to get confused here. He correctly states that a difference emerges between a “specifying” quantum and a quantum being specified, but he describes the latter as a “specific quantum” and an “intrinsic determinateness” (ansichseinde Bestimmtheit) when these terms actually characterise the specifying quantum. S. Kruck 2014, p. 125, and SL 329, 333-334 / LS 367, 371-372.
the measure now sets itself in relation to a quantum that is (or belongs to) something of its own and so falls outside the measure itself. In this way, the difference internal to the measure mutates logically into a relation between the measure and another quantum. Such a relation is thus not just an accidental feature of measure, but renders explicit the difference that is at the heart of the measure from the start.\(^{19}\)

A measure ‘in relation’ first specifies the quantum that it confronts by providing an external measure for it: one that Hegel calls a “rule” (Regel) or “standard” (Maßstab) (SL 337 / LS 375).\(^{20}\) Since this rule and the external quantum are initially just immediately other than one another, the former does not actively negate and change the latter (as the third form of measure will do) but simply stands next to it. Yet, as a measure, the rule must specify and limit the quantum in some way. So how does it do so? We learn in the account of quantity that a quantum as such — or, more precisely, a quantum as a number — is a determinate “amount” of featureless “units” (SL 202-203 / LS 213-14).\(^{21}\) The rule, therefore, must specify either the amount of the external quantum, or the units it comprises, or both. The amount, however, belongs to the quantum, since it makes the latter the quantum or number that it is, and so it falls outside the rule. Accordingly, the rule — unlike the first form of measure — does not determine how big something may be or what degree it may reach. It must, therefore, specify the quantum by providing the unit (Einheit) in terms of which the latter is to be measured. Now the rule, as a measure, is something specific and determinate, so the unit it provides for the quantum must also be determinate.\(^{22}\) This unit is thus not just a bare unit as such, but a determinate one, such as a foot or a metre, and the quantum, which stands

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\(^{19}\) S. SL 330 / LS 368: “The development of measure contains the differentiation of these moments [...]”; s. also Hartmann 1999, p. 151, and Kruck 2014, p. 129. Burbidge maintains that we move to a new measure in order to “improve the rigour of our measuring”, rather than by rendering explicit the difference that is implicit in the specific quantum (Burbidge 2006, p. 54). In my view, however, Hegel’s logic of measure is not (or not principally) about the ways in which we measure things, but it sets out the measures inherent in being itself.

\(^{20}\) The thought of a rule or standard is introduced by Hegel in the first sub-division of the first chapter on measure (1.A) — in which he examines the initial immediate measure, or “specific quantum” — but, strictly speaking, that thought does not belong there. A rule or standard is a measure that, unlike the “specific quantum”, is explicitly distinct from the quantum to which it relates, and so it belongs in the second sub-division of that chapter (1.B.a) (s. SL 333 / LS 371). Kruck appears to conflate the initial measure with the rule or standard in his discussion of 1.A — though he goes on to point out that the rule must also be conceived as “something for itself” that is distinct from the quantum for which it provides the rule; s. Kruck 2014, p. 126-127, 129.

\(^{21}\) S. Houlgate 2014, p. 20-22.

\(^{22}\) Measure is, at the start of its logical development, “its own determinateness [Bestimmtheit] within itself” (SL 333 / LS 371), and such determinateness remains a feature of measure throughout that development (until we reach the thought of “indifference”).
in relation to the rule, must in turn be a certain amount of such units. The rule specifies the quantum, therefore, by determining the latter to be an amount, not just of bare units, but of units of a specific character.\(^{23}\)

Note that, pace John Burbidge, Hegel is here not just describing a process of measuring in which we engage.\(^{24}\) He is arguing that being itself must produce measures and that these measures themselves serve to specify the magnitudes of other things. Yet insofar as they are no more than a rule or standard of measurement, such measures remain external to the quanta they specify. They can thus be replaced by other measures and so are “arbitrary” magnitudes (SL 334 / LS 372). Furthermore, due to their externality these measures are themselves quanta that contain their own amounts. They can thus be specified in turn in terms of other units, just as a foot can be determined as an amount of inches.\(^{25}\)

Hegel now points out, however, that measure must take a further, third form. This emerges as we continue to render explicit what is implicit in being a measure. As we have seen, the measure is not only the immediate unity of quantity and quality but a something (Etwas) in its own right. As such, it must be accompanied by, and directly related to, another something, and so, as Hegel puts it, it must have “in it this side of being-for-other” (SL 337 / LS 376). In accordance with the logic of “something”, however, the first something must also be open to being changed by the other to which it relates and so have what Hegel earlier, in the account of quality, called a “constitution” (Beschaffenheit).\(^{26}\) Since the other is here principally another quantum, the something must be open in particular to having its quantum changed by that other.

