A frequency-response-based method of sound velocity measurement in an impedance tube

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Abstract:

A stable and accurate new method for the measurement of the velocity of sound is proposed. The method is based on the characteristics of the frequency response measured at different positions in an impedance tube and it eliminates adverse effects caused by reflections from the transmitting transducer at the bottom of the impedance tube. A series of experiments is conducted, at different water temperatures, different positions in the impedance tube and under constant pressure, to validate the feasibility and stability of the new method. The new technique is also extended to hydrostatic pressure conditions with stable sound velocity. Our method generates an accurate measurement result in comparison to the estimated or average value obtained with currently existing methods. The novel method is suitable to be widely used in underwater acoustics.

Keywords: Underwater acoustics; Impedance tube; Sound velocity; Frequency response; Measurement method

1. Introduction

The velocity of sound is the most fundamental parameter in the general area of acoustics. In the context of underwater acoustics the velocity of sound was first determined by Colladon and Sturm by means of measurements in the Geneva Lake in 1827 [1]. The knowledge of the sound velocity enabled and prompted new technologies and applications in a wide area of fields. For example, at the beginning of the 20th century the distance from a ship to a lighthouse could now be determined by the difference between the transmission times of a ring of a bell transmitted in air and in water by measurements on the ship [2]. Moreover, the precise knowledge of the sound velocity is crucial to many fundamental technologies in the underwater detection of bodies in military applications, or in connection with tsunami warnings or applied technologies in the fishing industry.

The measurement of the sound velocity in an impedance tube is extremely important and fundamental in basic research. Measurement methods for the velocity of sound can be divided into the two general approaches, that is the standing wave method [3] and the pulse method [4]. The standing wave method is based on continuous wave interference, and the sound value is determined by measuring the time delay until the echo of the emitted signal arrives at the sending emitter. The standing wave method requires a high precision of length measurements, and hydrophones in the field will influence acoustic scattering. The pulse method requires accurate measurements of the transmission time and small inaccuracies will

result in substantial errors for the determined sound velocity [5]. Both methods are, in particular, not suitable for measurement conditions where the pressure varies during the measurement process.

The formula for the underwater acoustic transmission loss without vertical sound velocity gradients was explored by Weston [6]. The sound velocity in sea was measured by summarizing the measured data points by a series of successive linear line segments in 1989 [7]. Subsequently , sound wave propagation was measured accurately with phase-detection [8] and phase velocity [9]. Moreover the sound velocity in dissipate liquids [10] and shallow water [11] has been investigated. Diffraction effects were corrected in sound velocity and absorption measurements [12]. The sound velocity was measured in pure water at different temperature and pressure [13-16]. The China State Oceanic Administration [17] developed a method of frequency response function to calibrate the sound velocity and obtained a patent for their new technology. Gedanitz [18] introduced a new apparatus based on pulse-echo technique to measure sound velocity. Meanwhile, sound velocities in other liquids and liquid mixtures have further extensive applications [19-21] such as estimation of molecular radius.

Obviously, the sound velocity should be stable and accurate in applications. However, due to the sound velocity measurement being influenced by the reflection coefficient of the transmitting transducer at the bottom of the acoustic tube, the value fluctuates within a certain range. Therefore, the sound velocity is usually assumed 1500 m/s in most instances or measured with an average frequency difference from many unstable values. Aside from errors associated with the measurement of distance and time the reflection coefficient is the primary origin for the data scatter of the traditional method.

In this paper a new method for the measurement of the sound velocity is described. This new method is based on the frequency response in an impedance tube. The first step in the outline of our new method is the derivation of the theory which underlies our procedures and which can eliminate the reflection coefficient of the transmitting transducer at the bottom of the tube. Thereafter, numerical simulations are described to illustrate the application of this measurement method. Finally, the sound velocity is measured under conditions of different temperatures and pressures. Compared with traditional measurement methods the current method can obtain a stable and accurate value instead of fluctuating velocity values.

