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Supplier Competition with Option Contracts for Discrete Blocks of Capacity

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Abstract. When a firm faces an uncertain demand, it is common to procure supply using some type of option in addition to spot purchases. A typical version of this problem involves capacity being purchased in advance, with a separate payment made that applies only to the part of the capacity that is needed. We consider a discrete version of this problem in which competing suppliers choose a reservation price and an execution price for blocks of capacity, and the buyer, facing known distributions of demand and spot price, needs to decide which blocks to reserve. We show how to solve the buyer's (combinatorial) problem efficiently and also show that suppliers can do no better than offer blocks at execution prices that match their costs, making profits only from the reservation part of their bids. Finally we show that in an equilibrium the buyer selects the welfare maximizing set of blocks.

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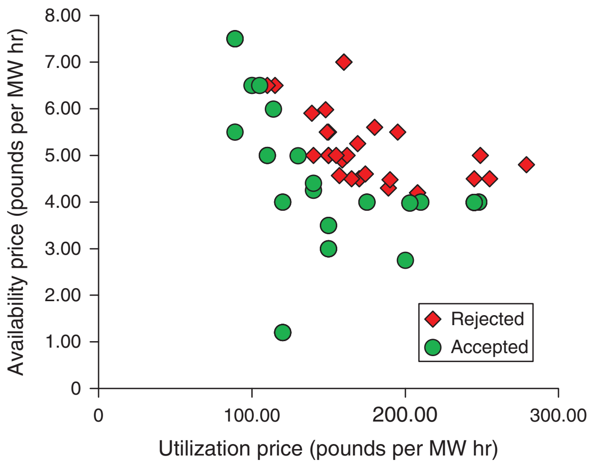
1. Introduction

Procurement of commodities plays a pivotal role in the success of global firms but brings with it the challenges of dealing with different sources of risk, such as supply unavailability, demand uncertainty, and price volatility. To manage these commodity risks, firms will use a portfolio of procurement arrangements. Buying from a spot market offers flexibility but is characterized by great price uncertainty (Seifert et al. 2004). An alternative is to use supply options to hedge against future price rises and low demand (Martínez-de-Albéniz and Simchi-Levi 2005, Kleindorfer 2008). Supply option contracts allow firms to adjust their procurement costs based on realized demand and spot price, but an upfront fee has to be paid to suppliers. By signing option contracts, the buyer's demand risk is mitigated by freely choosing the executed capacity after she knows the actual demand, and the supplier's financial risk is also diminished by receiving the reservation payment from the buyer in the early period. The underlying assumption is that the supplier only prepares the amount of capacity that is reserved, so this becomes the limit for later production.

The combination of spot purchases and supply options has seen widespread applications in capital-intensive industries, such as commodity chemicals, electric power, and semiconductors (Kleindorfer and Wu 2003, Wu and Kleindorfer 2005). The challenge of such hybrid commodity procurement is exacerbated if there are further restrictions that require buying firms to reserve capacity in *blocks*. This occurs when there are standard sizes for a production facility that needs to be built or made available in its entirety. An example of this sort occurs within the UK's system for purchasing Short Term Operating Reserve (STOR) for electricity supply (National Grid 2017). This is a scheme under which the National Grid maintains a reserve generation ability in case of sudden demand variations or plant failures. Part of the operating reserve is made up by contracts that are bid for within the STOR scheme. In this market, the bids come as blocks of capacity, so the National Grid determines the right set of blocks to reserve. Tenders are assessed on the basis of reservation prices (known as availability prices) and execution prices (known as utilization prices) together with a consideration of response times and geographical location. Figure 1 shows STOR bidding data from round 18 (2012: season 6.5) with accepted bids shown as circles (in green). This illustrates the portfolio selection and

Figure 1. (Color online) The STOR Bids Submitted in Round 18, Season 6.5



one can see that there is a curved boundary for the bids accepted.

The National Grid is not alone when it comes to capacity reservation in blocks. For example, Elia, Belgium's transmission system operator and a key player in the energy market, also purchases power reserve (in the forms of Strategic Reserve and Ancillary Services) to maintain the balance on the electricity system. Power plants are made available to be called on by Elia in case of demand surges or supply shortage at existing generation units. In the procurement auction, bidders are evaluated based on reservation price (€/MW/h) and variable activation price (€/MW/h) as well as other criteria such as compliance and reliability performances (Elia 2015).

Motivated by the STOR and Strategic Reserve examples, this paper studies an auction model where a single buyer, facing an uncertain future demand and volatile spot market, would like to determine an optimal portfolio of procurement strategies. The demand will be met using a combination of a spot market and supply options from multiple competing suppliers. In this model, each supplier dedicates a capacity block that is bid into the option market. The supplier bids consist of a reservation price and an execution price; given these bids, the buyer decides which blocks to reserve prior to knowing actual demand and spot price. When demand and spot price uncertainties are resolved, the buyer decides how much reserved capacity to use and how much to purchase from the spot market.

In this context, it is natural for the buyer to purchase from a portfolio of suppliers in the option market. If there is no uncertainty in demand, then the buyer will select the supplier with the lowest total of reservation and execution price. But if there is just a small chance of demand occurring, then it makes sense to use a supplier with a very low reservation price, even if the sum of reservation and execution prices is higher. When demand has a known distribution, then it will be best to reserve some supply from suppliers with low total cost (who will be used to meet demand that is relatively certain) and some further supply from low reservation cost suppliers (in order to meet higher demand on the few occasions that it occurs).

The same ideas arise when considering the mix of generation capacity in a wholesale electricity market. The base-load generation has the lowest overall cost, while other types of "peaking" generation have lower cost for a fixed amount of installed capacity but higher costs for generation. Peaking generation is an appropriate part of the portfolio mix for use when demand is high. Some form of capacity reservation mechanism operates in many electricity wholesale markets (UK Statutory Instruments 2014). These capacity auctions operate in parallel with a spot market; see Joskow (2008) for more information on capacity auctions.

Our model is distinguished by explicit consideration of nonscalable capacity, which must be purchased in blocks. The buyer's problem we consider involves a choice between "blocks" of capacity that are offered, where the buyer does not have the option of reserving only part of a block (this matches the STOR and Strategic Reserve examples above). Thus, the problem for the buyer is simply to select the right set of suppliers. We will assume that if capacity is not purchased in advance, it can also be purchased from a spot market. The spot market price is drawn from a known distribution, but its realization is unknown at the point when capacity is purchased. We are interested in the suppliers' optimal policies, where suppliers know their costs (both for reservation and execution) and want to determine their prices in a competitive market.

The main contributions of this paper are summarized as follows. First, we examine in depth a combinatorial auction problem in which competing suppliers each own a capacity block and are evaluated based on their two-dimensional prices by the buyer, who makes a binary choice on each block. To our knowledge, this paper is among the first to study this discrete form of capacity auction, and we establish results that serve as an important complement to the existing studies (see detailed discussions in Section 2). The buyer's problem is of interest in its own right, and our second contribution is to establish submodularity for the expected profit of the buyer as a function of block selection, a result we use in order to establish supply chain efficiency in equilibrium. Although maximizing a general submodular function is computationally hard, we show that the buyer's optimization problem can be solved very efficiently. Third, our results generate useful insights for both the bidders and the auctioneer. For example, setting the execution price at cost and making money only from the buyer's reservation payments is an optimal strategy for any bidder. This makes the decision making process much easier when bidding for the buyer's capacity procurement business. Moreover, we show that the full system efficiency can be achieved in equilibrium for this noncooperative game, which provides a theoretical support for these types of arrangements. We note that some of our results mirror those found in some similar studies, but the underlying driving forces are different as will be clear in later sections.

The remainder of the paper is organized as follows. After a literature review in the next section, we set up our model in Section 3 and describe the exact sequence of decisions that need to be made. In Section 4 we discuss the buyer's problem of choosing an optimal set of suppliers and show that one can efficiently identify an optimal solution when all the suppliers offer blocks of the same size. In Section 5 we turn to the problem faced by the suppliers. We show that (a) regardless of how the other suppliers bid, it is best for any supplier to bid his execution price at cost, making profits only on the reservation component in his bid, and (b) provided all the blocks are of an equal size, at any Nash equilibrium the buyer selects exactly those suppliers necessary to give an efficient outcome for the supply chain as a whole. We also characterize a class of equilibria for the case where suppliers' blocks are of different sizes. In Section 6, we discuss three extensions to show the limits of the results obtained for the baseline model. Finally, we make some concluding remarks in Section 7. All technical proofs are presented in the online appendix.

2. Literature Review

Several streams of literature are related to our problem. An auction in a multidimensional setting where players have private information about their own costs, is often treated as a type of mechanism design problem, where we ask what mechanism can be used by the auctioneer to ensure truthful revelation of costs and hence an efficient and effective choice between bidders. Chao and Wilson (2002) address some fundamental questions of an auction for power reserves in which the requirement is to specify both a scoring rule to determine which capacity is to be used and a payment rule for the winning suppliers. Schummer and Vohra (2003) develop an Expected Vickrey-Clarke-Groves (EVCG) mechanism that arranges the payments to give each supplier his contribution to the expected overall cost. There is a continuum of EVCG mechanisms with different amounts paid to the suppliers up front and after demand is realized, but they all have the characteristic of inducing truthful revelation of the actual costs (both for reservation and execution). Our approach is different from those of this literature: not only do we have a combinatorial style auction with blocks of capacity reserved from selected suppliers, but we also have suppliers who are paid exactly as they bid and a buyer who simply maximizes her own profit.

The transport area provides a number of applications for models in which capacity is purchased in advance. A freight forwarder usually books capacity (in the form of slots) from carriers for a particular route before knowing the future demand. Kasilingam (1996) and Hellermann (2006) give useful surveys of the market structure of the air cargo industry and the related literature. Several papers formulate a stylized game theoretic model between a carrier and a forwarder. Hellermann (2006) studies a capacity contract with a reservation price and an execution price like ours. Amaruchkul et al. (2011) extend the above models by examining how much information rent has to be paid to a forwarder who possesses private information.