Yet the first something is not merely a something, but also a measure. As such, it must limit and specify the quantitative change that the other brings about in it. In his account of quality Hegel argued that something is not completely at the mercy of other things, but has an intrinsic being or “determination” (Bestimmung) of its own that affects how other things affect and change it: “the determining from outside is at the same time determined by the

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\(^{23}\) There appears to be some confusion in Kruck’s account of the rule in 1.B.a, when he describes the rule as providing, for the quantum to which it relates, “the measure for determining the latter’s measure” (das Maß der Bestimmung von dessen Maß) (Kruck 2014, p. 130, my emphasis). On my view, the rule relates to a “mere quantum” (SL 336 / LS 375), not to a quantum that is also a measure, and it simply provides the unit of which that quantum is a certain amount.


\(^{25}\) S. SL 337 / LS 375. For the “externality” of quantity, s., for example, SL 185, 239, 314 / LS 192, 258, 350.

something’s own, immanent determination” (SL 125 / LS 121). We now see more clearly one of the things that this means: through the measure that it contains, something limits in a specific way — and so specifies — the changes in quantity to which it is subjected by another.

The measure has thus mutated, logically, from an external standard or rule into an explicitly “specifying measure” (SL 337 / LS 376). Accordingly, it now no longer relates only to a quantum that is outside and other than it, but in relating to another quantum it also relates to itself, to the quantum that it is: for it specifies the quantum within itself that comes from the other. In this respect the measure blends together the relation to another quantum that characterizes the second form of measure with the specifying of its own quantum that characterizes the first form. This third form of measure thus embodies more explicitly than either of its predecessors what it is to be a measure.

Note, too, that its relation to the “mere” quantum is more active and negative than in the case of the rule. The rule simply limits such a quantum to being an amount of these units, rather than those; it thereby remains external to that quantum and leaves the latter itself unaltered. By contrast, the new specifying measure limits the change that is imposed on it by the other quantum, and thereby negates and changes that change: it alters the quantum that it is given by the other. In this way, the something negates the mere quantum in two senses: it negates its own quantum insofar as the latter is determined by the other quantum, and so it negates that other quantum as well. It does so in a specific way that is governed by its own measure. In specifying the effect that another quantum has on it, therefore, something shows itself to be something of its own, something for itself. Hegel pulls these thoughts together in the following lines:

> Something, in so far as it is a measure within itself, has the magnitude of its quality altered from outside itself; it does not accept this externally imposed alteration as an arithmetical amount: its measure reacts against it, relates itself as something intensive [ein Intensives] to the amount and assimilates it in a distinctive way [auf eine eigentümliche Weise]; it alters the externally imposed alteration, makes this quantum into a different one and through this specifying

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27 Note that in the 1st edition of the Logic’s ‘doctrine of being’, Hegel continues to call this measure a “rule”; s. LS 1812 230: “In the rule, on the contrary […]” (In der Regel hingegen […]). In the 2nd edition this is changed to: “In the specifying measure, on the contrary […]” (In dem spezifizierenden Maße hingegen […] (SL 338 / LS 376).
shows itself to be being-for-self [Fürsichsein] in this externality (SL 337 / LS 376, translation altered).

It is crucial to recognise the complexity of the logical structure that Hegel is setting out here. There is one something in relation to another, the first of which is a measure, whereas the second is merely a quantum. The one that is a measure, however, is thereby itself a quantum. As a measure, therefore, it stands in a negative relation to both its own quantum and that of the other: it negates its own quantum, insofar as the latter is in turn determined by the other. More precisely, it negates the amount that is added to its own quantum by the other. Here we see the clear difference between a merely quantitative relation between quanta and the more nuanced relation between a quantum and a measure. If one bare quantum is added to another bare quantum, the latter increases by precisely what is added to it: add 2 to 3, and 3 becomes 5. Something with a measure, however, does not directly take on what is added to it: as Hegel puts it, it does not accept the “arithmetical amount” that is given to it. Rather, it accepts, and so increases by, an amount that has been specified by its measure. This additional amount remains a quantum, and is still dependent on the other quantum. Yet it is not completely dependent on the latter and is not a mere “quantum as such”, but it is a quantum “specified in a constant [konstante] manner” (SL 337 / LS 376). This moment of constant ‘specification’, Hegel notes, constitutes the “exponent” that governs the relation between the something and any quantum that changes it. If the same amount is added to different things, their specific measures or exponents will thus ensure that each in fact increases by a different amount.