2. Sound velocity measurement method

Figure 1 schematically illustrates a sound wave emitted from a sound source that is subsequently being reflected from a surface opposite the source. The incident sound wave can be described as:

$$P_i = P_{ia} \cdot e^{j(\omega t - kd)} \tag{1}$$

Where, d is the distance between the hydrophone and the reflective surface, P_{ia} is the amplitude of the incident sound wave, ω is the frequency $\omega=2\pi f$ and $k=2\pi f/c$ is the wave number [25].

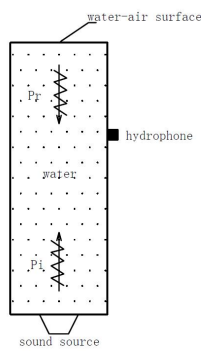


Figure 1 Sound velocity measurement principle

In a water-filled tube, with multiple

sound-wave reflections between top and bottom, the reflected sound pressure of the hydrophone after m reflections can be expressed as:

$$P_m(d) = A \cdot B = P_{ia} \cdot e^{j(\omega t - kd)} \times \left(1 + r \cdot e^{2kdj}\right) \sum_{n=0}^{m} \left(r_e \cdot r \cdot e^{2kLj}\right)^n \quad (2)$$

$$A = P_{ia} \cdot e^{j(wt-kd)} \left(1 + r \cdot e^{j \cdot 2kd}\right) \quad (3)$$

$$B = \sum_{n=0}^{m} (r_e \cdot r)^n \cdot e^{j \cdot 2nkL} \quad (4)$$

Where r is the reflection coefficient at the reflective surface at the top of the tube; r_e is the reflection coefficient of the sound source at the bottom of the tube; L is the length of the tube; φ_1 and φ_2 are their phases.

When the reflective surface is a water-air surface r_e= -1, such Eqs. 3) and (4) become

$$A = P_{ia} \cdot e^{j(wt-kd)} (1 - e^{j \cdot 2kd}) \quad (5)$$

$$B = \sum_{n=0}^{m} (r_e \cdot r)^n \cdot e^{2nkLj} \approx \frac{1}{1 - r_e \cdot e^{2nkLj}} \quad 6)$$

Then,

$$P_m(d) = A \cdot B \approx P_{ia} \cdot \frac{e^{j(wt-kd)} \cdot \left(1 - e^{j \cdot 2kd}\right)}{1 - r_e \cdot e^{j \cdot 2kL}} \quad (7)$$

$$\lg|P_m(d)| = \lg|P_{ia}| \left| \frac{1 - \cos 2kd}{1 - |r_e| \cos(2kL + \varphi_2)} \right| \quad (8)$$

The numerator and the denominator in equation (8) are periodic functions. Since $L>d$, the period of the numerator is longer than that of the denominator. When the reflection coefficient of the sound source at the bottom of the tube is not related to the frequency then the sound velocity can be calculated by the classic expression [26]:

$$c = 2L\Delta f \quad (9)$$

Where, Δf is the frequency difference between successive two minimum frequencies.

However, the frequency difference Δf is related to the reflection coefficient of the sound source at the bottom of the tube. The frequency difference Δf in the low frequency band is not the same as that in the high frequency band due to the reflection coefficient r_e. Obviously, the sound velocity will fluctuate with Δf. When the sound pressure from another hydrophone is measured at the same time, the pressure can be described as:

$$P_m(d') = A' \cdot B = P_{ia} \cdot e^{j(wt-kd')} \cdot \frac{(1 - e^{j \cdot 2kd'})}{1 - r_e \cdot e^{j \cdot 2kL}} \quad (10)$$

Where, d' is the distance from the other hydrophone to the reflective surface.