Problems of procurement and contract design for supply options have been considered by many authors; see, e.g., Barnes-Schuster et al. (2002) and Burnetas and Ritchken (2005). Detailed literature reviews can be found in Wu and Kleindorfer (2005) and Martínez-de-Albéniz and Simchi-Levi (2005). Recently, several papers have incorporated spot market and capacity limits into supply option models. Fu et al. (2010) consider a procurement problem where the spot price available to the buyer is random and may be correlated with the uncertain demand. Lee et al. (2013) analyze a model with a spot market and a capacity limit on each supplier. A similar model is also analyzed by Perakis and Zaretsky (2008), who consider a multiperiod version of the problem. These models are concerned with the buyer's purchasing decision instead of the bidding game between suppliers.

A closely related paper by Wu et al. (2002) considers a model with a spot market. In the model the buyer signs a contract in advance with a single supplier at a fixed reservation and execution price, but the decision on how much to purchase is delayed until the price in the spot market is known. Demand is determined by the buyer's utility and hence depends on the price paid. In this model the randomness occurs in the spot price, rather than directly in demand. With just a single supplier operating alongside a spot market, they show that it is best for the supplier to offer a contract with execution price equal to its cost. Wu and Kleindorfer (2005) extend this result to the case of multiple competing suppliers where again a spot market provides an alternative source of supply/demand for the buyer/suppliers. They show that a competitive equilibrium between the suppliers will deliver an efficient solution for the supply chain as a whole. Our model is different from this literature since we have uncertainty in demand as well as in the spot price and we also have a restriction that capacity is only available in discrete blocks—the buyer must reserve it all or none. We show that the results of Wu and Kleindorfer (2005) extend to this case with suppliers offering contracts with an execution price equal to cost and an equilibrium that is efficient for the whole supply chain. However, "the ultimate driver of [their] efficiency results is competition in the presence of a backup open spot market" (p. 460), while we identify a different driving force for the efficiency result, namely, the all-or-none nature of the buyer's decision. Furthermore, the equilibrium reservation prices and profit allocation are different, and also the existence of equilibrium is not guaranteed in their model (see Theorem 2 therein), whereas it is guaranteed in our model.

The model considered by Martínez-de-Albéniz and Simchi-Levi (2009) is also close to ours, with competing suppliers offering reservation and execution prices to a buyer who has to meet uncertain demand. Their model does not include a spot market, but a more significant difference is that they assume that each supplier has an (infinitely) scalable capacity, and the buyer can decide how *much* capacity to reserve from each supplier. The authors show that the suppliers will set reservation and execution prices that cluster together into groups of two or three suppliers. They also consider how much allocation inefficiency can incur because of supplier competition and show that if the demand distribution is log-concave, the inefficiency is never more than 25%. In our model, however, we assume that capacity comes as a block so that the buyer is faced with a combinatorial optimization problem. Such model differences result in contrasting findings as mentioned earlier.

Other research related to our work is the literature on combinatorial auctions, in which bidders can submit bids on bundles (or packages) of goods rather than just individual items. When the bids are in the form of price-quantity pairs, bidders engage in supply function competition (Klemperer and Meyer 1989, Vives 2011). The appealing characteristic of a combinatorial auction is that it can capture the complementarities or synergies among goods. Cramton et al. (2006) provide an excellent exposition on this topic from an interdisciplinary perspective of economics, operations research, and computer science. Combinatorial auctions have seen widespread applications in many areas, including industrial procurement, radio spectrum and transportation (Elmaghraby and Keskinocak 2004, Cramton et al. 2006). Despite the compelling motivation of allowing complementarity among goods, combinatorial auctions struggle with the winner determination problem and the extensive computational burdens for bidders because of the exploding number of bids involved in the bidding process. Our model shares a similar feature in that the auctioneer faces a combinatorial optimization problem (analogous to the winner determination problem in combinatorial auctions). However, we show that this optimization problem is easy to solve in our model because bidders each submit a single (two-part) bid rather than multiple bids for different bundles of goods. In addition, our focus is on understanding the competitive dynamics in a market where the single buyer reserves capacity in advance to hedge against future demand uncertainty and spot price uncertainty, an element missing in the combinatorial auction literature.

Finally, our paper is loosely related to a broad literature on cooperative games (Shapley 1971, Myerson 1991). Cooperative game theory abstracts away from the detailed actions of the players and focuses on what a group of players can achieve (Osborne and Rubinstein 1994, Peleg and Sudholter 2003). In contrast, our model of a price auction is a noncooperative game, where an outcome is a profile of pricing decisions made by the suppliers. Moreover, our model is a Stackelberg game, in which the suppliers are the leaders and the buyer is a follower. Nevertheless, the equilibria that occur in our model are in the *core* (one of the main solution concepts in cooperative games) for an appropriately constructed cooperative game, in which a coalition is formed between the buyer and a subset of suppliers.

3. The Model

We consider a supply chain with n suppliers and one buyer who faces random demand D. Before demand occurs, the buyer can reserve capacity that is offered in blocks by these n suppliers and pay a reservation price. After demand occurs, the buyer will meet the demand (up to the total amount of capacity reserved) and at this point pays an additional (execution) price for the capacity that is used. In addition to the reserved capacity, there is also a spot market from which the buyer can purchase to meet demand. We consider an open spot market where there is a much larger set of suppliers than are involved in bidding for contracts with this particular buyer; see Wu and Kleindorfer (2005) for a similar assumption. In the open spot market, no player can exercise market power to manipulate the (random) spot price P_0. Our model allows P_0 to be correlated with the demand D. Denote by $\bar{G}(t,p)$ the complementary cumulative probability for their joint distribution, i.e., $\bar{G}(t,p)=\Pr[D\geqslant t, P_0\geqslant p]$.

Decisions on the capacity to reserve are made prior to P_0 and D being realized, but the actual use of that capacity relies on there being sufficient demand and the spot market price being sufficiently high. The buyer is paid a price ρ for each unit of demand that can be met. We assume the upper bound of P_0 is no greater than ρ so that demand will be always fulfilled. This is without loss of generality because, once violated, we can define a new variable $\widetilde{P}_0=\min(\rho, P_0)$ and replace P_0 with $\widetilde{P}_0$; then all the results will follow.

Suppliers each dedicate a block of capacity to bid into the option market and do not use this capacity in the spot market; in Section 6 we relax this assumption by allowing suppliers to sell the unused capacity to the spot market. The suppliers each try to maximize their expected profits by choosing reservation prices r_i and execution prices p_i, given their reservation costs e_i, execution costs c_i, and block size K_i, where $i \in N = \{1, \ldots, n\}$. Note that both p_i and r_i are prices per unit, so the buyer needs to pay an amount $r_i K_i$ to reserve block i. If not all of the block is required when demand occurs and the spot price is realized, say, an amount w is needed, then the buyer pays an execution cost of $p_i w < p_i K_i$.

Overall, we analyze a two-stage model. In the first stage suppliers each simultaneously submit a bid consisting of a reservation price and an execution price. Given these supplier bids, the buyer decides which blocks to reserve. In the second stage, both demand and spot price are realized, and the buyer decides how much reserved capacity to execute and how much to purchase from the spot market. We can see that it is a Stackelberg game where the suppliers are leaders and the buyer is a follower. The suppliers compete in the option market, and we use the concept of Nash equilibrium to study the suppliers' bidding behavior.

For convenience of presentation, we assume that all execution prices $\{p_i\colon i \in N\}$ are distinct and label the bids so that $p_1 < p_2 < \cdots < p_n$. Suppliers will not offer an execution price higher than the unit revenue, so it is reasonable to assume $\rho \geqslant p_n$. We suppose that the joint distribution $\bar{G}$ and unit revenue ρ together with all the costs e_i and c_i $(i = 1, 2, \ldots, n)$ are common knowledge; see similar assumptions made by Martínez-de-Albéniz and Simchi-Levi (2009).

Given the set of bids $\mathscr{B} = \{(p_i, r_i, K_i)\colon i \in N\}$, the buyer decides which blocks to reserve. For any $S \subseteq N$, we denote by $S(\mathscr{B})^*$ the optimal set of bids for the buyer given that the choice is made from among bidders in S. Here the optimality is with respect to maximizing the total expected buyer's profit $\Pi_{\mathscr{B}}(X)$ from bidders in X; i.e.,

$$S(\mathscr{B})^* = \arg\max_{X \subseteq S} \Pi_{\mathscr{B}}(X). \tag{1}$$

When there is no confusion over the bids considered, we drop indication of $\mathscr{B}$ from notation (1).

The solution to the right-hand side of Equation (1) may not be unique. Since the buyer's choice has an impact on the suppliers' decisions, we need to give a definite description of the buyer's behavior when different choices yield the same expected profit for the buyer. As mentioned earlier, the problem faced by the suppliers and the buyer forms a Stackelberg game with multiple leaders and thus involves bilevel optimization (see, e.g., Dempe 2002). There are several alternative approaches to the issue of possible multiple solutions of the lower-level problem, "each requiring some assumptions about the level of cooperation between the players" (Dempe 2002, p. 28). Drawing on the bilevel programming literature, we will adopt the *optimistic* approach with the economic interpretation that the follower is willing to support the leaders. On the other hand, the Bertrand-Edgeworth rationing rules (see, e.g., Levitan and Shubik 1972) on market share in competitive pricing treat zero-profit strategies as a limit of iterative competition. Either approach leads us to the following assumption.

Assumption 1 (Tie-Breaking Rule). *If two sets of blocks give the same (maximum) expected profit, the buyer chooses the set of more blocks.*

Note that the above rule arises from a situation where a supplier has an incentive to constantly adjust his bidding so that his marginal contribution to the total expected profit of the buyer remains positive (to keep himself selected by the buyer) but is infinitesimally small. This type of situation in a supply chain has also been dealt with by Du et al. (2014, Section 2.2) using the optimistic approach.

