The Price of Capital and the Financial Accelerator

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Abstract

The price of capital is a key determinant of the *financial accelerator*, a transmission mechanism of shocks generated through the capital accumulation process of entrepreneurs that borrow in credit markets with frictions. This paper shows that the procedure of approximating the price of old capital by the net-of-depreciation price of new capital, as used in many articles since Bernanke et al. (1999), has profound implications when the capital depreciation rate is positive. When accounting for the appropriate price of capital, the effects of the *financial accelerator* are even stronger than originally assessed.

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1 Introduction

Frictions in financial and credit markets can create a powerful propagation and amplification channel for the transmission of various shocks to the real economy. In their seminal contribution Bernanke et al. (1999) (henceforth, BGG) design a general equilibrium model in which asymmetric information between borrowers and lenders arises from a costly state verification problem, as first studied in Townsend (1979). This credit/financial friction results in the so-called *financial accelerator*. Since in this setup entrepreneurs borrow in credit markets to finance their investment in capital, the strength of the *financial accelerator* as amplification mechanism crucially depends on the dynamics of the price of capital.

In this paper, we analytically show that the procedure of approximating the price of previously-installed capital by the net-of-depreciation price of new capital, as used in BGG, has important first-order effects on the solution of a model that assumes a positive depreciation rate of capital together with investment adjustment costs.[1]

In a nutshell, BGG set up the profit function of capital producers under the implicit assumption that the capital depreciation rate is zero. As we will demonstrate, only in that specific case approximating the price of previously-installed capital by the net-of-depreciation price of new capital, as suggested in BGG, is an innocuous assumption because it does not generate any first-order effects. However, since annual capital depreciation rate is about 10 percent, it is important to understand the profound (first-order) consequences of that approximation. After analytically proving this point, we investigate how the strength of the financial accelerator channel is quantitatively affected by the simplification. We conclude that when the appropriate, and non-approximated, price of capital is considered the financial accelerator effect is even stronger than originally assessed.

The paper is organized as follows. In Section 2 we review the baseline BGG's model. In Section 3 we demonstrate that approximating the price of previously-installed capital by the net-of-depreciation price of new capital has first-order effects when the capital depreciation rate is greater than zero. In Section 4 we present the quantitative implications of accounting for the equilibrium price of capital. In Section 5 we conclude with final remarks.

2 Review of the Financial Accelerator Framework

In order to contextualize the role played by the price of capital in determining the strength of the financial accelerator, in this section we briefly summarize Bernanke et al.

[1] There is a large number of papers that use the same approximation to address various questions. A non-exhaustive list includes: Bernanke and Gertler (1999), Hall and Vila (2002), Walentin (2005), Meier and Muller (2006), Gertler et al. (2007), Christensen and Dib (2008), Dmitriev and Hoddenbagh (2015) and Carlstrom et al. (2016), among others.

(1999)'s framework.

There are five types of agents in the economy: households, a fiscal and monetary authority, entrepreneurs, retailers, and capital-producing firms. Since the economic problems of households, fiscal and monetary authorities, entrepreneurs, and retailers do not affect at all our results, we omit their formal description. Instead, in the next section we formalize the economic problem of capital-producing firms, since it is at the core of our results.

Households Households are risk averse and infinitely lived. They get utility from consumption, leisure, and money holding. They work, consume, pay taxes, hold money, and invest their savings, in form of deposits, in a financial intermediary that pays the riskless rate of return. These deposits are transferred to entrepreneurs in the form of loanable funds. Households' problem is standard: they maximize the lifetime expected utility by choosing consumption, hours worked, money holding, and savings, subject to the budget constraint and taking all prices as given.

Fiscal and Monetary Authority The government is subject to a budget constraint that states that government expenditures are financed by lump-sum taxes and money creation. The government adjusts the mix of financing between money creation and taxes to support a Taylor-type nominal interest rate rule.

