Ambiguity, Reasoned Determination, and Climate-Change Policy

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Abstract

This paper examines climate-change benefit-cost analysis in the presence of scientific uncertainty in the form of ambiguity. The specific issue addressed is the robustness of benefit-cost analyses of climate-change policy alternatives to relaxation of Savage's original axioms. Two alternatives to subjective expected utility (SEU) are considered: maximin expected utility (MEU) and incomplete expected utility (IEU). Among other results, it is demonstrated that polar opposite recommendations can emerge in an ambiguous decision setting even if all agree on Society's rate of time preference, Society's risk attitudes, the degree of ambiguity faced, and the scientific primitives. We show that, for a simple numerical simulation of our model, an MEU decision maker favors policies which immediately tackle climate change while an IEU decision prefers "business as usual".

"Each agency shall assess the costs and benefits of the intended regulation, and recognizing that some costs and benefits are difficult to quantify, propose or adopt a regulation only upon a reasoned determination that the benefits of the intended regulation justify its cost." *Executive Order 12866 of the US President*

1 Introduction

This paper studies the meaning of "*a reasoned determination that the benefits of the intended regulation justify its cost*" in an uncertain (ambiguous) setting. The specific focus is on climate change. The highly imprecise nature of existing climate-science knowledge, the potential for fundamental but unknown irreversibilities in physical systems, the long-time lags involved, and the unpredictability of technological adaptation all ensure that probabilistic assessments for climate change are inherently subjective. Not only are probabilistic assessments subjective, they are widely disparate.

In its Fourth Assessment Report (AR4), the Intergovernmental Panel on Climate Change (IPCC) reported no fewer than 18 climate-sensitivity probability distributions while noting "no well-established formal way of estimating a single PDF" exists (IPCC 2007, Box 10.2, Figures 1 and 2). Six years on, the IPCC Fifth Assessment Report reported that "there does not exist at present a single agreed on and robust formal methodology to deliver uncertainty quantification estimates of future changes in all climate variables" (IPCC AR5, 2013, p. 1040). This ambiguity is especially pronounced for large global-temperature increases. Current knowledge is not data-based and relies instead on extrapolations from models for which many key components of climate-change processes are poorly understood. Many factors, including data limitations and poor understanding of geophysical responses, contribute to this ambiguity.[1]

Economists have responded to this widespread ambiguity by conducting benefit-cost analyses of climate-policy alternatives in a subjective-expected-utility (SEU) framework. The behavioral axioms underlying the SEU model have been widely criticized. And these criticisms have spawned an array of alternatives, many of which were developed expressly to accommodate *known* shortcomings of SEU theory in an ambiguous setting. A fundamental observation motivating this criticism is that oftentimes, when an objective probability distribution is not available, observed decision behavior contradicts both objective and subjective EU theory and, more generally, probabilistic sophistication (Machina and Schmeidler, 1992). The widespread ignorance and scientific uncertainty surrounding climate change ensure that current policy makers face an ambiguous decision situation not unlike betting on Ellsberg urns. The stakes, however, are immeasurably higher than in Ellsberg's (1961) thought experiments.

[1] IPCC (2007) Chapters 8 through 10 contain a particularly informative and detailed discussions concerning the causes and the presence of scientific uncertainty.

One alternative to accommodate behavioral sensitivity to ambiguity is to relax or alter Savage's sure-thing principle. The most popular models taking this approach include maximin expected utility (Gilboa and Schmeidler 1989), Choquet expected utility (Schmeidler 1989), and the smooth ambiguity model (Klibanoff, Marinacci, and Mukerji 2005). Another alternative is to relax Savage's completeness axiom (Aumann 1962, Bewley 1986). Von-Neumann and Morgenstern (1947), Aumann (1962), Bewley (1986), Schmeidler (1989), and more recently Galaabaatar and Karni (2013), all have questioned both its realism and its *normative* content. Aumann (1962, p. 446) wrote that "[o]f all the axioms of utility theory, the completeness axiom is perhaps the most questionable. Like others of the axioms, it is inaccurate as a description of real life; but unlike them, we find it hard to accept even from a normative viewpoint." Much earlier, von-Neumann and Morgenstern (1947) had recognized that "... it may even in a way be more realistic...to allow for cases where the individual is neither able to state which of two alternatives he prefers nor that they are equally desirable". Even Savage (1954) expressed ambivalence about the completeness axiom: "There is some temptation to explore the possibilities of analyzing preference among acts as a **partial ordering,...,** admitting that some pairs of acts are incomparable. This would seem to give expression to introspective sensations of indecision or vacillation, which we may be reluctant to identify with indifference." (emphasis in original)

This paper asks: What are the practical consequences for making a reasoned determination about the benefits and costs of alternative climate policies of considering alternatives to Savage's (1954) normative framework? The maximin expected utility (MEU) model (Gilboa and Schmeidler 1989) and the incomplete expected utility (IEU) model (Bewley 1986) are considered as exemplars. Both are early alternatives to SEU. Both offer the SEU model as an "in-between" case. And both represent complementary approaches to making decisions. Gilboa, Maccheroni, Marinacci, and Schmeidler (2010) have shown that MEU is a *subjectively rational* framework for making decisions and that IEU is an *objectively rational* framework. *Subjectively rational* requires making choices that others cannot convince a decision maker are wrong. *Objectively rational* requires making choices that a decision maker can convince others are correct.

The practical consequence of considering alternative normative frameworks is that polar opposite recommendations emerge from these alternatives even if all agree on Society's rate of time preference, Society's risk attitudes, the degree of ambiguity faced, and on the "scientific" primitives. IEU decision makers are more conservative than SEU or MEU decision makers in adopting policies to ameliorate climate change, while MEU decision makers are more progressive in adopting such policies. Consequently, an empirical chasm typically exists between the subjectively rational MEU alternative and its objectively rational IEU alternative. And so, where existing SEU analyses have narrowly focused debate on the two parameters of the constant relative risk-averse utility structure, the differences that emerge by different parametric choices are dwarfed by differences that emerge from using different normative alternatives. Simply put, the results from SEU benefit-cost analyses of climate-change policies lack normative robustness.

Making this point requires a formal framework. The model developed is intended to be as simple as possible while still preserving the uncertain, dynamic, and general-equilibrium nature of the decision environment. Our study is not the first to examine climate-change alternatives using non-expected utility preferences. A growing literature exists on incorporating ambiguity into economic analysis of climate change. Lange and Treich (2008), Millner et al. (2013) and Traeger (2014) use Klibanoff et al.'s (2005) smooth ambiguity model to evaluate alternative policies. Lange and Treich (2008) construct examples where increasing ambiguity aversion leads to a more stringent environmental policy and to where it has an ambiguous effect. Millner et al. (2013) characterize conditions under which optimal abatement increases with ambiguity aversion. The same authors also combine their preference model with the DICE integrated assessment model (Nordhaus, 2008) to investigate how ambiguity about climate sensitivity affects welfare analysis. Traeger (2014) establishes a relationship between the dynamic smooth ambiguity model and the model of intertemporal risk aversion. He also derives the stochastic social discount rate for various specifications of the intertemporal model. Asano (2010) deviates from the smooth ambiguity framework by developing a dynamic maximin expected utility model. He demonstrates that an increase in ambiguity brings forward the adoption of the optimal environmental policy.

In what follows, we first introduce the model. To crystallize the argument, a stylized world is assumed in which there exists common agreement on many hotly contested items. And so, we first introduce things on which we choose to pretend all agree. These include the physical technology for transferring consumption possibilities from one period to the next, the existing degree of ambiguity, Society's rate of time preference, and Society's risk attitudes. Then we turn to things on which there is potential disagreement. That disagreement is restricted to which rationality axioms to impose upon Society's decision makers. Three alternative sets, each resulting in a specific benefit-cost criterion, are presented. First, the differences are analyzed conceptually and then a quantitative analysis that relies heavily on previously-used parametrization is presented. The final section discusses the implications of the results.

2 The model

Much, if not most, existing economic climate-change analyses are in integrated-assessment model (IAM) form. Pioneered by Nordhaus (1991, 1993), these models integrate climate-science models with economic models of how climate change affects important economic variables. By their very essence, they are simultaneously complicated and deeply simplified. They are complicated because they integrate so many components into a common structure. They are simplified because they rely on tractably convenient assumptions about physical interrelationships, technological interactions, and economic behavior. Core contributions, in addition to Nordhaus (1991, 1993), include Stern (2007), Nordhaus (2008), and the United States Interagency Working Group on the Social Cost of Carbon (USIWGSCC) (2010). Pindyck (2013) presents a deeply critical review of the overall IAM effort.

We take another tack. We aim for simplicity to ensure that the origin of our results is transparent and can be grasped without detailed knowledge of existing IAM models. Although formalized in a very different way, what follows is more closely related to Weitzman (2007, 2009) and Pindyck (2012) which marry basic concepts from Ramsey-type growth theory with benefit-cost analyses in an SEU framework.

2.1 Commonly agreed ingredients

There are two periods and a single decision maker. The current period, 0, is nonstochastic but the future period, period 1, is uncertain. The decision maker's problem is how to allocate her current period wealth, w, between current period consumption, $c^0 \in \mathbb{R}_+$, and investments to generate consumption in period 1. Uncertainty is represented by a finite set of states $\Omega = \{1, 2, ..., S\}$. A natural intuitive interpretation of Ω is as cataloguing the range of possibilities for a key environmental variable such as the *climate-sensitivity parameter*[2] that is a crucial component of many IAMs. But we emphasize the "intuitive" nature of the interpretation because Ω can accommodate more general and realistic decision scenarios. $X \subset \mathbb{R}^S$ denotes the set of constant acts (elements of $\mathbb{R}^S$ taking the same value in each state), and $x \in X$ denotes the constant act taking the same real value, x, in each state of Nature.

There exists "scientific uncertainty" so that there is no common agreement upon a single probability measure to associate with Ω. Rather, the beliefs about Ω are characterized by a nonempty, nonsingleton, closed convex set Π which is a subset of the probability simplex $\Delta \subset \mathbb{R}^S_+$. Scientific uncertainty in the form of ambiguity is a core assumption of our model. Although it is not frequently maintained, it is realistic given the broadly divergent scientific findings regarding the likelihood of the degree of climate change and the associated welfare implications (IPCC 2007, 2013, and Heal and Millner, 2014, 2015).[3]

[2]Climate sensivity is usually expressed as the equilibrium change in the temperature from pre-industrial times that would eventually materialize if the atmposhperic CO_2 concentration were to double.

[3]Roe and Baker (2007) observe that uncertainty in climate projections is very significant and that it "has not narrowed appreciably over past 30 years" (p. 629). Allen and Frame (2007) go even further and argue that climate sensitivity may be "unknowable." Similarly, Pindyck (2013) writes that "...the physical mechanisms that determine climate sensitivity involve crucial feedback loops, and the parameter values that determine the strength (and even the sign) of those feedback loops are largely unknown, and for the foreseeable future may even be unknowable".