Now let us consider how the buyer makes a choice between two *equal-cardinality* sets of blocks, each giving the maximum expected profit. Interestingly, our results will hold under *any* rule for breaking such a tie as long as such a rule is known to the suppliers and satisfies the following desirable property of consistency:

Assumption 2 (Independence Axiom). *If two equal-cardinality sets of blocks give the same marginal expected profit in the optimal selection of the buyer, then her preference of one over another is independent of the availability of any additional block.*

It is natural to suppose that if set A of blocks is preferred to set B of blocks, then this is still the case when an additional block ℓ becomes available but is not part of the buyer's optimal selection. The Independence Axiom implies that if the additional block ℓ does become part of the optimal selection and both A and B would contribute the same marginal expected profit (albeit possibly smaller than when ℓ was unavailable), then the above property says that A is still preferred to B by the buyer. Such a property, which is an axiom in decision theory and also known as *Independence of Irrelevant Alternatives* in social choice theory proposed by Arrow (1953), is desirable to avoid strategic manipulation: if candidate x is preferred to candidate y, then the presence of a third candidate z must not make y preferable to x.

The Independence Axiom can be satisfied by many decision rules. A very simple one is that the buyer always prefers the set of blocks that is lexicographically largest when each set of blocks is represented by the sequence of its block indices in descending order. Another natural decision rule is to give preference to the selection with the best worst-case profit for the buyer. The worst case occurs when reserved blocks are not required because of low spot price or low demand. This leads to a decision rule that prefers sets of blocks with a low value of the sum of r_i values.

To further understand the dynamics in the Stackelberg game, we follow the standard backward induction approach: we first consider an optimal policy for the buyer and then turn to considering the optimal behavior and the equilibria for the suppliers.

4. The Buyer's Selection of Suppliers

We start with a general case in which blocks can be of any size. Later we will restrict our attention to the case with equal-size blocks, for which we are able to obtain richer results.

4.1. Capacity Blocks of General Sizes

The buyer makes a two-stage decision that involves the reservation choice prior to knowing actual demand and spot price and the execution decision when both demand and spot price are known. We begin with the analysis of the execution decision given the buyer's reservation choice.

Given bids $\mathcal{B} = \{(p_i, r_i, K_i)\colon\ i \in N\}$, suppose the buyer's reservation set is $\{(p_i, r_i, K_i)\colon i \in S \subseteq N\}$, where

$$S = \{j_1, \ldots, j_v\} \quad \text{with } j_1 < \cdots < j_v. \tag{2}$$

It is convenient to denote by Y_i the total capacity of the first i blocks in S. Thus

$$Y_i = \sum_{m=1}^{i} K_{j_m}, \quad i = 1, \ldots, v. \tag{3}$$

At the time when actual demand and spot price are known, the buyer can fulfill customer demand by using the reserved capacity and spot purchases. Our first observation, simple but important, is that once a set of blocks has been reserved (and reservation payments made), when demand occurs the blocks that are used will be those that have the cheapest execution prices. In other words, the blocks will be used in increasing order of execution price, so the buyer will first use block j_1, then j_2, etc. Not all the v blocks will necessarily be used since demand may not require it, or at a certain point it may be cheaper to purchase from the spot market.

For any realized demand t and spot price p_0, we denote by $x_i(t, p_0)$ the amount of capacity from supplier j_i that is used. In light of the above observation, we obtain

$$x_i(t, p_0) = \begin{cases} \min\{(t - Y_{i-1})_+, K_{j_i}\}, & \text{if } p_{j_i} \leqslant p_0, \\ 0, & \text{otherwise,} \end{cases}$$

where $(z)_+ = \max\{z, 0\}$, and the purchase amount from the spot market is $t - \sum_{i=1}^{v} x_i(t, p_0)$. The buyer's expected profit from reserving S in the option market (as well as purchasing in the spot market) is then given by

$$\Pi(S) = \underbrace{\sum_{i=1}^{v}((\rho - p_{j_i})\mathbb{E}_{D,P_0}[x_i(D, P_0)] - r_{j_i}K_{j_i})}_{\text{expected profit from reserving bids in } S} + \underbrace{\mathbb{E}_{D,P_0}\left[(\rho - P_0)\left(D - \sum_{i=1}^{v} x_i(D, P_0)\right)\right]}_{\text{expected profit from purchasing in spot market}}, \tag{4}$$

where the expectations are taken over D and P_0. We note that $\mathbb{E}_{D,P_0}[x_i(D, P_0)]$ measures the expected used capacity from supplier j_i. The first and second terms in $\Pi(S)$ represent the buyer's profits from purchasing in the option market and the spot market, respectively.

It is worth mentioning that our model also deals with the case where there is no spot market simply by setting the random variable P_0 to be equal to ρ. In this special case, the buyer will meet demand by using supply options only, and her expected profit becomes

$$\Pi(S) = \sum_{i=1}^{v}\big((\rho - p_{j_i})\mathbb{E}_D[\min\{(t - Y_{i-1})_+, K_{j_i}\}] - r_{j_i}K_{j_i}\big).$$

One can see that the consideration of a spot market does not structurally change the buyer's expected profit function because from the buyer's perspective the spot market can be thought of as a nonstrategic supplier who has unlimited capacity, zero reservation price, and (stochastic) execution price P_0.

One strategy for the buyer is to reserve no capacity and rely entirely on the spot market. We write W for the expected profit under this policy. Hence, we obtain

$$W = \mathbb{E}_{D,P_0}[(\rho - P_0)D].$$

Having defined W, we can reformulate the expected profit for the buyer:

$$\Pi(S) = \sum_{i=1}^{v} \mathbb{E}_{D,P_0}[(P_0 - p_{j_i})x_i(D, P_0) - r_{j_i}K_{j_i}] + W. \tag{5}$$

From Equation (5) we observe that $\mathbb{E}_{D,P_0}[(P_0 - p_{j_i}) \cdot x_i(D, P_0) - r_{j_i}K_{j_i}]$ measures the (expected) extra profit the buyer can make from reserving block j_i in comparison with the profit by relying on the spot market alone. Clearly, in the special case where P_0 is extremely small, then option contracts may not bring any value to the buyer, and thus the buyer may rely on the spot market only. Since W is a constant, the buyer essentially maximizes the sum of these additional profits by choosing the optimal (sub)set of suppliers in the option market.

We now explore the property of the set function $\Pi(X)$ with $X \subseteq N$. The following lemma shows the submodularity of $\Pi(X)$, which is related to the economic phenomenon of diminishing returns.

Lemma 1 (Submodularity for Buyer Profit). *The set function $\Pi(X)$ with $X \subseteq N$ is submodular.*

The submodularity of $\Pi(X)$ implies that the marginal contribution of a block to the buyer's expected profit is smaller when the existing set of blocks is larger; i.e., for any blocks i, j and subset S, we have $\Pi(S \cup \{i, j\}) - \Pi(S \cup \{j\}) \leqslant \Pi(S \cup \{i\}) - \Pi(S)$. We can see that $\Pi(X)$ is nonmonotone; i.e., the relationship $X' \subseteq X''$ does not necessarily imply $\Pi(X') \leqslant \Pi(X'')$ since the marginal contribution of a block could be negative if it is forced into the choice set.

The intuition of Lemma 1 is as follows. Submodularity concerns how the addition of a block, say i, to the existing blocks affects the buyer's expected profit. First note that the reservation payments for block i are the same, regardless of the existing blocks. Thus we focus on the effect on the buyer's expected profit (excluding reservation payments), termed as *the expected execution profit*. Depending on the relationship between p_i and the execution prices of existing blocks, we have two cases: (i) p_i is the largest, and then the addition of block i will not affect the rankings of the existing blocks, and (ii) p_i is not the largest, and then the existing blocks with execution prices greater than p_i will be pushed backward (note that we sort blocks in increasing order of execution price). This will affect the buyer's expected execution profit in three ways: (a) the buyer's profit is increased by meeting the demand using block i that was previously unmet; (b) the buyer's profit by using block i is larger, when satisfying certain demand that was previously satisfied by other blocks with larger execution prices; (c) the buyer's expected execution profit from those affected blocks is reduced because there is a smaller chance for them to be used when demand occurs. The proof of the lemma shows that with a larger existing set, the addition of block i will either make the positive effects (a) and (b) less prominent and/or make the negative effect (c) more prominent, depending on the positions of block i in the existing sets. As a result, the marginal value of block i is smaller when the existing set is larger, leading to the submodularity property of $\Pi(X)$.

The buyer's problem is to find a set $S \subseteq N$ that maximizes the submodular function $\Pi(S)$: $\max_{S\subseteq N} \Pi(S)$. In general, it is NP-hard to maximize a nonmonotone submodular function since it includes computing the maximum cut of a graph as a special case (Feige et al. 2011). Next we will show that for the case with equal-size blocks, an efficient algorithm can be developed to solve for the buyer's optimal choice set in polynomial time.

4.2. Capacity Blocks of an Equal Size

When all the suppliers have equal-size blocks, we can establish stronger results. Without loss of generality we take $K_i = 1$. In this case $\Pi(S)$ can be rewritten as

$$\Pi(S) = \sum_{i=1}^{v} \left(\int_{i-1}^{i} \left(\int_{p_{j_i}}^{\rho} \bar{G}(t,p)\,dp \right) dt - r_{j_i} \right) + W. \qquad (6)$$

In fact, this is a special case of a more general result when blocks have unequal sizes that we give as Lemma 5 in the appendix.

Suppose that a block with prices (p_k, r_k) is not in the optimal choice set of the buyer; then substituting this block for any of the accepted blocks (p_{j_i}, r_{j_i}) cannot improve $\Pi(S)$. Hence

$$r_k > r_{j_i} - \int_{i-1}^{i} \left(\int_{p_{j_i}}^{\rho} \bar{G}(t,p)\,dp \right) dt + \int_{i-1}^{i} \left(\int_{p_k}^{\rho} \bar{G}(t,p)\,dp \right) dt.$$

This shows that the point (p_k, r_k) lies above the function

$$\tilde{r}_i(y) = r_{j_i} - \int_{i-1}^{i} \left(\int_{p_{j_i}}^{\rho} \bar{G}(t,p)\,dp \right) dt + \int_{i-1}^{i} \left(\int_{y}^{\rho} \bar{G}(t,p)\,dp \right) dt,$$

which passes through the point (p_{j_i}, r_{j_i}) associated with block j_i. Observe that $\partial \tilde{r}_i(y)/\partial y < 0$ and it is increasing in y (since $\bar{G}(t,p)$ is decreasing in p), so $\tilde{r}_i(y)$ is a convex decreasing function. This implies that the rejected blocks all lie above the (convex) upper envelope of these $\tilde{r}_i(y)$ $(i = 1, \ldots, v)$ functions associated with all the accepted blocks. Indeed, we can see exactly this kind of behavior in the shape of the accepted block region that occurs in Figure 1.