The Entrepreneurial Sector Entrepreneurs are risk neutral, own the production technology, and have a constant probability of surviving to the next period. They acquire physical capital from capital-producing firms, which will be specified in the next section. Capital and hired labor are combined to produce output through a constant return to scale technology. Entrepreneurs finance capital through their net worth and borrowing. The lender-borrower relationship is characterized by a financial friction that originates from asymmetric information about the realized return of capital.

At the aggregate level, BGG show that the demand for capital is described by the expected gross return on holding a unit of capital from period t to $t+1$, which consists of the marginal product of capital (MPK) and capital gain, i.e.:

$$\mathbb{E}_t R^k_{t+1} = \mathbb{E}_t \left\{ \frac{MPK_{t+1} + \tilde{Q}_{t+1}}{Q_t} \right\}, \tag{1}$$

where $\mathbb{E}_t$ denotes the expectation operator conditional on the information available at time t, and Q_t and $\tilde{Q}_{t+1}$ represent the price of newly-installed and previously-installed capital respectively. The determination of these prices will be discussed in the next section. In equilibrium, MPK is equal to the rental rate of capital:

$$MPK_{t+1} = r_{t+1} = A_{t+1}\alpha K_{t+1}^{\alpha} L_{t+1}^{1-\alpha},$$

where A_t is the aggregate technology shock, K_t is the capital stock, L_t is labor input, and α is the capital share in output.[2]

The aggregate supply curve for investment finance is given by:

$$\mathbb{E}_t R_{t+1}^k = s\left(\frac{N_{t+1}}{Q_t K_{t+1}}\right) R_{t+1}, \tag{2}$$

where N_{t+1} is entrepreneurial net worth, R_{t+1} is the risk-free interest rate in the economy, and the function $s(\cdot)$ is the ratio of the costs of external and internal finance. Through this function, the investment supply curve in equation (2) incorporates the financial frictions that characterize the economy, and it endogenously creates an external-finance premium that follows from the conflict of interests between borrowers and lenders. Notice that the *financial accelerator* resulting from equation (2) crucially depends on the evolution of the prices of capital Q_t and $\tilde{Q}_t$.

Retail Sector and Price Setting In order to account for nominal rigidities, BGG assume the existence of a monopolistically competitive retail sector subject to a price-setting decision a là Calvo (1983).

The crucial ingredient of the model for our results is the determination of the prices of capital, Q_t and $\tilde{Q}_t$. Since, as suggested by the equilibrium conditions (1) and (2), these two prices are one of the key determinants of the strength of the *financial accelerator*, we now formally describe how the two prices are determined.

3 Capital-Producing Firm and the Price of Capital

In order to understand how the prices of capital are determined, it is useful to carefully describe the decentralized equilibrium of the economy, in the same spirit as Christiano and Fisher (1995), Christiano and Fisher (2003), Christiano and Davis (2006), and as assumed in BGG.[3] There are identical and perfectly competitive capital-producing firms that purchase investment goods, I_t, and old capital, K_t, to produce new capital, K_{t+1}, which will be sold

[2]We have implicitly assumed a Cobb-Douglas production function, $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$, as in BGG.
[3]See footnote 12, p. 1356.

to entrepreneurs, using the following homogenous technology:

$$K_{t+1} = \Phi\left(\frac{I_t}{K_t}\right) K_t + (1-\delta) K_t, \tag{3}$$

where $\Phi(\cdot)$ is an increasing and concave adjustment cost function that depends on the ratio of investment and capital, and δ is the depreciation rate of capital.