Because these two distances are different, the reflection coefficient r_e can be eliminated when equation (7) is divided by equation (10) .

$$\lg\left|\frac{P_m(d)}{P_m(d')}\right| = \lg\left|\frac{1 - e^{j \cdot 2kd}}{1 - e^{j \cdot 2kd'}}\right| = \frac{1}{2}\lg\left(\frac{1 - \cos(2kd)}{1 - \cos(2kd')}\right) \quad (11)$$

When the distance d in the numerator is bigger than the distance d' in the denominator, the sound velocity can be calculated by the frequency difference Δf from the minimum values of the numerator. The frequency difference Δf between the two successive minimum values should satisfy equation (12).

$$2\Delta kd = 2\pi \quad (12)$$

such that,

$$c = 2d\Delta f \quad (13)$$

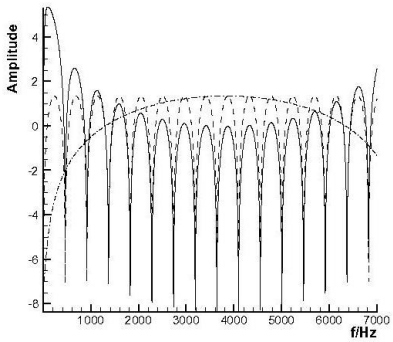
Therefore, the proposed method is not related to the reflection coefficient r_e of the transducer at the bottom of the tube. When the frequency difference Δf is a single stable and accurate value, the sound velocity is a precise value.

3. Signal simulations

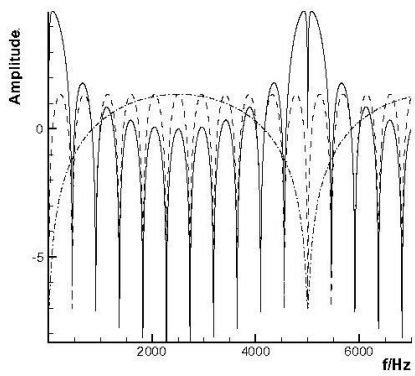
In equation (11), the numerator and the denominator are periodic functions. The period is the frequency difference Δf. It can be found that the accuracy of the sound velocity

directly relates to the stability of the frequency difference from equation (13). In order to verify the periodicity of the function and the stability of the frequency difference, this type is simulated.

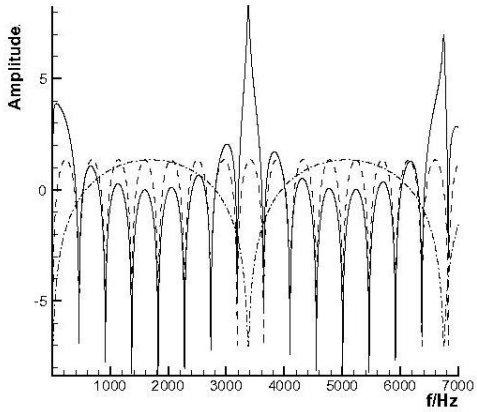
For our current experimental arrangement of hydrophones inside a water filled tube, the distance d of one hydrophone is 1.595 m. The distances d' of the other four hydrophones are respectively 0.095 m, 0.145 m, 0.215 m and 0.495 m. Assuming that the local sound velocity is 1450 m/s, the sound source signal is a continuous frequency sweep signal ranging from 1 Hz to 7000 Hz. The frequency resolution is 0.5 Hz. The signal simulation results are shown in Figure 2. In Figure 2, the solid line represents equation (11); the dashed line displays the numerator function of equation (11); the dash-dotted line shows the denominator function of equation (11).



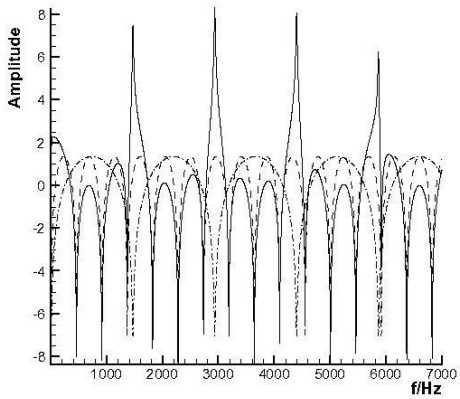
(a) d=1.595 m，d'=0.095 m



(b) d=1.595 m，d'=0.145 m



(c) d=1.595 m，d'=0.215 m



(d) d=1.595 m，d'=0.495 m

Figure 2 Signal simulations of hydrophones at different locations

From Figure 2, the successive minimum values of equation (11) are separated from one another by constant intervals. The period is the same as the period of the numerator function, but this is unrelated to the period of the function of the denominator. The maxima of equation (11) are characterized by smooth, continuous changes of their ordinate values while the minima display a sharp, cusp-like structure. Therefore, the minimum values are selected to reduce the error associated with determining the period of the displayed function.