We now consider the problem where the buyer is restricted to choose a subset with k elements. We have the following property for the buyer's optimal choice.

Lemma 2 (Buyer's Nested Choices). *When blocks are of an equal size, the optimal buyer's choice when restricted to at most k blocks can be chosen as a subset of the optimal buyer's choice when restricted to at most $k+1$ blocks.*

Lemma 2 shows that when suppliers offer equal-size blocks, the buyer's optimal decision is nested, in the sense that the optimal choice of k blocks is a subset of that of $k+1$ blocks. This result makes the buyer's problem quite straightforward to solve using a dynamic programming approach. We simply solve the problem with a fixed number k of blocks, starting with $k = 1$ and then increasing k by one at a time. At each stage there are less than n options to consider as we add each of the possible blocks into the reservation set one at a time. In fact, it can be shown that once we stop making an improvement by adding another block, then we have found an optimal solution.

The buyer's computational problem can be formulated as a shortest path problem, similar to the one in Schummer and Vohra (2003) for the minimum cost problem, which has a time complexity of $O(n^2 d_{\max}^4)$, where $d_{\max}$ is an upper bound of total demand. With time complexity of $O(n d_{\max})$, our dynamic programming approach is more efficient.

Lemma 1 has shown that the buyer's profit function $\Pi(X)$ is submodular. The next result establishes that with equal-size blocks, this property is inherited by the function $\Pi^*(X)$, which takes the best buyer profit given a set of available blocks $X \subseteq N$:

$$\Pi^*(X) = \max_{S \subseteq X} \Pi(S). \tag{7}$$

Theorem 1 (Submodularity for Optimal Buyer Profit). *When blocks are of an equal size, the set function $\Pi^*(X)$ with $X \subseteq N$ is submodular.*

Theorem 1 shows that the optimal buyer profit, as a set function, is also submodular. The theorem is quite complex to establish because we need to track the change of the buyer's optimal choice when an additional block is available. We offer some intuitive explanations here. With equal-size blocks, the selection of an additional block ℓ has rather limited effects on the buyer's choice over existing blocks (see Lemma 8 in the proof for details): first, any block that is not chosen in the absence of block ℓ will still not be chosen in the presence of block ℓ; second, there is at most one existing block that is chosen when block ℓ is unavailable but will not be chosen when block ℓ is available. Consequently, the additional value added by block ℓ occurs because (i) block ℓ has more competitive prices than the dropped block or (ii) the buyer simply requires it to meet certain demand (without affecting the existing blocks). The proof shows that if the existing set is larger, it is less likely that block ℓ will be chosen or used (even if it is chosen) by the buyer. In other words, there will be smaller incidence of block ℓ being more competitive, or there will be a smaller chance that block ℓ will be used by the buyer when demand occurs. As a consequence, the optimal buyer's profit function is also submodular.

The result of Theorem 1 does not carry over to the case with unequal-size blocks as demonstrated by the following example.

Example 1. *Submodularity fails when blocks do not have the same size*. Suppose that demand takes only the single value 10, the unit selling price $\rho = 50$, and the spot price is also assumed to be 50. We have 5 blocks available $\{a, b, c, g, h\}$ with (p_i, r_i, K_i) triples as follows: $a = b = c = (1, 10, 4)$ and $g = h = (1, 7, 5)$. Then $\Pi(\{a, b\}) = 49 \times 8 - (10 \times 8) = 312$, $\Pi(\{a, g\}) = 49 \times 9 - (10 \times 4) - (7 \times 5) = 366$, $\Pi(\{a, b, c\}) = 49 \times 10 - (10 \times 12) = 370$, $\Pi(\{a, b, g\}) = 49 \times 10 - (10 \times 8) - (7 \times 5) = 375$, $\Pi(\{g, h\}) = 49 \times 10 - (10 \times 7) = 420$. Hence we see that

$$\Pi^*(\{a, b, c\}) = \Pi(\{a, b, c\}) = 370,$$
$$\Pi^*(\{a, b, c, g\}) = \Pi^*(\{a, b, c, h\}) = \Pi(\{a, b, g\}) = 375,$$
$$\Pi^*(\{a, b, c, g, h\}) = \Pi(\{g, h\}) = 420.$$

Thus $\Pi^*(\{a, b, c\}) + \Pi^*(\{a, b, c, g, h\}) > \Pi^*(\{a, b, c, g\}) + \Pi^*(\{a, b, c, h\})$, contradicting submodularity. □

The reason why submodularity does not hold for the general case is as follows. Unlike the case with equal-size blocks, the buyer's optimal choice depends not only on bidding prices but also on block sizes, where the latter determine how a block fits with other blocks. As a result, the addition of a block may have substantial impact on the buyer's choice over existing blocks. Specifically, as opposed to Lemma 8, the previously unaccepted blocks may be chosen (e.g., because these blocks match the new block better), or there may be more than one block that will be deselected (e.g., because the new block has a large size). For example, as illustrated by the above example, when block h is added to the set $\{a, b, c, g\}$, the buyer's optimal choice switches from $\{a, b, g\}$ to $\{g, h\}$, leaving both a and b out. This is not surprising because the block size of h is 5, which together with g just matches the total demand. Therefore, if blocks are of different sizes, the marginal contribution of an additional block may be bigger when the existing set of blocks is larger since this may enable more ways of combining different blocks to better suit the demand. This explains the failure of submodularity for the optimal buyer's profit.

It is worth comparing the above theorem with Lemma 1. On the surface, Theorem 1 shows submodularity for the buyer's *optimal* expected profit function $\Pi^*(\cdot)$, which by definition is monotone since the buyer will not select an additional block if it decreases her expected profit. This is in contrast with profit function $\Pi(\cdot)$, which is not monotone. In contrast to $\Pi^*(\cdot)$, the submodularity of $\Pi(\cdot)$ does not rely on the assumption of equal-size blocks because blocks are forced into the buyer's choice set, and we do not need to consider the fit between different blocks. In a nutshell, the two results are driven by different driving forces as discussed earlier. From a technical point of view, the endogeneity of the buyer's decision greatly complicates the analysis. Since the buyer is a profit maximizer in our model, the theorem plays an important role in subsequent equilibrium analysis.

5. The Suppliers' Bidding Competition

After understanding the buyer's reservation behavior, we are now in a position to examine the best responses and the equilibria for suppliers.

5.1. Best Response Strategies

Let us start with an examination of a supplier's best response to the bids of the other suppliers. We look at how supplier ℓ responds to bids of suppliers $L = N \backslash \{\ell\}$ by a choice of (p_ℓ, r_ℓ). Using the notation in (1), we write N^* and L^*, respectively, for the optimal buyer's choice from the set of bids N and L. We write $\pi_\ell(p_\ell, r_\ell)$ for the expected profit for supplier ℓ if he makes an offer with execution price p_ℓ and reservation price r_ℓ, assuming a fixed set of bids by the suppliers L. We are interested in the optimal choices of p_ℓ and r_ℓ to maximize π_ℓ. Notice that for any given value of p_ℓ, it is optimal for supplier ℓ to set r_ℓ as high as possible, subject to the proviso that the bid ℓ is still chosen by the buyer. We have the following result for supplier ℓ's best response.

Theorem 2 (Best Response). *Given bids $\{(p_i, r_i, K_i)\colon i \in L\}$ it is optimal for supplier ℓ to choose $p_\ell^* = c_\ell$.*

Theorem 2 shows that in an optimal solution, suppliers charge only costs for their execution prices but make profits from the buyer's reservation payments. We find that setting $p_\ell^* = c_\ell$ will maximize the total supply chain surplus. Since the supplier's profit equals the total surplus less the buyer's original profit $\Pi_{\mathscr{B}}^*(L)$, such a bidding strategy must also maximize the supplier's profit. Let $\mathscr{B}' = \{(p_i, r_i, K_i)\colon i \in L\} \cup \{(c_\ell, e_\ell, K_\ell)\}$ so that block ℓ is offered at cost. The proof of the theorem reveals that for the optimal solution with $p_\ell^* = c_\ell$, we have $r_\ell^* = e_\ell + (\Pi_{\mathscr{B}'}^*(N) - \Pi_{\mathscr{B}}^*(L))/K_\ell$. Therefore, supplier ℓ's optimal expected profit is $\pi_\ell^* = \Pi_{\mathscr{B}'}^*(N) - \Pi_{\mathscr{B}}^*(L)$. This is the supplier ℓ's marginal contribution to the supply chain with the existing bids $\{(p_i, r_i, K_i)\colon i \in L\}$. Supplier ℓ is able to extract all the marginal surpluses because the buyer in our model makes an all-or-nothing decision, which significantly limits her choice flexibility. Given the reserved amount is predetermined (which equals block size K_ℓ), supplier ℓ does not need to use prices to intervene the buyer's decision on how *much* to reserve.