To model the (stochastic) consumption possibilities available for period 1, we borrow methods originally developed in the literatures on activity analysis, general equilibrium under uncertainty, and finance under uncertainty (Koopmans 1951, Debreu 1959, LeRoy and Werner 2001). Specifically, we assume that consumption in period 1 is achieved by diverting period 0 initial wealth, w, towards a stochastic production process. Thus, we view that diverted wealth as an input to that process that one can generically conceptualize as *effort.* That stochastic production process, which gives rise to period 1 consumption possibilities, involves allocating that period 0 effort across J distinct linear stochastic production activities.

To be specific, the stochastic period 1 output generated by operating the jth production activity with one unit of effort is $A_j \in \mathbb{R}^S_+$, $j = 1, 2, ..., J$.[4] If the decision maker allocates $h^j \in \mathbb{R}_+$ units of period 0 effort to the jth activity, the linearity of the production activity generates a period 1 stochastic consumption stream of $A_j h^j \in \mathbb{R}^S_+$. Thus, the stochastic period 1 consumption available from devoting $\sum_{j=1}^J h^j \in \mathbb{R}_+$ in period 0 to the J different production activities is

$$c^1 = \sum_{j=1}^{J} A_j h^j \in \mathbb{R}^S_+,$$

or in matrix notation

$$c^1 = Ah,$$

where $A = [A_1, A_2, ..., A_J]$ and $h = \left[h^1, ..., h^J\right]^\top$.[5]

2.2 Axiomatic alternatives

The decision-maker's preferences are defined over the two-period consumption stream $(c^0, c^1) \in \mathbb{R}_+ \times \mathbb{R}^S$. Three different decision paradigms are considered. Each is rationalized by a binary relation defined on $\mathbb{R}_+ \times \mathbb{R}^S$ and denoted by $\succ$ where $(y^0, y^1) \succ (q^0, q^1)$ is to be read as (y^0, y^1) is strictly preferred to (q^0, q^1) . Each binary relation is strictly increasing in the sense that $(y^0, y^1) - (q^0, q^1) \in$

The uncertainty surrounding the effect of climate change on various environmental and economic indicators is even greater than that pertaining to climate sensitivity. Nordhaus (2010) suggests that "understanding the market and nonmarket impacts of climate change continues to be the thorniest issue in climate-change economics" (p. 11722).

[4] For example, USIWGSCC considered five such trajectories in formulating its conclusions (USIWGSCC, 2010; Greenstone, Kopits, and Wolverton, 2013). Alternatively, one might think of each A_j intuitively as being the stochastic consumption trajectory consistent with a particular emissions pathway such as those given by the four Representative Concentration Pathways (RCPs) utilized in the IPCC AR5. But this should be done cautiously because the identification between the two is incomplete. The RCPs are internally consistent sets of forcing projections to be used in alternative climate-change models. Multiple socioeconomic scenarios can be consistent with a single RCP (Collins et al., 2013), and multiple RCPs can be consistent with a single socioeconomic scenario. So while there are 4 RCP's in AR5, *a priori*, there is no reason to restrict the column dimension of A to be 4 or smaller.

[5] All vectors in the paper without a transpose sign "$\top$" are in column form.

$\mathbb{R}_+^{1+S} \setminus \{0\} \implies (y^0, y^1) \succ (q^0, q^1)$. By this monotonicity assumption, it follows that the decision maker always combines production activities to ensure that period 1 stochastic consumption is financed at minimal period 0 cost. The period 0 minimal cost of assembling a period 1 stochastic consumption of $c^1 \in \mathbb{R}_+^S$ from the J production activities is given by the function $m : \mathbb{R}_+^S \to \bar{\mathbb{R}}_+$ defined as

$$\begin{aligned} m\left(c^1\right) &\equiv \min_{h \in \mathbb{R}_+^J} \left\{1^\top h : Ah \geq c^1\right\} \\ &= \max_{p \in \mathbb{R}^S} \left\{p^\top c^1 : p^\top A \leq 1^\top\right\}, \end{aligned}$$

if there exists h such that $Ah \geq c^1$ and ∞ otherwise. Here $1^\top$ denotes the J dimensional row vector with ones in each entry. Thus, economically efficient (that is, consistent with minimal period 0 cost) consumption possibilities associated with a period 0 expenditure of $(w - c^0)$ are given by

$$C\left(w - c^0\right) = \left\{c^1 : \left(w - c^0\right) \geq m\left(c^1\right)\right\}.$$

The fundamental distinction between the three paradigms lies in how each augments or alters a common set of axioms, maintained throughout the paper. The *common axioms* require that $\succ$ be *irreflexive, transitive, continuous, monotonic, and risk averse.*

The benchmark is the SEU model. It augments the common axioms by requiring that $\succ$ completely orders $\mathbb{R}_+ \times \mathbb{R}^S$ and satisfies Savage's sure-thing principle (independence). The two alternatives are the IEU model (Aumann 1962; Bewley 1986), and Gilboa and Schmeidler's (1989) MEU model. IEU augments the common axioms by imposing Savage's sure-thing principle, but it does not require that $\succ$ completely order $\mathbb{R}_+ \times \mathbb{R}^S$. MEU maintains complete ordering but relaxes independence to "certainty independence". Roughly speaking, certainty independence requires that mixing gambles with degenerate gambles (that is, gambles $x \in X$) preserves the preference ordering.

The specific functional forms were chosen to satisfy two criteria: first, to simplify comparisons across paradigms; and second, to simplify comparisons with existing $SEU-$based analyses of climate change. A decision maker with SEU preferences, denoted $\succ_{SEU}$, ranks alternative consumption bundles as:

$$\left(c^0, c^1\right) \succ_{SEU} \left(c^{0\prime}, c^{1\prime}\right) \Longleftrightarrow \delta u\left(c^0\right) + \sum_{s=1}^{S} \hat{\pi}_s u\left(c_s^1\right) > \delta u\left(c^{0\prime}\right) + \sum_{s=1}^{S} \hat{\pi}_s u\left(c_s^{1\prime}\right),$$

where $\hat{\pi} \equiv (\hat{\pi}_1, ..., \hat{\pi}_S) \in \Pi$ is a subjective probability distribution over Ω and δ measures the rate of time preference. In our formulation, δ is the reverse of the discount factor. It is commonly assumed that $\delta > 1$ so that more weight is placed on the present, period-0 consumption, than on the future, period-1 consumption. We maintain this assumption in the numerical part of the paper. However, none of the theoretical findings hinges on this assumption. We assume that $u(\cdot)$ is strictly concave and that it satisfies the standard Inada conditions.[6] An MEU decision maker, denoted $\succ_{MEU}$, ranks consumption according to

$$\left(c^0, c^1\right) \succ_{MEU} \left(c^{0\prime}, c^{1\prime}\right) \Longleftrightarrow \delta u\left(c^0\right) + \min_{\pi \in \Pi}\left\{\sum_{s=1}^{S} \pi_s u\left(c_s^1\right)\right\} > \delta u\left(c^{0\prime}\right) + \min_{\pi \in \Pi}\left\{\sum_{s=1}^{S} \pi_s u\left(c_s^{1\prime}\right)\right\},$$

and the decision maker has IEU preferences $\succ_{IEU}$ if:

$$\left(c^0, c^1\right) \succ_{IEU} \left(c^{0\prime}, c^{1\prime}\right) \Longleftrightarrow \delta u\left(c^0\right) + \sum_{s=1}^{S} \pi_s u\left(c_s^1\right) > \delta u\left(c^{0\prime}\right) + \sum_{s=1}^{S} \pi_s u\left(c_s^{1\prime}\right) \text{ for all } \pi \in \Pi.$$

In contrast to an SEU decision maker, both an MEU and an IEU decision makers have beliefs given by a set of probability distributions. However, the ways the latter two decision makers utilize their beliefs to compare different consumption streams stand in sharp contrast. An MEU decision maker evaluates each consumption stream using the probability distribution that yields the lowest expected utlity among all probability distributions from the set Π. Thus, an MEU decision maker exhibits complete pessimism for each consumption stream. In contrast, an IEU decision maker strictly prefers one consumption stream to another only if the former yields a strictly higher expected utility for all probability distributions in the set Π. Such unanimity favors the status quo consumption stream and it can be interpreted as optimism toward that status quo.

Given our desire to have the different decision makers agree as much as possible, our assumption that beliefs $\hat{\pi}$ of an SEU decision maker belong to set Π is natural. Because Π is convex, any $\hat{\pi}$ in its relative interior is a convex combination of other elements of Π. Hence, $\hat{\pi}$ can be derived via Bayesian calculation, where a prior over Π is used to calculate an "expected" probability measure. Thus, this specification accommodates Weitzman's (2007, 2009) Bayesian-updating-induced "tail fattening". Throughout the remainder of the paper, δ, u, and Π are assumed common across paradigms. Gilboa et al. (2010) show that commonality of δ, u, and Π across the MEU and IEU paradigms emerges from requiring *consistency* between objectively rational and subjectively rational choices and *caution.* Consistency requires that anything that is objectively rational must

[6] Assuming u is strictly concave and satisfies the standard Inada conditions

$$\lim_{c \to 0} u'\left(c\right) = \infty, \lim_{c \to \infty} u'\left(c\right) = 0,$$

opens the door, with a finite state space and positive probability measures, for the "Dismal Theorem" of Weitzman (2009). The consequences of the Dismal Theorem have been debated in a number of fora including Nordhaus (2011), Pindyck (2011), Weitzman (2011), and Millner (2013). Weitzman (2009, 2011) and Pindyck (2011) have argued for the introduction of a value of statistic life (VSL)-type parameter in SEU cost-benefit calculations as a device for closing the model in making practical policy evaluations when the CRRA specification is used in an SEU criterion function. Our model can certainly replicate the Dismal Theorem, but because our concern lies elsewhere, we avoid "closing the model" in this fashion.

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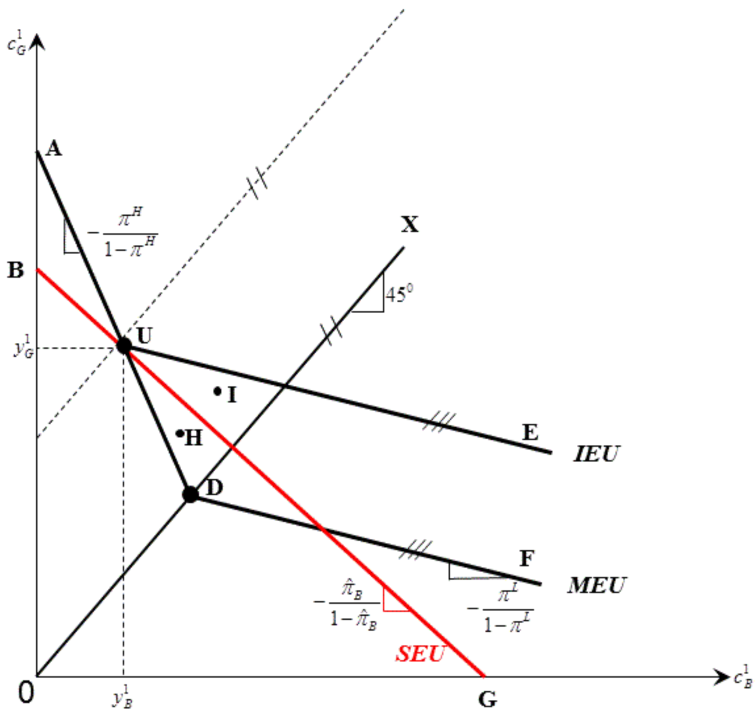

Figure 1. SEU, MEU and IEU indifference curves

1.pdf

also be subjectively rational. Caution requires that an uncertain act is preferred to a constant act only if it is never objectively rational to prefer the constant act.