This kind of measure, Hegel claims, explains why different bodies absorb in different ways the heat transferred to them. As the temperature of a “general medium” — say, the air — increases, particular bodies in the medium differ in the way they absorb it, “for through their immanent measure they determine the externally received temperature” (SL 338 / LS 377-378, translation altered). This “immanent measure” is their “specific heat” (spezifische Wärme), and it explains why, for example, a metal absorbs heat at a different rate from water. The third form of measure is thus not just a form of thought, but it underlies a significant phenomenon in nature.

IV. Measure as the Quantitative Relation between Qualities
After examining this “specifying measure”, Hegel proceeds to render explicit what is implicit in the latter, and he thereby again derives a new form of measure. He points out first that the merely external quantum we have been considering is not in fact purely quantitative after all, because it is itself “qualitatively different” from quality, that is, from the qualitative, specifying measure (SL 339 / LS 378, translation altered). This in turn reflects the fact that quantity as such is qualitatively different from quality: it is a further form of quality that no longer exhibits the characteristic logical structure of quality itself.28 For this very reason, however, the external quantum in the specifying measure is explicitly quantitative, not qualitative; this is why it is subject to specification by the measure and not the other way around. The quantum, as quantum, is thus only implicitly qualitative, and there is only an implicit qualitative difference between it and its specifying counterpart. When, however, that implicit qualitative difference is rendered explicit, or “posited in the immediacy of being” (SL 339 / LS 378-379), in accordance with speculative method,29 both sides in the relation have to be conceived as explicitly qualitative. That means in turn that the quantum on each side is not merely a quantum but the specific quantum of a quality.

This takes us to a new logical structure that must be carefully distinguished from its predecessors. Both the rule and the specifying measure confront a quantum that is, or belongs to, something other than the measure.30 Such a something in turn necessarily has a certain quality; indeed, in the case of the specifying measure Hegel states that the quantum belongs to a something with “the same quality” as the measure itself (which enables the former to act on the latter and the latter to specify the effect the former has on it) (SL 337 / LS 376). Yet in these two cases, the quantum specified by the measure is a matter of indifference to the quality of the thing with that quantum; it is not explicitly the thing’s measure and so in that sense is not itself explicitly ‘qualitative’. The quantum belongs to something with a quality, and implicitly constitutes its measure since it permits the thing to be what it is; yet it is explicitly a mere “measureless” (maßlos) quantum — the mere quantum that the rule and the specifying measure require as their logical counterpart (SL 337 / LS 376).31

28 S. LS 1812 234, and SL 239, 323 / LS 258, 360: “The externality of the determinateness is the quality of quantum”.

29 S. EL § 88 R.

30 S. SL 337 / LS 375-376.

31 Miller translates “maßlos” as “having no significance as a measure”.
In the new logical structure, by contrast, that quantum is now itself explicitly qualitative. This means not just that it belongs to something with a quality, but that it is explicitly specific to such quality. The quantum that is merely external to the specifying measure is now no longer just a quantum but the “quantum of a something and of its quality” (SL 339 / LS 379, my emphasis). Both sides of the relation, therefore, now have the same logical structure: each is explicitly quantitative and qualitative.

Note, however, that this shared structure does not eliminate the difference between the two sides. The reason why is that the external quantum becomes qualitative when we render explicit the implicit qualitative difference between it and its specifying counterpart. As Hegel puts it, it is “this difference between them” that is posited in the “immediacy of being” (SL 339 / LS 378-379). So, although the external quantum does, indeed, become qualitative, like its counterpart, it does so as it becomes explicitly different qualitatively from the latter. The two sides in the new logical structure must, therefore, have their own distinctive qualities, and the quantum that each is must be the specific quantum of that quality. It is, of course, possible, as a matter of fact, to encounter two related things with the same specific quantum and same quality, such as two equal amounts of water; but such a relation between things is not what is made necessary at this point by the logic of measure. What is made necessary here is a relation between two things, each of which has its specific quantum and the distinctive quality associated with the latter. In the new measure, therefore, two quanta now coincide with two different qualities in relation to one another.32

There is, however, a subtle logical difference between the things as qualitative and as quantitative. As qualitative, they are principally distinct from one another; indeed, Hegel states, “each is for itself [für sich] such a determinate being” (SL 339 / LS 379, translation altered).33 As such, therefore, they are not explicitly related to one another: they are not connected by their different qualities. In the previous “specifying” measure, however, measure took the form of the explicit relation between two quanta (in which one altered or “specified” the other). This remains the case in the new measure, since the latter simply renders explicit what is implicit in its predecessor. Accordingly, although the two things in this measure are, as qualitative, not explicitly related, they are explicitly related to one

32 This is not to deny that the two quanta may be the same, but in each case it is the specific quantum of its quality.

33 S. also SL 344 / LS 384.
another by their specific magnitudes. As Hegel puts it, “measure is thus the *immanent* quantitative relating of *two* qualities to each other” (SL 340 / LS 379).