From the simulations, it can be found that when the distance d is fixed, the frequency difference Δf from successive minimum values is constant. The frequency differences Δf of

the four groups are all 454.5 Hz. Therefore, the sound velocity can be calculated by equation (13).

$$c = 2d\Delta f = 1449.855\text{ms}^{-1} \qquad (14)$$

When the simulated frequency resolution is increased to 0.0001 Hz, the simulated sound velocity will increase to 1449.999826 m/s. This value is almost equal to that of the assumed local sound velocity. The deviation arises from the frequency resolution. It can be assumed that the sound velocity approaches the assumed value when the frequency resolution becomes sufficiently small.

4. Sound velocity measurement experiments

The experiments to corroborate our theoretical considerations outlined above were conducted at the Fluid and Acoustic Engineering Laboratory of Beihang University. The experimental arrangement is illustrated schematically in Figure 3. The tube used is manufactured from stainless steel (type: SUS321). Its inner radius is 60 mm and its outer radius is 120 mm. The water injected into the tube is cold boiled water. The water must be filtered and stand for 24 hours to eliminate the impurities and electrolytes. Five hydrophones are embedded in the tube which are not disassembled during operation. The distances of the five hydrophones on the wall are *ch1*=0.095 m, *ch2*=0.145 m, *ch3*=0.215 m, *ch4*=0.495 m, *ch5*=1.595 m. The effective length of the tube is *L*=5.000 m. The continuous sweep signal from the signal source (type: Agilent 33220A) is controlled by a computer. By means of a power amplifier (type: AR40AD1), signal from the signal source will be transmitted to the transducer at the bottom of the water-filled tube. After multiple reflections a standing wave field will have developed.

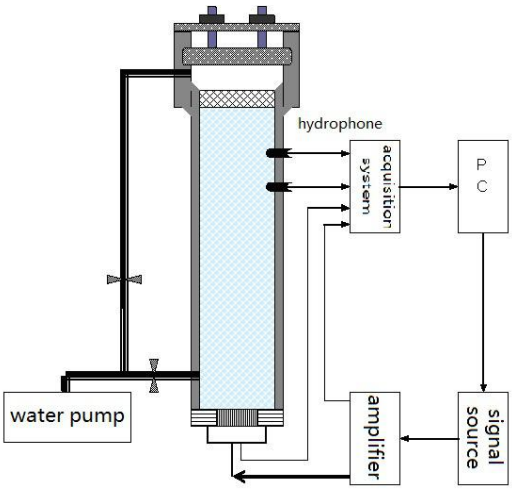
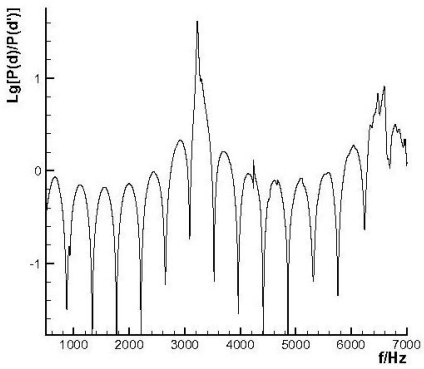
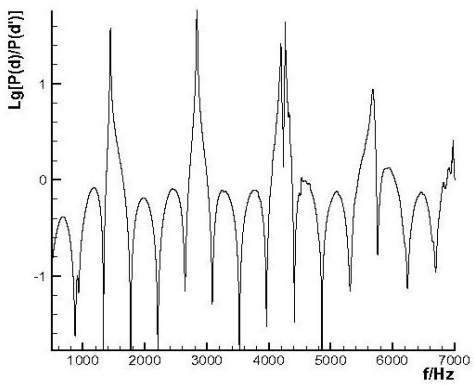