After the entrepreneurs have used the acquired capital for production purposes, they sell the used capital back to the capital-producing firms. Hence, there are two relevant prices in this setting: the price of newly-produced capital, Q_t, and the price of previously-installed capital, $\tilde{Q}_t$. The representative capital-producing firm buys used capital at the cost of $\tilde{Q}_t K_t$, invests a total amount I_t, and sells new capital for a revenue of $Q_t K_{t+1}$.The problem of the representative capital-producing firm is then to maximize profits given by:

$$\max_{K_t, I_t} Q_t K_{t+1} - I_t - \tilde{Q}_t K_t \tag{4}$$
$$\text{s.t.} \quad K_{t+1} = \Phi\left(\tfrac{I_t}{K_t}\right) K_t + (1-\delta) K_t.$$

The optimality conditions, which pin down the two equilibrium prices, are:

$$Q_t = \left(\Phi'\left(\frac{I_t}{K_t}\right)\right)^{-1}, \tag{5}$$

$$\tilde{Q}_t = \left[(1-\delta) + \Phi\left(\frac{I_t}{K_t}\right) - \Phi'\left(\frac{I_t}{K_t}\right)\frac{I_t}{K_t}\right] Q_t. \tag{6}$$

Notice that, while equation (5) is identical to the one derived in BGG, the price of previously-installed capital in equation (6) differs from theirs. Let us use $\bar{Q}_t$ to denote BGG's price of previously-installed capital. They derive the following zero-profit condition:[4]

$$Q_t \Phi\left(\frac{I_t}{K_t}\right) K_t + Q_t K_t - I_t - \bar{Q}_t K_t = 0, \tag{7}$$

which gives:

$$\bar{Q}_t = Q_t \Phi\left(\frac{I_t}{K_t}\right) - \frac{I_t}{K_t} + Q_t.$$

Four remarks are necessary at this point. First, notice that BGG's profits in (7) are computed implicitly assuming that the capital depreciation rate, δ, is equal to zero. In fact, if and only if $\delta = 0$ the profit function in equation (4) coincides with the one in equation (7). Second, notice that the steady state value of $\tilde{Q}_t$ is $\tilde{Q} = 1 - \delta$, whereas the steady state value of $\bar{Q}_t$ is $\bar{Q} = 1$.[5] Third, notice that if we solve for the price of previously-installed

[4] See footnote 13, p. 1357.

[5] Recall that the following relationships hold in the steady state of this model: $\Phi\left(\frac{I}{K}\right) = \delta$, $\Phi'\left(\frac{I}{K}\right) = 1$, and $\frac{I}{K} = \delta$.

capital by imposing the zero-profit condition with non-zero depreciation rate, using (4), we obtain $\tilde{Q}_t$, since by (5), $Q_t\Phi'\left(\frac{I_t}{K_t}\right) = 1$. This should not come as a surprise, since a homogenous technology as in (3) and a competitive capital-producing market imply that the first order conditions in (5) and (6) lead to profits being zero in equilibrium. Forth, as a consequence of all the previous points, whereas $\tilde{Q}_t$ is always the equilibrium price level, $\bar{Q}_t$ is the equilibrium price only when $\delta = 0$.

BGG argue that difference between $\bar{Q}_t$ and Q_t is of a second order, and therefore replace the quantity $(1-\delta)Q_{t+1}$ with $\tilde{Q}_{t+1}$ in the expression for the return of capital (1).[6] However, we argue that the proposed approximation is appropriate only in the special case when $\delta = 0$, which is also the only case that makes $\bar{Q}_t$ the correct equilibrium price.

To see this, linearize the general expression for $\tilde{Q}_t$, which is computed without assuming zero capital depreciation. By totally differentiating (6) around the steady state, and using (5), as well as the fact that in the steady-state $\Phi\left(\frac{I}{K}\right) = \delta$, $\Phi'\left(\frac{I}{K}\right) = 1$, and $\frac{I}{K} = \delta$, we have:

$$d\tilde{Q}_t = (1-\delta)dQ_t + \delta dQ_t.$$

As suggested by this expression, the approximation error $d\tilde{Q}_{t+1} - (1-\delta)dQ_{t+1}$ equals δdQ_{t+1}. If $\delta = 0$, then $\tilde{Q}_{t+1} = \bar{Q}_{t+1}$ and the approximation $\bar{Q}_{t+1} \approx (1-\delta)Q_{t+1}$ is indeed valid up to first order. However, more generally, when $\delta > 0$, then $\bar{Q}_{t+1}$ is not an equilibrium price, as it is computed by equating an incorrect profit equation to zero. In that case, then, using the approximation $(1-\delta)Q_{t+1}$ leads to a first-order error departure from the correct equilibrium price $\tilde{Q}_{t+1}$.