MEU and IEU preferences have kinked indifference curves. In contrast, SEU indifference curves are smooth and tangent to the fair-odds line defined by $\hat{\pi}$ in the neighborhood of X. Figure 1 illustrates in the space of period 1 consumption vectors. There, $\Omega = \{1, 2\}$. Consumption in state 1 is measured on the horizontal axis and consumption in state 2 on the vertical axis. The set $X \subset \mathbb{R}^2$ consists of the points on the 45^o degree line (the bisector) passing through the origin on which $c_1 = c_2$ for every point. For visual clarity, these indifference curves are drawn for risk-neutral (that is, u linear) preferences. The SEU indifference curve is the straight line passing through B, U, and G. The IEU indifference curve, when the initial allocation of period 1 consumption is at U, is the kinked line passing through A, U, and E. Finally, the kinked indifference curve for MEU preferences passes through points A, U, D, and F. The key difference between MEU and IEU preferences is that the indifference curves for the former are only kinked in the neighborhood of D.

MEU and IEU preferences reflect two different types of "conservative behavior". MEU preferences, being kinked in the neighborhood of riskless outcomes (point D in Figure 1), reflect conservatism in moving away from the riskless outcome. They have been offered, for example, as an explanation for individuals fully insuring outcomes at actuarially unfair odds. Away from the riskless outcome, alternatives are always evaluated, relative to the riskless outcome, in terms of the worst possible odds.

IEU preferences, on the other hand, are kinked at the current consumption point, which can occur anywhere. They, too, reflect conservative behavior, but of "the-devil-you-know" variety, and have been used to explain the status-quo bias and individuals refusing to trade or failing to mutually insure.

2.3 Cardinalizing preferences

Benefit-cost analysis requires a cardinal representation of the underlying social preference structure. Because IEU preferences are not complete, cardinalization is slightly complicated because they cannot be represented by a real-valued "welfare function" that ranks consumption bundles (c^0, c^1), $W : \mathbb{R}_+ \times \mathbb{R}^S \to \mathbb{R}$, such that $W(c^0, c^1) > W(c^{0\prime}, c^{1\prime}) \iff (c^0, c^1) \succ_{IEU} (c^{0\prime}, c^{1\prime})$. Thus, a more "primitive" functional representation of preferences is needed. Our specific choice is a willingness to pay measure defined in terms of period 0 consumption. Define $T : \mathbb{R}^S \times \mathbb{R}^S \to \bar{\mathbb{R}}$ by:

$$T(q; y) = \sup\left\{\beta \in \mathbb{R} : \left(q^0 - \beta, q^1\right) \succ \left(y^0, y^1\right)\right\},$$

if there exists $\beta \in \mathbb{R}$ such that $(q^0 - \beta, q^1) \succ (y^0, y^1)$ and $-\infty$ otherwise, where $q \equiv (q^0, q^1)$ and $y \equiv (y^0, y^1)$. $T(q; y)$ gives the largest decrease in period-0 consumption q^0 consistent with maintaining $(q^0 - \beta, q^1)$ strictly preferred to (y^0, y^1) and, thus measures the willingness to pay, from a starting point of y, to make the move $q - y$.

For any $\succ$ satisfying $q \succ y \implies q + \mathbb{R}^S \setminus \{0\} \succ y$, $T(q; y)$ satisfies $T(y; y) = 0$. It is also a complete functional representation of $\succ$ in that

$$T(q; y) > 0 \iff q \succ y, \tag{1}$$

so that knowledge of T is equivalent to knowledge of $\succ$. If one is willing to pay a positive amount to make the move $q - y$, q must be preferred to y. For SEU preferences:

$$\begin{aligned} T_{SEU}(q; y) &= \sup\left\{\beta : \delta u\left(q^0 - \beta\right) + \sum_{s=1}^{S} \hat{\pi}_s u\left(q_s^1\right) > \delta u\left(y^0\right) + \sum_{s=1}^{S} \hat{\pi}_s u\left(y_s^1\right)\right\} \\ &= q^0 - u^{-1}\left[u\left(y^0\right) - \frac{1}{\delta}\sum_{s=1}^{S} \hat{\pi}_s\left(u\left(q_s^1\right) - u\left(y_s^1\right)\right)\right], \end{aligned} \tag{2}$$

for MEU preferences

$$\begin{aligned} T_{MEU}(q; y) &= \sup\left\{\beta : \delta u\left(q^0 - \beta\right) + \min_{\pi \in \Pi}\left\{\sum_{s=1}^{S} \pi_s u\left(q_s^1\right)\right\} > \delta u\left(y^0\right) + \min_{\pi \in \Pi}\left\{\sum_{s=1}^{S} \pi_s u\left(y_s^1\right)\right\}\right\} \\ &= q^0 - u^{-1}\left[u\left(y^0\right) - \frac{1}{\delta}\left(\min_{\pi \in \Pi}\left\{\sum_{s=1}^{S} \pi_s u\left(q_s^1\right)\right\} - \min_{\pi \in \Pi}\left\{\sum_{s=1}^{S} \pi_s u\left(y_s^1\right)\right\}\right)\right], \end{aligned} \tag{3}$$

and for IEU preferences

$$\begin{aligned} T_{IEU}(q;y) &= \sup\left\{\beta : \delta u\left(q^0-\beta\right)+\sum_{s=1}^{S}\pi_s u\left(q_s^1\right) > \delta u\left(y^0\right)+\sum_{s=1}^{S}\pi_s u\left(y_s^1\right) \text{ for all } \pi\in\Pi\right\} \quad (4) \\ &= q^0-\left(u^{-1}\left[u\left(y^0\right)-\frac{1}{\delta}\min_{\pi\in\Pi}\sum_{s=1}^{S}\pi_s\left(u\left(q_s^1\right)-u\left(y_s^1\right)\right)\right]\right). \end{aligned}$$

Our choice of $T(q;y)$ as the device for cardinalizing preferences is to some extent arbitrary. While $T(q;y)$ is a function representation of $\succ$, it is not the only such possible measure. For example, Pindyck (2012) uses the willingness to pay definition, in our notation,

$$\sup\left\{\alpha>0 : \left((1-\alpha)q^0,(1-\alpha)q^1\right)\succ\left(y^0,y^1\right)\right\}$$

that corresponds to the percentage of q one would be willing to pay to forego the movement $y-q$. It, too, is a complete function representation of $\succ$. Thus, qualitative results obtained using either measure will be equivalent.

3 Benefit-cost analyses of alternative policies

Suppose that the decision maker initially is at (y^0,y^1) with $y^0=w-m(y^1)$ and is considering the alternative (q^0,q^1) with $q^0=w-m(q^1)$. The criterion for adopting $(w-m(q^1),q^1)$ requires

$$\left(w-m\left(q^1\right),q^1\right)\succ\left(w-m\left(y^1\right),y^1\right),$$

or equivalently

$$T\left(\left(w-m\left(q^1\right),q^1\right);\left(w-m\left(y^1\right),y^1\right)\right)>0.$$

Recalling that $T(y;y)=0$, this requires that

$$T\left(\left(w-m\left(q^1\right),q^1\right);\left(w-m\left(y^1\right),y^1\right)\right)-T\left(\left(w-m\left(y^1\right),y^1\right);\left(w-m\left(y^1\right),y^1\right)\right)>0,$$

which for marginal changes, converts to the (one-sided) directional derivative[7] of $T(\cdot;\cdot)$ evaluated at $(y;y)$ in the direction $(m(y^1)-m(q^1),q^1-y^1)$:

$$T^o\left(y;y;m\left(y^1\right)-m\left(q^1\right),q^1-y^1\right)=\lim_{\lambda\downarrow 0}\left[\begin{array}{c}\frac{T\left(w-m\left(y^1\right)+\lambda\left(m\left(y^1\right)-m\left(q^1\right)\right),y^1+\lambda\left(q^1-y^1\right);w-m\left(y^1\right),y^1\right)}{\lambda}\\ -\frac{T\left(w-m\left(y^1\right),y^1;w-m\left(y^1\right),y^1\right)}{\lambda}\end{array}\right]>0. \quad (5)$$

[7] The (one-sided) directional derivative of $T(d;f)$ evaluated at $(d;f)\in\mathbb{R}^{S+1}\times\mathbb{R}^{S+1}$ in the direction $n\in\mathbb{R}^{S+1}$ is given by

$$T^o(d;f;g)=\lim_{\lambda\downarrow 0}\frac{T(d+\lambda g;f)-T(d;f)}{\lambda}.$$

The use of one-sided directional derivatives in making this and other marginal arguments is necessitated by the non-smooth character of the preference maps associated with IEU and MEU preferences. This nonsmoothness, which has often been associated with market inertia, is fundamental to how these decisionmakers respond to ambiguity.

We proceed in stages. First, we derive the general result that IEU preferences have the most conservative (most difficult to satisfy) criterion for adoption. Then, to sharpen the analysis and to set the stage for our numerical analysis, we treat the case where $\Omega = \{B, G\}$ for "bad" and "good", respectively.

3.1 The general case

We demonstrate in the Appendix that

$$\begin{aligned}
T^o_{SEU}\left(y; y; m\left(y^1\right) - m\left(q^1\right), q^1 - y^1\right) &= \frac{\sum_s \hat{\pi}_s u'\left(y^1_s\right)\left(q^1_s - y^1_s\right)}{\delta u'\left(w - m\left(y^1\right)\right)} + \left(m\left(y^1\right) - m\left(q^1\right)\right), \qquad (6)\\
T^o_{MEU}\left(y; y; m\left(y^1\right) - m\left(q^1\right), q^1 - y^1\right) &= \frac{\min\limits_{\pi \in \Pi^{MEU}(y)} \left\{\sum_s \pi_s u'\left(y^1_s\right)\left(q^1_s - y^1_s\right)\right\}}{\delta u'\left(w - m\left(y^1\right)\right)} + \left(m\left(y^1\right) - m\left(q^1\right)\right),\\
T^o_{IEU}\left(y; y; m\left(y^1\right) - m\left(q^1\right), q^1 - y^1\right) &= \frac{\min\limits_{\pi \in \Pi} \left\{\sum_s \pi_s u'\left(y^1_s\right)\left(q^1_s - y^1_s\right)\right\}}{\delta u'\left(w - m\left(y^1\right)\right)} + \left(m\left(y^1\right) - m\left(q^1\right)\right),
\end{aligned}$$

where $\Pi^{MEU}(y) = \arg\min\limits_{\pi \in \Pi} \left\{\sum_{s=1}^S \pi_s u\left(y^1_s\right)\right\}$.