This measure is, more precisely, a *single* measure — “*ein* Maß” (SL 336 / LS 375) — that consists *in* a quantitative relation between qualities. Moreover, since each side of this relation is the “specific magnitude” of a quality, each is itself a measure in its own right (SL 339 / LS 379). The new measure we are considering is thus one measure that is a relation between two measures.34

Now, as we know, a measure as such contains a quantum in two senses: once as constituting the measure itself — as the quantum that is specific to, and sustains, the quality of the thing concerned — and once as an immediate, ‘external’ quantum that can change (and exceed the measure of the thing). Accordingly, the two quanta in the new measure must also be both kinds of quantum. In Hegel’s words, “the quantum in its dual character [*Doppelsein*] is both external and specific so that each of the distinct quantities possesses this twofold determination and is at the same time inseparably linked with the other” (SL 340 / LS 379). Each, therefore, must be a merely external, changeable quantum in relation to another such quantum, but each must also be a specific quantum that belongs specifically to this quality rather than that. Hegel argues that this requires the new measure to take *three* different forms, depending on which aspect of the measure is more to the fore. Two of these will mirror measures we have already encountered, whereas the third will be unique to this new measure and, indeed, will alone be the full realisation of the latter.

As just noted, each quality in the new measure has a quantum that belongs specifically to it (and so each has its own measure). Initially, however, this must itself be merely some simple, *immediate* quantum that is attached to the quality: as Hegel puts it, the two sides in the relation are “taken at first simply as determinacies of magnitude [*Größebestimmtheiten*]” (SL 341 / LS 380, translation altered). The new measure thus consists first in the relation between these magnitudes. It is a definite, fixed relation between them, because it has a determinate character of its own that makes it the measure it is; yet the two quanta in the relation, as simple, immediate *quanta*, are also inherently *changeable*. The distinctive “determinacy of the measure” (*Maßbestimmung*) thus resides in a fixed relation, or *direct ratio*, between two changeable quanta: so, as one increases, the measure requires the other to increase by a proportional amount.35 As an example of this measure, Hegel points to

34 S. SL 330, 339 / LS 368, 378.

35 For the term “*Maßbestimmung*”, s. SL 340 / LS 379. Miller translates it as “determination of measure”.

velocity, in which a certain quantum of space is traversed in a given time: say, two metres per second. The distance travelled can increase from two to four metres, but the measure is preserved insofar as the direct ratio between distance and time — the velocity — is preserved: so four metres are traversed in two seconds.\textsuperscript{36} Velocity might seem to be a purely arbitrary relation between distance and time, but it in fact combines the two aspects of measure noted above. The magnitudes of the distance and time are, indeed, simply given, and so arbitrary, and the velocity could just as easily have been another. Yet the velocity is the measure of a certain (uniform rectilinear) motion,\textsuperscript{37} and each magnitude, as a moment of that measure, is specific to its quality and stands in a fixed relation to its counterpart (even as it changes).

The two sides of the new measure must, however, be more clearly differentiated from one another than this, since their relation must also render explicit the difference within measure between the measure itself and the mere quantum. This difference is present in velocity, since, as in the rule, one of the qualities provides the “unit” through which the other’s amount is “specifically determined” (SL 341 / LS 380): velocity is distance-per-unit-of-time or time-per-unit-of-distance. Just as in the rule, however, the unit can itself be regarded as an amount, so each side remains a given quantum. To understand how measure and quantum can be explicitly distinguished in the new measure, therefore, we must look back to the measure that comes after the rule. In the latter, one moment remains a mere quantum, but the other is the “specifying” measure that explicitly negates and changes the first (rather than just subordinating it to a rule). It does so by asserting its distinctive quality — or, in Hegel’s words, “the qualitative moment” — against its counterpart (SL 338 / LS 377).