The optimal bidding strategy $p_\ell^* = c_\ell$ mirrors a result of Wu and Kleindorfer (2005). Our result complements theirs by extending the finding to a discrete version of the problem where each supplier's capacity comes as a block. The above result is in sharp contrast with that of Martínez-de-Albéniz and Simchi-Levi (2009), where $p_\ell^* = c_\ell$ does not hold and suppliers cannot take all the marginal surpluses. More specifically, the authors find that supplier ℓ will bid infinitely close to some other bid that affects how much the buyer will reserve from him (see Theorem 2 therein). In their model the buyer's decision on how much to reserve is endogenous and depends critically on supplier bids (see Proposition 1 therein). As discussed by the authors, the buyer's decision in their model extends the classical newsvendor decision to multiple suppliers. Analogous to the double marginalization effect in the selling-to-newsvendor setting, it is not in supplier ℓ's best interest to maximize the total supply chain surplus by setting $p_\ell^* = c_\ell$. Moreover, supplier ℓ cannot extract all the marginal surpluses. In a similar vein, we show that the result $p_\ell^* = c_\ell$ fails to hold when suppliers own more than one block or the buyer can reserve just part of a block (see Section 6).

5.2. Equilibria with Blocks of an Equal Size

Having established the best response for each supplier, we now investigate Nash equilibria among the n suppliers. We start with the problem where blocks are of an equal size and then consider the more general problem where suppliers may have blocks of unequal sizes.

Without loss of generality we take the block sizes as $K_i = 1$, $i = 1, 2, \ldots, n$. The suppliers are characterized by their costs: $\mathscr{C} = \{(c_1, e_1), \ldots, (c_n, e_n)\}$. As before we will assume $c_1 < c_2 < \cdots < c_n$ for convenience of presentation, and based on Theorem 2 we can assume that each supplier chooses an execution price $p_i = c_i$. Hence all the execution prices are distinct.

We first present a key characteristic of an equilibrium set of bids: at equilibrium the buyer's optimal expected profit remains the same when any individual block is removed.

Lemma 3. *Suppose all blocks are of an equal size and bids $\mathscr{B}$ form an equilibrium; then for any $i \in N$,*

$$\Pi_{\mathscr{B}}^*(N) = \Pi_{\mathscr{B}}^*(N \backslash \{i\}). \tag{8}$$

Lemma 3 shows that the buyer's expected profit will not be affected by removing any individual block at equilibrium. The intuition is that if the buyer makes a lower profit when some block i is absent, then supplier i can always increase his bidding price a little while ensuring that block i is still selected by the buyer.

Now we will establish that any equilibrium with $p_i^* = c_i$ remains an equilibrium after the unchosen suppliers have their reservation prices reduced to reservation costs. Moreover, this process does not change the buyer's choice set.

Lemma 4. *Suppose all blocks are of an equal size. Let bids $\mathscr{B} = \{(c_i, r_i)\colon i \in N\}$ be an equilibrium and $S \subseteq N$ be any subset of the blocks not selected by the buyer under $\mathscr{B}$. Then the bids*

$$\hat{\mathscr{B}} = \mathscr{B} \backslash \{(c_i, r_i)\colon i \in S\} \cup \{(c_i, e_i)\colon i \in S\},$$

also form an equilibrium. Moreover, $N(\hat{\mathscr{B}})^* = N(\mathscr{B})^*$, and thus $\Pi_{\hat{\mathscr{B}}}^*(N) = \Pi_{\mathscr{B}}^*(N)$.

With these lemmas we are ready to characterize the equilibrium for suppliers.

Theorem 3 (Nash Equilibrium for Equal-Size Blocks). *When all blocks are of an equal size, the bids $\mathscr{B}^* = \{(c_i, r_i^*)\colon i \in N\}$ form a Nash equilibrium, where $r_i^* = e_i + \Pi_{\mathscr{C}}^*(N) - \Pi_{\mathscr{C}}^*(N \backslash \{i\})$ for $i \in N$. Moreover, at any equilibrium with $p_i^* = c_i$, $i \in N$, the buyer chooses the supply chain optimal set $N(\mathscr{C})^*$, and supplier i makes a profit $\pi_i^* = \Pi_{\mathscr{C}}^*(N) - \Pi_{\mathscr{C}}^*(N \backslash \{i\})$.*

Theorem 3 shows that at equilibrium each supplier charges her execution costs only and adds a premium to her reservation costs. Thus, suppliers make profits from the buyer's reservation payments only. By signing an option contract, suppliers share some demand risk with the buyer. The result $p_i^* = c_i$ suggests that suppliers tend to eliminate the consequence of this risk because they always break even, regardless of how much reserved capacity is used by the buyer. Wu and Kleindorfer (2005) also show that execution price equals execution cost, but the reservation price presents a different structure from ours. Specifically, the authors show that any equilibrium must be symmetric for all the selected suppliers, where they enjoy the same contract price (see Theorem 2 therein). Most importantly, Theorem 3 implies that the existence of equilibrium is guaranteed in our model, whereas there may be no equilibrium in their model.

In sharp contrast, Martínez-de-Albéniz and Simchi-Levi (2009) show that in a continuous version of such a problem without capacity constraints, suppliers offer bids that cluster together in groups of two to three, a phenomenon termed as "cluster competition." The implication of their result is that suppliers compete with those with similar costs, and hence at equilibrium the market will be segmented by groups of similar technologies.

The equilibrium stated in Theorem 3 is not unique. This occurs because the unchosen suppliers can set their reservation prices to any values no less than their reservation costs (see Lemma 4). Although there may be multiple equilibrium bidding strategies, all equilibria lead to the same profit allocation: each supplier makes a profit equal to his marginal contribution to the supply chain optimal profit, and the buyer takes the remaining profit. More specifically, supplier i makes a nonnegative profit, i.e., $\pi_i^* = \Pi_{\mathscr{C}}^*(N) - \Pi_{\mathscr{C}}^*(N\backslash\{i\}) \geqslant 0$, and the buyer's profit is equal to the supply chain optimal profit less the sum of the chosen suppliers' profits, which is given by

$$\begin{aligned}\Pi_{\mathscr{B}^*}^* &= \Pi_{\mathscr{C}}^*(N) - \sum_{i\in N(\mathscr{C})^*} \pi_i^* \\ &= \Pi_{\mathscr{C}}^*(N) - \sum_{i\in N(\mathscr{C})^*} (\Pi_{\mathscr{C}}^*(N) - \Pi_{\mathscr{C}}^*(N\backslash\{i\})).\end{aligned}$$

It is clear that the buyer's equilibrium profit must be nonnegative since otherwise she would simply reserve no blocks. This also emerges from the monotonicity of the function $\Pi_{\mathscr{B}}^*(\cdot)$. The following example illustrates the profit allocation stated in the theorem.

Example 2. Suppose the random demand is given by $\Pr[D=i]=1/4$ for $i=0,1,2,3$. The retail price is $\rho=5$. The spot price is given by $\Pr[P_0=1.5]=\Pr[P_0=3.5] = 1/2$. For simplicity we assume D and P_0 are independent. Three suppliers each have a unit-block with respective execution costs $c_1=1$, $c_2=2$, and $c_3=3$, and their reservation costs are $e_1=e_2=e_3=0$. Thus we can write $\mathscr{C}=\{(1,0),(2,0),(3,0)\}$. Using Equation (4), we can calculate the supply chain profit $\Pi_{\mathscr{C}}(S)$ for any subset $S\subseteq N=\{1,2,3\}$. Take $\Pi_{\mathscr{C}}(N)$, for example, where all the three unit-blocks are chosen. For each combination of realized demand t and spot price p_0, we then have the buyer's execution decision as shown in Table 1.

Table 1. With All Bids Accepted the Buyer's Execution Decision for Different Values of Demand and Spot Price

t	p_0	$\Pr[t,p_0]$	$x_1(t,p_0)$	$x_2(t,p_0)$	$x_3(t,p_0)$	$t-\sum_{i=1}^3 x_i(t,p_0)$
0	1.5	1/8	0	0	0	0
1	1.5	1/8	1	0	0	0
2	1.5	1/8	1	0	0	1
3	1.5	1/8	1	0	0	2
0	3.5	1/8	0	0	0	0
1	3.5	1/8	1	0	0	0
2	3.5	1/8	1	1	0	0
3	3.5	1/8	1	1	1	0

Note that the last column contains the purchase amounts from the spot market. Using Equation (4) we obtain

$$\begin{aligned}\Pi_{\mathscr{C}}(N) = \tfrac{6}{8}(5-1) - 0 + \tfrac{2}{8}(5-2) - 0 + \tfrac{1}{8}(5-3) \\ -0 + \frac{1+2}{8}(5-1.5) = \tfrac{85}{16}.\end{aligned}$$

Similarly, we can calculate other supply chain profits as follows:

$$\begin{gathered}\Pi_{\mathscr{C}}(N\backslash\{1\}) = \tfrac{71}{16}, \quad \Pi_{\mathscr{C}}(N\backslash\{2\}) = \tfrac{80}{16}, \\ \Pi_{\mathscr{C}}(N\backslash\{3\}) = \tfrac{84}{16}; \quad \Pi_{\mathscr{C}}(\{1\}) = \tfrac{78}{16}, \\ \Pi_{\mathscr{C}}(\{2\}) = \tfrac{69}{16}, \quad \Pi_{\mathscr{C}}(\{3\}) = \tfrac{63}{16}.\end{gathered}$$

Therefore, the supply chain optimal set is N and in equilibrium bids $\mathscr{B}^*=\{(c_i,r_i^*)\colon i=1,2,3\}$ we have $r_1^*=7/8$, $r_2^*=5/16$, and $r_3^*=1/16$, which lead to the following profit allocation among the buyer and the three suppliers:

$$\Pi_{\mathscr{B}^*}^* = \tfrac{65}{16}, \quad \pi_1^* = \tfrac{7}{8}, \quad \pi_2^* = \tfrac{5}{16}, \quad \pi_3^* = \tfrac{1}{16}. \quad \square$$

The profit allocation in equilibrium can be quite diverse, depending on the values of system parameters. Consider two polar cases. First, in the case where suppliers engage in head-to-head competition so that each supplier's marginal contribution to the system is null, then the buyer will take the entire supply chain optimal profit, a result analogous to the (symmetric) Bertrand competition equilibrium. In the other polar case where suppliers complement each other in achieving system maximum, the total profit may be split between suppliers only. Thus it is possible for suppliers, as the Stackelberg leaders, to extract most of the supply chain profit, leaving the buyer a very small profit.