The intuition behind each adoption criterion is the same. Just as an investor should be willing to incorporate into his or her portfolio assets whose stochastically discounted return exceeds their acquisition cost (a familiar martingale pricing principle), alternative q should be adopted if its stochastically discounted marginal return exceeds its marginal cost. The differences between the adoption criteria lie in the probability measure associated with the stochastic discount factor, $\frac{u'\left(y^1\right)}{\delta u'\left(w - m\left(y^1\right)\right)} \in \mathbb{R}^S$. IEU chooses that measure pessimistically (relative to staying put). MEU chooses it optimistically (relative to staying put). SEU falls somewhere in between. More formally, it follows from (5) and (6) that

Proposition 1 *A policy $\left(w - m\left(q^1\right), q^1\right)$ is adopted under IEU only if it is adopted under both SEU and MEU.*

Figure 1 illustrates our result. The initial allocation y^1 is given by the point U. To focus attention on uncertain outcomes, suppose for the purposes of illustration that both the initial and alternative allocations are equally costly, $m\left(q^1\right) = m\left(y^1\right)$. Starting at y, a policy alternative represented by the point H will be accepted by the individual with MEU preferences but not by a decision maker with either SEU preferences or IEU preferences. Similarly, policy alternative I will be accepted by both MEU and SEU but not by an IEU decision maker.[8]

The reason that this occurs is that the MEU decision maker judges gambles such as H using the least-favorable odds for state 2 which now represents the "good" state of Nature because H

[8] Note that all three allocations, U, H, and I, involve some exposure to uncertainty.

returns a higher consumption in that state than in state 1. The SEU decision maker, on the other hand, judges policy H by a less pessimistic set of odds for state 2. Consequently, for that decision maker, H does not represent an attractive alternative relative to U, and it will not be adopted. The IEU decision maker also does not adopt H because her conservative behavior requires that H be attractive for both the most favorable and the least favorable odds for state 2. And so, H is rejected in favor of staying at U. The reasoning why policy alternative I is preferred to the initial allocation U by both MEU and SEU but not by an IEU decision maker is similar.

By recycling Figure 1, one easily sees that similar behavior will be exhibited in cases where the initial policy falls below X so that 1 instead of 2 is now the good state. The behavioral reason behind such decisions is encapsulated in the probability measures that support each initial point. For the IEU decision maker, the set of supporting probability measures is always Π, while for the SEU decision maker it is always the singleton set $\{\hat{\pi}\} \in \Pi$. Thus, while an SEU decision maker may accept an alternative using $\hat{\pi}$, there is no guarantee that both the least favorable and most favorable odds will judge the alternative as acceptable. Similarly, the MEU decision maker's supporting probabilities are characterized by the set $\Pi^{MEU}(y) \subseteq \Pi$. Sometimes, for example if the initial position involves no uncertainty, $\Pi^{MEU}(y) = \Pi$, both an MEU and IEU decision makers' conservatism will lead them to act identically. But more generally, the MEU decision maker's conservatism is more $X-$primordial in the sense that it always harkens back to those riskless acts as its ultimate goal. Thus, the MEU decision maker is willing to adopt alternatives that move towards that goal that an IEU decision maker, who simply hesitates to move, will shun. Generally, one cannot predict the relative behavior of SEU and MEU decision makers without prior knowledge of the initial position. For example, if the initial position is somewhere in X, alternatives exist which an SEU decision maker will adopt, but which an MEU decision maker will not. However, away from X just the opposite pattern may occur as Figure 1 illustrates.

3.2 Two-state example

There are two states $\Omega = \{B, G\}$, and the range of beliefs is given by

$$\Pi = \left\{(\pi_B, \pi_G) : \pi_B \in \left[\pi^L, \pi^H\right] \subset [0,1]\right\}$$

with $\hat{\pi}_B \in \left[\pi^L, \pi^H\right]$. There are two consumption pathways. One, referred to as "business-as-usual", is denoted by subscript u, and the other is a "climate-responsive" pathway, denoted by c. More formally,

$$A \equiv \begin{bmatrix} A_{Bu} & A_{Bc} \\ A_{Gu} & A_{Gc} \end{bmatrix} \equiv [A_u, A_c],$$

with

$$A_{Gu} > A_{Gc} > A_{Bc} > A_{Bu}. \tag{7}$$

2n

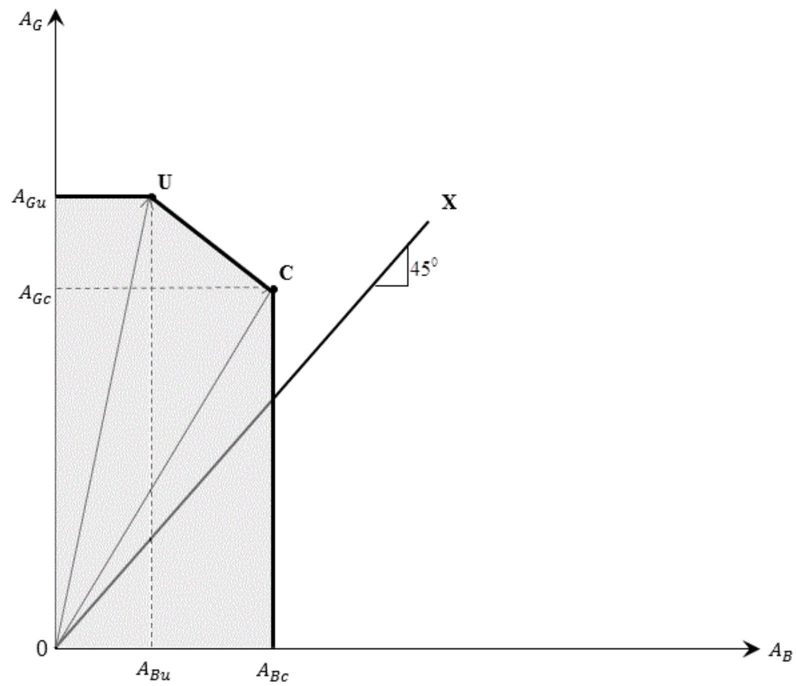

Figure 2. "Business-as-usual" and "climate-responsive" pathways

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Expression (7) requires that A_c attenuates the dispersion of consumption outcomes associated with A_u. If G occurs A_c sacrifices some consumption, which is balanced against a consumption gain if B occurs. Neither A_u nor A_c dominates the other, both are *technically* efficient, in the sense of Koopmans (1951). In the present context, (7) guarantees the existence of a positive solution to[9]

$$p^\top A = 1^\top.$$

These consumption pathways are illustrated in Figure 2. There the vertical axis measures outcomes in state G, mnemonically one can think of it as good in terms of climate outcomes and the horizontal axis measures outcomes in state B, mnemonically this is the bad state. The 45^o degree line passing through the origin and labelled X represents the set of constant acts.

The two vectors labelled U and C represent the two consumption pathways. Each pathway represents consumption possibilities in states B and G associated with one unit of foregone consumption in period 0. The pathway labeled U involves a relatively high consumption if the good climate state

[9]In a financial context, (7) rules out the presence of arbitrages (Ross 1976). Direct calculation reveals

$$p^\top = \left[\frac{A_{Gc} - A_{Gu}}{A_{Bu}A_{Gc} - A_{Bc}A_{Gu}}, \frac{A_{Bu} - A_{Bc}}{A_{Bu}A_{Gc} - A_{Bc}A_{Gu}}\right] > 0.$$

eventuates but low consumption otherwise. Intuitively, this is a business-as-usual practice that does relatively little to prepare for the potential effects of climate change. The pathway labeled C on the other hand rotates away from pathway U towards X. In intuitive terms, it is less "uncertain" than U. It manifests a technology that, compared to U, sacrifices G state consumption in return for higher B state consumption.

Restriction (7) ensures that neither of the two pathways dominates the other in both states. This is reflected in their relative lengths. One can envision a situation where U (C) was radially extended enough so that it dominated C (U) in both B and G. If that radially extended U (C) could be had for one unit of period 0 consumption, it would render C (U) redundant.

Normalizing period 0 expenditures on period 1 consumption to one, the range of period 1 feasible consumption choices is given by the points dominated by the convex combinations of U and C, the shaded trapezoid $0A_{Gu}UCA_{Bc}$ in Figure 2. The resulting range of choices mimics what one would obtain from a piecewise linear "transformation" curve that transforms consumption in state G into state B consumption. This, of course, reflects our model's ultimate roots in the general-equilibrium analysis of financial markets. Moreover, as expenditure on period 1 consumption increases, this "transformation" curve shifts out radially. Similarly, as expenditure decreases the curve contracts radially. And finally as more and more independent consumption pathways are added the transformation curve closer and closer approximates a smooth transformation curve.[10]

For convenience, units are calibrated so that $y^1 = A_u$. In words, business-as-usual represents the *status quo* pathway. Consequently,

$$m\left(y^1\right) = m\left(A_u\right) = p^\top A_u = 1. \tag{8}$$

The alternative, q^1, combines the "business as usual" and the "climate-responsive" pathways. Thus,

$$q^1 - y^1 = \varphi^c A_c + \varphi^u A_u, \tag{9}$$

where φ^c denotes the level at which pathway A_c is operated in q^1 and φ^u represents the change in pathway A_u involved in moving from y^1 to q^1.

Using (6), (8) and (9), the respective criteria are to adopt if:

$$\begin{aligned}
SEU &: \frac{\left(1-\hat{\pi}_B\right) u'\left(A_{Gu}\right)\left(\varphi^c A_{Gc}+\varphi^u A_{Gu}\right)+\hat{\pi}_B u'\left(A_{Bu}\right)\left(\varphi^c A_{Bc}+\varphi^u A_{Bu}\right)}{\delta u'(w-1)} > \varphi^c+\varphi^u, \\
MEU &: \frac{\left(1-\pi^H\right) u'\left(A_{Gu}\right)\left(\varphi^c A_{Gc}+\varphi^u A_{Gu}\right)+\pi^H u'\left(A_{Bu}\right)\left(\varphi^c A_{Bc}+\varphi^u A_{Bu}\right)}{\delta u'(w-1)} > \varphi^c+\varphi^u, \\
IEU &: \frac{\left(1-\pi^L\right) u'\left(A_{Gu}\right)\left(\varphi^c A_{Gc}+\varphi^u A_{Gu}\right)+\pi^L u'\left(A_{Bu}\right)\left(\varphi^c A_{Bc}+\varphi^u A_{Bu}\right)}{\delta u'(w-1)} > \varphi^c+\varphi^u.
\end{aligned}$$

[10] This of course manifests Houthhakker's famous demonstration that smooth isoquants can be closely approximated by Leontief technologies.