The consequences of replacing $\tilde{Q}_{t+1}$ with $(1-\delta)Q_{t+1}$ when the capital depreciation rate is not equal to zero are as follows. Computing the profit of the capital-producing firm in equation (4) and using the approximation $\tilde{Q}_{t+1} = (1-\delta)Q_{t+1}$, results in:

$$\pi_t^k = Q_t\Phi\left(\frac{I_t}{K_t}\right)K_t - I_t.$$

Notice that the extra-profits resulting from the approximation, i.e. $Q_tK_t\Phi\left(\frac{I_t}{K_t}\right) - I_t$ are not zero even at first order. In fact, totally differentiating that expression around the steady state, we obtain that, up to first order, the extra profits are equal to $\delta K dQ_t$. This should not come as a surprise, since the approximation $\tilde{Q}_t = \bar{Q}_t = (1-\delta)Q_t$ is valid only when $\delta = 0$.

[6]It is important to understand that the factor $(1-\delta)$ is included to take into account the fact that $\tilde{Q}_t$ has a steady state level of $(1-\delta)$, whereas Q_t and $\bar{Q}_t$ have both a steady state level of 1.

The Price of Capital in the Equilibrium System In line with the findings in the previous section, the solution of the model can be found by augmenting the system of linearized equations with the linearized version of the equilibrium condition for the return to capital in equation (1) as a function of $\tilde{Q}_t$, and of the equilibrium price of capital $\tilde{Q}_t$ in equation (6). The linearized versions of the two altered equilibrium conditions are:

$$\mathbb{E}_t r^k_{t+1} = (1-\epsilon)(y_{t+1} - k_{t+1} - x_{t+1}) + \epsilon \tilde{q}_{t+1} - q_t, \tag{8}$$

$$\tilde{q}_t = -\frac{\delta\varphi}{(1-\delta)} i_t - \frac{\delta\varphi}{(1-\delta)} k_t + q_t, \tag{9}$$

where X_t is the gross markup of retail goods over wholesale goods, $\epsilon = \frac{1-\delta}{(1-\delta)+\alpha Y/(XK)}$, $\varphi = \frac{(\Phi(I/K)^{-1})'}{(\Phi(I/K)^{-1})''}$, and Y, X, K, I denote respective steady state values.[7]

4 Quantitative Effects

We now investigate the quantitative implications of explicitly taking into account the correct price of old capital compared to approximating it by the net-of-depreciation price of new capital. We calibrate the model as in BGG, who fix the depreciation rate of capital to $\delta = 0.025$. As expected, we observe important quantitative differences between the solution of the model computed using BGG's approximated price of old capital and the model that uses the equilibrium price of old capital $\tilde{q}_t$.

The financial accelerator is able to propagate and amplify both real and nominal shocks to the economy. We first analyze the impulse response functions to a one standard-deviation technology shock (Figure 1a) and monetary policy shock (Figure 1b).[8] The dashed lines represent the impulse response functions when BGG's approximation for the price of capital is used, whereas the solid lines represent the impulse response functions when the equilibrium price of capital, $\tilde{q}_t$, is used instead. As displayed in the figures, the impulse response functions implied by the two approaches are quite different. This evidence supports our analytical results; in fact, if using $\tilde{q}_t$ did not have any first-order effect, the impulse response functions across the two approaches would be exactly identical, since the model is solved up to a first-order approximation.