Theorem 3 also shows that the supply chain is self-coordinated in the sense that the buyer's equilibrium choice of suppliers maximizes the system profit. This result resonates with that of Wu and Kleindorfer (2005) who show that the two-part options are efficient in coordinating the supply chain, but the driving forces are different. Wu and Kleindorfer show that "the ultimate driver of [their] efficiency result is competition in the presence of a backup open spot market" (p. 460), whereas in our model the efficiency result does not depend on the existence of a spot market. In fact, we can show that all the results remain intact when spot market is absent. Instead, the main driver in our model lies in the all-or-nothing nature of the buyer's decision. The efficiency result is in contrast to that of the equivalent noncombinatorial problem analyzed by Martínez-de-Albéniz and Simchi-Levi (2009) where equilibria may have a total loss up to 25% (and even more when the demand distribution is not log-concave). The efficiency loss comes from the fact that $p_\ell^* = c_\ell$ may not be optimal for supplier ℓ: since the buyer's reservation quantity is endogenously determined by the prices, it is not in each supplier's best interest to maximize the supply chain profit by setting $p_\ell^* = c_\ell$. As a consequence, the results in Lemmas 3 and 4 do not hold in their model either. The efficiency loss result is analogous to the double marginalization effect that occurs in the classical selling-to-newsvendor setting in the sense that suppliers have to use prices to influence the buyer's quantity decision (Lariviere and Porteus 2001).

Theorem 3 also applies in the case that the spot price is correlated with the demand. In this case the set of blocks chosen at equilibrium, $N(\mathscr{C})^*$, can vary according to the correlation. We illustrate this in the following example.

Example 3. Following Ritchken and Tapiero (1986), we suppose the random demand D and the spot price P_0 follow a bivariate lognormal distribution, which is parameterized by μ_d, μ_p, σ_d, σ_p, and r, where r is a proxy for the correlation between D and P_0. A positive (negative) value of r indicates a positive (negative) correlation. For this example, we use the following parameter values: $\mu_d = 2$, $\mu_p = 1$, $\sigma_d = 0.6$, $\sigma_p = 0.35$, and $\rho = 6$. Consistent with the fact that a higher demand often leads to a higher spot price, we focus on the case of positive correlation with r increasing from 0 to 0.9 with a step size 0.1 (we do not include $r = 1$ because the joint probability density function for this instance involves infinity). Four suppliers each have a unit-block with respective execution costs $c_1 = 0.5$, $c_2 = 1.3$, $c_3 = 1.8$, and $c_4 = 2.2$ and reservation costs $e_1 = 2$, $e_2 = 1.5$, $e_3 = 1$, and $e_4 = 0.5$. Thus we can write $\mathscr{C} = \{(0.5, 2), (1.3, 1.5), (1.8, 1), (2.2, 0.5)\}$.

Using Equation (4), we can numerically calculate the supply chain profit $\Pi_{\mathscr{C}}(S)$ for any subset $S \subseteq N = \{1, 2, 3, 4\}$. Table 2 shows the buyer's optimal choice, the profit split among players, and the value of option contracts in equilibrium.

The second column of Table 2 reveals that when the correlation is high enough (i.e., $r \geqslant 0.6$) the buyer's optimal choice switches from $\{1, 3, 4\}$ to $\{1, 2, 3, 4\}$. It is to be expected that high correlation makes option contracts more attractive since the high demand cases when having more options is helpful are likely to involve high spot prices, making it unattractive to meet demand through the spot market. Related to this observation, the last column shows that the value of option contracts increases with the correlation. Similar results have been found in Ritchken and Tapiero (1986). We also observe from Table 2 that both the supply chain optimal profit and the buyer's profit decrease in the correlation (see columns SC and Buyer). This arises from the existence of the term W in the buyer (or supply chain) profit, where $W = \mathbb{E}_{D, P_0}[(\rho - P_0)D]$). On the other hand we note that individual suppliers' profits may not be monotone in the correlation (see columns S1–S4). □

5.3. Equilibria with Blocks of Unequal Sizes

In this subsection, we show how to construct an equilibrium when suppliers have blocks of unequal sizes. Let $\mathscr{C} = \{(c_i, e_i, K_i): i \in N\}$, and $N(\mathscr{C})^* = \{j_1, \ldots, j_m\}$, which is an optimal buyer's choice when each supplier offers at cost.

Following the result of Theorem 2, we focus on the bidding strategies with execution price equal to execution cost. We adjust suppliers' reservation prices

Table 2. Effect of Correlation on Buyer's Optimal Choice, Profit Allocation, and Value of Options

r	Optimal choice	SC	S1	S2	S3	S4	Buyer	W	Option value
0	$\{1,3,4\}$	28.101	0.286	0.000	0.033	0.217	27.565	27.512	0.589
0.1	$\{1,3,4\}$	27.565	0.288	0.000	0.038	0.222	27.017	26.970	0.595
0.2	$\{1,3,4\}$	27.017	0.288	0.000	0.038	0.227	26.464	26.416	0.601
0.3	$\{1,3,4\}$	26.457	0.288	0.000	0.037	0.228	25.903	25.850	0.607
0.4	$\{1,3,4\}$	25.883	0.288	0.000	0.037	0.226	25.332	25.272	0.612
0.5	$\{1,3,4\}$	25.297	0.288	0.000	0.037	0.225	24.748	24.682	0.615
0.6	$\{1,2,3,4\}$	24.703	0.293	0.005	0.042	0.228	24.135	24.079	0.624
0.7	$\{1,2,3,4\}$	24.098	0.302	0.014	0.050	0.235	23.497	23.464	0.634
0.8	$\{1,2,3,4\}$	23.478	0.309	0.021	0.057	0.240	22.851	22.835	0.643
0.9	$\{1,2,3,4\}$	22.841	0.314	0.026	0.061	0.244	22.197	22.193	0.648

by following a recursive procedure. Define $\{\mathscr{B}^{(k)}: k = 0, \ldots, m\}$ recursively as follows:

$$\mathscr{B}^{(0)} = \mathscr{C}; \quad \mathscr{B}^{(k)} = \mathscr{B}^{(k-1)} \backslash \{(c_{j_k}, e_{j_k}, K_{j_k})\} \cup \{(c_{j_k}, r^*_{j_k}, K_{j_k})\}, \quad k = 1, \ldots, m,$$

where

$$r^*_{j_k} = (\Pi^*_{\mathscr{B}^{(k-1)}}(N) - \Pi^*_{\mathscr{B}^{(k-1)}}(N \backslash \{j_k\}))/K_{j_k} + e_{j_k}. \tag{9}$$

At the initial step ($k = 0$), we set price to be cost for all blocks. Thus, solving the buyer's problem is equivalent to solving the supply chain optimal problem. In the next, we adjust the reservation prices for the blocks in $N(\mathscr{C})^*$ one at a time. Specifically, at step $k > 0$, we keep increasing the reservation price of block j_k until it is dropped by the buyer. This leads to the maximum allowable increase $(\Pi^*_{\mathscr{B}^{(k-1)}}(N) - \Pi^*_{\mathscr{B}^{(k-1)}}(N \backslash \{j_k\}))/K_{j_k}$. Thus, Equation (9) gives the maximum reservation price $r^*_{j_k}$. It is easy to see that $r^*_{j_k} \geqslant e_{j_k}$, and hence no supplier will make a loss by using the above bidding strategies. At the end of the final step m, the procedure returns a set of bids $\mathscr{B}^{(m)} = \{(c_i, e_i, K_i): i \in N \backslash N(\mathscr{C})^*\} \cup \{(c_i, r^*_i, K_i): i \in N(\mathscr{C})^*\}$, and it can be shown to form an equilibrium in the following theorem.

Theorem 4 (Nash Equilibrium for Unequal-Size Blocks). *Even when blocks are not necessarily of an equal size, the set of bids $\mathscr{B}^{(m)}$, defined above, forms an equilibrium.*

Theorem 4 states that, in the above equilibrium, the suppliers who are not in the supply chain optimal set, offer prices equal to their costs, but the suppliers in the supply chain optimal set add a margin to their reservation costs. The equilibrium constructed in the procedure ensures that the buyer's optimal choice matches the supply chain optimal set so that even with different block sizes, the supply chain optimal result can still be achieved. It also implies that there is always a Nash equilibrium for the case with general-size blocks.

Note that in the construction we have assumed a particular order for the blocks $\{j_1, \ldots, j_m\}$ in $N(C)^*$. As the example below demonstrates, different orders may give different equilibria. Therefore, we can expect there to be multiple equilibria with different profit allocations, despite the fact that in this class of equilibria the buyer's optimal choice is the same.

Example 4. Demand is fixed at 10, the retail price is 10, and spot prices are assumed to be 10. There are four blocks with (c_i, e_i, K_i) triples as follows: $a = (0, 3, 3)$, $b = (0, 1.5, 7)$, $c = (0, 3, 2)$, and $d = (0, 3, 8)$. The supply chain optimal solution is $\{a, b\}$, which gives $\Pi(\{a, b\}) = 80.5$. We also have

$$\Pi^*(\{b, c, d\}) = \Pi(\{b, c\}) = 73.5,$$
$$\Pi^*(\{a, c, d\}) = \Pi(\{c, d\}) = 70,$$
$$\Pi^*(\{a, b, c, d\}) = \Pi(\{a, b\}) = 80.5,$$
$$\Pi^*(\{a, b, c\}) = \Pi^*(\{a, b, d\}) = \Pi(\{a, b\}) = 80.5.$$

One can see that the marginal contribution (to the optimal supply chain profit) of each of c and d is zero. The marginal contribution of a is $80.5 - 73.5 = 7$, whereas the marginal contribution of the supplier with bid b is $80.5 - 70 = 10.5$. Note that when a is absent, block b is still chosen, whereas when b is absent, a is not chosen. The marginal contribution of 10.5 (of supplier b) is actually the joint contribution of both a and b.