In writing the IEU criterion, we assume that

$$q_G^1 - y_G^1 = \varphi^c A_{Gc} + \varphi^u A_{Gu} < 0.$$

This ensures that q^1 requires foregoing state G consumption relative to y^1. The analysis extends to a much broader class of policies, but this choice, which is maintained hereafter, focuses attention on the type of choices of most current interest.

Figure 3 illustrates the decision environment. The ordered pairs of pathways or policy alternatives, (φ^u, φ^c), on the negatively sloped 45^o line emanating from the origin leave period 0 costs unchanged. Pairs above it increase costs, and pairs below decrease them. Attention is restricted to the policy pairs on or above the zero-net-cost line, so that q^1 is at least as costly as y^1. Again, the model permits more general analysis, but the current pragmatic debate is about *costly* climate-policy alternatives. Pairs satisfying

$$\varphi^c \theta_G + \varphi^u = 0,$$

where $\theta_G \equiv A_{Gc}/A_{Gu}$ involve period 1 $G-$state consumption remaining constant. These are illustrated by the negatively sloped ray emerging from the origin labelled θ_G. By (7),

$$\theta_G < 1.$$

The cone defined by θ_G and the 45^o line, which delimits the alternative policies under consideration, is referred to as the "policy cone".[11]

Proposition 2 *(1) If allocation $(w - m(q^1), q^1)$ satisfies the IEU adoption criterion, then it also satisfies the SEU and MEU criteria. If allocation $(w - m(q^1), q^1)$ satisfies the SEU adoption criterion, then it also satisfies the MEU criterion. (2) Any "spread" of Π (or increase in ambiguity), which is represented by a change in beliefs from Π to $\Pi' \supseteq \Pi$, makes it less likely for the IEU benefit-cost criterion for adoption to be met and more likely for the MEU benefit-cost criterion to be met. (3) In the case of complete ambiguity, $\Pi = [0,1]$, the IEU benefit-cost criterion for adoption is never satisfied.*

Proposition 2 reflects the behavioral differences inherent in the three preference structures. An IEU decision maker, relative to the other decision makers, manifests a preference for the status quo. This is an immediate consequence of her inability to compare all potential outcomes. Her valuation of any move from A_u is necessarily lower than that of either the MEU or SEU decision maker. At the other extreme is the MEU decision maker. Her pessimism predisposes her to believe that B is likely to occur. Thus, in evaluating gains and losses, she heavily discounts G state losses

[11] The general analysis, of course, applies to all policies in (φ^c, φ^u) space. We leave it to the interested reader to extend our arguments to other pairs.

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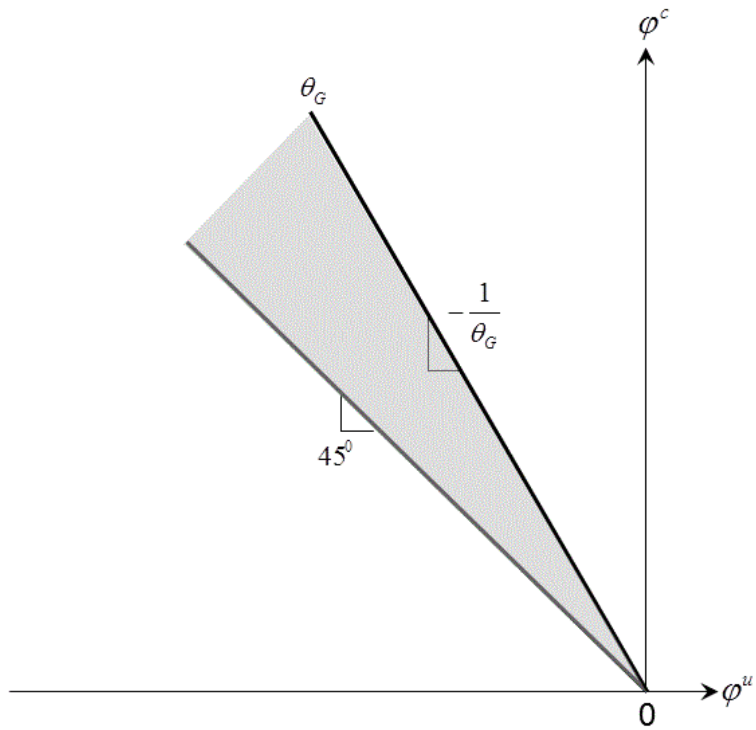

Figure 3. Policy Cone

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in favor of B state gains and finds A_c more attractive than the other decision makers. The two forms of pessimism, one about changes from the *status quo* (IEU) and the other about the *status quo* (MEU) clash. As a result, the objectively rational IEU decision maker will not adopt the alternative policy in instances where the subjectively rational MEU decision maker would.

SEU and MEU decision makers are "overly rational". They can rank all possible uncertain alternatives. IEU decision makers are rational, but their ability to make comparisons is limited. Consequently, their evaluation of future consumption streams is more guarded than those of either an SEU or IEU decision maker.

When ambiguity is extreme, here approximated by setting $\Pi = [0,1]$, the IEU criterion for adoption is never satisfied. The objectively rational decision maker always stays put. The subjectively rational MEU decision maker adopts if:

$$\frac{u'(A_{Bu})(\varphi^c A_{Bc} + \varphi^u A_{Bu})}{\delta u'(w-1)} > \varphi^c + \varphi^u.$$

Because u is strictly concave, *there always exists a critical level of* A_{Bu} *satisfying this criterion.* Therefore, if A_u involves a bad-enough outcome in state B, an MEU decision maker will always adopt the alternative in the presence of extreme ambiguity because he or she places all of the decision weight on that poor outcome.

4n

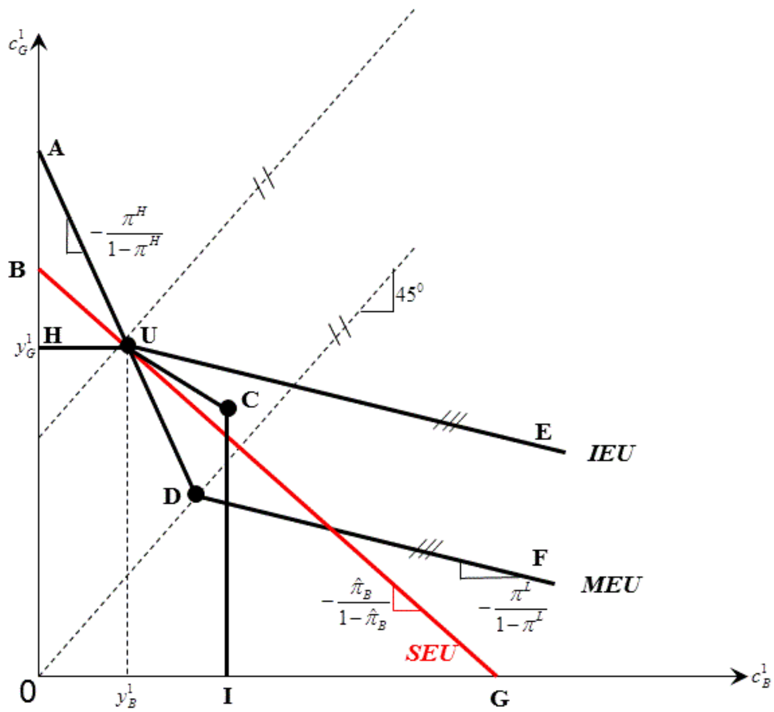

Figure 4. SEU, MEU and IEU adoption criteria

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More generally, for a fixed amount of ambiguity, $(\pi^H - \pi^L)$, the difference between the MEU and the IEU stochastically discounted marginal benefits,

$$\frac{(\pi^H - \pi^L)\left[u'(A_{Bu})(\varphi^c A_{Bc} + \varphi^u A_{Bu}) - u'(A_{Gu})(\varphi^c A_{Gc} + \varphi^u A_{Gu})\right]}{\delta u'(w-1)} > 0,$$

measures the gap between the two decision criteria. It becomes arbitrarily large, *ceteris paribus,* as A_{Bu} becomes arbitrarily small.[12] Thus, *subjectively rational and objectively rational benefit-cost criteria diverge the most precisely when the possible consequences of inaction (as captured by A_{Bu}) are the largest.*[13]

Figure 4, which combines Figures 1 and 2, illustrates Proposition 2. For visual clarity, linear u is assumed. Point U represents A_u. Point C represents A_c. Trapezoid $OHUCI$ represents the set of feasible activities for $w - c^0 = 1$. In the illustrated case, both the SEU and MEU criteria for adopting A_c are met. The IEU criterion is not.

An increase in ambiguity is visually represented by the IEU and MEU indifference curves becoming more kinked (closer to right angles). Complete ambiguity corresponds to the case where

[12] The reasoning here parallels that behind Weitzman's (2009). As $A_{Bu} \to 0$, the marginal utility loss associated with the bad outcome becomes infinitely large.

[13] Please see the Introduction for definitions of subjective and objective rationality.

the kinks are right angles. Under complete ambiguity, an IEU decision maker will not adopt any pathway that requires sacrificing any consumption in state G. The MEU decision maker, on the other hand, is willing to adopt any marginal change that increases consumption in state B.

An obvious question that arises in evaluating Proposition 2 is how it extends to a larger state and action space. Without doubt enriching both complicates the analysis. The peculiar strength of the discrete two-state case is its ability to cleanly sort outcomes into either "good" or "bad". When there are more states, there are more potential outcomes, and this ability is necessarily diminished. The key analytic question, however, is whether those outcomes can be rank ordered (as, for example, in majorization analysis or in rank-dependent expected utility analysis). If they can, our results should be relatively robust because the concavity of $T(\cdot)$ ensures that its superdifferentials are cyclically monotone (Rockefellar 1970) which, in turn, guarantees a patterned manner in which to assess outcomes. Models formulated in terms of random variables continuously distributed along a finite or infinite support on $\mathbb{R}$, which segment into "good tails" and "bad tails" (for example, Weitzman 2009, Pindyck 2012), impose that rank ordering by construction. Consequently, our results should readily extend to that setting with relatively minor changes.

4 Application to climate change with numerical simulations

We now turn to a quantitative analysis of the model. In common with much of the applied macroeconomics and finance literatures, the default specification for u in climate-change analyses is the CRRA form

$$u(y) = \frac{y^{1-n}}{1-n},$$

where n is the Pratt-Arrow coefficient of relative risk aversion. To ensure comparability of our results, we also adopt that specification.