Now, as we know, every measure is a quantum that is specific to, and so one with, a quality. The identity of quantum and quality can, however, be more or less explicit. In the first, immediate measure (described in 1.A of the section on measure in the Logic), the quantum to which a quality is attached is itself simply immediate: it is just “some determinate quantum”, or range of quanta, that sustains “some determinate quality” (SL 333 / LS 371, translation altered). Yet at the end of the account of quantity, just before the transition to

\textsuperscript{36} S. SL 342 / LS 381-2. Note that space (or distance) and time are regarded here as qualities whose quanta stand in a certain relation; s. SL 341 / LS 380. In the philosophy of nature, however, the ‘quality’ of space is itself understood to be “pure quantity”; s. EPN § 254 R.

\textsuperscript{37} S. Biard \textit{et al} 1981, p. 247.
measure, the quantum proves to be explicitly qualitative by raising itself to a *power* of itself: for, in doing the latter, it relates to *itself* in becoming another quantum and thereby exhibits the distinctive quality of “self-relation” that characterizes “being-for-self”.\(^{38}\) It follows that a quantum is most explicitly one with quality as a *measure*, when it is not just “some determinate quantum” but one raised to a power. Logically, therefore, the specifying measure that explicitly differentiates itself from and acts on the mere quantum must change the latter “in accordance with a power-determination [*Potenzenbestimmung*]” (SL 338 / LS 377, translation altered) — though Hegel does not explain exactly how this might manifest itself in specific heat.\(^{39}\)

Since the two quanta in the new measure must also be distinguished from (and related to) one another as explicit measure and mere quantum, the former must also determine the latter by raising itself to a power of itself. This yields the second form of the new measure. Hegel’s example is Galileo’s law of falling bodies, according to which distance is proportional to the *square* of the time, or (as Hegel expresses it) \(s = at^2\) (SL 342 / LS 381).

The distance and the time needed to traverse it are both changeable quanta, but, as in the case of simple velocity, their relation to one another is once again fixed. In this case, however, to calculate the distance travelled in an increased time — say, in three seconds, rather than one — the initial distance-per-second is multiplied not just by the new time, but by the square of the new time. So, as Hegel explains in an addition to his *Encyclopaedia Philosophy of Nature*, if “the body falls a little more than 15 feet in the first second”, “in two seconds, the body falls, not twice but four times the distance, i.e. 60 feet; in three seconds it falls 9 x 15 feet, and so on” (EPN § 267 A).\(^{40}\) This relation between distance and the square of the time does not characterise all movement, but it is the distinctive measure of freely falling, and thereby uniformly accelerating, bodies (on a planet or moon); and Hegel claims that it is logically necessary that there be a measure with this form.\(^{41}\)

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\(^{39}\) In fact specific heat or ‘heat capacity’ itself decreases at low temperatures ‘in accordance with a power-determination’; s. Valla\^nce Group: “the Debye heat capacity decreases as \(T^3\) at low temperatures, in agreement with experimental observation”.

\(^{40}\) S. Houlgate 2005, p. 138-144.

\(^{41}\) Another possible example of this form of measure is “\(e = mc^2\)”, though of course we cannot know what Hegel himself would have thought about the latter.
The new measure that has arisen at this point in the *Logic* thus takes the form of two distinct relations between quanta. Yet Hegel argues that it also takes the form of a third relation, in which, indeed, this new measure is most fully realised. This third relation renders explicit the fact that the measure is a relation between *two measures*. The two sides are thus not just quanta, and are not just related as quantum and its specifying counterpart, but both are quanta raised to a power — though each is raised to a different power, and so “specified” in a different way, by the different quality to which it belongs. The example Hegel gives of this third form of the new measure is Kepler’s third law of planetary motion: the principle that the squares of the orbital periods of any two planets are proportional to the cubes of their mean distances from the sun, or (in Hegel’s expression) $s^3 = at^2$ (SL 342 / LS 381). Once again, not all motion is subject to this law, but the law is a specific measure of planetary motion, and it exemplifies a form of measure that, in Hegel’s view, is made necessary logically by the nature of measure itself.

Hegel calls the measure we have been considering in this section “the realized measure” (SL 340 / LS 380). This measure realizes itself most fully, however, only in the third of the three forms that it takes: for only in this case are the two measures related to one another explicitly as quanta that are “qualitatively determined” and so as *measures*. The “higher realization of the qualifying of the quantitative”, Hegel states, “is that in which both sides are related to each other in higher determinations of powers (as is the case in $s^3 = at^2$)” (SL 342 / LS 381).

V. Contingency in Measure

It can be tempting to admire the consistency of Hegel’s derivation of categories in the *Logic*, but to wonder what the point of it all is. In Hegel’s view, however, the point is one of great significance. Logic shows not just that certain categories are conceivable, but that they — and the corresponding ways of being — are logically necessary. It does so by demonstrating that

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42 S. SL 341-342 / LS 381 (on the “extensive” and “intensive”). Note that, in Hegel’s view, the difference between the “extensive” and “intensive” also explains which quantum must be the power-determination in the second form of this measure.