Figure 3 Sound velocity measurement system

Band-pass filtering is used in the experimental sampling processing. The sweep signal ranges from 500 Hz to 7000 Hz. The frequency resolution is 0.5 Hz. The sampling time of the single frequency is 5 s. The sampling frequency is 300 kHz. The sound velocity is measured at water temperatures of *T*=276 K and *T*=294 K. The sound pressure signals of two hydrophones are chosen to calculate the sound velocity by equation (11). The results are shown in Figure 4.



(a) *d*=1.595 m *d'*=0.215 m



(b) d=1.595 m d'=0.495 m

Figure 4 Signals of water temperature T=276 K

It can be seen from Figure 4 that the frequency difference Δf is constant at 438.0000 Hz (accuracy of the sound signal is 0.0001 Hz) whether d'=0.215 m or d'=0.495 m when d =1.595 m. This is consistent with the conclusion of the simulations that the frequency difference Δf is only related to the distance d. Therefore, the sound velocity can be calculated by equation (13) and this yields

$$c = 2d\Delta f = 1397.22 \pm 0.00032\ \mathrm{ms}^{-1} \quad (15)$$

Table 1 frequency difference measured with the traditional method and proposed method（Hz）

traditional	140.0009	139.5009	138.5001	139.0010	139.0009	139.0010
method	140.0011	139.0009	138.5010	139.5008	139.2009	
proposed	439.0000	439.0000	439.0000	439.0000	439.0000	439.0000
method	439.0000	439.0000	439.0000	439.0000	439.0000	

From the signals in Figure 4, the sound pressure of the hydrophone *ch4* is analyzed by the traditional method [27]. The results are shown in Figure 5. The frequency differences Δf obtained with the traditional method are listed in Table 1. The average value of the frequency difference is 139.20086 Hz with standard deviation σ=0.51002 Hz. Therefore, the sound velocity according to the traditional method [26] follows as

$$c = 2L\Delta\overline{f} = 1392.0086 \pm 5.1002\mathrm{ms}^{-1} \quad (16)$$

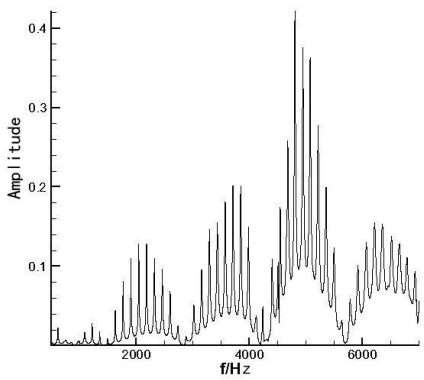


Figure 5 Traditional measurement method

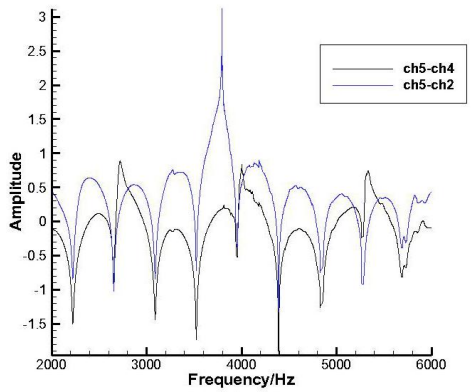


Figure 6 Signals of water temperature *T*=294 K

The velocity from this novel method in equation (16) is a single stable and accurate value due to a single stable frequency difference instead of the average frequency difference. In order to verify the reliability of this new method, the sound velocity is measured under higher temperature T=294 K, which is shown in Figure 6. The frequency difference is also very stable at 439.0000 Hz when the distance d is 1.597 m.

$$c = 2d_5\Delta f = 1402.166 \pm 0.00032\mathrm{ms}^{-1} \quad (17)$$

Table 2 Experiment pressures

Initial pressure (MPa)	Final pressure (MPa)	Average pressure (MPa)	Sampling time (h)
1.782	1.747	1.765	14
3.380	3.348	3.364	14
4.555	4.325	4.440	14

Meanwhile, the velocity is measured at a water temperature T=294 K after the water is injected to the sealed water-filled tube by a water pump to increase the pressure. Due to sealing problems the pressure decreases during the sampling-time interval of 14 hours. The initial pressure and the final pressure are shown in Table 2.