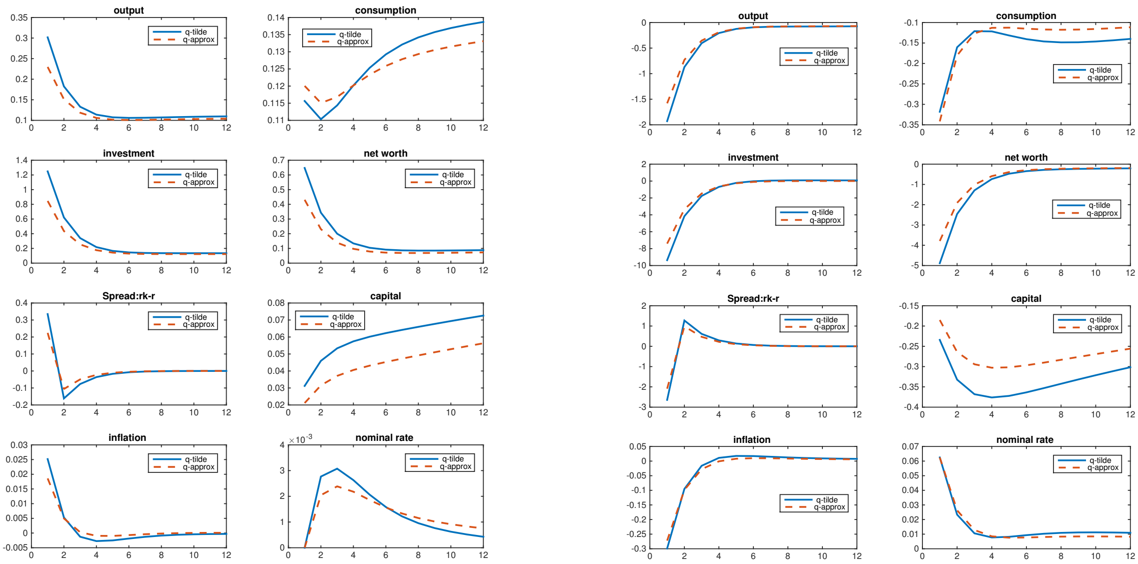
As Figure 1a displays, accounting for the equilibrium price of old capital results in much larger responses of investment, output, net worth, and rental rate of capital to a technology

[7] It is straightforward to incorporate the equilibrium price of previously-installed capital in (9) and the associated return to capital in (8) in the set of linearized equilibrium condition described in BGG (p.1361). Codes for solving the model with the equilibrium price of capital, $\tilde{q}_t$, are available at the website: http://www.robertopancrazi.com/BGG99_qtilde.txt and are based on the codes in Cesa-Bianchi and Fernandez-Corugedo (2014).

[8] The replication files for these figures are available at the website: http://www.robertopancrazi.com/research.html.

shock than in BGG. It appears that the financial accelerator is even stronger than reported in the original paper. A similar result applies to a monetary policy shock, as displayed in Figure 1b; a shock that increases the nominal interest rate has larger real effects when the equilibrium price of capital is considered rather than its approximation as in BGG. The differences in the responses of consumption, capital, and net worth are particularly noticeable.[9]

Figure 1 – Impulse response function: Benchmark model



(a) Technology shock

(b) Monetary shock

Note: These plots display the IRFs to a one-standard deviation technology shock (left panel) and to a one-standard deviation monetary policy shock (right panel). The solid blue lines correspond to the model solved including the equilibrium price of capital $\tilde{q}_t$, whereas the dashed blue lines correspond to the model solved including the approximated price of capital as in Bernanke et al. (1999).

5 Conclusions

This paper shows that approximating the price of previously-installed capital by the net-of-depreciation price of new capital, as suggested in Bernanke et al. (1999), has distorting first-order effects on the equilibrium of an economy with positive capital depreciation rate. After proving this point, we show the set of equilibrium conditions that take into account the appropriate equilibrium price of used capital. We then quantify the effects of accounting for the equilibrium price of capital and we conclude that the financial accelerator mechanism is *even stronger* than originally assessed. We believe, therefore, that this paper transmits a positive message and that it will be beneficial to researchers who use this approach to study financial frictions.

[9]Similar results are obtained when we apply the same strategy to model with delayed investment extension described in BGG in section 4.2.1.

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