44 It is clear from Hegel’s account of the realized measure that his interest in Galileo and Kepler arises not just from their importance for science but from the fact that their laws express necessary measures; s. SL 343 / LS 383. On the relative significance of Kepler and Newton, in Hegel’s view, s. Houlgate 2005, p. 155-156.
they are inherent in thought — and being — itself; and it does this by rendering explicit what is implicit in the category of pure being and the subsequent categories that arise. The logic of measure thus shows that certain forms of measure belong of necessity to the very fabric of being. The occurrence of these forms of measure in nature is then confirmed by the examples that Hegel provides.45

Hegel’s logic of measure is thus not just a “reconstruction” of concepts from the history of philosophy or science; nor is it just a critique of inadequate ways of thinking “determinately” about reality.46 It is a positive metaphysics that discloses the measures there must be in the world. This in turn means that, for speculative philosophy, certain forms of motion and the laws that govern them are not just contingent, but exemplify being’s very own measures. This, however, is not to deny that there is contingency in the world. Indeed, Hegel argues that such contingency is actually an integral feature of measure itself.

It has been noted above that in a measure the quantum has a “dual character” (SL 340 / LS 379): it is, on the one hand, a mere quantum and, on the other, a quantum that is specific to a certain quality (and thus the measure of the thing concerned). This dual character is evident in the fact that the very first measure (in 1.A) is a quantum that sets a limit to the changes it can undergo as a mere quantum; and it becomes explicit in the relation between, first, the rule and the quantum and, second, the specifying measure and the quantum. As we have just seen, this dual character also manifests itself in the new “realized” measure by requiring the latter to take three forms: the relation between two quanta, the relation between a quantum and a “specifying” quantum in the form of a power-determination, and the relation between two power-determinations. The dual character, however, also manifests itself in the fact that the last two relations themselves coincide with relations between simple, given quanta; and it is this fact that places contingency at the heart of measure.

A simple quantum is by its nature contingent, since it is simply and immediately what it is and could just as well be different; there is thus contingency in the fact that there is this much water, rather than that much, in the sea. There is also contingency in the fact that something has its measure in this immediate quantum — that water boils at 100° C, rather than 60° C — and the fact that the first form of realized measure, velocity, has this

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45 In speculative logic these are just examples, but in the philosophy of nature Hegel argues that they are themselves made necessary by the logic of nature (that is, of space and time); s. SL 342 / LS 382; EPN § 267 R, § 270 R, and Houlgate 2005, p. 138-153.

46 For Karin de Boer, by contrast, Hegel’s logic as a whole is such a reconstruction, and for Robert Pippin the doctrine of being as a whole is such a critique. S. de Boer 2010, p. 40-41, and Pippin 1989, p. 191-201.
magnitude. These measures, as measures, are logically necessary; but they necessarily contain contingency, because the quanta they involve must be immediately given and thus be beyond explanation by logic. This is not, of course, to deny that natural science might be able to explain why water boils at 100°C or an object travels at a certain speed, but logic alone cannot do so.47

Now logic, as we have seen, requires the realized measure to take two further forms beyond that exemplified by velocity. In these forms, however, the quanta concerned are not just immediate, and so contingent, but one has, or both have, a character that is made necessary by logic itself (more specifically by the logic of quantity). In the ‘Galilean’ realized measure one quantum is thus a power-determination, and in the ‘Keplerian’ measure both quanta are. The ‘Keplerian’ measure in particular is, therefore, explicitly the relation between two specifying quanta, or measures, rather than between two merely immediate and contingent quanta.

Yet a measure as such is both a measure and an immediate quantum at the same time. This should thus be the case in both the last two forms of realized measure; and indeed closer attention shows that it is. One might think that such immediacy can be found in the particular powers to which quanta are raised in those measures, but Hegel argues in the philosophy of nature that these powers are themselves necessary, so I will leave them to one side here.48

There is, however, an element of immediacy and contingency in those measures in another sense: for each power is itself the power of an immediate quantum. These immediate quanta are represented in the two laws that exemplify the two measures by $s$ and $t$, that is, distance and time; any power of $s$ and $t$ must, therefore, also be an immediate quantum, and, accordingly, the measure itself — $s = at^2$ or $s^3 = at^2$ — must be a direct ratio between such quanta. This ratio in turn must have an exponent that is found by dividing one side of the ratio by the other, and this exponent is the ineliminable element of contingent immediacy in the realized measure. It is what Hegel calls the “empirical coefficient” in such a measure and it is represented in his expressions of the two laws of motion by $a$ (SL 346 / LS 386).49