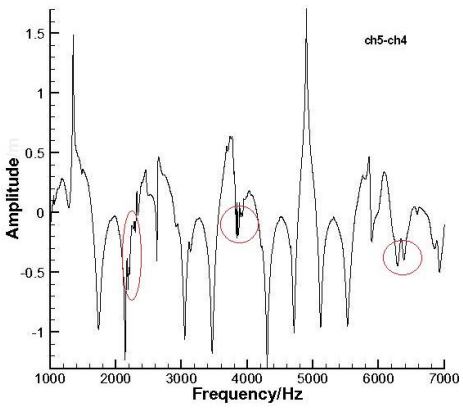


Figure 7 Signals of water pressure 1.765 MPa

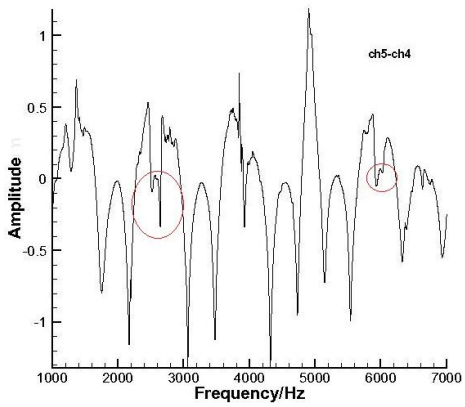


Figure 8 Signals of water pressure 3.364 MPa

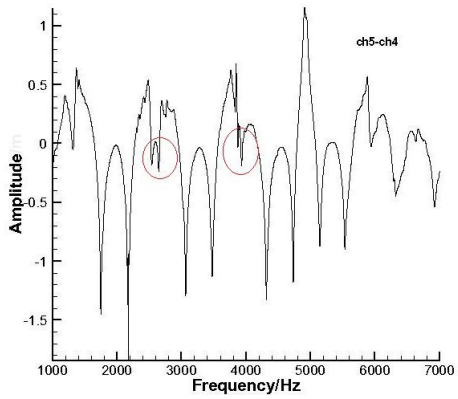


Figure 9 Signals of water pressure 4.440 MPa

The experimental results for different pressures values are shown in Figure 7- Figure 9. Some minimum values are saw-tooth waves as shown with red circles which can lead to a bigger error. These values should be used to get the frequency difference. Moreover, the distance d changes from 1.595 m to 1.730 m due to the injected water. The sound velocities are shown in Table 3.

Table 3 Sound velocity with pressure

Pressure (MPa)	Frequency difference (Hz)				Sound velocity (m/s)
1.765	408.9984	409.0047	408.0046	408.0043	1413.4204±1.9919
3.364	415.9995	415.005	412.0048	412.0048	1431.5872±7.1264
4.440	414.0028	418.0056	412.0051	416.0041	1435.9152±8.9349

5. Error analysis

The measured values with traditional method and new method in this paper are lower than empirical values in open water. In this part the deviation from empirical value is discussed. From equation (13), it can be found that the deviations are associated with the distance d

and the frequency difference Δf. However, experimental conditions such as water temperature and wall thickness will also affect the result. The influences of these factors are as follows:

5.1 The influence of the temperature

The fact that the sound velocity changes with the temperature has been widely recognized. If the sampling time is too long and the temperature cannot be controlled stably, the sound velocity will change. According to the empirical formula in Ref. [27], the sound velocity for distilled water can be estimated as

$$c = 1402.7 + 4.88t - 0.0482t^2 + 1.35\times10^{-6}t^3 + (15.9 + 2.8\times10^{-2}t + 2.4\times10^{-4}t^2)(P\times10^{-7}) \quad (18)$$

Where, P is the current pressure and t is the current temperature in degrees centigrade.