- If we start with block a, then the following bids form an equilibrium: $a = (0, 3 + 7/3)$, $b = (0, 1.5 + 3.5/7)$, $c = (0, 3)$, and $d = (0, 3)$. In this equilibrium, the buyer will choose blocks $\{a, b\}$. Supplier a's profit is 7, which equals his marginal contribution. Supplier b's profit is 3.5, which equals the joint contribution of suppliers a and b less the contribution of supplier a.
- If we start with block b, then the following bids form an equilibrium: $a = (0, 3)$, $b = (0, 1.5 + 10.5/7)$, $c = (0, 3)$, and $d = (0, 3)$. In this equilibrium, the buyer will choose blocks $\{a, b\}$. Supplier b makes a profit of 10.5. Profit of supplier a is 0, which is less than his marginal contribution.

We have constructed two equilibria by changing the orders of blocks. In both equilibria the buyer's optimal choice is $\{a, b\}$, but the profit splits are different. In contrast to the outcomes in Theorem 3, suppliers may make a profit that is less than their marginal contribution. □

The equilibria in Theorem 4 are reminiscent of the core of a cooperative game. It is interesting to establish the connection between our results and the core of the corresponding cooperative game (M, v), which is characterized by (i) the set of all players is $M = N \cup \{B\}$, where B represents the buyer and $N = \{1, \ldots, n\}$ is the set of n suppliers, and (ii) the value/characteristic function is given by, for any $S \subseteq M$, $v(S) = \Pi^*_{\mathscr{C}}(S \backslash \{B\})$ if $B \in S$, and $v(S) = 0$ if $B \notin S$. It is clear that, if the buyer is included in the set S, $v(S)$ is the optimal supply chain profit when blocks $S \backslash \{B\}$ are available. Reflecting the fact that only when suppliers are paired with the buyer can there be positive profits, the optimal supply chain profit will be zero when the buyer is not included in the set.

Cooperative game theory abstracts away from the detailed strategies of players and focuses on the formation and the division schemes of coalitions. Following the literature of cooperative game theory (Osborne and Rubinstein 1994), we concentrate on the case in which a grand coalition forms so that the players in M cooperate successfully. An *allocation* $\mathbf{a} = (a_1, \ldots, a_n, a_B)$ is a vector of values or profits assigned to all players, where a_i denotes supplier i's profit and a_B denotes the buyer's profit.

The *core* of the cooperative game (M, v) is the set of allocations for which no coalition is able to improve its aggregate profit by walking away from the allocation policy. An allocation a is in the core if it satisfies the following conditions: (i) $\sum_{i\in M} a_i = v(M)$ and (ii) $a(S) \geqslant v(S)$ for all $S \subseteq M$, where the total value assigned to a subset $S \subseteq M$ is $a(S) = \sum_{i\in S} a_i$. A condition equivalent to (ii) is (ii′) $a(S) \leqslant v(M) - v(M\backslash S)$ for all $S \subseteq M$, meaning that no subset of players can make a profit larger than their marginal contribution as a whole.

Having set up the cooperative game (M, v), we are now ready to establish the connection between the equilibria and the core of the cooperative game.

Proposition 1. *The profit allocations of the equilibria in Theorem 4 are in the core of the cooperative game (M, v).*

Though the profit splits for the constructed equilibria are in the core of the corresponding cooperative game, the core contains solutions that are not equilibria, as we show in the following example.

Example 5. Demand is fixed at 15, the retail price is $\rho = 10$, and spot prices are assumed to be 10. There are four blocks with (c, e, K) triples as follows: $i = (0, 4, 5)$, $j = k = (0, 3, 5)$, and $l = (0, 6, 8)$. With this setup, we can show that the supply chain optimal set is $\{i, j, k\}$, and it leads to the optimal supply chain profit of 100. Now consider a set of bids $i = (0, 9.8)$, $j = (0, 3.6)$, $k = (0, 3.8)$, and $l = (0, 6)$, where the entries in each pair represent the execution price and the reservation price, respectively. Given these bids, the buyer's optimal choice is still $\{i, j, k\}$, and the profit split among players is as follows: supplier i makes a profit of 29, supplier j makes a profit of 3, supplier k makes a profit of 4, supplier l makes a profit of 0, and the buyer's profit is 64. We can write the corresponding allocation as $\mathbf{a} = (29, 3, 4, 0, 64)$. It can be checked that this allocation is in the core of the corresponding cooperative game as previously defined. However, we can show that supplier j will have an incentive to deviate from the bid $(0, 3.6, 5)$. This implies that a set of bids that lead to an allocation in the core may not form an equilibrium. □

The discussion above shows the limit of the connections between the noncooperative game and cooperative game for our model setup. The solution concept of a Nash equilibrium involves a single player (a supplier) being unable to claim a greater allocation without the buyer's choice (which also equates to the winning coalition) being changed. But in this equilibrium concept an increase in the supplier's profit will automatically lead to a reduction in the buyer's profit (the buyer being a Stackelberg follower), and this is the key difference between our model and the cooperative game.

6. Extensions

The model so far has assumed that suppliers do not have access to the spot market, the buyer is restricted to reserving a whole block at a time, and each supplier has a single block. In this section, we examine the impact of each of these assumptions.

Extension 1: Supplier Participation in the Spot Market

In our baseline model, we consider the case where suppliers dedicate blocks of capacity that are bid into the option market with unused capacity being wasted. In some cases, however, if the buyer does not use all of the reserved capacity, suppliers may sell the unused capacity to the spot market, which therefore provides suppliers with a second source of revenue. We investigate whether the consideration of suppliers' participation in the spot market will change the main results derived for the baseline model.

As before, given bids $\mathcal{B} = \{(p_i, r_i, K_i)\colon i \in N\}$, suppose the buyer's reservation set is $S = \{j_1, \ldots, j_v\}$. For any demand t and spot price p_0, recall that the buyer's used amount from supplier j_i is given by $x_i(t, p_0)$. For simplicity, we assume the spot market has perfect liquidity so that suppliers have complete access to the spot market. Thus, supplier j_i will sell $K_{j_i} - x_i(t, p_0)$ to the spot market, and any supplier $m \notin S$ will sell K_m to the spot market.

Note that the buyer's problem remains the same as in the baseline model, so all the results for the buyer's optimal choice will carry over to this extended case. We now consider the suppliers' problem and show how the results of Theorems 2 and 3 carry over to this setting.

Proposition 2. *Suppose that suppliers may sell their unused capacities to the spot market; then* (a) *given bids $\{(p_i, r_i, K_i)\colon i \in L\}$ it is optimal for supplier ℓ to choose $p_\ell^* = c_\ell$;* (b) *when all blocks are of an equal size, the bids $\mathcal{B}^* = \{(c_i, r_i^*)\colon i \in N\}$ form a Nash equilibrium, where $r_i^* = e_i + \Pi_{\mathscr{C}}^*(N) - \Pi_{\mathscr{C}}^*(N\backslash\{i\})$ for $i \in N$. Moreover, at any equilibrium bids with $p_i^* = c_i$, $i \in N$, the buyer chooses the supply chain optimal set $N(\mathscr{C})^*$; supplier $i \in N_{\mathscr{C}}^*$ makes a profit $\pi_i^* = \Pi_{\mathscr{C}}^*(N) - \Pi_{\mathscr{C}}^*(N\backslash\{i\}) + \mathbb{E}_{D, P_0}[(P_0 - c_i)_+ \cdot (K_i - \min((D - Y_{w-1})_+, K_i))]$, where w is the position of block i within the set $N_{\mathscr{C}}^*$ and Y_{w-1} is the cumulative capacity of the first $w-1$ blocks of $N_{\mathscr{C}}^*$; and supplier $i \notin N_{\mathscr{C}}^*$ makes a profit $\pi_i^* = \mathbb{E}_{D, P_0}[(P_0 - c_i - e_i)_+ \cdot K_i]$.*

Part (a) of Proposition 2 shows that the result of Theorem 2 still holds when suppliers can participate in the spot market. One can see from the proof of the proposition in the appendix that although supplier ℓ's best response remains unchanged, his expected profit is increased by the additional profit obtained by selling the unused capacity to the spot market. Similarly, part (b) of Proposition 2 extends Theorem 3 to the setting where suppliers can participate in the spot market. The equilibrium bidding strategies remain unchanged, but each supplier's equilibrium profit is increased by the additional profit obtained by selling the unused capacity to the spot market.

The above results demonstrate the limited impact of suppliers' participation in the spot market on the main results derived in the paper. Since competition occurs in the option market only, suppliers' participation in the spot market simply increases each supplier's profit and does not change the competitive dynamics in the option market. Therefore, the equilibrium bidding strategies remain unchanged.

Extension 2: Partial Reservation

In this extension, the buyer is not restricted to reserving a whole block. As in the baseline model, each supplier owns a single block and the blocks can be of different sizes. Every supplier chooses an execution price and a reservation price for all elements of his block.

One of the key results obtained for our baseline model, Theorem 2, is that a best response for each supplier i is to set $p_i^* = c_i$ and the suppliers make profits only through reservation payments. For this extension, however, this result fails, as we demonstrate with the following example.

Suppose the random demand is given by $\Pr[D=i] = 1/5$ for $i = 1,\ldots,5$, the retail price is $\rho = 15$, and spot prices are 15. Supplier ℓ has a block of size $K_\ell = 3$ and both the execution and reservation costs are $c_\ell = e_\ell = 2$. The other bids $\{(p_i, r_i, K_i)\colon\ i = 1,2\}$ are $\mathscr{B}_0 = \{(1,3,1),(4,1,1)\}$. We investigate the best response of supplier ℓ in choosing (p_ℓ, r_ℓ). Denote $\mathscr{B}' = \mathscr{B}_0 \cup \{(2,2,3)\}$, in which supplier ℓ bids at cost.