Note, however, that this “coefficient” is not only a function of $s$ and $t$, but also the immediate quantitative determinacy of the measure: it is the quantum that gives the measure its distinctive empirical character. As such, it is the moment of fixed immediacy or “being-


“for-self” in the measure that limits the change of s and t (SL 346 / LS 386) — though, of course, it can itself change and so give rise to a different empirical measure. The change of s and t is thus limited in two ways by the two aspects of the measure. On the one hand, it is limited by the logical form of the measure: so in the ‘Galilean’ measure, the distance travelled by a falling body must be proportional to the square of the time that has passed, or \( s = t^2 \). On the other hand, it is also limited by the “empirical coefficient” in the measure: so \( s = a \times t^2 \) (where \( a \) is the distance the body falls in the first unit of time).

Freely falling bodies thus always fall in accordance with Galileo’s law, but their rate of fall is also governed by a particular number that is not determined by the logic of measure (or by the nature of space and time) and so, from the point of view of speculative philosophy, is immediate and contingent. In the philosophy of nature Hegel reminds us that, leaving aside the effects of air resistance, bodies on the same planet fall at the same rate. Their rate of fall may differ, however, from planet to planet and moon to moon: the same body in the same initial unit of time may fall a different distance on a different planet. The ground of the distinctive immediacy that determines the way bodies fall is thus to be found in the terrestrial (or lunar) body to which they belong.

A similar empirical coefficient governs the movement of the planets in our solar system. Each planet lies at a different mean distance from the sun and has a different orbital period; but in each case the cube of the distance is proportional to the square of the period, and in each case the exponent of the direct ratio between the two sides is the same, namely approximately twenty-five. Planets belonging to a different solar system will also obey Kepler’s third law (unless contingencies in the system intervene), and the ratio between the cubes of their mean distances from their sun to the squares of their orbital periods — when these are converted into simple numbers — will also be governed by an exponent; but the numerical value of that exponent may differ from that of our solar system.

VI. Transition to Real Measure


51 The surface gravity on the Moon, for example, is 0.16 of that on Earth, and on Mars it is 0.38. S. Sparrow 2006, p. 72, 88.

52 The mean distance of Mars from the sun is 227.9 million kilometres and its orbital period is 687 earth days. If one divides the cube of the former by the square of the latter, the result is 25.079. The mean distance of Jupiter from the sun is 778.3 million kilometres and its orbital period is 11.86 earth years. In this case, the cube of the former divided by the square of the latter yields the result, 25.158. S. Sparrow 2006, p. 88, 140.
The measure that governs the fall of a body or the orbit of a planet is thus not just one single measure but comprises two relations: a ‘specifying’ relation between powers (or a power and a quantum) and a direct relation between amounts. These two relations are, however, independent of one another: the fact that a body is subject to Galileo’s law does not determine how far it should fall in the first period of time, and the fact that $s^3 = at^2$ does not require $a$ to have one value rather than another. In that sense, the two relations constitute two distinct measures governing falling or orbiting bodies. Yet these two distinct measures actually constitute one measure, since any ‘specifying’ relation between quanta of qualities is inseparable from a direct relation between quanta and the latter will always have a given exponent: that is to say, whenever $s = at^2$ or $s^3 = at^2$, $a$ must have some particular numerical value.

The realized measure is thus not simply a relation between two quanta (one or both of which is a power-determination), but it is also the relation between, and indeed unity of, two different relations between those quanta (SL 347 / LS 387-388). Implicit in this measure so conceived, Hegel argues, is a new measure that he calls a “real measure”. When that new measure is considered in its initial immediacy, however, it must be a relation between two immediate, direct ratios, rather than between a direct ratio and a ratio involving powers (as in the realized measure). As Hegel puts it, “since the sides which now constitute the measure relation are themselves measures, but at the same time real somethings, their measures are, in the first place, immediate measures and the relations in them are direct relations” (SL 348 / LS 388-389, translation altered). The real measure, therefore, is a ratio in which the two sides are no longer just quanta — whether simple or raised to a power — but ratios. Since, however, the latter are direct ratios, their sides are once again — like the sides of the simplest realized measure — quanta of given qualities. Density, which Hegel cites as an example of such a direct ratio, is thus the ratio between a quantum of mass and a quantum of volume.\(^{53}\)

The real measure is therefore exemplified in nature by the direct ratio between two densities, each of which is itself the direct ratio between a mass and a volume. The ratio