In climate-change analyses n plays two roles. One is to measure how individuals assess period 1 risks. Another emerges from the inequality-measurement literature (Atkinson, 1970). In that context, n measures attitudes towards intertemporal consumption inequality (see, for example, Dasgupta, 2007). Thus, the choice of n has proven controversial. In the risk literature, it is widely believed that n should fall somewhere between 1 and 4. However, the macroeconomic literature surrounding the equity premium paradox suggests it may be much higher. Rather than fix it at a single level, we vary it over the alternatives $\{1.5, 1.75, 2.00, 2.25, 2.50\}$, which covers most of the moderate alternatives in the climate-change literature (Nordhaus, 2008; Weitzman, 2007; Dasgupta, 2007). We also normalize the wealth level so that $u'(w-1) = 1$. For this parametrization, the generic benefit-cost criterion[14] for adopting q^1 is to adopt if

$$\frac{(1-\pi_B)\left(\theta_G\varphi^c+\varphi^u\right)A_{Gu}^{1-n}+\pi_B\left(\theta_B\varphi^c+\varphi^u\right)A_{Bu}^{1-n}}{\delta}>\varphi^c+\varphi^u, \tag{10}$$

where $\theta_B \equiv A_{Bc}/A_{Bu}$, $\pi_B = \hat{\pi}_B$ for SEU, $\pi_B = \pi^H$ for MEU, and $\pi_B = \pi^L$ for IEU.

The time span between period 0 and period 1 is set at 100 years. This is in line with projections from many IAMs, but about 100 years less than Weitzman's (2009) calculations. To accommodate our discrete state space, the continuous range of temperature change[15] is broken into two alternatives. State B corresponds to a temperature increase that exceeds 5^0C. That size of increase can be considered high but not extremely high. For example, IPCC AR5 (2013) reports that equilibrium climate sensitivity is "*likely* in the range of 1.5 to 4.5 C^0 with high confidence ... and very unlikely greater than 6^0 with medium confidence". State G corresponds to a temperature change with a relatively small impact.

The current approach to incorporating uncertainty into many IAMs is to assign probability distributions to key parameters, such as equilibrium climate-sensitivity, and then perform Monte Carlo simulations (see, for example, Stern, 2007 and Pindyck, 2013). For example, USIWGSCC (2010), recognizing the existence of varying estimated probability distributions (it reports 8), used the Roe and Baker (2007) probability distribution calibrated for consistency with IPCC AR 4 (2007).[16] Cruder approaches to consolidating different estimated probability distributions, such as simple averaging, are also common (Weitzman, 2009; Pindyck, 2012).

A central problem in identifying a probability structure for climate sensitivity is that the actual sensitivity of the real climate system is not directly measurable. Effort, therefore, has concentrated on relating the standard climate-sensitivity measure to observable quantities. This can be achieved either directly or through a model (IPCC AR4, 2007). The result is widespread variability across studies in how empirical probability distributions are estimated.

Given such widespread ambiguity, our analysis questions the integrity of selecting a single prior to evaluate uncertainty. Instead of a single prior, we rely on a range drawn from IPCC AR 5 (2013). Using information available in Chapter 12 (especially Figures 12.8, 12.36, 12.37, and 12.40 and the surrounding discussion), we set $\pi^H = 0.12, \pi^L = 0.04$, and $\hat{\pi}_B = 0.075$.[17]

Economic studies of the damage due to climate change typically relate realized temperature

[14] By generic, we mean for an arbitrary $\pi \in \Delta$.

[15] Here, as elsewhere in the numerical illustration, we use the term "temperature increase" in the same sense that it is used in the definition of climate sensitivity to denote the increase in global-mean temperature since pre-industrial times.

[16] White House (2010) also considered three alternatives to the Roe and Baker (2007) distribution.

[17] AR5 projects that the globally averaged surface temperature will increase by $1.4°C$ to $5.8°C$ over the period 1990 to 2100 under the IPCC business as usual emissions scenario (see Table 1.1 in IPCC, 2013). These findings are consistent with a comparison of models that was conducted by the USGCRP (2009).

change measured in degrees Centigrade, T, to GDP loss or consumption loss using a loss function specification, $D(T)$, with $D(0) = 1$ (no damage) and D declining in T. Specifications differ. Nordhaus (2008), for example, uses an inverse quadratic specification, while Pindyck (2012) uses an exponential loss function.[18] We use Weitzman's (2012) reactive damage function:

$$D(T) = \frac{1}{1 + (0.049T)^2 + (0.16T)^{6.75}}. \tag{11}$$

$D(T)$ equals 0.78 for a $5^0 C$ temperature increase.

The consumption pathways in A could be based upon scenarios from other climate-change studies. But, in practice, those scenarios are often highly speculative and, at best, only based on poorly understood physical relationships that have even less understood large potential feedbacks. Thus, to keep the analysis as simple and transparent as possible, we calibrate A. In the calibration, we set $\theta_G = 0.98$ and $\theta_B = 1.2$. The parameters θ_B and θ_G represent the extent to which A_c attenuates the dispersion in outcomes associated with A_u. This attenuation effect is important because it determines the mutual insurance properties of A_c and A_u. Roughly speaking, our calibration requires that A_c avoids a loss of 20% of consumption if state B materializes at the cost of 2% of consumption if state G materializes. Thus, the probability of B occurring that would make the expected value of the implied attenuation effect zero is approximately 11.5%.

The annual growth rate for A_{Gu} is set to 3%. This is consistent with global output projections adjusted for population growth. The IMF's global growth projection for 2015 was around 3.8% (IMF, 2014). UN (2004) estimates an average annual population growth rate of 0.77% for the period $2000-2050$. However, that 3% is about 1% higher than the per-capita consumption growth rate used, for example, in USIWGSCC (2010). For a 100-year time horizon, that choice yields $A_{Gu} = 1.03^{100} = 19.22$, and $A_{Bu} = D(5^0 C) \cdot A_{Gu} = .78 \cdot 1.03^{100} = 14.99$. The annual rate of time preference, denoted by d, varies between 1.00 and 1.04. Hence, δ varies in the range $[1.00^{100}, 1.04^{100}] = [1.00, 50.51]$. Table 1 summarizes the parameter values.

Table 1. The values of model parameters.

[18] See also Dietz and Stern (2014) for a discussion and analysis based on various damage functions.

Time horizon	100 years
Temperature Change, $D(T)$	$T = 5^0 C, \qquad D(T) = .78$
Average probability of B	$\hat{\pi}_B = 0.075$
Lower probability of B	$\pi_L = 0.04$
Upper probability of B	$\pi_H = 0.12$
Mitigation	$\theta_G = .98, \theta_B = 1.2$
Degree of relative risk aversion	$n \in \{1.5, 1.75, 2.00, 2.25, 2.50\}$
Annual growth rate	3%
Annual rate of time preference	$d \in [1.00, 1.04]$

Suppose first that the alternative policy simply replaces business as usual, A_u, with A_c, that is, $q^1 = A_c$. For the parameter values in Table 1, the generic benefit-cost criterion is to adopt if

$$(1 - \pi_B)(0.98\varphi^c + \varphi^u) \cdot 19.22^{1-n} + \pi_B (1.2\varphi^c + \varphi^u) \cdot 14.99^{1-n} > (\varphi^c + \varphi^u) d^{100}. \tag{12}$$

When $q^1 = A_c$, some manipulation establishes

$$\tilde{\pi}_B \equiv \frac{.02 \cdot 19.22^{1-n}}{(2 \cdot 14.99^{1-n} + .02 \cdot 19.22^{1-n})},$$

as the lower bound for π_B requiring adoption. Calculated $\tilde{\pi}_B$ ranges from a low of .06444 ($n = 2.5$) to a high of .08115 ($n = 1.5$). For the tabulated values in Table 1, *the IEU decision maker never adopts A_c* regardless of risk aversion. On the other hand, *the MEU decision maker always adopts A_c*. The SEU decision maker, with $\hat{\pi}_B$ equal to .075, does not adopt for lower levels of risk aversion ($n = 1.5, 1.75$), but does for higher levels of risk aversion. Whether the SEU decision maker adopts or not depends critically upon where $\hat{\pi}_B$ is set. The closer $\hat{\pi}_B$ approaches $\pi^L\left(\pi^H\right)$, the closer the SEU decision maker's behavior approaches that of the IEU (MEU) decision maker.

For the more general case where q^1 can represent a mixture of A_u and A_c, the policy cone in our calibration corresponds to (φ^c, φ^u) satisfying (to six digits)

$$-1.020408 \cdot \varphi^u \geq \varphi^c \geq -\varphi^u.$$

Taking $n = 2$ and $d = 1.02$ in (12) and performing the calculation results in the following adoption criteria

$$\begin{aligned} \varphi^c &< -1.000095 \cdot \varphi^u, \qquad MEU, \\ \varphi^c &< -1.000005 \cdot \varphi^u, \qquad SEU, \text{ and} \\ \varphi^c &< -0.999935 \cdot \varphi^u, \qquad IEU. \end{aligned} \tag{13}$$

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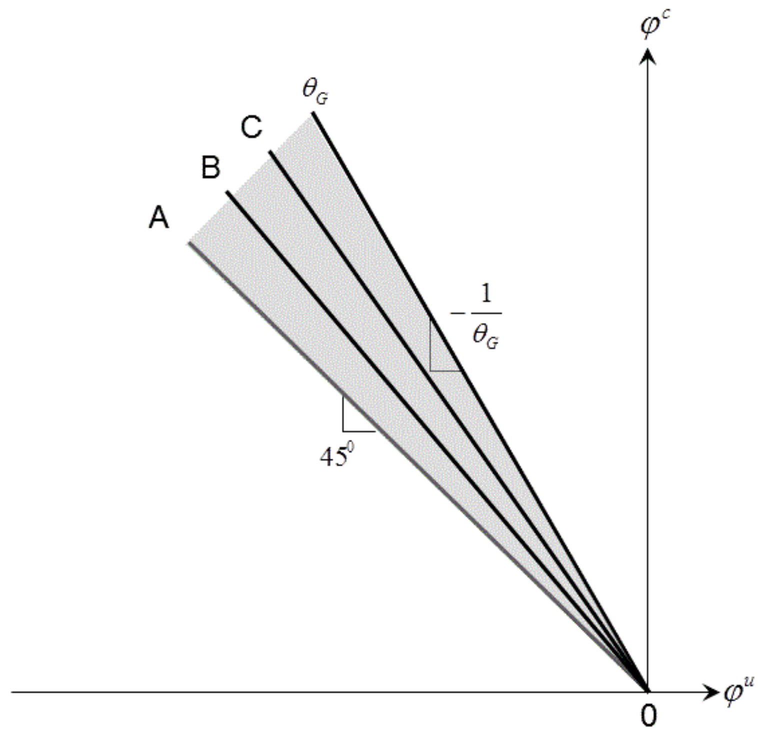

Figure 5. Policies that meet SEU, MEU, and IEU criteria

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For this parametrization, *none of the policy alternatives in the policy cone satisfy the* IEU *criterion* for adoption. Some alternatives satisfy both the SEU and the MEU adoption criteria. Figure 5 illustrates. The (φ^c, φ^u) alternatives satisfying the SEU criterion fall in the cone between the rays OA and OB. And the set satisfying the MEU criterion is given by the cone between OA and OC. Both are proper subsets of the policy cone.