From the empirical formula in equation (18) one finds that when the temperature changes by about ΔT=2 K then the error is of the order of 1%. The temperature at the top of the water-filled tube being different from that at the bottom is a common occurrence. In order to obtain accurate velocity values it is therefore crucial to control the ambient temperature in the experiments to avoid temperature gradients.

However, a temperature control was not feasible for our experimental arrangement. In order to alleviate the problems associated with development of temperature gradients, it is suggested that the sampling interval should be shorter to minimize the temperature bias. To solve this problem between the frequency difference and the time in the sampling process, the variable frequency resolution can be used for the different range, which means smaller frequency resolution near the sharp points and bigger frequency resolution in the rest parts.

5.2 The influence of the thickness and the elasticity of the wall

Since the impedance of the water-filled tube is similar to the water impedance, the wall cannot be regarded as the rigid boundary. The sound field in the tube will be influenced by the wall [26]. Therefore, the sound velocity will be affected. Moreover, the standing wave field will be changed due to the sound velocity and the wave-number change. All these issues can affect the accuracy of the measurement. The sound velocity in the tube will be lower than the standard velocity from the empirical formula [27] in the open water. If the wall is thicker, the sound velocity will be closer to the standard value. In Ref. [23], the deviation from the empirical value would be 2.5% or 3.5% when the ratio (inner radius/tube thickness) is 1 or 0.5. Therefore, the measured value is much lower than the standard value in the tube.

5.3 The influence of the pressure

It is impossible to seal the tube completely. As shown in Table 2, the pressure decreases over longer time intervals or with higher initial pressure. In our impedance tube, the pressure will decrease at most 5% under highest pressure 5 MPa. This will affect slightly the velocity according to the empirical formula in equation (18).

5.4 The influence of the frequency resolution

From equation (13) the error of the frequency difference will change the velocity. In Table 1, higher frequency resolution results in a more accurate value. In our experiments the frequency resolution will change the velocity at most ±3.5‰ due to high frequency resolution.

5.5 The influence of the distance

The frequency difference Δf is only determined by d. In figure 2 and figure 4, different values of d' cannot influence Δf.

Meanwhile, the frequency difference Δf will decrease with the increase of d. Therefore, there should be an optimal d to measure Δf in the five values in consideration of different factors such as measurement error, hydrophone installation position and practical application.

5.6 The influence of the other factors

Some other factors may also affect the velocity. For example, when the environment vibrates, the water-air surface is not a plane. Air bubbles [27] and impurities [28] will lead to scattering, even the water quality can influence the sound velocity. According to Ref. [29], the natural frequency of vibration, f, of water in a cylindrical containment can be evaluated by equation (19). The design frequency band of our tube is 400 Hz ~ 8000 Hz. The minimum frequency (500 Hz) in the experiments is kept apart from the natural frequency.

$$f = \frac{2n-1}{4l}\sqrt{\frac{E}{\rho}} = 74.16\text{Hz} \qquad (19)$$

Where, n is the natural frequency order, l is the length of the water column, E is the elastic modulus and ρ is the density of the water.

In our tube, cold boiled water is used to replace the pure water. The boiled water was allowed to settle for at least 24 hours to eliminate bubbles and electrolytes. After the water was injected to the tube the experiments did not commence for another 24-hour settling period for the same purpose.

6. Conclusion

A new method for the measurement of the sound velocity has been described. The signals of two hydrophones are utilized to eliminate the reflection coefficient of the transducer at the bottom of a water-filled tube. By means of simulations and experiments with different temperatures and pressures, it has been shown that the method is suitable to obtain a stable and accurate value for the sound velocity in Part 4. The velocity will increase with higher temperature or pressure in these experiments which is consistent with the theoretical predictions. Some improvements of the experiment were suggested in the error analyses for the future research. This method offers a single accurate sound velocity, which can be widely used for the basic work of underwater acoustics.

Acknowledgments

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