\(^{53}\) Ruschig points out that density can be expressed as g/cm\(^3\) and so still involves a power-determination (Ruschig 1997, p. 42). Yet density is nonetheless a direct ratio between mass and volume. If the density remains constant and the volume increases, the mass increases in proportion to the volume, not to the square or cube of the volume. By contrast, when the rate of fall of a body is constant, the distance travelled is proportional to the square of the time elapsed. Density thus exemplifies one ‘side’ of a real measure, whereas Galileo’s law of fall exemplifies the second form of realized measure. For an insightful account of real measure in the Logic, s. Schick 2014.
between the density of a substance and that of a reference substance (usually water or air) is called “specific gravity”. Specific gravity thus exemplifies the real measure, while simple density exemplifies the ratio that constitutes one side of such a measure.

**VII. Conclusion**

Readers will note that the various measures examined by Hegel are exemplified by very different natural phenomena: specific heat, the laws of gravitational motion, density (or specific gravity). Hegel, however, is not claiming that there is a necessary natural connection between these phenomena, or that one gives rise to the other in nature; that is something for natural science to consider. His claim is that implicit in the logical structure of each measure exemplified by such a phenomenon is the logical structure of a new measure, and that one measure thus makes another necessary logically. It is the intrinsic logic of measure, therefore, that generates the sequence of measures that Hegel discusses.

Measure is initially (in 1.A) the immediate *unity* of a quantum (or range of quanta) with a quality. This measure, however, also contains the *difference* between itself and the quantum of the thing by setting a limit to the latter beyond which the thing ceases to be. When this difference is rendered explicit, two new forms of measure are generated. In the first (in 1.B.a), the measure relates, as a mere *quantum*, to another quantum by providing the unit of which the other is the amount. In the second (in 1.B.b), the measure relates, as a specifying measure, to another quantum by limiting the changes that the latter can bring about in it. The mere quantum in this relation is, however, itself implicitly qualitative, since, as a *quantum*, it is qualitatively distinct from the explicitly *qualitative* measure. This points logically to a new, “realized” measure in which both sides have an explicit and distinct quality, but in which they are related by their quanta (see 1.B.c). Those quanta in turn are, in the first form of realized measure, mere quanta (for example, in simple velocity); but then, in the other two forms, at least one of them is an explicitly qualitative, “specified” quantum. As Hegel argues in the account of quantity, such a quantum is one that raises itself to a *power* of itself: for in so doing it exhibits the distinctive quality of “self-relation” that belongs to “being-for-self” (SL 321-323 / LS 359-361). The second and third forms of realized measure are thus relations, respectively, between a quantum and a power-determination and between two power-determinations. Each relation, however, coincides with a direct ratio between two mere quanta, and the realized measure thereby proves to be a relation between, or unity of, two different *relations between* the quanta concerned.
This leads logically to the real measure, in which one measure is the relation between two relations, which in their initial immediacy are direct ratios, such as densities. The development of measure then leads to the explicit unity or “combination” of such ratios (because the real measure is two as one), and to a series of such combinations (since each ratio, as the relation between quanta, is subject to the logic of the quantum and the one [Eins], and so is not just one of two but one of many). The real measure thus proves to be an explicit unity, but is then dispersed into many different unities.54

The subsequent development of measure leads via further complex measures, such as “elective affinity” and the “nodal line”, to the transition from measure to essence.55 That development is too detailed to summarise here. What needs to be borne in mind throughout, however, is that it continues to be guided by the logic of measure, rather than the natural phenomena that, in Hegel’s view, exemplify each measure. It is certainly tempting to think that Hegel considers elective affinities, only because they were the subject of scientific concern at the time he was writing the Logic. In truth, however, he considers these affinities, and all the other measures, because the logic of measure itself requires him to.56 This logic is one to which Kant and Spinoza were both blind, but in Hegel’s view it is at work in both thought and being. It determines us to think about measures in certain ways (though such thought needs time and history to become fully explicit), and it also determines there to be certain measures in the world.57

Bibliography


57 Hegel claims that the “free development” of the different forms of measure occurs more in nature than in the realm of spirit and, indeed, that “laws” (Gesetze) expressing measures, such as those of Galileo and Kepler, are to be found only “in the sphere of mechanics”. Certain forms of political constitution have their own measure in that they are “suitable only for states of a certain size”, but they are not governed by clear “laws of measure”. S. SL 331-332 / LS 369-370, and Rinaldi 1992, p. 173-174.


http://vallance.chem.ox.ac.uk/pdfs/EinsteinDebye.pdf, visited on 17 August 2015.