The parameter φ^u measures the change in the level at which A_u is used under the new policy alternative. As a practical matter, most climate-change policy discussions involve moving away from A_u to A_c. Thus, φ^u is expected to be negative. The IEU adoption criterion is met in this case only if the increase in φ^c less than matches the decline in φ^u. The objectively rational IEU decision maker only adopts moves towards A_c that reduce period 0 costs! He or she needs to be compensated for decreased period 1 consumption in state G by increased current period consumption. Future sacrifice only comes if there is a clear myopic benefit. Both the SEU and the MEU, on the other hand, are willing to trade some increase in current period cost for the benefit associated with the alternative. But, as our conceptual results imply, the SEU decision maker is willing to absorb a smaller current cost burden than the MEU decision maker.

For each decision maker, there are two effects involved. One is the pure income effect of moving from A_c towards A_u. At the margin, MEU evaluates the expectation of that income effect to be positive (.021488 in undiscounted terms). Both the SEU and the IEU decision makers evaluate its expectation to be negative ($-.13702$ and $-.249104$, respectively). The second effect is the insurance effect associated with moving towards the less dispersed A_c. For all decision makers, this is positive. The MEU decision maker, with a positive income effect and a positive insurance effect, adopts. For SEU, the insurance effect, in this case, is positive enough to counteract the negative income effect for small enough moves in the direction of $A_{c.}$ For IEU, it is not. The difference in each case reduces to which probability each decision maker uses to evaluate both the income effect and the risk effect.

Table 2 summarizes our general adoption results that are obtained by varying n and d. Each cell reports the benefit-cost criteria that satisfy inequality (13) for some (φ^u, φ^c) combinations in the policy cone. The fractions reported in parentheses represent the percentage of the policy cone that satisfies (13), as defined by the ratio

$$\left(\frac{((1-\pi_B)\,19.22^{1-n} + \pi_B 14.99^{1-n} - d^{100})}{(0.98\,(1-\pi_B)\,19.22^{1-n} + 1.2\pi_B 14.99^{1-n} - d^{100})} - 1\right) / \left(\frac{1}{\theta_G} - 1\right),$$

for each of the respective π_B. So, for example, when $n = 2.0$ and $d = 1.0$, the IEU benefit-cost criterion is never satisfied in the policy cone. The SEU criterion is satisfied for .197% of the policy cone, and the MEU criterion is satisfied for 3.552% of the policy alternatives.

A particularly stark result emerges from Table 2. *The IEU criterion is never met for any parameter values in the policy cone, but the MEU criterion is always satisfied for some alternatives in the policy cone.* So, an MEU decision maker is always willing to accept some costly policy alternatives in this calibration of our model, but an IEU one never will.[19] An SEU decision maker who is relatively risk tolerant ($n = 1.5$ and $n = 1.75$) will never accept any alternative in the policy cone, while an SEU decision maker who is relatively risk averse ($n = 2.00$, $n = 2.25$, and $n = 2.5$) will accept some alternatives in the policy cone. However, for $\hat{\pi}_B = .075$, the percentage of the policy cone meeting the SEU criterion is quite small (compared to MEU) and never exceeds .25%.

Table 2. Benefit-cost Adoption Criteria Satisfaction

[19] There are no parameterizations in our grid for which either MEU or SEU criteria are satisfied for all points in the policy cone.

	$n = 1.5$	$n = 1.75$	$n = 2.00$	$n = 2.25$	$n = 2.5$
$d = 1.00$	MEU (0.1397)	MEU (0.0681)	MEU (0.03552) SEU (0.00197)	MEU (0.01894) SEU (0.00246)	MEU (0.01016) SEU (0.00193)
$d = 1.01$	MEU (0.0433)	MEU (0.0233)	MEU (0.01267) SEU (0.00070)	MEU (0.00688) SEU (0.00089)	MEU (0.00373) SEU (0.00071)
$d = 1.02$	MEU (0.0153)	MEU (0.0085)	MEU (0.00467) SEU (0.00026)	MEU (0.00255) SEU (0.00033)	MEU (0.00139) SEU (0.00026)
$d = 1.03$	MEU (0.0056)	MEU (0.0032)	MEU (0.00175) SEU (0.00010)	MEU (0.00096) SEU (0.00012)	MEU (0.00052) SEU (0.00010)
$d = 1.04$	MEU (0.0021)	MEU (0.0012)	MEU (0.00067) SEU (0.00004)	MEU (0.00037) SEU (0.00005)	MEU (0.00020) SEU (0.00004)

Not surprisingly, the percentage of alternatives judged acceptable by either MEU or SEU declines as d rises. This behavior reflects the decision maker's increasing unwillingness to trade today's consumption for future consumption gains associated with increasing d. On the other hand, SEU and MEU acceptance behavior react differently qualitatively to changing n. For MEU, as risk aversion increases, holding d constant, the percentage of policy alternatives satisfying the adoption criteria declines. For the SEU decision maker, the percentage increases, peaking in each instance (albeit at very small levels) at $n = 2.5$ and then decreasing.

4.1 Sensitivity analysis

The relative attractiveness of A_c as an alternative to A_u naturally depends upon how it attenuates the latter's outcome variability. This attenuation is captured parametrically by θ_B and θ_G. Increasing θ_B provides additional protection against B relative to A_u. Decreasing θ_G increases the period 1 cost of implementing A_c if G occurs. Simultaneously increasing θ_B and decreasing θ_G trades decreased returns in G for additional protection against B. For example, if θ_B is increased and θ_G is decreased by the same differentially small amount ($\varepsilon > 0$), the marginal change in the generic benefit-cost adoption criterion is

$$\frac{\varphi^c}{\delta}\left[\pi_B A_{Bu}^{1-n} - (1-\pi_B) A_{Gu}^{1-n}\right]\varepsilon,$$

which is increasing in π_B. Consequently, the MEU decision maker would perceive such a change as being more attractive than either the SEU or the IEU decision makers.

To investigate these effects, set $\theta_B = 1.15$ which corresponds to A_c avoiding a loss of 15% of consumption if state B materializes (in contrast to 20% in our calculations above). *Ceteris paribus*, this makes activity A_c less attractive for all decision makers. Setting $n = 2.00$, $d = 1.02$, and

$\theta_G = 0.98$, results in the following respective benefit-cost criteria:

$$\begin{aligned} \varphi^c &< -1.000040 \cdot \varphi^u, && MEU, \\ \varphi^c &< -0.999971 \cdot \varphi^u, && SEU, \text{ and} \\ \varphi^c &< -0.999917 \cdot \varphi^u, && IEU. \end{aligned} \tag{14}$$

The MEU criterion is satisfied for some alternatives in the policy cone, both the SEU and IEU criteria are never satisfied. Thus, changing θ_B from 1.2 to 1.15 qualitatively changes the SEU decision maker's behavior. While some alternatives in the policy cone are attractive for $\theta_B = 1.2$, A_u is always the preferred option under $\theta_B = 1.15$.

Increasing the attractiveness of activity A_c, by changing θ_B from 1.20 to 1.25, does not lead to qualitative changes in decision maker's behavior. Setting $\theta_B = 1.25$, $n = 2.00$, $d = 1.02$, and $\theta_G = 0.98$, the respective benefit-cost criteria become

$$\begin{aligned} \varphi^c &< -1.000151 \cdot \varphi^u, && MEU, \\ \varphi^c &< -1.000040 \cdot \varphi^u, && SEU, \text{ and} \\ \varphi^c &< -0.999954 \cdot \varphi^u, && IEU. \end{aligned} \tag{15}$$

The MEU and SEU criteria are satisfied for some alternatives in the policy cone while the IEU criterion is never satisfied.

We also ran two additional simulations for parameter θ_G : $\theta_G = 0.97$ and $\theta_G = 0.99$. The first increases the period 1 cost of activity A_c if G occurs, and the second decreases it. The other parameters were set at $n = 2.00$, $d = 1.02$, and $\theta_B = 1.2$. The SEU criterion is satisfied for some alternatives in the policy cone under $\theta_G = 0.98$ but never satisfied for $\theta_G = 0.97$. The MEU criterion is satisfied for some alternatives in the policy cone while the IEU criterion is never satisfied when $\theta_G = 0.97$. When θ_G is set to 0.99, implying that the alternative to A_c only incurs a 1% loss if G eventuates, the IEU decision maker's benefit-cost adoption criterion is satisfied for some alternatives in the policy cone. As one would expect, the MEU and SEU criteria are also met for some alternatives in the policy cone under this scenario. Thus, if costs associated with the alternative activity are sufficiently small, the alternative can prove attractive to all decision makers.

In truth, little to nothing is known about the true form of $D(T)$. Faced with this ignorance, modelers have treated specification selection for $D(T)$ more as a matter of analytic convenience than hard science. For example, Pindyck (2013, p.867) writes: "When it comes to the damage function, however, we know almost nothing, so developers of IAMs can do little more than make up functional forms and corresponding parameter values. And that is pretty much what they have done." Similarly, a reviewer has reacted to our setting $D(T)$ to 0.78 for a 5^0C temperature increase with disbelief noting that many scientists think such a temperature increase may be civilization-ending.

To investigate how the choice of $D(T)$ may affect decision criteria, we set $n = 2.00$ and $d = 1.02$ and then calculate the level of $D(T)$ that would convince an IEU decision maker to move away from A_u toward the alternative. The resulting value is $D(T) = .42$, which implies a 58% consumption loss if the bad state eventuates (the temperature change is 5^0 C), and is substantially smaller than the value in Table 1, which implies a 22% loss. In other words, damages have to be approximately 2.5 times as large as those implied by Table 1 before adoption occurs. It is hard to call such a loss anything other than truly catastrophic. It is important to note that this calculation, which uses π^L, effectively sets the relevant subjective probability at approximately 4%, which is orders of magnitude higher than current scientific predictions about catastrophic outcomes. If π^L is set to .01, which is still extremely high for a truly catastrophic loss, the IEU decision maker would only abandon A_u if $D(T) < 0.1$. Losses in state B would have to exceed 90% relative to state G.

5 Concluding Remarks

The model is intentionally simplified. And while the goal is not practical policy advice, such advice is important, and many economists want to provide it. The key issue confronting economists is whether policy to control greenhouse gases should be immediately stringent or increase abatement gradually. Our results show that the policy advice offered depends crucially upon the normative framework.

Regardless of risk attitudes and concern about future generations, the objectively rational IEU framework suggests caution in adopting policies to mitigate the effects of climate change, the subjectively rational MEU framework is far more proactive. The SEU framework falls between those poles. Whether it supports either immediate stringency or gradualism depends crucially, as is already well-known, upon risk attitudes and concerns about future generations.