The buyer's optimal solution includes all the three blocks with five units, which are used according to increasing order of execution prices in uncertainty realization. Therefore, recalling that the bids are in unit prices, we obtain the buyer's maximum expected profit as follows:

$$\Pi^*_{\mathscr{B}'}(N) = \left(\tfrac{5}{5}(15-1)-3\right) + \left(\sum_{k=2}^{4}\frac{k}{5}(15-2) - 2\times 3\right) + \left(\tfrac{1}{5}(15-4)-1\right) = 29.6.$$

Note that if supplier ℓ is unavailable, the buyer will choose both blocks in $\mathscr{B}_0$, making a profit of

$$\Pi^*_{\mathscr{B}_0}(N\backslash\{\ell\}) = \tfrac{5}{5}(15-1) - 3 + \tfrac{4}{5}(15-4) - 1 = 18.8.$$

Given that supplier ℓ bids at cost, we would expect that the maximum profit supplier ℓ could achieve is $29.6 - 18.8 = 10.8$ (and this is the case for the baseline model). However, in the extended model, the optimal profit of supplier ℓ is less than 10.8.

We show that it is suboptimal for supplier ℓ to choose $p_\ell^* = 2$. First we observe that to make nonnegative expected profit in this case, supplier ℓ has to set his reservation price $r_\ell \geqslant 2$. Since from supplier ℓ the buyer can reserve 0, 1, 2, or 3 units to maximize her expected profit, her objective is to maximize the following function:

$$f(r_\ell) = \max\{f_0(r_\ell), f_1(r_\ell), f_2(r_\ell), f_3(r_\ell)\}, \qquad r_\ell \geqslant 2,$$

where $f_i(r_\ell)$ $(i = 0,\ldots,3)$ are respective expected profits when reserving i units from supplier ℓ in addition to the two units from the other two suppliers, which can be easily calculated as follows:

$$f_0(r_\ell) = 18.8, \quad f_1(r_\ell) = 27 - r_\ell, \quad f_2(r_\ell) = 32.6 - 2r_\ell, \quad f_3(r_\ell) = 35.6 - 3r_\ell.$$

Therefore, we have

$$f(r_\ell) = \begin{cases} f_3(r_\ell), & 2 \leqslant r_\ell \leqslant 3; \\ f_2(r_\ell), & 3 < r_\ell \leqslant 5.6; \\ f_1(r_\ell), & 5.6 < r_\ell \leqslant 8.2; \\ f_0(r_\ell), & r_\ell > 8.2. \end{cases}$$

Accordingly, the expected profit $\pi_\ell(r_\ell)$ of supplier ℓ is as follows:

$$\pi_\ell(r_\ell) = \begin{cases} 3(r_\ell - 2), & 2 \leqslant r_\ell \leqslant 3; \\ 2(r_\ell - 2), & 3 < r_\ell \leqslant 5.6; \\ r_\ell - 2, & 5.6 < r_\ell \leqslant 8.2; \\ 0, & r_\ell > 8.2. \end{cases}$$

Therefore, at $r_\ell^* = 5.6$, the expected profit $\pi_\ell(r_\ell)$ is maximized to $\pi_\ell^* = 7.2$.

On the other hand, if the bids of supplier ℓ are set at $(p'_\ell, r'_\ell) = (15, 0)$, then the buyer will reserve all five units from the three suppliers with expected profit of 18.8, which gives supplier ℓ an expected profit

$$\pi'_\ell = \left(\tfrac{3}{5} + \tfrac{2}{5} + \tfrac{1}{5}\right)(p'_\ell - c_\ell) + 3(r'_\ell - e_\ell) = 9.6.$$

Consequently, we obtain $\pi_\ell^* < \pi'_\ell$.

In this extension, the buyer can determine the reservation quantity, which depends on supplier bids (see a similar result in Proposition 1 of Martínez-de-Albéniz and Simchi-Levi 2009). The suppliers' profits depend on both the buyer's quantity and the prices, and it turns out that charging execution costs may be suboptimal for suppliers. In fact, this extended model reduces to the model of Martínez-de-Albéniz and Simchi-Levi when the spot market is absent and each supplier has a very large capacity.

Extension 3: Multiple Blocks with Common Owner

In this section, each supplier owns multiple unit-blocks and can choose different prices for different unit-blocks. As before the buyer can freely choose among the offered blocks. We will show that the key result $p_i^* = c_i$ does not hold in this extension either.

We will continue to use the example as given in Extension 2. For this extension, we can think of a single supplier ℓ owning three unit-blocks with costs that are identical across these unit-blocks. In contrast with the partial reservation case, supplier ℓ can choose different prices for the three unit-blocks. We denote by $\{(p_{\ell 1}, r_{\ell 1}), (p_{\ell 2}, r_{\ell 2}), (p_{\ell 3}, r_{\ell 3})\}$ the bidding strategy of supplier ℓ.

On one hand, if we impose the constraints of $p^*_{\ell 1} = p^*_{\ell 2} = p^*_{\ell 3} = 2$, by assuming without loss of generality that $r^*_{\ell 1} \leqslant r^*_{\ell 2} \leqslant r^*_{\ell 3}$, we find that as in the case of Extension 2, it is optimal for supplier ℓ to set $r^*_{\ell 1} = r^*_{\ell 2} = r^*_{\ell 3} = 5.6$, and the buyer will reserve two unit-blocks of supplier ℓ in addition to the two blocks of the other two suppliers. Thus supplier ℓ's expected profit is $\pi^*_\ell = 7.2$ as in Extension 2.

On the other hand, as we have shown in the example for Extension 2, if supplier ℓ sets $(p_{\ell j}, r_{\ell j}) = (15, 0)$ for all $j = 1, 2, 3$, then he will have an expected profit of $\pi_\ell = 9.6$. Actually, he can even do better by setting $p'_{\ell 1} = 2$, $r'_{\ell 1} = 8.2$, $p'_{\ell 2} = p'_{\ell 3} = 15$, and $r'_{\ell 2} = r'_{\ell 3} = 0$. The buyer will then still reserve all five blocks of the three suppliers, which gives supplier ℓ an expected profit

$$\begin{aligned}\pi'_\ell &= \tfrac{4}{5}(p'_{\ell 1} - c_\ell) + r'_{\ell 1} - e_\ell + \tfrac{2}{5}(p'_{\ell 2} - c_\ell) + r'_{\ell 2} - e_\ell \\ &\quad + \tfrac{1}{5}(p'_{\ell 3} - c_\ell) + r'_{\ell 3} - e_\ell \\ &= 8.2 - 2 + \frac{2+1}{5}(15 - 2) + 2(0 - 2) = 10.\end{aligned}$$

Since $\pi^*_\ell < \pi'_\ell$, it is suboptimal to force $p^*_{\ell 1} = p^*_{\ell 2} = p^*_{\ell 3} = 2$.

The reason why the result $p^*_i = c_i$ does not hold in this extension is because, in addition to the external competition between suppliers, there is also internal competition between blocks within each supplier. As shown in Martínez-de-Albéniz and Simchi-Levi (2009), firms compete with others with similar technologies, leading to the cluster competition in groups of two or three at equilibrium. In a similar vein, suppliers in this extended model tend to diversify their blocks to mitigate the internal competition between their own blocks. Consequently, setting $p^*_i = c_i$ may not be optimal for supplier i.

7. Conclusions

We have considered a situation in which suppliers compete by offering option contracts to a buyer who faces demand uncertainty and spot price volatility. In this model each supplier has a block of capacity and sets two prices: a reservation price and an execution price. The buyer needs to choose which blocks of capacity to reserve in advance of knowing the customer demand and spot price. Thus the buyer's problem becomes combinatorial: choosing the right subset of suppliers to select, which is the key difference from the existing literature.

We have analyzed the buyer's problem and shown how it can be efficiently solved. The buyer's capacity reservation problem is interesting in its own right, though our primary interest is in the suppliers' decisions given that the buyer behaves optimally. In the case where suppliers have equal-size blocks, we are able to develop an efficient algorithm to solve for the optimal buyer choice in polynomial time. Regarding the suppliers' bidding behavior, we find that if the competing suppliers know the other bids that the buyer may choose from, then it is optimal for each supplier to bid their real execution costs and make profit only from the reservation margin. However, the result does not hold when the buyer is able to reserve just a portion of a block or if a single supplier owns more than one block that can be offered at different prices.

Our model combines features from two models in the literature. It has limited capacity and a spot market as occur in Wu and Kleindorfer (2005), at the same time it includes the uncertain demand that characterizes the model of Martínez-de-Albéniz and Simchi-Levi (2009). In addition we have a block structure, and it is this characteristic that enables us to recover Wu and Kleindorfer's result that execution prices are set at costs. We have also shown that the equilibrium results remain intact when the spot market is present. Without the spot market, similar to Martínez-de-Albéniz and Simchi-Levi (2005, 2009), it is demand uncertainty and supplier heterogeneity (in terms of cost structure) that lead to a portfolio effect of the buyer's choice. We show that when the buyer is restricted to reserving complete blocks only, the system efficiency can be achieved, a result in contrast with that of Martínez-de-Albéniz and Simchi-Levi (2009).

By using a submodularity result on the buyer's optimal profit as a function of the available set of supplier bids, we are able to analyze the equilibrium behavior for the suppliers. When the blocks are of equal size, the equilibrium is essentially unique: the buyer choice at equilibrium matches that which achieves the maximum overall profit in the supply chain, and each supplier makes a profit equal to its contribution (i.e., the difference between the overall supply chain profit when that supplier is present and when he is absent). But this equilibrium result depends on the fact that each supplier having the same capacity. In the case that suppliers have blocks of different sizes, the equilibrium is no longer unique, and we characterize a set of equilibria that imply different profit values for the suppliers, depending on which equilibrium occurs. For this set of equilibria, again, the supply chain achieves its maximum overall profit.

Our model starts with the assumption that suppliers do not participate in the spot market, which fits some settings better than others. This assumption is suitable when it is difficult for suppliers to sell their excess capacity to the spot market at the last minute. In the UK's STOR market, for example, service providers must be able to deliver full MW within 240 minutes or less from receiving instructions from National Grid (National Grid 2017). In other settings, however, production or delivery lead times may be short, and suppliers may be able to sell the unused capacity to the spot market. For this setting, we have relaxed the assumption of nonparticipation of suppliers in the spot market and shown that the main results remain qualitatively unchanged.

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