A crucial point to understand is that the perceived gap between the IEU and MEU recommendations is in an important sense *science-based.* It results from the lack of agreement among professional scientists on the likelihood of the degree of climate change. To be objectively rational in the sense of Gilboa et al. (2010), a policy recommendation in our setting needs to satisfy the IEU criterion, which boils down to requiring unanimity across different probability structures. Because there is such widespread disagreement in the scientific community, the objectively rational suggestion is effectively “wait and see”. Subjective rationality, on the other hand, in this setting essentially requires that the decision maker adhere to the ‘one-percent doctrine’, famously attributed to Dick Cheney: if there is even a small chance of a catastrophic outcome, it should be treated as though it were a near certainty.

That leaves us on familiar ground. Stern’s (2007) IAM-based analysis advocated immediate and drastic policy action. This contrasted dramatically with other IAM-based studies that had concluded a more gradual approach was appropriate. Stern's (2007) results were quickly traced to what were argued to be 'extreme' choices for δ and the curvature of u. Multiple authors classed these as 'ethical choices' and criticized them on that basis. Weitzman (2007, 2009), while criticizing Stern (2007) on similar grounds, noted that support for some of Stern's (2007) recommendations might be found in his 'dismal theorem', which effectively buttresses the 'one-percent doctrine'. Even more recently, Pindyck (2013) has written that IAM-based analysis has created "...a perception of knowledge and precision, but that perception is illusory and misleading." Our results reinforce Pindyck's (2013) criticism by showing that changes in normative assumptions profoundly change policy suggestions. In other words, economic-policy suggestions hinge crucially upon *ethical choices.*

That finding emphasizes that, in another sense, the gap between the objectively rational IEU and the subjectively rational MEU is not science based. Rather it emerges from different, *and fundamentally normative (ethical)*, assumptions about what characterizes rational behavior for a decision maker. The same is true for SEU. It rests on a distinct viewpoint as to what is rational behavior. Because those viewpoints differ and those differences turn out to have deep implications for policy recommendations, it is hard to accept any of those recommendations (be it from IEU, MEU, or SEU) as truly science-based. Instead, following Gilboa (2009), they are perhaps more properly recognized as rhetorical devices marshalled by economists to support different policy positions

We carry no brief for any of the approaches. Our intent is not to criticize the independence axiom or any of the alternatives as decision rules. That has been done elsewhere (see, for example, Al-Najjar, 2013).[20] Rather, in an atmosphere where the SEU criteria seem to have been uncritically accepted, our goal is to identify what happens if alternatives are considered. Thus, borrowing Weitzman's phrasing, we envision the analysis here as investigating what *"..the discipline-imposing form of ..."* $IEU(MEU)$ *"... might offer by way of guidance for coherently thinking about ..."* climate change. When contrasted with SEU, they offer radically different recommendations.

There obviously remain shortcomings. One is that only two alternatives to SEU have been considered. "Smooth ambiguity" models, in particular, have proved popular in climate-change analyses (see, for example, Lange and Treich (2008) and Millner et al. (2013)). One clear reason for their popularity is that they permit analysis using standard calculus-based manipulation rather than requiring the use of super and sub differentials. The smooth ambiguity model, as does SEU, "falls between" the MEU and IEU preference structures. Unlike SEU, however, the smooth ambiguity

[20] There also exists a burgeoning experimental literature that elicits perceptions and attitudes to ambiguity. Camerer and Wember (1992) and Trautmann and van de Kuilen (2015) provide an early and recent reviews of this literature, respectively. A plethora of studies finds that decision-makers are sensitive to ambiguity and attitudes to ambiguity vary considerably over decision-makers and choice environments. Based on a rather exhaustive review of the literature, Trautmann and van de Kuilen (2015) conclude that ambiguity aversion is most widespread in the domain of moderate-likelihood gains while ambiguity seeking is typical in the domains of low likelihoods or losses.

model is structured to permit discrimination *within its parametrization* between ambiguity aversion and risk aversion. But, as Epstein (1999) shows, that requires postulating a notion of "ambiguity neutrality" and then measuring ambiguity aversion relative to that norm,[21] and that requires yet another ethical judgment. At one extreme of the smooth ambiguity model is a completely ambiguity-averse decision maker with MEU preferences. At the other extreme is the decision maker with IEU preferences. Thus, our results can be used to illuminate the scope of policy prescriptions for different parameterizations of the smooth ambiguity model.

Another, closely related, challenge is illustrated by our "generic benefit-cost criterion". Any of the reported results can be rationalized in a Bayesian SEU framework by an appropriate choice of priors over Π. This is well-known, and is true of any smooth ambiguity model or multiple-prior representation. For example, our numerical IEU results can be rationalized in a Bayesian framework by specifying a degenerate prior over Π that placed all the weight on π^L. Similarly, the MEU numerical results can be rationalized by a Bayesian prior that placed all the weight on π^H.

Thus, one could explain any of our results by a proper choice of priors. *But that is not how they were derived.* Rather they were derived by considering alternatives to axioms that are fundamental to SEU modelling and then considering the alternative in *the same decision setting as faced by the SEU decision maker.* The MEU thinkers are not modeled as hysterics. They are rational individuals whose preferences satisfy a weakened version of the SEU axioms. Similarly, IEU thinkers are not modeled as myopic. Rather, they are rational but realize that they may not be able to compare everything. And in an applied policy setting where the choice of utility structures, damage functions, and probability distributions by highly trained economists is routinely driven by computational tractability and not reality, that type of rationality is not without its own appeal.

Note also that the choice of a prior over Π to rationalize policy prescriptions in a Bayesian SEU framework will be a function of the proposed policy (or, in decision-theoretic terms, act). This is illustrated very effectively by the two-color Ellsberg experiment. Consider a bet on an ambiguous urn with black and white balls in unknown proportions. A decision maker with MEU preferences will rely on the lowest possible likelihood of drawing a black ball when betting on black and will rely on the lowest possible likelihood of drawing a white ball when betting on white. Since these two probability distributions are different for an MEU decision maker, the latter corresponds to a different SEU decision maker on each of the two choice occasions. Thus, in general, one cannot pick a "right" SEU decision maker with unique "right" beliefs to model an MEU decision-maker.

This discussion brings us to another important point. Any suitably smooth (super or sub differentiably smooth that is) welfare structure can be approximated by a local expected utility function in the sense of Machina (1982) or a local probability transformation in the sense of Quig-

[21] This point, as Epstein (1999) emphasizes, echoes Yaari's (1969) earlier demonstration that risk aversion is fundamentally a comparative notion requiring comparing "more risk averse" behavior to "risk-neutral" behavior.

gin and Chambers (2003). The latter implies that any suitably smooth structure can be locally approximated by local "risk-neutral probabilities" of the type encountered in the finance literature. Operationally, these "risk-neutral probabilities" are defined by the superdifferentials of $T(q; y)$ in q^1. Thus, welfare comparisons can be supported by an appropriate choice of "risk-neutral probabilities" and thus reduce to risk-neutral benefit-cost analyses. That implies that the exercise of cost-benefit analysis under SEU degenerates to choosing the appropriate "risk-neutral probabilities" to assess benefits and costs. In other words, a general representation of $T(q; y)$ can be used to do cost-benefit analysis in terms of these "risk-neutral probabilities". It's only a slight exaggeration to say from this perspective that much of the controversy about climate-change policy degenerates to a single point. One side, those in favor of immediate action, believes those risk-neutral probabilities should be set near one and the other, those in favor of delay, believes they should be set to zero. Our analysis illustrates this deeper point in a more structured way by using familiar parametrization to show how crucially "scientific" economic results depend upon that axiomatic setting and are not robust to its relaxation.

We envision a number of avenues for future research. The present model does not treat learning opportunities, technological change, or other dynamic considerations. Exploring how such factors interact with ambiguity in different decision paradigms is crucial to determining practical policy advice. Another noteworthy direction involves an examination of climate-change policies in a framework with mulitple decision-makers (representing, for example, different countries) differentiated by perceptions and attitudes to ambiguity. A more refined quantitative analysis of these extensions of the present model, with a more detailed modeling of the physical processes governing climate change and the associated ambiguity, could yield new and important insights.

6 Appendix: Derivation of (6)

Using (2) and (5), we obtain

$$
\begin{aligned}
& T^o_{SEU}\left(y; y; m\left(y^1\right) - m\left(q^1\right), q^1 - y^1\right) \\
= & \lim_{\lambda \downarrow 0}\left[\frac{w - m\left(y^1\right) + \lambda\left(m\left(y^1\right) - m\left(q^1\right)\right) - u^{-1}\left[u\left(w - m\left(y^1\right)\right) - \frac{\sum_{s=1}^S \hat{\pi}_s\left(u\left(y^1_s + \lambda\left(q^1_s - y^1_s\right)\right) - u\left(y^1_s\right)\right)}{\delta}\right]}{\lambda}\right] \\
= & \left(m\left(y^1\right) - m\left(q^1\right)\right) + \lim_{\lambda \downarrow 0}\left[\frac{w - m\left(y^1\right) - u^{-1}\left[u\left(w - m\left(y^1\right)\right) - \frac{\sum_{s=1}^S \hat{\pi}_s\left(u\left(y^1_s + \lambda\left(q^1_s - y^1_s\right)\right) - u\left(y^1_s\right)\right)}{\delta}\right]}{\lambda}\right] \\
= & \left(m\left(y^1\right) - m\left(q^1\right)\right) + \lim_{\lambda \downarrow 0}\left[\frac{M(\lambda)}{\lambda}\right],
\end{aligned}
$$

where

$$
M(\lambda) \equiv w - m\left(y^1\right) - u^{-1}\left[u\left(w - m\left(y^1\right)\right) - \frac{1}{\delta}\sum_{s=1}^S \hat{\pi}_s\left(u\left(y^1_s + \lambda\left(q^1_s - y^1_s\right)\right) - u\left(y^1_s\right)\right)\right].
$$

It follows from the preceding expression that

$$
\left[u\left(w - m\left(y^1\right)\right) - u\left[w - m\left(y^1\right) - M(\lambda)\right]\right] = \frac{1}{\delta}\sum_{s=1}^S \hat{\pi}_s\left(u\left(y^1_s + \lambda\left(q^1_s - y^1_s\right)\right) - u\left(y^1_s\right)\right).
$$

Dividing by λ and taking limits on both sides, we obtain

$$
u'\left(w - m\left(y^1\right)\right)\lim_{\lambda \downarrow 0}\frac{M(\lambda)}{\lambda} = \frac{1}{\delta}\sum_{s=1}^S \hat{\pi}_s u'\left(y^1_s\right)\left(q^1_s - y^1_s\right).
$$

Hence,

$$
T^o_{SEU}\left(y; y; m\left(y^1\right) - m\left(q^1\right), q^1 - y^1\right) = \frac{\sum_{s=1}^S \hat{\pi}_s u'\left(y^1_s\right)\left(q^1_s - y^1_s\right)}{\delta u'\left(w - m\left(y^1\right)\right)} + \left(m\left(y^1\right) - m\left(q^1\right)\right).
$$

The directional derivatives for MEU and IEU preferences can be obtained similarly.

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